ASSIGNMENT 1

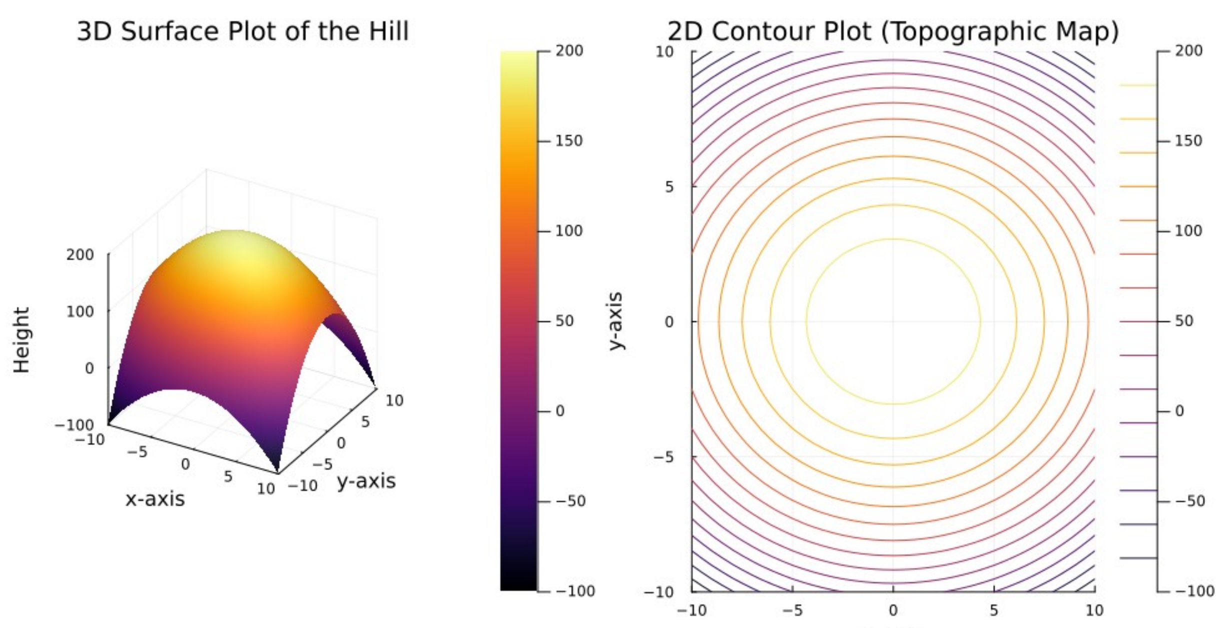
https://github.com/sengarharshit

Question 1: Hill Height Scalar Field

The height of a hill is given by the function: $h(x,y)=200-x^2-2y^2$

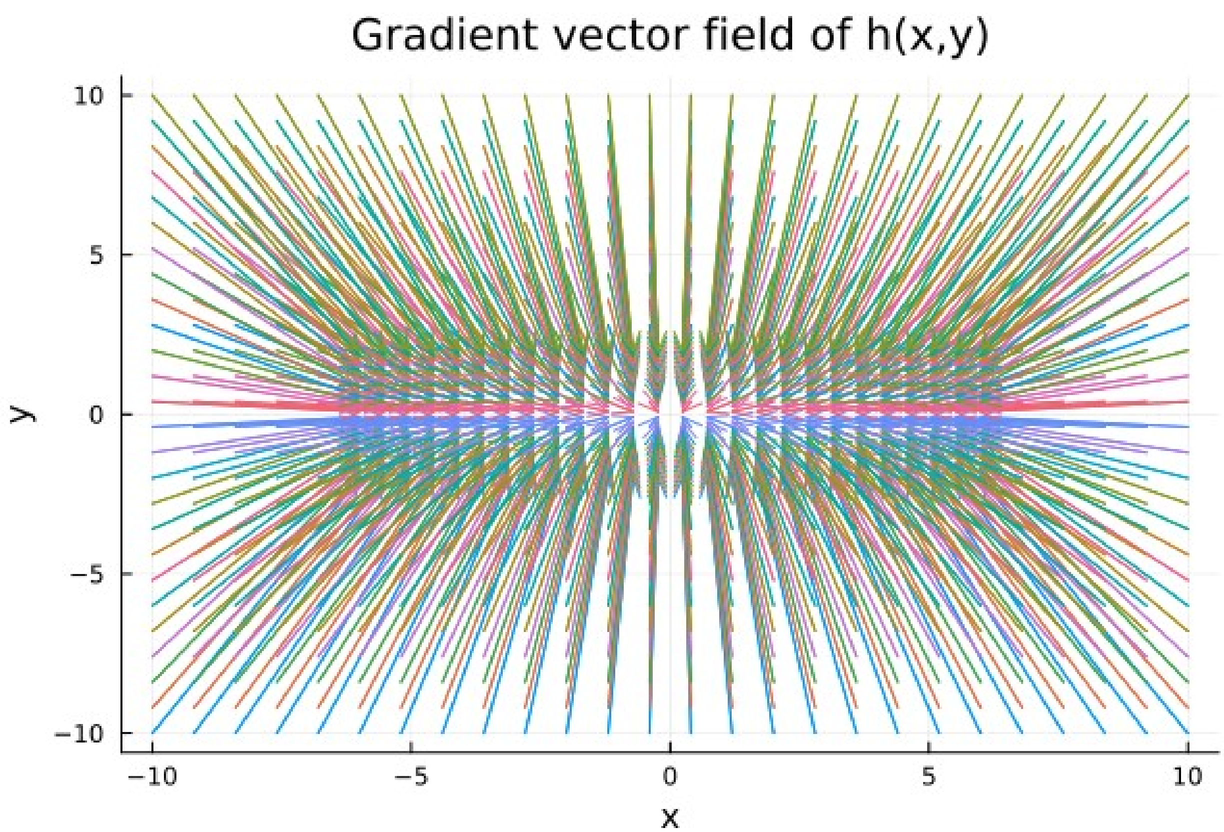
(a) Visualization of the scaler field as 3D surface plot which will show the shape of the hill and 2D contour plots which will show the lines of constant height.

```
using Plots
using CalculusWithJulia
h(x, y) = 200 - x^2 - 2*y^2
x = -10:0.5:10
y = -10:0.5:10
p1 = surface(
    x, y, h,
    title="3D Surface Plot of the Hill",
    xlabel="x-axis", ylabel="y-axis", zlabel="Height"
)
p2 = contour(
    x, y, h,
    title="2D Contour Plot (Topographic Map)",
    fill=false,
    xlabel="x-axis", ylabel="y-axis"
)
plot(p1, p2, layout=(1, 2), size=(1000, 500))
```



(b) Plotting the gradient of the scalar field using the automatic gradient calculation tool available in Julia

```
Xgrid = -10:0.8:10
Ygrid = -10:0.8:10
U = [gradient(u -> h(u[1],u[2]),[x,y])[1] for x in Xgrid, y in Ygrid]
V = [gradient(u -> h(u[1],u[2]),[x,y])[2] for x in Xgrid, y in Ygrid]
X = [x for x in Xgrid, y in Ygrid]
Y = [y for x in Xgrid, y in Ygrid]
scale = 0.20
quiver(X,Y, quiver = (U.*=scale,V.*=scale), arrowsize=0.15,  title =
"Gradient vector field of h(x,y)", xlabel = "x", ylabel = "y")
```



(c) Determination of the gradient vector and plotting the obtained gradient vector field from manual calculation

Manual calculation:

