CSCI 3656: Numerical Computation **Pourna Sengupta**
Homework 3 September 25, 2020

Python Code:

```
"""
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used http://hplgit.github.io/Programming-for-Computations/pub/p4c/._p4c-solarized-
Python031.html
https://en.wikibooks.org/wiki/Python_Programming/Basic_Math
to help write code
"""
import sys
import math
import numpy
import matplotlib.pyplot as plt


"Define Newton's Method Function"
"f = intial function f(x) = 0"
"df = derivative of f(x) f'(x)"
"p0 = given initial approximation p_0"
"tol = tolerance -> stopping criteria for |f(x) < tol|"
"n = max number of iterations of newton's method n "
def newtonsMethod(f, df, p0, tol, n):

    "create empty arrays to store values"
    astore = []
    bstore = []

    "new variable fx to represent f(x) at each iteration of p"
    "f(p0) = f(p_0) initial approximation value"
    "set f(x) = f(p_0)"
    fx = f(p0)

    "new variable i iteration counter"
    i = 0


    "RUN NEWTON'S METHOD FOR N ITERATIONS"
    "while loop with 2 conditions"
    "|fNorm| > tol and i < n"
```

```
    "fNorm must be greater than the tolerance"
    "iteration must be less than max iterations"
    while abs(fx) > tol and i < n:
        "solve p_i = p_o - (f(p_0)/f'(p_0))"
        "set p_i as new value"
        "p0 = p0 - float(fx)/df(p0)"

        try:
            p0 = p0 - float(fx)/(df(p0))
        except ZeroDivisionError:
            print("Zero Derivative for x = %f", p0)
            sys.exit("Solution not found")

        "update value of fx"
        "fx = f(p_0)"
        fx = f(p0)

        "append values to arrays"
        astore.append(p0)
        bstore.append(fx)

        "update iterator"
        i +=1


    if abs(fx) > tol:
        "update iterator if solution is found"
        "or max number of iterations reached"
        i = -1

    return p0, i, astore, bstore

def f(p0):
    return (1)/(1 + math.exp(p0)) - 1/2

def df(p0):
    return -(math.exp(p0))/(1 + math.exp(p0))**2

solution, zeroI, avalues, bvalues = newtonsMethod(f, df, p0 = 0.25, tol=1.0e-14, n=100)

"if the solution is found"
if zeroI > 0:
    print ("Number of iterations: %d" % (1+2*zeroI))
    print ("Solution: %f" %(solution))
else:
    print("Solution not found")
```

```
a = numpy.array([i for i in avalues])
b = numpy.array(bvalues)

fig = plt.figure()
plt.plot(a, b, label = 'Newtons Method')
plt.legend()

def f1(x):
    return numpy.int((1)/(1 + math.exp(x)) - 1/2)

def derive(x):
    h = 0.000000001
    return (f1(x+h) - f1(x)/h)

def tanLine():
    x = numpy.linspace(-5, 5, 100)
    y = x**2 + 2;
    plt.plot(x, y, 'b-', 'LineWidth', 2);
    plt.grid(True)



    xTangent = -4.5;

    slope = 2 * xTangent;

    yTangent = xTangent**2 + 2;

    plt.plot(xTangent, yTangent, 'r*', 'LineWidth', 2, 'MarkerSize', 10);

    yTangentLine = slope * (x - xTangent) + yTangent;
    plt.plot(x, yTangentLine, 'b-', 'LineWidth', 2);
```

Code Output (graphs attached to questions 2 and 4):

```
In [95]: runfile('/Users/pournasengupta/Dropbox/Fall2020/CSCI 3656/Homework 3/newtons-
method.py', wdir='/Users/pournasengupta/Dropbox/Fall2020/CSCI 3656/Homework 3')
Number of iterations: 7
Solution: 0.000000
```

Graphs were being funky, so I eventually gave up. I understand the concept but am having trouble coding it, so I drew it out to show that I understood the concept.

