CSCI 2824
Spring 2020

Write **clearly**:

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Section number: 002
Assignment: Homework 4

Read the following:

- This cover sheet must be included as the first page for all written homework submissions to CSCI 2824.
- Fill out all of the fields above.
- Submit your written homework assignment to the electronic dropbox. You will receive graded feedback through the same mechanism.
- If you type up your homework assignment using MS Word or LaTeX, then you can earn two extra credit points per homework assignment. You **must** use properly formatted equations and nice-looking text in order to be eligible for this extra credit point. If you type it up and do not format equations properly or do not use the cover sheet (for example), you might still lose the style/neatness points.
- By submitting this assignment, you are agreeing that you have abided by the **CU Honor Code**, and that this submission constitutes your own original work.

CSCI 2824 – Spring 2020 **Homework 4**

This assignment is due on Friday, February 14 to Gradescope by 11:59pm. You are expected to write up your solutions neatly and **use the coverpage**. Remember that you are encouraged to discuss problems with your classmates, but you must work and write your solutions on your own.

Important: On the **official CSCI 2824 cover page** of your assignment clearly write your full name, the lecture section you belong to (001 or 002), and your student ID number. You may **neatly** type your solutions for +2 extra credit on the assignment. You will lose *all* 5 style/neatness points if you fail to use the official cover page.

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(1) You have constructed an intelligent knowledge-based system which answers queries by drawing conclusions from its knowledge base. Information is stored as predicate logical statements. The system has access to all necessary rules of inference, and can use them to draw solutions.

After some training, the system has the following knowledge base:

(i) Every student must join at least one of the following clubs:

  (i) Dance club

  (ii) Aerobics club

(ii) Every student is either in cooking club or they are not in aerobics club (or they are both in cooking and not in aerobics).

(iii) If a student is in the Bowling club, they cannot join the Cooking club.

(iv) If a student did not join the Bowling club, then they must have joined the Football club.

(v) No student in the Football club failed to join the Gardening club.

(vi) The student Hermoine is not in Dance club.

(1a) For each of (i)-(vi) above, translate the English sentence of the knowledge base of the club into propositional functions.

(1b) Your system is now queried with "Are all students in the Bowling club?" What does it conclude, and why? Include a full logical equivalence proof of your system's answer.

(1c) Your system is now queried with "What clubs did Hermoine join?" What does it conclude, and why? Include a full logical equivalence proof of your system's answer.

Solution:

(a) x: domain of all students
y: domain of all clubs
R(x,y): "student **'x'** joined club **'y'**"

(i) $\forall x(R(x, dance) \vee R(x, aerobics))$

$(ii)\forall x(R(x, cooking) \vee \neg R(x, aerobics))$

$(iii)\forall x(R(x, bowling) \rightarrow \neg R(x, cooking))$

$(iv)\forall x(\neg R(x, bowling) \rightarrow R(x, football))$

$(v)\forall x(R(x, football) \rightarrow R(x, gardening))$

$(vi) \neg R(Hermoine, dance)$

(b) "Are all students in the Bowling club?"

-	Claim	Justification
(1)	(i) $\forall x(R(x, dance) \vee R(x, aerobics))$	given
(2)	(ii) $\forall x(R(x, cooking) \vee \neg R(x, aerobics))$	given
(3)	(iii) $\forall x(R(x, bowling) \rightarrow \neg R(x, cooking))$	given
(4)	(iv) $\forall x(\neg R(x, bowling) \rightarrow R(x, football))$	given
(5)	(v) $\forall x(R(x, football) \rightarrow R(x, gardening))$	given
(6)	(vi) $\neg R(Hermoine, dance)$	given
(7)	R(Hermoine, dance) $\vee R(Hermoine, aerobics)$	Universal Instantiation of (1)
(8)	R(Hermoine, aerobics)	Disjuntive Syllogism of (6) and (7)
(9)	R(Hermoine, cooking) $\vee \neg R(Hermoine, aerobics)$	Universal Instantiation of (2)
(10)	R(Hermoine, aerobics) $\rightarrow R(Hermoine, cooking)$	Relation by Implication of (9)
(11)	R(Hermoine, cooking)	Modus Ponens of (8) and (10)
(12)	R(Hermoine, bowling) $\rightarrow \vee \neg R(Hermoine, cooking)$	Universal Instantiation of (3)
(13)	$\neg R(Hermoine, bowling \rightarrow R(Hermoine, cooking)$	Contraposition of (12)
(14)	$\neg R(Hermoine, bowling)$	Modus Tollens of (11) and (12)
(15)	$\exists x(\neg R(x, bowling))$	Existential Generalization of (13)