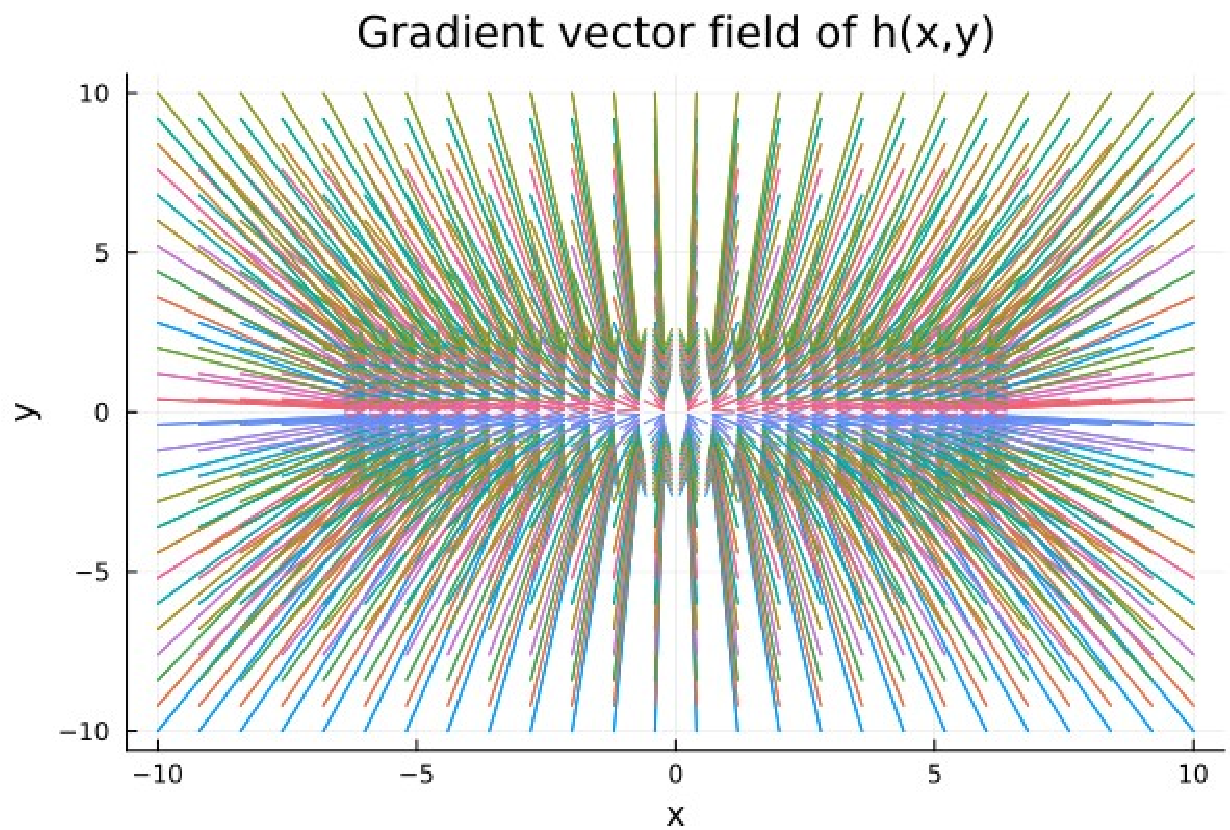
The gradient of a scalar field $h(x,y)=200-x^2-2y^2$ is a vector field defined as:

$$\nabla h(x,y)=\frac{\partial h}{\partial x}e_1+\frac{\partial h}{\partial y}e_2$$

$$\nabla h(x,y)=-2xe_1-4ye_2$$

```
using Plots
scale=0.20
Xgrid = -10:0.8:10
Ygrid = -10:0.8:10
function ObtainedVectorField(x, y)
    v = -2 * x
    u = -4 * y
    return [v,u]
end
V=[ObtainedVectorField(x, y)[1] for x in Xgrid, y in Ygrid]
U=[ObtainedVectorField(x, y)[2] for x in Xgrid, y in Ygrid]

X = [x for x in Xgrid, y in Ygrid]
Y=[y for x in Xgrid, y in Ygrid]
quiver(X,Y, quiver = (V.*=scale,U.*=scale), arrowsize=0.15, title =
"Gradient vector field of h(x,y)", xlabel = "x", ylabel = "y")
```



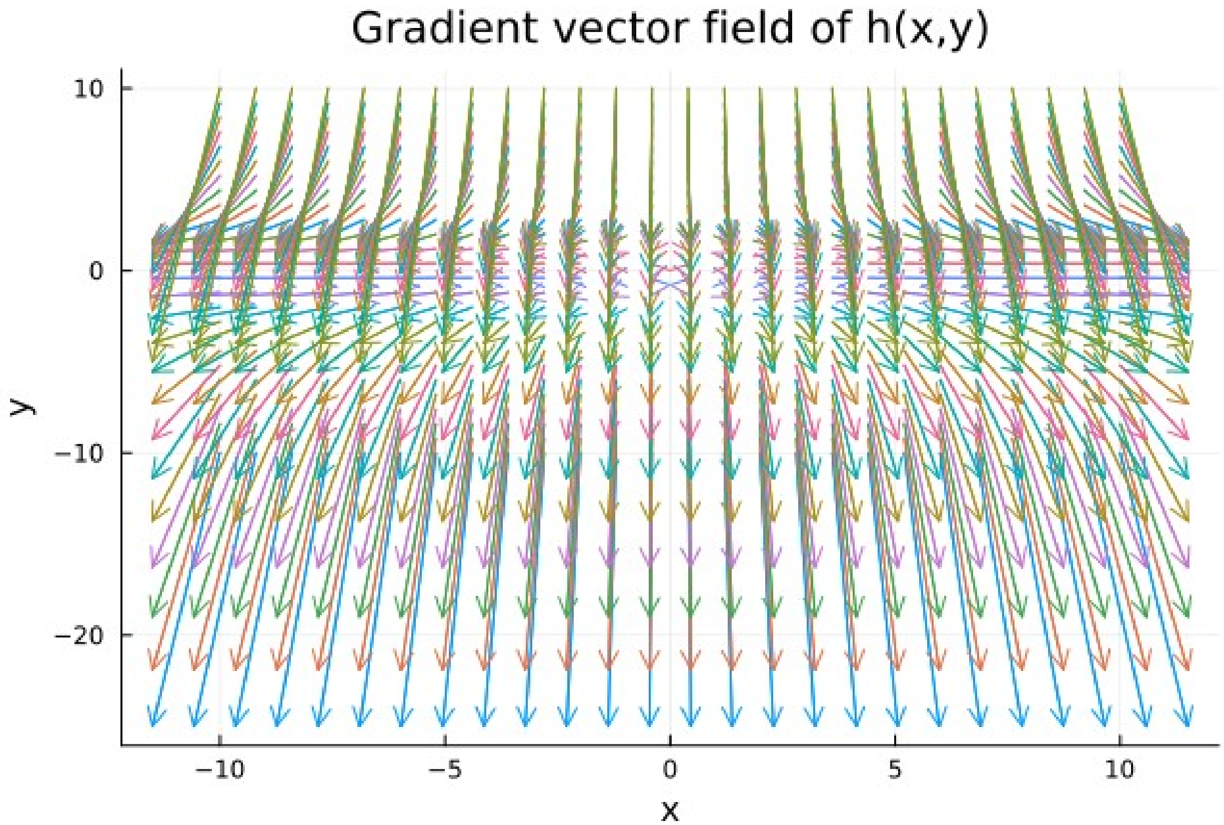
Quesiton 2: Velocity of water particles in a river