Question 1:

$f'(x) = \frac{d}{dx}(\frac{1}{1+e^x} - \frac{1}{2})$

$= \frac{d}{dx}(\frac{1}{1+e^x}) - \frac{d}{dx}(\frac{1}{2}$

explonent rule
$= \frac{d}{dx}((1+e^x)^-1)$

chain rule
$= -\frac{1}{(1+e^x)^2}\frac{d}{dx}(1+e^x)$

take derivative
$= -\frac{1}{(1+e^x)^2} * e^x$

simplify
$= \frac{-e^x}{(1+e^x)^2}$

Answer: $f'(x) = \frac{-e^x}{(1+e^x)^2}$

Question 2:

Algorithm for Newton's Method Approximation

Given: f(x) = 0 and p_0 (initial approximation)

find the approximate solution p or print failure message
tol for tolerance
n for max number of iterations

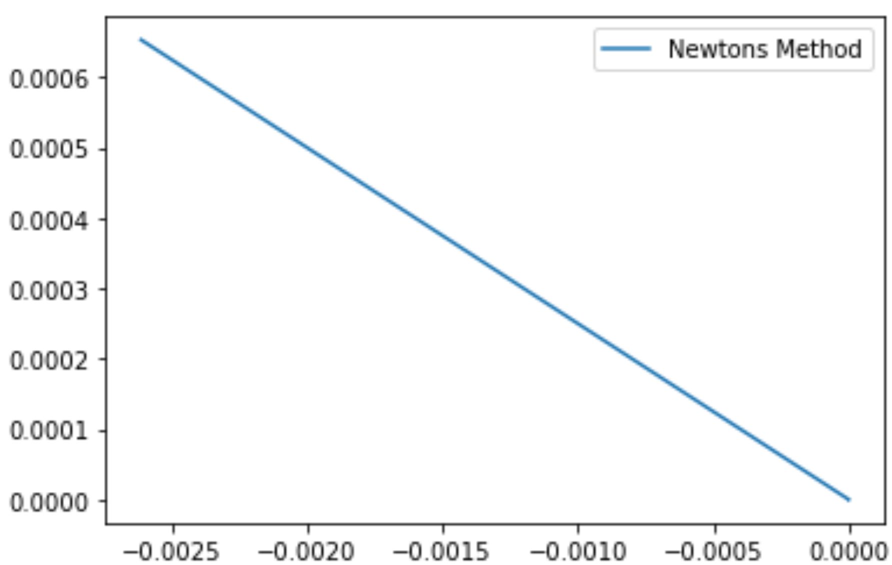
$i = 1$
$while(i =< n)$
  $p_i = p_0 - \frac{f(p_0)}{f'(p_0)}$
  $if|p_i - p_0| < tol$
    $printp_i$
  $i++$
  $(p_0 = p_i)$
method failed after n iterations

$f(x) = \frac{1}{1+e^x} - \frac{1}{2}$

in python: (1) / (1 + math.exp(x)) - (1)/(2)

