Chapter 20

Planning Accelerated Life Tests

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Based on the authors' text *Statistical Methods for Reliability Data*, John Wiley & Sons Inc. 1998.

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19h 16min

Planning Accelerated Life Tests
Chapter 20 Objectives

- Outline reasons and practical issues in planning ALTs.
- Describe criteria for ALT planning.
- Illustrate how to evaluate the properties of ALTs.
- Describe methods of constructing and choosing among ALT plans
  - One-variable plans.
  - Two-variable plans.
- Present guidelines for developing practical ALT plans with good statistical properties.

Possible Reasons for Conducting an Accelerated Test

Accelerated tests (ATs) are used for different purposes. These include:

- ATs designed to identify failure modes and other weaknesses in product design.
- ATs for improving reliability
- ATs to assess the durability of materials and components.
- ATs to monitor and audit a production process to identify changes in design or process that might have a seriously negative effect on product reliability.

Motivation/Example

Reliability Assessment of an Adhesive Bond

- **Need:** Estimate of the B10 of failure-time distribution at 50°C (expect $\geq$ 10 years).
- Constraints
  - 300 test units.
  - 6 months for testing.
- 50°C test expected to yield little relevant data.

Model and Assumptions

- Failure-time distribution is loglocation-scale
$$\Pr(T \leq t) = F(t; \mu, \sigma) = \Phi\left[\frac{\log(t) - \mu}{\sigma}\right]$$
- $\mu = \mu(x) = \beta_0 + \beta_1 x$, where
$$x = \frac{11605}{\texttt{temp}\,°\text{C} + 273.15}.$$
- σ does not depend on the experimental variables.
- Units tested simultaneously until censoring time t_c.
- Observations statistically independent.

Assumed Planning Information for the Adhesive Bond Experiment

The objective is finding a test plan to estimate B10 with good precision.

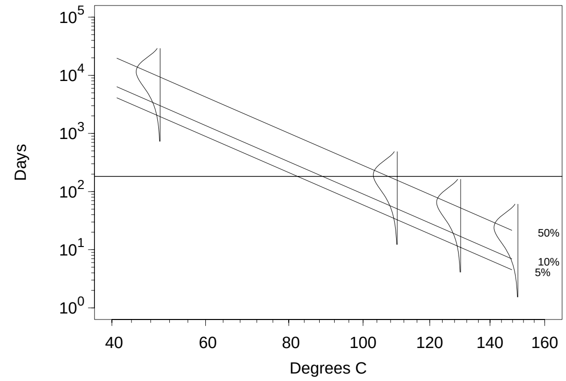
- Weibull failure-time distribution with same shape parameter at each level of temperature σ and location scale parameter $\mu(x) = \beta_0 + \beta_1 x$, where x is °C in the Arrhenius scale.
- .1% failing in 6 months at 50°C.
- 90% failing in 6 months at 120°C.

Result: Defines failure probability in 6 months at all levels of temperature. If σ is given also, defines all model parameters.

Engineers' Originally Proposed Test Plan for the Adhesive Bond

Temp °C	Allocation Proportion π_i	Allocation Number n_i	Failure Probability p_i	Expected Number Failing $E(r_i)$
50			0.001	
110	1/3	100	0.60	60
130	1/3	100	1.00	100
150	1/3	100	1.00	100

Adhesive Bond
Engineers' Originally Proposed Test Plan
$n = 300$, $\pi_i = 1/3$ at each 110°C, 130°C, 150°C



Critique of Engineers' Original Proposed Plan

- Arrhenius model in doubt at high temperatures (above 120°C).
- Question ability to extrapolate to 50°C.
- Data much above the B10 are of limited value.

