MONASH University

MONASH
INFORMATION
TECHNOLOGY

FIT2004
Algorithms and Data Structures

Ian Wern Han Lim
lim.wern.han@monash.edu

Referencing materials by
Nathan Companez, Aamir Cheema, Arun Konagurthu and Lloyd Allison

GROUP OF EIGHT AUSTRALIA

Faculty of Information Technology, Monash University

Ready?

Agenda

- Minimum Spanning Tree (MST)

Agenda

- Minimum Spanning Tree (MST)
  - Prim's algorithm
  - Kruskal's algorithm

Let us begin…

Minimum Spanning Tree

What is it?

Minimum Spanning Tree
What is it?

- A tree

Minimum Spanning Tree

What is it?

- A tree
- Spanning every vertex

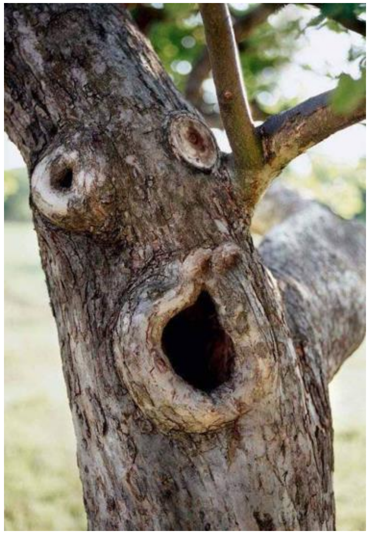
- A tree
- Spanning every vertex
- Minimum total edges

Minimum Spanning Tree

What is it?

- A tree
- Spanning every vertex
  - Minimum number of edges to connect all vertex? True or False?
  - Maximum number of edges in graph without cycle? True or False?
- Minimum total edges weight

Minimum Spanning Tree
What is it?



- A tree
  - No cycle
  - Undirected
- Spanning every vertex
  - Minimum number of edges to connect all vertex
  - Maximum number of edges in graph without cycle
- Minimum total edges weight

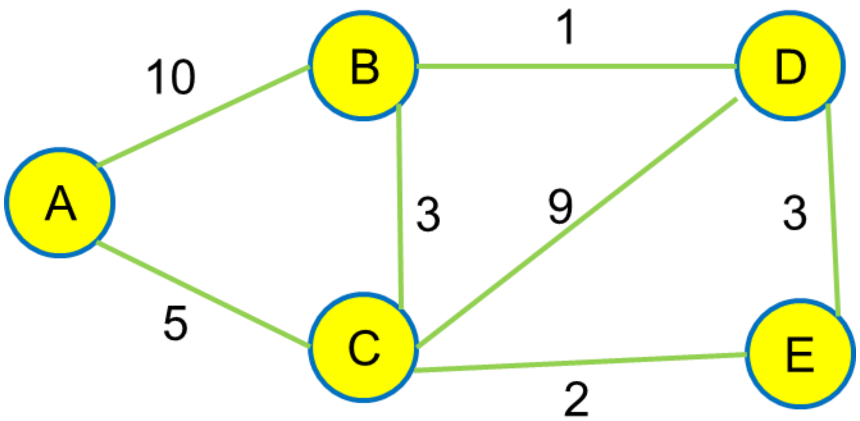
- A tree
  - No cycle
  - Undirected
- Spanning every vertex
  - Minimum number of edges to connect all vertex
  - Maximum number of edges in graph without cycle
- Minimum total edges weight

Spanning Tree:

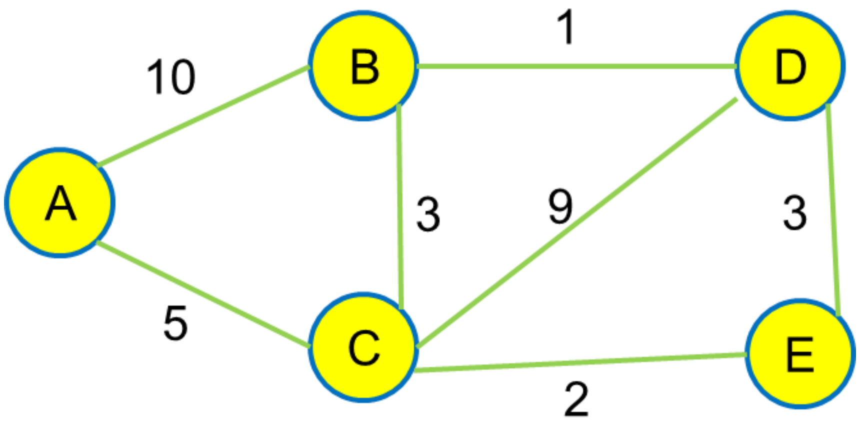
- A spanning tree of a general undirected weighted graph G is a tree that spans G (i.e., a tree that includes every vertex of G) and is a subgraph of G (i.e., every edge in the spanning tree belongs to G).

What is it?

- Let say we have a graph

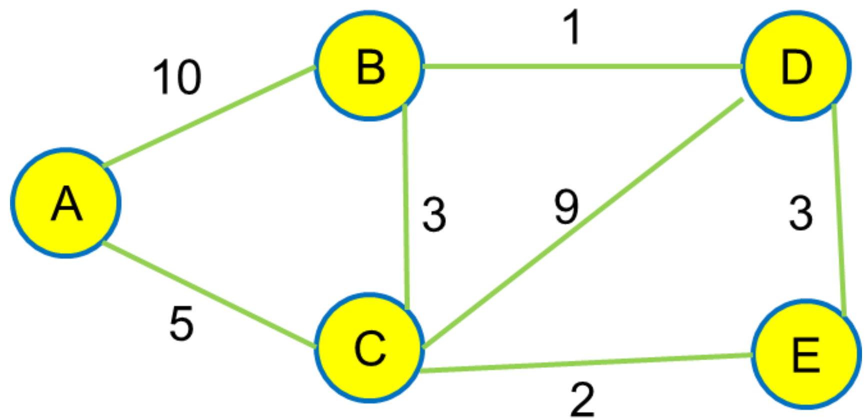


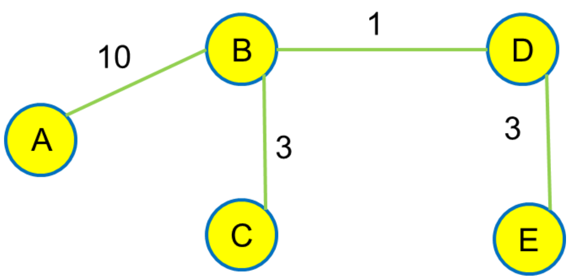
- Let say we have a graph
  - Can you form spanning trees?



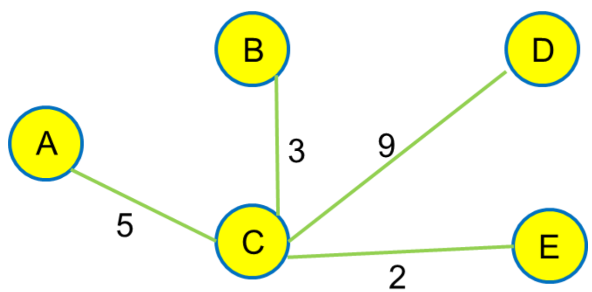
Minimum Spanning Tree
What is it?

- Let say we have a graph
  - Can you form spanning trees?

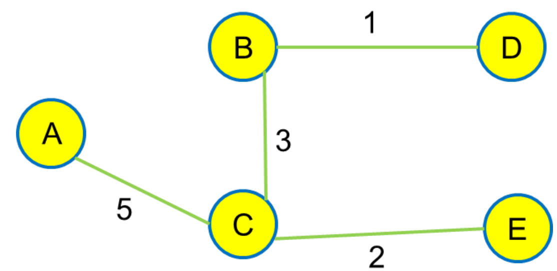




Spanning Tree 1



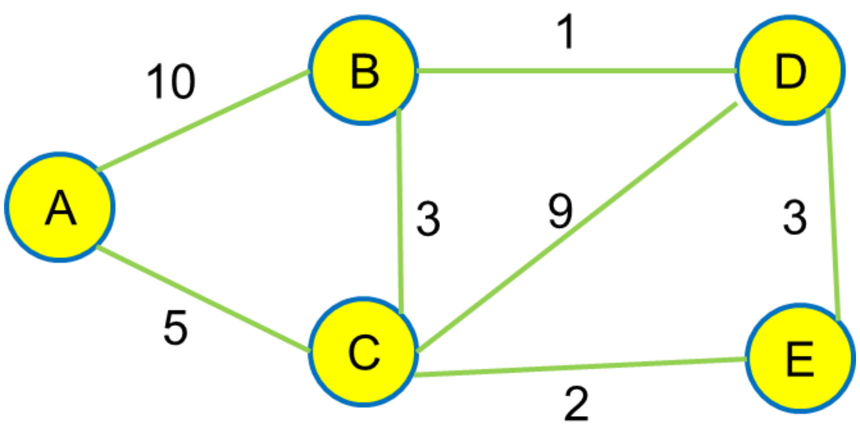
Spanning Tree 2

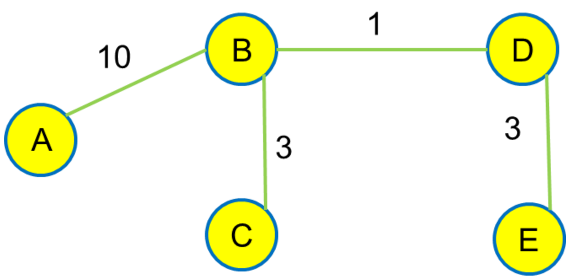


Spanning Tree 3

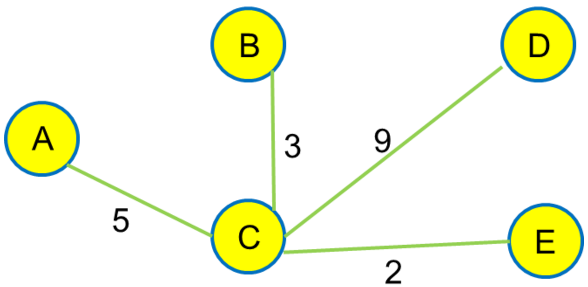
Minimum Spanning Tree
What is it?

- Let say we have a graph
  - Can you form spanning trees?
  - Which is the minimum?

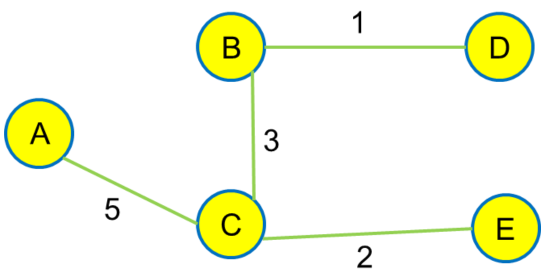




Spanning Tree 1



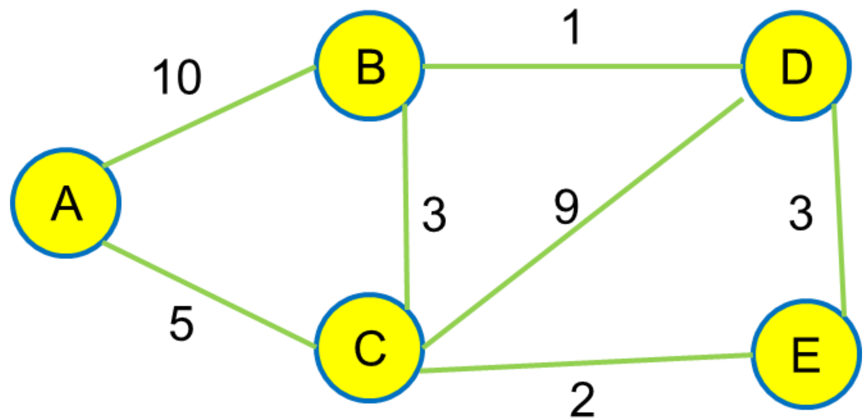
Spanning Tree 2

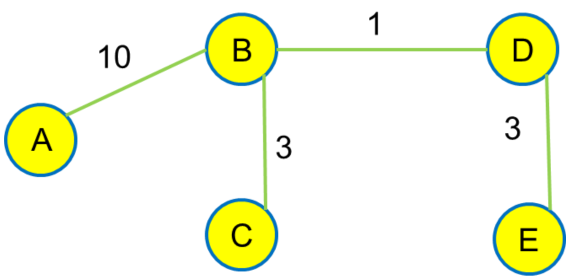


Spanning Tree 3

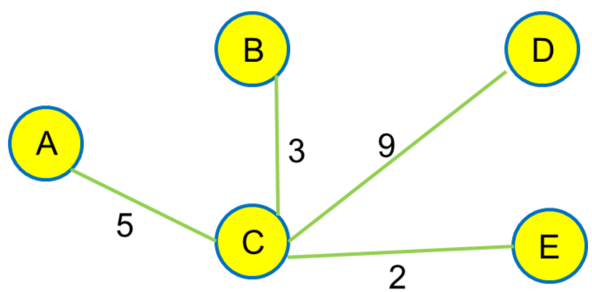
Minimum Spanning Tree
What is it?

- Let say we have a graph
  - Can you form spanning trees?
  - Which is the minimum?
    - Tree 1 = 10 + 1 + 3 + 3
    - Tree 2 = 5 + 3 + 9 + 2
    - Tree 3 = 5 + 3 + 2 + 1

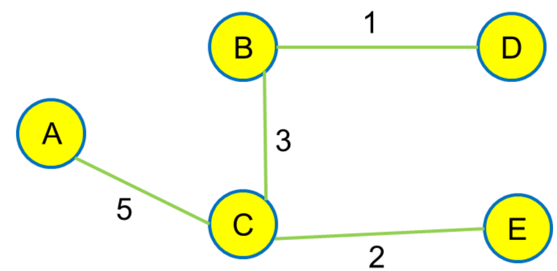




Spanning Tree 1



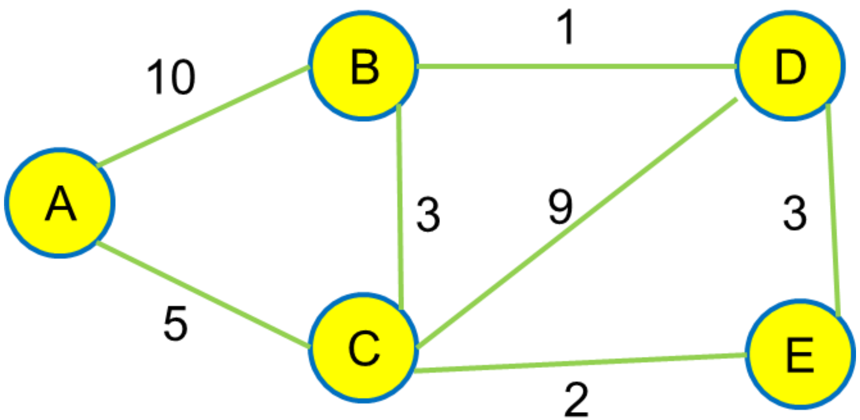
Spanning Tree 2

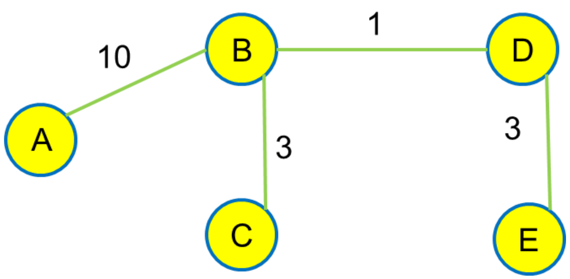


Spanning Tree 3

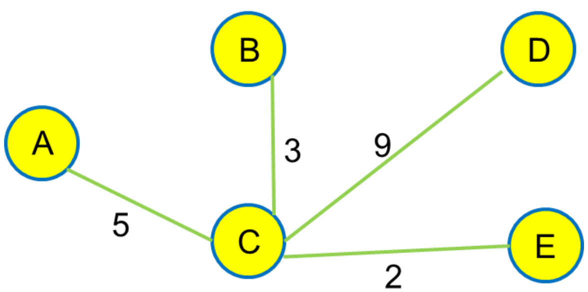
Minimum Spanning Tree
What is it?

- Let say we have a graph
  - Can you form spanning trees?
  - Which is the minimum?
    - Tree 1 = 10 + 1 + 3 + 3 = 17
    - Tree 2 = 5 + 3 + 9 + 2 = 19
    - Tree 3 = 5 + 3 + 2 + 1 = 11

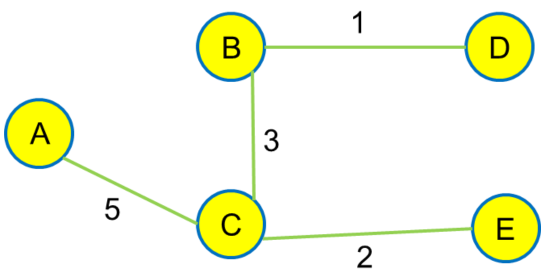




Spanning Tree 1



Spanning Tree 2

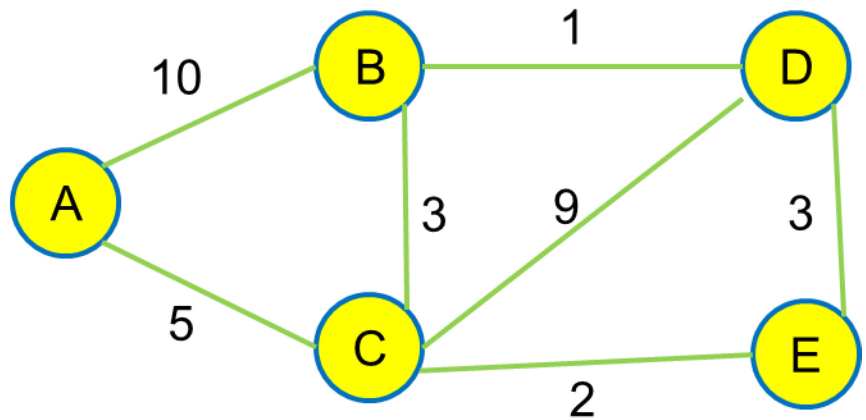


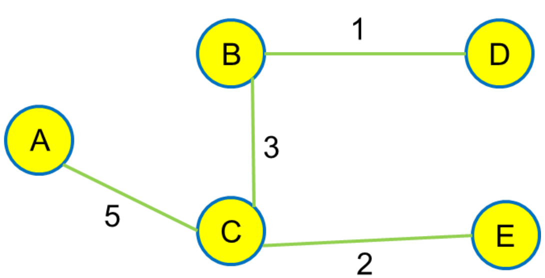
Spanning Tree 3

Minimum Spanning Tree
What is it?

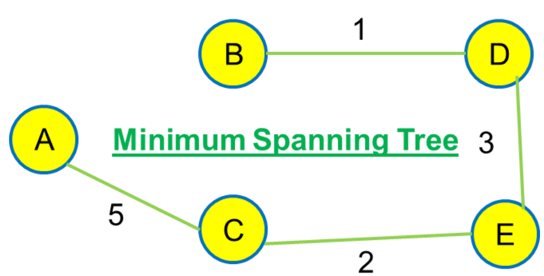
- Let say we have a graph
  - Can you form spanning trees?
  - Which is the minimum?
    - Tree 1 = 10 + 1 + 3 + 3 = 17
    - Tree 2 = 5 + 3 + 9 + 2 = 19
    - Tree 3 = 5 + 3 + 2 + 1 = 11
    - Tree 4 = 5 + 2 + 3 + 1 = 11

Not unique





Spanning Tree 3



Spanning Tree 4

MONASH University

Questions?

Minimum Spanning Tree
How to build it?

- Prim’s
- Kruskal’s

Minimum Spanning Tree
How to build it?

- Prim's
  - Growing of tree
- Kruskal's
  - Merging of trees

How to build it?

- Prim's
  - Growing of tree
- Kruskal's
  - Merging of trees

- Both are greedy

Minimum Spanning Tree

How to build it?

- Prim’s
  - Growing of tree
- Kruskal’s
  - Merging of trees

- Both are greedy
  - Choose local optimal
  - Believe to get global optimal

- Prim's
  - Growing of tree
- Kruskal's
  - Merging of trees

- Both are greedy
  - Choose local optimal
  - Believe to get global optimal
  - We will learn to prove it later

Minimum Spanning Tree
How to build it?

- Prim's
  - Growing of tree
  - Very similar to Dijkstra's. Can be known a Prim-Dijkstra
- Kruskal's
  - Merging of trees
- Both are greedy
  - Choose local optimal
  - Believe to get global optimal
  - We will learn to prove it later

Minimum Spanning Tree
How to build it?

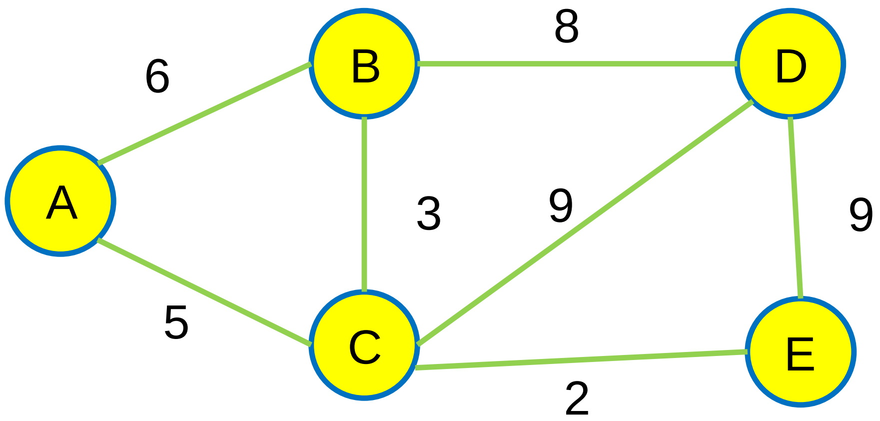
- Prim's
  - Growing of tree
  - Very similar to Dijkstra's. Can be known a Prim-Dijkstra
    Instead of nearest vertex from source, it is nearest vertex from tree!
- Kruskal's
  - Merging of trees
- Both are greedy
  - Choose local optimal
  - Believe to get global optimal
  - We will learn to prove it later

Questions?

