MONASH
INFORMATION
TECHNOLOGY

FIT2004
Algorithms and Data Structures

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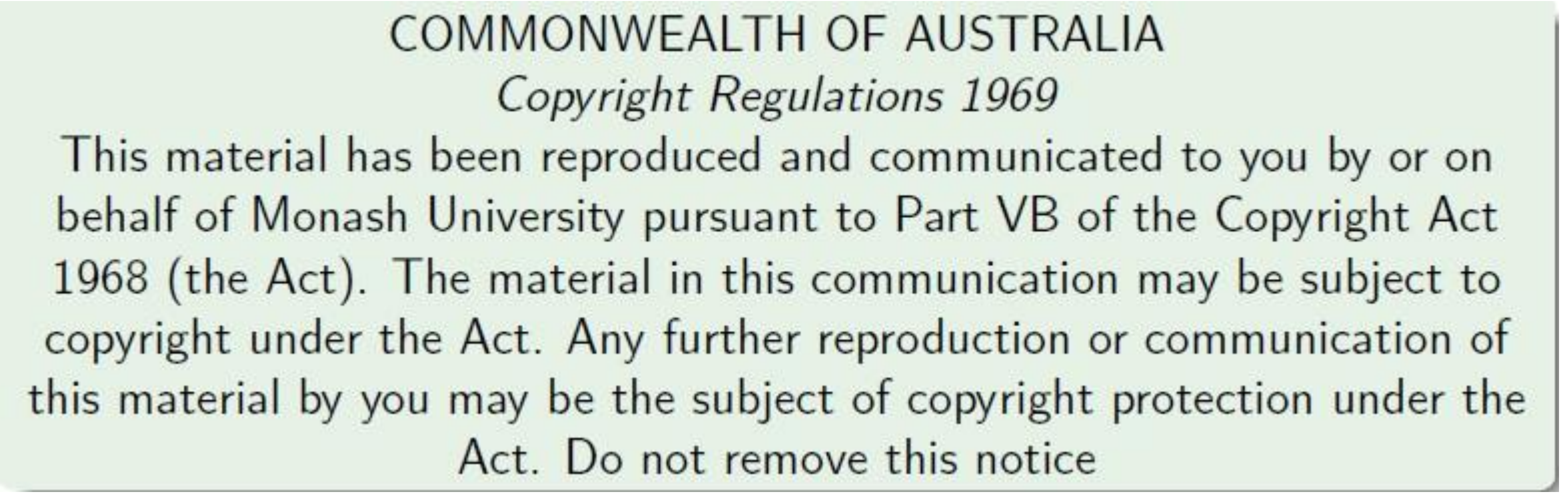
Referencing materials by
Nathan Companez, Aamir Cheema, Arun Konagurthu and Lloyd Allison

Faculty of Information Technology, Monash University



Ready?

Agenda

- Complexity Analysis
  - Time
  - Space
- Sorting Algorithms
  - Comparison based
    - Selection
    - Insertion
  - Non-comparison based (the IMBA ones)
    - Counting
    - Radix

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Let us begin…

Algorithm Analysis

- Correctness
- Complexity

Algorithm Analysis

- Correctness
  - Loop invariant
  - Termination

  } Last lecture
- Complexity

Algorithm Analysis

- Correctness
  - Loop invariant
  - Termination
- Complexity
  - Time
  - Space

Algorithm Analysis

- Correctness
  - Loop invariant
  - Termination
- Complexity
  - Time
    - Best
    - Worst (big focus here)
    - Lower bound aka big Omega
    - Output sensitive
  - Space
    - Total
    - Auxiliary

Questions?

Complexity

Time

- Best
- Worst

Complexity
Time

- Best
- Worst

- You know what are they

Complexity

Time

- Best
- Worst
  - Focus!

- You know what are they

Complexity

Time

- Now let us have some recap with some functions

Complexity

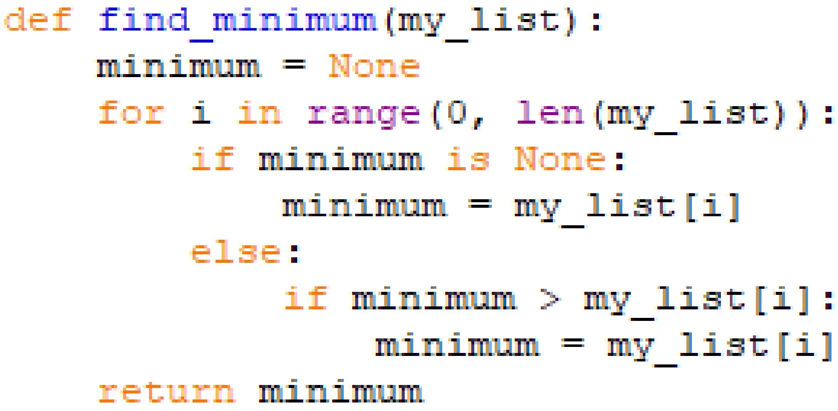
Time

- Now let us have some recap with some functions
  - Minimum
  - Binary search
  - Heap sort

Complexity

Minimum

- Consider the code
- What is the time complexity?



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Complexity
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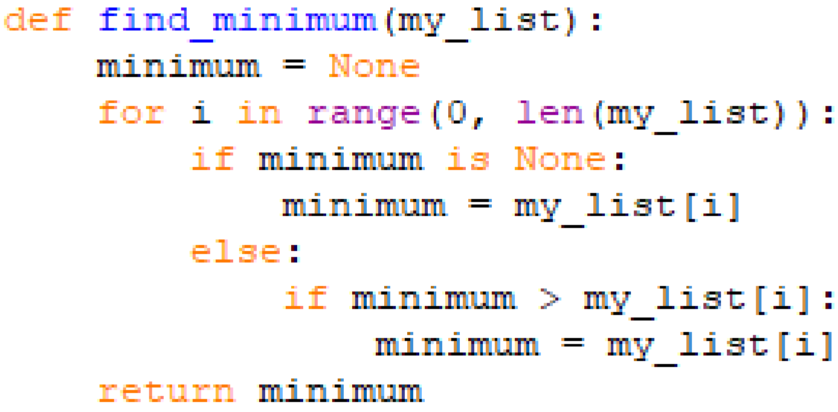

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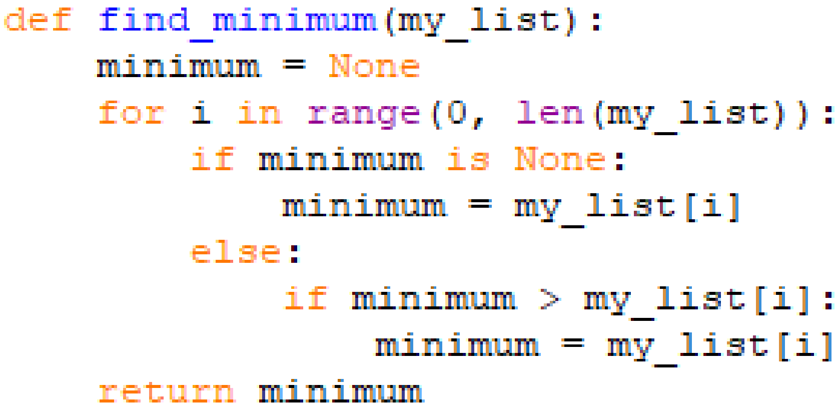
  - Both are O(N) because…



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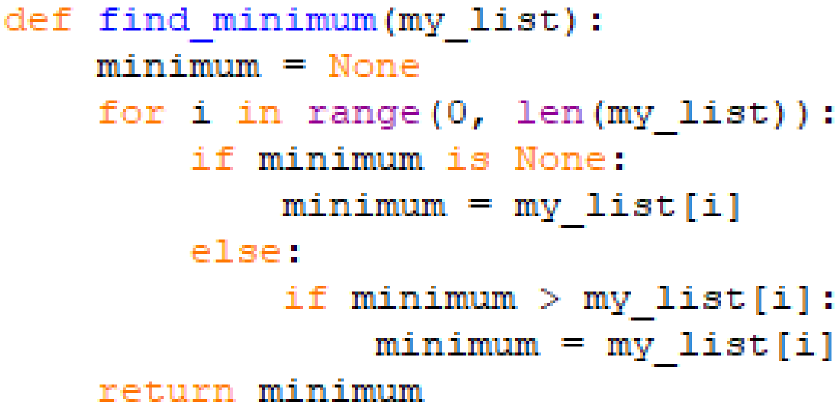

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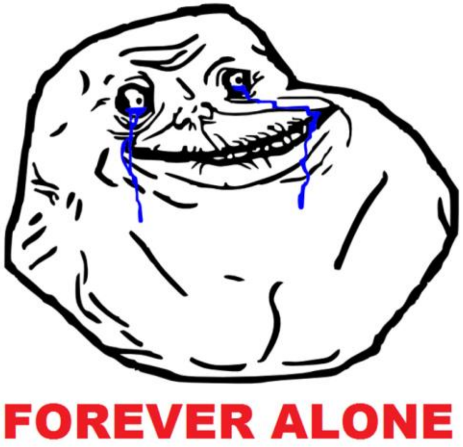
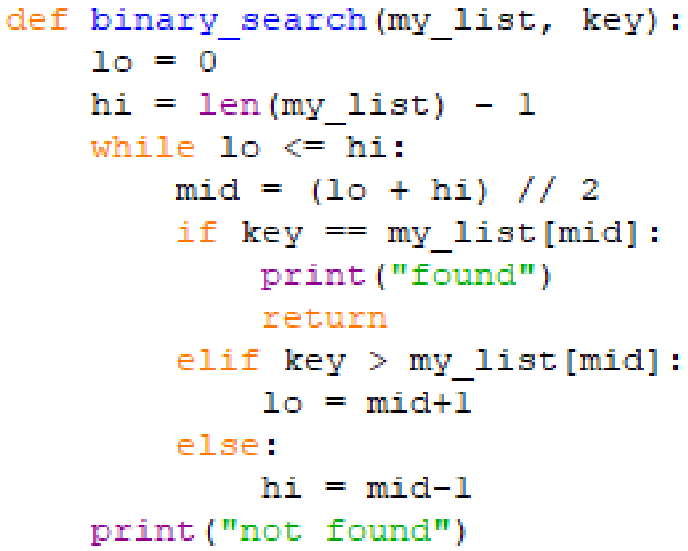


- Remember we can't say best O(1) when list have 1 item
  - Need to be for a list of size N

Complexity

Binary search

- Consider the code
- What is the time complexity?



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Complexity

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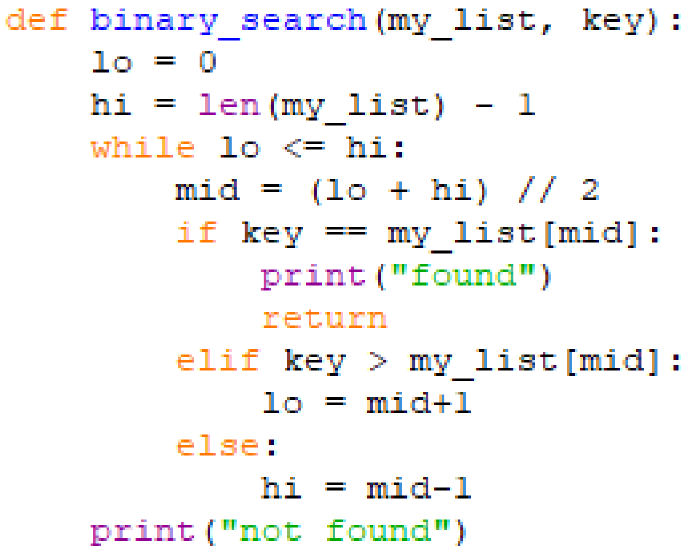

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Complexity

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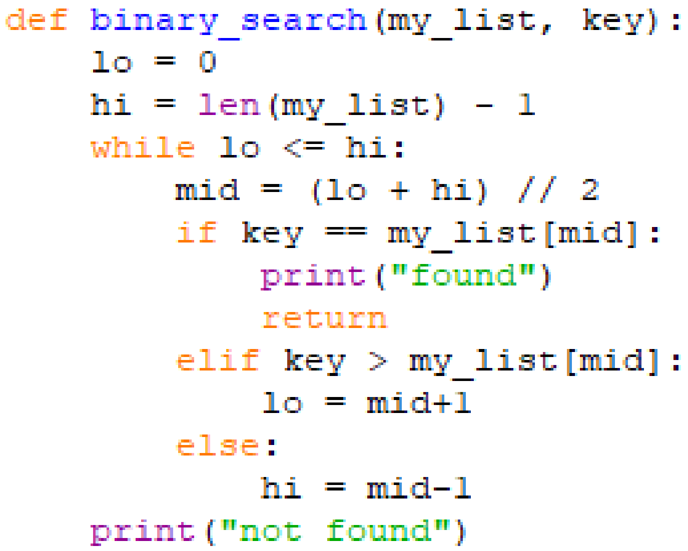
  - Worst O(log N)



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Complexity

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- Consider the code
- What is the time complexity?
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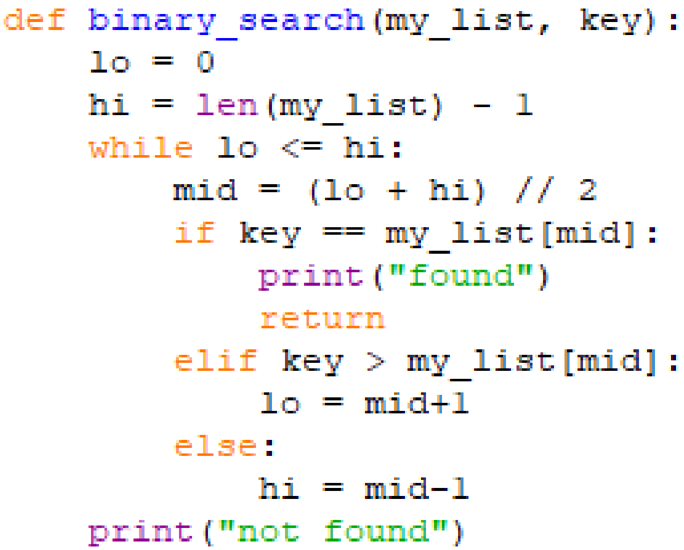

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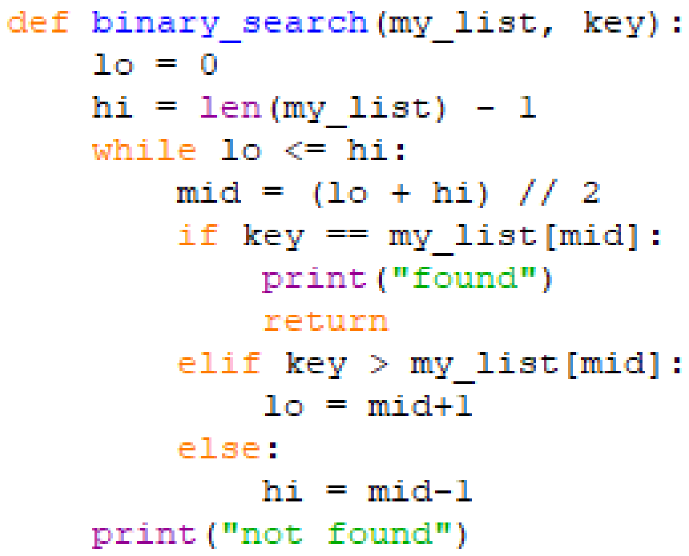
Complexity

Binary search

- How can we show worst is O(log N)?

- Search space



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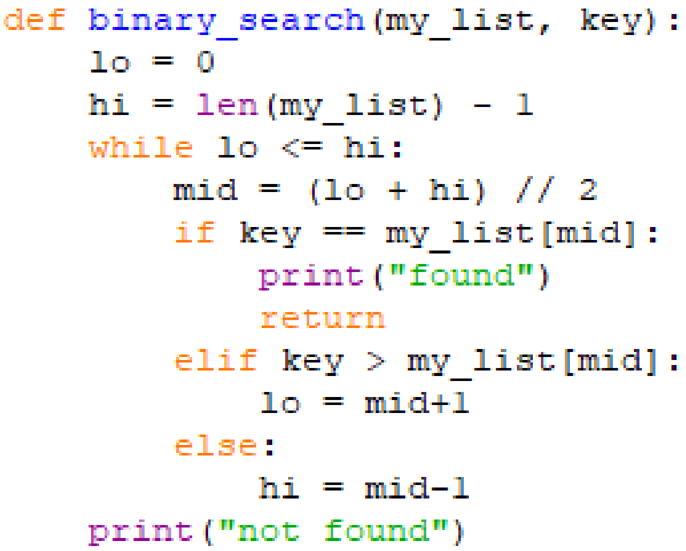
Complexity

Binary search

- How can we show worst is O(log N)?

- Search space
  - Initially N



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Complexity

Binary search

- How can we show worst is O(log N)?
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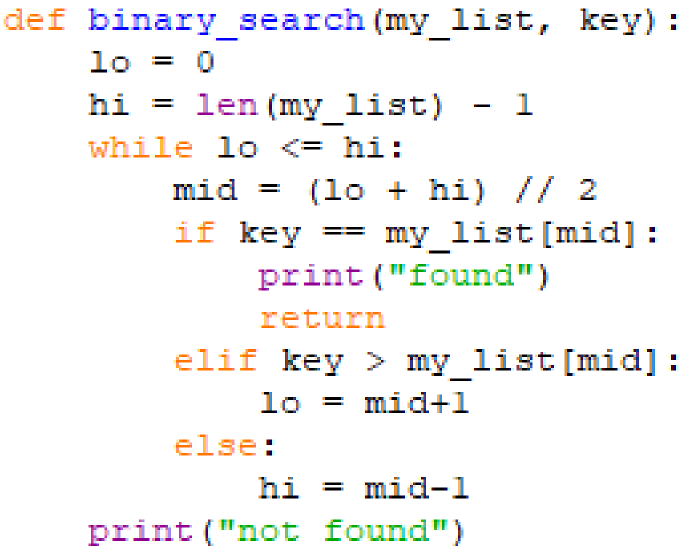
  - Initially = N
  - 1st iteration = N/2
  - 2nd iteration = N/4



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  - Initially = N
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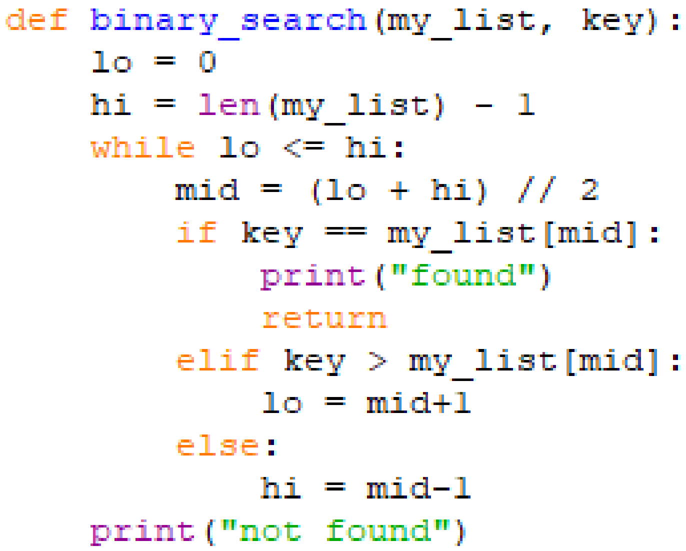
  - …
  - Last iteration = 1



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  - 1st iteration = N/2^1
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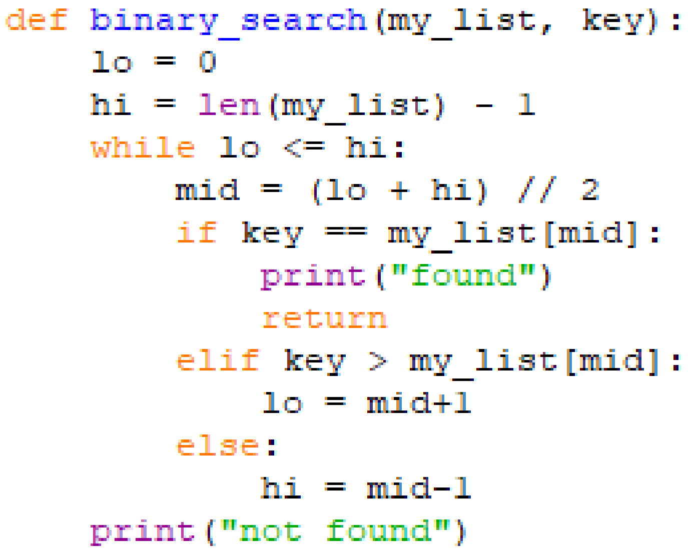

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  - 1st iteration = N/2^1
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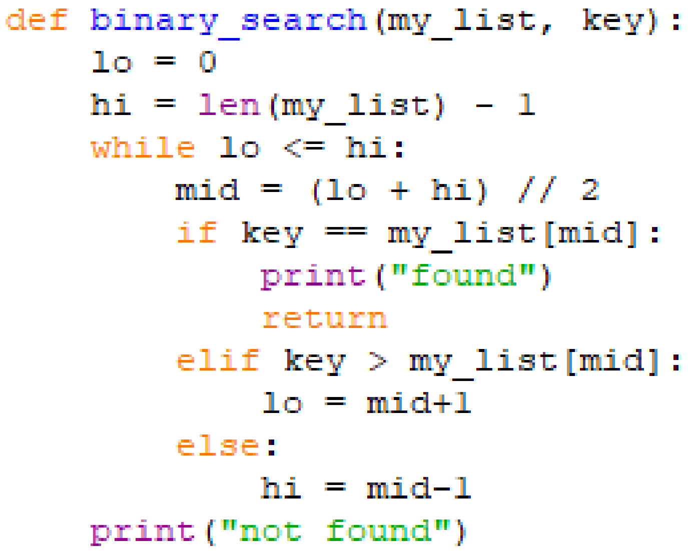
  - Thus N = 2^k
    - Which give us k = log N
    - Worst case is when we reach height k, which is log N



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Complexity

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- So we know time complexity pretty well now
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  - Worst case

Complexity

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- But we have more!

Complexity
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  - Worst case
- But we have more!
  - Lower bound (big omega)
  - Output-sensitive

Questions?

Complexity

Time – Lower Bound

- We know for a given problem, there can be a lot of solutions or algorithms….

Complexity

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- We know for a given problem, there can be a lot of solutions or algorithms….

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- If we are to print items in a list, we don’t have a choice but to print through every item in the list. Thus $\Omega(N)$ for list printing.

Complexity

Time – Lower Bound

- We know for a given problem, there can be a lot of solutions or algorithms….
  - Known or unknown

- The lower bound (aka big-omega) is the best complexity we can achieve for a given problem irregardless of the solution or algorithm…
  - Opposite of big-O

- If we are to print items in a list, we don't have a choice but to print through every item in the list. Thus $\Omega(N)$ for list printing.

Complexity

Time – Lower Bound

- So… what is the lower bound for the sorting algorithms that we have learnt?
  - Bubble
  - Insertion
  - Selection
  - Quick
  - Merge

Complexity

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- These are all comparison based
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- We will see more of this later

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Time – Output Sensitive

- What is it?

Complexity

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Complexity

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- Given two values x and y
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- What is our complexity here?

Complexity

Time – Output Sensitive

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- Approach 01
  - Loop through the entire list
  - If item > x and item < y, print item

Complexity

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  - This gives O(N) complexity
    - Looping through the list

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  - This isn’t output sensitive, x and y value doesn’t matter

Complexity

Time – Output Sensitive

- Given a sorted array of unique numbers
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- Find all numbers greater than x but smaller than y

- Approach 02
  - Binary search to find smallest number greater than x
  - Linear search from x till reach a greater number or equal than y

Complexity

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    - O(log N) for binary search
    - O(W) for printing the values where O(W) is O(y-x)

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    - O(W) for printing the values where O(W) is O(y-x)
    - Why?

Complexity

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    - O(log N) for binary search
    - O(W) for printing the values where O(W) is O(y-x)
    - Why? W can be as big as N!

Complexity

Time – Output Sensitive

- Output-sensitive complexity is only relevant when the output-size may vary
  - Not sorting
  - Not finding minimum

Complexity

Time – Output Sensitive

- Output-sensitive complexity is only relevant when the output-size may vary
  - Not sorting
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- If you look at your assignment, certain question have additional complexity – that is dependent on the output!

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Questions?

Complexity

Space

- What is it?

