MONASH University

MONASH
INFORMATION
TECHNOLOGY

FIT2004
Algorithms and Data Structures

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Referencing materials by
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GROUP OF EIGHT AUSTRALIA

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Ready?

Agenda

- Quick-select

Agenda

- Quick-select
  - K-th order statistics

Agenda

- Quick-select
  - K-th order statistics
  - Using it to find the median

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- Quick-select
  - K-th order statistics
  - Using it to find the median
    - For Quick sort pivot?
    - Median of median?

Let us begin…

K-the Order Statistics

What is it?

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What is it?

- Given an unsorted array
- Find the k-th smallest elements in the array

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- Example: 21,84,16,14,79,51,66,21,54,32
- If k=1, we want the smallest item
  – 14

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- Given an unsorted array
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- Example: 21,84,16,14,79,51,66,21,54,32
- If k=1, we want the smallest item
  - 14
- If k=2, want the 2 smallest items
  - 16,14 (note: order is not important)

K-the Order Statistics

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- Given an unsorted array
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- Example: 21,84,16,14,79,51,66,21,54,32
- If k=1, we want the smallest item
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- If k=2, want the 2 smallest items
  - 16,14 (note: order is not important)
- If k=5, we want the 5 smallest items
  - 21,16,14,21,32

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- Example: 21,84,16,14,79,51,66,21,54,32
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- If k=2, want the 2 smallest items
  - 16,14 (note: order is not important)
- If k=5, we want the 5 smallest items
  - 21,16,14,21,32
  - Isn't this partition?

K-the Order Statistics

What is it?

- Given an unsorted array
- Find the k-th smallest elements in the array
  - First quartile (Q1); k=N/4
  - Median; k=N/2
  - Third quartile (Q3); k= 3N/4

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  - Median; k=N/2
  - Third quartile (Q3) = k= 3N/4

- But how can we get it?

Questions?

K-the Order Statistics

Getting it

- Sort the list

K-the Order Statistics

Getting it

- Sort the list
- Then slice the list for the k-th we want

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Getting it

- Sort the list
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- Complexity is high!

K-the Order Statistics

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- Sort the list
  - Sorting gives us O(NM log N)
  - Where N is number of item in list
  - Where M is the comparison cost
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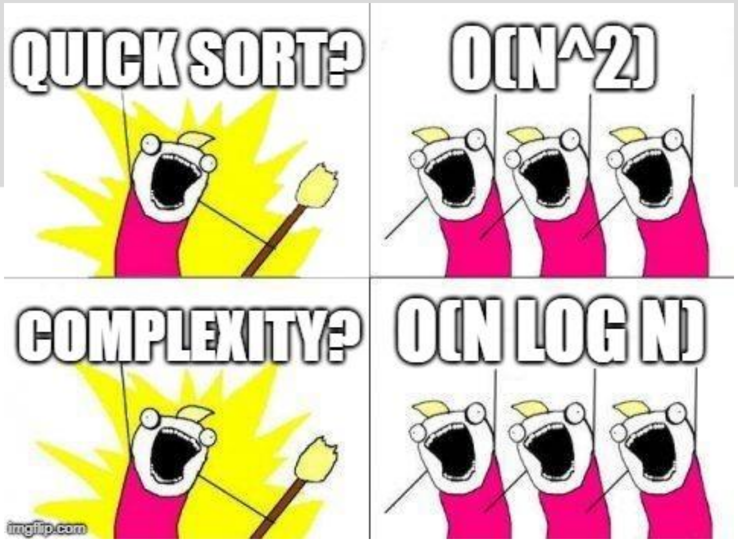

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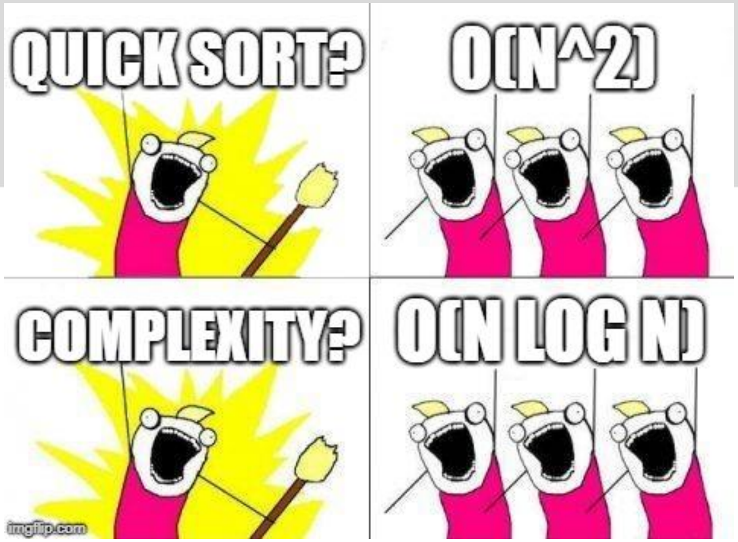

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20	80	90	10	30	50	70	60

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Pivot X

In Sorted position X

Others X

In sorted position (at index 4, i.e., 4th smallest)

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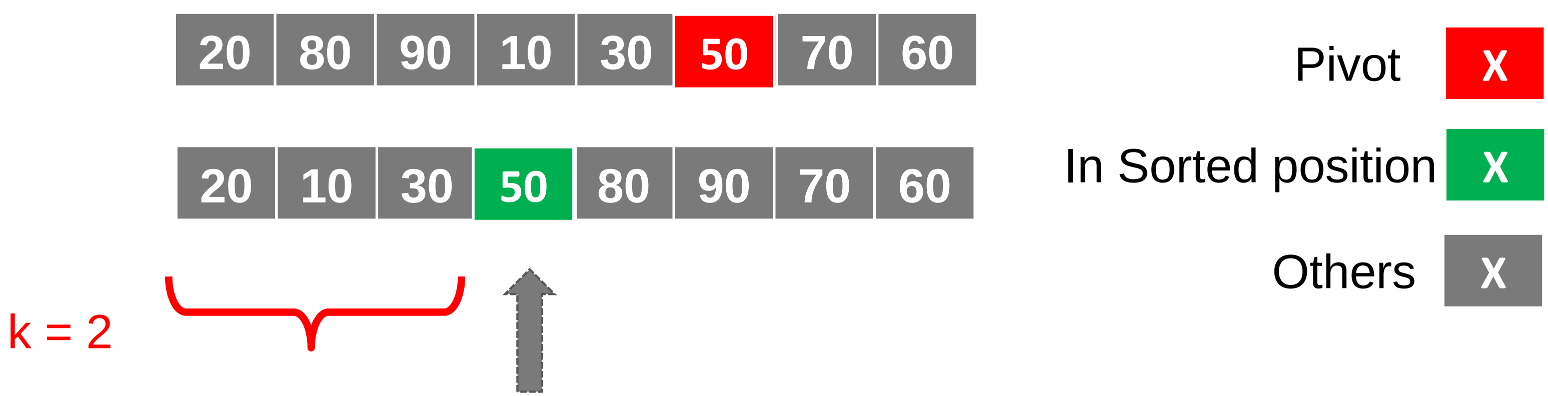

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k = 6

In sorted position (at index 4, i.e., 4th smallest)

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Pivot	X
In Sorted position	X
Others	X

In sorted position (at index 4, i.e., 4th smallest), return everything on the left of the index…

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- Allow us to find what we want without sorting!

Questions?

- So we will now
  - Use quick select to find the median as a pivot
  - Use quick sort to sort

Quick Sort
With quick select

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  - Use quick select to find the median as a pivot
    - Note: Since quick-select do perform the partition as well, we can avoid doing partition in the quick sort phase itself!
  - Use quick sort to sort

Quick Sort
With quick select

- So we will now
  - Use quick select to find the median as a pivot
    - Worst case complexity of O(N^2) when pivot != k till the last final iteration…
    - Note: Since quick-select do perform the partition as well, we can avoid doing partition in the quick sort phase itself!
  - Use quick sort to sort

