

**Chapter 18**  
**Accelerated Test Models**

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19h 16min

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**Chapter 18**  
**Accelerated Test Models**  
**Objectives**

- Describe motivation and applications of accelerated reliability testing.
- Explain the connections between degradation, physical failure, and acceleration of reliability tests.
- Examine the basis for temperature and humidity acceleration.
- Examine the basis for voltage and pressure stress acceleration.
- Show how to compute time-acceleration factors.
- Review other accelerated test models and assumptions.

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**Accelerated Tests Increasingly Important**

**Today's** manufactures need to develop newer, higher technology products in record time while improving productivity, reliability, and quality.

Important issues:

- Rapid product development.
- Rapidly changing technologies.
- More complicated products with more components.
- Higher customer expectations for better reliability.

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**Need for Accelerated Tests**

Need timely information on high reliability products.

- Modern products designed to last for years or decades.
- Accelerated Tests (ATs) used for timely assessment of reliability of product components and materials.
- Tests at high levels of use rate, temperature, voltage, pressure, humidity, etc.
- Estimate life at **use conditions**.

**Note:** Estimation/prediction from ATs involves **extrapolation**.

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**Applications of Accelerated Tests**

Applications of Accelerated Tests include:

- Evaluation the effect of stress on life.
- Assessing component reliability.
- Demonstrating component reliability.
- Detecting failure modes.
- Comparing two or more competing products.
- Establishing safe warranty times.

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**Methods of Acceleration**

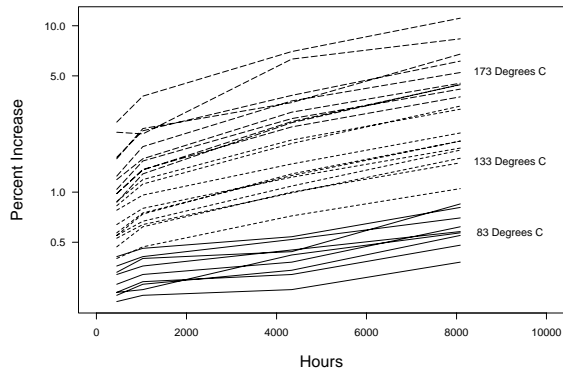
Three fundamentally different methods of accelerating a reliability test:

- Increase the use-rate of the product (e.g., test a toaster 200 times/day). Higher use rate reduces test time.
- Use elevated temperature or humidity to increase rate of failure-causing chemical/physical process.
- Increase stress (e.g., voltage or pressure) to make degrading units fail more quickly.

Use a **physical/chemical** (preferable) or **empirical** model relating degradation or lifetime to **use** conditions.

18 - 6

### Change in Resistance Over Time of Carbon-Film Resistors (Shiomi and Yanagisawa 1979)



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### Accelerated Degradation Tests (ADTs)

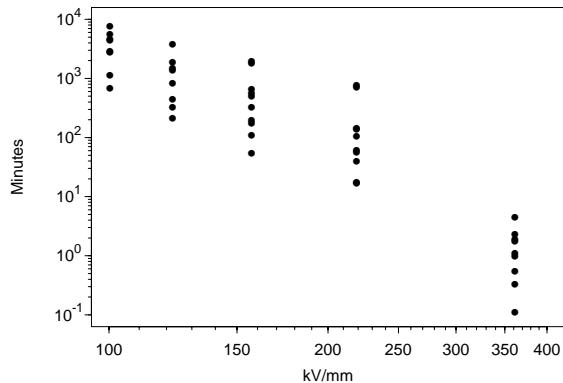
**Response:** Amount of degradation at points in time.

**Model components:**

- Model for degradation over time.
- A definition of **failure** as a function of degradation variable.
- Relationship(s) between degradation model parameters (e.g., chemical process reaction rates) and acceleration variables (e.g., temperature or humidity).

18 - 8

### Breakdown Times in Minutes of a Mylar-Polyurethane Insulating Structure (from Kalkanis and Rosso 1989)



18 - 9

### Accelerated Life Tests (ALTs)

**Response:**

- Failure time (or interval) for units that fail.
- Censoring time for units that do not fail.

