

Institut National des Télécommunications et des Technologies de l'Information et  
de la Communication d'Oran



Module : Radiocommunications Numériques

# TP : N°03

## Optimum Receivers for AWGN Channels

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**Estimation du temps nécessaire : 10h**

**Objective :**

Comprendre la génération et l'analyse des signaux passe bande : Modulations numériques.  
Démodulation, détection d'enveloppe, récepteur Optimal pour un canal AWGN

**Se préparer au TP :**

Il est recommandé de réviser les points suivants : lowpass equivalent, signal analytique, représentation vectorielle des signaux sur une base orthonormée, produit scalaire pour vecteurs ou fonctions/signaux, distance euclidienne, détection optimal pour un canal AWGN.

Voir les annexes.

**Travail demandé :**

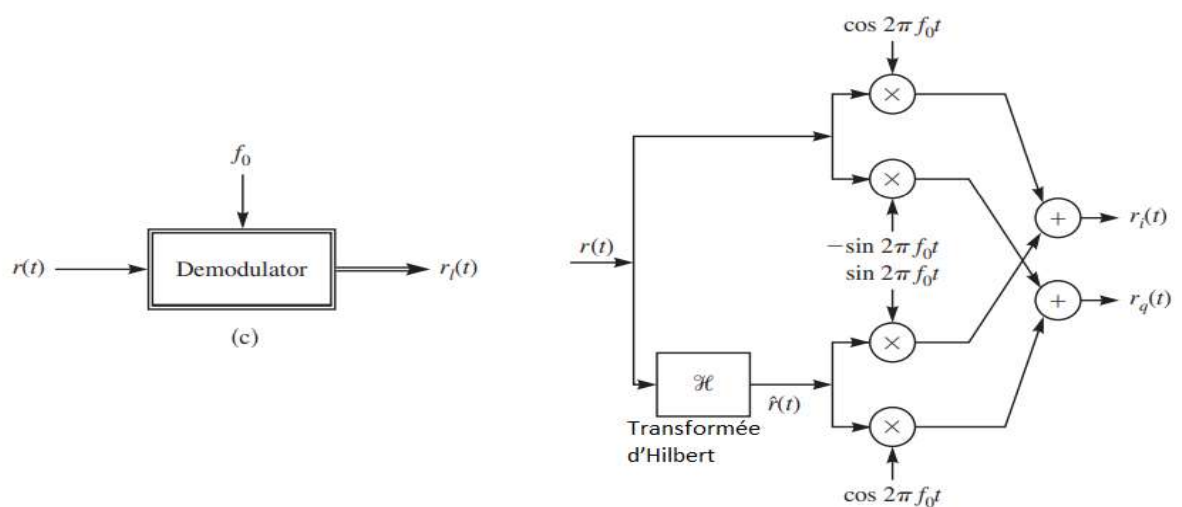
Remarque : mettez votre script principal dans un fichier « .m » et Commentez tous les ligne de code que vous introduisez. En cas de plusieurs fichiers, mettez-les dans un seul répertoire qui porte votre nom et l'intitulé du TP.

Assurez-vous que l'exécution des fichiers « .m » fonctionne comme prévue (surtout la clarté de l'affichage).

1. Préparer un signal binaire « s » contenant 100 points et générer le signal modulé en 4-QAM en bande de base. Tracer le signal (I ou Q) temporel, le diagramme de constellation et le spectre.
2. Introduire la modulation QAM sur porteuse ( $f_c=2 \times$  le débit symbole) et afficher les formes d'onde avec le spectre correspondant. Comparer les résultats trouvés dans 1 et 2.
3. Que pouvez-vous dire sur l'enveloppe du signal ?
4. Faire passer le signal QAM passe bande dans un canal AWGN et observer l'effet du canal dans les graphes cités. Justifier l'affichage.