Quick Sort

With quick select

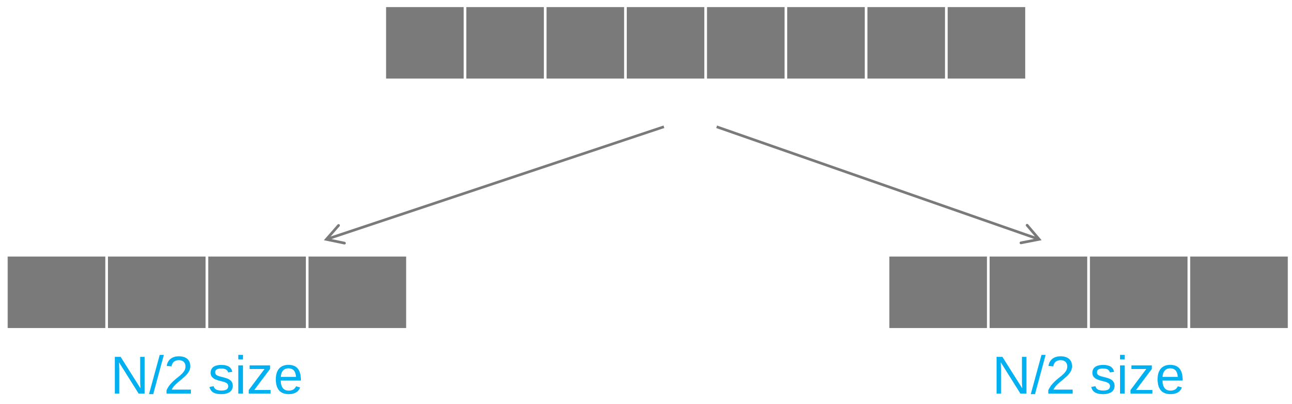
Quick Sort
With quick select

N^2

Quick Sort

With quick select

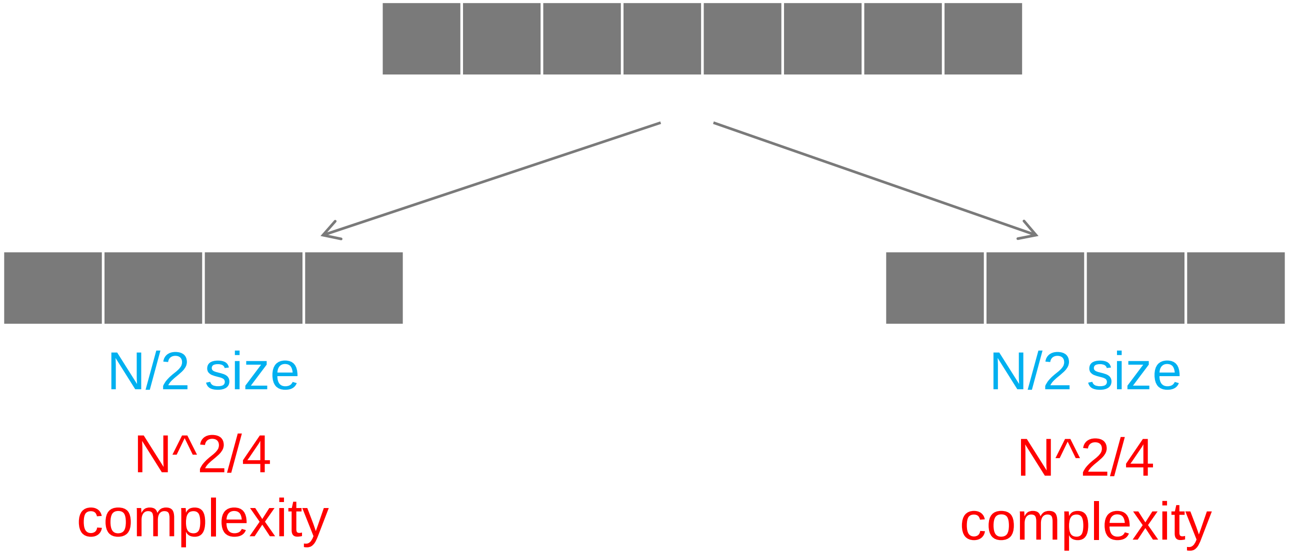
N^2



Quick Sort

With quick select

N^2



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With quick select

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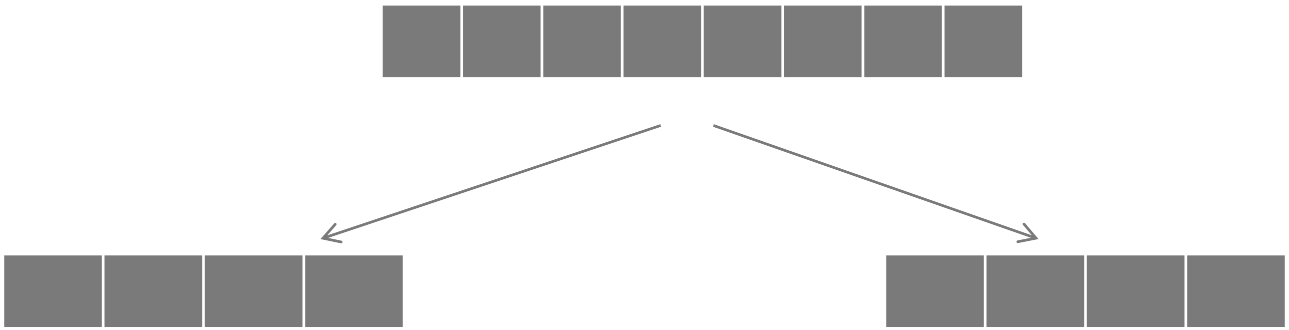


N/2 size

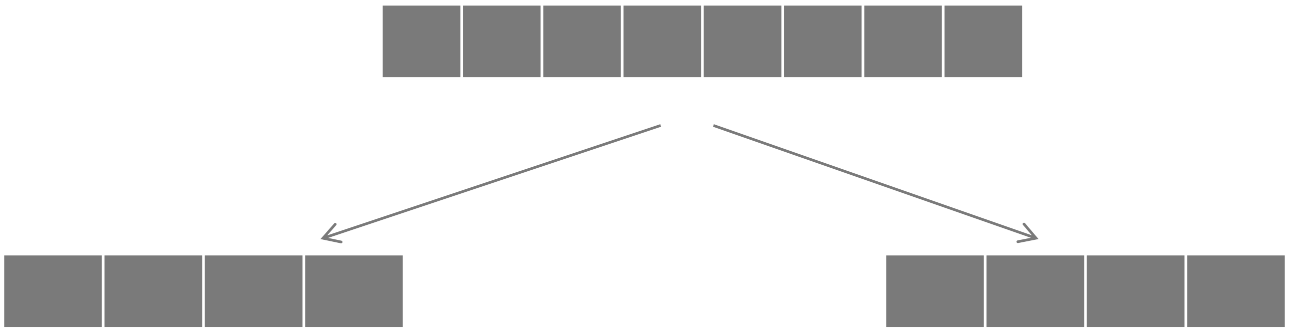
N^2/4
complexity

+

N/2 size

N^2/4
complexity

Quick Sort
With quick select



N^2

$N^2/2$

N/2 size

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N/2 size

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Quick Sort

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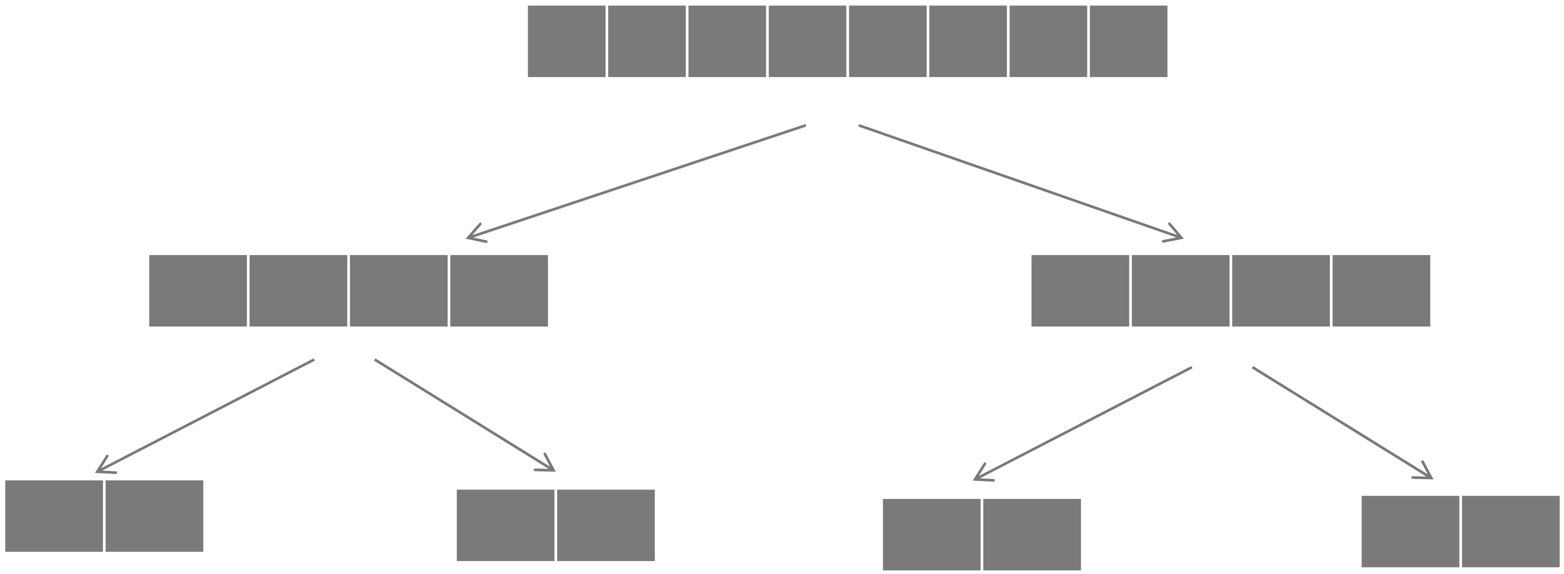

N^2

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Quick Sort

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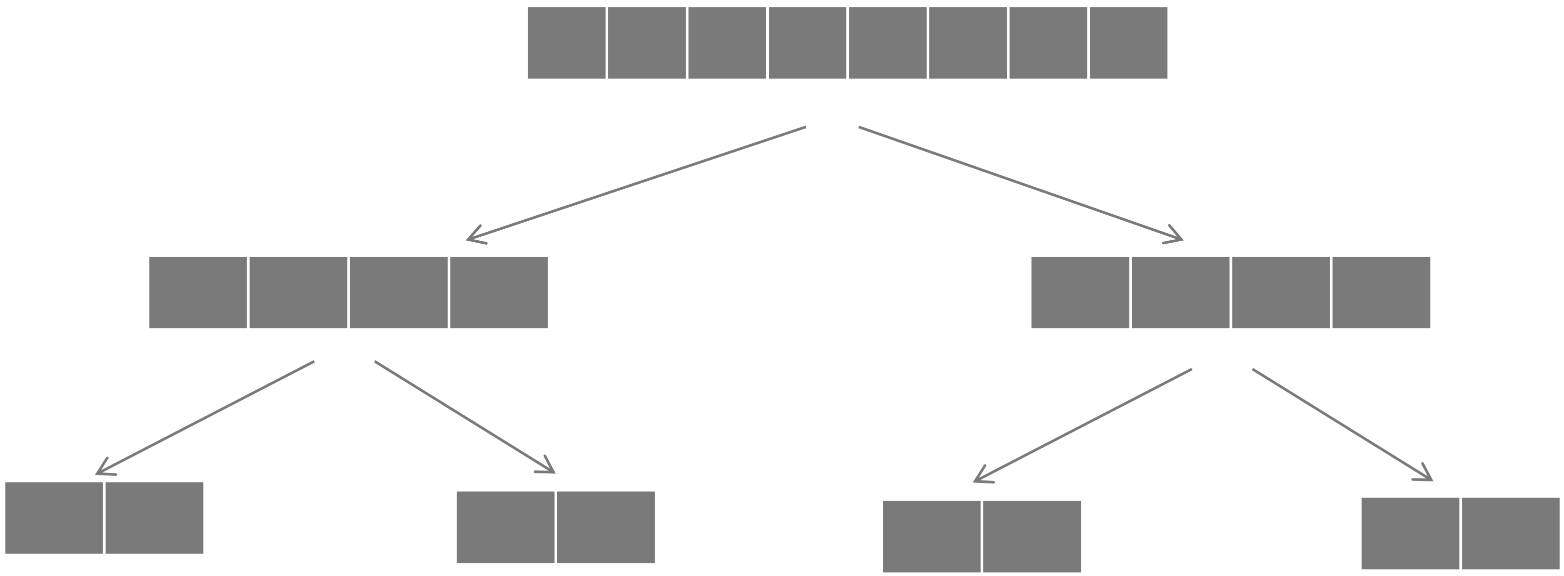

