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CSCI 3104, Algorithms **Escobedo & Jahagirdar**
Homework 2A (45 points) **Summer 2020, CU-Boulder**

Advice 1: For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.

Advice 2: Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.

Instructions for submitting your solution:

- The solutions **should be typed**, we cannot accept hand-written solutions. Here's a short intro to **Latex**.
- In this homework we denote the asymptomatic *Big-O* notation by $\mathcal{O}$ and *Small-O* notation is represented as o.
- We recommend using online Latex editor **Overleaf**. Download the **.tex** file from Canvas and upload it on overleaf to edit.
- You should submit your work through **Gradescope** only.
- If you don't have an account on it, sign up for one using your CU email. You should have gotten an email to sign up. If your name based CU email doesn't work, try the identikey@colorado.edu version.
- Gradescope will only accept **.pdf** files (except for code files that should be submitted separately on Canvas if a problem set has them) and **try to fit your work in the box provided**.
- You cannot submit a pdf which has less pages than what we provided you as Gradescope won't allow it.

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Piazza threads for hints and further discussion

Piazza Threads
Question 1
Question 2
Question 3
Question 4
Question 5

Recommended reading: Divide Conquer; Recurrence Relations: Ch. 2 2.3; Ch. 4 4.1, 4.2, 4.3, 4.4, 4.5. The three methods for solving recurrences from **4.3 - 4.5** is important.

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1. *(5 × 4 = 20 pts) Solve the following recurrence relations. For each case, show your work.*

   (a) $T(n) = 3T(n-1) + 1$ *if* $n > 1$ *and* $T(1) = \Theta(1)$

T(n-1) = 3(3T(n-2)+1)+1
T(n-1) = $3^2T(n-2) + 3 + 1$

$T(n-2) = 3(3^2T(n-3) + 3 + 1) + 1$
$T(n-2) = 3^3T(n-3) + 3^2 + 3 + 1$

$T(n-3) = 3(3^3T(n-4) + 3^2 + 3 + 1) + 1$
$T(n-3) = 3^4T(n-4) + 3^3 + 3^2 + 3 + 1$

$T(n) = 3^kT(n-k) + 3^{k-1} + 3^{k-2} + ... + 3 + 1$
$T(1) = \Theta(1)$
$k = n - 1$
$T(n) = 3^{n-1}T(1) + 3^{n-2} + 3^{n-3} + ... + 3 + 1$
$T(n) = \Theta(3^n)$

(b) $T(n) = 2T(\frac{n}{3}) + \Theta(n)$ *if* $n > 1$ *and* $T(1) = \Theta(1)$

Case 3 of the Master's Theorem
1. $n^{\log_b(a)} = n^{\log_3(2)}$
$\log_3(3) = 1$
$\log_3(2) < 1$

2. f(n) = n

3. $n > n^{\log_3(2)}$
$n = n^{(\log_3(2)+\epsilon)}$
$\epsilon = 1 - \log_3(2)$
therefore, $\epsilon > 1$

$af(n/b) \leq cf(n)$
$a(n/b) \leq c(n)$
$n * 2/3 \leq cn$
$2/3 \leq c < 1$
$c = \frac{2.1}{3}$

$T(n) = \Theta(f(n))$
$T(n) = \Theta(n)$

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(c) $T(n) = 3T(\frac{n}{2}) + \Theta(n)$ *if* $n > 1$ *and* $T(1) = \Theta(1)$

Case 3 of the Master's Theorem

1. $n^{\log_b(a)} = n^{\log_2(3)}$

$\log_2(2) = 1$

2. f(n) = n

3. f(n) = n = $n^{(\log_2(3-\epsilon))}$

$3 - \epsilon = n$

T(n) = $\Theta(f(n))$

$T(n) = \Theta(n^{(\log_2(3)})$

(d) $T(n) = 4T(\frac{n}{2}) + \Theta(n^2)$ *if* $n > 1$ *and* $T(1) = \Theta(1)$

Case 2 of Master's Theorem
$n^{log_b(a)} = n^{log_2(4)}$

$f(n) = \Theta(n^2) n^{\log_b(a)}$
$f(n) = n^{\log_2(4)}$
$f(n) = n^2$
$\Theta(n^2) = n^2$
$f(n) = \Theta(n^{\log_b(a)}) = \Theta(n^2)$
$T(n) = \Theta(n^2 log(n))$

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2. *(5 pts) Can master's theorem be applied to solve the following recurrence* $T(n) = 8T(\frac{n}{2}) + n^3 \log(n)$. *If* $n > 1$ *and* $T(1) = \Theta(1)$
Prove that the above recurrence cannot be solved using Master's theorem by showing that none of the three conditions required to apply the theorem are satisfied by the recurrence.

a = 8, b = 2, f(n) = $n^3 log(n)$
1.$n^{\log_b(a)} = n^{\log_2(8)} = n^3$
2.$f(n) = n^{log_b(a)}$
3.$n^3 log(n) = n^3$
Case 1: *doesnotapply*
$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n^3 \log(n)}{n^{3-\epsilon}} = \lim_{n\to\infty} \frac{\log(n)}{n^{-\epsilon}} = \lim_{n\to\infty} n^{\epsilon} \log(n) = \infty.$$

Case 2: does not apply since $n^3 log(n)$ does not equal $\Theta(n^3)$
$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n^3 \log(n)}{n^3} = \lim_{n\to\infty} \log(n) = \infty.$$
$n^3 log(n)$ grows faster than n^3 so it is not $\Theta(n^3)$

Case 3: does not apply since there is no $\epsilon > 0$ that makes $n^(2 + \epsilon)$ the asymptotic lower bound for f(n)
$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n^3 \log(n)}{n^{3+\epsilon}} = \lim_{n\to\infty} \frac{\log(n)}{n^{\epsilon}} = \lim_{n\to\infty} \frac{\frac{1}{n}}{\epsilon\, n^{\epsilon-1}} = \lim_{n\to\infty} \frac{1}{\epsilon\, n^{\epsilon}} = 0$$

Since none of the cases work, the Master's Theorem is not satisfied by the recurrence relation.