Traitement coté récepteur :

5. Pour extraire le signal équivalent en bande de base, déterminer les expressions de  $r_i(t)$  et  $r_q(t)$  puis implémenter le traitement illustré dans le schéma suivant :



6. Quelle est l'utilité du traitement de l'étape (5) ? (Voir annexe 02)
7. Concevoir puis implémenter le détecteur optimal adéquat à ce signal. (Voir annexe 02)  
Tracer la constellation et déterminer son taux d'erreurs.

**8. Challenge 1:**

Introduire les lignes de code manquantes pour les fonctions se trouvant dans les fichiers « char2psk.m » et « psk2char.m », afficher les graphes nécessaires et expliquer fonctionnement des deux fonctions.

Réorganiser votre script en fonctions, dont chacune représente un bloc dans le schéma de bloc d'une chaîne de transmission. (en Matlab, il faut définir chaque fonction dans un fichier « .m » séparé, puis vous mettez tous les fichiers dans un seul répertoire)

9. Rédiger une conclusion générale qui résume ce que vous avez appris dans ce travail.

## Annexe 01

### Some useful commands:

```
x = hilbert( xr ) % returns the analytic signal, x , from a real data sequence, xr
fe=8000;          % define a Sampling Freq => Ts=1/8000 = 125 µs
s=[1:1:10];      % Example of Data
x=dec2bin(s);
x=x(:);
y=bin2dec(x(1:1:end,1)); % one column amplitudes
pulse= ones(50,1) ;% Rect pulse of T = 50*1/8000= 6.3 ms
Fc=4000; t=[0: 1/8000: (50-1)*1/8000];
cos_wave=cos(2*pi*Fc*t);
sin_wave=sin(2*pi*Fc*t);
sig =conv2(pulse, y);
bas_bnd_sig = sig (:);
plot(bas_bnd_sig) % Adding noise
Noisy_bas_bnd_sig= Bas_bnd_sig + 0.2*randn(size(Bas_bnd_sig));
[Psd1,freq] = pwelch(Bas_bnd_sig(1:end,1),512,0,512,fe); plot (freq, 10*log10(Psd1))
%Eye Diagram
Start_eye_data =1; % Offset in starting of display eyediagram unit:symbol period.
End_eye_data = 100; % Number of symbols displayed in the eyediagram
Delay = 0.9; % Channel transmission time. Unit: symbol period.
Nb_Pt_Pls = 50 %samples per impulse
P=3;          % Number of symbol period to be displayed in eye diagram
eyediagram(N_Out_Chans(round((Start_eye_data+Delay)*Nb_Pt_Pls):End_eye_data*Nb_Pt_Pls,:),P*Nb_Pt_Pls,P);
delayed_out=[zeros(10,1); Bas_bnd_sig];
clear all; close all; clc

%QPSK
%Set the modulation order
```

## Annexe 01

```
M = 16;

%Generate random data symbols.

data = randi([0 M-1],1000,1);

%Modulate the data symbols.

txSig = pskmod(data,M,pi/M);

%Pass the signal through white noise and plot its constellation.

rxSig = awgn(txSig,20);

scatterplot(txSig)

scatterplot(rxSig)

numErrs = symerr(dataIn,dataOut)
%Modulate and demodulate the data using Gray and natural binary encoded data.

symgray = pskmod(data,M,phz, 'gray');
mapgray = pskdemod(symgray,M,phz, 'gray');

symbin = pskmod(data,M,phz, 'bin');
mapbin = pskdemod(symbin,M,phz, 'bin');
pxx = pwelch(x) returns the (PSD) estimate of x.
```

When  $x$  is a matrix, the PSD is computed independently for each **column** and stored in the corresponding column of  $pxx$ .

