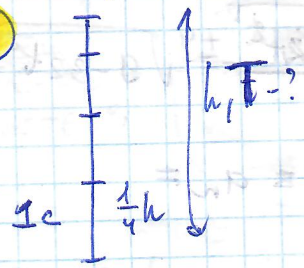
Серие 1

(1.)



$$S(\Delta t) = \int_{t_1}^{t_2} gt\,dt = \frac{1}{2}gt^2\Big|_{t_1}^{t_2} = \frac{1}{2}g\left(t_2^2 - t_1^2\right)$$

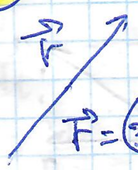
$$\frac{1}{4}S = \frac{1}{4}\int_{0}^{t_2} gt\,dt = \frac{1}{2}\frac{1}{4}gt^2\Big|_{0}^{t_2} = \frac{1}{8}gt_2^2$$

$$\begin{cases} \frac{1}{2}g(t_2^2 - t_1^2) = \frac{1}{8}gt_2^2 \\ t_2 - t_1 = 1 \end{cases} \qquad \begin{cases} 3t_2^2 - 4t_1^2 = 0 \\ t_1 = t_2 - 1 \end{cases}$$

$$3t_2^2 - 4(t_2-1)^2 = 0$$

$$-t_2^2 + 8t_2 + 4 = 0 \Rightarrow \left[\begin{array}{l} t = 0,54 \\ t = 7,46 \text{ — ответ} \end{array}\right.$$

(2)



а) найти траекторию

б) найти ускорение и скорость

$\vec{r} = (3 - 6t^2)\vec{i} + 2t\vec{j}$ в) касательное и норм. ускорения.

$j(t) = 2t$

$i(t) = 3 + 6t^2$

$\Rightarrow$ Траектория: $t = \frac{j(t)}{2}$

$i(j) = 3 + 6\frac{j(t)^2}{4}$

б) скорость:

$\dot{j}(t) = 2$

$\dot{i}(t) = 12t$

$|v| = \sqrt{4 + 144t^2}$

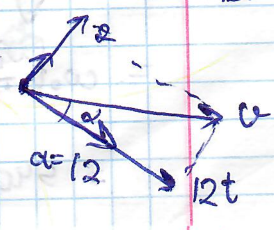
ускорение:

$\ddot{j}(t) = 0$

$\ddot{i}(t) = 12$

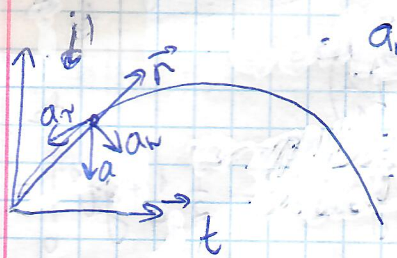
$|a| = \sqrt{0 + 144} = 12$

$\alpha = \frac{2}{12t}$



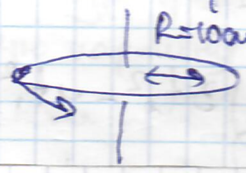
в) Нормальное ускорение:

$$a_n = \frac{\vartheta^2}{R} = \frac{144t^2+4}{}$$



3.

$$\varphi = A + Bt + Ct^3 \Rightarrow \omega = B + 3Ct^2 = 302\ c^{-1}$$



$A = 3$ рад

$B = 2$ рад/с

$C = 1$ рад/с³

$$\varepsilon = \frac{d\omega}{dt} = 3\cdot 2t = 60c^{-2}$$

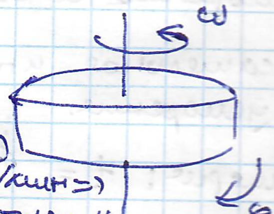
$t = 10c$

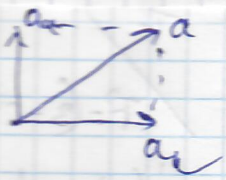
$$a_\tau = \varepsilon R = 60c^{-2} \cdot 0{,}1м = 6м/c^2$$

$$a_n = \omega^2 R = 302^2 c^{-2} \cdot 0{,}1м \approx 9000м/c^2$$

$$a = \sqrt{(9000м/c^2)^2 + (6м/c^2)^2} \approx 9000м/c^2$$

4.





t – ?

N – ?

$\nu = 180$ об/мин $\Rightarrow$

$\Rightarrow \omega_0 = 2\pi \cdot 180$ 1/рад

$\varepsilon = 3рад/c^2$

$$t = -\frac{\omega_0}{\varepsilon} = 60c$$

$$N = \int_0^t \omega(t)dt = \int_0^t (\omega_0 + \varepsilon t)dt =$$

$$= \omega_0 t + \frac{\varepsilon t^2}{2}\Big|_0^t = \omega_0 t + \frac{\varepsilon t^2}{2} = 60\cdot 180 - \frac{3}{2}3600 =$$

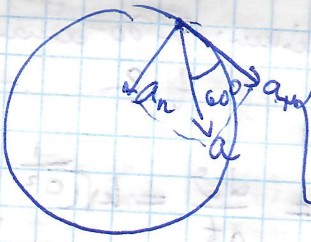
$= 5400$ оборотов

5

$\varepsilon - ?$

$t = 2c$

$\alpha = 60^\circ$



$$\begin{cases} a \cdot \cos 60^\circ = a_\tau = \dfrac{d\vartheta}{dt} = \dfrac{d(\omega R)}{dt} = \varepsilon R \\ a \sin 60^\circ = a_n = \omega_1^2 R \\ \omega_1 = \varepsilon t \end{cases}$$

$$a \cdot \frac{1}{2} = \varepsilon R \quad ; a\frac{\sqrt{3}}{2} = \omega_1^2 R \ ; \ \omega_1 = \varepsilon t$$

$$a = \frac{2\omega_1^2 R}{\sqrt{3}} = \frac{2\varepsilon^2 t^2 R}{\sqrt{3}}$$

$$\frac{2\varepsilon^2 R t^2}{2\sqrt{3}} = \varepsilon R$$

$$\frac{\varepsilon t^2}{\sqrt{3}} = 1 \qquad \varepsilon = \frac{\sqrt{3}}{t^2} = \frac{\sqrt{3}}{4c^2} = 0{,}43 c^{-2}$$

6

$V = a\sqrt{x}$

$a = const > 0$

$t_0 \to x = 0$

Найти: а) $\vartheta(t)$

$a(t)$

в) $\langle \vartheta \rangle$ за s метров

$$\vartheta = a\sqrt{x}$$

$$\frac{dx}{dt} = a\sqrt{x}$$

$$\int dx \frac{1}{\sqrt{x}} = \int a dt$$

$$\int x^{-1/2} dx = at$$

$$2x^{1/2} = at \Rightarrow x(t) = \left(\frac{1}{2}at\right)^2$$

$$2\sqrt{x} = at \qquad x(t) = \frac{a^2t^2}{4}$$

$$\vartheta(t) = x(t)' = \frac{1}{4}a^2 \cdot \frac{1}{2}t = \frac{1}{8}a^2 t$$

$$a(t) = \vartheta(t)' = \frac{1}{8}a^2$$

$$S = x_0 + \frac{1}{8}a^2t^2 + \frac{1}{8}\frac{a^2t^2}{2} =$$

$$= \frac{2a^2t^2 + a^2t^2}{16} = \frac{3a^2t^2}{16}$$

$$\Rightarrow \langle \vartheta \rangle = \frac{S}{t} = \frac{3}{16}a^2 t$$

7 $a = k\sqrt{\vartheta}$ Найти: S - до остановки

t - ?