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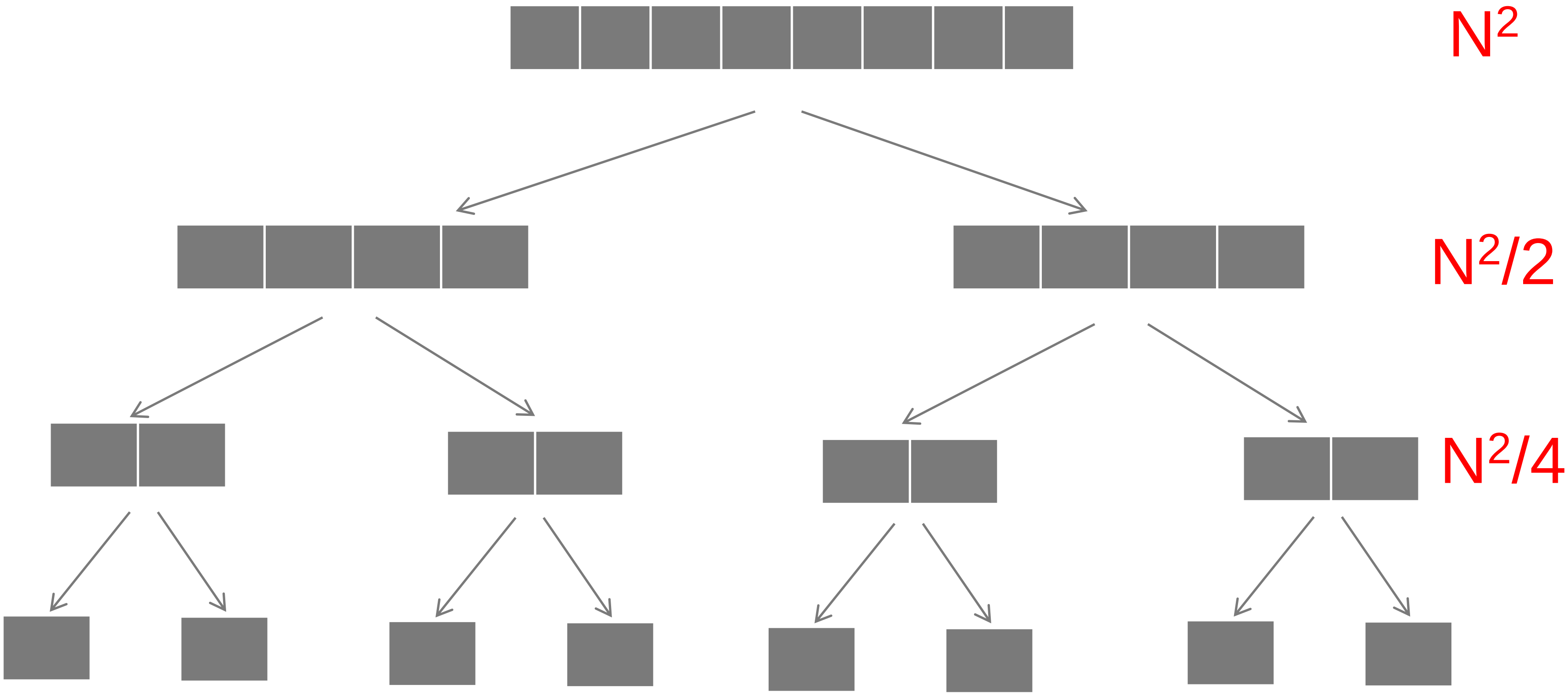

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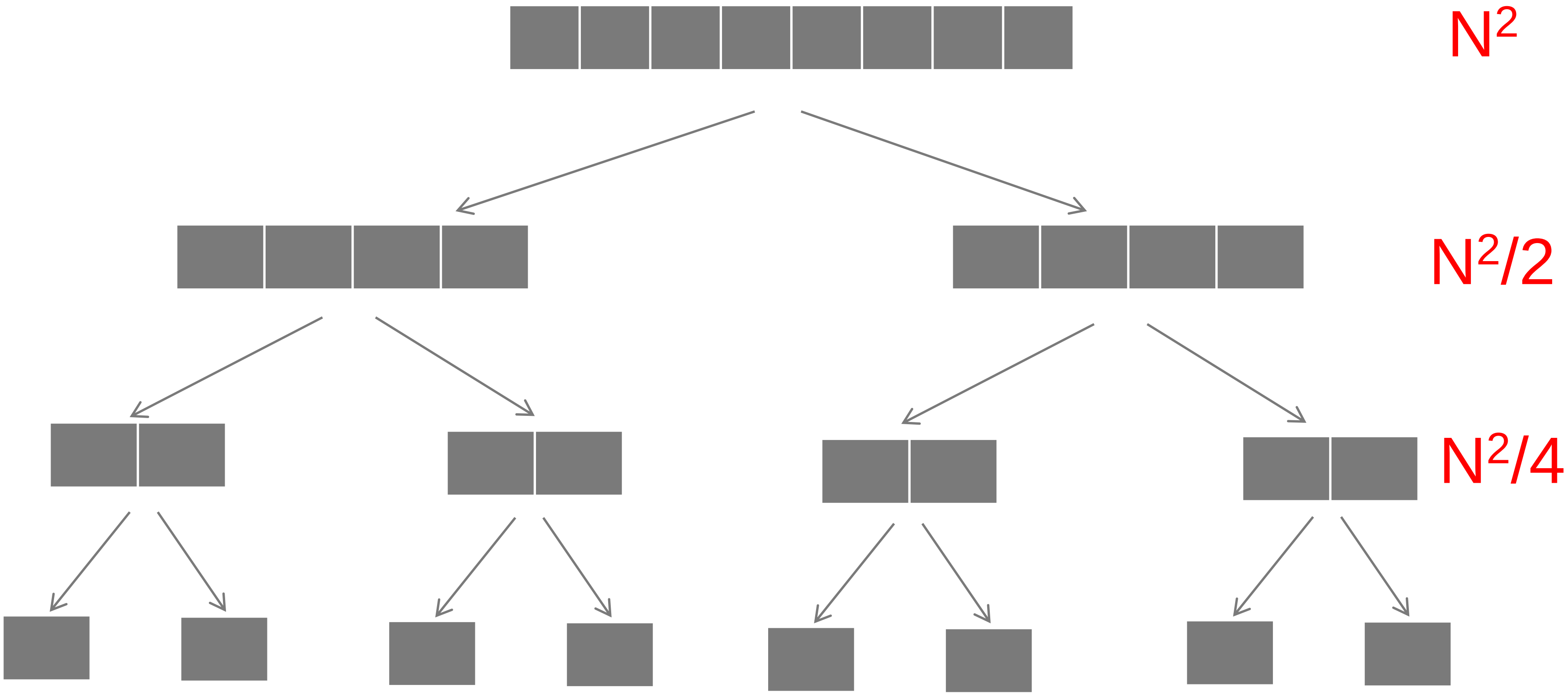