Complexity

Space

- How much space is used

Complexity

Space

- How much space is used

- Consider our functions earlier…

Complexity

Space -- minimum

- How much space is used

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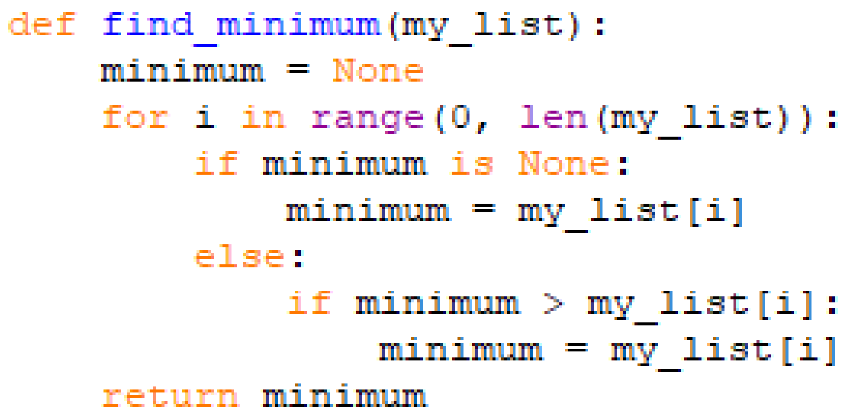

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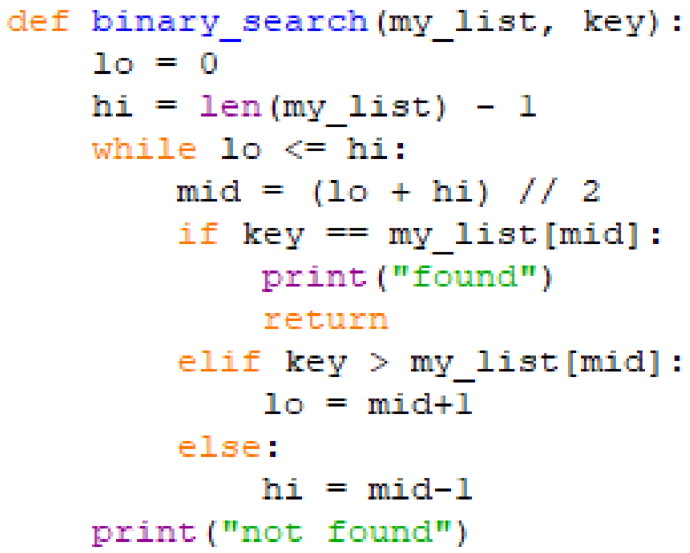

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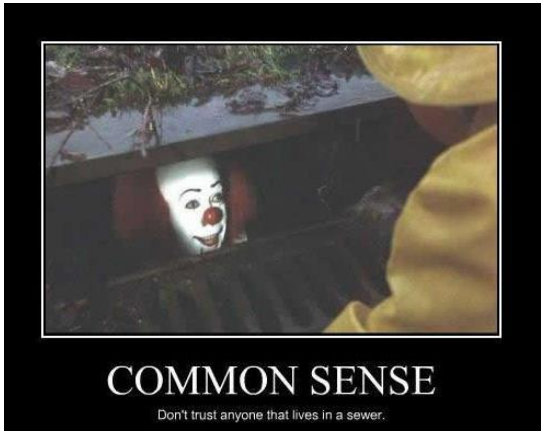
Complexity

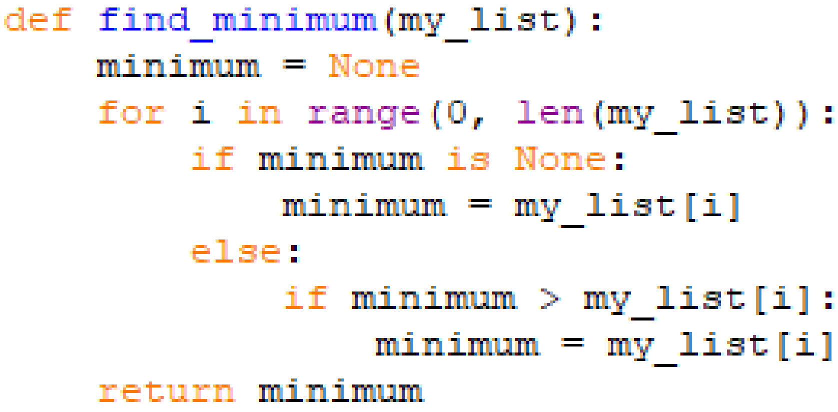
Space

- How much space is used

- Consider our functions earlier…

- We need O(N) space to for the input list





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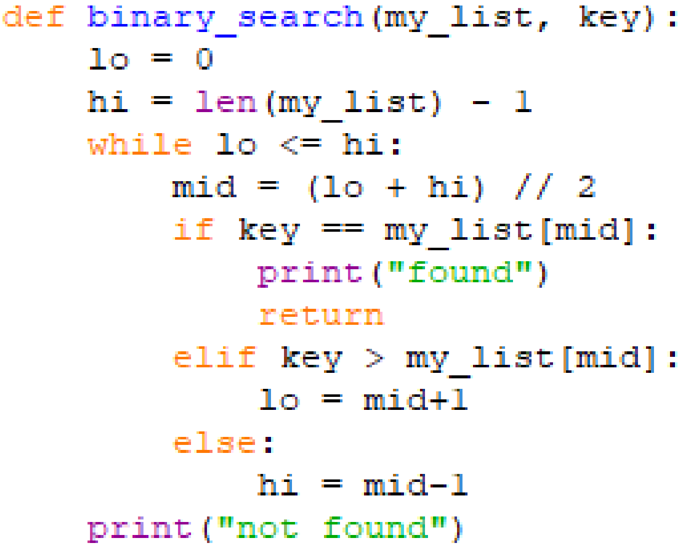

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Auxiliary Space

- What is this now then?

Complexity

Auxiliary Space

- What is this now then?
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Complexity

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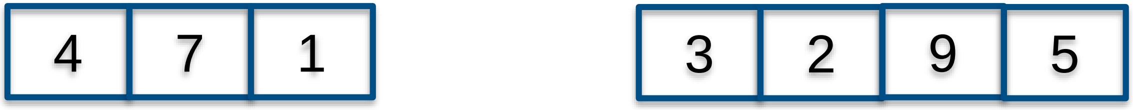
- Remember the merge sort’s merge operation?



4 7 1

3 2 9 5

Complexity

Auxiliary Space

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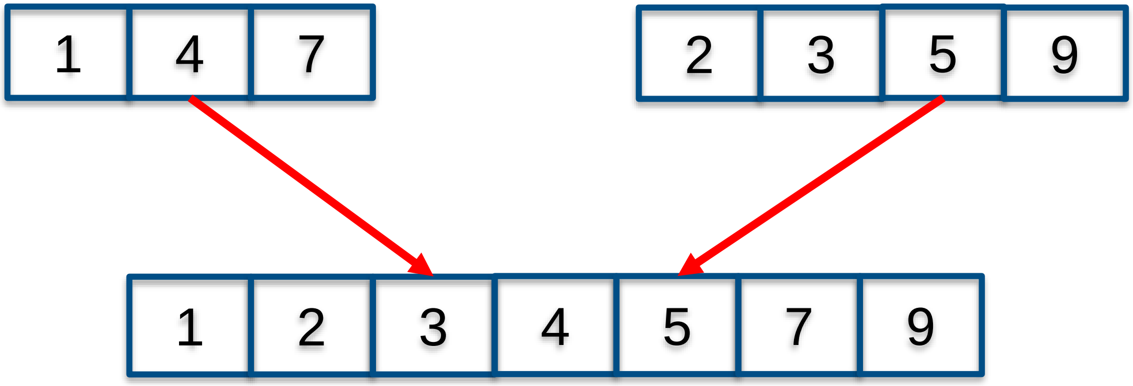

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2 3 5 9

1 2 3 4 5 7 9

Complexity

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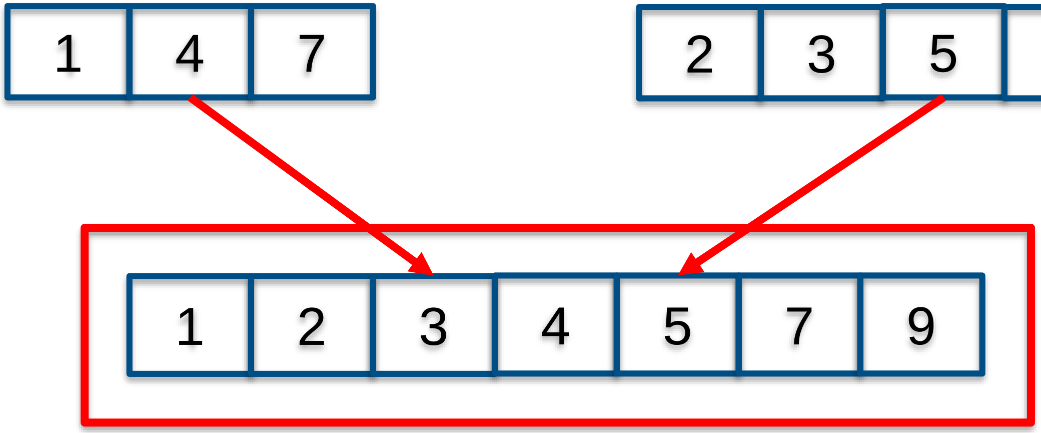

Additional space

Complexity

Auxiliary Space

- What is this now then?
- Additional space required in addition to the input

- Remember the merge sort’s merge operation?
  - Space complexity = 2N = O(N)
  - Auxiliary space = 2N – N = O(N)

Complexity

Auxiliary Space

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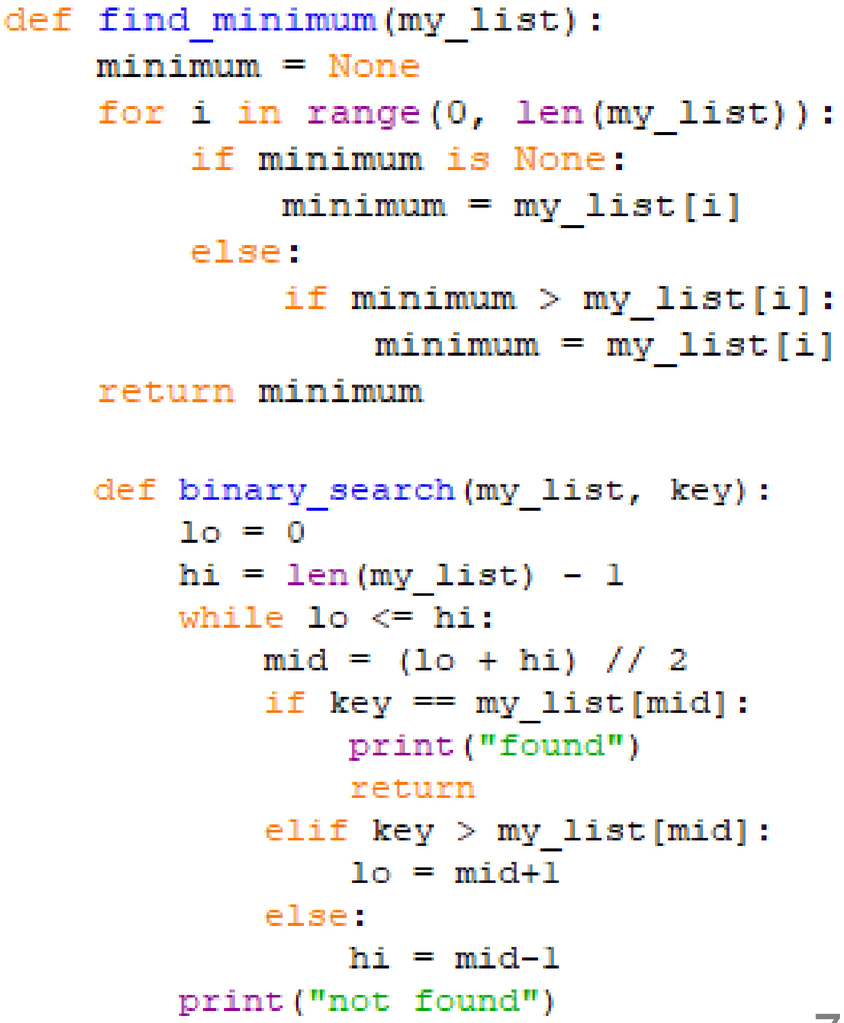
- So what is the auxiliary space complexity for these then?

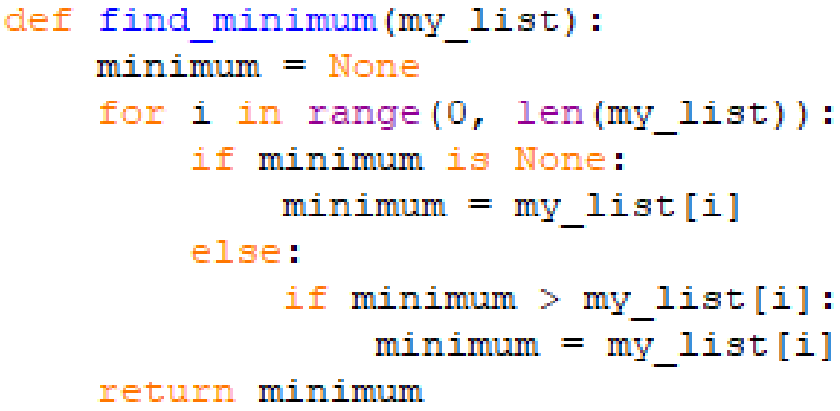


```
def find_minimum(my_list):
    minimum = None
    for i in range(0, len(my_list)):
        if minimum is None:
            minimum = my_list[i]
        else:
            if minimum > my_list[i]:
                minimum = my_list[i]
    return minimum

    def binary_search(my_list, key):
        lo = 0
        hi = len(my_list) - 1
        while lo <= hi:
            mid = (lo + hi) // 2
            if key == my_list[mid]:
                print("found")
                return
            elif key > my_list[mid]:
                lo = mid+1
            else:
                hi = mid-1
        print("not found")
```

Complexity
Auxiliary Space

- So what is the auxiliary space complexity for these then?
  - Both are O(1)
  - Do not require additional space



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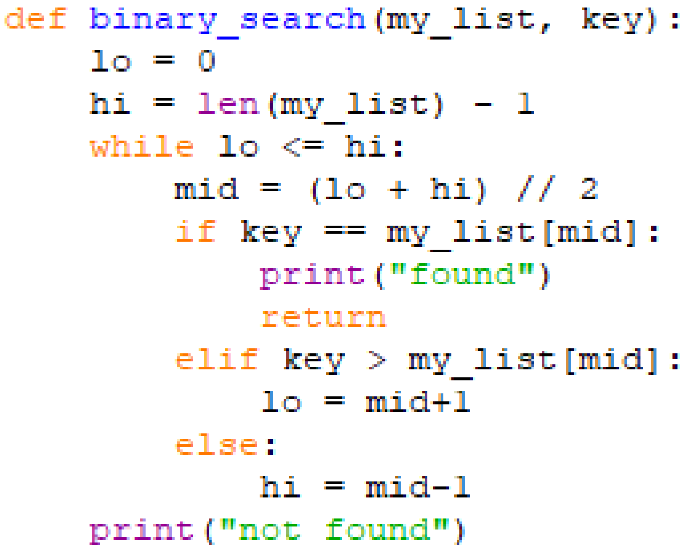

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Complexity

Auxiliary Space

- So what is the auxiliary space complexity for these then?
  - Both are O(1)
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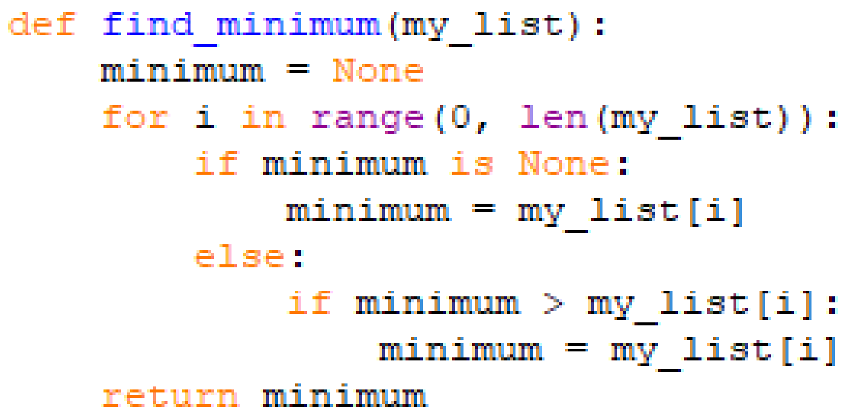
- Known as in-place
  - Can process in the input itself!



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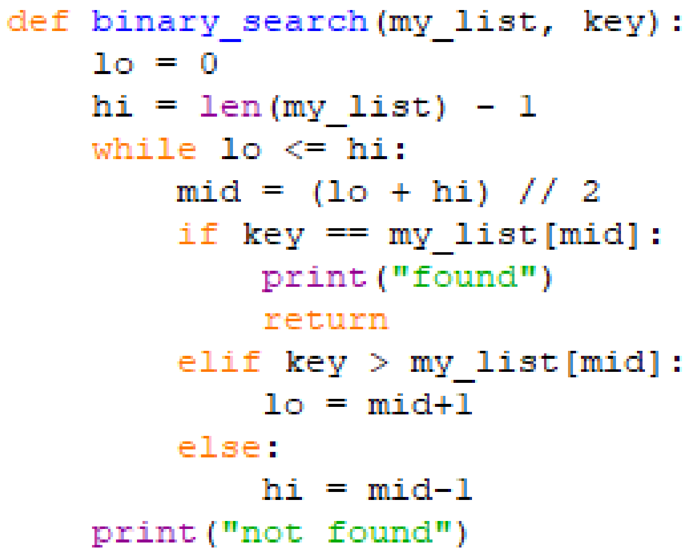

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Complexity

Auxiliary Space

- So what is the auxiliary space complexity for these then?
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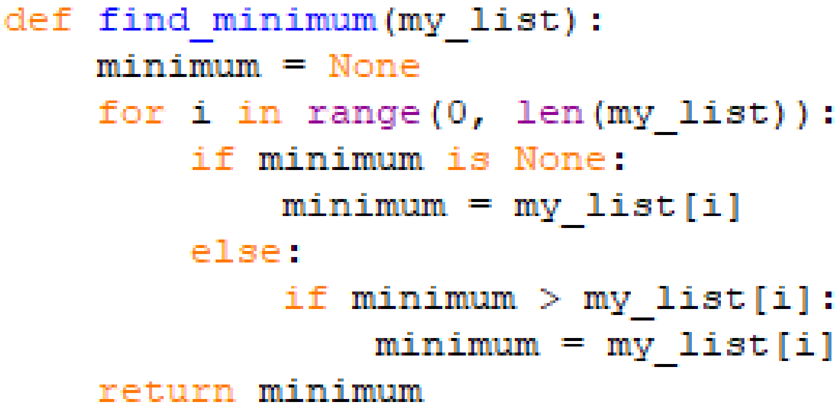
- Known as in-place
  - Can process in the input itself!
  - Auxiliary space of O(1)



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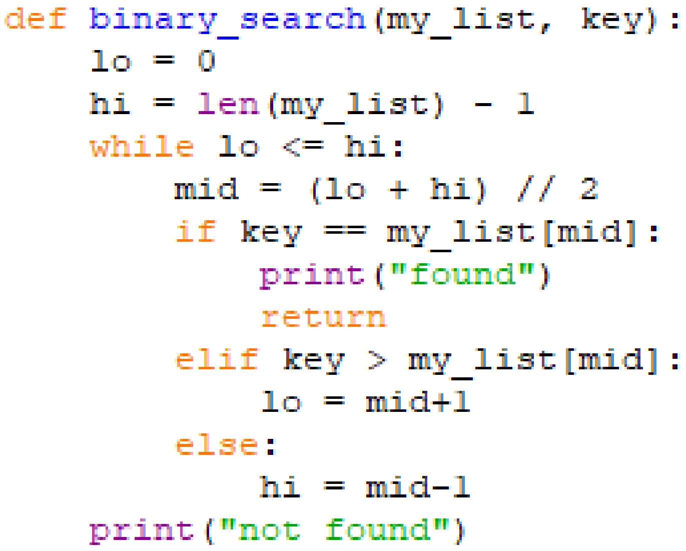

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Questions?

Thank You

Sorting

- We are back to sorting!
  - Bubble
  - Insertion
  - Selection
  - Merge
  - Quick

Sorting

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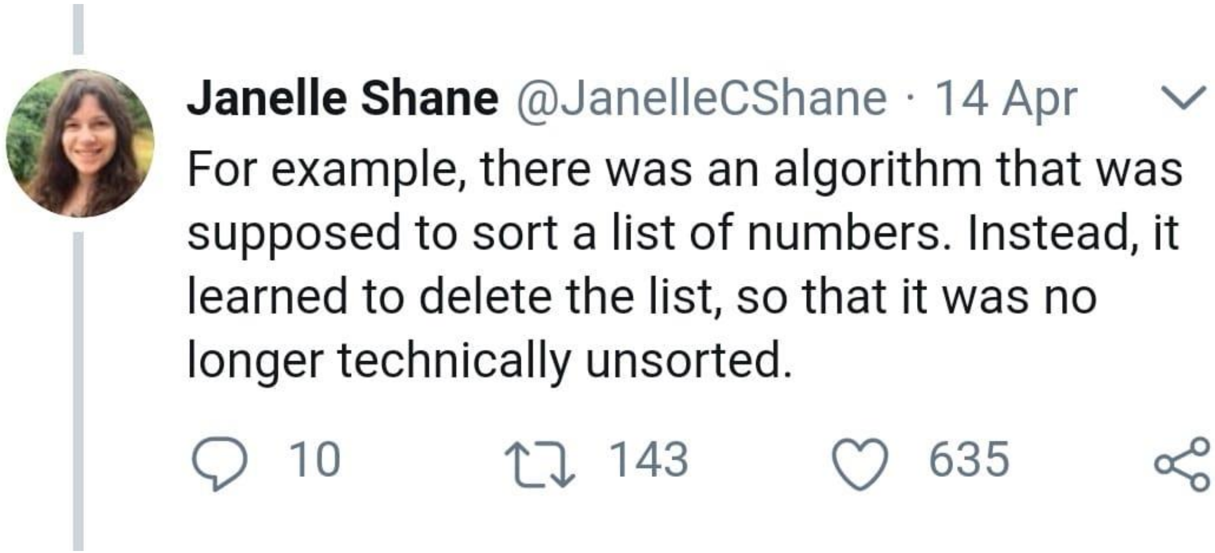

Janelle Shane @JanelleCShane · 14 Apr

For example, there was an algorithm that was supposed to sort a list of numbers. Instead, it learned to delete the list, so that it was no longer technically unsorted.

Sorting

- We are back to sorting!
  - Bubble
  - Insertion
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  - Merge
  - Quick

- All of these are known as comparison based sorting. Why?

Sorting

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- All of these are known as comparison based sorting. Why? Because we compare between items to know if a < b or b > a

Sorting

- We are back to sorting!
  - Bubble
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  - Selection
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- All of these are known as comparison based sorting. Why? Because we compare between items to know if
  a < b or b > a
- Now let us analyze them based on what we have learnt!

Questions?

