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INFORMATION
TECHNOLOGY

FIT2004
Algorithms and Data Structures

Ian Wern Han Lim
lim.wern.han@monash.edu

Referencing materials by
Nathan Companez, Aamir Cheema, Arun Konagurthu and Lloyd Allison

GROUP OF EIGHT AUSTRALIA

Faculty of Information Technology, Monash University

Ready?

Agenda

- More shortest distance algorithms

Agenda

- More shortest distance algorithms
  - Remember we can get the path through back tracking

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- Bellman-Ford
- Floyd-Warshall

Agenda

- More shortest distance algorithms
  - Remember we can get the path through back tracking

- Bellman-Ford
- Floyd-Warshall
  - Warshall’s algorithm for transitive closure

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Let us begin…

Dijkstra's Algorithm

A recap

- Let us recap Dijkstra

Dijkstra's Algorithm

A recap

- Shortest distance algorithm

Dijkstra’s Algorithm
A recap

- Shortest distance algorithm
  - Similar to BFS
    - Is BFS when the graph is unweighted

Dijkstra's Algorithm
A recap

- Shortest distance algorithm
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    - Is BFS when the graph is unweighted
  - Uses a priority queue
    - Implemented with a min-heap

Dijkstra’s Algorithm
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    - Is BFS when the graph is unweighted
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    - Implemented with a min-heap
  - What is the complexity?

Dijkstra's Algorithm
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  - What is the complexity? O(E log V)

Dijkstra's Algorithm

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- Dijkstra is a…

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  - Dynamic programming algorithm
  - Greedy algorithm

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- Dijkstra is a…
  - Dynamic programming algorithm
  - Greedy algorithm
    - Might not work when there is a negative edge

Questions?

Bellman Ford

Another shortest distance…

- Bellman-Ford isn’t greedy

Bellman Ford

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- 2 main components

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- Bellman-Ford isn’t greedy
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    - Yes, we consider all edges
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- 2 main components
  - Distance calculation
  - Check for negative cycle

Bellman Ford

Another shortest distance…

- Distance calculation

- Check for negative cycle

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- Distance calculation

- Check for negative cycle
  - Check if there's a negative cycle…

- Distance calculation

- Check for negative cycle
  - Check if there's a negative cycle… If we have a negative cycle, there won't be a shortest distance… Why?

Another shortest distance…

- Negative cycle is bad…

Questions?

Bellman Ford

Another shortest distance…

- Distance calculation
  - Here we loop |V| - 1 times

- Check for negative cycle
  - Check if there's a negative cycle… If we have a negative cycle, there won't be a shortest distance… Why?

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    Going from vertex u to vertex v, you can only go through a maximum of |V| - 1 edges without a cycle…

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- Check for negative cycle
  - Check if there's a negative cycle… If we have a negative cycle, there won't be a shortest distance… Why?
  - Then here, we repeat the process ONE MORE TIME. Why?
    If a cycle exist, this additional traversal will form the biggest cycle!

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- Let's look at the algorithm first, then I will explain from there…

Bellman Ford

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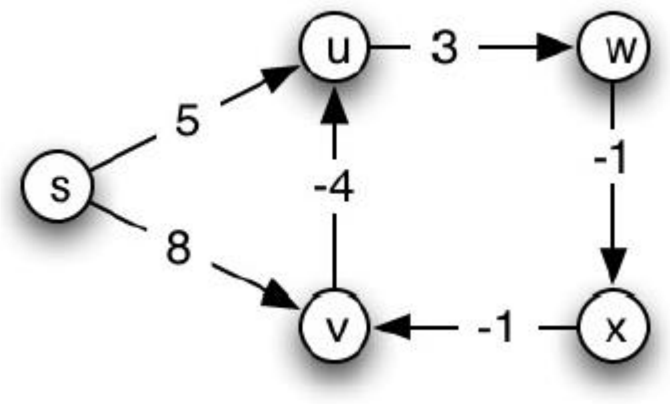
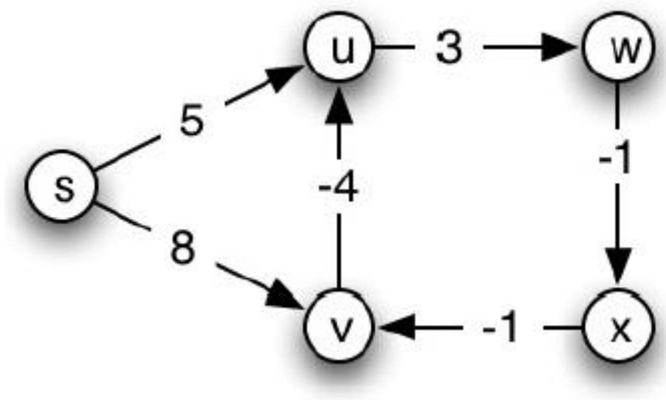
- Let us try with an example first…

