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MONASH INFORMATION TECHNOLOGY

FIT2004 Algorithms and Data Structures

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Referencing materials by
Nathan Companez, Aamir Cheema, Arun Konagurthu and Lloyd Allison

GROUP OF EIGHT AUSTRALIA

Faculty of Information Technology, Monash University

COMMONWEALTH OF AUSTRALIA

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Ready?

Agenda

- String retrieval

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- String retrieval
- Tries and suffix tries

Let us begin…

Introduction

String retrieval

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- Anything can be represented as a string

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- Anything can be represented as a string
  - DNA sequence
  - Images (RGB)
  - Keys
  - … and many more!

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  - Sort the strings
  - Binary search for what you want

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    - O(MN) using radix sort
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    - O(M log N)… again O(M) for string comparison
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Questions?

Tries

Efficient string retrieval

- When we search, we would need to go through every character of a string. Thus, we can use a special data structure that organise it according to characters…

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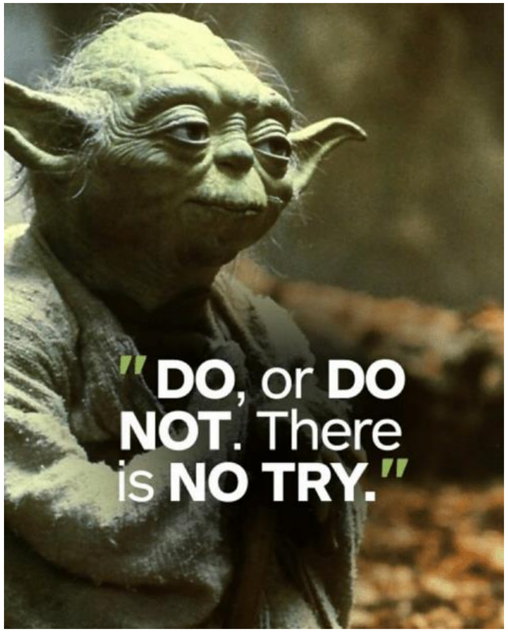

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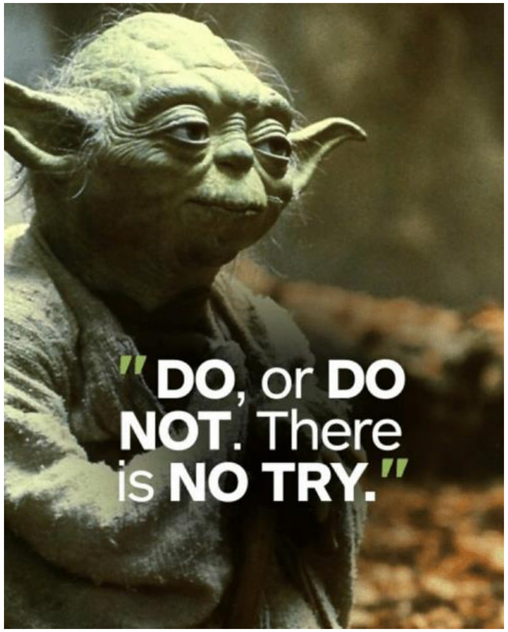
  - A tree
  - M-child per node



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  - A tree
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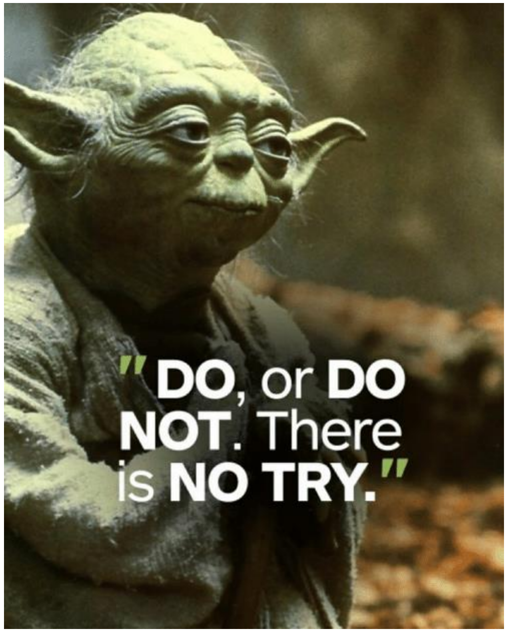
    M = number of unique character



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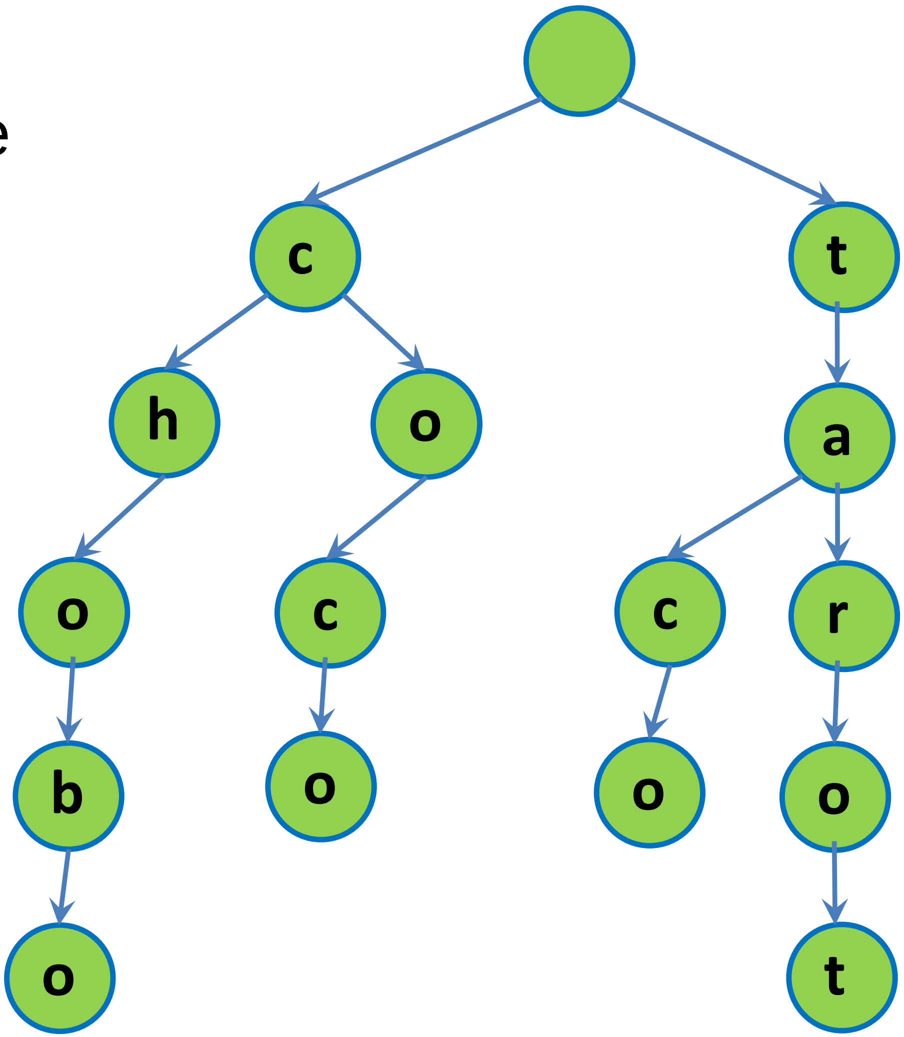

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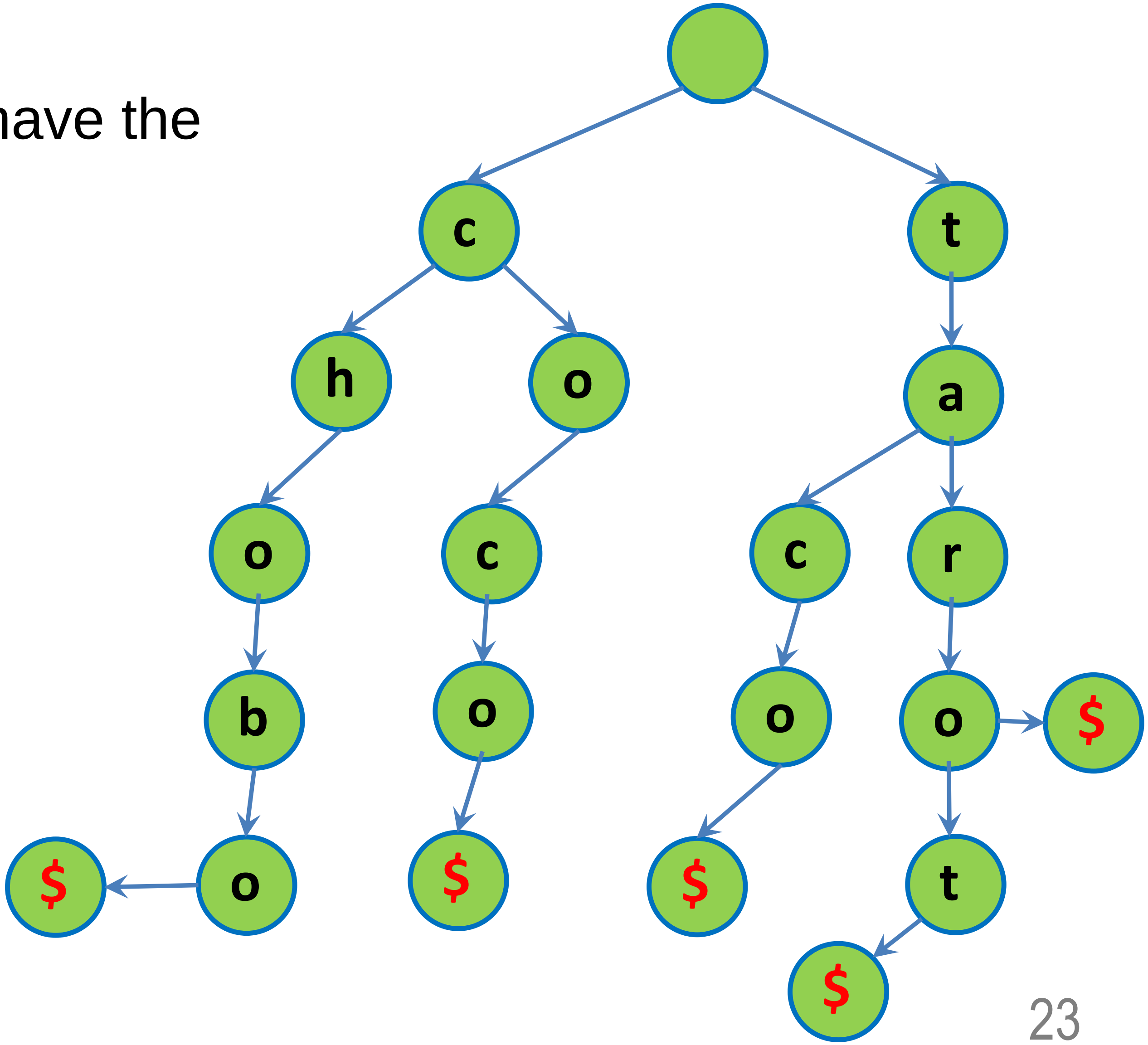

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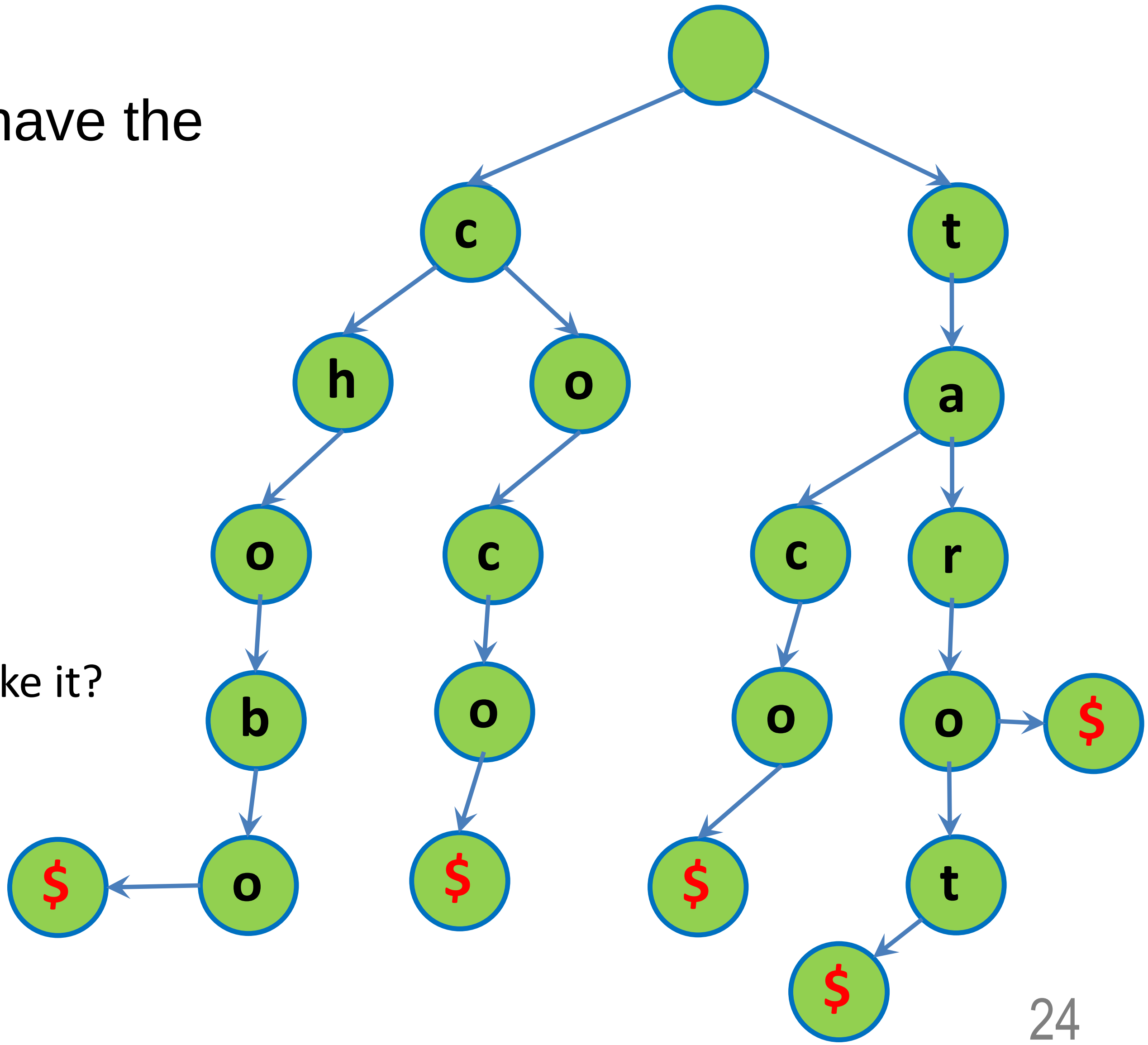

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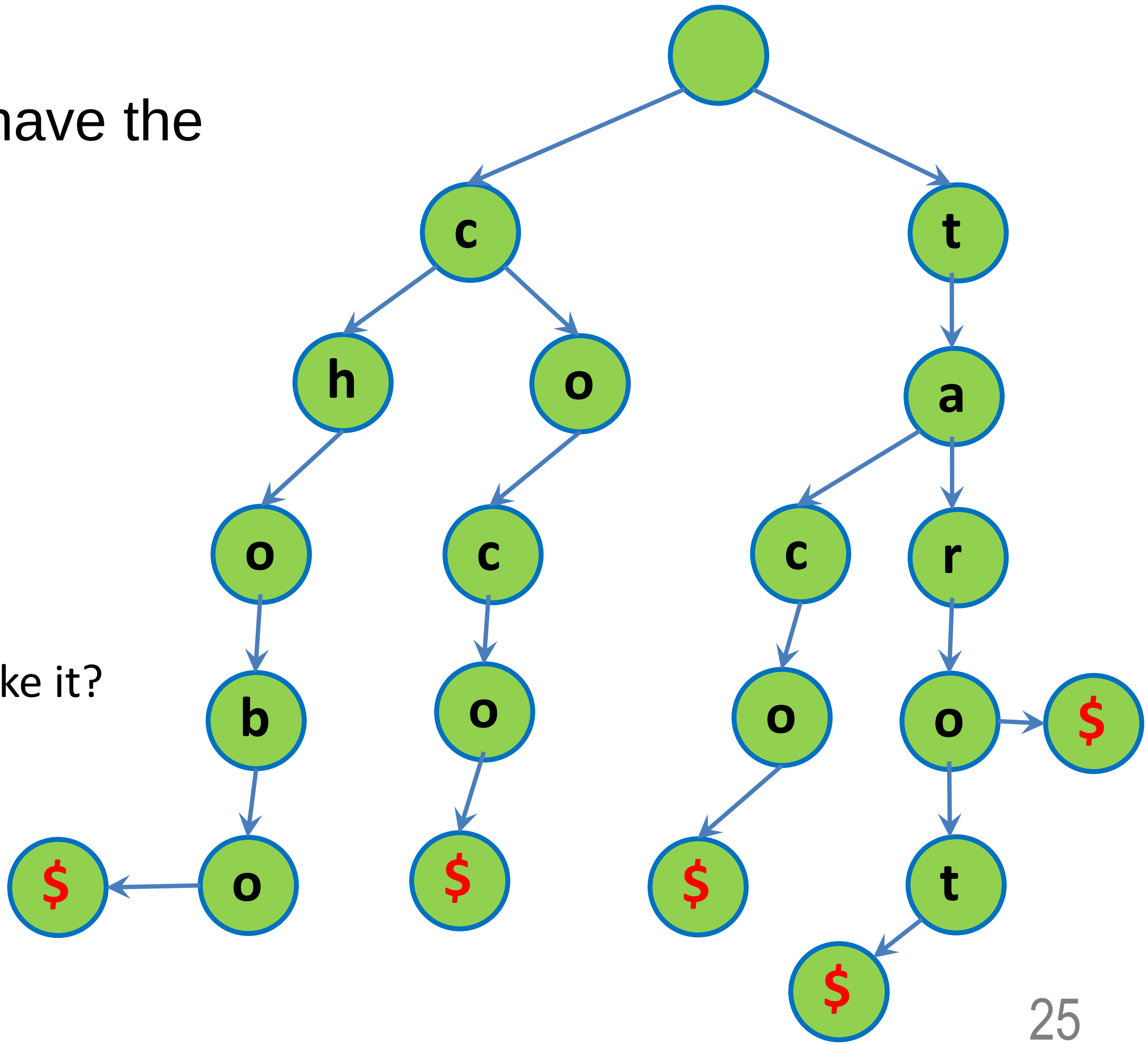
    - Step by step…



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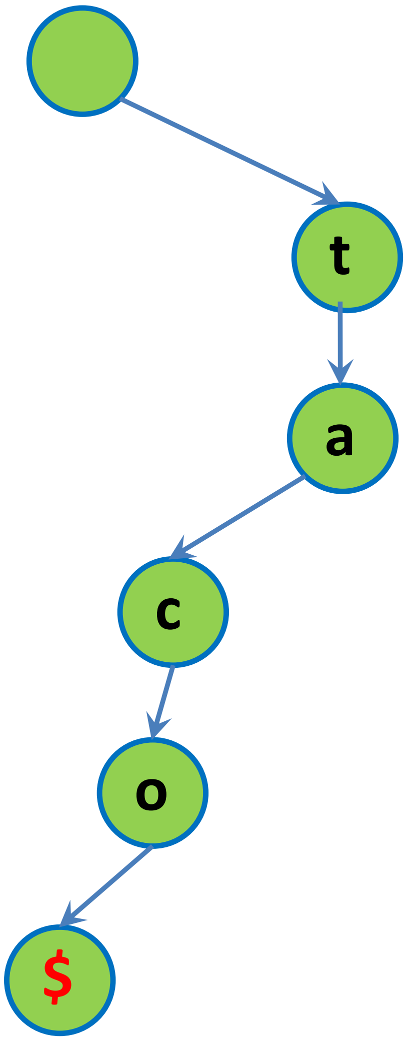

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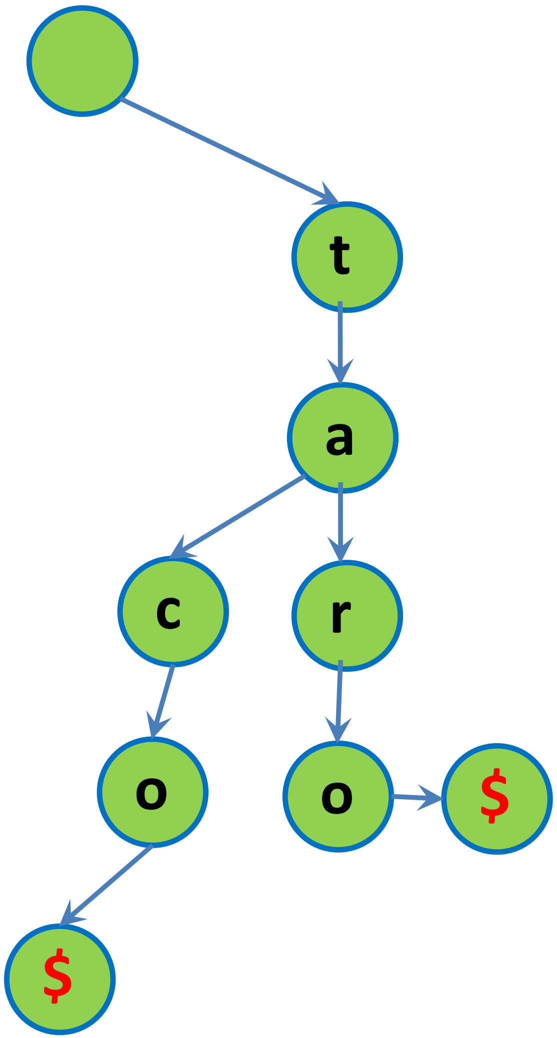

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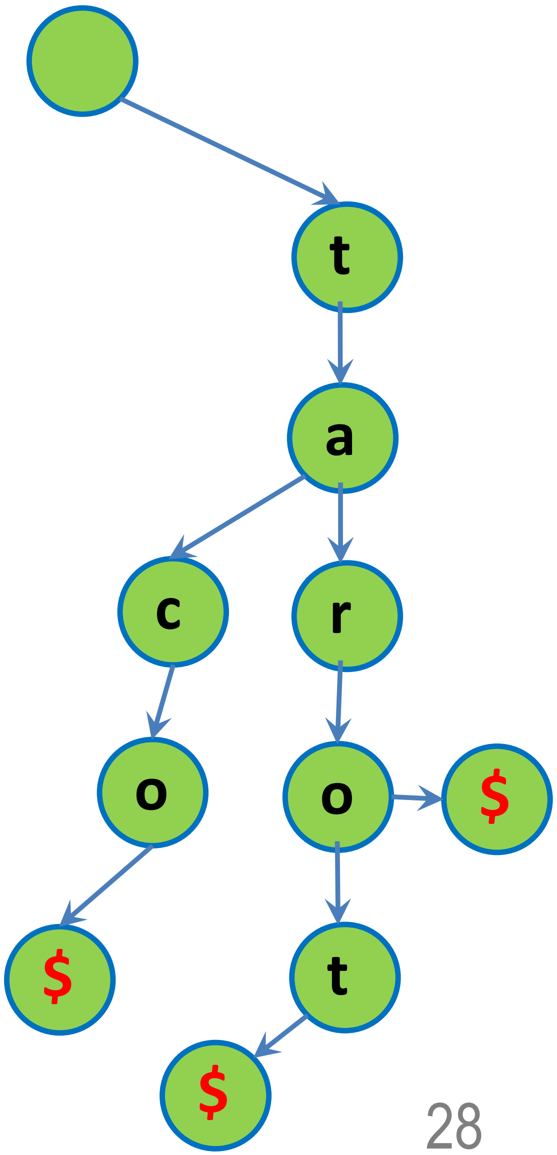

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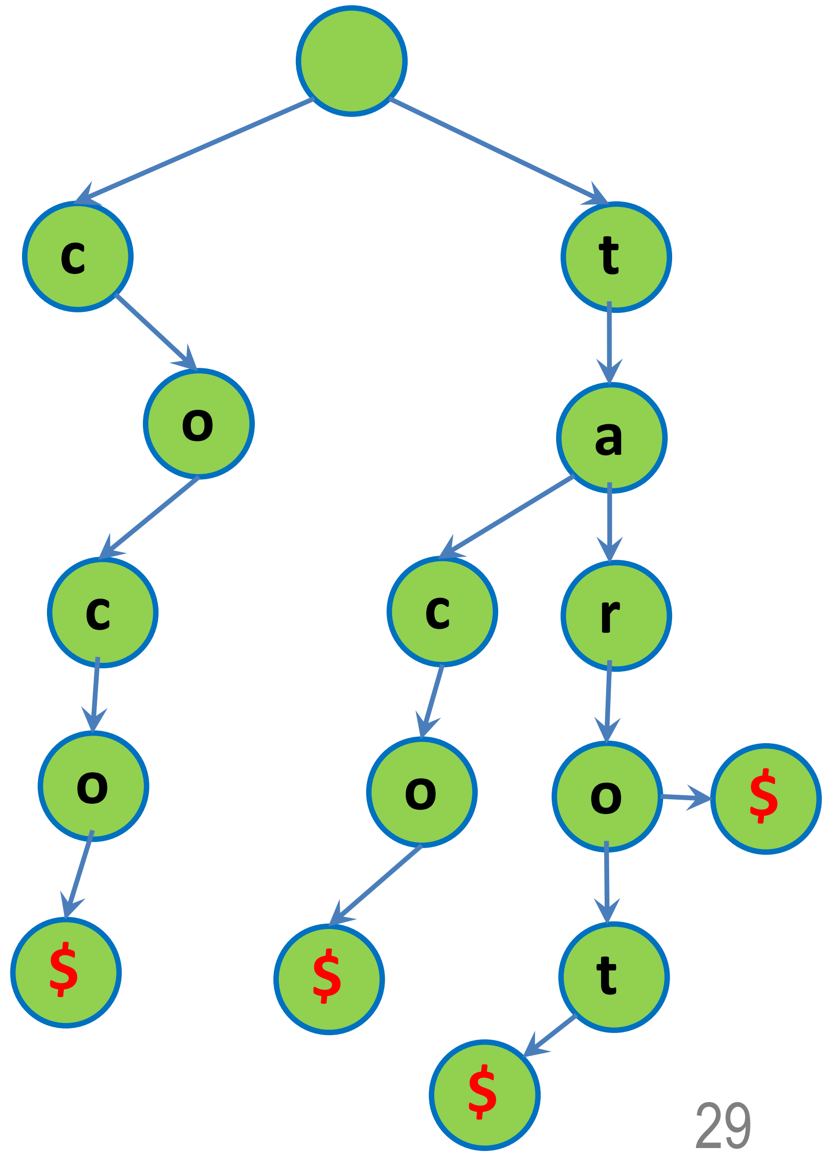

Tries

Efficient string retrieval

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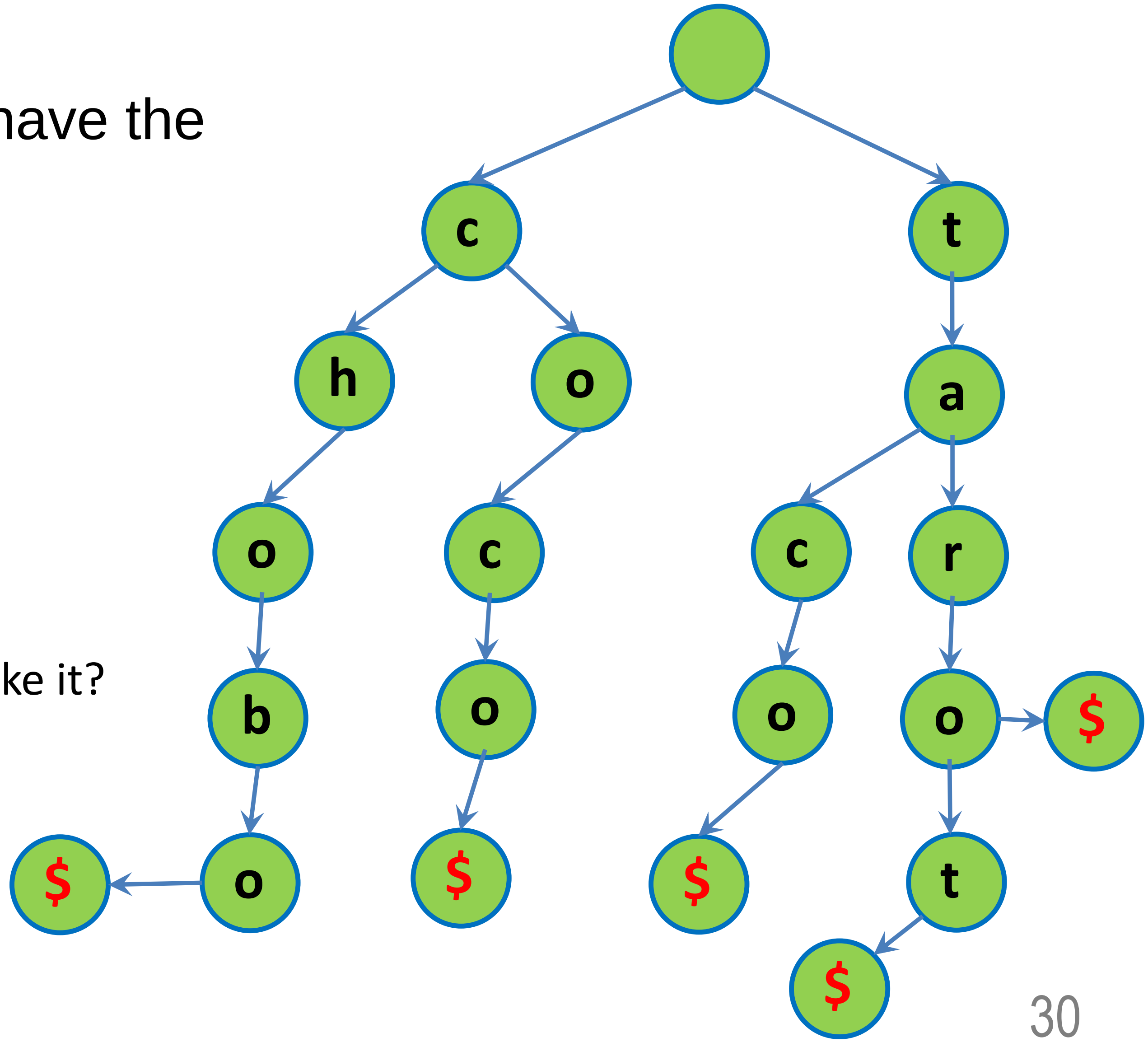

