A logistic regression-based pairwise comparison method to aggregate preferences

J. P. Arias-Nicolás, C. J. Pérez, and J. Martín *

Department of Mathematics, University of Extremadura, Avda. de Elvas s/n, 06071 Badajoz, Spain.

Abstract

In a group decision making process, several individuals or a committee have the responsibility to choose the best alternative from a set. The problem addressed in this paper is how to aggregate personal preferences to arrive at an optimal group decision. New technologies allow individuals that may seldom or never meet to make group decisions. This paper proposes a methodology to obtain the group preference ordering in two steps. Firstly, each individual studies the problem isolated, and then, in a possibly virtual meeting, the group must agree on the preferences on some pairs of alternatives. Then, the group criterion is achieved by using a logistic regression model within the pairwise comparison framework proposed here. Properties of the procedure are studied and two illustrative examples are presented.

Key words: e-democracy, group decision making, group preference aggregation, logistic regression, pairwise comparison

1 Introduction

The emergence of e-society has provided new ways for stakeholder interaction, and means for the decision maker to communicate risks, conduct stakeholder surveys, and elicit stakeholder preferences with respect to the alternatives in a

* Corresponding author. Fax: +34 924272911
Email addresses: jparias@unex.es (J. P. Arias-Nicolás), carper@unex.es (C. J. Pérez), jrmartin@unex.es (J. Martín).

decision process. The advent of new technologies has brought the possibility of supporting geographically and temporally dispersed decision making and has huge potential in advancing decision making processes in a democratic society. See the editorial in French (2003) for a substantive vision of e-democracy.

If group members have different viewpoints, some method of aggregating preferences and reconciling differences will be needed. Searching for reasonable compromises between, often contradictory, demands and wishes has attracted the attention of many researchers in the field of group decision making. In the last years, there has been an important increase in the development of group decision support systems based on e-negotiation. Ríos Insua et al. (2003) outline concepts and describe an architecture underlying an electronic negotiation system, pointing out the implications for such a system within e-democracy, whereas Ríos et al. (2005) describe a web-based system to support groups in elaborating participatory budgets.

In this paper we focus our interest in value-oriented methods, i.e., the ones that require the decision maker to state preferences among criteria or alternatives. Then, the problem consists of finding the best alternative (or a ranking of all of them) considering multiple conflicting criteria or objectives for the members of the group. These methods comprise the Analytic Hierarchy Process (AHP), (Saaty, 1980), Multi-Attribute Utility Theory (MAUT), (Keeney & Raiffa, 1976), and the UTilités Additives method (UTA), (Jacquet-Lagrèze & Siskos, 1982), among others. Specifically, the UTA method aims at inferring one or more additive value functions from a given ranking on a reference set. It uses special linear programming techniques to assess these functions so that the ranking(s) obtained through these functions on the reference set is (are) as consistent as possible with the given one. In this paper, using a reference set as in UTA, a method to rank group alternatives based on pairwise comparison is proposed. This technique consists of adjusting a logistic regression model to values obtained through a pairwise comparison framework. This method can be easily implemented as a web-based model.

The outline of the paper is as follows. In Section 2, a multi-participant decision problem is described. In Section 3, the proposed method is formulated. In order to show how the proposed method is applied in practice, two simple but illustrative examples are presented in Section 4. Finally, a discussion is presented in Section 5.

2 Problem description

Decision making is often a group activity. Committees, boards of directors, electorates, and others are usually responsible for decisions. Obviously, among group members there are usually many differences of preferences and beliefs. Moreover, Internet allows to support larger groups that could be located in different cities or countries, the setting typically involved in e-democracy.

Suppose that n individuals are jointly responsible to rank actions from a finite set $\mathcal{A}$. Assume that each individual is able to rank the actions if he is the only responsible to make decisions. Let u_i be the utility function that models preferences for the individual i ($i = 1, 2, \ldots, n$), defined by the following weak preference ordering $\succeq_i$

$$a_k \succeq_i a_l \Longleftrightarrow u_i(a_k) \geq u_i(a_l),$$

where $a_k \succeq_i a_l$ means that the i-th individual believes a_k to be at least as good as a_l. Similarly, the symbol $\succ_i$ denotes strict preference.