Sorting

Selection Sort

- Correctness
- Complexity

Sorting

Selection Sort

- Correctness
  - Loop invariant
  - Termination
- Complexity
  - Time
  - Space

Sorting

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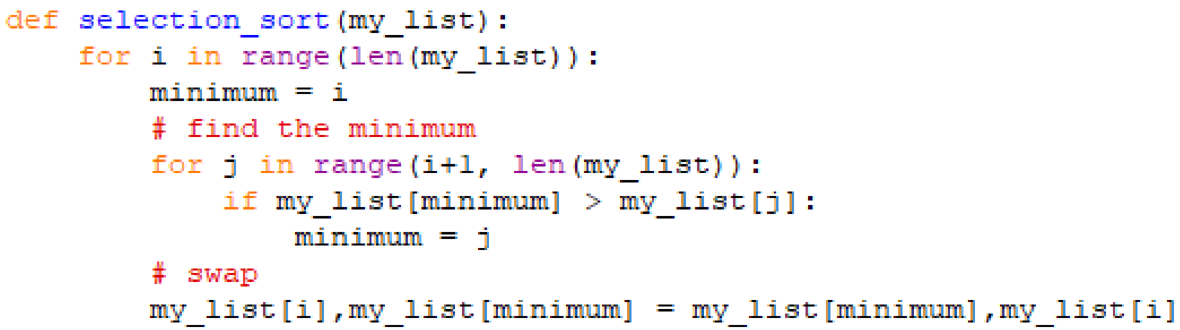

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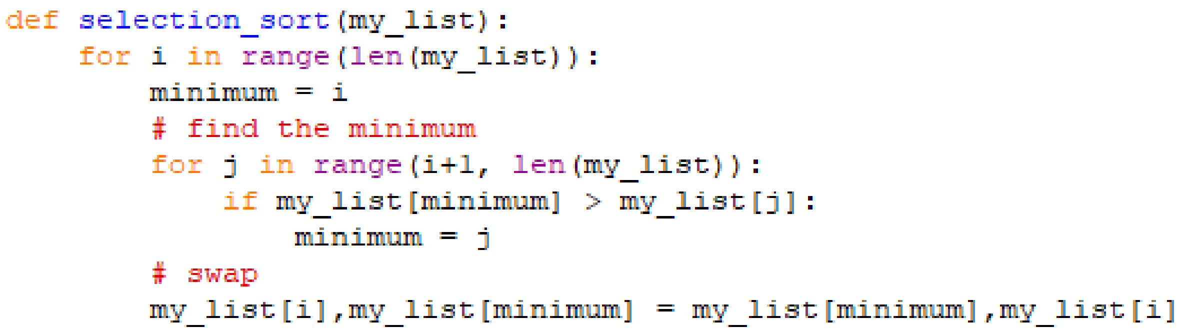
        my_list[i],my_list[minimum] = my_list[minimum],my_list[i]
```

Sorting

Selection Sort

- Correctness
  - Loop invariant
  - Termination



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Sorting

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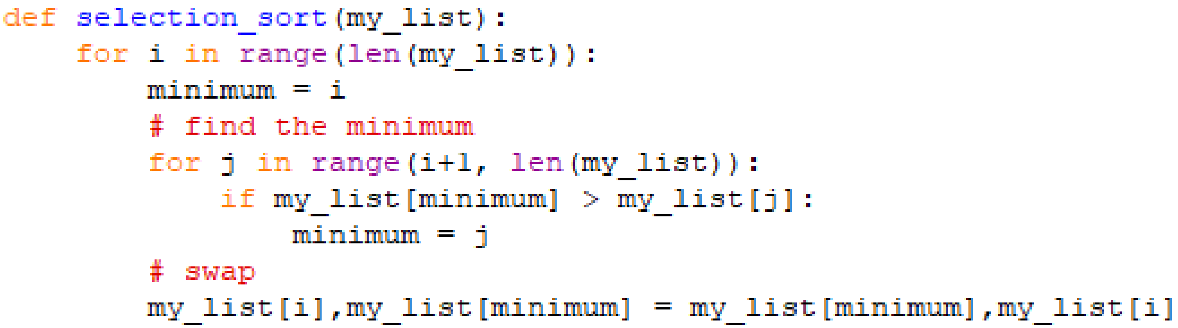

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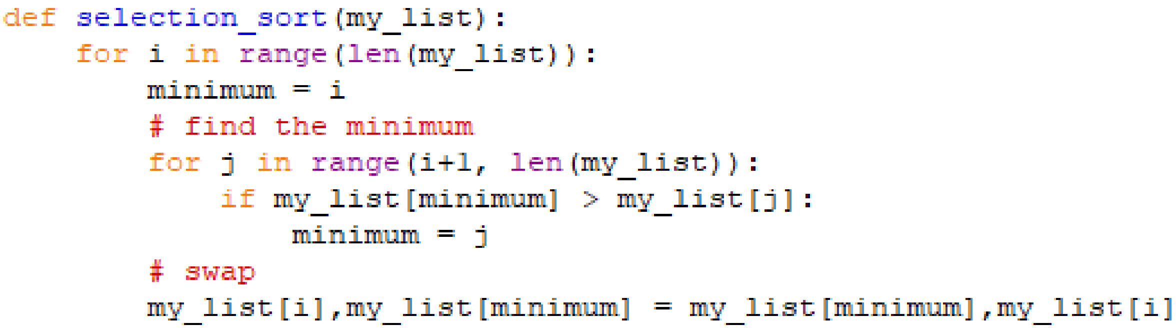
Sorting

Selection Sort

- Correctness
  - Loop invariant
    - my_list[0…i-1] is sorted
    - my_list[0…i-1] <= my_list[i…N]
  - Termination



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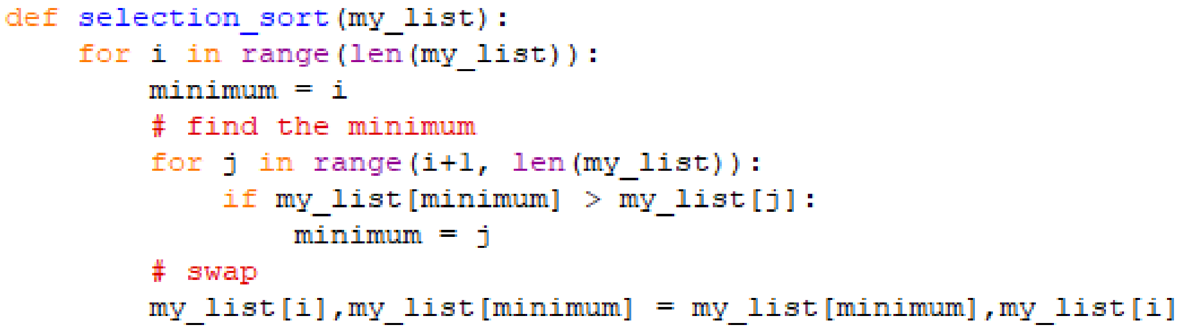

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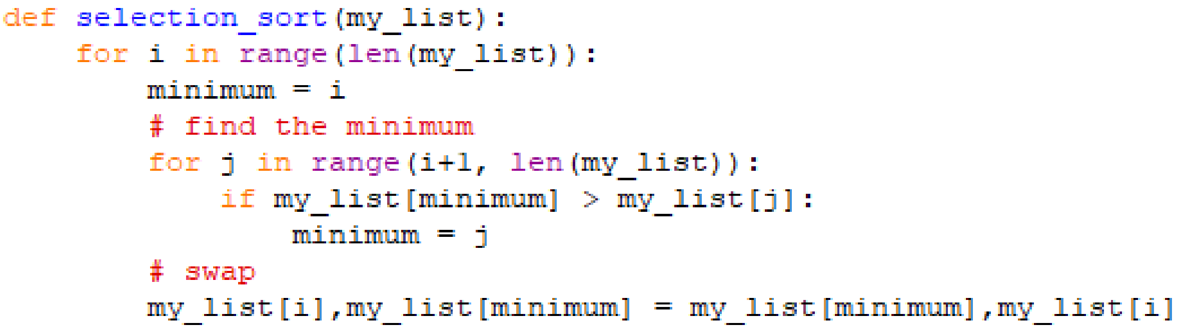
    - i and j always increment and both reach the end of the list



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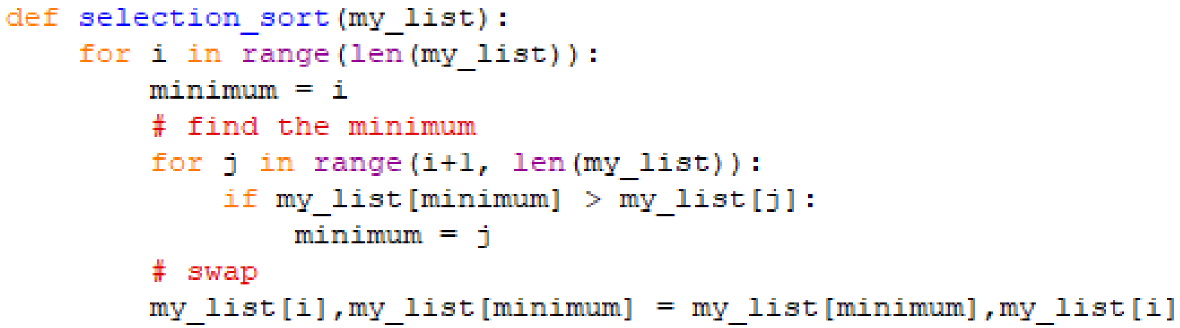
  - So why is it working then?



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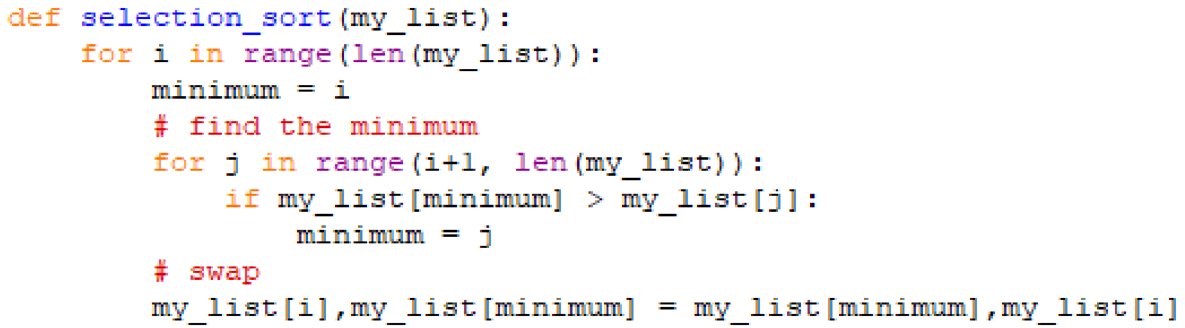
    - my_list[0…i-1] <= my_list[i…N]
  - Termination
    - i and j always increment and both reach the end of the list
  - So why is it working then?
    - i keep increment till n and we know from invariant 0…i-1 is sorted, thus we will sort the entire list!



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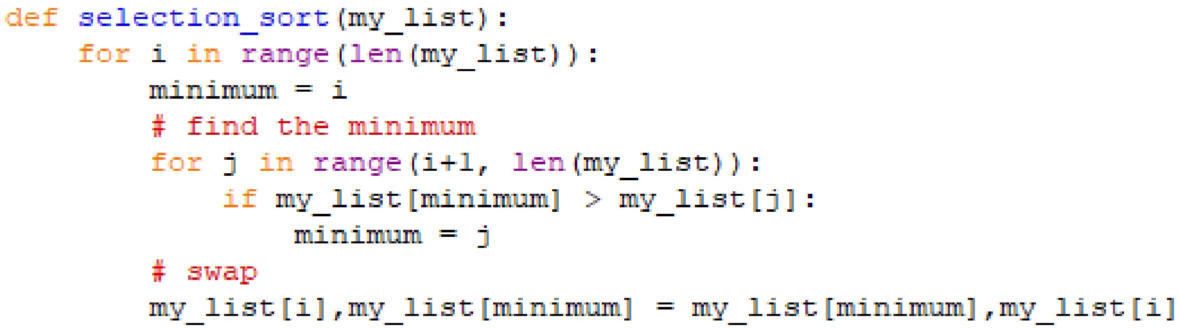
Sorting

Selection Sort

- Correctness
- Complexity
  - Time
  - Space



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Sorting

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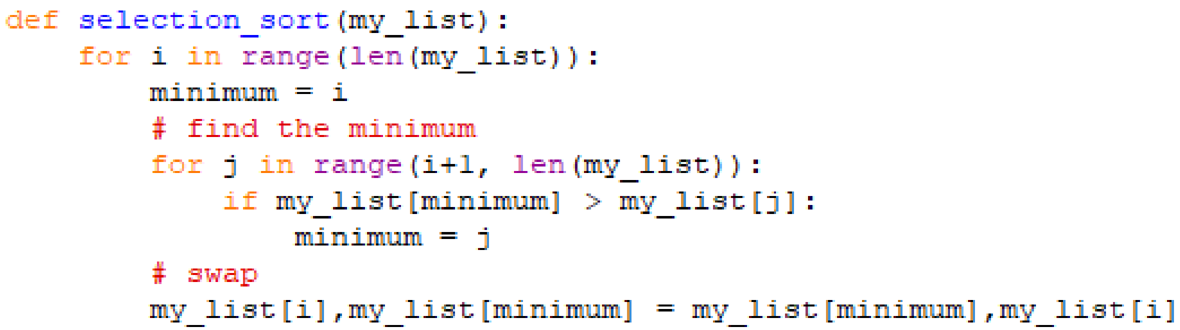

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Sorting

Selection Sort

- Correctness
- Complexity
  - Time
    - Best = O(N^2)
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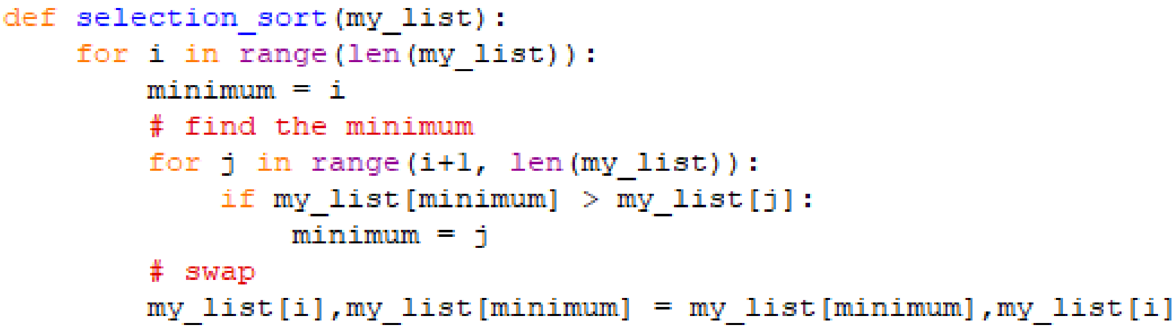
  - Space



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Sorting
Selection Sort

- Correctness
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    - Best = O(N^2) because no matter what we have to find the minimum and cant terminate earlier!
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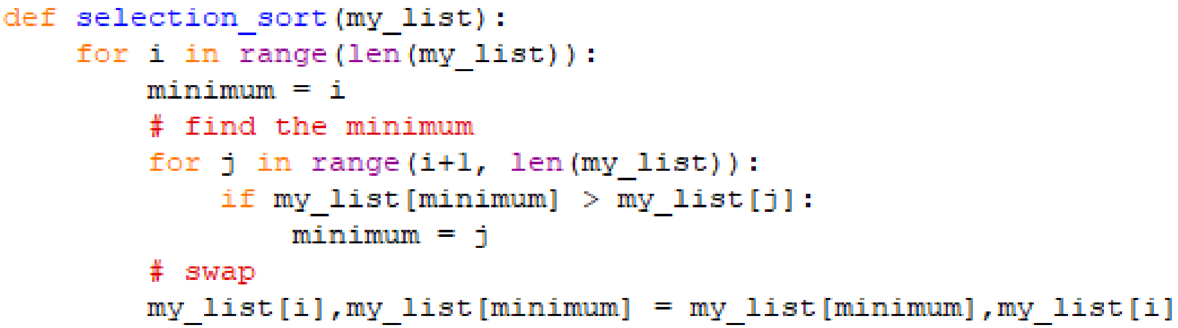
  - Space



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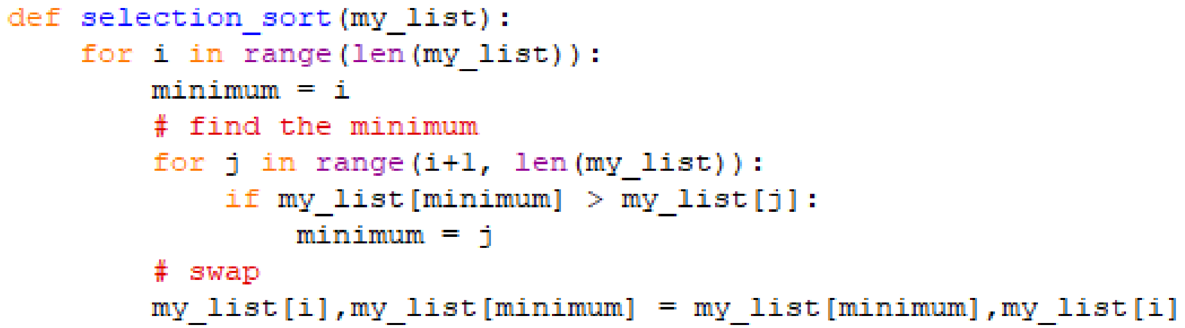

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Sorting

Selection Sort

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    - Best = O(N^2) because no matter what we have to find the minimum and cant terminate earlier!
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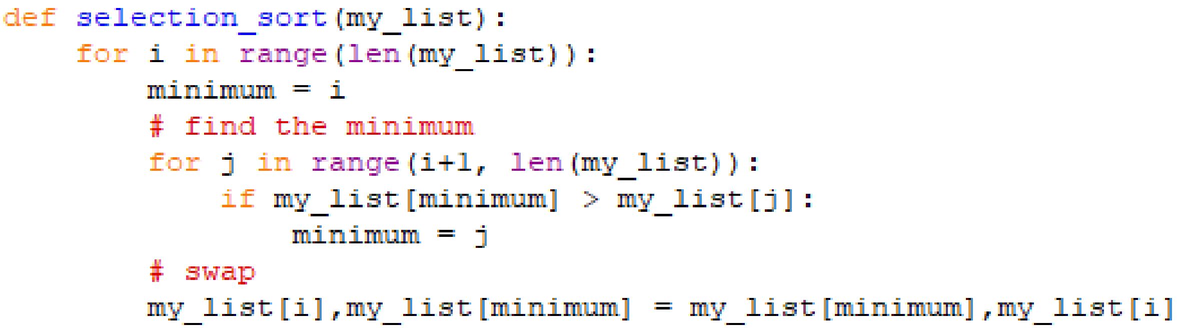
    - O(N) for the input list
    - Auxiliary?



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Sorting
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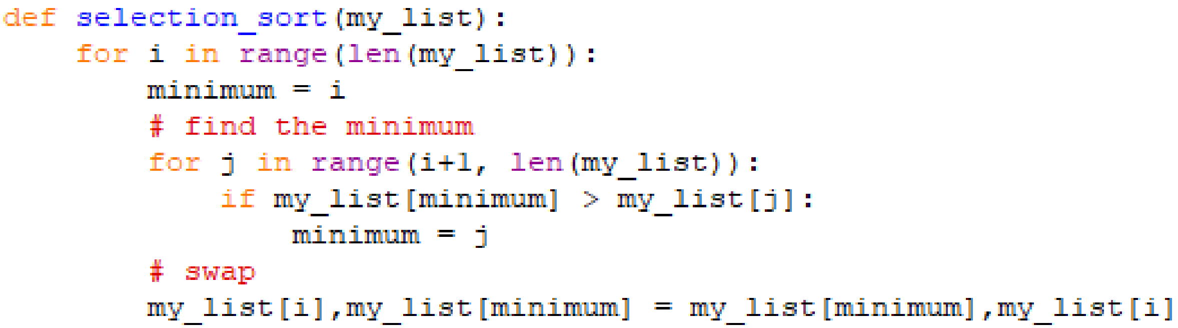

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Sorting

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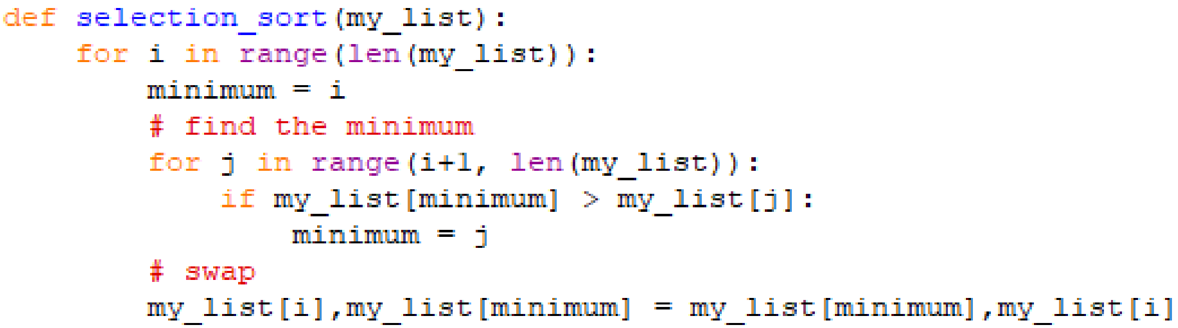
    - Worst = O(N^2)
  - Space
    - O(N) for the input list
    - Auxiliary? O(1) in place



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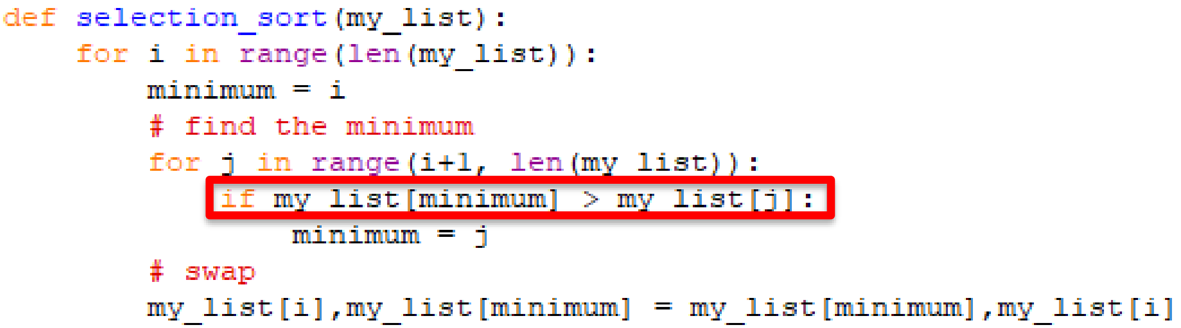
Sorting
Selection Sort

- Correctness
- Complexity
  - Time
    - Best = O(N^2) because no matter what we have to find the minimum and cant terminate earlier!
    - Worst = O(N^2)
    - But what if I tell you comparing the items have a cost of O(k)
      - Like comparing between words, you need to compare the alphabets



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Sorting
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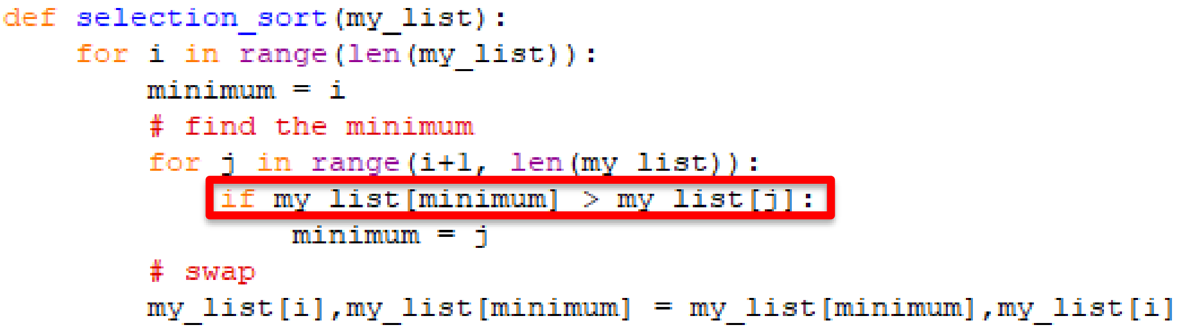
      - Like comparing between words, you need to compare the alphabets
      - We know complexity is based on comparison O(N^2) comparisons…



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Sorting

Selection Sort

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  - Time
    - Best = O(N^2) because no matter what we have to find the minimum and cant terminate earlier!
    - Worst = O(N^2)
    - But what if I tell you comparing the items have a cost of O(k)
      - Like comparing between words, you need to compare the alphabets
      - We know complexity is based on comparison O(N^2) comparisons…
      - So our final complexity is O(kN^2)

Sorting
Selection Sort

- Correctness
- Complexity
- Stable?

Sorting

Selection Sort

- Correctness
- Complexity
- Stable?
  - Relative ordering doesn’t change

Sorting

Selection Sort

- Correctness
- Complexity
- Stable?
  - Relative ordering doesn’t change
  - Is it stable?

Sorting

Selection Sort

- Correctness
- Complexity
- Stable?
  - Relative ordering doesn't change
  - Is it stable? No! but why?

Sorting

Selection Sort

- Correctness
- Complexity
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  - Is it stable? No! but why?
  - [4a, 2, 3, 4b, 1]

Sorting
Selection Sort

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- Stable?
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  - Minimum is 1, so we swap

Sorting

Selection Sort

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- Complexity
- Stable?
  - Relative ordering doesn’t change
  - Is it stable? No! but why?
  - [4a, 2, 3, 4b, 1]
  - Minimum is 1, so we swap
  - [1, 2, 3, 4b, 4a]

Sorting
Selection Sort

- Correctness
- Complexity
- Stable?
  - Relative ordering doesn’t change
  - Is it stable? No! but why?
  - [4a, 2, 3, 4b, 1]
  - Minimum is 1, so we swap
  - [1, 2, 3, 4b, 4a]
  - Now we see that 4a is behind 4b!

MONASH University

Questions?

Sorting

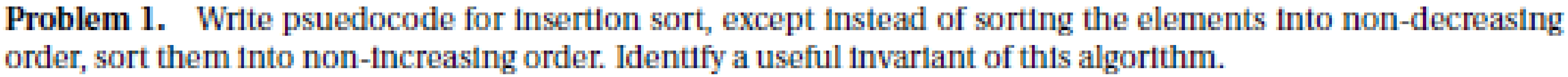
Insertion Sort

- Correctness
- Complexity

Sorting

Insertion Sort

- Correctness
- Complexity

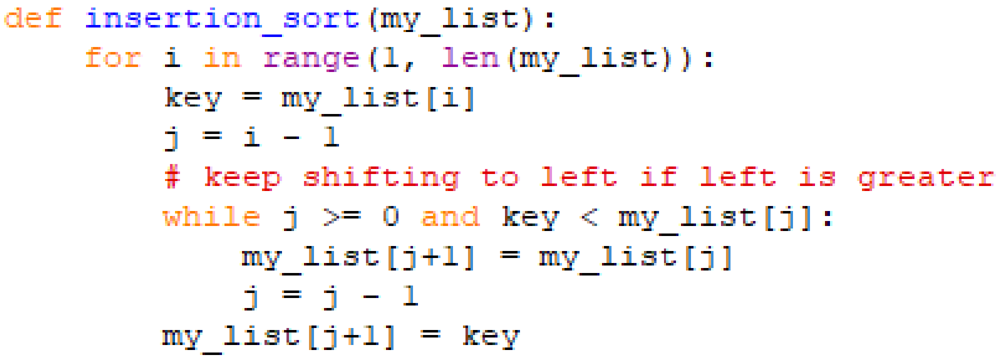


Problem 1. Write psuedocode for insertion sort, except instead of sorting the elements into non-decreasing order, sort them into non-increasing order. Identify a useful invariant of this algorithm.

Sorting

Insertion Sort

- Correctness
- Complexity



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        j = i - 1
        # keep shifting to left if left is greater
        while j >= 0 and key < my_list[j]:
            my_list[j+1] = my_list[j]
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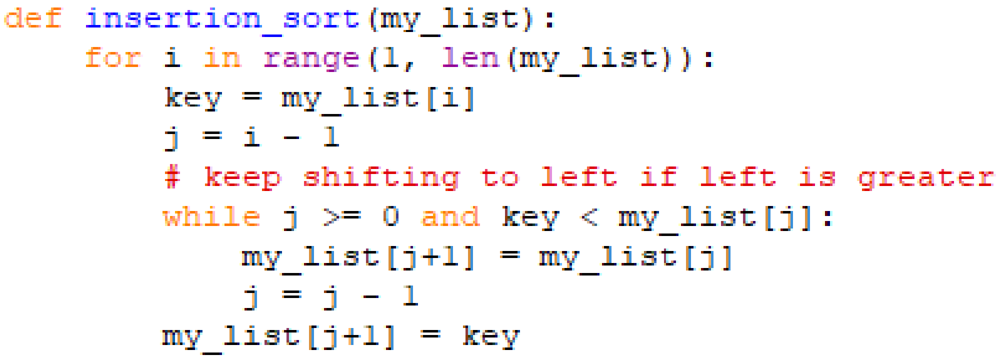
        my_list[j+1] = key
```