Vector field of velocity: $f = x\,e_1 - y^2 e_2$

(a) Plotting the above mentioned vector field

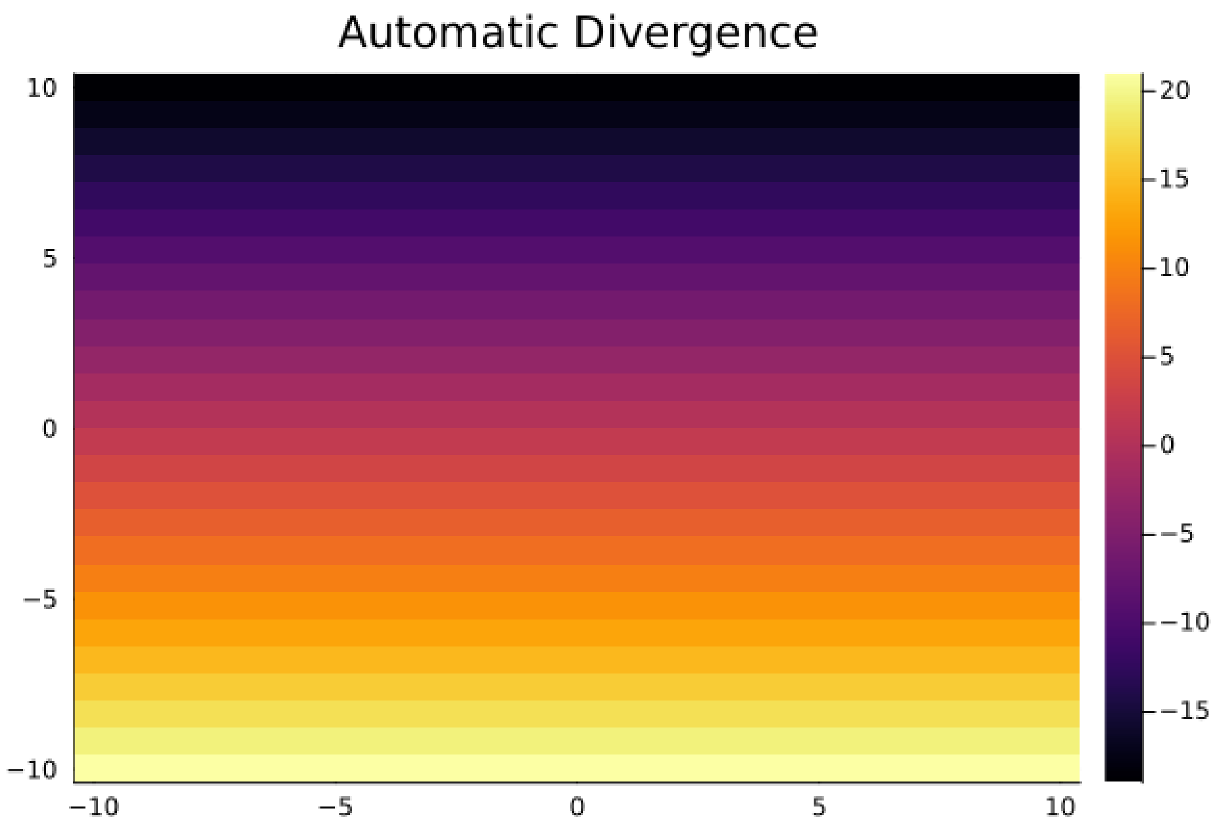
```
using Plots
scale=0.15
Xgrid = -10:0.8:10
Ygrid = -10:0.8:10
function ObtainedVectorField(x, y)
    v = x
    u = -y^2
    return [v,u]
end
V=[ObtainedVectorField(x, y)[1] for x in Xgrid, y in Ygrid]
U=[ObtainedVectorField(x, y)[2] for x in Xgrid, y in Ygrid]

X = [x for x in Xgrid, y in Ygrid]
Y = [y for x in Xgrid, y in Ygrid]
quiver(X,Y, quiver = (V.*=scale,U.*=scale), arrowsize=0.15, title =
"Gradient vector field of h(x,y)", xlabel = "x", ylabel = "y")
```



(b) Plotting the divergence of the vector field using automatic divergence calculation

```
using Plots
Xgrid = -10:0.8:10
Ygrid = -10:0.8:10
function VelocityVector(x,y)
    v=x
    u=-y^2
    return [v,u]
end
div(x,y)=divergence(u -> VelocityVector(u[1],u[2]),[x,y])
p1 = heatmap(Xgrid, Ygrid, div,title="Automatic Divergence")
plot(p1)
```



Plotting the divergence of the vector field using manual divergence calculation

The divergence of a 2D vector field $f = x\,e_1 - y^2\,e_2$ is given by:

$$\operatorname{div}(f) = \nabla \cdot f = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}$$

For the given field, $P(x, y) = x$ and $Q(x, y) = -y^2$.

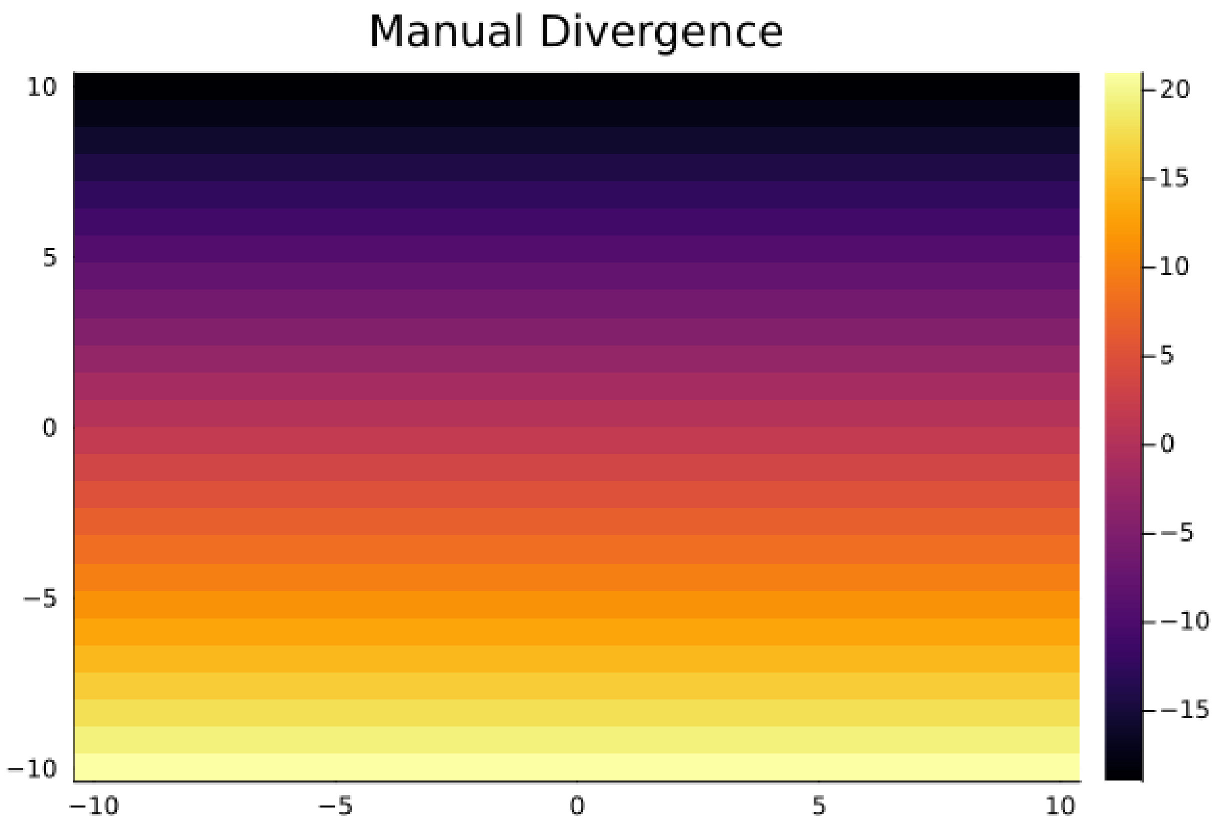
$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial x}(x) = 1$$