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3. *(5 × 2 = 10 pts) Consider the following functions. For each of them, determine how many times is 'hi' printed in terms of the input n (i.e in Asymptotic Notation of n). You should first write down a recurrence and then solve it using the* ***recursion tree method****. That means you should write down the first few levels of the recursion tree, specify the pattern, and then solve.*

(a)

```
1 def fun(n) {
2     if (n > 1) {
3         print( 'hi' 'hi' 'hi' )
4         fun(n/3)
5         fun(n/3)
6     }
7 }
```

Recurrence: T(n) = 2T(n/3)+1
$T(n) \to 1$
(branches into)
$T(n/3)$ and $T(n/3) \to 2$
(branches into)
$T(n/3^2)$ and $T(n/3^2)$ and $T(n/3^2)$ and $T(n/3^2) \to 2^2$
(branches into)
$T(n/3^k) \to 2^k$
$n/3^k = 1$ and $n = 3^k$
$k = \log_3(n)$
Each call prints 'hi' 3 times. Therefore, the number of times 'hi' is printed is $3 * (2^{\log_3(n)+1} - 1)$.

(b)

```
1 def fun(n) {
2     if (n > 1) {
3         for i=1 to n {
4             print( 'hi' 'hi' )
5         }
6         fun(n/3)
7         fun(n/3)
8     }
9 }
```

Recurrence: T(n) = 2T(n/3)+n
$n \to n$
(branches into)
$n/3$ and $n/3 \to 2n/3$
(branches into)
$n/3^2$ and $n/3^2$ and $n/3^2$ and $n/3^2 \to 2^2 * n/3^2$
(branches into)
$n/3^k \to 2^k * n/3^k$
$k = \log_3(n)$
Each call prints 'hi' 2 times. Therefore, the number of times 'hi' is printed is $2 * (n \log_3(n) + n^{\log_3(2)})$.

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4. *(5 pts) Let* $T(n) = 3T(\frac{n}{3}) + n$*, where* $T(n)$ *is constant when* $n \leq 3$*. Using unrolling, determine tight asymptotic bounds for* $T(n)$*. That is, find a function* $g(n)$ *such that* $T(n) \in \Theta(g(n))$*. Clearly show all your work*

T(n) = 3T(n/3) + n
= 3(3T(n/9) + n/3) + n
= 9T(n/9) + 2n
= 27T(n/27) + 3n

T(n) = $3^k T(n/3^k) + kn$
$n/3^k = 1$
$k = \log_3(n)$

T(n) = nT(1) + $n\log_2(n)$
$T(n) = n\log_2(n) + n$
$T(n) = \Theta(n\log(n))$

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5. *(5 pts) Consider the following pseudo-code.*

```
def fun(A[1,2,3....n]) {
    if A.length == 0:
        return 0
    count = 1
    for i=1 to n:
        for j=1 to n:
            count++
    return 1 + fun(A[1,2,3.....n-2])
}
```

Find a recurrence for the worst-case runtime complexity of this algorithm. Then solve your recurrence and get a tight bound on the worst-case runtime complexity.

Recurrence Relation: T(n) = O(1) + O(1) + O(n^2) + $O(1) + T(n-2)$
$= T(n-2) + O(n^2)$
$so, T(n) = T(n-2) + n^2$

Solve the Recurrence Relation
T(n) = T(n-2) + n^2
$T(n-4) = T(n-4) + (n-2)^2$
$T(n-6) = T(n-6) + (n-4)^2$
$T(n) = T(n-4) + (n-2)^2 + n^2$
$T(n) = T(n-6) + (n-4)^2 + (n-2)^2 + n^2$
$T(n) = T(n-2k) + \sum_{i=0}^{k=1} (n-2i)^2$
$T(n) = T(n-2k) + \frac{1}{3}k(4k^2 - 6k(n+1) + 3n^2 + 6n + 2)$

Find number of iterations (or k)
n - 2k = 1
k = $\frac{(n-1)}{2}$

T(n) = T(1) + $\frac{1}{3}$ ($\frac{(n-1)}{2}$) ($4(\frac{(n-1)}{2})^2 - 6((n-1)\frac{}{2})(n+1) + 3n^2 + 6n + 2$)
$T(n) = 1\frac{}{6}$ ($n^3 + 3n^2 + 2n - 6$)
$T(n) = \Theta(n^3)$

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6. ***Extra Credit (5% of total homework grade)*** *For this extra credit question, please refer the leetcode link provided below or click here. Multiple solutions exist to this question ranging from brute force to the most optimal one. Points will be provided based on Time and Space Complexities relative to that of the most optimal solution.*

   Please provide your solution with proper comments which carries points as well.

   https://leetcode.com/problems/maximum-subarray/

   ```
   Replace this text with your source code inside of the .tex document
   ```