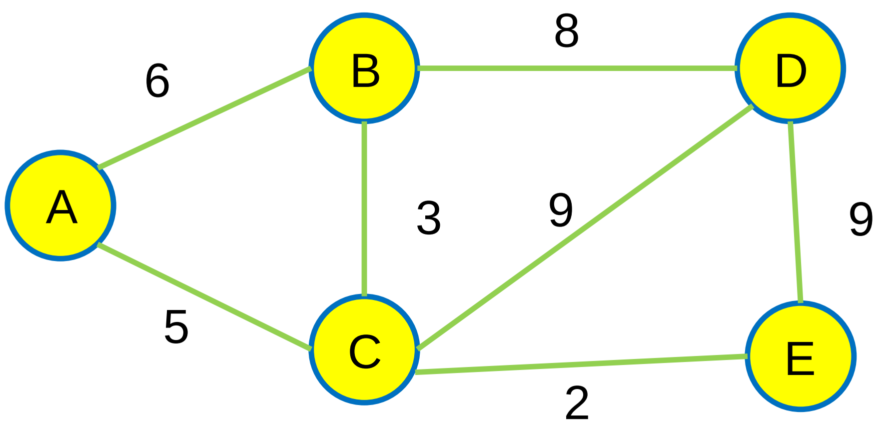
Prim’s Algorithm
Growing of MST

- Very similar to Dijkstra
  - Choosing nearest vertex to tree

- Very similar to Dijkstra
  - Choosing nearest vertex to tree
  - Let us try it out



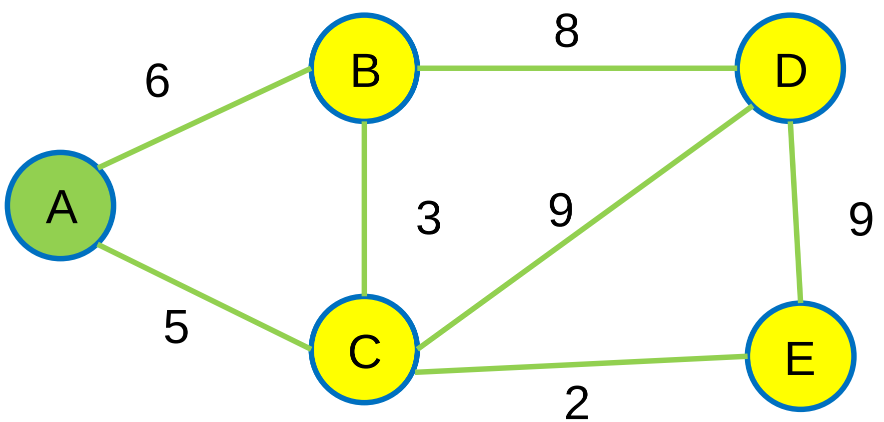
- Very similar to Dijkstra
  - Choosing nearest vertex to tree
  - Let us try it out



A	B	C	D	E
0	inf	inf	inf	inf

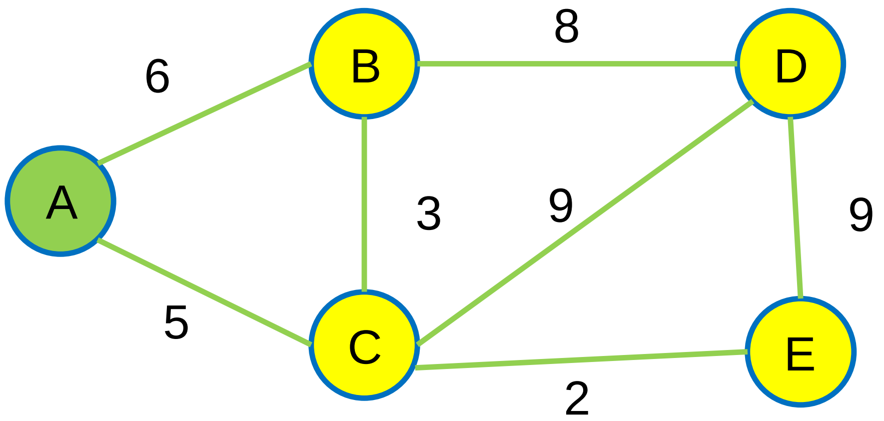
Prim's Algorithm
Growing of MST

- Very similar to Dijkstra
  - Choosing nearest vertex to tree
  - Let us try it out
  - Start from A



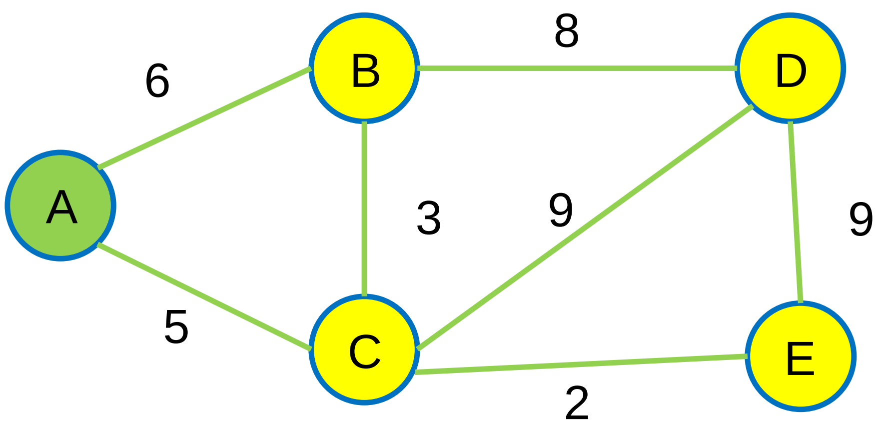
A	B	C	D	E
0	inf	inf	inf	inf

- Very similar to Dijkstra
  - Choosing nearest vertex to tree
  - Let us try it out
  - Start from A
  - Update adjacent B and C



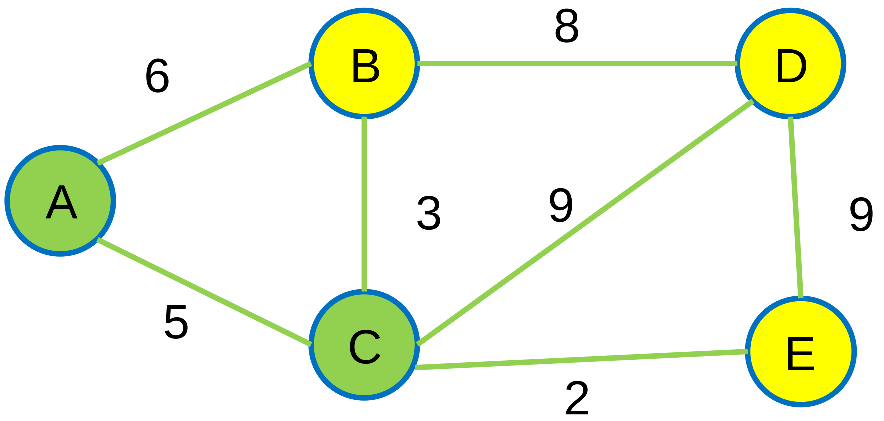
A	B	C	D	E
0	6, A	inf	inf	inf

- Very similar to Dijkstra
  - Choosing nearest vertex to tree
  - Let us try it out
  - Start from A
  - Update adjacent B and C



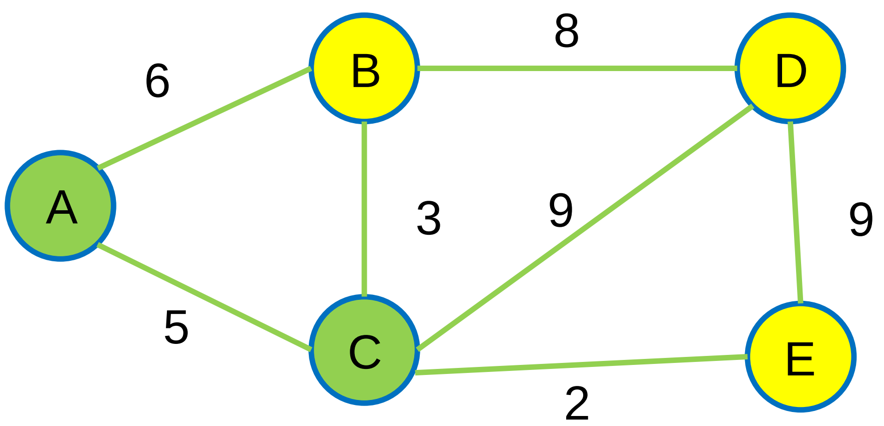
A	B	C	D	E
0	6, A	5, A	inf	inf

- Very similar to Dijkstra
  - Choosing nearest vertex to tree
  - Let us try it out
  - Start from A
  - Update adjacent B and C
  - Choose closest C



A	B	C	D	E
0	6, A	5, A	inf	inf

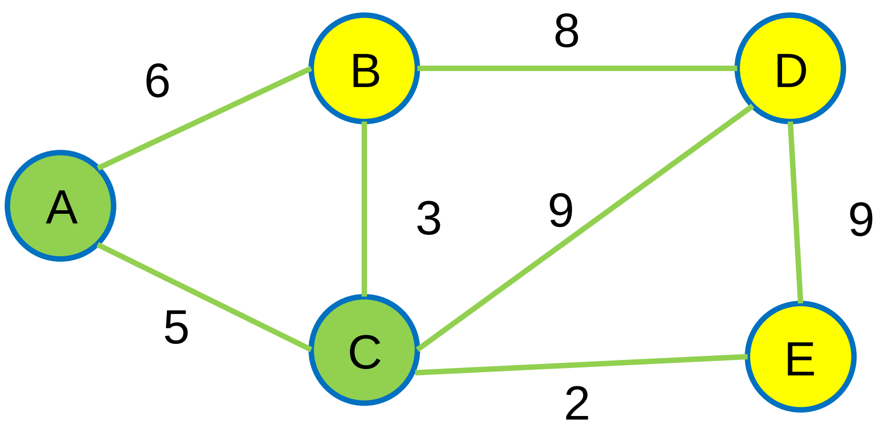
- Very similar to Dijkstra
  - Choosing nearest vertex to tree
  - Let us try it out
  - Start from A
  - Update adjacent B and C
  - Choose closest C
  - Update adjacent B, D, E



A	B	C	D	E
0	6	5, A	inf	inf

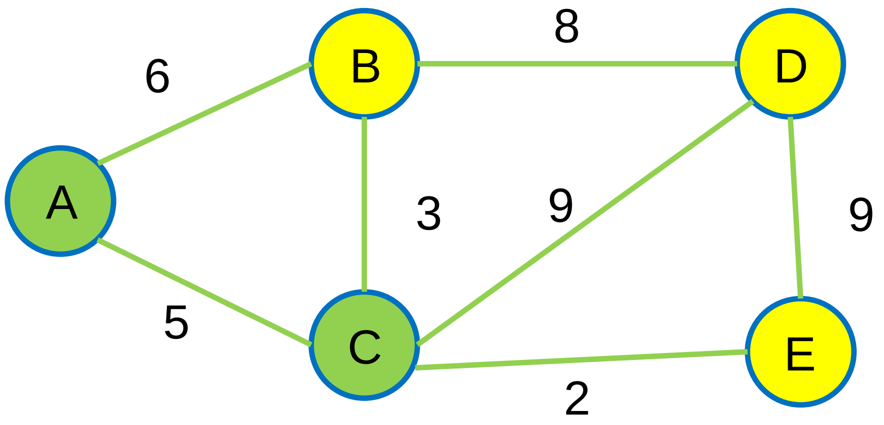
MONASH University

- Very similar to Dijkstra
  - Choosing nearest vertex to tree
  - Let us try it out
  - Start from A
  - Update adjacent B and C
  - Choose closest C
  - Update adjacent B, D, E



A	B	C	D	E
0	6vs3	5, A	inf	inf

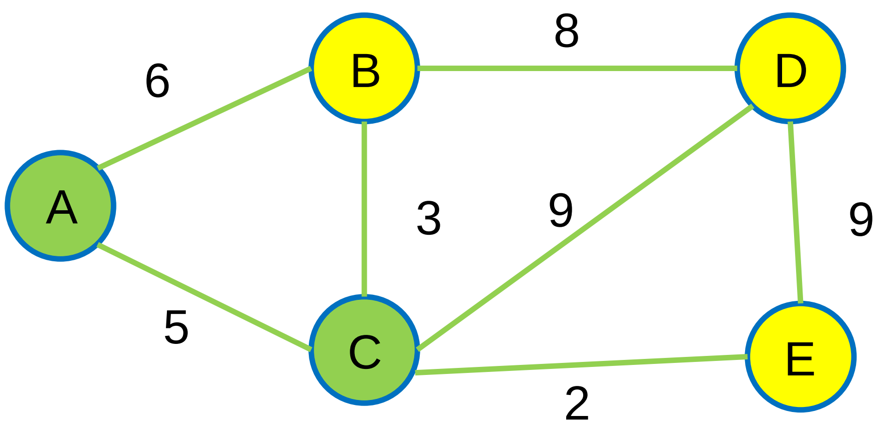
- Very similar to Dijkstra
  - Choosing nearest vertex to tree
  - Let us try it out
  - Start from A
  - Update adjacent B and C
  - Choose closest C
  - Update adjacent B, D, E



A	B	C	D	E
0	3, C	5, A	inf	inf

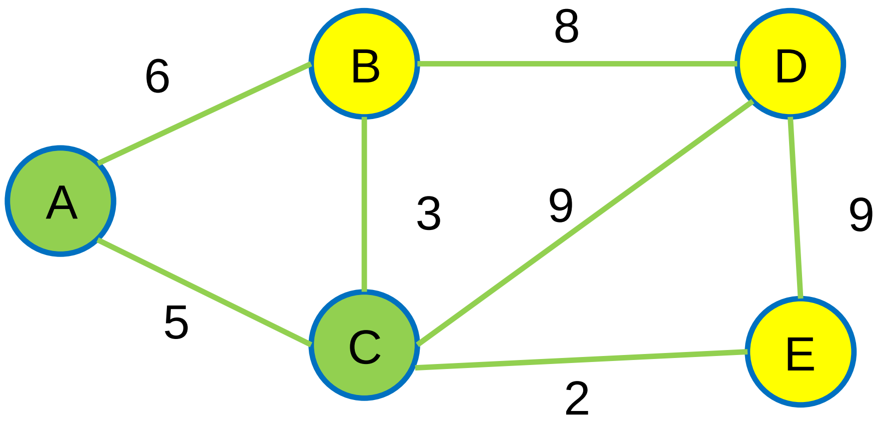
Prim's Algorithm
Growing of MST

- Very similar to Dijkstra
  - Choosing nearest vertex to tree
  - Let us try it out
  - Start from A
  - Update adjacent B and C
  - Choose closest C
  - Update adjacent B, D, E



A	B	C	D	E
0	3, C	5, A	9, C	inf

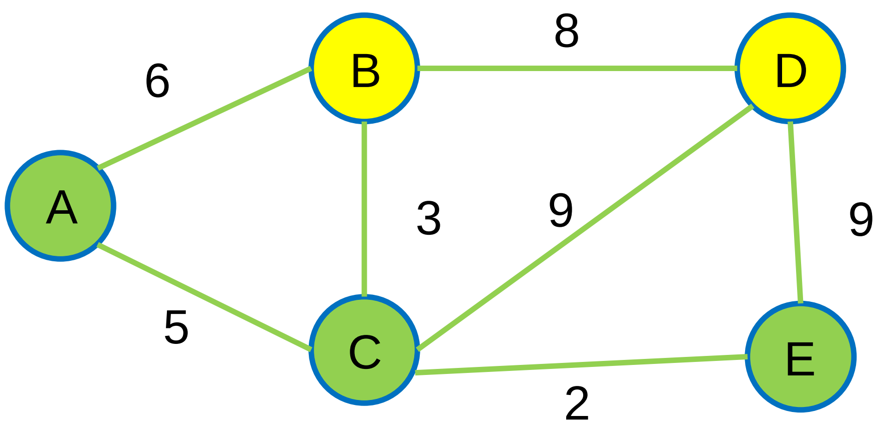
- Very similar to Dijkstra
  - Choosing nearest vertex to tree
  - Let us try it out
  - Start from A
  - Update adjacent B and C
  - Choose closest C
  - Update adjacent B, D, E



A	B	C	D	E
0	3, C	5, A	9, C	2, C

Prim's Algorithm
Growing of MST

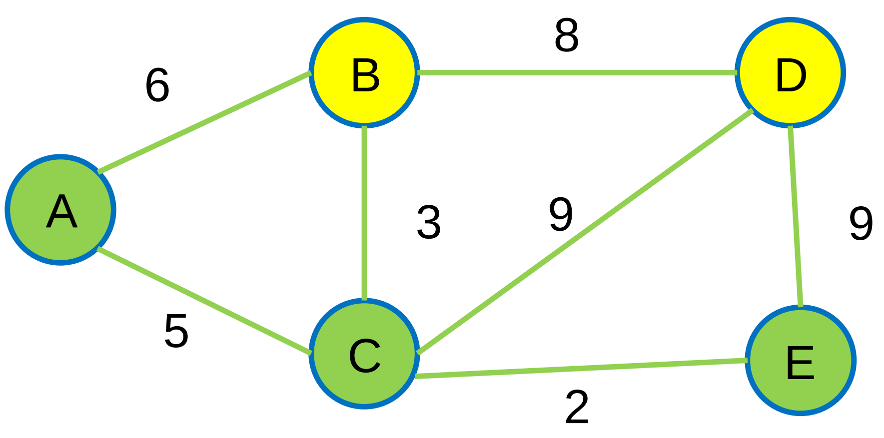
- Very similar to Dijkstra
  - Choosing nearest vertex to tree
  - Let us try it out
  - Start from A
  - Update adjacent B and C
  - Choose closest C
  - Update adjacent B, D, E
  - Choose closest E



A	B	C	D	E
0	3, C	5, A	9, C	2, C

MONASH University

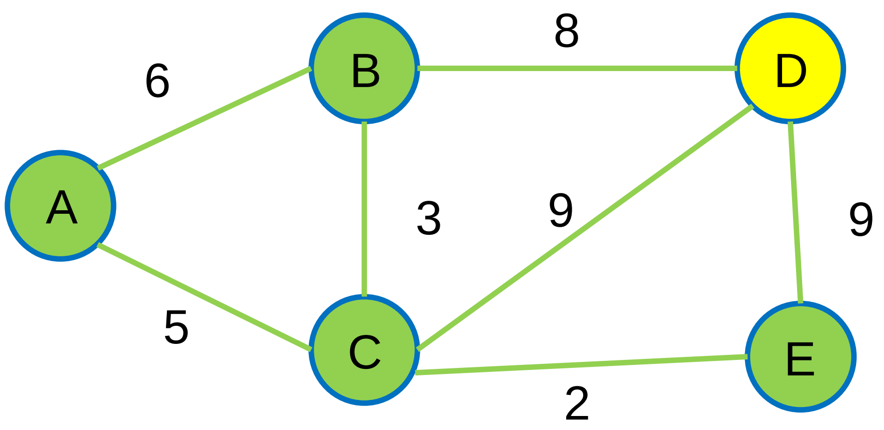
- Very similar to Dijkstra
  - Choosing nearest vertex to tree
  - Let us try it out
  - Start from A
  - Update adjacent B and C
  - Choose closest C
  - Update adjacent B, D, E
  - Choose closest E
  - Update adjacent D



A	B	C	D	E
0	3, C	5, A	9, C	2, C

Prim's Algorithm
Growing of MST

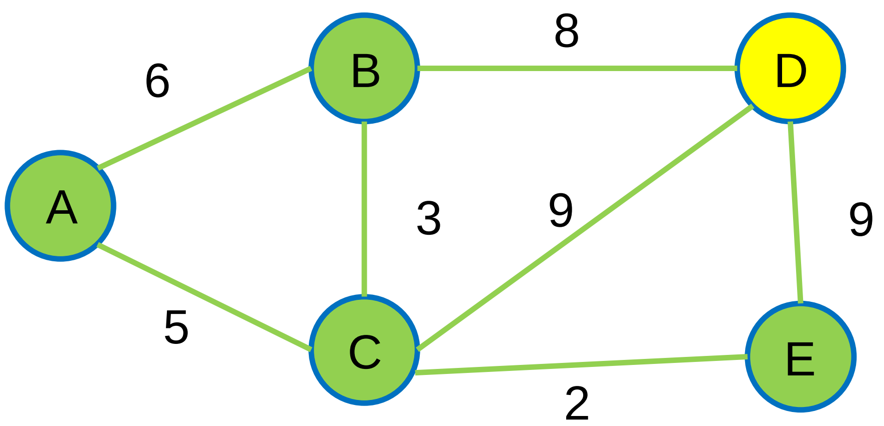
- Very similar to Dijkstra
  - Choosing nearest vertex to tree
  - Let us try it out
  - Start from A
  - Update adjacent B and C
  - Choose closest C
  - Update adjacent B, D, E
  - Choose closest E
  - Update adjacent D
  - Choose closest B



A	B	C	D	E
0	3, C	5, A	9, C	2, C

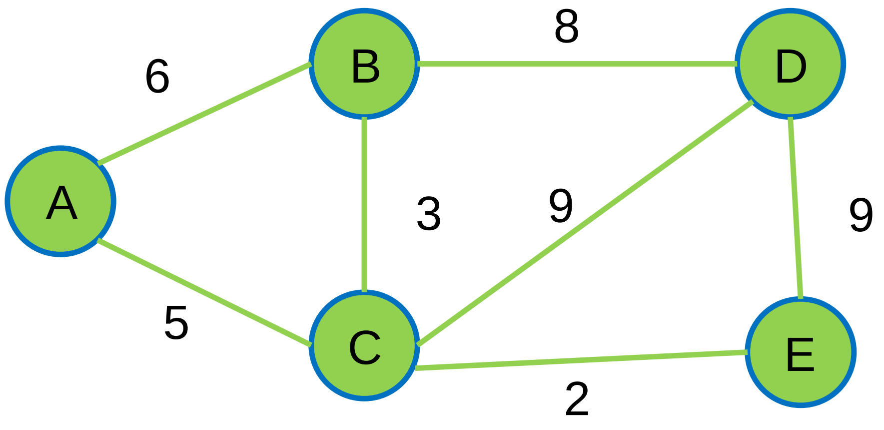
Prim's Algorithm
Growing of MST

- Very similar to Dijkstra
  - Choosing nearest vertex to tree
  - Let us try it out
  - Start from A
  - Update adjacent B and C
  - Choose closest C
  - Update adjacent B, D, E
  - Choose closest E
  - Update adjacent D
  - Choose closest B
  - Update adjacent D



