MAT4378

Assignment 1

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1 Exercises

Exercise 1

(Use a hand held calculator to answer this question). To collect data in an introductory statistics course, Professor Agresti gave the students a questionnaire. One question asked whether the student was a vegetarian. Of 25 students, 0 answered 'yes'. They were not a random sample, but let us use these data to illustrate inference for a proportion. Assume that it is a binomial expriment. Let π denote the population proportion who would say 'yes'.

a) *Find the 95% Wald confidence interval por π. Is it believable? (When the observation falls at the boundary of the sample space, often Wald methods do not provide sensible answers)*

We recall that the 95% Wald confidence interval por π is given by

$$\hat{\pi} \pm z_{0.975}\sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n}} \qquad \hat{\pi} = \frac{w}{n}.$$

We have that $\hat{\pi} = \frac{0}{25} = 0$, which implies that the 95% Wald CI is $\{0\}$. This result is not beliveable. It is very hard to believe that there are not vegetarian people, and taking into account that what we are trying to estimate is precisely the proportion of vegetarian people, this confidence interval should not be taken into account.

b) *Find the 95% Wilson confidence interval for π and also the 95% Agresti-Coull confidence interval for π.*

– The 95% Wilson CI is given by the expression

$$\frac{w + Z^2_{0.975}/2}{n + Z^2_{0.975}} \pm \frac{Z_{0.975}\sqrt{n}}{n + Z^2_{0.975}}\sqrt{\hat{\pi}(1-\hat{\pi}) + \frac{Z^2_{0.975}}{4n}}$$

Substituting, we get that the endpoints of this CI are

$$\frac{1.96^2}{2(25 + 1.96^2)} \pm \frac{1.96^2\sqrt{25}}{25 + 1.96^2}\sqrt{\frac{1.96^2}{100}}$$

=(0,0.1332)

– The 95% Agresti-Coull CI is expressed as

$$\frac{w + Z^2_{0.975}/2}{n + Z^2_{0.975}} \pm z_{0.975}\sqrt{\frac{\frac{w+Z^2_{0.975}/2}{n+Z^2_{0.975}}(1 - \frac{w+Z^2_{0.975}/2}{n+Z^2_{0.975}})}{n + Z^2_{0.975}}}$$

Plugging our parameters into the expression we get

$$\frac{1.96^2/2}{25 + 1.96^2} \pm 1.96\sqrt{\frac{\frac{1.96^2/2}{25+1.96^2}\left(1 - \frac{1.96^2/2}{25+1.96^2}\right)}{25 + 1.96^2}} = (-0.0244, 0.1576)$$

Exercise 2

Refer to Question 1. Use R to compute the three 95% confidence intervals for π: Wald, Wilson and Agresti-Coull.

```
library(binom)
alpha<-0.95
binom.confint(x=0,n=25,alpha,
              c("agresti-coull","asymptotic","wilson"))
```

```
##          method x  n mean       lower     upper
## 1 agresti-coull 0 25    0 -0.02439494 0.1575872
## 2    asymptotic 0 25    0  0.00000000 0.0000000
## 3        wilson 0 25    0  0.00000000 0.1331923
```

Exercise 3

Consider a binomial experiment with n trials and a probability of success π. Using calculus, it is easier to derive the maximum of the log of the likelihood function

$$\log(\mathscr{L}) = w\log(\pi) + (n-w)\log(1-\pi)$$

rather that the likelihood function itself. Here w is the observed number of successes. Both functions attain their maximum at the same value, so it is sufficient to do either.

a) *One can usually determine the point at which the maximum of a log likelihood occurs by solving the likelihood equation. This is the equation resulting from differentiating $\log(\mathscr{L})$ with respect to the parameter, and setting the derivative to zero. Find the likelihood equation for estimating π, and show that its solution is $\hat{\pi} = \frac{w}{n}$ = sample proportion.*