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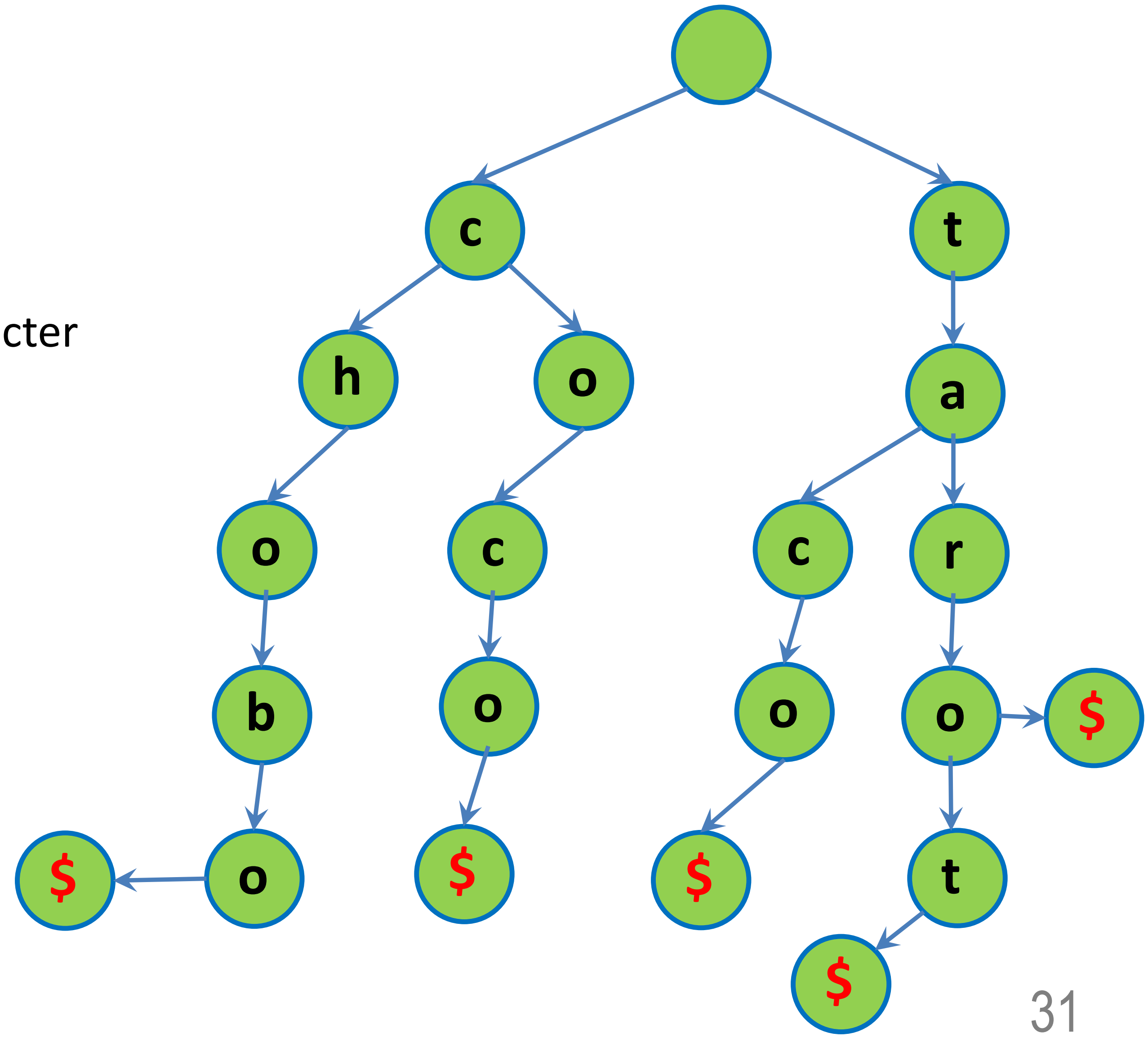
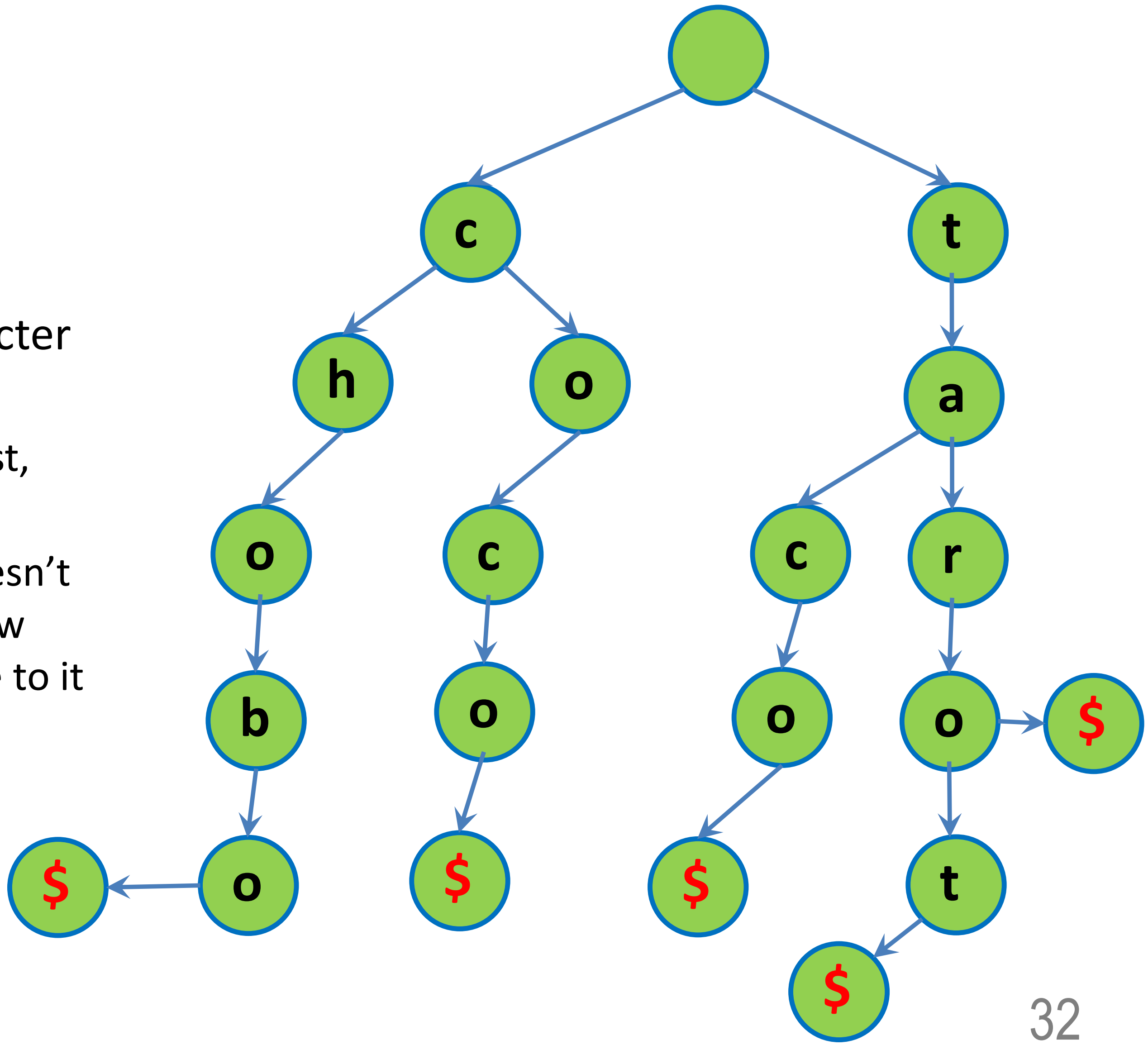
Efficient string retrieval

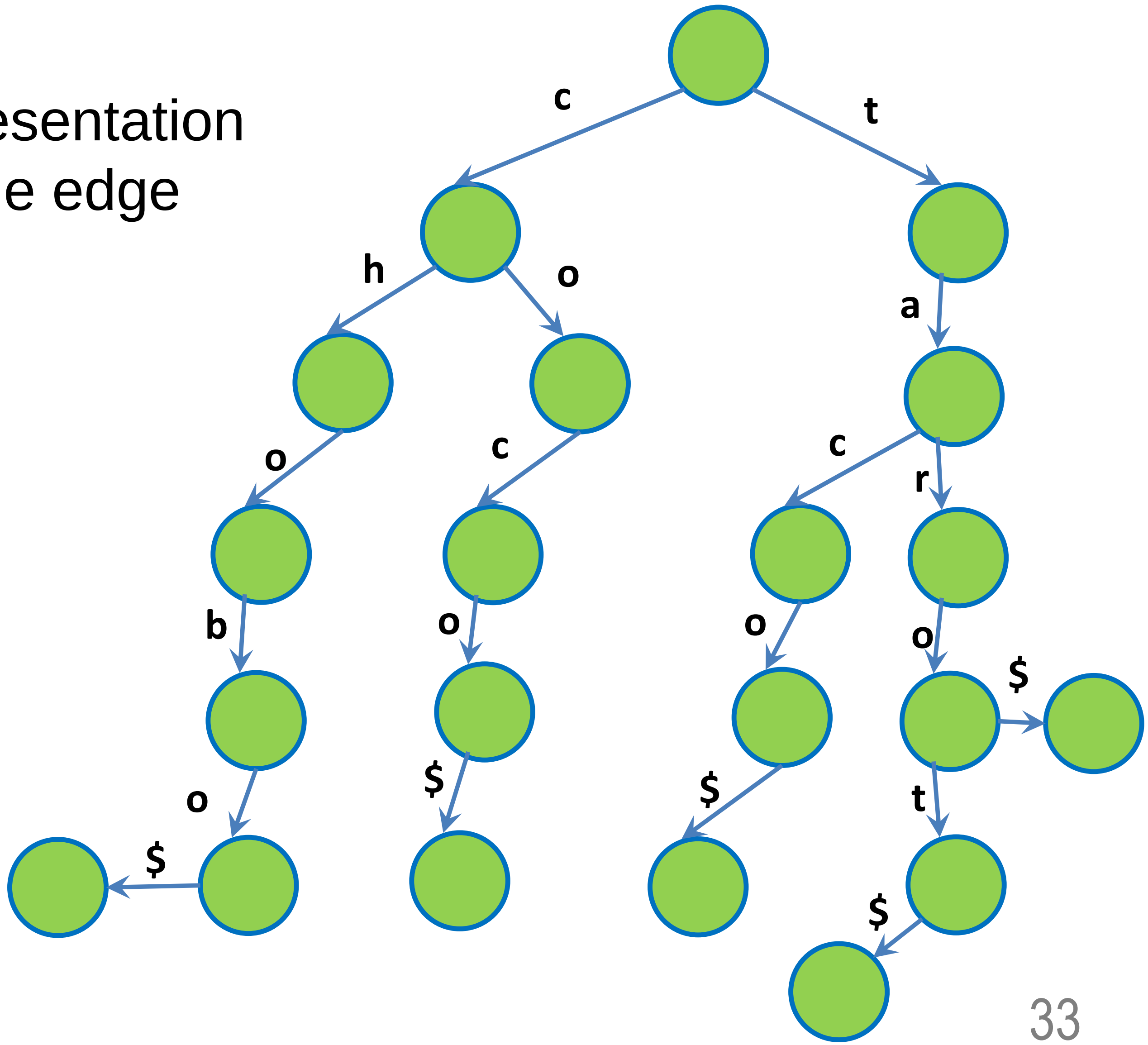
- So steps?
  - For each word
  - Start from root
  - Go through character by character



- So steps?
  - For each word
  - Start from root
  - Go through character by character
    - If character exist, follow through
    - If character doesn't exist, create new node and move to it



- The proper representation is character at the edge



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Efficient string retrieval

- The proper representation is character at the edge
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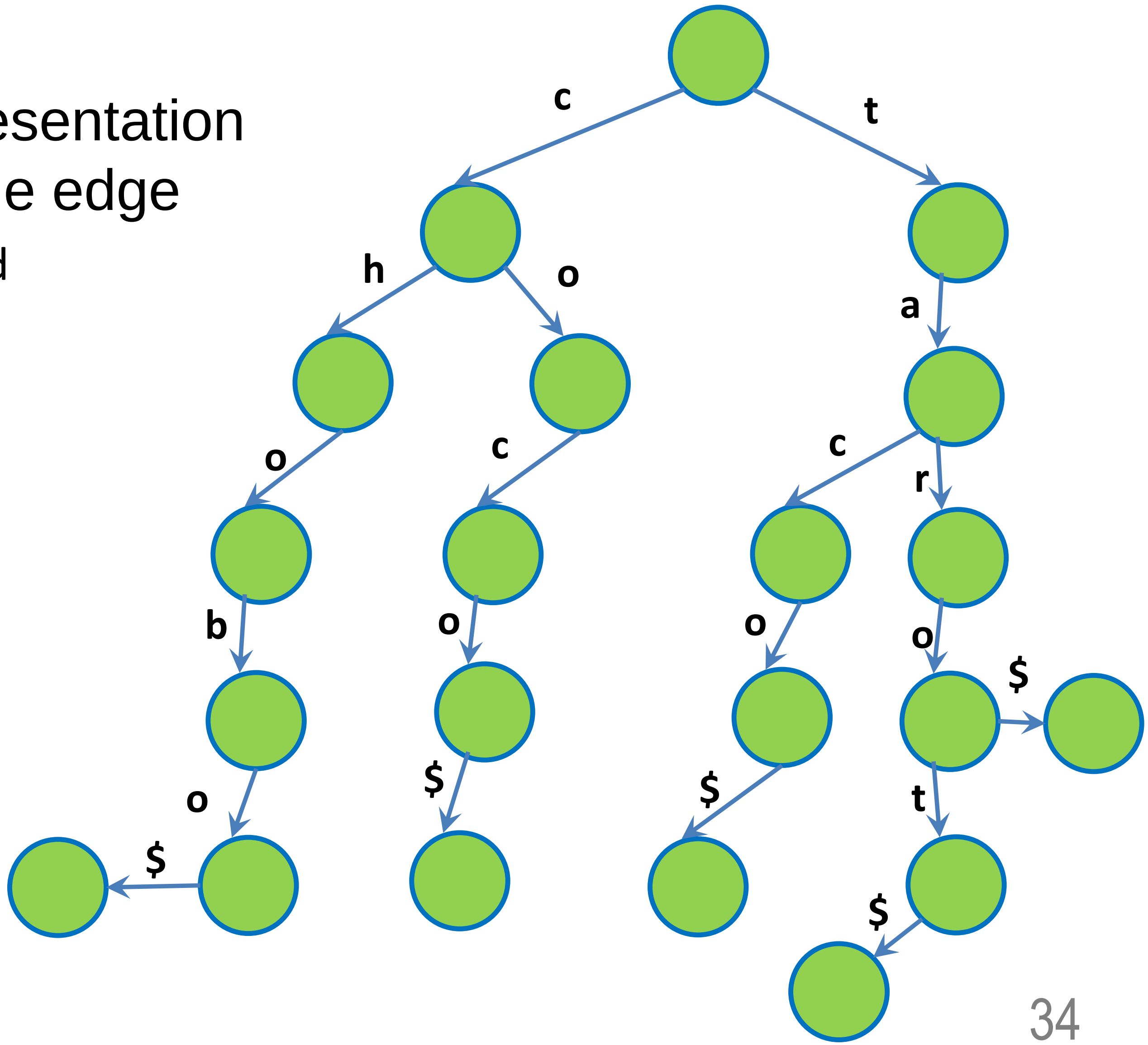

- The proper representation is character at the edge
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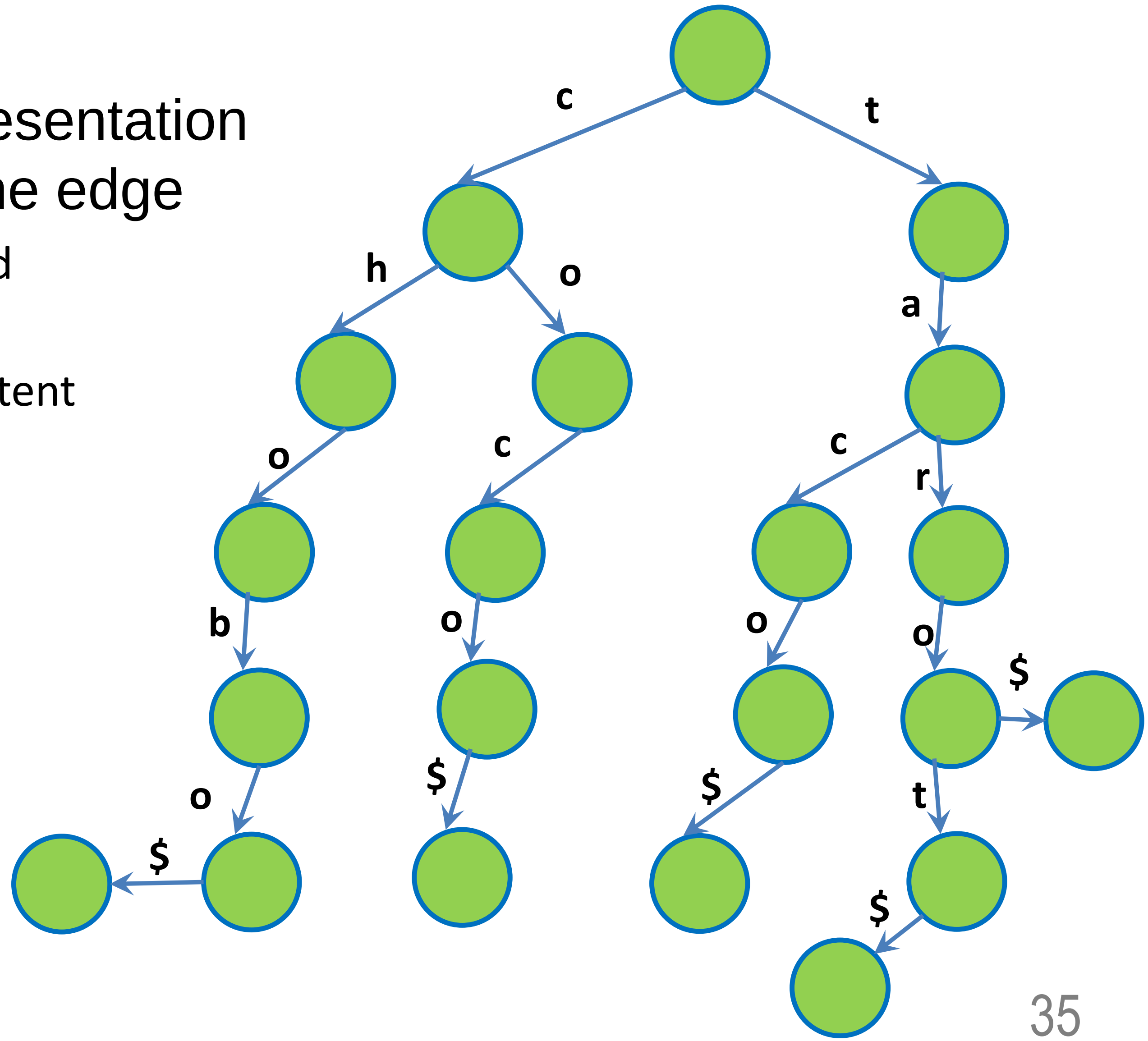
  - This is also consistent with the graph representation

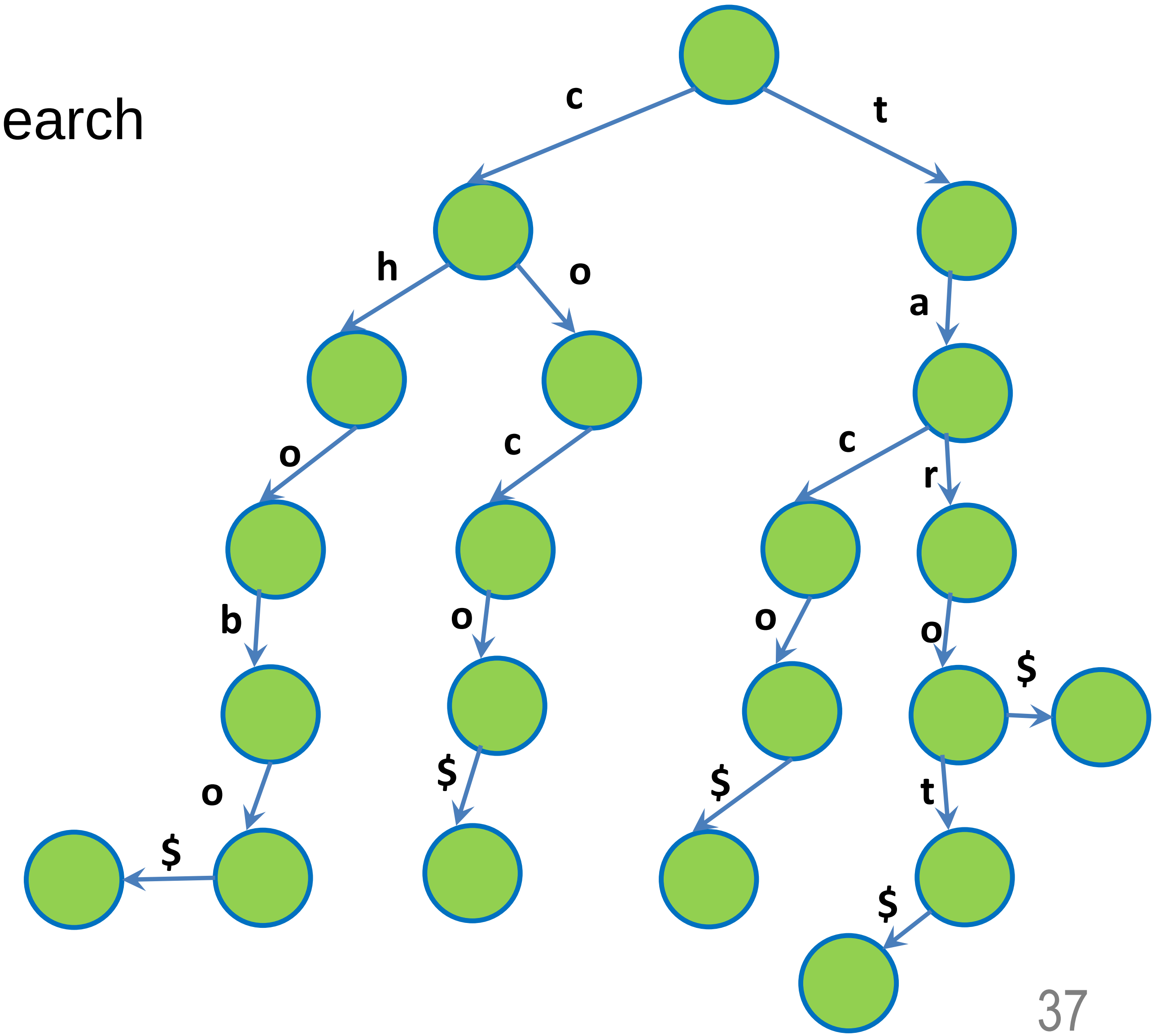


Questions?

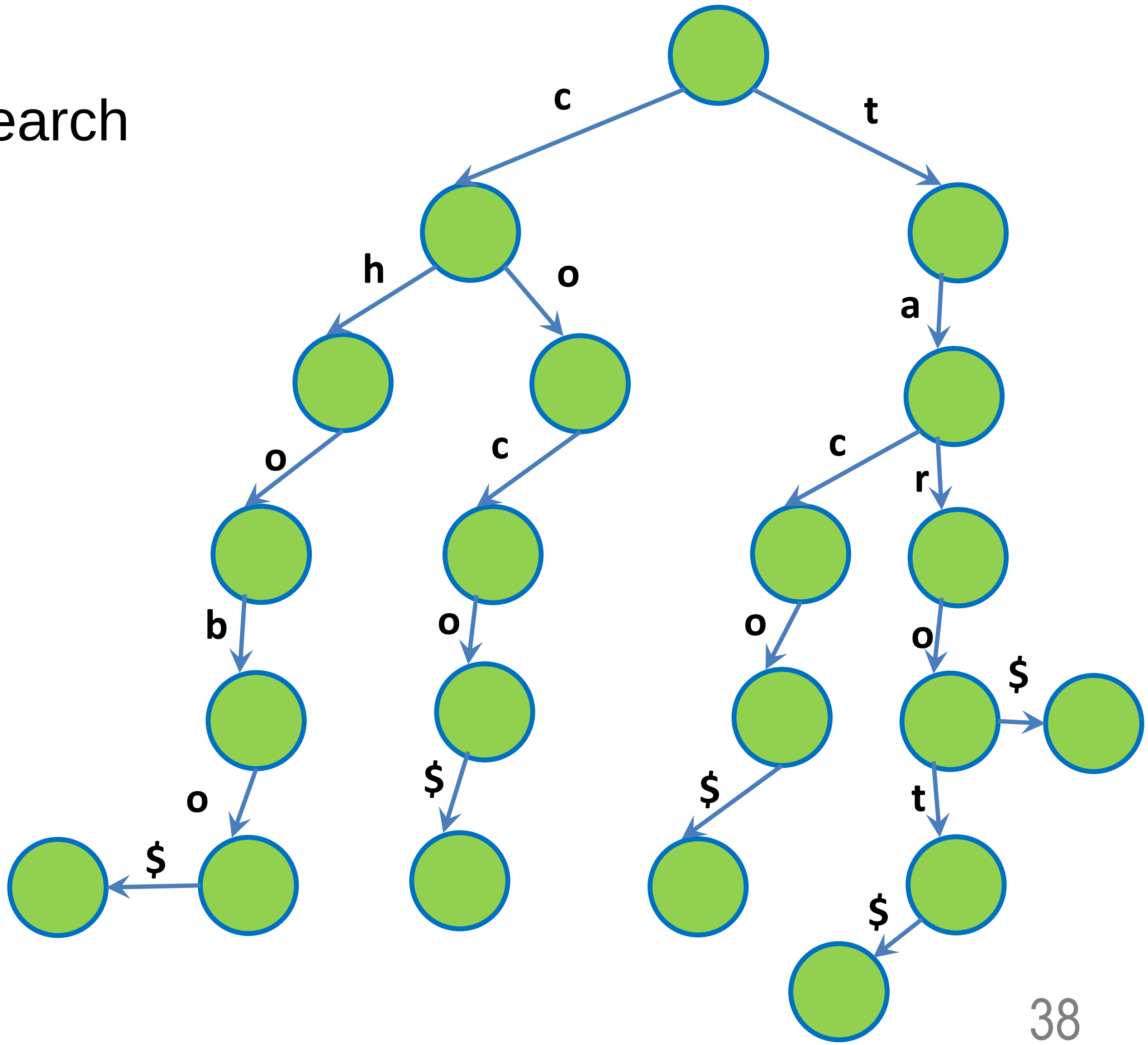
Tries

Efficient string retrieval

- So how do we search for retrieval?