Bellman Ford

Another shortest distance…



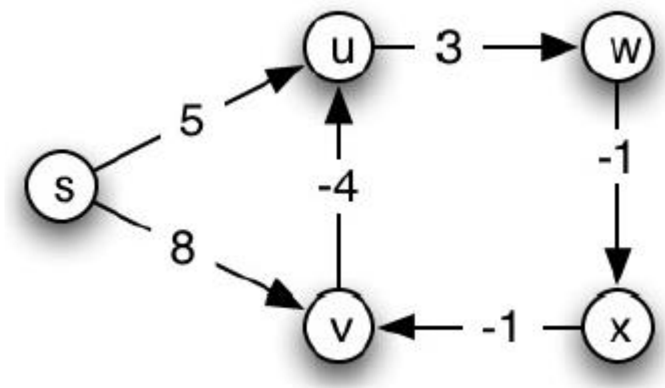
Another shortest distance…



s
u
v
w
x

Bellman Ford

Another shortest distance…



s
u
v
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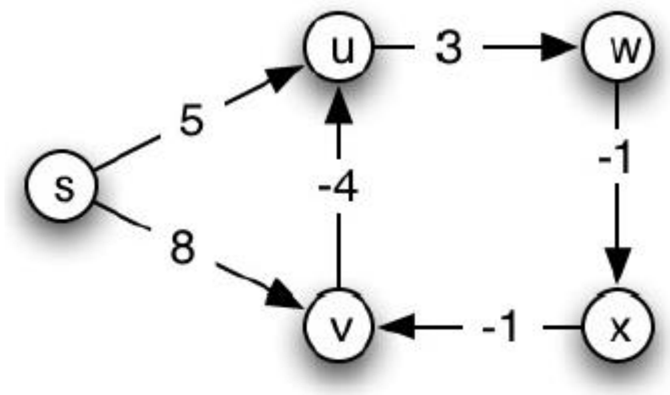
Bellman Ford