   We have that
   $$\frac{\partial \log(\mathscr{L})}{\partial \pi} = \frac{w}{\pi} + (w-n)\frac{1}{1-\pi}.$$
   As a consequence, the likelihood equation is
   $$\frac{w}{\pi} + (w-n)\frac{1}{1-\pi} = 0.$$
   We now proceed to prove that $\hat{\pi}$ is the solution to that equation. Let π_0 be the solution of the likelihood equation. We thence have that
   $$\frac{w}{\pi_0} = \frac{n-w}{1-\pi_0} \Rightarrow w - w\pi_0 = n\pi_0 - w\pi_0 \Rightarrow \pi_0 = \frac{w}{n} = \hat{\pi}$$

b) *Prove that $\hat{\pi}$ is indeed the value that maximizes the log likelihood function over the interval $0 \le \pi \le 1$. Hint: Consider the three cases: (i) $0 < w < n$; (ii) $w = 0$, (iii) $w = n$.*

   We start considering the cases $w = 0$ ($\Leftrightarrow \hat{\pi} = 0$) and $w = n$ ($\Leftrightarrow \hat{\pi} = 1$). In the first case, we have that $\log(\mathscr{L}) = n\log(1-\pi)$, which is negative in all its domain but in $\pi = \hat{\pi} = 0$, so $\hat{\pi}$ is the maximum of $\log(\mathscr{L})$. If $w = n$, $\log(\mathscr{L}) = n\log(\pi)$ gets maximized at $\pi = \hat{\pi} = 1$ because the log function is increasing in its domain.

   We now suppose that $0 < w < n$. We are going to calculate the second derivative of $\log(\mathscr{L})$ with respect to π:
   $$\frac{\partial^2 \log(\mathscr{L})}{\partial \pi^2} = -\frac{w}{\pi^2} + (w-n)\frac{1}{(1-\pi)^2} < 0 \qquad (\text{note } w-n<0)$$
   $\frac{\partial^2 \log(\mathscr{L})}{\partial \pi^2}$ is a concave function in $(0,1)$, and as a consequence $\hat{\pi}$ is the global maximum and is, indeed, the value that maximizes the likelihood function.

Exercise 4

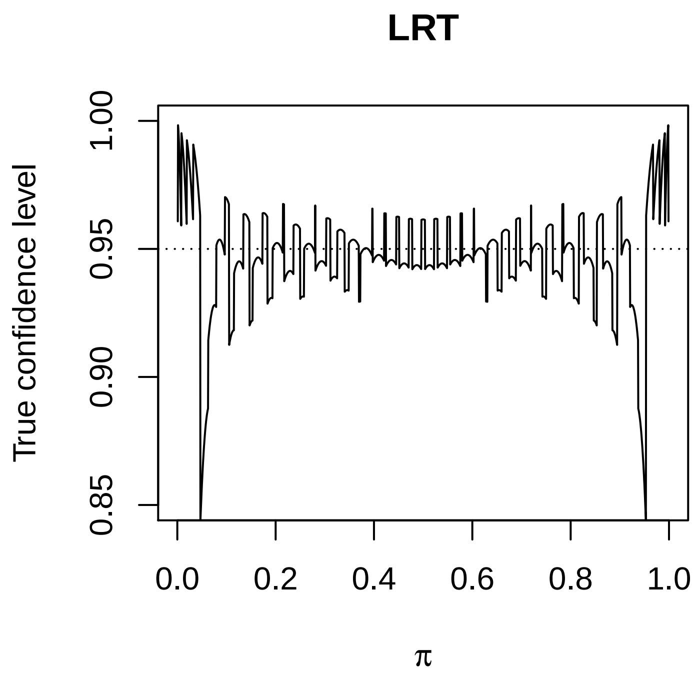
Exercise 13 in Section 1.3 of the textbook.

a) *Verify the* 95% *LR confidence interval is* $0.1456 < \pi < 0.7000$ *when* $n = 10$ *and* $w = 4$. *Note that binom.confit() calculates this interval using the methods='lrt'*

```
library(binom)
alpha<-0.05
binom.confint(x=4,n=10,1-alpha,
              "lrt")
```

```
##   method x  n mean     lower     upper
## 1    lrt 4 10  0.4 0.1456425 0.7000216
```

b) *Construct a plot of the true confidence levels similar to those in Figure 1.3. Use* $n = 40$ *and* $\alpha = 0.05$ *and vary* π *from 0.001 to 0.999 by 0.0005.*



c) *Compare the LR interval's true confidence level to those of the four other intervals discussed in Section 1.1.2. Which of these intervals is best? Explain.*