To set the DC in the middle :

```
[Psd1,freq] = pwelch(Bas_bnd_sig(1:end,1),512,0,512,fe); plot (freq, 10*log10(Psd1))
plot(psd(spectrum.welch,pb_sig(:),'Fs',fe,'CenterDC',true));
```

```
x2_2bit_words=reshape(x2, 20, 2)

00 -> 1 + j;   01 -> 1 - j;   10 -> -1+j;  11 -> -1-j

qam_clpx_symb=zeros(20,1);

for i = 1:20

switch bin2dec(x2_2bit_words(i,:))

case 0

qam_clpx_symb(i)= 1+j

case 1

qam_clpx_symb(i)= 1-j
```

## Annexe 01

case 2

```
qam_clpx_symb(i)= -1+j
```

case 3

```
qam_clpx_symb(i)= -1-j
```

end

end

```
qam_I=real(qam_clpx_symb);
```

```
qam_Q=imag(qam_clpx_symb);
```

```
qam_Q=cos_wave'*imag(bas_bnd_qam)';
```

```
qam_basband_I=pulse* qam_I'
```

```
qam_basband_Q= pulse*qam_Q'
```

```
#qam_basband_I= qam_basband_I(:) # to plot
```

```
#qam_basband_Q= qam_basband_Q(:)
```

```
qam_bandPas= cos_wave'* qam_I' - sin_wave'*qam_Q'
```

#Demodulation:

```
r_anlrc_x2 = hilbert( qam_bandPas) ;
```

```
r_=real(r_anlrc_x2);
```

```
r_hlb=imag(r_anlrc_x2);
```

```
>> r_Q=zeros(size(r_));
```

```
>> r_I=zeros(size(r_));
```

```
for j =[1:20]
```

```
  r_I(:,j)= cos_wave'.*r_(:,j) + sin_wave'.*r_hlb(:,j);
```

```
  r_Q(:,j)= -sin_wave'.*r_(:,j) + cos_wave'.*r_hlb(:,j);
```

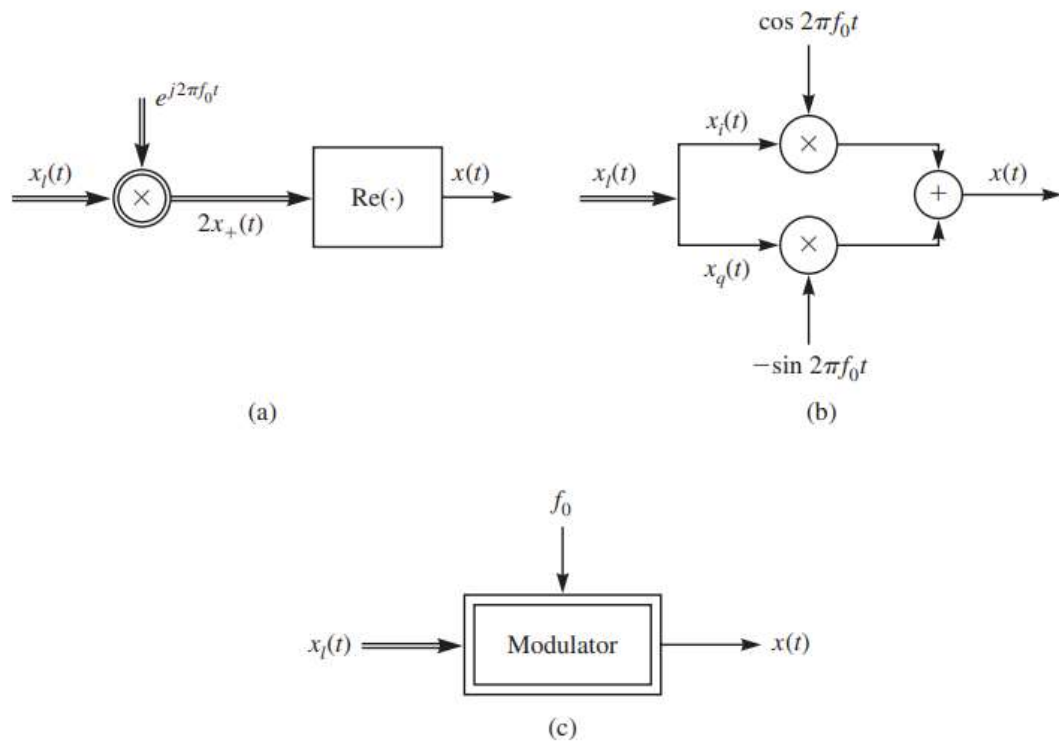
end

```
filtred_r_Q= conv(r_Q(:,1),pulse);
```

```
figure; [pxx,f] = pwelch(x,500,300,500,fs, 'centered', 'power'); plot(f,10*log10(pxx))
```