$$\frac{\partial Q}{\partial y} = \frac{\partial}{\partial y}(-y^2) = -2y$$

$$div(f) = 1 - 2y$$

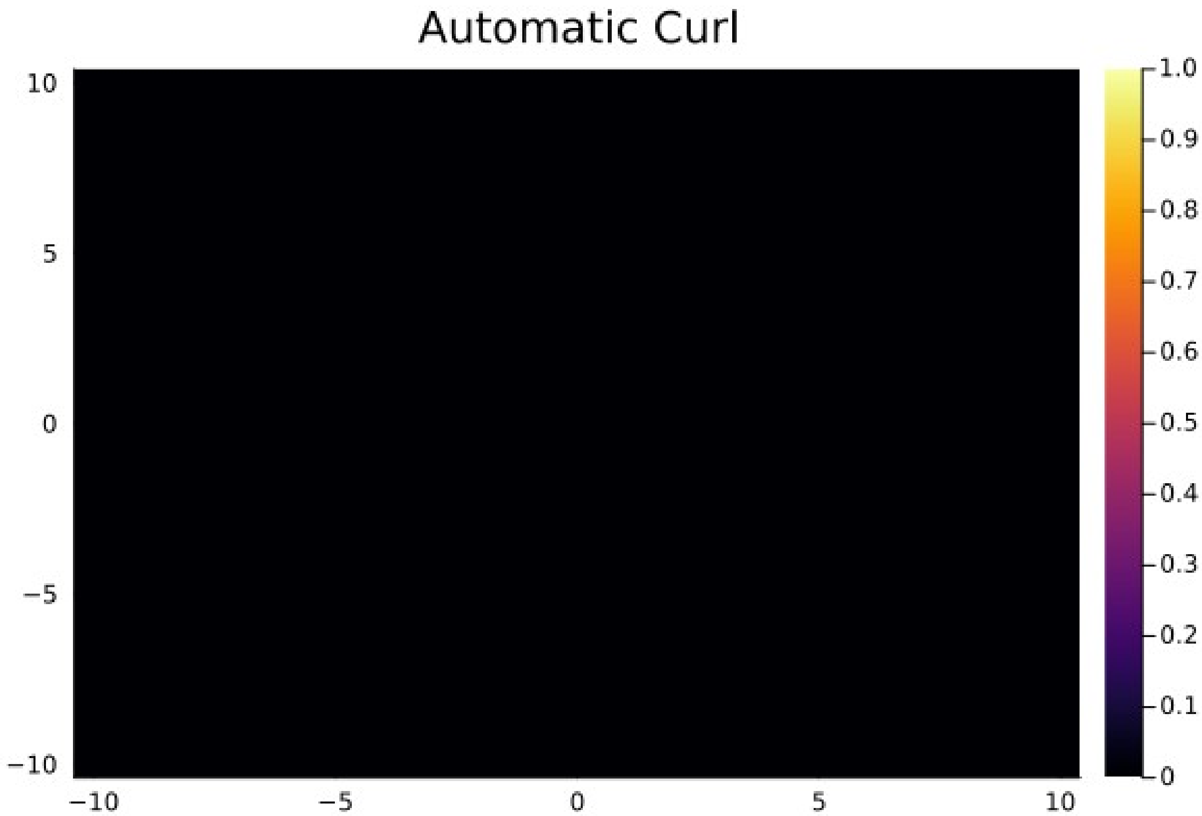
```
using Plots
Xgrid = -10:0.8:10
Ygrid = -10:0.8:10
function ObtainedDivergence(x, y)
    return 1-(2*y)
```

```
end
plot(heatmap(Xgrid,Ygrid,ObtainedDivergence, title = "Manual
Divergence"))
```



(c) Determination of the curl of the vector field using automatic curl calculation

```
using Plots
using CalculusWithJulia
Xgrid = -10:0.8:10
Ygrid = -10:0.8:10
function VelocityVector(x,y)
    v=x
    u=-y^2
    return [v,u]
end
curlof(x,y)=curl(u -> VelocityVector(u[1],u[2]),[x,y])
p1 = heatmap(Xgrid, Ygrid, curlof,title="Automatic Curl")
plot(p1)
```



Automatic Curl
10
5
0
−5
−10
−10
−5
0
5
10
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

Determination of the curl of the vector field using manual curl calculation

The curl of a 2D vector field $v(x, y) = P(x, y) e_1 + Q(x, y) e_2$ is a scalar quantity given by:

$$\operatorname{curl}(v) = \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$$

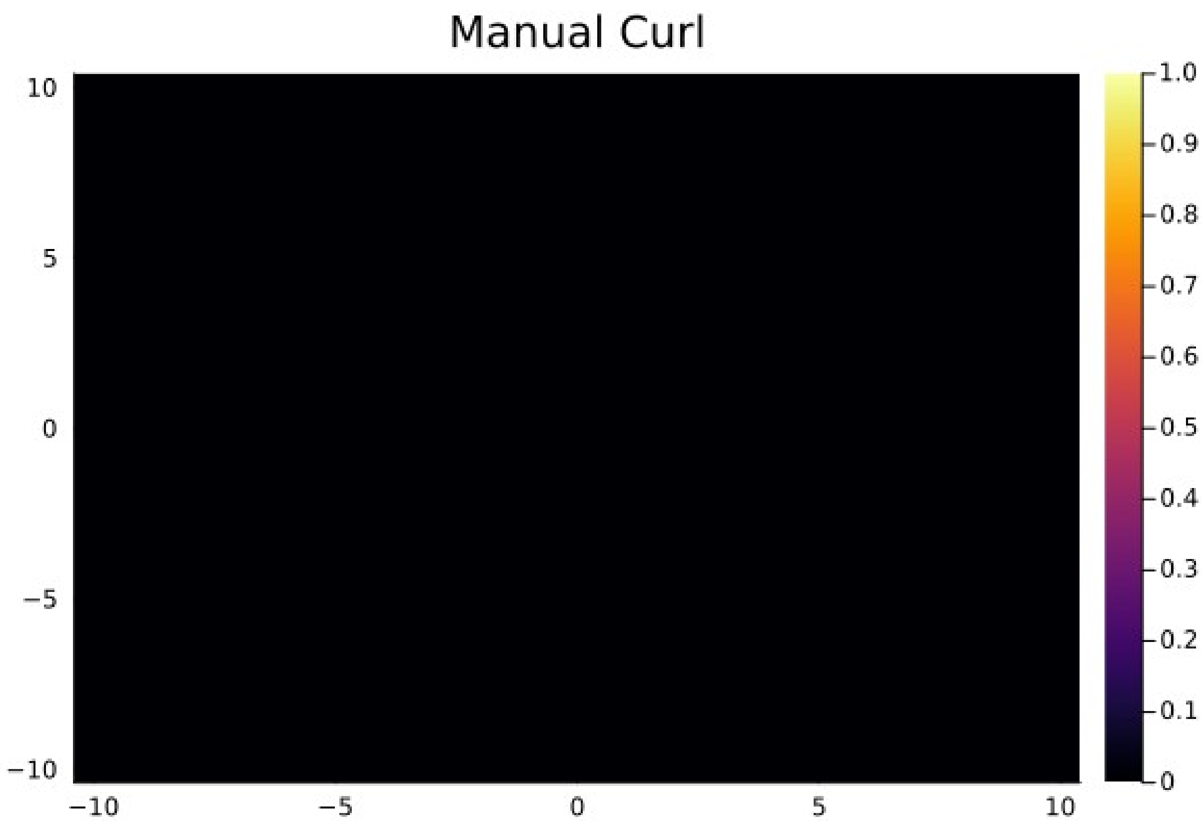
For the given field, $P(x, y) = x$ and $Q(x, y) = -y^2$.

$$\frac{\partial Q}{\partial x} = \frac{\partial}{\partial x}(-y^2) = 0$$

$$\frac{\partial P}{\partial y} = \frac{\partial}{\partial y}(x) = 0$$

Therefore, the curl is: $\operatorname{curl}(v) = 0 - 0 = 0$

```
using Plots
Xgrid = -10:0.8:10
Ygrid = -10:0.8:10
function ObtainedCurl(x, y)
    return 0
end
plot(heatmap(Xgrid,Ygrid,ObtainedCurl, title = "Manual Curl"))
```