**Model Components:**

- Constant-stress time-to-failure distribution.
- Relationship(s) between one (or more) of the constant-stress model parameters and the accelerating variables.

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### Use-Rate Acceleration

**Basic idea:** Increase use-rate to accelerate failure-causing wear or degradation.

**Examples:**

- Running automobile engines or appliances continuously.
- Rapid cycling of relays and switches.
- Cycles to failure in fatigue testing.

**Simple assumption:** Useful if life adequately modeled by cycles of operation. Reasonable if cycling simulates actual use and if test units return to steady state after each cycle.

**More complicated situations:** Wear rate or degradation rate depends on cycling frequency or product deteriorates in stand-by as well as during actual use.

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### Elevated Temperature Acceleration of Chemical Reaction Rates

- The **Arrhenius** model **Reaction Rate**,  $\mathcal{R}(\text{temp})$ , is

$$\mathcal{R}(\text{temp}) = \gamma_0 \exp\left(\frac{-E_a}{k_B(\text{temp}^\circ\text{C} + 273.15)}\right) = \gamma_0 \exp\left(\frac{-E_a \times 11605}{\text{temp K}}\right)$$

where  $\text{temp K} = \text{temp}^\circ\text{C} + 273.15$  is temperature in degrees Kelvin and  $k_B = 1/11605$  is Boltzmann's constant in units of electron volts per K. The reaction activation energy,  $E_a$ , and  $\gamma_0$  are characteristics of the product or material being tested.

- The reaction rate **Acceleration Factor** is

$$\begin{aligned} \mathcal{AF}(\text{temp}, \text{temp}_U, E_a) &= \frac{\mathcal{R}(\text{temp})}{\mathcal{R}(\text{temp}_U)} \\ &= \exp\left[E_a \left(\frac{11605}{\text{temp}_U \text{ K}} - \frac{11605}{\text{temp K}}\right)\right] \end{aligned}$$

- When  $\text{temp} > \text{temp}_U$ ,  $\mathcal{AF}(\text{temp}, \text{temp}_U, E_a) > 1$ .

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### Acceleration Factors for the SAFT Arrhenius Model

- Table 18.2 gives the **Temperature Differential Factors (TDF)**

$$\text{TDF} = \left( \frac{11605}{\text{temp}_{\text{Low}} \text{ K}} - \frac{11605}{\text{temp}_{\text{High}} \text{ K}} \right).$$

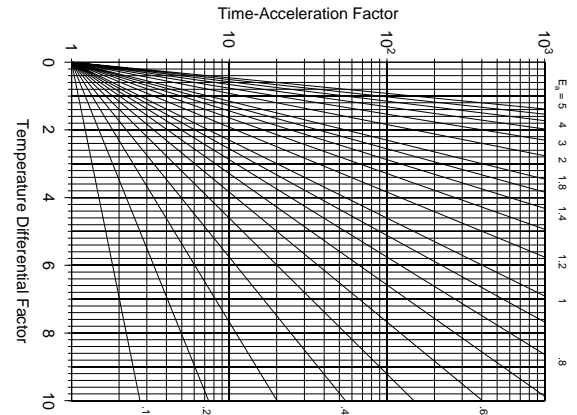
- Figure 18.3 gives

$$\mathcal{AF}(\text{temp}_{\text{High}}, \text{temp}_{\text{Low}}, E_a) = \exp(E_a \times \text{TDF})$$

- We use  $\mathcal{AF}(\text{temp}) = \mathcal{AF}(\text{temp}, \text{temp}_U, E_a)$  when  $\text{temp}_U$  and  $E_a$  are understood to be, respectively, product use temperature and reaction-specific activation energy.

18 - 13

**Time-Acceleration Factor as a Function of Temperature Differential Factor (Figure 18.3)**



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### Nonlinear Degradation Reaction-Rate Acceleration

- Consider the simple chemical degradation path model

$$\mathcal{D}(t; \text{temp}) = \mathcal{D}_{\infty} \times \{1 - \exp[-\mathcal{R}_U \times \mathcal{AF}(\text{temp}) \times t]\}$$

where  $\mathcal{R}_U$  is the rate reaction at use temperature ( $\text{temp}_U$ ) and for  $\text{temp} > \text{temp}_U$ ,  $\mathcal{AF}(\text{temp}) > 1$ .