Now, a preference order for the group as a whole must be prescribed. Let $\succeq_g$ denote the group preference ordering. The voting system or procedure whereby $\succeq_1, \succeq_2, \ldots, \succeq_n$ are combined to give $\succeq_g$ is often referred to as the constitution of the group. Arrow (1951) suggested that there should be several axioms encoding minimum requirements of justice, fairness and rationality for this constitution, although he proved that they are mutually contradictory. See Kelly (1978) and French (2006), for more information on this topic.

The main objective in this paper is to determine a preference relation reflecting the group's opinion $\succeq_g$. The proposed method is based on two assumptions:

1. each individual is able to determine his own preference order $\succeq_i$, and
2. the group as a whole is able to compare some pairs of alternatives.

The first assumption will easily hold in applications. Although the best case is obtained when the utility u_i is known, it suffices that each individual is able to order the alternatives, see Remark 6 in Section 3. The second assumption requires a meeting, possibly through Internet, to agree on the definition of $\succeq_g$ for the proposed pairs. When a consensus to order pairs of alternatives is not reached in the negotiation process, an external arbitration mechanism must be carried out.

The next section describes a logistic regression-based pairwise comparison method to estimate group preferences for all the alternatives in the problem based on the above two assumptions.

3 Pairwise comparison method

Let a and b denote two alternatives in a reference set $\mathcal{S} \subset \mathcal{A}$. For each decision maker $i = 1, 2, \ldots, n$, define:

$$X_i^{ab} = u_i(a) - u_i(b).$$

Thus, $a \succeq_i b$ if and only if $X_i^{ab} \geq 0$. Let $\boldsymbol{X}^{ab} = (X_1^{ab}, X_2^{ab}, \ldots, X_n^{ab})$ be a vector that represents the individual preferences of the decision makers between a and b. The binary variable Y^{ab} will represent the group preference between a and b, i.e.,

$$Y^{ab} = \begin{cases} 1 & \text{if } a \succeq_g b \\ 0 & \text{if } a \prec_g b \end{cases}$$

¿From the previous definition, it is deduced that, if two actions a and b are indifferent for the group, i.e., $a \succeq_g b$ and $b \succeq_g a$, then $Y^{ab} = Y^{ba} = 1$. Their indifference is denoted by $a \sim_g b$.

Suppose that $\mathcal{S} \subset \mathcal{A}$ is a set containing m alternatives for which the individual and group preferences are known, possibly, after a virtual meeting. When a consensus to order the alternatives in $\mathcal{S}$ is not fully reached in the negotiation process, an external arbitration mechanism must be carried out. This requirement provides information about the group preferences. Therefore, the aggregation process is not only based on the individual preferences, but also on the group preferences. Note that nothing is known about the group preferences in the set $\mathcal{A} - \mathcal{S}$. Then, the proposed method is applied to make a global alternative ranking for the group as a whole. Note that it is not necessary to order all the pairs in $\mathcal{S} \times \mathcal{S}$, but only to provide a set of ordered pairs, see Remark 1.

Without loss of generality, we shall assume that the m ordered alternatives in

$\mathcal{S}$ are:

$$a_1 \succeq_g a_2 \succeq_g \cdots \succeq_g a_m.$$

By comparing every pair of alternatives in $\mathcal{S}$, $m(m-1)/2$ data vectors are obtained, i.e:

$$(y^{a_k a_l}, x_1^{a_k a_l}, x_2^{a_k a_l}, \ldots, x_n^{a_k a_l}), \quad k > l = 1, 2, \ldots, m-1,$$

which will be used to estimate the parameter $\boldsymbol{\beta} = (\beta_1, \beta_2, \ldots, \beta_n)$ in a logistic regression model. The estimation can be obtained by using the maximum likelihood method and it is denoted $\hat{\boldsymbol{\beta}}$. See, e.g., McCullagh and Nelder (1989) for a full description. Some background on logistic regression is summarized in the appendix.

