Basic User Manual

for

CSP-Rules V2.1

Denis Berthier

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Books by Denis Berthier:
Le Savoir et l'Ordinateur, Éditions L'Harmattan, Paris, November 2002.
Méditations sur le Réel et le Virtuel, Éditions L'Harmattan, Paris, May 2004.
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Pattern-Based Constraint Satisfaction and Logic Puzzles (First Edition), Lulu.com, November 2012.
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Table of Contents

1. Introduction

1.1 What is CSP-Rules?

A finite binary Constraint Satisfaction Problem (CSP) is defined by a finite set of variables (hereafter called the CSP-Variables), each with a finite domain; the problem is to find a value in each of their domains, in such a way that these values satisfy a set of pre-defined binary constraints.

CSP-Rules is a *pattern-based* (or *rule-based*) solver of finite binary CSPs. In the CSP-Rules approach, *a possible value of a CSP-Variable is called a candidate and a binary constraint is represented by a contradiction link between two candidates*.

CSP-Rules is inherently associated with the approach to CSP solving defined and largely illustrated in my book "Pattern-Based Constraint Satisfaction and Logic Puzzles" ([PBCS]). This approach can be traced back, in the particular Sudoku context, to my much older book "The Hidden Logic of Sudoku" ([HLS]). CSP-Rules is best considered as a companion to [PBCS] and [HLS]. The books develop the theoretical part and the software is the proof that the theories are widely applicable.

In the [PBCS] approach, pertaining to the very general "progressive domain restriction" family of CSP solving techniques, the domain of each CSP-Variable is represented by a monotonously decreasing set of candidates and the fundamental concept is that of a *resolution rule*, i.e. a rule of the form "pattern => elimination of a candidate" or (more rarely in practice) "pattern => assertion of a value for a CSP-Variable". Here, "pattern" is a clearly defined set of logical conditions, based only on the fixed set of CSP-Variables, the fixed links (i.e. direct binary contradictions) between candidates and on the current situation (i.e. the remaining candidates for each variable). This pattern must be precisely defined so as to imply the conclusion in an otherwise context-independent way (i.e. in a way that does not depend on any CSP-Variable, link, candidate or value not explicitly mentioned in the pattern). Of course, the rule itself must be provable from the CSP axioms.

Considering this definition, an implementation in terms of rules of a forward-chaining inference engine seemed relevant (in spite of possible *a priori* performance issues). Both the oldest and the current implementations of CSP-Rules are therefore based on CLIPS, the most widely used (and tested) inference engine. CLIPS is written in C, is highly portable, has been public since 1996; it implements the RETE algorithm with drastic performance improvements – and it's open source and free.

One essential aspect of the [PBCS] approach is the introduction of additional CSP-Variables: the set of CSP-Variables is extended beyond the natural, originally given ones. As a simple illustration of this idea in the Sudoku case, in addition to the "natural" rc CSP-Variables (represented by the cells of the standard Sudoku grid), I introduced (in [HLS1]) new rn, cn and bn variables, represented by the cells of the three additional spaces of my "Extended Sudoku Board" (a copy of which is present in the "Publications" folder of CSP-Rules.)

A priori, CSP-Rules deals only with binary constraints, but the applications studied in [PBCS] and included in the CSP-Rules-V2.1 package (namely: solvers for Latin Squares, Sudoku, Futoshiki, Kakuro, Map Colouring, Numbrix, Hidato and Slithering) show that many types of non-binary constraints can be efficiently transformed into binary ones by adding problem-specific CSP-Variables, thus making them amenable to the CSP-Rules treatment.

Patterns in CSP-Rules can take many forms, but the most powerful generic ones are various kinds of chains (bivalue-chains, t-whips, whips, g-whips…). *A chain is defined as a continuous sequence of candidates, where continuity means that each candidate is linked to the previous one*. In the context of logic games, these chains can be considered as logical abstractions of the universal, spontaneous practice of human solvers wondering "can this candidate be true?" and checking for a possible contradiction implied by such an hypothesis; but they also suggest much more constrained ways of proceeding. In particular, the continuity condition is a very strong guide for a human solver looking for chain patterns. More generally, *the absence of OR-branching in any of the CSP-Rules chain patterns means that each of them supports a single stream of reasoning*.

Instead of having zillions of application-specific rules (like e.g. most existing rule-based Sudoku solvers), the resolution backbone of CSP-Rules consists of only a few types of universal rules – though it remains perfectly compatible with the addition of any number of application-specific rules (see the Slitherlink chapter).

One more essential aspect of the CSP-Rules resolution paradigm is its insistence on using the "simplest-first strategy". Indeed, much of my approach can be considered as a detailed study into possible meanings of "*simplest-first search*". At each step of the resolution process, the simplest available rule is applied. Here, "simplest" is not to be understood as it is generally done in the world of AI, i.e. in an abstract logical way that has never had any real application. Simplest is defined precisely in terms of the patterns making the conditions of the rules. In case of chains, simplicity is easy to define: a chain (of any type) is simpler than another chain (of any type) if it is shorter, where the length of a chain is defined as the number of CSP-Variables its definition involves. For chains of same lengths but of different types, it's also easy to define their relative simplicity (see chapter 4). As a result, for each family of rules, an intrinsic rating of the difficulty of instances can be defined and it can often be obtained at the end of a single resolution path. The simplest-first strategy is intimately related to the *confluence property* (see [PBCS]).

1.2 The contents of CSP-Rules

CSP-Rules consists of a generic part (in the folder "CSP-Rules-Generic") together with a few application-specific parts aimed at solving some familiar logic puzzles. The application-specific parts are integral members of CSP-Rules. They were chosen in order to illustrate how, by the proper choice of additional CSP-Variables, the generic concepts can be used in so different CSPs as the above-mentioned ones, including some non-binary ones.

The generic part consists of powerful generic resolution rules together with a general mechanism for managing the whole system, including the outputting of solutions in an easy-to-understand, universal notation. The generic part cannot be run alone. It requires an application-specific part to feed it with problem instances in the proper format.

Each application-specific part consists of a specific interfacing with the generic part of CSP-Rules, specific commands for launching the resolution process (they will be reviewed in the application-specific chapters in Part II of this Manual), plus a configuration file allowing the user to select general settings and which families of rules he wants to apply. It often also contains application-specific resolution rules (a few of which may be rewritings of the basic generic rules for better performance).

1.3 Scope of this Manual

All the general ideas, the precise concepts and the generic resolution rules, plus the specific applications and associated specific rules that materialised into CSP-Rules were introduced years ago in a series of books and articles starting with the first edition of [HLS] ([HLS1], May 2007) and ending in the second edition of [PBCS] ([PBCS2], July 2015).

All my publications related to the pattern-based resolution of finite CSPs are an integral part of CSP-Rules. You can find them in the "Publications" folder, together with the present Manual. At least some general understanding of PBCS is a pre-requisite for a deep understanding of CSP-Rules and of this Manual; [PBCS] is also enough, in the sense that there's no other prerequisite.

It must thus be clear that *this "Basic User Manual for CSP-Rules V2.1" is designed as a supplement to [PBCS], although only a general understanding of [PBCS] is required*.

"Basic" means that it explains how to install CSP-Rules and how to use it for solving instances of the specific types of puzzles that are already part of CSP-Rules.

It does not explain the CSP-Rules technical design, i.e. how the concepts introduced in [HLS] or [PBCS] are implemented in CSP-Rules, using the CLIPS language. [A possible future "Advanced User Manual" will explain how to add new rules to an existing application and a possible future "Programmer Manual" will explain how to create new applications or variants of the existing ones. Adding new rules is already easily possible by copying and adapting the existing ones.]

V2.0 was the stable version of CSP-Rules used to solve all the puzzles presented in [PBCS2]. It has also solved millions of Sudoku puzzles and thousands of puzzles of each of the other types defined in [PBCS2]. No software can ever be claimed to be bug-free, but V2.0 was probably as bug-free as it could be (in particular, bugs that would illegitimately eliminate a candidate are in general found after trying less than ten puzzles). This is no legal guarantee and there will be no reward other than eternal gratitude for reporting a bug ;) Version V2.1 is the first public distribution and it's only a small variant of V2.0 (with: easier installation procedure; easier selection of rules in the configuration files; optionally, more compact output thanks to "blocked" variants of some existing rules; zero change in the existing rules themselves but addition of a few typed-chains in order to comply with continued requests from readers of [HLS] to re-introduce my older "2D-chains" (as special cases of the general chain rules); addition of "oddagons").

"Supplement" means that this Manual is not fully self-contained; it does not repeat the general resolution paradigm defined and explained in [PBCS]. Nor does it repeat the detailed definitions of the different patterns or their resolution power. General concepts of the resolution paradigm or of whips, g-whips, braids, … are defined in [PBCS] and that is where you should look for full information about them. However, in order to avoid any ambiguity about CSP-Rules, I'll repeat (in the next two sections) the general motivations for the approach adopted in [PBCS] and some remarks about the rating of instances of a CSP. For an *informal* description of the main chain-pattern (whips), I'll also briefly give a graphico-symbolic representation of t-whips and whips in section 1.7, showing how they are related.

1.4 Motivations for the [PBCS] approach (adapted from [PBCS])

Since the 1970s, when it was identified as a class of problems with its own specificities, Constraint Satisfaction has quickly evolved into a major area of Artificial Intelligence (AI). Two broad families of very efficient algorithms (with many freely available implementations) have become widely used for solving its instances: general purpose structured search of the "problem space" (e.g. depth-first DFS, breadth-first BFS) and more specialised "constraint propagation" techniques (that must generally be combined with structured search according to various recipes). SAT solvers are also publicly available.

You may therefore wonder why you would want to use the computationally much harder techniques inherent in the approach introduced in [PBCS]. It should be clear from the start that there's no reason at all if speed is your first or only criterion. In particular, if your goal is to generate instances of a CSP (which requires a fast solver), CSP-Rules is not a viable choice.

But, instead of just wanting a final result obtained by any available and/or efficient method, you can easily imagine additional requirements of various types and you may thus be interested in *how* the solution was reached, i.e. in the *resolution path*. Whatever meaning is associated with the quoted words below, there are several inter-related families of requirements you can consider:

– the solution must be built by "constructive" methods, with no "guessing";

– the solution must be obtained by "pure logic";

– the solution must be "pattern-based", "rule-based";

– the solution must be "understandable", "explainable";

– the solution must be the "simplest" one satisfying the above requirements.

Vague as they may be, such requirements are quite natural for logic puzzles and in many other conceivable situations, e.g. when you want to ask explanations about the solution or parts of it.

Starting from the above vague requirements, Part I of [PBCS] (generalising the [HLS] approach) elaborated a formal interpretation of the first three, leading to a very general, pattern-based resolution paradigm belonging to the classical "progressive domain restriction" family and resting on the notions of a *resolution rule* and a *resolution theory* (i.e. a set of resolution rules).

Then, in relation with the last purpose of finding the "simplest" solution, [PBCS] introduced ideas that, if read in an algorithmic perspective, should be considered as defining a new kind of search, "*simplest-first search*" – indeed various versions of it, based on different notions of logical simplicity. However, instead of such an algorithmic view (or at least before it), a pure logic one was systematically adopted, because:

– it's consistent with the previous purposes,

– it conveys clear non-ambiguous semantics (and it therefore includes a unique complete specification for possibly multiple types of implementations),

– it allows a deeper understanding of the general idea of "simplest-first search", in particular of how there can be various underlying concrete notions of logical simplicity and how these have to be defined by different kinds of resolution rules associated with different types of chain patterns.

1.5 Simplest-first search and the rating of instances (adapted from [PBCS])

In the [PBCS] context and in conjunction with the "simplest solution" requirement, there appeared the question of rating and/or classifying the instances of a (fixed size) CSP according to their "difficulty" – a much more difficult topic than just solving them. The main families of resolution rules introduced in [PBCS] are various kinds of chains with no OR-branching [which would amount to adding hidden levels of T&E] and they go by couples, corresponding to two different linking properties, namely "T-whips" and "T-braids", where the T parameter refers to the "elementary" patterns allowed to appear in the chain as its building blocks (i.e. mere candidates for whips, g-candidates for g-whips, Subsets for S-whips, inner whips for W-whips…). For each T, there are two ratings, defined in pure logic ways:

– one based on T-braids, allowing a smooth theoretical development and having good abstract computational properties; much time was devoted to prove the *confluence property* of all the T-braid resolution theories introduced in [PBCS] (when T itself has the confluence property), because it justifies a "*simplest-first" resolution strateg*y (and the associated "simplest-first search" algorithms that may implement it) and it allows to find the "simplest" resolution path and the corresponding rating by *following only one resolution path*, a computationally noticeable advantage; the T-braid rating has the fundamental property of being invariant under CSP isomorphisms.

– one based on T-whips, providing in practice a good, much easier to compute approximation of the first when it is combined with the "simplest-first" strategy. (The quality of the approximation was studied in detail and precisely quantified in the Sudoku case, but it also appeared in intuitive form in all our other examples.)

[PBCS] explained in which restricted sense all the above ratings are compatible. But it also showed that each of them corresponds to a different legitimate "pure logic view" of simplicity – so that defining logical simplicity is not straightforward.

It must also be noted that all of the above ratings are ratings of the hardest-step. One might want to rate full resolution paths instead, but nobody has ever made any consistent proposal of how to approach such a problem.

1.6 What can one do with CSP-Rules?

Basically, CSP-Rules can solve and rate instances of a finite CSP (provided that this type of CSP has been interfaced to the generic part of CSP-Rules) – with some mild restrictions on their complexity. It will display the full resolution path, in an easy-to-read formal syntax clearly stating the justification of each step.

In chapter 11 of [PBCS], the scope of various resolution rules was analysed in terms of a search procedure with no "guessing" – Trial-and-Error (T&E) – and of the T&E-depth necessary to solve an instance; in and of itself (and contrary e.g. to the maximum depth reached by a DFS procedure), *this T&E-depth defines a broad, intrinsic and universal classification of the difficulty of all the instances of any finite binary CSP of any size* (and a – slow – T&E simulator is therefore included in CSP-Rules).

For instances in T&E(1) and T&E(2) (i.e. requiring no more than one or two levels of Trial-and-Error), there are universal "pure logic" ratings based on two T-braids families, respectively the B and the $B_?B$ ratings. [In the present case, universality must be understood in the sense that these two ratings assign a finite value to all of the corresponding instances, but not in the sense that they could provide a unique notion of simplicity.]

The above-mentioned universal T&E classification is an intrinsic property of all the instances of any finite binary CSP. CSP-Rules cannot be blamed for being unable to solve in "pure logic" ways some exceptionally hard instances that are intrinsically beyond the capabilities of its relatively simple chain patterns.

Being in T&E(0) means being solvable by the most elementary resolution rules. Being in T&E(1) amounts to being solvable by braids (and very often by whips). Being in T&E(2) requires not only braids but also much more complex B-braids (not coded in CSP-Rules) or, alternatively, bi-braid contradictions and B*-Braids. However, at the lower end of T&E(2), there is a small layer of puzzles in gT&E(1) = T&E(whips[1], 1) that is worth mentioning. Being in T&E(n) for larger n requires still more complex tri-braid (quadri-braid…) contradictions and $B^{*\cdots\cdots}$-braids.

As a result of this intrinsic T&E stratification, the solution of some exceptionally hard instances cannot be found if the resolution rules one has selected are not powerful enough. For the hardest instances, CSP-Rules may be able to find only a broad classification – e.g. membership in B_pB for instances in T&E(2) – instead of a pattern-based solution or a more precise rating. This can nevertheless be very useful if one wants to check the power of a new resolution rule R and asks: can R simplify the problem enough to make it belong to a lower B_qB after being applied? In case a puzzle cannot be solved by the chain-patterns, CSP-Rules also includes a simulated DFS procedure (very slow for that kind of algorithm) that can solve almost anything (if you allow it enough time).

In order to give a general idea of the power of whips and braids, it has been found that only one minimal Sudoku puzzle in about 70,000,000 is in T&E(2). This means that almost all of them are in T&E(0 or 1) and can therefore be solved by braids (and, most of the time, by whips).

Another way of considering the limits of CSP-Rules is, the various degrees of difficulty of puzzles you can find in newspapers correspond to walking to your garden versus to your nearest neighbour's home. The most extreme known instances correspond to going to far-away galaxies. CSP-Rules lets you travel this galaxy.

It may also be the case that some problem instance (considered as an ill-posed problem in the context of logic games) has several solutions. This is not a real problem for CSP-Rules. But as it can only do (constructive) pure logic deductions, it can only prove properties that are common to all the solutions (i.e. eliminate candidates that are false in all the solutions or find values that are true in all the solutions). Inconsistency of an instance is not a problem either: if it can be proven with the selected rules, CSP-Rules will prove it. Notice that proving inconsistency can be as hard as finding a solution.

Finally, remember that CSP-Rules was designed as a software for research, with the purpose of providing the "simplest" pattern-based solution as defined in [PBCS]. This is *per se* a much harder problem than just finding a solution and computation times cannot be expected to match those of the simpler problem (typically small fractions of a second). But I must also recall that my project was not a programming one, the only purpose of CSP-Rules was to validate the wide range of applicability and the resolution power of the rules defined in [PBCS] (plus alternative ones that I did not keep). Definitely, these rules could be implemented much more efficiently than they currently are. Efficient programming has never been my purpose in writing CSP-Rules, even though I've done a few speed and memory optimisations within the Rete algorithm paradigm underlying CLIPS.

1.7 A quick graphical introduction to the most basic chain rules

This section is not intended to be a summary of [PBCS], but only a graphical introduction to the most elementary chain rules. The central pattern in CSP-Rules is that of a whip. Except possibly bivalue-chains (that can be better understood without any reference to whips), all the chain rules in CSP-Rules can be considered as variants (either special cases or generalisations) of whips. Whence the necessity to understand whips. Whips have a very important property: they are continuous sequences of candidates, where "continuous" means that each candidate is linked to the previous one in the sequence.

First, we shall need some graphical conventions:

- I'll represent a CSP-Variable as a thick vertical straight line with a subscripted name above it, starting with V, e.g. V_1. You must understand that this is a purely abstract representation, that we could as well decide to represent a CSP-Variable by an oval surface, and that it is not supposed to represent real lines (rows or columns) in puzzles residing in a square or rectangular grid. To be concrete, in Sudoku, such a thick vertical line can represent a CSP-Variable of any of the four types (rc, rn, cn, or bn). In other terms, if you prefer, it represents any of the 324 (rc, rn, cn, or bn) cells of the Extended Sudoku Board.

- I'll represent candidates for V_i as doubly subscripted letters near this line (the first subscript being equal to the subscript of the CSP-Variable, i). Z, the name I usually choose for the target of a chain, will be a candidate for any CSP-Variable other than the V_i's. Some distinguished candidate for V_i may also be named R_i.
- I'll represent any other (non-CSP) link between two candidates as a thin line joining them. Remember that a link means a direct contradiction between two candidates.

1.7.1 Whips[1]

Simple as they may seem (they are the simplest pattern one can imagine, apart from Singles), whips[1] are an extremely powerful pattern. Whips[1] involve a single CSP-Variable.

Whips[1] can be considered as the first step of almost all the chain rules defined in [PBCS]. I strongly recommend that you look for whips[1] in different puzzles, not only in the most common one, Sudoku. The more examples you see from different perspectives, the more you will understand the magic of whips[1].

In Sudoku, rules called Intersections, Interactions, or also Pointing and Claiming, were known much before I introduced whips[1] (indeed much before I heard of Sudoku). As was proven in detail in [HLS] (with all the necessary graphics), it is easy to check that whips[1] are equivalent to these rules; i.e. they allow the same eliminations. (If you are not yet familiar with whips, I suggest that you prove this equivalence for yourself and you work on whips[1] as long as needed to understand them perfectly before trying to understand longer ones. A good test for this is whether you understand all of this section.)

However, there are three major differences between the usual viewpoint in Sudoku and the one introduced by whips[1]:

- whips[1] are defined in a universal way, meaningful for any binary CSP (and I have shown in [PBCS] that they are indeed very useful in many different types of CSPs, independently of having an underlying grid structure or not); try the various applications included in CSP-Rules to see how they appear in them;
- whips[1] make a strict difference between CSP-Variables and links (a difference totally blurred in Sudoku); but they treat all the types of CSP-Variables the same way and they treat all the kinds of links the same way; for instance, in Sudoku, they don't make any difference between numbers, rows, columns and blocks (more precisely, between rc, rn, cn and bn CSP-Variables);
- whips[1] can easily be generalised to longer chains, based on the same reasoning, as shown in the next subsection; it's therefore necessary to understand them perfectly in the precise context of any specific application before looking for longer ones.

***The whip[1] rule**: if a candidate Z is linked to all the remaining candidates for a CSP-Variable V_1 (which implies that Z is not a candidate for V_1), then Z can be deleted*. The proof is obvious: if Z was True, V_1 could have no value.

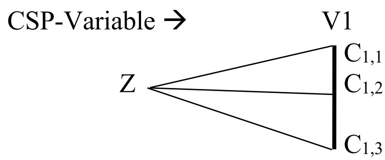


***Figure 1.1.** Symbolic representation of a whip[1]*

The elimination described in the whip[1] rule will be written symbolically as: $V_1\{C_{1,1}\ .\}$ => not Z. Here, $C_{1,1}$ is called the left-linking candidate and $C_{1,2}$ or $C_{1,3}$ the z-candidates (because they are linked to Z); in the present case, their roles could be interchanged.

In practice, when there is an underlying rectangular grid, a more specific notation (the nrc notation) will be applied and the eliminations will appear in still a more readable form, such as: $r_3n_4\{c_5\ .\}$ => $r_4c_6 \neq 4$. Here CSP-Variable V_1 is r_3n_4, it is supposed to have candidates only from the set $\{c_4, c_5, c_6\}$ ("in the intersection of row r_3 with block b_2"), which implies that any candidate "with" number 4 in block b_2 but not in row r_3 can be eliminated. This is the case for target Z = $n_4r_4c_6$.

1.7.2 A partial whip[1] and a whip[2]

It often happens that a candidate Z is linked to all the candidates for a CSP-Variable V_1, except to one of them. In and of itself, this does not allow any elimination; but it can be used to build whips[2].

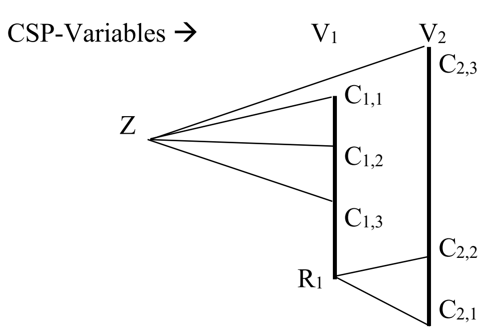


***Figure 1.2.** Symbolic representation of a whip[2]. A partial-whip[2] would have in addition one pending candidate for V_2: R_2 (as in Figure 1.3)*

In Figure 1.2, CSP-Variable V_1 has four remaining candidates, the first three of which (the same $C_{1,1}$, $C_{1,2}$, $C_{1,3}$ as before in Figure 1.1) are linked to Z and the fourth of which (R_1) remains pending. This makes a pattern for what I called a partial-whip[1] in [PBCS], where R_1 is called a right-linking candidate. CSP-Variable V_2 (different from V_1) is supposed to have three remaining candidates ($C_{2,1}$, $C_{2,2}$ and $C_{2,3}$), the first two of which are linked to R_1 and the last of which is linked to Z.