- So how do we search for retrieval?
  - Search for “coco”



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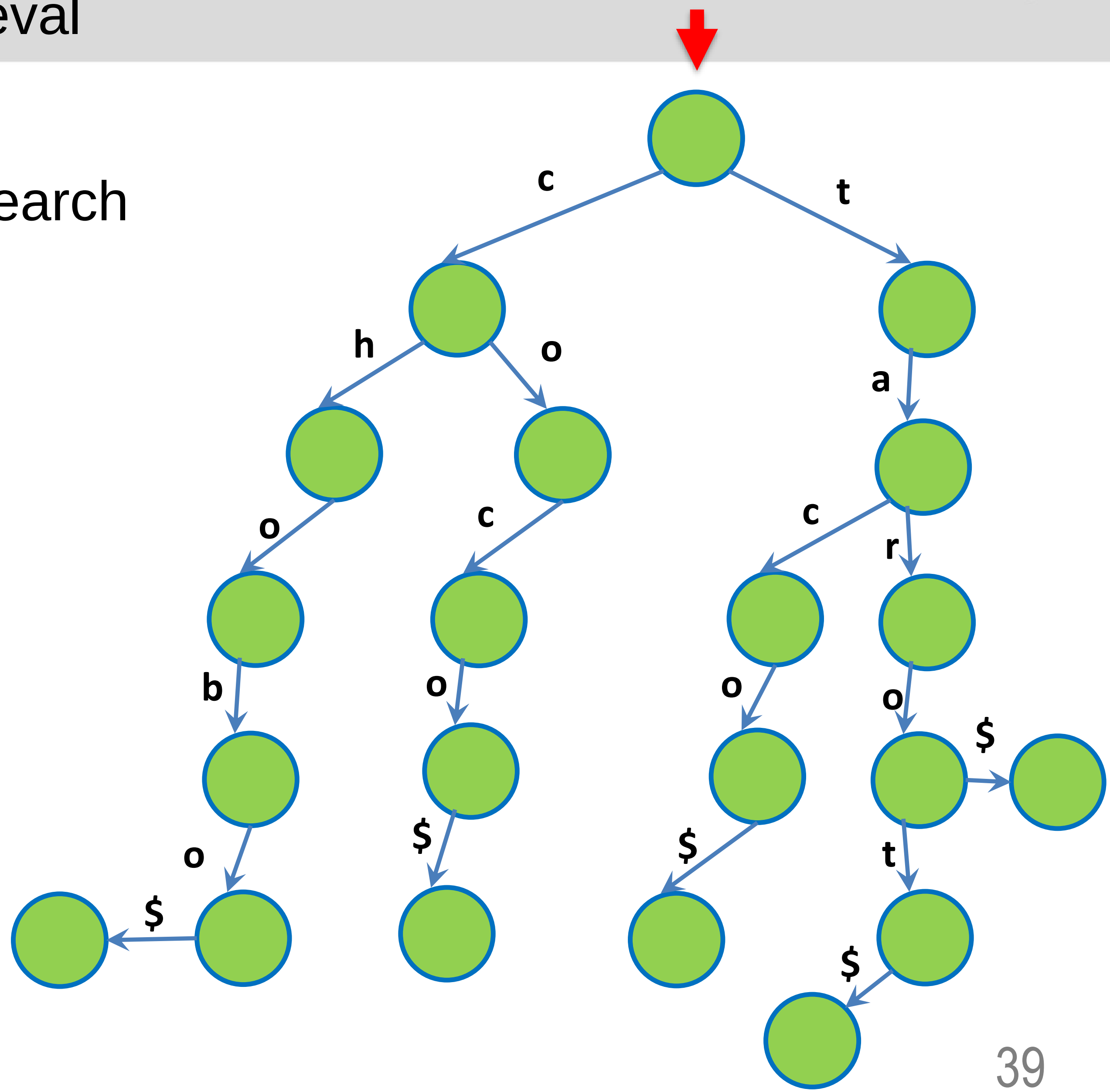

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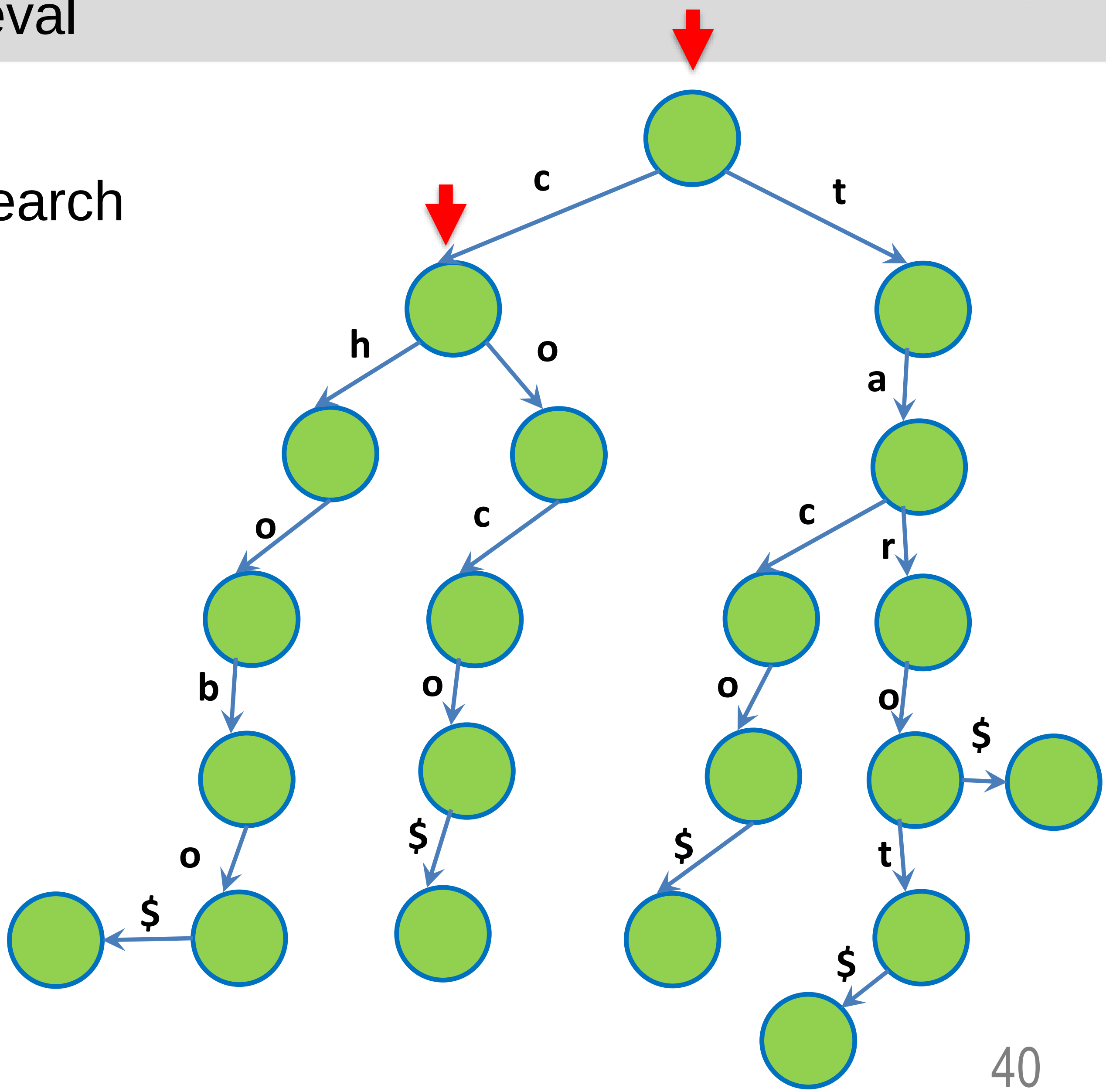

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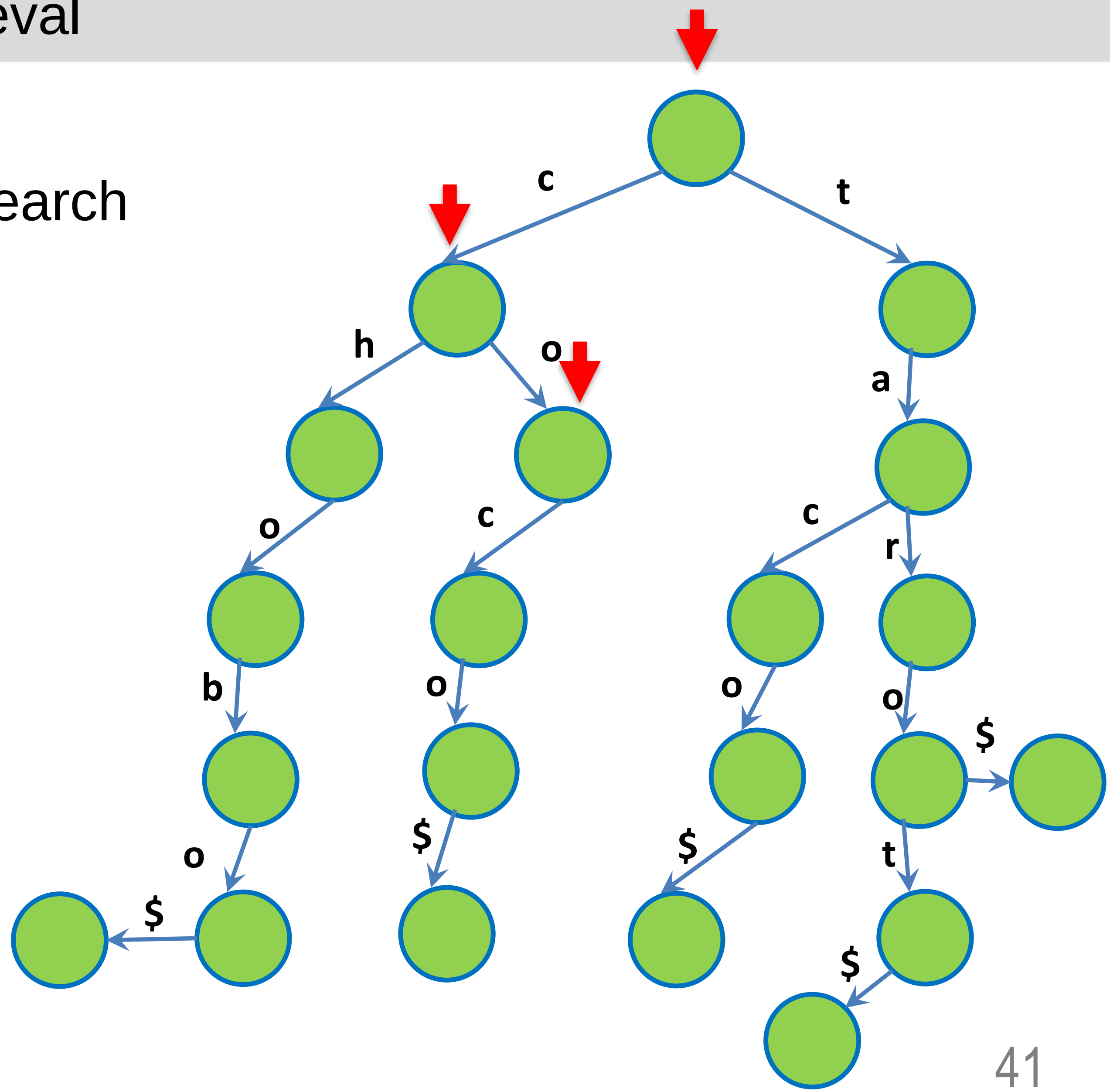

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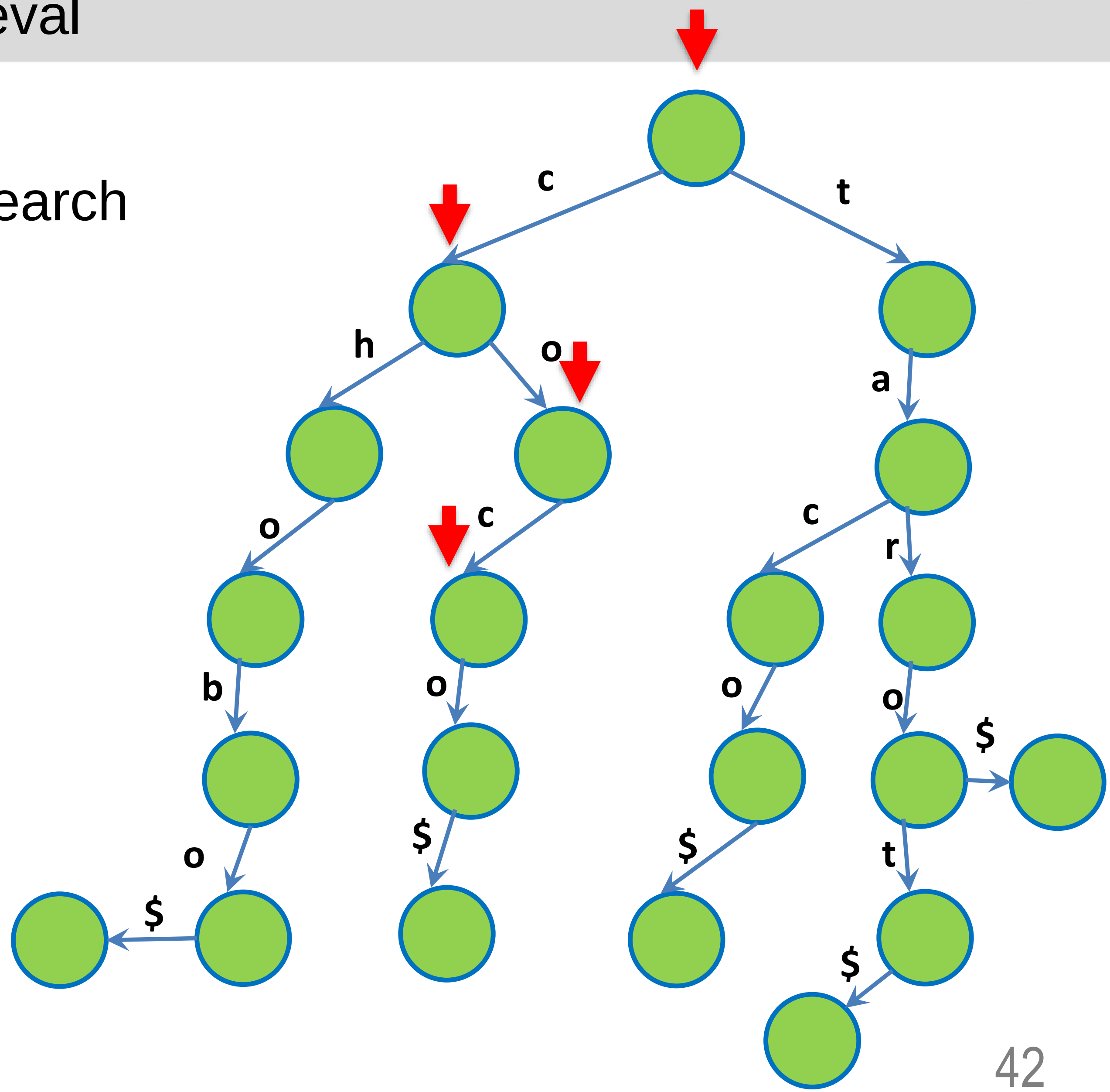

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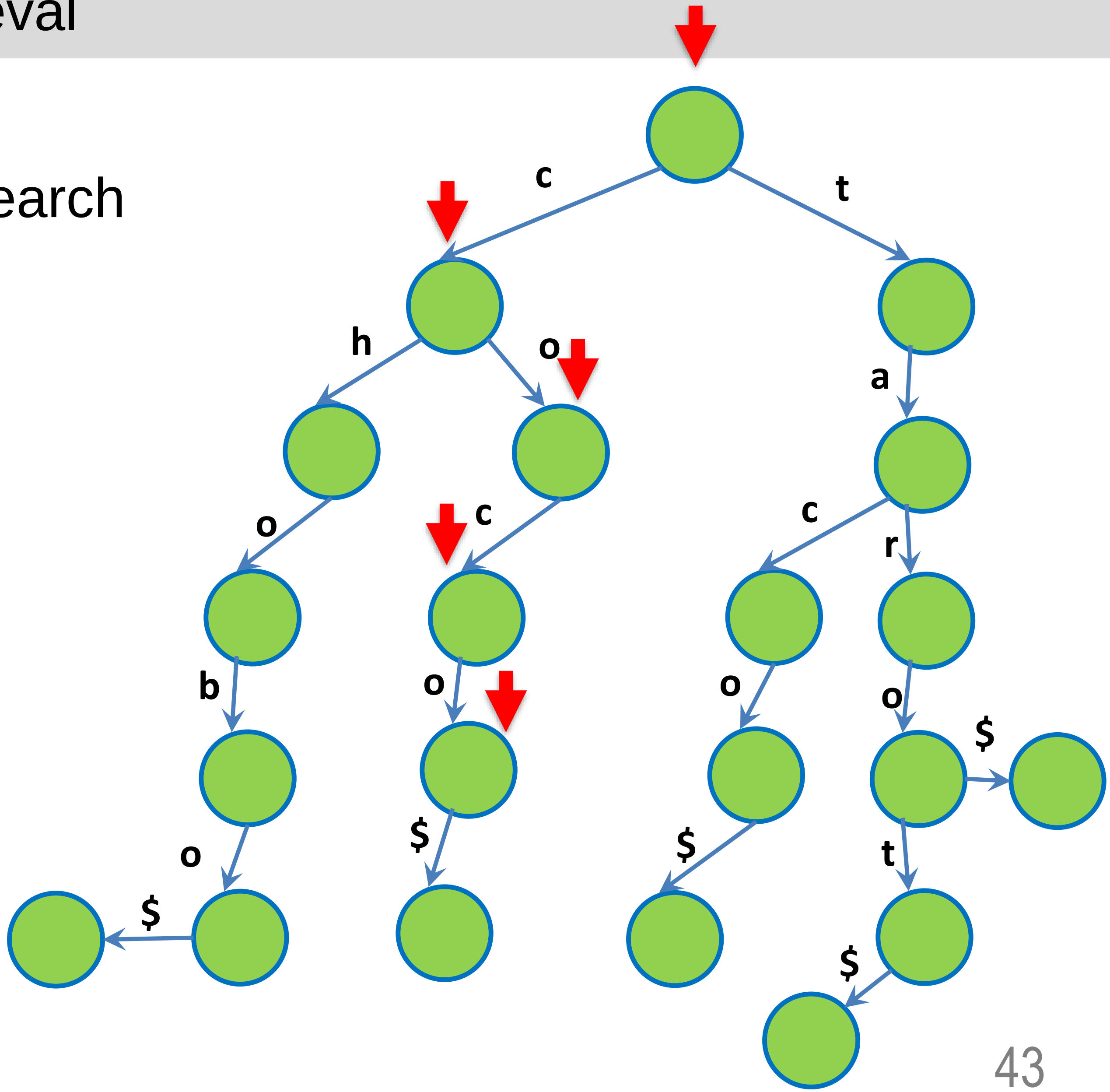
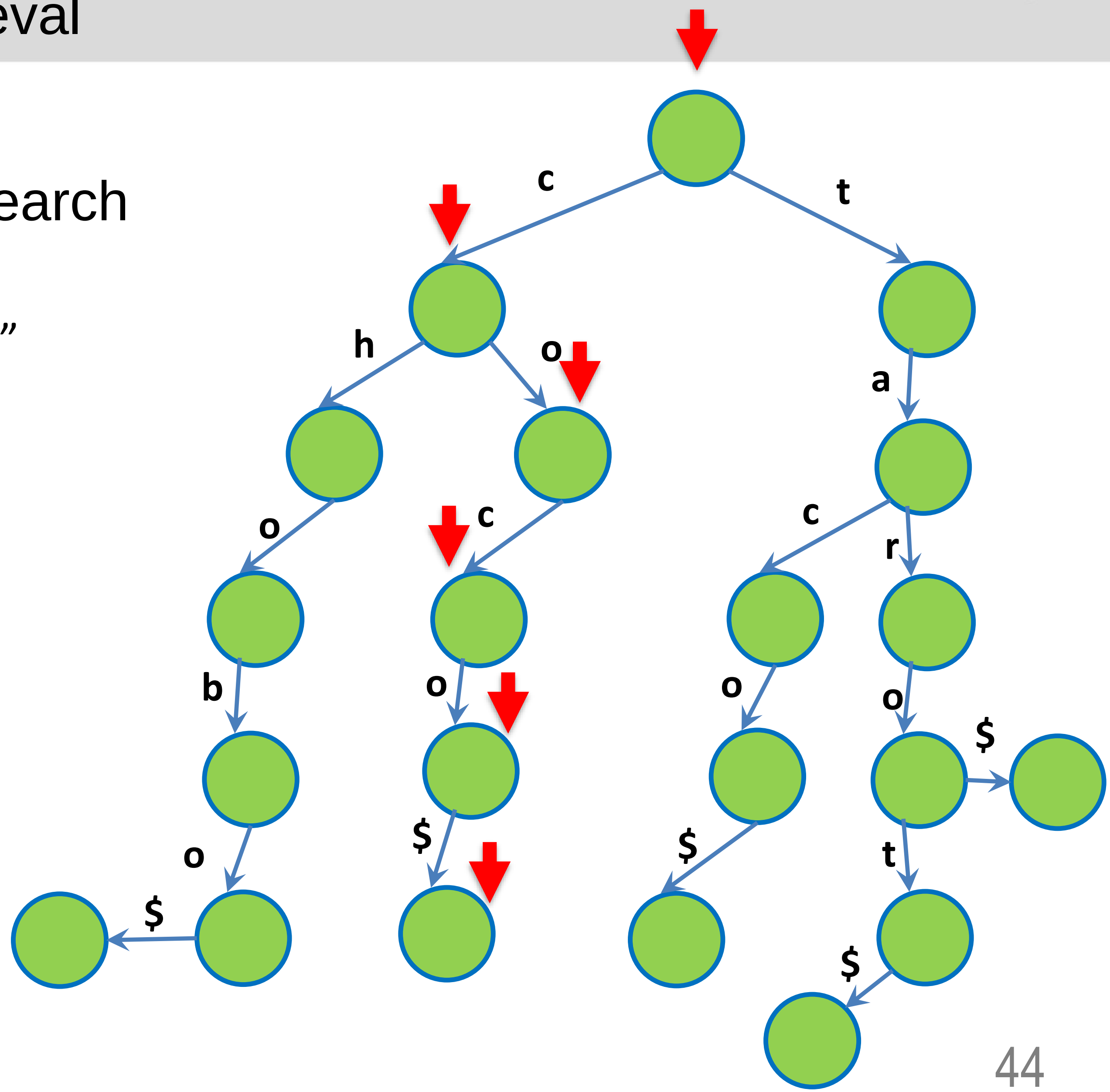


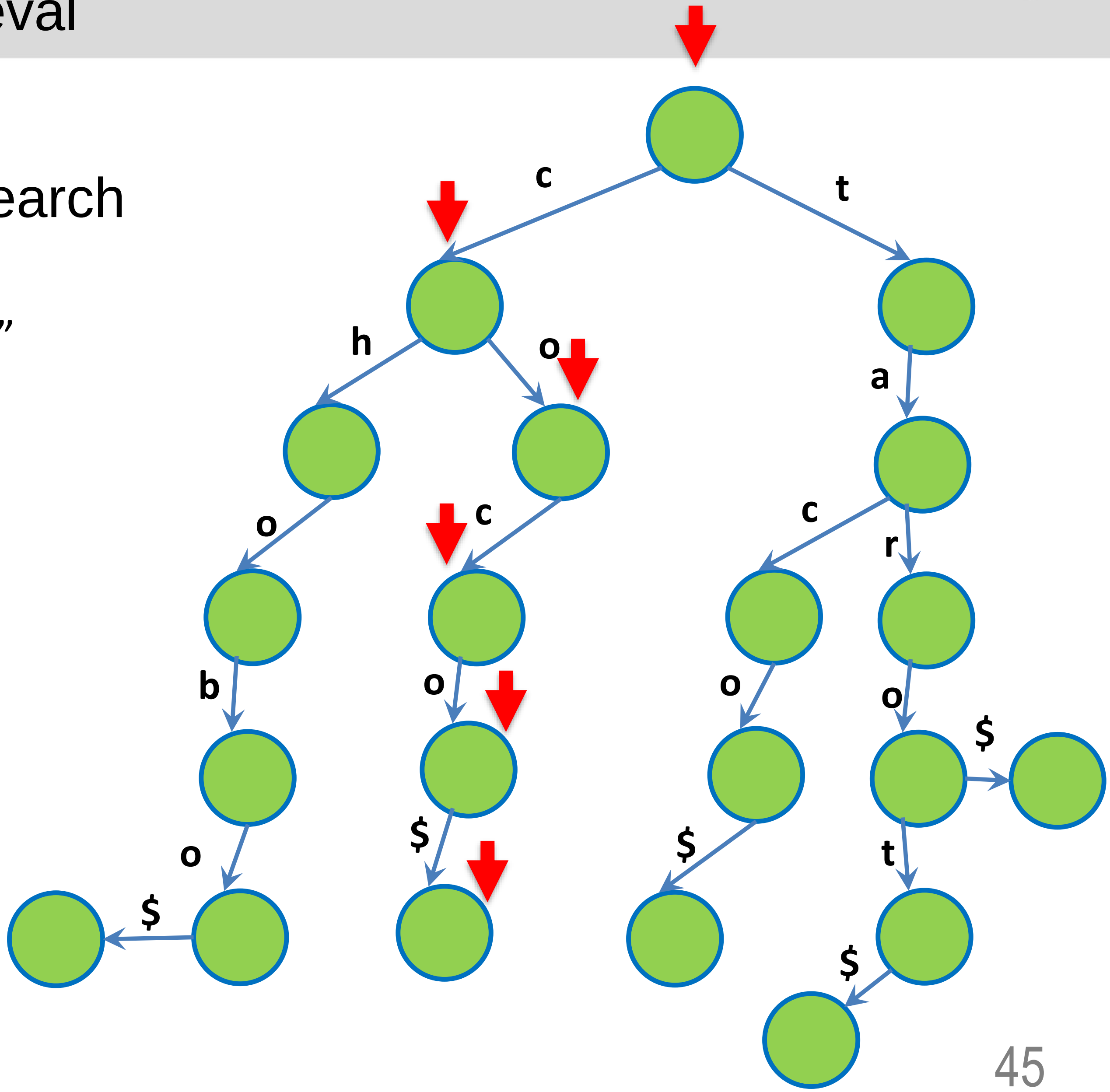
- So how do we search for retrieval?
  - Search for “coco$”



Tries

Efficient string retrieval

- So how do we search for retrieval?
  - Search for “coco$” so we found it!



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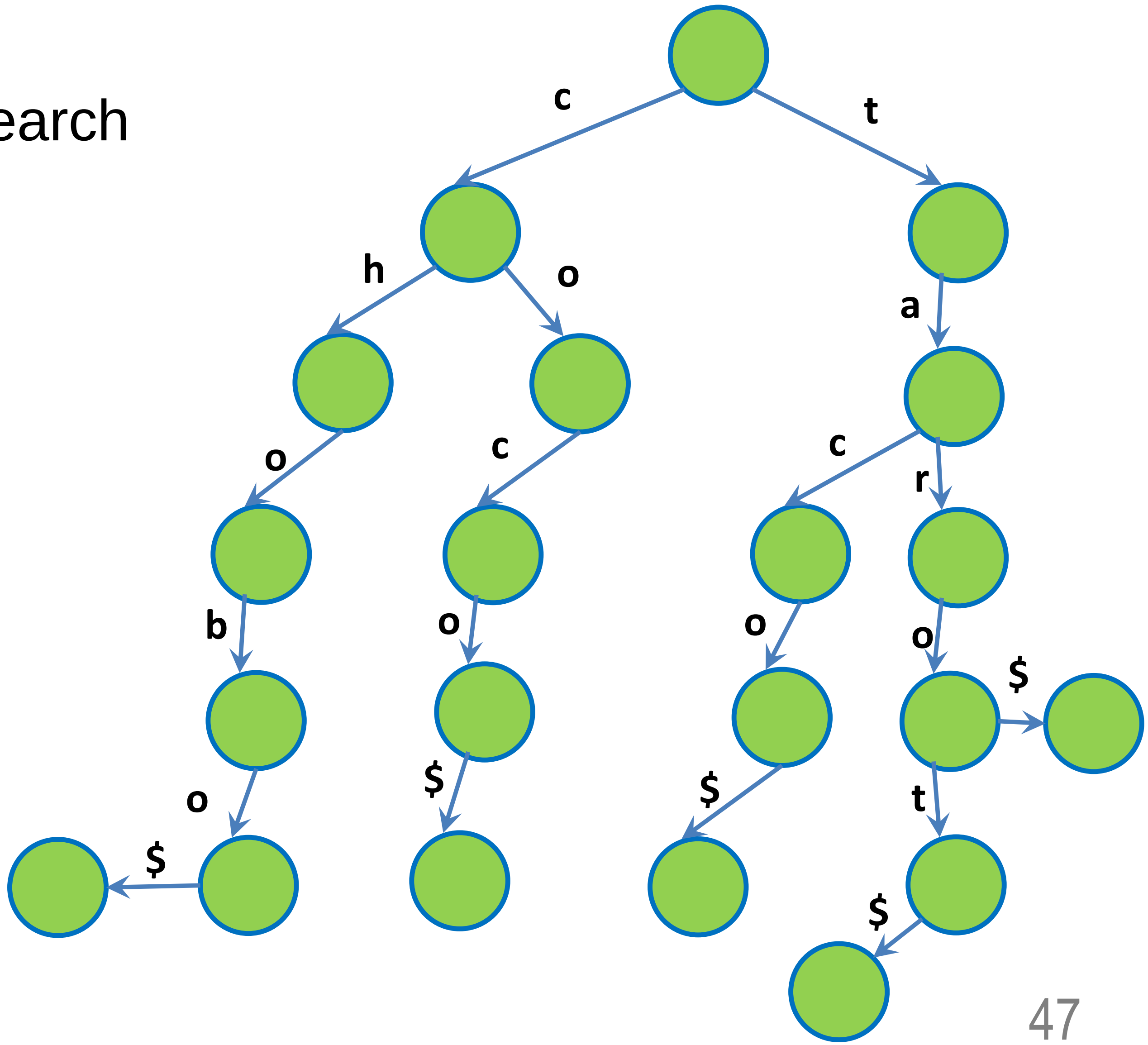
Questions?

Tries

Efficient string retrieval

- So how do we search for retrieval?
  - Search for “tarot”



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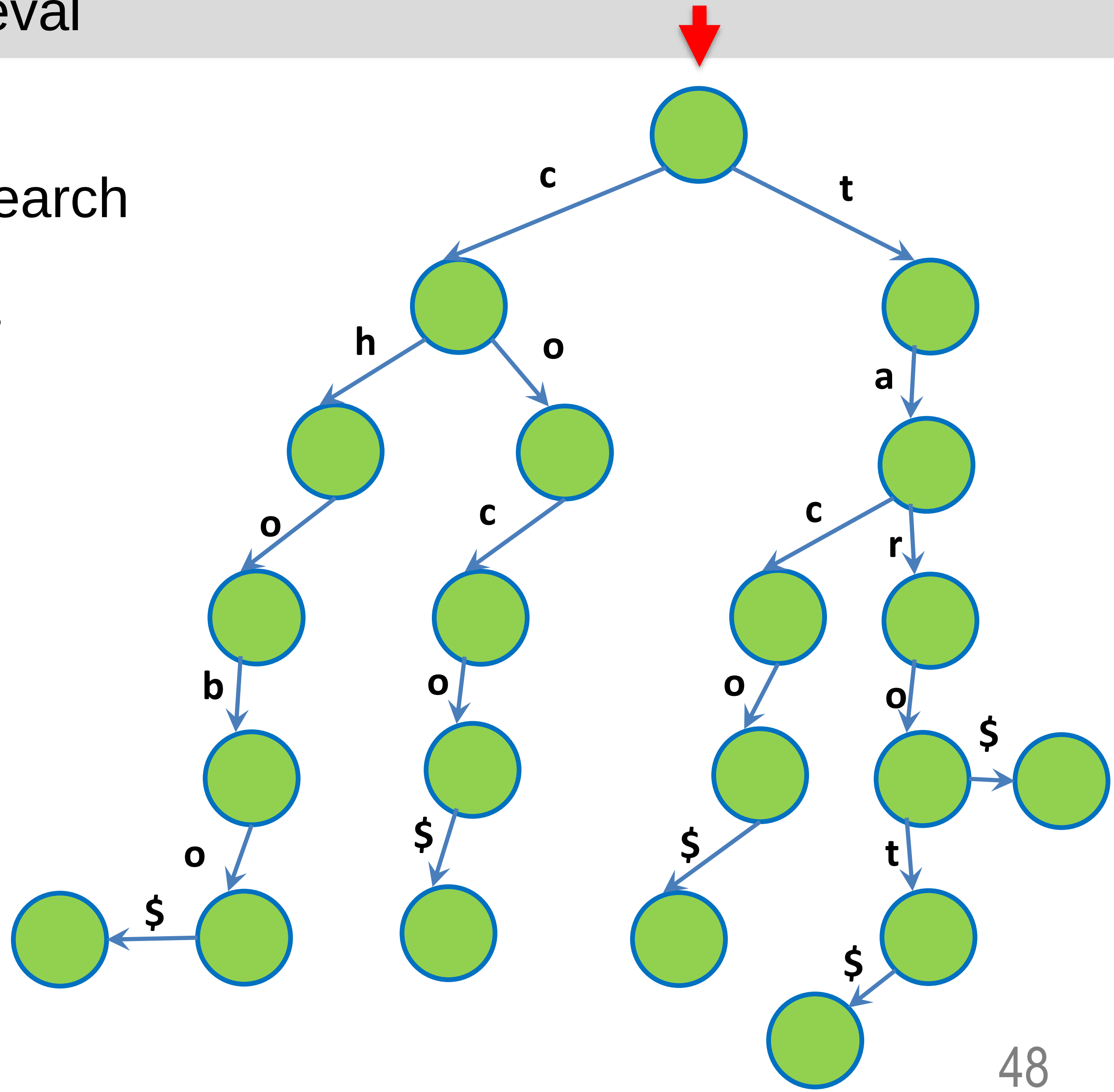