Now, let π^{ab} denote the probability that a is at least as preferred by the group as b. It can be expressed as:

$$\pi^{ab} = P[a \succeq_g b] = \frac{e^{\boldsymbol{\beta}' \boldsymbol{X}^{ab}}}{1 + e^{\boldsymbol{\beta}' \boldsymbol{X}^{ab}}}.$$

Note that for all actions $a \in \mathcal{S}$, it holds that $a \succeq_i a$ and, equivalently, $X_i^{aa} = u_i(a) - u_i(a) = 0$, for $i = 1, 2, \ldots, n$. Therefore, $\pi^{aa} = \frac{1}{2}$, since we use a logistic regression model without an intercept term β_0. Thus, given $a, b \in \mathcal{S}$, $a \succeq_g b$ if and only if $\pi^{ab} \geq \frac{1}{2}$ and $b \succ_g a$ if and only if $\pi^{ab} < \frac{1}{2}$. The alternatives are indifferent when $\pi^{ab} = \frac{1}{2}$.

The probability π^{ab} is estimated through

$$\hat{\pi}^{ab} = \frac{e^{\hat{\boldsymbol{\beta}}' \boldsymbol{X}^{ab}}}{1 + e^{\hat{\boldsymbol{\beta}}' \boldsymbol{X}^{ab}}},$$

where $\hat{\boldsymbol{\beta}}$ represents the previously obtained estimation for $\boldsymbol{\beta}$.

Then, a group preference induced by the logistic regression method is obtained and it is denoted by $a \succeq_{\hat{g}} b$ for all $a, b \in \mathcal{A}$. By using this method, it is possible to order any alternative $b \in \mathcal{A}$ with unknown position in the group ranking. In order to obtain optimal alternatives, an exhaustive search can be performed if the set of alternatives is small. Otherwise, stochastic optimization techniques can be used, see, e.g., Pirlot (1996).

Remark 1. It is not necessary to make a full pairwise comparison in $\mathcal{S} \times \mathcal{S}$. It is only required to rank a set of ordered pairs of alternatives. The pairs of alternatives to be compared should be properly chosen to provide all the necessary information for the group constitution. Considerations on the number of ordered pairs required can be deduced from Freeman (1987) and Perduzzi et al. (1996). When using all the pairs in $\mathcal{S} \times \mathcal{S}$ some redundant information can be obtained.

Remark 2. All Arrow's axioms are satisfied except binary relevance and Pareto's principle for strict preference.

1. *Weak ordering.* The preferences $\succeq_1, \succeq_2, \ldots, \succeq_n$ should be weak orders. The order $\succeq_{\hat{g}}$ induced by the logistic regression method is also a weak order.
2. *Non-triviality.* In order to apply the method, there should be at least two members of the group and three alternatives.
3. *Universal domain.* The method has been designed so that $\succeq_{\hat{g}}$ is defined whatever $\succeq_1, \succeq_2, \ldots, \succeq_n$ and $\succeq_g$ over $S \neq \emptyset$ may be.
4. *Binary relevance.* Let $\succeq_1, \succeq_2, \ldots, \succeq_n$ be a set of individual preference orders over a set of alternatives $\mathcal{A}$. Let $\succeq_1', \succeq_2', \ldots, \succeq_n'$ be another set of individual preference orders over a set of alternatives $\mathcal{A}'$. Suppose that alternatives a and b belong to $\mathcal{A} \cap \mathcal{A}'$ and further that $\succeq_1, \succeq_2, \ldots, \succeq_n$ and $\succeq_1', \succeq_2', \ldots, \succeq_n'$ are identical on $\{a, b\}$, i.e. $\forall i$,

$$a \succeq_i b \Leftrightarrow a \succeq_i' b$$

$$b \succeq_i a \Leftrightarrow b \succeq_i' a$$

   Then, the constitution should lead to the same group preference between a and b.
   $\succeq_{\hat{g}}$ might not satisfy the Binary relevance axiom, as $\mathcal{S}$ and $\mathcal{S}'$ can provide different information and the induced group orders are not necessarily the same. Even in the case the information is the same, this axiom is not usually held.
5. *Pareto's principle for strict preferences.* If every individual holds $a \succ_i b$, then the group holds $a \succ_{\hat{g}} b$.
   The ordering $\succeq_{\hat{g}}$ induced by the logistic regression method does not always satisfy this axiom. If this axiom does not hold, then at least one component in the parameter $\boldsymbol{\beta}$ is negative. On the other hand, if all the components in the parameter $\boldsymbol{\beta}$ are non-negative, then Pareto's principle is satisfied. Therefore, by applying the method subject to the constraint that all the components in $\boldsymbol{\beta}$ are positive, this axiom holds.