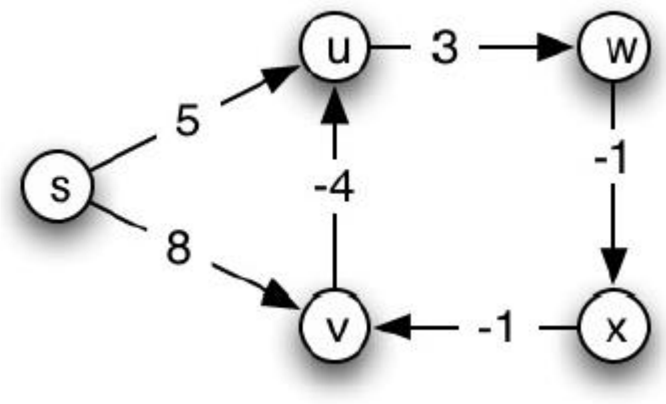
Another shortest distance…



	i=0
s	0
u	inf
v	inf
w	inf
x	inf

Bellman Ford

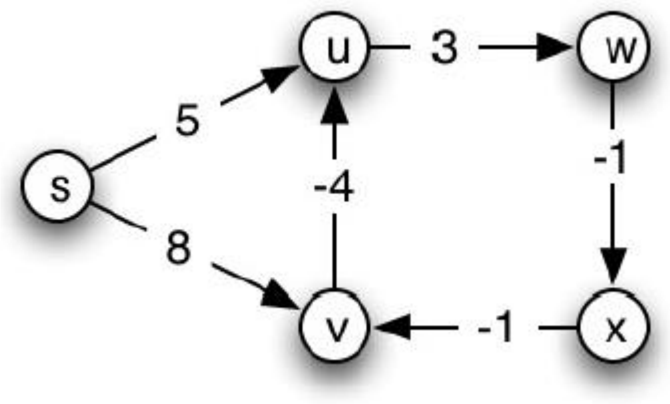
Another shortest distance…



	i=0	i=1
s	0	0
u	inf	5s
v	inf	8s
w	inf	inf
x	inf	inf

Bellman Ford

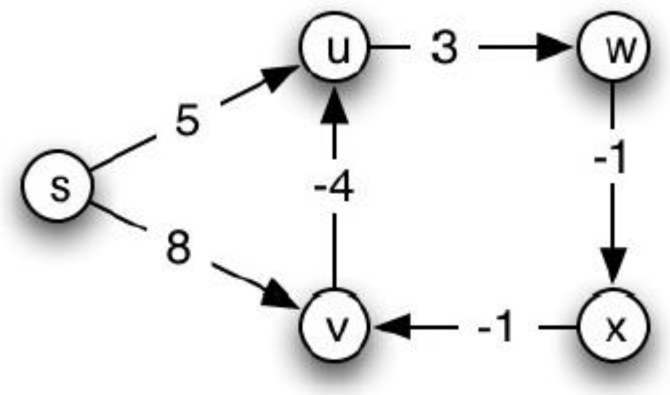
Another shortest distance…



	i=0	i=1	i=2
s	0	0	0
u	inf	5s	4v
v	inf	8s	8s
w	inf	inf	8u
x	inf	inf	inf

Bellman Ford

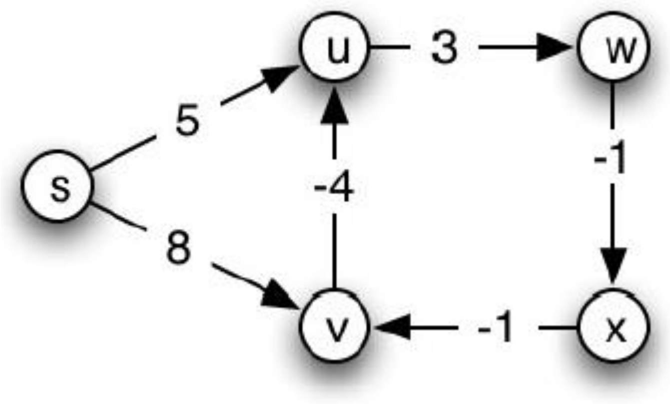
Another shortest distance…



	i=0	i=1	i=2	i=3
s	0	0	0	0
u	inf	5s	4v	4v
v	inf	8s	8s	8s
w	inf	inf	8u	7u
x	inf	inf	inf	7w

Bellman Ford

Another shortest distance…



	i=0	i=1	i=2	i=3	i=4
s	0	0	0	0	0
u	inf	5s	4v	4v	4v
v	inf	8s	8s	8s	6x
w	inf	inf	8u	7u	7u
x	inf	inf	inf	7w	6w

Bellman Ford

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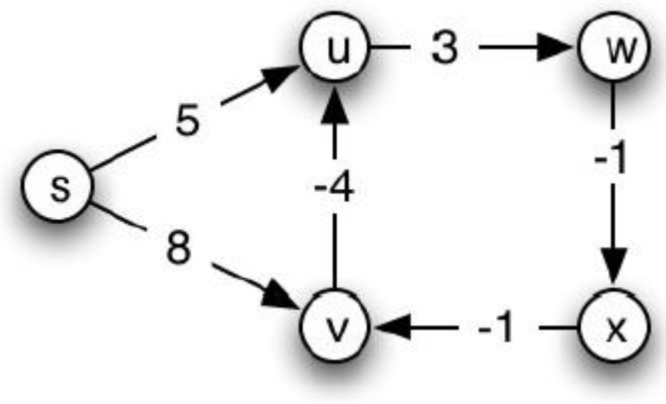

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Bellman Ford

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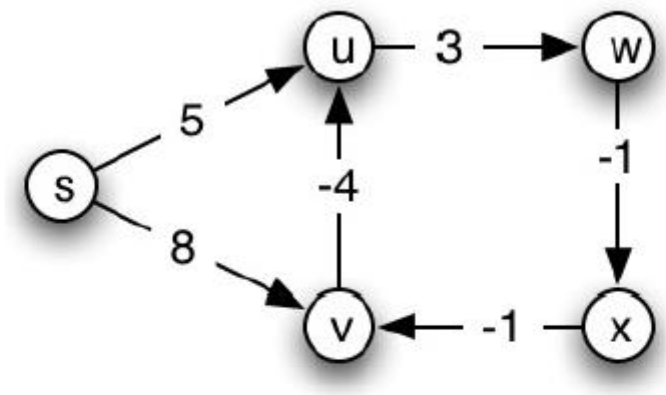

	i=0	i=1	i=2	i=3	i=4	Checking	
s	0	0	0	0	0	0	
u	inf	5s	4v	4v	4v	2v	Break
v	inf	8s	8s	8s	6x	5x	Break too
w	inf	inf	8u	7u	7u	7u	
x	inf	inf	inf	7w	6w	6w	

