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INFORMATION
TECHNOLOGY

FIT2004
Algorithms and Data Structures

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Referencing materials by
Nathan Companez, Aamir Cheema, Arun Konagurthu and Lloyd Allison

GROUP OF EIGHT AUSTRALIA

Ready?

Agenda

- Complexity Analysis
  - Big-O
  - Recurrence relation

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  - Big-O
  - Recurrence relation

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- Complexity Analysis
  - Big-O
  - Recurrence relation

Covered in Tutorial 02 using tutorial questions as case study

Let us begin…

- You have done time complexity last time

Recap

- You have done time complexity last time
- You know what Big-O is

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What is the complexity of an algorithm in Big-O notation that runs in $30N \log (N^2) + 10 \log N + 8N$?

A. $O(N \log N)$
B. $O(N \log (N^2))$
C. $O(N \log (N^2) + N + \log N)$
D. Option D because

What is your answer?

Complexity Analysis
Recap

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Complexity Analysis

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A. **O(N log N)**

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C. **O(N log (N^2) + N + log N)**

D. **Option D because**

Complexity Analysis

Recap

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What is the complexity of an algorithm in Big-O notation that runs in $30N \log (N^2) + 10 \log N + 8N$?

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B. $O(N \log (N^2))$

C. $O(N \log (N^2) + N + \log N)$

D. Option D because

Log N^2 = 2 log N

Complexity Analysis
Recap

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- If I have a list, and I want to sort it with bubble sort…

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- If I have a list, and I want to sort it with bubble sort…
  - Best case?
  - Worst case?

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- If I have a list, and I want to sort it with bubble sort…
  – Best case? O(1) when list is empty
  – Worst case? O(N^2) when list is sorted in reverse order

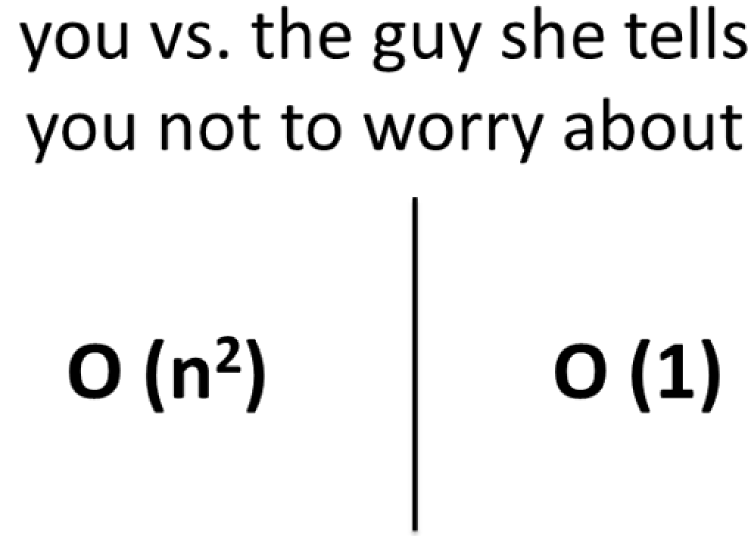
- You have done time complexity last time
- You know what Big-O is

- If I have a list, and I want to sort it with bubble sort…
  – Best case? O(1) when list is empty
  – Worst case? O(N^2) when list is sorted in reverse order

- You have done time complexity last time
- You know what Big-O is

- If I have a list, and I want to sort it with bubble sort…
  - Best case? O(N) when list is sorted and we can terminate earlier
  - Worst case? O(N^2) when list is sorted in reverse order

- You have done time complexity last time
- You know what Big-O is



- You have done time complexity last time
- You know what Big-O is

- So what’s new?

Recap

- You have done time complexity last time
- You know what Big-O is

- So what's new?
  - Space complexity
  - Time complexity for recursion

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- So what’s new?
  - Space complexity
  - Time complexity for recursion
    - By solving recurrence relation

Recap

- You have done time complexity last time
- You know what Big-O is

- So what's new?
  - Space complexity
    - We'll leave this for Lecture 02 when we go through some sorting algorithms
  - Time complexity for recursion
    - By solving recurrence relation

Questions?

- This is asked a lot
  - Final exam

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  - Final exam

- Helps you figure out the complexity of recursive functions

Complexity Analysis

Recurrence relation

- Look at the algorithm written in Python

```python
def power(x,n):
    """
    Returns x^n calculated recursively
    """
    if n == 0:
        return 1
    elif n == 1:
        return x
    else:
        return x * power (x,n-1)
```

Complexity Analysis
Recurrence relation

- Look at the algorithm written in Python

- Note: Can you explain what this function do? … and reason out the complexity?
  - Using FIT1008 knowledge

```python
def power(x,n):
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    """
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```

- Look at the algorithm written in Python
- What is the recurrence relation?