- LRT and Wald: undoubtedly, LRT is more preferable than Wald. For small values of n Wald is always below the 0.95 true confidence level and has a hideous behavior for extreme values of π.
- LRT and Agresti-Coull: as we can see from the plot and the Figure 1.3, LRT is further from 0.95 than Agresti-Coull when π is close to 0 or 1, and their performance is quite similar in other cases.
- LRT and Wilson: they are both similar. However, LRT is much harder to compute than Wilson, so we prefer the latter.

- LRT and Clopper-Pearson: Clopper-Pearson has better performance than LRT in all cases. In fact, Clopper-Pearson seems to be the best interval. It is similar to Wilson and Agresti-Coull. However, they are are sometimes liberal and sometimes conservative, whereas Clopper-Pearson is conservative. As we know, we prefer a conservative interval over a liberal.

Exercise 5

Exercise 15 in Section 1.3 of the textbook.

a) *We would like a confidence interval to be as short as possible with respect to its expected length while having the correct confidence level. Why?*

   The most important thing for us is to have the correct confidence level, because it is something that we are choosing. When we fix a confidence level we are deciding how rigorous (or flexible) we want to be with our analysis. Once we have that, naturally the shorter the confidence interval is, the better for us, because we know that the parameter that we are trying to get is closer to our estimation.

b) *For $n = 40$, $\pi = 0.16$, and $\alpha = 0.05$, find the expected length for the Wald interval and verify it is* 0.2215.

   We are going to use the following formula:

$$\sum_{w=0}^{n} L(\hat{\pi}) \binom{n}{w} \pi^w (1-\pi)^{n-w},$$

   where $L(\hat{\pi})$ is the interval length. Taking into account the expression of the Wald confidence interval, then we have that $L(\hat{\pi}) = 2\sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n}}$. Substituting, we have that our expected length is

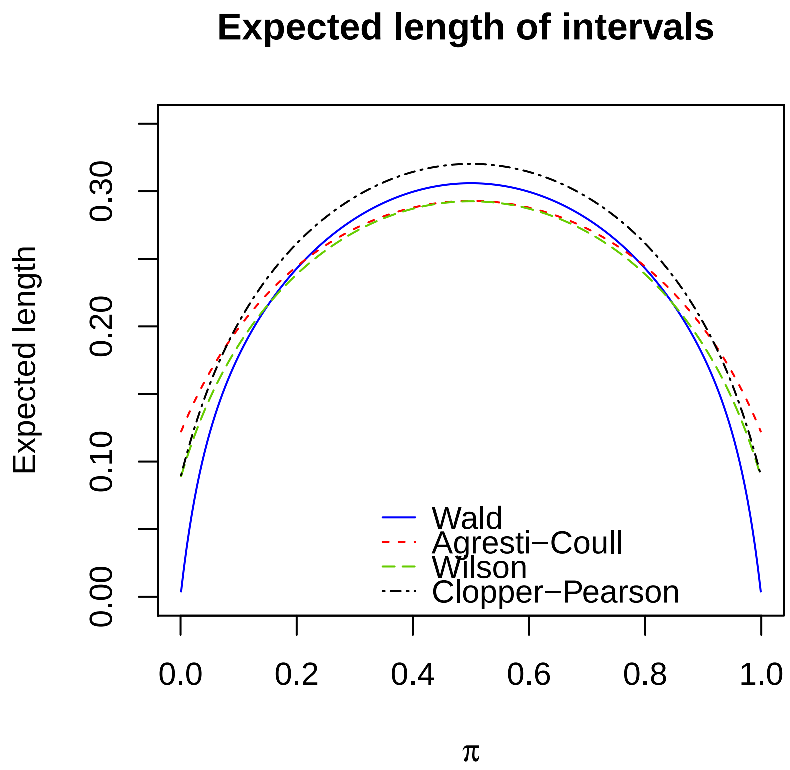
$$2z_{0.975} \sum_{w=0}^{40} \sqrt{\frac{w(1-\frac{w}{40})}{40^2}} \binom{40}{w} \pi^w (1-\pi)^{40-w}$$

```
n<-40
w<-0:n
pi<-0.16

expected.length<-2*1.96*sum(choose(n,w)*pi^(w)*(1-pi)^(n-w)*
                             sqrt((1-w/n)*w/(n^2)))
expected.length

## [1] 0.2215288
```