A	B	C	D	E
0	3, C	5, A	8, B	2, C

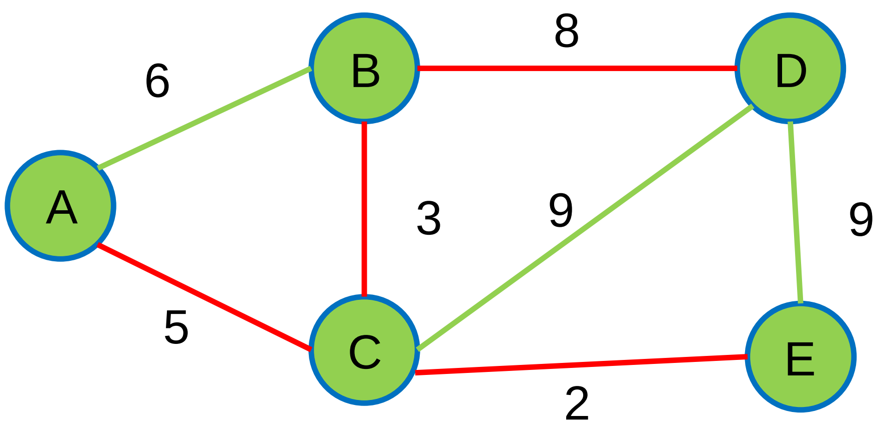
- Very similar to Dijkstra
  - Choosing nearest vertex to tree
  - Let us try it out
  - Start from A
  - Update adjacent B and C
  - Choose closest C
  - Update adjacent B, D, E
  - Choose closest E
  - Update adjacent D
  - Choose closest B
  - Update adjacent D
  - Choose closest D



A	B	C	D	E
0	3, C	5, A	8, B	2, C

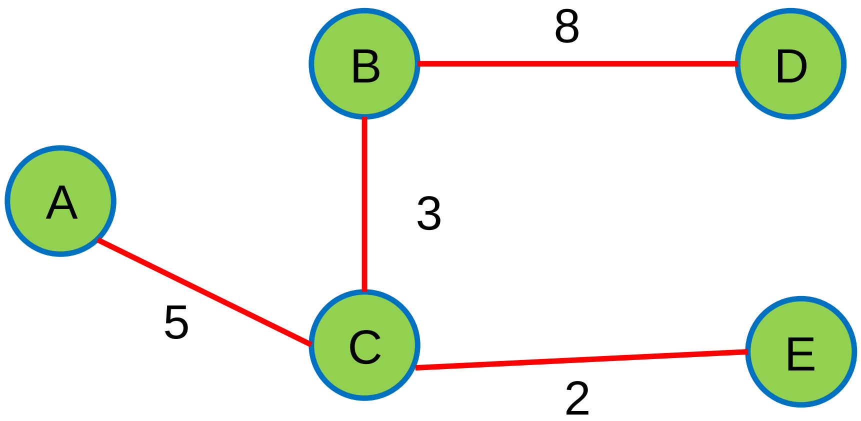
Prim's Algorithm
Growing of MST

- Very similar to Dijkstra
  - Choosing nearest vertex to tree
  - Let us try it out
  - Start from A
  - Update adjacent B and C
  - Choose closest C
  - Update adjacent B, D, E
  - Choose closest E
  - Update adjacent D
  - Choose closest B
  - Update adjacent D
  - Choose closest D
  - Have all of the edges



A	B	C	D	E
0	3, C	5, A	8, B	2, C

- Very similar to Dijkstra
  - Choosing nearest vertex to tree
  - Let us try it out
  - Start from A
  - Update adjacent B and C
  - Choose closest C
  - Update adjacent B, D, E
  - Choose closest E
  - Update adjacent D
  - Choose closest B
  - Update adjacent D
  - Choose closest D
  - Have all of the edges



A	B	C	D	E
0	3, C	5, A	8, B	2, C

Prim’s Algorithm

Growing of MST

- So take Dijkstra
  - Modify the distance update/ calculation for edge <u,v,w>
    - Instead of v.distance = u.distance + w
    - Change to v.distance = w

- So take Dijkstra
  - Modify the distance update/ calculation for edge <u,v,w>
    - Instead of v.distance = u.distance + w
    - Change to v.distance = w
    - Perform relaxation only if distance is smaller

- So take Dijkstra
  - Modify the distance update/ calculation for edge <u,v,w>
    - Instead of v.distance = u.distance + w
    - Change to v.distance = w
    - Perform relaxation only if distance is smaller

- So what is the complexity?

Prim’s Algorithm

Growing of MST

- So take Dijkstra
  - Modify the distance update/ calculation for edge <u,v,w>
    - Instead of v.distance = u.distance + w
    - Change to v.distance = w
    - Perform relaxation only if distance is smaller

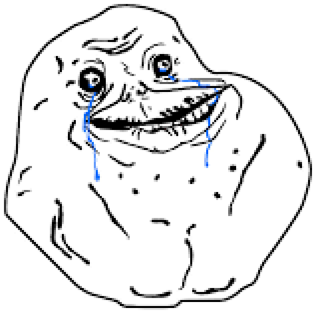
- So what is the complexity?
  - Same as Dijkstra O(V log V + E log V)
  - Thus O(E log V)

Questions?

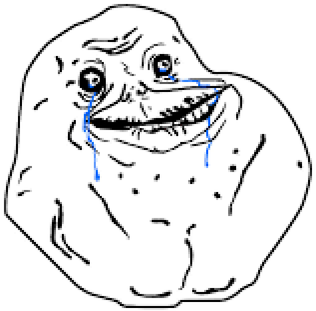
Kruskal's Algorithm
Combining (Union of) Trees

Kruskal's Algorithm
Combining (Union of) Trees

- Imagine every vertex is a tree
  - Only 1 node #ForeverAlone

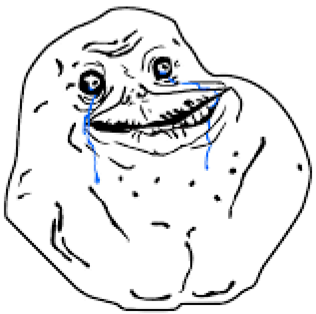


- Imagine every vertex is a tree
  - Only 1 node #ForeverAlone
  - Trees are connected by edges



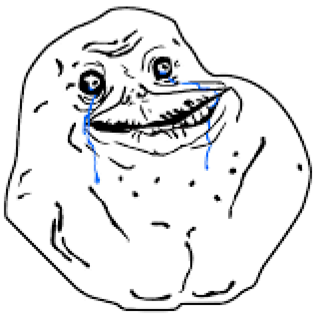
Kruskal's Algorithm
Combining (Union of) Trees

- Imagine every vertex is a tree
  - Only 1 node #ForeverAlone
  - Trees are connected by edges
    - Adding edge <u,v,w> combine the trees of vertex u and vertex v



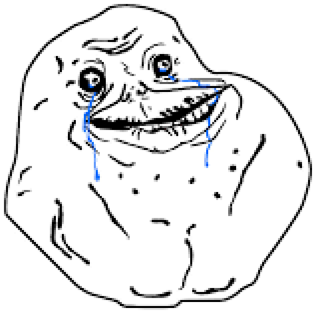
Kruskal's Algorithm
Combining (Union of) Trees

- Imagine every vertex is a tree
  - Only 1 node #ForeverAlone
  - Trees are connected by edges
    - Adding edge <u,v,w> combine the trees of vertex u and vertex v
    - Only add if vertex u and vertex v are not in the same tree. Why?



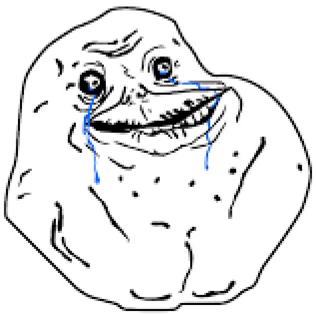
Kruskal’s Algorithm
Combining (Union of) Trees

- Imagine every vertex is a tree
  - Only 1 node #ForeverAlone
  - Trees are connected by edges
    - Adding edge <u,v,w> combine the trees of vertex u and vertex v
    - Only add if vertex u and vertex v are not in the same tree. Why? NO CYCLE



Kruskal's Algorithm
Combining (Union of) Trees

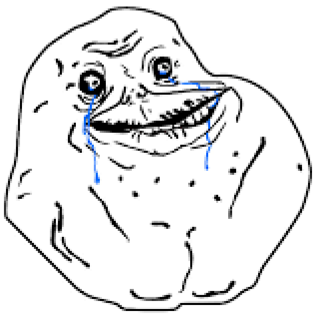
- Imagine every vertex is a tree
  - Only 1 node #ForeverAlone
  - Trees are connected by edges
    - Adding edge <u,v,w> combine the trees of vertex u and vertex v
    - Only add if vertex u and vertex v are not in the same tree. Why? NO CYCLE



- So how do we do it?

Kruskal's Algorithm
Combining (Union of) Trees

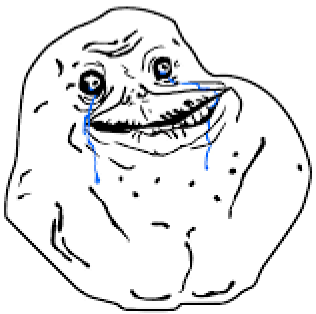
- Imagine every vertex is a tree
  - Only 1 node #ForeverAlone
  - Trees are connected by edges
    - Adding edge <u,v,w> combine the trees of vertex u and vertex v
    - Only add if vertex u and vertex v are not in the same tree. Why? NO CYCLE



- So how do we do it?
  - Take add edges
  - Sort it
  - Then go through the edges one by one

Kruskal’s Algorithm
Combining (Union of) Trees

- Imagine every vertex is a tree
  - Only 1 node #ForeverAlone
  - Trees are connected by edges
    - Adding edge <u,v,w> combine the trees of vertex u and vertex v
    - Only add if vertex u and vertex v are not in the same tree. Why? NO CYCLE

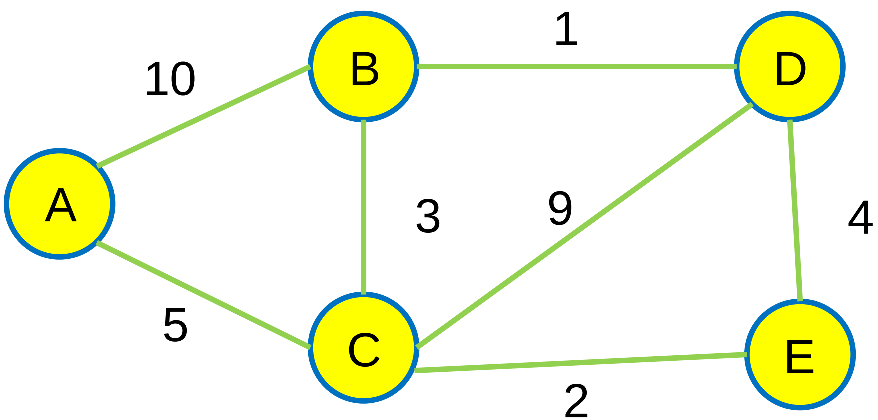


- So how do we do it?
  - Take add edges
  - Sort it
  - Then go through the edges one by one
  - Let us visualize it…

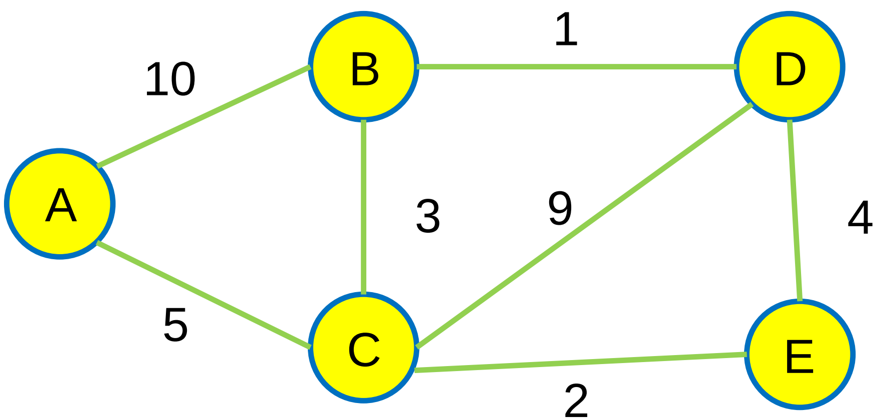
Kruskal’s Algorithm

Combining (Union of) Trees

- Look at the graph



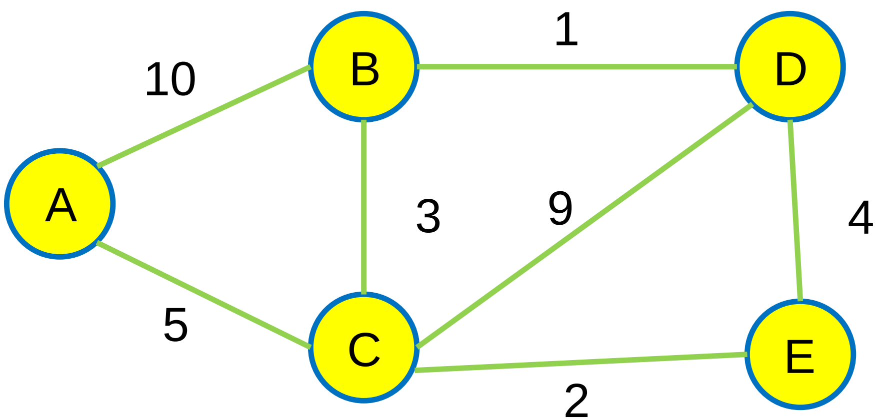
- Look at the graph
  - Take all the edges



AB	AC	BC	BD	CD	CE	DE
10	5	3	1	9	2	4

Kruskal's Algorithm
Combining (Union of) Trees

- Look at the graph
  - Take all the edges
  - Sort it

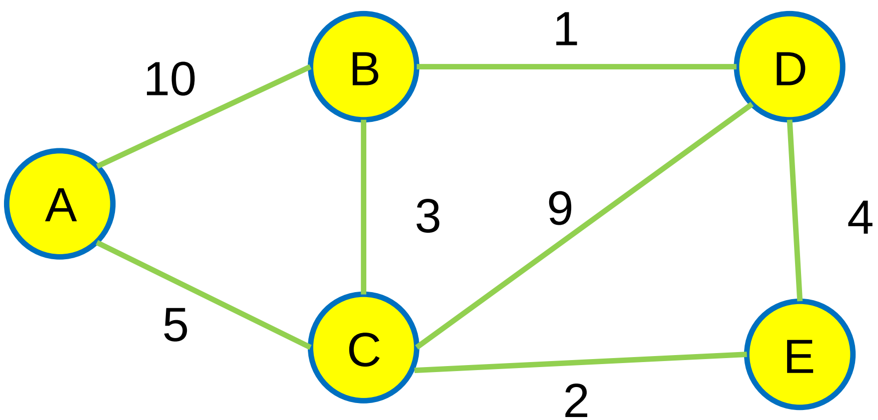


AB	AC	BC	BD	CD	CE	DE
10	5	3	1	9	2	4

BD	CE	BC	DE	AC	CD	AB
1	2	3	4	5	9	10

Kruskal's Algorithm
Combining (Union of) Trees

- Look at the graph
  - Take all the edges
  - Sort it
  - Go through the edges one by one

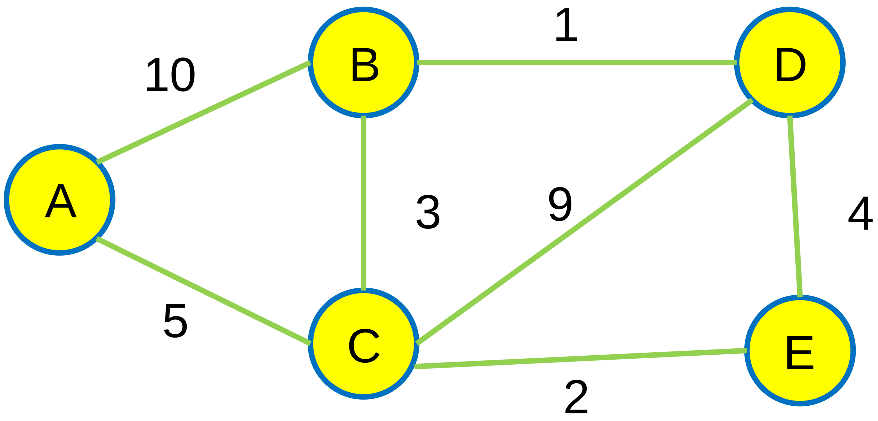


AB	AC	BC	BD	CD	CE	DE
10	5	3	1	9	2	4

BD	CE	BC	DE	AC	CD	AB
1	2	3	4	5	9	10

Kruskal's Algorithm
Combining (Union of) Trees

- Look at the graph
  - Take all the edges
  - Sort it
  - Go through the edges one by one



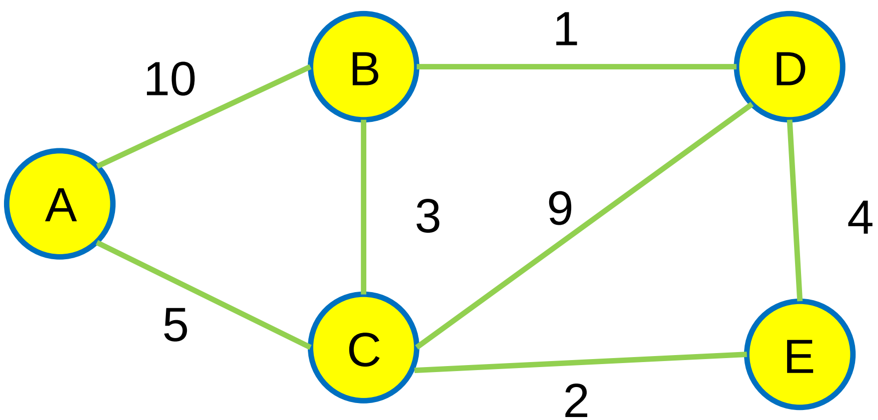
AB	AC	BC	BD	CD	CE	DE
10	5	3	1	9	2	4

BD	CE	BC	DE	AC	CD	AB
1	2	3	4	5	9	10

Kruskal's Algorithm

Combining (Union of) Trees

- Look at the graph
  - Take all the edges
  - Sort it
  - Go through the edges one by one
    - Add if u and w not same tree



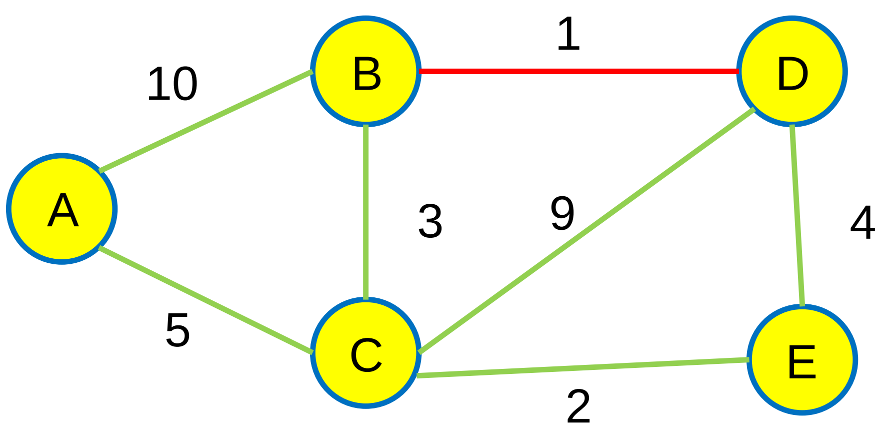
AB	AC	BC	BD	CD	CE	DE
10	5	3	1	9	2	4

BD	CE	BC	DE	AC	CD	AB
1	2	3	4	5	9	10

Kruskal's Algorithm

Combining (Union of) Trees

- Look at the graph
  - Take all the edges
  - Sort it
  - Go through the edges one by one
    - Add if u and w not same tree

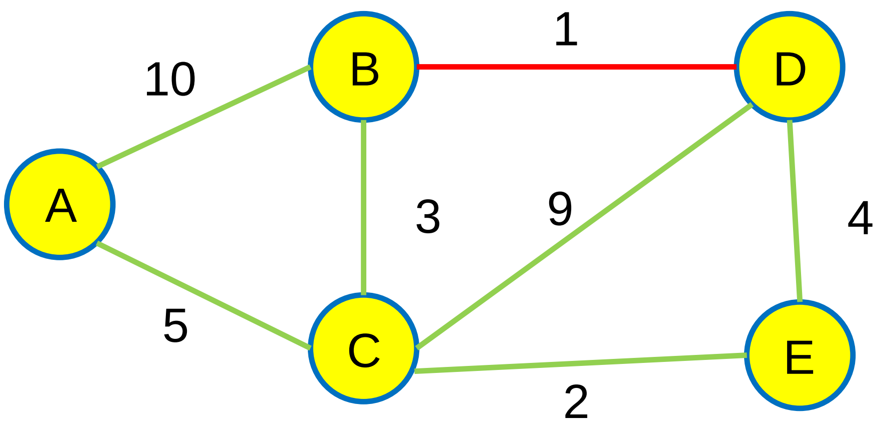


AB	AC	BC	BD	CD	CE	DE
10	5	3	1	9	2	4

BD	CE	BC	DE	AC	CD	AB
1	2	3	4	5	9	10

Kruskal’s Algorithm
Combining (Union of) Trees

- Look at the graph
  - Take all the edges
  - Sort it
  - Go through the edges one by one
    - Add if u and w not same tree



