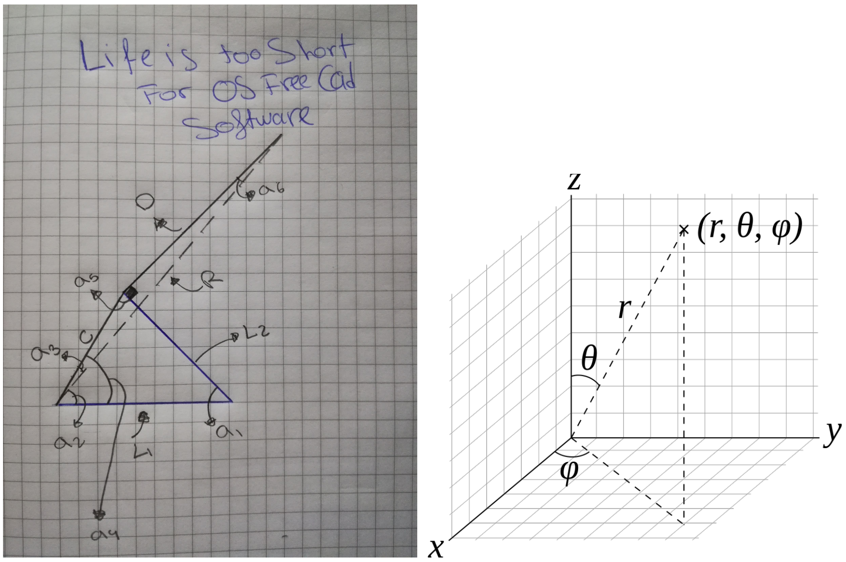

(a) x,y (b) x,y,z

Figure 1: 2 axis gimbal geometrical representation

From the servos and the sensor we get 2 angles and distance. the angle of the servo moving the sensor in the x/y plane can be used directly as the ϕ angle of the spherical coordinates. The radius and the θ angle will be calculated by the formulas below.

By applying the cosin rule on the L_1 L_2 C triangle:

$$C^2 = L_1^2 + L_2^2 - 2L_1L_2\cos a_1 \tag{1}$$

By applying the sine rule on the L_1 L_2 C triangle:

$$\frac{L_1}{\sin a_5} = \frac{C}{\sin a_1} \tag{2}$$

$$a_5 = \arcsin\left(\frac{L_1.\sin a_1}{C}\right) \tag{3}$$

By Combining (1) and (3):

$$a_5 = \arcsin\left(\frac{L_1.\sin a_1}{\sqrt{L_1^2 + L_2^2 - 2L_1.L_2.\cos a_1}}\right) \tag{4}$$

By applying the cosin rule on the R C D triangle:

$$R^2 = C^2 + D^2 - 2.C.D\cos(a_5 + 90) \tag{5}$$

From the Triangle C L_2 L_1

$$a_4 = 180 - a_5 - a_1 \tag{6}$$
$$a_2 = a_4 - a_3 \tag{7}$$

From (6) and (7)

$$a_2 = 180 - a_5 - a_1 - a_3 \tag{8}$$

By applying the sine rule on the R C D triangle:

$$\frac{R}{\sin(a_5 + 90)} = \frac{D}{\sin(a_3)} \tag{9}$$
$$a_3 = \arcsin\left(\frac{D.\sin(a_5 + 90)}{R}\right) \tag{10}$$

From (8) and (10)

$$a_2 = 180 - a_5 - a_1 - \arcsin\left(\frac{D.\sin(a_5 + 90)}{R}\right) \tag{11}$$

From Fig 1(b)

$$\theta = 90 - a_2 \tag{12}$$

From (12) and (11)

$$\theta = a_5 + a_1 + \arcsin\left(\frac{D.\sin(a_5 + 90)}{R}\right) - 90 \tag{13}$$

Finally we get from (13)

$$x = R.\sin(\theta).\cos(\phi) \tag{14}$$
$$y = R.\sin(\theta).\sin(\phi) \tag{15}$$
$$z = R.\cos(\theta) \tag{16}$$