Worst-case cost at level k: $N^2/2^k$

Quick Sort

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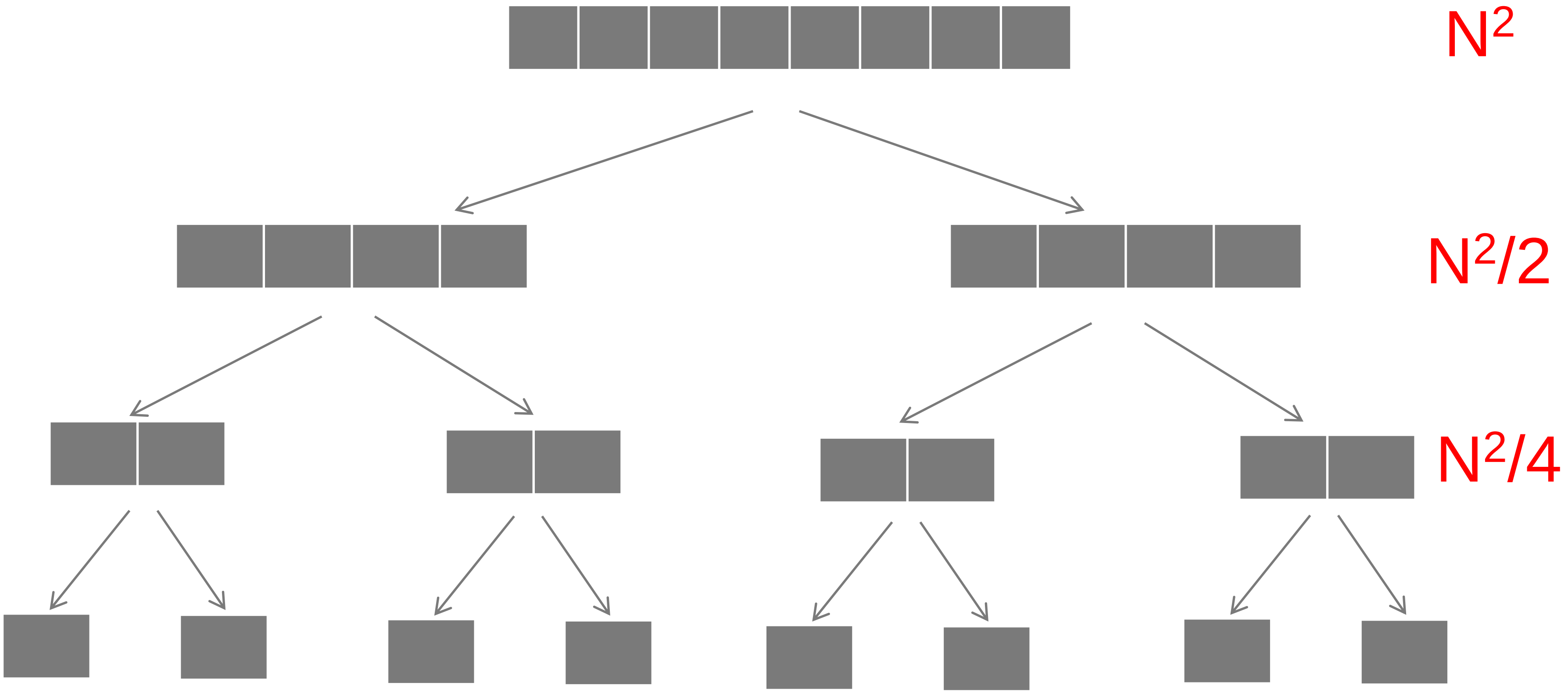

Worst-case cost at level k: $N^2/2^k$

Total cost: $N^2 + N^2/2 + N^2/4 + \ldots + 1 = N^2(1 + ½ + ¼ + \ldots) = O(N^2)$

Quick Sort

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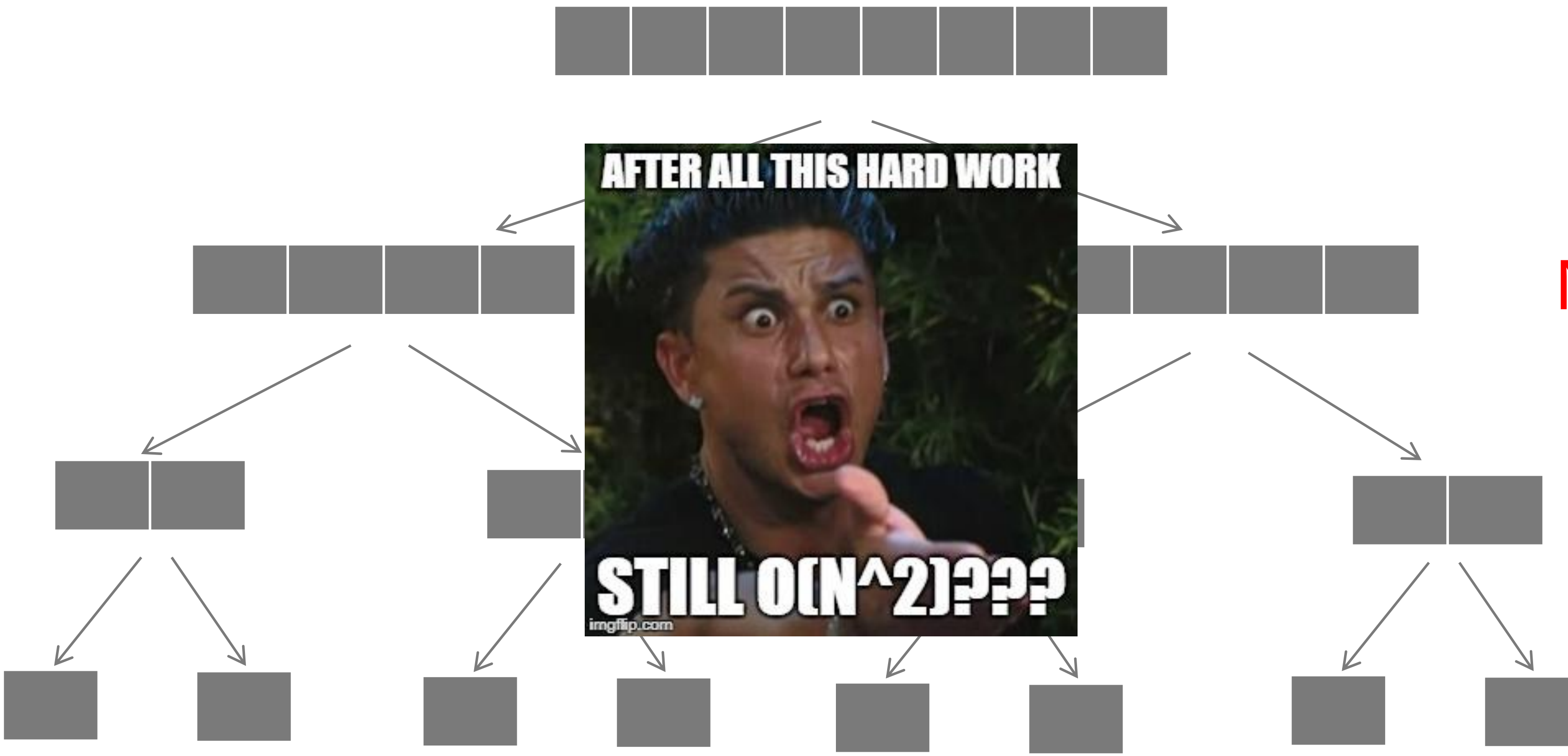

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- So what is the complexity?
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  - Why not we adjust it now, to not find the median but find just “good enough” known as the median of median…

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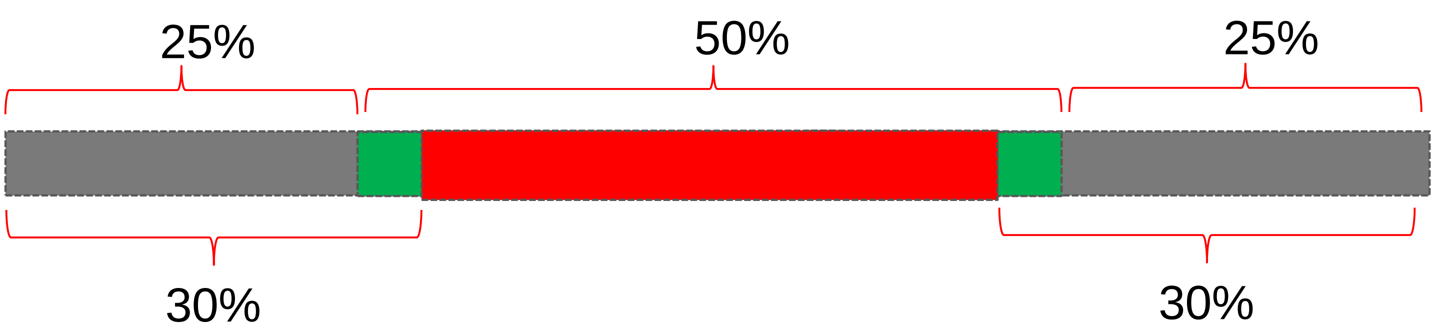

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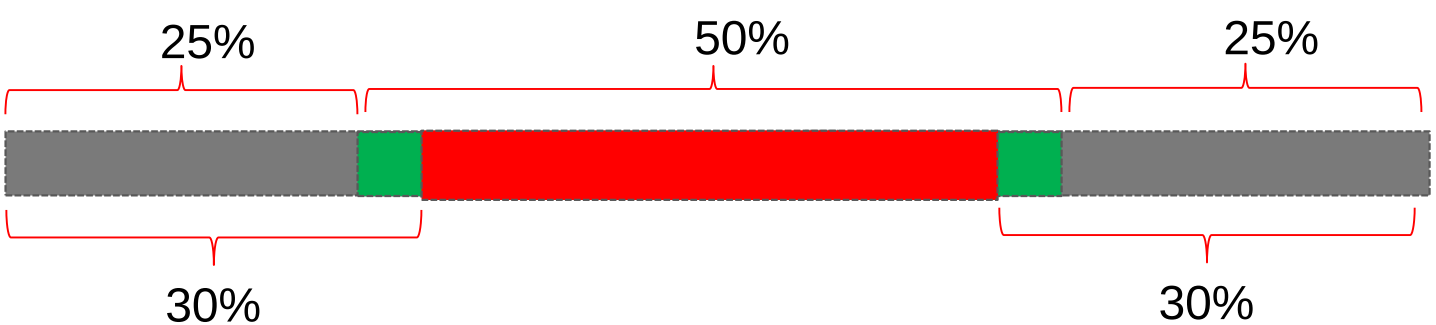