Tries

Efficient string retrieval

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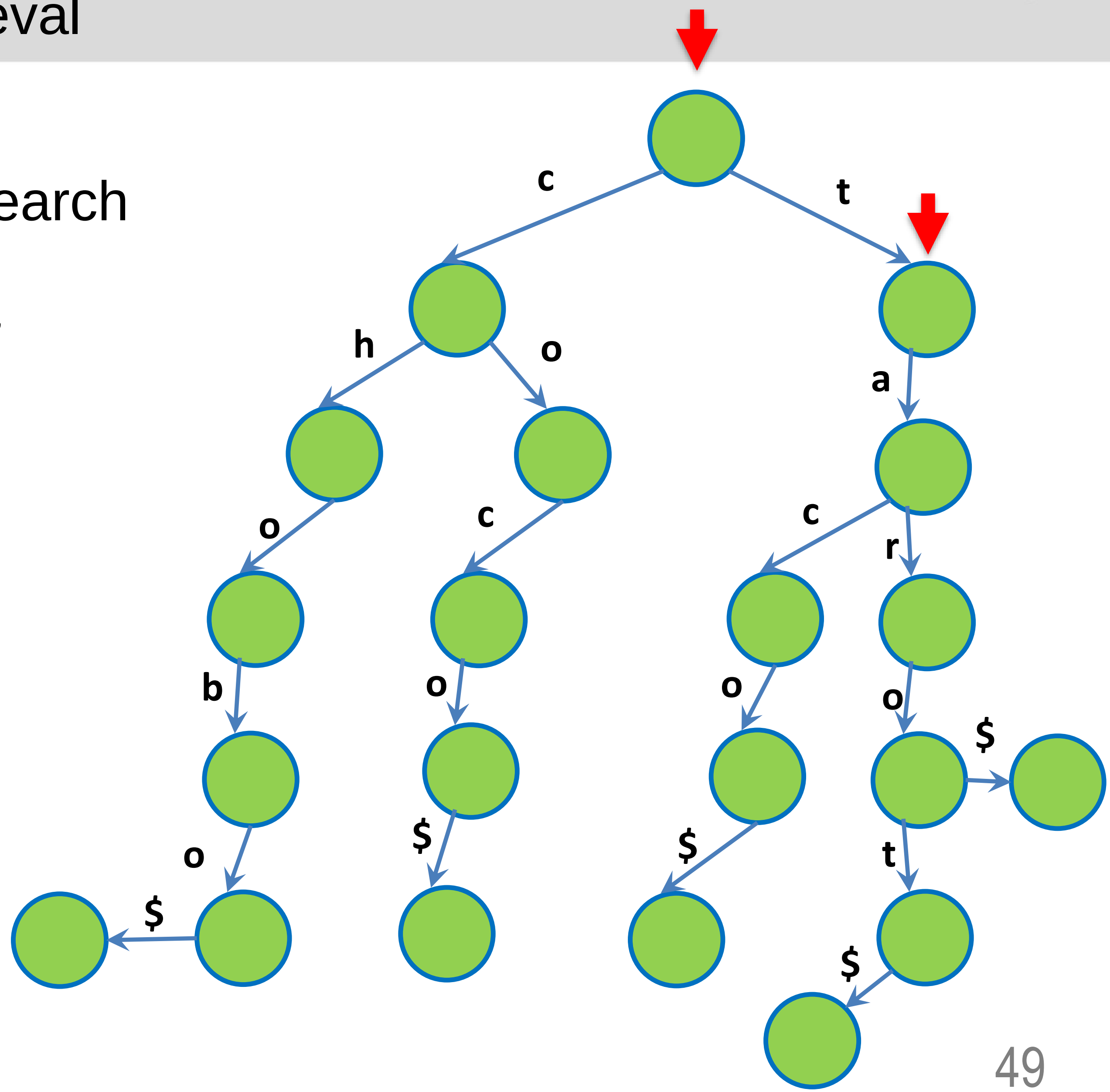

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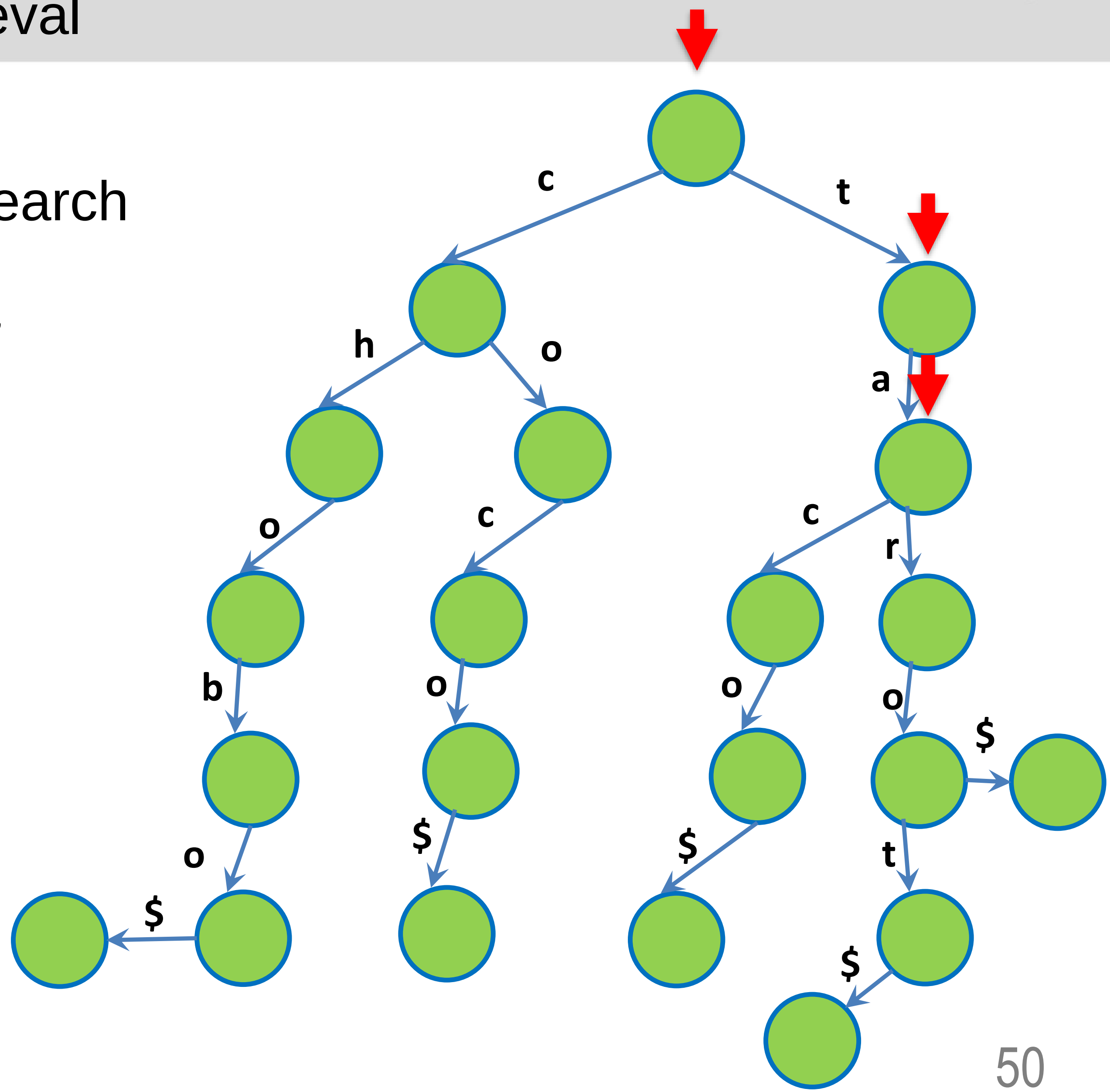

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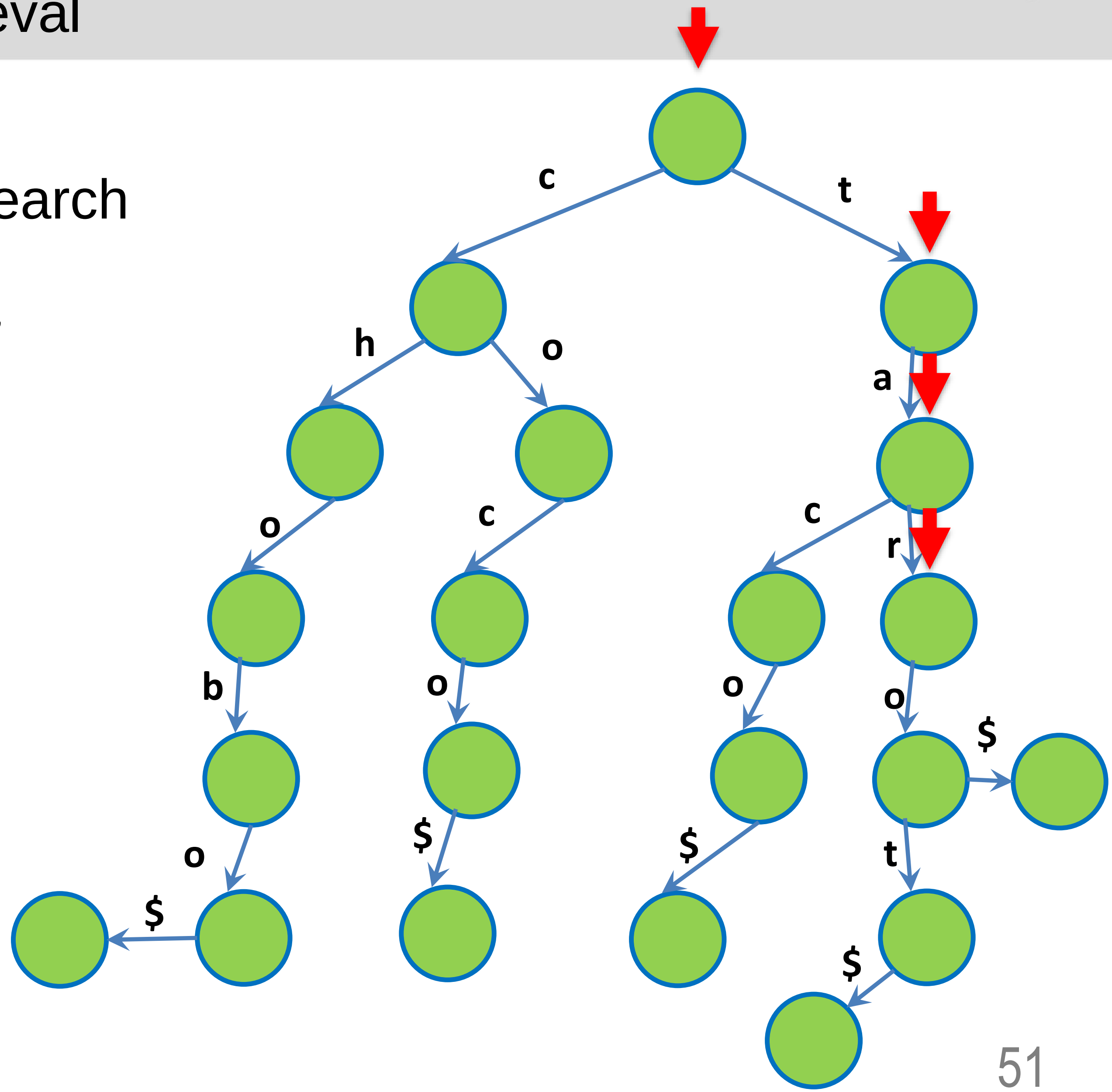

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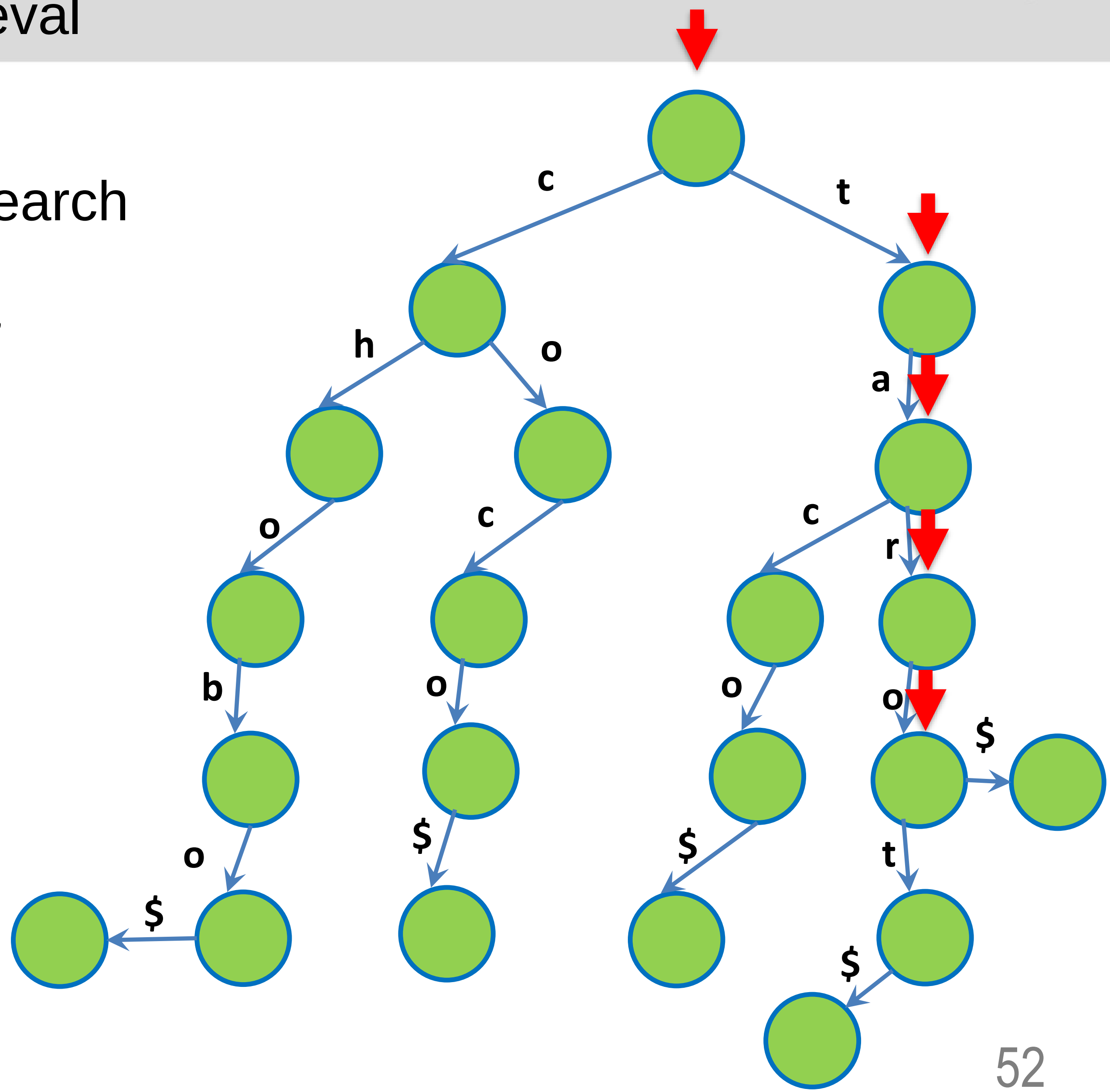

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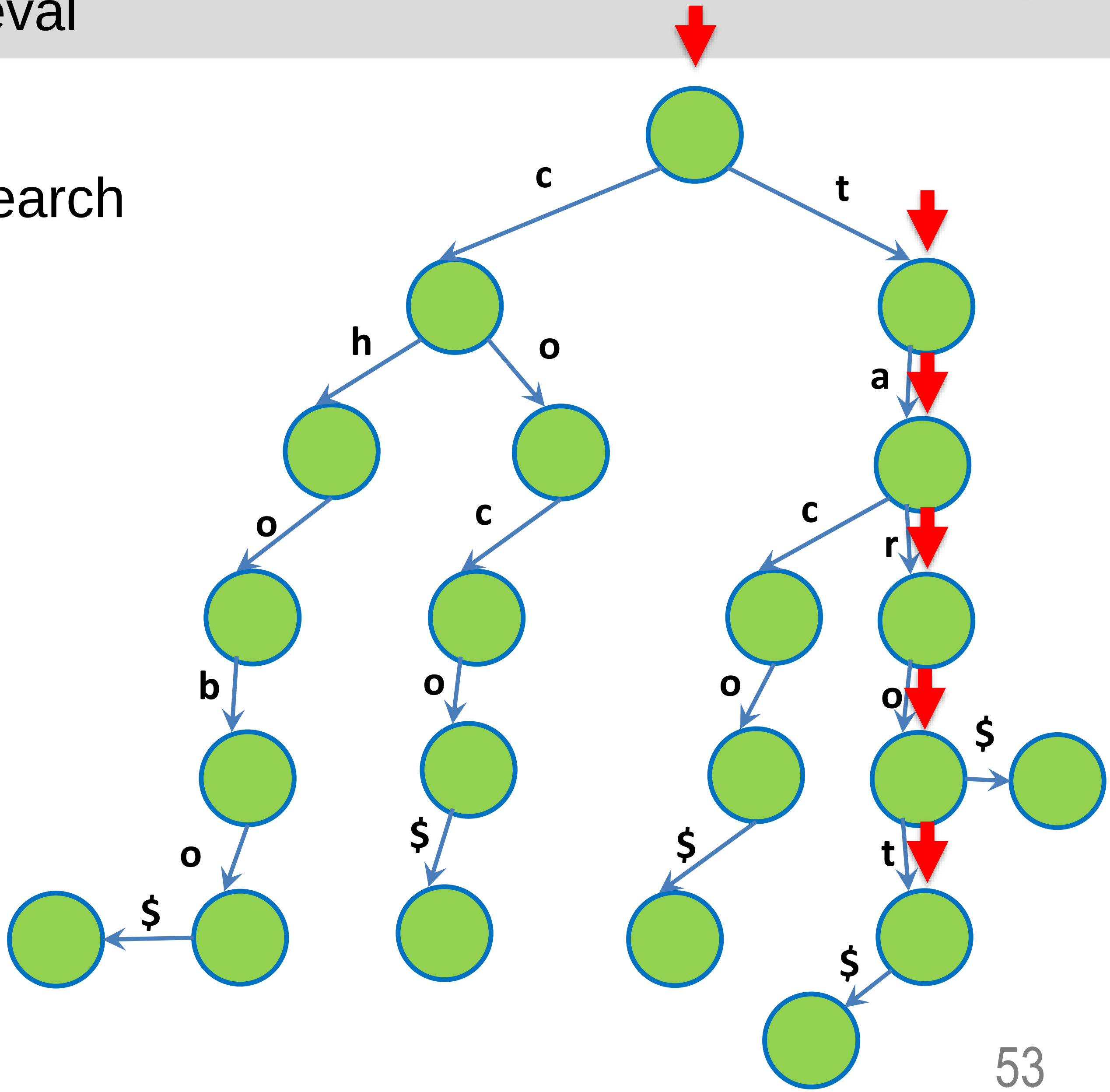

Tries

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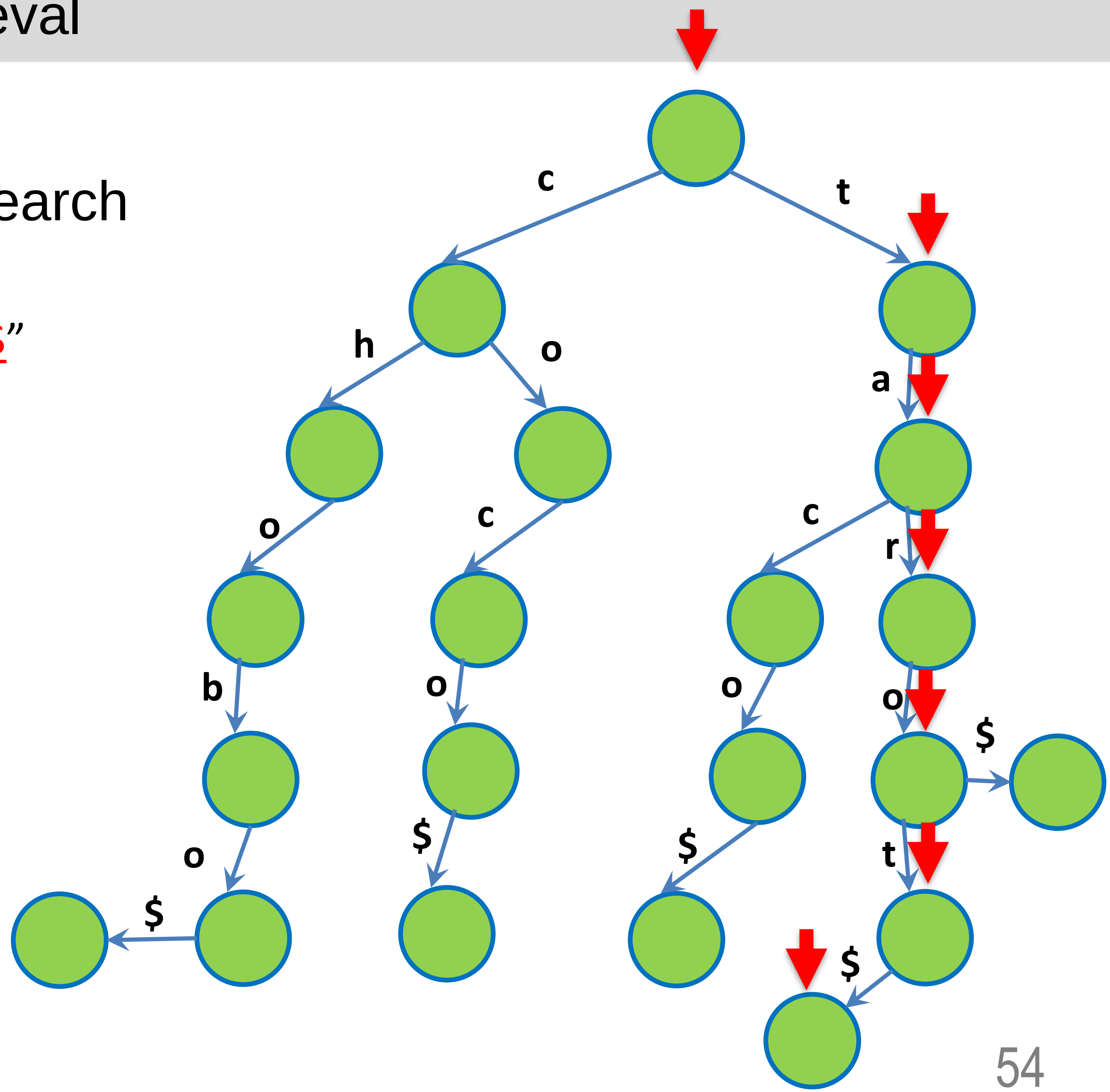
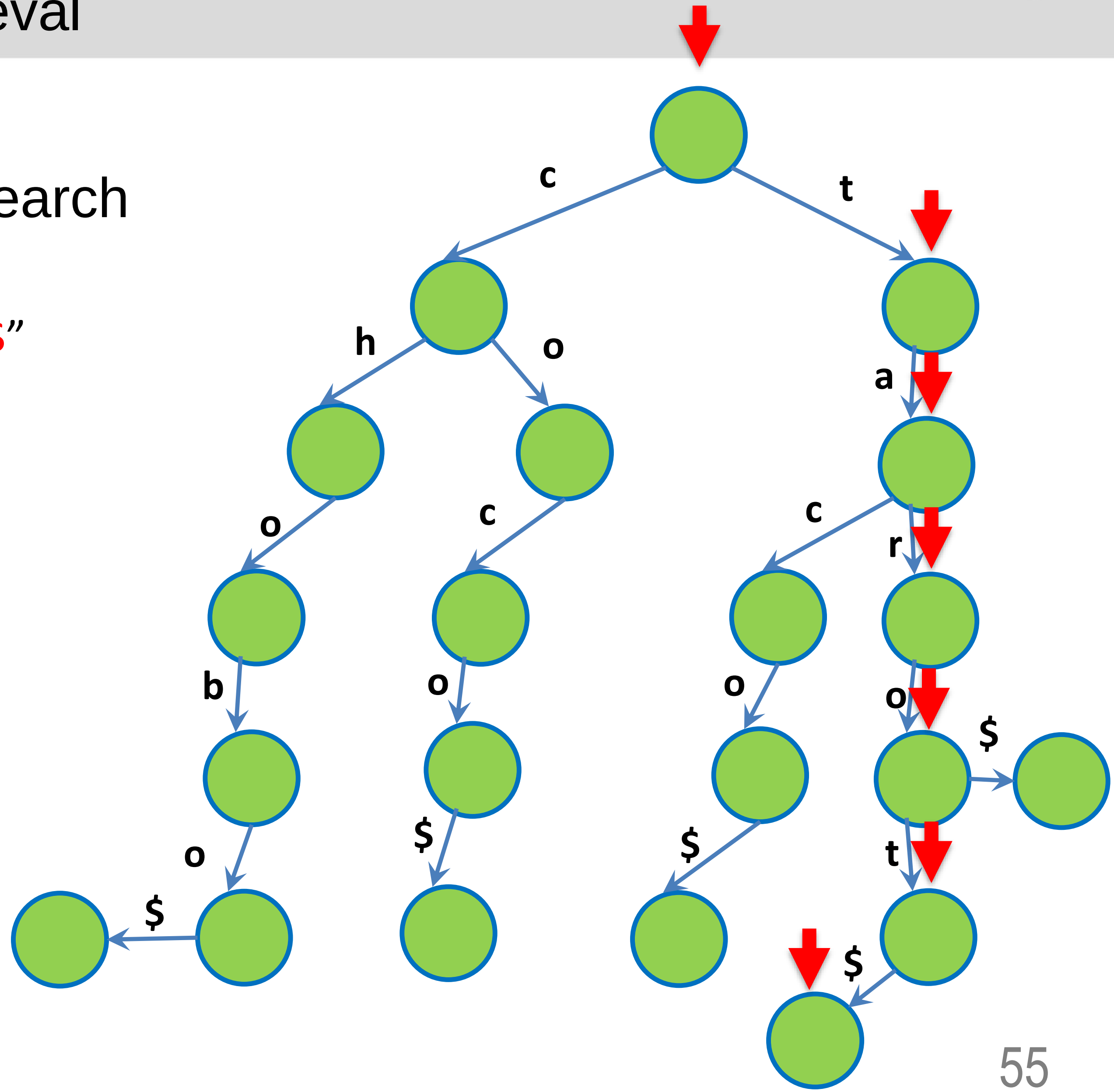


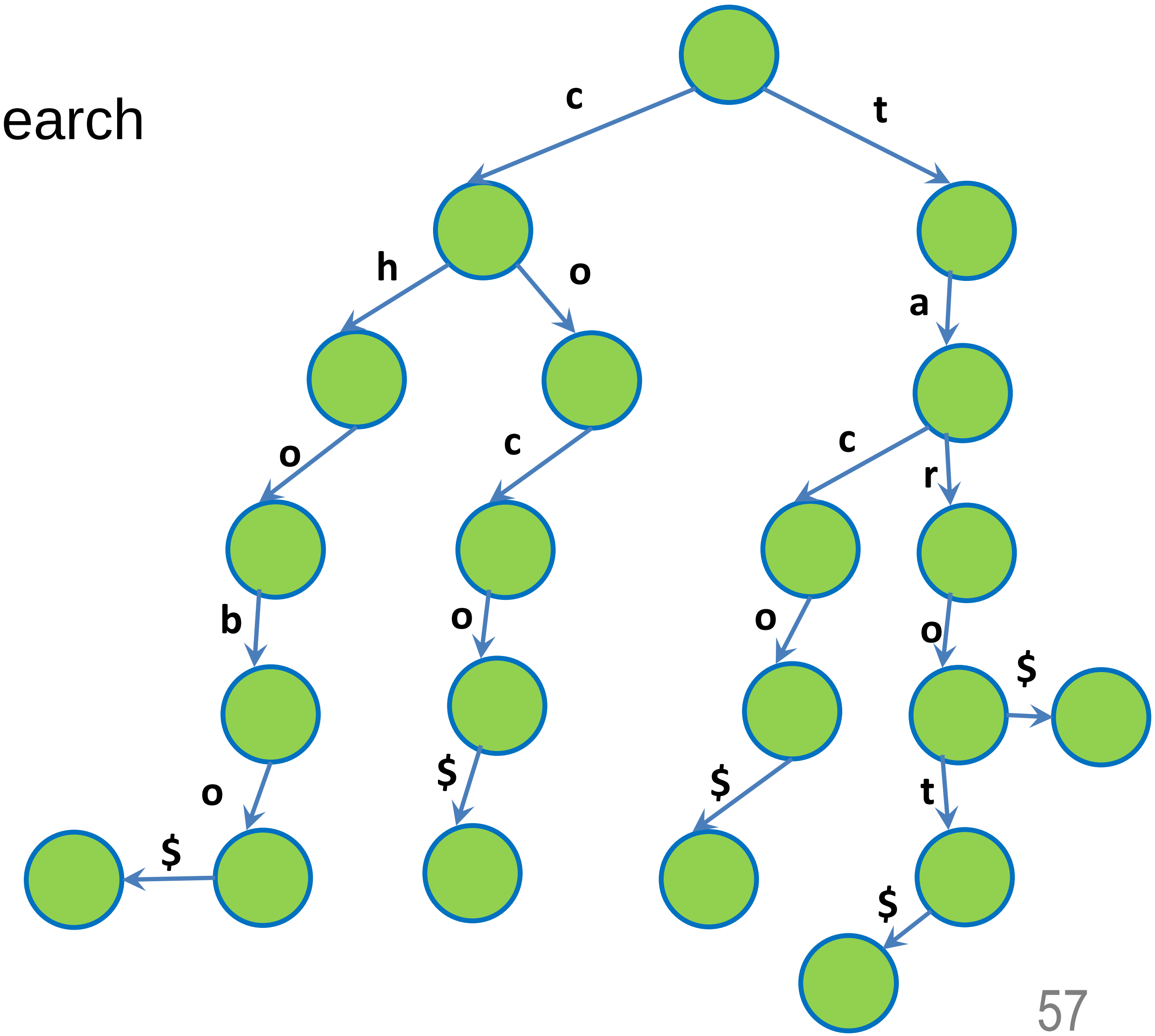
Tries
Efficient string retrieval

- So how do we search for retrieval?
  - Search for “tarot$”
    Found!



Questions?