const $k \geq 0$

$\boxed{t_0 \to \vartheta_0}$

$$a = \frac{d\vartheta}{dt} = k\sqrt{\vartheta}$$

$$d\vartheta \frac{1}{\sqrt{\vartheta}} = k\,dt$$

$$\int_{\vartheta_0}^{0} \frac{1}{\sqrt{\vartheta}}\,d\vartheta = k\int_0^t dt$$

$$2\vartheta^{1/2} = kt \Rightarrow \vartheta = \frac{1}{4}k^2t^2$$

$$\frac{dx}{dt} = \frac{1}{4}k^2t^2$$

$$\int_0^x dx = \frac{1}{4}k^2\int_0^t t^2\,dt = \frac{1}{4}k^2\frac{t^3}{3} = x$$

$$X$$

$$a = \frac{1}{2}k^2t$$

$$\vartheta(t) = \vartheta_0 - at =$$

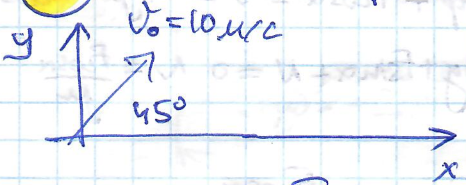
$$= -\frac{1}{2}k^2t + \vartheta_0 = 0$$

$$\Rightarrow t = \frac{2\vartheta_0}{k^2} \Rightarrow S = \vartheta_0\frac{2\vartheta_0}{k^2} -$$

$$- \frac{1}{2}k^2\frac{4\vartheta_0^2}{k^4}\cdot\frac{1}{2} =$$

$$= \frac{2\vartheta_0^2}{k^2} - \frac{1}{4}\cdot\frac{4\vartheta_0^2}{k^2} = \frac{\vartheta_0^2}{k^2} = S$$

1 Кривизна – ? Серия 2



$$K = \frac{1}{R} = \frac{y''_x}{(1 - y'^2_x)^{3/2}} \Rightarrow$$

$$\begin{cases} x(t) = \cos 45^\circ v_0 t = \frac{1}{\sqrt{2}} v_0 t \\ y(t) = \sin 45^\circ v_0 t - gt^2/2 = \frac{1}{\sqrt{2}} v_0 t - gt^2/2 \end{cases} \Rightarrow$$

$$t = \frac{\sqrt{2}x}{v_0}$$

$$\Rightarrow y(x) = \text{tg}\,\alpha\, x - \frac{g}{2v_0^2\cos^2\alpha}x^2 \Rightarrow$$

$$x(1с) = \frac{1}{\sqrt{2}} 10 \text{ м/с} \cdot 1с = 7{,}07 \text{м}$$

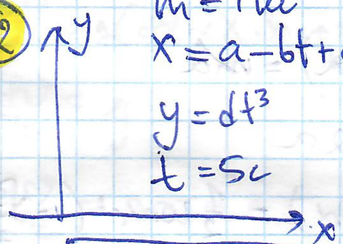
$$\Rightarrow y(x)' = \text{tg}\,\alpha - \frac{g}{\cancel{2}v_0^2\cos^2\alpha}\cancel{2}x = 1 - \frac{10 \text{ м/с}^2}{100 \text{ м}^2/\text{с}^2 \cdot \frac{1}{2}} 7{,}07\text{м} = -0{,}414$$

$$y(x)'' = -\frac{g}{v_0^2\cos^2\alpha} = -\frac{10 \text{ м/с}^2}{100 \text{ м}^2/\text{с}^2 \cdot \frac{1}{2}} = -0{,}2 \text{ м}^{-1}$$

$$\Rightarrow k = \frac{1}{R} = \frac{-\frac{g}{v_0^2\cos^2\alpha}}{\left(1 - \left(\text{tg}\,\alpha - \frac{g}{v_0^2\cos^2\alpha}x\right)^2\right)^{3/2}} = \frac{-0{,}2\text{м}^{-1}}{(1 - 0{,}414^2)^{3/2}} = -0{,}26 \text{ м}^{-1}$$

$$\Rightarrow R = 3{,}85 \text{ м}$$

2



$m = 1$ кг

$x = a - bt + ct^3$ $c = 1$ м/с³ $d = 2$ м/с³

$y = dt^3$

$t = 5$с

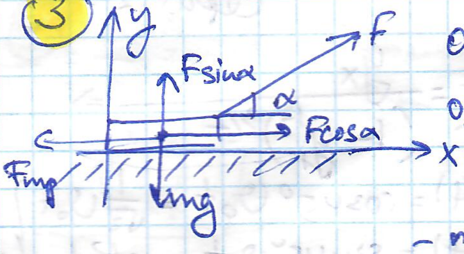
$$x'' = (-b + 3ct^2)' = 6ct = 6 \cdot 1 \text{м/с}^3 \cdot 5\text{с} = 30 \text{ м/с}^2$$

$$y'' = (3dt^2)' = 6dt = 6 \cdot 2\text{м/с}^3 \cdot 5\text{с} = 60 \text{м/с}^2$$

$$a = \sqrt{x''^2 + y''^2} = \sqrt{(30 \text{ м/с}^2)^2 + (60 \text{ м/с}^2)^2} = 67 \text{ м/с}^2$$

$$F = ma = 67 \text{м/с}^2 \cdot 1\text{кг} = 67 \text{ Н}$$

③



OX: $-F_{тр} + F\cos\alpha = 0 = -N\mu + F\cos\alpha$

OY: $-mg + F\sin\alpha + N = 0 \quad N = \frac{F\cos\alpha}{\mu}$

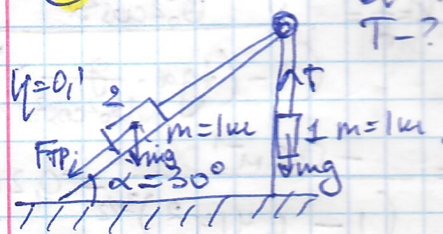
$$-mg + F\sin\alpha + \frac{F\cos\alpha}{\mu} = 0$$

$$F = \frac{mg}{\left(\sin\alpha + \frac{\cos\alpha}{\mu}\right)} = \frac{mg\mu}{\sin\alpha\,\mu + \cos\alpha}$$

$\alpha \to \max?$ $(\cos\alpha + \sin\alpha\,\mu)' = -\sin\alpha + \cos\alpha\,\mu = 0$

$\Rightarrow \alpha = \operatorname{arctg}(\mu)$

④



a – ?
T – ?

Тело1: $mg - T = ma$

Тело2: $-\underset{\mu}{F_{тр}} - \sin\alpha\, mg + T = ma$

$-mg\cos\alpha\,\mu$

$T = m(g-a)$

$-mg(\cos\mu + \sin\alpha) + m(g-a) = ma$

$g(1 - \cos\alpha\mu - \sin\alpha) = 2a$

$a = \frac{1}{2}g(1 - \cos\alpha\,\mu - \sin\alpha) = \frac{1}{2}\,10м/с^2\left(1 - \frac{\sqrt{3}}{2}0{,}1 - \frac{1}{2}\right) =$