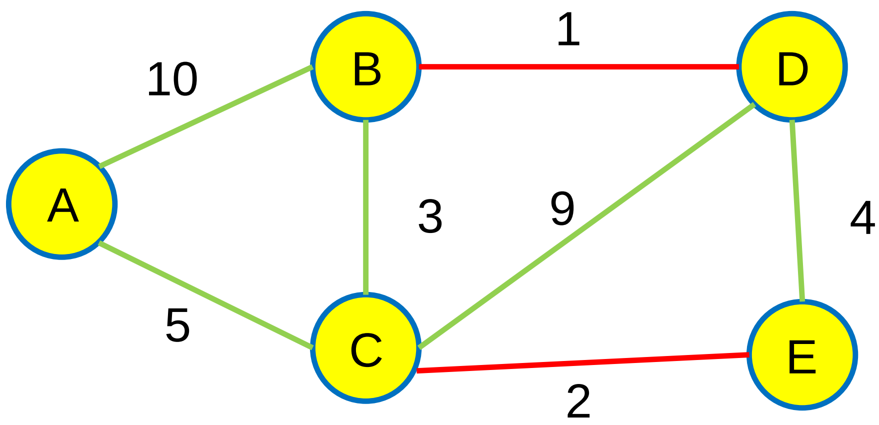
AB	AC	BC	BD	CD	CE	DE
10	5	3	1	9	2	4

BD	CE	BC	DE	AC	CD	AB
1	2	3	4	5	9	10

Kruskal's Algorithm

Combining (Union of) Trees

- Look at the graph
  - Take all the edges
  - Sort it
  - Go through the edges one by one
    - Add if u and w not same tree



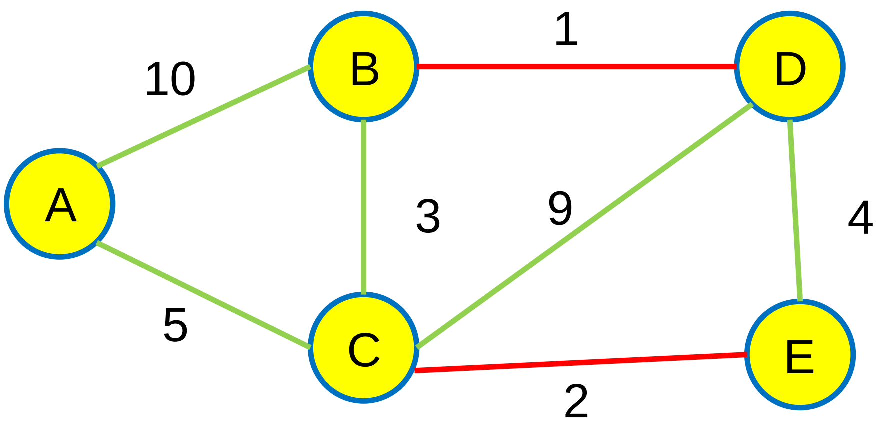
AB	AC	BC	BD	CD	CE	DE
10	5	3	1	9	2	4

BD	CE	BC	DE	AC	CD	AB
1	2	3	4	5	9	10

Kruskal's Algorithm
Combining (Union of) Trees

MONASH University

- Look at the graph
  - Take all the edges
  - Sort it
  - Go through the edges one by one
    - Add if u and w not same tree

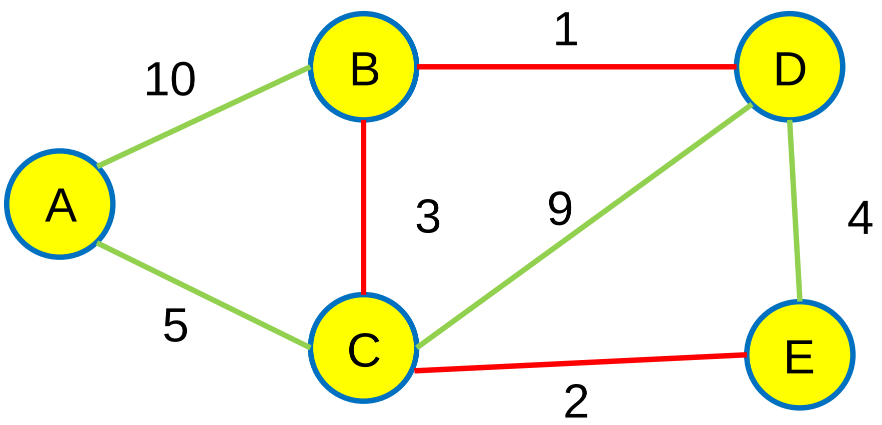


AB	AC	BC	BD	CD	CE	DE
10	5	3	1	9	2	4

BD	CE	BC	DE	AC	CD	AB
1	2	3	4	5	9	10

Combining (Union of) Trees

- Look at the graph
  - Take all the edges
  - Sort it
  - Go through the edges one by one
    - Add if u and w not same tree

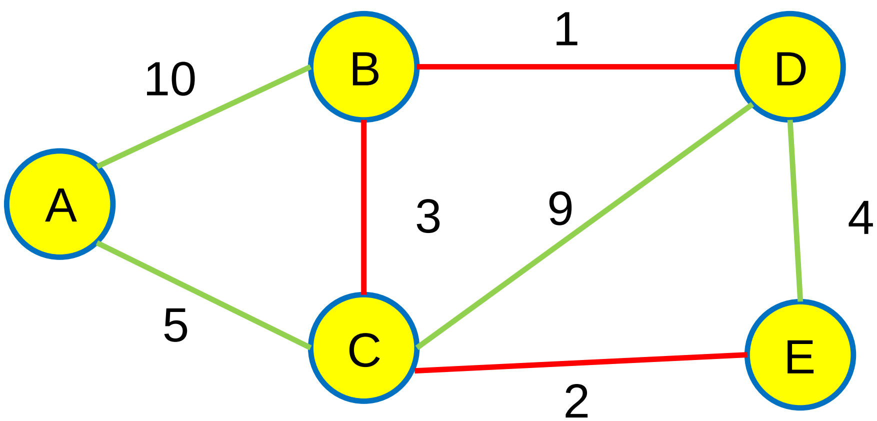


AB	AC	BC	BD	CD	CE	DE
10	5	3	1	9	2	4

BD	CE	BC	DE	AC	CD	AB
1	2	3	4	5	9	10

Kruskal's Algorithm
Combining (Union of) Trees

- Look at the graph
  - Take all the edges
  - Sort it
  - Go through the edges one by one
    - Add if u and w not same tree

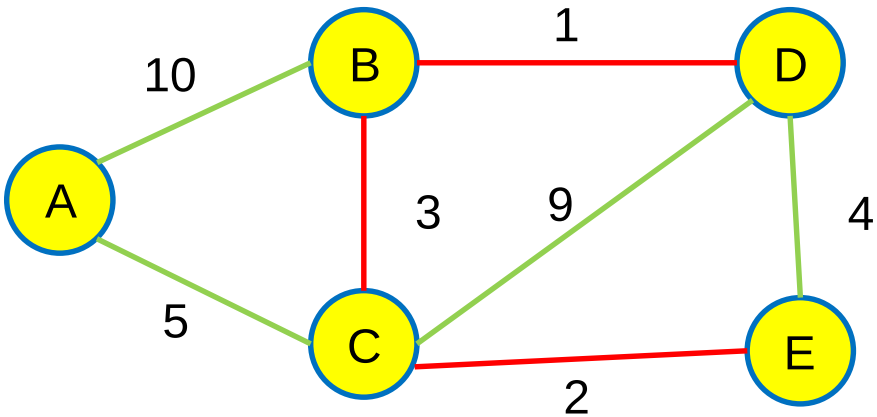


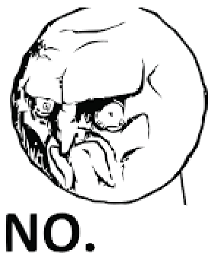
AB	AC	BC	BD	CD	CE	DE
10	5	3	1	9	2	4

BD	CE	BC	DE	AC	CD	AB
1	2	3	4	5	9	10

Kruskal's Algorithm
Combining (Union of) Trees

- Look at the graph
  - Take all the edges
  - Sort it
  - Go through the edges one by one
    - Add if u and w not same tree



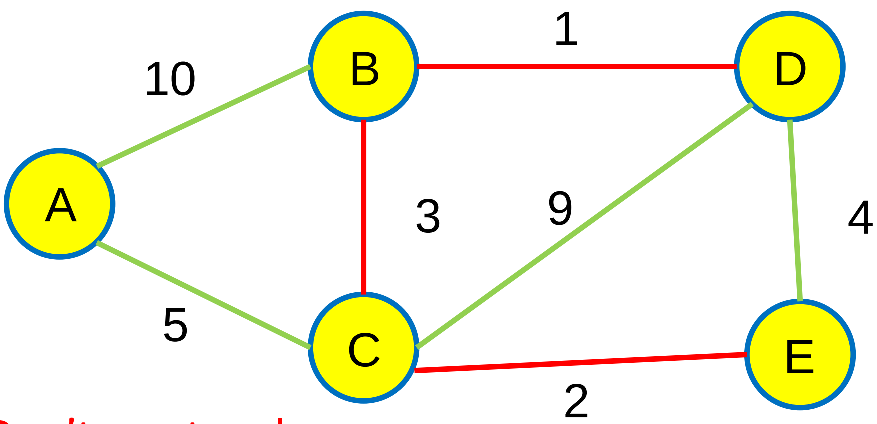


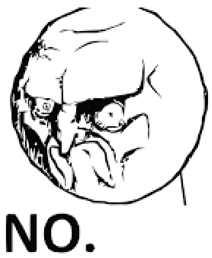
AB	AC	BC	BD	CD	CE	DE
10	5	3	1	9	2	4

BD	CE	BC	DE	AC	CD	AB
1	2	3	4	5	9	10

Kruskal's Algorithm
Combining (Union of) Trees

- Look at the graph
  - Take all the edges
  - Sort it
  - Go through the edges one by one
    - Add if u and w not same tree. Don't want cycle



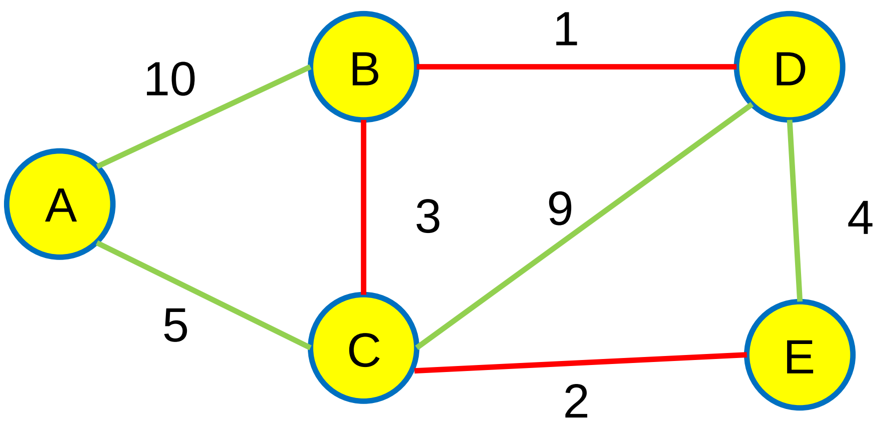


AB	AC	BC	BD	CD	CE	DE
10	5	3	1	9	2	4

BD	CE	BC	DE	AC	CD	AB
1	2	3	4	5	9	10

Kruskal's Algorithm
Combining (Union of) Trees

- Look at the graph
  - Take all the edges
  - Sort it
  - Go through the edges one by one
    - Add if u and w not same tree

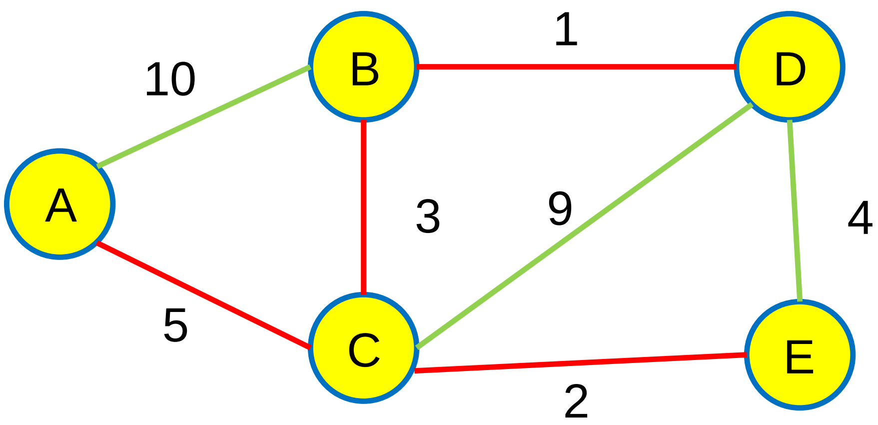


AB	AC	BC	BD	CD	CE	DE
10	5	3	1	9	2	4

BD	CE	BC	DE	AC	CD	AB
1	2	3	4	5	9	10

Kruskal's Algorithm
Combining (Union of) Trees

- Look at the graph
  - Take all the edges
  - Sort it
  - Go through the edges one by one
    - Add if u and w not same tree

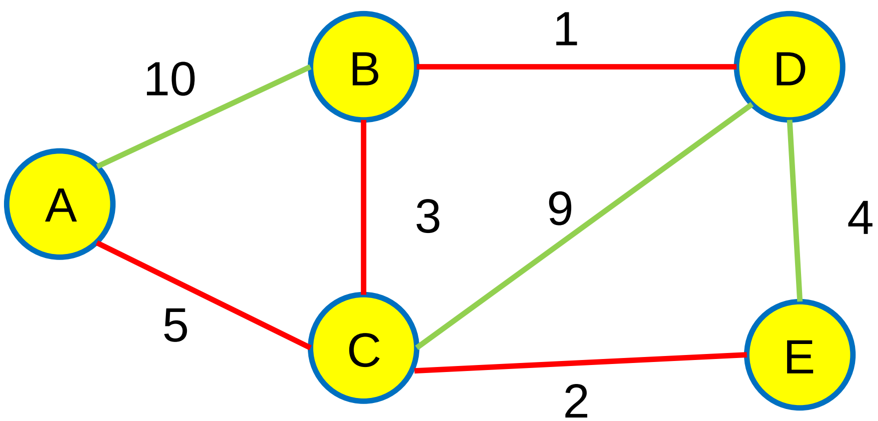


AB	AC	BC	BD	CD	CE	DE
10	5	3	1	9	2	4

BD	CE	BC	DE	AC	CD	AB
1	2	3	4	5	9	10

Kruskal's Algorithm
Combining (Union of) Trees

- Look at the graph
  - Take all the edges
  - Sort it
  - Go through the edges one by one
    - Add if u and w not same tree



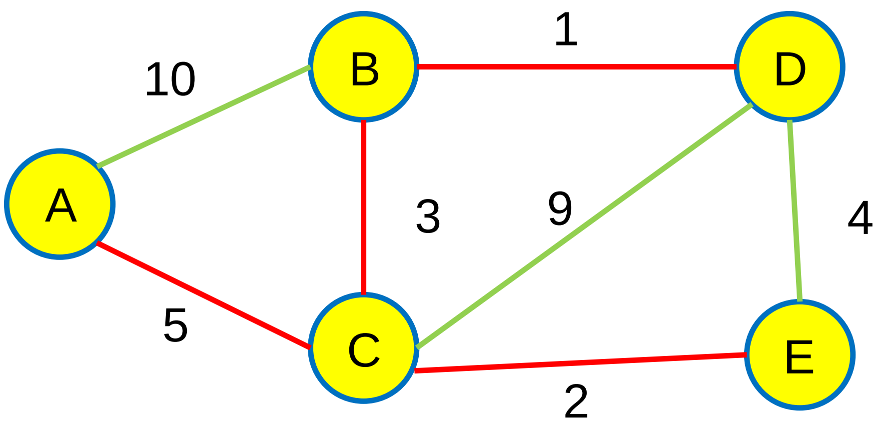
AB	AC	BC	BD	CD	CE	DE
10	5	3	1	9	2	4

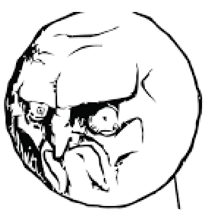
BD	CE	BC	DE	AC	CD	AB
1	2	3	4	5	9	10

Kruskal's Algorithm
Combining (Union of) Trees

MONASH University

- Look at the graph
  - Take all the edges
  - Sort it
  - Go through the edges one by one
    - Add if u and w not same tree



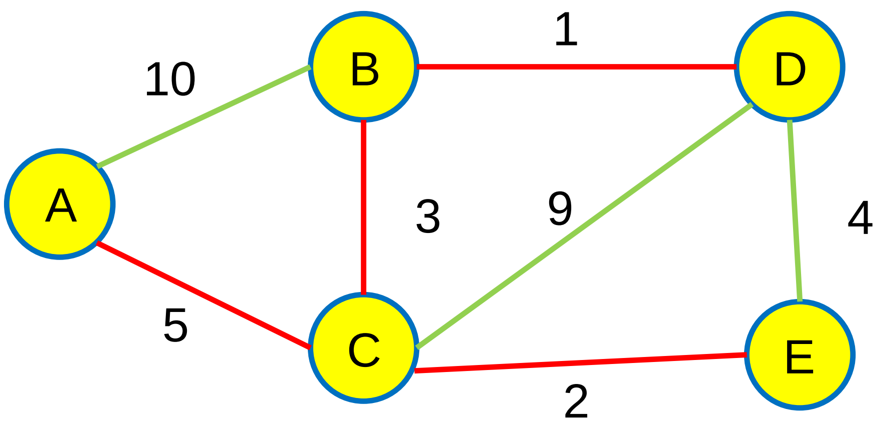


NO.

AB	AC	BC	BD	CD	CE	DE
10	5	3	1	9	2	4

BD	CE	BC	DE	AC	CD	AB
1	2	3	4	5	9	10

- Look at the graph
  - Take all the edges
  - Sort it
  - Go through the edges one by one
    - Add if u and w not same tree

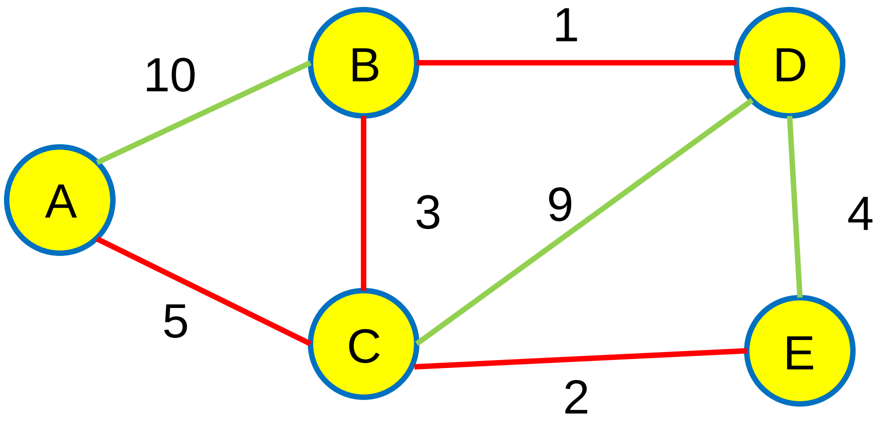


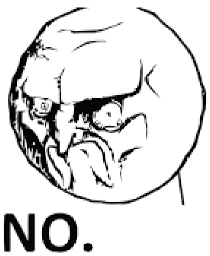
AB	AC	BC	BD	CD	CE	DE
10	5	3	1	9	2	4

BD	CE	BC	DE	AC	CD	AB
1	2	3	4	5	9	10

Kruskal's Algorithm
Combining (Union of) Trees

- Look at the graph
  - Take all the edges
  - Sort it
  - Go through the edges one by one
    - Add if u and w not same tree



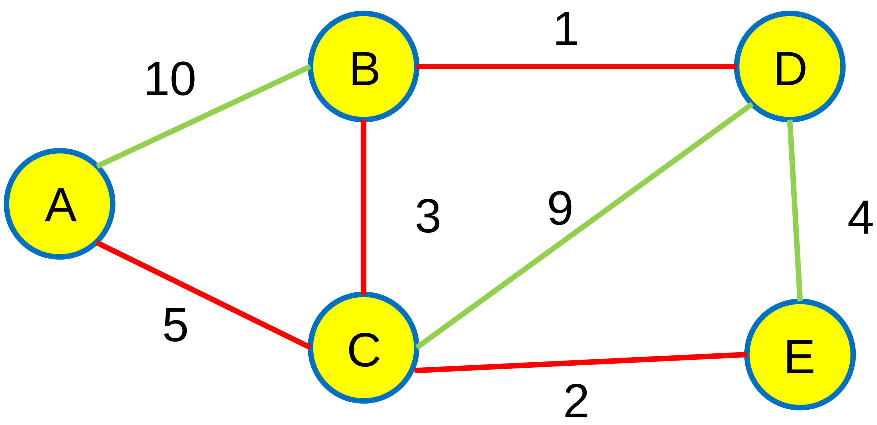


AB	AC	BC	BD	CD	CE	DE
10	5	3	1	9	2	4

BD	CE	BC	DE	AC	CD	AB
1	2	3	4	5	9	10

Kruskal's Algorithm
Combining (Union of) Trees

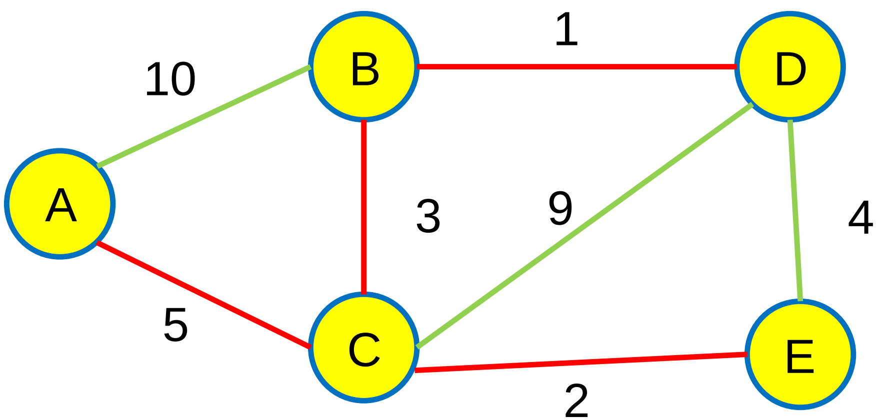
- Look at the graph
  - Take all the edges
  - Sort it
  - Go through the edges one by one
    - Add if u and w not same tree
  - And we are done!