- So how do we search for retrieval?
  - Search for "cow"



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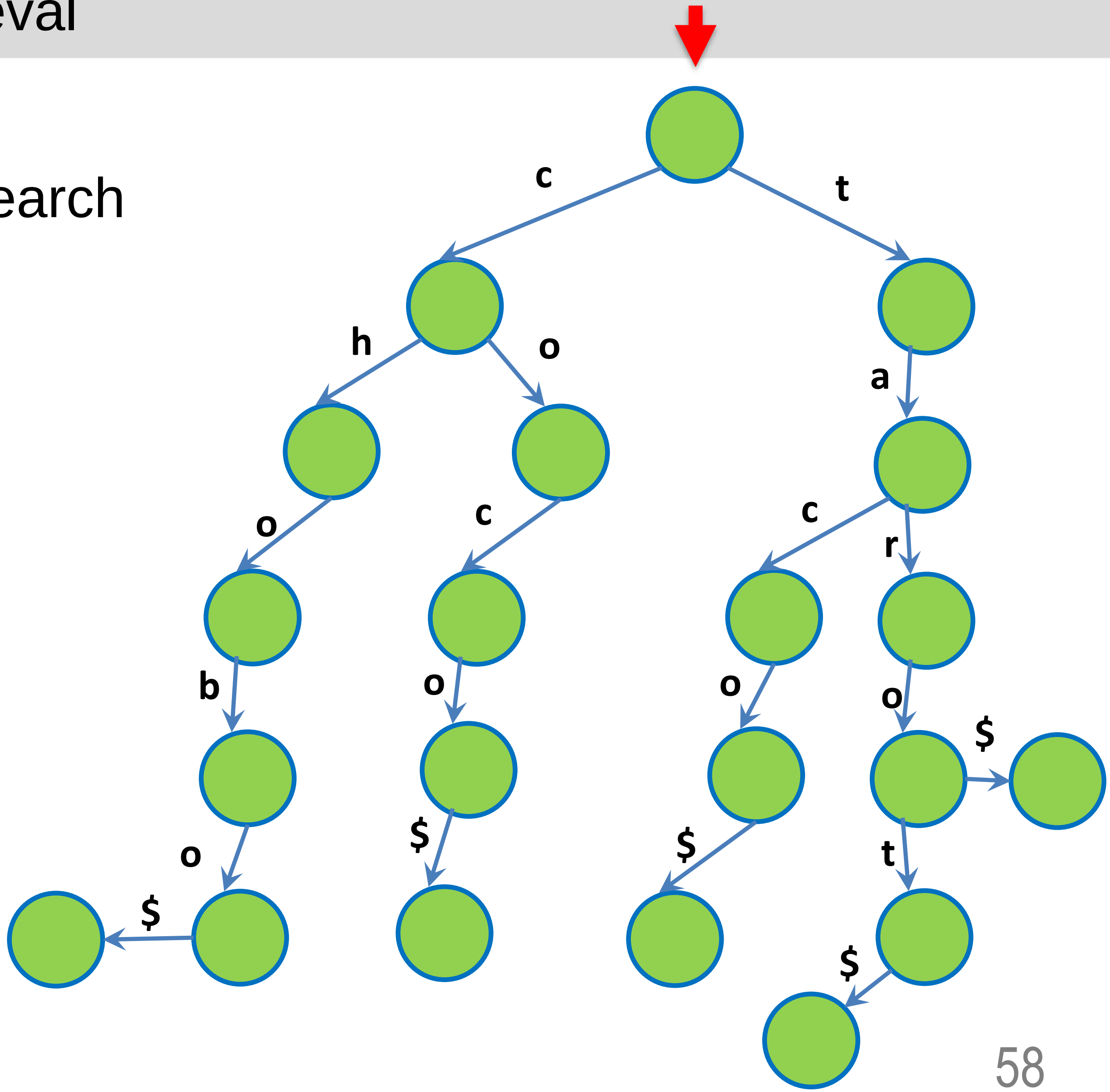

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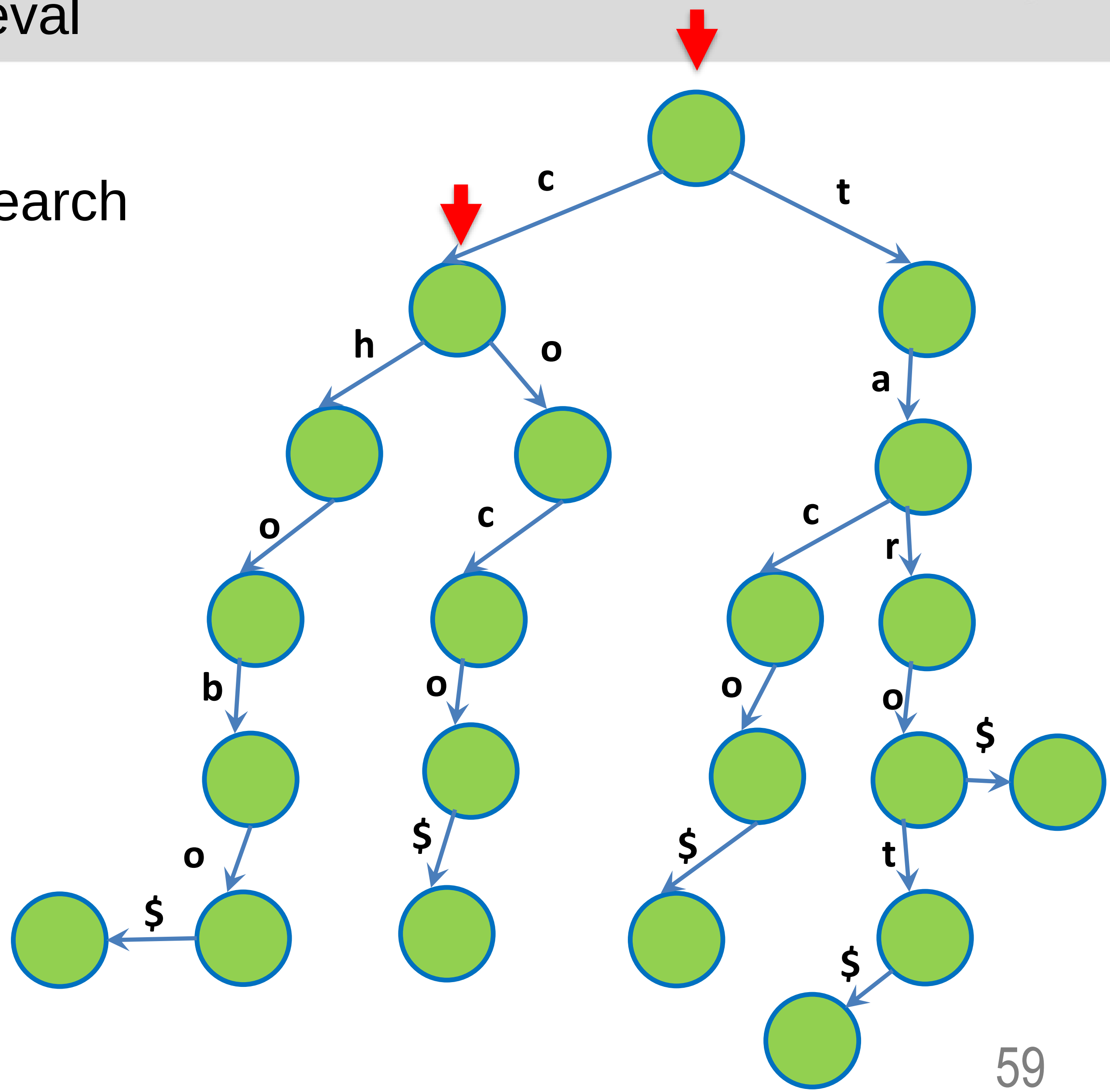

Tries

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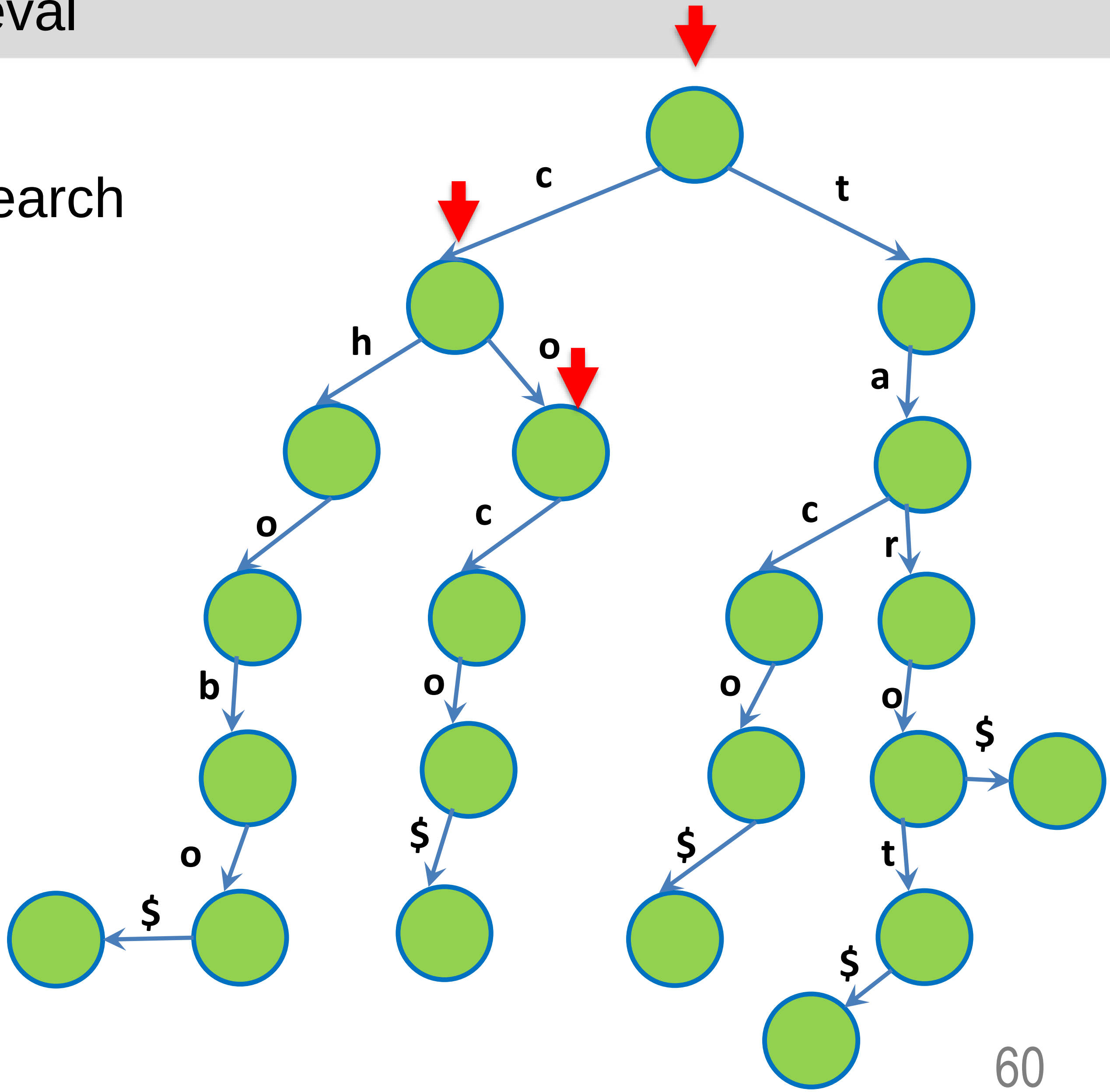

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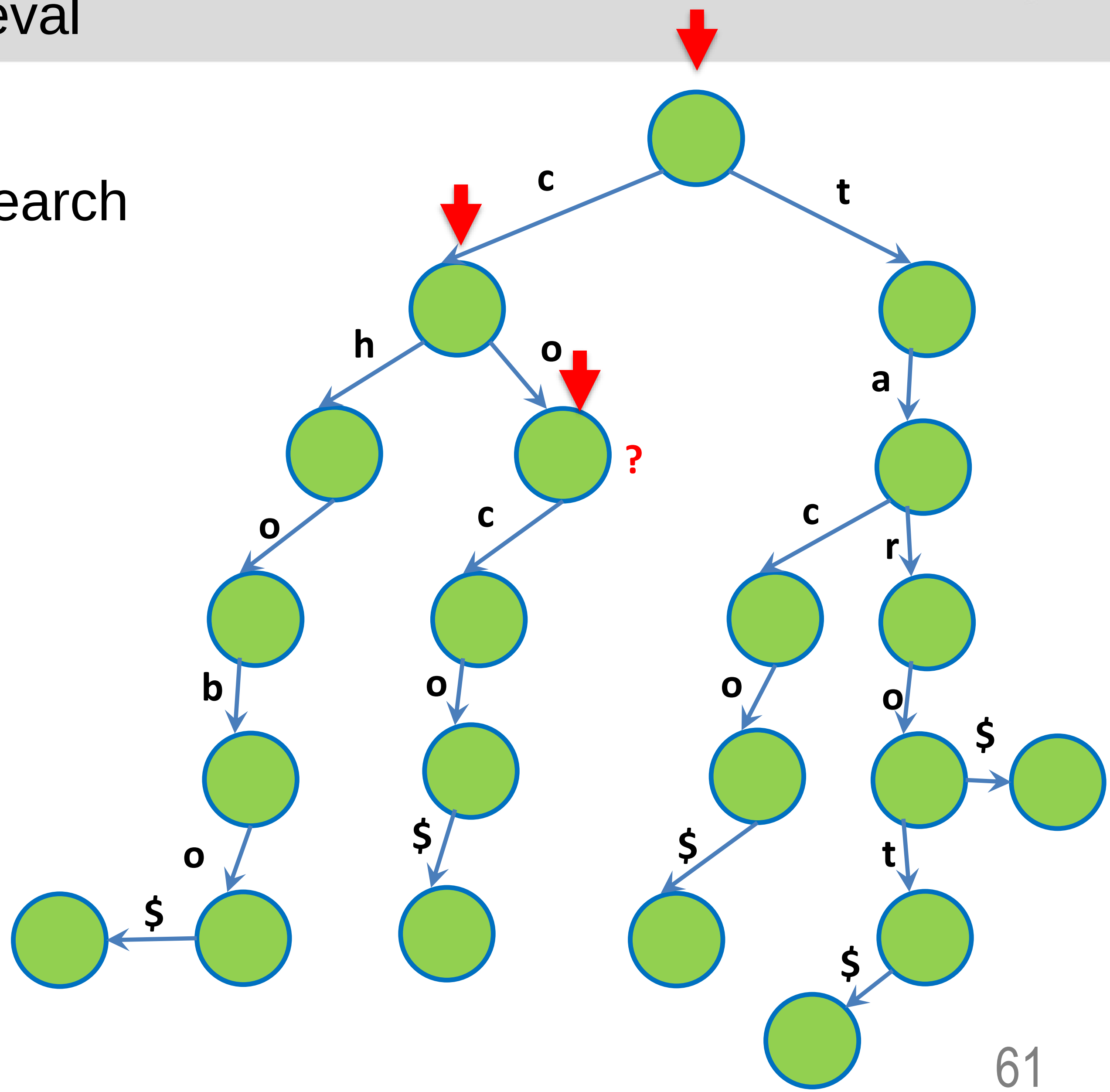

Tries

Efficient string retrieval

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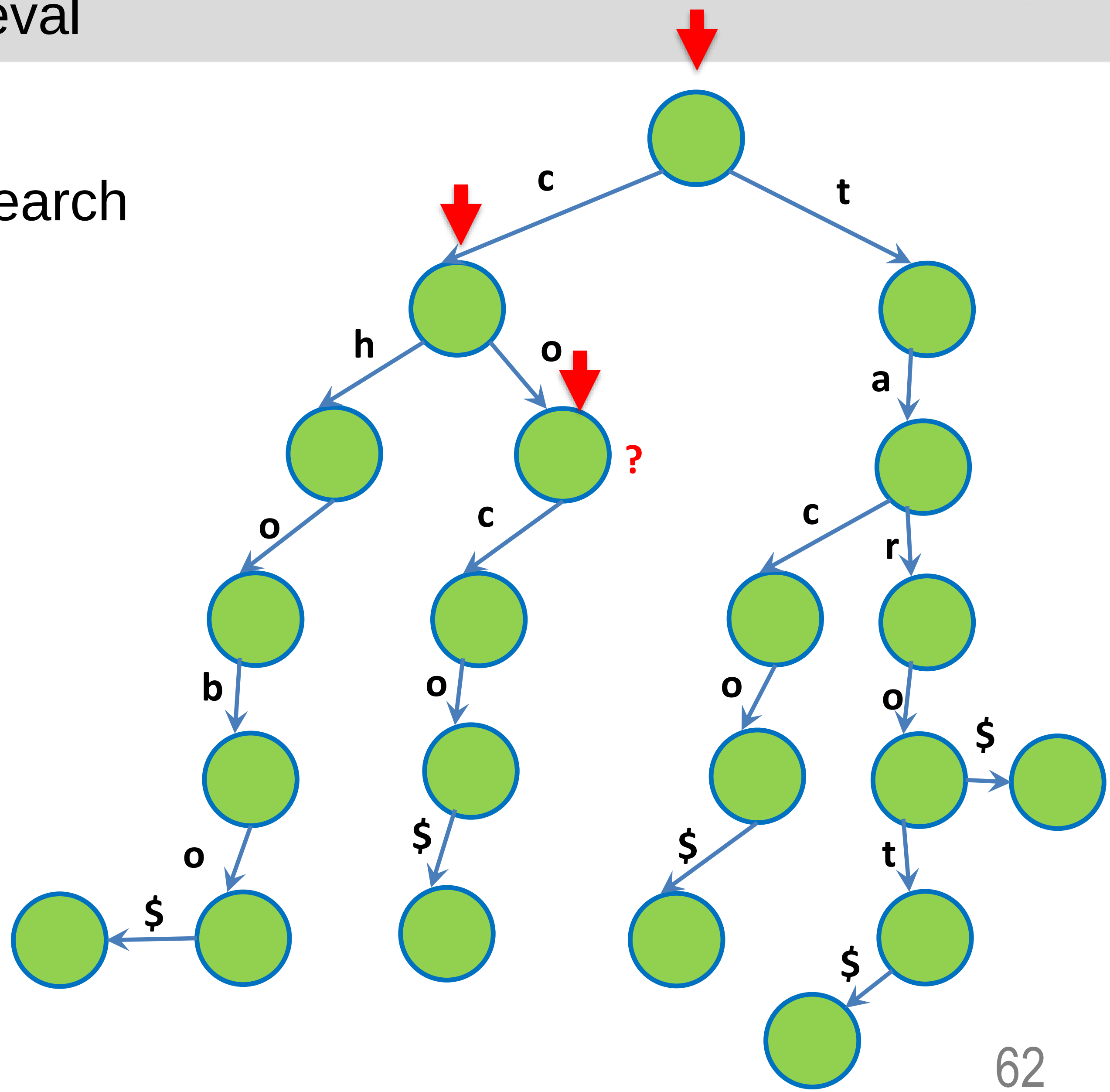
  - Search for “cow”
    Not found T.T



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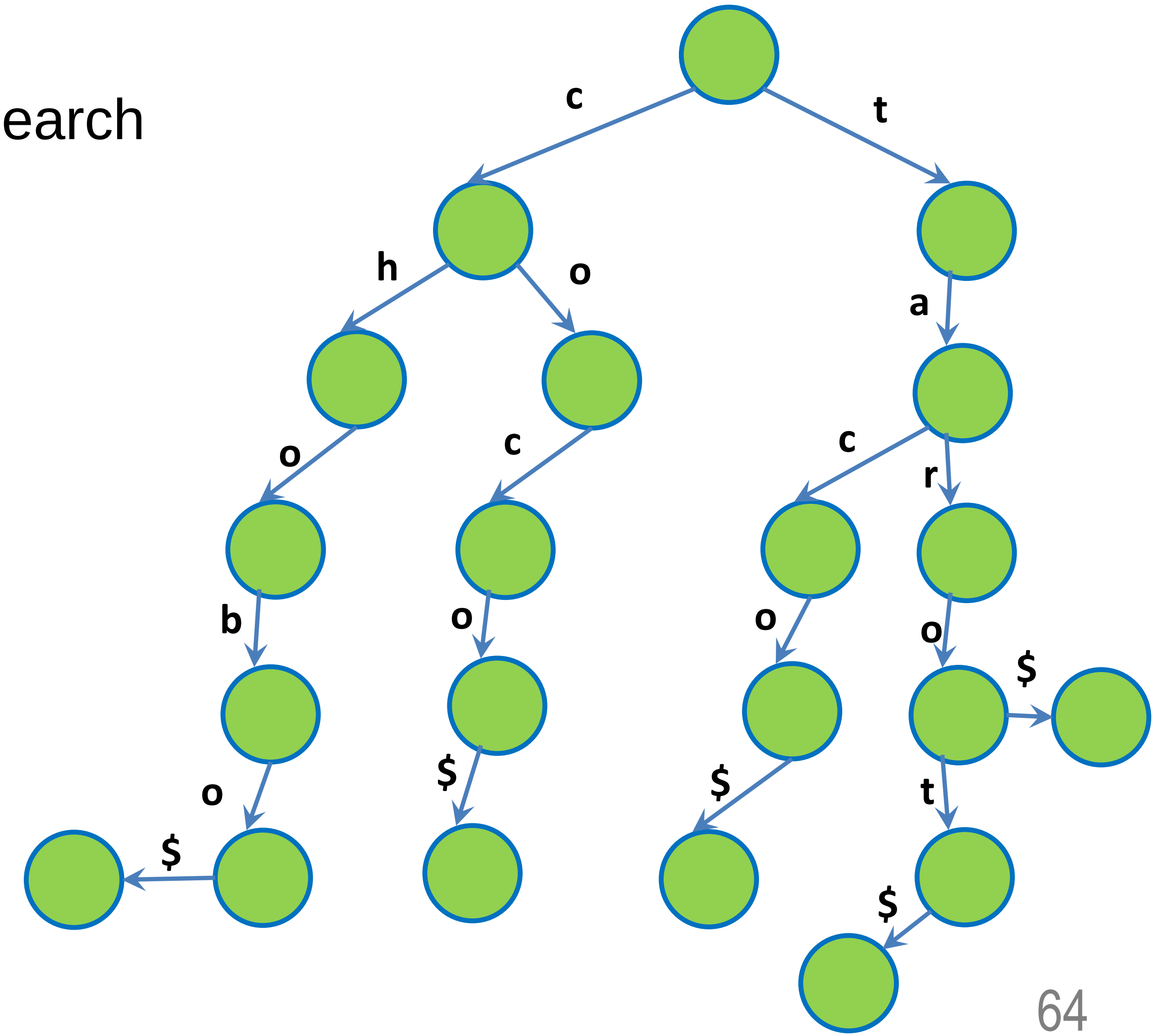

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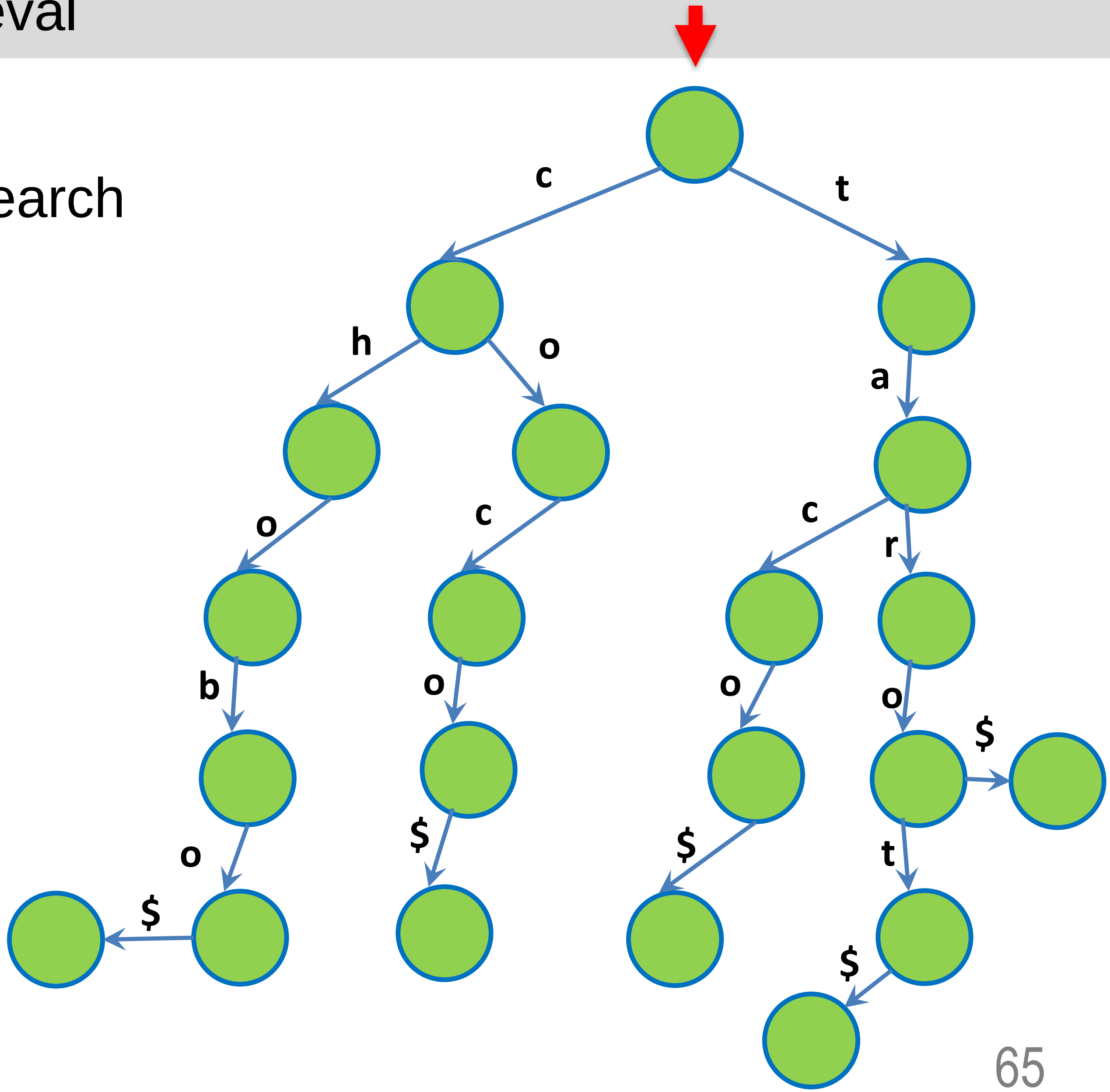

Tries

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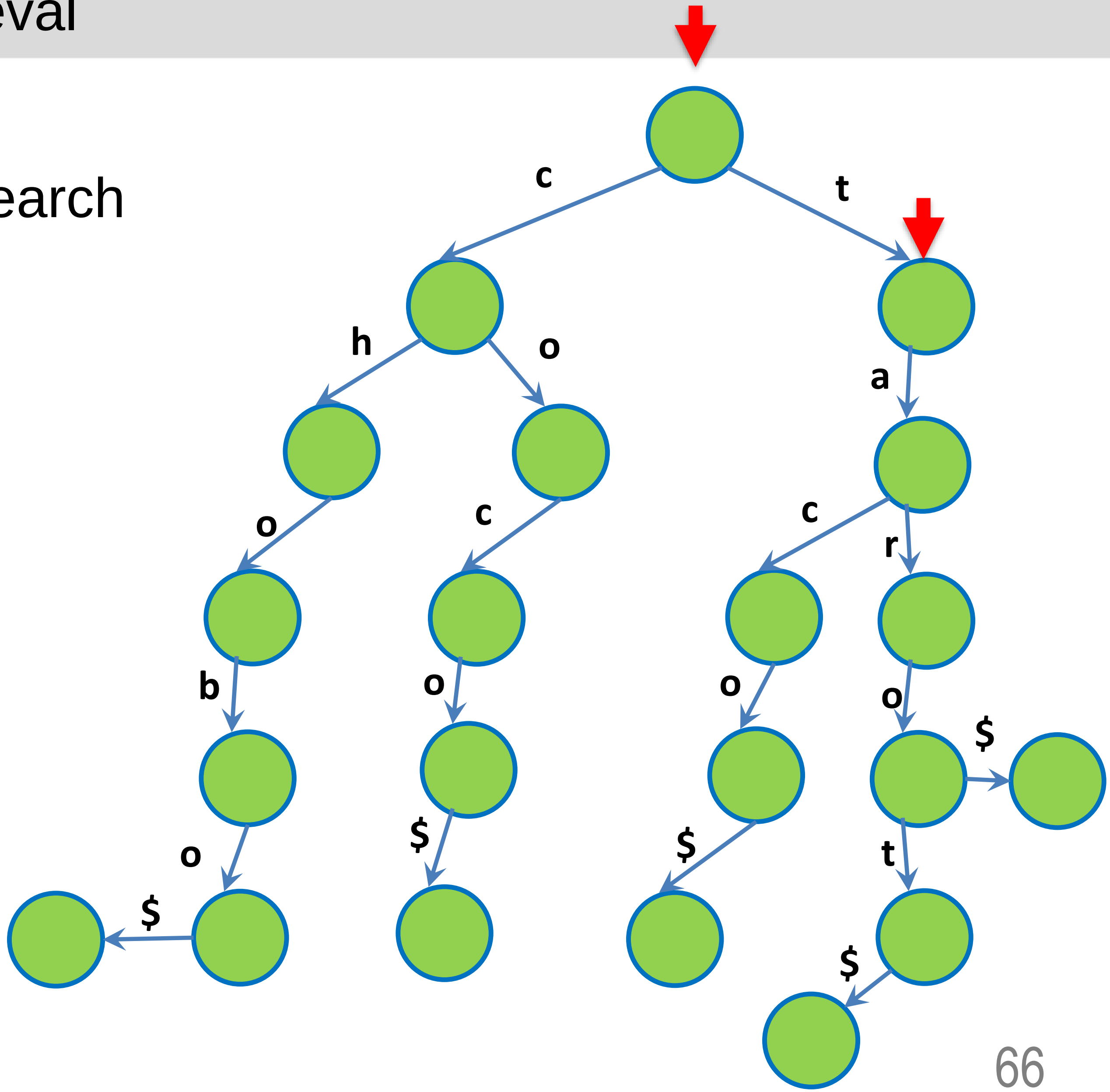

Tries
Efficient string retrieval

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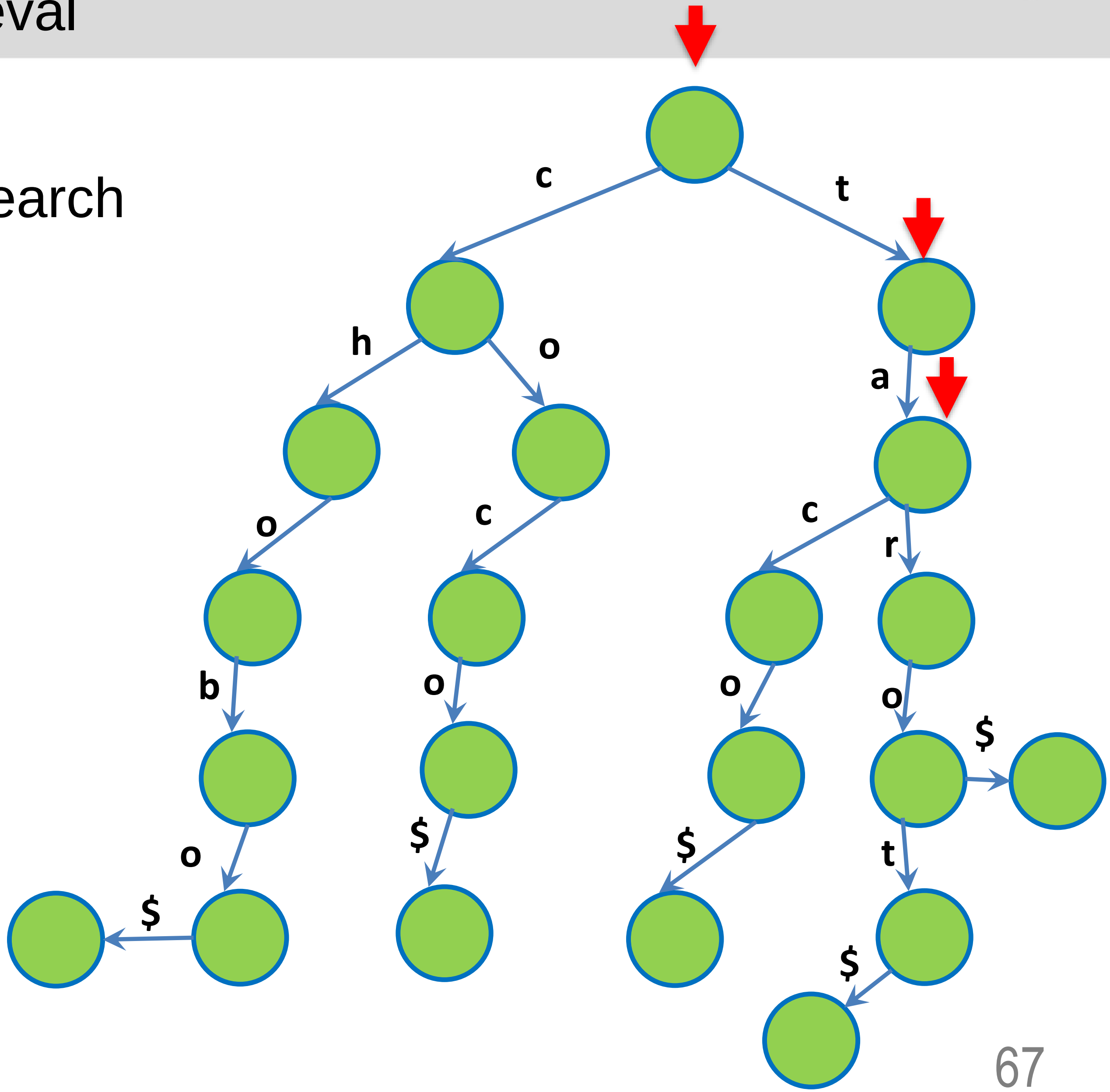