Suggestion for improvement:

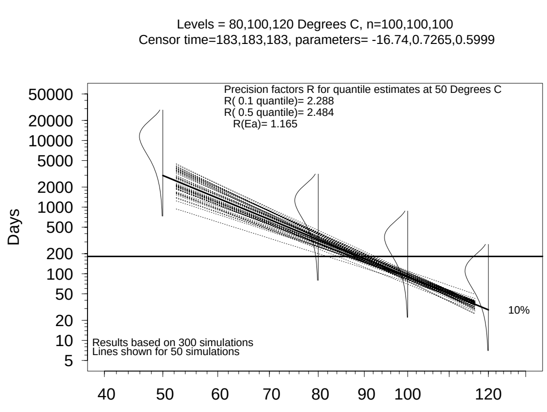
- Test at lower more realistic temperatures (even if only small fraction will fail).
- Larger allocation to lower temperatures.

Engineers' Modified Traditional ALT Plan with a Maximum Test Temperature of 120°C

Temp °C	Allocation Proportion π_i	Allocation Number n_i	Failure Probability p_i	Expected Number Failing $E(r_i)$
50	0			
80	1/3	100	.04	4
100	1/3	100	.29	29
120	1/3	100	.90	90

For this plan and the Weibull-Arrhenius model, Ase$[\log(\widehat{t}_{.1}(50))] =$ .4167. The asymptotic precision factor for a 95% confidence interval of $t_{.1}(50)$ is $R = \exp(1.96 \times \text{Ase}) = 2.26$.

Simulation of Engineers' Modified Traditional ALT Plan



Methods of Evaluating Test Plan Properties

Assume inferences needed on a function $g(\boldsymbol{\theta})$ (one-to-one and all the first derivatives with respect to the elements of $\boldsymbol{\theta}$ exist, and are continuous).

- Properties depend on test plan, model and (unknown) parameter values. **Need planning values**.
- **Asymptotic variance of $g(\widehat{\boldsymbol{\theta}})$**

$$\text{Avar}[g(\widehat{\theta})] = \left[\frac{\partial g(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right]' \Sigma_{\widehat{\boldsymbol{\theta}}} \left[\frac{\partial g(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right].$$

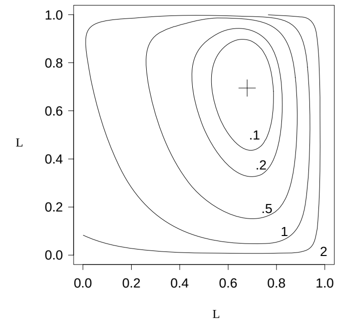
  Simple to compute (with software) and general results.

- Use Monte Carlo simulation. Specific results, provides picture of data, requires much computer time.

Statistically Optimum Plan for the Adhesive Bond

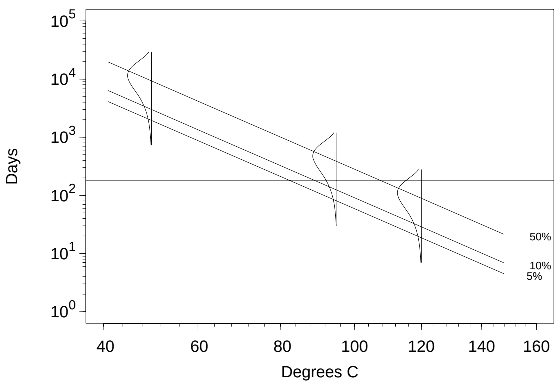
- **Objective:** Estimate B10 at 50°C with minimum variance.
- **Constraint:** Maximum testing temperature of 120°C.
- **Inputs:** Failure probabilities $p_U = .001$ and $p_H = .90$.

