ML Mathematics

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Linear Algebra

2D → Line ; 3D → plane ; nD → Hyperplane^n $(n>5)$

$A = \begin{bmatrix} & \\ & \end{bmatrix}_{m\times n}$ m → no of rows ; n → no of col^n.

→ Unit matrix → its a square matrix and all diagonal elements are 1

Ex : $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$ ✓ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}_{3\times 2}$ ✗

i.e. $I_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$

$A \times I = A$ $\{ A \in \mathbb{R}^{m\times n}$

→ Matrix Addition

i) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 4 \\ 4 & 6 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} = N/A$

iii) $\begin{bmatrix} 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ {∵ should have same order

→ Matrix Multiplication

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} * \begin{bmatrix} 1 \\ 2 \end{bmatrix}_{2\times 1} \Rightarrow \begin{bmatrix} 1\times 1 + 2\times 2 \\ 3\times 1 + 4\times 2 \end{bmatrix}_{2\times 1} = \begin{bmatrix} 5 \\ 11 \end{bmatrix}_{2\times 1}$

→ Transpose :-

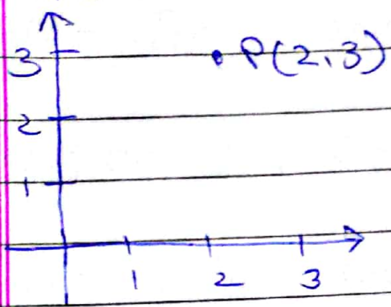
$A^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

i.e. $(A_{ij})^T = A_{ji}$; $\cancel{((A_{ij})^T)^T} = (A^T)^T = A$

$(A+B)^T = A^T + B^T$
$(A\times B)^T = B^T \times A^T$

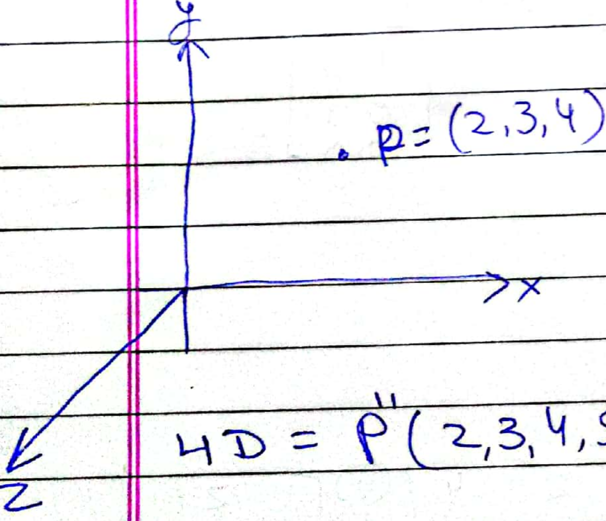
} $AB \neq BA$
$A+B = B+A$

* Vector:



$P = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ = column vector

→ 2D



$P' = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

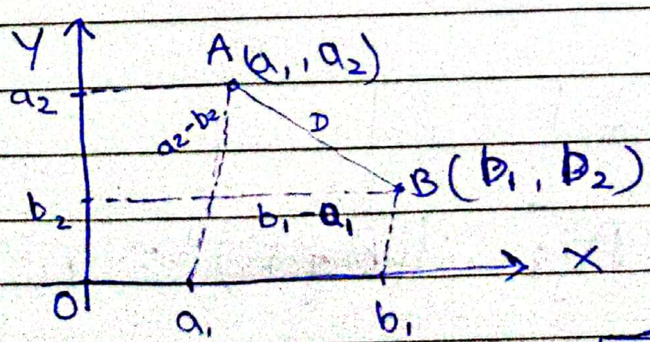
→ 3D

$4D = P''(2,3,4,5) = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$

→ Types of vector → Column vector: by default representation of point

→ Row vector.

* Distance b/w 2 points (Euclidean Distance



$D^2 = (b_1 - a_1)^2 + (a_2 - b_2)^2$

$D = \sqrt{(b_1 - a_1)^2 + (a_2 - b_2)^2}$

$D = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$

Now, $A = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ $B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ calculate distance?

$D = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$

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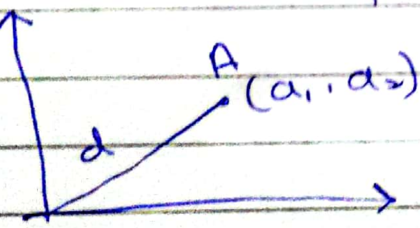
→ Generalize formula of distance :-

$$D_{pq} = \sqrt{(a_1-b_1)^2+(a_2-b_2)^2 \dots + (a_n-b_n)^2}$$

$$D_n = \text{sqrt}\left(\sum_{i=1}^{n} (a_i+b_i)^2\right)$$

→ Distance of point from origin



$$d = \sqrt{(a_1-0)^2+(a_2-0)^2}$$

$$d_1 = \sqrt{a_1^2+a_2^2}$$

$$n\text{-}D = \sqrt{a_1^2+a_2^2+\dots a_n^2} = \sqrt{\sum_{i=1}^{n} a_i^2}$$