$f'(x) = -\frac{e^x}{(1+e^x)^2}$

in python: -(math.exp(x))/((1 + math.exp(x)))**2

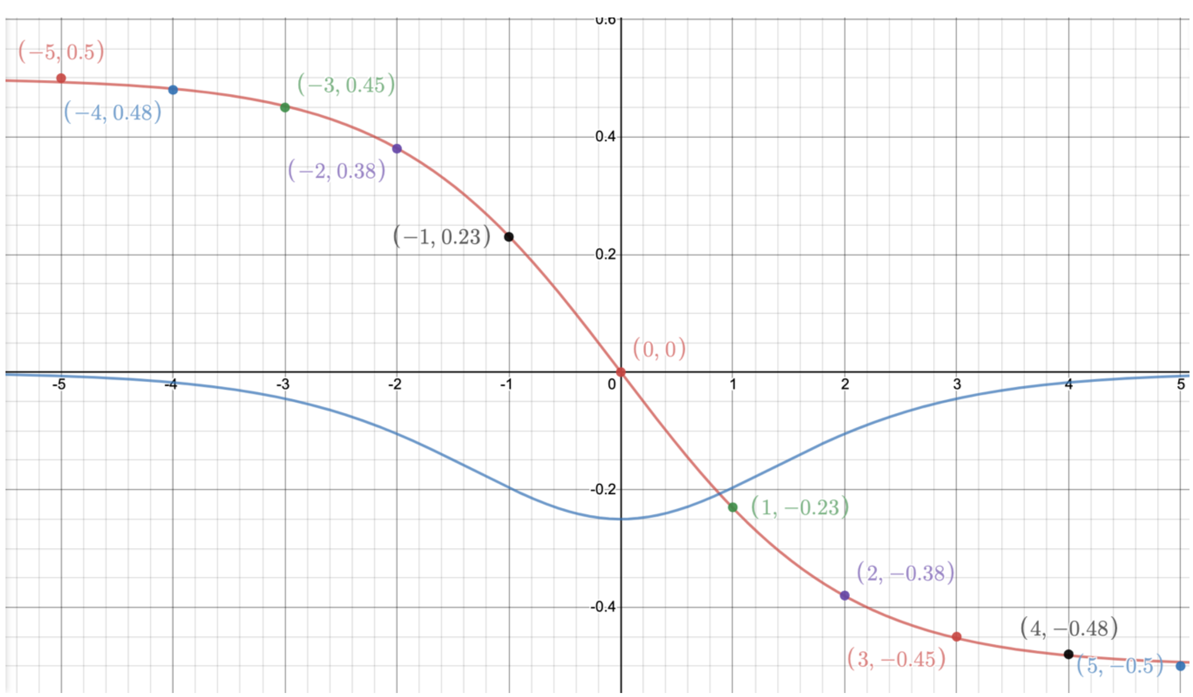


Question 3:

x	f(x)
-5	0.5
-4	0.48
-3	0.45
-2	0.38
-1	0.23
0	0
5	-0.5
4	-0.48
3	-0.45
2	-0.38
1	-0.23

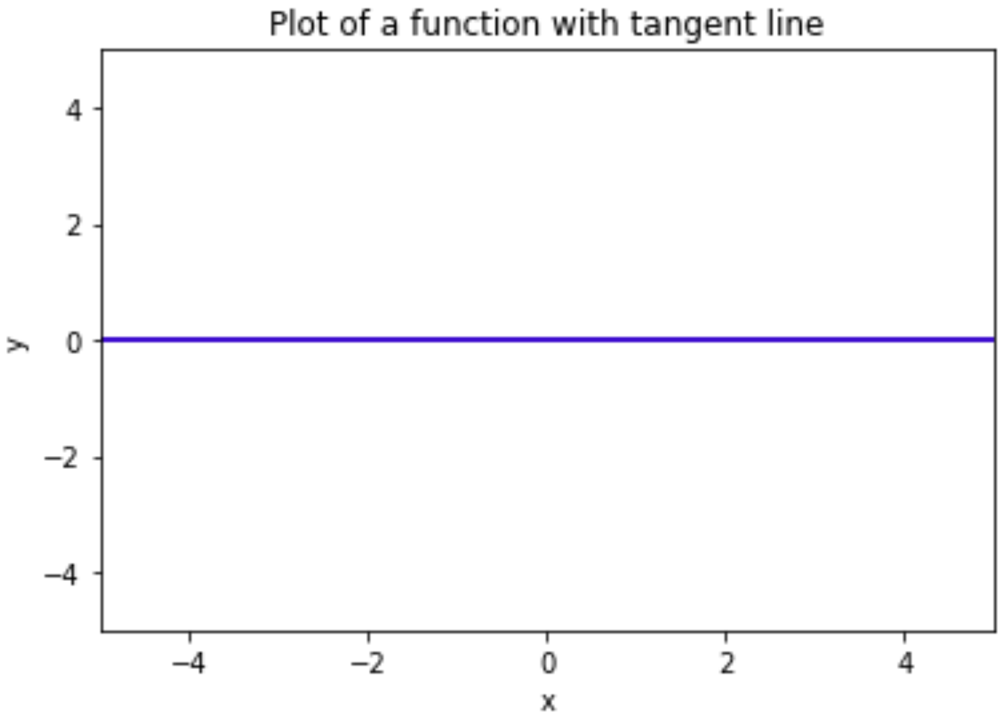
1 $$f(x) = \frac{1}{1+e^{x}} - \frac{1}{2}$$

2 $$f'(x)$$



Question 4:

Newton's Method will converge for any initial guess in the intervals x_0 ϵ[-2, 0] or x_0 ϵ[0, 2]



Bonus Question 2:

To find the convergence of f'(x), I used both the root and ratio tests. Both showed convergence, as the limit L was less than 1. The ratio test showed that the interval [-0.625, 0) and (0, 0.625] converged. My interval is much larger than the interval found through the ratio test but from my graphs, it makes sense that the initial guesses converge within such small intervals.