- For  $\mathcal{D}_{\infty} > 0$ , failure occurs when  $\mathcal{D}(T; \text{temp}) > \mathcal{D}_f$ . Equating  $\mathcal{D}(T; \text{temp})$  to  $\mathcal{D}_f$  and solving for failure time,  $T(\text{temp})$ , gives

$$T(\text{temp}) = \frac{T(\text{temp}_U)}{\mathcal{AF}(\text{temp})} = \frac{\left[ -\frac{1}{\mathcal{R}_U} \log \left( 1 - \frac{\mathcal{D}_f}{\mathcal{D}_{\infty}} \right) \right]}{\mathcal{AF}(\text{temp})}$$

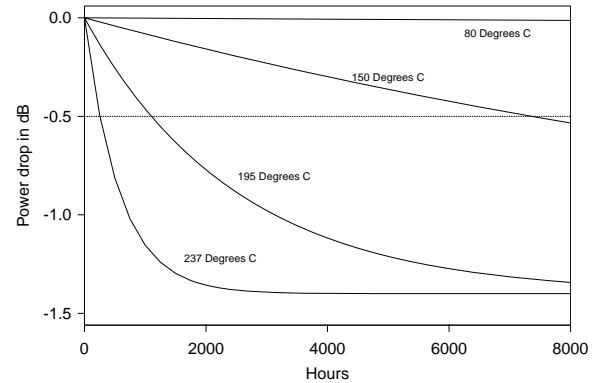
where  $T(\text{temp}_U)$  is failure time at use conditions.

- This is an SAFT model.

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### SAFT Model from Nonlinear Degradation Paths

$$\mathcal{D}(t; \text{temp}) = \mathcal{D}_{\infty} \times \{1 - \exp[-\mathcal{R}_U \times \mathcal{AF}(\text{temp}) \times t]\}$$



18 - 16

### The Arrhenius-Lognormal Regression Model

The Arrhenius-lognormal regression model is

$$\Pr[T \leq t; \text{temp}] = \Phi_{\text{nor}} \left[ \frac{\log(t) - \mu}{\sigma} \right]$$

where

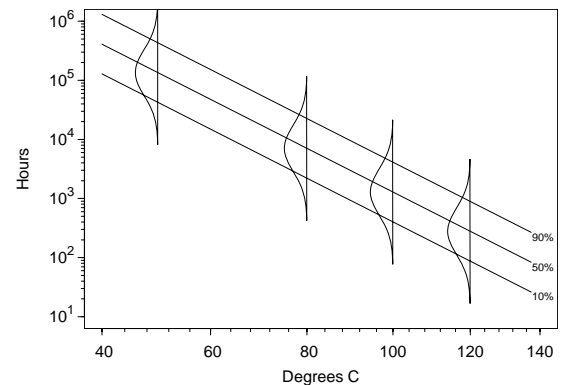
- $\mu = \beta_0 + \beta_1 x$ ,
- $x = 11605/(\text{temp K}) = 11605/(\text{temp } ^\circ\text{C} + 273.15)$
- and  $\beta_1 = E_a$  is the effective activation energy in electron volts (eV).
- $\sigma$  is constant
- This implies that

$$t_p(\text{temp}_U) = t_p(\text{temp}) \times \mathcal{AF}(\text{temp})$$

18 - 17

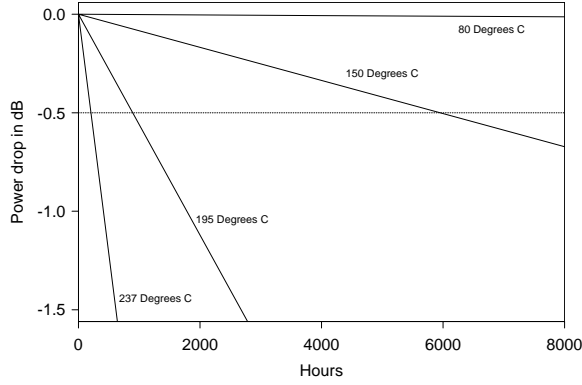
### Example Arrhenius-Lognormal Life Model

$$\log[t_p(\text{temp})] = \beta_0 + \beta_1 x + \Phi_{\text{nor}}^{-1}(p)\sigma$$