c) *For $n = 40$ and $\alpha = 0.05$ construct a similar plot to Figure 1.3, but now using the expected length for the y-axis on the plots. Compare the expected lenghts among the intervals.*

   The plot is shown in the following page and the code that generated it in the appendix. We can note some things:

   1. At extreme values of π Agresti-Coull has the largest expected length. On the contrary, Wald has the smallest. We already knew that: in Exercise 1 we saw how under the null $\pi = 0$ (or 1) the length of the interval was 0.
   2. At non-extreme values Clopper-Pearson and Wald have expected length considerably larger than Agresti-Coull and Wilson. Clopper-Pearson has the largest expected length.



d) *Using Figure 1.3 and the plot from (c), which interval is best? Explain.*

We remember that the best intervall will be as short as possible while mantaining the correct confidence level. In Exercise 4 we had determined that the best interval was Clopper-Pearson, with its performance being close to Wilson and Agresti-Coull. With the plot from (c) we see that we might have to change our conclusions: we chose Clopper-Pearson for "philosophical" reasons (we prefere conservative intervals over liberal ones), but now we see that the length of Clopper-Pearson is too big in comparison with Wilson/Agresti-Coull (most of the times Wald has the shortest length but its true confidence level is too bad). The best interval is going to be, as a consequence, Wilson or Agresti-Coull. Taking into account that in the discussión of the textbook Wilson was referred as slightly better, and that its expected length is shorter most of the times and never larger, Wilson definitely becomes the best interval.

e) *Outline the steps that would be needed to find the estimated expected length using Monte Carlo Simulation.*

1. For each π in seq(from = 0.001, to = 0.999, by = 0.0005): calculate N samples from B(n,π) using rbinom()
2. Calculate the $(1-\alpha)100\%$ interval confidence. Let $\{I_{\pi,i}\}_1^N$ be the set of intervals associated to π.
3. Calculate the length of the intervals (this can also be done without calculating the interval explicitly).
4. For each π, the mean of the lengths of $\{I_{\pi,i}\}_1^N$ will be a good estimator of the expected length.

In the appendix we will show one example of this.

Exercise 6

Exercise 17 in Section 1.3 of the textbook. First of all, let

$$n_1 = 26 \quad w_1 = 22 \qquad n_2 = 16 \quad w_2 = 10$$

a) *Calculate the Wald and Agresti-Caffo confidence intervals for the difference in probabilities of success conditioning on the strategy. Interpret the results.*

Let $\alpha = 0.05$.

- Wald: the 95% Wald confidence interval for $\pi_1 - \pi_2$ is

$$\hat{\pi_1} - \hat{\pi_2} \pm z_{0.975} S\{\hat{\pi_1} - \hat{\pi_2}\}$$

We then proceed to compute every one of the terms in that formula:

$$\hat{\pi_1} = \frac{22}{26} = \frac{11}{13}; \quad \hat{\pi_2} = \frac{10}{16} = \frac{5}{8}; \quad S\{\hat{\pi_1}\} = \sqrt{\frac{11(1-\frac{11}{13})}{13 \times 26}} = 0.0708; \quad S\{\hat{\pi_2}\} = \sqrt{\frac{5(1-\frac{5}{8})}{8 \times 16}} = 0.1210$$