6. *No dictatorship.* There is no individual whose preferences automatically become the group preferences independently of the preferences of the other members. It is assumed that there is no dictator when defining the group preferences in $\mathcal{S}$.

Remark 3. Observe that $\hat{\pi}^{ab} \geq \frac{1}{2}$ is equivalent to $\hat{\boldsymbol{\beta}}' \boldsymbol{X}^{ab} \geq 0$. Thus, we could consider

$$u_g(\cdot) = \sum_{i=1}^{n} \hat{\beta}_i u_i(\cdot)$$

as a generalized group additive utility function, since the parameters $\hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_n$ can be negative. The maximum likelihood estimate $\hat{\boldsymbol{\beta}}$ tries to achieve values of $\hat{\pi}^{ab}$ close to y^{ab}. This implies that, in general, $\hat{\boldsymbol{\beta}}$ does not try to estimate the parameters of the group utility function, even if estimations are constrained to $\mathbb{R}^n_+$. It estimates values $\hat{\boldsymbol{\beta}}$ that provides probabilities as near as possible to 0 or 1 for $\hat{\pi}^{ab}$.

Remark 4. If there is a group additive utility i.e., $a \succeq_g b$ if and only if $\lambda_1 u_1(a) + \lambda_2 u_2(a) + \ldots + \lambda_n u_n(a) \leq \lambda_1 u_1(b) + \lambda_2 u_2(b) + \ldots + \lambda_n u_n(b)$, then $\succeq_g$ is equivalent to $\succeq_{\hat{g}}$.

The proof is as follows. Let $\hat{\beta}_i(K) = K\lambda_i$, $K = 1, 2, \ldots,$ for $i = 1, 2, \ldots, n$. Then, for all $a, b \in \mathcal{A}$,

$$\lim_{K \to \infty} \frac{e^{\hat{\boldsymbol{\beta}}(\boldsymbol{K})' \boldsymbol{X}^{ab}}}{1 + e^{\hat{\boldsymbol{\beta}}(\boldsymbol{K})' \boldsymbol{X}^{ab}}} = \begin{cases} 0 & \text{if } \sum_{i=1}^{n} \lambda_i u_i(a) < \sum_{i=1}^{n} \lambda_i u_i(b) \\ 1/2 & \text{if } \sum_{i=1}^{n} \lambda_i u_i(a) = \sum_{i=1}^{n} \lambda_i u_i(b) \\ 1 & \text{if } \sum_{i=1}^{n} \lambda_i u_i(a) > \sum_{i=1}^{n} \lambda_i u_i(b) \end{cases}.$$

For $i = 1, 2, \ldots, n$, $\hat{\beta}_i(K) = K\lambda_i$, $K = 1, 2, \ldots,$ is a sequence of parameters that tends to the maximum likelihood estimator of β_i. Moreover, when applying logistic regression, large value estimations for β_i, $i = 1, 2, \ldots, n$, are obtained, as in the first example presented in Section 4. Obviously, with these parameters, it holds that $a \succeq_g b$ if and only if $a \succeq_{\hat{g}} b$, for all $a, b \in \mathcal{A}$.

Remark 5. Should there be an i such that $\forall a, b \in \mathcal{S}$, $a \succeq_g b \iff a \succeq_i b$, that is, the decision maker i would be a dictator over $(\mathcal{S}, \succeq_g)$, then he would be also a dictator over $(\mathcal{A}, \succeq_{\hat{g}})$. So, prior to the estimation of the regression parameters, it is necessary to check that there is no dictator over $(\mathcal{S}, \succeq_g)$ to satisfy Arrow's axiom number 6.

