# Bits and Bytes

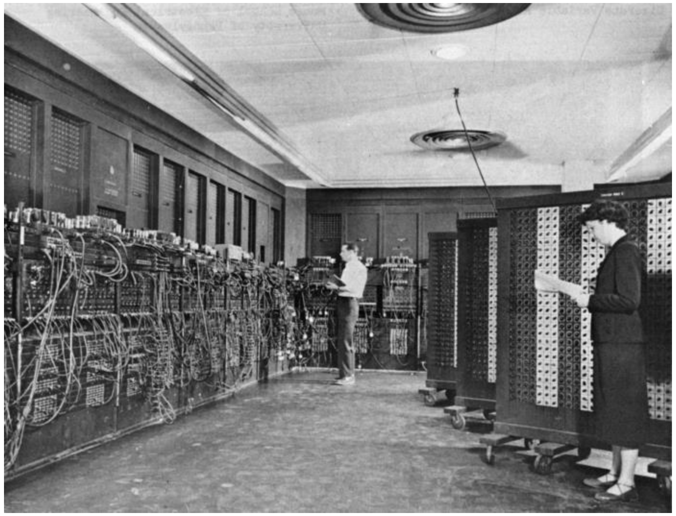
Reading: CS:APP Chapter 2.1

**These slides adapted from materials provided by the textbook authors.**

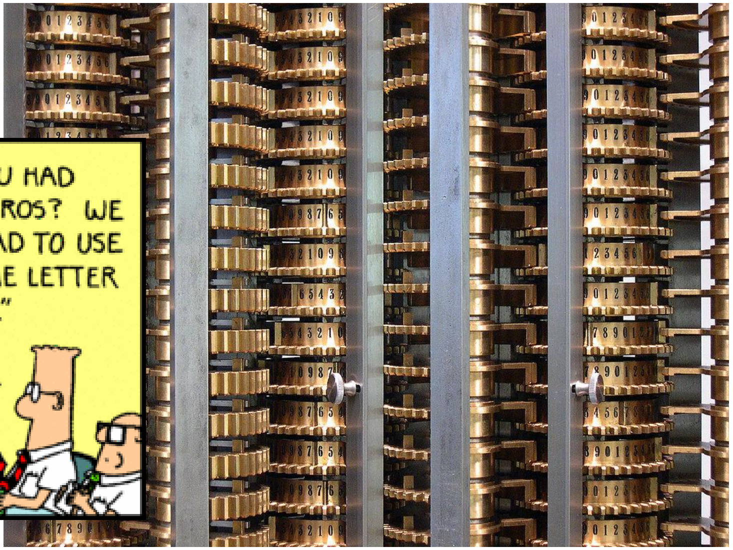
# Bits, Bytes, and Integers

- **Early Computers**
- **Representing information as bits**
- **Bit-level manipulations**

# Early Computers Used Many Bases

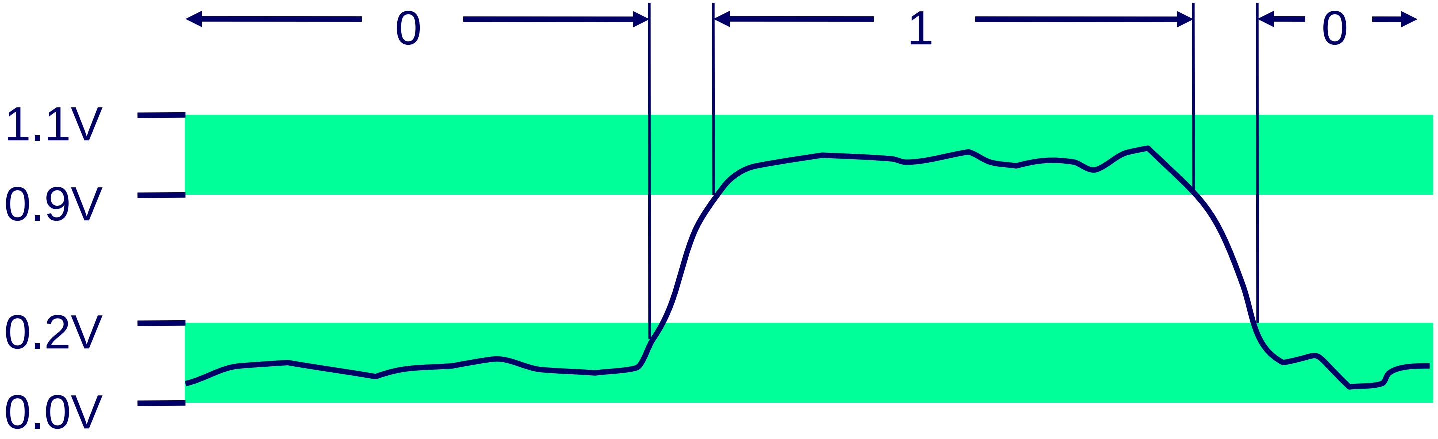


- **Babbage 1830's difference engine used decimal**
- **Fowler's wooden calculating machines in 1840's used Ternary**
- **ENIAC used 10's complement**
  **https://en.wikipedia.org/wiki/Method_of_complements**