Contour Plot Showing $\log_{10}\{\text{Avar}[\log(\widehat{t}_{.1})]/\min \text{Avar}[\log(\widehat{t}_{.1})]\}$ as Function of ξ_L, π_L to Find the Optimum ALT Plan

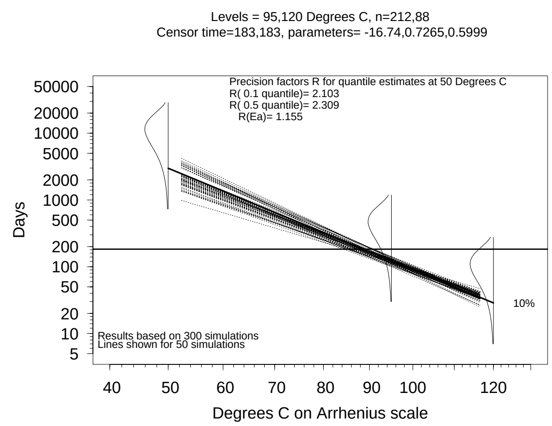


Adhesive Bond Weibull Distribution Statistically Optimum Plan

Allocations: $\pi_{\textbf{Low}} = .71$ **at** 95°C, $\pi_{\textbf{High}} = .29$ **at** 120°C



Simulation of the Weibull Distribution Statistically Optimum Plan



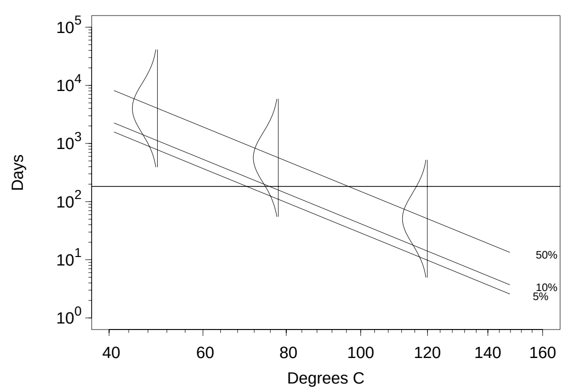
Weibull Distribution Statistically Optimum Plan

Temp °C	Allocation Proportion π_i	Allocation Number n_i	Failure Probability p_i	Expected Number Failing $\text{E}(r_i)$
50			.001	
95	.71	213	.18	38
120	.29	87	.90	78

For this plan and the Weibull-Arrhenius model, $\text{Ase}[\log(\widehat{t}_{.1}(50))] = .3794$

Adhesive Bond Lognormal Distribution Statistically Optimum Plan

Allocations: $\pi_{\textbf{Low}} = .74$ **at** 78°C, $\pi_{\textbf{High}} = .26$ **at** 120°C



Lognormal Distribution
Statistically Optimum Plan

Temp °C	Allocation Proportion π_i	Allocation Number n_i	Failure Probability p_i	Expected Number Failing $E(r_i)$
50			.001	
78	.74	233	.13	30
120	.26	77	.90	69

For this plan and the Lognormal-Arrhenius model, $\text{Ase}[\log(\hat{t}_{.1}(50))] = .2002$

Critique of the Statistically Optimum Plan

- Still too much temperature extrapolation (to 50°C).
- Only two levels of temperature.
- Optimum Weibull and lognormal plans quite different
  - ▶ 95°C and 120°C for Weibull versus.
  - ▶ 78°C and 120°C for lognormal.

In general, optimum plans not robust to model departures.

Want a Plan That

- Meets practical constraints and is intuitively appealing.
- Is robust to deviations from assumed inputs.
- Has reasonably good statistical properties.

Criteria for Test Planning

Subject to constraints in time, sample size and ranges of experimental variables,

- Minimize $\text{Var}[\log(\hat{t}_p)]$ under the assumed model.
- Maximize the determinant of the Fisher information matrix.
- Minimize $\text{Var}[\log(\hat{t}_p)]$ under more general or higher-order model(s) (for robustness).
- Control the expected number of failures at each experimental condition (since a small expected number of failures at critical experimental conditions suggests potential for a failed experiment).

Types of Accelerated Life Test Plans

- **Optimum plans**—Maximize statistical precision.
- **Traditional plans**—Equal spacing and allocation; may be inefficient.
- **Optimized (best) compromise plans**—require at least 3 levels of the accelerating variable (e.g., 20% constrained at middle) and optimize lower level and allocation.