ANSWER: Hermoine did not join the bowling club. Therefore, there is at least one student not in the bowling club. $\exists x(\neg R(x, bowling))$

(c) "What clubs did Hermoine join?"

-	Claim	Justification
(1)	(i) $\forall x(R(x, dance) \vee R(x, aerobics))$	given
(2)	(ii) $\forall x(R(x, cooking) \vee \neg R(x, aerobics))$	given
(3)	(iii) $\forall x(R(x, bowling) \rightarrow \neg R(x, cooking))$	given
(4)	(iv) $\forall x(\neg R(x, bowling) \rightarrow R(x, football))$	given
(5)	(v) $\forall x(R(x, football) \rightarrow R(x, gardening))$	given
(6)	(vi) $\neg R(Hermoine, dance)$	given
(7)	R(Hermoine, dance) $\vee R(Hermoine, aerobics)$	Universal Instantiation of (1)
(8)	R(Hermoine, aerobics)	Disjunctive Syllogism (6) and (7)
(9)	R(Hermoine, cooking) $\vee \neg R(Hermoine, aerobics)$	Universal Instantiation of (2)
(10)	R(Hermoine, aerobics) $\rightarrow R(Hermoine, cooking)$	Relation by Implication of (4)
(11)	R(Hermoine, cooking)	Modus Ponens of (8) and (10)
(12)	R(Hermion, bowling) $\rightarrow \neg R(Hermoine, cooking)$	Universal Instantiation of (3)
(13)	$\neg R(Hermoine, bowling)$	Modus Ponens of (11) and (12)
(14)	$\neg R(Hermoine, bowling) \rightarrow R(Hermoine, football)$	Universal Instantiation of (4)
(15)	R(Hermoine, football)	Modus Ponens of (13) and (14)
(16)	R(Hermoine, football) $\rightarrow R(Hermoine, gardening)$	Universal Instantiation of (5)
(17)	R(Hermoine, gardening)	Meodus Ponens of (15) and (16)

ANSWER: Hermoine joined the aerobics, cooking, football, and gardening club. She did not join the dance or the bowling club.

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(2) For each of the following claims, prove or disprove the claim:

(a) For the domain of all 3-digit numbers n, if the sum of the digits of n is divisible by 9, then n is divisible by 9.

(b) For natural numbers n, $n^2 + n + 17$ is always prime.

Solution:

(a) **Direct Proof:**
n = abc
n = 100a + 10b + c
a + b + c = 9k
100a + 10b + c = 9k + 99a + 9b
n = 9k + 99a + 9b
n = 9k + 9(11a + b)

ANSWER: Lets say n = abc, where a is the hundreds place digit, b is the tenths place digit, and c is the ones place digit. The proof above shows that if n = 100a + 10b + c. If the sum of the digits of n are divisible by 9, where the sum is equal to some number k, we can say, a + b + c = 9k. With some basic match, we can substitute n into the second equation and find that n = 9k + 9(11a + b). This tells us that n is divisible by 9 when the sum is equal to 9k + 9(11a + b). This means that for some sum k with the the additive (11a + b) and multiplier 9, both the sum and n are divisible by 9.

(b) **Proof by Contradiction:**
n = 17 and $n^2 + n + 17$ = k
$(17)^2 + (17) + 17 = 289$

ANSWER: 289 is not a prime number and therefore the claim "For natural numbers n, $n^2 + n + 17$ is always prime" is false.

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(3) Prove that n^3 is odd if and only if $n + 1$ is even.

Solution:

Proof by Contradiction:
let (n + 1) be odd

(n + 1) = 2k + 1 where k ∈ (Z)
$n = 2k$
n^3 = $8k^3$

Therefore, n^3 is always even, if (n + 1) is odd, as shown by the multiplier 8.