It is easy to see that if Z was True, V_1 would have only one remaining possible value (R_1), which would imply that $C_{2,1}$ and $C_{2,2}$ would be eliminated, leaving $C_{2,3}$ as the only possible value for V_2. But $C_{2,3}$ is made impossible by Z. This proves that candidate Z is impossible and can be eliminated. This can be written symbolically as: whip[2]: $V_1\{C_{1,1}\ R_1\}$ — $V_2\{C_{2,1}\ .\}$ => not Z, where $C_{2,1}$ is chosen as the second left-linking candidate. $C_{2,2}$ is called a t-candidate, because it is linked to a previous right-linking candidate. In this case, the roles of $C_{2,1}$ and $C_{2,2}$ could be interchanged.

This is exactly what the whip[2] pattern is: a simple extension of the whip[1].

1.7.3 A partial-whip[2] and a whip[3]

It's now easy to generalise this to longer whips.

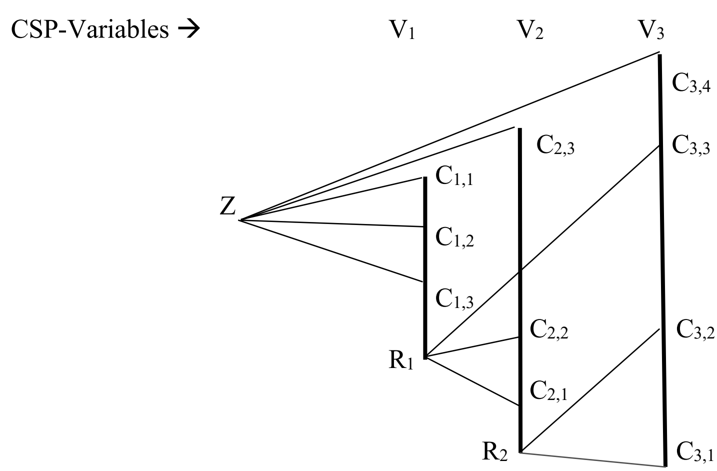


Figure 1.3. *Symbolic representation of a whip[3]*

Consider first the partial-whip[2] obtained from Figure 1.2 by adding a pending candidate R_2 to its last CSP-Variable, V_2. Figure 1.3 gives a symbolic representation for a whip[3]. A new CSP-Variable is added to the right and all its candidates are linked either to Z or to the right-linking candidates for the previous CSP-Variables, i.e. to R_1 or to R_2. $C_{3,1}$ is chosen as the left-linking candidate (linked to R_2), $C_{3,2}$ and

$C_{3,3}$ are t-candidates (linked to a previous right-linking one). $C_{3,4}$ is a z-candidate (linked to the target Z). There are none in this graphics, but V_2 also could have z-candidates and V_3 could have more than one. It might also be the case that the last CSP-Variable V_3 has no candidate linked to Z, if all its candidates are linked to a previous right-linking candidate.

The whip[3] in Figure 1.3 can be written symbolically as:
whip[3]: $V_1\{C_{1,1}\ R_1\}$ — $V_2\{C_{2,1}\ R_2\}$ — $V_3\{C_{3,1}\ .\}$ => not Z

1.7.4 A partial-whip[2] and a t-whip[3]

There is a second way a partial-whip[n-1] can be used for eliminations. The first way we have seen in section 1.7.3 is by extending its tail so as to obtain a whip[n]. The other way is by adding it a head so as to turn it into a t-whip[3]; and the head has to be a partial-whip[1].

In Figure 1.4, the whip[2] of Figure 1.2 has first been transformed into the same partial-whip[2] as in Figure 1.3, by adding a pending candidate R_2 to CSP-Variable V_2. Its ex-target Z has been renamed R_0 (because it will not be the target of the t-whip[3]). A partial-whip[1] for some target Z and CSP-Variable V_0 has been added to the left. The pending candidate for the unique CSP-Variable V_0 of this new partial-whip[1] must be R_0. And, in order to have a complete t-whip[3], its target Z must be linked to the last pending candidate of the previous partial-whip[2], namely R_2. I leave it as an easy exercise for the reader to prove that in this situation, Z can be eliminated.

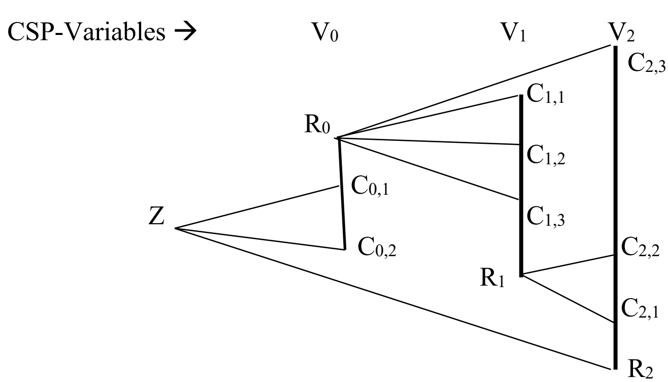


Figure 1.4. *Symbolic representation of a t-whip[3]*

The t-whip[3] in Figure 1.3 can be written symbolically as:
t-whip[3]: $V_0\{C_{0,1}\ R_0\}$ — $V_1\{C_{1,1}\ R_1\}$ — $V_2\{C_{2,1}\ .\}$ => not Z

Some users prefer t-whips because the main part of their construction does not depend on their final target(s). Unfortunately, they are less powerful than whips (as is obvious from their construction). Anyway, understanding the *simple relationship between partial-whips[n-1], whips[n] and t-whips[n]* is important for a human solver, because it means that, when he has found a partial-whip, he can try to extend it both ways. It is technically important also in the implementation of CSP-Rules, because the partial-whip structures are indeed shared between whips and t-whips.

1.7.5 Partial g-whips[1] and g-whips[2]

Whips[1] allow another kind of generalisation. In Figure 1.2, suppose that, instead of being a candidate, R_1 is a g-candidate for V_1, i.e. a particular group of candidates for V_1 such that each of these candidates is linked to both $C_{2,1}$ and $C_{2,2}$. It is clear that the same reasoning as above still applies (using reasoning by cases for $C_{1,4}$) and the same conclusion allows to eliminate Z. This is what a g-whip[2] is. The same remarks apply to R_2 and R_3. Again, I will not dwell on details or on extension to longer g-whips, but I'll refer you instead to [PBCS]. The existence of g-whips in a particular CSP is conditioned on the existence of whips[1].

Finally, braids and g-braids allow another type of extension, where the continuity condition is relaxed: in Figure 1.3, imagine that, instead of being linked to the pending candidate R_1 for the previous CSP-Variable V_1, "left linking candidates" $C_{2,1}$ and $C_{2,2}$ for V_2 were linked to Z; or that, instead of being linked to the pending candidate R_2 for the previous CSP-Variable V_2, "left linking candidates" $C_{3,1}$ and $C_{3,2}$ for V_3 were linked to Z or to a more distant pending candidate, e.g. R_1. It's easy to see that the proof for the elimination of Z remains valid. We have lost the simple form of continuity between the candidates making a single-threaded whip, but we have some "braided" continuity instead. However, this easy modification drastically increases the computational complexity, making the statistical approximation results of braids by whips invaluable in practice.

1.8 General warnings

- CSP-Rules interface is text-only and it is not interactive. One types a simple command line in a simple Lisp-like syntax asking for some solution and the output is the full resolution path in plain text. No blinking, no colours, no frills, no nothing. If you cannot do without a graphical interface, you would be better inspired to look for something else or to propose your own graphical extensions via GitHub, because I am not going to write one. CSP-Rules was designed as a research and teaching tool; several times, I gave my AI students a part of SudoRules plus some (generally vague and incomplete) definitions of rules found on websites and I asked them to formalise them, first in English, then in First Order Logic and then to code them in CLIPS. I make it public now because of an increasing number of readers of [PBCS] and [HLS] insisting on giving it a try.

- CSP-Rules hasn't been designed for fast solving but for ease of implementation (for me). If what you need is a fast solver, you are not looking at the right place. Even the included DFS or T&E simulators are very slow compared to ones programmed in an optimal way in C. Thinking specifically of Constraint Satisfaction, there are much faster (and free) generic CSP solvers – though none (as far as I know) that will output a readable resolution path. Although they all use the same rules, there are also differences of speed between the various existing CSP-Rules applications, e.g. between KakuRules (fast) and SlitherRules (slow).
- As any knowledge-based system, CSP-Rules may require a fair amount of memory for hard puzzles. However, all the examples appearing in [PBCS] have been solved with only 16 GB of RAM and most of them require much less.
- If you are a "normal" puzzle solver, i.e. you are not submitting to CSP-Rules puzzles that any player would not even consider solving himself, you can completely forget the above-mentioned limitations of CSP-Rules in terms of speed or memory. Instead, you can play with the various combinations of rules and see what they allow. The possibilities are almost unlimited.

1.9 Selected generalities about CLIPS

Although the reader of this Manual does not need to know much about CLIPS, it may be useful when typing a user function to remember a few basic facts. Don't worry if you have never used any programming language: you don't have to understand this section in order to use CSP-Rules at the ordinary user level; you can totally skip it. The only two things you will have to do is to use a text editor to delete semi-colons in the configuration files and to type something like:

(solve "4...3.......6..8..........1....5..9..8....6...7.2........1.27..5.3....4.9........"),

where the string is some more or less standard representation of a puzzle. (See the application-specific chapters for details.)

What follows is more for programmers of classical languages, so that they don't get lost with the specificities of CLIPS.

- If you have a look at the resolution rules coded in CSP-Rules, you will see that the CLIPS rules syntax is an elaborated version of First Order Logic. The main part of CSP-Rules is made of rules: ***the rules are the program***; it is the role of the CLIPS inference engine to apply properly these rules. The parts of CSP-Rules that are not made of rules are of two kinds: output functions and interfaces of the various applications that feed the givens of a problem instance to the rules of CSP-Rules.
- CLIPS general syntax, as you will mainly see it in the user functions, is Lisp-like. Lisp is a functional language. A function is not called as f(x) but as (f x): opening parentheses are before the function symbol.
- Variables names always start with "?". Global variables names always start with "?*" and always end with "*".

- The last argument of a function may be a list; in the function definition it has to be written with leading characters "$?" as in "$?rest" (the rest of the arguments); in a call to the function, all the arguments remaining after those that are associated with individually named variables are automatically made into a list and assigned to variable $?rest. It is important to remember this, because many user functions in CSP-Rules will use this very convenient possibility. There can be only one list variable in the definition of a function and it can only be the last argument.
- Any positive number of consecutive spaces, tabs, carriage returns or line feeds is equivalent to a single space; this has the advantage of allowing nice formatting of the programs or the arguments to functions, at no cost for the programmer.
- A semi-colon is a reserved character. Anything in a line after a semi-colon is a comment and can be considered as non-existent as far as the effective code is concerned. It's common usage (but not mandatory) to use three semi-colons for comments at the start of a line. Comments can appear at almost any place inside a function:

```
;;; Use this function to solve a puzzle
(solve
   ;;; I am a general comment about function solve
   9 ; I am the size of the grid
   Hidato ; I am the name of the type of game
   ; argument3 ; in this version of the software, I am no longer an argument
   1 5 6 12 ; we are the arguments making the last variable (a list)
)
```

There are very strange consequences of this rule when a semi-colon appears inside double quotes; but this will never happen in CSP-Rules (because it leads to what I consider unpredictable behaviour). Just for fun, you can play with this:

```
(printout t "; I am a string" crlf)
```

By the way, if you are blocked after typing something like that (i.e. if the CLIPS prompt does not reappear), try typing several closing parentheses. If it does not work, try typing a double quote (only one) and then as many closing parentheses as necessary. This will avoid to quit CLIPS in a savage way – i.e. with a control-c in Unix –, the proper way being by typing "(quit)".

As a general recommendation, if you have to type anything in the CLIPS interpreter, it is much better to type it first in any text editor and then to paste it into CLIPS: it's very easy to miss or add some double quote or some semi-colon or some parenthesis and it may lead you to a blocked situation similar to the above-mentioned one; once a line is entered into CLIPS, there's no way to go back.

- There are six CLIPS commands that it may occasionally be useful to know:

  1) (rules) will output the set of current rules in CLIPS (the knowledge base);

  2) (facts) will output the set of current facts in CLIPS (the facts base);

3) (clear) will empty CLIPS of all its facts and rules. You can use this instead of quitting CLIPS before loading another set of rules. It is equivalent to quitting and re-launching, but in practice I usually prefer quitting and re-launching, especially after an exceptionally hard puzzle that required large amounts of memory.

4) (reset) will clear all the facts but not the rules. In CSP-Rules, the behaviour of (reset) is fixed so that it does not change the values of the global variables and it is important to not change this. A reset is done automatically by CSP-Rules every time a "solve" function is called and it involves releasing memory no longer used. Memory is managed in a very safe way in CLIPS: many times, I've left a single instance of CLIPS run for full days to solve hundreds of thousands after hundreds of thousands of puzzles without ever needing to quit and restart it.

5) (release-mem) allows to release the memory asked by CLIPS during resolution. This may be useful after a very hard puzzle has been solved. But this is also done automatically before each new puzzle is solved.

6) (dribble "filename) allows to redirect all the output (normally sent to the Terminal / Command Window) to the file named "filename".

Part One

GENERALITIES

2. Installing and launching CSP-Rules

CSP-Rules is coded as a knowledge-based system and it requires an independent inference engine to run its sets of rules. The “programming” language used by CSP-Rules is that of the CLIPS inference engine, with a Lisp-like syntax. The present Manual will not explain how to use CLIPS beyond launching it – the only thing strictly necessary to run CSP-Rules. Indeed, in order to make CSP-Rules easy to use for anyone without requiring preliminary compilations (which can be quite challenging on Windows), executable versions of CLIPS for Unix (including MacOS) or Windows environments are provided with CSP-Rules-V2.1 (in the “CLIPS” folder).

2.1 Installing CSP-Rules (machine independent)

Create a “CSP-Rules” folder at any reasonable place in your file system, e.g. within a “Tools” directory inside your home directory. Download CSP-Rules-V2.1.zip from the GitHub repository: goto https://github.com/denis-berthier/CSP-Rules-V2.1/master, click the “Code” button to open its menu and then select “Download ZIP”; move the downloaded file into your “CSP-Rules” folder, unzip it (by double-clicking it) and rename it “CSP-Rules-V2.1” (i.e. delete the final “master” part, if any).

At this point, you will need your preferred text editor to modify some files.

Modify as follows each of the configuration files of the predefined applications (i.e. all the files inside the “CSP-Rules-V2.1” folder with names ending in “config.clp”). The three modifications listed below are in the upper block of each configuration file, under the local banner “INSTALLATION ONLY”. The four lines are marked with an ending “<<<<<<<<<<<” sign. You can modify one of the configuration files and then copy/paste this whole upper block onto the corresponding blocks of the other files so as to replace them.

1 – mandatory in odd cases) If you are using any odd system (other than MacOS, Unix or recent Windows) that doesn’t use “/” as its directory symbol, then define it here, e.g.

```
 (bind ?*Directory-symbol* "\")
```

2 – mandatory) Change the definition of the the ?*CSP-Rules* global variable, so that it coincides with the absolute path to your previously defined CSP-Rules

folder (including the final directory symbol "/", otherwise it will not work); for instance:

- on a Unix system:

```
(defglobal ?*CSP-Rules* =
"/Users/<your_home_directory>/Tools/CSP-Rules/")
```

Beware: Clips does NOT understand the Unix "~" as a shortcut to your home directory.

- on a Windows system:

```
(defglobal ?*CSP-Rules* =
"c:/Users/<your_home_directory>/Tools/CSP-Rules/")
```

With this way of proceeding, CSP-Rules can be run from any folder.

3 – optional) If necessary (i.e. if you want to use a release of CLIPS different from the r770 one provided with CSP-Rules), change the CLIPS version number (this is only used for keeping track of which release of CLIPS you have used during resolution and it will appear only in the CSP-Rules banners); it will not change anything else if you forget to do this:

```
(defglobal ?*Clips-version* = "6.32-r770")
```

4 – optional) Describe, with all the details you like, the machine you will use to run CSP-Rules. This will only be used for keeping track of this information and for printing it at the end of each resolution path, in the CSP-Rules banner:

```
(defglobal ?*Computer-description* =
"MacBookPro Retina Mid-2012 i7 2.7GHz 16GB, MacOS 10.5.4")
```

Et voilà. CSP-Rules is ready to run.

2.2 Launching a CSP-Rules application

Each application consists of two parts: a directory named *App*Rules-V2.1 plus a configuration file named *App*Rules-V2.1-config.clp. If you frequently use several different configurations for the same application, there is no restriction to having several different configuration files for it. My recommendation is to always name them according to the pattern *App*Rules-V2.1-config-xxx.clp, where xxx can be any mnemonic for your configuration.

To launch and use an application, there are four steps:

1) Launch Clips by double-clicking the proper application in the CLIPS subdirectory of CSP-Rules-V2.1, i.e. "clips" on a Unix (including MacOS) system and "clips.exe" on a Windows system. The starting of CLIPS is immediate. This is indicated by the standard CLIPS prompt: "CLIPS>".

2) In the relevant config file, specify which options and which resolution rules you want to use [how to make such choices consistently will be seen in the relevant chapters of Part II]. A choice by default is already made in the included configuration files, so that you can start using CSP-Rules without any delay.

3) Inject the configuration file into CLIPS. There are two ways to "inject" it:

- either copy the full content of the file and paste it directly into CLIPS (this works perfectly on MacOS, but I did not try on other systems);
- or type in Clips the following command line (with the relevant changes for the parts in italics):

```
(batch
"/Users/<your_home_directory>/Tools/CSP-Rules/CSP-Rules-V2.1/AppRules-config.clp")
```

Both ways amount to applying successively in Clips all the "constructs" and commands included in the configuration file. The configuration file sets a few global parameters and then loads all the files corresponding to the application and the chosen resolution rules, with the chosen options.

At this point, after some amount of useless messages (including normal warnings about rules being redefined), the CSP-Rules banner should appear, with the first line recalling which application you are using, based on which version of CSP-Rules, which version of CLIPS (if you have configured it during installation) and which resolution theory. The banner is followed by the standard CLIPS prompt ("CLIPS>"). Here you can see my standard – and the default – configuration for SudoRules: whips + Subsets + Finned Fish (the ".s" means the default option for rules, "speed", has been kept):

```
*********************************************************************************
*** SudoRules 20.1.s based on CSP-Rules 2.1.s, using CLIPS 6.32-r770, config = W+SFin
*** Copyright Denis Berthier
*** Running on MacBookPro Retina Mid-2012 i7 2.7GHz 16GB 1600MHz DDR3, MacOS 10.15.4
*********************************************************************************
CLIPS>
```

The information thus displayed may seem useless to you now, but if you solve hundreds or thousands of puzzles, keeping track of it together with the actual resolution paths may spare you the necessity to re-run them in case you don't remember.

CSP-Rules is now ready to solve puzzles for you. If you want to check which rules have been loaded, you can type in the CLIPS command: "(rules)". The last line of the output will be the total number of rules. Notice that what is called a resolution rule in [PBCS] is generally implemented via several CLIPS rules (generally an activation rule, a tracking rule, one or more pattern definition/extension rules and one or more rule-application rules), so that what you get here is the number of CLIPS rules, not the number of resolution rules in the sense of [PBCS].

4) Decide which puzzle P you want to solve and type something like:

```
(solve "…………the_puzzle………")
```

where "…………the_puzzle………" must be the proper representation of P for this type of puzzle in CSP-Rules. Depending on the application, one or more additional parameters may be needed, such as size of the problem, variant of the puzzle… More application-specific commands will be described in part II, but function "solve", or some form of it, is the basic command for all the applications. Go directly to the application-specific chapter for details about its syntax and a direct use of CSP-Rules.

The result will be the full resolution path for the chosen puzzle. Note that the computation times can be extremely different for different types of puzzles and for different instances of a given type. A good way of getting used to this is to try progressively harder puzzles.

Step 4 can be repeated as many times as you want, as long as you don't change the type of puzzle, the options or the set of resolution rules you want to apply. The "cleaning" of the CSP-Rules "memory" between the resolution of two puzzles is automatic, but you can apply it yourself by typing "(reset)" and then "(release-mem)" when the CLIPS prompt re-appears.

However, ***if you want to change the set of resolution rules or the options or the type of puzzle (and for some applications such as Sudoku, the size of puzzles), you have to re-start from step 1*** – because, for efficiency reasons, only the rules selected in the configuration file are loaded when you "inject" this file into CLIPS at step 3.

3. The generic resolution rules present in CSP-Rules

CSP-Rules V2.1 includes most of the generic resolution rules mentioned in [PBCS], plus (some generic form of) the more specific rules ("2D-chains") previously defined in [HLS]. It also includes rules specific to some applications, that will be discussed in the relevant chapters, in Part Two of this Manual.

3.1 Basic generic resolution rules

The first generic resolution rules present in CSP-Rules V2.1 constitute the universal Basic Resolution Theory (BRT) defined in [PBCS] and, as such, they cannot be turned off. They are:

- ECP (Elementary Constraints Propagation); their conclusions are so obvious that they are generally not printed (unless one explicitly sets global variable ?*print-details* to TRUE in the configuration file);
- Singles;
- Rules for managing the resolution process, such as contradiction detection, solution detection, … (see [PBCS] for details). They act unnoticed by the ordinary user.

Note also that each application may (and some do effectively) overwrite the generic version of ECP and/or Singles or even whips[1], for efficiency reasons. All this is transparent to the ordinary user.

3.2 Ordinary generic resolution rules

First, note that Subset rules (and their variants, such as Finned Fish in Sudoku) are not provided in a generic form in CSP-Rules, although they could be programmed as set covering rules. But such a general form would be very inefficient (and it would cause quick memory overload). Instead, they have application-specific versions, but there are generic hooks to deal with them (see the relevant chapters).

As a result, the main types of resolution rules introduced in [HLS] and [PBCS] that are present in generic form in CSP-Rules V2.1 are various kinds of chains. They are:

- bivalue-chains, up to length 20;

- typed bivalue-chains, up to length 20;
- z-chains, up to length 20;
- typed-z-chains, up to length 20;
- oddagons, up to length 15;
- t-whips, up to length 36; notice that t-whips are a special case of whips, they are less powerful, but they can be built independently of the target z;
- typed t-whips, up to length 36;
- whips, up to length 36;
- typed whips, up to length 36;
- g-bivalue chains, up to length 20;
- g2-whips, up to length 36;
- g-whips, up to length 36;
- braids, up to length 36;
- g-braids, up to length 36;
- forcing-whips and forcing-g-whips, up to length 36;
- forcing-braids and forcing-g-braids, up to length 36.

I set the above-mentioned maximum lengths for each kind of chains because I never needed anything close to so long chains when solving millions of puzzles of different types. In practice, you'll never see so long chains.

All of the above chain rules, except oddagons and g2-whips, come in two independent versions: speed and memory, corresponding to different optimisations. g2-whips have only the memory version, because they are supposed to be used mainly when g-links haven't yet been defined. Forcing-whips, forcing-g-whips, forcing-braids and forcing-g-braids have apparently only one version, but they are built on the chosen speed or memory versions of whips, g-whips, braids and g-braids, so that, in practice, they do have two versions also.