Bellman Ford

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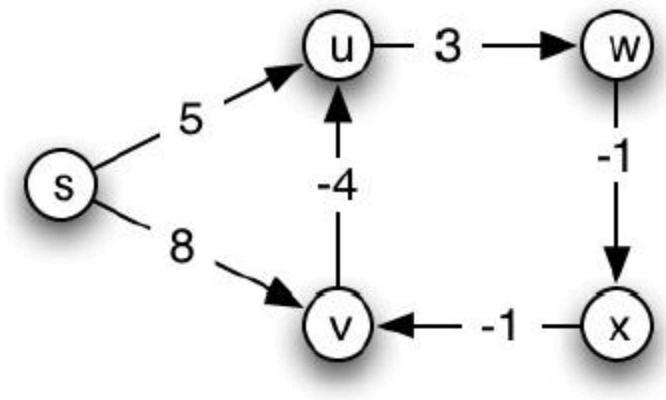

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Bellman Ford

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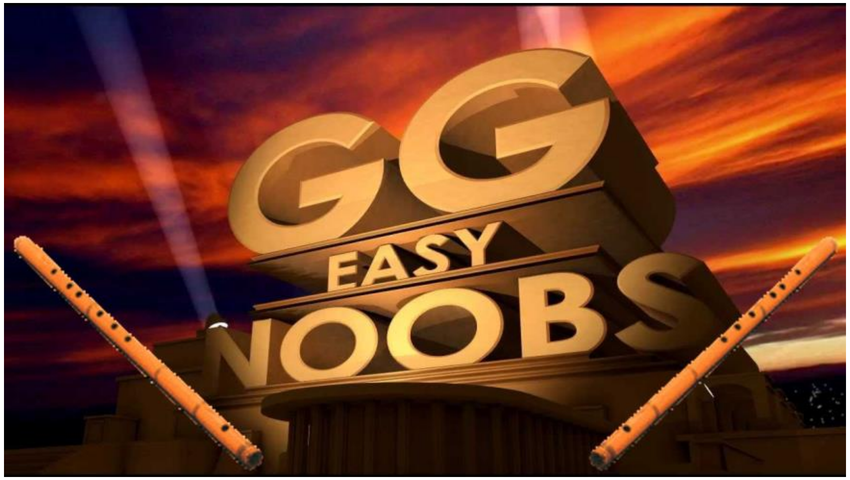

Questions?

Transitive Closure

Reachability

- Given a graph G=(V,E)

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    - A -> B, B -> C therefore A -> C

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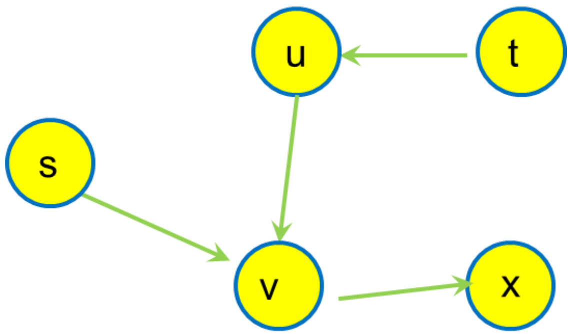

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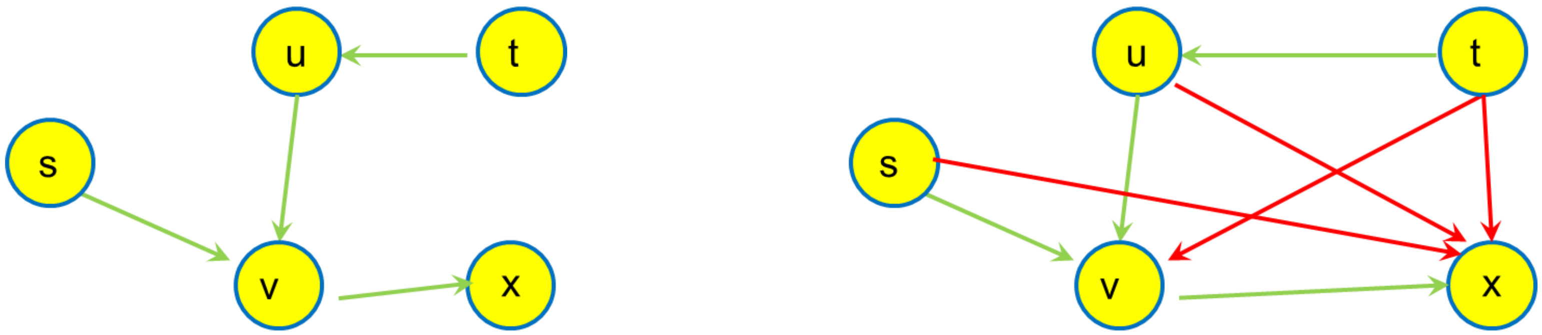

Questions?