Sorting

Insertion Sort

- Correctness
  - Loop invariant
  - Termination
- Complexity



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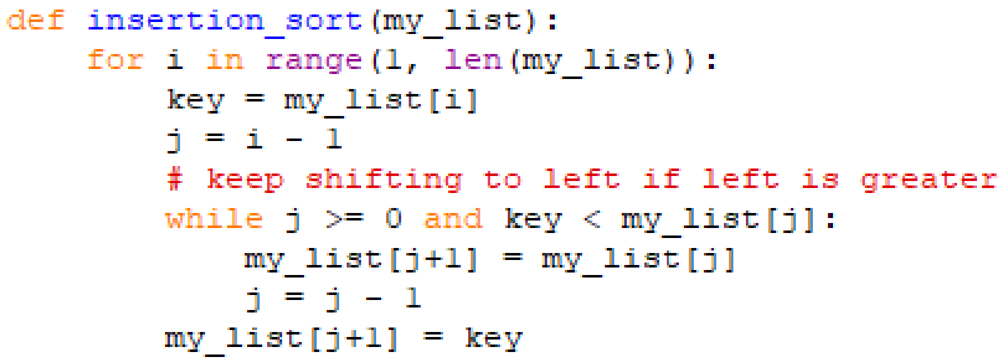
```

Sorting

Insertion Sort

- Correctness
  - Loop invariant
  - Termination
    - Simple, I skip this
- Complexity



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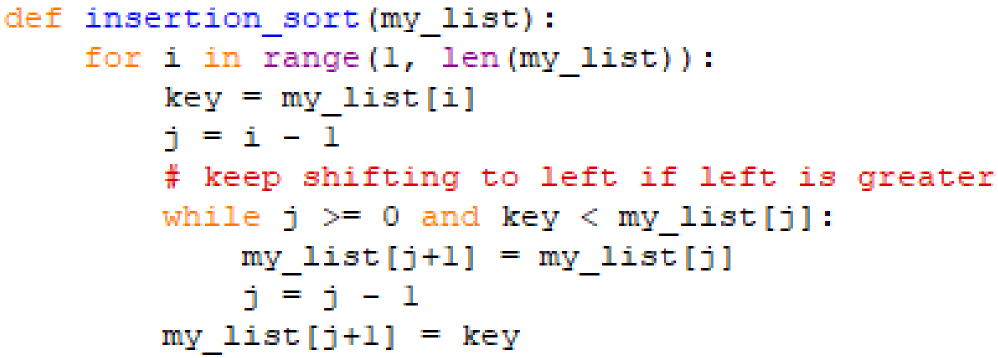
            j = j - 1
        my_list[j+1] = key
```

Sorting

Insertion Sort

- Correctness
  - Loop invariant
    - my_list[0…i-1] sorted
  - Termination
    - Simple, I skip this
- Complexity



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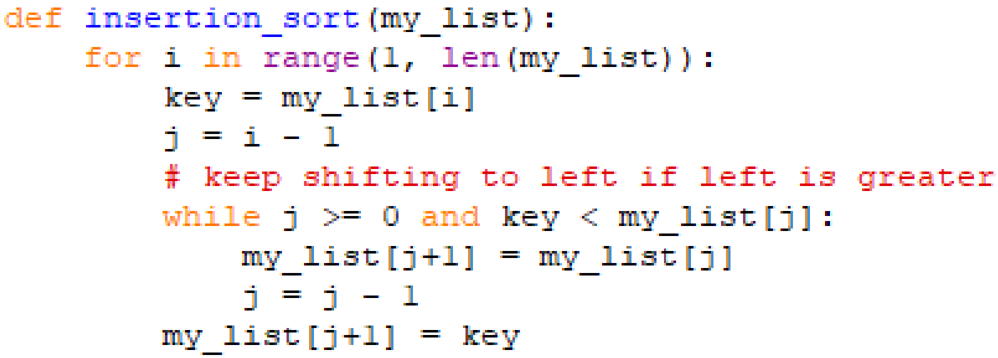
```

Sorting

Insertion Sort

- Correctness
- Complexity
  - Best
  - Worst



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            j = j - 1
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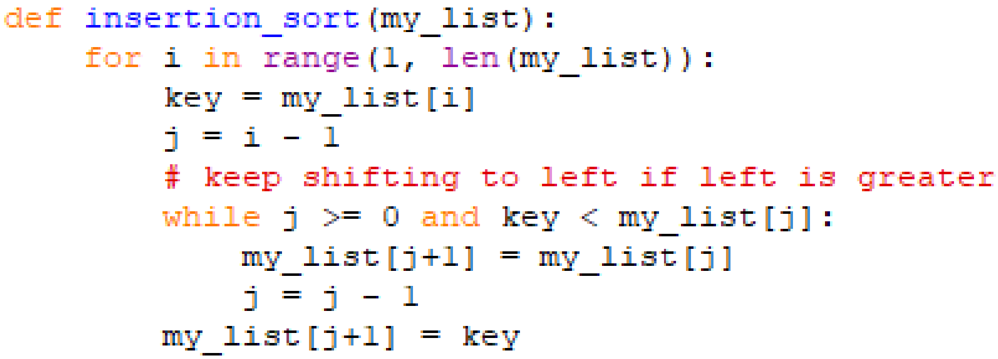
```

Sorting
Insertion Sort

- Correctness
- Complexity
  - Best O(N) comparison
    - Each loop only look and compare with left item once
  - Worst



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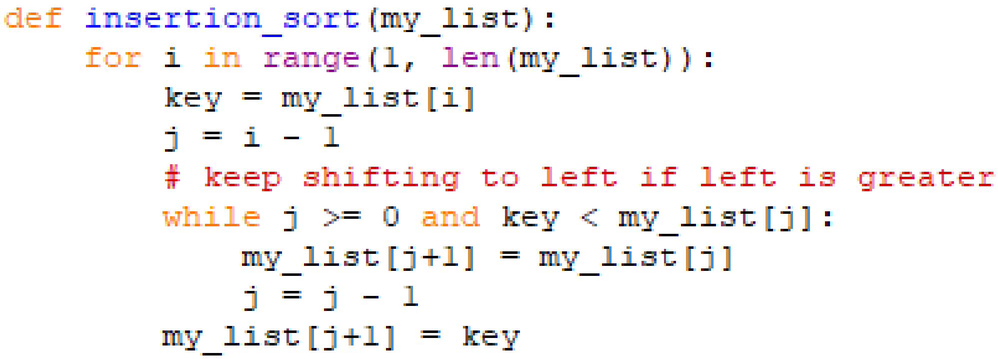
```

Sorting

Insertion Sort

- Correctness
- Complexity
  - Best O(N) comparison
    - Each loop only look and compare with left item once
  - Worst O(N^2)
    - Each loop keep look left, compare and swap till beginning of list



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Sorting

Insertion Sort

- Correctness
- Complexity
  - Best O(N) comparison
    - Each loop only look and compare with left item once
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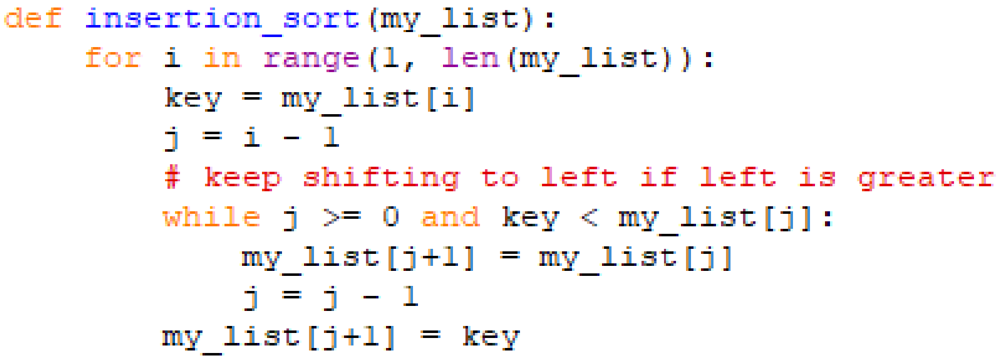
    - Each loop keep look left, compare and swap till beginning of list
  - So if O(k) is the comparison cost, when we have O(kN^2) worst case!



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Sorting

Insertion Sort

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  - Best O(N) comparison
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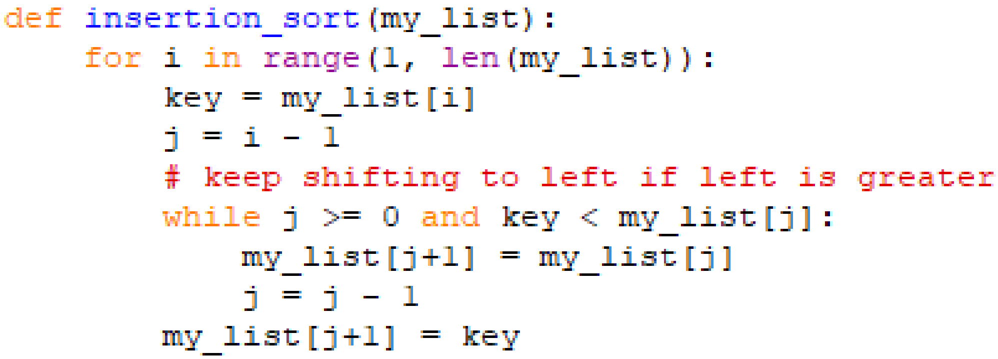
  - What about space?



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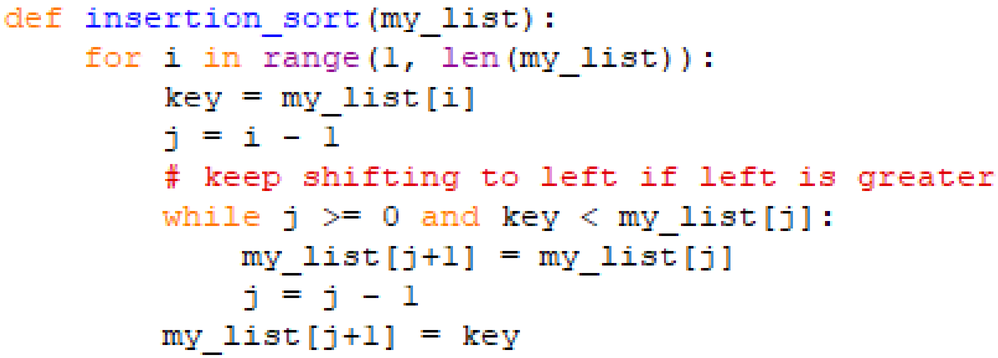
Sorting

Insertion Sort

- Correctness
- Complexity
  - Best O(N) comparison
    - Each loop only look and compare with left item once
  - Worst O(N^2)
    - Each loop keep look left, compare and swap till beginning of list
  - So if O(k) is the comparison cost, when we have O(kN^2) worst case!
  - What about space?
    - O(N) for the input list
    - O(1) auxiliary cause it is in-place



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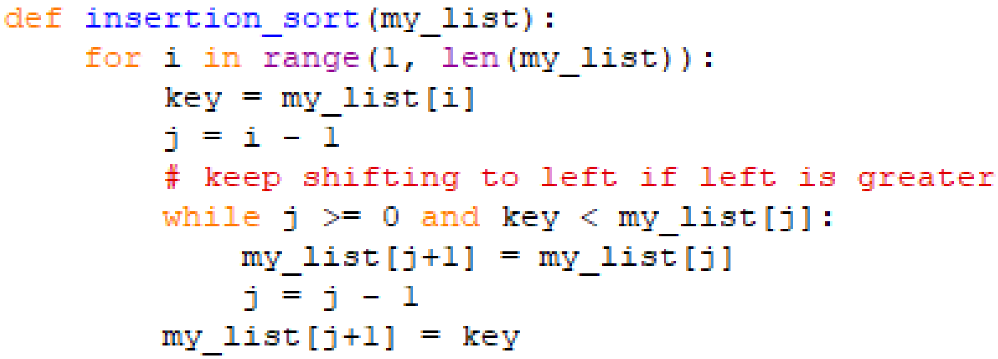
```

Sorting

Insertion Sort

- Correctness
- Complexity
- Stability



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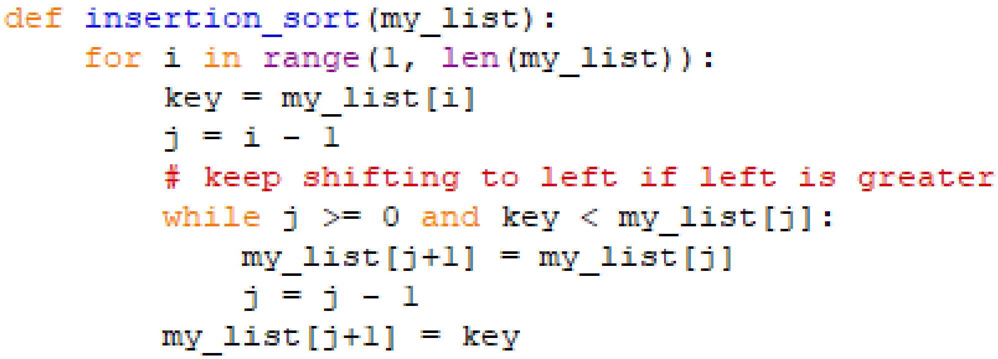
```

Sorting

Insertion Sort

- Correctness
- Complexity
- Stability
  - Yes



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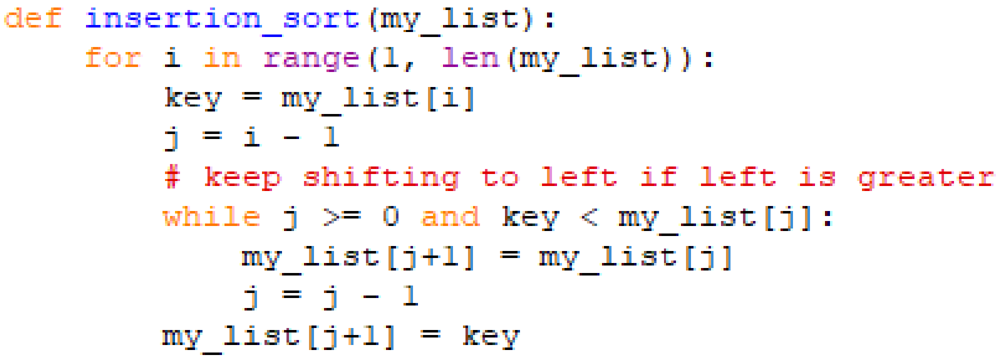
Sorting

Insertion Sort

- Correctness
- Complexity
- Stability
  - Yes
  - Don’t swap if value is the same



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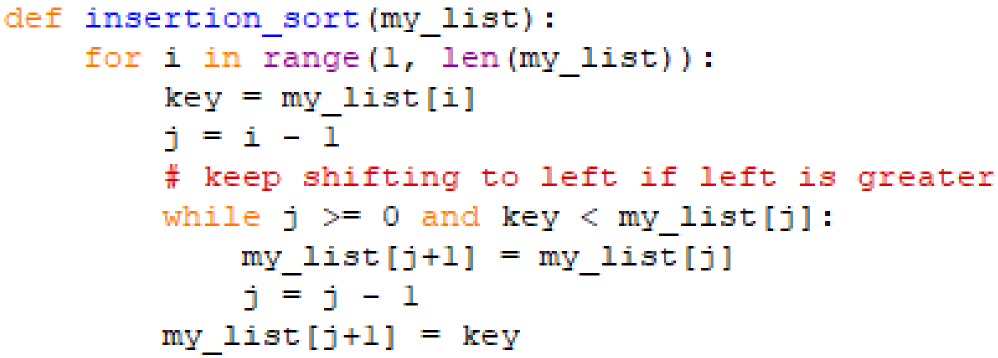
        my_list[j+1] = key
```

Sorting

Insertion Sort

- Correctness
- Complexity
- Stability
  - Yes
  - Don’t swap if value is the same
  - Most shifting will ensure stability



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```

Questions?

Summary

Sorting

	Best	Worst	Average	Stable?	In-place?
Selection Sort	$O(N^2)$	$O(N^2)$	$O(N^2)$	No	Yes
Insertion Sort	O(N)	$O(N^2)$	$O(N^2)$	Yes	Yes
Heap Sort	O(N log N)	O(N log N)	O(N log N)	No	Yes
Merge Sort	O(N log N)	O(N log N)	O(N log N)	Yes	No
Quick Sort	O(N log N)	$O(N^2)$ – can be made O(N log N)	O(N log N)	Depends	No

Summary

Sorting

	Best	Worst	Average		
Selection Sort	$O(N^2)$	$O(N^2)$	$O(N^2)$		
Insertion Sort	$O(N)$	$O(N^2)$	$O(N^2)$		
Heap Sort	O(N log N)	O(N log N)	O(N log N)		
Merge Sort	O(N log N)	O(N log N)	O(N log N)	Yes	No
Quick Sort	O(N log N)	$O(N^2)$ – can be made O(N log N)	O(N log N)	Depends	No

Summary

Auxiliary for Recursion

- The recursion stack takes up memory!!!

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  - So that is why it isn’t in place!

Summary

Auxiliary for Recursion

- The recursion stack takes up memory!!!
  - So that is why it isn't in place!
  - If I have recursion log N times, then I take O(log N) space for the recursion alone!
  - If each recursion is k, then my total space is O(k log N)!!!

Summary

Auxiliary for Recursion

- The recursion stack takes up memory!!!
  - So that is why it isn't in-place!
    - Iterative is easier to get in-place
  - If I have recursion log N times, then I take O(log N) space for the recursion alone!
  - If each recursion is k, then my total space is O(k log N)!!!

Summary

Sorting

	Best	Worst	Average	Stable?	In-place?
Selection Sort	$O(N^2)$	$O(N^2)$	$O(N^2)$	No	Yes
Insertion Sort	$O(N)$	$O(N^2)$	$O(N^2)$	Yes	Yes
Heap Sort	$O(N \log N)$	$O(N \log N)$	$O(N \log N)$	No	Yes
Merge Sort	$O(N \log N)$	$O(N \log N)$	$O(N \log N)$	Yes	No
Quick Sort	$O(N \log N)$	$O(N^2)$ – can be made $O(N \log N)$	$O(N \log N)$	Depends	No

Complexity

Time – Lower Bound

- So… what is the lower bound for the sorting algorithms that we have learnt?
  - Bubble
  - Insertion
  - Selection
  - Quick
  - Merge

- These are all comparison based
- Ω(N log N)
- We will see more of this later

Questions?

Have a break!

Sorting

Non-Comparison

- We can sort without comparing elements in a list!

Sorting

Non-Comparison

- We can sort without comparing elements in a list!
  - Counting sort
  - Radix sort

Questions?

Counting Sort

- Very simple concept
- I am sure we all know this…

- Now let us begin with a list

4	2	1	3	1	4	5

Counting Sort

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- I am sure we all know this…

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4	2	1	3	1	4	5

- What is the maximum number?

Counting Sort

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4	2	1	3	1	4	5

- What is the maximum number?
  - 5 but how do we know?

Counting Sort

- Very simple concept
- I am sure we all know this…

- Now let us begin with a list

4	2	1	3	1	4	5

- What is the maximum number?
  - 5 but how do we know? Loop through the list in O(N)

Counting Sort

- Our input

4	2	1	3	1	4	5

- We know max is 5

Counting Sort

- Our input

4	2	1	3	1	4	5

Anyone noticed
the list is crooked?
#OCDtrigger

- We know max is 5

Counting Sort

- Our input

4	2	1	3	1	4	5

- We know max is 5

0	1	2	3	4	5

Counting Sort

- Our input

4	2	1	3	1	4	5

- We know max is 5

0	1	2	3	4	5

Counting Sort

- Out input

4	2	1	3	1	4	5

- We know max is 5

0	1	2	3	4	5
0	0	0	0	0	0

Counting Sort

- Our input

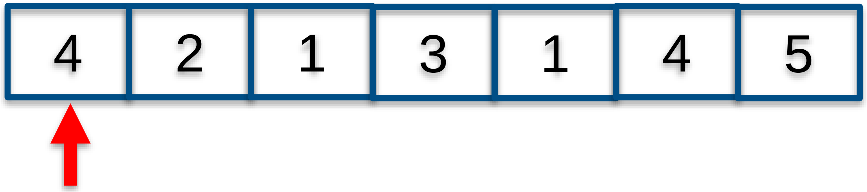
4	2	1	3	1	4	5

- We know max is 5

0	1	2	3	4	5
0	0	0	0	0	0

Counting Sort

- Our input



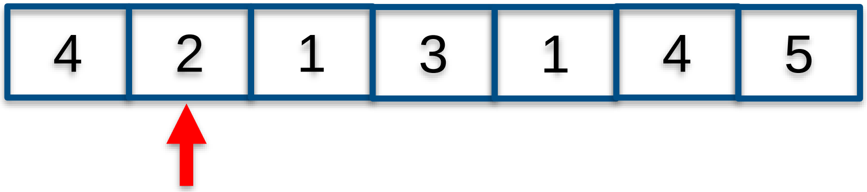
4	2	1	3	1	4	5

- We know max is 5

0	1	2	3	4	5
0	0	0	0	1	0

Counting Sort

- Our input



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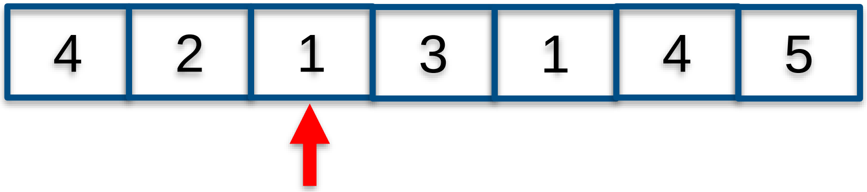
Counting Sort

- Out input



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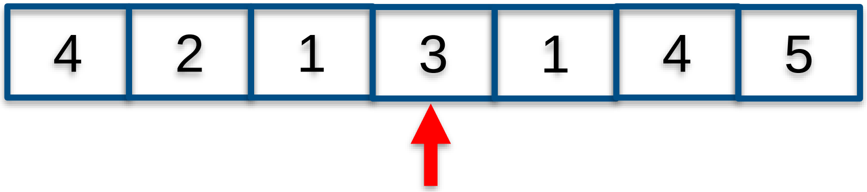
Counting Sort

- Our input



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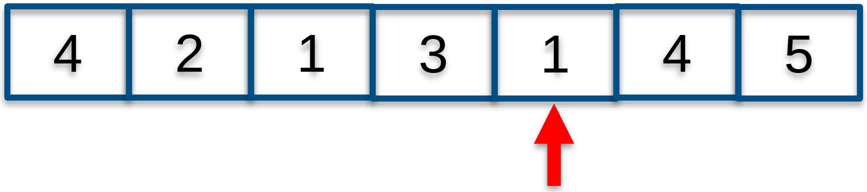
Counting Sort

- Our input



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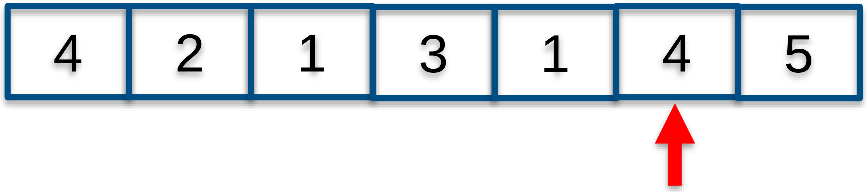
Counting Sort

- Our input



4	2	1	3	1	4	5

- We know max is 5

0	1	2	3	4	5
0	2	1	1	2	0

Counting Sort

- Our input

4	2	1	3	1	4	5

- We know max is 5

0	1	2	3	4	5
0	2	1	1	2	1

Counting Sort

- Our input

4	2	1	3	1	4	5

- We know max is 5

0	1	2	3	4	5	ItemID
0	2	1	1	2	1	Frequency

Counting Sort

- Our input

4	2	1	3	1	4	5

- We know max is 5

0	1	2	3	4	5	ItemID
0	2	1	1	2	1	Frequency

- So how do we sort it now then?