```python
def power(x,n):
    """
    Returns x^n calculated recursively
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        return 1
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        return x
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```

- Look at the algorithm written in Python
- What is the recurrence relation?
  - Base case
  - Recursive case

```python
def power(x,n):
    """
    Returns x^n calculated recursively
    """
    if n == 0:
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        return x
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        return x * power (x,n-1)
```

Complexity Analysis

Recurrence relation

- Look at the algorithm written in Python
- What is the recurrence relation? T(N)
  - Base case
  - Recursive case

```python
def power(x,n):
    """
    Returns x^n calculated recursively
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        return 1
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        return x
    else:
        return x * power (x,n-1)
```

Complexity Analysis

Recurrence relation

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- Look at the algorithm written in Python
- What is the recurrence relation? T(N)
  - Base case
    T(0) = a
  - Recursive case

```python
def power(x,n):
    """
    Returns x^n calculated recursively
    """
    if n == 0:
        return 1
    elif n == 1:
        return x
    else:
        return x * power (x,n-1)
```

Complexity Analysis
Recurrence relation

- Look at the algorithm written in Python
- What is the recurrence relation? T(N)
  - Base case
    T(0) = a
    T(1) = b
  - Recursive case

```python
def power(x,n):
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- Look at the algorithm written in Python
- What is the recurrence relation? T(N)
  - Base case
    T(0) = a
    T(1) = b
    Constant operating cost…
  - Recursive case

```python
def power(x,n):
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    if n == 0:
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Complexity Analysis

Recurrence relation

- Look at the algorithm written in Python
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  - Base case
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  - Recursive case
    T(N) = T(N-1)

```python
def power(x,n):
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```

- Look at the algorithm written in Python
- What is the recurrence relation? T(N)
  - Base case
    T(0) = a
    T(1) = b
    Constant operating cost…
  - Recursive case
    T(N) = T(N-1) * X?

```python
def power(x,n):
    """
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    """
    if n == 0:
        return 1
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```

- Look at the algorithm written in Python
- What is the recurrence relation? T(N)
  - Base case
    T(0) = a
    T(1) = b
    Constant operating cost…
  - Recursive case
    T(N) = T(N-1) + c
    Cause of constant operating cost in multiplying…

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def power(x,n):
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- Look at the algorithm written in Python
- What is the recurrence relation? T(N)
  - Base case
    T(0) = a
    T(1) = b
    Constant operating cost…
  - Recursive case (general case)
    T(N) = T(N-1) + c
    Cause of constant operating cost in multiplying…

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Complexity Analysis

Recurrence relation

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```

Complexity Analysis

Recurrence relation

- $T(0) = a$
- $T(1) = b$
- $T(N) = T(N-1) + c$

- Now solve it for the complexity

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def power(x,n):
    """
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        return x
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```

- $T(0) = a$
- $T(1) = b$
- $T(N) = T(N-1) + c$

- Now solve it for the complexity
  - $T(N) = T(N-1) + c$
  - so $T(N-1) = T(N-2) + c$

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Complexity Analysis
Recurrence relation

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def power(x,n):
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```

- Now solve it for the complexity
  - $T(N) = T(N-1) + c$
  - so $T(N-1) = T(N-2) + c$
  - $T(N) = T(N-2) + c + c$
  - $T(N) = T(N-3) + c + c + c$

Complexity Analysis

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Known as telescoping

Complexity Analysis
Recurrence relation

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  – so $T(N-1) = T(N-2) + c$
  – $T(N) = T(N-2) + c + c$
  – $T(N) = T(N-3) + c + c + c$
  – generalized into $T(N) = T(N-k) + kc$

Complexity Analysis

Recurrence relation

- $T(0) = a$
- $T(1) = b$
- $T(N) = T(N-1) + c$

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- Now solve it for the complexity
  - $T(N) = T(N-1) + c$
  - so $T(N-1) = T(N-2) + c$
  - $T(N) = T(N-2) + c + c$
  - $T(N) = T(N-3) + c + c + c$
  - generalized into $T(N) = T(N-k) + kc$
  - Base case when $N=0$ or $N=1$

Complexity Analysis
Recurrence relation

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def power(x,n):
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```

- T(0) = a
- T(1) = b
- T(N) = T(N-1) + c

- Now solve it for the complexity
  - T(N) = T(N-1) + c
  - so T(N-1) = T(N-2) + c
  - T(N) = T(N-2) + c + c
  - T(N) = T(N-3) + c + c + c
  - generalized into T(N) = T(N-k) + kc
  - Base case when N=0 or N=1
  - N-k = 0, therefore base case when k=N. we replace into above…

Complexity Analysis

Recurrence relation

- $T(0) = a$
- $T(1) = b$
- $T(N) = T(N-1) + c$
- Now solve it for the complexity
  - $T(N) = T(N-1) + c$
  - so $T(N-1) = T(N-2) + c$
  - $T(N) = T(N-2) + c + c$
  - $T(N) = T(N-3) + c + c + c$
  - generalized into $T(N) = T(N-k) + kc$
  - Base case when N=0 or N=1
  - $N-k = 0$, therefore base case when $k=N$. we replace into above…
  - $T(N) = T(N-N) + Nc = T(0) + Nc = a + Nc$