General Guidelines for Planning ALTs
(Suggested from Optimum Plan Theory)

- Choose the highest level of the accelerating variable to be as high as possible.
- Lowest level of the accelerating variable can be optimized.
- Allocate more units to lower levels of the accelerating variable.
- Test-plan properties and optimum plans depend on unknown inputs.

Practical Guidelines for Compromise ALT Plans

- Use three or four levels of the accelerating variable.
- Limit high level of the accelerating variable to maximum reasonable condition.
- Reduce lowest level of the accelerating variable (to minimize extrapolation)—subject to seeing some **action**.
- Allocate more units to lower levels of the accelerating variable.
- Use statistically optimum plan as a starting point.
- Evaluate plans in various meaningful ways.

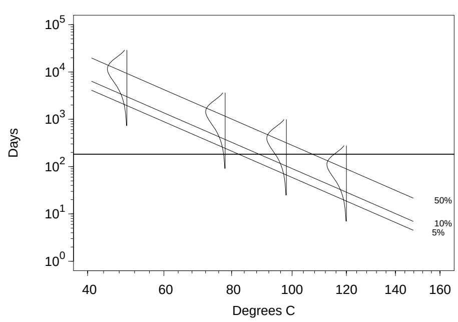
Adjusted Compromise Weibull ALT Plan for the Adhesive Bond (20% Constrained Allocation at Middle)

Temp °C	Allocation Proportion π_i	Allocation Number n_i	Failure Probability p_i	Expected Number Failing $E(r_i)$
50			.001	
78	.52	156	.03	5
98	.20	60	.24	14
120	.28	84	.90	76

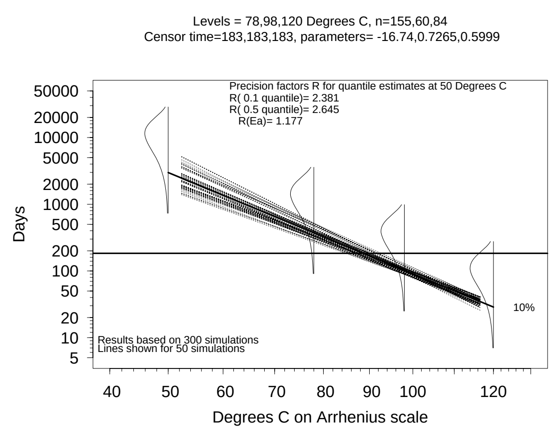
For this plan with the Weibull-Arrhenius model, $\text{Ase}[\log(\widehat{t}_{.1}(50))] = .4375$.

Adhesive Bond Adjusted Compromise Weibull ALT Plan

$\pi_{\text{Low}} = .52$, $\pi_{\text{Mid}} = .20$, $\pi_{\text{High}} = .28$



Simulation of the Adhesive Bond Compromise Weibull ALT Plan



Basic Issue 1: Choose Levels of Accelerating Variables

Need to Balance:

- **Extrapolation in the acceleration variable** (assumed temperature time relationship).
- **Extrapolation in time** (assumed failure-time distribution).

Suggested Plan:

- Middle and high levels of the acceleration variable—expect to interpolate in time.
- Low level of the acceleration variable—expect to extrapolate in time.

Basic Issue 2: Allocation of Test Units

- Allocate more test units to low rather than high levels of the accelerating variable.
  - ▶ Tends to equalize the number of failures at experimental conditions.
  - ▶ Testing more units near the use conditions is intuitively appealing.
  - ▶ Suggested by statistically optimum plan.
- Need to constrain a certain percentage of units to the middle level of the accelerating variable.

Properties of Compromise ALT Plans Relative to Statistically Optimum Plans

- Increases asymptotic variance of estimator of B10 at 50°C by 33% (if assumptions are correct).