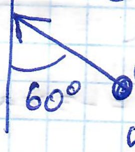
$= 2{,}05\,м/с^2$

$T = 1кг\,(10м/с^2 - 2{,}05\,м/с^2) = 7{,}95Н$

5

$t - ?$ $\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = d\vec{p}/dt$

$\langle F_{уд} \rangle = 20Н$ $p = 6 м/с \cdot \sin 60° \cdot 0{,}15 м =$

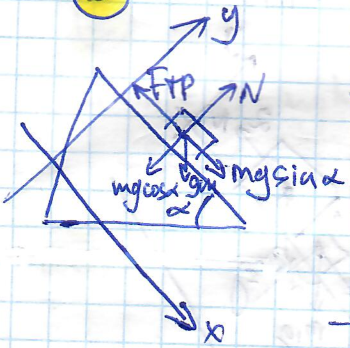


$v = 6 м/с$ $= 0{,}78 м/с м$

$m = 150г$ $\Rightarrow \langle F_{уд} \rangle = \frac{p}{t}$

$t = \frac{p}{\langle F_{уд} \rangle} = \frac{0{,}78 м/с \cdot м}{20Н} = 0{,}039с$

6



$\mu = \gamma x$ $\gamma = const$ $S - ?$

$\begin{cases} -F_{тр} + mg\sin\alpha = ma_x & :0x \\ N - mg\cos\alpha = 0 & :0y \end{cases}$

$-mg\cos\mu + mg\sin\alpha = max$

$\Rightarrow a_x = g(\cos\alpha\mu + \sin\alpha)$

$\ddot{x} = g(\cos\alpha\gamma \cdot x + \sin\alpha)$

$\frac{dv}{dt} = g\cos\alpha\gamma \cdot x + g\sin\alpha$ $\frac{dx}{dt} = v$

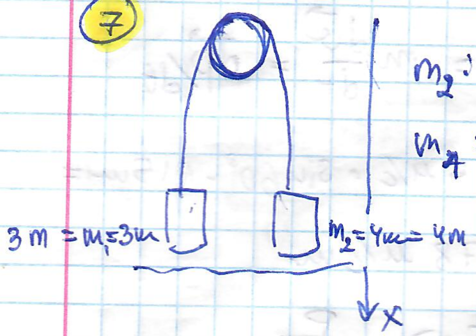
$\frac{dv}{\frac{dx}{v}} = g\cos\alpha\gamma x + g\sin\alpha$ $dt = \frac{dx}{v}$

$\frac{v\,dv}{dx} = g\cos\alpha\gamma x + g\sin\alpha$ $\int_0^t v\,dv = \int_0^x (g\cos\alpha\gamma x + g\sin\alpha)dx$

$\frac{v^2}{2} = g\frac{\cos\alpha\gamma x^2}{2} + g\sin\alpha x = 0$ т.к. в итоге тело останов.

$\Rightarrow x = \frac{2\operatorname{tg}\alpha g}{x}$

7



$m_2: \; 4mg - T = 4ma$

$m_1: \; 3mg - T = -3ma$

$T = 4mg - 4ma$

$3mg - 4mg + 4ma = -3ma$

$3g - 4g = -7a$

$7a = g \Rightarrow a = \frac{1}{7}g$

$\Rightarrow \frac{3м}{2} = \frac{1}{7}\frac{gt^2}{2} \Rightarrow \frac{21}{g} = t^2 \Rightarrow t = \sqrt{\frac{21}{10}} = 1{,}44c$

8

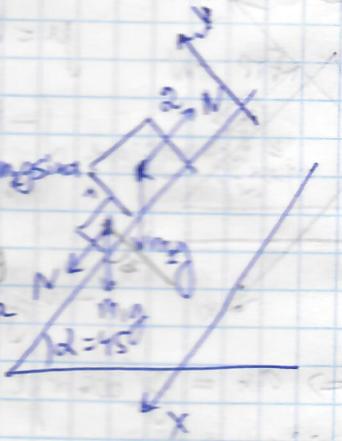
$\mu_2 = 0{,}5 \quad m_2 = 2кг$

$\mu_1 = 0{,}7 \quad m_1 = 1кг$

$2\begin{cases} ox: g\sin\alpha - F_{тр2} - N = m_2 a \\ oy: -g\cos\alpha\, m_2 + N_2 = 0 \end{cases}$ $N_2 = mg\sin\alpha$

$N_{01}\mu = \cos\alpha\,\mu mg$

$1\begin{cases} ox: g\sin\alpha\, m_1 - F_{тр1} + N = m_1 a \\ oy: -g\cos\alpha + N_{01} = 0 \end{cases}$



$\begin{cases} m_2 g\sin\alpha - m_2 g\cos\alpha\,\mu_2 - N = m_2 a \\ m_1 g\sin\alpha - m_1 g\cos\alpha\,\mu_1 + N = m_1 a \end{cases}$

$N = m_2 g\sin\alpha - m_2 g\cos\alpha\,\mu_2 - m_2 a = N$

$m_1 g\sin\alpha - m_1 g\cos\alpha\,\mu_1 + m_2 g\sin\alpha - m_2 g\cos\alpha\,\mu - m_2 a = m_1 a$

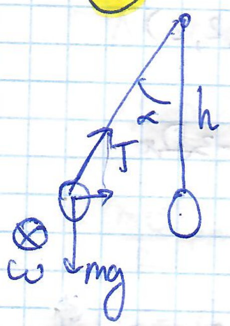
$a(m_1 + m_2) = g\,(m_1(\sin\alpha - \cos\alpha\,\mu_1) + m_2(\sin\alpha - \cos\alpha\,\mu_2))$

$a = \frac{g(m_1(\sin\alpha - \cos\alpha\,\mu_1) + m_2(\sin\alpha - \cos\alpha\,\mu_2))}{(m_1 + m_2)} = 3м/с$

$\Rightarrow N = m_2 g(\sin\alpha - \cos\alpha\,\mu_2) - m_2 a = 1\ Н$

Работа и Энергия Серия 3

1



$$a_n = \omega^2 R$$

$$\begin{cases} a_n = \sin\alpha\, T = \omega^2 R \\ mg = \cos\alpha\, T \end{cases} \qquad T = \frac{\omega^2 R}{\sin\alpha}$$

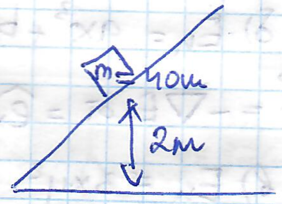
$$mg = \omega^2 R \cdot \frac{1}{tg\alpha} = \omega^2 \cancel{R}\, \frac{h}{\cancel{R}}$$

$$tg\alpha = \frac{R}{h} \qquad mg = \omega^2 h$$

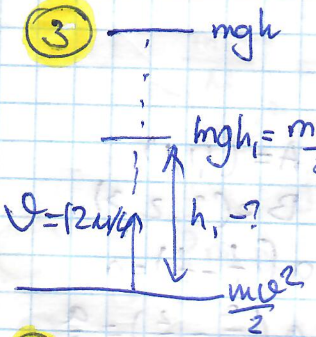
$$h = \frac{mg}{\omega^2}$$

2 $m = 40$ т, $h = 2$ м

$$\Delta E_n = A = mgh = 10 \cdot 40 \cdot 2 \text{ МДж} = 800 \text{ МДж}$$



3



$$E_{полн} = \frac{m\vartheta^2}{2}$$

$$\frac{m\vartheta^2}{4} = mgh_1 = \frac{m\vartheta_1^2}{2} \quad \text{т.к. энергия сохраняется}$$

$$\Rightarrow h_1 = \frac{\vartheta^2}{4g} = \frac{144 \text{ м}^2/\text{с}^2}{4 \cdot 10 \text{ м/с}^2} = 3{,}6 \text{ м}$$

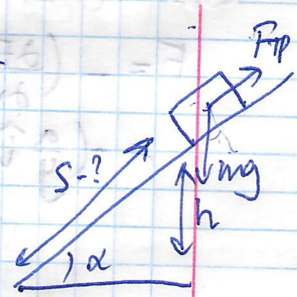
4

$$\mu = \gamma x \qquad mgh - \int_0^S F_{тр}\, dx = 0 \qquad \frac{h}{S} = \sin\alpha \Rightarrow$$