A small part of the resolution rules also come in two slightly different versions (which may make a total of four different versions for some of them): an ordinary one that works and prints the output as described in [PBCS] (and as in V2.0 – i.e. only one elimination at a time) and a new "blocked" one that finds all the potential targets for an instantiation of the rule before eliminating all of them in a single sweep. In this new behaviour, rules will no longer seem to be "interrupted" by simpler ones (and sometimes to re-start later). After experimenting with this behaviour, I made it the default one in version V2.1, but each configuration file allows to revert to the previous [PBCS] (and V2.0) behaviour if that is what you prefer. Rules amenable to this new "blocked" behaviour are whips[1], all the (typed or not) bivalue-chains, all the (typed or not) t-whips and all the Subset rules in any of the coded applications; they can be independently reset to the old behaviour. Application-specific rules for Slitherlink were coded from the start with this new behaviour and they have no other version.

3.3 A word on typed chains

Typed bivalue-chains, typed z-chains, typed t-whips, typed whips and typed g-whips were not discussed in [PBCS], but they were introduced in [HLS] under other names. In Sudoku, they are the "2D" counterparts of bivalue-chains, z-chains, t-whips, whips and g-whips (see section 6.4).

The difference of any of these typed chains with its untyped version is, all its CSP-Variables must have the same type (e.g. rc, rn, cn or bn in Sudoku); however, typed-chains of different types can appear in a resolution path – unless this possibility is further restricted, using a simple generic mechanism (see the application-specific chapters for details). Obviously, the notion of a typed-chain is less general than that of a chain of the same kind (i.e. bivalue-chain, z-chain, t-whip, whip or g-whip), but it may be used for many different purposes, depending on the applications and on one's goals.

In a resolution path, a typed chain (e.g. a typed t-whip) is printed as a chain of the same kind (a t-whip), with its type appended between its kind and its length (e.g. t-whip-rc[5]); the word "typed" does not appear at the start, because it would be redundant with the explicit naming of the type at the end.

3.4 Resolution rules for simulating T&E (Trial and Error)

CSP-Rules V2.1 also provides a rule-based version of T&E at depths 1, 2 and 3. (Notice that it's very inefficient, compared to what an optimised C version could do.) This is mainly designed:

- to check (e.g. before launching the actual resolution) whether a puzzle is in T&E(1), i.e. whether it can be solved by braids (and then, probably by whips),
- to check (e.g. before computing its $B_?B$ classification) whether a puzzle is in T&E(2), i.e. whether it could be solved by B-braids,
- to find the $B_?B$ classification of a puzzle in T&E(2),
- to find the $S_?B$ classification of a puzzle in T&E(S, 1), when application-specific Subset rules are defined in some application.

In principle, T&E can be used in combination with any resolution theory T (i.e. any set of ordinary resolution rules), provided that T has the confluence property, in order to check whether a puzzle is in T&E(T, 1), T&E(T, 2)… Notice however that using T&E(T, 1) amounts to solving hundreds of puzzles and it can take much time if T is complex. T&E(T, 2) will roughly take almost the squared time of T&E(T, 1).

In practice, the configuration files consider using T&E for only three purposes:

- finding the T&E-depth of a puzzle P, i.e. its membership in T&E(BRT, k) = T&E(k), the useful result being when k = 1 or k = 2 (k = 0 is trivial). k = 1 ensures the existence of a solution with braids (thanks to the T&E vs braids theorem). Most of the time in this case, there is also a solution using only whips. But sometimes, braids are really necessary; such puzzles are generally very hard and you can expect very long resolution times and memory requirements with braids; it's often faster to look for a solution with g-whips – though being in T&E(1) is not a guarantee for a g-whip solution;
- finding the smallest k such that P is in T&E(S_k), i.e. it's solvable by S_k-braids;
- finding the smallest k such that P is in T&E(B_k), i.e. it's solvable by B_k-braids.

3.5 Resolution rules for simulating DFS (depth-first-search)

DFS is known to be the fastest method for solving 9×9 Sudoku puzzles. CSP-Rules V2.1 also provides a (very inefficient) rule-based version of DFS. Originally, this was mainly designed for obtaining solutions faster than with the true rule-based methods. DFS can be combined with any choice of resolution rules, but it is generally faster if one only allows whips[1] and possibly whips[2] or whips[3], depending on the application. Notice that the computation time of DFS increases very fast (i.e. exponentially, for all practical purposes) with the size of the puzzle and does not depend in any measurable way on its W, B, gW, gB… complexity.

3.6 Hooks for introducing new resolution rules

Although CSP-Rules-V2.1 does not have generic Subset rules, it provides hooks for dealing with CSPs defined on rectangular grids with rows and columns, as is often the case for logic puzzles. In particular, it defines the *nrc-notation* for the output of every rule, as a specialisation of the general CSP-Rules output mechanism. All the applications discussed in this Manual use this nrc-notation (or some direct variant of it when there is no underlying rectangular grid, such as in MapRules).

4. Simplest-first strategy, saliences, ratings

As recalled in the introduction, given a fixed set of resolution rules, the resolution process of CSP-Rules is guided by the "simplest-first" strategy.

This strategy could be implemented in many different ways in CSP-Rules, but I've chosen to use CLIPS "saliences" (i.e. priorities between rules) in a systematic way. The use of too many saliences in the knowledge base (i.e. the set of rules) of a knowledge-based system is generally not very well considered in the AI community, but this is the most versatile way of allowing additional (application-specific or user-specific) rules to be finely inscribed at the proper level of the complexity hierarchy if an occasional user wants to write some rule(s) of his own, without requiring a deep understanding of the system. Moreover, from a more theoretical point of view, it allows to make this hierarchy explicit and thus to keep totally separate:

- the "*knowledge level*" (in the sense of [Newell 1982]), defined by the textual and/or logical formulations of the resolution rules, as written in [HLS] or [PBCS];
- the "*symbol level*" (also in the sense of [Newell 1982]), defined by their implementation as a set of CLIPS rules;
- the "*strategic meta-level*" (introduced in all the modern methodologies of Rule-Based systems development) of how the rules defined at the knowledge level are to be used in a resolution process; as said before, this meta-level is implemented in CLIPS as a set of saliences.

As the main interest of this point is for the user who wants to extend some existing CSP-Rule application or to add a new one, details will only be given in a more advanced manual. What I'll say in this chapter intends mainly to explain why some patterns appear before other ones in the resolution paths, in (the frequent) cases when different possibilities are available.

General priorities between existing resolution rules have been described in [PBCS]. They are organised by levels, each level being defined by the number of CSP-Variables involved in the defining pattern (i.e. the conditions of the rule).

Within each level, priorities are mainly based on the generalisation relation, where a special case is given higher priority (otherwise, it would never appear in the resolution paths). For every $k > 1$, one has:

biv-chain[k] > z-chain[k] > t-whip[k] > whip[k] > g-whip[k] > braid[k] > g-braid[k] > forcing-whip[k] > forcing-braid[k]

For the typed-chains, which were not considered in [PBCS], the most natural place is to insert them just before their untyped version. The place of forcing-whips and forcing-braids was not mentioned in [PBCS], but they naturally come at the end of this hierarchy.

Notice that z-chains[k] are placed before t-whips[k], though none is more general than the other. The reason is, although they depend on their target, z-chains are simpler to find than t-whips.

Notice also that g-whips[k] are placed before braids[k], though none is more general than the other. There are three reasons: they respect the continuity condition, they are computationally simpler patterns and they are statistically more likely to produce results not reachable by whips.

CSP-Rules allows an easy addition of application-specific saliences at each level, before or after the generic saliences. In general, application-specific rules based on k CSP-Variables should be given higher priority than the generic rules with the same number of CSP-Variables. Otherwise, they might never appear in the resolution paths, because most application-specific rules turn out to be particular cases of the generic chain rules. This assertion is not strictly true for Subsets, but it is true in most of the cases (for details, see the subsumption theorems for Subsets in [PBCS]).

If two rules with the same priorities are available, which of the two is used first is decided randomly (this refers to conceptual randomness, i.e. you cannot rely on any assumption about how the choice will be made, but if you run the same example with the same rules, you will get the same resolution path; if you want different paths, you may try to vary the rules, e.g. add or delete some types of special cases. Often, I do this by activating or de-activating z-chains and/or t-whips. In case the application has typed-chains, they can also be used that way. Notice that having many different kinds of chains has a cost in terms of memory and computation time and it is not recommended for the very hard puzzles.

Finally, CSP-Rules is not intended to be a normative system. As an automatic solver, it must have some strategy; it finds a resolution path according to the selected rules and the priorities defined by its strategy. It doesn’t mean that you are wrong if you find a different path.

Part Two

RUNNING SPECIFIC APPLICATIONS

5. General structure of the configuration files

A user has two independent means of interacting with an application of CSP-Rules: its configuration file and its user functions (or commands).

The functions available to the user may differ from application to application, but there's always some form of a "solve" function (with its own list of arguments); it is the most basic function for solving an instance of the CSP under consideration. See the relevant chapter for each application.

As for the configuration files, they all have the same structure and they are almost self-explanatory. For definiteness, I'll take here the SudoRules configuration file as an example and I'll only add a few comments at the relevant places. For clarity, parts of this chapter copied from the SudoRules configuration file use a different, machine-like, font.

In the configuration file, the user chooses general options and which rules he wants to use; at the end of the configuration file, CLIPS is asked to load the rules corresponding to the user choices. For efficiency reasons, non-selected rules are not loaded, except the rules implied by the selected ones (such as braids => whips). The user functions are totally independent of these choices and they work the same way for any of them; they will be described in each application-specific chapter. If you have no experience in programming, don't let these files impress you: the only thing you may have to do is to use a text editor to delete a few semi-colons.

5.1 The first part of the configuration file is for installation

The first part of the configuration file is for installation and needs to be changed once and only once, as explained in chapter 2. If you have already done the installation, you can proceed directly to section 5.3 below. Otherwise, check the four lines marked with the ending signs "   <<<<<<<<<"). This part is identical for all the applications and can be copied from the one to the next.

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;
;;; INSTALLATION ONLY:
;;; Define environment variables: OS, installation directory and inference engine
;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
```

```
;;; Default setting is for Unix and MacOS,
;;; but it should also work for recent versions of Windows:
(defglobal ?*Directory-symbol* = "/")

;;; Define your general CSP-Rules installation directory (including the ending
directory symbol /).
;;; This is the directory in which the CSP-Rules-V2.1 version is installed, not the
CSP-Rules-V2.1 directory.
;;; By defining the path in an absolute way, you will be able to launch CSP-Rules-V2.1
from anywhere.
;;; You need to write something as follows.
;;; For Unix (including MacOS):
 (defglobal ?*CSP-Rules* = "/Users/berthier/Documents/Projets/CSP-Rules/") ; <<<<<<<<
;;; For Windows:
; (defglobal ?*CSP-Rules* = "c:/Users/berthier/Documents/Projets/CSP-Rules/") ; <<<<<<

;;; compatibility with JESS is no longer guaranteed and CLIPS is the default inference
engine
;;; the version of CLIPS used may be defined here (used only for displaying it in the
banner)
(defglobal ?*Clips-version* = "6.32-r770");           <<<<<<<<<<<<<<<<<<<<<<<<<<<<<


;;; Description of the computer used for the resolution
(defglobal ?*Computer-description* =
  "MacBookPro Retina Mid-2012 i7 2.7GHz 16GB 1600MHz DDR3, MacOS 10.15.4")     <<<<<<<
```

5.2 The second part of the configuration file must not be changed

The next part of a configuration file is application specific in its details but has the same structure in all the applications. It just elaborates the user data defined in the first part and it pre-loads some global variables necessary for the user's upcoming choices. It should not be modified. For Sudoku, it looks like this:

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;
;;; Define the application
;;; Define useful directories and load all the globals
;;; (they must be available before choosing the configuration of rules)
;;;
;;; don't change any of the following
;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

;;; an ?*Application* must be defined as the name of the application (here,
SudoRules), not as the name of the puzzle (Sudoku)
;;; this name must coincide with the leading part of the name of the directory for the
application inside the CSP-Rules-V2.1 directory
;;; the version number of the ?*Application* must also be defined
```

```
;;; this allows to have several versions of the same application based on the same version of CSP-Rules
;;; for historical reasons, SudoRules version number inside CSP-Rules-V2.1 is not 2.1 but 20.1
;;; (there were many versions of SudoRules before the development of a generic CSP-Rules core)
(defglobal ?*Application* = "SudoRules")
(defglobal ?*Application-VersionNumber* = 20.1)
(defglobal ?*CSP-Rules-current-version* = (str-cat ?*CSP-Rules* "CSP-Rules-V2.1" ?*Directory-symbol*))
(defglobal ?*CSP-Rules-Generic-Dir* = (str-cat ?*CSP-Rules-current-version* "CSP-Rules-Generic" ?*Directory-symbol*))
(defglobal ?*CSP-Rules-Generic-Loader* = (str-cat ?*CSP-Rules-Generic-Dir* "CSP-Rules-Generic-Loader.clp"))

(defglobal ?*Application-Dir* = (str-cat ?*CSP-Rules-current-version* ?*Application* "-V" ?*Application-VersionNumber* ?*Directory-symbol*))
(defglobal ?*Application-Loader* = (str-cat ?*Application-Dir* ?*Application* "-Loader.clp"))

;;; load declarations for the global variables necessary for the upcoming choices
(load (str-cat ?*CSP-Rules-Generic-Dir* "GENERAL" ?*Directory-symbol* "globals.clp"))
(load (str-cat ?*Application-Dir* "GENERAL" ?*Directory-symbol* "globals.clp"))
```

5.3 The third part of the configuration file is application specific

The next part of a configuration file is application specific and most of the time, it will be non-existent. In SudoRules, it looks like this:

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;
;;; General application-specific choices
;;; Define grid size and related parameters
;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

;;; because ordinary Sudoku puzzles are 9x9 (i.e. segment-size = 3),
;;; SudoRules has its own way of dealing with the size of puzzles
;;; and this can only be done here; maximum segment size is 9
; (bind ?*segment-size* 4);                                 <<<<<<<<<<<<<<<<<<<<<<
```

5.4 The fourth part of the configuration file defines the configuration

The next part of a configuration file is the most important one for the user. It allows you to define precisely which sets of rules you want to apply and with which options. With all the comments they include, most subparts are self-explanatory. As recalled in the banner of this part of the configuration file, it is important to have semi-colons deleted in only one section or sub-section of rules at a time.

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;
;;;                           DEFINE YOUR CONFIGURATION HERE:
;;;                               Choose general settings
;;;          Define your resolution theory (i.e. your set of resolution rules)
;;;              Simply delete the leading semicolon of the proper line(s)
;;;                             IN ORDER TO AVOID ERRORS,
;;;                DELETE SEMI-COLONS IN ONLY ONE SECTION OF RULES AT A TIME
;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;
```

5.4.1 Choose general settings

There are three kinds of settings:

- The rule optimisation type: speed or memory. Default is “speed” and should not be changed unless you face a memory overflow problem for some hard puzzle. Notice that resolution will be slower if “memory” is chosen.
- The “blocked” behaviour for some rules (Whips[1], Subsets, bivalue-chains, t-whips and their typed versions) is now the default behaviour in CSP-Rules and in all the applications: when a pattern in the above families of rules is found, all its targets are eliminated as a group before other, simpler rules, are allowed to fire. However, the user may choose, independently for each of the above families of rules, to revert to the old behaviour (until V2.0 and as in [PBCS]), where candidates were eliminated one by one. The same choice applies to typed and untyped versions of the rules.
- The detail of the output. The names of the control variables are self-explanatory, except maybe ?*print-levels*: it allows to print a line each time the solver enters a new level of the solving process, i.e. when it activates a new type of rules or a longer length for a type already activated. For efficiency reasons, all the rules selected by the user in this configuration file (plus implied rules) are loaded at the start, but they are effectively activated only when necessary, i.e. when all the simpler rules have produced all that they can. (Once activated, they remain activated until the end.) This has the advantage of limiting memory requirements. For the hardest puzzles, it may be useful to display tracking information when nothing seems to happen (no elimination) for a long time; it shows that CSP-Rules is still working and it gives an idea of how many CLIPS facts it’s dealing with.

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;
;;; 0) Choose general settings
;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
```

```
;;; DON'T CHANGE ANYTHING IN THIS SECTION UNLESS YOU HAVE SOME REASON
;;; Possibly change the type of optimisation for the chain rules.
;;; Default is pre-defined as SPEED.
;;; Don't change this unless you meet a memory overflow problem.
; (bind ?*chain-rules-optimisation-type* MEMORY)

;;; In the previous standard behaviour of CSP-Rules, when a pattern could have
produced more than one elimination,
;;; the activation of a simpler rule by the first elimination could prevent further
potential eliminations.
;;; This default behaviour is now changed for Whips[1], bivalue-chains (typed or not),
t-Whips (typed or not) and Subsets.
;;; But CSP-Rules allows to revert to the previous behaviour,
;;; independently for Whips[1], for bivalue-chains and t-Whips of any length and for
Subsets.
;;; Un-comment the relevant line(s) below if you want these rules to be "interrupted"
as the other chain rules:
; (bind ?*blocked-Whips[1]* FALSE)
; (bind ?*blocked-bivalue-chains* FALSE)
; (bind ?*blocked-t-Whips* FALSE)
; (bind ?*blocked-Subsets* FALSE)

;;; Choose what's printed as the output.
;;; The default combination is what has been used in [PBCS].
;;; Changes below will print more or less details.
; (bind ?*print-init-details* TRUE)
; (bind ?*print-ECP-details* TRUE)
; (bind ?*print-actions* FALSE)
; (bind ?*print-levels* FALSE)
; (bind ?*print-solution* FALSE)
```

5.4.2 Choose which resolution method you want to apply

There are four kinds of choices that should not be mixed, i.e. un-commented lines should appear in only one of the following four sub-sections:

1) Ordinary resolution rules, where several groups of options appear (my personal most usual choice is selected by default);
2) T&E configurations, where different typical options are proposed;
3) DFS (depth-first-search) options (with or without whips[1 or 2]);
4) Rules based on indirect binary contradictions (not in this public V2.1 version).

The four possibilities are described in sections 5.4.3 to 5.4.6 below. Remember that *selection is done by deleting the semi-colon at the start of a line*, as in the “my standard config” part in section 5.4.3 below.

5.4.3 Choose ordinary resolution rules

First, let me state the rule loading dependencies that will be automatically implemented at load time. Here, "A => B" means "if A is loaded, then B will be loaded". For chain rules, these implications can be read length by length (e.g. if braids are activated up to length 10, whips will be activated up to length 10 or more if they have been separately activated with a longer maximum length). There is no implication other than those listed here; in particular, other special cases of selected rules (such as bivalue-chains, t-whips, typed versions of the selected rules…) are not loaded unless explicitly enabled. In addition, for each specific application, you will be able to activate only rules meaningful for it (e.g. you will not be able to activate g-whips in LatinRules or Subsets in MapRules).

- Subsets => Subsets[4] => Subsets[3] => Subsets[2] => Whips[1]
- FinnedFish => FinnedFish[4] => FinnedFish[3] => FinnedFish[2]
- FinnedFish[k] => Subset[k]

- z-Chains[k] => Bivalue-Chains[k]
- Typed-z-Chains[k] => Typed-Bivalue-Chains[k]

- G-Whips (or G2-Whips) => Whips
- Braids => Whips
- G-Braids => Braids and G-Whips

- Forcing-Whips => Whips
- Forcing-G-Whips => G-Whips and Forcing-Whips
- Forcing-Braids => Braids and Forcing-Whips
- Forcing-G-Braids => G-Braids and Forcing-G-Whips

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;
;;; 1) ordinary resolution rules
;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;; My standard config and its usual variants
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
```

```
;;; My most usual rules (config = W+S+Fin), with unrestricted lengths:
;;; Sudoku-specific:
 (bind ?*Subsets* TRUE)
 (bind ?*FinnedFish* TRUE)
;;; generic:
 (bind ?*Bivalue-Chains* TRUE)
 (bind ?*Whips* TRUE)

;;; Some additional rules I use frequently:
; (bind ?*z-Chains* TRUE)
; (bind ?*t-Whips* TRUE)
; (bind ?*G-Whips* TRUE)

;;; Some optional intermediary Typed Chains, allowing more varied resolution paths:
;;; (remember that whips[1] cannot be type-restricted)
; (bind ?*Typed-Bivalue-Chains* TRUE)
; (bind ?*Typed-z-Chains* TRUE)
; (bind ?*Typed-t-Whips* TRUE)
; (bind ?*Typed-Whips* TRUE)
;;; Choose stricter type restrictions in the above Typed Chains.
;;; The same type restrictions will apply to all the typed-chains.
;;; Type restrictions correspond to working in only some of the four 2D-spaces,
;;; i.e. using only part of the Extended Sudoku Board.
;;; BEWARE: type restrictions defined by global variable ?*allowed-csp-types*
;;; will apply only if ?*restrict-csp-types-in-typed-chains* is set to TRUE.
; (bind ?*restrict-csp-types-in-typed-chains* TRUE)
; (bind ?*allowed-csp-types* (create$ rc))

;;; Some additional rules I use occasionally:
; (bind ?*G2-Whips* TRUE)
; (bind ?*G-Bivalue-Chains* TRUE)
; (bind ?*Braids* TRUE)
; (bind ?*G-Braids* TRUE)

; (bind ?*Oddagons* TRUE)

;;; Some additional rules I almost never use:
; (bind ?*Forcing-Whips* TRUE)
; (bind ?*Forcing-G-Whips* TRUE)
; (bind ?*Forcing-Braids* TRUE)
; (bind ?*Forcing-G-Braids* TRUE)

;;; Setting ?*All-generic-chain-rules* to TRUE will activate all the generic chain
rules listed above,
;;; (which doesn't include the Subset rules),
;;; with the max-lengths as specified below (but automatically modified for
consistency).
;;; It is NOT RECOMMENDED to use this possibility, unless you know what you are doing
;;; Many complex rules are loaded and memory overflow problems may appear.
; (bind ?*All-generic-chain-rules* TRUE)
```

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;; Sudoku-specific rules: uniqueness and "exotic" patterns
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

;;; U-resolution rules for uniqueness.
;;; BEWARE: don't activate the following uniqueness rules,
;;; if you are not sure that the puzzle has a unique solution.
;;; The result would be undefined.
; (bind ?*Unique-Rectangles* TRUE)
; (bind ?*BUG* TRUE)

;;; Exotic patterns
;;; Belt (sk-loop) and J-Exocet rules fall under the category of what I called exotic
patterns,
;;; because they are very specialised and rarely present in a puzzle -
;;; a name that has immediately been adopted on the Sudoku forums

; (bind ?*Belt4* TRUE)
; (bind ?*Belt6* TRUE)

; (bind ?*J-Exocet* TRUE)
; (bind ?*J2-Exocet* TRUE)
; (bind ?*J3-Exocet* TRUE)
; (bind ?*J4-Exocet* TRUE)
; (bind ?*J5-Exocet* TRUE)


;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;; Change the default maximal lengths of the chain patterns
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

;;; Don't change these lengths unless you have some reason:

; (bind ?*bivalue-chains-max-length* 20)
; (bind ?*z-chains-max-length* 20)
; (bind ?*t-whips-max-length* 36)
; (bind ?*whips-max-length* 36)
; (bind ?*g2whips-max-length* 36)
; (bind ?*g-bivalue-chains-max-length* 20)
; (bind ?*gwhips-max-length* 36)
; (bind ?*braids-max-length* 36)
; (bind ?*gbraids-max-length* 36)