Counting Sort

- Our input

- We know max is 5

0	1	2	3	4	5	ItemID
0	2	1	1	2	1	Frequency

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Counting Sort

- Our input

- We know max is 5

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Counting Sort

- Our input

- We know max is 5

0	1	2	3	4	5	ItemID
0	2	1	1	2	1	Frequency

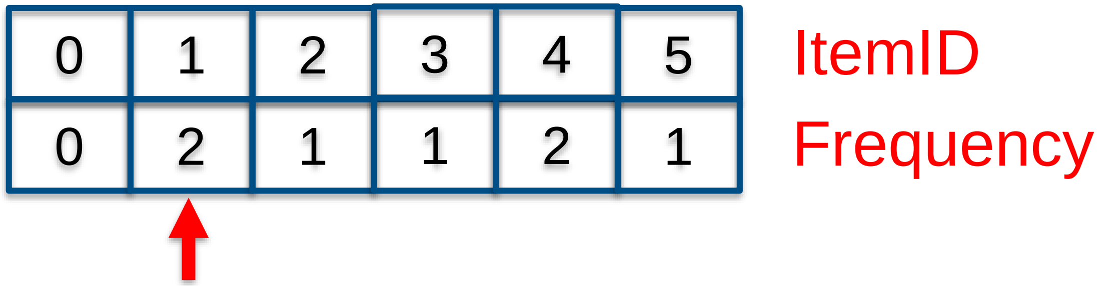
- So how do we sort it now then?

Counting Sort

- Our input

1	1					

- We know max is 5



0	1	2	3	4	5	ItemID
0	2	1	1	2	1	Frequency

- So how do we sort it now then?

Counting Sort

- Our input

1	1	2				

- We know max is 5

0	1	2	3	4	5	ItemID
0	2	1	1	2	1	Frequency

- So how do we sort it now then?

Counting Sort

- Our input

1	1	2	3			

- We know max is 5

0	1	2	3	4	5	ItemID
0	2	1	1	2	1	Frequency

- So how do we sort it now then?

Counting Sort

- Our input

1	1	2	3	4	4	

- We know max is 5

0	1	2	3	4	5	ItemID
0	2	1	1	2	1	Frequency

- So how do we sort it now then?

Counting Sort

- Our input

1	1	2	3	4	4	5

- We know max is 5

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0	2	1	1	2	1	Frequency

- So how do we sort it now then?

Counting Sort

- Our input

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- We know max is 5

0	1	2	3	4	5	ItemID
0	2	1	1	2	1	Frequency

- So how do we sort it now then?

Counting Sort
Complexity

- Time?

Counting Sort

Complexity

- Time?
  - Find the maximum O(N)

Counting Sort

Complexity

- Time?
  - Find the maximum O(N)
  - Build the count-array O(M) where M is the max

Counting Sort

Complexity

- Time?
  - Find the maximum O(N)
  - Build the count-array O(M) where M is the max
  - Go through input list and update the count-array

Counting Sort

Complexity

- Time?
  - Find the maximum O(N)
  - Build the count-array O(M) where M is the max
  - Go through input list and update the count-array
    - How to make it fast?

Counting Sort

Complexity

- Time?
  - Find the maximum O(N)
  - Build the count-array O(M) where M is the max
  - Go through input list and update the count-array
    - How to make it fast?

0	1	2	3	4	5	Index
0	2	1	1	2	1	Frequency

Counting Sort

Complexity

- Time?
  - Find the maximum O(N)
  - Build the count-array O(M) where M is the max
  - Go through input list and update the count-array
    - How to make it fast?
    - Therefore this is O(N) since we can have O(1) access to the count-array

Counting Sort

Complexity

- Time?
  - Find the maximum O(N)
  - Build the count-array O(M) where M is the max
  - Go through input list and update the count-array
    - How to make it fast?
    - Therefore this is O(N) since we can have O(1) access to the count-array
  - Loop through count-array to rebuild the original list O(M)

Counting Sort

Complexity

- Time?
  - Find the maximum O(N)
  - Build the count-array O(M) where M is the max
  - Go through input list and update the count-array
    - How to make it fast?
    - Therefore this is O(N) since we can have O(1) access to the count-array
  - Loop through count-array to rebuild the original list O(M)
  - Total = O(N + M + N + M) = O(N+M)

Counting Sort

Complexity

- Time?
  - Find the maximum O(N)
  - Build the count-array O(M) where M is the max
  - Go through input list and update the count-array
    - How to make it fast?
    - Therefore this is O(N) since we can have O(1) access to the count-array
  - Loop through count-array to rebuild the original list O(M)
  - Total = O(N + M + N + M) = O(N+M)

  - So we want M << N for this to be good
    - Else even N log N < M

Counting Sort
Complexity

- Time?
  - Find the maximum O(N)
  - Build the count-array O(M) where M is the max
  - Go through input list and update the count-array
    - How to make it fast?
    - Therefore this is O(N) since we can have O(1) access to the count-array
  - Loop through count-array to rebuild the original list O(M)
  - Total = O(N + M + N + M) = O(N+M)

  - So we want M << N for this to be good
  - If we are doing alphabets only, then the M = 26 for the 26 character (after ascii conversion + maths)

MONASH University

Questions?

Counting Sort

Complexity

- Space?

Counting Sort

Complexity

- Space?
  - Input list O(N)
  - Count-array O(M)

Counting Sort

Complexity

- Space?
  - Input list O(N)
  - Count-array O(M)

  - Total = O(N + M)
  - Auxiliary = O(M)

Questions?

Counting Sort

- Live programming session
- Let us try to code this since it is simple…

Counting Sort

- Live programming session
- Let us try to code this since it is simple…

- I will start writing the first part
  - You try to add in your own codes and compare at each step

Questions?

Counting Sort

Issue…

- Now imagine the following:

200	151	291	981	369	421	671

Counting Sort

Issue…

- Now imagine the following:

200	151	291	981	369	421	671

  – What is my complexity?

Counting Sort

Issue…

- Now imagine the following:

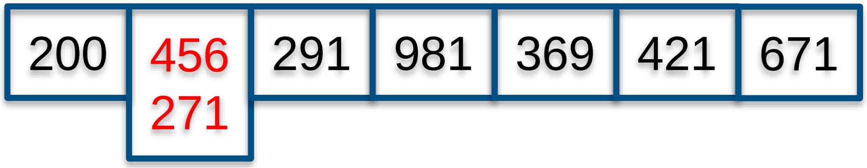
200	151	291	981	369	421	671

  - What is my complexity?
    - Time…
    - Space…

Counting Sort

Issue…

- Now imagine the following:



200	456 271	291	981	369	421	671

  - What is my complexity?
    - Time…
    - Space…

  - What if one of the value is LARGE

Counting Sort

Issue…

- Now imagine the following:

200	456 271	291	981	369	421	671

  - What is my complexity?
    - Time…
    - Space…

  - What if one of the value is LARGE

M is large!!!

Counting Sort

Issue…

- Now imagine the following:

200	151	291	981	369	421	671

  – What is my complexity?
    - Time…
    - Space…

  – Let us leave it at it is first…

Questions?

Counting Sort

- Stable?

Counting Sort

- Stable?
  - No
  - We only remember the frequency

Counting Sort

- Stable?
  - No
  - We only remember the frequency

- But can we make it stable?

Counting Sort

- Stable?
  - No
  - We only remember the frequency

- But can we make it stable?
  - Yes but at the cost of memory

Counting Sort

4a	2	1a	3	1b	4b	5

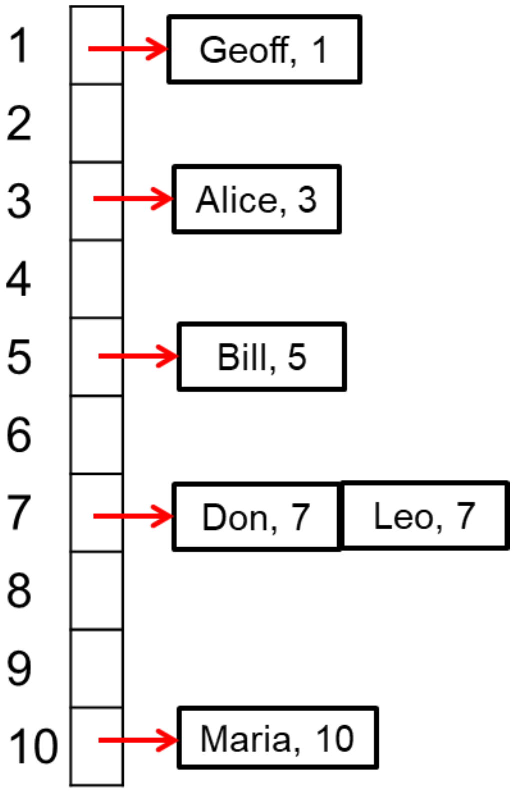
0	1	2	3	4	5	Index
	1a	2	3	4a	5	Frequency
	1b			4b		

Counting Sort

- Stable?
  - No
  - We only remember the frequency
- But can we make it stable?
  - Yes but at the cost of memory
  - Similar to separate chaining

Counting Sort

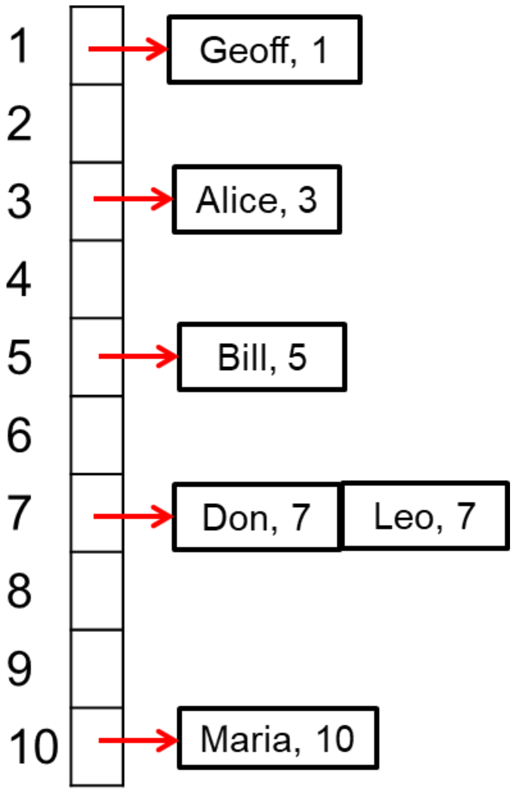
- Stable?
  - No
  - We only remember the frequency
- But can we make it stable?
  - Yes but at the cost of memory
  - Similar to separate chaining



Marks	3	5	7	1	7	10
Name	Alice	Bill	Don	Geoff	Leo	Maria

Counting Sort

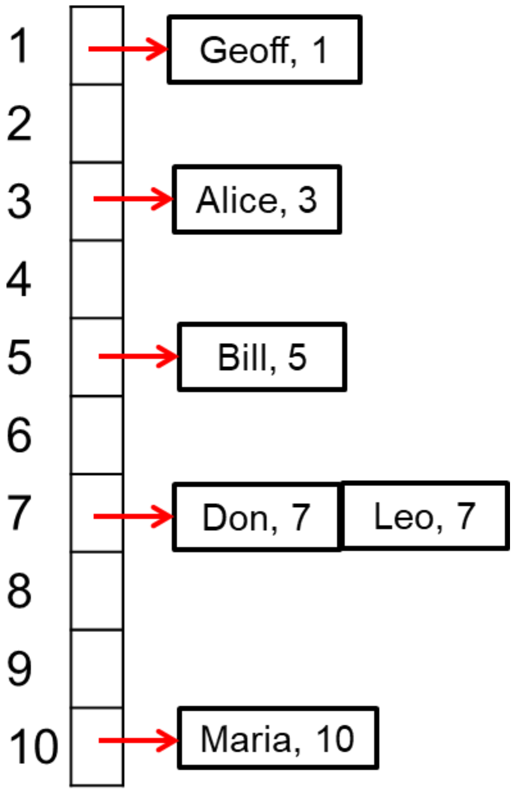
- Stable?
  - No
  - We only remember the frequency
- But can we make it stable?
  - Yes but at the cost of memory
  - Similar to separate chaining
  - At most we have N items only anyways
    - So it is O(M + N) space still



Marks	3	5	7	1	7	10
Name	Alice	Bill	Don	Geoff	Leo	Maria

Counting Sort

- Stable?
  - No
  - We only remember the frequency
- But can we make it stable?
  - Yes but at the cost of memory
  - Similar to separate chaining
  - At most we have N items only anyways
    - So it is O(M + N) space still
    - Can you see why?



Marks	3	5	7	1	7	10
Name	Alice	Bill	Don	Geoff	Leo	Maria

Counting Sort

- Stable?
  - No
  - We only remember the frequency
- But can we make it stable?
  - Yes but at the cost of memory
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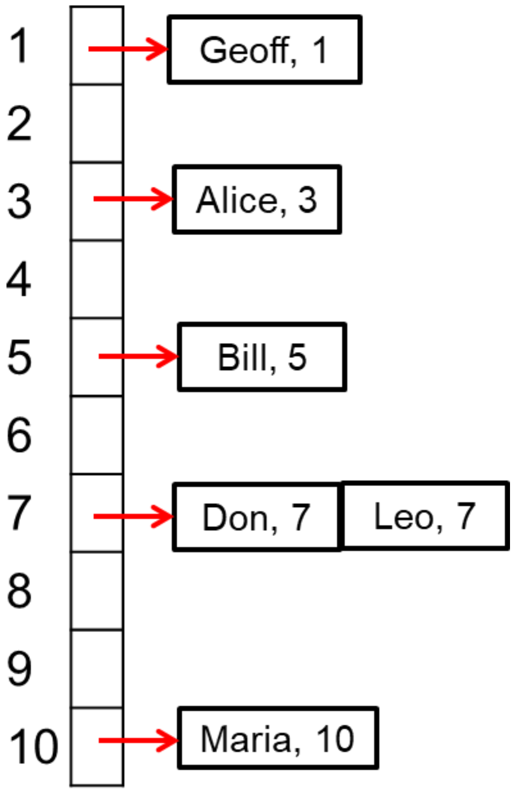
    - Can you see why?



1 → Geoff, 1

2

3 → Alice, 3

4

5 → Bill, 5

6

7 → Don, 7 → Leo, 7

8

9

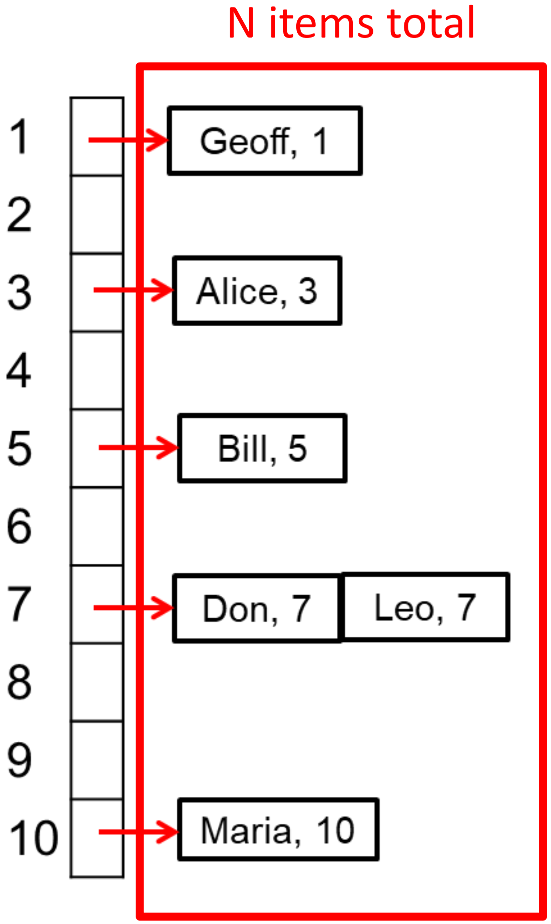
10 → Maria, 10

Marks	3	5	7	1	7	10
Name	Alice	Bill	Don	Geoff	Leo	Maria

N items

Counting Sort

- Stable?
  - No
  - We only remember the frequency
- But can we make it stable?
  - Yes but at the cost of memory
  - Similar to separate chaining
  - At most we have N items only anyways
    - So it is O(M + N) space still
    - Can you see why?



Marks	3	5	7	1	7	10
Name	Alice	Bill	Don	Geoff	Leo	Maria

N items

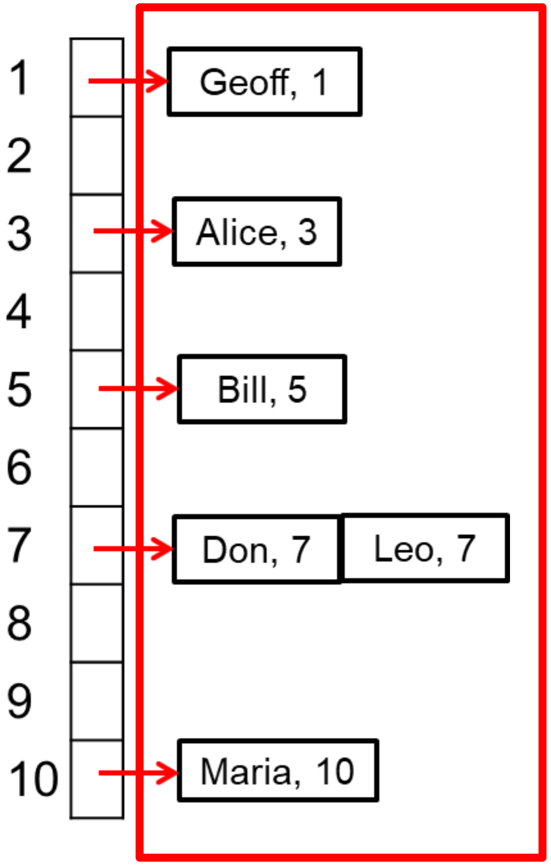
Counting Sort

- Stable?
  - No
  - We only remember the frequency

- But can we make it stable?
  - Yes but at the cost of memory
  - Similar to separate chaining
  - At most we have N items only anyways
    - So it is O(M + N) space still
    - Can you see why?

Not O(N*M)

N items total



Marks	3	5	7	1	7	10
Name	Alice	Bill	Don	Geoff	Leo	Maria

N items

Counting Sort

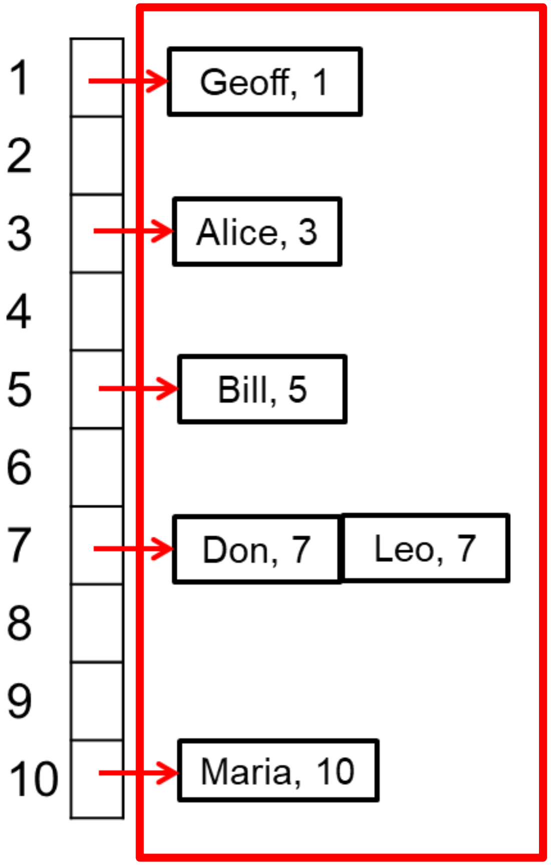
- Stable?
  - No
  - We only remember the frequency

VERY COMMON MISCONCEPTION

- But can we make it stable?
  - Yes but at the cost of memory
  - Similar to separate chaining
  - At most we have N items only anyways
    - So it is O(M + N) space still
    - Can you see why?

Not O(N*M)

N items total



Marks	3	5	7	1	7	10
Name	Alice	Bill	Don	Geoff	Leo	Maria

N items

Questions?