18 - 18

### SAFT Model from Linear Degradation Paths



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### Linear Degradation Reaction-Rate Acceleration

If  $\mathcal{R}_U \times \mathcal{AF}(\text{temp}) \times t$  is small so that  $\mathcal{D}(t)$  is small relative to  $\mathcal{D}_\infty$ , then

$$\begin{aligned}\mathcal{D}(t; \text{temp}) &= \mathcal{D}_\infty \times \{1 - \exp[-\mathcal{R}_U \times \mathcal{AF}(\text{temp}) \times t]\} \\ &\approx \mathcal{D}_\infty \times \mathcal{R}_U \times \mathcal{AF}(\text{temp}) \times t = \mathcal{R}_U^+ \times \mathcal{AF}(\text{temp}) \times t\end{aligned}$$

is approximately linear in  $t$ .

- Also some degradation processes are linear in time:

$$\mathcal{D}(t; \text{temp}) = \mathcal{R}_U \times \mathcal{AF}(\text{temp}) \times t.$$

- Failure occurs when  $\mathcal{D}(T; \text{temp}) > \mathcal{D}_f$ . Equating  $\mathcal{D}(T; \text{temp})$  to  $\mathcal{D}_f$  and solving for failure time,  $T(\text{temp})$ ,

$$T(\text{temp}) = \frac{T(\text{temp}_U)}{\mathcal{AF}(\text{temp})}$$

where  $T(\text{temp}_U) = \mathcal{D}_f / \mathcal{R}_U$  is failure time at use conditions.

- This is an SAFT model and, for example,  $T(\text{temp}_U) \sim \text{WEIB}(\mu, \sigma)$  implies  $T(\text{temp}) \sim \text{WEIB}[\mu - \log(\mathcal{AF}(\text{temp})), \sigma]$ .

18 - 20

### Non-SAFT Degradation Reaction-Rate Acceleration

Consider the more complicated chemical degradation path

$$\begin{aligned}\mathcal{D}(t; \text{temp}) &= \mathcal{D}_{1\infty} \times \{1 - \exp[-\mathcal{R}_{1U} \times \mathcal{AF}_1(\text{temp}) \times t]\} \\ &\quad + \mathcal{D}_{2\infty} \times \{1 - \exp[-\mathcal{R}_{2U} \times \mathcal{AF}_2(\text{temp}) \times t]\}\end{aligned}$$

$\mathcal{R}_{1U}, \mathcal{R}_{2U}$  are the rates of the reactions contributing to failure.

This is **not** an SAFT model. Temperature affects the two degradation processes differently, inducing a nonlinearity into the acceleration function relating times at two different temperatures.

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### Voltage Acceleration and Voltage Stress Inverse Power Relationship

- Depending on the failure mode, voltage can be raised to:

- Increase the strength of electric fields. This can accelerate some failure-causing reactions.
- Increase the stress level (e.g., volt = voltage stress relative to declining **voltage strength**).

- An **empirical** model for life at volt relative to use conditions  $\text{volt}_U$  is

$$T(\text{volt}) = \frac{T(\text{volt}_U)}{\mathcal{AF}(\text{volt})} = \left(\frac{\text{volt}}{\text{volt}_U}\right)^{\beta_1} T(\text{volt}_U)$$

where  $\mathcal{AF}(\text{volt}) = \mathcal{AF}(\text{volt}, \text{volt}_U, \beta_1)$ ,

$$\mathcal{AF}(\text{volt}) = \mathcal{AF}(\text{volt}, \text{volt}_U, \beta_1) = \frac{T(\text{volt}_U)}{T(\text{volt})} = \left(\frac{\text{volt}}{\text{volt}_U}\right)^{-\beta_1}$$

and  $\beta_1$  is a material characteristic.  $T(\text{volt}), T(\text{volt}_U)$  are the failure times at increased voltage and use conditions.