; (bind ?*typed-bivalue-chains-max-length* 20)
; (bind ?*typed-z-chains-max-length* 20)
; (bind ?*typed-t-whips-max-length* 36)
; (bind ?*typed-whips-max-length* 36)

; (bind ?*forcing-whips-max-length* 36)
; (bind ?*forcing-gwhips-max-length* 36)
; (bind ?*forcing-braids-max-length* 36)
; (bind ?*forcing-gbraids-max-length* 36)
```

5.4.4 Decide to use T&E for computing some classification of a puzzle

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;
;;; 2) Choose typical T&E config options, for various predefined purposes
;;;
;;; DON'T FORGET TO DISABLE ALL THE RULES IN THE OTHER SECTIONS
;;; BEFORE ACTIVATING T&E
;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;; Un-comment the proper line below to change the level of details you want to be
printed:
; (bind ?*print-actions* FALSE)
; (bind ?*print-levels* FALSE)
; (bind ?*print-details* TRUE)
; (bind ?*print-solution* FALSE)
; (bind ?*print-hypothesis* FALSE)
; (bind ?*print-phase* TRUE)

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;; 2a) for checking membership in T&E(k) or gT&E(k), k = 1,2,3
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

;;; Choose one of the following 3 depths of T&E:
;;; - depth 2 is enough for all the 9x9 Sudokus
;;; - but deeper T&E is often required for larger Sudokus or for Sukakus

; (bind ?*TE1* TRUE) ;;; for T&E at level 1
; (bind ?*TE2* TRUE) ;;; for T&E at level 2
; (bind ?*TE3* TRUE) ;;; for T&E at level 3

;;; In addition to the previous choice, you can give priority to bivalue candidates:
; (bind ?*special-TE* TRUE)

;;; For gT&E(k) instead of T&E(k), activate the next two lines:
; (bind ?*Whips* TRUE)
; (bind ?*whips-max-length* 1)

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;; 2b) For computing the SpB classification
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

;;; Remember that whips[1] are always activated before Subsets,
;;; even if you don't activate them explicitly here.
;;; But you can choose to activate only them, to get gT&E (as in 2a)
; (bind ?*Whips* TRUE)
; (bind ?*whips-max-length* 1)

;;; choose which Subsets[p] and FinnedFish[p] are activated:
```

```
; (bind ?*Subsets* TRUE)
; (bind ?*Subsets[2]* TRUE)
; (bind ?*Subsets[3]* TRUE)
; (bind ?*Subsets[4]* TRUE)
; (bind ?*FinnedFish* TRUE)
; (bind ?*FinnedFish[2]* TRUE)
; (bind ?*FinnedFish[3]* TRUE)
; (bind ?*FinnedFish[4]* TRUE)

;;; choose one of the following forms of T&E(1, Sp or SpFin)
; (bind ?*TE1* TRUE) ;;; for T&E at level 1
;;; For T&E at level 1, with priority for bivalue variables, add the following:
; (bind ?*special-TE* TRUE)
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;; 2c) for computing the BpB classification
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

;;; choose p (here p = 3):
; (bind ?*Whips* TRUE)
; (bind ?*Braids* TRUE)
; (bind ?*whips-max-length* 3)
; (bind ?*braids-max-length* 3)

;;; choose one of the following forms of T&E(1)
; (bind ?*TE1* TRUE) ;;; for T&E at level 1
;;; For T&E at level 1, with priority for bivalue variables, add the following:
; (bind ?*special-TE* TRUE)
```

5.4.5 Decide to try DFS

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;
;;; 3) Choose DFS (dept-first search) options
;;;
;;; DON'T FORGET TO DISABLE ALL THE RULES IN THE OTHER SECTIONS
;;; BEFORE ACTIVATING DFS
;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

;;; DFS can be used to provide a relatively fast solution

; (bind ?*print-actions* FALSE)
; (bind ?*print-levels* FALSE)
; (bind ?*print-details* TRUE)
; (bind ?*print-solution* FALSE)
; (bind ?*print-hypothesis* FALSE)
; (bind ?*print-solution* FALSE)
; (bind ?*print-phase* TRUE)
```

```
;;; To activate DFS:
; (bind ?*DFS* TRUE)
;;; To activate priority for bivalue cells, activate this line, in addition to the above line:
; (bind ?*special-DFS* TRUE)

;;; Activate short whips for combining whips[1] or whips[2] with DFS:
;;; this often gives a faster result (but not with larger whips)
; (bind ?*Whips* TRUE)
; (bind ?*whips-max-length* 1)
; (bind ?*whips-max-length* 2)
```

5.4.6 Decide to use rules based on intermediate binary contradictions

This sub-section starts with the following heading:

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;
;;; 4) rules based on intermediate binary contradictions
;;;
;;; DON'T FORGET TO DISABLE ALL THE RULES IN THE OTHER SECTIONS
;;; BEFORE ACTIVATING ANY RULE BELOW
;;; ACTIVATE RULES FROM ONLY ONE OF THE FOLLOWING SUB-SECTIONS AT A TIME
;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;
;;; BEWARE: this section should not be used unless one has a deep understanding
;;; of these rules, as described in chapter 12 of [PBCS]
```

As this part of the configuration file relies on advanced rules and procedures, it will be discussed only later, namely in chapter 12 on Advanced resolution techniques. It should normally be of interest only to people dealing with the very hardest puzzles. The techniques appearing here often suppose more involvement on the part of the user than merely typing a command such as: (solve "….the puzzle…."). This is not included in the present public version of CSP-Rules.

Any Sudoku puzzle found in the newspapers or obtained from a bottom-up, top-down or controlled-bias generator should be solvable by the rules described in the previous parts of the configuration file. In standard Sudoku, there is about only one puzzle in 70,000,000 that would benefit from using the techniques described in this part. However, the situation may be different for other puzzles or for larger Sudokus.

At the end of this sub-section, the following banner marks the end of what the user may change:

```
;;;
;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;
;;;                             end of allowed changes
;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
```

5.5 The final part of the configuration file must not be changed

After the configuration has been defined by the user, the final part ensures the proper loading of the relevant rules. In Sudoku, it looks like this:

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;
;;; AUTOMATIC CONFIGURATION AND LOADING (DON'T CHANGE ANYTHING BELOW THIS LINE)
;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

;;; because grid size may have been changed in this file,
;;; redefine the associated internal factors;
;;; this has to be done BEFORE loading
(redefine-internal-factors)
;;; now, load all
;;; Notice that the generic loader also loads the application-specific files
(if (and (or ?*G-Bivalue-Chains* ?*G-Whips* ?*G-Braids*) (> ?*segment-size* 4))
    then (printout t
       "BEWARE: g-labels, g-bivalue-chains, g-whips and g-braids are not managed" crlf
       "for segment size larger than 4, i.e. grid size larger than 16" crlf)
    else (batch ?*CSP-Rules-Generic-Loader*)
)
```

6. SudoRules and LatinRules

SudoRules is the pattern-based solver of Sudoku puzzles based on CSP-Rules. The pattern-based approach of a CSP was first described in [HLS] for Sudoku and then in [PBCS] for the general CSP. I've written several versions of SudoRules before I started a total re-writing of all the general rules in SudoRules into a generic form, thus making what is now the generic part of CSP-Rules.

As a result of this historical development, together with a much broader and deeper study of the Sudoku puzzles, SudoRules is the most developed application of CSP-Rules. It has many more user functions than the other applications. It has been used to solve and rate millions of puzzles. One thing to be reminded is, SudoRules does not include the zillion Sudoku-specific rules one can find in Sudoku forums (most of the time in very imprecise and ambiguous forms) or in a typical Sudoku solver. Most of these rules are subsumed by the generic ones. My initial goals were, and remain, to prove the real applicability and to study the range of the generic resolution rules introduced in [PBCS]. (And, needless to repeat it, these goals included ease of programming for me; they did not include speed of resolution.)

Notice that none of the functions defined below takes any argument for grid size. The default puzzle size in SudoRules is set to 9. But the size can be changed in the SudoRules configuration file. The main reasons for this choice are history and the fact that almost all the Sudokus one meets in nature are 9×9. There is also a technical reason: g-whips don't work for large grid sizes and the corresponding rules should not be loaded when one tries to solve such puzzles. The same remarks (except the technical one) apply to LatinRules.

The first four sections are for Sudoku, the fifth for Latin Squares.

In this chapter, "number" or "digit" means:

- a member of {1, 2, …, 9} if grid size is 9×9,
- a member of {1, 2, …, 9, A, B, C, D, E, F, G} if grid size is 16×16,
- a member of {1, 2, …, 9, A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P} if grid size is 25×25,
- a member of {1, 2, …, 9, A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z, 0} if grid size is 36×36 – but this large size may be in practice beyond the upper limit for SudoRules, depending on how much RAM you have.

6.1 Solving a puzzle given in the standard line format

“***solve***” is the first and simplest user function, in SudoRules as in all the CSP-Rules applications. In a first contact with SudoRules, this is indeed the only user function you need to know. The syntax of “solve” is very simple:

```
(solve ?string)
```

where “?string” is a string of 81 characters (between double quotes) representing the puzzle in the classical “line format”. The rc-cells in the grid are supposed to be numbered from left to right and from top to bottom. A digit at the nth place in the string represents a given for the nth cell; a dot (sometimes a 0 for 9×9 puzzles) represents the absence of a given (it is there to mark a place in the string). Same example, with dots and with 0s:

```
(solve
"4...3.......6..8..........1....5..9..8....6...7.2........1.27..5.3....4.9........")
(solve
"400030000000600800000000001000050090080000600070200000000102700503000040900000000")
```

“***solve-knowing-solution***” is a variant of “solve” that can be used when the solution is already known:

```
(solve-knowing-solution ?puzzle-string ?sol-string)
```

where both “?puzzle-string” and “?sol-string” are strings of 81 characters representing the puzzle and the solution, as before. Example:

```
(solve-knowing-solution
"4...3.......6..8..........1....5..9..8....6...7.2........1.27..5.3....4.9........"
“468931527751624839392578461134756298289413675675289314846192753513867942927345186”)
```

The presence of this function may be surprising at first: why should one want to solve a puzzle if he already knows the solution? The reason is, the solution may be known (e.g. after applying a depth-first-search program) without a readable resolution path and/or without the rating being known.

In this function, the solution is only used to avoid building chains (whips,…) that are known in advance to be unable to lead to any elimination: candidates known to be in the solution are not tried as targets for building such chains on them.

The resolution path and the final result are exactly the same as those of “solve”, even if the puzzle has no solution or multiple solutions, except that useless calculations are avoided. This may be occasionally useful for the hardest puzzles, to save computation time and/or memory.

“***solve-string-of-candidates***” and “***solve-Sukaku***” are a variant of “solve” that can be used when a resolution state (a state including both values and candidates) is given instead of an original puzzle (with values only). The name “Sukaku” has been introduced recently in the Sudoku world for naming such situations; I therefore added “solve-Sukaku” as a name variant of “solve-string-of-candidates”:

```
(solve-string-of-candidates ?string)
(solve-Sukaku ?string)
```

where "?string" is a sequence of 729 digits or dots, such that the nth group of nine characters represents the candidates present in the nth rc-cell. There is much redundancy in this representation of a resolution state, because the presence of a digit in an rc-cell is doubly represented: by its value and by its position in the sequence. A sequence of 0s and 1s would be enough but it would also be less readable. Anyway, that is the form in which it came to existence and I kept it as is.

Example of an extremely difficult Sukaku found by Tarek, a famous puzzle creator (SudoRules will not solve it without DFS):

```
(solve-Sukaku
"..3.5678.1.3456789123456789.23456789...4.678912.4567891234567891.34567891..45.789123.
567891.3...7..12345.7..123....891234.678912.4...8.123..6.891........12345.789123456789
1234567..12345678.123456.8912345678912345678.123.56.89123456..9123456789.23456789...4.
678912.4567891.3.567891.34567891..45.789.234567891..4.678912.456789.23..6.891234.67891
23456789123.567891.3...7.912345.789123.567891234.678912.4..78.123456.891234.6...123456
789123.56..91234567.91234..7..123456.891234.6...12345678.1234567891...5678912.45678912
345678912345678.12.45678..23.567891.3456789123456789123.567891.....7.912.45.789123..6.
891234.678912.4..78..23.567891.3...7.912345.789123.56..91234567.9123456789123456.89123
4.6...12345678.1234567891234567.912345.7..")
```

Notice that a Sukaku not originating from a real Sudoku puzzle (contrary to the above example) can be much harder than a Sudoku puzzle: all the known Sudoku puzzles are at worst in T&E(2), but some Sukakus are known to be in T&E(4).

"solve-sukaku" can typically be used when a resolution state of a Sudoku puzzle is obtained after the application of some basic rules and one asks: "what comes next?".

6.2 Solving Sudoku puzzles given in other formats

6.2.1 The list format for Sudoku

Often, a puzzle is given in a more user-friendly format than the line format, such as:

```
. . 1   2 . .   . . 3
. . .   . . 4   2 . .
5 . .   . . .   . 6 .
7 . .   6 . .   . 1 .
. . .   . 3 .   . . .
. 8 .   . . 9   . . 4
. 3 .   . . .   . . 6
. . 7   1 . .   . 2 .
4 . .   . . 7   5 . .
```

"***solve-grid-from-list***", also named "***solve-grid-as-list***", allows to solve puzzles given in this format. This function has a unique argument $?rest; as explained in the introduction, all the arguments given to it are interpreted as a single list. In this list format, make sure to keep at least one space between any two symbols (digits or dots): in a list, “7 5” is two elements, but “75” is only one (and a meaningless one in Sudoku).

```
(solve-grid-from-list
. . 1  2 . .  . . 3
. . .  . . 4  2 . .
5 . .  . . .  . 6 .
7 . .  6 . .  . 1 .
. . .  . 3 .  . . .
. 8 .  . . 9  . . 4
. 3 .  . . .  . . 6
. . 7  1 . .  . 2 .
4 . .  . . 7  5 . .
)
```

A still more user-friendly format (for the same puzzle) found on the Sudoku forums would be as shown below. This is the original reason why I introduced the list format. Unfortunately, there can be no way of giving the puzzle in this form directly to SudoRules, because “|” is a reserved symbol in CLIPS (it means logical “or” in some situations); in particular, it cannot appear in the argument of any function. However, the user can easily transform this format into the previous one, either manually or with any text editor.

```
+-------+-------+-------+
| . . 1 | 2 . . | . . 3 |
| . . . | . . 4 | 2 . . |
| 5 . . | . . . | . 6 . |
+-------+-------+-------+
| 7 . . | 6 . . | . 1 . |
| . . . | . 3 . | . . . |
| . 8 . | . . 9 | . . 4 |
+-------+-------+-------+
| . 3 . | . . . | . . 6 |
| . . 7 | 1 . . | . 2 . |
| 4 . . | . . 7 | 5 . . |
+-------+-------+-------+
```

6.2.2 The list format for Sukaku

Often, a Sukaku puzzle is given in a more user-friendly format than the line format and function ***“solve-Sukaku-as-list”*** allows to deal with it. This function has a single argument $?list representing the list of possible candidates for each cell. The candidates for each cell, are glued in a single symbol. The syntax should be clear from the following example:

```
(solve-Sukaku-as-list
      1 9 6    2    4   7     3    5   8
      8 2 7    3    5   1     4    6   9
      3 4 5    68   9   68    27   27  1
      7 8 24   1    6   49    29   3   5
      5 6 1    89   2   3     89   4   7
      9 3 24   578  78  458   1    28  6
      6 7 89   589  3   2     58  1   4
      4 5 3    6789 1   689   678 789 2
      2 1 89   4    78  56    56   789 3
)
```

6.2.3 The sdk format for files

It may sometimes be useful to have a puzzle displayed on 9 lines at the top of a file and then to write information about it and its resolution path(s) in the same file. The nine-line format is also easier to visualise than the one-line format. Function "***solve-sdk-grid***" reads the first nine symbols of the first nine lines of a file and solves the corresponding puzzle (its argument ?file-name is the path to the file; see section 6.3 for an example of path):

```
(solve-sdk-grid ?file-name)
```

For the puzzle in the previous example, the nine-line format at the top of the file is similar to the list format, but with all the spaces in each line deleted (including any spaces or tabs at the start of the lines); it's also similar to the line format, with carriage returns added after each group of nine symbols. It does not accept anything before the nine lines or before the content of each of them.

```
..12....3
.....42..
5......6.
7..6...1.
....3....
.8...9..4
.3......6
..71...2.
4....75..
```

6.2.3 The tatham format

A slightly different format is used by Tatham, a famous creator of many types of puzzles (see https://www.chiark.greenend.org.uk/~sgtatham/puzzles/). His puzzles are generally relatively easy but they may be interesting for beginners and beyond. The "Tatham format" for Sudoku is more compact for printing than the standard line format and as such, it's interesting, though it's harder to read: every sequence of zeros or dots in the line format is replaced by a lower case letter: "a" means one dot, "b" means two dots, …

"***solve-tatham***" is a variant of "solve" that can deal with the Tatham format:

```
(solve-tatham ?tatham-str)
```

As you may have guessed, "?tatham-str" is the Tatham representation of a puzzle as a string, i.e. between double quotes. Example:

```
(solve-tatham "2a9a15a4c58d9a3i1e4c127c7e9i3a4d17c2a54a6a8")
```

The underscores ("_") that, for some unknown reason, appear in the format when copied from the Tatham website don't add anything to the string and could as well be deleted. The following is equivalent to the above:

```
(solve-tatham
"2a9a1_5a4c5_8d9a3i1e4c1_2_7c7e9i3a4d1_7c2a5_4a6a8")
```

Both forms are equivalent to:

```
(solve
  "2.9.15.4...58....9.3.........1.....4...127...7.....9.........3.4....17...2.54.6.8")
```

If you want to see how a puzzle given in Tatham format looks like in the familiar line format, the function "tatham-to-sudoku-string" allows this.

```
(tatham-to-sudoku-string "2a9a1_5a4c5_8d9a3i1e4c1_2_7c7e9i3a4d1_7c2a5_4a6a8")
```

gives:

```
"2.9.15.4...58....9.3.........1.....4...127...7.....9.........3.4....17...2.54.6.8"
```

Indeed, (solve-tatham ?tatham-str) is defined as:
(solve (tatham-to-sudoku-string ?tatham-str))

6.3 Solving collections of puzzles

When studying Sudoku puzzles, one often has to deal with a long collection of puzzles written in a text file, with one puzzle in the line format per line. The functions in this section allow to deal with such cases. The first 81 characters of each line must be digits or dots; whatever comes after them is not taken into account. This allows to keep in the same file additional information about each puzzle, such as their "name" if they have one, their author, the date they were discovered, their SER (Sudoku Explainer Rating, …). Fundamentally, each of the following functions repeatedly calls the "solve" function on puzzles in the list, as many times as specified by the argument(s). It also keeps track of the total solving time and it prints it at the end.

"***solve-nth-grid-from-text-file***" allows to pick one puzzle from the list and to solve it:

```
(solve-nth-grid-from-text-file ?file-name ?nb)
```

where "?file-name" is the path to the file and "?nb" is the place of the puzzle of interest in the list. ?nb may not be larger than the number of lines in the file. Example:

```
(solve-nth-grid-from-text-file "/Users/My-User-name/Documents/MySudokuPuzzles.txt" 56)
```

It may also be useful to check what there's on a particular line in a file. This can be done with function display-nth-line-from-text-file:

```
(display-nth-line-from-text-file ?file-name ?nb)
```

The name of function "***solve-n-grids-after-first-p-from-text-file***" is sufficiently explicit to need no explanation:
`(solve-n-grids-after-first-p-from-text-file ?file-name ?p ?n)`
where "?file-name" is as before, ?p is the number of lines that must be skipped and "?n" is the number of puzzles that must be solved. Example:

```
(solve-n-grids-after-first-p-from-text-file
"/Users/User-name/Documents/Sudoku/MySudokuPuzzles.txt" 12 56)
```

If you want to solve all the puzzles in the file, set ?p to 0 and ?n to the length of the file.

It is very convenient to be able to keep track of global information about a collection of puzzles, as an additional first line in the same file. For this reason, SudoRules includes variants of the previous two functions that allow to work as if the first line of a file did not exist; they take the same arguments are before:

```
(display-nth-effective-line-from-titled-text-file ?file-name ?nb)
(solve-nth-grid-from-titled-text-file ?file-name ?nb)
(solve-n-grids-after-first-p-from-titled-text-file ?file-name ?p ?n)
```

6.4 For the readers of [HLS]

All the chain names used in [HLS] have been changed in [PBCS] and in this Manual. The naming convention in [HLS] was too dependent on the Sudoku grid structure and could not be made generic. The new naming convention is obviously simpler, as shown by the following table (where lengths of chains are random).

Name in [HLS]	**Name in [PBCS]**	Remarks
xy5-chain	**biv-chain-rc[5]**	
hxy-uu5-chain	**biv-chain-uu[5]**	for uu = rn, cn or bn
nrc5-chain	**biv-chain[5]**	
xyz6-chain	**z-chain-rc[6]**	
hxyz-uu7-chain	**z-chain-uu[6]**	for uu = rn, cn or bn
xyt-rc6-chain	**t-whip-rc[6]**	
hxyt-uu6-chain	**t-whip-uu[6]**	for uu = rn, cn or bn
nrct6-chain	**t-whip[6]**	t-whips are slightly more general than nrct-chains

xyzt-rc8-chain	**whip-rc[8]**	whips-rc are slightly more general than xyzt-chains
hxyzt-uu8-chain	**whip-uu[8]**	for uu = rn, cn or bn whips-uu are slightly more general than xyzt-uu-chains
nrczt8-chain	**whip[8]**	whips are slightly more general than nrczt-chains

6.5 LatinRules

LatinRules is a stripped-down version of SudoRules (eliminating any reference to blocks and g-labels) and it hasn't given rise to similar large scale studies. It has the same user functions as Sudoku, with the same syntax.

I keep LatinRules separate from SudoRules mainly for practical reasons: this way, it will be easier to code variants for Sudoku without needing to put (Latin Squares vs Sudoku) conditions everywhere blocks are mentioned. The absence of blocks means no Subset rules for blocks, no whips[1], no g-labels, no g-whips or g-braids, no g-Subsets (such as Finned Fish), no g-anything. For Sudoku players, the interactions between blocks and rows or columns make an important part of the excitement. Blocks are what makes Sudoku a unique logic game, with unmatched success.

6.6 How different selections of rules change the resolution path

I'll now use a moderately difficult puzzle to show how the various selections of rules made possible in the configuration file can give very different resolution paths. The puzzle (cbg#109) is:

```
. . . 4 . . . 8 9
. 5 . . . . 1 . .
. . 9 . 3 . . . 6
. 7 1 . 6 . 9 . 3
. . . . . 2 4 . .
9 . . . . 5 . . 1
3 . . . . . 6 . .
. 9 . . 4 . . 1 .
8 1 5 . . 3 . . .
```

All the set of rules I'll consider will have Subsets and Finned Fish, but this is not really relevant. All the presentations will follow the same schema: first, the selected set of options from the SudoRules configuration file (any option not listed is supposed to be inactive) and then the resolution path. Notice that all the patterns that can be "blocked" are, i.e. they allow several targets at a time.

All the resolution paths have the same straightforward start, ending in some resolution state RS1:

```
singles ==> r4c4 = 8, r4c6 = 4, r6c5 = 7, r6c4 = 3, r9c4 = 6, r8c7 = 3, r2c8 = 3, r6c7
= 8
whip[1]: r9n7{c9 .} ==> r8c9 ≠ 7, r7c8 ≠ 7, r7c9 ≠ 7
whip[1]: r9n4{c9 .} ==> r7c9 ≠ 4, r7c8 ≠ 4
whip[1]: c7n5{r3 .} ==> r3c8 ≠ 5
whip[1]: c1n4{r3 .} ==> r3c2 ≠ 4, r2c3 ≠ 4
whip[1]: b6n2{r6c8 .} ==> r9c8 ≠ 2, r3c8 ≠ 2, r7c8 ≠ 2
hidden-pairs-in-a-row: r5{n3 n8}{c2 c3} ==> r5c3 ≠ 6, r5c2 ≠ 6
;;; Resolution state RS1
```

This part of the resolution paths will not be repeated; instead, I'll write only the part after resolution state RS1. "stte" means "singles to the end".

6.6.1 Using whips

Let us first use the following configuration with Whips:

```
(bind ?*Subsets* TRUE)
(bind ?*FinnedFish* TRUE)
(bind ?*Bivalue-Chains* TRUE)
(bind ?*Whips* TRUE)
```

We get a resolution path in W5:

```
;;; Resolution state RS1
biv-chain[3]: r3c2{n2 n8} - r5c2{n8 n3} - b1n3{r1c2 r1c3} ==> r1c3 ≠ 2
biv-chain[3]: r5n7{c9 c8} - r3c8{n7 n4} - b9n4{r9c8 r9c9} ==> r9c9 ≠ 7
biv-chain[4]: r5c5{n1 n9} - r9n9{c5 c8} - r7c8{n9 n5} - c5n5{r7 r1} ==> r1c5 ≠ 1
whip[4]: c6n9{r2 r7} - r7c8{n9 n5} - c9n5{r8 r5} - c9n7{r5 .} ==> r2c6 ≠ 7
biv-chain[5]: r5c1{n5 n6} - c2n6{r6 r1} - b2n6{r1c6 r2c6} - c6n9{r2 r7} - r7c8{n9 n5}
==> r5c8 ≠ 5
biv-chain[4]: r2n4{c1 c9} - c9n7{r2 r5} - r5n5{c9 c1} - r4c1{n5 n2} ==> r2c1 ≠ 2
biv-chain[5]: c6n6{r1 r2} - c6n9{r2 r7} - r7c8{n9 n5} - b6n5{r4c8 r5c9} - r5c1{n5 n6}
==> r1c1 ≠ 6
whip[5]: c1n7{r3 r8} - r8c6{n7 n8} - c5n8{r7 r2} - c3n8{r2 r5} - c3n3{r5 .} ==>
r1c3 ≠ 7
whip[5]: r1n6{c3 c6} - r1n1{c6 c1} - r1n7{c1 c7} - r3c8{n7 n4} - c1n4{r3 .} ==>
r2c1 ≠ 6
whip[4]: r9c5{n2 n9} - c6n9{r7 r2} - r2n8{c6 c3} - r2n6{c3 .} ==> r2c5 ≠ 2
whip[4]: c3n7{r8 r2} - c9n7{r2 r5} - r5n5{c9 c1} - c1n6{r5 .} ==> r8c1 ≠ 7
whip[1]: b7n7{r8c3 .} ==> r2c3 ≠ 7
naked-triplets-in-a-column: c1{r4 r5 r8}{n2 n5 n6} ==> r3c1 ≠ 2, r1c1 ≠ 2
biv-chain[3]: c1n2{r4 r8} - r7c2{n2 n4} - b4n4{r6c2 r6c3} ==> r6c3 ≠ 2
whip[4]: c9n8{r8 r7} - c9n5{r7 r5} - r4c8{n5 n2} - c1n2{r4 .} ==> r8c9 ≠ 2
biv-chain[5]: c6n6{r1 r2} - c6n9{r2 r7} - r7c8{n9 n5} - r8c9{n5 n8} - r8c6{n8 n7} ==>
r1c6 ≠ 7
```

```
biv-chain[3]: r2c1{n4 n7} - r1n7{c1 c7} - r3c8{n7 n4} ==> r3c1 ≠ 4
singles ==> r2c1 = 4, r3c8 = 4, r9c9 = 4
finned-x-wing-in-columns: n2{c9 c3}{r2 r7} ==> r7c2 ≠ 2
singles ==> r7c2 = 4, r6c3 = 4
biv-chain[3]: r9n2{c7 c5} - r1c5{n2 n5} - b3n5{r1c7 r3c7} ==> r3c7 ≠ 2
biv-chain[3]: r2n7{c4 c9} - r5c9{n7 n5} - r8n5{c9 c4} ==> r8c4 ≠ 7
biv-chain[3]: r2n7{c4 c9} - c9n2{r2 r7} - r7c3{n2 n7} ==> r7c4 ≠ 7
whip[1]: b8n7{r8c6 .} ==> r3c6 ≠ 7
hidden-triplets-in-a-block: b2{r2c4 r1c5 r3c4}{n2 n5 n7} ==> r3c4 ≠ 1, r2c4 ≠ 9
whip[1]: b2n1{r3c6 .} ==> r7c6 ≠ 1
naked-pairs-in-a-row: r2{c4 c9}{n2 n7} ==> r2c3 ≠ 2
whip[1]: c3n2{r8 .} ==> r8c1 ≠ 2
stte
```

6.6.2 Using t-whips

Let us now replace whips by the *a priori* less powerful t-whips:

```
(bind ?*Subsets* TRUE)
(bind ?*FinnedFish* TRUE)
(bind ?*Bivalue-Chains* TRUE)
(bind ?*t-Whips* TRUE)
```

We get a resolution path in tW5; this is a (probably rare) case where using t-whips instead of whips does not require longer chains:

```
;;; Resolution state RS1
biv-chain[3]: r3c2{n2 n8} - r5c2{n8 n3} - b1n3{r1c2 r1c3} ==> r1c3 ≠ 2
biv-chain[3]: r5n7{c9 c8} - r3c8{n7 n4} - b9n4{r9c8 r9c9} ==> r9c9 ≠ 7
biv-chain[4]: r5c5{n1 n9} - r9n9{c5 c8} - r7c8{n9 n5} - c5n5{r7 r1} ==> r1c5 ≠ 1
t-whip[4]: c6n9{r2 r7} - r7c8{n9 n5} - c9n5{r8 r5} - c9n7{r5 .} ==> r2c6 ≠ 7
biv-chain[5]: r5c1{n5 n6} - c2n6{r6 r1} - b2n6{r1c6 r2c6} - c6n9{r2 r7} - r7c8{n9 n5}
==> r5c8 ≠ 5
biv-chain[4]: r2n4{c1 c9} - c9n7{r2 r5} - r5n5{c9 c1} - r4c1{n5 n2} ==> r2c1 ≠ 2
biv-chain[5]: c6n6{r1 r2} - c6n9{r2 r7} - r7c8{n9 n5} - b6n5{r4c8 r5c9} - r5c1{n5 n6}
==> r1c1 ≠ 6
t-whip[5]: c3n3{r1 r5} - r5c2{n3 n8} - r3n8{c2 c6} - r8c6{n8 n7} - r7n7{c6 .} ==>
r1c3 ≠ 7
t-whip[5]: r9c5{n2 n9} - c6n9{r7 r2} - c6n6{r2 r1} - r1c3{n6 n3} - r1c2{n3 .} ==>
r1c5 ≠ 2
singles ==> r1c5 = 5, r3c7 = 5
biv-chain[4]: r1c7{n2 n7} - c9n7{r2 r5} - r5n5{c9 c1} - r4c1{n5 n2} ==> r1c1 ≠ 2
t-whip[4]: c5n2{r9 r2} - c5n8{r2 r7} - r8n8{c6 c9} - r8n5{c9 .} ==> r8c4 ≠ 2
t-whip[5]: r1n6{c3 c6} - r1n1{c6 c1} - r1n7{c1 c7} - b3n2{r1c7 r2c9} - r2n4{c9 .} ==>
r2c1 ≠ 6
t-whip[4]: r9c5{n2 n9} - c6n9{r7 r2} - r2n6{c6 c3} - r2n8{c3 .} ==> r2c5 ≠ 2
whip[1]: c5n2{r9 .} ==> r7c4 ≠ 2
t-whip[4]: c1n7{r3 r8} - c1n6{r8 r5} - r5c8{n6 n7} - c9n7{r5 .} ==> r2c3 ≠ 7
whip[1]: c3n7{r8 .} ==> r8c1 ≠ 7
naked-triplets-in-a-column: c1{r4 r5 r8}{n2 n5 n6} ==> r3c1 ≠ 2
biv-chain[3]: c1n2{r4 r8} - r7c2{n2 n4} - b4n4{r6c2 r6c3} ==> r6c3 ≠ 2
```

```
biv-chain[5]: r6n2{c2 c8} - r4c8{n2 n5} - r7c8{n5 n9} - r9n9{c8 c5} - c5n2{r9 r7} ==>
r7c2 ≠ 2
singles ==> r7c2 = 4
hidden-single-in-a-block ==> r6c3 = 4
biv-chain[5]: c1n2{r8 r4} - r4c8{n2 n5} - r7c8{n5 n9} - r9n9{c8 c5} - c5n2{r9 r7} ==>
r7c3 ≠ 2
naked-single ==> r7c3 = 7
whip[1]: b7n2{r8c3 .} ==> r8c9 ≠ 2
t-whip[4]: c6n9{r2 r7} - r7c8{n9 n5} - r7c4{n5 n1} - r5c4{n1 .} ==> r2c4 ≠ 9
naked-triplets-in-a-row: r2{c1 c4 c9}{n4 n7 n2} ==> r2c3 ≠ 2
stte
```

6.6.3 Using typed-whips

Let us now replace the originally chosen whips by typed-whips (nrczt-chains in [HLS]) and (of course) also bivalue-chains by typed-bivalue-chains.

```
(bind ?*Subsets* TRUE)
(bind ?*FinnedFish* TRUE)
(bind ?*Typed-Bivalue-Chains* TRUE)
(bind ?*Typed-Whips* TRUE)
```

We get a resolution path in TyW9; i.e. this time, we need longer chains:

```
;;; Resolution state RS1
whip-cn[5]: c3n3{r1 r5} - c2n3{r5 r1} - c2n6{r1 r6} - c2n4{r6 r7} - c2n2{r7 .} ==>
r1c3 ≠ 2
whip-rc[6]: r9c7{n7 n2} - r9c5{n2 n9} - r9c8{n9 n4} - r3c8{n4 n7} - r3c7{n7 n5} -
r1c7{n5 .} ==> r9c9 ≠ 7
whip-cn[5]: c6n9{r2 r7} - c8n9{r7 r9} - c8n4{r9 r3} - c8n7{r3 r5} - c9n7{r5 .} ==>
r2c6 ≠ 7
whip-cn[7]: c2n6{r6 r1} - c6n6{r1 r2} - c6n9{r2 r7} - c8n9{r7 r9} - c8n4{r9 r3} -
c8n7{r3 r5} - c8n6{r5 .} ==> r6c3 ≠ 6
whip-cn[7]: c8n9{r7 r9} - c8n4{r9 r3} - c8n7{r3 r5} - c8n6{r5 r6} - c2n6{r6 r1} -
c6n6{r1 r2} - c6n9{r2 .} ==> r7c5 ≠ 9
whip-cn[7]: c8n9{r7 r9} - c8n4{r9 r3} - c8n7{r3 r5} - c8n6{r5 r6} - c2n6{r6 r1} -
c6n6{r1 r2} - c6n9{r2 .} ==> r7c4 ≠ 9
whip-bn[4]: b5n1{r5c5 r5c4} - b5n9{r5c4 r5c5} - b8n9{r9c5 r7c6} - b8n1{r7c6 .} ==>
r1c5 ≠ 1
whip-cn[7]: c6n6{r1 r2} - c6n9{r2 r7} - c8n9{r7 r9} - c8n4{r9 r3} - c8n7{r3 r5} -
c8n6{r5 r6} - c2n6{r6 .} ==> r1c3 ≠ 6
whip-cn[7]: c6n6{r1 r2} - c6n9{r2 r7} - c8n9{r7 r9} - c8n4{r9 r3} - c8n7{r3 r5} -
c8n6{r5 r6} - c2n6{r6 .} ==> r1c1 ≠ 6
whip-cn[7]: c8n6{r5 r6} - c2n6{r6 r1} - c6n6{r1 r2} - c6n9{r2 r7} - c8n9{r7 r9} -
c8n7{r9 r3} - c8n4{r3 .} ==> r5c8 ≠ 5
whip-rc[7]: r4c1{n2 n5} - r5c1{n5 n6} - r8c1{n6 n7} - r1c1{n7 n1} - r3c1{n1 n4} -
r3c8{n4 n7} - r5c8{n7 .} ==> r2c1 ≠ 2
whip-cn[7]: c6n9{r2 r7} - c8n9{r7 r9} - c8n4{r9 r3} - c8n7{r3 r5} - c8n6{r5 r6} -
c2n6{r6 r1} - c6n6{r1 .} ==> r2c6 ≠ 8
whip-rn[8]: r3n5{c7 c4} - r8n5{c4 c9} - r5n5{c9 c1} - r4n5{c1 c8} - r4n2{c8 c1} -
r3n2{c1 c2} - r3n8{c2 c6} - r8n8{c6 .} ==> r3c7 ≠ 7
```

```
whip-rc[9]: r3c2{n2 n8} - r5c2{n8 n3} - r1c2{n3 n6} - r2c3{n6 n7} - r1c1{n7 n1} -
r1c6{n1 n7} - r3c6{n7 n1} - r3c4{n1 n5} - r3c7{n5 .} ==> r3c1 ≠ 2
whip-cn[8]: c9n8{r8 r7} - c9n5{r7 r5} - c1n5{r5 r4} - c8n5{r4 r7} - c5n5{r7 r1} -
c7n5{r1 r3} - c7n2{r3 r1} - c1n2{r1 .} ==> r8c9 ≠ 2
whip-rc[5]: r8c6{n7 n8} - r3c6{n8 n1} - r7c6{n1 n9} - r7c8{n9 n5} - r8c9{n5 .} ==>
r1c6 ≠ 7
whip-rn[4]: r2n4{c1 c9} - r9n4{c9 c8} - r9n7{c8 c7} - r1n7{c7 .} ==> r2c1 ≠ 7
whip-rn[6]: r7n7{c6 c3} - r2n7{c3 c9} - r5n7{c9 c8} - r5n6{c8 c1} - r5n5{c1 c9} -
r8n5{c9 .} ==> r8c4 ≠ 7
whip-rc[6]: r9c5{n9 n2} - r8c4{n2 n5} - r8c9{n5 n8} - r8c6{n8 n7} - r7c4{n7 n1} -
r5c4{n1 .} ==> r5c5 ≠ 9
singles ==> r5c5 = 1, r5c4 = 9
biv-chain-rc[4]: r2c4{n7 n2} - r8c4{n2 n5} - r8c9{n5 n8} - r8c6{n8 n7} ==> r3c6 ≠ 7,
r7c4 ≠ 7
whip-rc[5]: r3c2{n2 n8} - r5c2{n8 n3} - r1c2{n3 n6} - r1c6{n6 n1} - r3c6{n1 .} ==>
r6c2 ≠ 2
biv-chain-rc[4]: r4c1{n2 n5} - r5c1{n5 n6} - r6c2{n6 n4} - r7c2{n4 n2} ==> r8c1 ≠ 2
biv-chain-rc[4]: r8c9{n5 n8} - r8c6{n8 n7} - r8c1{n7 n6} - r5c1{n6 n5} ==> r5c9 ≠ 5
singles ==> r5c9 = 7, r5c8 = 6, r5c1 = 5, r4c1 = 2, r4c8 = 5, r7c8 = 9, r6c3 = 4, r6c2
= 6, r6c8 = 2, r1c6 = 6, r2c6 = 9, r9c5 = 9, r1c1 = 1, r7c2 = 4
whip[1]: b7n2{r8c3 .} ==> r2c3 ≠ 2
whip[1]: r9n2{c9 .} ==> r7c9 ≠ 2
naked-pairs-in-a-row: r3{c1 c8}{n4 n7} ==> r3c4 ≠ 7
hidden-single-in-a-block ==> r2c4 = 7
whip-rc[3]: r8c4{n5 n2} - r7c5{n2 n8} - r7c9{n8 .} ==> r7c4 ≠ 5
biv-chain-rc[4]: r3c2{n2 n8} - r2c3{n8 n6} - r2c1{n6 n4} - r2c9{n4 n2} ==> r3c7 ≠ 2
stte
```

6.6.4 Using typed-t-whips

Let us restrict still more the patterns we allow, replacing the initial whips by typed-t-whips and (of course) also bivalue-chains by typed-bivalue-chains.

```
(bind ?*Subsets* TRUE)
(bind ?*FinnedFish* TRUE)
(bind ?*Typed-Bivalue-Chains* TRUE)
(bind ?*Typed-t-Whips* TRUE)
```

We get a resolution path in TytW9; this might have been conjectured in view of the W and tW ratings being identical, but the equality of the TyW and TytW ratings does not follow *a priori* from the equality of the W and tW ratings:

```
;;; Resolution state RS1
t-whip-cn[5]: c3n3{r1 r5} - c2n3{r5 r1} - c2n6{r1 r6} - c2n4{r6 r7} - c2n2{r7 .} ==>
r1c3 ≠ 2
t-whip-cn[7]: c6n9{r2 r7} - c8n9{r7 r9} - c8n4{r9 r3} - c8n7{r3 r5} - c8n6{r5 r6} -
c2n6{r6 r1} - c6n6{r1 .} ==> r2c6 ≠ 7, r2c6 ≠ 8
t-whip-cn[7]: c8n6{r5 r6} - c2n6{r6 r1} - c6n6{r1 r2} - c6n9{r2 r7} - c8n9{r7 r9} -
c8n4{r9 r3} - c8n7{r3 .} ==> r5c8 ≠ 5
t-whip-rc[7]: r5c9{n7 n5} - r5c1{n5 n6} - r5c8{n6 n7} - r3c8{n7 n4} - r9c8{n4 n9} -
r9c5{n9 n2} - r9c7{n2 .} ==> r9c9 ≠ 7
```

```
t-whip-cn[7]: c2n6{r6 r1} - c6n6{r1 r2} - c6n9{r2 r7} - c8n9{r7 r9} - c8n4{r9 r3} -
c8n7{r3 r5} - c8n6{r5 .} ==> r6c3 ≠ 6
t-whip-cn[7]: c8n9{r7 r9} - c8n4{r9 r3} - c8n7{r3 r5} - c8n6{r5 r6} - c2n6{r6 r1} -
c6n6{r1 r2} - c6n9{r2 .} ==> r7c4 ≠ 9, r7c5 ≠ 9
t-whip-cn[4]: c6n1{r3 r7} - c6n9{r7 r2} - c4n9{r2 r5} - c4n1{r5 .} ==> r1c5 ≠ 1
t-whip-cn[7]: c6n6{r1 r2} - c6n9{r2 r7} - c8n9{r7 r9} - c8n4{r9 r3} - c8n7{r3 r5} -
c8n6{r5 r6} - c2n6{r6 .} ==> r1c1 ≠ 6, r1c3 ≠ 6
t-whip-bn[9]: b1n4{r2c1 r3c1} - b3n4{r3c8 r2c9} - b9n4{r9c9 r9c8} - b9n7{r9c8 r9c7} -
b3n7{r1c7 r3c8} - b6n7{r5c8 r5c9} - b6n5{r5c9 r4c8} - b6n2{r4c8 r6c8} - b4n2{r6c3 .}
==> r2c1 ≠ 2
t-whip-rc[9]: r3c2{n8 n2} - r7c2{n2 n4} - r6c2{n4 n6} - r1c2{n6 n3} - r1c3{n3 n7} -
r7c3{n7 n2} - r8c3{n2 n6} - r8c1{n6 n7} - r8c6{n7 .} ==> r3c6 ≠ 8
singles ==> r2c5 = 8, r3c2 = 8, r5c2 = 3, r5c3 = 8, r1c3 = 3
t-whip-rn[4]: r9n2{c9 c5} - r9n9{c5 c8} - r7n9{c8 c6} - r7n8{c6 .} ==> r7c9 ≠ 2
t-whip-cn[5]: c5n2{r9 r1} - c5n5{r1 r7} - c8n5{r7 r4} - c8n2{r4 r6} - c2n2{r6 .} ==>
r7c4 ≠ 2
t-whip-rn[6]: r1n6{c2 c6} - r1n1{c6 c1} - r1n7{c1 c7} - r9n7{c7 c8} - r9n4{c8 c9} -
r2n4{c9 .} ==> r2c1 ≠ 6
t-whip-cn[6]: c1n7{r3 r8} - c1n6{r8 r5} - c1n5{r5 r4} - c8n5{r4 r7} - c9n5{r7 r5} -
c9n7{r5 .} ==> r2c3 ≠ 7
whip[1]: c3n7{r8 .} ==> r8c1 ≠ 7
naked-pairs-in-a-block: b1{r1c2 r2c3}{n2 n6} ==> r3c1 ≠ 2, r1c1 ≠ 2
hidden-pairs-in-a-row: r3{n2 n5}{c4 c7} ==> r3c7 ≠ 7, r3c4 ≠ 7, r3c4 ≠ 1
whip[1]: b2n1{r3c6 .} ==> r7c6 ≠ 1
naked-pairs-in-a-block: b2{r1c5 r3c4}{n2 n5} ==> r2c4 ≠ 2
biv-chain-bn[4]: b4n6{r6c2 r5c1} - b7n6{r8c1 r8c3} - b7n7{r8c3 r7c3} - b7n4{r7c3 r7c2}
==> r6c2 ≠ 4
hidden-single-in-a-block ==> r6c3 = 4
hidden-single-in-a-block ==> r7c2 = 4
biv-chain-rn[3]: r7n2{c5 c3} - r2n2{c3 c9} - r3n2{c7 c4} ==> r8c4 ≠ 2, r1c5 ≠ 2
singles ==> r1c5 = 5, r3c4 = 2, r3c7 = 5
finned-x-wing-in-rows: n2{r2 r8}{c9 c3} ==> r7c3 ≠ 2
stte
```

6.6.5 Using type-restricted typed-whips

You thought it was the end of it? It's not! With respect to the choices of section 6.6.3, we can also restrict the types of the typed-whips and (of course) also those of the typed-bivalue-chains. In Sudoku, the natural restriction is to allow only type rc, i.e. patterns that lie totally in the "natural" rc-space. This amounts to not taking bilocality into consideration. For some players, bilocality, i.e. a number being present in only two places in a row (or column or block), is more difficult to spot than bivalue rc-cells. When additional t- or z- candidates need to be taken into account, as in whips, the difference in difficulty may be still greater. It is therefore a reasonable idea to start looking for whips or t-whips in the rc-space.

