Chapter 21

Analysis of Accelerated Degradation Data

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Based on the authors' text *Statistical Methods for Reliability Data,* John Wiley & Sons Inc. 1998.

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14h 54min

Chapter 21
Analysis of Accelerated Degradation Data
Objectives

- Show how accelerated degradation tests can be used to assess and improve product reliability.

- Present models, methods of analysis, and methods of inference for accelerated degradation tests.

- Show how to analyze data from accelerated degradation tests.

- Compare accelerated degradation test methods with traditional accelerated life test methods using failure-time data.


Background

Today's manufacturers face strong pressure to:

- Develop newer, higher technology products in record time.

- Improve productivity, product field reliability, and overall quality.

- Increased need for **up-front** testing of materials, components and systems.

- Accelerated degradation tests can be useful for such **up-front** testing.

Percent Increase in Resistance Over Time for Carbon-Film Resistors
(Shiomi and Yanagisawa 1979)
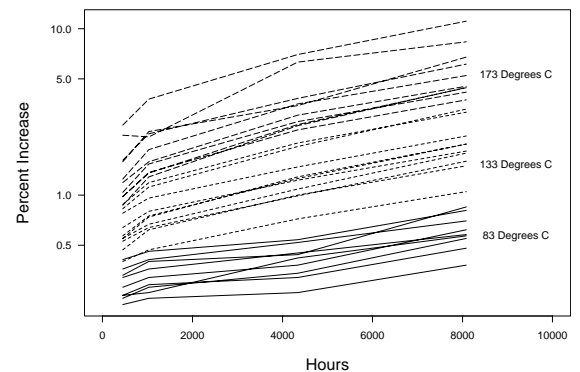
Advantages of Using Degradation Data Over Failure-Time Data

- Degradation is natural response for some tests.

- Useful reliability inferences even with 0 failures.

- More justification and credibility for extrapolative acceleration models.
  (Modeling closer to physics-of-failure)

- Can be more informative than failure-time data.
 (Reduction to failure-time data loses information)

Limitations of Degradation Data

- Degradation data may be difficult or impossible to obtain.

- Some degradation measurements are destructive (destructive degradation tests).

- Obtaining degradation data may have an effect on future product degradation (e.g., taking apart a motor to measure wear).

- Substantial measurement error can diminish the information in degradation data.

- Analyses more complicated; requires statistical methods not yet widely available.
  (Modern computing capabilities should help here)

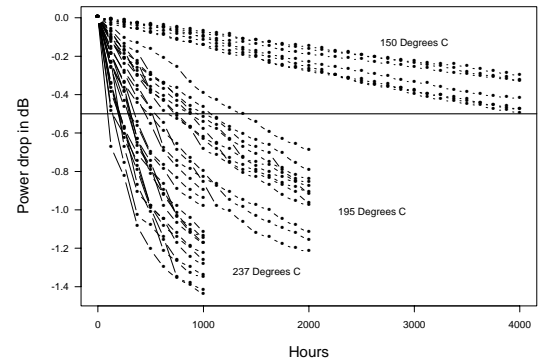- Degradation level may not correlate well with failure.

Percent Increase in Operating Current for GaAs Lasers Tested at $80°C$
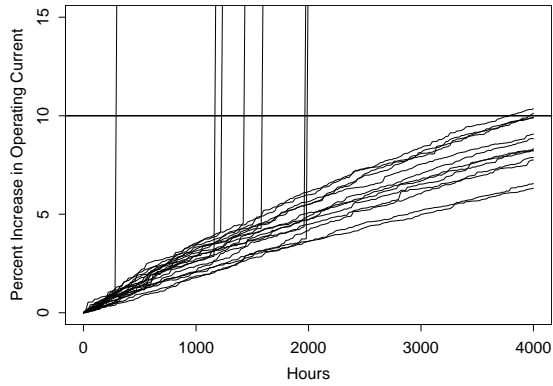
Percent Increase in Operating Current

Hours

Device-B Power Drop Accelerated Degradation Test Results at $150°C$, $195°C$, and $237°C$ (Use Conditions $80°C$)

Power drop in dB

150 Degrees C

195 Degrees C

237 Degrees C

Hours

Device-B Power Drop
Simple One-Step Chemical Reaction
Leading to Failure

- $A_1(t)$ is the amount of harmful material available for reaction at time t

- $A_2(t)$ is proportional to the amount of failure-causing compounds at time t.