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Complexity Analysis

Recurrence relation

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- T(0) = a
- T(1) = b
- T(N) = T(N-1) + c
- Now solve it for the complexity
  - T(N) = T(N-1) + c
  - so T(N-1) = T(N-2) + c
  - T(N) = T(N-2) + c + c
  - T(N) = T(N-3) + c + c + c
  - generalized into T(N) = T(N-k) + kc
  - Base case when N=0 or N=1
  - N-k = 0, therefore base case when k=N. we replace into above…
  - T(N) = T(N-N) + Nc = T(0) + Nc = a + Nc = O(N), eliminating the constant

```python
def power(x,n):
    """
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    if n == 0:
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```

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Questions?

- Let us try another one
- Come up with the recurrence relation

```
def power_squaring(x,n):
    """
    Returns x^n
    via exponential by squaring
    """
    if n == 0:
        return 1
    elif n == 1:
        return x
    elif n%2 == 0:
        return power (x*x, n//2)
    elif n%2 == 1:
        return power (x*x, n//2) * n
```

- Let us try another one
- Come up with the recurrence relation

- Fun fact, this was a programming question for the exam…

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def power_squaring(x,n):
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Complexity Analysis
Recurrence relation

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- Do you understand the code?

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        return 1
    elif n == 1:
        return x
    # when power is even
    # N^4 = N^2 * N^2
    elif n%2 == 0:
        return power (x*x, n//2)
    # when the power is odd
    # N^9 = N^4 & N^4 * N
    elif n%2 == 1:
        return power (x*x, n//2) * n
```

Complexity Analysis

Recurrence relation

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- Come up with the recurrence relation

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```

Should be *x

Complexity Analysis

Recurrence relation

- T(0) = a

```python
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```

Complexity Analysis

Recurrence relation

- T(0) = a
- T(1) = b

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Complexity Analysis
Recurrence relation

- T(0) = a
- T(1) = b
- T(N) = T(N//2) + c when N even

```python
def power_squaring(x,n):
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```

Complexity Analysis

Recurrence relation

- T(0) = a
- T(1) = b
- T(N) = T(N//2) + c when N even
- T(N) = T(N//2) + d when N odd

```
def power_squaring(x,n):
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Complexity Analysis

Recurrence relation

- T(0) = a
- T(1) = b
- T(N) = T(N//2) + c when N even
- T(N) = T(N//2) + d when N odd

  – Not that it isn’t T(N) = T(N//2) * N
    WHY?

```python
def power_squaring(x,n):
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    if n == 0:
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    elif n == 1:
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Complexity Analysis
Recurrence relation

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- T(0) = a
- T(1) = b
- T(N) = T(N//2) + c when N even
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- Now you solve it…

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Complexity Analysis

Recurrence relation

- T(0) = a
- T(1) = b
- T(N) = T(N//2) + c when N even
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- Now you solve it…
  Let me take the even one to start

```python
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Complexity Analysis

Recurrence relation

- T(0) = a
- T(1) = b
- T(N) = T(N//2) + c when N even
- T(N) = T(N//2) + d when N odd

- Now you solve it…
  Let me take the even one to start
  – T(N) = T(N//2) + c
  – T(N) = T(N//4) + 2c
  – T(N) = T(N//8) + 3c

```python
def power_squaring(x,n):
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    Returns x^n
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    if n == 0:
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Complexity Analysis
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Questions?

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  - O(N)
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- But we also saw how their complexity differs
  - O(N) = normal power
  - O(log N) = exponential by squaring

Questions?

- So now you know why functions have such complexity?

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- Some of the other common ones…

Recurrence and complexity

Recurrence relation:

$T(N) = T(N/2) + c$

$T(1) = b$

Example algorithm?

Binary search

Solution:

$O(\log N)$

Recurrence and complexity

Recurrence relation:

$T(N) = T(N-1) + c$

$T(1) = b$

Example algorithm?

Linear search

Solution:

$O(N)$

Recurrence and complexity

Recurrence relation:

T(N) = 2*T(N/2) + c*N

T(1) = b

Example algorithm?

Merge sort

Solution:

O(N log N)

Recurrence and complexity

Recurrence relation:

$T(N) = T(N-1) + c*N$

$T(1) = b$

Example algorithm?

Selection sort

Solution:

$O(N^2)$

Recurrence and complexity

Recurrence relation:

$T(N) = 2*T(N-1) + c$

$T(0) = b$

Example algorithm?

Naïve recursive Fibonacci

Solution:

$O(2^N)$

- So now you know why functions have such complexity?
- Some of the other common ones…

- Exam would also ask you to proof by induction for complexity…

- So now you know why functions have such complexity?
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- Exam would also ask you to proof by induction for complexity…
  - We will discuss more in the tutorial
  - Now, let us try to make my life difficult in the zoom session later this week…

Questions?

Thank You