The proof is as follows. Consider, without loss of generality, that $i = 1$. For $K = 1, 2, ...,$ let $\hat{\beta}_1(K) = K$, and let $\hat{\beta}_i(K) = 0$, $i = 2, 3, \ldots, n$. Then, for all $a, b \in \mathcal{S}$,

$$\lim_{K\to\infty} \frac{e^{\hat{\boldsymbol{\beta}}(\boldsymbol{K})'\boldsymbol{X}^{ab}}}{1 + e^{\hat{\boldsymbol{\beta}}(\boldsymbol{K})'\boldsymbol{X}^{ab}}} = \begin{cases} 0 & \text{if} \quad u_1(a) < u_1(b) \\ 1/2 & \text{if} \quad u_1(a) = u_1(b) \\ 1 & \text{if} \quad u_1(a) > u_1(b) \end{cases} .$$

For $i = 1, 2, \ldots, n$, $\hat{\beta}_i(K)$, $K = 1, 2, \ldots,$ is a sequence of parameters that tends to the maximum likelihood estimator of β_i. Moreover, for these parameters, $a \succeq_{\hat{g}} b$ if and only if $a \succeq_1 b$, for all $a, b \in \mathcal{A}$. So, individual 1 is a dictator over $(\mathcal{A}, \succeq_{\hat{g}})$.

Remark 6. It is actually not necessary to know the utility values for the individuals. The method is also applicable when we only have

$$X_i^{ab} = \begin{cases} 1 & \text{if } a \succeq_i b \\ 0 & \text{if } a \prec_i b \end{cases}$$

for $i = 1, 2, \ldots, n$.

Remark 7. $\succeq_{\hat{g}}$ is invariant with respect to positive affine transformations of the utility functions u_i.

The proof is as follows. Let $u_i' = \alpha_i u_i + \gamma_i$, $\alpha_i > 0$, $i = 1, 2, \ldots, n$. Then, $X_i'^{ab} = \alpha_i X_i^{ab}$, for all $a, b \in \mathcal{A}$. If $\hat{\beta}_1, \ldots, \hat{\beta}_n$ are the components of the maximum likelihood estimator for the model with utility functions u_i, then $\hat{\beta}_1/\alpha_1, \ldots, \hat{\beta}_n/\alpha_n$ are the components of the maximum likelihood estimator for the model with utility functions u_i'. Therefore, $\hat{\pi}'^{ab} = \hat{\pi}^{ab}$ and $\succeq_{\hat{g}}$ is not affected by positive affine transformations of the utility functions.

In order to show how the proposed method is applied in practice, the next section presents two examples.

4 Illustrative examples

Three town councillors constitute a committee to analyze several offers for a specific service for the city. From other experiences, the group ranking of seven offers is known, i.e., $\mathcal{S}$ is composed of 7 alternatives for which the group preferences are known. Each town councillor has his own utility for this type of offers which can be known for some new offer. However, the order in the group ranking for these new offers is unknown. The focus is on making a group ranking including the new offers.

Example 1. Assume there is an additive group utility function. The group preference, based on this group utility function, for the different projects are $a_1 \succeq_g a_2 \succeq_g a_3 \succeq_g a_4 \succeq_g a_5 \succeq_g a_6 \succeq_g a_7$, and the individual utilities are presented in Table 1.

Actions	a_1	a_2	a_3	a_4	a_5	a_6	a_7
u_1	0.240	0.202	0.128	0.156	0.126	0.084	0.074
u_2	0.040	0.118	0.094	0.054	0.052	0.076	0.034
u_3	0.799	0.615	0.665	0.503	0.552	0.449	0.476

Table 1
Individual utilities for seven ordered alternatives.

For illustration, three new alternatives for which their positions in the group ranking are unknown are considered. The new alternatives with the individual utilities are given in Table 2.

Actions	u_1	u_2	u_3
b_1	0.082	0.010	0.516
b_2	0.168	0.076	0.600
b_3	0.168	0.004	0.691

Table 2
Individual utilities for three new alternatives.

By applying the method presented in Section 3, the parameter estimates are found to be $\hat{\beta}_1 = 9.362 \cdot 10^{15}$, $\hat{\beta}_2 = 1.195 \cdot 10^{16}$, and $\hat{\beta}_3 = 5.282 \cdot 10^{15}$. By using these values, the probabilities π^{ab} for all possible pairs of alternatives can be estimated. Table 3 shows these estimations.