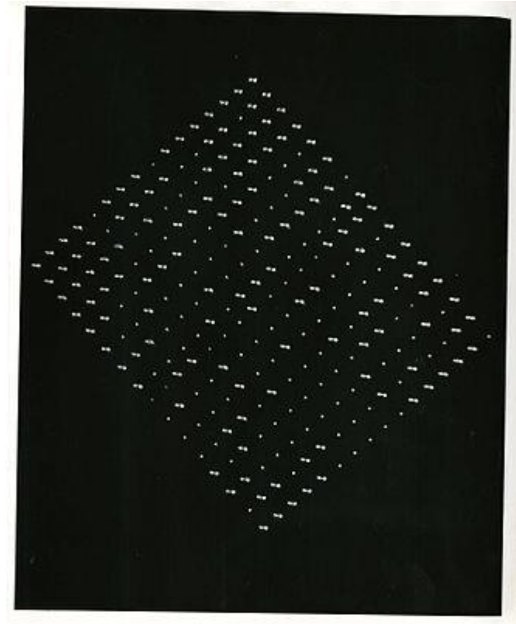


# Everything is bits

- **Each bit is 0 or 1**
- **By encoding/interpreting sets of bits in various ways**
  - Computers determine what to do (instructions)
  - … and represent and manipulate numbers, sets, strings, etc…
- **Why bits?  Electronic Implementation**
  - Easy to store with bi-stable elements
  - Reliably transmitted on noisy and inaccurate wires

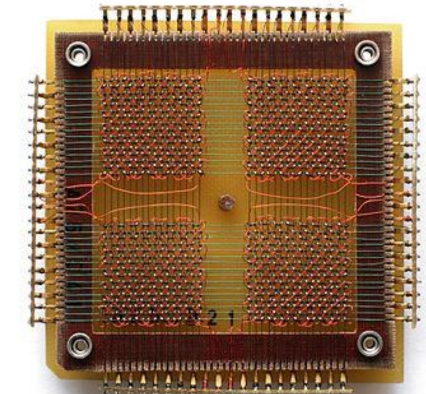


# Bits in Real Life



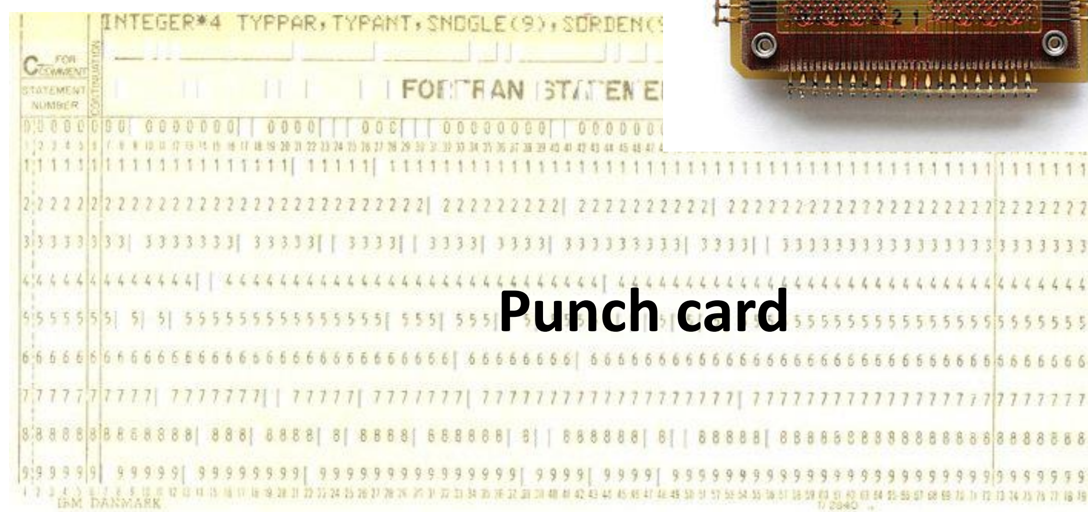
Williams Tube

Core Memory



Paper Tape & TTY

Jaquard Loom




Punch card

(All images from Wikimedia)