BD	CE	BC	DE	AC	CD	AB
1	2	3	4	5	9	10

Kruskal's Algorithm
Combining (Union of) Trees

- Look at the graph
  - Take all the edges
  - Sort it
  - Go through the edges one by one
    - Add if u and w not same tree
  - And we are done!
  - But how do we implement it?



BD	CE	BC	DE	AC	CD	AB
1	2	3	4	5	9	10

- Look at the graph
  - Take all the edges
  - Sort it
  - Go through the edges one by one
    - Add if u and w not same tree
  - And we are done!
  - But how do we implement it?

- But how do we implement it?

- But how do we implement it?
  - Take the list of edges and sort

Kruskal's Algorithm
Combining (Union of) Trees

- But how do we implement it?
  - Take the list of edges and sort.
    - Easy… just use quicksort

Kruskal’s Algorithm
Combining (Union of) Trees

- But how do we implement it?
  - Take the list of edges and sort.
    - Easy… just use quicksort
    - Why not Counting or Radix?

Combining (Union of) Trees

- But how do we implement it?
  - Take the list of edges and sort.
    - Easy… just use quicksort
  - Check if vertex u and vertex v in <u,v,w> is in the same tree

Kruskal's Algorithm
Combining (Union of) Trees

- But how do we implement it?
  - Take the list of edges and sort.
    - Easy… just use quicksort
  - Check if vertex u and vertex v in <u,v,w> is in the same tree
    - HOW?

Kruskal’s Algorithm
Combining (Union of) Trees

- But how do we implement it?
  - Take the list of edges and sort.
    - Easy… just use quicksort
  - Check if vertex u and vertex v in <u,v,w> is in the same tree
    - HOW?
    - We use set! Any set data structure

Combining (Union of) Trees

- But how do we implement it?
  - Take the list of edges and sort.
    - Easy… just use quicksort
  - Check if vertex u and vertex v in <u,v,w> is in the same tree
    - HOW?
    - We use set! Any set data structure
      - Built-in Python Set (which is based on Dictionary/ Hashtable)

Combining (Union of) Trees

- But how do we implement it?
  - Take the list of edges and sort.
    - Easy… just use quicksort
  - Check if vertex u and vertex v in <u,v,w> is in the same tree
    - HOW?
    - We use set! Any set data structure
      - Built-in Python Set (which is based on Dictionary/ Hashtable)
      - Disjoint-Set (which you learn in FIT3155)

Combining (Union of) Trees

- But how do we implement it?
  - Take the list of edges and sort.
    - Easy… just use quicksort
  - Check if vertex u and vertex v in <u,v,w> is in the same tree (Find)
    - HOW?
    - We use set! Any set data structure
      - Built-in Python Set (which is based on Dictionary/ Hashtable)
      - Disjoint-Set (which you learn in FIT3155)
  - If not the same set, you joint them with the edge (Union)

- But how do we implement it?
  - Take the list of edges and sort.
    - Easy… just use quicksort
  - Check if vertex u and vertex v in <u,v,w> is in the same tree (Find)
    - HOW?
    - We use set! Any set data structure
      - Built-in Python Set (which is based on Dictionary/ Hashtable)
      - Disjoint-Set (which you learn in FIT3155)
  - If not the same set, you joint them with the edge (Union)
  - Thus, known as Union-Find

Kruskal’s Algorithm
Combining (Union of) Trees

- But how do we implement it?
  - Take the list of edges and sort.
    - Easy… just use quicksort
  - Check if vertex u and vertex v in <u,v,w> is in the same tree (Find)
    - We use set! Any set data structure
      - Built-in Python Set (which is based on Dictionary/ Hashtable)
      - Disjoint-Set (which you learn in FIT3155)
  - If not the same set, you joint them with the edge (Union)
  - Thus, known as Union-Find

  - Complexity?

- But how do we implement it?
  - Take the list of edges and sort.
    - Easy… just use quicksort
    - This is O(E log E)
  - Check if vertex u and vertex v in <u,v,w> is in the same tree (Find)
    - We use set! Any set data structure
      - Built-in Python Set (which is based on Dictionary/ Hashtable)
      - Disjoint-Set (which you learn in FIT3155)
  - If not the same set, you joint them with the edge (Union)
  - Thus, known as Union-Find
  - Complexity?

Kruskal's Algorithm
Combining (Union of) Trees

- But how do we implement it?
  - Take the list of edges and sort.
    - Easy… just use quicksort
    - This is O(E log E)
  - Check if vertex u and vertex v in <u,v,w> is in the same tree (Find)
    - We use set! Any set data structure
      - Built-in Python Set (which is based on Dictionary/ Hashtable)
      - Disjoint-Set (which you learn in FIT3155)
    - This is O(1)
  - If not the same set, you joint them with the edge (Union)
    - This is O(V) for now
  - Thus, known as Union-Find

  - Complexity?

MONASH University

- But how do we implement it?
  - Take the list of edges and sort.
    - Easy… just use quicksort
    - This is O(E log E)
  - Check if vertex u and vertex v in <u,v,w> is in the same tree (Find)
    - We use set! Any set data structure
      - Built-in Python Set (which is based on Dictionary/ Hashtable)
      - Disjoint-Set (which you learn in FIT3155)
    - This is O(1)
  - If not the same set, you joint them with the edge (Union)
    - This is O(V) for now
  - Thus, known as Union-Find

  - Complexity?

For each edge

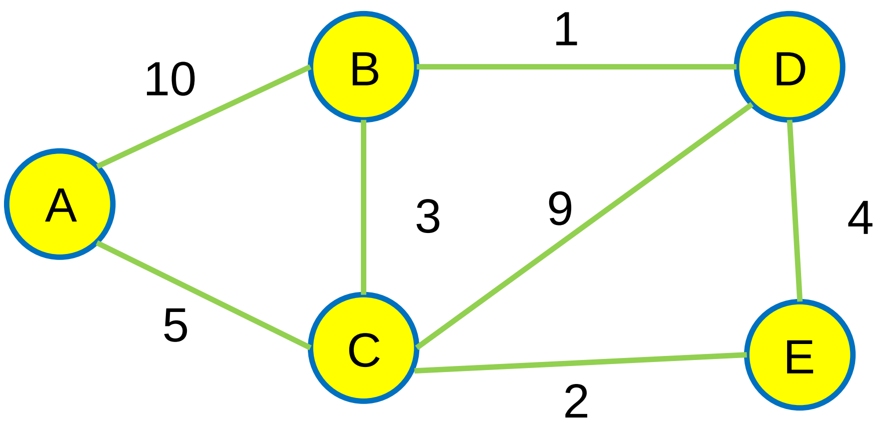
Kruskal's Algorithm
Combining (Union of) Trees

- But how do we implement it?
  - Take the list of edges and sort.
    - Easy… just use quicksort
    - This is O(E log E)
  - Check if vertex u and vertex v in <u,v,w> is in the same tree (Find)
    - We use set! Any set data structure
      - Built-in Python Set (which is based on Dictionary/ Hashtable)
      - Disjoint-Set (which you learn in FIT3155)
    - This is O(1)
  - If not the same set, you joint them with the edge (Union)
    - This is O(V) for now
  - Thus, known as Union-Find
  - Complexity? O(E log E + E(1+V)) = O(EV)