$\hat{\pi}^{ab}$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	b_1	b_2	b_3
a_1	0.5	1	1	1	1	1	1	1	1	1
a_2	0	0.5	1	1	1	1	1	1	1	1
a_3	0	0	0.5	1	1	1	1	1	1	1
a_4	0	0	0	0.5	1	1	1	1	0	0
a_5	0	0	0	0	0.5	1	1	1	0	0
a_6	0	0	0	0	0	0.5	1	1	0	0
a_7	0	0	0	0	0	0	0.5	1	0	0
b_1	0	0	0	0	0	0	0	0.5	0	0
b_2	0	0	0	1	1	1	1	1	0.5	1
b_3	0	0	0	1	1	1	1	1	0	0.5

Table 3
Estimations of π^{ab} for all possible pairs of alternatives.

Therefore, the new group ranking is given by:

$$a_1 \succeq_{\hat{g}} a_2 \succeq_{\hat{g}} a_3 \succeq_{\hat{g}} b_2 \succeq_{\hat{g}} b_3 \succeq_{\hat{g}} a_4 \succeq_{\hat{g}} a_5 \succeq_{\hat{g}} a_6 \succeq_{\hat{g}} a_7 \succeq_{\hat{g}} b_1.$$

Since group preferences have been obtained by using an additive group utility, then large regression values are obtained according to Remark 4. This fact explains also why values of $\hat{\pi}^{ab}$ are equal to 0, 0.5 or 1.

Example 2. We present now an example in which there is a dictator. The group preferences for seven offers are $a_1 \succeq_g a_2 \succeq_g a_3 \succeq_g a_4 \succeq_g a_5 \succeq_g a_6 \succeq_g a_7$. Individual utilities are in Table 4. Decision maker 3 is a dictator on $\mathcal{S}$. The positions in the group ranking are unknown for the following three alternatives whose individual utilities are given in Table 5.

Actions	a_1	a_2	a_3	a_4	a_5	a_6	a_7
u_1	0.201	0.601	0.501	0.401	0.710	0.381	1.101
u_2	0.476	0.229	0.327	0.258	0.203	0.386	1.020
u_3	0.780	0.650	0.513	0.400	0.323	0.290	0.100

Table 4
Individual utilities for seven ordered alternatives.

Actions	u_1	u_2	u_3
b_1	0.082	0.010	0.590
b_2	0.168	0.076	0.600
b_3	0.168	0.004	0.891

Table 5
Individual utilities for three new alternatives.

The parameter estimates are $\hat{\beta}_1 = 0.5505$, $\hat{\beta}_2 = -0.5850$, and $\hat{\beta}_3 = 10^7$. By using these values, the probabilities π^{ab} for all possible pairs of alternatives are estimated and the new group ranking is given by:

$$b_3 \succeq_{\hat{g}} a_1 \succeq_{\hat{g}} a_2 \succeq_{\hat{g}} b_2 \succeq_{\hat{g}} b_1 \succeq_{\hat{g}} a_3 \succeq_{\hat{g}} a_4 \succeq_{\hat{g}} a_5 \succeq_{\hat{g}} a_6 \succeq_{\hat{g}} a_7.$$

Note that the final order is the same as the one for decision maker 3. Remark 5 supports this fact.

In these examples, only three new alternatives have been used for illustrative purpose. In real problems, the set $\mathcal{A} - \mathcal{S}$ will usually be much larger than $\mathcal{S}$. As it has been shown with these examples, the proposed method is easily applied in practice and its usefulness relies on the information that can be obtained about the opinion of the group.

5 Discussion

Decision making in a multiple-participant context is a general problem receiving increasing interest. Group Decision Support Systems play a decisive role in situations where many persons are involved, each having his own perception on the context and the decision problem to be addressed. The proposed method is based on logistic regression and allows to derive a group ranking for all possible alternatives.

In order to implement the proposed method, an interactive group support system is outlined. The proposed methodology is described in the following steps and represented in Figure 1.