# For example, can count in binary

- **Base 2 Number Representation**
  - Represent $15213_{10}$ as $11101101101101_2$
  - Represent $1.20_{10}$ as $1.0011001100110011[0011]..._2$
  - Represent $1.5213 \times 10^4$ as $1.1101101101101_2 \times 2^{13}$

| Octal | Decimal | Binary |
|---|---|---|
| 0 | 0 | 0000 |
| 1 | 1 | 0001 |
| 2 | 2 | 0010 |
| 3 | 3 | 0011 |
| 4 | 4 | 0100 |
| 5 | 5 | 0101 |
| 6 | 6 | 0110 |
| 7 | 7 | 0111 |

- **Octal**
  - Octal (base-8) uses digits 0, 1, 2, 3, 4, 5, 6, 7
  - Represent $11101101101101_2$
    $\rightarrow$ $11\_101\_101\_101\_101_2$
    $\rightarrow$ $035555_8$
  - In C/C++ leading zero (0) indicates octal rather than decimal
  - Octal still used when manipulating file access (r/w/x)

# Encoding Byte Values

- **Hexadecimal $00_{16}$ to $FF_{16}$**
  - Base 16 number representation
  - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
  - Convert $11101101101101_2$
    $11\_1011\_0110\_1101_2$
    $3\ B\ \ 6\ D_{16}$
  - Write $FA1D37B_{16}$ in C as
    - 0xFA1D37B
    - 0xfa1d37b

- **Byte = 8 bits = 2 hex digits**
  - Binary $00000000_2$ to $11111111_2$
  - Decimal: $0_{10}$ to $255_{10}$

| Hex | Decimal | Binary |
|---|---|---|
| 0 | 0 | 0000 |
| 1 | 1 | 0001 |
| 2 | 2 | 0010 |
| 3 | 3 | 0011 |
| 4 | 4 | 0100 |
| 5 | 5 | 0101 |
| 6 | 6 | 0110 |
| 7 | 7 | 0111 |
| 8 | 8 | 1000 |
| 9 | 9 | 1001 |
| A | 10 | 1010 |
| B | 11 | 1011 |
| C | 12 | 1100 |
| D | 13 | 1101 |
| E | 14 | 1110 |
| F | 15 | 1111 |

# Example Data Representations

| C Data Type | Typical 32-bit | Typical 64-bit | x86-64 |
|---|---|---|---|
| `char` | 1 | 1 | 1 |
| `short` | 2 | 2 | 2 |
| `int` | 4 | 4 | 4 |
| `long` | 4 | 8 | 8 |
| `float` | 4 | 4 | 4 |
| `double` | 8 | 8 | 8 |
| `long double` | – | – | 10/16 |
| pointer | 4 | 8 | 8 |

# Today: Bits, Bytes, and Integers

- Representing information as bits
- **Bit-level manipulations**
  - Boolean operations
  - Bit vectors
  - Bit vectors as sets

# Boolean Algebra

- **Developed by George Boole in 19th Century**
  - Algebraic representation of logic
    - Encode "True" as 1 and "False" as 0
- **Similar rules to integer numbers, but not exactly same**
  - a(b+c) = ab + ac
  - a+(bc) = (a+b)(a+c)

**And, &, ***

| A | B | A&B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

**Or, |, +**

| A | B | A \| B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

**Xor, ^ , ⊕**

| A | B | A^ B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

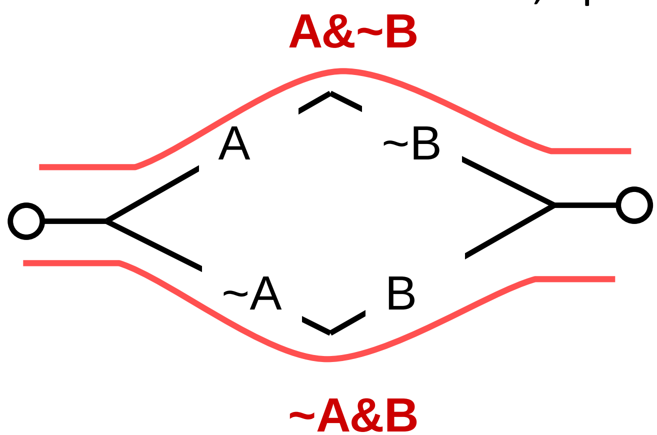
**Not, ~**

| A | ~A |
|---|---|
| 0 | 1 |
| 1 | 0 |

# Application of Boolean Algebra

## Applied to Digital Systems by Claude Shannon

- 1937 MIT Master's Thesis
- Reasoned about networks of relay switches
  - Encode closed switch as 1, open switch as 0