- Chemical reaction:

$$A_1 \xrightarrow{k_1} A_2$$
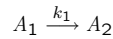
- Power drop proportional to $A_2(t)$

- The rate equations for this reaction are

$$\frac{dA_1}{dt} = -k_1 A_1 \quad \text{and} \quad \frac{dA_2}{dt} = k_1 A_1$$

Device-B Power Drop
Simple One-Step Chemical Reaction
Leading to Failure (continued)

- Solution to differential equations:

$$\begin{aligned} A_1(t) &= A_1(0)\exp(-k_1 t) \\ A_2(t) &= A_2(0) + A_1(0)[1 - \exp(-k_1 t)] \end{aligned}$$

where $A_1(0)$ and $A_2(0)$ are initial conditions.

- If $A_2(0) = 0$, then $\mathcal{D}_\infty = \lim_{t\to\infty} A_2(t) = A_1(0)$ and the solution for $A_2(t)$ (the function of primary interest) can be reexpressed as

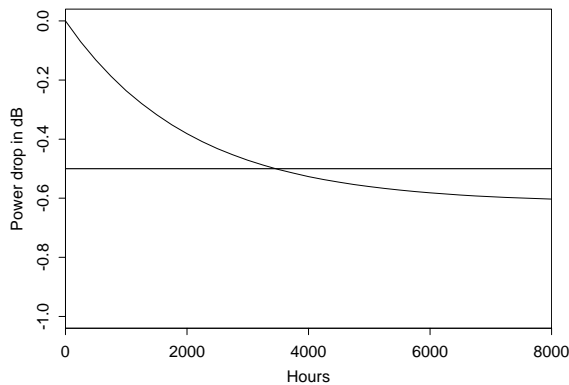$$\begin{aligned} A_2(t) &= A_1(t)[1 - \exp(-k_1 t)] \\ \mathcal{D}(t) &= \mathcal{D}_\infty[1 - \exp(-\mathcal{R} t)] \end{aligned}$$

where $\mathcal{D}(t) = A_2(t)$ is the degradation at time t and $\mathcal{R} = k_1$ is the reaction rate.

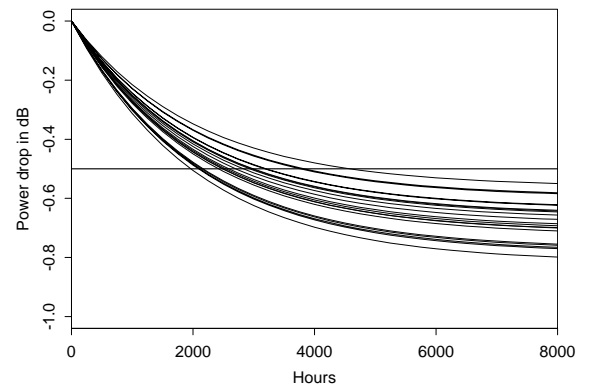- A simple 1-step diffusion process has the same solution.

Device-B Power Drop
$\mathcal{D}(t) = \mathcal{D}_\infty[1 - \exp(-\mathcal{R} t)]$
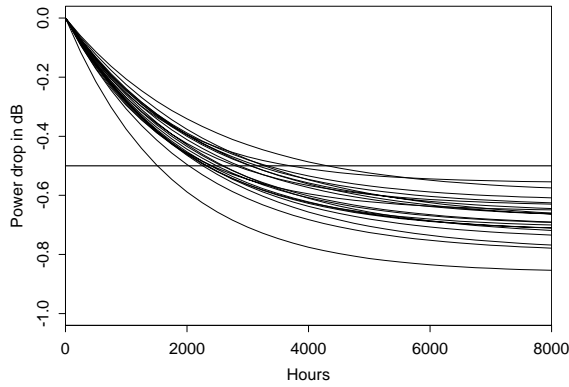## Fixed $\mathcal{D}_\infty$ and Rate $\mathcal{R}$