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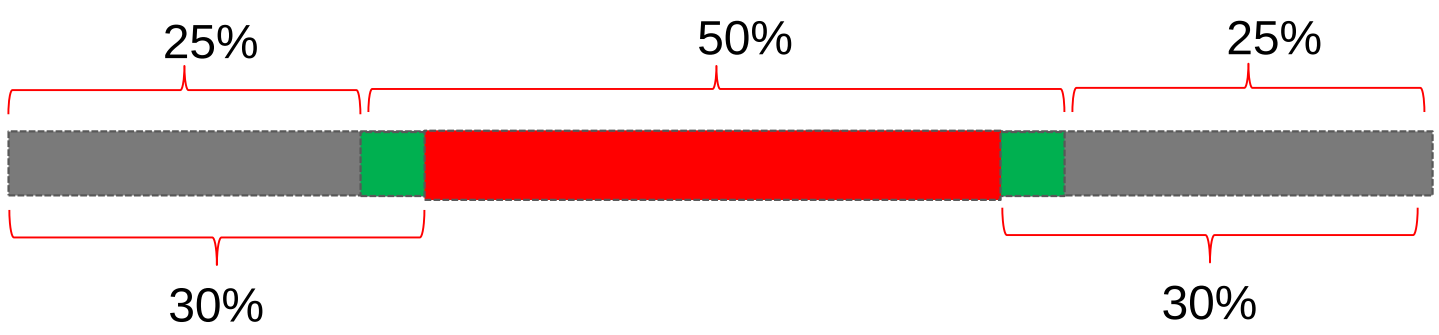

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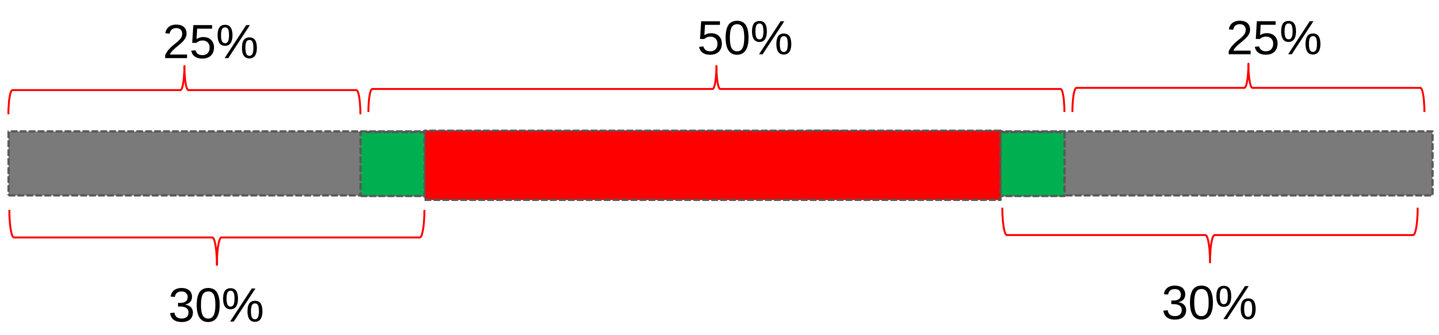

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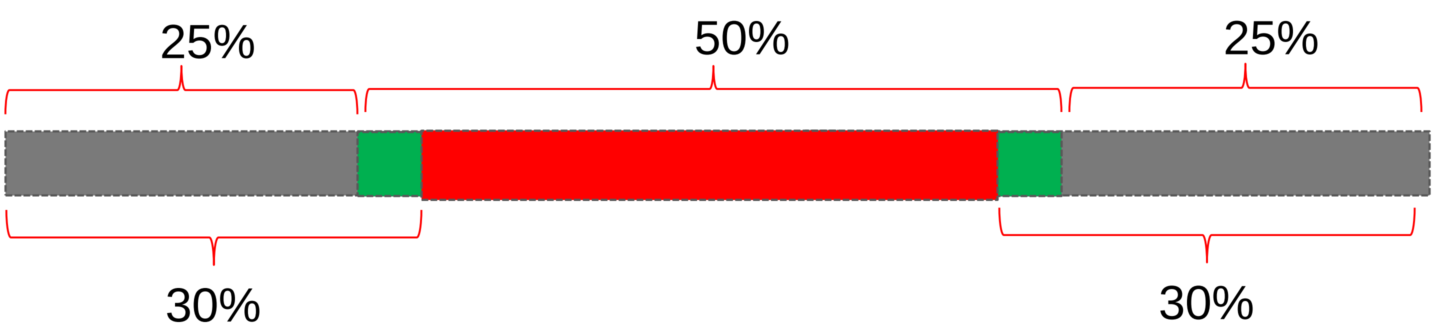

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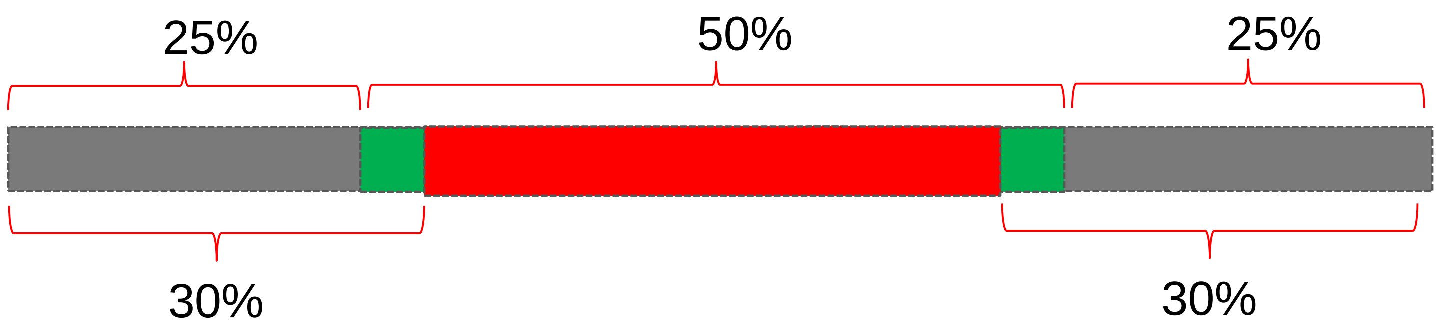

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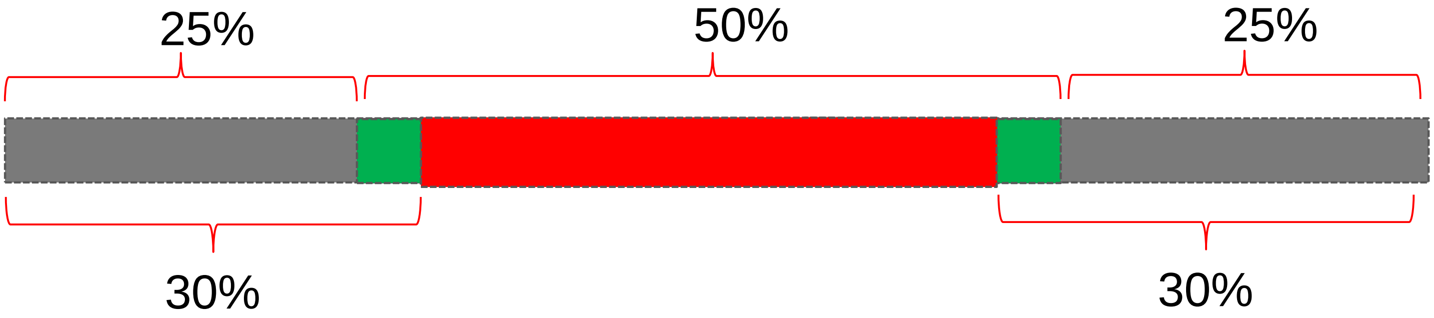

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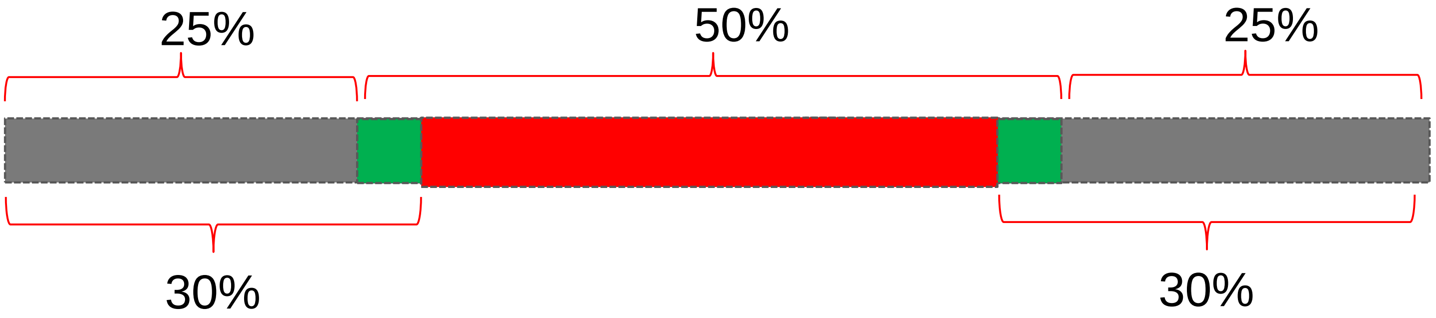

- In reality, random pivot works well due to probability…

Questions?