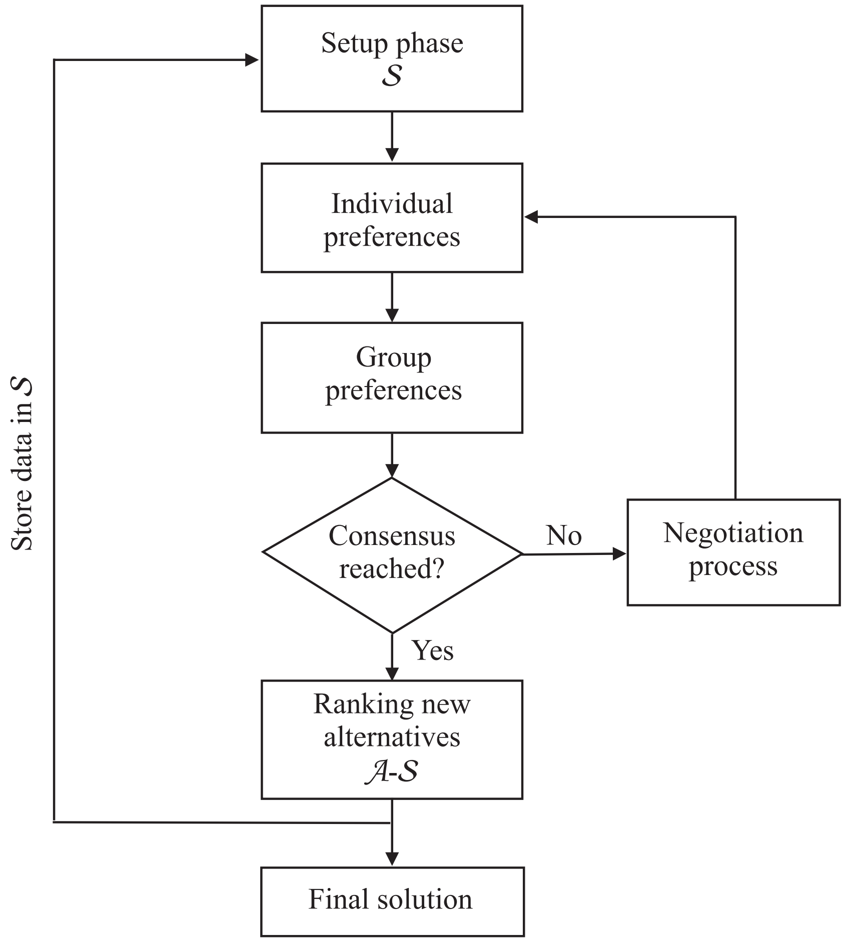

Fig. 1. Scheme of the proposed methodology

1. *Setup phase.* This stage involves the selection of alternatives in $\mathcal{S}$. It is possible to use fictitious alternatives in $\mathcal{S}$, as in Jacquet-Lagrèze and Siskos (1982), which are easily judged to attain the group ranking. For the more general case, the group preference order for the alternatives in $\mathcal{S}$ has to be set by the members of the group in step 3.
2. *Assessing individual preferences.* The decision makers must study the problem individually and provide a utility value for each alternative in

$\mathcal{S}$. It is also possible to provide only the preference relations among alternatives. In this case, the preference strength among alternatives will not be taken into account.

3. *Assessing group preferences.* If there is no historical data and no fictitious alternatives are used, this is the most critical part of the framework. The group as a whole has to determine the preference relations among the alternatives in $\mathcal{S}$. The assumptions presented in Section 3 are reasonable for many group decision problems. If a consensus to order the alternatives in $\mathcal{S}$ is not reached, a negotiation procedure must be established among the members of the group. This negotiation process allows the members of the group to arrive at an agreement and the members are allowed to modify their individual and group preferences by continuing the process from step 2. If a consensus is not reached after the negotiation process, then an external arbitration mechanism must be carried out.
4. *Ranking new alternatives.* In this phase, the preference aggregation for the group can be performed. New alternatives in $\mathcal{A} - \mathcal{S}$ for which nothing is known about their group preferences can be evaluated by means of the proposed aggregation method to arrive at a global group ranking. Besides, the results can be stored as historical data in the setup phase for other similar problems.