Power drop in dB

Hours

Device-B Power Drop
$\mathcal{D}(t) = \mathcal{D}_\infty[1 - \exp(-\mathcal{R} t)]$
Variability in Asymptote $\mathcal{D}_\infty$

Power drop in dB

Hours

Device-B Power Drop

$$\mathcal{D}(t) = \mathcal{D}_\infty[1 - \exp(-\mathcal{R}\,t)]$$

Variability in Asymptote $\mathcal{D}_\infty$ and Rate $\mathcal{R}$

Model for Degradation Data

- **Actual degradation path model:** Actual path of unit ith at time t_{ij} is

$$\mathcal{D}_{ij} = \mathcal{D}(t_{ij}, \beta_{1i}, \ldots, \beta_{ki})$$

- **Path parameters:** $\beta_{1i}, \ldots, \beta_{ki}$ may be random from unit-to-unit or fixed in the population/process.

- **Sample path model:** Sample degradation path of unit ith at t_{ij} (the jth inspection time for unit i) is

$$y_{ij} = \mathcal{D}_{ij} + \epsilon_{ij}, \quad \epsilon_{ij} \sim \mathrm{NID}(0, \sigma_\epsilon^2), \quad i = 1, \ldots, n, \quad j = 1, \ldots, m_i.$$

- Can use transformations on the response, time, or random parameters, as suggested by physical/chemical theory, past experience, or the data.

Acceleration of Degradation

- The **Arrhenius** model describing the effect that temperature has on the **rate** of a simple one-step chemical reaction is

$$\mathcal{R}(\texttt{temp}) = \gamma_0 \exp\left(\frac{-E_a}{k_{\mathsf{B}}(\texttt{temp} + 273.15)}\right)$$

where temp is temperature in °C and $k_{\mathsf{B}} = 8.6 \times 10^{-5}$ is Boltzmann's constant in units of electron volts (eV) per °C.

- The pre-exponential factor γ_0 and the reaction activation energy E_a are characteristics of the product or material.

- The **Acceleration Factor** between temp and $\texttt{temp}_U$ is

$$\mathcal{AF}(\texttt{temp}) = \mathcal{AF}(\texttt{temp}, \texttt{temp}_U, E_a) = \frac{\mathcal{R}(\texttt{temp})}{\mathcal{R}(\texttt{temp}_U)}$$

When $\texttt{temp} > \texttt{temp}_U$, $\mathcal{AF}(\texttt{temp}, \texttt{temp}_U, E_a) > 1$.

Arrhenius Model Temperature Effect on Time to an Event

- Re-expressing the single-step chemical reaction degradation path model to allow for acceleration:

$$\mathcal{D}(t; \texttt{temp}) = \mathcal{D}_\infty \times \{1 - \exp[-\{\mathcal{R}_U \times \mathcal{AF}(\texttt{temp})\} \times t]\}$$

where $\mathcal{R}_U$ is the rate reaction at $\texttt{temp}_U$.

- Failure defined by $\mathcal{D}(t) < \mathcal{D}_{\mathsf{f}}$.

- Equating $\mathcal{D}(T; \texttt{temp})$ to $\mathcal{D}_{\mathsf{f}}$ and solving for T gives the failure time at temperature temp as
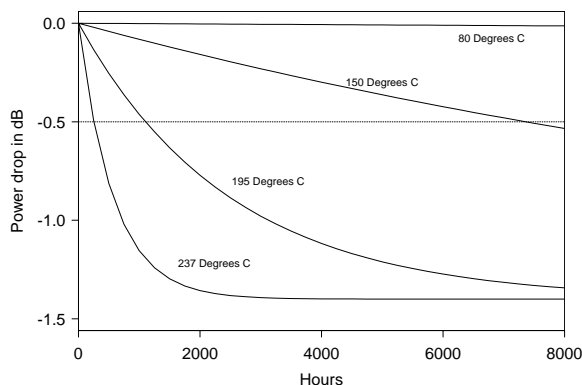
$$T(\texttt{temp}) = \frac{\left[-\frac{1}{\mathcal{R}_U}\log\left(1 - \frac{\mathcal{D}_{\mathsf{f}}}{\mathcal{D}_\infty}\right)\right]}{\mathcal{AF}(\texttt{temp})} = \frac{T(\texttt{temp}_U)}{\mathcal{AF}(\texttt{temp})}$$

- Thus the simple degradation process induces a Scale Accelerated Failure Time (SAFT) model.