Counting Sort

- Stable?
  - No
  - We only remember the frequency

- But can we make it stable?
  - Yes but at the cost of memory
  - Similar to separate chaining
  - There is another way, refer to Nathan's amazing slide

Stable Counting Sort (Method 1)

Input

(3,a)	**(1,p)**	**(3,c)**	**(7,f)**	**(5,g)**	**(3,b)**	**(7,d)**	**(8,w)**

Construct count:

- For each key in input,
- count[key] += 1

count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

Output

1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	**(1,p)**	**(3,c)**	**(7,f)**	**(5,g)**	**(3,b)**	**(7,d)**	**(8,w)**

Construct count:
- For each key in input,
- count[key] += 1

Construct position:
- Initialise first position as a 1

count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	1
2	0
3	0
4	0
5	0
6	0
7	0
8	0

Output

1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	**(1,p)**	**(3,c)**	**(7,f)**	**(5,g)**	**(3,b)**	**(7,d)**	**(8,w)**

Construct count:
- For each key in input,
- count[key] += 1

Construct position:
- Initialise first position as a 1
- position[i] = position[i-1] + count[i-1]

count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	1
2	2
3	0
4	0
5	0
6	0
7	0
8	0

Output

1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	(1,p)	(3,c)	(7,f)	(5,g)	(3,b)	(7,d)	(8,w)

Construct count:

- For each key in input,
- count[key] += 1

Construct position:

- Initialise first position as a 1
- position[i] = position[i-1] + count[i-1]

count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	1
2	2
3	2
4	0
5	0
6	0
7	0
8	0

Output

1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	**(1,p)**	**(3,c)**	**(7,f)**	**(5,g)**	**(3,b)**	**(7,d)**	**(8,w)**

Construct count:

- For each key in input,
- count[key] += 1

Construct position:

- Initialise first position as a 1
- position[i] = position[i-1] + count[i-1]

count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	1
2	2
3	2
4	5
5	0
6	0
7	0
8	0

Output

1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	**(1,p)**	**(3,c)**	**(7,f)**	**(5,g)**	**(3,b)**	**(7,d)**	**(8,w)**

Construct count:

- For each key in input,
- count[key] += 1

Construct position:

- Initialise first position as a 1
- position[i] = position[i-1] + count[i-1]

	count	position
1	1	1
2	0	2
3	3	2
4	0	5
5	1	5
6	0	0
7	2	0
8	1	0

Output

1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	**(1,p)**	**(3,c)**	**(7,f)**	**(5,g)**	**(3,b)**	**(7,d)**	**(8,w)**

Construct count:

- For each key in input,
- count[key] += 1

Construct position:

- Initialise first position as a 1
- position[i] = position[i-1] + count[i-1]

count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	1
2	2
3	2
4	5
5	5
6	6
7	0
8	0

Output

1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	(1,p)	(3,c)	(7,f)	(5,g)	(3,b)	(7,d)	(8,w)

Construct count:

- For each key in input,
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Construct position:

- Initialise first position as a 1
- position[i] = position[i-1] + count[i-1]

	count	position
1	1	1
2	0	2
3	3	2
4	0	5
5	1	5
6	0	6
7	2	6
8	1	0

Output

1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	**(1,p)**	**(3,c)**	**(7,f)**	**(5,g)**	**(3,b)**	**(7,d)**	**(8,w)**

Construct count:

- For each key in input,
- count[key] += 1

Construct position:

- Initialise first position as a 1
- position[i] = position[i-1] + count[i-1]

	count
1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

	position
1	1
2	2
3	2
4	5
5	5
6	6
7	6
8	8

Output

1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	**(1,p)**	**(3,c)**	**(7,f)**	**(5,g)**	**(3,b)**	**(7,d)**	**(8,w)**

Construct count:

- For each key in input,
- count[key] += 1

Construct position:

- Initialise first position as a 1
- position[i] = position[i-1] + count[i-1]

Construct output

- Go through input, looking at each (key, val)
- Set output[position[key]] to the (key, val) pair from input
- Increment position[key]

count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	1
2	2
3	2
4	5
5	5
6	6
7	6
8	8

Output

1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	(1,p)	(3,c)	(7,f)	(5,g)	(3,b)	(7,d)	(8,w)

Construct count:
- For each key in input,
- count[key] += 1

Construct position:
- Initialise first position as a 1
- position[i] = position[i-1] + count[i-1]

Construct output
- Go through input, looking at each (key, val)
- Set output[position[key]] to the (key, val) pair from input
- Increment position[key]

	count		position
1	1	1	1
2	0	2	2
3	3	3	2
4	0	4	5
5	1	5	5
6	0	6	6
7	2	7	6
8	1	8	8

Output

1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	(1,p)	(3,c)	(7,f)	(5,g)	(3,b)	(7,d)	(8,w)

Construct count:

- For each key in input,
- count[key] += 1

Construct position:

- Initialise first position as a 1
- position[i] = position[i-1] + count[i-1]

Construct output

- Go through input, looking at each (key, val)
- Set output[position[key]] to the (key, val) pair from input
- Increment position[key]

	count	position
1	1	1
2	0	2
3	3	3
4	0	5
5	1	5
6	0	6
7	2	6
8	1	8

Output

	(3,a)						
1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	**(1,p)**	**(3,c)**	**(7,f)**	**(5,g)**	**(3,b)**	**(7,d)**	**(8,w)**

Construct count:
- For each key in input,
- count[key] += 1

Construct position:
- Initialise first position as a 1
- position[i] = position[i-1] + count[i-1]

Construct output
- Go through input, looking at each (key, val)
- Set output[position[key]] to the (key, val) pair from input
- Increment position[key]

	count		position
1	1	1	2
2	0	2	2
3	3	3	3
4	0	4	5
5	1	5	5
6	0	6	6
7	2	7	6
8	1	8	8

Output

(1,p)	**(3,a)**						
1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	(1,p)	(3,c)	(7,f)	(5,g)	(3,b)	(7,d)	(8,w)

Construct count:

- For each key in input,
- count[key] += 1

Construct position:

- Initialise first position as a 1
- position[i] = position[i-1] + count[i-1]

Construct output

- Go through input, looking at each (key, val)
- Set output[position[key]] to the (key, val) pair from input
- Increment position[key]

count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	2
2	2
3	4
4	5
5	5
6	6
7	6
8	8

Output

(1,p)	(3,a)	(3,c)					
1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	**(1,p)**	**(3,c)**	**(7,f)**	**(5,g)**	**(3,b)**	**(7,d)**	**(8,w)**

Construct count:

- For each key in input,
- count[key] += 1

Construct position:

- Initialise first position as a 1
- position[i] = position[i-1] + count[i-1]

Construct output

- Go through input, looking at each (key, val)
- Set output[position[key]] to the (key, val) pair from input
- Increment position[key]

count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	2
2	2
3	4
4	5
5	5
6	7
7	6
8	8

Output

(1,p)	**(3,a)**	**(3,c)**			**(7,f)**		
1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	(1,p)	(3,c)	(7,f)	(5,g)	(3,b)	(7,d)	(8,w)

Construct count:

- For each key in input,
- count[key] += 1

Construct position:

- Initialise first position as a 1
- position[i] = position[i-1] + count[i-1]

Construct output

- Go through input, looking at each (key, val)
- Set output[position[key]] to the (key, val) pair from input
- Increment position[key]

count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	2
2	2
3	4
4	5
5	6
6	7
7	6
8	8

Output

(1,p)	(3,a)	(3,c)		(5,g)	(7,f)		
1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	**(1,p)**	**(3,c)**	**(7,f)**	**(5,g)**	**(3,b)**	**(7,d)**	**(8,w)**

Construct count:

- For each key in input,
- count[key] += 1

Construct position:

- Initialise first position as a 1
- position[i] = position[i-1] + count[i-1]

Construct output

- Go through input, looking at each (key, val)
- Set output[position[key]] to the (key, val) pair from input
- Increment position[key]

count

1	1
2	0
3	3
4	0
5	1
6	0
7	2
8	1

position

1	2
2	2
3	5
4	5
5	6
6	7
7	6
8	8

Output

(1,p)	**(3,a)**	**(3,c)**	**(3,b)**	**(5,g)**	**(7,f)**		
1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	**(1,p)**	**(3,c)**	**(7,f)**	**(5,g)**	**(3,b)**	**(7,d)**	**(8,w)**

Construct count:

- For each key in input,
- count[key] += 1

Construct position:

- Initialise first position as a 1
- position[i] = position[i-1] + count[i-1]

Construct output

- Go through input, looking at each (key, val)
- Set output[position[key]] to the (key, val) pair from input
- Increment position[key]

	count	position
1	1	2
2	0	2
3	3	5
4	0	5
5	1	6
6	0	7
7	2	7
8	1	8

Output

(1,p)	**(3,a)**	**(3,c)**	**(3,b)**	**(5,g)**	**(7,f)**	**(7,d)**	
1	2	3	4	5	6	7	8

Stable Counting Sort (Method 1)

Input

(3,a)	(1,p)	(3,c)	(7,f)	(5,g)	(3,b)	(7,d)	(8,w)

Construct count:

- For each key in input,
- count[key] += 1

Construct position:

- Initialise first position as a 1
- position[i] = position[i-1] + count[i-1]

Construct output

- Go through input, looking at each (key, val)
- Set output[position[key]] to the (key, val) pair from input
- Increment position[key]

	count		position
1	1	1	2
2	0	2	2
3	3	3	5
4	0	4	5
5	1	5	6
6	0	6	7
7	2	7	7
8	1	8	9

Output

(1,p)	(3,a)	(3,c)	(3,b)	(5,g)	(7,f)	(7,d)	(8,w)
1	2	3	4	5	6	7	8

Questions?

Counting Sort

- Stable?
  - No
  - We only remember the frequency

- But can we make it stable?
  - Yes but at the cost of memory
  - Similar to separate chaining
  - There is another way, refer to Nathan’s amazing slide
  - Are the complexity the same?

Questions?

Have a break again!

Counting Sort

Remember this issue…

- Now imagine the following:

200	151	291	981	369	421	671

  - What is my complexity?
    - Time…
    - Space…
  - Let us leave it at it is first…

Counting Sort

Remember this issue…

- Now imagine the following:

200	151	291	981	369	421	671

  - What is my complexity?
    - Time…
    - Space…
  - Let us leave it at it is first… We shall resolve this now…

Counting Sort

Remember this issue…

- Now imagine the following:

200	151	291	981	369	421	671

  - What is my complexity?
    - Time…
    - Space…
  - Let us leave it at it is first… We shall resolve this now…

Questions?

Radix Sort

A different outlook…

- With this input…

200	151	291	981	369	421	671

Radix Sort

A different outlook…

- With this input…

200	151	291	981	369	421	671

  - What if we view it differently?

Radix Sort

A different outlook…

- With this input…

200	151	291	981	369	421	671

  - What if we view it differently?

200
151
291
981
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421

Radix Sort

A different outlook…

- With this input…

200	151	291	981	369	421	671

  - What if we view it differently? How would we sort it?

200
151
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981
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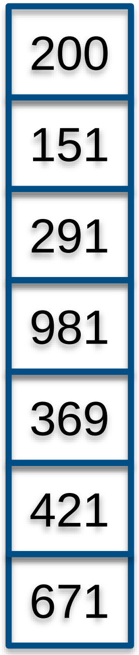
421

Radix Sort

A different outlook…

- With this input…
  - What if we view it differently? How would we sort it?



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Radix Sort

A different outlook…

- With this input…
  - What if we view it differently? How would we sort it?
    - Right most digit = least significant
    - Left most digit = most significant

200
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Radix Sort

A different outlook…

- With this input…
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Radix Sort

A different outlook…

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981
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Radix Sort

A different outlook…

- With this input…
  - What if we view it differently? How would we sort it?
    - Right most digit = least significant
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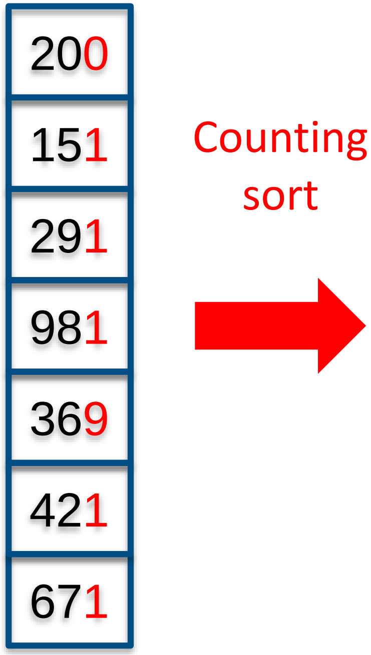

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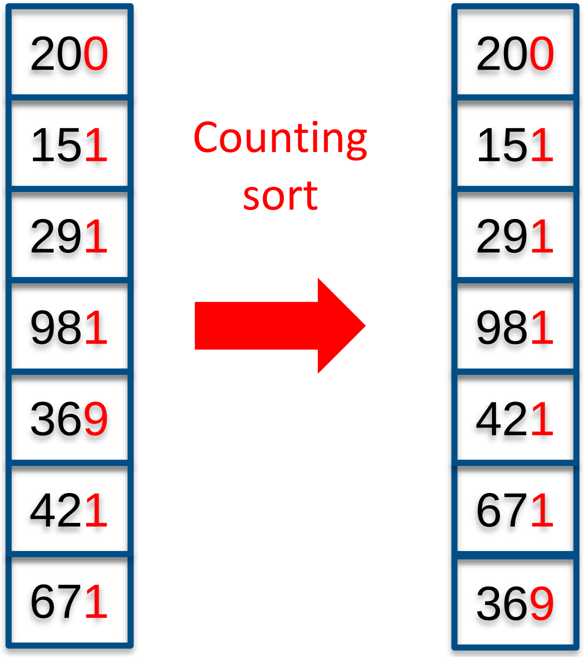
Counting sort

Radix Sort

A different outlook…

- With this input…
  - What if we view it differently? How would we sort it?
    - Right most digit = least significant
    - Left most digit = most significant



200
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Counting sort

200
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Radix Sort

A different outlook…

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  - What if we view it differently? How would we sort it?
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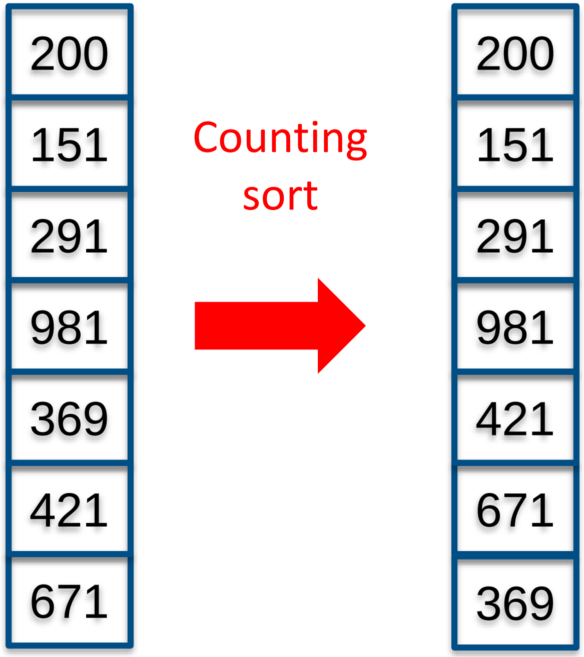

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Counting sort

200
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Radix Sort

A different outlook…

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  - What if we view it differently? How would we sort it?
    - Right most digit = least significant
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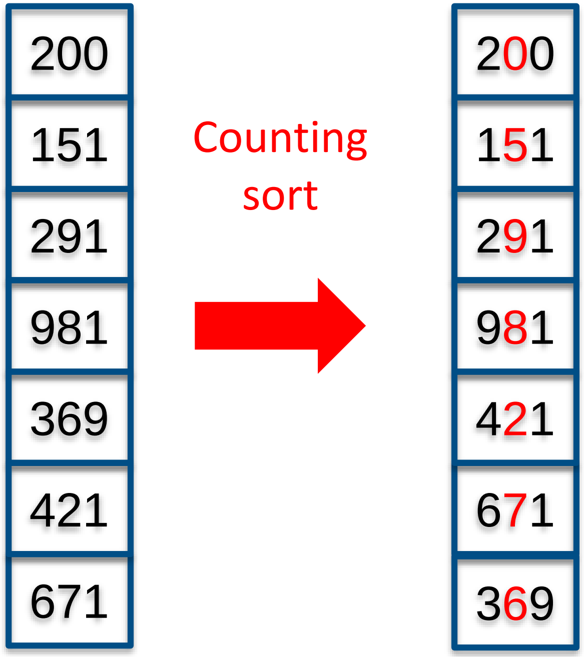
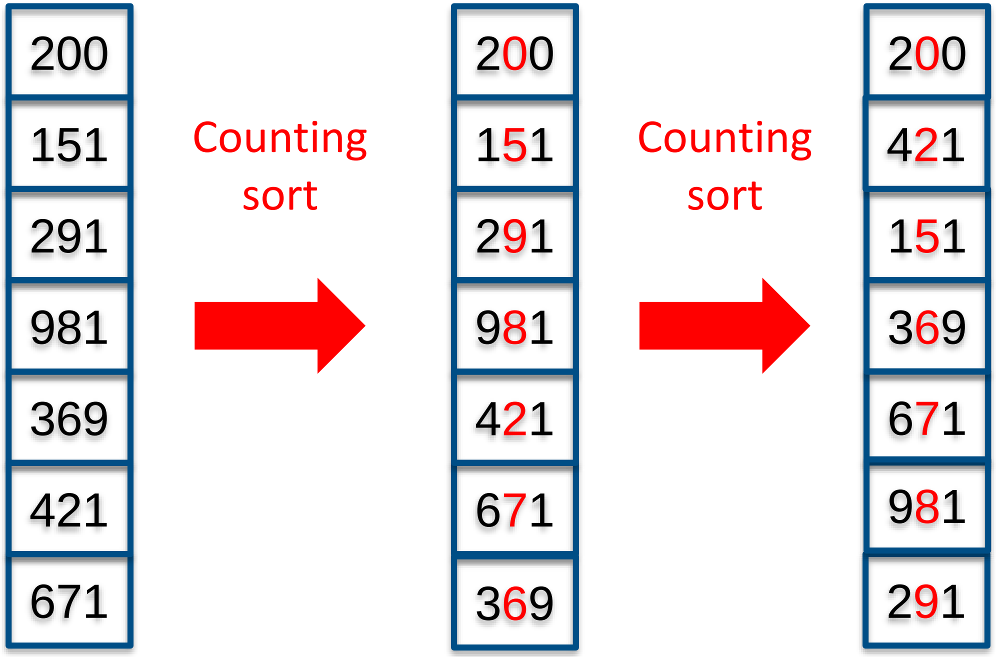


Radix Sort

A different outlook…

- With this input…
  - What if we view it differently? How would we sort it?
    - Right most digit = least significant
    - Left most digit = most significant

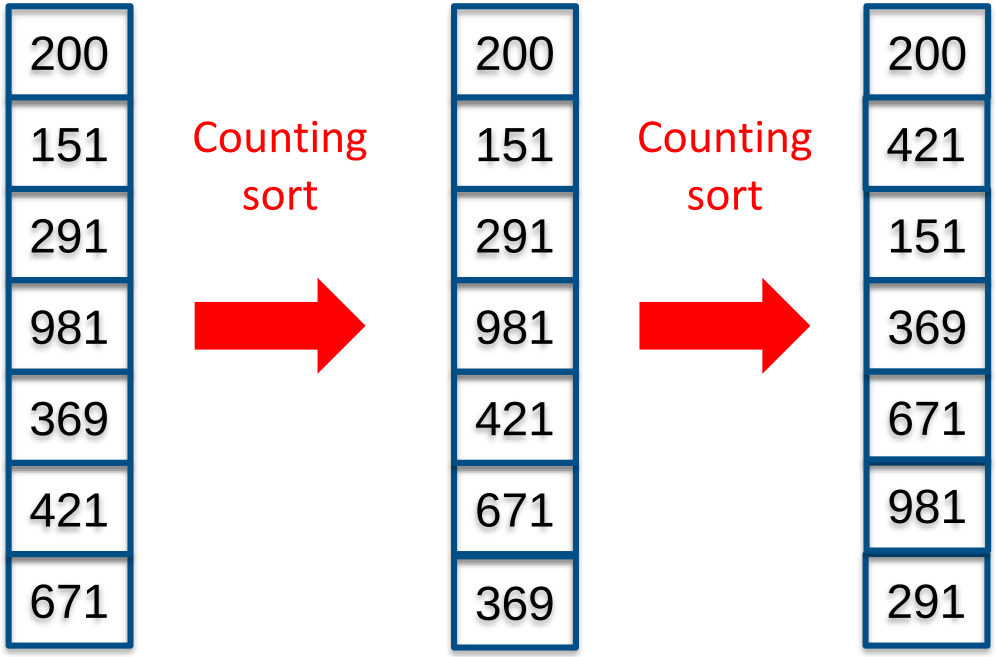


200	Counting sort	200	Counting sort	200
151		151		421
291		291		151
981		981		369
369		421		671
421		671		981
671		369		291

Radix Sort

A different outlook…

- With this input…
  - What if we view it differently? How would we sort it?
    - Right most digit = least significant
    - Left most digit = most significant



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Counting sort

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Counting sort

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Radix Sort

A different outlook…

- With this input…
  - What if we view it differently? How would we sort it?
    - Right most digit = least significant
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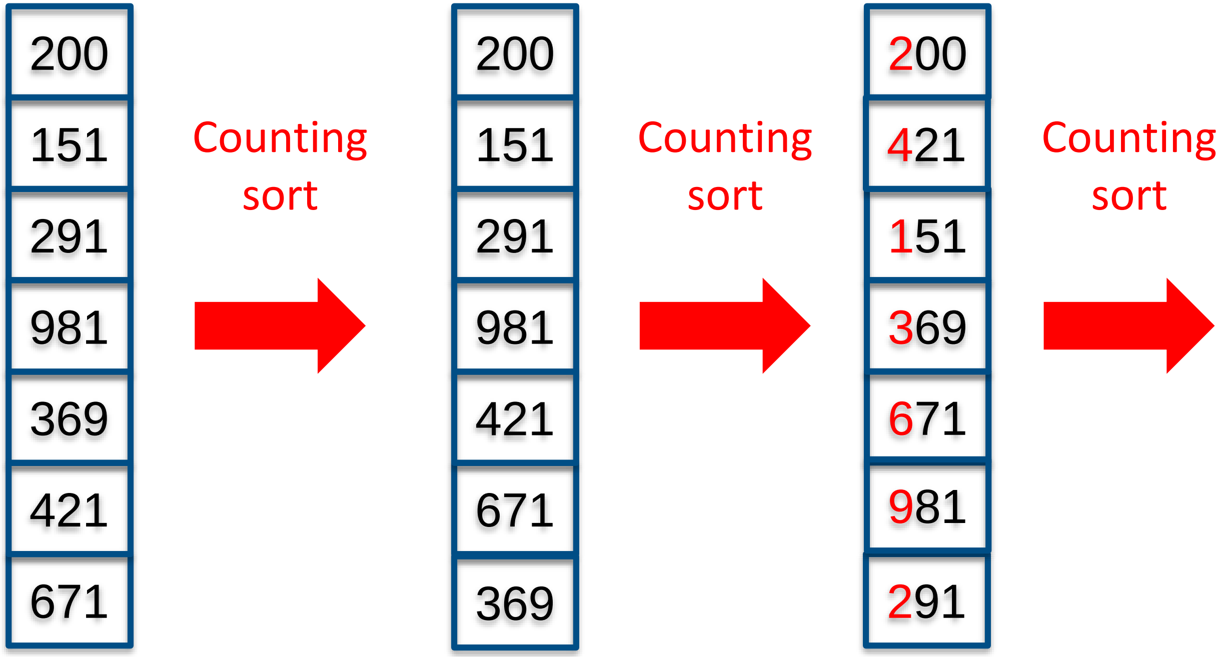
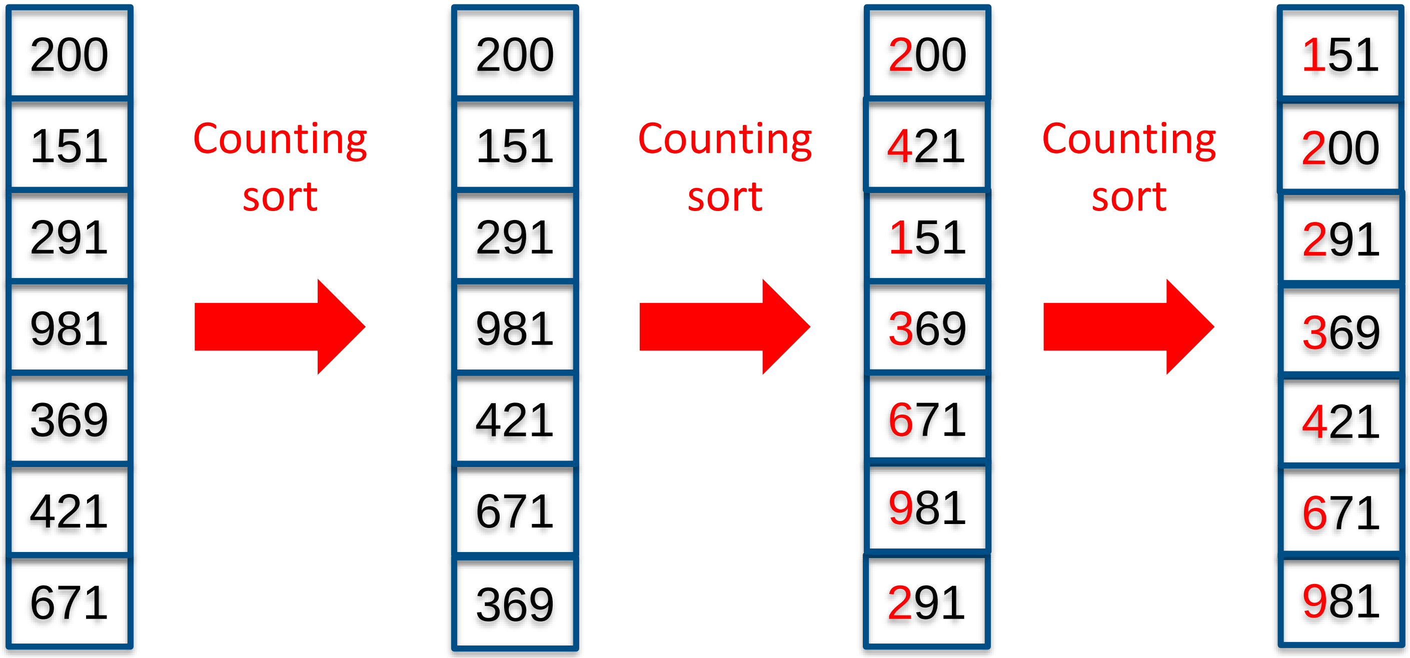


200	Counting sort	200	Counting sort	200	Counting sort
151		151		421	
291		291		151	
981		981		369	
369		421		671	
421		671		981	
671		369		291	

Radix Sort

A different outlook…

- With this input…
  - What if we view it differently? How would we sort it?
    - Right most digit = least significant
    - Left most digit = most significant