And so $S\{\hat{\pi_1} - \hat{\pi_2}\} = \sqrt{0.0708^2 + 0.1210^2} = 0.1402$. Plugging everything into the expression we get that the 95% Wald confidence interval for $\pi_1 - \pi_2$ is

$$\frac{23}{104} \pm 1.96 \times 0.1402 = (-0.0536, 0.4960)$$

- Agresti-Caffo: the 95% Agresti-Caffo confidence interval for $\pi_1 - \pi_2$ is

$$\tilde{\pi_1} - \tilde{\pi_2} \pm z_{0.975} \sqrt{\frac{\tilde{\pi_1}(1-\tilde{\pi_1})}{n_1 + 2} + \frac{\tilde{\pi_2}(1-\tilde{\pi_2})}{n_2 + 2}}$$

where $\tilde{\pi_i} = \frac{w_i + 1}{n_i + 2}$. We have that

$$\tilde{\pi_1} = \frac{22+1}{26+2} = \frac{23}{28} \qquad \tilde{\pi_2} = \frac{10+1}{16+2} = \frac{11}{18},$$

and hence the interval is

$$\left(\frac{23}{28} - \frac{11}{18}\right) \pm 1.96 \sqrt{\frac{23(1-\frac{23}{28})}{28 \times 28} + \frac{11(1-\frac{11}{18})}{18 \times 18}} = (-0.0559, 0.4765)$$

In both cases $0 \in$ CI, so we cannot reject the null hypothesis with the confidence level fixed beforehand. However in the two CIs, 0 is very close to the left endpoint, and this is something to take into account. We will experience this in the following sections.

b) *Perform a score test, Pearson chi-square test, and LRT to test for the equality of the success probabilities.*

Let us first indicate that our null hypothesis, H_0, is $\pi_1 = \pi_2 = \overline{\pi}$. H_1, on the contrary, is $\pi_1 \neq \pi_2$. We now indicate the statistics for the tests:

- Score test: the statistic for the Score test is

$$Z_0 = \frac{\hat{\pi_1} - \hat{\pi_2}}{\sqrt{\overline{\pi}(1-\overline{\pi})(\frac{1}{n_1} + \frac{1}{n_2})}}$$

- Pearson chi-square test: the statistic for the Pearson chi-square test is

$$X^2 = \sum_{j=1}^{2} \frac{(w_j - n_j\overline{\pi})^2}{n_j\overline{\pi}(1-\overline{\pi})}$$

– LRT test: the statistic for the LRT test is

$$-2\log(\Lambda) = -2\left[w_1\log\left(\frac{\pi}{\hat{\pi_1}}\right) + (n_1 - w_1)\log\left(\frac{1-\pi}{1-\hat{\pi_1}}\right) + w_2\log\left(\frac{\pi}{\hat{\pi_2}}\right) + (n_2 - w_2)\log\left(\frac{1-\pi}{1-\hat{\pi_2}}\right)\right]$$

For simplicity, we will compute it using R:

```
n1<-26
n2<-16
w1<-22
w2<-10
pi1<-w1/n1
pi2<-w2/n2
pi<-(w1+w2)/(n1+n2)
score.test<-(pi1-pi2)/sqrt(pi*(1-pi)*(1/n1+1/n2))
pearson.test<-(w1-n1*pi)^2/(n1*pi*(1-pi))+(w2-n2*pi)^2/(n2*pi*(1-pi))
lrt.test<--2*(w1*log(pi/pi1)+(n1-w1)*log((1-pi)/(1-pi1))+
                   w2*log(pi/pi2)+(n2-w2)*log((1-pi)/(1-pi2)))
score.test

## [1] 1.634146

pearson.test

## [1] 2.670433

lrt.test

## [1] 2.610628
```

We reject the Score test if $|Z_0| > Z_{0.975} = 1.96$, and we reject the Pearson chi-square and the LRT test if the statistic is greater than $\chi_{1,0.950} = 3.8415$. As a consequence, with the confidence level that we have chosen **we cannot reject** H_0 in any case.

c) *Estimate the relative risk and calculate the corresponding confidence interval for it. Interpret the results.*