```
(bind ?*Subsets* TRUE)
(bind ?*FinnedFish* TRUE)
(bind ?*Typed-Bivalue-Chains* TRUE)
(bind ?*Typed-Whips* TRUE)
```

```
(bind ?*restrict-csp-types-in-typed-chains* TRUE)
(bind ?*allowed-csp-types* (create$ rc))
```

We get a resolution path in rc-W12, i.e. we need significantly longer chains:

```
;;; Resolution state RS1
whip-rc[6]: r9c7{n7 n2} - r9c5{n2 n9} - r9c8{n9 n4} - r3c8{n4 n7} - r3c7{n7 n5} -
r1c7{n5 .} ==> r9c9 ≠ 7
whip-rc[7]: r3c2{n2 n8} - r5c2{n8 n3} - r1c2{n3 n6} - r2c3{n6 n7} - r2c1{n7 n4} -
r3c1{n4 n1} - r1c1{n1 .} ==> r1c3 ≠ 2
whip-rc[7]: r8c6{n7 n8} - r3c6{n8 n1} - r7c6{n1 n9} - r7c8{n9 n5} - r8c9{n5 n2} -
r9c9{n2 n4} - r2c9{n4 .} ==> r2c6 ≠ 7
whip-rc[10]: r3c2{n2 n8} - r5c2{n8 n3} - r1c2{n3 n6} - r2c3{n6 n7} - r2c9{n7 n4} -
r9c9{n4 n2} - r9c5{n2 n9} - r5c5{n9 n1} - r5c4{n1 n9} - r2c4{n9 .} ==> r2c1 ≠ 2
whip-rc[10]: r5c5{n1 n9} - r9c5{n9 n2} - r2c5{n2 n8} - r3c6{n8 n7} - r8c6{n7 n8} -
r7c5{n8 n5} - r7c8{n5 n9} - r7c6{n9 n1} - r7c4{n1 n7} - r8c4{n7 .} ==> r1c5 ≠ 1
whip-rc[12]: r3c2{n2 n8} - r5c2{n8 n3} - r1c2{n3 n6} - r2c3{n6 n7} - r1c1{n7 n1} -
r1c6{n1 n7} - r3c6{n7 n1} - r3c4{n1 n5} - r1c5{n5 n2} - r9c5{n2 n9} - r7c6{n9 n8} -
r8c6{n8 .} ==> r3c1 ≠ 2
whip-rc[8]: r3c8{n7 n4} - r2c9{n4 n2} - r1c7{n2 n5} - r1c5{n5 n2} - r9c5{n2 n9} -
r2c5{n9 n8} - r3c6{n8 n1} - r3c1{n1 .} ==> r3c7 ≠ 7
whip-rc[10]: r9c9{n2 n4} - r2c9{n4 n7} - r5c9{n7 n5} - r5c1{n5 n6} - r8c1{n6 n7} -
r2c1{n7 n4} - r3c1{n4 n1} - r1c1{n1 n2} - r1c5{n2 n5} - r1c7{n5 .} ==> r8c9 ≠ 2
whip-rc[5]: r8c6{n7 n8} - r3c6{n8 n1} - r7c6{n1 n9} - r7c8{n9 n5} - r8c9{n5 .} ==>
r1c6 ≠ 7
whip-rc[7]: r8c6{n7 n8} - r8c9{n8 n5} - r8c4{n5 n2} - r2c4{n2 n9} - r5c4{n9 n1} -
r5c5{n1 n9} - r9c5{n9 .} ==> r7c4 ≠ 7
whip-rc[9]: r3c2{n2 n8} - r5c2{n8 n3} - r1c2{n3 n6} - r1c6{n6 n1} - r3c6{n1 n7} -
r8c6{n7 n8} - r7c6{n8 n9} - r7c8{n9 n5} - r8c9{n5 .} ==> r6c2 ≠ 2
biv-chain-rc[4]: r4c1{n2 n5} - r5c1{n5 n6} - r6c2{n6 n4} - r7c2{n4 n2} ==> r8c1 ≠ 2
biv-chain-rc[4]: r8c9{n5 n8} - r8c6{n8 n7} - r8c1{n7 n6} - r5c1{n6 n5} ==> r5c9 ≠ 5
naked-single ==> r5c9 = 7
whip[1]: c9n5{r8 .} ==> r7c8 ≠ 5
singles ==> r7c8 = 9, r9c5 = 9, r5c5 = 1, r5c4 = 9, r2c6 = 9, r1c6 = 6, r6c2 = 6, r5c1
= 5, r4c1 = 2, r4c8 = 5, r6c3 = 4, r5c8 = 6, r6c8 = 2, r7c2 = 4, r1c1 = 1
whip[1]: b7n2{r8c3 .} ==> r2c3 ≠ 2
whip[1]: r9n2{c9 .} ==> r7c9 ≠ 2
naked-pairs-in-a-row: r3{c1 c8}{n4 n7} ==> r3c6 ≠ 7, r3c4 ≠ 7
hidden-single-in-a-block ==> r2c4 = 7
whip-rc[3]: r7c9{n5 n8} - r7c5{n8 n2} - r8c4{n2 .} ==> r7c4 ≠ 5
biv-chain-rc[4]: r3c2{n2 n8} - r2c3{n8 n6} - r2c1{n6 n4} - r2c9{n4 n2} ==> r3c7 ≠ 2
stte
```

Now, you can ask: can we go still further and restrict the allowed patterns to t-whips in rc-space only? We can, but not in a very useful way for this puzzle! The only thing CSP-Rules finds in rc-space after RS1 is this t-whip:

```
t-whip-rc[7]: r5c9{n7 n5} - r5c1{n5 n6} - r5c8{n6 n7} - r3c8{n7 n4} - r9c8{n4 n9} -
r9c5{n9 n2} - r9c7{n2 .} ==> r9c9 ≠ 7
```

Nevertheless I am sure that, after playing with SudoRules and the various rule combinations it allows, you will be able to find examples where a t-whip solution in rc-space does not require chains much longer than when you allow the full power of all the 3D-chains. Puzzles of medium difficulty are the best candidates.

7. FutoRules

FutoRules is the pattern-based solver of Futoshiki puzzles based on CSP-Rules. Futoshiki is played on a square grid. Grid size is unrestricted, but in practice, it will often be at most 9. FutoRules does not have any *a priori* restriction.

7.1 The configuration file

The generic part looks much like the generic Sudoku part described in chapter 5. In particular, it allows to select g-whips and g-braids.

The Futoshiki specific part deals with allowing (or not): 1) the presence of givens in the cells (i.e. dealing with "impure" or "pure" Futoshiki); 2) the rules specifically related to sequences of inequalities and 3) the Subsets rules. By default:

- Futoshiki is pure (i.e. it does not have givens in the cells, it has only inequality symbols); but this can be changed in the configuration file;
- all the rules related to sequences of inequalities are disabled by default; but this can be changed in the configuration file, and this is indeed changed in my standard configuration, because they are as natural to Futoshiki as Subsets are to Sudoku;
- the option for Subsets is necessarily FALSE by default in CSP-Rules; it can be set to TRUE in the configuration file and it is indeed TRUE in my standard config.

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;; My standard config and its usual variants
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

 ;;; change the default type of Futoshiki; pure by default (i.e. there is no given in
the cells)
; (bind ?*pure-Futoshiki* FALSE)                    <<<<<<<<<<<<<<<<<<<<<<<<<<<<<

;;; My most usual rules:
(bind ?*asc-chains* TRUE) ; enable ascending chains
(bind ?*str-asc-chains* TRUE) ; enable strong ascending chains
(bind ?*hills-and-valleys* TRUE) ; enable hills and valleys
(bind ?*Subsets* TRUE)
(bind ?*Bivalue-Chains* TRUE)
(bind ?*Whips* TRUE)

;;; Some additional rules I use frequently:
; (bind ?*t-Whips* TRUE)...
```

7.2 The user functions

FutoRules has only two user functions.

1) Function “***solve***” has four arguments, with the following syntax:
`(solve ?nb-rows ?givens-str ?horiz-ineq-str ?verti-ineq-str)`
where:

- ?nb-rows is the number of rows (or columns),
- ?givens-str is a string representing the sequence of givens in the cells (with the same conventions as in SudoRules: a dot represents no given):
  ".."
- ?horiz-ineq-str is a string representing the sequence of horizontal inequality signs,
- ?verti-ineq-str is a string representing the sequence of vertical inequality signs.

Example (Figure 7.1, H9305 from ATK:
http://www.atksolutions.com/games/futoshiki.html, solvable in S+W6):

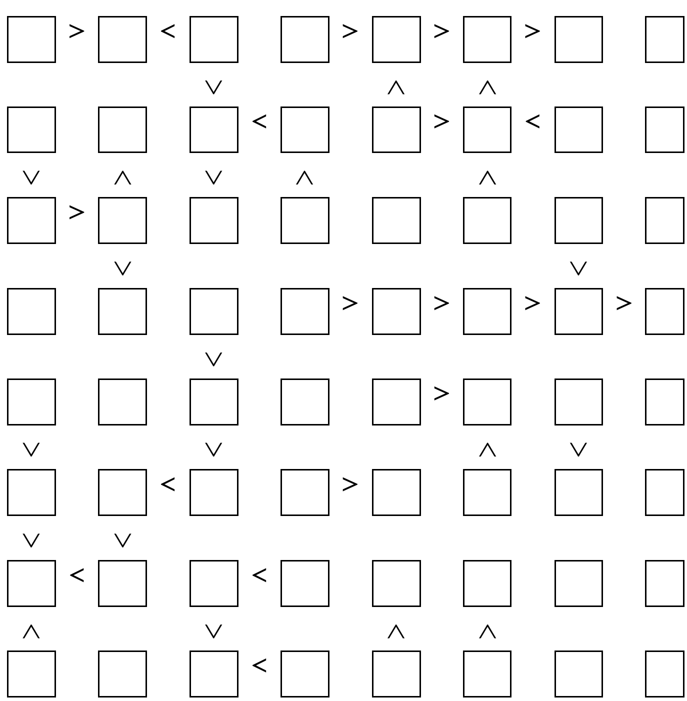


Figure 7.1. *Futoshiki puzzle H9305 from ATK*

To solve this 8x8 pure Futoshiki puzzle, the call to function “solve” should be:

```
(solve 8
   "................................................................"
   "><->>>---<-><->--------->>>>---->---<->---<-<------<----"
   "->-->><-<>-->->>->>->-<-----<-----<<<--<-<-->->---------"
)
```

The first argument is 8, the number of rows and columns of the puzzle. The second argument is as in Sudoku, with a dot representing no given in the corresponding cell; in “pure” Futoshiki (no givens), this string reduces to a sequence of n^2 dots, as in this example.

The last two arguments require more explanations. On each row of the puzzle, there are n cells and there are at most n-1 inequality signs. If “-” is used to represent the absence of an inequality between two cells, then a row can be represented as a sequence of n-1 signs, looking like this (for the first row): ><->>>-

The whole set of horizontal left to right inequalities, from top row to bottom row, will be a set of n similar lines:

```
><->>>-
--<-><-
>------
--->>>>
---->--
-<->---
<-<----
--<----
```

Stick these n lines together (by deleting the carriage returns), put them between double quotes and you get the first argument:

```
“><->>>---<-><->--------->>>>---->---<->---<-<------<----"
```

Similarly, write the set of top to bottom vertical inequalities, from left column to right column, as n lines of n-1 signs:

```
->-->><
-<>-->-
>>->>->
-<-----
<-----<
<<--<-<
-->->--
-------
```

Stick these n lines together (by deleting the carriage returns), put them between double quotes and you get the second argument:

```
"->-->><-<>-->->>->>->-<-----<-----<<<--<-<-->->---------"
```

2) Function “***solve-tatham***” has two arguments, with the following syntax:
`(solve-tatham ?n ?tatham-str)`

Suppose you generate a Futoshiki puzzle on the Tatham webpage for Futoshiki (where it’s named “unequal”
https://www.chiark.greenend.org.uk/~sgtatham/puzzles/js/unequal.html)
You will get a puzzle like in Figure 7.2 (this is a 7×7 “Tricky” one):

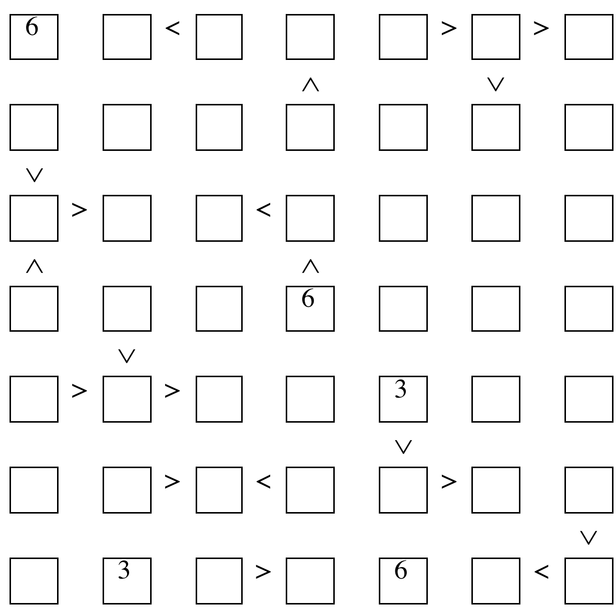


Figure 7.2. *“Tricky”Futoshiki puzzle from Tatham*

Now, suppose you click “link to this puzzle by game ID”. Your browser will give you an address that looks like this:

```
https://www.chiark.greenend.org.uk/~sgtatham/puzzles/js/unequal.html#7:6,0,0L,0,0R,0RD
,0,0D,0,0,0U,0,0,0,0R,0,0,0L,0,0,0,0U,0D,0,6U,0,0,0,0R,0R,0,0,3D,0,0,0,0R,0,0L,0R,0,0D
,0,3,0R,0,6,0,0L,
```

What I call the tatham string format for a Futoshiki puzzle is the part of this address following the second colon symbol, considered as a string. Notice that the address also encodes the size of the puzzle (between the “#” and the second “:”), but I prefer not to consider it as part of the representation of the puzzle, because grid size information is usually separate in the other CSP-Rules applications. This size will be given as the value for argument ?n. Thus, the tatham string format for the above 7x7 Futoshiki puzzle is:

```
“6,0,0L,0,0R,0RD,0,0D,0,0,0U,0,0,0,0R,0,0,0L,0,0,0,0U,0D,0,6U,0,0,0,0R,0R,0,0,3D,0,0,0
,0R,0,0L,0R,0,0D,0,3,0R,0,6,0,0L,”
```

This string can be considered as having 7×7 small parts, each followed by a comma. The nth part corresponds to the content of the nth cell (a digit, including 0), plus optionally one U (greater than the cell upwards), one D (greater than the cell downwards), one L (greater than the cell on the left) and/or one R (greater than the cell on the right); the symbols U, D, L, R express inequalities between cells. All four of them may appear in the same small part of the string, which will then be a hill, both horizontally and vertically.

Not surprisingly, function “solve-tatham” for this puzzle is written as:

```
(solve-tatham 7
“6,0,0L,0,0R,0RD,0,0D,0,0,0U,0,0,0,0R,0,0,0L,0,0,0,0U,0D,0,6U,0,0,0,0R,0R,0,0,3D,0,0,0
,0R,0,0L,0R,0,0D,0,3,0R,0,6,0,0L,”)
```

FutoRules has a function “tatham-to-FutoRules-strings” that outputs a list of the three string arguments to function “solve”; “solve-tatham” is the proper combination of those two functions. The command:

```
(tatham-to-FutoRules-strings 7
“6,0,0L,0,0R,0RD,0,0D,0,0,0U,0,0,0,0R,0,0,0L,0,0,0,0U,0D,0,6U,0,0,0,0R,0R,0,0,3D,0,0,0
,0R,0,0L,0R,0,0D,0,3,0R,0,6,0,0L,”)
```

outputs a list of the three strings:

```
("6.......................6.......3..........3..6.."
 "-<-->>------>-<--------->>-----><->--->--<"
 "-><------>--------<-<-------->->---------->")
```

Instead of “solve-tatham”, function “solve” can be directly called using the above three strings, with the same result as the above call to “solve-tatham”:

```
(solve 7
   "6.......................6.......3..........3..6.."
   "-<-->>------>-<--------->>-----><->--->--<"
   "-><------>--------<-<-------->->---------->"
)
```

Remarks:

- the above Tatham puzzle provides an example of Triplets in Futoshiki;
- the Tatham classification of puzzles is unclear to me. I would have thought that “Recursive” meant the puzzles were not solvable without T&E(2); but FutoRules has solved some “Recursive” ones using only whips, which implies they are in T&E(1). On the other hand, most “Extreme” puzzles should be solvable by FutoRules without requiring T&E but a few are not.

7.3 Some specific notations in the resolution path

As it has application-specific rules, FutoRules has some specific notation for them. All the examples in this section are taken from the resolution path of the Tatham puzzle in section 7.2.

Ascending chains (respectively strong ascending chains) appear as follows:

```
asc[3]: r3c3<r2c3<r1c3<r1c2 ==> r3c3 ≠ 7, r3c3 ≠ 6, r3c3 ≠ 5, r2c3 ≠ 7, r2c3 ≠ 6, r2c3
≠ 1, r1c3 ≠ 7, r1c3 ≠ 1, r1c2 ≠ 3, r1c2 ≠ 1
str-asc[1]: r4c2<r4c1 ==> r4c1 ≠ 3
```

The number between square brackets is the length of the chain, measured by the number of < signs. It should not mislead you about the complexity of the pattern. I've shown in [PBCS] that, as far as eliminations are concerned, an ascending chain is equivalent to a series of whips[1]. Ascending chains are an example of an application-specific rule that can be interpreted as a macro-rule, i.e. a sequence of simpler ones. These rules are activated by default in my usual configuration, because they provide more compact, better looking resolution paths, although they add no resolution power.

Hills, valleys, strong hills and strong valleys also have their own notation, similar to that of ascending chains, with the same convention about the pseudo-length.

```
str-hill[3]: r5c3<r5c4>r5c5>r5c6 ==> r5c4 ≠ 4
```

But they can only be reduced to much more complex rules (S_p-whips) and they do add to resolution power. However, as they rely on a very natural pattern, they are also activated by default in my usual configuration:

```
;;; My most usual rules:
  (bind ?*asc-chains* TRUE) ; enable ascending chains
  (bind ?*str-asc-chains* TRUE) ; enable strong ascending chains
  (bind ?*hills-and-valleys* TRUE) ; enable hills and valleys
  (bind ?*Subsets* TRUE)
  (bind ?*Bivalue-Chains* TRUE)
  (bind ?*Whips* TRUE)

;;; Some additional rules I use frequently:
; (bind ?*z-Chains* TRUE)
; (bind ?*t-Whips* TRUE)
; (bind ?*G-Whips* TRUE)
```

8. KakuRules

KakuRules is the pattern-based solver of Kakuro puzzles based on CSP-Rules. Kakuro can be played on any rectangular grid, but most of the grids we meet in practice are square. KakuRules requires a square grid (but a rectangular one can be straightforwardly transformed into a square one, by adding dummy black cells with no content).

Kakuro has Subsets and g-whips and both patterns are noteworthy. The theory of g-labels in Kakuro has been developed in full detail in [PBCS], with real examples only in [PBCS2] (due to a stupid bug in the implementation of glabels in KakuRules at the time of [PBCS1]). It is quite complex but, as in Sudoku or even more, it leads to one of the most fascinating applications of g-whips (see examples in [PBCS2]).

Remarks and warnings:

1) KakuRules does not have any version of the rules related to "surface sums", as they are explained in [PBCS]. It does not mean you cannot use KakuRules in case you notice almost closed areas in the grid. After properly using the surface sums, you can try to solve each resulting small puzzle separately and then inject the partial result(s) into the global puzzle. You can also apply KakuRules directly to the global puzzle, but then it may sometimes require (much) longer chains than if you first use the "surface sums". I make no statement about whether the presence of surface sums is a good thing or not in a Kakuro puzzle; every player will have his own opinion on them. I'm just warning: don't expect KakuRules to deal with them as such by itself.

2) In the CSP-Rules approach in general and in KakuRules in particular, additional CSP-Variables are added to the "natural" ones. In KakuRules, this means variables related to the sums in the black cells, i.e. variables intended to keep track of the *combinations* allowed in each horizontal or vertical sector. It is natural to attach these additional CSP-Variables to the black cells and to count the first (totally black) row and the first (totally black) column as parts of the Kakuro grid. This justifies the default numbering I've adopted for rows and columns, i.e. 1 for the first (black) row (or column). Similarly, the size or the grid (number of rows or columns) includes the first (black) row or column. The configuration file allows to change the numbering of the rows and columns (i.e. to start at 0 instead of 1) when printing the resolution path, but this does not change the size of the grid parameter required by function "solve".

3) KakuRules has an interesting application of Typed-Chains (see section 8.3), totally different from the SudoRules one.

8.1 The configuration file

There is nothing much to say about the KakuRules configuration file. As said above, you can change the numbering of the rows and columns (setting 0 instead of the default 1 for the first all-black row and column). You can also disable the initial data consistency check (not recommended).

Kakuro has the most beautiful applications of g-whips. Don't miss trying them.

8.2 The user functions

K	17	42				8	5		
7			13		6 20			41	
9				19 13					13
35							12 11		
13			12 3			24			
	5 6				17	9 8			19
13				11			3 8		
9			12	29 15					
	23					12			
		16					4		

Figure 8.1. *ATK Kakuro puzzle H2340*

Basically, there is only one user function: "***solve***", with syntax:

```
(solve ?n $?givens)
```

?n is the grid size (including the first row or column).

?givens is a sequence representing all the row and column data: first all the rows from top to bottom, then all the columns from left to right. The idea for this format, with separate data for the horizontal and vertical parts, was inspired by crosswords.

It has some syntactic sugar (the separate lines, the single and double slashes) for better clarity and for easy checks of consistency of the horizontal data with the vertical ones.

Once again, I'll use an example from the ATK website, (Figure 8.1, puzzle H2340, a puzzle that can be solved in W6), to illustrate the proper syntax. For this puzzle, the proper call to "solve" is as follows.

As size is n, there are n-1 lines of horizontal data, followed by n-1 lines of vertical data. In the horizontal data part, there's no line corresponding to the first (black) row; in the vertical data part, there's no line corresponding to the first (black) column: they would contain no information.

```
(solve 10
;;; horizontal data:
7 . . B B 6 . . B B /
9 . . . 19 . . . . B /
35 . . . . . B 12 . . /
13 . . 12 . . 24 . . . /
B 5 . . B B 9 . . B /
13 . . . 11 . . 3 . . /
9 . . B 29 . . . . . /
B 23 . . . . 12 . . . /
B B 16 . . B B 4 . . //

;;; vertical data:
17 . . . . 6 . . B B /
42 . . . . . . . . B /
B 13 . . 3 . . 12 . . /
B B 13 . . B B 15 . . /
B 20 . . . 17 . . . B /
8 . . B B 8 . . B B /
5 . . 11 . . 8 . . B /
B 41 . . . . . . . . /
B B 13 . . 19 . . . . //
)
```

Notice that the spaces between each entry are mandatory (writing "BB" instead of "B B" will produce an error message).

Each line must have n+1 symbols, each of which can only be:

- a dot, representing a white cell,
- an integer between 3 and 45, representing a sum constraint (horizontal if it's in the horizontal data part, vertical if it's in the vertical data part),
- symbol "B" representing a black cell with no horizontal (respectively vertical) sum constraint if it's in the horizontal (resp. vertical) data part,

- a slash, denoting the end of a line (it must appear at the end of the line and it may only appear there),
- a double slash, denoting the end of the horizontal and vertical parts (it must appear at these places in the sequence and it may only appear there).