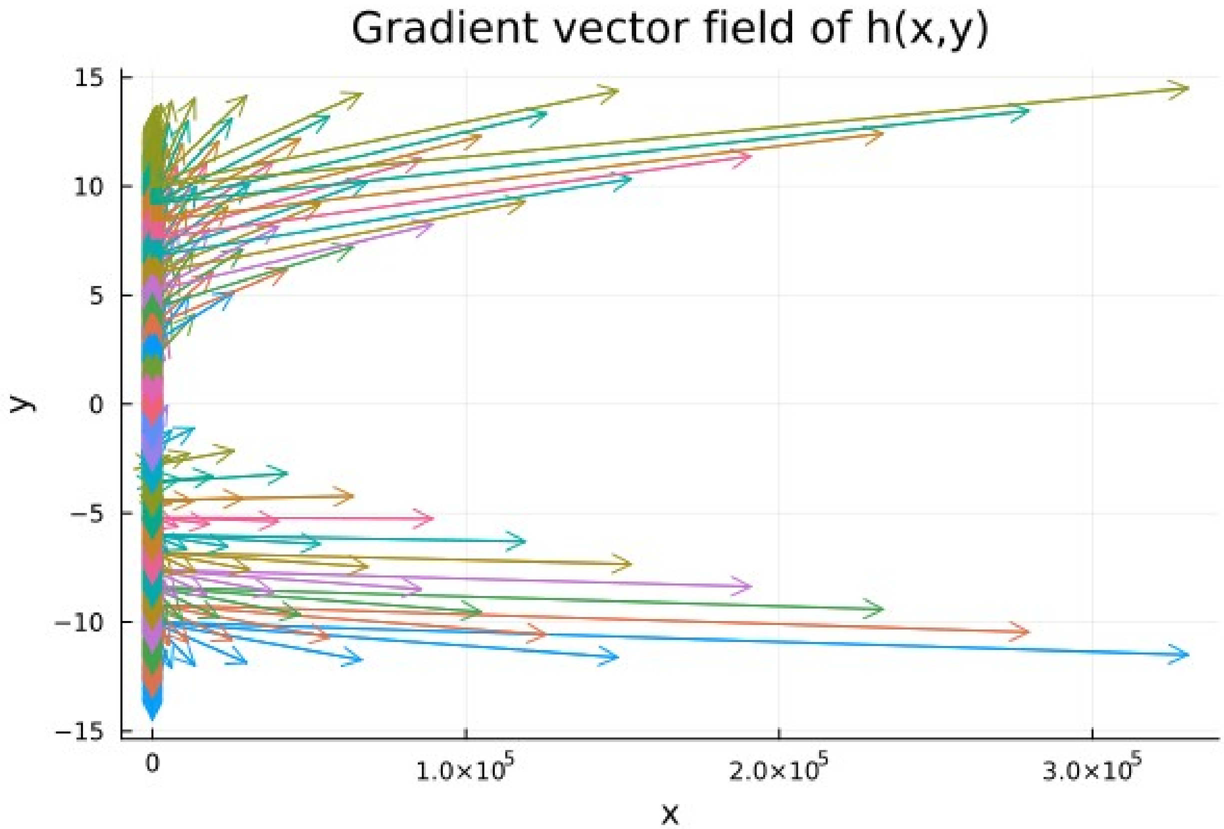
Quesiton 3: Velocity of water particles in a river

Vector field of velocity: $v(x,y) = e^x y^2 e_1 + (x+2y)e_2$

(a) Plotting the above mentioned vector field

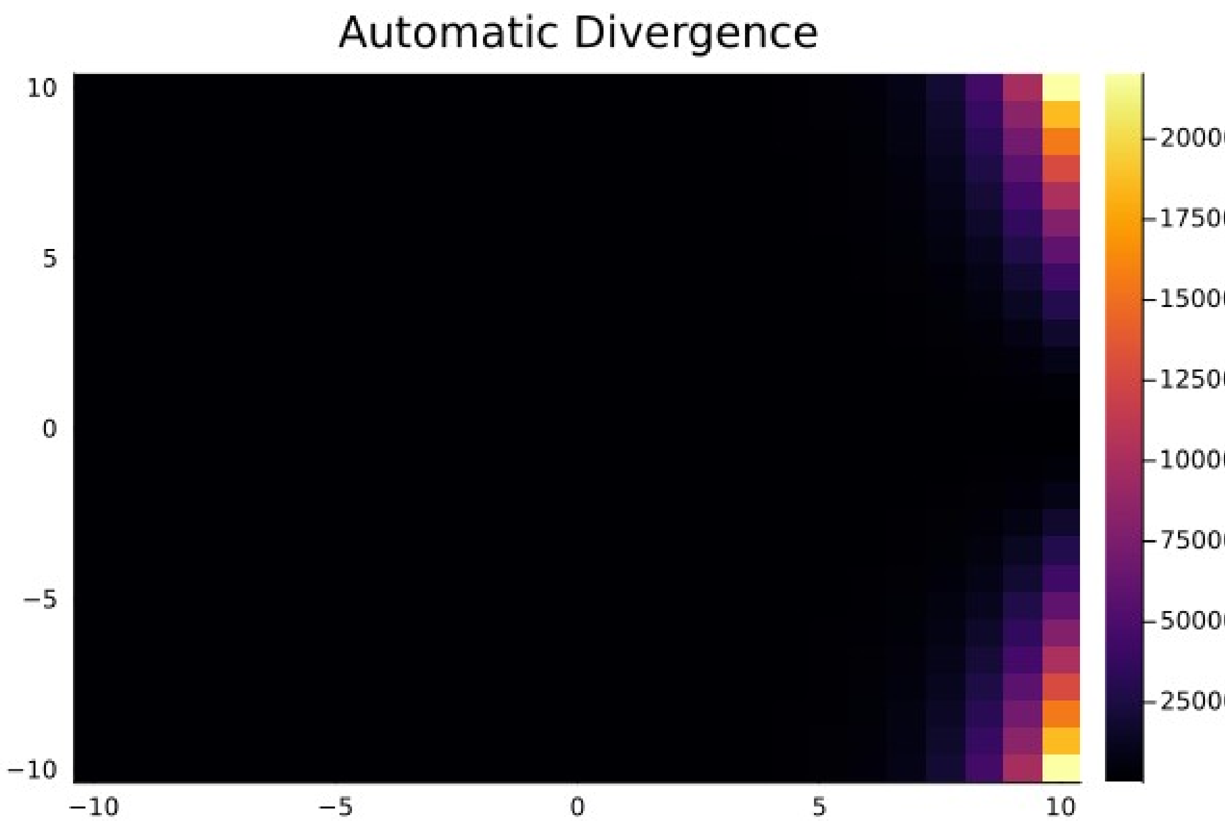
```
using Plots
scale=0.15
Xgrid = -10:0.8:10
Ygrid = -10:0.8:10
function ObtainedVectorField(x, y)
    v = (exp(1)^x)*(y^2)
    u = x+(2*y)
    return [v,u]
end
V=[ObtainedVectorField(x, y)[1] for x in Xgrid, y in Ygrid]
U=[ObtainedVectorField(x, y)[2] for x in Xgrid, y in Ygrid]

X = [x for x in Xgrid, y in Ygrid]
Y = [y for x in Xgrid, y in Ygrid]
quiver(X,Y, quiver = (V.*=scale,U.*=scale), arrowsize=0.15, title =
"Gradient vector field of h(x,y)", xlabel = "x", ylabel = "y")
```



(b) Plotting the divergence of the vector field using automatic divergence calculation

```
using Plots
using CalculusWithJulia
Xgrid = -10:0.8:10
Ygrid = -10:0.8:10
function VelocityVector(x,y)
    v= (exp(1)^x)*(y^2)
    u= 2
    return [v,u]
end
div(x,y)=divergence(u -> VelocityVector(u[1],u[2]),[x,y])
p1 = heatmap(Xgrid, Ygrid, div,title="Automatic Divergence")
plot(p1)
```



Plotting the divergence of the vector field using manual divergence calculation

The divergence of a 2D vector field $v = e^x y^2 e_1 + (x + 2y) e_2$ is given by:

$$\operatorname{div}(f) = \nabla \cdot f = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}$$

For the given field, $P(x, y) = e^x y^2$ and $Q(x, y) = x + 2y$.