(Recall: the null hypothesis now is H_0 : RR= 1) We will do these calculations with R. We remember that $\hat{RR} = \frac{\hat{\pi_1}}{\hat{\pi_2}}$ and that the $(1-\alpha)100\%$ Wald Confidence interval for RR is given by

$$exp\left[\log\left(\frac{\hat{\pi_1}}{\hat{\pi_2}}\right) \pm Z_{1-\frac{\alpha}{2}}\sqrt{\frac{1}{w_1} - \frac{1}{n_1} + \frac{1}{w_2} - \frac{1}{n_2}}\right]$$

We will set $\alpha = 0.05$, which gives $Z_{0.975} = 1.96$:

```
n1<-26
n2<-16
w1<-22
w2<-10
pi1<-w1/n1
pi2<-w2/n2
RRh<-pi1/pi2
lower.RR<-exp(log(RRh)-1.96*sqrt(1/w1-1/n1+1/w2-1/n2))
upper.RR<-exp(log(RRh)+1.96*sqrt(1/w1-1/n1+1/w2-1/n2))

lower.RR

## [1] 0.8954039
```

```
RRh

## [1] 1.353846

upper.RR

## [1] 2.047009
```

The interpretation of $\hat{RR}$, according to the results that we obtained, is that the risk of scoring a field goal is approximately 1.3 times easier without a timeout than with a timeout. However, the confidence interval for RR is (0.8954,2.0470), which does not let us reject the null hypothesis in favour of the alternative.

d) *Estimate the odds ratio and calculate the correspoding confidence interval for it. Interpret the results.*

We are going to proceed as in the latter section. We recall that $\hat{OR} = \frac{\hat{\pi_1}/(1-\hat{\pi_1})}{\hat{\pi_2}/(1-\hat{\pi_2})}$ and that the 95% Wald Confidence interval for OR is given by

$$exp\left[\log\left(\frac{\hat{\pi_1}/(1-\hat{\pi_1})}{\hat{\pi_2}/(1-\hat{\pi_2})}\right) \pm Z_{0.975}\sqrt{\frac{1}{w_1}+\frac{1}{n_1-w_1}+\frac{1}{w_2}+\frac{1}{n_2-w_2}}\right]$$

```
n1<-26
n2<-16
w1<-22
w2<-10
pi1<-w1/n1
pi2<-w2/n2
ORh<-(pi1/(1-pi1))/((pi2)/(1-pi2))
lower.OR<-exp(log(ORh)-1.96*sqrt(1/w1+1/(n1-w1)+1/w2+1/(n2-w2)))
upper.OR<-exp(log(ORh)+1.96*sqrt(1/w1+1/(n1-w1)+1/w2+1/(n2-w2)))

lower.OR

## [1] 0.7591298

ORh

## [1] 3.3

upper.OR

## [1] 14.34537
```

We have obtained $\hat{OR} = 3.3$, which can be interpreted as: the estimated odds of a field goal are 3.3 without a timeout than with a timeout. Again, our confidence interval is $(0.7591, 14.3454)$, and we are not in a position to reject the null hypothesis.

e) *Is there sufficient evidence to conclude that icing the kicker is a good strategy to follow? Explain.*

In every single one of the test and confidence intervals we have done we have not been able to reject the null hypothesis ($H_0 : \pi_1 = \pi_2$ or equivalently H_0 : RR$= 1$, OR$= 1$,...). However we have always been very close to doing it. Besides, in some cases we know we have been conservative:

in the textbook we can find, talking about the OR confidence interval, that "Lui and Lin (2003) show the interval to be conservative with respect to its true confidence level, where the degree of its conservativeness is dependent on π_1, π_2, and $n+$. The true confidence level is a little above the stated level most of the time". Nevertheless, the rule is clear: first, we fix a level of confidence and, if the statistic (respectively, the confidence interval) exceeds the quantil of the distribution it follows (resp., does not contain the value that defines the null hypothesis) we reject H_0. In other case, we do not reject H_0. As a consequence, **we do not have sufficient evidence to conclude that icing the kicker is a good strategy to follow**.

2 Appendix

The 2 following codes are based on textbook and professor codes.