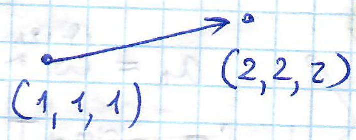
$$\int_0^S F_{тр}\, dx = \int_0^S mg\cos\alpha\, \gamma x\, dx = mg\cos\alpha\, \gamma \frac{S^2}{2}$$

$$mg\sin\alpha S - mg\cos\alpha\, \gamma \frac{S^2}{2} = 0$$

$$2\sin\alpha S - \cos\alpha\, S^2 = 0 \qquad S = 2\, tg\alpha$$



5. $U = -x^2 - y^2 + z^2$



$A = -\Delta E_{пот} = E_B - E_A =$

$= -2^2 - 2^2 + 2^2 - (-1^2 - 1^2 + 1^2) = -4 - (-1) = -3$ Дж

6. а) $E_п = a\,xyz$, $a = const$

$\vec{F} = -\vec{\nabla} E_{пот} = \hat{e_x}\frac{\partial axyz}{\partial x} + \hat{e_y}\frac{\partial axyz}{\partial y} + \hat{e_z}\frac{\partial axyz}{\partial y} =$

$= \hat{e_x}\, ayz + \hat{e_y}\, axz + \hat{e_z}\, axy$

б) $E_п = ax^3 + bxy^3 + cz^3$

$\vec{F} = -\vec{\nabla} E_{пот} = \hat{e_x}\, 3ax^2 + \hat{e_y}\, 3by^2 + \hat{e_z}\, 3cz^2$

в) $E_п = 3xy^2$

$\vec{F} = -\vec{\nabla} E_{пот} = \hat{e_x}\, 3y^2 + \hat{e_y}\, 6xy$

7. $U = a\left(\frac{x}{y} - \frac{y}{z}\right)$ $F - ?$ A_{AB} $A = (1,1,1)$

$B = (2,2,3)$

$-\Delta E_{п\,AB} = A = \frac{a}{3}$

$E_{пA} = a\left(\frac{1}{1} - \frac{1}{1}\right) = 0$

$A = \Delta F \cdot l \Rightarrow \Delta F = \frac{A}{l} = \frac{a}{3\sqrt{6}}$

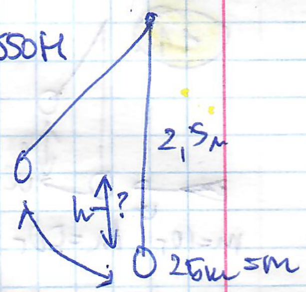
$E_{пB} = a\left(\frac{2}{2} - \frac{2}{3}\right) = \frac{a}{3}$

$l = \sqrt{(1-2)^2 + (1-2)^2 + (1-3)^2} = \sqrt{1+1+4} = \sqrt{6}$

$F = -\left(\frac{\partial E_п}{\partial x}\hat{i} + \frac{\partial E_п}{\partial y}\hat{j} + \frac{\partial E_п}{\partial z}\hat{k}\right) =$

$= -\left(\frac{a}{y}\hat{i} - a\left(\frac{x}{y^2} + \frac{1}{z}\right)\hat{j} + a\frac{y}{z^2}\hat{k}\right)$

8



$$T = mg + \frac{m\vartheta^2}{2}$$

$$N - Mg = am = \frac{m\vartheta^2}{R}$$

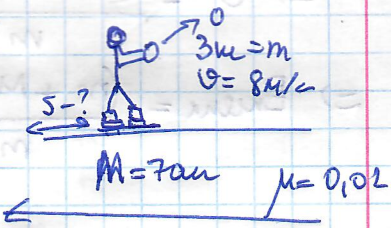
$$N \leq 550H \quad , N = \frac{m\vartheta^2}{R} + mg$$

$$\frac{m\vartheta^2}{R} + mg = 550H$$

$$\vartheta^2 = (550H - 250H) \cdot \frac{2{,}5м}{25кг} = 30\,H\,\frac{м}{кг}$$

$$mgh = \frac{m\vartheta^2}{2} \Rightarrow h = \frac{\vartheta^2}{2g} = \frac{30\,H\frac{м}{кг}}{2 \cdot 10м/с^2} = 1{,}5м$$

9



$$p_0 = 0 = p_ч + m_к$$

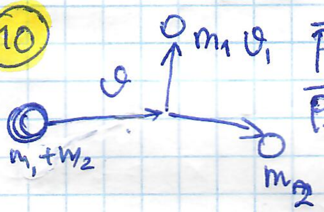
$$p_ч = MU \quad p_к = -m\vartheta$$

$$MU - m\vartheta = 0$$

$$U = \frac{m\vartheta}{M} \Rightarrow \frac{MU^2}{2} = F_{тр} S$$

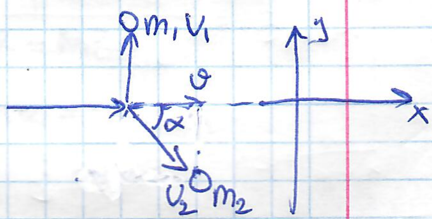
$$S = \frac{MU^2}{2Mg\mu} = \frac{m^2\vartheta^2}{M^2}\frac{M}{2Mg\mu} = \frac{(3кг)^2(8м/с)^2}{2(70кг)^2\, 10м/с^2 \cdot 0{,}02} = 0{,}05м$$

10



$$\vec{P}_0 = (m_1+m_2)\vartheta$$

$$\vec{P}_3 = \vec{P}_1 + \vec{P}_2$$



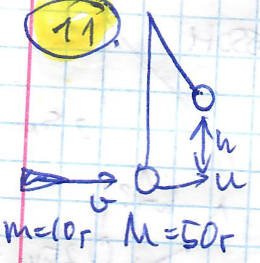
$$\vartheta_{2y} = \frac{v_1 m_1}{m_2}$$

$$P_{1x} = 0 \quad P_{1y} = P_1$$

$$P_{2x} = P_0 \quad P_{2y} = P_{1y} = P_1 \qquad \vartheta_{2x} = \frac{(m_1+m_2)\vartheta}{m_2}$$

$$\Rightarrow \vartheta_2 = \sqrt{\frac{(m_1+m_2)^2\vartheta^2}{m_2^2} + \frac{\vartheta_1^2 m_1^2}{m_2^2}} \qquad tg\,\alpha = \frac{v_1 m_1}{(m_2+m_1)\vartheta}$$

11



По з-ну сохранения импульса:

$p_0 = m\vartheta \qquad p_0 = p_1$

$p_1 = (m+M)u \qquad u = \frac{m\vartheta}{M+m}$

$\frac{m\vartheta^2}{2} = \frac{u^2(m+M)}{2} + E_{mепл}$

По з-ну сохр. энергии

$\frac{u^2(M+m)}{2} = (m+M)gh \Rightarrow u^2 = 2gh \Rightarrow u = \sqrt{2gh}$

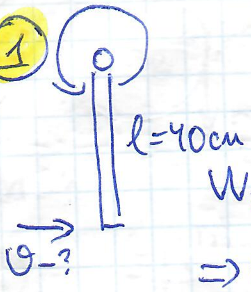
$\Rightarrow \frac{\vartheta m}{M+m} = u = \sqrt{2gh} \Rightarrow \vartheta = \frac{(m+M)\sqrt{2gh}}{m}$

