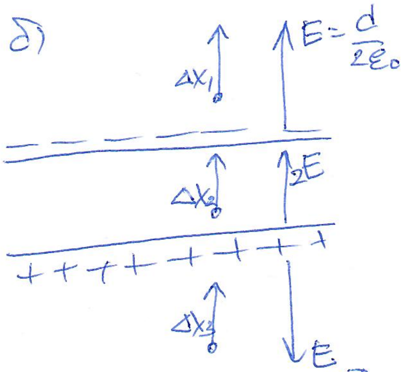
1

а)

$$\Delta\varphi = \int_1^2 E\,dx = \int_1^2 \frac{\sigma}{2\varepsilon_0}\,dx = \frac{\sigma\Delta x}{2\varepsilon_0}$$

б)

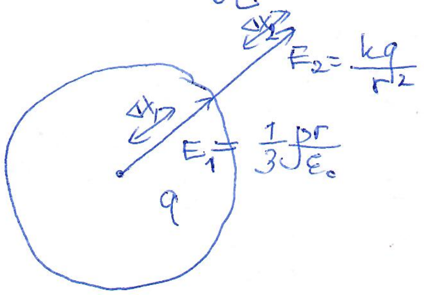


по аналогии с а): $\Delta\varphi_1 = \frac{\sigma}{2\varepsilon_0}\Delta x_1$

$$\Delta\varphi_2 = \frac{\sigma}{\varepsilon_0}\Delta x_2$$

$$\Delta\varphi_3 = \frac{\sigma\Delta x_3}{2\varepsilon_0}$$

в)



$$\Delta\varphi_1 = \int_1^2 E_1\,dr = \int_1^2 \frac{1}{3}\frac{\rho r}{\varepsilon_0}\,dr = \int_1^2 \frac{1}{3}\frac{q r}{\frac{4}{3}\pi r^3\varepsilon_0}\,dr =$$

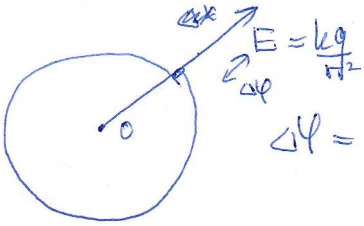
$$\rho = \frac{q}{\frac{4}{3}\pi r^3}$$

$$= \int_1^2 \frac{q\,dr}{4\pi r^2\varepsilon_0} = -\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) =$$

$$= -\frac{q}{4\pi\varepsilon_0}\frac{r_1 - r_2}{r_1 r_2} = \frac{q(r_2 - r_1)}{4\pi\varepsilon_0 r_1 r_2}$$

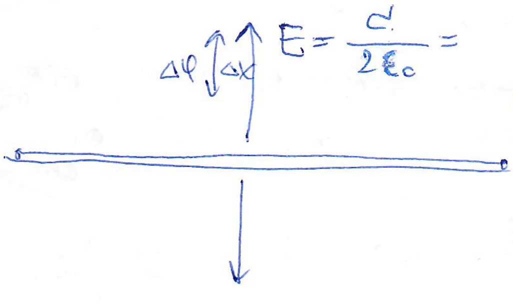
$$\Delta\varphi_2 = \int_1^2 E_2\,dr = \int_1^2 \frac{kq}{r^2}\,dr = kq\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

г)



$$\Delta\varphi = \int_1^2 E\,dr = \int_1^2 \frac{kq}{r^2}\,dr = kq\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

д)



$$\Delta\varphi = \int_1^2 E\,dr = \frac{\sigma}{2\varepsilon_0}\Delta x$$

② а) $\varphi = a(x^2 - y^2)$ $\quad E = -\operatorname{grad}\varphi \Rightarrow$

$$\Rightarrow E = -\frac{\partial\varphi}{\partial x}\vec{i} - \frac{\partial\varphi}{\partial y}\vec{j} - \frac{\partial\varphi}{\partial z}\vec{k} = -(2ax\vec{i} + 2ay\vec{j} + 0) = -2a(x\vec{i} + y\vec{j})$$

б) $\varphi = axy$ $\quad \Rightarrow E = -\frac{\partial\varphi}{\partial x}\vec{i} - \frac{\partial\varphi}{\partial y}\vec{j} = -ay\vec{i} - ax\vec{j} = -a(x\vec{i} + y\vec{j})$