Code ef Ex. 4, section b

```
n<-40
alpha<-0.95
w<-0:n
pi.seq<-seq(from = 0.001, to = 0.999, by = 0.0005)
LRT.int<-binom.confint(x = w, n = n, conf.level = alpha, methods = "lrt")

lower.LRT<-LRT.int$lower
upper.LRT<-LRT.int$upper
save.true.conf<-matrix(data = NA, nrow = length(pi.seq), ncol = 2)
counter<-1
for(pi in pi.seq) {
  pmf<-dbinom(x = w, size = n, prob = pi)
  # LRT
  save.LRT<-ifelse(test = pi>lower.LRT, yes =
                     ifelse(test = pi<upper.LRT, yes = 1, no = 0), no = 0)
  LRT<-sum(save.LRT*pmf)
  save.true.conf[counter,]<-c(pi, LRT)
  counter<-counter+1

}

# Plots
plot(x = save.true.conf[,1], y = save.true.conf[,2], main = "LRT",
     xlab = expression(pi),
     ylab = "True confidence level", type = "l", ylim = c(0.85,1))
abline(h = alpha, lty = "dotted")
```

(

Code of Ex. 5, section c

```
alpha<-0.05
n<-40
w<-0:n
pi.hat<-w/n

# Clopper-Pearson - This is a little more complicated due to the y = 0 and n cases
lower.CP<-numeric(n+1)  # This initializes a vector to save the lower bounds into
```