18 - 22

### Inverse Power Relationship-Weibull Model

The inverse power relationship-Weibull model is

$$\Pr[T \leq t; \text{volt}] = \Phi_{\text{sev}} \left[ \frac{\log(t) - \mu}{\sigma} \right]$$

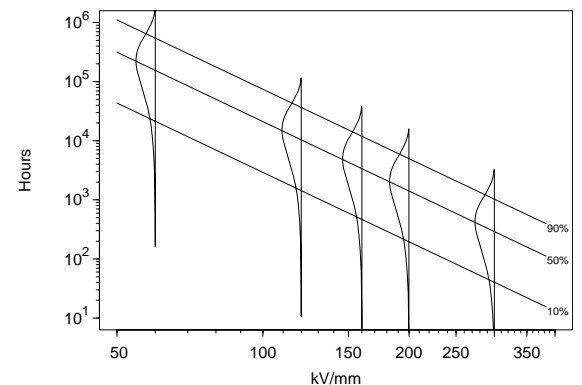
where

- $\mu = \beta_0 + \beta_1 x$ , and
- $x = \log(\text{volt})$ , where  $\text{volt}$  = voltage stress.
- $\sigma$  assumed to be constant.

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### Example Weibull Inverse Power Relationship Between Life and Voltage Stress

$$\log[t_p(\text{volt})] = \beta_0 + \beta_1 x + \Phi_{\text{sev}}^{-1}(p)\sigma$$



18 - 24

### Other Commonly Used Life-Stress Relationships

Other commonly used SAFT models have the simple form:

$$T(x) = T(x_U)/\mathcal{AF}(x)$$

where  $\mathcal{AF}(x) = \mathcal{AF}(x, x_U, \beta_1) = \exp[\beta_1(x - x_U)]$ .  $\beta_1$  is a material characteristic. Examples include:

- Cycling rate:  $x = \log(\text{frequency})$ .
- Current density:  $x = \log(\text{current})$ .
- Size:  $x = \log(\text{thickness})$ .
- Humidity 1:  $x = \log(\text{RH})$ ,  $\text{RH} > 0$ .
- Humidity 2:  $x = \log[\text{RH}/(100 - \text{RH})]$ ,  $0 < \text{RH} < 100$ .
- Logit:  $x = \log[\text{RH}/(1 - \text{RH})]$ ,  $0 < \text{RH} < 1$ .

Some of these models are empirical. For a location-scale time-to-failure distribution  $\mu = \beta_0 + \beta_1 x$ .

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### Eyring Temperature Relationship

- Arrhenius relationship obtained from **empirical** observation.
- Eyring developed **physical theory** describing the effect that temperature has on a reaction rate:

$$\mathcal{R}(\text{temp}) = \gamma_0 \times A(\text{temp}) \times \exp\left(\frac{-E_a}{k_B \times \text{temp K}}\right)$$

- $A(\text{temp})$  is a function of temperature depending on the specifics of the reaction dynamics;  $\gamma_0$  and  $E_a$  are constants.
- Applications in the literature have used  $A(\text{temp}) = (\text{temp K})^m$  with a fixed value of  $m$  ranging between  $m = 0$  (Boccaletti et al. 1989),  $m = .5$  (Klinger 1991a), to  $m = 1$  (Nelson 1990a and Mann Schafer and Singpurwalla 1974). Difficult to identify  $m$  from limited data.
- Eyring showed how to include other accelerating variables.

18 - 26

### The Eyring Regression Model (e.g., for Weibull or Lognormal Distributions)

The Eyring temperature-acceleration regression model is

$$\Pr(T \leq t; \text{temp}) = \Phi\left[\frac{\log(t) - \mu}{\sigma}\right]$$

where

- $\mu = -m \log(\text{temp } ^\circ\text{C} + 273.15) + \beta_0 + \beta_1 x$ .
- $x = 11605/(\text{temp } ^\circ\text{C} + 273.15)$ .
- $\beta_1 = E_a$  is the activation energy.
- $m$  is usually given;  $\sigma$  is constant, but usually unknown.
- With  $m > 0$ , Arrhenius provides a useful first order approximation to the Eyring model, with conservative extrapolation to lower temperatures.