③ $\varphi = \alpha(xy - z^2)$

$M = \{2, 1, -3\}$

$\vec{a} = \vec{i} + 3\vec{k}$

$$\vec{E} = -\operatorname{grad}\varphi \Rightarrow \vec{E} = -\left(\frac{\partial\varphi}{\partial x}\vec{i} + \frac{\partial\varphi}{\partial y}\vec{j} + \frac{\partial\varphi}{\partial z}\vec{k}\right) =$$

$$= -(\alpha y\vec{i} + \alpha x\vec{j} - 2\alpha z\vec{k}) \Rightarrow$$

$$E_M = \frac{\vec{E}_M \cdot \vec{a}}{|a|} = \frac{(\alpha\vec{i} + 2\alpha\vec{j} + 6\alpha\vec{k})(\vec{i} + 3\vec{k})}{\sqrt{1+9}} = \frac{\alpha y - 6\alpha z}{\sqrt{10}} = \frac{\alpha 19}{\sqrt{10}}$$

④ а) $\vec{E} = \vec{a}(y\vec{i} + x\vec{j})$ $\quad \Rightarrow \varphi = -\left(\int_0^x ay\,dx + \int_0^y ax\,dy\right) = -ayx - ayx = -2ayx$

б) $E = 2axy\vec{i} + a(x^2 + y^2)\vec{j} \Rightarrow \varphi = -\left(\int_0^x 2axy\,dx + \int_0^y a(x^2 + y^2)\,dy\right) =$

$$= -\left(\frac{2ayx^2}{2} + ax^2y + \frac{ay^3}{3}\right) = -\left(2ax^2y + \frac{1}{3}ay^3\right)$$

⑤ $E - ?$ $\quad \varphi = \vec{a}\vec{r}$

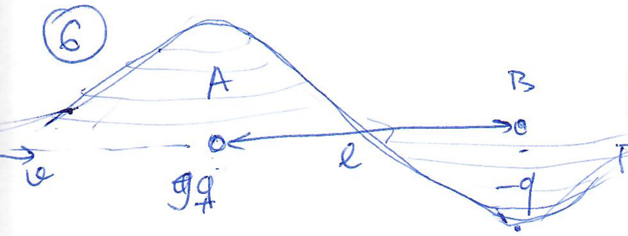
$\vec{a} = (a_x; a_y; a_z)$

$\vec{r} = (x, y, z)$

$$E = -\operatorname{grad}\varphi = -\left(\frac{\partial\vec{a}\vec{r}}{\partial x}\vec{i} + \frac{\partial\vec{a}\vec{r}}{\partial y}\vec{j} + \frac{\partial\vec{a}\vec{r}}{\partial z}\vec{k}\right) =$$

$$= -(a_x + a_y + a_z) = -\vec{a}$$

⑥

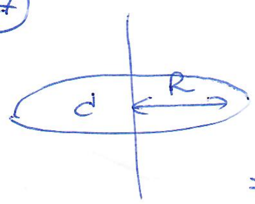


(кинетическая) Потенциальная энергия частицы $E_k = \frac{m\upsilon^2}{2}$

Потенциал частицы на бесконечности равен нулю.,

$$\varphi_A = \frac{9kq^2}{l} \quad \varphi_B = -\frac{kq^2}{l} \quad \Delta\varphi = \frac{8kq^2}{l} \Rightarrow \upsilon = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{16kq^2}{lm}}$$

⑦

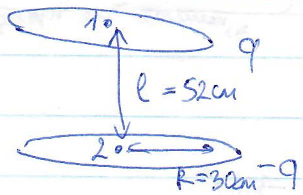


$$d\varphi = \frac{k\,dq}{dr} \quad \sigma = \frac{dq}{dS} \Rightarrow dq = dS\sigma \Rightarrow \varphi = \int d\varphi = \int \frac{k\sigma\,dr\,2\pi r}{\sqrt{r^2 + l^2}} =$$

$$= \frac{1}{4\pi\varepsilon_0}\sigma 2\pi \int_0^R \frac{1}{\sqrt{r^2 + l^2}}dr = \frac{1}{2\varepsilon_0} 2\int_0^R \frac{dr(r^2 + l^2)}{\sqrt{r^2 + l^2}} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2 + l^2} - l\right)$$

$$|\vec{E}| = |-\operatorname{grad}\varphi| = \frac{\partial\varphi}{\partial l} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{l}{\sqrt{R^2 + l^2}}\right)$$

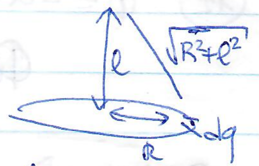
8



$\varphi_1 = \varphi_{k1} + \varphi_{k2}$ $\varphi_2 = \varphi_{k1}' + \varphi_{k2}'$

$\varphi_{k1} = \frac{kq}{R}$ $\varphi_1 = \frac{kq}{R} - \frac{kq}{\sqrt{R^2+l^2}}$

$\varphi_{k2} = \frac{kq}{\sqrt{R^2+l^2}}$ $\varphi_2 = -\frac{kq}{R} + \frac{kq}{\sqrt{R^2+l^2}}$