$\Rightarrow \vartheta^2 = \frac{(M+m)^2 2gh}{m^2}$

$\Rightarrow E_{mепл} = \frac{(m+M)^2 2gh}{m^2} - (m+M)gh$

Динамика вращательного движения Серия 4

1

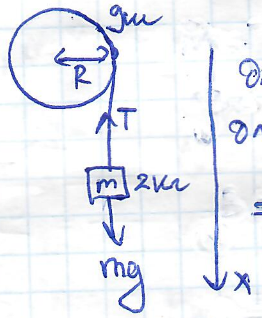


$$\frac{I\omega^2}{2}+mgh=0 \qquad \omega=\frac{v}{R}$$

$$W_k=\frac{I\omega^2}{2}=\frac{Iv^2}{2R^2}=\frac{m\frac{1}{3}l^2v^2}{2l^2}=\frac{mv^2}{6}\Rightarrow$$

$$\Rightarrow v^2=6gl \qquad v=\sqrt{6gl}=4{,}8\ \text{м/с}$$

2



для груза: $ma=mg-T \qquad T=m(g-a)$

для барабана: $M=TR=I\beta \qquad I=MR^2\cdot\frac{1}{2}$

$$\Rightarrow \beta=\frac{TR}{I}=\frac{m(g-a)\cancel{R}}{MR^{\cancel{2}}\cdot\frac{1}{2}}=\frac{2m(g-a)}{MR}$$

$$a=R\beta \qquad \beta=\frac{a}{R}$$

$$\frac{2m(g-a)}{MR}=\frac{a}{R}$$

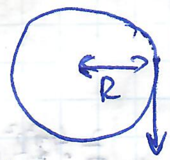
$$2m(g-a)R=MRa$$

$$2mgR-2maR=MRa$$

$$2mgR=a(MR+2mR)$$

$$a=\frac{2mgR}{(M+2m)R}=\frac{2\cdot 2\text{кг}\cdot 10\text{м/с}^2}{(9\text{кг}+4\text{кг})}=3{,}3\ \text{м/с}^2$$

3



$R=0{,}2м$

$M_{тр}=4{,}9Нм$

$\varepsilon=100\ рад/с$

$F=98{,}1Н$

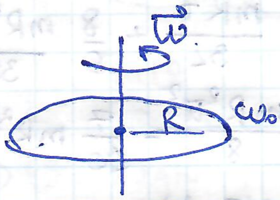
$M=?$

$$M=F\cdot R-M_{тр}=I\beta$$

$$I=\frac{FR-M_{тр}}{\beta}=\frac{mR^2}{2}$$

$$m=\frac{2(FR-M_{тр})}{R^2\beta}=7{,}36кг$$

4



μ ; t, N – ?

$$L_0=I\omega=mR^2\omega$$

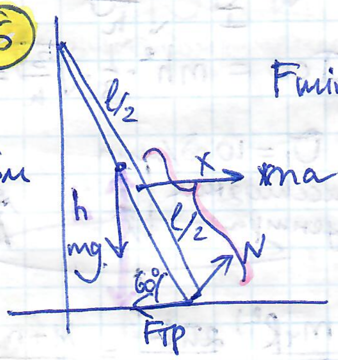
$$\Delta L=\frac{mR^2\omega_0}{t}=M_{тр}=F_{тр}R=N\mu R=$$

$$mg\mu R=\frac{mR^2\omega_0}{t}\Rightarrow t=\frac{R\omega_0}{g\mu}$$

$$\varphi=2\pi N=\frac{\varepsilon t^2}{2}$$

$$N=\frac{\varepsilon t^2}{4\pi}=\frac{\omega_0 t}{4\pi}=\frac{\omega_0^2 R}{4\pi g\mu}$$

5



$l=5м$

$F_{min}=173Н$

h – ?

$$h/x=\sin\alpha$$

$$M_{тяж}=mg\cdot x\cos\alpha$$

$$M_{и}=ma\cdot\frac{l}{2}\sin\alpha$$

$$M_{тяж}=M_{и}$$

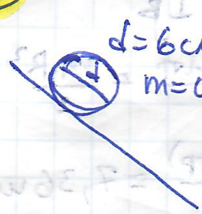
$$mgx\cos\alpha=ma\frac{l}{2}\sin\alpha=F\frac{l}{2}\sin\alpha$$

$$mgx\cos\alpha=F\,l/_2\sin\alpha$$

$$h=\sin\alpha\, x=\frac{1}{2}x=\frac{1}{2}\frac{Fl}{2}\sin\alpha\frac{1}{\cos\alpha\, mg}=\frac{Fl}{4}\operatorname{tg}\alpha\frac{1}{mg}=$$

$$=\frac{Fl\operatorname{tg}\alpha}{4mg}=\frac{173Н\cdot\sqrt{3}\cdot 5м}{4\cdot 10кг\cdot 9{,}8м/с^2}=3{,}8м$$

6



$d = 6$ см
$m = 0{,}25$ кг

$W_к = \frac{m\upsilon^2}{2} + \frac{I\omega^2}{2} =$

$\omega = \frac{2\pi R}{4}$ $\quad = \frac{4\pi^2 R^4}{16}\cdot\frac{m}{2} + \frac{2}{5}\cdot\frac{4\pi^2 R^2}{16}\cdot\frac{m}{2}R^2 =$

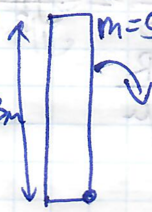
$\upsilon = \omega R = \frac{2\pi R^2}{4}$

$= 4\frac{mR^4\pi^2}{32} + \frac{8}{5}\frac{mR^4\pi^2}{32} =$

$= \frac{mR^4\pi^2}{8} + \frac{2}{5}\frac{mR^4\pi^2}{8} = \left(1+\frac{2}{5}\right)\frac{mR^2\pi^2}{8} = \frac{7}{40}mR^2\pi^2 =$

$= \frac{7}{40}\cdot 0{,}25\text{кг}\cdot 0{,}06^2\text{м}^2\cdot 3{,}14^2 = 0{,}0015$ Дж

7



$m = 50$ кг
$h = 3$ м
υ — ?
L = ?

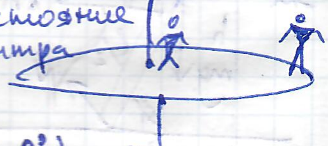
$E_п = \frac{mgh}{2} \to E_{к\,вращ} = \frac{I\omega^2}{2}$

$\frac{mgh}{2} = \frac{I\omega^2}{2} = \frac{mh^2\omega^2}{6}$ $\quad \omega = \sqrt{\frac{mgh}{I}} = \sqrt{\frac{3g}{h}}$

$\omega h = \upsilon = h\sqrt{\frac{3g}{h}}$; $\quad L = I\omega = mh^2\cdot\frac{1}{3}\cdot\sqrt{\frac{3g}{h}} = 474$ м²кг

$\upsilon = 9{,}48$ м

8



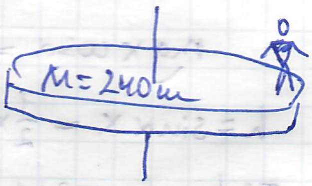
$I_п = \frac{MR^2}{2}$ $\quad I_ч = m_ч l^2$, l — расстояние до центра; $\quad \nu_1 = 10$ об/мин

$M = const \Rightarrow (I_п + I_ч)\omega_1 = I_п\omega_2$

$\omega_2 = \frac{(I_п + I_ч)\omega_1}{I_п} = \frac{(MR^2 + m_ч R^2)}{MR^2}\cdot\frac{2\pi R}{10} = \frac{M + m_ч}{M}\cdot\frac{2\pi R}{10}$