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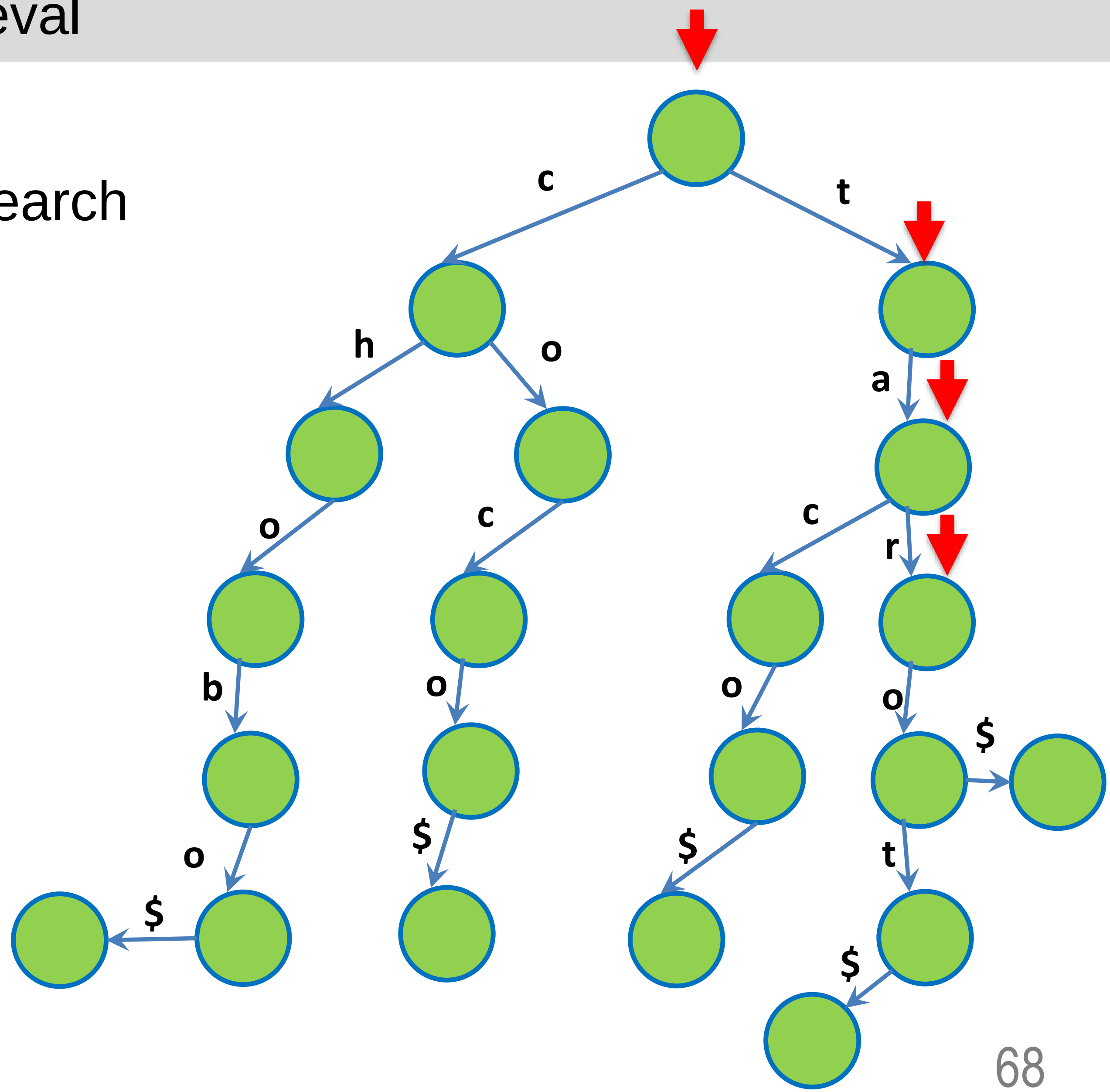
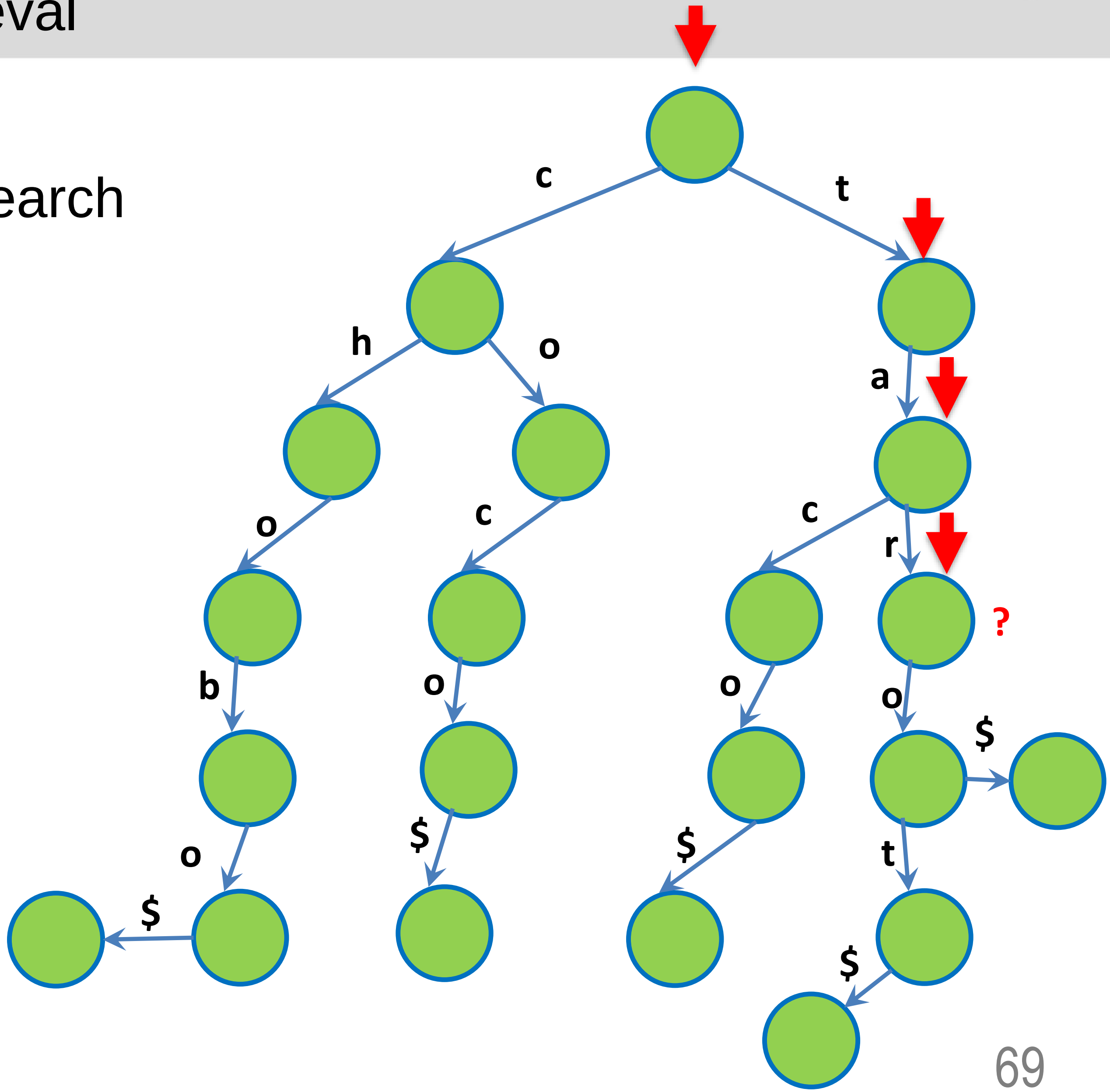


- So how do we search for retrieval?
  - Search for “tar”

Tries

Efficient string retrieval

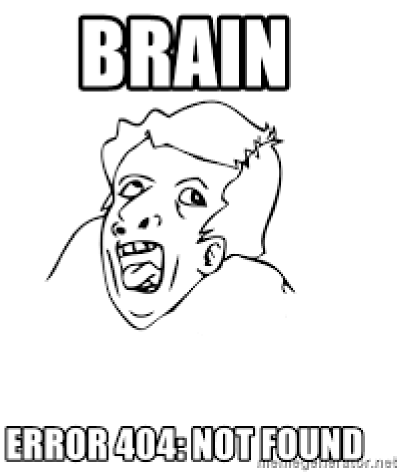
- So how do we search for retrieval?
  - Search for “tar$”

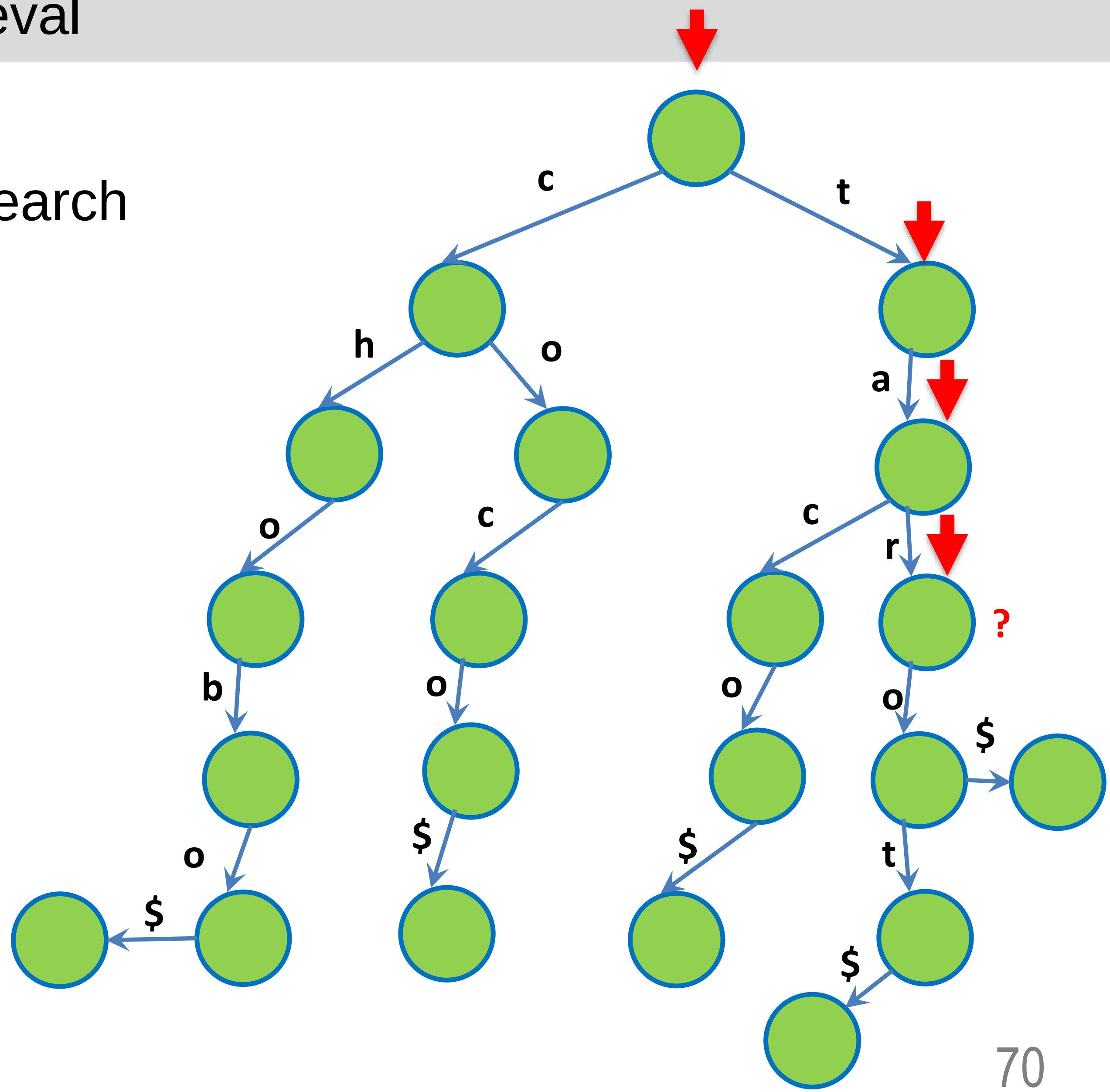


Tries

Efficient string retrieval

- So how do we search for retrieval?
  - Search for “tar$”
    Not found =(





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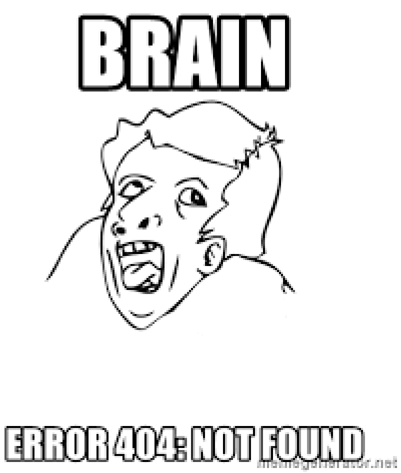
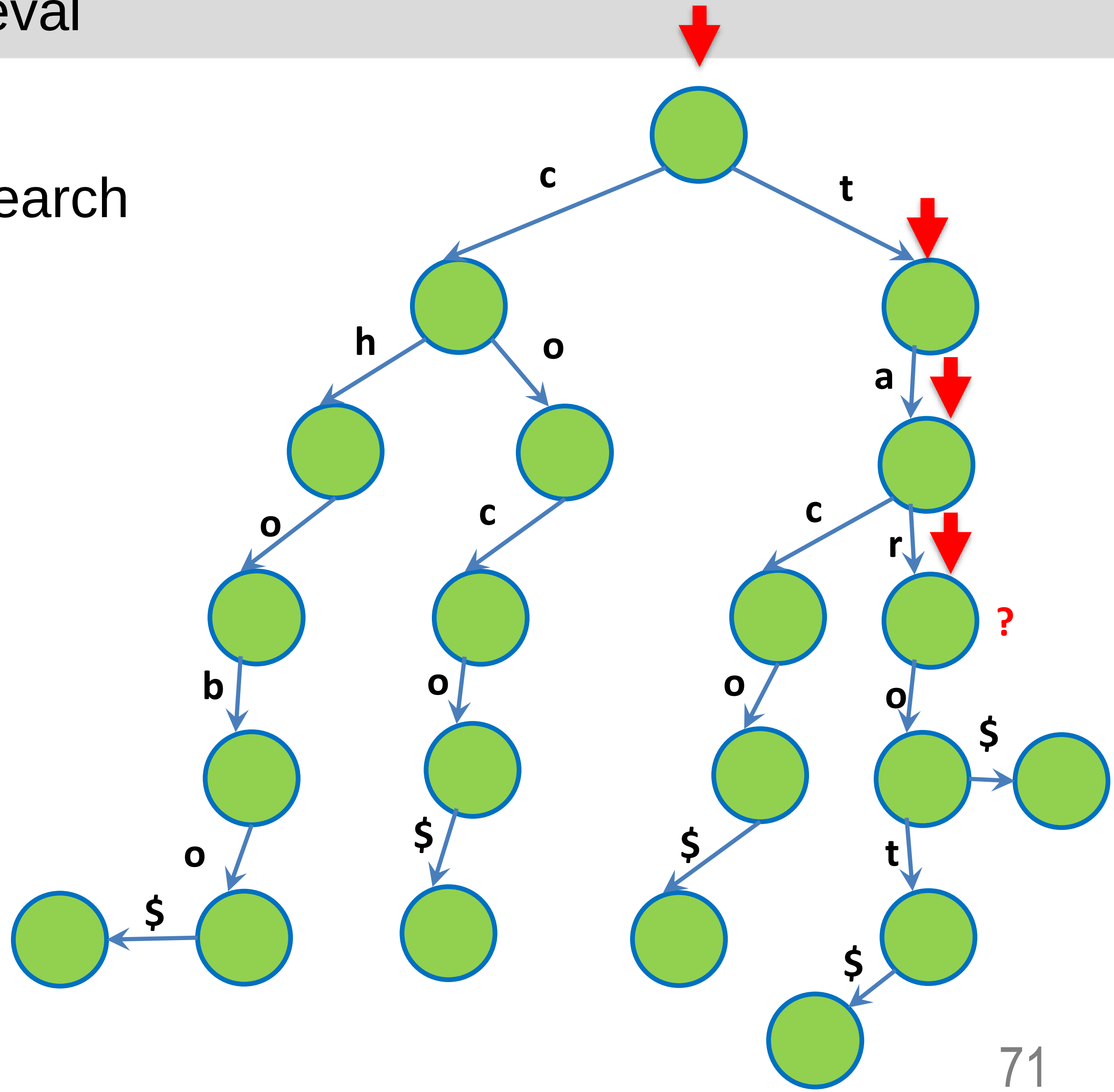
  - Search for “tar$” Not found =(Need those $$$



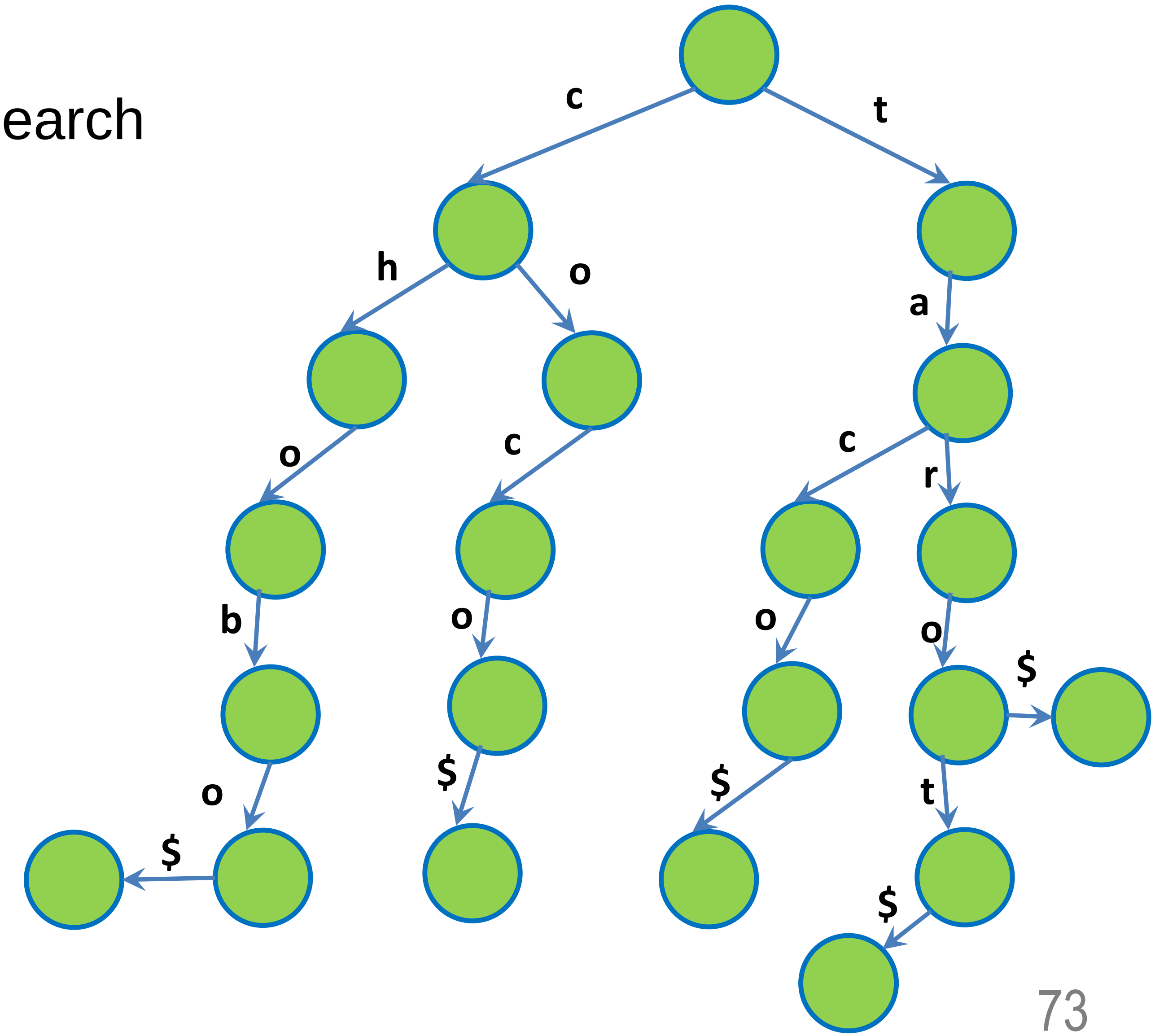


Questions?

Tries

Efficient string retrieval

- So how do we search for retrieval?
  - Complexity?



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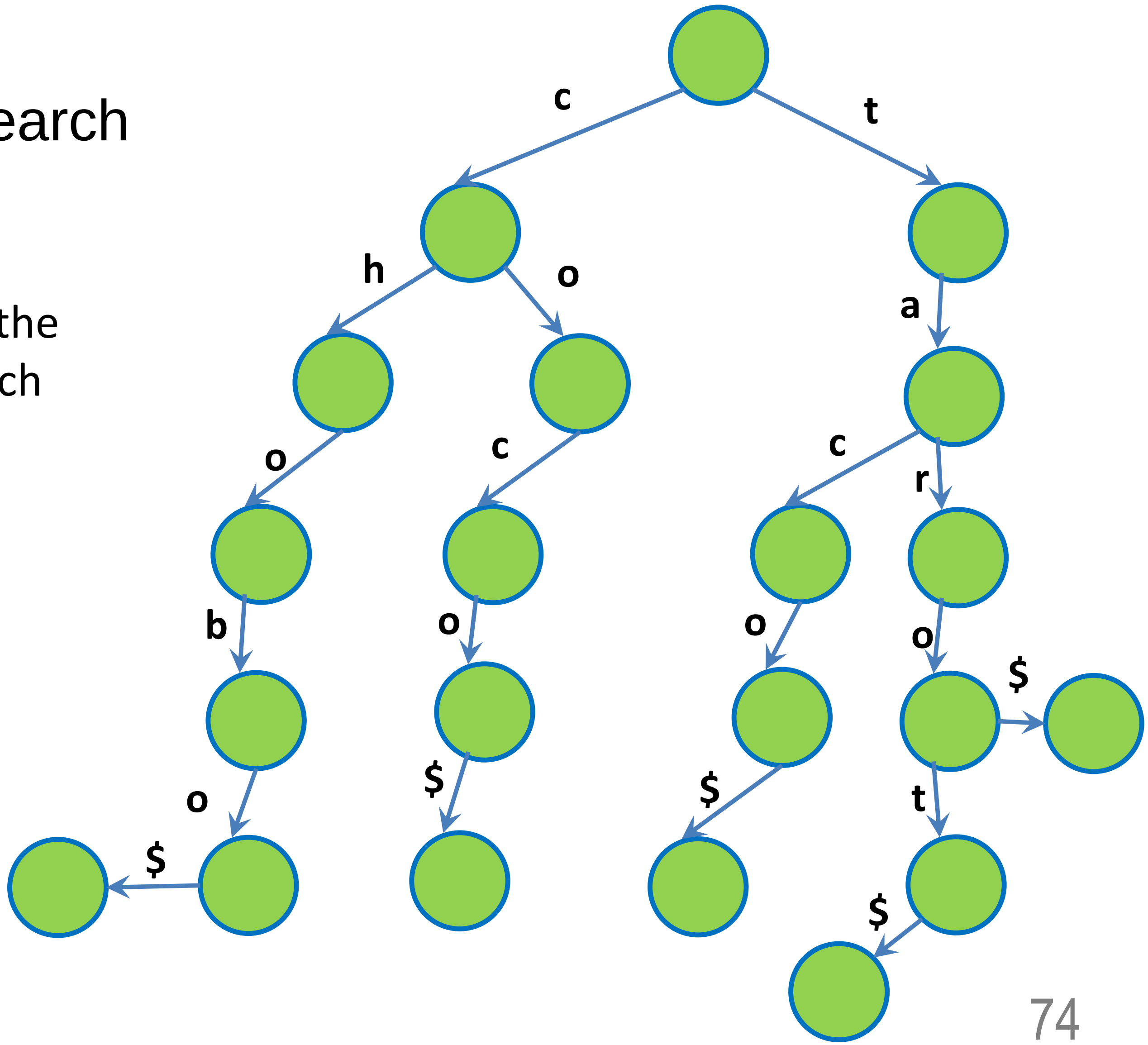

Tries

Efficient string retrieval

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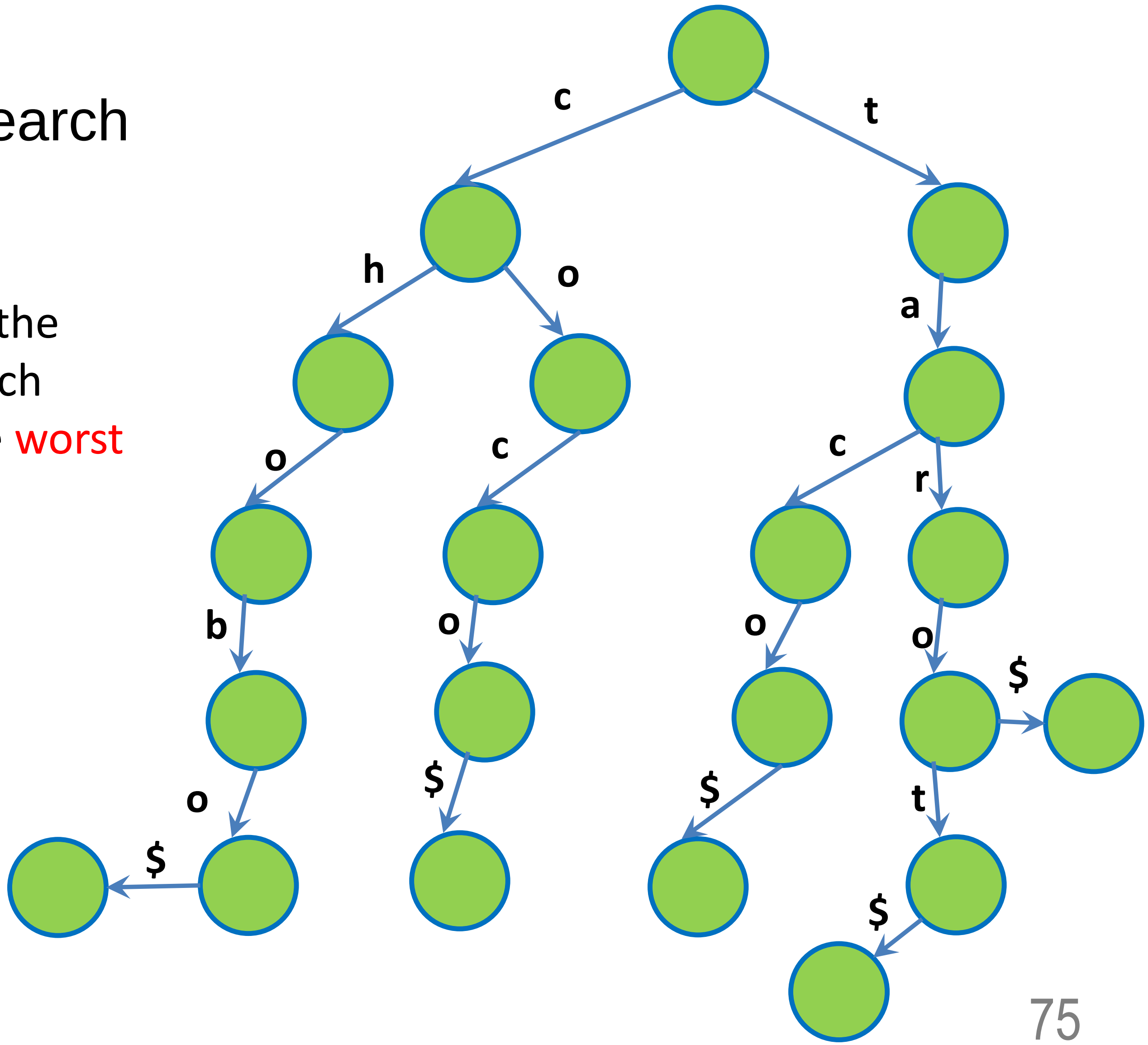

Tries

Efficient string retrieval

- So how do we search for retrieval?
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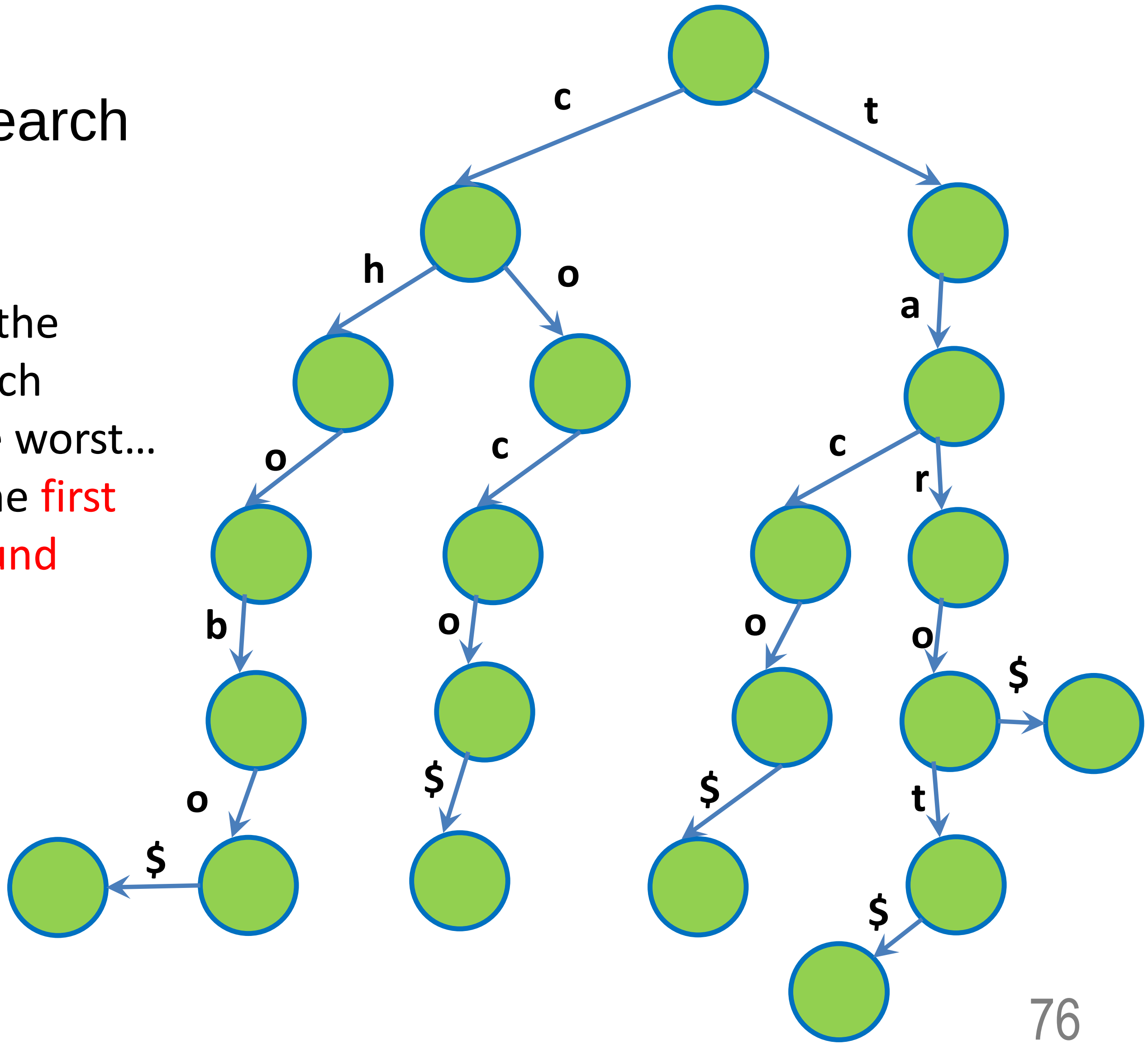
    O(M) where M is the length of the search string… This is the worst…
    O(1) best when the first character isn't found



Questions?