$\int d\varphi = \int \frac{k\,dq}{R}$

$\varphi = \frac{kQ}{R}$

$\Rightarrow \Delta\varphi_{21} = \varphi_2 - \varphi_1 = 2kq\cdot\frac{1}{\sqrt{R^2+l^2}} - 2kq\frac{1}{R}$

9

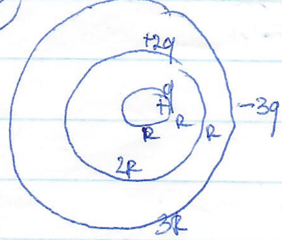
Чтобы перенести заряд из бесконечности в центр шара надо совершить работу равную потенциалу центра шара. или $A = \varphi = \int_{\infty}^{0} E\,dr$.

$E(r) = \int_0^R E_{пл}(l)\,dr = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{r}{\sqrt{r^2+R^2}}dr = \frac{\sigma}{2\varepsilon_0}\sqrt{r^2+R^2}$

$E_{пластинки} = \frac{1}{2}\frac{\sigma}{\varepsilon_0}\left(1 - \frac{l}{\sqrt{l^2+R^2}}\right)$ (из задачи 7) $\Rightarrow A = \int_{\infty}^{0} \frac{\sigma}{2\varepsilon_0}\sqrt{r^2+R^2}\,dr =$

(интегралы я считала в WolframAlpha, извините :( ) $= \frac{\sigma}{4\varepsilon_0}\left(r\sqrt{r^2+R^2} + R^2\ln\left(\sqrt{r^2+R^2}+r\right)\right)$

10



Посчитаем φ как работу по переносу заряда из ∞.

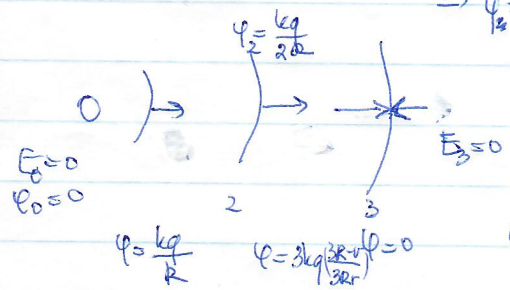
По Т. Гаусса $\Phi_{вн} = E\cdot S = \frac{q}{\varepsilon_0}$ $E = \frac{q}{\varepsilon_0 S}$, причем q равен 0, $q+2q-3q$

значит поле снаружи нет $\Rightarrow \varphi = 0$

Перенос заряда $+q$ с 3 на 2:

$\Phi = ES = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{\varepsilon_0 S}$, где $q = 3q \Rightarrow E = \frac{3kq}{r^2}$

$\Rightarrow \varphi_2 = -\int_{3R}^{r} E\,dr = -\int_{3R}^{r}\frac{3kq}{r^2}dr = +3kq\left(\frac{1}{r} - \frac{1}{3R}\right) = +3kq\left(\frac{3R-r}{3Rr}\right) = \varphi(r)$



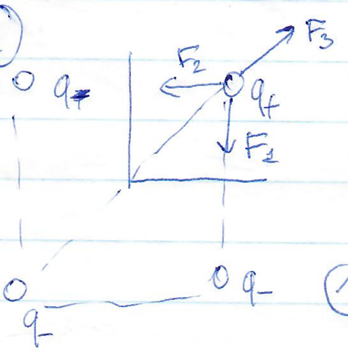
Значит $\varphi_2 = 3kq\frac{3R-2R}{6R^2} = \frac{1}{2R}kq$ при $r \in [2R;3R]$

и на участке при $r \in [R;2R]$

$\varphi = -\int_{2R}^{r}\frac{kq}{r^2}dr = kq\left(\frac{2R-r}{2Rr}\right) = \varphi(r) \Rightarrow \varphi_3 = kq\frac{R}{2R^2} = \frac{kq}{R}$

$r \in [R;2R]$

11



$|F_1| = \frac{kq^2}{2r^2}$ $|F_1 + F_2| = |F_1|\cdot\sqrt{2}$

$F = |F_1|\sqrt{2} - |F_3| = \frac{\sqrt{2}kq^2}{2r^2} = \frac{kq^2}{4r^2} = \frac{kq^2}{r^2}\left(\frac{\sqrt{2}}{2} - \frac{1}{4}\right)$

12   $\varphi = -x^3ab$ $\nabla^2\varphi = -\frac{\rho}{\varepsilon_0} = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = \frac{\partial(-3abx^2)}{\partial x} =$

$= -6abx$

$\Rightarrow \rho = 6abx\varepsilon_0$