What function "solve" deals with could be called "pure" Kakuro, in the same sense as we spoke of "pure" Futoshiki. KakuRules also has a user function, "***solve-partly-solved-puzzle***", for solving a puzzle with data in the white cells, with syntax:
`(solve-partly-solved-puzzle ?n $?givens)`

This function may be useful when you find an almost closed surface, you apply the surface sums rule, you solve some of the small sub-puzzle(s) and you want to inject the partial results thus obtained into the global puzzle.

"solve-partly-solved-puzzle" seems to have the same syntax as "solve", but the $?givens argument must contain additional information, for expressing the givens in the white cells. This information will appear after the horizontal and vertical parts. For the above puzzle (which has no white cell data), the proper call would be:

```
(solve-partly-solved-puzzle 10
;;; horizontal data:
7 . . B B 6 . . B B /
9 . . . 19 . . . . B /
35 . . . . . B 12 . . /
13 . . 12 . . 24 . . . /
B 5 . . B B 9 . . B /
13 . . . 11 . . 3 . . /
9 . . B 29 . . . . . /
B 23 . . . . 12 . . . /
B B 16 . . B B 4 . . //

;;; vertical data:
17 . . . . 6 . . B B /
42 . . . . . . . . B /
B 13 . . 3 . . 12 . . /
B B 13 . . B B 15 . . /
B 20 . . . 17 . . . B /
8 . . B B 8 . . B B /
5 . . 11 . . 8 . . B /
B 41 . . . . . . . . /
B B 13 . . 19 . . . . //

;;; white cells data:
. . . . . . . . .
. . . . . . . . .
. . . . . . . . .
. . . . . . . . .
. . . . . . . . .
. . . . . . . . .
```

```
. . . . . . . . .
. . . . . . . . .
. . . . . . . . .
)
```

The horizontal and vertical parts are the same as in function “solve”. The white cells part contains the already solved cells (none in the present case). The above calls to “solve” and to “solve-partly-solved-puzzle” for the same puzzle are equivalent. Try them to check that they give exactly the same resolution path. Of course, it would be particularly absurd in practice to use “solve-partly-solved-puzzle” in case no white cell is solved.

Notice that the white cells data part has n-1 lines of n-1 symbols each; it totally discards the first (black) row and column. Each line corresponds to a row in the grid (except the first black one) and its content corresponds to the n-1 cells of this row in the puzzle (excluding the first black cell). Each symbol may only be a dot or a non-zero digit. Non-zero digits may only appear at a place corresponding to a white cell.

It you want to try a real partly solved puzzle, you can change the white part in the above puzzle, by adding a single given, into the following. You will notice that the result is now in W5 instead of W6:

```
;;; white cells data
. . . . . . . . .
. . . . . . . . .
. . . . . . . . .
. . . . . . . . .
. . . . . . . . .
. . . . . . . . .
. 8 . . . . . . .
. . . . . . . . .
. . . . . . . . .
```

8.2.1 An example from [PBCS2]

In order to end this section with a very good example of whips, g-whips, surface sums, …, you can play with the puzzle discussed in [PBCS2], by trying different sets of rules (showing that it is in W10 but in gW8, comparing the different resolution times and memory requirements), using surface sums, … It’s a hard puzzle from ATK (Figure 8.2).

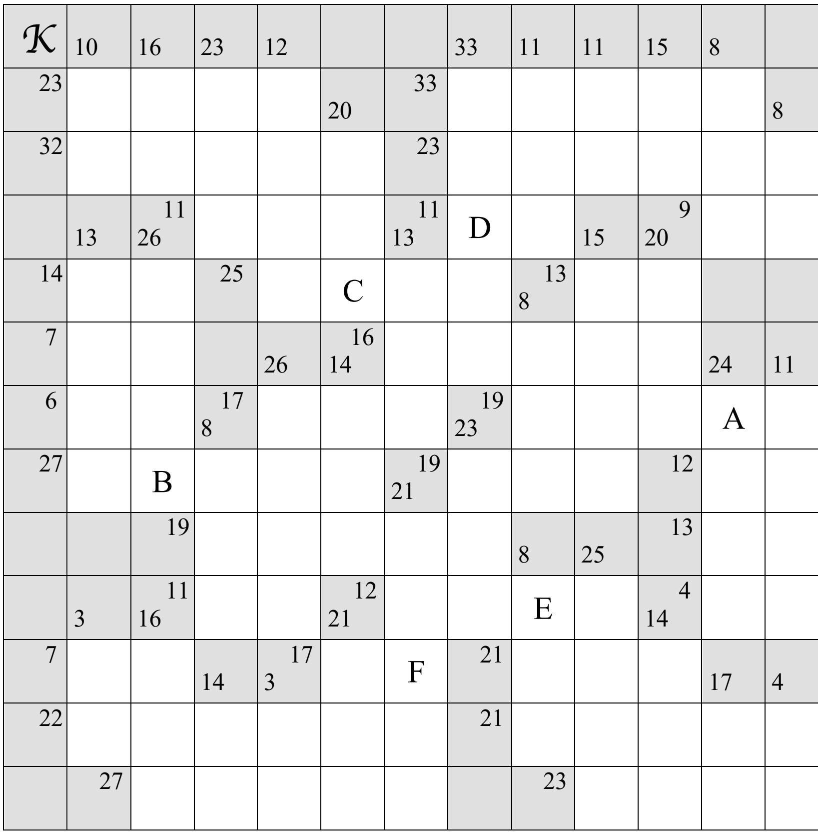


Figure 8.2. *ATK Kakuro puzzle H83722*

To make it easier to try it for yourself, here is the proper call to function "solve":

```
(solve 13
;;; horizontal data:
23 . . . . B 33 . . . . . B /
32 . . . . . 23 . . . . . . /
B B 11 . . . 11 . . B 9 . . /
14 . . 25 . . . . 13 . . B B /
7 . . B B 16 . . . . . B B /
6 . . 17 . . . 19 . . . . . /
27 . . . . . 19 . . . 12 . . /
B B 19 . . . . . B B 13 . . /
B B 11 . . 12 . . . . 4 . . /
7 . . B 17 . . 21 . . . B B /
22 . . . . . . 21 . . . . . /
B 27 . . . . . B 23 . . . . //
```

```
;;; vertical data:
10 . . 13 . . . . B 3 . . B /
16 . . 26 . . . . B 16 . . . /
23 . . . B B 8 . . . 14 . . /
12 . . . . 26 . . . . 3 . . /
B 20 . . . 14 . . . 21 . . . /
B B B 13 . . . 21 . . . . . /
33 . . . . . 23 . . . B B B /
11 . . . 8 . . . 8 . . . B /
11 . . 15 . . . . 25 . . . . /
15 . . 20 . . . B B 14 . . . /
8 . . . B 24 . . . . 17 . . /
B 8 . . B 11 . . . . 4 . . //
)
```

8.3 Typed-Chains in KakuRules

In chapter 6, we have seen how the generic Typed-Chains can be applied to SudoRules, when we adopt a typing system based on the four kinds of CSP-Variables specific to Sudoku, namely the "natural" rc ones and the additional rn, cn and bn ones, as I introduced them in [HLS] and as they are represented by the four 2D spaces of the Extended Sudoku Board. However, such a typing system would not be very interesting in Kakuro, because the most important interactions between CSP-Variables of types hrc (or vrc) and rc (i.e. interactions between the still possible combinations for a sector and the still possible values in the white cells of this sector) would be discarded in the resulting Typed-Chains. Instead of this, I've therefore defined a totally different typing system, where a type corresponds to a fixed (vertical or horizontal) sector.

For each horizontal sector, represented by its controller (black) cell r_ic_k and named correspondingly hr_ic_k, the following three kinds of CSP-Variables are assigned the hr_ic_k type:

- hr_ic_k (i.e. the variable representing the still possible combinations for this sector)
- $r_{i'}c_{k'}$ for any white rc-cell $r_{i'}c_{k'}$ in the sector
- $r_{i'}n_{k'}$ for any rn-cell $r_{i'}n_{k'}$ in the sector and any mandatory number in this sector.

Of course, there is a corresponding definition for each vertical sector.

The attentive reader will have noticed that each CSP-Variable for a white cell is assigned two types, one corresponding to its horizontal sector and one corresponding to its vertical sector. However, this is not a problem for CSP-Rules because the notion of "type" used in the Typed-Chains is not a function but a predicate.

Remark: few Kakuro puzzles are solvable using only Typed-Chains. But those that are may define an interesting level of easy puzzles: the human player does not have to look for complex interactions between values in different sectors. The only interactions between different sectors will appear as candidates in the white cells are effectively deleted or turned into values. Everything harder happens in a single sector.

As such typed rules for Kakuro were not mentioned in [PBCS], time has come for an example. Once more, I'll take one from the ATK website, their puzzle M42942 (see Figure 8.3).

K	7	16	30	33	6	20		23	38
28							12		
39							9 11		
	39	17 23			11 15				
31						23 27			
16			19 13				14 27		
22				15 18					
19					11 13			5	6
5			32						
9			38						

Figure 8.3. *ATK Kakuro puzzle M42942*

The calling function from CSP-Rules is:

```
(solve 10
;;; horizontal data
28 . . . . . . 12 . . /
39 . . . . . . 9 . . /
B B 17 . . 11 . . . . /
31 . . . . . 23 . . . /
16 . . 19 . . . 14 . . /
22 . . . 15 . . . . . /
19 . . . . 11 . . B B /
```

```
5 . . 32 . . . . . . /
9 . . 38 . . . . . . //

;;; vertical data
7 . . 39 . . . . . . /
16 . . 23 . . . . . . /
30 . . . . 13 . . B B /
33 . . . . . 18 . . . /
6 . . 15 . . . 13 . . /
20 . . . 27 . . . . . /
B B 11 . . 27 . . . . /
23 . . . . . . 5 . . /
38 . . . . . . 6 . . //
)
```

This puzzle has a solution in W3 or in TyW5, as shown in the following two resolution paths. Note that, in the first resolution path, where untyped chains are activated, I’ve activated both typed and untyped chains, in order to show how they easily live together. Also, this allows to have a common path at the start, until resolution state RS1:

```
*****************************************************************************************
***  KakuRules 2.1.s based on CSP-Rules 2.1.s, config = W+S
***  using CLIPS 6.32-r770
***  Running on MacBookPro Retina Mid-2012 i7 2.7GHz 16GB, 1600MHz DDR3, MacOS 10.15.4
*****************************************************************************************
       Uppermost (black) row and leftmost (black) column have index 1
singles ==> r10c10 = 5, r10c9 = 3, r9c2 = 4, r6c3 = 7, r6c2 = 9, vr4c3 = 123467, r7c3
= 6, hr7c1 = 679, r7c2 = 7, r7c4 = 9, vr6c4 = 49, r8c4 = 4, hr9c1 = 14, r9c3 = 1,
vr8c9 = 23, r9c9 = 2, vr8c10 = 15, r9c10 = 1, hr9c4 = 125789, ==> r4c9 = 1, r4c8 = 2,
vr3c8 = 29, r5c8 = 9
323 candidates, 768 csp-links and 1672 links. Density = 3.22%
naked-pairs-in-verti-sector: c10{r4 r7}{n3 n5} ==> r6c10 ≠ 5, r3c10 ≠ 5, r3c10 ≠ 3,
r2c10 ≠ 5, r2c10 ≠ 3
biv-chain-vr1c6[2]: vr1c6{n15 n24} - r3c6{n5 n4} ==> r2c6 ≠ 4
biv-chain-vr1c6[2]: vr1c6{n24 n15} - r3c6{n4 n5} ==> r2c6 ≠ 5
biv-chain-vr1c9[2]: vr1c9{n123458 n123467} - r5c9{n8 n6} ==> r3c9 ≠ 6, r6c9 ≠ 6
singles ==> vr1c9 = 123458, r5c9 = 8, r6c9 = 5, r5c10 = 6, r3c10 = 7, hr3c8 = 27, r3c9
= 2, hr6c8 = 59, r6c10 = 9, r2c10 = 8, hr2c8 = 48, r2c9 = 4, r7c9 = 3, r7c10 = 5, r7c8
= 4, r4c10 = 3, r4c7 = 5, vr6c8 = 4689
z-chain-hr8c1[2]: r8c3{n3 n2} - hr8c1{n3457 .} ==> r8c5 ≠ 3
whip-hr8c1[2]: r8c3{n2 n3} - hr8c1{n2458 .} ==> r8c5 ≠ 2
naked-single ==> vr7c5 = 567
whip-vr4c6[2]: r7c6{n2 n1} - vr4c6{n249 .} ==> r6c6 ≠ 2, r5c6 ≠ 2
whip-hr8c6[2]: hr8c6{n56 n29} - r8c8{n6 .} ==> r8c7 ≠ 9
whip-hr8c6[2]: hr8c6{n56 n38} - r8c8{n6 .} ==> r8c7 ≠ 8
whip-hr8c6[2]: hr8c6{n38 n56} - r8c8{n8 .} ==> r8c7 ≠ 6
z-chain-vr5c7[2]: r8c7{n5 n2} - r7c7{n2 .} ==> vr5c7 ≠ 24678
whip-hr8c1[2]: hr8c1{n3457 n2467} - r8c2{n5 .} ==> r8c5 ≠ 6
naked-pairs-in-verti-sector: c5{r8 r9}{n5 n7} ==> r10c5 ≠ 7
```

```
singles ==> r10c5 = 6, vr8c6 = 58, r10c6 = 8, r10c8 = 9, r10c7 = 7, r9c8 = 8, r8c8 =
6, r9c6 = 5, r9c7 = 9, r9c5 = 7, r8c5 = 5, hr8c6 = 56, r8c7 = 5, vr5c7 = 24579, r7c7 =
2, r7c6 = 1, r6c7 = 4, r8c2 = 8, hr8c1 = 2458, r8c3 = 2
biv-chain-hr5c1[2]: r5c3{n4 n3} - hr5c1{n45679 n35689} ==> r5c5 ≠ 4
biv-chain-hr5c1[2]: hr5c1{n45679 n35689} - r5c3{n4 n3} ==> r5c5 ≠ 3
whip-hr6c4[2]: hr6c4{n469 n478} - r6c6{n6 .} ==> r6c5 ≠ 8
whip-vr4c6[2]: vr4c6{n168 n159} - r6c6{n6 .} ==> r5c6 ≠ 9
x-wing-in-horiz-sectors: n9{r4 r5}{c4 c5} ==> r6c5 ≠ 9, r3c5 ≠ 9, r3c4 ≠ 9, r2c5 ≠ 9,
r2c4 ≠ 9
biv-chain-hr6c4[2]: hr6c4{n478 n469} - r6c5{n7 n6} ==> r6c6 ≠ 6
biv-chain-vr4c6[2]: vr4c6{n159 n168} - r6c6{n9 n8} ==> r5c6 ≠ 8
naked-pairs-in-horiz-sector: r5{c2 c6}{n5 n6} ==> r5c5 ≠ 6, r5c5 ≠ 5, r5c4 ≠ 6
whip-vr1c2[2]: vr1c2{n34 n16} - r3c2{n4 .} ==> r2c2 ≠ 6
whip-vr1c2[2]: vr1c2{n34 n25} - r3c2{n4 .} ==> r2c2 ≠ 5
whip-vr1c2[2]: vr1c2{n25 n34} - r3c2{n5 .} ==> r2c2 ≠ 4
;;; resolution state RS1
```

Here is now the end of the resolution path after state RS1, in case untyped chains are activated; you will notice an untyped bivalue-chains[3]; in this puzzle (where Subsets are activated) this is the only interaction between two sectors based on a rule involving several sectors (indeed only two, as the first two CSP-Variables are of the same type: vr1c7).

```
;;; resolution state RS1
biv-chain[3]: vr1c7{n569 n578} - r3n9{c7 c3} - r2c3{n9 n7} ==> r2c7 ≠ 7 ; <<<<<<<<<<<<<
whip-vr1c7[2]: vr1c7{n569 n578} - r2c7{n9 .} ==> r3c7 ≠ 8
x-wing-in-horiz-sectors: n8{r3 r4}{c4 c5} ==> r5c5 ≠ 8, r5c4 ≠ 8, r2c5 ≠ 8, r2c4 ≠ 8
singles ==> hr5c1 = 45679, r5c3 = 4, r10c3 = 3, hr10c1 = 36, r10c2 = 6, r5c2 = 5, r5c6
= 6
naked-single ==> vr4c6 = 168, r6c6 = 8, hr6c4 = 478, r6c5 = 7, r5c5 = 9, r5c4 = 7,
r2c4 = 6, r3c4 = 8, r4c4 = 9, r4c5 = 8
hidden-pairs-in-horiz-sector: r3{n7 n9}{c3 c7} ==> r3c7 ≠ 6
singles to the end
----------
-176428-48
-698547-27
---98-5213
-54796-986
-97-784-59
-769-12435
-8245-56--
-41-759821
-63-687935
```

Finally, here is the alternative end of the resolution path after state RS1, in case only typed chains are activated:

```
;;; resolution state RS1
z-chain-hr2c1[3]: r2c3{n7 n9} - r2c7{n9 n8} - r2c4{n8 .} ==> hr2c1 ≠ 123589
z-chain-hr2c1[3]: r2c3{n7 n9} - r2c7{n9 n6} - r2c4{n6 .} ==> hr2c1 ≠ 134569
```

```
whip-hr2c1[3]: r2c3{n9 n7} - r2c4{n7 n8} - r2c7{n8 .} ==> hr2c1 ≠ 134578
whip-hr2c1[3]: r2c4{n8 n7} - r2c3{n7 n9} - r2c7{n9 .} ==> hr2c1 ≠ 124579
ctr-to-horiz-sector  ==> r2c5 ≠ 5
whip-hr2c1[4]: r2c3{n7 n9} - hr2c1{n124678 n123679} - r2c4{n8 n6} - r2c7{n6 .} ==>
r2c5 ≠ 7
whip-hr2c1[4]: hr2c1{n123679 n124678} - r2c3{n9 n7} - r2c4{n7 n6} - r2c7{n6 .} ==>
r2c5 ≠ 8
whip-vr1c5[2]: vr1c5{n36789 n45789} - r2c5{n6 .} ==> r3c5 ≠ 4
whip-hr2c1[5]: hr2c1{n124678 n123679} - r2c5{n4 n6} - r2c4{n6 n7} - r2c3{n7 n9} -
r2c7{n9 .} ==> r2c2 ≠ 3
singles to the end
```

Which of the two resolution paths is simpler for a human solver is a matter of taste. As should be expected, using only less powerful chain rules (Typed-Chains) requires some form of compensation and this is provided in the present case by using longer chains.

9. SlitherRules

SlitherRules is the pattern-based solver of Slitherlink puzzles based on CSP-Rules. Slitherlink is played on a rectangular grid and SlitherRules does allow the numbers of rows and columns to be different (and function "solve" will therefore require two size arguments, nb-rows and nb-columns). However, most Slitherlink puzzles found on the web are square.

There are no Subsets in Slitherlink.

Computationally, the main characteristic of Slitherlink is its large fan-out or branching factor: most candidates are linked to many ones. This explains the reputation of Slitherlink as a hard kind of puzzle for automated solvers. For the human player, he has to do many obvious eliminations between each interesting step. (It is worth trying to solve manually some of these puzzles if you want to really understand what I mean here). When such steps are done by an automated solver and each step is printed out, the resolution path seems extremely boring.

As a result of the large fan-out, each resolution state may lead to many different ones via Elementary Constraint Propagation rules. For the same reason, each chain of length n-1 can give rise to many chains of length n and this can lead very fast to memory overload. Therefore, by default, chains are limited to length 5 in the Slitherlink configuration file. For large grids, this may still be too large. For small grids, if you have enough memory, you can try to extend this to 7 if 5 is not enough to solve the puzzle. Be warned, SlitherRules is the slowest part of CSP-Rules.

Slitherlink does have whips[1], indeed lots of them, as you will see in the resolution paths. Therefore, it has g-labels and it could have g-whips. However, due to the already very large branching factor, I haven't coded g-labels; as a result, g-whips or any g-something cannot be activated. You may want to try g2-whips, a special case of g-whips.

Of all the CSP-Rules applications described in this Manual, SlitherRules has the largest number of application-specific resolution rules, the main purpose being to shorten the resolution paths. A complete and detailed description of each of them is available in [PBCS2]; it is largely inspired by what I found in [slinker www] and in the Wikipedia page (which seems to be a copy of [slinker www], without the proper credit); what I introduced is only some rational organisation and a systematic naming convention for the rules.

Using SlitherRules as an assistant theorem prover, I also showed that most (but not all) of these situation-specific rules are equivalent to sequences of Singles and whips[1] (with a few of them needing slightly longer whips).

One change to be noticed with respect to version 1.0 used in [PBCS2] is the higher saliences given to Quasi-Loops and Colour rules. The reason is, these rules are very general and very easy to use. They have been raised to just before the whips[1]. As a result, the resolution paths may be different from those given in [PBCS2].

Other changes are the introduction of isolated-HV-chains and extended Quasi-Loops (see section 9.5) and of pre-computed backgrounds for square grids (making initialisation times much shorter).

9.1 The configuration file

After the generic settings that are the same in all the CSP-Rules applications, you can choose application-specific ones:

H, V, I, P and B singles are elementary rules for Singles applied to the H, V, I, P and B CSP-Variables. Contrary to "normal" Single rules, these are not printed by default, but they can be activated individually. If you set them to TRUE here, long sequences of them will appear:

```
;;; As H/V-singles, I-singles, P-singles and B-singles are trivial rules that
;;; appear very often, their output is not printed by default.
;;; Printing can be enabled here:
 (bind ?*print-HV-single* TRUE)
 (bind ?*print-I-single* TRUE)
 (bind ?*print-P-single* TRUE)
 (bind ?*print-B-single* TRUE)
```

You can also decide how the final output will appear:

```
;;; By default, the final output is not printed in any form.
;;; But you can independently choose to print it in two forms: HV borders and in/out
cells.
;;; (However, IO will be effectively printed only if rules for Colours are activated.)
 (bind ?*print-IO-solution* TRUE)
 (bind ?*print-HV-solution* TRUE)
```

Global variable ?*print-IO-solution* controls whether the solution will be printed in the form of inner and outer cells (if ?*Colours* has been set to TRUE), as in the following, where an "x" stands for an inner cell and an "o" for an outer cell.

```
oxoxo
xxxxx
xxoox
xxxoo
xoxxo
```

Global variable ?*print-HV-solution* controls whether the solution will be printed in the (standard) form of borders around the cells, as in the following standard form:

```
.   .___.   .___.   .
    | 3 |   | 3 | 2
.___.   .___.   .___.
| 2   0           2 |
.   .   .___.___.   .
|       | 3     |   |
.   .   .___.   .___.
| 1         | 2   1
.   .___.   .___.   .
| 3 | 3 |     3 |
.___.   .___.___.   .
```

When a complete solution is not found, the partial solution can be printed (default option) or not. Before printing, it requires some filling of the yet undecided cells.

```
;;; Global variable ?*Final-fill* is used to add dummy values in the undecided cells
;;; when it hasn't found a solution, so as to be able to print the final state
;;; It is TRUE by default, but it can be changed here:
; (bind ?*Final-fill* FALSE)
```

The next choices are about application-specific rules. These rules are separated into four groups: quasi-loops and extended-loops, colours, rules that are equivalent to series of Singles and whips[1], and rules that are not (which is in which group is made clear in [PBCS2]). The four groups are disabled by default in CSP-Rules and they can be separately activated; I did not consider useful to add a possibility for individually activating each rule.