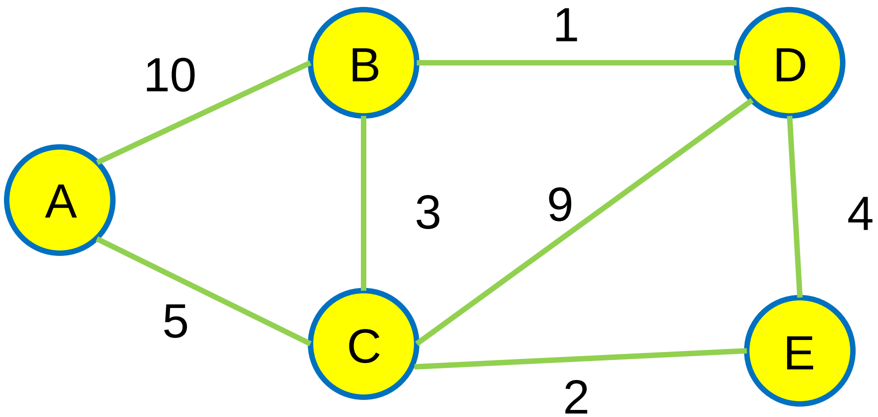
For each edge

Kruskal's Algorithm
Combining (Union of) Trees

- Union-Find with sets

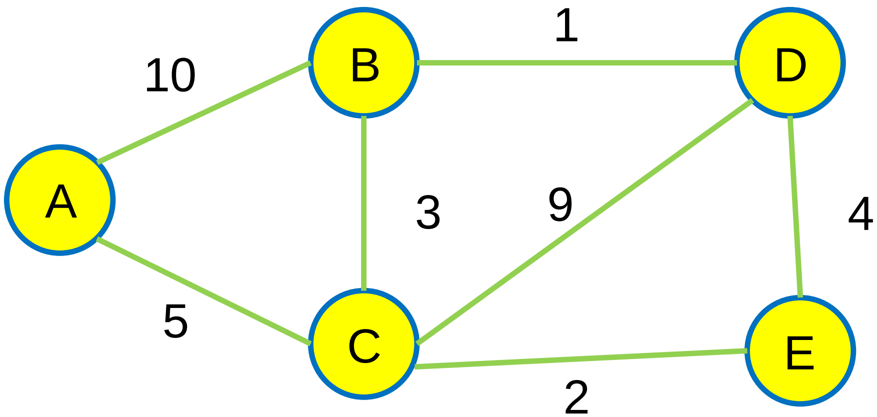


- Union-Find with sets



A	B	C	D	E
1	2	3	4	5

- Union-Find with sets



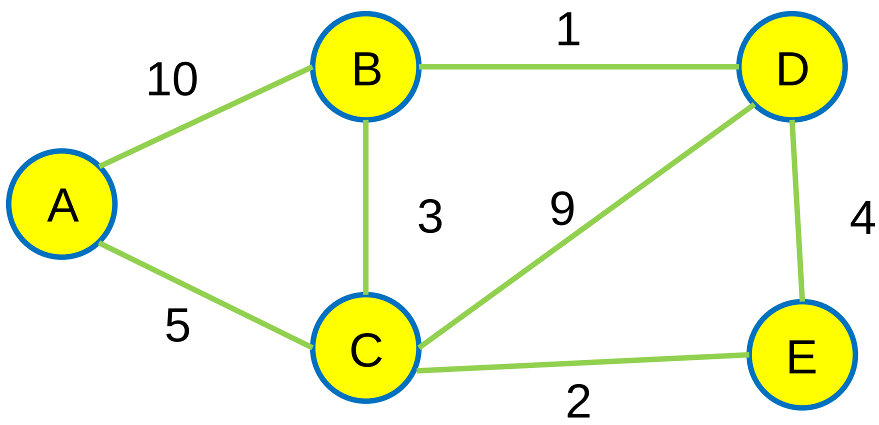
1	2	3	4	5
A	B	C	D	E

A	B	C	D	E
1	2	3	4	5

Kruskal's Algorithm

Combining (Union of) Trees

- Union-Find with sets



1	2	3	4	5
A	B	C	D	E

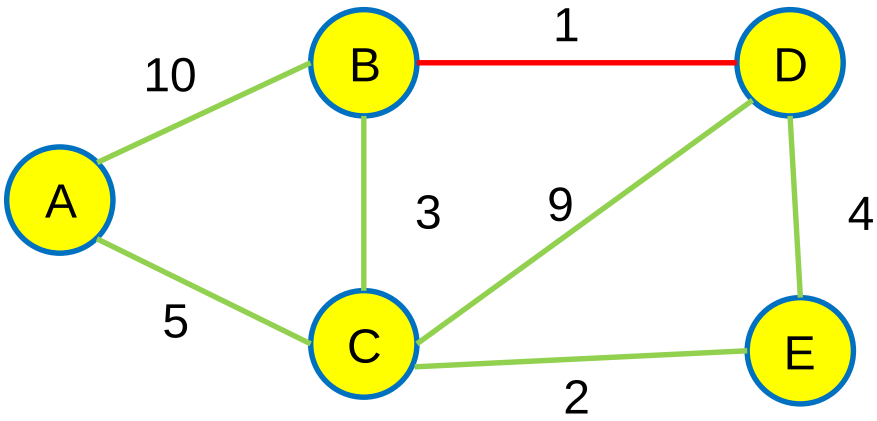
Set Array

A	B	C	D	E
1	2	3	4	5

Map Array

Kruskal’s Algorithm
Combining (Union of) Trees

- Union-Find with sets



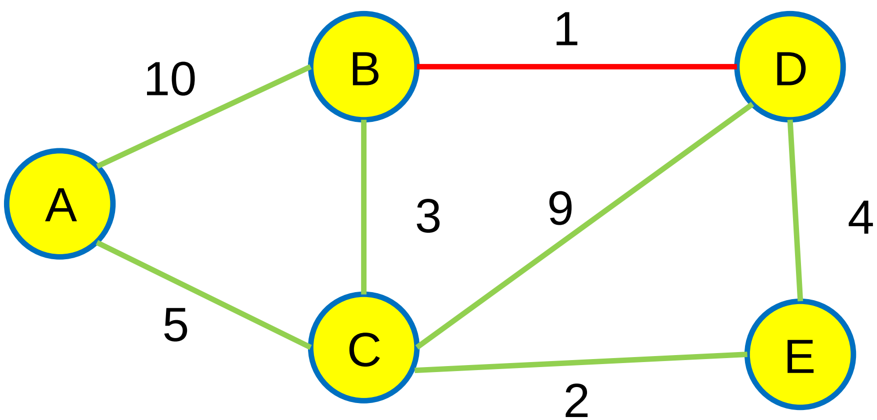
1	2	3	4	5
A	B	C	D	E

Set Array

A	B	C	D	E
1	2	3	4	5

Map Array

- Union-Find with sets



1	2	3	4	5
A	B	C	D	E

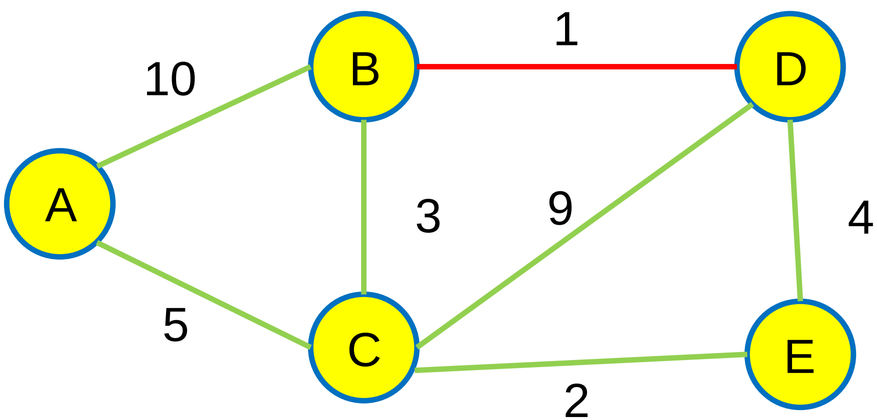
Set Array

A	B	C	D	E
1	2	3	4	5

Map Array

Kruskal's Algorithm
Combining (Union of) Trees

- Union-Find with sets



1	2	3	4	5
A	B,D	C		E

Set Array

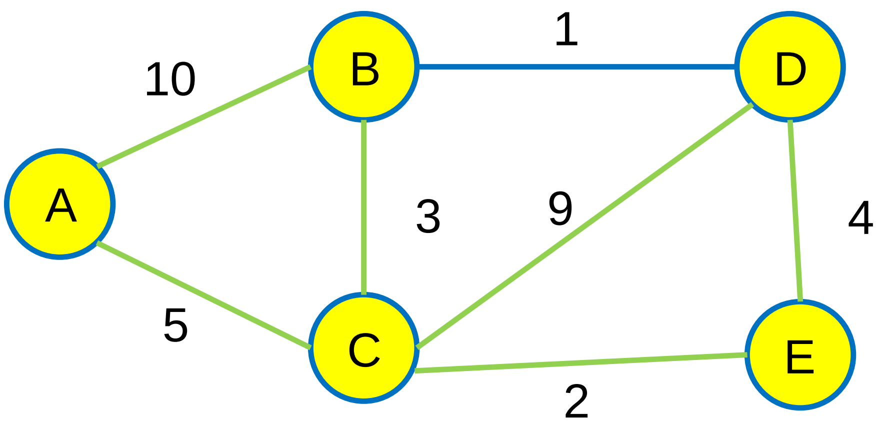
A	B	C	D	E
1	2	3	2	5

Map Array

Kruskal's Algorithm

Combining (Union of) Trees

- Union-Find with sets



1	2	3	4	5
A	B,D	C		E

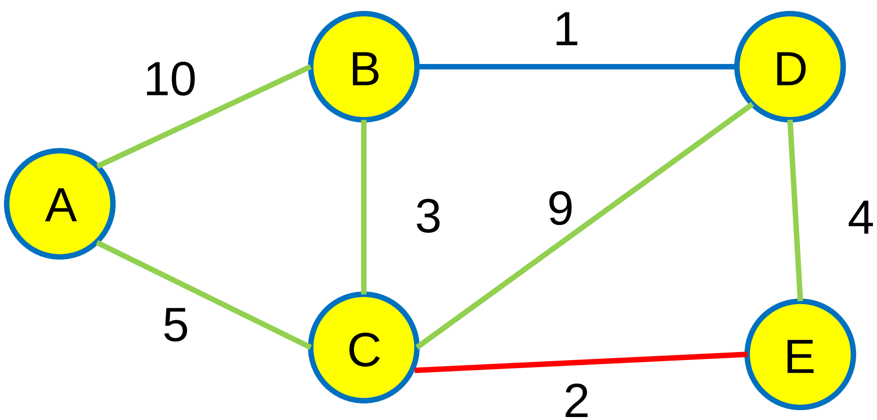
Set Array

A	B	C	D	E
1	2	3	2	5

Map Array

Kruskal's Algorithm
Combining (Union of) Trees

- Union-Find with sets



1	2	3	4	5
A	B,D	C		E

Set Array

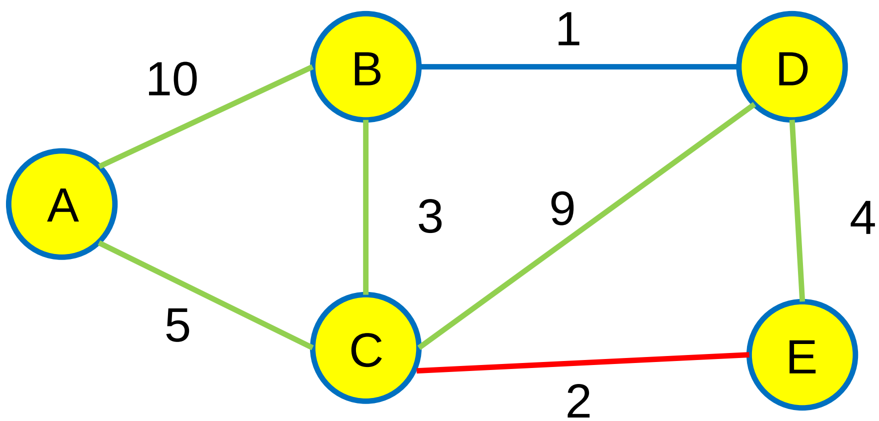
A	B	C	D	E
1	2	3	2	5

Map Array

Kruskal's Algorithm

Combining (Union of) Trees

- Union-Find with sets



1	2	3	4	5
A	B,D	C		E

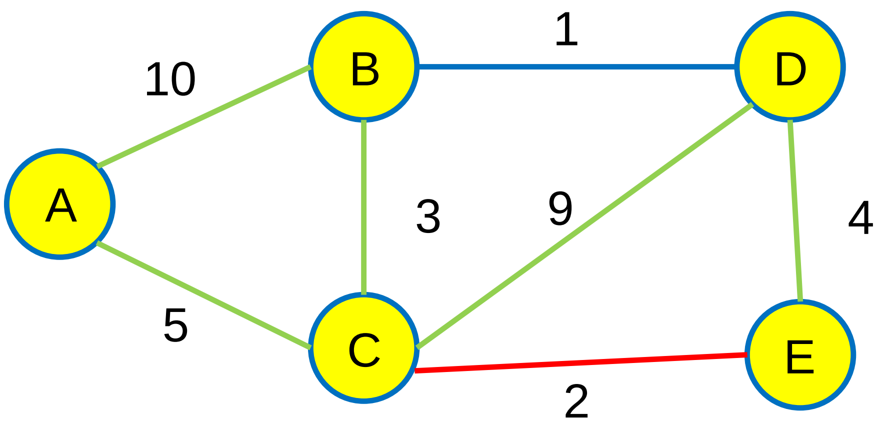
Set Array

A	B	C	D	E
1	2	3	2	5

Map Array

Kruskal's Algorithm
Combining (Union of) Trees

- Union-Find with sets



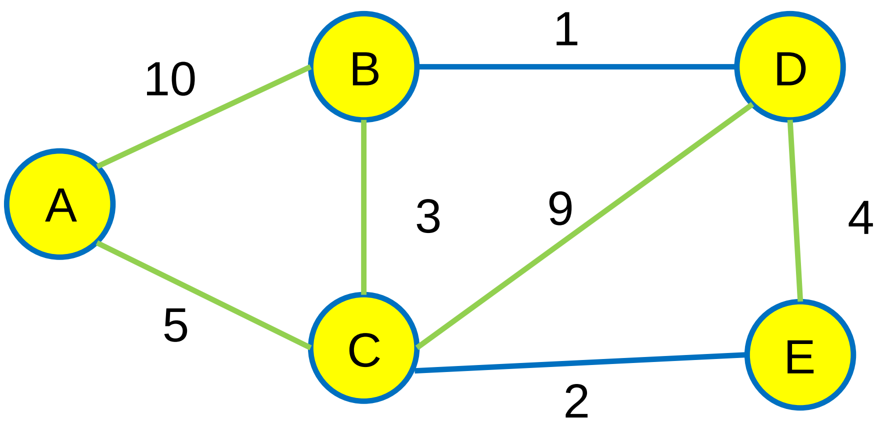
1	2	3	4	5
A	B,D	C,E		E

Set Array

A	B	C	D	E
1	2	3	2	3

Map Array

- Union-Find with sets



1	2	3	4	5
A	B,D	C,E		

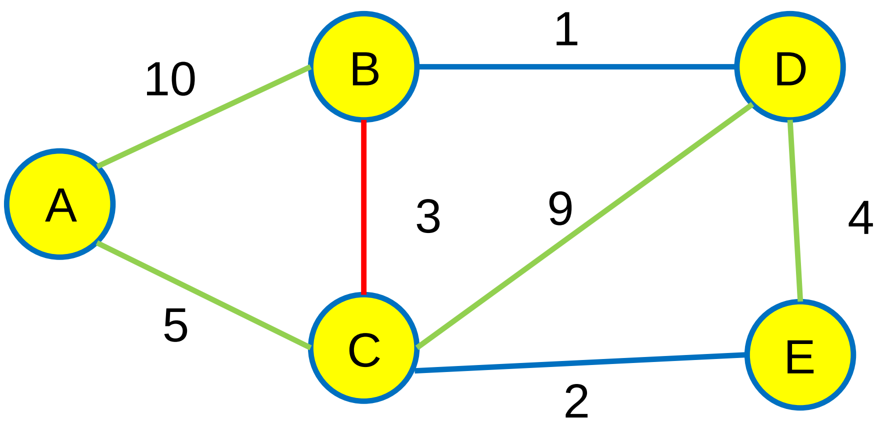
Set Array

A	B	C	D	E
1	2	3	2	3

Map Array

Combining (Union of) Trees

- Union-Find with sets



1	2	3	4	5
A	B,D	C,E		

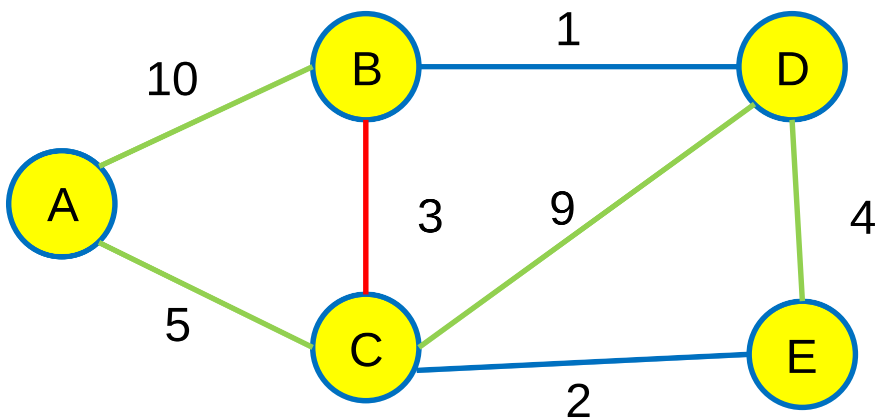
Set Array

A	B	C	D	E
1	2	3	2	3

Map Array

Kruskal's Algorithm
Combining (Union of) Trees

- Union-Find with sets



1	2	3	4	5
A	B,D	C,E		

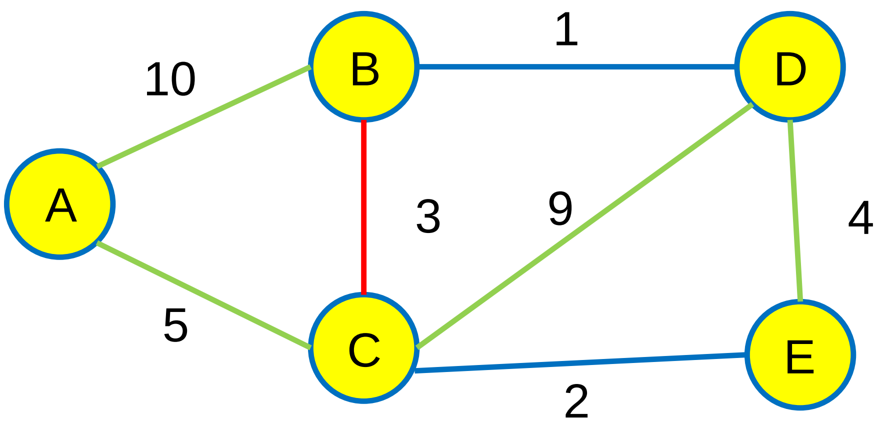
Set Array

A	B	C	D	E
1	2	3	2	3

Map Array

Kruskal's Algorithm
Combining (Union of) Trees

- Union-Find with sets



1	2	3	4	5
A	B,D	C,E		

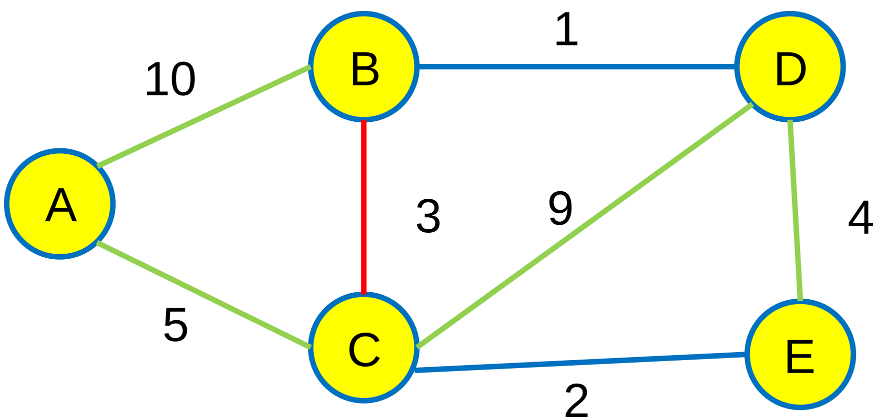
Set Array

A	B	C	D	E
1	2	3	2	3

Map Array

Kruskal's Algorithm
Combining (Union of) Trees

- Union-Find with sets



1	2	3	4	5
A	B,D	C,E		

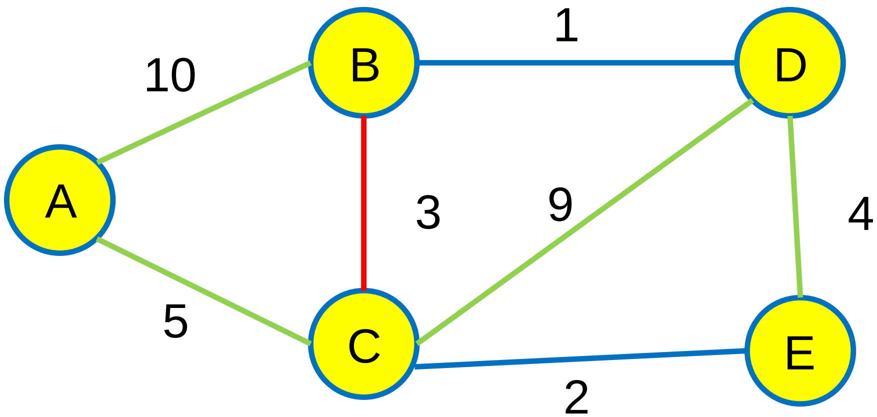
Set Array

A	B	C	D	E
1	2	3	2	3

Map Array

Combining (Union of) Trees

- Union-Find with sets



1	2	3	4	5
A	B,D,C,E			

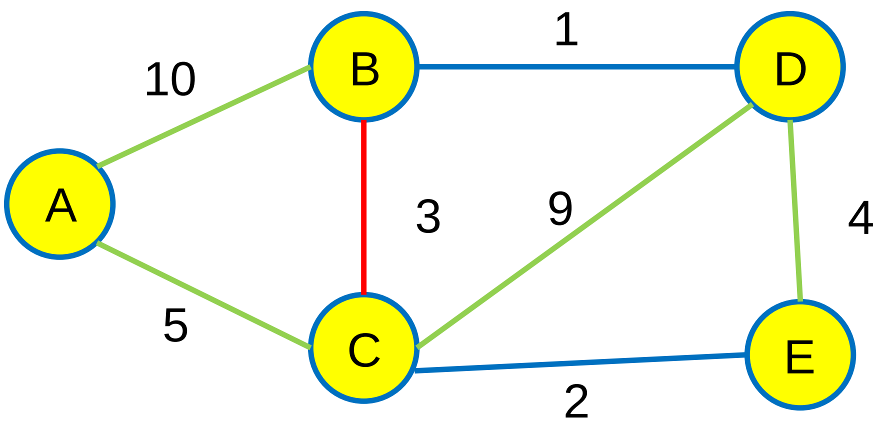
Set Array

A	B	C	D	E
1	2	3	2	3

Map Array

Kruskal's Algorithm
Combining (Union of) Trees

- Union-Find with sets



1	2	3	4	5
A	B,D,C,E			

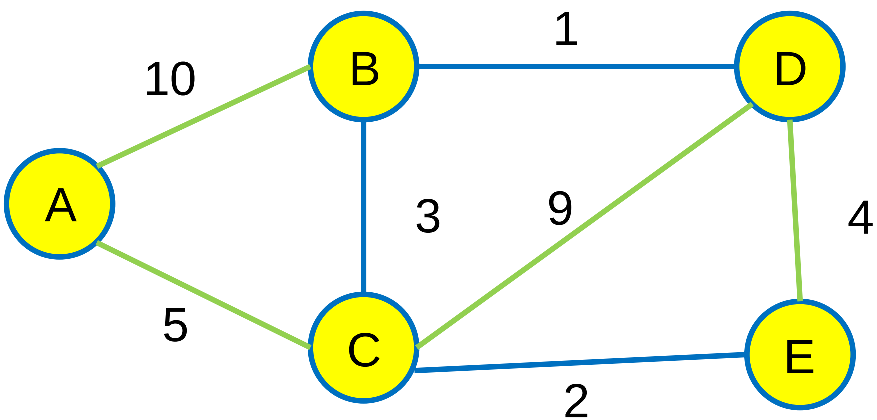
Set Array

A	B	C	D	E
1	2	2	2	2

Map Array

Kruskal's Algorithm
Combining (Union of) Trees

- Union-Find with sets



1	2	3	4	5
A	B,D,C,E			

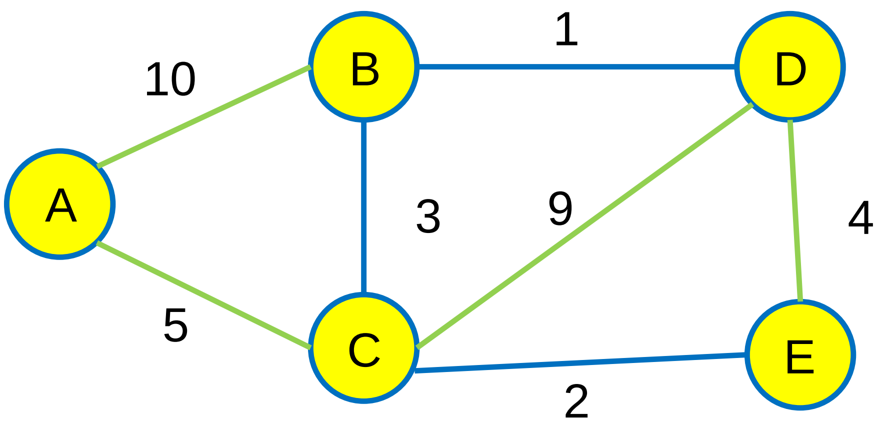
Set Array

A	B	C	D	E
1	2	2	2	2

Map Array

Combining (Union of) Trees

- Union-Find with sets
  … and so on
  you get the idea…



1	2	3	4	5
A	B,D,C,E			

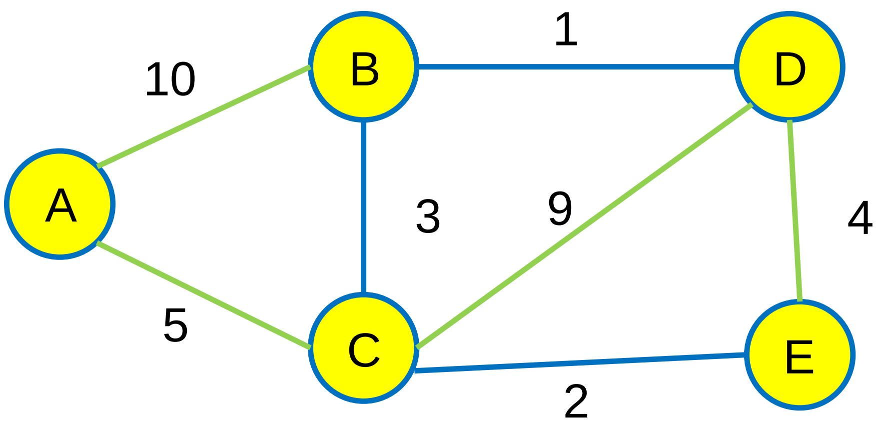
Set Array

A	B	C	D	E
1	2	2	2	2

Map Array

Kruskal's Algorithm
Combining (Union of) Trees

- Union-Find with sets
  - Check the set for vertex
  - Merge vertex set
    - Smaller set -> bigger set
    - Update the map array…



1	2	3	4	5
A	B,D,C,E			

Set Array

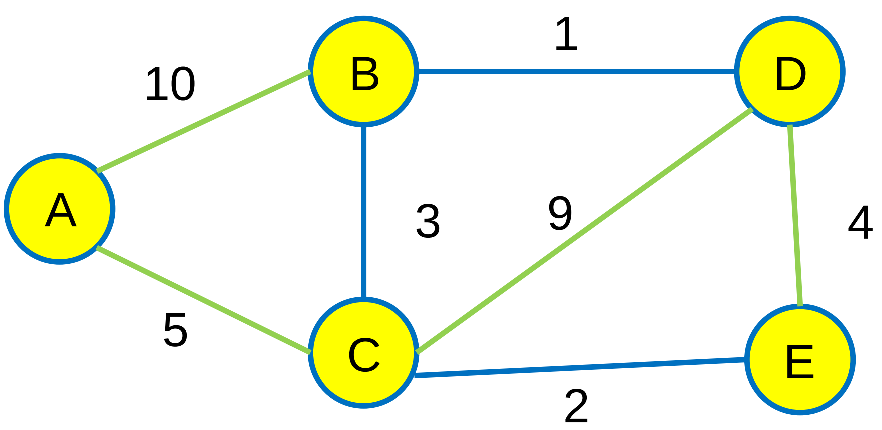
A	B	C	D	E
1	2	2	2	2

Map Array

Kruskal's Algorithm

Combining (Union of) Trees

- Union-Find with sets
  - Check the set for vertex
  - Merge vertex set
    - Smaller set -> bigger set
    - Update the map array…
  - Repeat…



1	2	3	4	5
A	B,D,C,E			

Set Array

A	B	C	D	E
1	2	2	2	2

Map Array

Kruskal’s Algorithm
Combining (Union of) Trees

MONASH University

- But how do we implement it?
  - Take the list of edges and sort.
    - Easy… just use quicksort
    - This is O(E log E)
  - Check if vertex u and vertex v in <u,v,w> is in the same tree (Find)
    - We use set! Any set data structure
      - Built-in Python Set (which is based on Dictionary/ Hashtable)
      - Disjoint-Set (which you learn in FIT3155)
    - This is O(1)
  - If not the same set, you joint them with the edge (Union)
    - This is O(V) for now
  - Thus, known as Union-Find
  - Complexity? O(EV) but this is O(E log V) amortized

For each edge

Kruskal's Algorithm
Combining (Union of) Trees

- But how do we implement it?
  - Take the list of edges and sort.
    - Easy… just use quicksort
    - This is O(E log E)
  - Check if vertex u and vertex v in <u,v,w> is in the same tree (Find)
    - We use set! Any set data structure
      - Built-in Python Set (which is based on Dictionary/ Hashtable)
      - Disjoint-Set (which you learn in FIT3155)
    - This is O(1)
  - If not the same set, you joint them with the edge (Union)
    - This is O(V) for now
  - Thus, known as Union-Find
  - Complexity? O(EV) but this is O(E log V) amortized (worst case merge 2 same size)

For each edge

Questions?

- This is a week of content itself for FIT3155
  - Though, probably the shortest and easiest one to learn

- This is a week of content itself for FIT3155
  - Though, probably the shortest and easiest one to learn

- Union by size
- Union by height/ rank

- This is a week of content itself for FIT3155
  - Though, probably the shortest and easiest one to learn

- Union by size
- Union by height/ rank

- Done by using an array, called the parent array

Union-Find

- This is a week of content itself for FIT3155
  - Though, probably the shortest and easiest one to learn

- Union by size
- Union by height/ rank

- Done by using an array, called the parent array
  - Index of the parent, as positive value

- This is a week of content itself for FIT3155
  - Though, probably the shortest and easiest one to learn

- Union by size
- Union by height/ rank

- Done by using an array, called the parent array
  - Index of the parent, as positive value
  - Size or height, as negative value

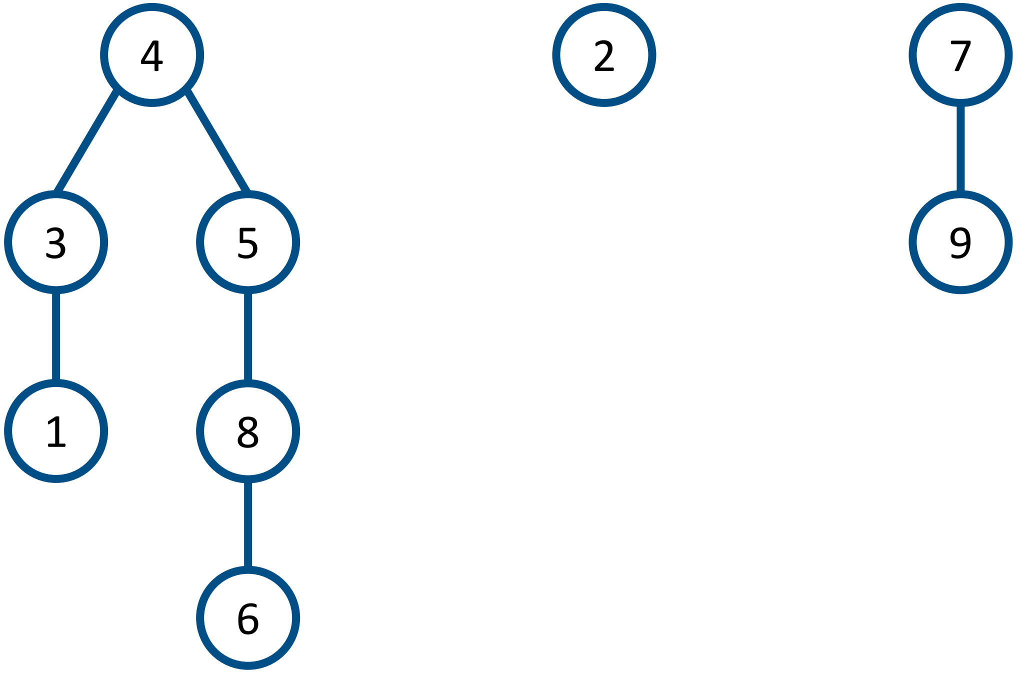
MONASH University

- This is a week of content itself for FIT3155
  - Though, probably the shortest and easiest one to learn

- Union by size
- Union by height/ rank

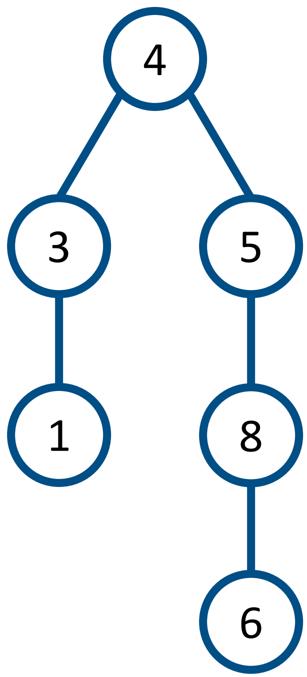
- Done by using an array, called the parent array
  - Index of the parent, as positive value
  - Size or height, as negative value but only at the root

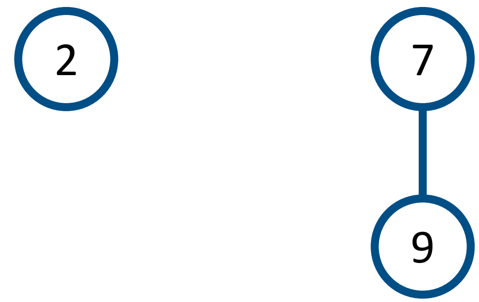
Kruskal's
Union-Find



Kruskal's

Union-Find





	1	2	3	4	5	6	7	8	9
	3	-1	4	-6	4	8	-2	5	7

- Why such an implementation?