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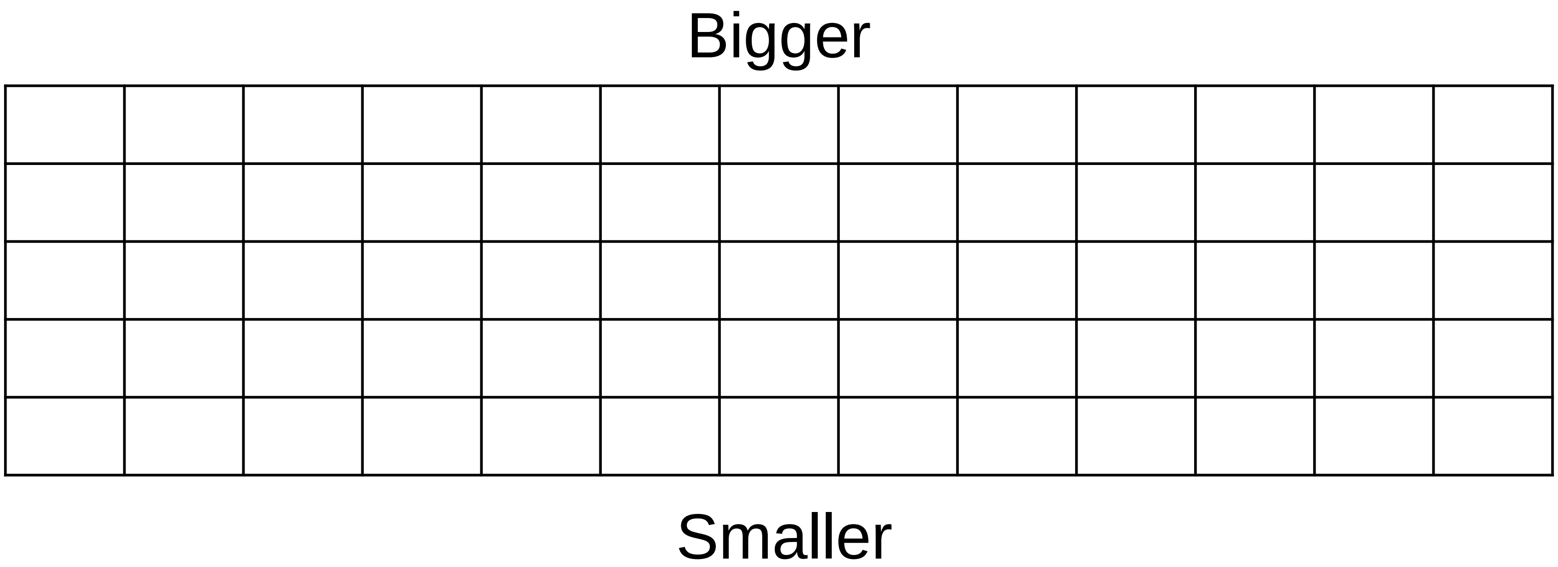
- And this ensure the worst case or quick-sort is O(N log N)

Median of medians (not examinable)

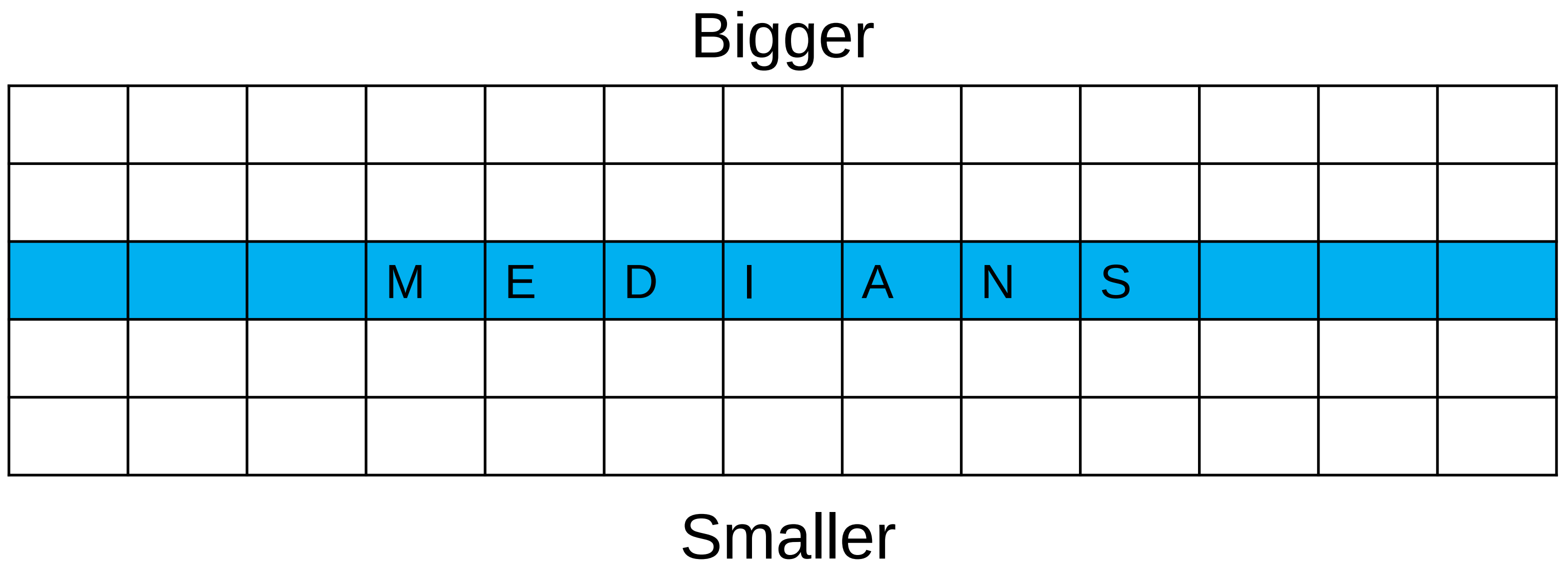
- Sort groups of size five



Median of medians (not examinable)

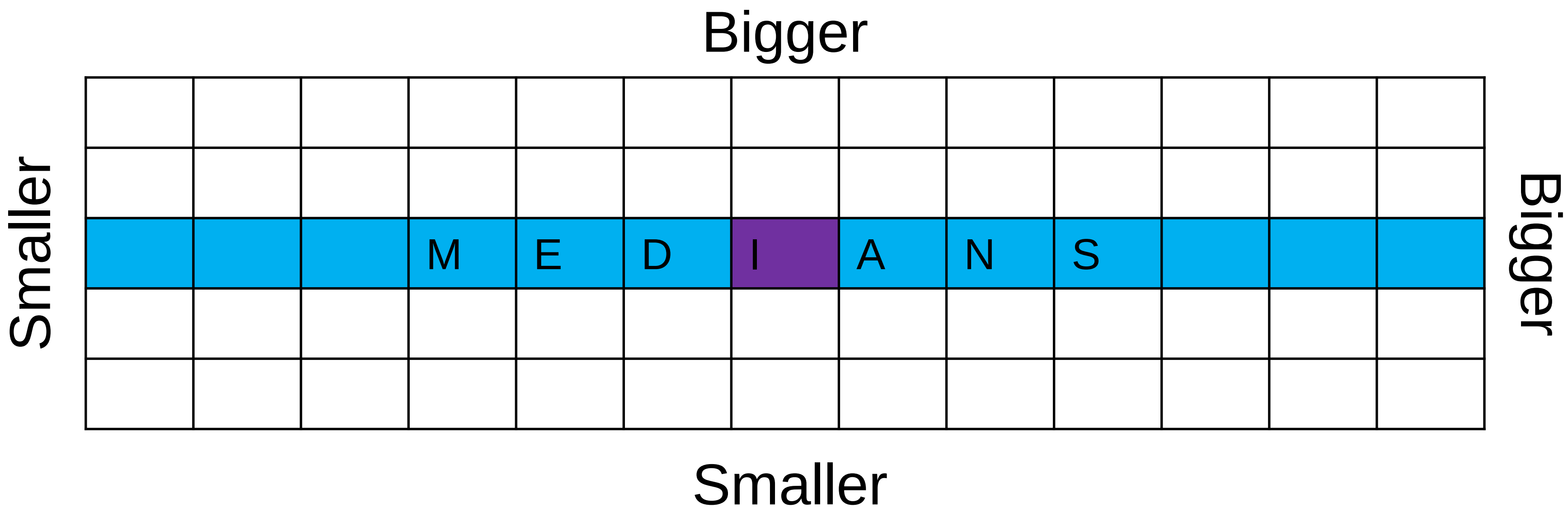
Sort groups of size five

Find the medians



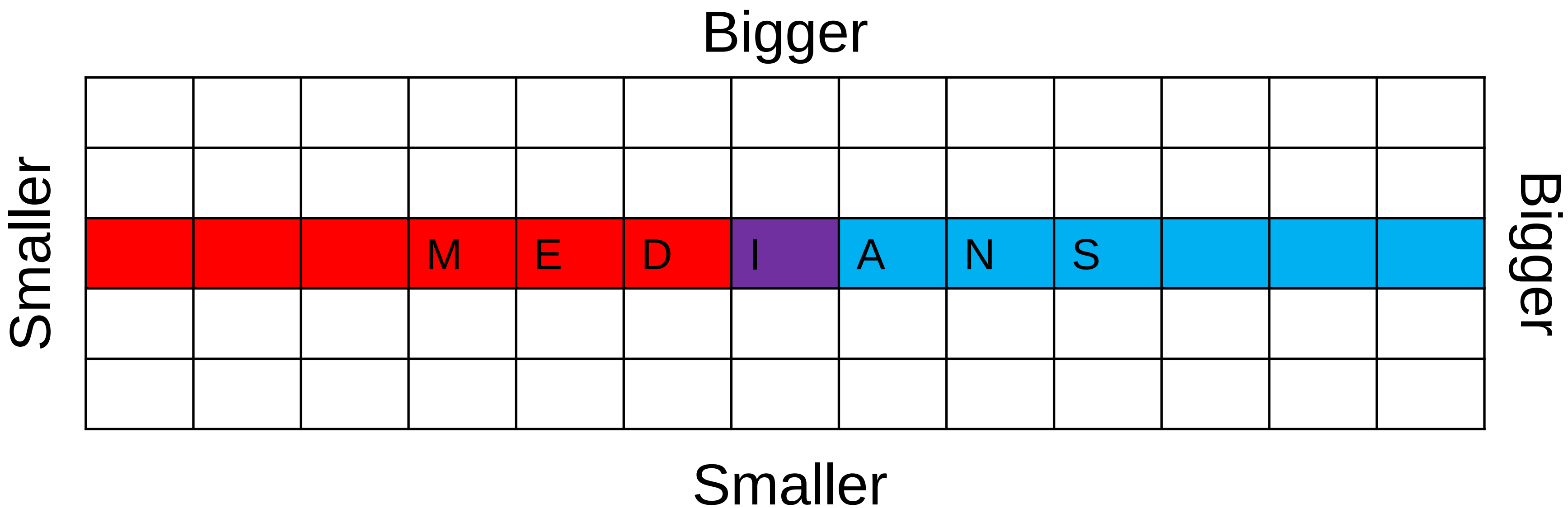
Median of medians (not examinable)