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### Humidity Acceleration Models

- Useful for accelerating failure mechanisms involving **corrosion** and certain other kinds of **chemical degradation**.
- Often used in conjunction with elevated temperature.
- Most humidity models have been developed empirically.
- Empirical and limited theoretical results for corrosion on thin films (Gillen and Mead 1980, Peck 1986, and Klinger 1991b) suggest the use of RH instead of  $P_v$  (vapor pressure) as the independent (or experimental) variable in humidity relationships when temperature is also to be varied.
- RH is the preferred variable because the change in life, as a function of RH, does not depend on temperature. That is,

$$\frac{\partial^2 \text{Life}}{\partial \text{RH} \partial \text{temp}} = 0$$

or no **statistical interaction**.

18 - 28

### Humidity Regression Relationships

- Consider the Weibull/lognormal lifetime regression model

$$\Pr(T \leq t; \text{humidity}) = \Phi_{\text{sev}}\left[\frac{\log(t) - \mu}{\sigma}\right]$$

where  $\mu = \beta_0 + \beta_1 x_1$  and  $\sigma$  is constant. Letting  $0 < \text{RH} < 1$  denote **relative humidity**, possible humidity relationships are:

- ▶  $x_1 = \text{RH}$  [Intel, empirical].
- ▶  $x_1 = \log(\text{RH})$  [Peck, empirical].
- ▶  $x_1 = \log[\text{RH}/(1 - \text{RH})]$  [Klinger, corrosion on thin films].
- For temperature and humidity acceleration, possible relationships include

$$\begin{aligned} \mu &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 \\ \text{or } \mu &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 \\ \text{or } \mu &= \beta_0 + \beta_2 x_2 + \beta_3 x_1 x_2 \end{aligned}$$

where  $x_2 = 11605/(\text{temp } ^\circ\text{C} + 273.15)$ .

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### Temperature/Humidity Acceleration Factors with RH versus temp K No Interaction

- Peck's relationship

$$\begin{aligned} \mathcal{AF}(\text{temp}, \text{RH}) &= \frac{\mathcal{R}(\text{temp}, \text{RH})}{\mathcal{R}(\text{temp}_U, \text{RH}_U)} \\ &= \left(\frac{\text{RH}_U}{\text{RH}}\right)^{\beta_1} \exp\left[E_a \left(\frac{11605}{\text{temp}_U \text{ K}} - \frac{11605}{\text{temp K}}\right)\right]. \end{aligned}$$

- Klinger's relationship

$$\begin{aligned} \mathcal{AF}(\text{temp}, \text{RH}) &= \frac{\mathcal{R}(\text{temp}, \text{RH})}{\mathcal{R}(\text{temp}_U, \text{RH}_U)} \\ &= \left[\left(\frac{\text{RH}_U}{1 - \text{RH}_U}\right) \left(\frac{1 - \text{RH}}{\text{RH}}\right)\right]^{\beta_1} \times \\ &\quad \exp\left[E_a \left(\frac{11605}{\text{temp}_U \text{ K}} - \frac{11605}{\text{temp K}}\right)\right]. \end{aligned}$$