However it also,

- Reduces low test temperature to 78°C (from 95°C).
- Uses three levels of accelerating variable, instead of two levels.
- Is more robust to departures from assumptions and uncertain inputs.

Generalizations and Comments

- Constraints on test positions (instead of test units): Consider replacement after $100p\%$ failures at each level of accelerating variable.
- Continue tests at each level of accelerating variable until at least $100p\%$ units have failed.
- Include some tests at the use conditions.
- Fine tune with computer evaluation and/or simulation of user-suggested plans.
- Desire to estimate reliability (instead of a quantile) at use conditions.
- Need to quantify robustness.

ALT with Two or More Variables

- Moderate increases in two accelerating variables may be safer than using a large amount of a single accelerating variable.
- There may be interest in assessing the effect of nonaccelerating variables.
- There may be interest in assessing joint effects of two more accelerating variables.

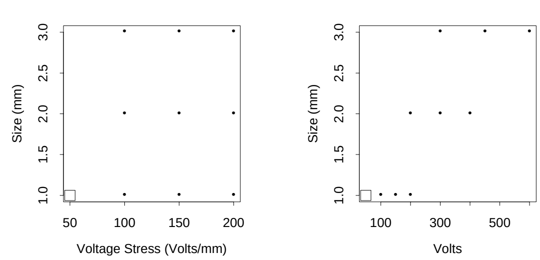
Choosing Experimental Variable Definition to Minimize Interaction Effects

- Care should be used in **defining** experimental variables.
- Guidance on variable definition and possible transformation of the response and the experimental models should, as much as possible, be taken from **mechanistic** models.
- Proper choice can reduce the occurrence or importance of statistical interactions.
- Models without statistical interactions simplify modeling, interpretation, explanation, and experimental design.
- Knowledge from mechanistic models is also useful for **planning** experiments.

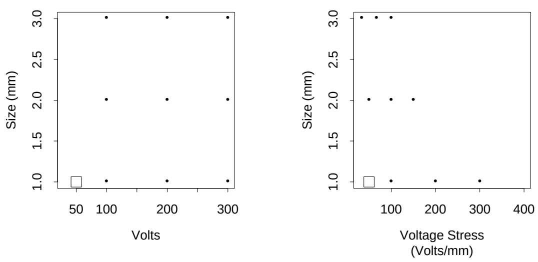
Examples of Choosing Experimental Variable Definition to Minimize Interaction Effects

- For humidity testing of corrosion mechanism use RH and temperature (not vapor pressure and temperature)
- For testing dielectrics, use size and volts stress (e.g., mm and volts/mm instead of mm and volts)
- For light exposure, use aperture and total light energy (not aperture and exposure time)
- To evaluate the adequacy of large-sample approximations with censored data, use % failing and expected number failing (not % failing and sample size).

Comparison of Experimental Layout with Volts/mm Versus Size and Volts Versus Size



Comparison of Experimental Layout with Volts versus Size and Volts/mm versus Size



Insulation ALT From Chapter 6 of Nelson (1990) and Escobar and Meeker (1995)

- Engineers needed rapid assessment of insulation life at use conditions.
- 1000/10000 hours available for testing.
- 170 test units available for testing.
- Possible experimental variables:
  - VPM (`Volts/mm`) [accelerating].
  - THICK (`cm`) [nonaccelerating].
  - TEMP (°C) [accelerating].