- Sort groups of size five
- Find the medians
- Find the median of those!
- (Note that the groups of 5 are not actually sorted, just shown here in sorted order for clarity)



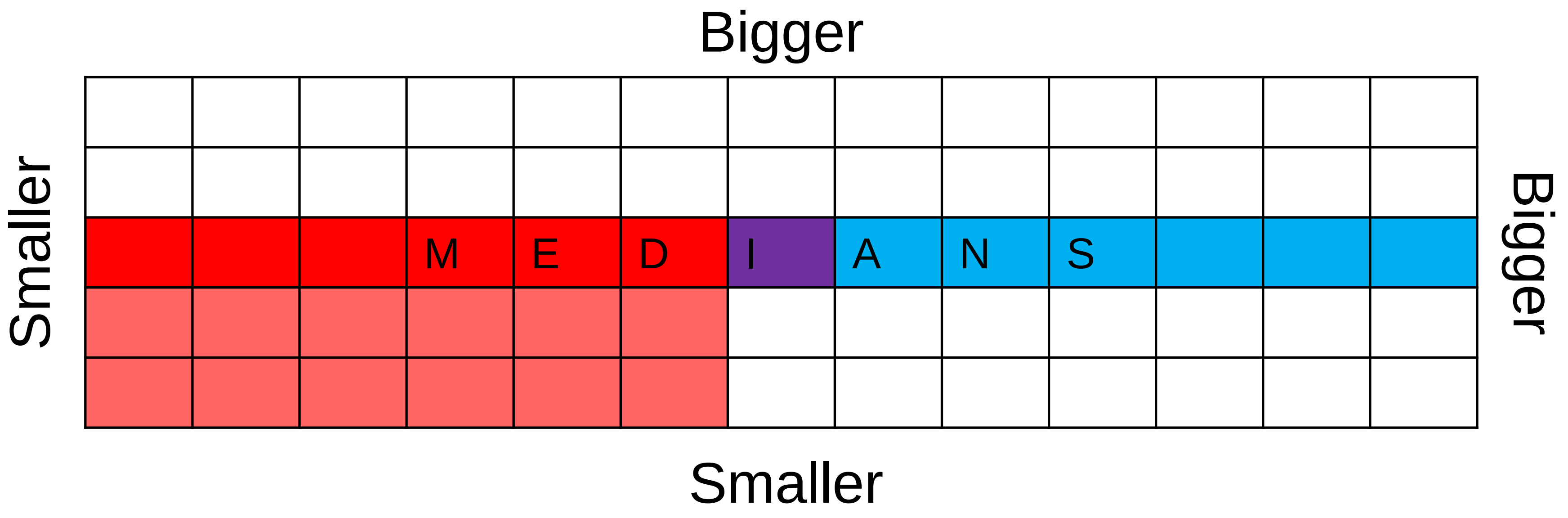
Median of medians (not examinable)

- Median of medians is bigger than half the medians



Median of medians (not examinable)

- Median of medians is bigger than half the medians
- So it is bigger than all the red values as well



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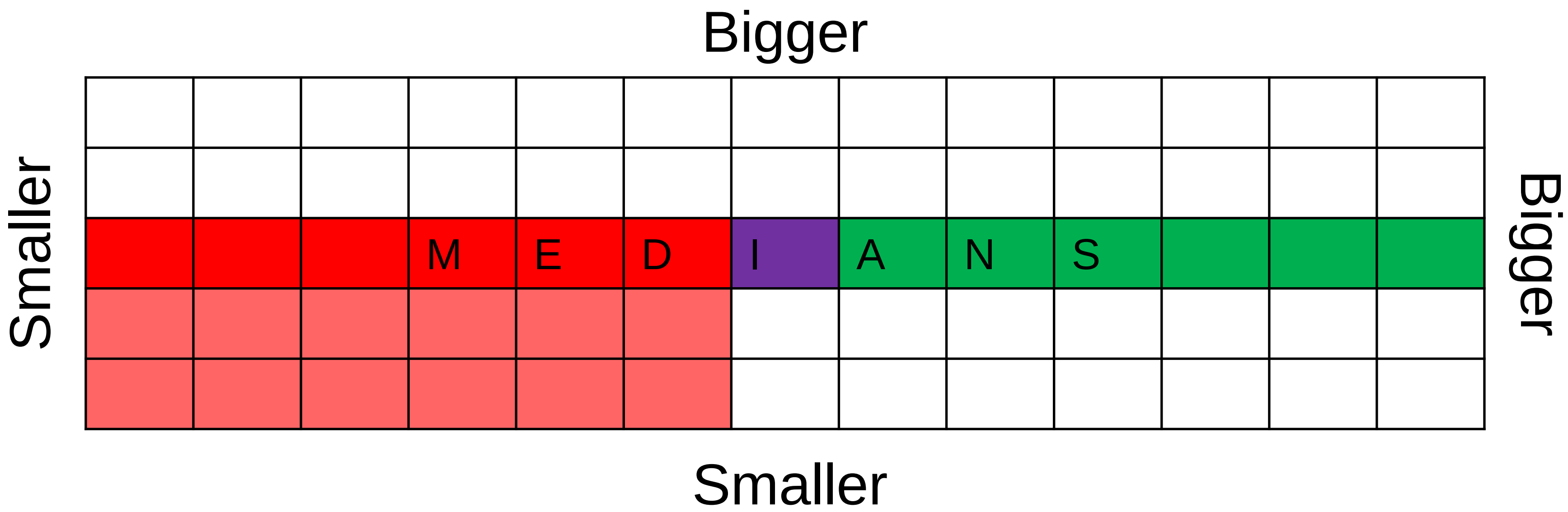

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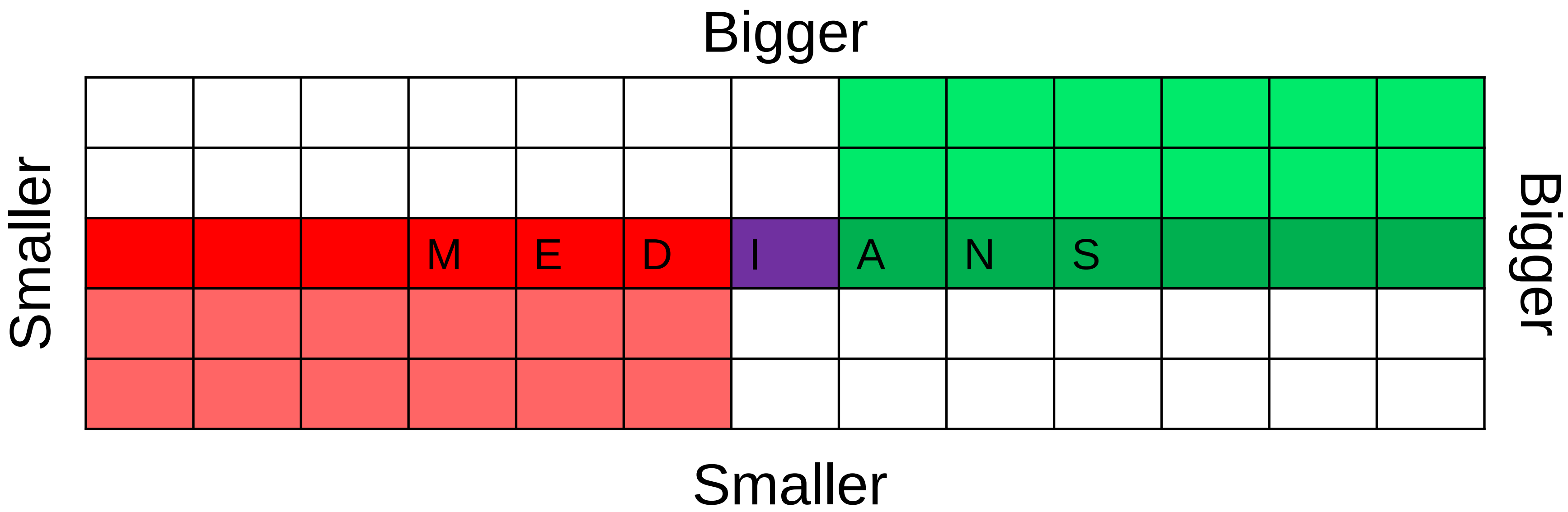
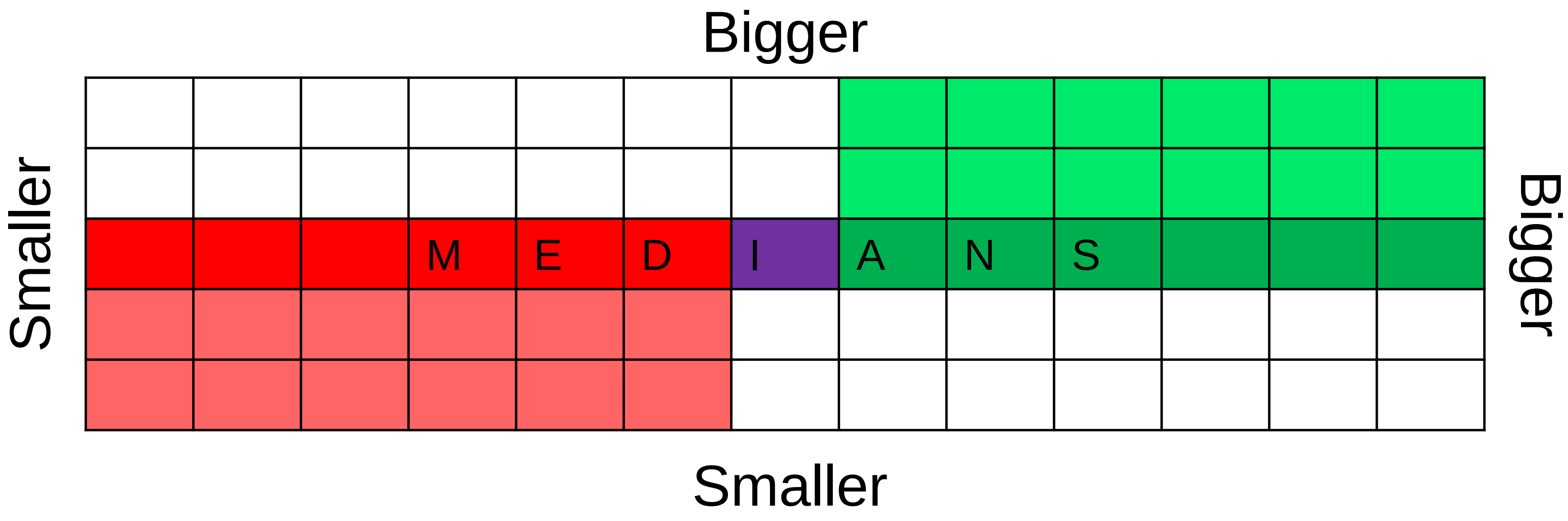