18 - 30

<div data-bbox="331 69 518 94" data-label="Section-Header"> <h3>Thermal Cycling</h3> </div> <div data-bbox="103 144 737 588" data-label="List-Group"> <ul style="list-style-type: none"> <li>• Fatigue is an important failure mechanism for many products and materials.</li> <li>• Mechanical expansion and contraction from thermal cycling can lead to fatigue cracking and failure.</li> <li>• Applications include: <ul style="list-style-type: none"> <li>▶ Power-on/power-off cycling of electronic equipment and effect on component encapsulment and solder joints.</li> <li>▶ Take-off power-thrust in jet engines and its effect on crack initiation and growth in fan disks.</li> <li>▶ Power-up/power-down of nuclear power plants and effect on the growth of cracks in heat generator tubes.</li> <li>▶ Thermal inkjet printhead delamination could be caused by temperature cycling during normal use.</li> </ul> </li> </ul> </div> <div data-bbox="683 619 730 636" data-label="Page-Footer"> <p>18 - 31</p> </div>	<div data-bbox="1078 48 1391 71" data-label="Section-Header"> <h3>Coffin-Manson Relationship</h3> </div> <div data-bbox="915 117 1549 168" data-label="List-Group"> <ul style="list-style-type: none"> <li>• The <b>Coffin-Manson</b> relationship says that the <b>typical</b> number of cycles to failure is</li> </ul> </div> <div data-bbox="1162 178 1321 231" data-label="Equation-Block"> <math display="block">N = \frac{\delta}{(\Delta\text{temp})^{\beta_1}}</math> </div> <div data-bbox="937 237 1549 342" data-label="Text"> <p>where <math>\Delta\text{temp}</math> is the temperature range and <math>\delta</math> and <math>\beta_1</math> are properties of the material and test setup. This power-rule relationship explains the effect that temperature range has on thermal-fatigue life. For some metals, <math>\beta_1 \approx 2</math>.</p> </div> <div data-bbox="915 359 1549 464" data-label="List-Group"> <ul style="list-style-type: none"> <li>• Letting <math>T</math> be the random number of cycles to failure (e.g., <math>T = N \times \epsilon</math> where <math>\epsilon</math> is a random variable), the acceleration factor when <math>\Delta\text{temp}</math>, relative to the number of cycles when <math>\Delta\text{temp}_U</math>, is</li> </ul> </div> <div data-bbox="1026 478 1456 539" data-label="Equation-Block"> <math display="block">\mathcal{AF}(\Delta\text{temp}) = \frac{T(\Delta\text{temp}_U)}{T(\Delta\text{temp})} = \left( \frac{\Delta\text{temp}}{\Delta\text{temp}_U} \right)^{\beta_1}</math> </div> <div data-bbox="915 560 1549 611" data-label="List-Group"> <ul style="list-style-type: none"> <li>• There may be a <math>\Delta\text{temp}</math> threshold below which little or no fatigue damage is done during thermal cycling.</li> </ul> </div> <div data-bbox="1497 619 1542 636" data-label="Page-Footer"> <p>18 - 32</p> </div>
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<div data-bbox="196 766 651 789" data-label="Section-Header"> <h3>Generalized Coffin-Manson Relationship</h3> </div> <div data-bbox="103 840 737 1092" data-label="List-Group"> <ul style="list-style-type: none"> <li>• Empirical evidence has shown that effect of temperature cycling can depend importantly on <math>\text{temp}_{\text{max K}}</math>, the maximum temperature in the cycling (e.g., if <math>\text{temp}_{\text{max K}}</math> is more than .2 or .3 times a metal's melting point).</li> <li>• The effect of temperature cycling can also depend on the cycling rate (e.g., due to heat buildup).</li> <li>• An <b>empirical</b> extension of the Coffin-Manson relationship is</li> </ul> </div> <div data-bbox="190 1104 670 1161" data-label="Equation-Block"> <math display="block">N = \frac{\delta}{(\Delta\text{temp})^{\beta_1}} \times \frac{1}{(\text{freq})^{\beta_2}} \times \exp\left(\frac{E_a \times 11605}{\text{temp}_{\text{max K}}}\right)</math> </div> <div data-bbox="125 1167 737 1220" data-label="Text"> <p>where <math>\text{freq}</math> is the cycling frequency, and <math>E_a</math> is an activation energy.</p> </div> <div data-bbox="103 1239 737 1291" data-label="List-Group"> <ul style="list-style-type: none"> <li>• Caution must be used when using such a model outside the range of available data and past experience.</li> </ul> </div> <div data-bbox="683 1318 730 1335" data-label="Page-Footer"> <p>18 - 33</p> </div>	<div data-bbox="1076 890 1395 913" data-label="Section-Header"> <h3>Other Topics in Chapter 18</h3> </div> <div data-bbox="915 966 1549 1165" data-label="List-Group"> <ul style="list-style-type: none"> <li>• Other accelerated degradation models and relationships to accelerated time models.</li> <li>• Discussion of stress-cycling models.</li> <li>• Other models for two or more experimental factors.</li> </ul> </div> <div data-bbox="1497 1318 1542 1335" data-label="Page-Footer"> <p>18 - 34</p> </div>
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