Illustration of the Effect of Arrhenius Temperature Dependence on the Degradation Caused by a Single-Step Chemical Reaction

$$\mathcal{D}(t; \texttt{temp}) = \mathcal{D}_\infty \times \{1 - \exp[-\{\mathcal{R}_U \times \mathcal{AF}(\texttt{temp})\} \times t]\}$$

Device-B Power Drop Degradation Model and Parameters

- **Basic parameters:** $\mathcal{R}_U = \mathcal{R}(80)$, $\mathcal{D}_\infty$, E_a.

- **Estimation parameters:**
  $\beta_1 = \log[\mathcal{R}(195)]$, $\beta_2 = \log(-\mathcal{D}_\infty)$, and $\beta_3 = E_a$.

- Assume that (β_1, β_2) follow a bivariate normal distribution.

- Assume that activation energy $\beta_3 = E_a$ is a fixed (but unknown) characteristic of Device-B.

- **Variability in path model parameters:** $(\beta_1, \beta_2, \beta_3) \sim \mathrm{MVN}(\boldsymbol{\mu_\beta}, \boldsymbol{\Sigma_\beta})$ [but $\mathrm{Var}(\beta_3){=}0$].
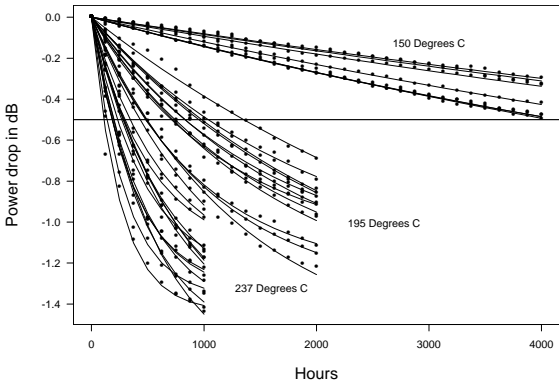
Device-B Power Drop Data
Approximate ML Estimates
(Computed with Program of Pinheiro and Bates 1995)

$$\hat{\boldsymbol{\mu}}_\beta = \begin{pmatrix} -7.572 \\ .3510 \\ .6670 \end{pmatrix}, \qquad \hat{\boldsymbol{\Sigma}}_\beta = \begin{pmatrix} .15021 & -.02918 & 0 \\ -.02918 & .01809 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
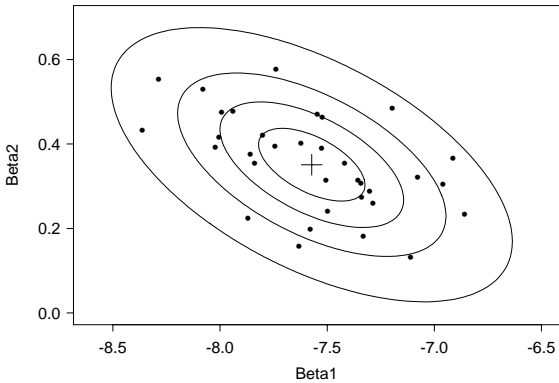
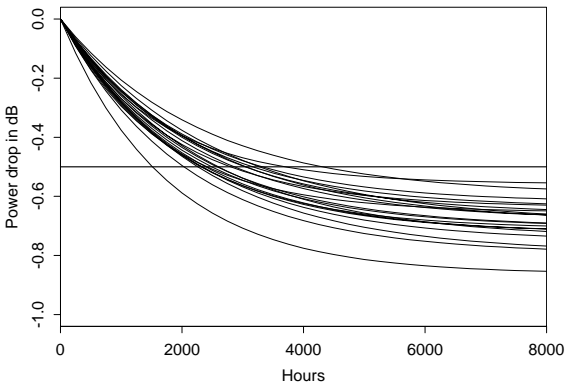$\hat{\sigma}_\epsilon = .0233,$