Direct Proof:
let (n + 1) be even

(n + 1) = 2k
n = 2k - 1
$n^3 = (2k - 1)^3$
$n^3 = 8k^3 - 12k^2 + 6k - 1$

Therefore, n^3 is always odd, if (n + 1) is even, as shown by the additive -1.

Proved: (n+1) must be even for n^3 to be odd.

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(4) Let p and q be integers and define $r = pq + p + q$. Prove that r is even if and only if p and q are both even.

Solution:

Proof by Contradiction:
let p and q be odd where
p = 2k + 1
q = 2m + 1

r = pq + p + q
r = (2k + 1)(2m + 1) + (2k + 1) + (2m + 1)
r = 4km + 2k + 2m + 1 + 2k + 1 + 2m + 1
r = 4km + 4k + 4m + 3 r = 4(km + k + m) + 3 Therefore, r is always odd, if p and q are odd, as shown by the additive 3.

Direct Proof:
let p and q be even where
p = 2k
q = 2m

r = pq + p + q
r = (2k)(2m) + (2k) + (2m)
r = 4km + 2k + 2m
r = 2(2km + k + m)
Therefore, r is always even, if p and q are even, as shown by the multiplier 2.

Proved: p and q must be even for r to be even.

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(5) Fully list the power set of the set $A = \{\{beagle,\ westie\}, 3, \alpha\}$.

Solution: P(A) = $\{\{\emptyset\}, \{3\}, \{\alpha\}, \{3, \alpha\}, \{\{beagle\}, \{westie\}\}, \{\{beagle, westie\}, 3\}, \{\{beagle, westie\}, \alpha\}$, $\{\{beagle, westie\}, 3, \alpha\}\}$

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(6) For sets A and B, the cardinality of their union is given by: $|A \cup B| = |A| + |B| - |A \cap B|$. Derive the following analogous rule for the union of three sets:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

[Hint: Start by treating $(B \cup C)$ as one set and then apply the two-set rule given above, along with set identities from Table 1 in Section 2.2.]

Solution:

Direct Proof:

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\begin{array}{ll}
\mid A \cup B \mid = \mid A \mid + \mid B \mid - \mid A \cap B \mid & (1)\ \mathit{given} \\
\mid A \cup B \cup C \mid & (2)\ \mathit{given} \\
\equiv \mid (A \cup B) \cup C \mid & (3)\ \mathit{association} \\
\equiv \mid A \cup B \mid + \mid C \mid - \mid (A \cup B) \cap C \mid & (4)\ \mathit{apply}(1) \\
\equiv \mid A \mid + \mid B \mid - \mid A \cap B \mid + \mid C \mid - \mid (A \cup B) \cap C \mid & (5)\ \mathit{apply}(1) \\
\equiv \mid A \mid + \mid B \mid + \mid C \mid - \mid A \cap B \mid - [\mid A \cap C \mid \cup \mid B \cap C \mid] & (6)\ \mathit{distributivelaws} \\
\equiv \mid A \mid + \mid B \mid + \mid C \mid - \mid A \cap B \mid - [\mid A \cap C \mid + \mid B \cap C \mid - (\mid A \cap C \mid \cap \mid B \cap C \mid)] & (7)\ \mathit{apply}(1) \\
\equiv \mid A \mid + \mid B \mid + \mid C \mid - \mid A \cap B \mid - \mid A \cap C \mid - \mid B \cap C \mid + [\mid A \cap C \mid \cap \mid B \cap C \mid] & (8)\ \mathit{distribution}(7) \\
\equiv \mid A \mid + \mid B \mid + \mid C \mid - \mid A \cap B \mid - \mid A \cap C \mid - \mid B \cap C \mid + \mid A \cap B \cap C \mid & (9)\ \mathit{absorption}(8)
\end{array}
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NB: Remember that to prove a claim, you must prove it *in general* (i.e., for all cases), and to disprove a claim, you should present a counterexample. If you prove a claim, be sure to indicate whether you are using a Direct Proof, a Contrapositive Proof, a Proof by Cases, a Proof by Contradiction, or a Proof by Construction (existence proof). If you use a Proof by Cases, be sure to indicate what each case is, and in each case, which type of proof you are using.