Multiple Variable ALT Model and Assumptions

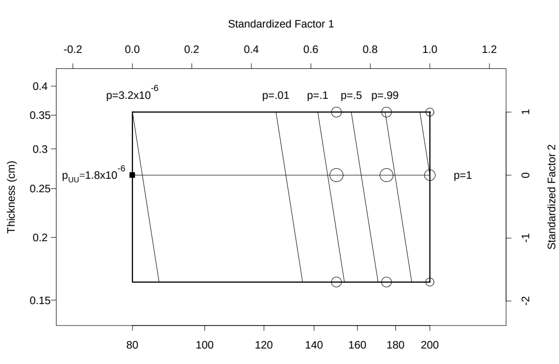
- Failure-time distribution
$$\Pr(T \le t) = F(t;\mu,\sigma) = \Phi\left[\frac{\log(t)-\mu}{\sigma}\right].$$
- $\mu = \mu(\boldsymbol{x})$ is a function of the accelerating (or other experimental) variables.
- σ does not depend on the experimental variables.
- Units tested simultaneously until censoring time t_c.
- Observations statistically independent.

Models Used in Examples

$$\begin{aligned}
\mu &= \beta_0 + \beta_1 \log(\text{VPM}) \\
\mu &= \beta_0 + \beta_1 \log(\text{VPM}) + \beta_2 \log(\text{THICK}) \\
\mu &= \beta_0 + \beta_1 \log(\text{VPM}) + \beta_2 \left[\frac{11605}{\texttt{temp}\,{}^\circ\text{C} + 273.15}\right]
\end{aligned}$$

σ constant.

Insulation ALT 3×3 VPM $\times$ THICK Factorial Test Plan



The ALT Design Problem

- Design test plan to estimate life at the **use conditions** of $\text{VPM}_U = 80$ volts/mm, $\text{THICK}_U = 0.266$ `cm`, $\text{TEMP}_U = 120$ °C.
- Interest centers on a quantile in lower tail of life distribution, $t_p = \exp\left[\mu(\boldsymbol{x}_U) + \Phi^{-1}(p)\sigma\right]$.
- Need to choose levels of the accelerating variable(s) $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_k$ and allocations $\pi_1, \cdots, \pi_k$ to those conditions. Equal allocation can be a poor choice.

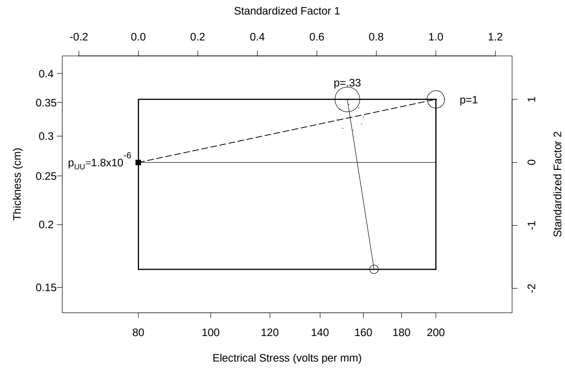
Multi-Variable Experimental Region

- Maximum levels for all variables:
  $\text{VPM}_H = 200$ volts/mm
  $\text{THICK}_H = 0.355$ cm
  $\text{TEMP}_H = 260\,°\text{C}$.

- Explicit minimum levels for all experimental variables:
  $\text{VPM}_A = 80$volts/mm
  $\text{THICK}_A = 0.163$cm
  $\text{TEMP}_A = 120°\text{C}$
  (also stricter implicit limits for VPM and TEMP).

- May need to restrict highest combinations of accelerating variables; e.g., constrain by equal failure-probability line (by using a maximum failure probability constraint p^* or equivalently a standardized censored failure time ζ^* constraint).

Insulation ALT VPM $\times$ THICK Optimum Test Plan



Degenerate and Nondegenerate Test Plans to Estimate t_p

Degenerate plans:

- Test all units at $\boldsymbol{x}_U$.

- Test two (or more) combinations of the experimental variables on a line with slope s passing through $\boldsymbol{x}_U$.

Nondegenerate practical plans:

- Test at three (or more) noncollinear combinations of the experimental variables in the plane.

Optimum Degenerate Plan: Technical Results

- When acceleration does not help sufficiently, it is optimum to test all units at the use conditions.

- Otherwise there is at least one optimum degenerate test plan in the $x_1 \times x_2$ plane.