Transitive Closure
Reachability

- Given a graph G=(V,E)
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Transitive Closure

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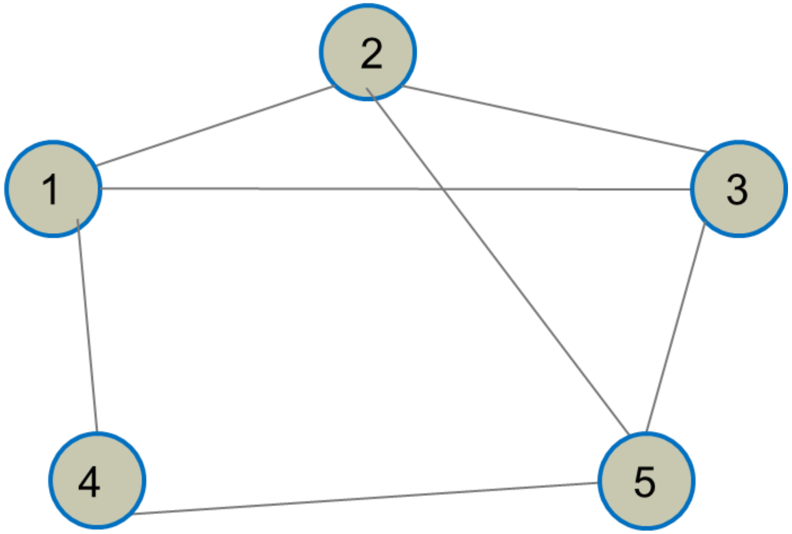

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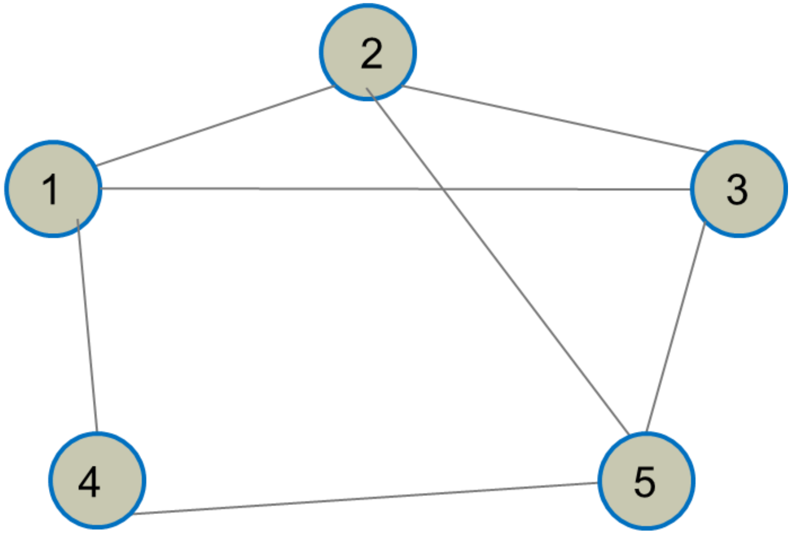

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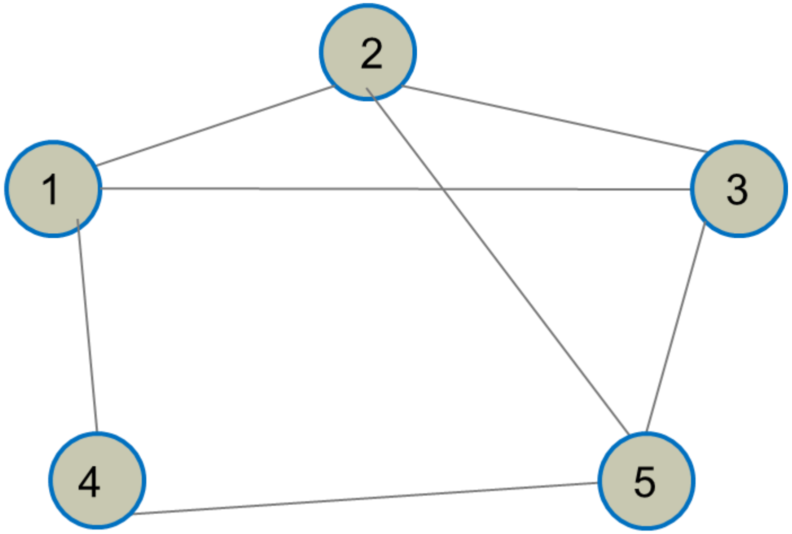