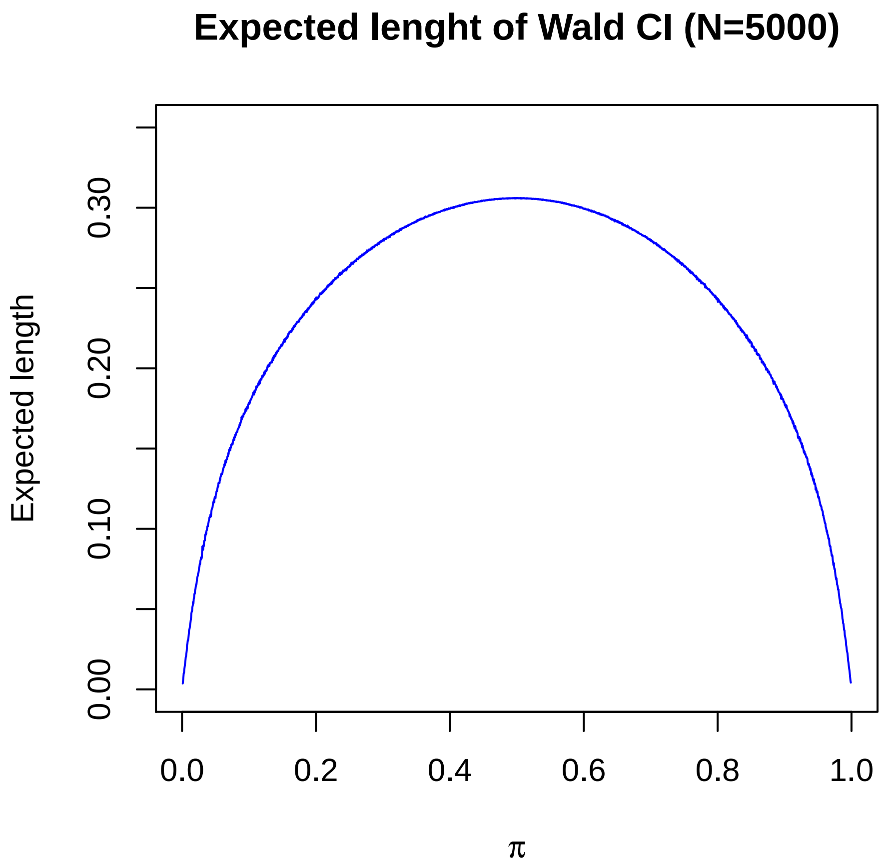
```r
upper.CP<-numeric(n+1)  # This initializes a vector to save the upper bounds into

# y = 0
w0<-0  # Set here for emphasis
lower.CP[1]<-0
upper.CP[1]<-qbeta(p = 1-alpha/2, shape1 = w0+1, shape2 = n-w0)

# y = n
wn<-n  # Set here for emphasis
lower.CP[n+1]<-qbeta(p = alpha/2, shape1 = wn, shape2 = n-wn+1)
upper.CP[n+1]<-1

# y = 1, ..., n-1
w.new<-1:(n-1)
lower.CP[2:n]<-qbeta(p = alpha/2, shape1 = w.new, shape2 = n-w.new+1)
upper.CP[2:n]<-qbeta(p = 1-alpha/2, shape1 = w.new+1, shape2 = n-w.new)


# All pi's
pi.seq<-seq(from = 0.001, to = 0.999, by = 0.0005)

# Save expected interval lengths in a matrix
save.exp.length<-matrix(data = NA, nrow = length(pi.seq), ncol = 5)

# Create counter for the loop
counter<-1

# Loop over each pi that the expected interval length is calculated on
for(pi in pi.seq) {

  # Wald
  wald<-2*1.96*sum(choose(n,w)*pi^(w)*(1-pi)^(n-w)*
                   sqrt((1-w/n)*w/(n^2)))

  # Agresti-Coull
  pi.tilde<-(w+1.96^2/2)/(n+1.96^2)
  ac<-2*(1.96)*sum(sqrt((pi.tilde*(1-pi.tilde))/(n+1.96^2))
                  *choose(n,w)*pi^(w)*(1-pi)^(n-w))

  # Wilson
  wilson<-2*(1.96*sqrt(n))/(n+1.96^2)*sum(sqrt((w/n)*(1-w/n)+(1.96^2)/(4*n))
                                          *choose(n,w)*pi^(w)*(1-pi)^(n-w))

  # Clopper-Pearson
  aux<-numeric(n+1)
  for(i in 0:40) {
    aux[i+1]<-(upper.CP[i+1]-lower.CP[i+1])*choose(n,i)*pi^(i)*(1-pi)^(n-i)
  }
  cp<-sum(aux)

  save.exp.length[counter,]<-c(pi,wald,ac,wilson,cp)
  counter<-counter+1

}
```

```
# Plots
x11(width = 7, height = 6, pointsize = 12)
plot(x = save.exp.length[,1], y = save.exp.length[,2],
     main = "Expected length of intervals ",
     xlab = expression(pi),
     ylab = "Expected length", type = "l",
     lty=1,col="blue",ylim = c(0,0.35))
points(x = save.exp.length[,1], y = save.exp.length[,3],type = "l",
       lty=2, col="red",ylim = c(0,0.35),pch=20)
points(x = save.exp.length[,1], y = save.exp.length[,4],type = "l",
       lty=5, col="chartreuse3",ylim = c(0,0.35))
points(x = save.exp.length[,1], y = save.exp.length[,5],type = "l",
       lty=6, col="black",ylim = c(0,0.35))
legend(x="bottom", legend = c("Wald", "Agresti-Coull","Wilson","Clopper-Pearson"),
       lty = c(1,2,5,6),
       bty = "n", col = c("blue", "red","chartreuse3","black"))
```

Monte Carlo Simulation for expected length of interval

We are going to compute the expected length of the Wald interval, as an example of the steps outlined in 5e. We will do 5000 simulations.

```
N<-5000
n<-40
pi.seq<-seq(from=0.001,to=0.999,by=0.0005)
w<-matrix(data=NA,nrow=N,ncol=length(pi.seq))
counter<-1
for(pi in pi.seq){
  w[,counter]<-rbinom(N,n,pi)
  counter<-counter+1
}
w<-2*1.96*sqrt(w/n*(1-w/n)/n)
#Note that in the last step we are computing directly the length of the Wald interval.
length<-colMeans(w)
x11(width = 7, height = 6, pointsize = 12)
plot(x = pi.seq, y = length, main = "Expected lenght of Wald CI (N=5000)",
     xlab = expression(pi),
     ylab = "Expected length", type = "l",
     lty=1,col="blue",ylim = c(0,0.35))
```



Expected lenght of Wald CI (N=5000)
Expected length
0.00
0.10
0.20
0.30
0.0
0.2
0.4
0.6
0.8
1.0
π