$\nu_2 = \frac{\cancel{2\pi R}\, M\cdot\nu_1}{(M+m_ч)\cancel{2\pi R}} = 6{,}25$

9



$m = 60$ кг

$t = 0$: $L_{чел} = 0$ $\quad L_{пл} = 0$

$L_ч + L_п = 0 = const = L_{чн} + L_{пн}$

$L_{чн} = I_ч\omega_ч$ $\quad L = -I_п\omega_п$

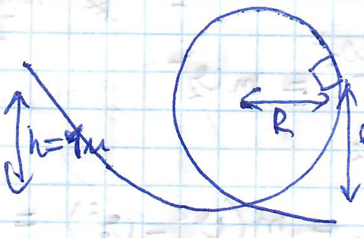
$I_ч\omega_ч = I_п\omega_п$ $\quad \omega = \frac{\Delta\varphi}{\Delta t}$

$I_ч\frac{\Delta\varphi_ч}{\Delta t} = I_п\frac{\Delta\varphi_п}{\Delta t}$ $\quad mR^2 2\pi = \frac{1}{2}MR^2\Delta\varphi_п$ $\quad \Delta\varphi_п = \frac{2m\pi}{\frac{1}{2}M} = \pi = 180°$

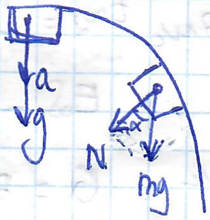
Динамика Серия 5.

Законы Сохранение.

1



$R = 0{,}4$м



$mg\cos\alpha + N = ma = m\frac{\vartheta_1^2}{R}$

$\Rightarrow N = m\frac{\vartheta_1^2}{R} - mg\cos\alpha$

Если N станет = 0 до верхней точки $\Rightarrow$ шарик упадёт

$mgh = \frac{m\vartheta^2}{2}$

$\Rightarrow \vartheta = \sqrt{2gh}$

$N = m\frac{\vartheta_1^2}{R} - mg\cos\alpha = 0$

$\vartheta_1^2 = g\cos\alpha R$, но $\cos\alpha = \frac{l-R}{R}$

$\vartheta_1^2 = gR\frac{l-R}{R} = g(l-R)$

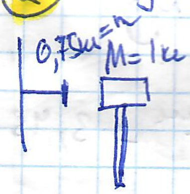
Из з-на Сохранение энергии

$mgh = \frac{m\vartheta_1^2}{2} + mgl \Rightarrow 2gh = \vartheta_1^2 + 2gl \Rightarrow \vartheta_1^2 = 2g(h-l)$

$\Rightarrow g(l-R) = 2g(h-l)$

$l = \frac{2h+R}{3} \Rightarrow \frac{2\cdot 1м + 0{,}4м}{3} = 0{,}8м$

2 η - ?



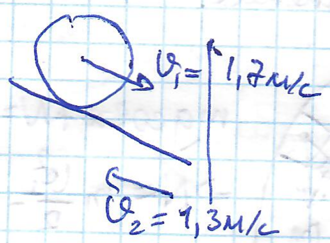
$\eta = \frac{A_n}{A_{зат}} =$ $A_з = \frac{M\vartheta^2}{2}$ $A_n = \frac{(m+M)u^2}{2}$

$M\vartheta = (m+M)u \Rightarrow u = \frac{M\vartheta}{(m+M)}$

$\Rightarrow A_n = \frac{\cancel{(m+M)}u^2\vartheta^2}{2(m+M)^2} = \frac{M^2\vartheta^2}{2(M+m)}$

$\Rightarrow \eta = \frac{\frac{M^2\vartheta^2}{2(m+M)}}{\frac{M\vartheta^2}{2}} = \frac{M}{m+M} = \frac{1кг}{1{,}75кг} = 0{,}93$

(4)



$m_{об} = 1,2 кг$

$Q - ?$

$$E_{об_1} = \frac{m v_1^2}{2} + \frac{I\omega_1^2}{2} = \frac{m_1 v_1^2}{2} + \frac{m}{2} R^2 \frac{v_1^2}{R^2} = \frac{2m v_1^2}{2}$$

$$E_{об_2} = \frac{m v_2^2}{2} + \frac{I\omega_2^2}{2} = m v_2^2$$

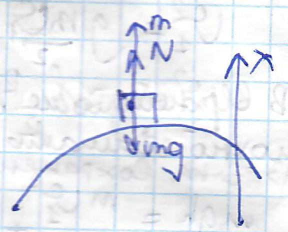
$$E_{об_1} - Q = E_{об_2}$$

$$Q = E_{об_1} - E_{об_2} = m(v_1^2 - v_2^2) =$$

$$= 1,2 кг \cdot ((1,7 м/с)^2 - (1,3 м/с)^2) = 1,44 Дж$$

(6) На какую часть уменьшается вес тела вследствие вращения Земли?

На тело на земле действует центростремительное ускорение



$$ma + N - mg = 0$$

$$a = \frac{v^2}{R} = \omega^2 R$$

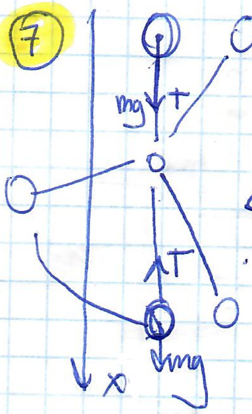
$$\Rightarrow F_{тяж} = N = m(g - a)$$

$$\frac{\Delta F_{тяж}}{F_{тяж}} = \frac{mg - N}{mg} = \frac{mg - mg + ma}{mg} = \frac{ma}{mg} =$$

$$= \frac{a}{g} = \frac{\omega^2 R}{g} = \frac{(7,29 \cdot 10^{-5} с^{-1})^2 \, 6371 км}{9,8 м/с^2} = 3,45 \, 10^{-5} =$$

$$= 0,034\ \%$$

7



m-? в верхней точке:

$mg + T_1 = ma$ $\quad T_1 = m(a-g)$

$\Delta T = 10H$ в нижней точке: $\quad$ Тут T_1 может быть $=0 \Rightarrow a=g$

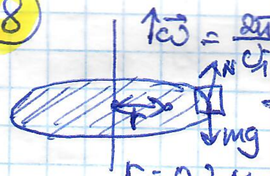
$-T_2 + mg = -ma$ $\quad T_2 = m(g-a)$

$\Delta T = T_2 - T_1 = m(g-a-a+g) =$

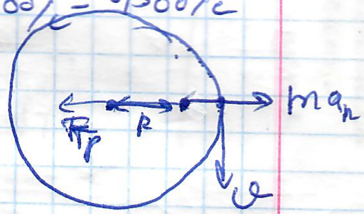
$= \Delta T = m2(g-a) = 10H \Rightarrow m = \frac{\Delta T}{2(g-a)} =$

$= \frac{\Delta T}{2g} = 0{,}5$ кг

8



$\omega = \frac{2\pi}{\nu_1}R$ $\quad \nu_1 = 30$ об/мин $= \frac{30}{60}$ об/с $= 0{,}5$ об/с



Тело не будет соскальзывать с диска, если

$r = 0{,}2$ м

μ - ?