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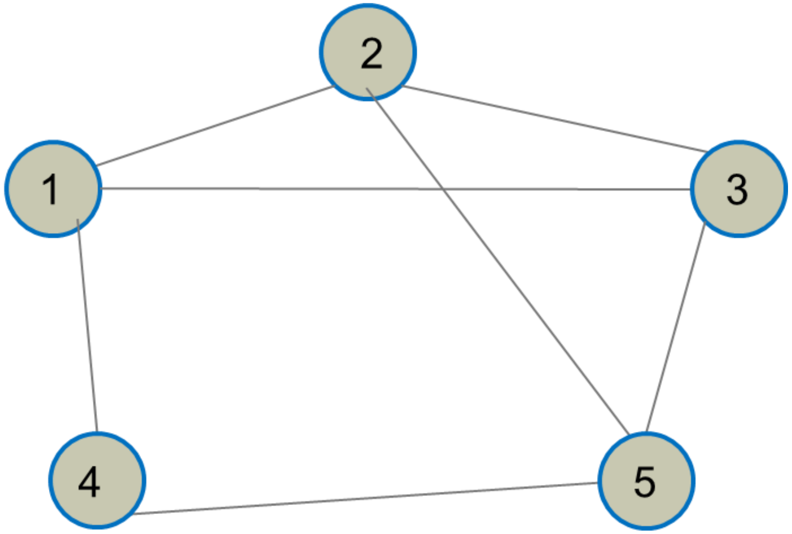

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- What is the complexity?
  - Time = O(V^3)
  - Space = O(V^2) for the matrix

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Questions?

Floyd-Warshall

Shortest distance

- So we know the Warshall's algorithm

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All-Pair shortest distance

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Floyd-Warshall
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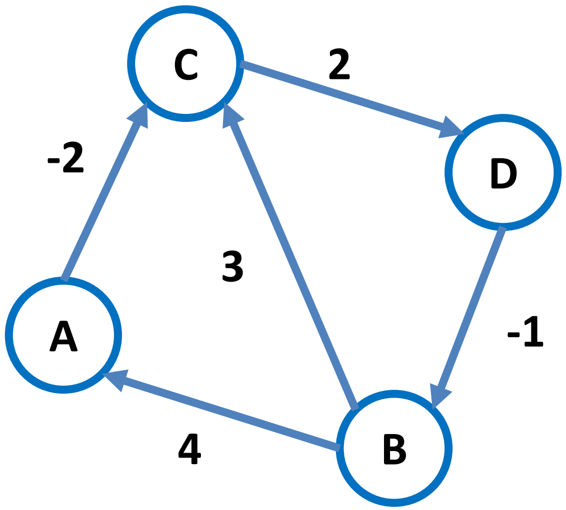
    O(V) * O(VE) = O(V^2 E) = O(V^4)

- Floyd-Warshall can do it quicker!

Questions?

- Now let us do it manually…



Floyd-Warshall

All-Pair shortest distance

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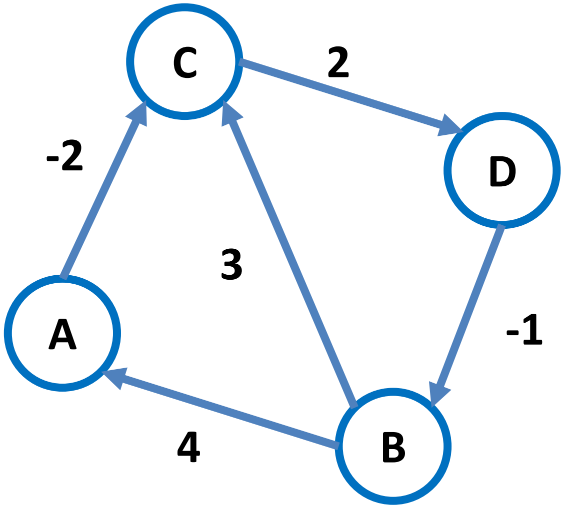
	A	B	C	D
A				
B				
C				
D				



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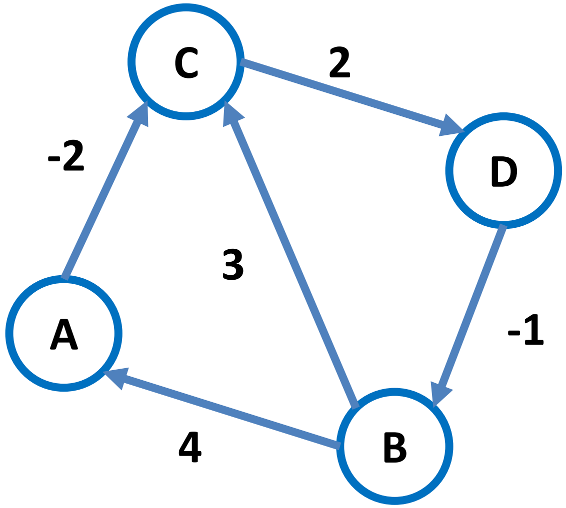
A	0			
B		0		
C			0	
D				0