% Obtain the DC-centered power spectrum using Welch's method. Use a segment length of 500 samples with 300 overlapped samples and a DFT length of 500 points. Plot the resul

**Modulator:**



**FIGURE 2.1-5**

A complex (a) and real (b) modulator. A general representation for a modulator is shown in (c).

### 4.3–4 Demodulation and Detection

ASK, PSK, and QAM have one- or two-dimensional constellations with orthonormal basis of the form

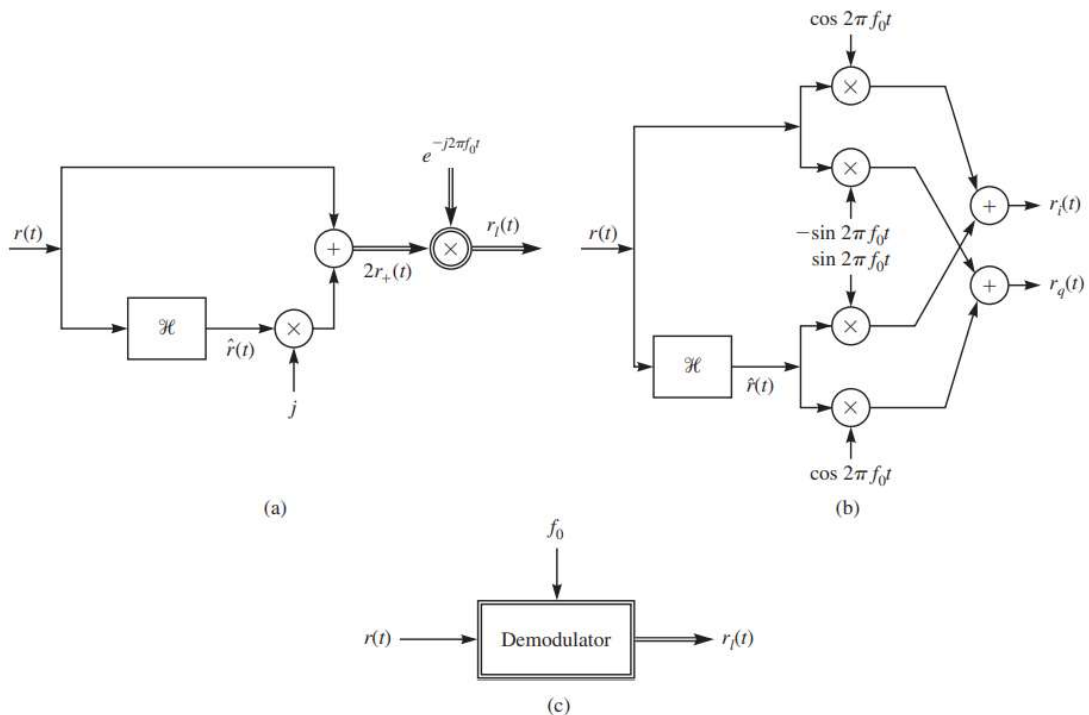
$$\begin{aligned}\phi_1(t) &= \sqrt{\frac{2}{\mathcal{E}_g}} g(t) \cos 2\pi f_c t \\ \phi_2(t) &= -\sqrt{\frac{2}{\mathcal{E}_g}} g(t) \sin 2\pi f_c t\end{aligned}\quad (4.3-36)$$

for PSK and QAM and

$$\phi_1(t) = \sqrt{\frac{2}{\mathcal{E}_g}} g(t) \cos 2\pi f_c t \quad (4.3-37)$$

for ASK. The optimal detector in these systems requires filters matched to  $\phi_1(t)$  and  $\phi_2(t)$ . Since both the received signal  $r(t)$  and the basis functions are high frequency bandpass signals, the filtering process, if implemented in software, requires high sampling rates.