Loglikelihood $= 1201.8.$

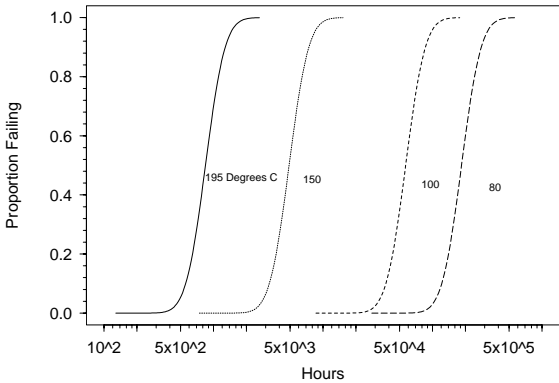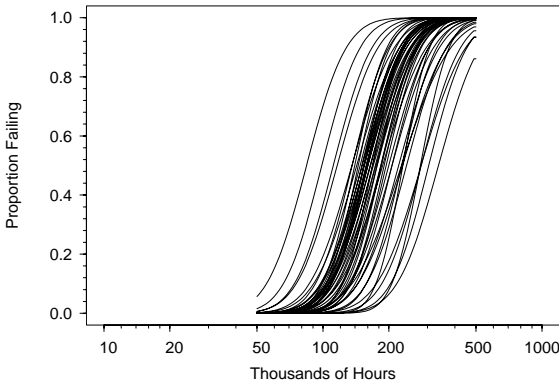## Device-B Power Drop Observations and Fitted Degradation Model for the $i = 1, \dots, 34$ Sample Paths

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Plot of $\beta_1 = \log[\mathcal{R}(195)]$ Versus $\beta_2 = \log(-\mathcal{D}_\infty)$ for the $i = 1, \dots, 34$ Sample Paths from Device-B
$$\hat{\rho}_{\beta_1\beta_2} = -.56$$

Device-B Power Drop
$$\mathcal{D}(t) = \mathcal{D}_\infty[1 - \exp(-\mathcal{R}\,t)]$$
Variability in Asymptote $\mathcal{D}_\infty$ and rate $\mathcal{R}$
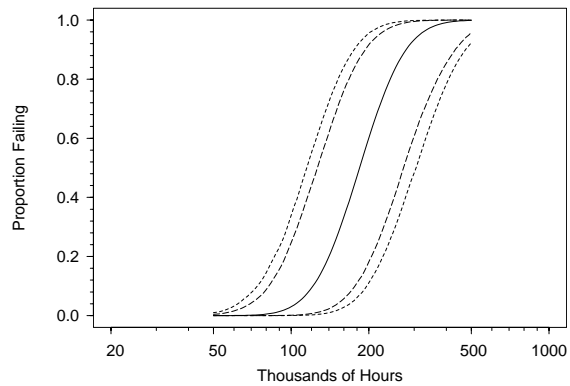

Estimates of the Device-B Life Distributions at 80, 100, 150, and $190°C$, Based on the Degradation Data

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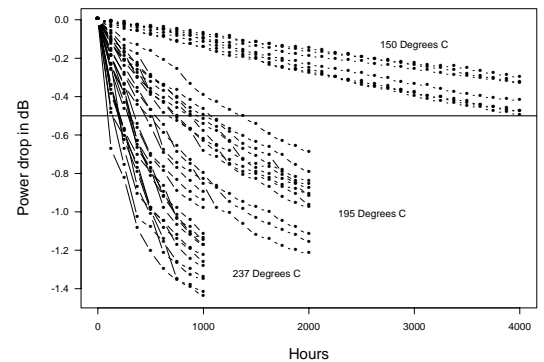
Bootstrap Sample Estimates of $F(t)$ at $80°C$

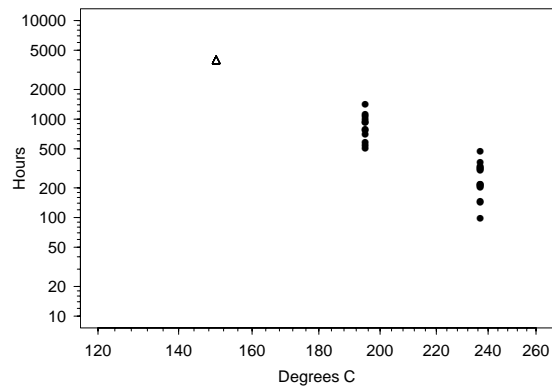## 80% and 90% Bias-Corrected Percentile Bootstrap Confidence Intervals for $F(t)$ at $80°$C