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	A	B	C	D
A	0		-2	
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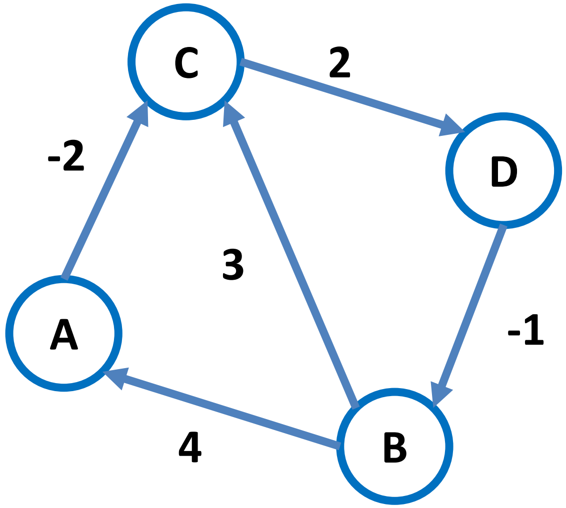
C			0	2
D		-1		0



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- Now let us do it manually…
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  - Infinity for all of the non-edges

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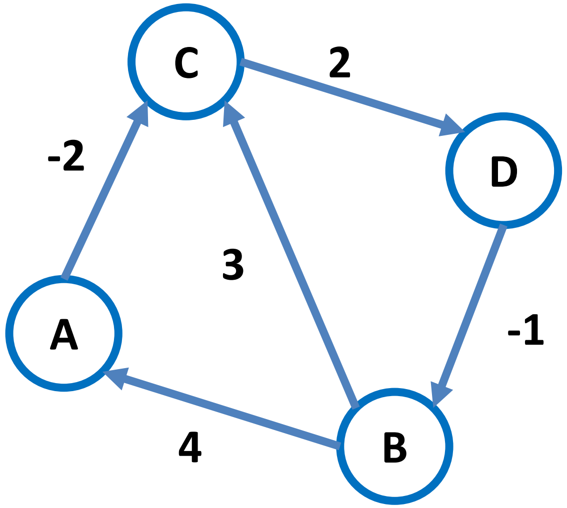
D	inf	-1	inf	0



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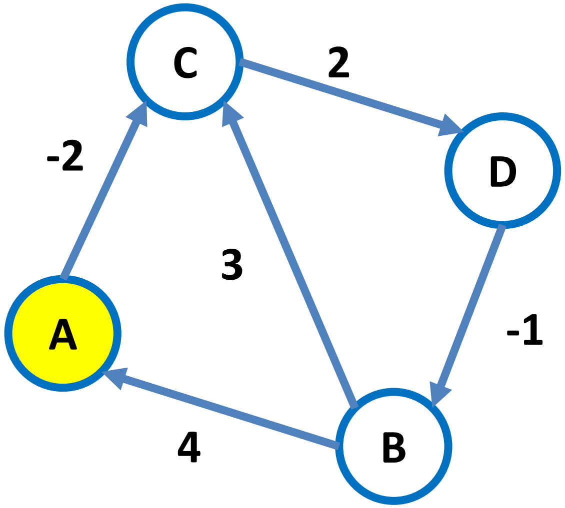

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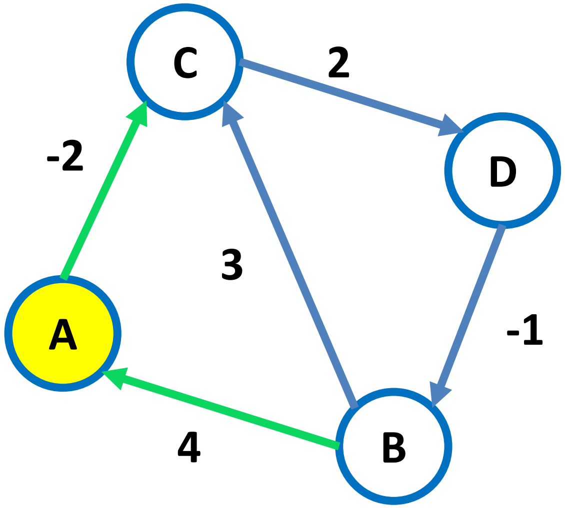