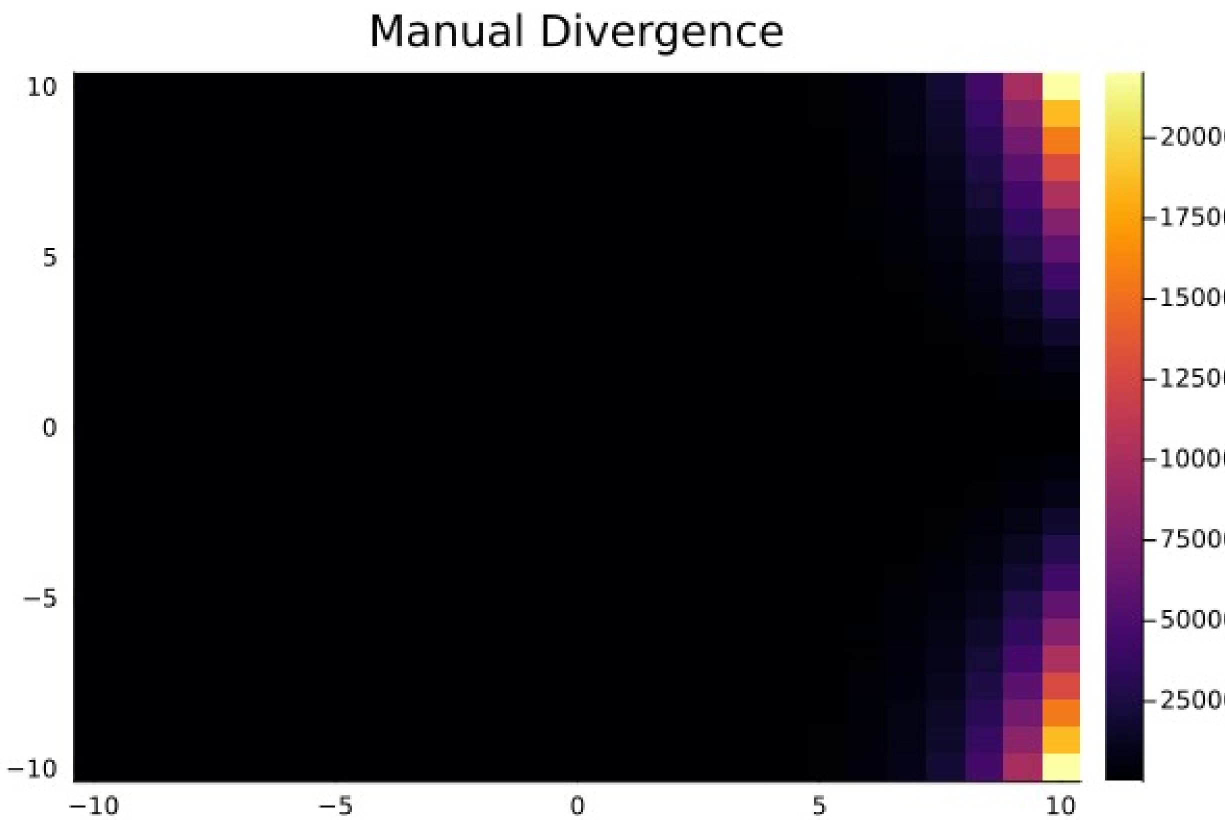
$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial x}(e^x y^2) = e^x y^2$$

$$\frac{\partial Q}{\partial y} = \frac{\partial}{\partial y}(x + 2y) = 2$$

$$div(f) = e^x y^2 + 2$$

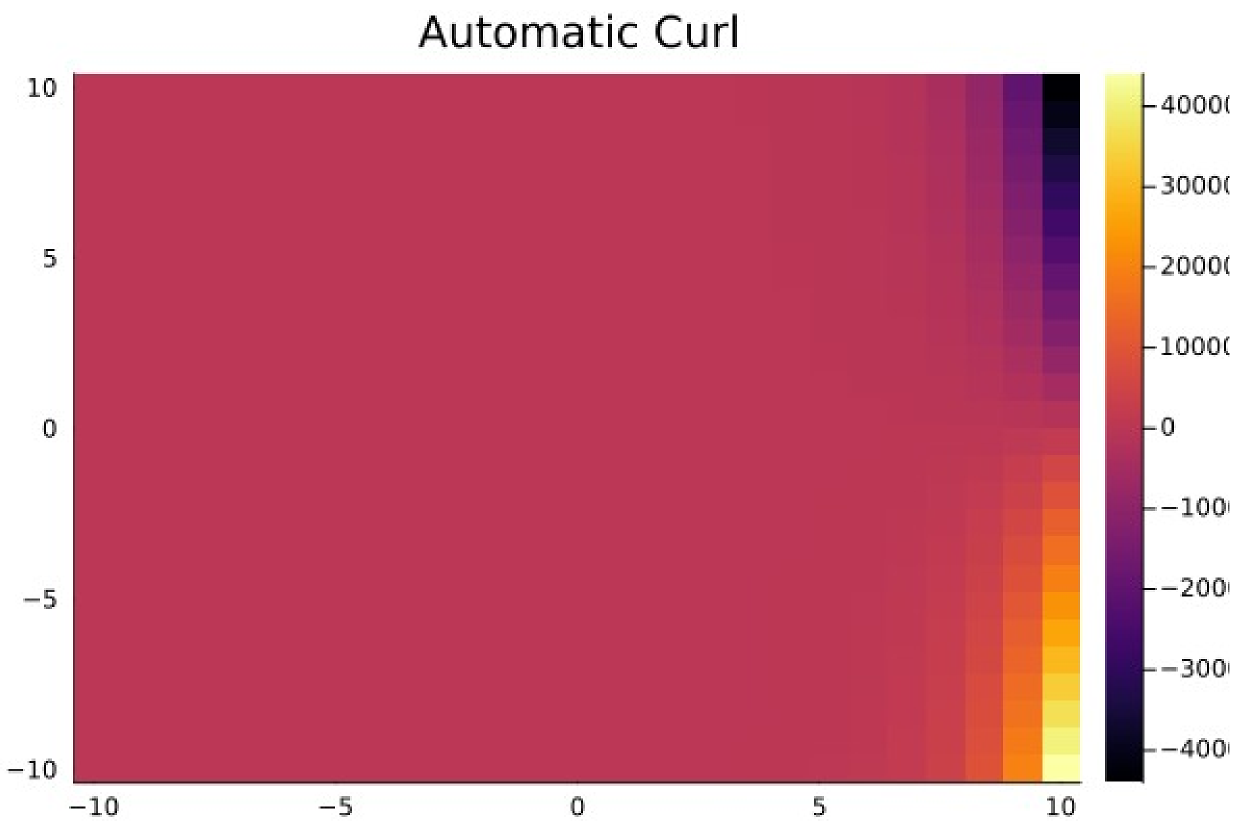
```
using Plots
Xgrid = -10:0.8:10
Ygrid = -10:0.8:10
function ObtainedDivergence(x, y)
    return (e^x)*(y^2)+2
```

```
end
plot(heatmap(Xgrid,Ygrid,ObtainedDivergence, title = "Manual
Divergence"))
```



(c) Determination of the curl of the vector field using automatic curl calculation

```
using Plots
using CalculusWithJulia
Xgrid = -10:0.8:10
Ygrid = -10:0.8:10
function VelocityVector(x,y)
    v=(e^x)*(y^2)
    u=x+(2*y)
    return [v,u]
end
curlof(x,y)=curl(u -> VelocityVector(u[1],u[2]),[x,y])
p1 = heatmap(Xgrid, Ygrid, curlof,title="Automatic Curl")
plot(p1)
```