Proportion Failing vs Thousands of Hours

Device-B Power Drop Accelerated Degradation Test Results at $150°$C, $195°$C, and $237°$C (Use Conditions $80°$C)

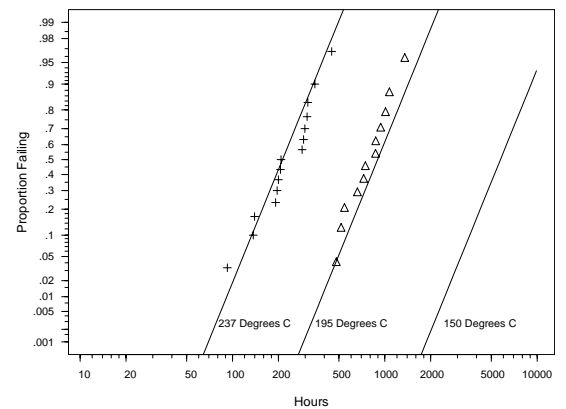Power drop in dB vs Hours; 150 Degrees C, 195 Degrees C, 237 Degrees C

Scatterplot of Device-B Failure-Time Data with Failure Defined as Power Drop Below $-.5$ dB

Hours vs Degrees C
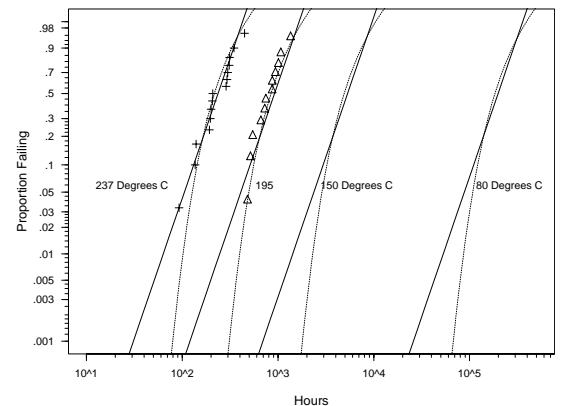
Lognormal-Arrhenius Model Fit to the Device-B Failure-Time Data

Proportion Failing vs Hours; 237 Degrees C, 195 Degrees C, 150 Degrees C

Lognormal-Arrhenius Model Fit to the Device-B Failure-Time Data with Degradation Model Estimates

Proportion Failing vs Hours; 237 Degrees C, 195, 150 Degrees C, 80 Degrees C
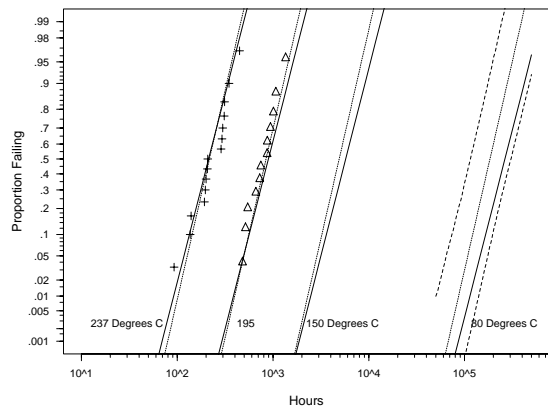
Weibull-Arrhenius Model Fit to the Device-B Failure-Time Data with Degradation Model Estimates

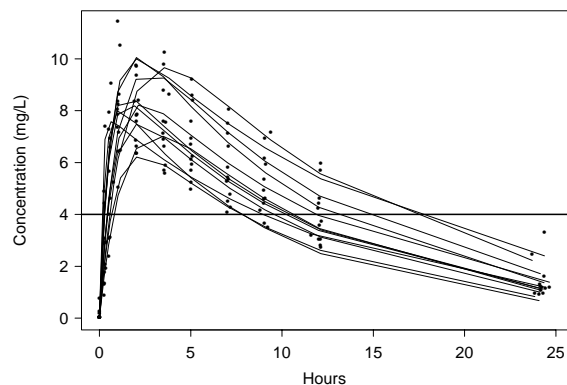Proportion Failing vs Hours; 237 Degrees C, 195, 150 Degrees C, 80 Degrees C