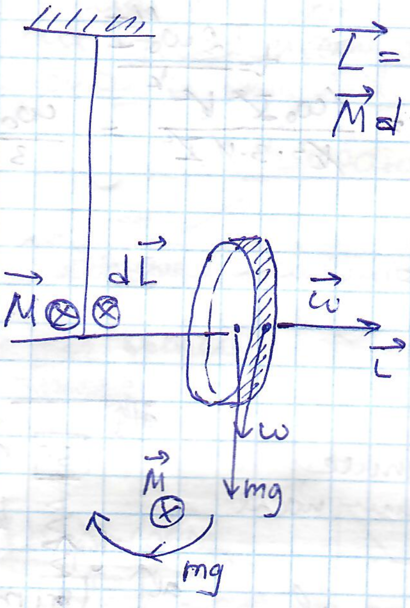
$F_{тр} = ma_n$

$a_n = \frac{v^2}{R} = \omega^2 R = \frac{4\pi^2}{\nu_1^2}R$ $\quad F_{тр} = mg\mu$

$\mu = \frac{F_{тр}}{mg} = \frac{ma_n}{mg} = \frac{4\pi^2}{\nu_1^2 g}R^3 =$

$= \frac{4\cdot 3{,}14^2}{(0{,}5\ об/с)^2\cdot 10\ м/с^2}(0{,}2\ м)^3 = 0{,}13$

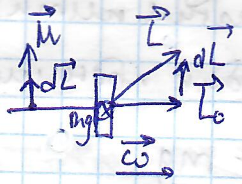
Серия Динамика.
Законы Сохранения



$$\vec L = I\vec\omega$$

$$\vec M dt = d\vec L = \Delta\vec L = \vec L - \vec L_0$$

$$\vec L = \vec L_0 + \Delta\vec L$$



Прецессия — поворот оси вращения.

Гироскопический эффект — поворот оси вращения вокруг себя.

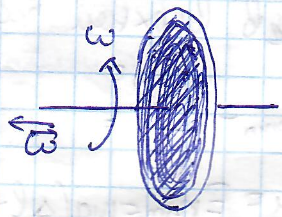
5.

$M \sim \sqrt{\omega}$

$M = -k\sqrt{\omega}$

$\omega_0 = 45$ рад/с

$\langle\omega\rangle$ — ?



$$M = I\varepsilon$$

$$-k\sqrt{\omega} = I\frac{d\omega}{dt}$$

$$-\frac{k}{I}dt = \frac{d\omega}{\sqrt{\omega}} = \frac{d\omega}{\omega^{\frac{1}{2}}} = \omega^{-\frac{1}{2}}d\omega$$

$$-\frac{k}{I}t = -\frac{k}{I}\int_0^t dt = \int_{\omega_0}^{0}\omega^{-\frac{1}{2}}d\omega = 2\omega^{\frac{1}{2}}\Big|_{\omega_0}^{0} = 0 - 2\omega_0^{\frac{1}{2}}$$

$$2\omega_0^{1/2} = \frac{k}{I}t \qquad t = \frac{2\omega_0^{1/2}}{k}I \qquad \omega_0 = \left(\frac{k}{2I}t\right)^2 = \frac{d\varphi}{dt}$$

$$\int_0^{\varphi} d\varphi = \int_0^{t} \frac{k^2}{4I^2} t^2 dt$$

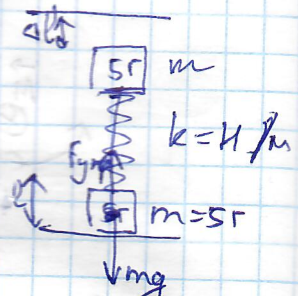
$$\varphi = \frac{t^3}{3}\frac{k^2}{4I^2}$$

$$t = \frac{2\omega_0^{1/2} I}{k}$$

$$\langle \omega \rangle = \frac{\varphi}{t} = \frac{t^{\cancel{3}2} k^2}{3\cdot 4 I^2 \cancel{k}} = \frac{\cancel{4}\omega_0 \cancel{I^2} \cancel{k^2}}{\cancel{k^2}\cdot 3\cdot \cancel{4} \cancel{I^2}} = \frac{\omega_0}{3} = 15 \text{ рад/с}$$

$\boxed{\text{рубежка}}$ = 10 вопросов
+ 5 задач
+ 5 тестовых вопросов

(3) Каково мин. сжатие т.а. кубики подскочили после пережигания нити?

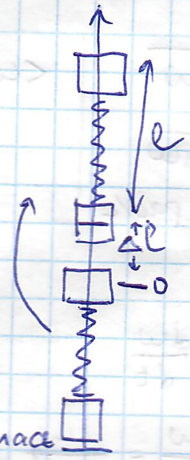


$\exists$ кубик поднимется на Δl

$k\Delta l \geq mg$

Fупр ≥ mg
↓
тогда подскочит

в верхней точке:



$$E_{пот_0} = \frac{k\Delta l^2}{2} =$$

$$E_{пот_1} = mg(\Delta l + l) +$$

$$E_{пат\,пружины} = \frac{kl^2}{2}$$

она перерастянулась т.к. она подняла нижнее тело

$$\begin{cases} \frac{k\Delta l^2}{2} = mg(\Delta l + l) + \frac{kl^2}{2} \\ k\Delta l = mg \end{cases}$$

Сорие колебание

①

$\nu = 50 \text{ Гц}$

$x_m = 1{,}8 \text{ м}$

$\varphi_0 = 30^\circ$

$x(t) = ?$

$x(t) = x_m \cos(\omega t + \varphi_0) \Rightarrow$

$= x_m \cos(2\pi\nu t + \frac{\pi}{6})$

②

$T = 0{,}5 \text{ с}$

$a_{max} = 15{,}8 \text{ м/с}^2$

$A - ?$

$v = \dot{x} = \frac{dx}{dt} = -x_m \omega \sin(\omega t + \varphi_0)$

$a = \ddot{x} = \frac{d^2x}{dt^2} = -x_m \omega^2 \cos(\omega t + \varphi_0)$

$-x_m \omega^2 \underbrace{\cos(\omega t + \varphi_0)}_{= x(t)} = 15{,}8 = a_{max}$

$x = \frac{a_{max}}{-\omega^2}$ $\boxed{\ddot{x} = -\omega^2 x}$ уравнение гармонических колебаний

$x_m \cos(\omega t + \varphi) = \frac{a_{max}}{-\omega^2}$

решение уровнение $\boxed{x(t) = x_m \cos(\omega t + \varphi_0)}$

$x_m = \frac{a_{max} \cos(\omega t + \varphi)}{-\omega^2} =$

$= \frac{a_{max} \cos(\frac{2\pi}{T} t + \varphi)}{-(\frac{2\pi}{T})^2} = \frac{\cos(4\pi t + 0)\, a_{max}}{-16\pi^2} =$

$=$

(10) $\nu = 10$ Гц. $\quad 2\pi\nu = \omega$

$\varphi_0 = \pi$

cos

$x = 0$

$t = ?$

$$x = x_{max}\cos(2\pi\nu t + \pi) = 0$$

$$\cos(2\pi\nu t + \pi) = 0$$

$$2\pi\nu t + \pi = \frac{3\pi}{2}$$

$$2\pi\nu t = \frac{\pi}{2} \quad t = \frac{1}{4\nu} = \frac{1}{40} \text{ c}$$

(4) $F_{max} = 16$ Н

$m = 0{,}4$ кг

$x_m = 0{,}1$ м

$$= x_m\cos(\omega t + \varphi_0)$$

$$F = ma = -\underbrace{m\omega^2}_{k} x = -kx =$$

$$= m\omega^2 x_m \cos(\omega t + \varphi)$$

$$m\omega^2 x_m \Rightarrow \omega = \sqrt{\frac{F_{max}}{x_{ax}\, m}}$$

$E_к$ - Кинетическая энергия К или Т

$E_п$ - Потенциальная энергия П или U

(6.) φ_0

$\frac{T}{П} - ?$

$$E_к = T = \frac{m\vartheta^2}{2} = \frac{(-x_m\omega\sin(\omega t + \varphi_0))^2 m}{2} =$$