Median of medians (not examinable)

- Median of medians is greater than 30% and also less than 30%, so its in the middle 40%
- The worst split we can get using the MoM is 70:30!
- However, we did need to find the exact median of n/5 items… how?



Quicksort with O(N log N) in worst-case

Median_of_medians(list[1..n])

divide into sublists of size 5

medians = [median of each sublist]

use quickselect to find the median of **medians**

Quicksort with O(N log N) in worst-case

```
Median_of_medians(list[1..n])
    if n <= 5
        use insertion sort to find the median, and return it
    divide into sublists of size 5
    medians = [median of each sublist]
    use quickselect to find the median of medians
```

Quicksort with O(N log N) in worst-case

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Median_of_medians(list[1..n])
    if n <= 5
        use insertion sort to find the median, and return it
    divide into sublists of size 5
    medians = [median of each sublist]
    return quickselect(medians, (len(medians)+1)/2)
```

Quicksort with O(N log N) in worst-case

This call uses quickselect!
But with a weaker pivot

Quickselect(list, lo, hi, k)

    if lo > hi
        return array[k]
    pivot = **median_of_medians**(list, lo, hi, k)
    mid = *partition*(array, lo, hi, pivot)
    if mid > k
        return *quickselect*(array, lo, mid-1, k)
    elif k > mid
        return *quickselect*(array, mid+1, hi, k)
    else
        return array[k]

Quick Sort
with O(N log N) in worst-case

- Wait what?
  - Median of median calls quick select
  - Quick select calls median of median

Quick Sort
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  - This is called co-recursion…

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Quickselect(list, lo, hi, k)

    if lo > hi
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    mid = *partition*(array, lo, hi, pivot) **(70:30 pivot in worst)**
    if mid > k
        return *quickselect*(array, lo, mid-1, k) **(n/7 in worst)**
    elif k > mid
        return *quickselect*(array, mid+1, hi, k) **(n/7 in worst)**
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Quicksort with O(N log N) in worst-case

Quickselect time complexity recurrence

$$T(n) = T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right) + an$$

- $T\left(\frac{n}{5}\right)$ for recursing on the list of the medians of groups of 5 (inside the call to median of medians)
- $T\left(\frac{7n}{10}\right)$ for the main recursive call, which is guaranteed to have split the list at least 30:70 (because the pivot was selected by MoM)
- an for the linear time partition algorithm + time to find medians of groups of five

Solving this give linear time!

So armed with a linear time quickselect, we can now quicksort in NlogN worst case…

Questions?

Quick Select

and average case complexity

- Note: NOT EXAMINABLE for math approach

Quick Select
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- What algorithm is quick select?

Quick Select
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  - Partial sorting with partition
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$$T_N = N + 1 + \begin{cases} T(N-i) & \text{if } i < k, \\ 1 & \text{if } i = k, \\ T(i-1) & \text{if } i > k \end{cases} .$$

Quick Select
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  - i is the rank of the pivot
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Quick Select

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- What algorithm is quick select?
  - With this, we can now figure out the average complexity
  - This is the expected run time on average

$$T_N = N + 1 + \frac{1}{N}\left(\sum_{i=1}^{k-1} T(N-i) + \sum_{i=k+1}^{N} T(i-1) + 1\right)$$

  - What does this means?

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  - What does this means?
    - Every pivot case
    - Sum it all up
    - Get the average

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    - Multiply both side with N

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Quick Select

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- Then have the left side of T_N

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Quick Select
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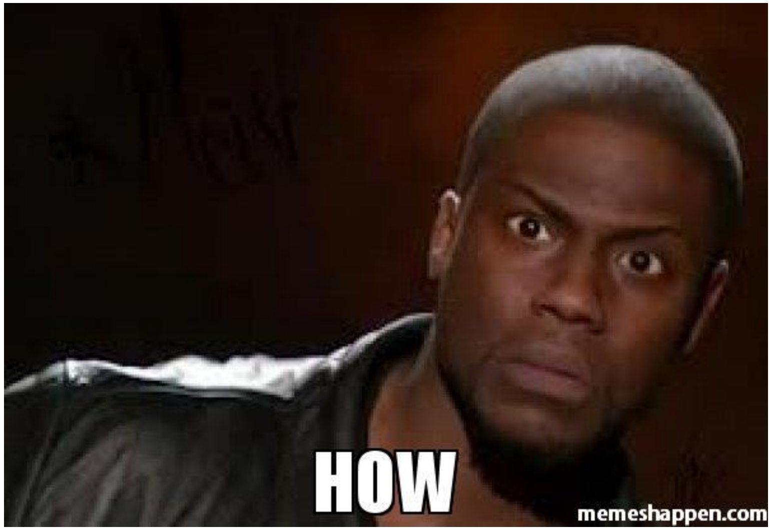

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$$T_N < 3N = O(N)$$

Questions?

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Bonus
Online algorithms

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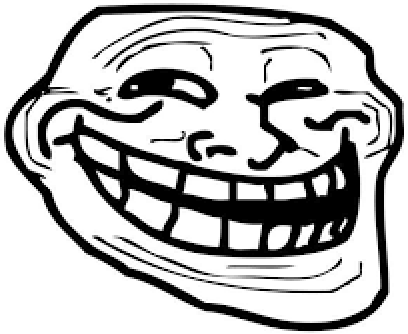

problem?

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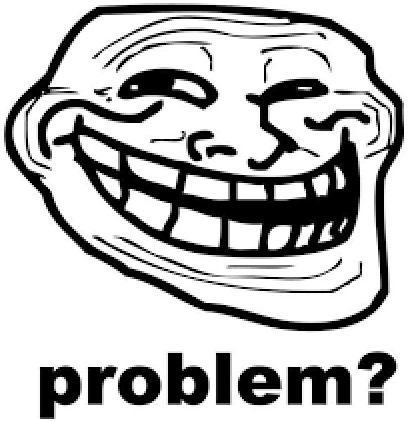

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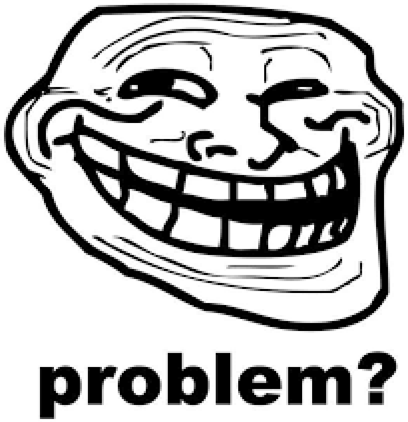

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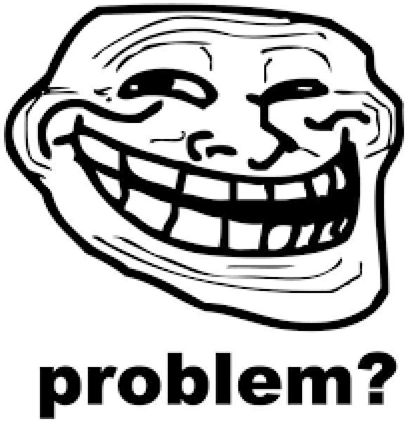

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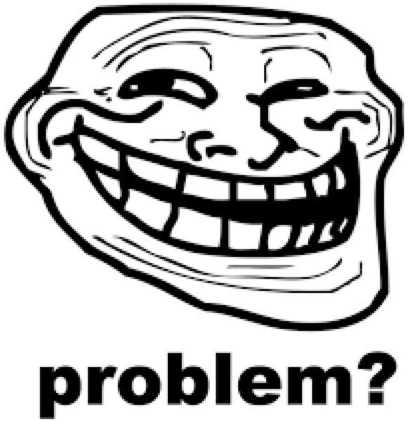
    - Insertion sort
    - What about k-th order statistic?
      - Does quick select still work?



Bonus
Online algorithms

- From your Tutorial 03 Question 08

Problem 8. Devise an efficient online algorithm[1] that finds the smallest k elements of a sequence of integers. Write psuedocode for your algorithm. [Hint: Use a data structure that you have learned about in a previous unit]

- From your Tutorial 03 Question 08
  - Using quick select?
  - Using a new approach?

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Questions?

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Thank You