- Why such an implementation?
  - On find(u)
    - Loop till we reach the root of u (having negative number)

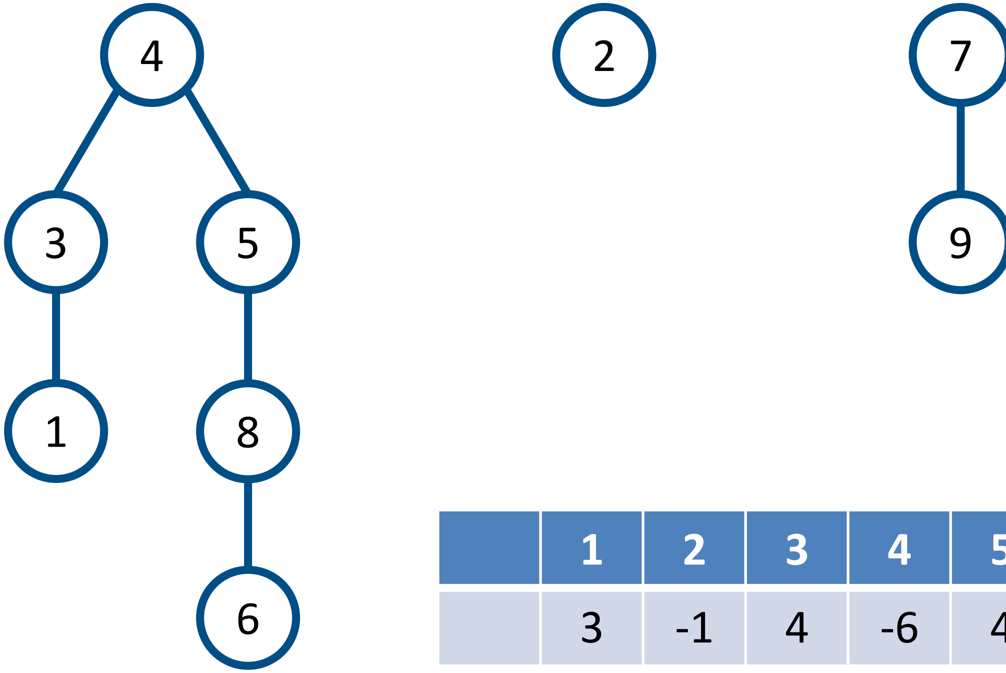
- Why such an implementation?
  - On find(u)
    - Loop till we reach the root of u (having negative number)
  - On find(v)
    - Loop till we reach the root of v (having negative number)

- Why such an implementation?
  - On find(u)
    - Loop till we reach the root of u (having negative number)
  - On find(v)
    - Loop till we reach the root of v (having negative number)

  - Remember: roots always store the size (as a negative number)

Kruskal's

Union-Find



	1	2	3	4	5	6	7	8	9
	3	-1	4	-6	4	8	-2	5	7

– Remember: roots always store the size (as a negative number)

- Why such an implementation?
  - On find(u)
    - Loop till we reach the root of u (having negative number)
  - On find(v)
    - Loop till we reach the root of v (having negative number)

  - If both u and v have the same root…

- Why such an implementation?
  - On find(u)
    - Loop till we reach the root of u (having negative number)
  - On find(v)
    - Loop till we reach the root of v (having negative number)
  - If both u and v have the same root…
    - They are in the same team/ set/ tree

- Why such an implementation?
  - On find(u)
    - Loop till we reach the root of u (having negative number)
  - On find(v)
    - Loop till we reach the root of v (having negative number)
  - If both u and v have the same root…
    - They are in the same team/ set/ tree
    - We can't perform union(u, v)

MONASH University

- Why such an implementation?
  - On find(u)
    - Loop till we reach the root of u (having negative number)
  - On find(v)
    - Loop till we reach the root of v (having negative number)

  - If both u and v have the same root…
    - They are in the same team/ set/ tree
    - We can't perform union(u, v)
  - If both u and v have different root…

- Why such an implementation?
  - On find(u)
    - Loop till we reach the root of u (having negative number)
  - On find(v)
    - Loop till we reach the root of v (having negative number)
  - If both u and v have the same root…
    - They are in the same team/ set/ tree
    - We can't perform union(u, v)
  - If both u and v have different root…
    - They are in different team/ set/ tree

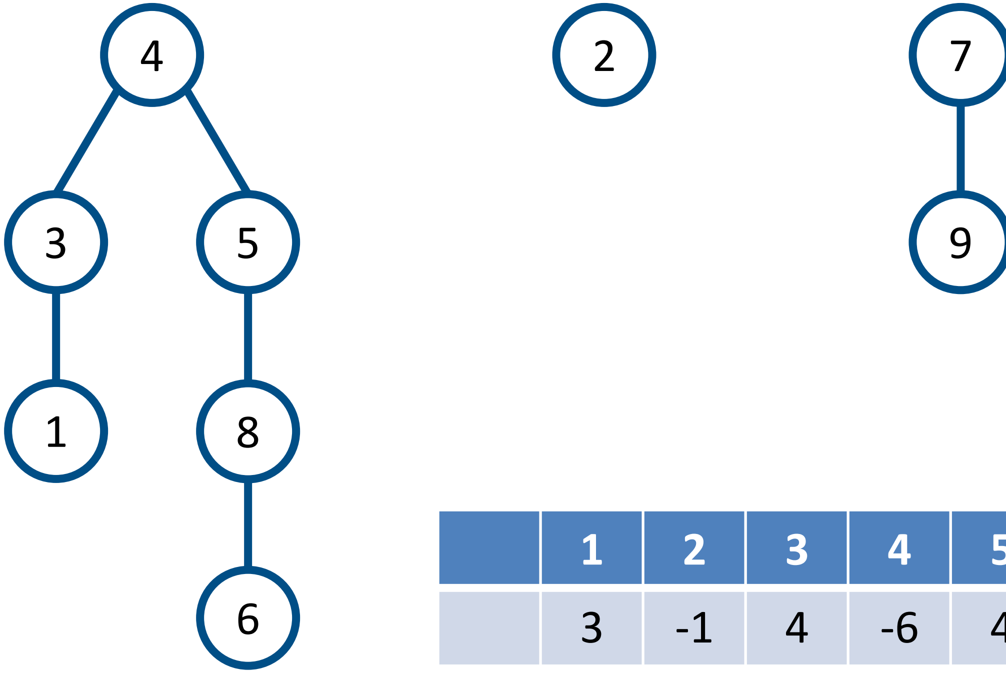
- Why such an implementation?
  - On find(u)
    - Loop till we reach the root of u (having negative number)
  - On find(v)
    - Loop till we reach the root of v (having negative number)
  - If both u and v have the same root…
    - They are in the same team/ set/ tree
    - We can't perform union(u, v)
  - If both u and v have different root…
    - They are in different team/ set/ tree
    - Then we can perform union(u, v)

MONASH University

- Why such an implementation?
  - On find(u)
    - Loop till we reach the root of u (having negative number)
  - On find(v)
    - Loop till we reach the root of v (having negative number)
  - If both u and v have the same root…
    - They are in the same team/ set/ tree
    - We can't perform union(u, v)
  - If both u and v have different root…
    - They are in different team/ set/ tree
    - Then we can perform union(u, v)
    - If tree with u has more items than tree with v, root of u becomes parent of root of v
    - … vice versa

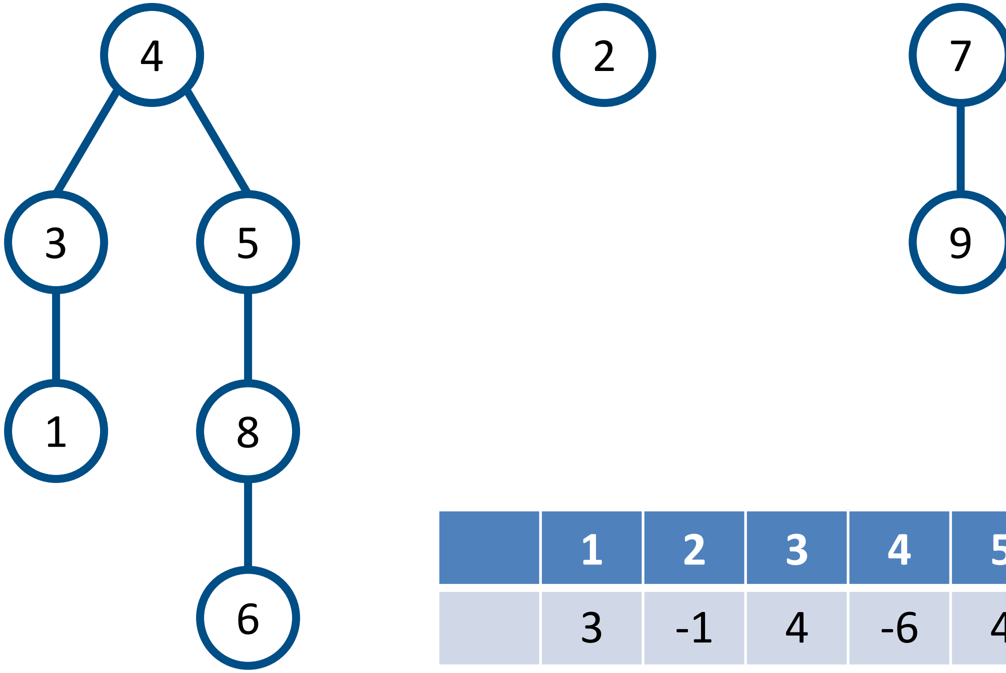
Questions?

Kruskal’s
Union-Find



	1	2	3	4	5	6	7	8	9
	3	-1	4	-6	4	8	-2	5	7

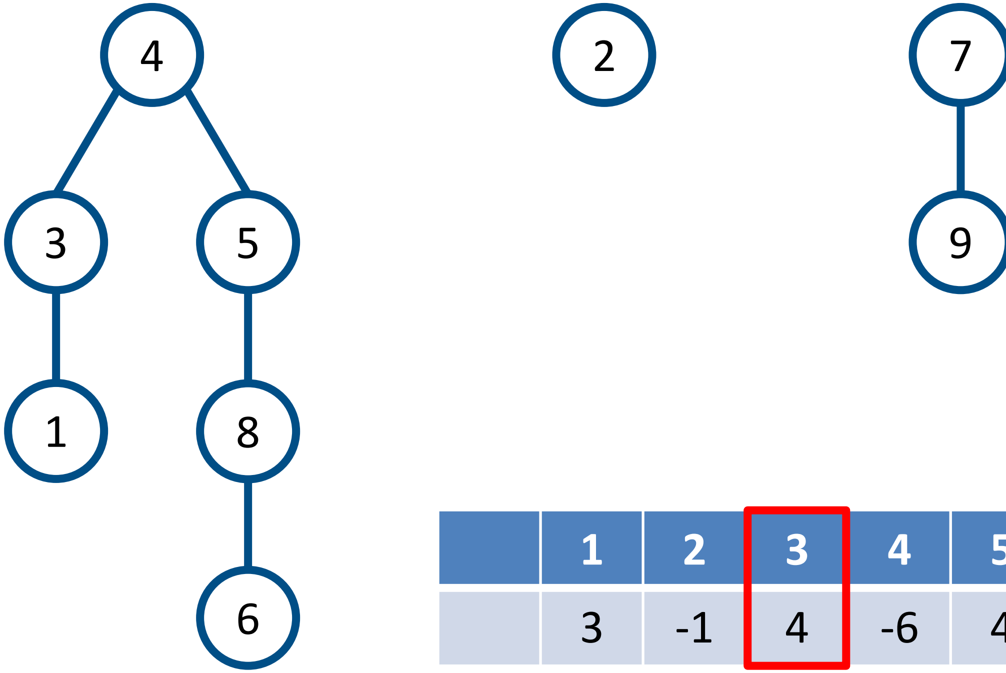
– Union(3,8)



	1	2	3	4	5	6	7	8	9
	3	-1	4	-6	4	8	-2	5	7

- Union(3,8)
  - Find(3)
  - Find(8)

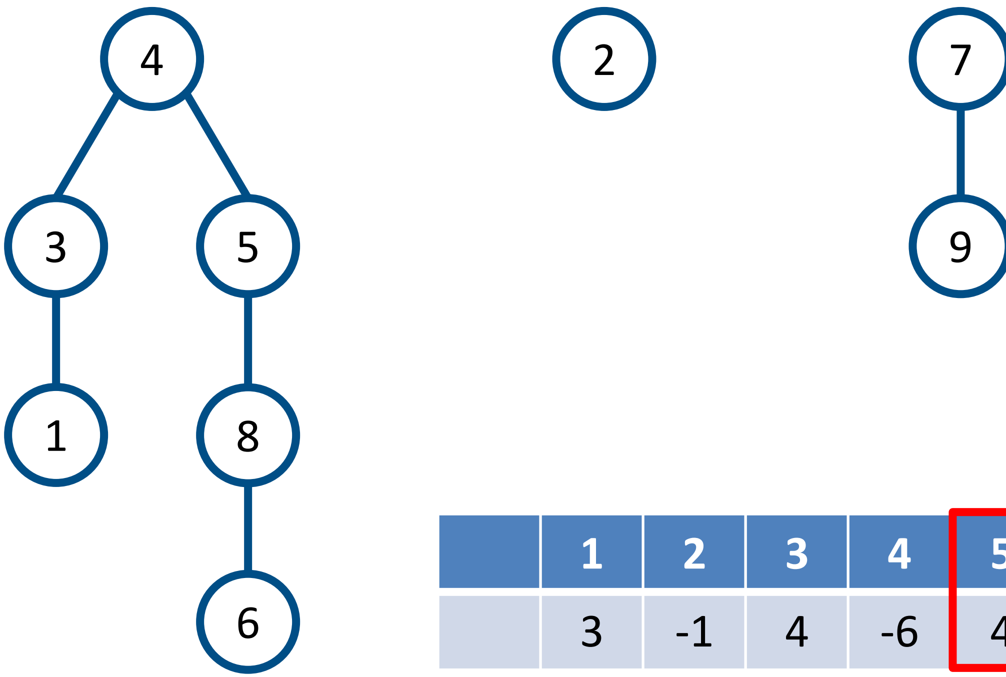
Kruskal's
Union-Find



	1	2	3	4	5	6	7	8	9
	3	-1	4	-6	4	8	-2	5	7

- Union(3,8)
  - Find(3) -> 4

Kruskal's
Union-Find

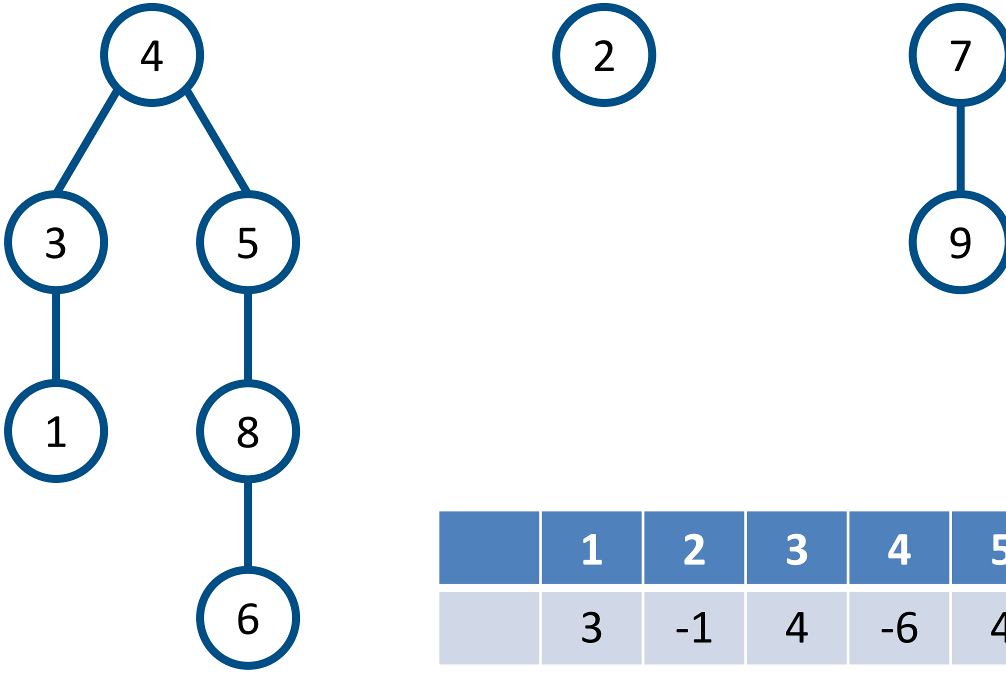


	1	2	3	4	5	6	7	8	9
	3	-1	4	-6	4	8	-2	5	7

- Union(3,8)
  - Find(3) -> 4
  - Find(8) -> 4

Kruskal's

Union-Find

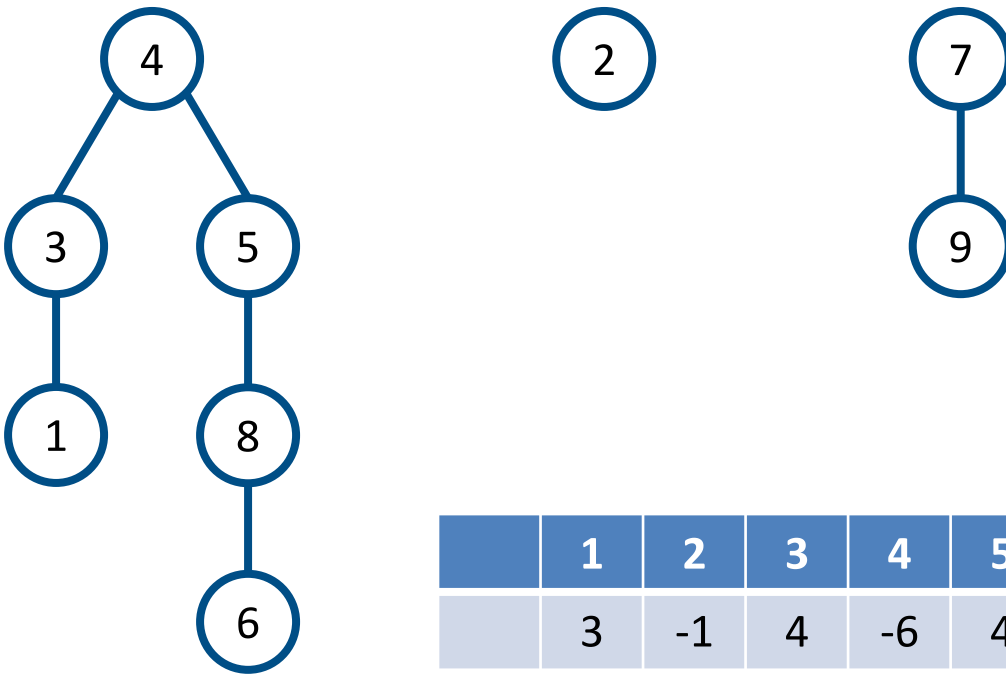


	1	2	3	4	5	6	7	8	9
	3	-1	4	-6	4	8	-2	5	7

- Union(3,8), can't perform the union
  - Find(3) -> 4
  - Find(8) -> 4

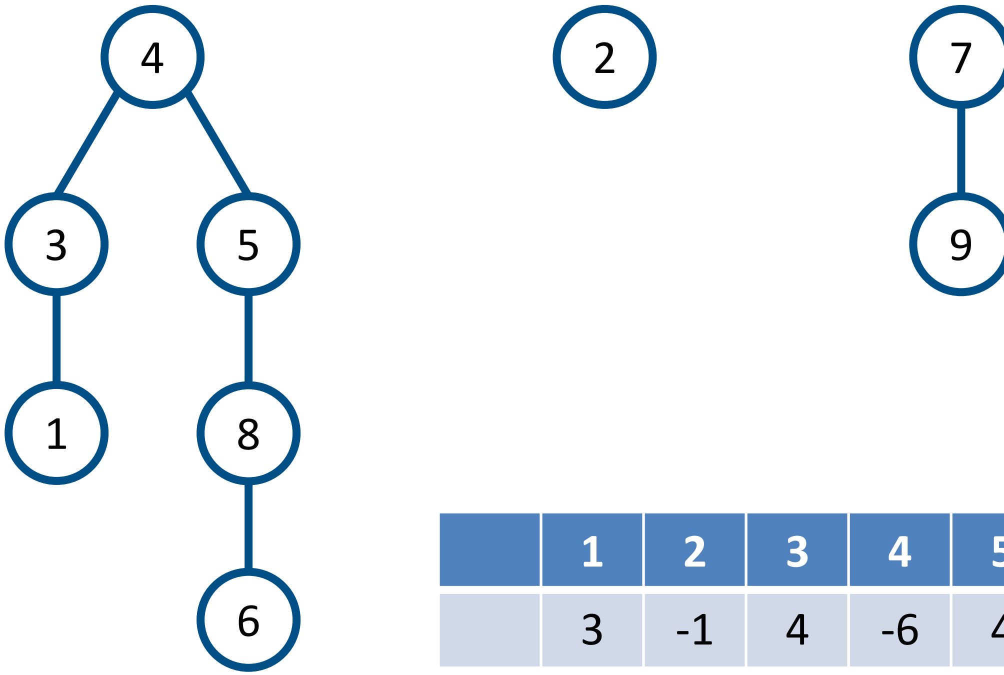
Questions?