- Some units tested at highest levels of accelerating variables.

- Optimum degenerate plan corresponds to a single-variable optimum.

Splitting Degenerate Plans

- It is possible to **split** a degenerate plan into a nondegenerate optimum test plan (maintaining optimum $\text{Var}[\log(\widehat{t}_p)]$).

- Use secondary criteria to chose **best** split plan.

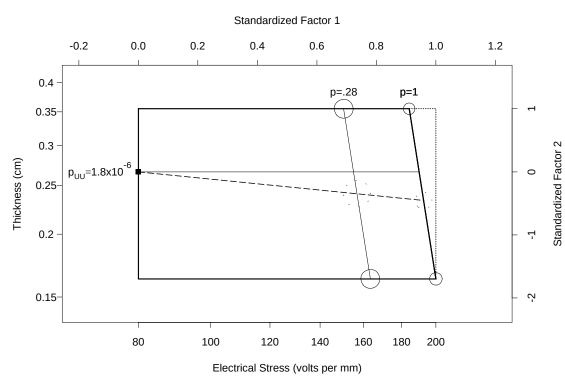
- Split $\boldsymbol{x}_i = (x_{1i}, x_{2i})'$ with allocation π_i into
  $\boldsymbol{x}_{i1} = (x_{1i1}, x_{2i1})'$ and $\boldsymbol{x}_{i2} = (x_{1i2}, x_{2i2})'$
  with allocations π_{i1} and π_{i2} (where $\pi_{i1} + \pi_{i2} = \pi_i$)

$$\pi_{i1}\boldsymbol{x}_{i1} + \pi_{i2}\boldsymbol{x}_{i2} = \pi_i\boldsymbol{x}_i.$$

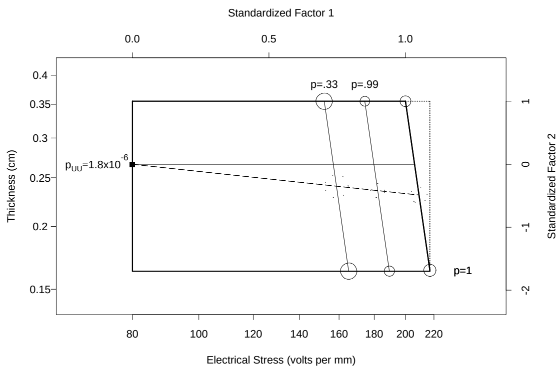
- Can introduce a p^* constraint
  [or a ζ^* constraint where $p^* = \Phi(\zeta^*)$].

- Can also split **compromise** plans and maintain $\text{Var}[\log(\widehat{t}_p)]$.

Insulation ALT VPM $\times$ THICK Optimum Test Plan with p^*/ζ^* constraint



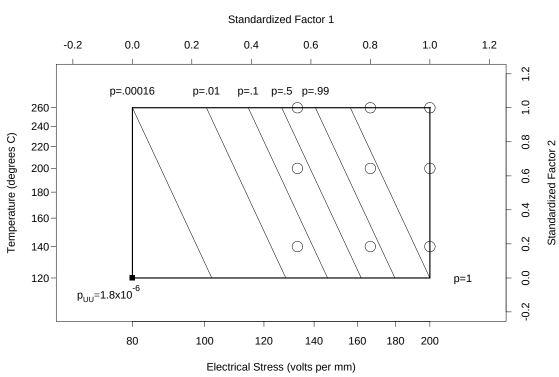
Insulation ALT VPM $\times$ THICK 20% Compromise Test Plan with p^*/ζ^* constraint