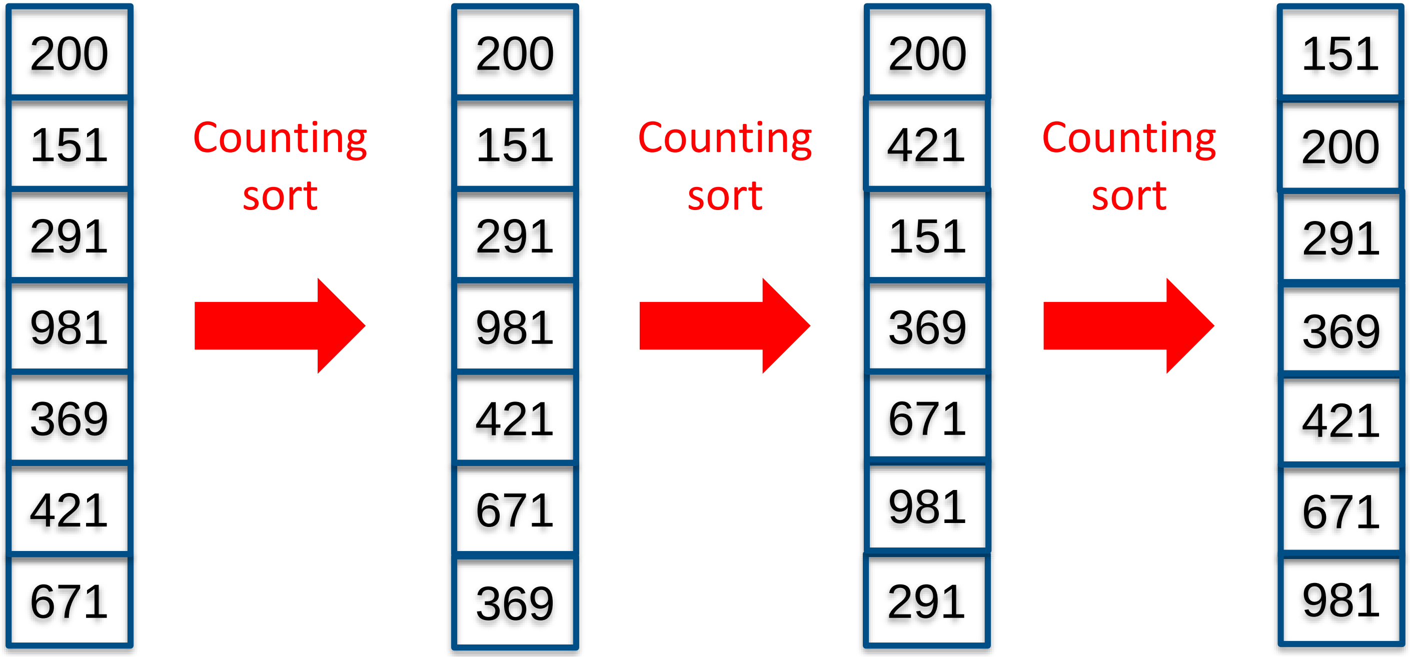


200		200		200		151
151	Counting sort	151	Counting sort	421	Counting sort	200
291		291		151		291
981		981		369		369
369		421		671		421
421		671		981		671
671		369		291		981

Radix Sort

A different outlook…

- With this input…
  - What if we view it differently? How would we sort it?
    - Right most digit = least significant
    - Left most digit = most significant



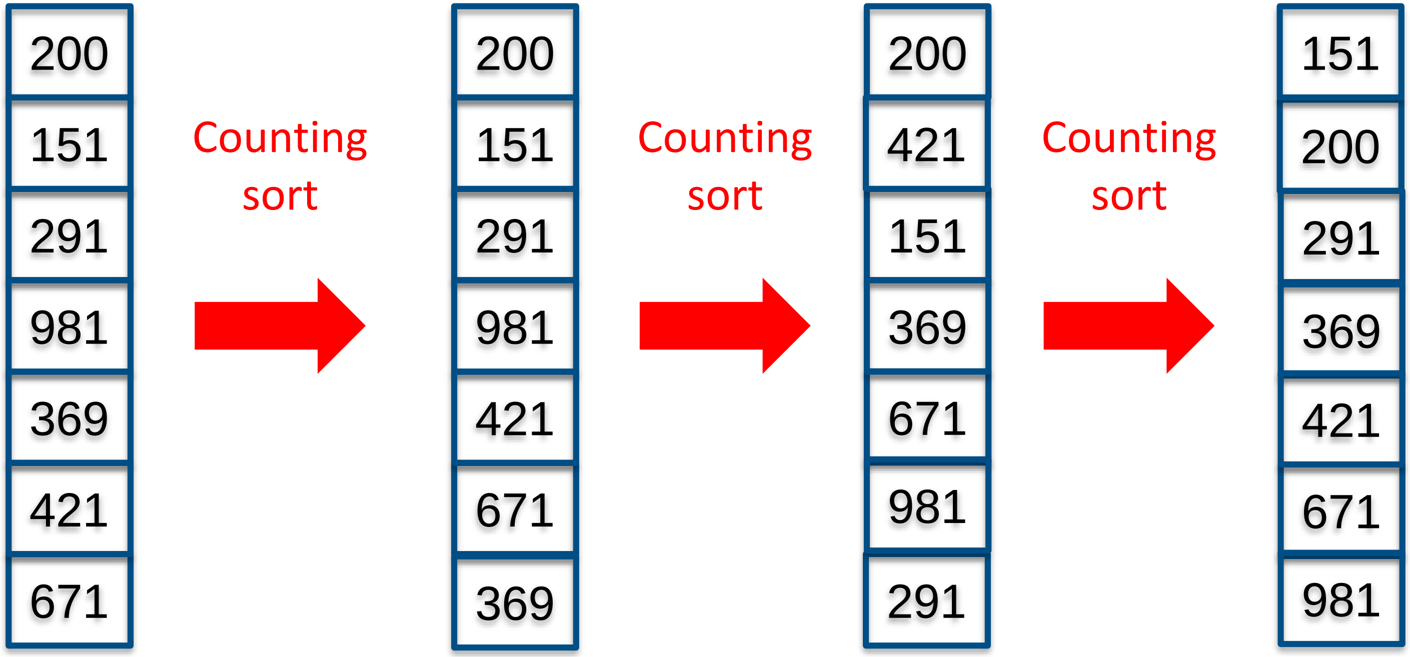
200		200		200		151
151	Counting sort	151	Counting sort	421	Counting sort	200
291		291		151		291
981		981		369		369
369		421		671		421
421		671		981		671
671		369		291		981

Questions?

Radix Sort

A different outlook…

- With this input…
  - What if we view it differently? How would we sort it?
  - But the sorting need to be stable



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Counting sort

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Counting sort

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Counting sort

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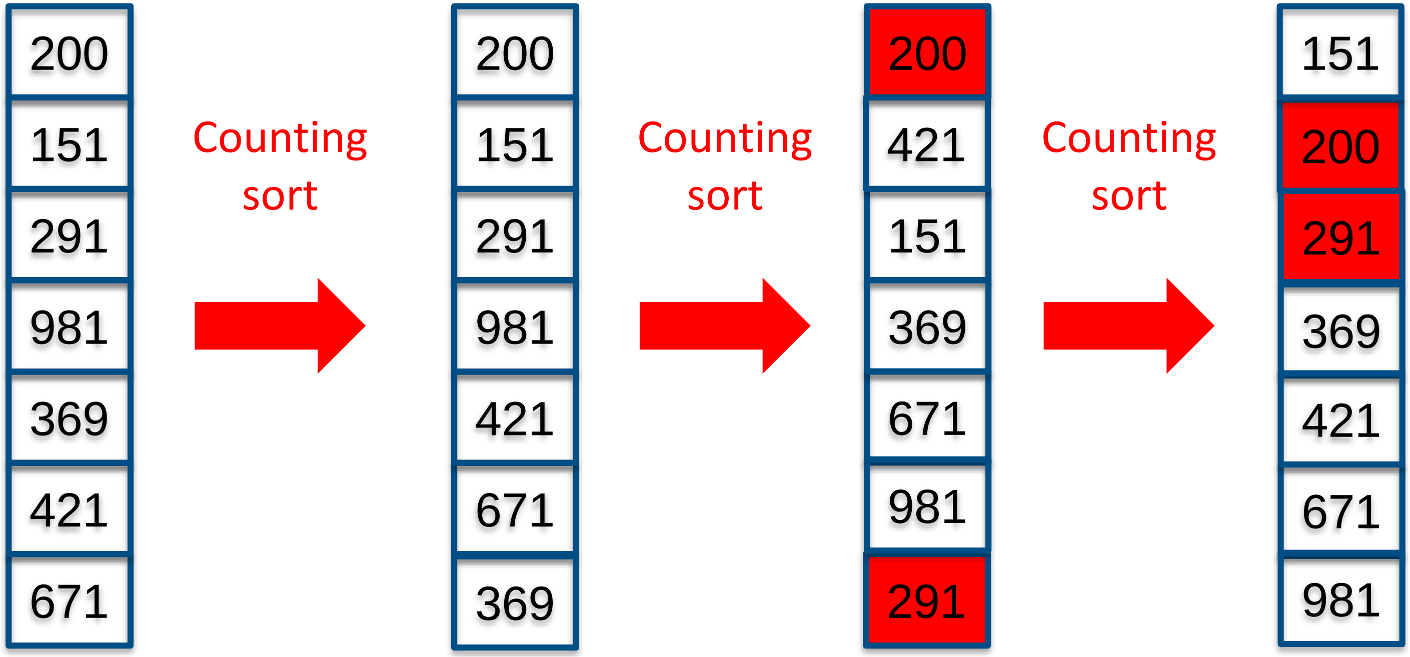
Radix Sort

A different outlook…

- With this input…
  - What if we view it differently? How would we sort it?
  - But the sorting need to be stable



200	200	200	151
151	151	421	200
291	291	151	291
981	981	369	369
369	421	671	421
421	671	981	671
671	369	291	981

Counting sort

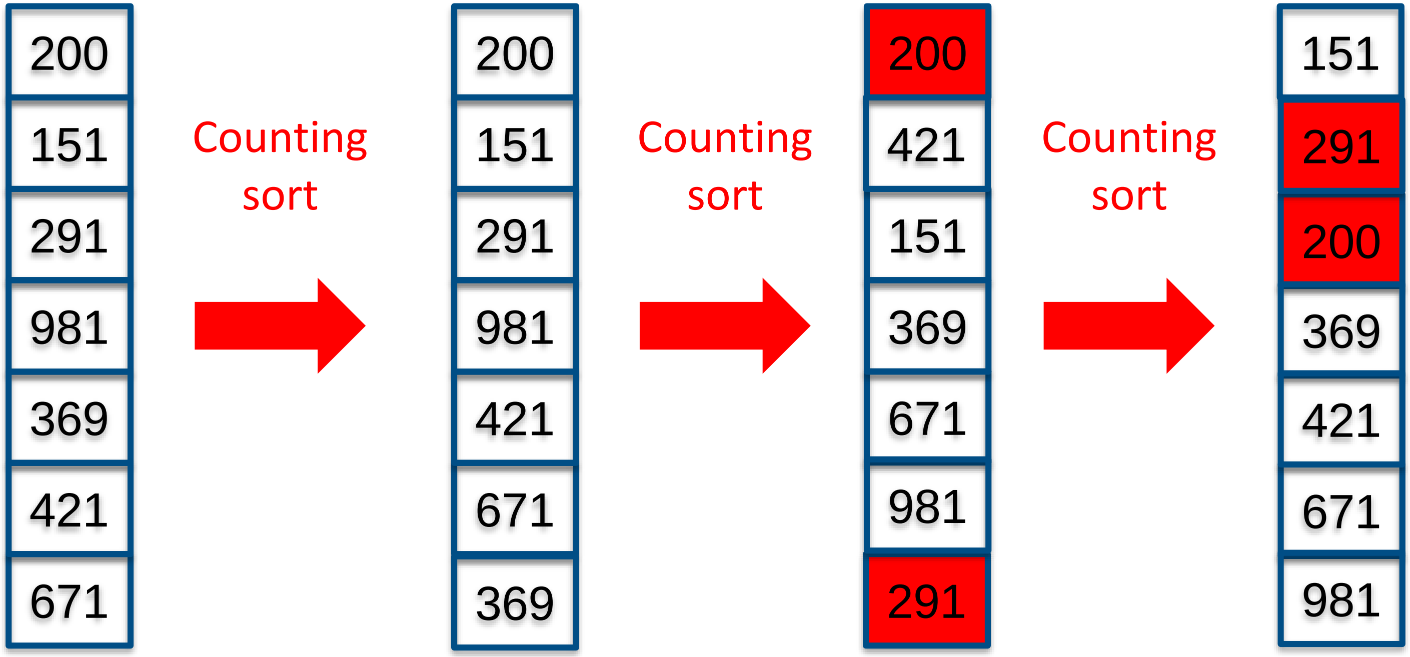
Counting sort

Counting sort

Radix Sort

A different outlook…

- With this input…
  – What if we view it differently? How would we sort it?
  – But the sorting need to be stable, if not…



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Counting sort

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Counting sort

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Counting sort

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It’s a disastah!

Questions?

Radix Sort

Complexity

- What is the complexity?
  - Time
  - Space

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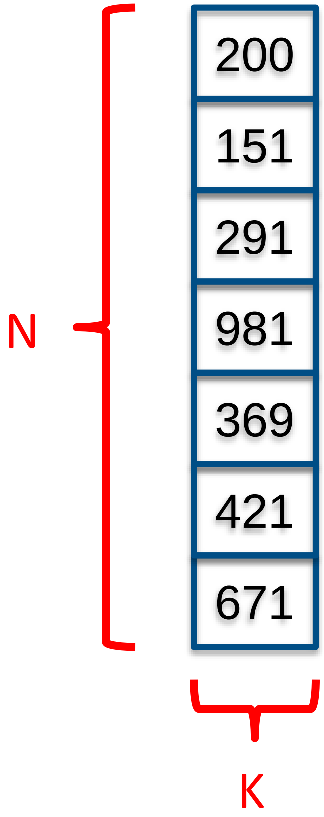
Radix Sort

Complexity

- What is the complexity?
  - Time
  - Space



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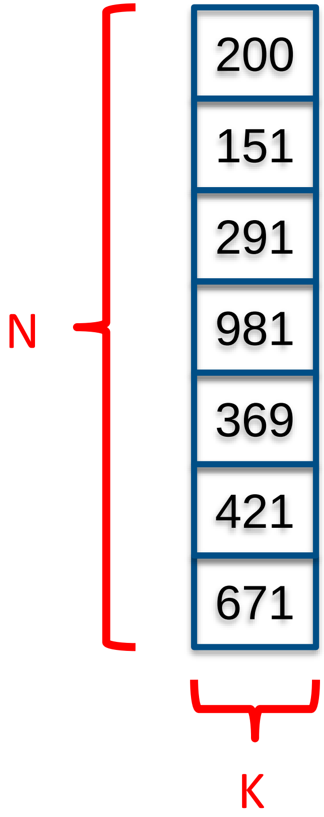
N

K

Radix Sort

Complexity

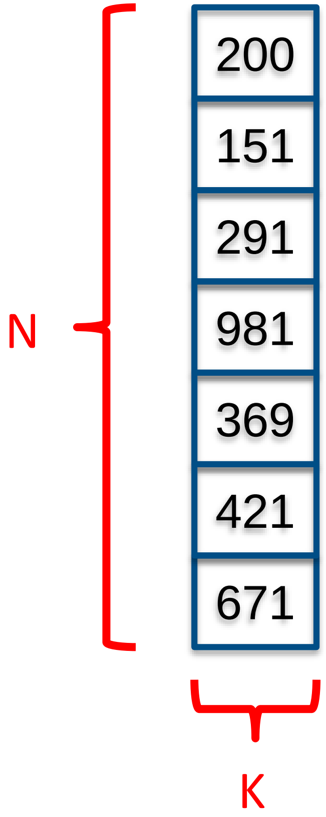
- What is the complexity?
  - Time
    - O(KN)?
  - Space



Radix Sort

Complexity

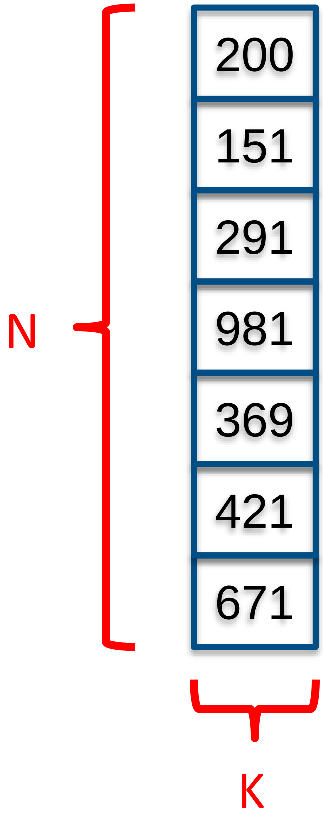
- What is the complexity?
  - Time
    - O(KN) + O(KM)
      where M is the number of unique characters
  - Space



Radix Sort

Complexity

- What is the complexity?
  - Time
    - O(KN) + O(KM)
      where M is the number of unique characters
    - Why? Recall counting sort, we account for the max
  - Space



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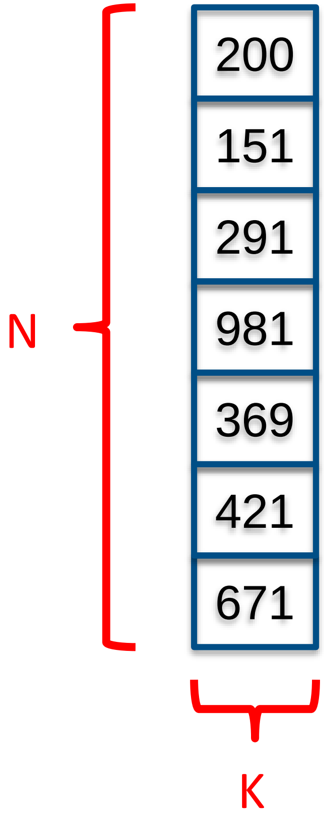
Radix Sort

Complexity

- What is the complexity?
  - Time
    - O(KN) + O(KM)
      where M is the number of unique characters
    - Why? Recall counting sort, we account for the max giving us O(N+M)
  - Space



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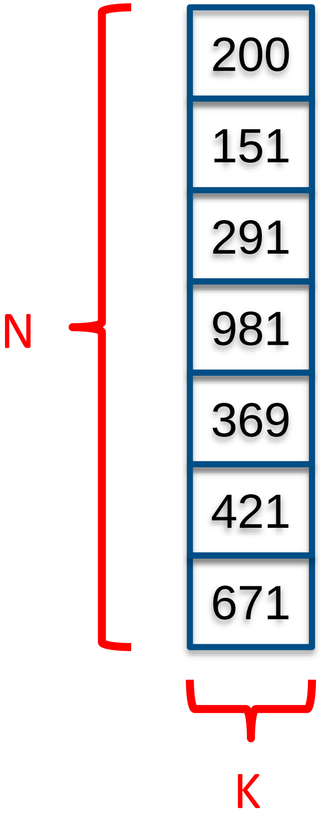
Radix Sort

Complexity

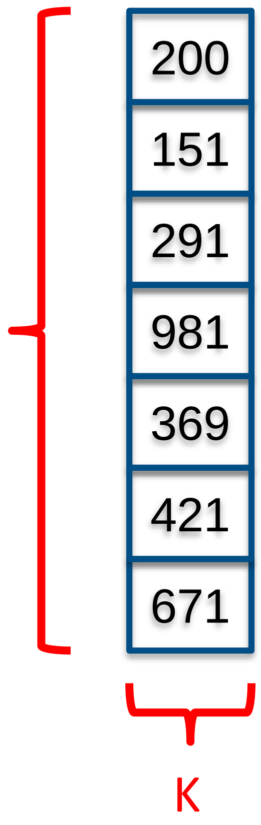
- What is the complexity?
  - Time
    - O(KN) + O(KM)
      where M is the number of unique characters
    - Why? Recall counting sort, we account for the max giving us O(N+M)
    - Then we have K columns
  - Space



Radix Sort

Complexity

- What is the complexity?
  - Time
    - O(KN) + O(KM)
      where M is the number of unique characters
    - Why? Recall counting sort, we account for the max giving us O(N+M)
    - Then we have K columns
      giving us O(K) * O(N+M)
  - Space



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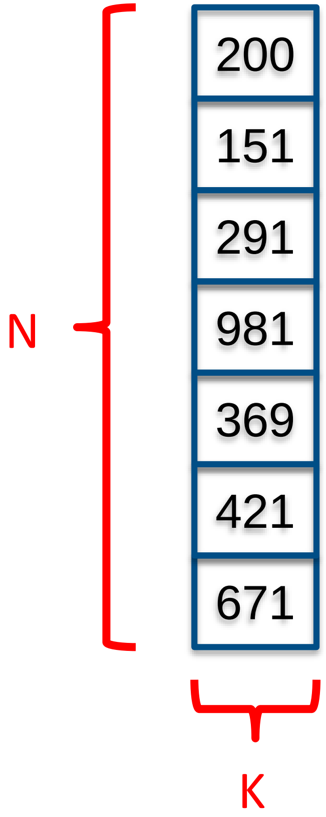
Radix Sort

Complexity

- What is the complexity?
  - Time
    - O(KN + KM)
      where M is the number of unique characters
    - Why? Recall counting sort, we account for the max giving us O(N+M)
    - Then we have K columns
      giving us O(K) * O(N+M)
  - Space



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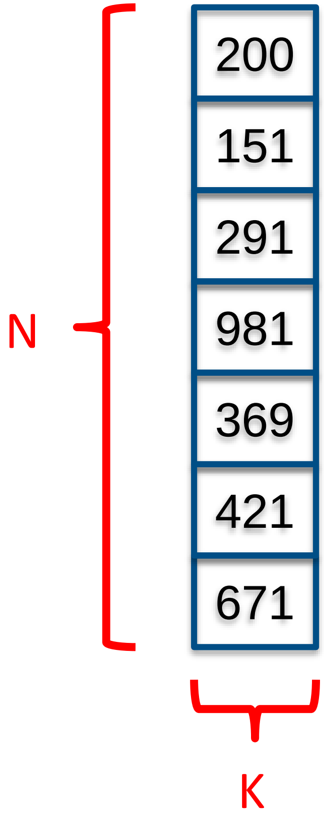
Radix Sort

Complexity

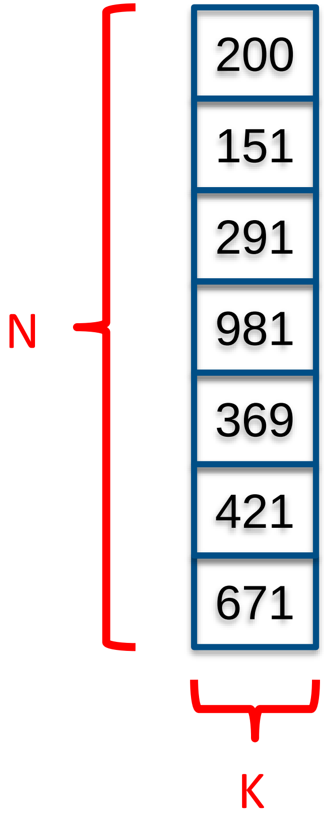
- What is the complexity?
  - Time
    - O(KN + KM)
      where M is the number of unique characters
    - Why? Recall counting sort, we account for the max giving us O(N+M)
    - Then we have K columns
      giving us O(K) * O(N+M)
  - Space
    - Input is O(KN)
    - Each counting sort needs O(M+N)



Radix Sort

Complexity

- What is the complexity?
  - Time
    - O(KN + KM)
      where M is the number of unique characters
    - Why? Recall counting sort, we account for the max giving us O(N+M)
    - Then we have K columns
      giving us O(K) * O(N+M)
  - Space
    - Input is O(KN)
    - Each counting sort needs O(M+N)
    - Total is O(KN + M + N)



N

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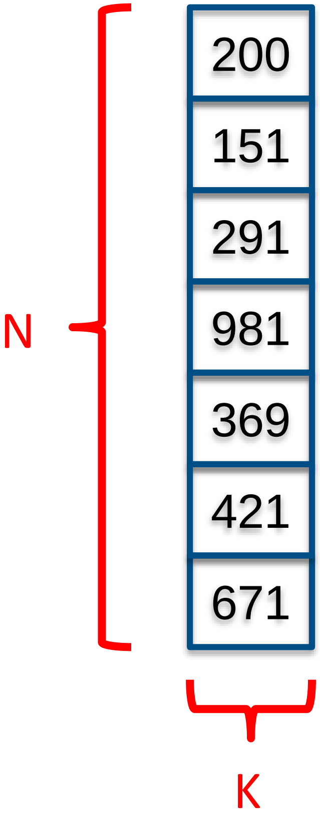
K

Radix Sort

Complexity

- What is the complexity?
  - But we know M = 10 for 0, 1, …, 9
  - Time
    - O(KN + KM)
      where M is the number of unique characters
    - Why? Recall counting sort, we account for the max giving us O(N+M)
    - Then we have K columns giving us O(K) * O(N+M)
  - Space
    - Input is O(KN)
    - Each counting sort needs O(M+N)
    - Total is O(KN + M + N)



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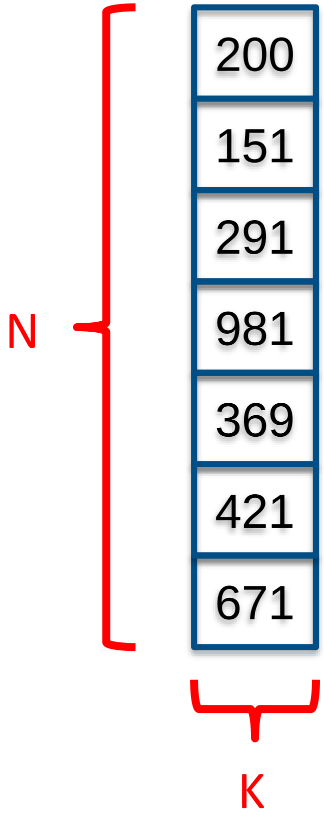
Radix Sort