The growth of the Internet and web technologies offer new possibilities for direct participation in democratic processes. Research and development activities in the field of e-democracy aim at evaluating how interactive decision analytic tools can help to develop inclusive e-democratic systems. Democratic processes in which individuals and stakeholder groups may participate through Internet leading up to a group decision are specially interesting. By using web-based technologies, the decision groups can become larger and separate in space, and the decisions can be separated in time. These facts bring many advantages to the decision making process. A natural framework consists in a decision process that would be carried out by an analyst on a master system for the owner of the decision making problem (the elected representative of a council, the president of a company, the owner of a business organization,...). By using a Graphic User Interface (GUI), each decision maker connects to the server to include his preferences and to receive information about the state of the process. There exists a continuous interaction among members of the group. If a negotiation round is necessary to reach a consensus solution for the group preferences, then this negotiation process is also supported through the web in a possibly asynchronous way. The external mediator, if necessary, participates in this step. When the final solution is obtained, it is ready to use and it can also be stored in a database for a possible future use.

Acknowledgments

The authors would like to thank the reviewers for comments and suggestions which have highly improved not only the readability of the paper but also its content. This research was partially supported by *Ministerio de Educación y Ciencia*, Spain (Project TSI2004-06801-C04-03).

Appendix A. Logistic Regression Model

This appendix is presented to introduce the notation and uses of the logistic regression model. Logistic regression is part of a category of statistical models called Generalized Linear Models (GLM). Excellent treatments of GLM are presented, for example, in Agresti (1996) and Ryan (1997).

Binary logistic regression is a variation of ordinary regression, useful when the dependent variable Y is restricted to take two values, which usually represent the occurrence (coded as 1) or non-occurrence (coded as 0) of some outcome event. It produces a formula that predicts the probability of the event occurrence π as a function of independent variables $X_1, X_2, \ldots, X_n$. The independent or predictor variables can take any form (continuous, discrete, dichotomous,...) and no assumption is necessary about their distribution.

Then, the expression for π is given by:

$$\pi = \frac{e^{\beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n}}{1 + e^{\beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n}}.$$

One of the main uses of logistic regression is the prediction of group membership.

References

Agresti, A. (1996), *An Introduction to Categorical Data Analysis*, Wiley.

Arrow, K. J. (1951), *Social Choice and Individual Values*, Wiley.

Freeman, D. H. (1987), *Applied categorical data analysis*, Marcel Dekker.

French, S. (2003), 'The Challenges in Extending the MCDA Paradigm to e-Democracy', *Journal of Multi-Criteria Decision Analysis* **12**, 63–233.

French, S. (2006), 'Web-enabled Strategic GDSS, e-Democracy, and Arrow's theorem: A Bayesian Perspective', *Decision Support Systems* . To appear.

Jacquet-Lagrèze, E. & Siskos, Y. (1982), 'Assessing a set of additive utility functions for multicriteria decision making: The uta method', *European Journal of Operational Research* **10**(2), 151–164.

Keeney, R. L. & Raiffa, H. (1976), *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*, Wiley.

Kelly, F. S. (1978), *Arrow Impossibility Theorems*, New York Academic Press.

McCullagh, P. & Nelder, J. A. (1989), *Generalized Lineal Models*, Monographs on Statistics and Applied Probability, Chapman & Hall.

Perduzzi, P., Concato, J., Kemper, E., Holford, T. R. & Feinstein, A. R. (1996), 'A simulation study of the number of events per variable in logistic regression analysis', *Journal of Clinical Epidemiology* **49**, 1373–1379.

Pirlot, M. (1996), 'General local search methods', *European Journal of Operational Research* **92**, 493–511.

Ryan, T. P. (1997), *Modern Regression Methods*, Wiley.

Ríos Insua, D., Holgado, J. & Moreno, R. (2003), 'Multicriteria e-negotiation systems for e-democracy', *Journal of Multi-Criteria Decision Analysis* **12**(2-3), 213–218.

Ríos, J., Ríos Insua, D., Fernández, E. & Rivero, J. A. (2005), *TCGOV 2005*, IFIP International Federation for Information Processing, chapter Participatory Budget Formation Throught the Web, pp. 268–276.

Saaty, T. L. (1980), *The Analytic Hierarchy Process*, McGraw-Hill.