$$= \frac{m\omega^2 x_m^2 \sin^2(\omega t + \varphi_0)}{2}$$

$$dA = -dE_{пот}$$

$$E_{пот} = -\int F dx = -\int -x_m\omega^2 m\cos(\omega t + \varphi_0)\, dx =$$

$$F = am = -x_m\cos(\omega t + \varphi_0)\omega^2 m$$

$$a = -x_m\cos(\omega t + \varphi_0)\omega^2$$

$$= -\int -\omega^2 m x\, dx =$$

$$= +\frac{1}{2}x^2\omega^2 m$$

$$\Rightarrow E_{пот} = \frac{x^2 m \omega^2}{2} = \frac{x_m^2 \cos^2(\omega t + \varphi_0)\,\omega^2 m}{2}$$

$$E_к = \frac{m\omega^2 x_m^2 \sin^2(\omega t + \varphi_0)}{2}$$

$$\Rightarrow \frac{E_к}{E_{пот}} = \frac{\sin^2(\omega t + \varphi_0)}{\cos^2(\omega t + \varphi_0)} = tg^2(\omega t + \varphi_0)$$

Переписывания рубежки на 52ой неделе.

Разбор задач.

① $T = \frac{m\vartheta^2}{2} + \frac{I\omega^2}{2} = \frac{m\vartheta^2}{2} + \frac{xm\ldots}{\ldots}$

$$\frac{T_{шара}}{T_{цилиндра}} = \frac{1+\frac{2}{5}}{1+\frac{1}{2}} = \frac{14}{15}$$

Как относятся полная энергия катящ... шара и цилиндра равной массы и радиу...

$J_ш = \frac{2}{5} mR^2$

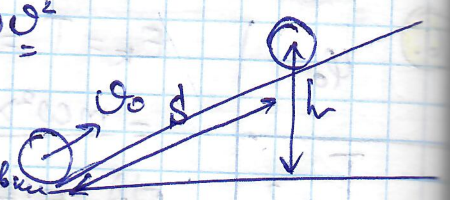
$\omega R = \vartheta$

$I_{цил} = \frac{1}{2} mR^2$

② Какой путь до остановки пройдет ~~плос...~~ центр шара катящегося по наклонной ~~плоск~~ с $\angle = \alpha$. Начальная скорость – ϑ_0.

$$E_к = \frac{m\vartheta_0^2}{2} + \frac{2}{5}\frac{m(R^2\omega^2)\vartheta^2}{2} =$$

$= \frac{7}{5} m\vartheta_0^2$ = полная энергия в точке остановки



$$\sin\alpha \cdot S m g = \frac{7}{5}\frac{m\vartheta_0^2}{2} \qquad S = \frac{7}{5}\frac{m\vartheta_0^2}{2mg\sin\alpha} = \frac{7}{10}\frac{\vartheta_0^2}{g\sin\alpha}$$

③

$x(t) = A\sin(\omega t + \varphi_0)$

$A = 0{,}1м \quad T = 5с$

$a_{max} = ? \quad \vartheta_{max} = ?$

$T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T}$

$\vartheta(t) = \overset{\circ}{x}(t) =$

$= A\omega\cos(\omega t + \varphi_0)$

$\vartheta(t) = max$, если $\cos(\omega t + \varphi_0) = 1$

$\Rightarrow \vartheta(t)_{max} = A\omega = 2\pi\frac{A}{T} = 2 \cdot 3{,}14 \cdot \frac{0{,}1м}{5с} = 0{,}125 м/с$

$a(t) = \overset{\circ\circ}{x}(t) = -A\omega^2\sin(\omega t + \varphi_0) \Rightarrow$

$\Rightarrow a_{max} = A\omega^2 =$, когда $\sin(\omega t + \varphi_0) = -1$

$= \frac{0{,}1м}{25с^2} \cdot 4 \cdot 3{,}14^2 = 0{,}157 м/с^2$

⑤

$x = 50\sin(\pi/3\, t) \Rightarrow \vartheta = \dot{x} = 50\frac{\pi}{3}\cos(\frac{\pi}{3}t)$

$a = \ddot{x} = -50(\frac{\pi}{3})^2\sin(\pi/3\, t)$

$\cap$

$[-50(\frac{\pi}{3})^2; 50(\frac{\pi}{3})^2]$

$\vartheta_{max} = 50\frac{\pi}{3} \Rightarrow$

$\Rightarrow E_{полн} = E_{k\,max} = \frac{\vartheta_{max}^2 m}{2} =$

$= \frac{(50\frac{\pi}{3})^2 2кг \cdot м^2/с^2}{2} =$

$= 2741 Дж$

$\Rightarrow F \in [50(\frac{\pi}{3})^2 m; 50(\frac{\pi}{3})^2 m] \Rightarrow$

$\Rightarrow \Delta F = 2 \cdot 50(\frac{\pi}{3})^2 m = 2 \cdot 50 \cdot (\frac{3{,}14}{3})^2 2кг\,м/с = 219 Н.$

⑦

$x(t) = A\sin(\omega t)$

$T = 6с$

$x(\Delta t) = A\sin(\omega\Delta t) = \frac{1}{2}A$

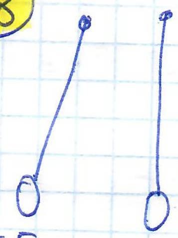
$\omega = \frac{2\pi}{T} \quad \sin(\frac{2\pi}{T}\Delta t) = \frac{1}{2}$

$\frac{2\pi}{T}\Delta t = \frac{\pi}{6}$

$\Delta t = \frac{T}{2\pi} \cdot \frac{\pi}{6} = 0{,}5с$

$\varphi_0) = 1$

8



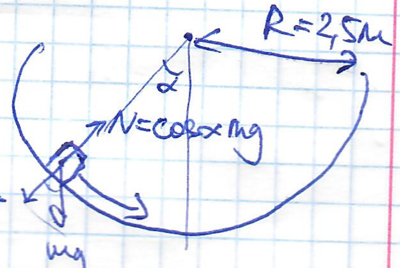
$T = 5c$ $T = 6c$

$k = -1 \Rightarrow t_{min} = 0{,}5\,c$

$$x_1(t) = A_1 \sin(\omega_1 t + \varphi_1)$$
$$x_2(t) = A_2 \sin(\omega_2 t + \varphi_2)$$
$$T = \frac{2\pi}{T} \qquad \sin(\omega_1 t + \varphi) = \sin(\omega_2 t + \varphi)$$
$$\varphi_1 = \varphi_2 \qquad \omega_1 t + \varphi = \omega_2 t + \varphi + 2\pi k$$
$$\frac{2\pi}{T_1} t = \frac{2\pi}{T_2} t + 2\pi k$$
$$\frac{2}{T_1} t = \frac{2}{T_2} t + 2k$$
$$t = \frac{2k}{\left(\frac{2}{T_1} - \frac{2}{T_2}\right)} = \frac{\cancel{2}\, k T_1 T_2}{\cancel{2}(T_2 - T_1)} =$$
$$= k \cdot (-30\,c)$$

$^n =$

9 Т.к. трения нет этот кубик эквивалентен мат. маятнику, т.е. колеблется гармоническим образом



$$\cancel{x(t)} = \Rightarrow T = \frac{2\pi}{\omega} = \sqrt{\frac{R}{g}}\, 2\pi =$$
$$= 2\sqrt{\frac{2{,}5\ м}{9{,}8\ м/с^2}} \cdot 3{,}14 = 3{,}17\,c$$