Kruskal's
Union-Find



	1	2	3	4	5	6	7	8	9
	3	-1	4	-6	4	8	-2	5	7

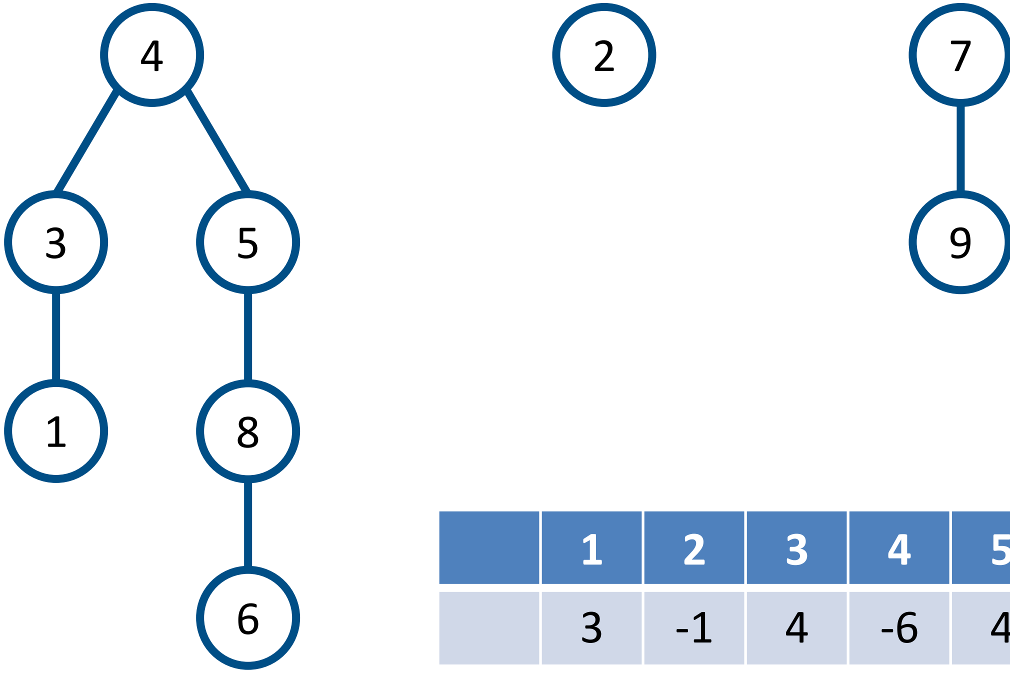
– Union(9,8)



	1	2	3	4	5	6	7	8	9
	3	-1	4	-6	4	8	-2	5	7

- Union(9,8)
  - Find(9)

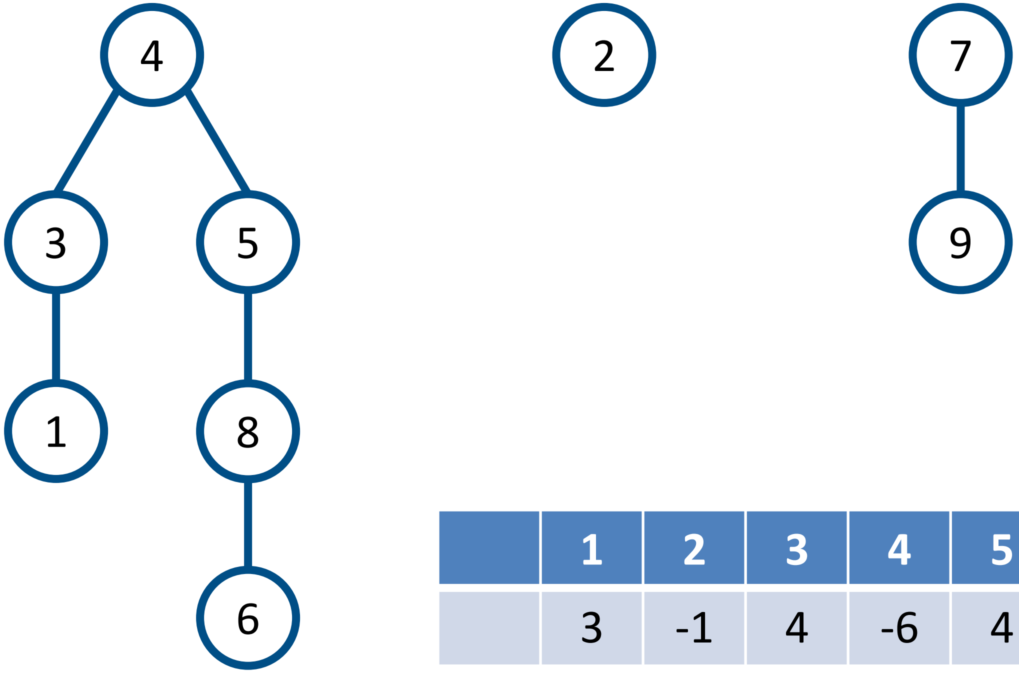
Kruskal's
Union-Find



	1	2	3	4	5	6	7	8	9
	3	-1	4	-6	4	8	-2	5	7

– Union(9,8)
  - Find(9) -> 7

Kruskal's
Union-Find

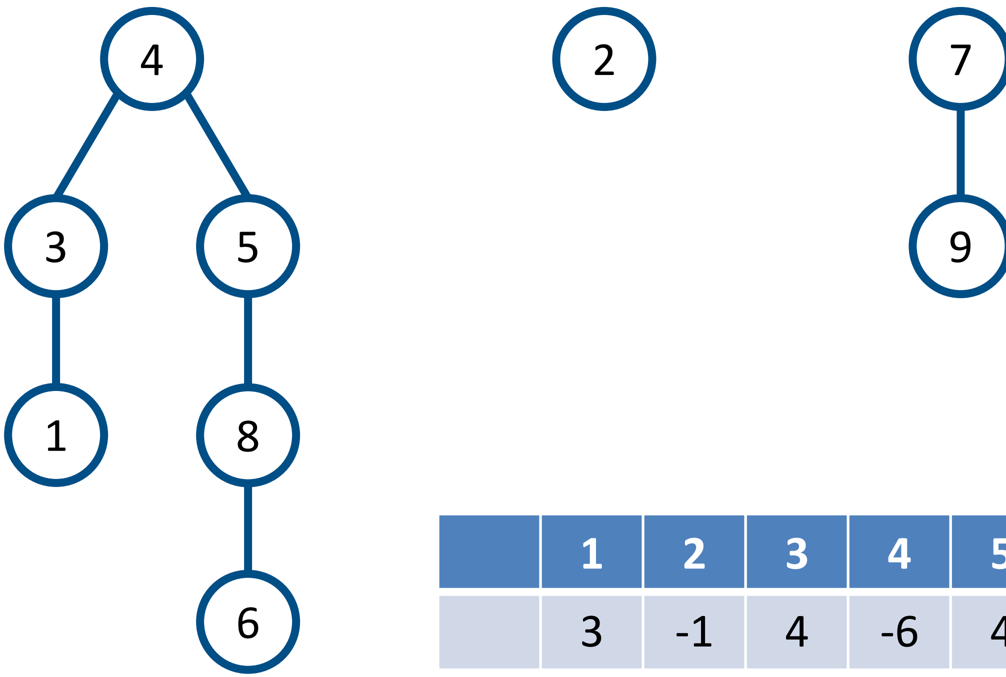


	1	2	3	4	5	6	7	8	9
	3	-1	4	-6	4	8	-2	5	7

- Union(9,8)
  - Find(9) -> 7
  - Find(8)

Kruskal's

Union-Find

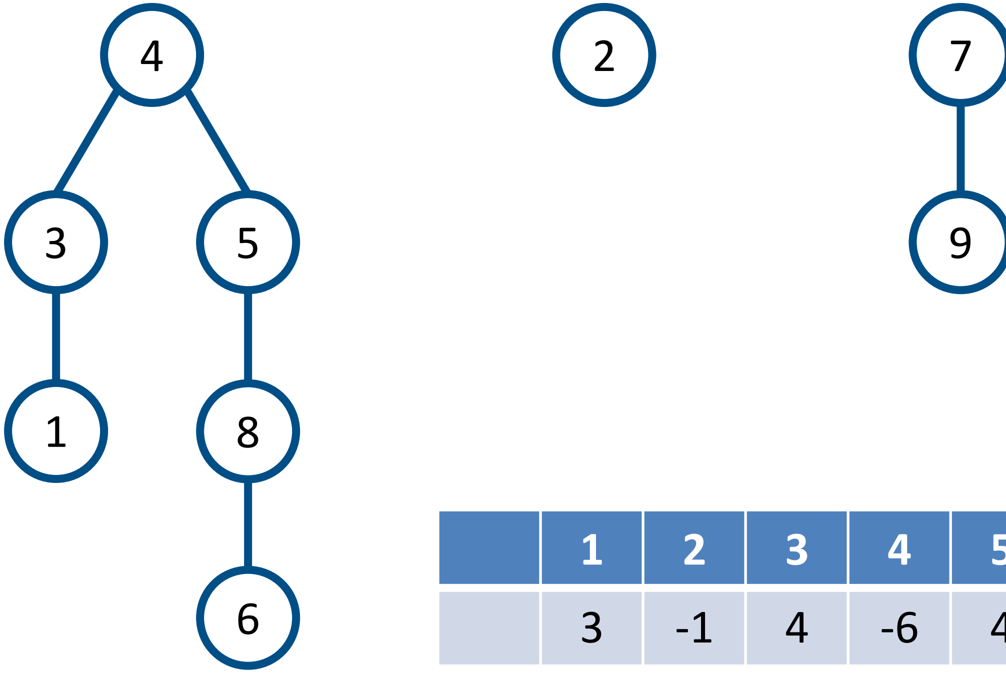


	1	2	3	4	5	6	7	8	9
	3	-1	4	-6	4	8	-2	5	7

- Union(9,8)
  - Find(9) -> 7
  - Find(8) -> 4

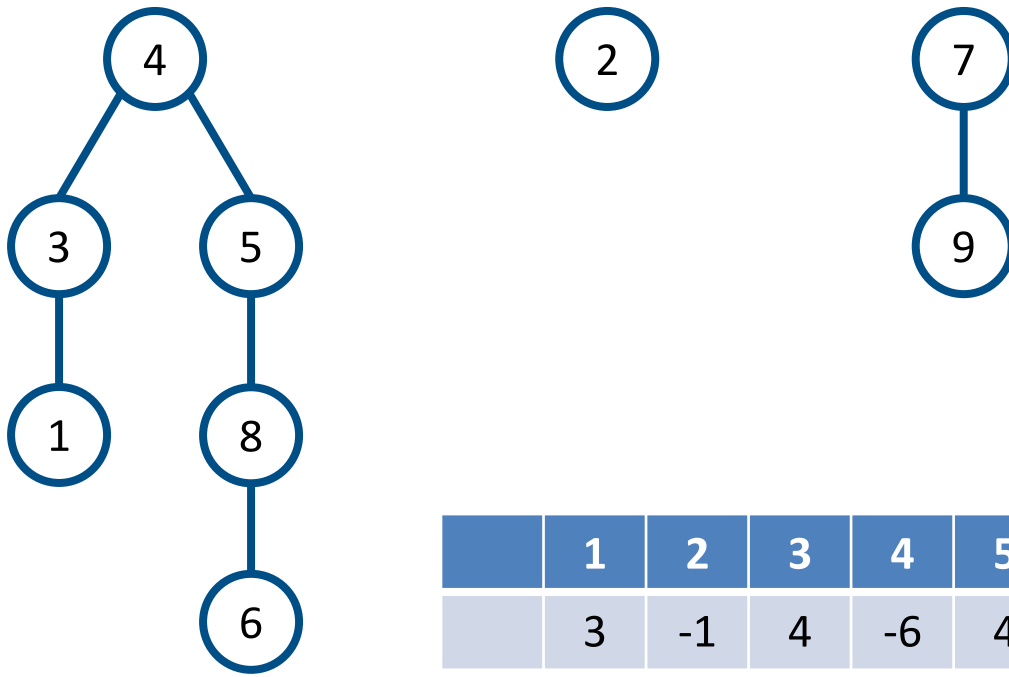
Kruskal’s

Union-Find



	1	2	3	4	5	6	7	8	9
	3	-1	4	-6	4	8	-2	5	7

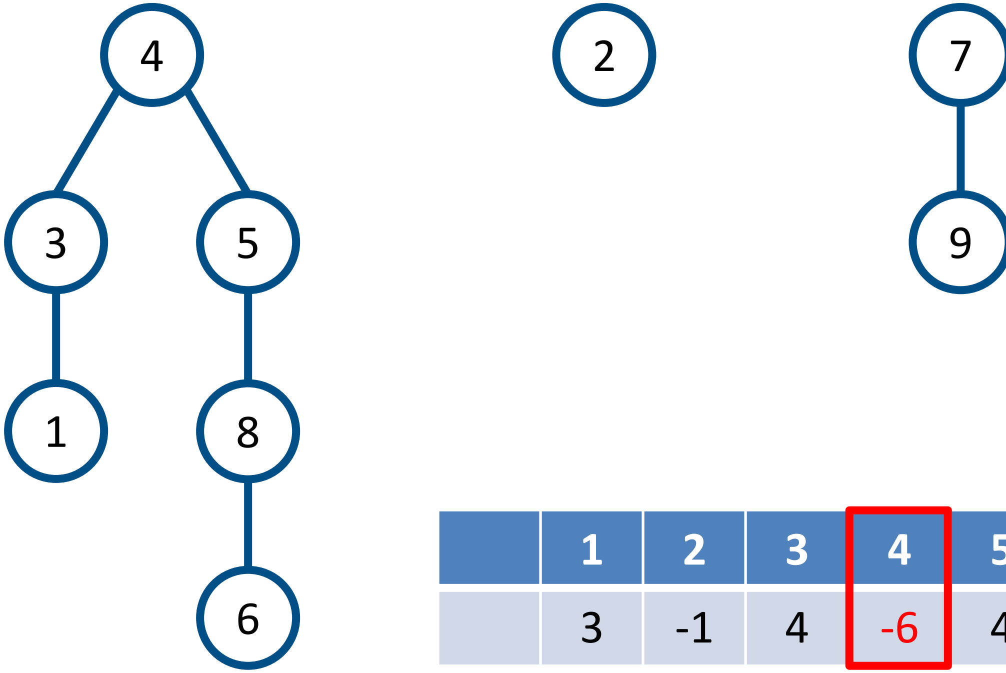
– Union(9,8), different tree so we can perform union
  - Find(9) -> 7
  - Find(8) -> 4



	1	2	3	4	5	6	7	8	9
	3	-1	4	-6	4	8	-2	5	7

- Union(9,8), different tree so we can perform union
  - Find(9) -> 7, size of 2
  - Find(8) -> 4,

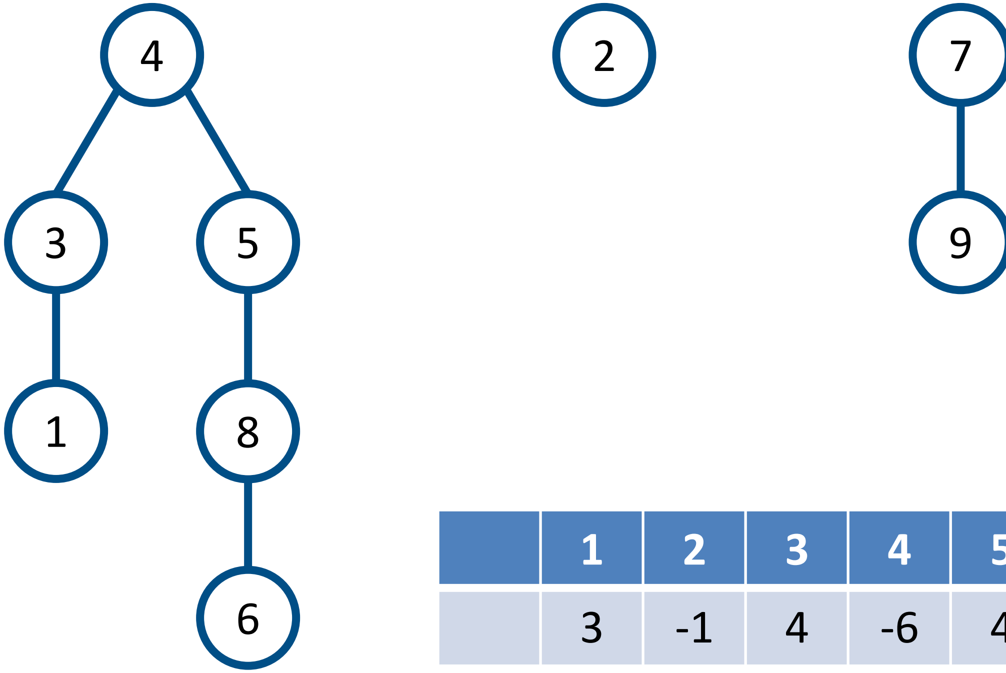
Kruskal's
Union-Find



	1	2	3	4	5	6	7	8	9
	3	-1	4	-6	4	8	-2	5	7

– Union(9,8), different tree so we can perform union
  - Find(9) -> 7, size of 2
  - Find(8) -> 4, size of 6

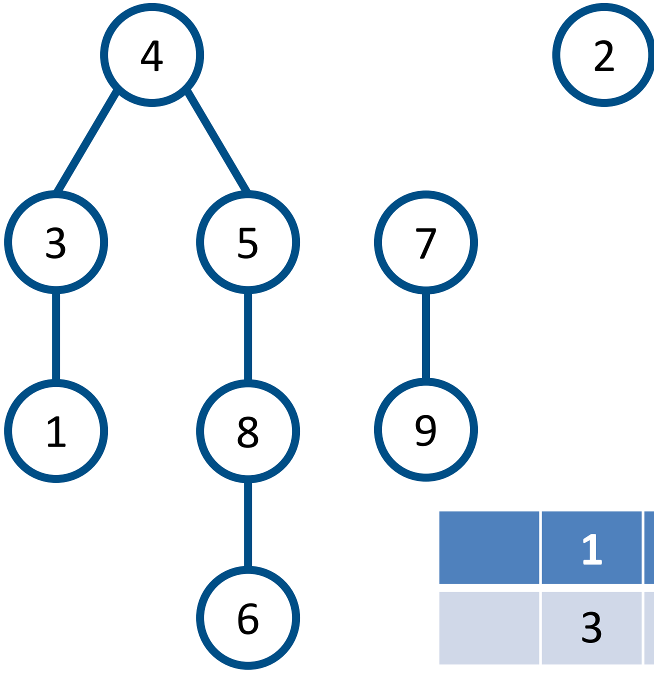
Kruskal's
Union-Find



	1	2	3	4	5	6	7	8	9
	3	-1	4	-6	4	8	-2	5	7

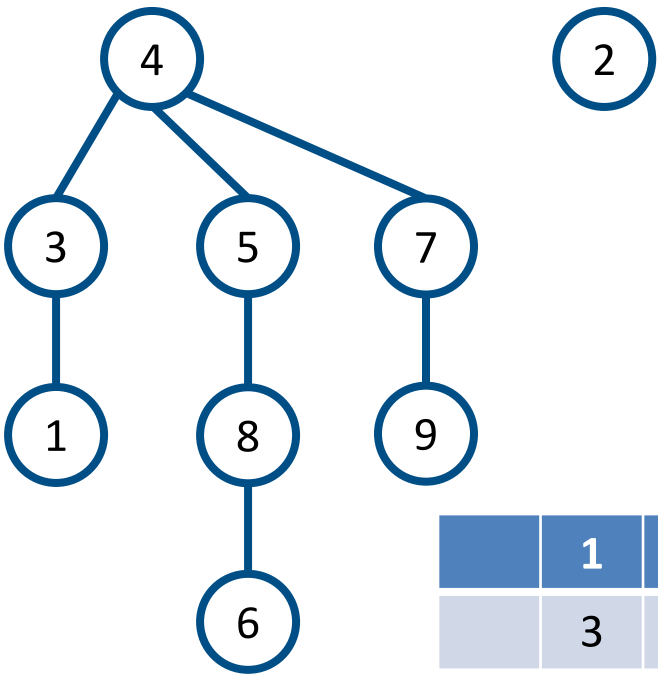
- Union(9,8), different tree so we can perform union
  - Find(9) -> 7, size of 2, smaller tree so merge to bigger tree
  - Find(8) -> 4, size of 6

Kruskal's
Union-Find



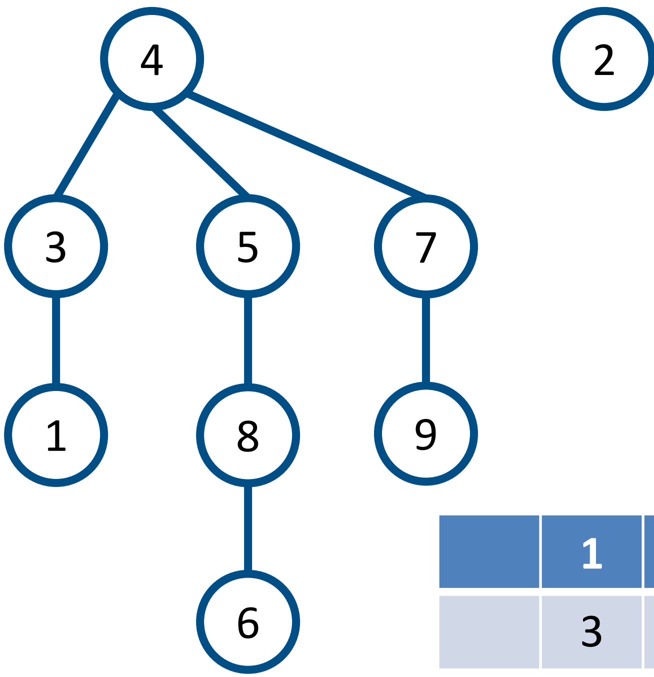
	1	2	3	4	5	6	7	8	9
	3	-1	4	-6	4	8	-2	5	7

- Union(9,8), different tree so we can perform union
  - Find(9) -> 7, size of 2, smaller tree so merge to bigger tree
  - Find(8) -> 4, size of 6



	1	2	3	4	5	6	7	8	9
	3	-1	4	-8	4	8	4	5	7

- Union(9,8), different tree so we can perform union
  - Find(9) -> 7, size of 2, smaller tree so merge to bigger tree
  - Find(8) -> 4, size of 6, size updated to 8



	1	2	3	4	5	6	7	8	9
	3	-1	4	-8	4	8	4	5	7

- Union(9,8), different tree so we can perform union
  - Find(9) -> 7, size of 2, smaller tree so merge to bigger tree
  - Find(8) -> 4, size of 6, size updated to 8

Questions?

Prim’s and Kruskal’s

Does it work?

- For a graph, can we always find the MST?

Prim's and Kruskal's
Does it work?

- For a graph, can we always find the MST?
- Time to prove it on the whiteboard…
  - Known as proof by contradiction…

Questions?

Prim's and Kruskal's

Does it work?

Prim's and Kruskal's

Does it work?

- For negative edges?

Prim’s and Kruskal’s

Does it work?

- For negative edges?
  - Prim’s work fine cause it will choose the negative one from the tree
  - Kruskal’s work fine cause the negative edges is sorted forward

Prim’s and Kruskal’s

Does it work?

- For negative edges?
  - Prim’s work fine cause it will choose the negative one from the tree
  - Kruskal’s work fine cause the negative edges is sorted forward

- For negative cycles?

Prim’s and Kruskal’s

Does it work?

- For negative edges?
  - Prim’s work fine cause it will choose the negative one from the tree
  - Kruskal’s work fine cause the negative edges is sorted forward

- For negative cycles?
  - Yes we chose the smallest edges without form cycles!

Prim's and Kruskal's
Does it work?

- For negative edges?
  - Prim's work fine cause it will choose the negative one from the tree
  - Kruskal's work fine cause the negative edges is sorted forward
- For negative cycles?
  - Yes we chose the smallest edges without form cycles!
- Can you prove that the greediness is correct?

Prim's and Kruskal's
Does it work?

- For negative edges?
  - Prim's work fine cause it will choose the negative one from the tree
  - Kruskal's work fine cause the negative edges is sorted forward
- For negative cycles?
  - Yes we chose the smallest edges without form cycles!
- Can you prove that the greediness is correct?
  - Yes…
  - I will now work both out on the whiteboard

- For negative edges?
  - Prim's work fine cause it will choose the negative one from the tree
  - Kruskal's work fine cause the negative edges is sorted forward
- For negative cycles?
  - Yes we chose the smallest edges without form cycles!
- Can you prove that the greediness is correct?
  - Yes…
  - I will now work both out on the whiteboard
  - Invariant: The selected edges will be part of the final MST

Questions?

Thank You