**Connection when**

**A&~B | ~A&B**

**= A^B**

- Confusingly, there are multiple notations used:
  - C: P & Q, P | Q, P^Q, ~P
  - ECE-land: PQ, P+Q, $p \oplus q$ , $\bar{P}$
  - Symbolic logic-land': $p \oplus q \quad = \quad (p \vee q) \wedge \neg(p \wedge q)$

# General Boolean Algebras

- **Operate on Bit Vectors**
  - Operations applied bitwise

```
  01101001      01101001      01101001
& 01010101    | 01010101    ^ 01010101    ~ 01010101
  01000001      01111101      00111100      10101010
```

- **All of the Properties of Boolean Algebra Apply**

# Example: Representing & Manipulating Sets

- **Representation**
  - Width w bit vector represents subsets of {0, …, w–1}
  - $a_j = 1$ if $j \in A$

    - 01101001 { 0, 3, 5, 6 }
    - *76543210*

    - 01010101 { 0, 2, 4, 6 }
    - *76543210*

- **Operations**
  - & Intersection 01000001 { 0, 6 }
  - | Union 01111101 { 0, 2, 3, 4, 5, 6 }
  - ^ Symmetric difference 00111100 { 2, 3, 4, 5 }
  - ~ Complement 10101010 { 1, 3, 5, 7 }

# Bit-wise Ops Demo w/ `gdb`

# Bit-Level Operations in C

- **Operations &, |, ~, ^ Available in C**
  - Apply to any “integral” data type
    - long, int, short, char, unsigned
  - View arguments as bit vectors
  - Arguments applied bit-wise
- **Examples (Char data type)**
  - ~0x41 ⇒ 0xBE
    - $\sim 01000001_2 \Rightarrow 10111110_2$
  - ~0x00 ⇒ 0xFF
    - $\sim 00000000_2 \Rightarrow 11111111_2$
  - 0x69 & 0x55 ⇒ 0x41
    - $01101001_2 \,\&\, 01010101_2 \Rightarrow 01000001_2$
  - 0x69 | 0x55 ⇒ 0x7D
    - $01101001_2 \mid 01010101_2 \Rightarrow 01111101_2$

# Contrast: Logic Operations in C

- **Contrast to Logical Operators**
  - &&, ||, !
    - View 0 as “False”
    - Anything nonzero as “True”
    - Always return 0 or 1
    - Early termination
- **Examples (char data type)**
  - !0x41 ⇒ 0x00
  - !0x00 ⇒ 0x01
  - !!0x41 ⇒ 0x01
  - 0x69 && 0x55 ⇒ 0x01
  - 0x69 || 0x55 ⇒ 0x01
  - p && *p (avoids null pointer access)

# Contrast: Logic Operations in C

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**Watch out for && vs. &, || vs. |, and = vs. == ... one of the more common oopsies in C programming**

# Masks and Shifting Bit Vectors

- **Bit vectors are commonly used for *masks***

- **Typically involves shifting bit vectors**
  - $1011\_1110_2 << 3$ becomes $1111\_0000_2$
  - $1011\_1110_2 >> 3$ becomes $0001\_0111_2$

    or $1111\_0111_2$

  - **Logical or arithmetic shift depends on the "integer representation", but masking idiom is very common**

# Bit-wise Programming Idioms

### Extract Last Byte

- **Task:** Given hex value like 0xb01dface, extract last byte ('ce')

- 0xb01dface & 0xff

### Extract All but Last Digit

- **Task:** Given hex value like 0xb01dface, extract last byte ('ce'), e.g. 0xb01dfa00

- 0xb01dface & ~0xff

# Shift Operations

- **Left Shift: `x << y`**
  - Shift bit-vector **x** left **y** positions
    - – Throw away extra bits on left
    - Fill with 0's on right
- **Right Shift: `x >> y`**
  - Shift bit-vector **x** right **y** positions
    - Throw away extra bits on right
  - Logical shift
    - Fill with 0's on left
  - Arithmetic shift
    - Replicate most significant bit on left
- **Undefined Behavior**
  - Shift amount < 0 or ≥ word size

| Argument x | 01100010 |
|---|---|
| << 3 | 00010000 |
| Log. >> 2 | 00011000 |
| Arith. >> 2 | 00011000 |

| Argument x | 10100010 |
|---|---|
| << 3 | 00010000 |
| Log. >> 2 | 00101000 |
| Arith. >> 2 | 11101000 |

Bryant and O'Hallaron, Computer Systems: A Programmer's Perspective, Third Edition