Plasma Concentrations of Indomethicin Following Intravenous Injection
Fitted Biexponential Model
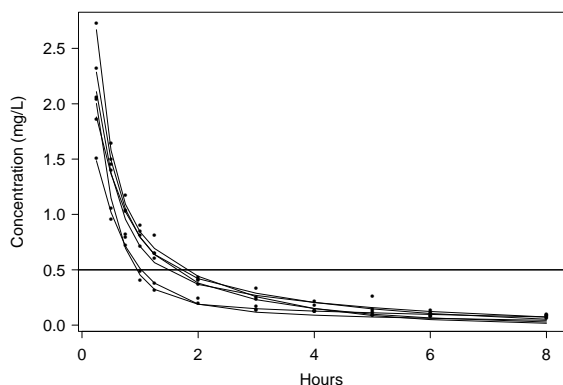
Theophylline Serum Concentrations
Fitted Curves for a First-Order Compartment Model

Approximate Accelerated Degradation Analysis

The simple method for degradation data analysis extends directly to accelerated degradation analysis.

- For each sample path one uses the algorithm described to predict the failure times.

- These data can be analyzed using the methods to analyze ALT data.

- It is important to remember, however, that such an analysis has the same limitations described in for the simple analysis of degradation data.

Sliding Metal Wear Data Analysis

- An experiment was conducted to test the wear resistance of a particular metal alloy.

- The sliding test was conducted over a range of different applied weights in order to study the effect of weight and to gain a better understanding of the wear mechanism.

- The predicted pseudo failure times were obtained by using ordinary least squares to fit a line through each sample path on the log-log scale and extrapolating to the time at which the scar width would be 50 microns.
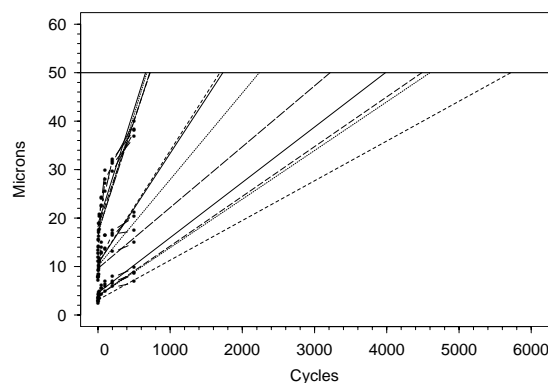
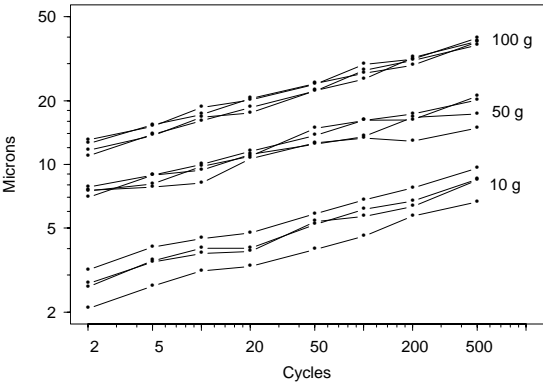## Scar Width Resulting from a Metal-to-Metal Sliding Test for Different Applied Weights
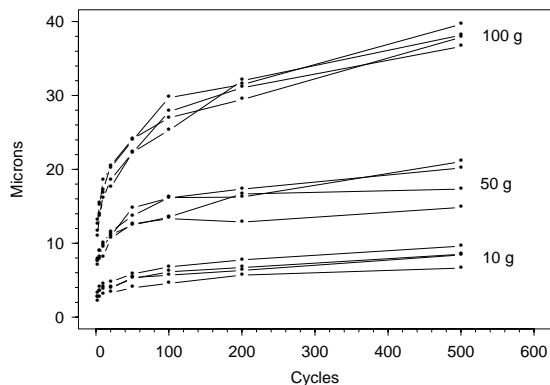
Metal-to-Metal Sliding Test for Different Applied Weights
Extrapolation to Failure Definition
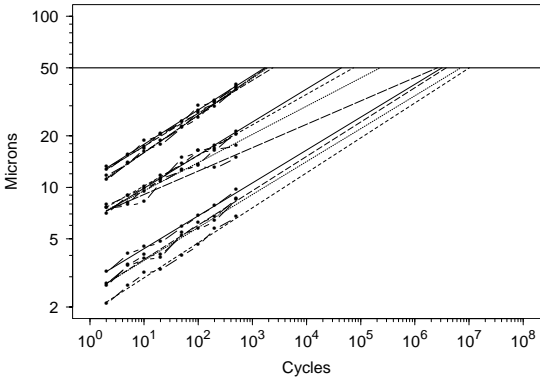### (Using linear regression on linear axes)