- How to implement it?

- How to implement it? With OOP!

Tries
Efficient string retrieval

- How to implement it? With OOP!
  - Node class

```
class Trie:
    def __init__(self):
        self.root = Node()

class Node:
    def __init__(self, data=None):
        self.data = data
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```

  - Then we need to code the traversal from the root
    - If a link exist, travel through it
      This is O(1) due to the array data structure

Questions?

- Benefits?

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- Disadvantage?
  - At times can be slower than hash table
  - Wasted space if the self.link array is left empty most of the time

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Questions?

Usage?

- Height of the trie = length of the longest string

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  - Useful for auto correct/ auto complete
  - And many other applications!

Questions?

Suffix Trie

For suffixes

- Same as a trie
- But for suffixes

Suffix Trie

For suffixes

- Can you make a suffix trie for apple?

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- List all the suffixes
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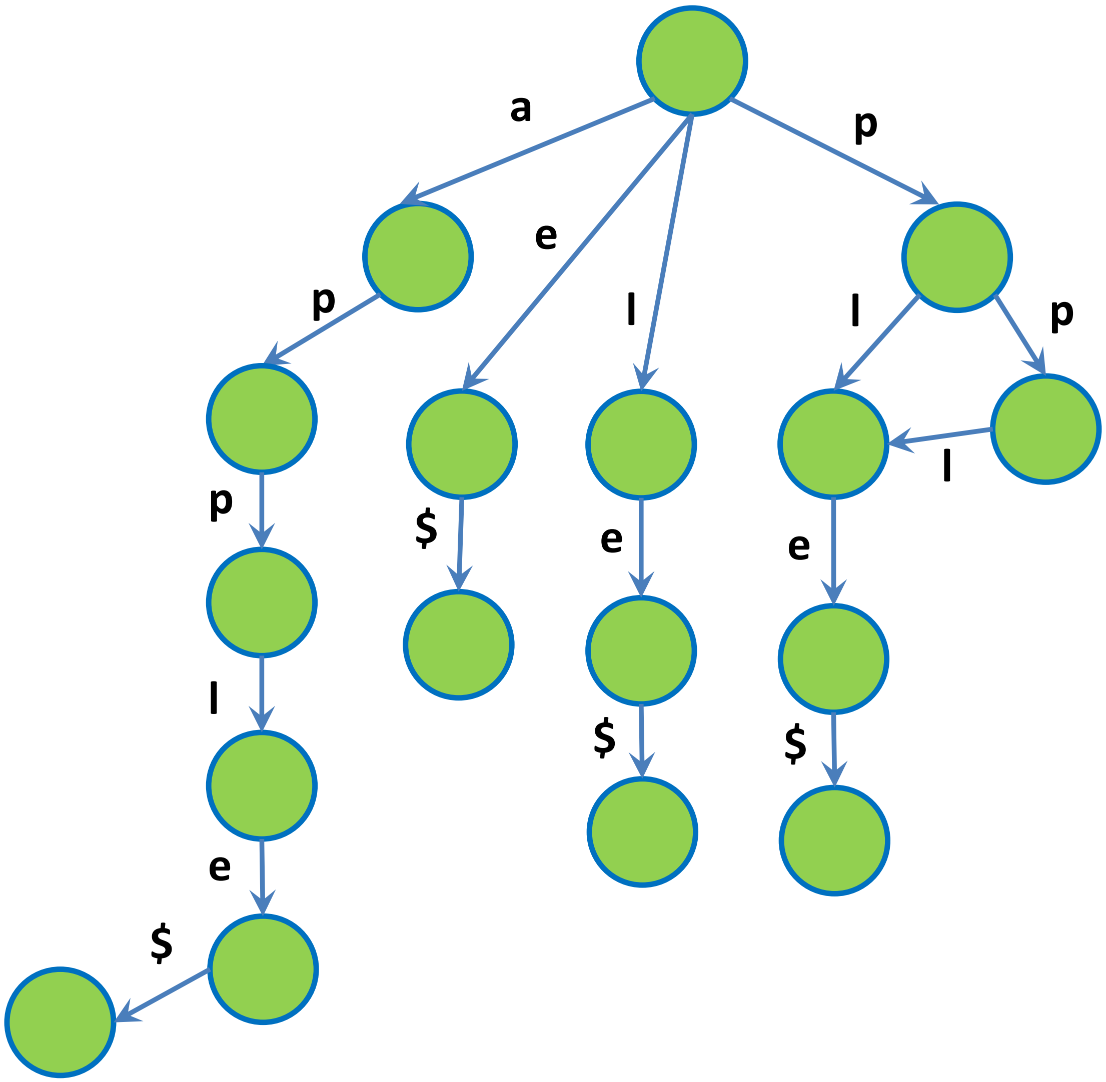

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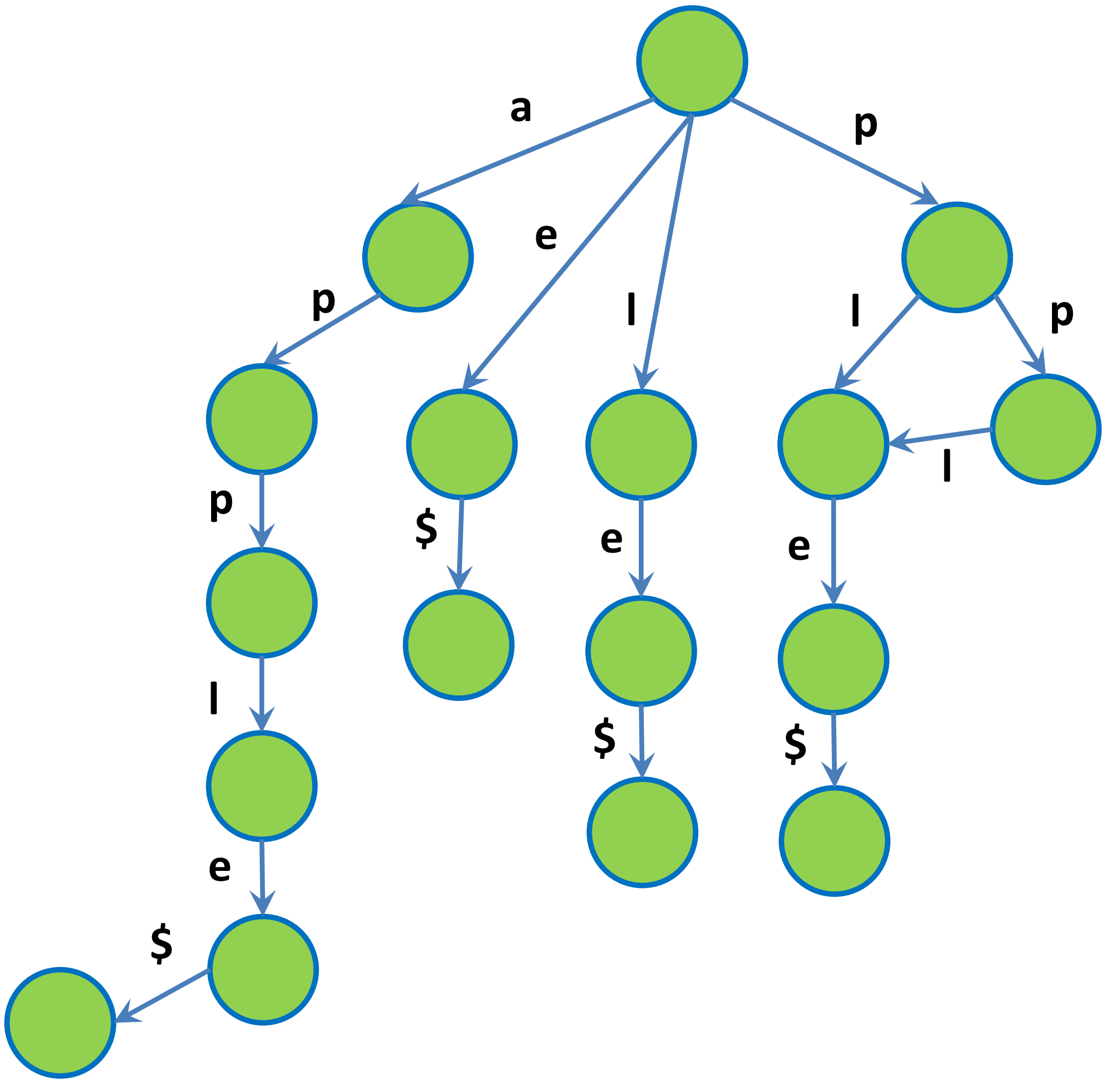
  NO! CYCLE!
  so this is wrong…



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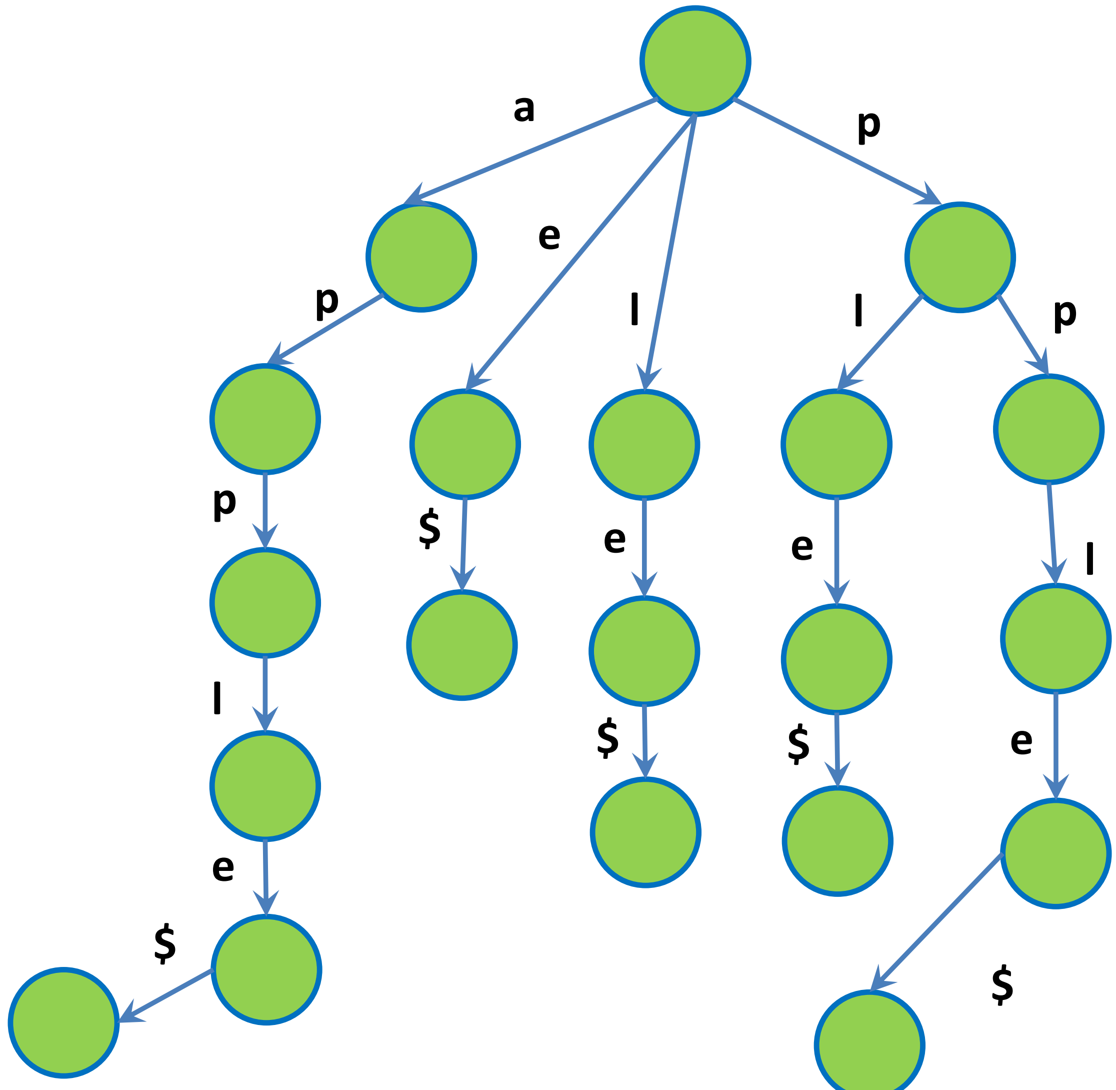

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Questions?

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Suffix Trie
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Suffix Trie
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Suffix Trie

Applications?

- Space complexity?
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  - Have N number of leaves!

Questions?

Suffix Tree
A tree, not a trie

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Suffix Trie　　　　Suffix Tree

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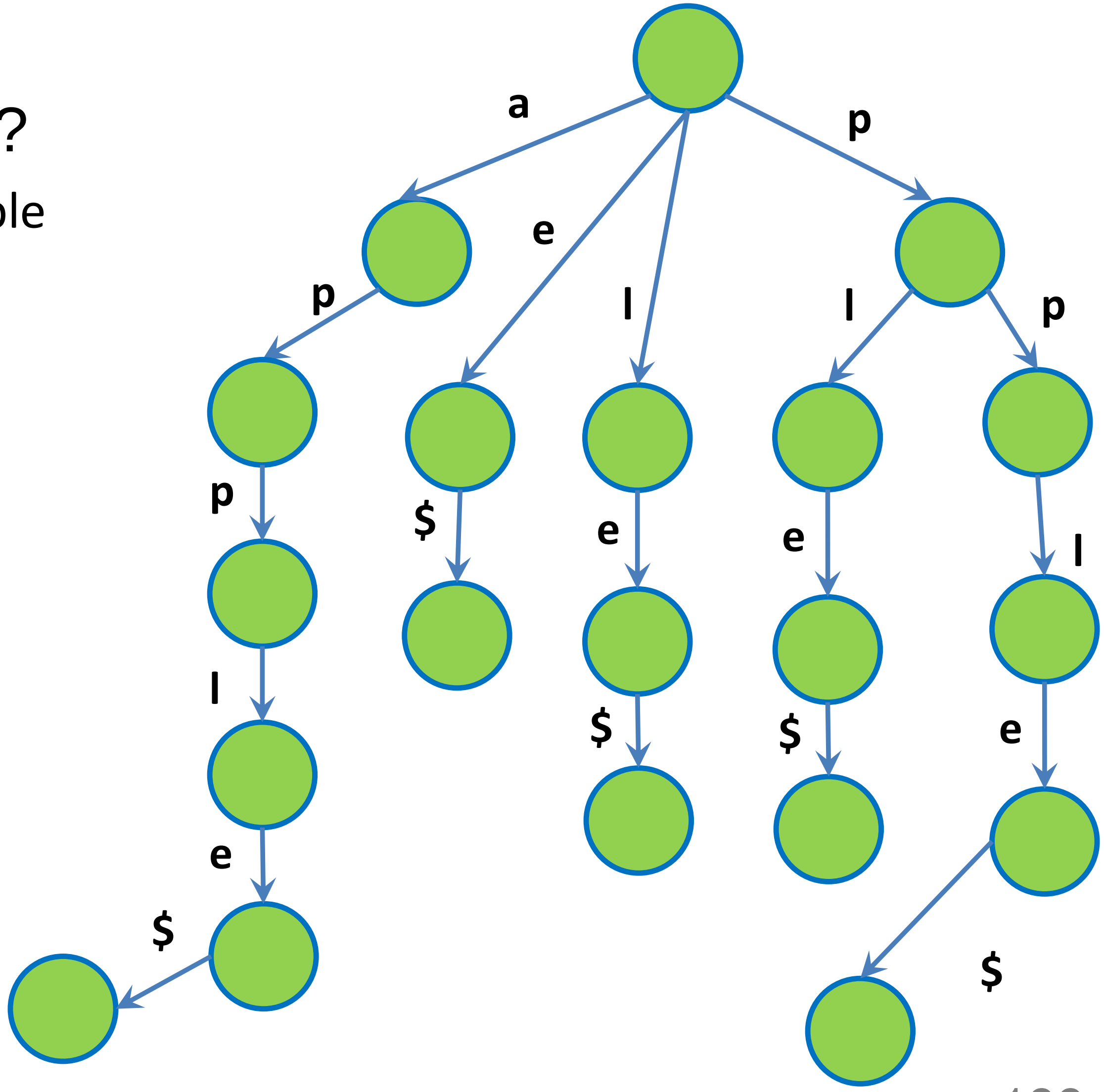

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- What is a suffix tree?
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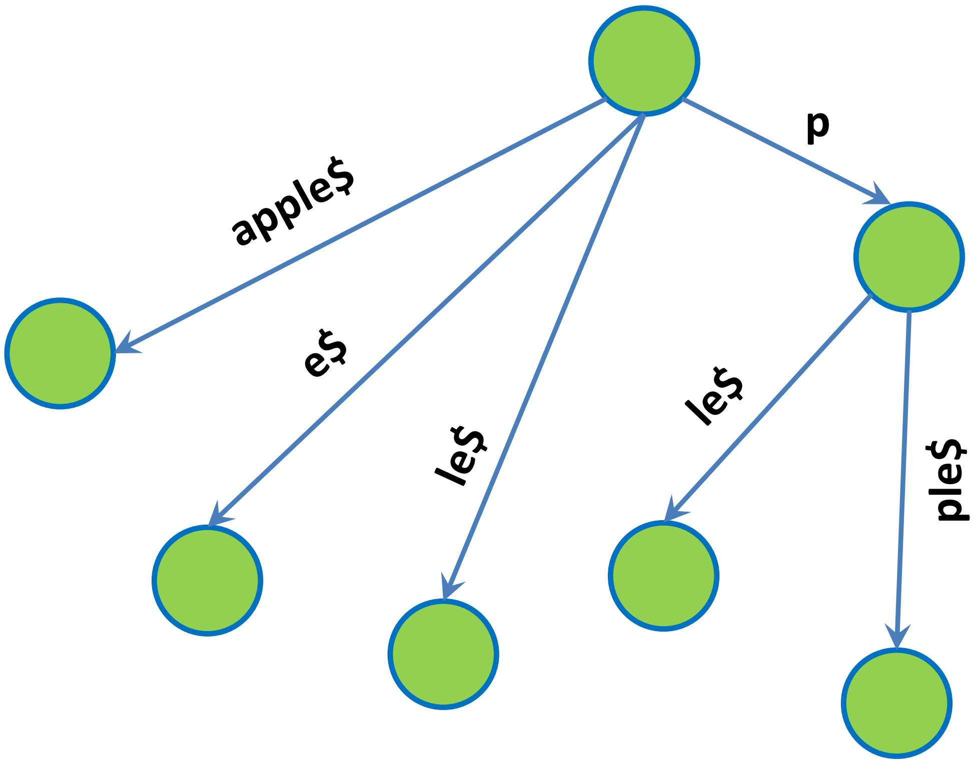