To alleviate this requirement, we can first demodulate the received signal to obtain its lowpass equivalent signal and then perform the detection on this signal. The process of demodulation was previously discussed in Section 2.1–2 and the block diagram of the demodulator is repeated in Figure 4.3–9.



**FIGURE 4.3–9**

Complex (a) and real (b) demodulators. A general representation for a demodulator is shown in (c).



It is important to note that the demodulation process is an invertible process. We have seen in Section 4.1–1 that invertible preprocessing does not affect optimality of the receiver. Therefore, the optimal detector designed for the demodulated signal performs as well as the optimal detector designed for the bandpass signal. The benefit of the demodulator-detector implementation is that in this structure the signal processing required for the detection is done on the demodulated lowpass signal, thus reducing the complexity of the receiver.

Recall from Equations 2.1–21 and 2.1–24 that  $\mathcal{E}_x = \frac{1}{2}\mathcal{E}_{x_l}$  and  $\langle x(t), y(t) \rangle = \frac{1}{2} \text{Re} [\langle x_l(t), y_l(t) \rangle]$ . From these relations the optimal detection rule

$$\hat{m} = \arg \max_{1 \leq m \leq M} \left( \mathbf{r} \cdot \mathbf{s}_m + \frac{N_0}{2} \ln P_m - \frac{1}{2} \mathcal{E}_m \right) \quad (4.3-38)$$

can be written in the following lowpass equivalent form

$$\hat{m} = \arg \max_{1 \leq m \leq M} \left( \text{Re} [\mathbf{r}_l \cdot \mathbf{s}_{ml}] + N_0 \ln P_m - \frac{1}{2} \mathcal{E}_{ml} \right) \quad (4.3-39)$$

or, equivalently,

$$\hat{m} = \arg \max_{1 \leq m \leq M} \left( \text{Re} \left[ \int_{-\infty}^{\infty} r_l(t) s_{ml}^*(t) dt \right] + N_0 \ln P_m - \frac{1}{2} \int_{-\infty}^{\infty} |s_{ml}(t)|^2 dt \right) \quad (4.3-40)$$

The ML detection rule is obviously

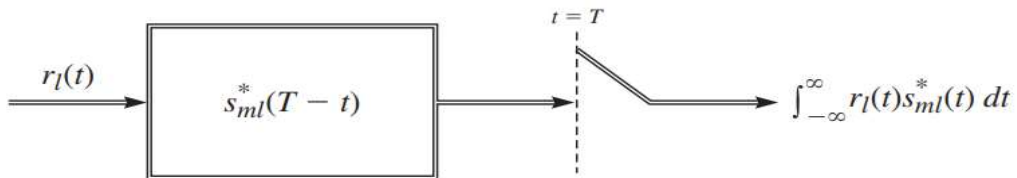
$$\hat{m} = \arg \max_{1 \leq m \leq M} \left( \text{Re} \left[ \int_{-\infty}^{\infty} r_l(t) s_{ml}^*(t) dt \right] - \frac{1}{2} \int_{-\infty}^{\infty} |s_{ml}(t)|^2 dt \right) \quad (4.3-41)$$

$$\hat{m} = \arg \max_{1 \leq m \leq M} \left( \text{Re} \left[ \int_{-\infty}^{\infty} r_l(t) s_{ml}^*(t) dt \right] - \frac{1}{2} \int_{-\infty}^{\infty} |s_{ml}(t)|^2 dt \right) \quad (4.3-41)$$

Equations 4.3–39 to 4.3–41 are baseband detection rules after demodulation.