## Scar Width Resulting from a Metal-to-Metal Sliding Test for Different Applied Weights (Using log-log Axes)

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Metal-to-Metal Sliding Test for Different Applied Weights
Extrapolation to Failure Definition
### (Using linear regression on log-log axes)

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Metal-Wear Failure Times in Hours

| Grams	Pseudo Failure Times			
100	724	718	659	677
50	3216	1729	2234	1689
10	3981	4600	5718	4487

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Pseudo Failure Time to 50 Microns Scar Width Versus Applied Weight for the Metal-to-Metal Sliding Test

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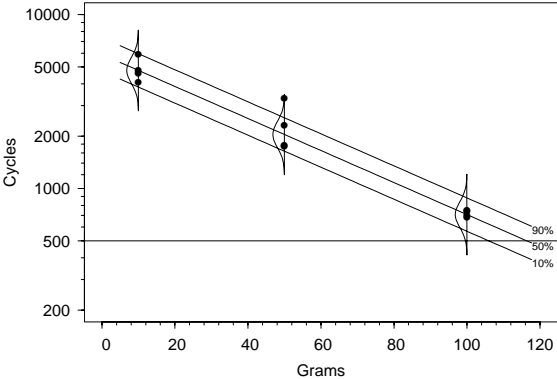
Lognormal Probability Plot Showing the ML Estimates of Time to 50 Microns Width for Each Weight

21 - 41
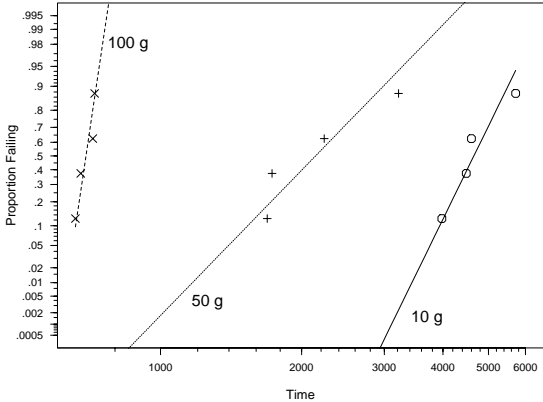
Lognormal Probability Plot Showing the Lognormal Regression Model ML Estimates of Time to 50 Microns Width for Each Weight

21 - 42
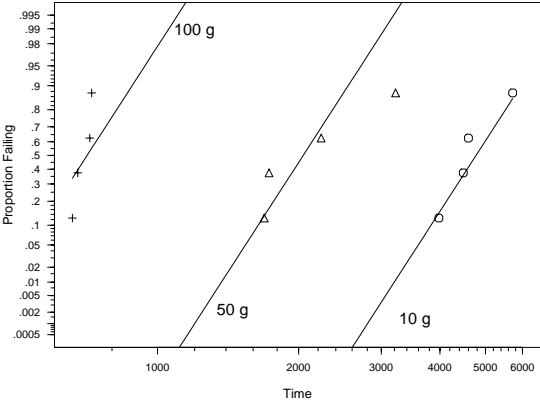
Other Topics in Chapter 21

- Choice of parameter transformation in the estimation/bootstrap procedure.

- Stochastic process degradation models.

Test planning case study in Chapter 22.