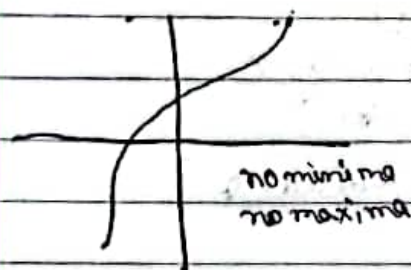
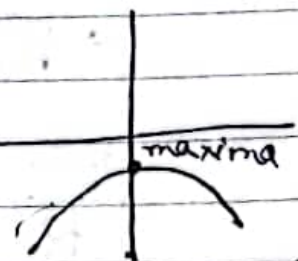
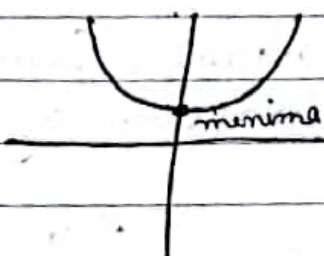
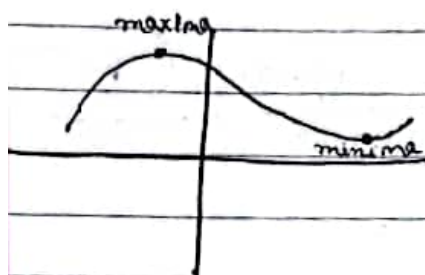
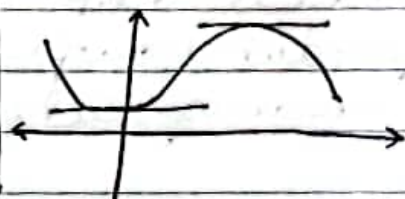


# \* Gradient Descent \*



Now lets create slope line in maxima & minima



slope is always '0' (zero)

$f(x) = x^2 - 3x + 2$ , find minima of  $f(x)$

$\therefore$  slope of  $f(x) = 0$  @ minima

$$\frac{df}{dx} = 0$$

$$\therefore \frac{d x^a}{dx} = a x^{a-1}$$

$$2x - 3 = 0$$

$$\therefore \frac{d a x^b}{dx} = a b x^{b-a}$$

$$x = 3/2 = 1.5$$

@  $x = 1.5$  slope of  $f(x)$  is '0'

So, this  $x = 1.5$  can be minima or maxima

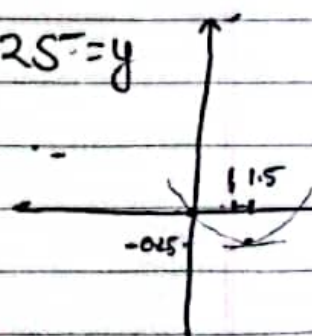
$$f(x) = x^2 - 3x + 2$$

$$f(1.5) = (1.5)^2 - 3 \times 1.5 + 2 = -0.25 = y$$

$$f(1) = 1^2 - 3 \times 1 + 2 = 0$$

$$\text{so, } f(1) > f(1.5)$$

$\therefore$  so  $f(1.5)$  is minima

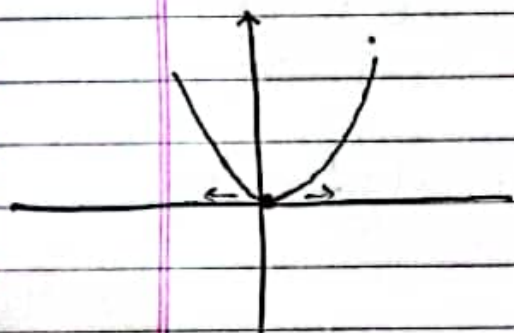


In linear eqn we try to minimize

$$m^*, c^* = \arg \min_{m, c} \left\{ \sum (y_i - (mx_i + c))^2 \right\}$$

ie.,  $f(m, c) = \sum (y_i - (mx_i + c))^2$

to perform this  $f(m, c)$  with very fast iterative computation we use gradient descent method



- minima Right side : slope is (+)ve
- minima left side : slope is (-)ve
- As you move towards minima  
→ slope reduce
- moving away from minima  
→ slope increases

So in G.P. :-

→ pick @ random an initial point =  $x_0$

→ pick another  $x_{i+1}$ , such that  $x_{i+1}$  is closer to  $x^*$

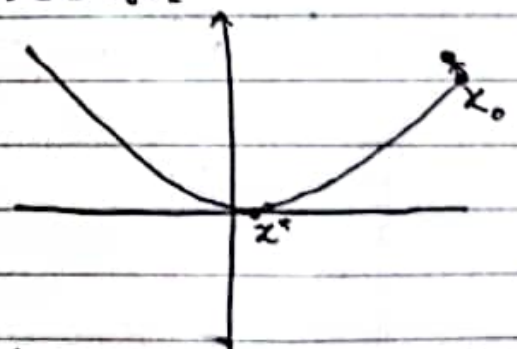
∴  $x_{i+1} = (+)x_i - (+ve)$  for +ve  $x_i$

$x_{i+1} = (-)x_i - (-ve)$  for +ve  $x_i$

$\underbrace{x_i}_{x_i} \quad \underbrace{[slope] x_i}_{[slope] x_i}$

∴  $x_{i+1} = x_i - \eta \left[ \frac{df}{dx} \right]_{x_i}$  ∵  $\eta$  = learning rate

→ Terminate → slope = 0 or  
 $x_k - x_{k+1}$  is very very slow.





In linear regression Sum of square error:

$$\rightarrow m^*, c^* = \arg \min \{ f(m, c) \}$$

$$\because f(m, c) = \sum (y_i - (mx_i + c))^2$$

$\rightarrow$  pick initial  $m_1$  &  $c_1 = 0$

$$m_{i+1} = m_i - \eta \left[ \frac{\partial f}{\partial m} \right]_{m_i} \quad \because \partial (\text{dom})$$

$$c_{i+1} = c_i - \eta \left[ \frac{\partial f}{\partial c} \right]_{c_i} = d (\text{differentiation})$$

should assume value of  $\eta = 0.001$

$$\left[ \frac{\partial f}{\partial m} \right]_{m_i} = \nabla_m f = \frac{2}{N} \sum_{j=1}^n (y_i - (mx_i + c)) \cdot x_i$$

$$\nabla_c f = \frac{1}{N} \sum_{i=1}^N (y_i - (mx_i + c)) \cdot (-1)$$

so,

$$\nabla_m f = \frac{2}{N} \sum_{j=1}^N \{ (y_i - \hat{y}_i) \cdot (-x_i) \} \quad \because mx_i + c = \hat{y}$$