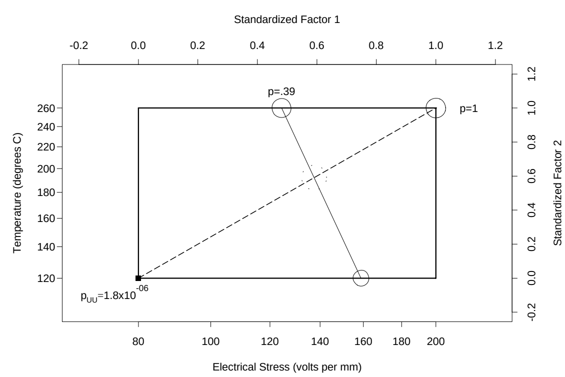
Comparison of Test Plans and Properties for the VPM$\times$THICK ALT

Plan	No Interaction Model V[log($\widehat{t_p}$)]	No Interaction Model $\lvert F\rvert$	Interaction Model V[log($\widehat{t_p}$)]	Interaction Model $\lvert F\rvert$
3×3 Factorial from Nelson (1990)	144	2.4×10^{-3}	145	1.2×10^{-5}
Optimum degenerate No ζ^*	80.1	0.0	∞	0.0
Optimum split No ζ^*	80.1	7.3×10^{-4}	∞	0.0
Optimum degenerate $\zeta^* = 2.5454$	131	0.0	∞	0.0
Optimum split $\zeta^* = 2.5454$	131	1.6×10^{-3}	138	1.7×10^{-5}
20% Compromise degenerate $\zeta^* = 4.04$	96.1	0.0	9710	0.0
20% Compromise split $\zeta^* = 4.04$	96.1	7.0×10^{-3}	102	1.2×10^{-4}

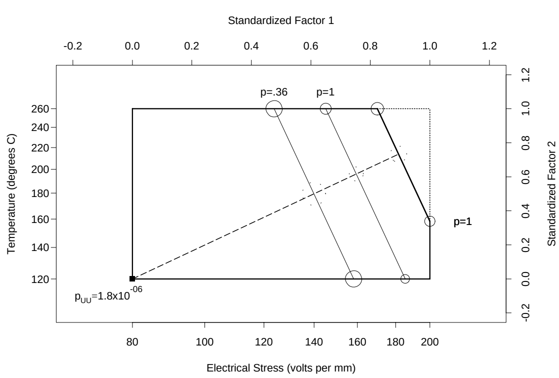
Insulation ALT VPM $\times$ TEMP 3×3 Factorial Test Plan



Insulation ALT VPM $\times$ TEMP Optimum Test Plan



Insulation ALT VPM $\times$ TEMP 20% Compromise Test Plan with p^*/ζ^* constraint



Comparison of Test Plan Properties for the VPM$\times$TEMP ALT

Plan	No Interaction Model V[log($\widehat{t_p}$)]	No Interaction Model $\lvert F\rvert$	Interaction Model V[log($\widehat{t_p}$)]	Interaction Model $\lvert F\rvert$
3×3 Factorial Adapted from Nelson (1990)	77.3	1.7×10^{-3}	349	2.7×10^{-6}
Optimum degenerate No ζ^*	50.5	0.0	∞	0.0
Optimum split No ζ^*	50.5	1.3×10^{-3}	∞	0.0
20% Compromise degenerate No ζ^*	54.7	0.0	1613	0.0
20% Compromise split No ζ^*	54.7	2.0×10^{-3}	430	3.0×10^{-6}
20% Compromise degenerate $\zeta^* = 5.0$	77.7	0.0	5768	0.0
20% Compromise split $\zeta^* = 5.0$	77.7	1.2×10^{-3}	324	1.7×10^{-6}

Extensions of Results to Other Problems

- With one **accelerating** and several other **regular** experimental variables, replicate single-variable ALT at each combination of the **regular** experimental variables.
- Can use a fractional factorial for the **regular** experimental variables.
- If the approximate effect of a **regular** experimental variable is known, can **tilt** factorial to improve precision.
- With two or more **accelerating** variables, our results show how to **tilt** the traditional factorial plans to restrict extrapolation and maintain statistical efficiency.