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
 ;;; My standard config and its usual variants
 ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;; Without Loops, most puzzles cannot be solved;
;;; they are therefore always activated in my usual configuration.
;;; By default, ?*loops-max-length* is set to 300;
;;; it may have to be increased for very large puzzles (size > 15); change it here:
; (bind ?*loops-max-length* 300)
 (bind ?*Loops* TRUE)
;;; extended-Loops are useful in some circumstances
;;; their max-length is the same as Loops.
; (bind ?*xtd-Loops* TRUE)

;;; Propagation of colours (or in/out positions) is also a very general and useful
method.
 (bind ?*Colours* TRUE)
```

```
;;; By default, all the application-specific rules are FALSE
;;; They are enabled in  my standard configuration:
 (bind ?*W1-equiv-patterns* TRUE)
 (bind ?*non-W1-equiv-patterns* TRUE)

;;; generic:
 (bind ?*Bivalue-Chains* TRUE)
 (bind ?*bivalue-chains-max-length* 5)
 (bind ?*Whips* TRUE)
 (bind ?*whips-max-length* 5)

;;; Some additional rules I use frequently:
; (bind ?*z-Chains* TRUE)
; (bind ?*z-chains-max-length* 5)
; (bind ?*t-Whips* TRUE)
; (bind ?*t-whips-max-length* 5)

;;; Some rules I use occasionally:
; (bind ?*G2-Whips* = TRUE)
; (bind ?*g2whips-max-length* 5)

;;; Some rules I almost never use:
; (bind ?*Braids* TRUE)
; (bind ?*braids-max-length* 5)
```

9.2 The user functions

Slitherlink has only two user functions, "solve" and "solve-tatham", plus an auxiliary function for changing formats.

Example of using "solve" for a 10×10 puzzle:

```
(solve 10 10
2 . . 3 . 3 2 . . .
. 3 . 3 . . . 3 2 2
. . . 2 2 . 1 2 2 .
. 1 . 2 . . . . . 3
. . 2 2 . . . 2 . .
3 3 . . 3 1 3 . 2 .
2 . . . . . . . . 3
. 3 1 . . 2 . . 1 .
2 1 2 2 1 1 2 . . 3
. . . 1 . . . . 1 .
)
```

The syntax for "***solve***" is:

```
(solve ?nb-rows ?nb-columns $?givens)
```

where ?nb-rows (respectively ?nb-columns) is the number of rows (resp. columns) and $?givens is the sequence of givens in the cells (as usual, a dot denotes no given). $?nb-givens must have exactly nb-rows × nb-columns symbols. As usual, the givens may (but don't have to) be arranged in an easy to read format.

"***solve-tatham***" is a syntactic variant of "solve", based on the tatham format. The syntax is:

```
(solve-tatham ?nb-rows ?nb-columns ?tatham-str)
```

where ?nb-rows (respectively ?nb-columns) are as before and ?tatham-str is the string representation of the puzzle in the Tatham format for Slitherlink. For the above puzzle, it would be

```
2b3a32d3a3c322c22a122b1a2e3b22c2b33b313a2a2h3a31b2b1a2122112b3c1d1a
```

In the Tatham format, each letter stands for the corresponding number of dots. Take the Tatham string, replace each letter by the right number of dots, insert a space between any two symbols (re-format it to make it look nicer) and you get the sequence used in the "solve" function. We have that:

```
(solve-tatham 10 10
  "2b3a32d3a3c322c22a122b1a2e3b22c2b33b313a2a2h3a31b2b1a2122112b3c1d1a ")
```

is equivalent to:

```
(solve 10 10
   (tatham-to-csp-rules-list
      "2b3a32d3a3c322c22a122b1a2e3b22c2b33b313a2a2h3a31b2b1a2122112b3c1d1a"))
```

If you want to use the Tatham generator of Slitherlink puzzles, you should be aware that most of its "recursive" puzzles will require the use of T&E or DFS. The most entertaining Slitherlink puzzles I've found on the web are at https://www.puzzle-loop.com; but you have to copy them manually.

9.3 A lot of application-specific rules

Of all the applications of CSP-Rules, SlitherRules is the one for which I've coded the largest number of application-specific rules. Part of this work was somehow frustrating, as most of these rules can be replaced by whips[1] and they did not increase the resolution power of SlitherRules. But it made the output look much nicer, as it involved rules familiar to the players and it shortened the resolution paths by directly asserting several H or V lines. It also allowed to single out a few rules that did add some resolution power.

Slitherlink illustrates on a much larger scale than Futoshiki the idea of a *macro-rule*, i.e. a resolution rule that can be reduced in some resolution theory T to a sequence of rules in T. Although macro-rules don't extend the resolution power of T, they can lead to shorter (and therefore more readable) resolution paths.

With this large number of application-specific rules, there appeared a strong need to test them individually. Slitherlink has therefore a "TEST" directory that reproduces the structure of the "SPECIFIC" directory: for each file for an application-specific rule, there is a file for testing it, with the same name. Load only the W1-equiv and non-W1-equiv rules (no Loops, Colours or Whips), set Final-Fill and Print-HV-solution to TRUE, choose the rule you want to test, go to the corresponding file in “TEST”, launch the examples there and check the result.

9.4 An experience in assisted theorem proving

I’ve shown in [PBCS2] that most of the classical resolution rules specific to Slitherlink can be considered as macro-rules in W_1 (or in W_k, for some k>1, in a few cases), i.e. they can be reduced to sequences of Singles and whips[1] (or longer whips) – highlighting as a result those that are irreducibly Slitherlink-specific. For this purpose, I’ve used SlitherRules as an assistant theorem prover. This version of CSP-Rules allows you to easily reproduce all these proofs: activate only whips (and none of the Loops, Colours, W1-Equiv or Non-W1-Equiv patterns), open the file in “SlitheRules-V2.1/SPECIFIC/TESTS” corresponding to the rule you want to prove and paste into CSP-Rules one of the “(solve ….)” commands present in this file. You should obtain all the eliminations/assertions of H and V variables done by the rule (plus some for the P and B CSP-Variables).

The general idea of these assisted proofs was to create a pattern representing the situation one wants to deal with, on a sufficiently large grid to ensure that no unwanted corner or border condition creeps in, and then run SlitherRules on it. The result is a resolution path, each step of which is a short whip, with precise rows and columns, but this can easily be turned manually into a general proof: change the values of rows and columns in the given pattern into variable names, and then change accordingly all the row and columns in the resolution path into variables relative to the preceding ones. For instance, if the original pattern involves row 5 and column 6 and a row 6 and column 4 appear somewhere in the resolution path, first change “row 5” into variable ?row, “column 6” into variable ?col and then change “row 6” into (+ ?row 1) and column 4 into (- ?col 2). This is exactly what I’ve done for generating the proofs that appear in [PBCS2] (where I’ve also pruned the resolution path of its unnecessary steps). The final step involves checking that the tentative proof thus obtained is valid.

As the essential parts of these proofs have been published in [PBCS2], I leave it to the reader as an easy exercise to reproduce them for himself.

9.5 Isolated-HV-chains and Extended-Loops

In section 17.10.2 of [PBCS2], I mentioned the possibility of extending the definition of Quasi-Loops. I now have added this extension to SlitherRules-V2.1. Here are the relevant definitions:

- two undecided H or V lines with a common Point are said to have an *isolated junction* (at this Point) if the other two lines from this Point are decided and FALSE; as a result, the two given lines can only be TRUE or FALSE at the same time; they behave as a single line.

- a sequence of n H/V lines is called an *isolated-HV-chain[n]* if all these H/V lines are undecided and any two contiguous H/V lines in the sequence have an isolated junction; as an obvious result, all the lines of this isolated-HV-chain[n] can only be TRUE or FALSE at the same time;
- an *Extended-Loop[n]* is defined as a Quasi-Loop[p] plus an isolated-HV-chain[q], such that p + q = n and the two chains meet at their two ends, making a closed loop; as a result, depending on the total number of lines touching the given cells, the whole isolated-HV-chain (i.e. all of its HV-lines) can be declared TRUE or FALSE; this is the Extended-Loop resolution rule.

Let's take the following example (puzzle I.4 from Mebane – see the "Examples" folder for details of the resolution paths):

```
. 3 . . 2 . . 2 . .
3 0 . 2 0 2 . 1 2 .
. 3 . . 3 . . 0 . .
. . . . . . . . . .
. . 3 . . . 1 3 0 .
. 3 1 0 . . . 2 . .
. . . . . . . . . .
. . 1 . . 0 . . 2 .
. 3 3 . 1 2 2 . 0 2
. . 1 . . 2 . . 3 .
```

If we apply all the rules in SlitherRules, except the Extended-Loops, we reach the following partial solution:

```
.   .———.   .   .———.———.———.———.———.———.
    | 3 |       | 2           2         |
.———.   .———.———.   .   .———.———.———.   .
| 3   0       2   0   2 |     1   2 |   |
.———.   .———.———.   .———.   .   .   .   .
    | 3 |       | 3 |         0     |   |
.   .———.   .   .———.   .   .   .———.   .
                                    |   |
.———.———.———.   .   .   .   .———.   .   .
|         3 |             1 | 3   0     |
.   .———.———.   .   .   .   .———.   .....
|   | 3   1   0               2 |   :   :
.   .———.   .   .   .   .   .   .———.....
|       |                           :   :
.................   .   .   .   .———.....
:   :     1     :     0         | 2     |
.................   .   .———.———.   .   .
|   | 3 | 3 |     1   2 | 2       0   2 |
.................———.———.   .———.   .———.
:   :     1     :     2     |   | 3 |
.................———.———.———.   .———.   .
```

You can see three (and only three) isolated-HV-chains (you could argue there are indeed six chains, as they can be reversed):

- isolated-HV-chains [2]: Hr6c10-Vr6c10
- isolated-HV-chains [2]: Hr11c2-Hr11c3
- isolated-HV-chains [4]: Hr8c3-Hr8c4-Vr8c5-Hr9c4.

If we activate xtd-Loops, the part of the resolution rule up to the previous partial solution is unchanged, but then an Extended-Loop appears and it's enough to allow a full solution using a few more standard Loops (see the "Examples/ Slitherlink/Mebane" folder for details). The partial-loop part has length 42 and the isolated-HV-chain part has length 2 (hence the total length, 44):

```
XTD-LOOP[44]:      Hr7c9-Vr6c9-Hr6c8-Vr5c8-Hr5c8-Vr4c9-Hr4c9-Vr3c10-Vr2c10-Hr2c9-Hr2c8-
Hr2c7-Vr2c7-Hr3c6-Vr3c6-Hr4c5-Vr3c5-Hr3c4-Hr3c3-Vr3c3-Hr4c2-Vr3c2-Hr3c1-Vr2c1-Hr2c1-
Vr1c2-Hr1c2-Vr1c3-Hr2c3-Hr2c4-Vr1c5-Hr1c5-Hr1c6-Hr1c7-Hr1c8-Hr1c9-Hr1c10-Vr1c11-
Vr2c11-Vr3c11-Vr4c11-Vr5c11- ==> Hr6c10-Vr6c10 = 0
```

Notice the convention for writing the Extended-Loop: on the left is the partial-loop made of decided TRUE H/V-lines; on the right is the isolated-HV-chain made of undecided H/V-lines; they all become decided by the conclusion of the rule (i.e. they are all set to FALSE in the present case).

```
.   .———.   .   .———.———.———.———.———.———.
    | 3 |       | 2         2         |
.———.   .———.———.   .   .———.———.———.   .
| 3   0       2   0   2 |     1   2 |   |
.———.   .———.———.   .———.   .   .   .   .
    | 3 |       | 3 |         0     |   |
.   .———.   .   .———.   .   .   .———.   .
                                |       |
.———.———.———.   .   .   .   .———.   .   .
|         3 |             1 | 3   0     |
.   .———.———.   .   .   .   .———.   .   .
|   | 3   1   0               2 |       |
.   .———.   .   .   .   .   .   .———.   .
|       |                           |   |
.   .———.   .   .   .   .   .   .———.   .
|   |     1           0         | 2     |
.   .   .———.   .   .   .———.———.   .   .
|   | 3 | 3 |     1   2 | 2       0   2 |
.   .———.   .———.———.———.   .———.   .———.
|         1           2     |   | 3 |
.———.———.———.———.———.———.———.   .———.   .
```

10. HidatoRules (Numbrix and Hidato)

HidatoRules is the pattern-based solver of Hidato and Numbrix puzzles based on CSP-Rules. Both games consist of finding a path, using only adjacent white cells for each elementary step. The only difference between Numbrix and Hidato lies in the meaning of “adjacent”: in Numbrix two cells are adjacent if and only if they touch each other by one side in the same row or column; in Hidato , they may also touch each other diagonally, i.e. by a corner.

10.1 The configuration file

Numbrix® and Hidato® have Subsets, but their implementation is greedy for memory and I often use only Subsets[2].

They also have whips[1] and therefore g-labels, g-whips,… But these are not coded in this version of HidatoRules, for the same reason they are not coded in SlitherRules: both applications have a large branching factor and g-whips would soon lead to memory overflow. Even g2-whips are not safe from this problem; you can try them, if the puzzle has small size (see the last but one example in this chapter).

10.2 The user functions

HidatoRules has a single user function, “***solve***”. The syntax is:

```
(solve
   ?game ?model ?grid-size ?nb-white-cells $?data-sequence)
```

where:

- ?game is either Numbrix or Hidato
- ?model is either topological or geometric (see [PBCS] for the difference)
- ?grid-size is the number of rows (or columns)
- ?nb-white-cells is the number of white cells
- $?data-sequence is a sequence of ?grid-size × ?grid-size symbols; each symbol is either a dot or a “B” (for black cell) or a number between 1 and ?nb-white-cells (both included); no number can be repeated in the sequence. If there are B’s, their number must be equal to ?grid-size × ?grid-size - ?nb-white-cells.

Numbrix example, from the https://parade.com/member/marilynvossavant/ website:

```
(solve Numbrix topological 9 81
69 .  67 .  65 .  61 .  57
.  .  .  .  .  .  .  .  .
77 .  .  .  .  .  .  .  55
.  .  .  .  .  .  .  .  .
19 .  .  .  .  .  .  .  45
.  .  .  .  .  .  .  .  .
3  .  .  .  .  .  .  .  39
.  .  .  .  .  .  .  .  .
5  .  7  .  11 .  31 .  33
)
```

Hidato example, from the https://www.smithsonianmag.com/games/ website:

```
(solve Hidato topological 10 78
B  B  .  .  65 .  .  62 B  B
B  .  19 .  .  66 .  .  .  B
.  21 .  B  .  .  B  .  58 59
.  .  12 B  .  .  B  69 .  56
.  10 .  74 .  .  .  1  .  .
.  26 .  5  4  76 .  .  54 52
28 .  B  .  .  .  78 B  .  .
.  .  .  B  B  B  B  48 47 .
B  .  .  .  .  39 .  43 .  B
B  B  33 35 37 40 42 .  B  B
)
```

Even small puzzles can be very hard. Example III.4 from the https://mellowmelon.files.wordpress.com/2012/05/pack03hidato_v3.pdf website (a puzzle in W8 or g2W7):

Generally speaking, the geometric model is more powerful, but it may lead faster to memory overflow for hard puzzles (because it has more links and therefore a larger fan-out). As an example of the difference, the following (hard) puzzle by Evert can only be solved with the geometric model, in W14. It can also illustrate the difference in computation time when you add special case patterns (e.g. z-chains, t-whips…) to only whips.

```
(solve Hidato geometric 9 81
.  .  .  .  77 .  28 .  .
.  13 .  .  .  .  .  21 .
.  .  .  15 16 .  31 .  .
.  6  .  .  73 .  19 .  .
.  58 .  .  .  .  36 .  33
60 .  62 .  71 .  .  .  .
.  .  .  .  64 .  .  .  43
.  54 .  .  .  68 .  .  .
.  .  .  .  .  .  .  .  .
)
```

11. MapRules

MapRules is the pattern-based solver of map colouring problems based on CSP-Rules. It has neither Subsets nor g-labels, which drastically restricts the possibilities for complex patterns. As the least developed application of CSP-Rules, it is intended only to provide one more illustration of the generic rules. A full solver of colouring problems should include pattern representation of graph concepts similar to those necessary for proving the four colour theorem.

11.1 The configuration file

There isn't much to say about the MapRules configuration file. My standard config is as follows.

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;; my standard config
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

 (bind ?*Bivalue-Chains* TRUE)
 (bind ?*Whips* TRUE)
; (bind ?*Braids* TRUE)

; (bind ?*bivalue-chains-max-length* 20)
; (bind ?*whips-max-length* 36)
; (bind ?*braids-max-length* 36)
```

11.2 The user functions

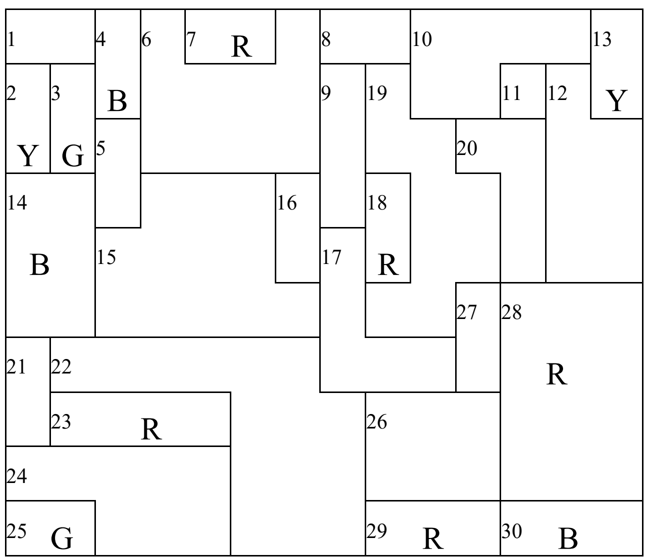
MapRules has a single user function: "***solve***" with syntax:

```
(solve ?nb-colours ?nb-countries ?givens $?list)
```

- ?nb-colours is the number of allowed colours;
- ?nb-countries is the number of countries to be coloured;
- ?givens is a string of given colours for some of the countries; its length must be equal to ?nb-countries; a country with no pre-defined colour appears as a dot;
- $?list is a sequence of adjacency data; it is composed of at most ?nb-count - 1 sub-sequences, separated by colons; each sub-sequence is a sequence of country numbers, meaning that the first country in the sub-sequence has a common border with each of the other countries in the same sub-sequence; if c1 < c2 and a

common border between c1 and c2 has already been declared in the sub-sequence starting with c1, it does not have to be declared again in the sub-sequence starting with c2 (which also implies that there may be no sequence starting with c2).

Example (the same example as in [PBCS], an "unreasonable" one from the https://www.chiark.greenend.org.uk/~sgtatham/puzzles/js/map.html website, (apparently the only place where this kind of colouring puzzle can be found); this example requires a whip[7]:




```
(solve 4 30
".YGB..R.....YB...R....R.G..RRB"
1 2 3 4 :
2 3 14 :
3 4 5 14 :
4 5 6 :
5 6 14 15 :
6 7 8 9 15 16 :
8 9 10 19 :
9 16 17 18 19 :
10 11 12 13 19 20 :
11 12 20 :
12 13 20 28 :
14 15 21 22 :
15 16 17 22 :
16 17 :
17 18 19 22 26 27 :
18 19 :
```

```
19 20 27 :
20 28 :
21 22 23 24 :
22 23 24 26 29 :
23 24 25 :
24 25 :
26 27 28 29 : 2
7 28 :
28 30 :
29 30
)
```

There are 30 countries (named 1, …30) and only 4 colours are allowed. The given colours appear in large capital letters in the Figure and they correspond to the sequence appearing as the third argument in the function call.

As for the rest of the arguments (considered as a single list by CLIPS), the first sub-sequence of adjacency data "1 2 3 4" means that country 1 has a common border with each of countries 2, 3 and 4 and none other. You can notice that there is no sub-sequence starting from country 7; the reason is, it has a common border with country 6 only and this has already been declared in the sequence for country 6. Readability aside, all this could be written in a single line (but be careful to keep the ":" separators, whichever way you write these data).

Colours in the data and the solution are displayed as B, Y, R, G, because these are the default names for the first four colours (the full list is B,Y, R, G, O, V, I, 8, 9, 10, …, if you need more than four colours.

I wanted to write a "solve-tatham" function for MapRules, especially as this is the only website I know of that produces those puzzles. But I've been unable to understand the tatham format and I've received no answer to an email I sent them. If anyone has information about this, I would be glad to add such a function, which would allow testing many more examples than those one can only copy manually.

12. Miscellanea

12.1 Examples

CSP-Rules includes a small set of examples, all of which appear in the “Examples” folder (as expected), which has one sub-folder for each application. The purpose is not to provide an extensive set of cases, but to give you a more concrete idea of the various puzzles and their resolution paths, with more cases than can be mentioned in book form. Nor is the purpose to replace the original websites; it is to direct you to them for more examples of the same kinds.

All these examples have been run with CSP-Rules V2.1 on a MacBookPro from mid-2012 with 16 GB of RAM, so that you can be sure the resolution times you will get on your machines are much smaller than those appearing at the end of the paths (unless your PC is still older). Don’t consider these runtimes as absolutes, because some puzzles were solved while I was running other processes and may therefore be higher than they could have been if they had run alone. Nevertheless, they provide useful approximative indications about the relative times you can expect for each puzzle.

Unless otherwise stated, all the examples have been run with the “standard” configuration of their application.

The “Examples” folder for each application includes several folders, with names corresponding to the origins of the puzzles. Thus, an “ATK” folder means “from the “atksolutions.com” website; a “Puzzle-loop” folder means “from the “puzzle-loop.com” website… Instead of a website, it can be the name of a puzzle creator, e.g. “Tarek”. It can also be both, e.g. “Tatham”, which means both the person Tatham and the website “tatham.com”. If you compare the puzzles from different sources, you will generally observe that they have different characteristics. For instance, the Slitherlink puzzles from puzzle-loop.com slither more than those on grids of same size from kakuro-online.com and they therefore make a more intensive use of the Quasi-loop rules.

See the “README” file of the “Examples” folder of each application for more specific details.

The resolution paths given in all the examples are the raw output from CSP-Rules, with all the printout options on. Of course, if such a path was to be published in a book, some manual cleaning would be necessary. I am thinking of Slitherlink in particular (but not only), where interminable sequence of whips[1] appear. Most of these could be considered as trivial and merely deleted, keeping only assertions of H, V and P values. Similarly, long sequences of colour rules could be replaced by a single line “colours => xxx in, yyy out, … Hij = 0, … Vkl =1…”

12.2 Reporting bugs

Any bug found in CSP-Rules should preferably be reported via GitHub.

12.3 Improving CSP-Rules

The present manual does not say anything about how to modify CSP-Rules. Until I have written further manuals, you can contact me via GitHub if you want to discuss some improvements, extensions or modifications.

One way of improving the speed of the generic part and of all the applications would be to extend CLIPS. CLIPS lacks complex data structures such as (sparse) binary matrices or low density graphs that would allow to have a more efficient implementation of binary links with a quick access to them.

See the “CONTRIBUTING” file on github.com for other suggestions.

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