Determination of the curl of the vector field using manual curl calculation ## The curl of a 2D vector field $f(x,y)=v=e^x y^2 e_1+(x+2y)e_2$ is a scalar quantity given by:

$$\mathrm{curl}(f)=\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}$$

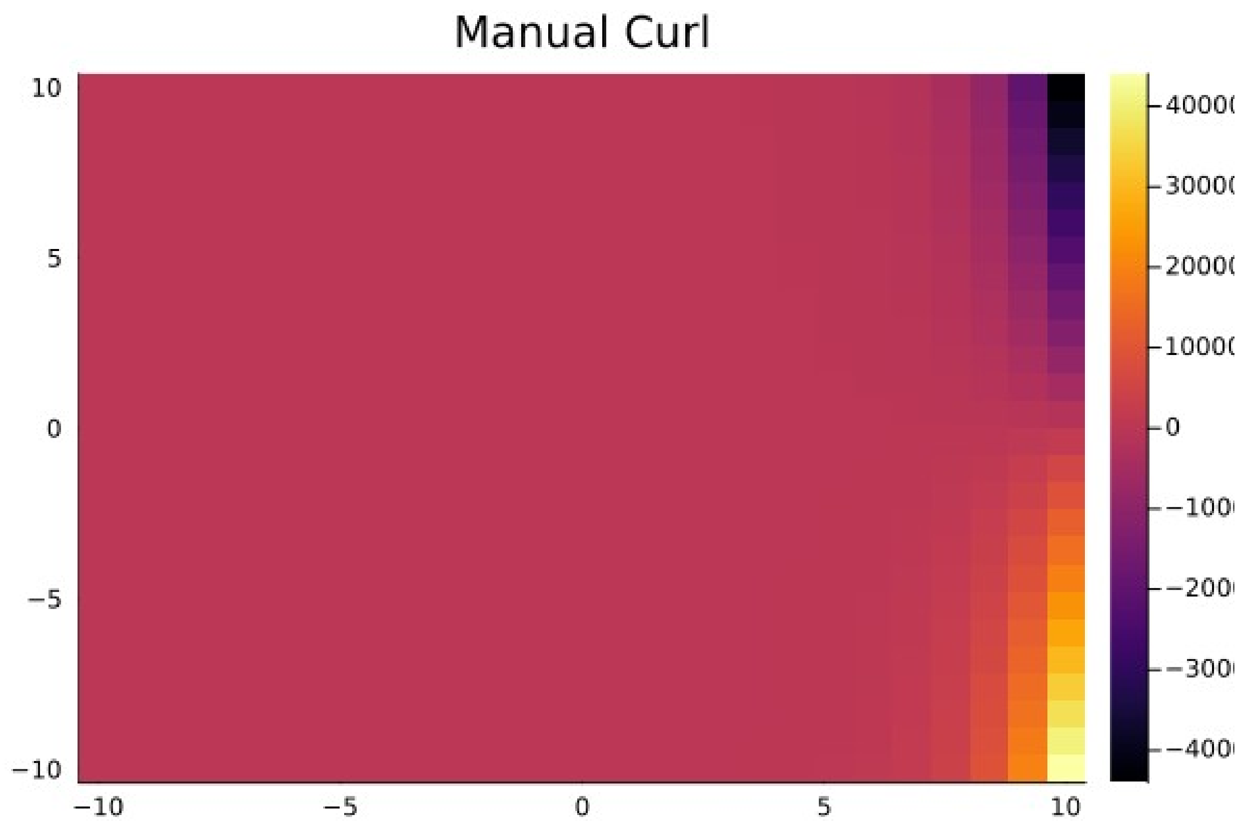
For the given field, $P(x,y)=e^x y^2$ and $Q(x,y)=x+2y$.

$$\frac{\partial Q}{\partial x}=\frac{\partial}{\partial x}(x+2y)=1$$

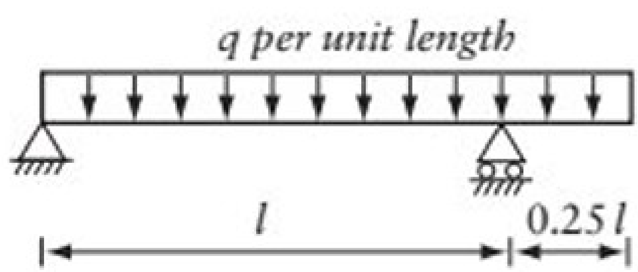
$$\frac{\partial P}{\partial y}=\frac{\partial}{\partial y}(e^x y^2)=2ye^x$$

Therefore, the curl is: $\mathrm{curl}(f)=1-2ye^x$

```
using Plots
Xgrid = -10:0.8:10
Ygrid = -10:0.8:10
function ObtainedCurl(x, y)
    return 1-(2*y*e^x)
end
plot(heatmap(Xgrid,Ygrid,ObtainedCurl, title = "Manual Curl"))
```



Question 4:



Structural Analysis

1. **Support Reactions**:
   - Sum of moments about A ($\sum M_A = 0$):

$$R_B \cdot l - (q \cdot 1.25l) \cdot \left(\frac{1.25\,l}{2}\right) = 0$$

$$R_B = 0.78125\,q\,l$$

   - Sum of vertical forces ($\sum F_y = 0$):

$$R_A + R_B - 1.25\,q\,l = 0$$

$$R_A = 1.25\,q\,l - 0.78125\,q\,l \Longrightarrow R_A = 0.46875\,q\,l$$

2. **Shear Force (V) and Bending Moment (M) Equations**:
   - For section AB ($0 \le x \le l$):

$$V(x) = R_A - q\,x = 0.46875\,q\,l - q\,x$$

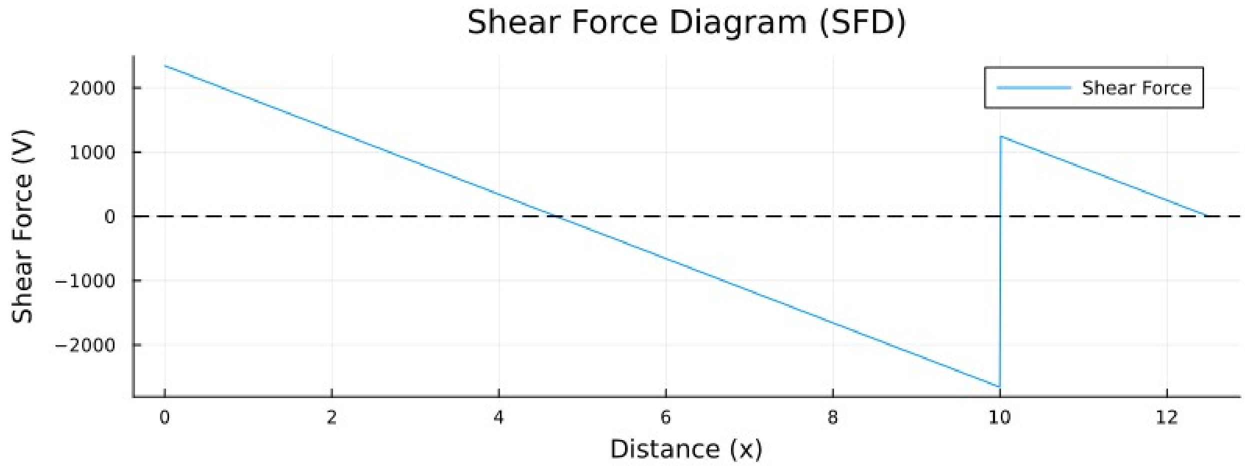
$$M(x) = R_A\,x - \frac{q\,x^2}{2} = 0.46875\,q\,l\,x - \frac{q\,x^2}{2}$$