Complexity

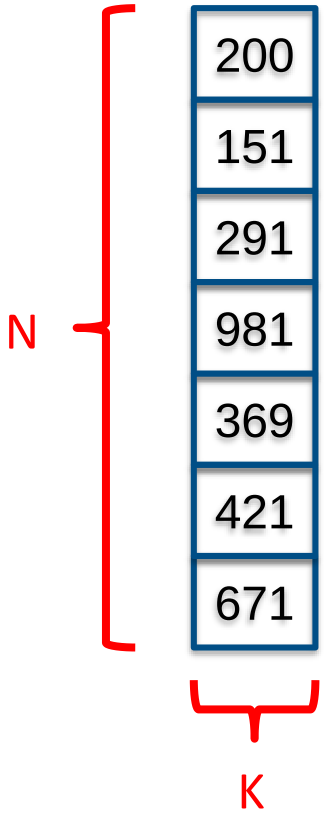
- What is the complexity?
  - But we know M = 10 for 0, 1, …, 9
  - Time
    - O(KN + KM) ≈ O(KN)
      where M is the number of unique characters
    - Why? Recall counting sort, we account for the max giving us O(N+M)
    - Then we have K columns
      giving us O(K) * O(N+M)
  - Space
    - Input is O(KN)
    - Each counting sort needs O(M+N)
    - Total is O(KN + M + N) ≈ O(KN)
    - Auxiliary is O(M + N) ≈ O(N)



Radix Sort

Complexity

- What is the complexity?
  - Better than merge sort O(k N log N)!
  - But we know M = 10 for 0, 1, …, 9
  - Time
    - O(KN + KM) ≈ O(KN)
      where M is the number of unique characters
    - Why? Recall counting sort, we account for the max giving us O(N+M)
    - Then we have K columns
      giving us O(K) * O(N+M)
  - Space
    - Input is O(KN)
    - Each counting sort needs O(M+N)
    - Total is O(KN + M + N) ≈ O(KN)
    - Auxiliary is O(M + N) ≈ O(N)



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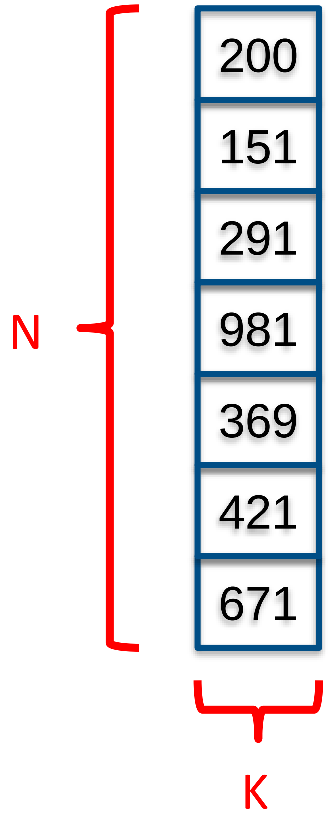
K

Radix Sort

Complexity

- What is the complexity?
  - Better than merge sort O(k N log N)!
  - But we know M = 10 for 0, 1, …, 9
  - Time
    - O(KN + KM) ≈ O(KN)
      where M is the number of unique characters
    - Why? Recall counting sort, we account for the max giving us O(N+M)
    - Then we have K columns
      giving us O(K) * O(N+M)
  - Space
    - Input is O(KN)
    - Each counting sort needs O(M+N)
    - Total is O(KN + M + N) ≈ O(KN)
    - Auxiliary is O(M + N) ≈ O(N) <- why no K? Come ask me if interested…



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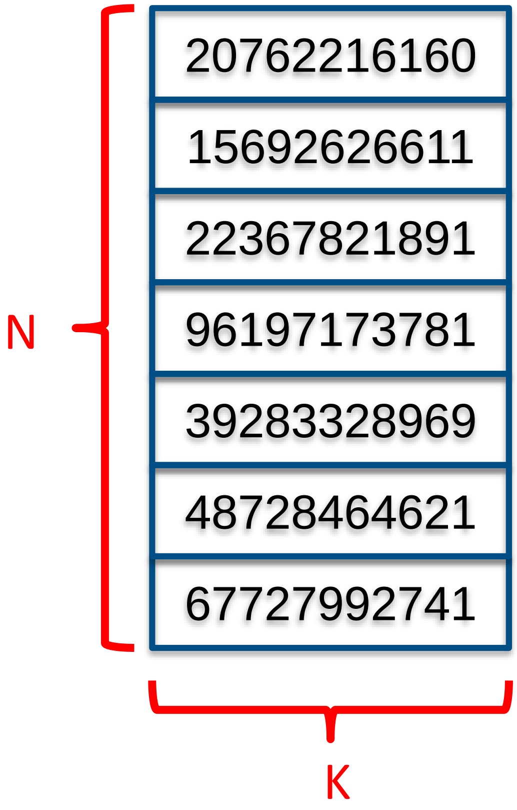
Questions?

Radix Sort

Complexity

- What is the complexity?
  - What if k is bigger?
  - But we know M = 10 for 0, 1, …, 9
  - Time
    - O(KN + KM) ≈ O(KN)
      where M is the number of unique characters
    - Why? Recall counting sort, we account for the max giving us O(N+M)
    - Then we have K columns
      giving us O(K) * O(N+M)
  - Space
    - Input is O(KN)
    - Each counting sort needs O(M+N)
    - Total is O(KN + M + N) ≈ O(KN)
    - Auxiliary is O(M + N) ≈ O(N)



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96197173781

39283328969

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67727992741

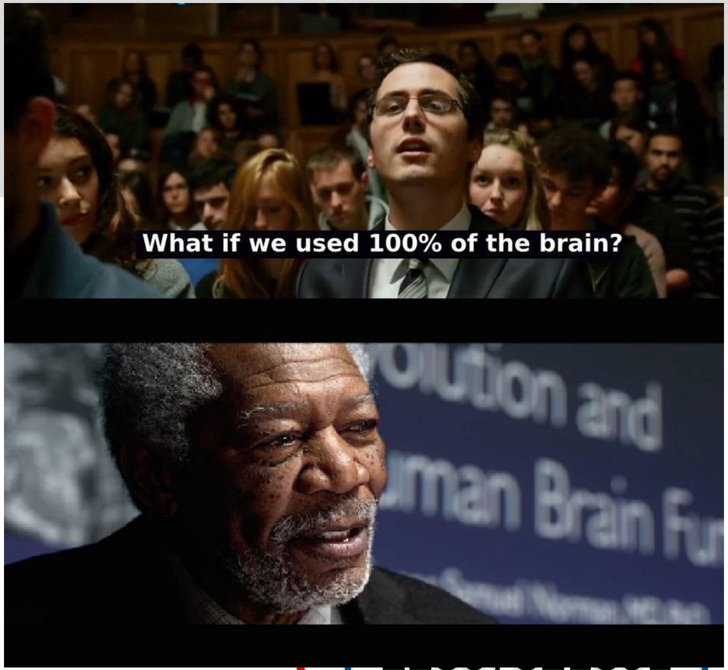
N

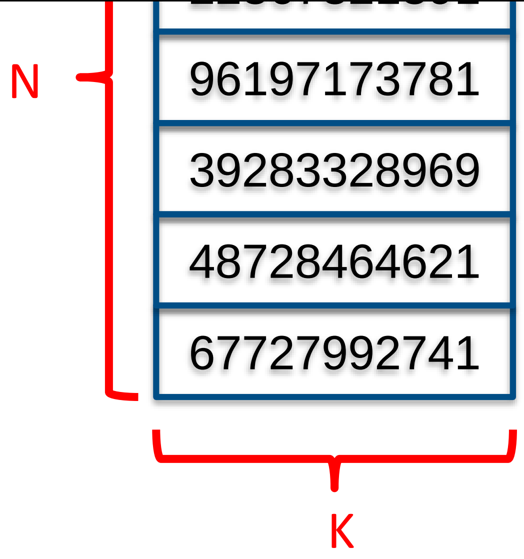
K

Radix Sort

Complexity

- What is the complexity?
  - What if k is bigger?
  - But we know M = 10 for 0, 1, …, 9
  - Time
    - O(KN + KM) ≈ O(KN)
      where M is the number of unique characters
    - Why? Recall counting sort, we account for the max giving us O(N+M)
    - Then we have K columns
      giving us O(K) * O(N+M)
  - Space
    - Input is O(KN)
    - Each counting sort needs O(M+N)
    - Total is O(KN + M + N) ≈ O(KN)
    - Auxiliary is O(M + N) ≈ O(N)





96197173781

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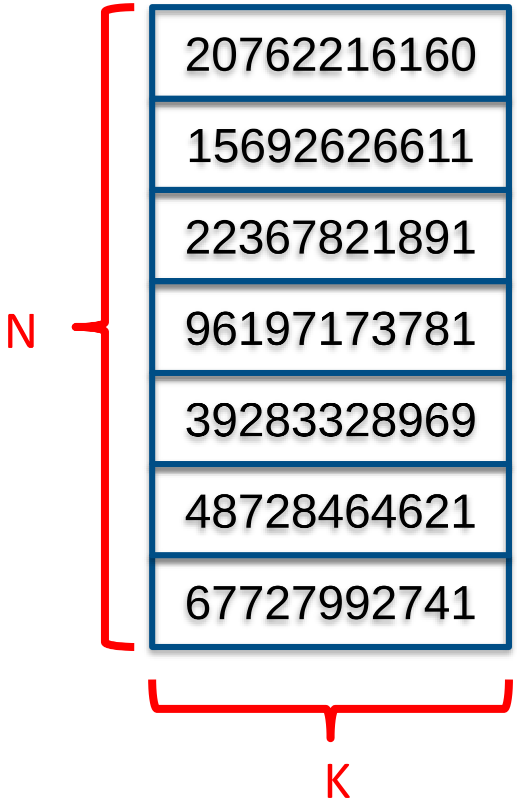
48728464621

67727992741

Radix Sort

Complexity

- What is the complexity?
  - What if k is bigger?
  - We increase M = 100 for 0, 1, …, 99
  - Time
    - O(KN + KM) ≈ O(KN)
      where M is the number of unique characters
    - Why? Recall counting sort, we account for the max giving us O(N+M)
    - Then we have K columns
      giving us O(K) * O(N+M)
  - Space
    - Input is O(KN)
    - Each counting sort needs O(M+N)
    - Total is O(KN + M + N) ≈ O(KN)
    - Auxiliary is O(M + N) ≈ O(N)



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Radix Sort

Complexity

- Time complexity is O(KN +KM)
- Space complexity is O(KN + M + N)

Radix Sort

Complexity

- Time complexity is O(KN +KM)
- Space complexity is O(KN + M + N)

- M is the base

Radix Sort

Complexity

- Time complexity is O(KN +KM)
- Space complexity is O(KN + M + N)

- M is the base
  - For decimal numbers, it is 10 from 0 to 10

Radix Sort

Complexity

- Time complexity is O(KN +KM)
- Space complexity is O(KN + M + N)

- M is the base
  - For decimal numbers, it is 10 from 0 to 10
  - For binary numbers,

Radix Sort

Complexity

- Time complexity is O(KN +KM)
- Space complexity is O(KN + M + N)

- M is the base
  - For decimal numbers, it is 10 from 0 to 10
  - For binary numbers, it is 2 from 0 to 1

Radix Sort

Complexity

- Time complexity is O(KN +KM)
- Space complexity is O(KN + M + N)

- M is the base
  - For decimal numbers, it is 10 from 0 to 10
  - For binary numbers, it is 2 from 0 to 1
  - We can increase the M, to reduce the K?

Radix Sort

Complexity

- Time complexity is O(KN +KM)
- Space complexity is O(KN + M + N)

- M is the base
  - For decimal numbers, it is 10 from 0 to 10
  - For binary numbers, it is 2 from 0 to 1
  - We can increase the M, to reduce the K?

  - If we deal with the English alphabet, this would be 26 from a to z

baihns
hnmapg
lhhang
uhnagh
banana
trolls
hahaha

Radix Sort

Complexity

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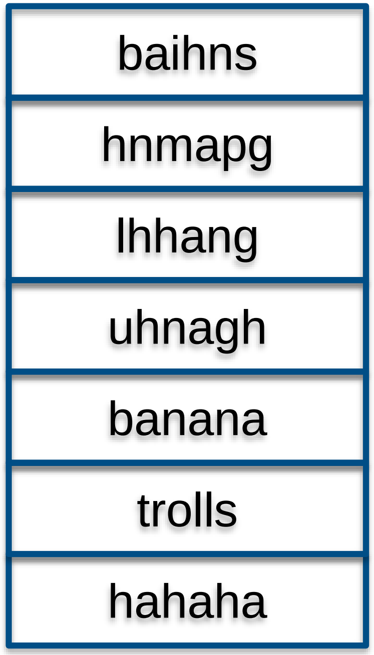

baihns

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trolls

hahaha

  - If we deal with the English alphabet, this would be 26 from a to z

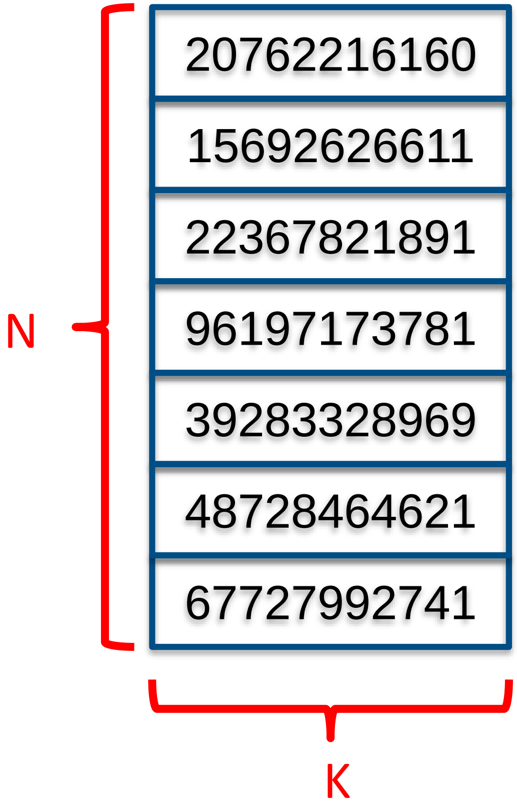
  - Nathan did a good analysis on it

Questions?

Radix Sort

TL;DR

- So you know radix sort
- What have you notice?



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15692626611

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96197173781

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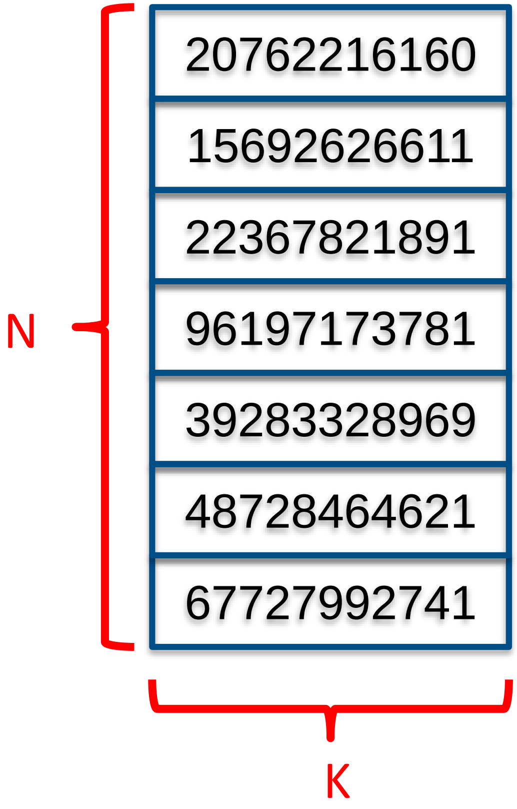
K

Radix Sort

TL;DR

- So you know radix sort
- What have you notice?
  - It is counting sort really, done multiple times



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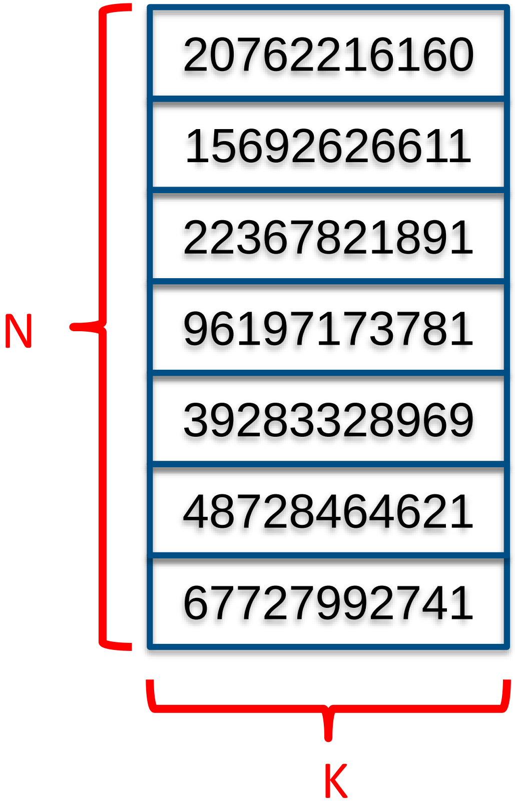
Radix Sort

TL;DR

- So you know radix sort
- What have you notice?
  - It is counting sort really, done multiple times
  - Usually least significant (right) to most significant (left)



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Radix Sort

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- So you know radix sort
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    - We can reduce this by increasing the base
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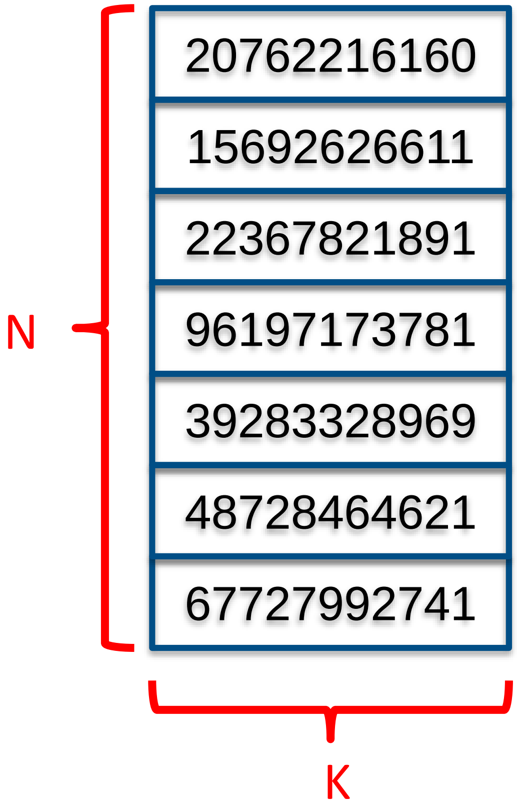

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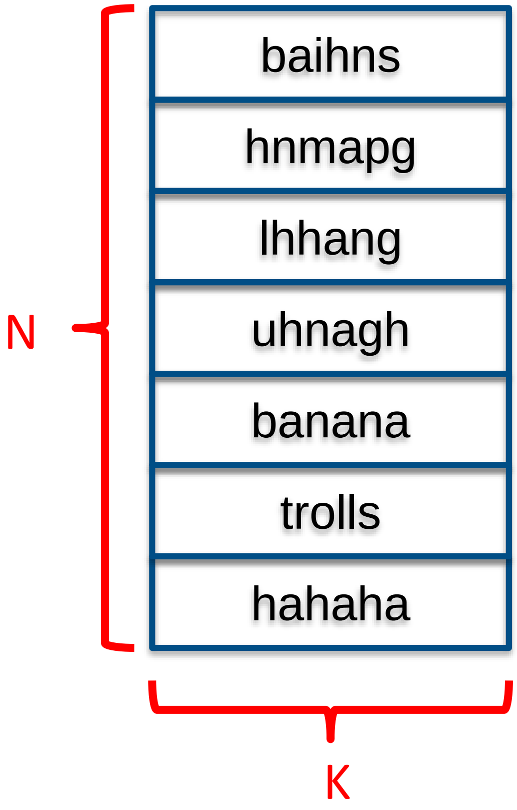
Radix Sort

TL;DR

- So you know radix sort
- What have you notice?
  - It is counting sort really, done multiple times
    - We can reduce this by increasing the base
    - Works well for characters as well
  - Usually least significant (right) to most significant (left)



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Radix Sort

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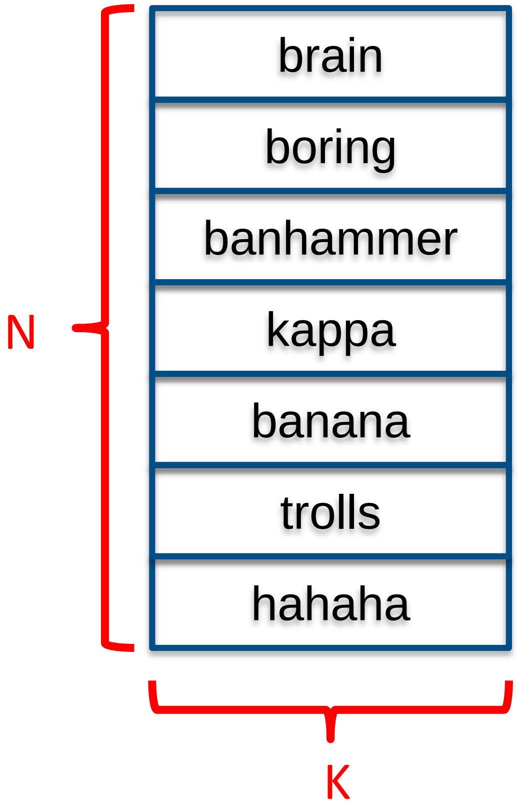
- But what if they are not the same length?



Radix Sort

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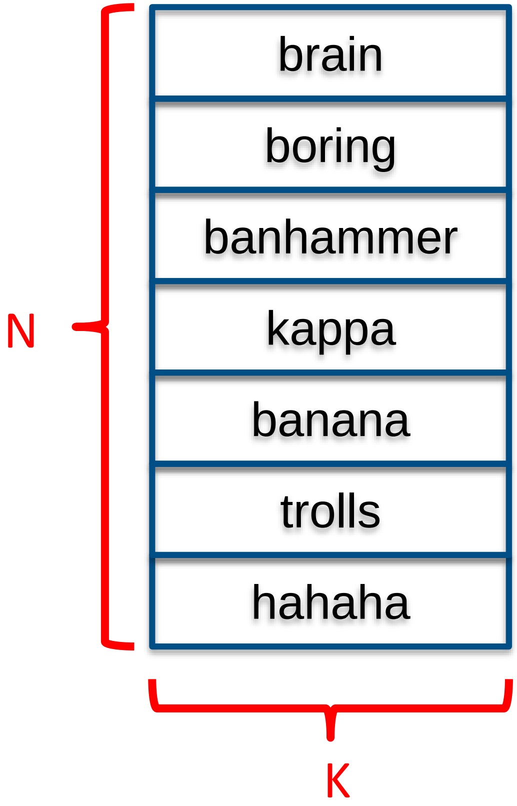
- But what if they are not the same length?
  - Left-aligned?
  - Right-aligned?



brain

boring

banhammer

kappa

banana

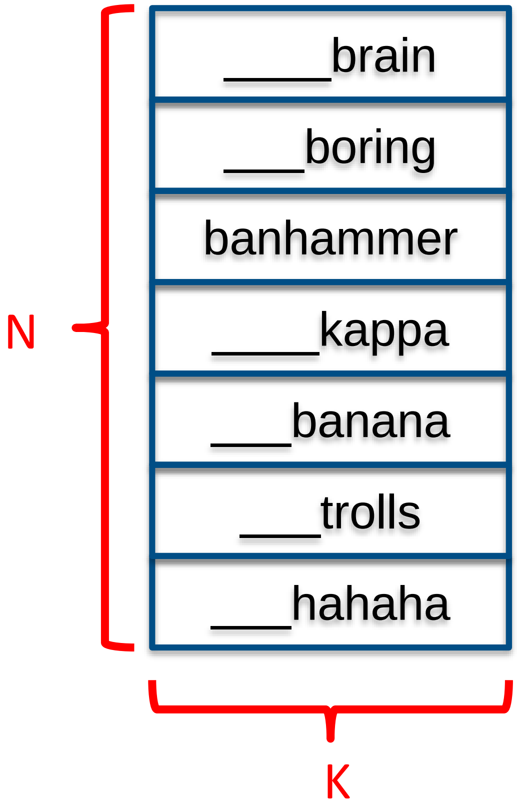
trolls

hahaha

Radix Sort

TL;DR

- So you know radix sort
- What have you notice?
  - It is counting sort really, done multiple times
    - We can reduce this by increasing the base
    - Works well for characters as well
  - Usually least significant (right) to most significant (left)
- But what if they are not the same length? Add spaces!
  - Left-aligned?
  - Right-aligned?



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banhammer

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___trolls

___hahaha

N

K

Questions?

Thank You