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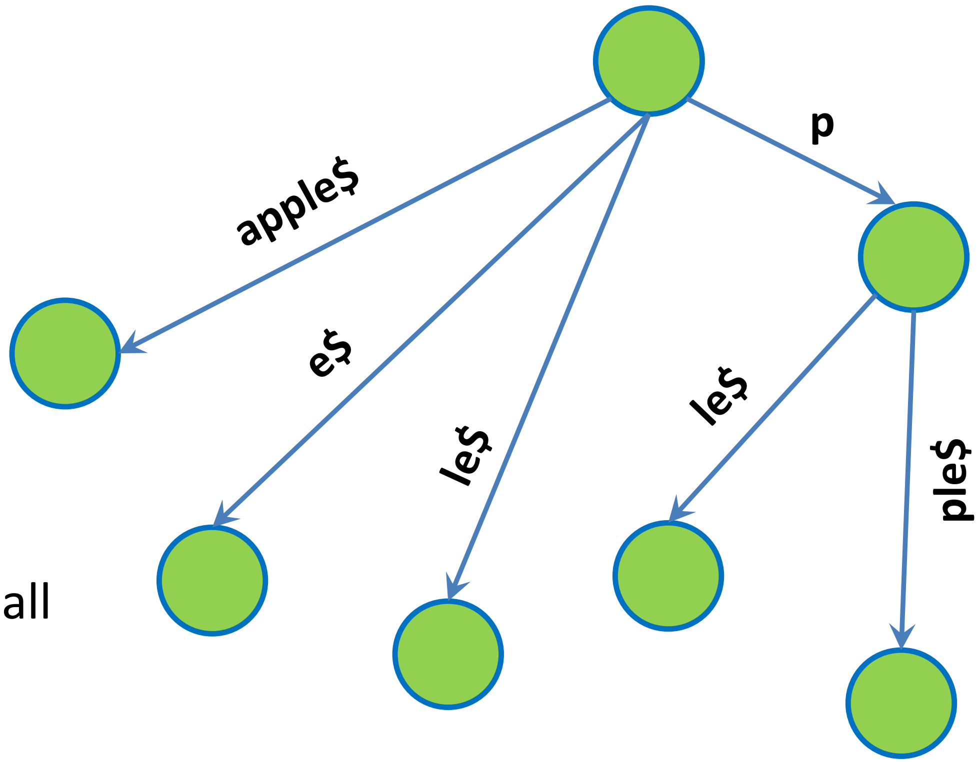

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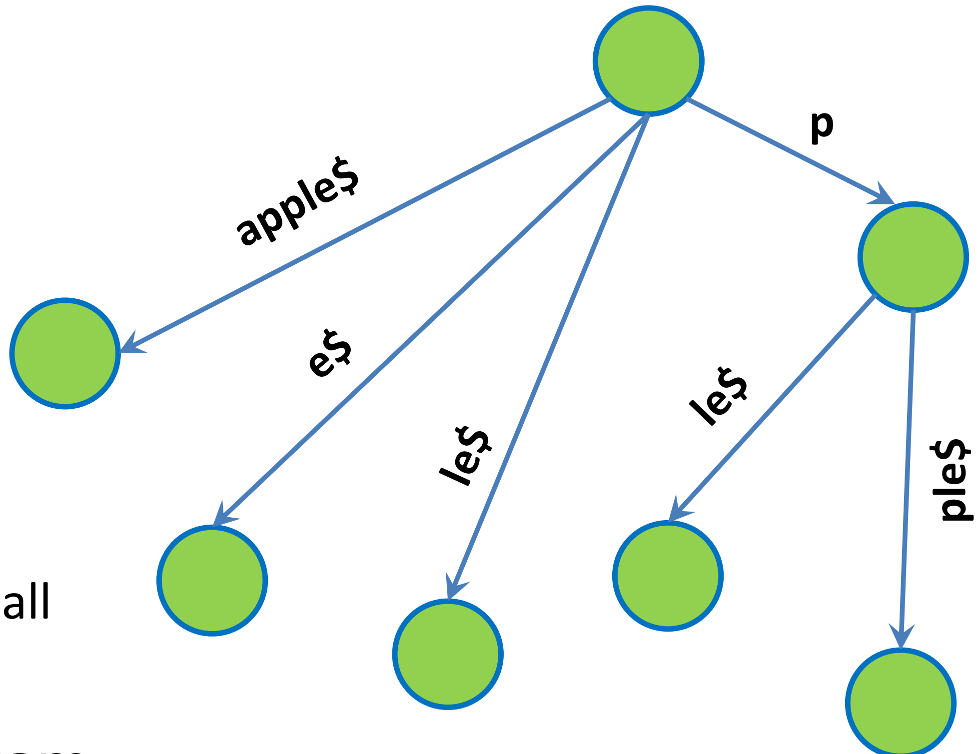

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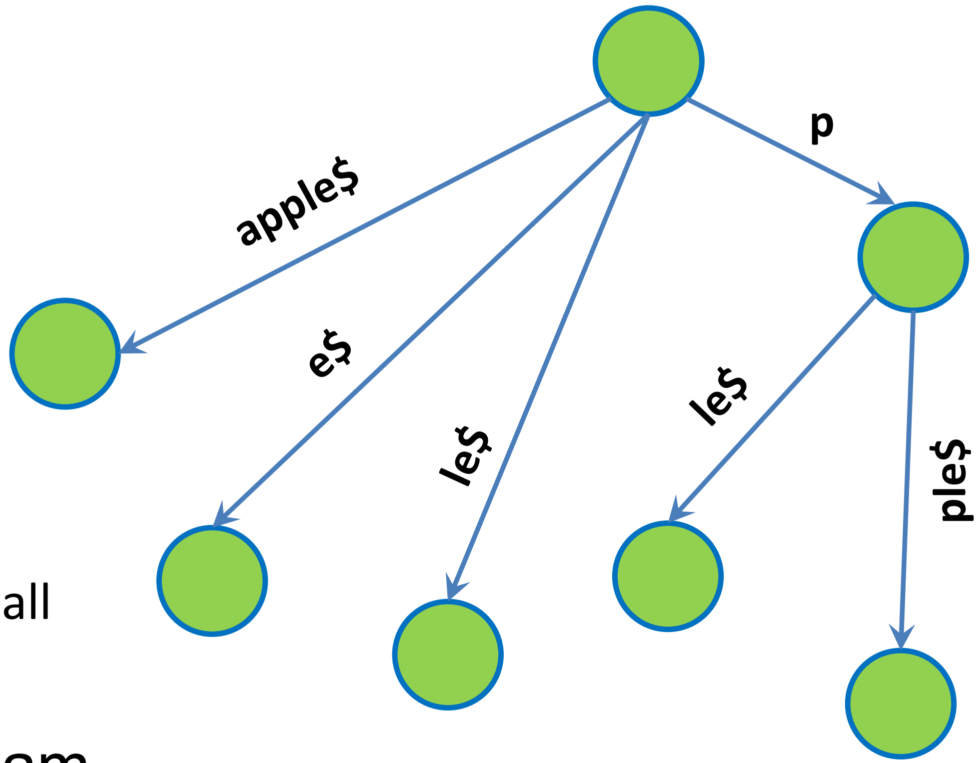
- Note: Some like to separate out the $ node



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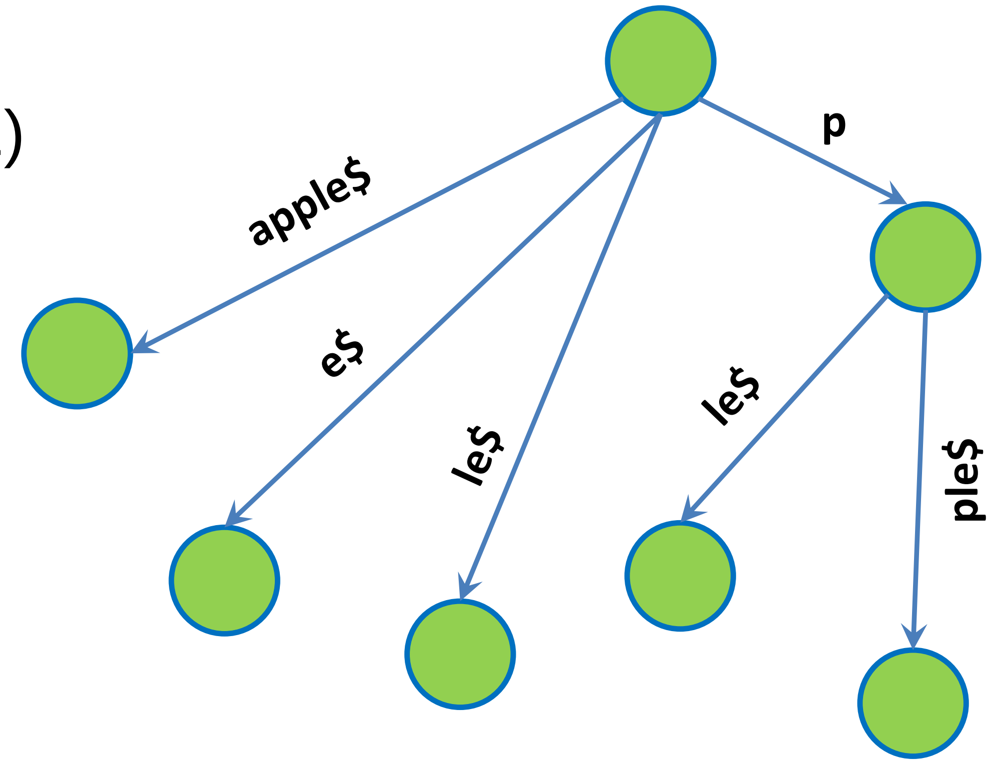

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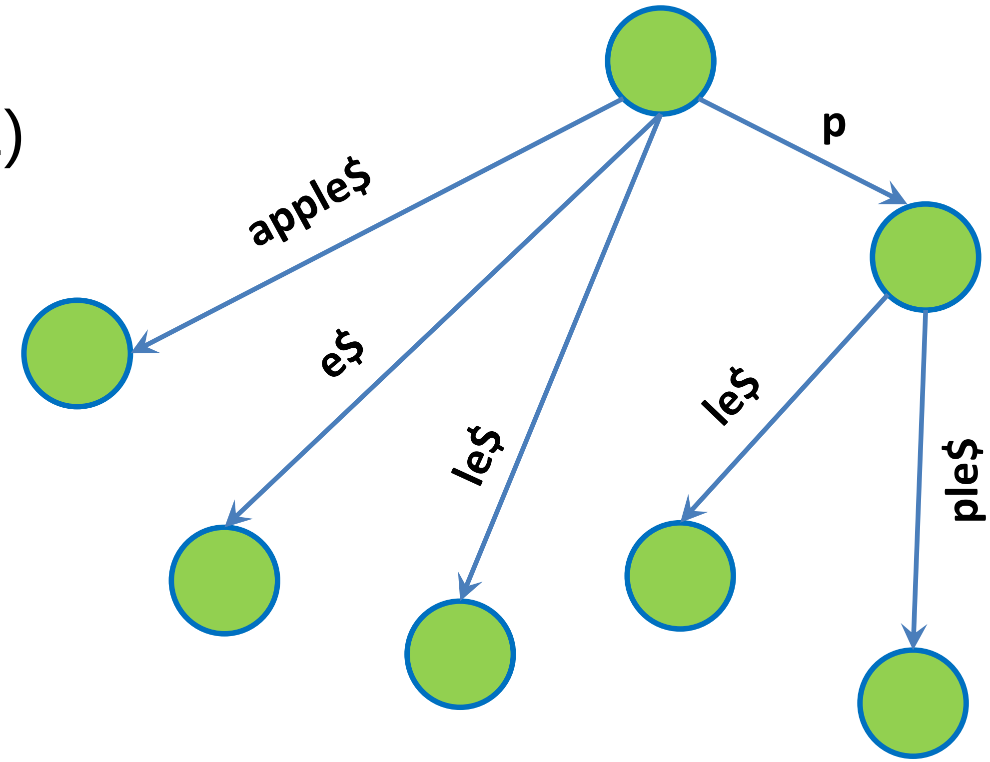

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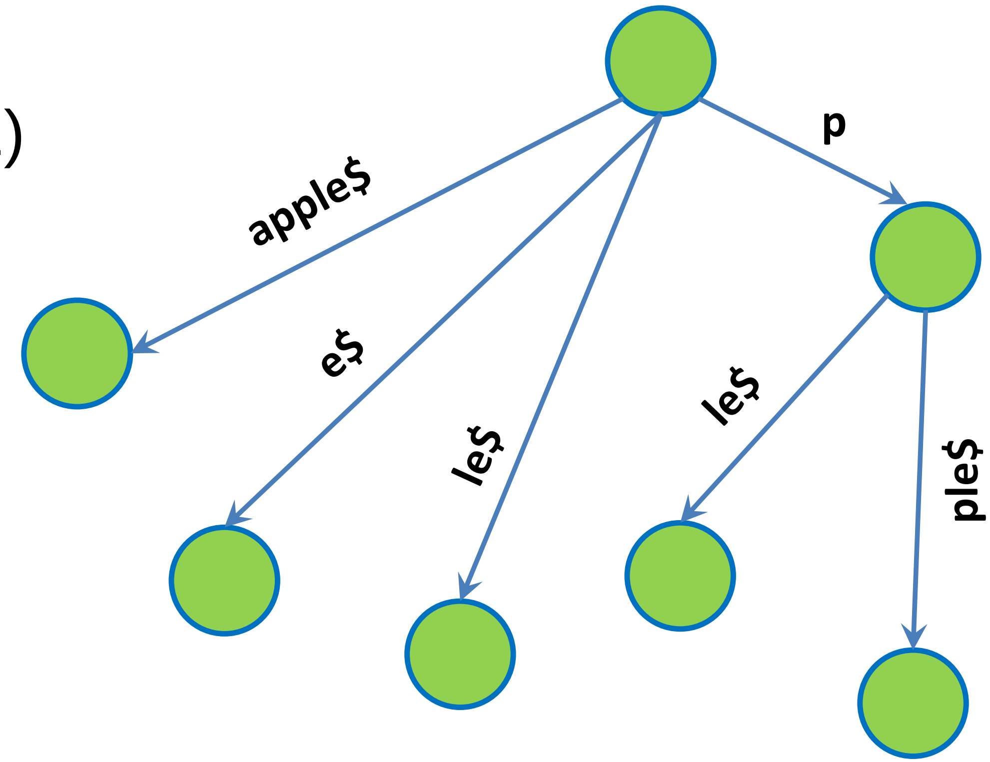

a	p	p	l	e	$
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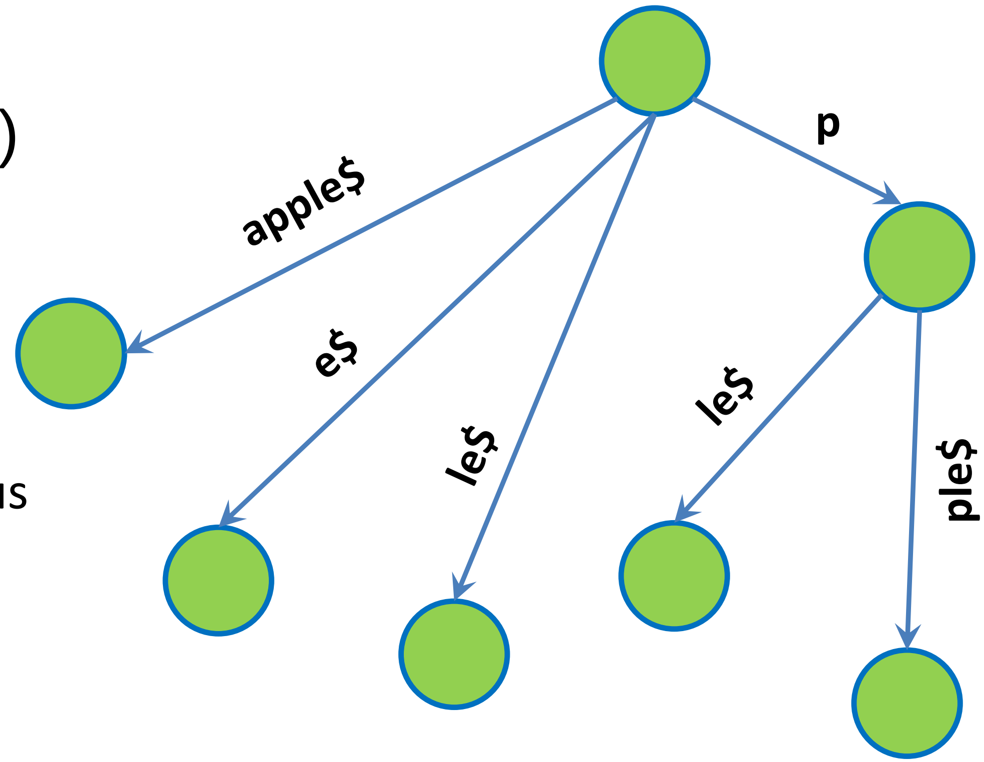

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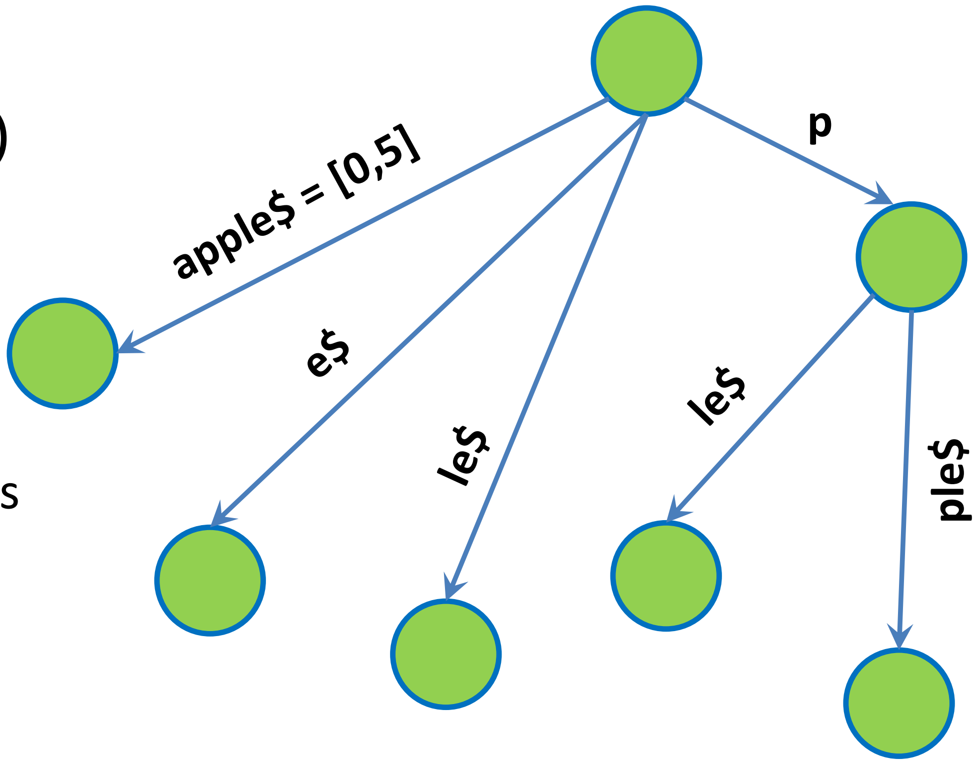

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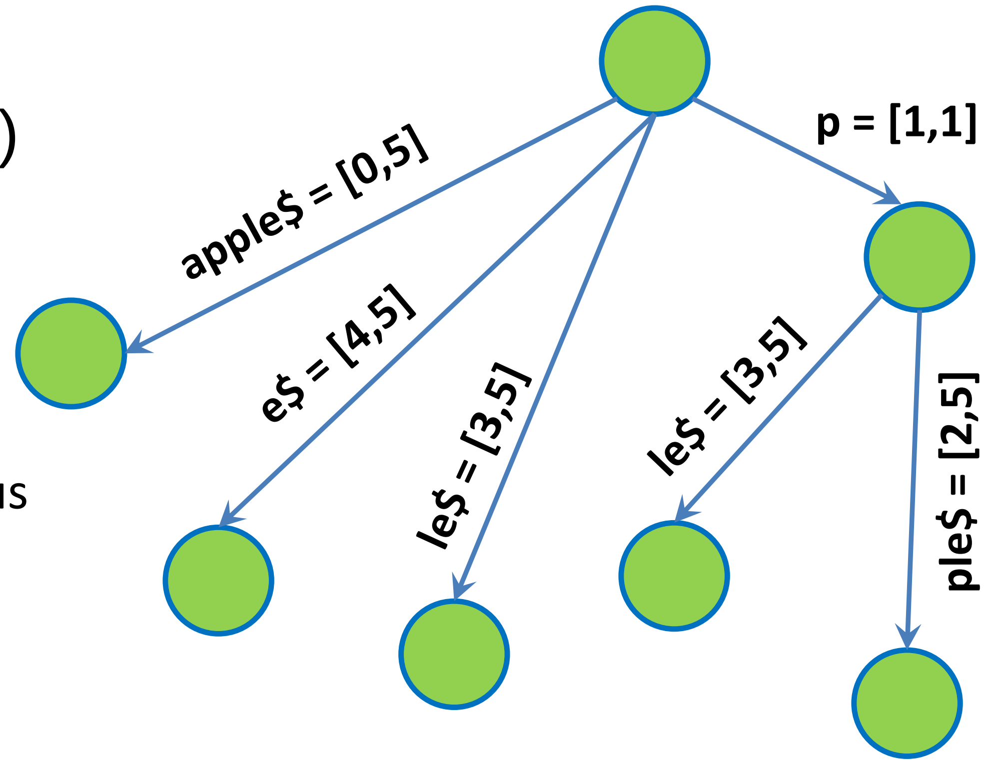

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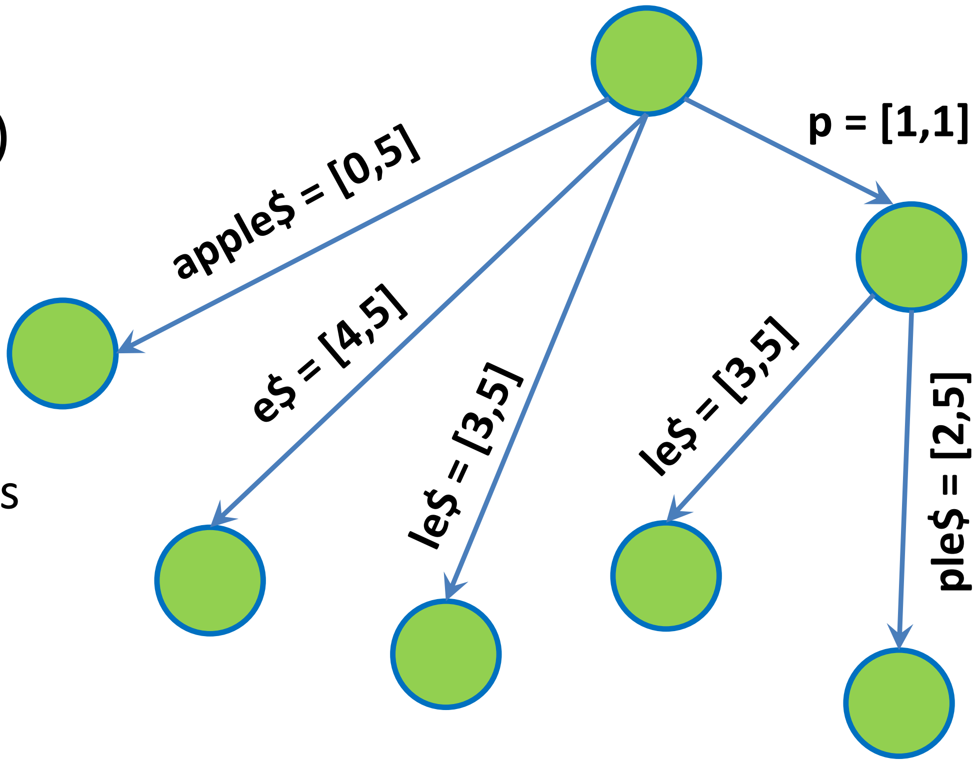

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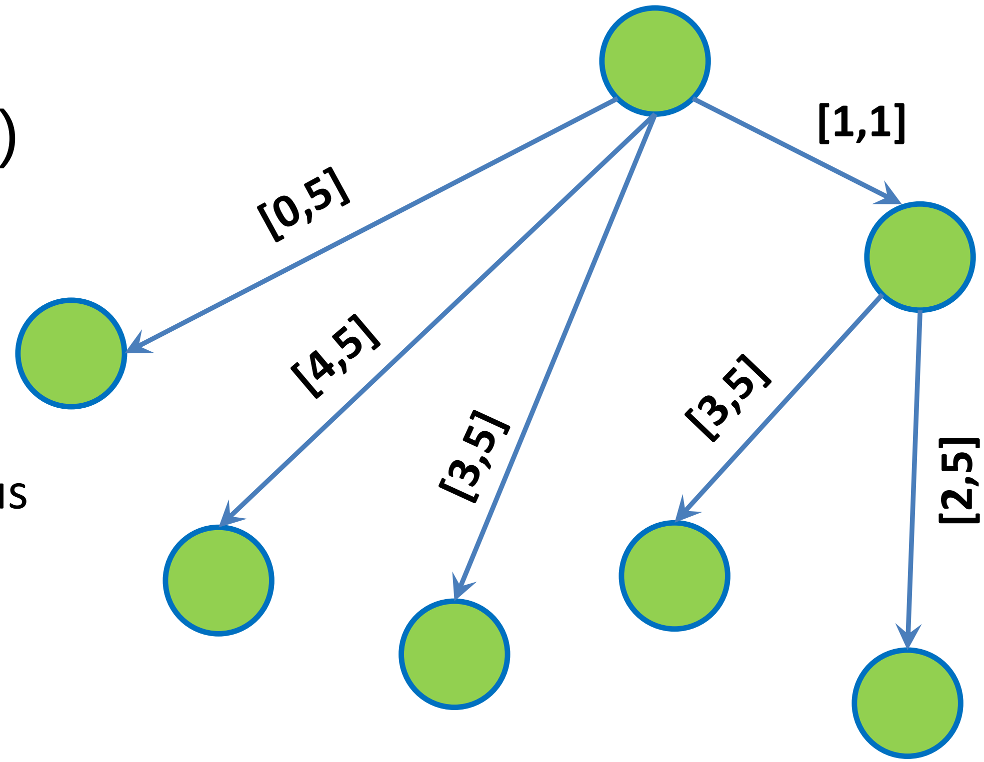

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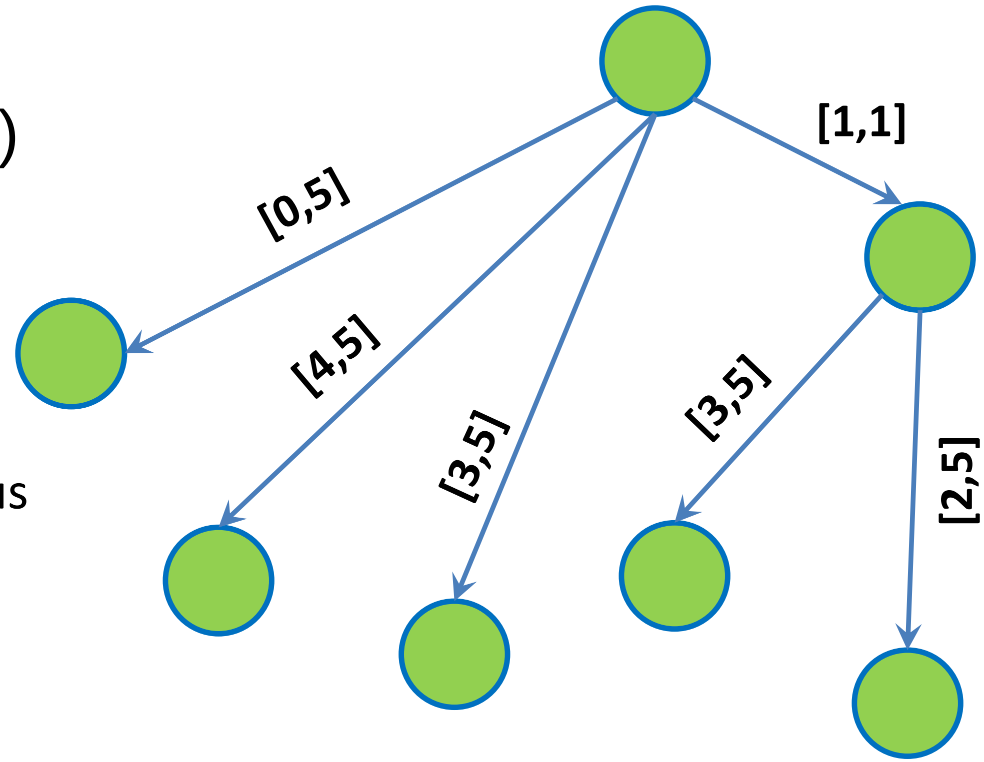

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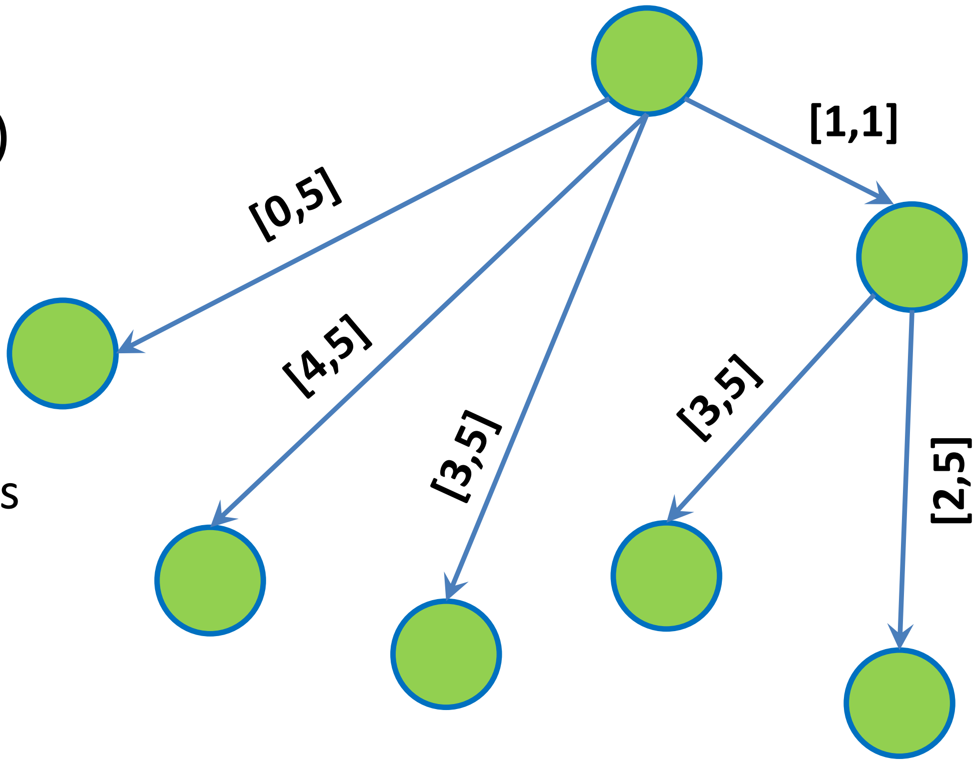

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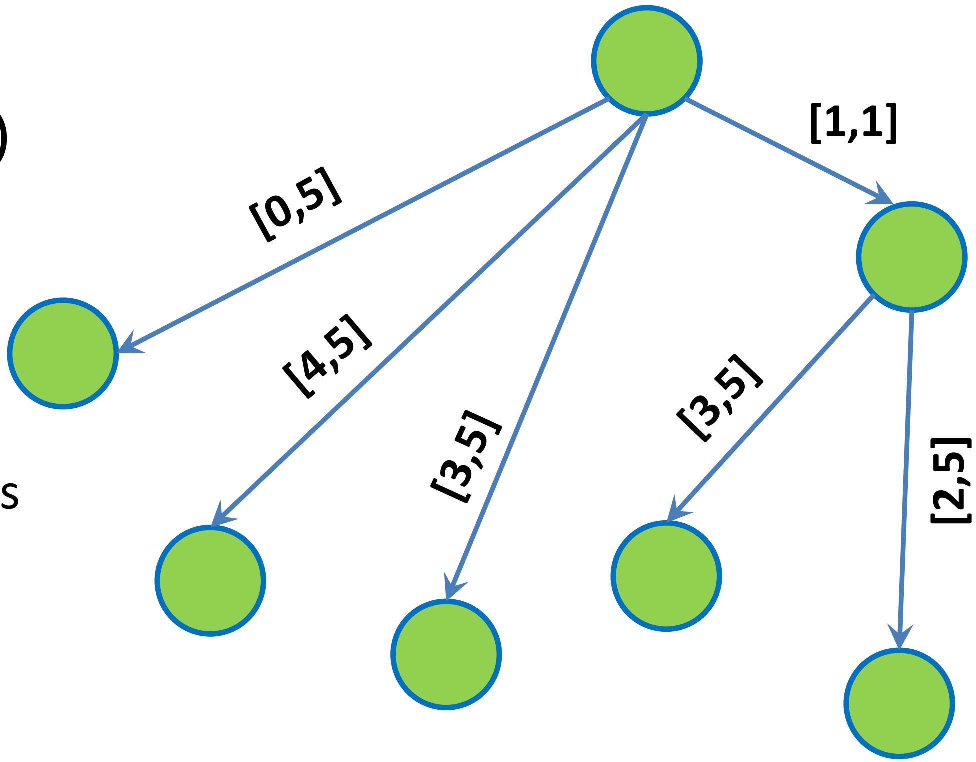

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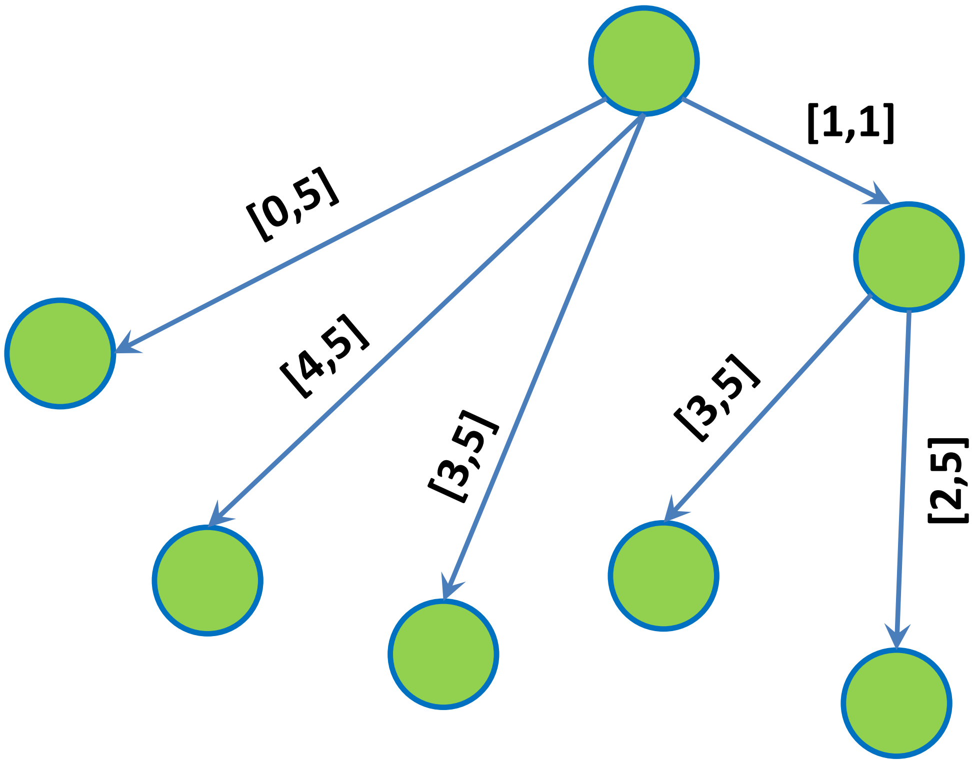

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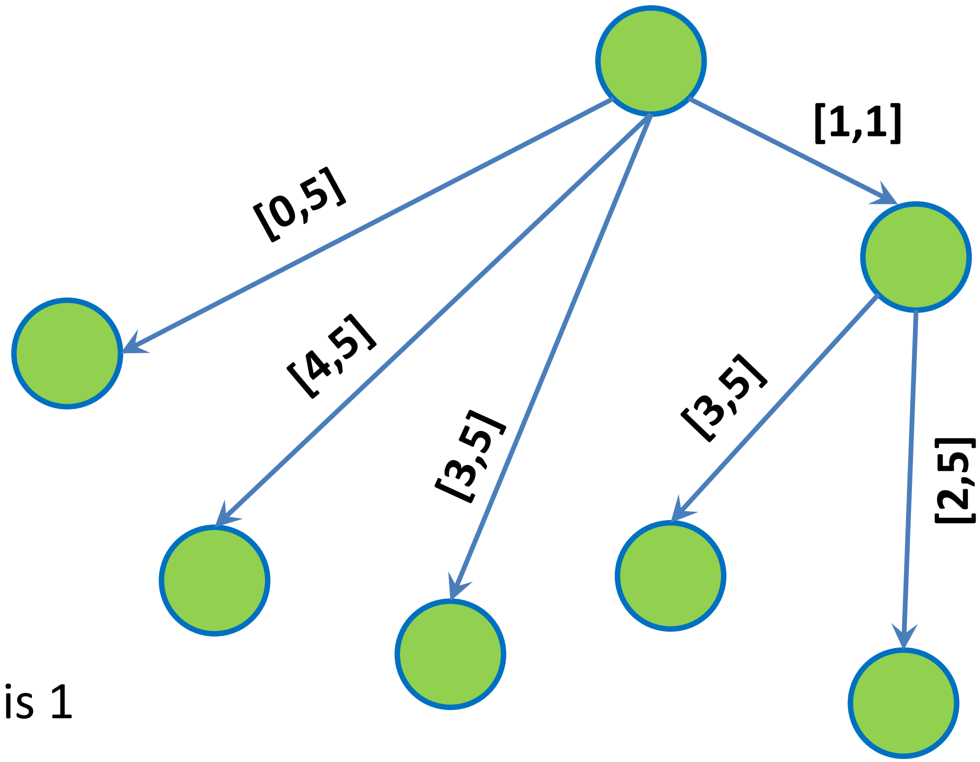

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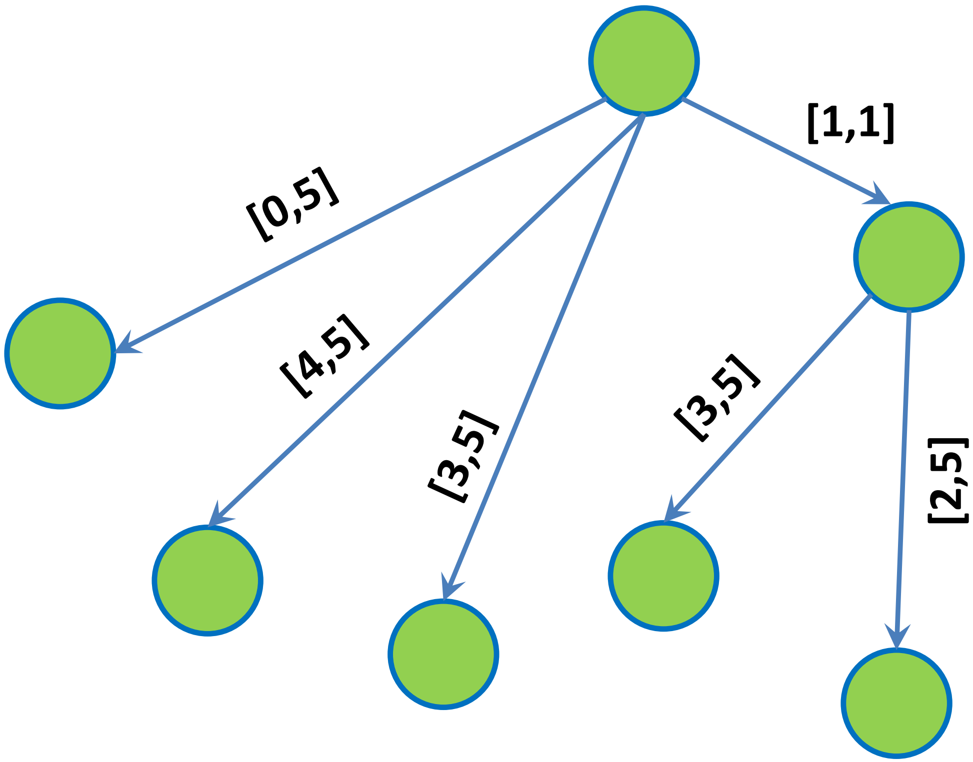

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We learn the hax called
Ukkonen's algorithm (1995)
in FIT3155 to do in O(N)

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Trie

Data Structure via OOP

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- As a class activity
- … and some of the same functions
  - Better than you searching online and not understanding what is happening
  - But 2 implementation
    - Iterative
    - Recursive (efficient)

Questions?

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Thank You