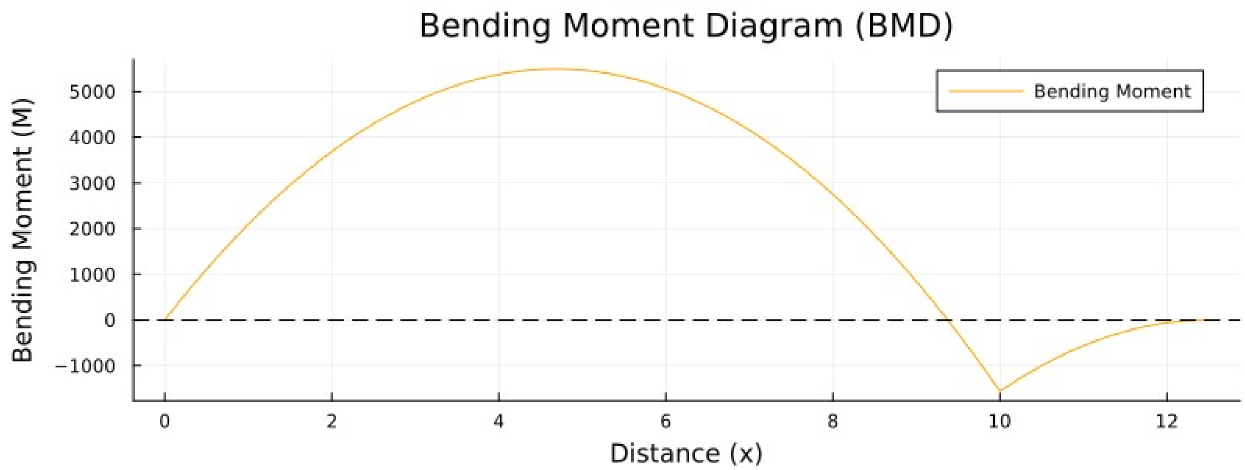
   - For section BC ($l < x \le 1.25\,l$):

$$V(x) = R_A + R_B - q\,x = 1.25\,q\,l - q\,x$$

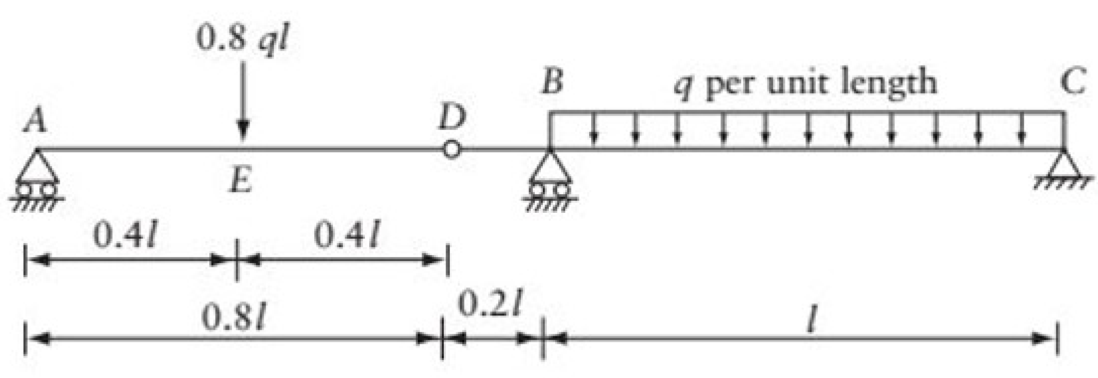
$$M(x) = R_A\,x + R_B(x - l) - \frac{q\,x^2}{2} = 1.25\,q\,l\,x - 0.78125\,q\,l^2 - \frac{q\,x^2}{2}$$

```
l = 10.0
q = 500.0
R_A = 0.46875 * q * l
R_B = 0.78125 * q * l
x_vals = range(0, 1.25 * l, length=1000)
V = zeros(1000)
M = zeros(1000)
for (i) in 1 : 1000
    x=x_vals[i]
    if 0 <= x <= l
        V[i] = R_A - q * x
        M[i] = R_A * x - q * x^2 / 2
    else
        V[i] = R_A + R_B - q * x
        M[i] = R_A * x + R_B * (x - l) - q * x^2 / 2
    end
end
p_sfd = plot(x_vals, V,
             title="Shear Force Diagram (SFD)",
             xlabel="Distance (x)",
             ylabel="Shear Force (V)",
             label="Shear Force",
             legend=:topright)
hline!([0], color=:black, linestyle=:dash, label="")
p_bmd = plot(x_vals, M,
             title="Bending Moment Diagram (BMD)",
             xlabel="Distance (x)",
             ylabel="Bending Moment (M)",
             label="Bending Moment",
             color=:orange)
hline!([0], color=:black, linestyle=:dash, label="")
plot(p_sfd, p_bmd, layout=(2, 1), size=(800, 600))
```





Question 5:



Structural Analysis

1. **Analysis at Hinge D**:
   - **Member AD (Considering the left portion of hinge D)**:
   - $\sum M_D = 0$:

$$R_A \cdot (0.4l + 0.4l) - 0.8ql \cdot (0.4l) = 0$$

$$R_A = 0.4\,q\,l$$

2. **Support Reactions**:
   - $\sum M_B = 0$:

$$R_C \cdot l - (q \cdot l) \cdot \frac{l}{2} - 0.4\,q\,l \cdot l + 0.8\,q\,l \cdot (0.6\,l) = 0$$

$$R_C = 0.42\,q\,l$$

   - $\sum F_y = 0$:

$$R_B + R_C + R_A - (q \cdot l) - 0.8\,q \cdot l = 0$$

$$R_B = -0.4\,q\,l + q\,l - 0.42\,q\,l + 0.8\,q \cdot l \Longrightarrow R_B = 0.98\,q\,l$$

3. **SFD and BMD Equations**:
   - **AE ($0 \le x \le 0.4\,l$)**:

$$V(x) = R_A = 0.4\,q\,l$$

$$M(x) = R_A\,x = 0.4\,q\,l\,x$$

   - **ED ($0.4\,l < x \le 0.8\,l$)**:

$$V(x) = R_A - 0.8\,q\,l = -0.4\,q\,l$$

$$M(x) = R_A\,x - 0.8\,q\,l\,(x - 0.4\,l)$$

   - **DB ($0.8\,l < x \le 1.0\,l$)**: (No new loads, equations are the same as ED)

$$V(x) = -0.4\,q\,l$$

$$M(x) = R_A\,x - 0.8\,q\,l\,(x - 0.4\,l)$$

   - **BC ($1.0\,l < x \le 2.0\,l$)**:

$$V(x) = R_A - 0.8\,q\,l + R_B - q\,(x - 1.0\,l)$$

$$M(x) = R_A\,x - 0.8\,q\,l\,(x - 0.4\,l) + R_B(x - 1.0\,l) - \frac{q\,(x - 1.0\,l)^2}{2}$$

```julia
using Plots
l = 10.0
q = 500.0
R_A = 0.4 * q * l
R_B = 0.98 * q * l
R_C = 0.42 * q * l
P = 0.8 * q * l
x_vals = range(0, 2*l , length=1000)
V = zeros(1000)
M = zeros(1000)
for (i) in 1:1000
    x=x_vals[i]
    if  0<= x <= 0.4*l
        V[i] = R_A
        M[i] = R_A * x
    elseif  x <= l
        V[i] = R_A - P
        M[i] = R_A * x - P * (x - 0.4*l)
    else
        V[i] = R_A - P + R_B - q * (x - l)
```

```
        M[i] = R_A * x - P * (x - 0.4*l) + R_B * (x - l) - q * (x -
l)^2 / 2
    end
end
p_sfd = plot(x_vals, V, label="Shear Force",title="Shear Force Diagram
(SFD)", xlabel="Distance (x)", ylabel="Shear Force (V)")
hline!([0], color=:black, linestyle=:dash,label="Zero line")
p_bmd = plot(x_vals, M, label="Bending Moment",title="Bending Moment
Diagram (BMD)", xlabel="Distance (x)", ylabel="Bending Moment
(M)",color=:orange)
hline!([0], color=:black, linestyle=:dash,label="Zero line")
plot(p_sfd, p_bmd, layout=(2, 1), size=(800, 600))
```