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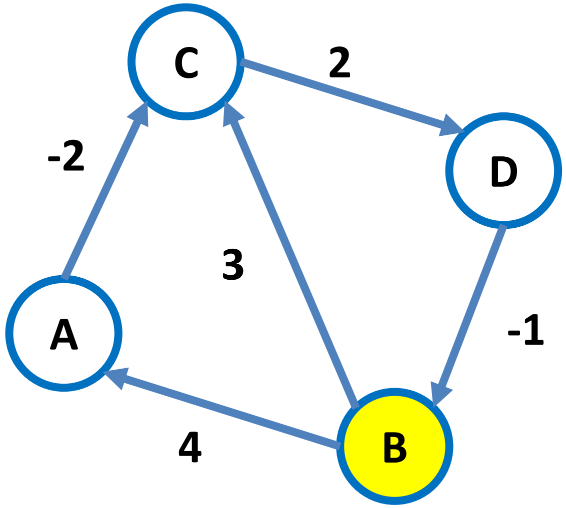

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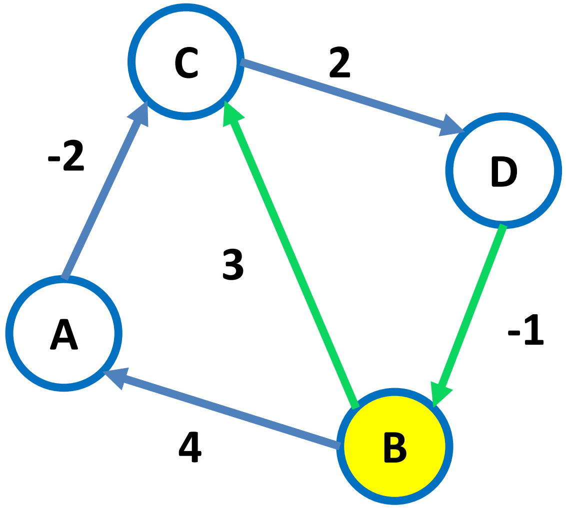

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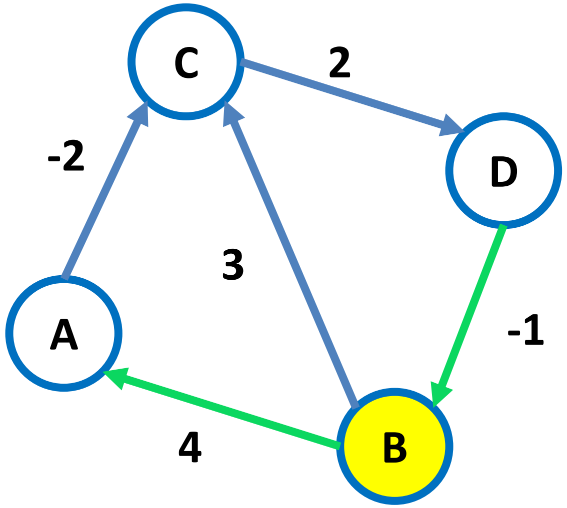

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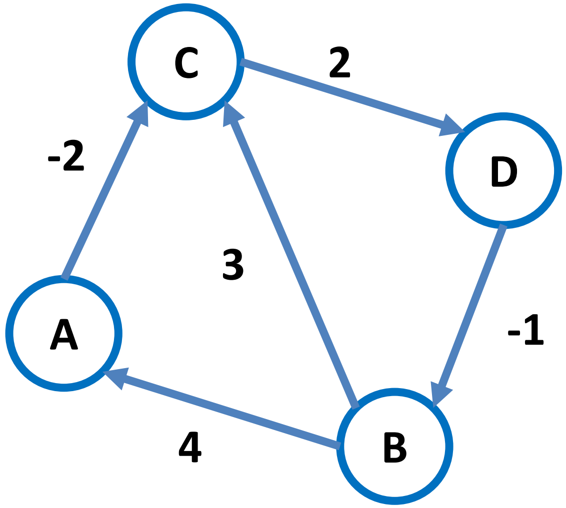

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  - What is the meaning of the outer loop?
    - As we increment, we find the minimum distance going through vertex k. Thus, we would have the minimum through every vertex, updating as needed!

Questions?

Floyd-Warshall

All-Pair shortest distance

- What is the complexity?

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  - We know about cycle by looking at the diagonal going from vertex u back to vertex u
  - So if we have negative cycle, the diagonal will tell us! If the value is negative =(

Questions?

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- Weighted graph?

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Thank You