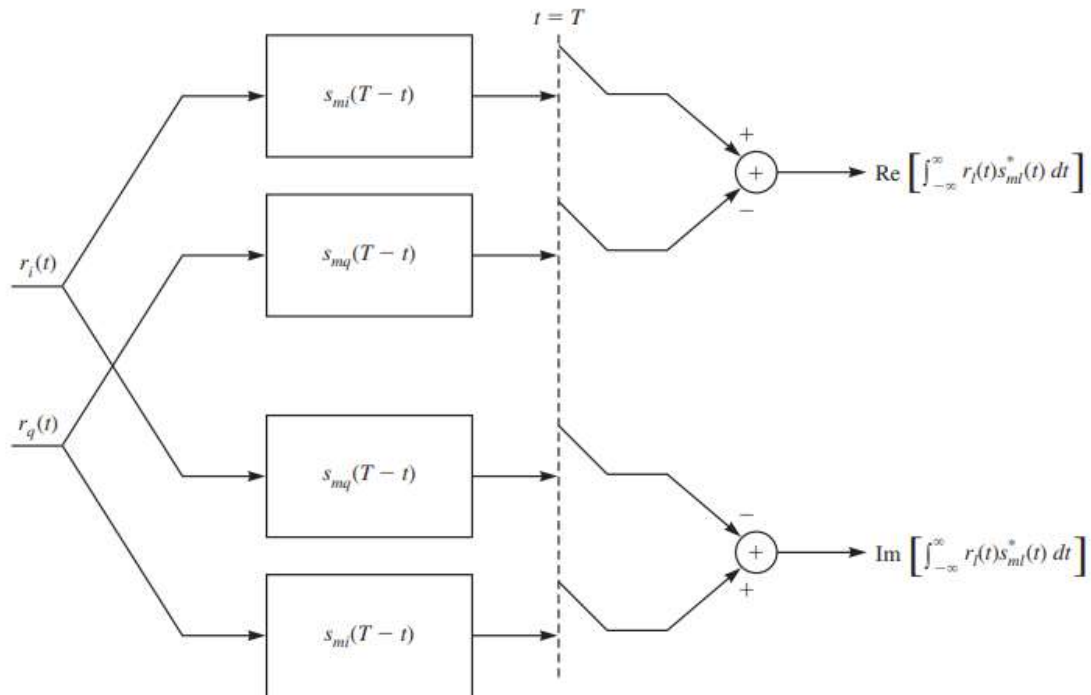
The implementation of Equations 4.3–39 to 4.3–41 can be done either in the form of a correlation receiver or in the form of matched filters where the matched filters are of the form  $s_{ml}^*(T - t)$  or  $\phi_{jl}^*(T - t)$ . Figure 4.3–10 shows the schematic diagram for a complex matched filter, and Figure 4.3–11 illustrates the detailed structure of a complex matched filter in terms of its in-phase and quadrature components. Note that for ASK, PSK, and QAM we have  $s_{ml}(t) = A_m g(t)$ , where  $A_m$  is in general a complex number (real for ASK). Therefore  $\phi_1(t) = g(t)/\sqrt{\mathcal{E}_g}$  serves as the basis function, and the signal points are represented by complex numbers of the form  $A_m \sqrt{\mathcal{E}_g}$ . Also note that for PSK detection the last term in Equation 4.3–41 can be dropped.

Throughout this discussion we have assumed that the receiver has complete knowledge of the carrier frequency and phase. This requires full synchronization between the



**FIGURE 4.3–10**

Complex lowpass equivalent matched filter.

**FIGURE 4.3–11**

Equivalent lowpass matched filter.

transmitter and the receiver. In Section 4.5 we will study the case where the carrier generated at the receiver is not in phase coherence with the transmitter carrier.