Fuzzy Controller for the Inverted Pendulum

Part 3: Yamakawa's Fuzzy Controller Design

Design

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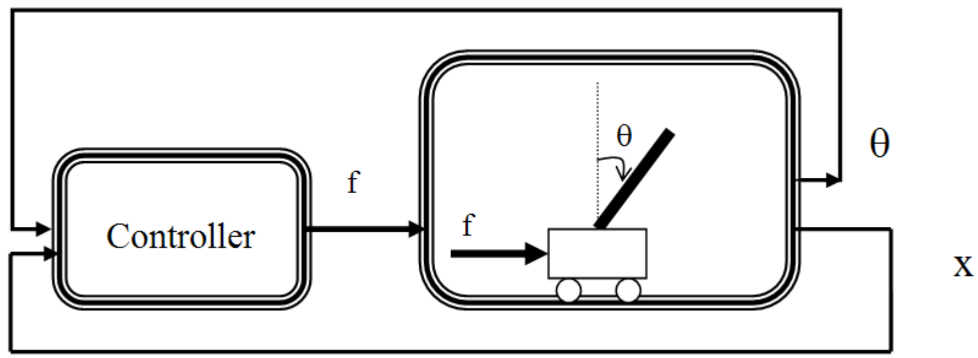
Recall: Inverted Pendulum Problem

Inverted Pendulum Controller



<u>Sensed values:</u>

X – position of object with respect to the horizontal axis

θ - angle of pole relative to the vertical axis

<u>Derived values:</u>

X' - Velocity along the x-axis

θ' - Angular velocity

<u>Input variables:</u> sensed and derived values

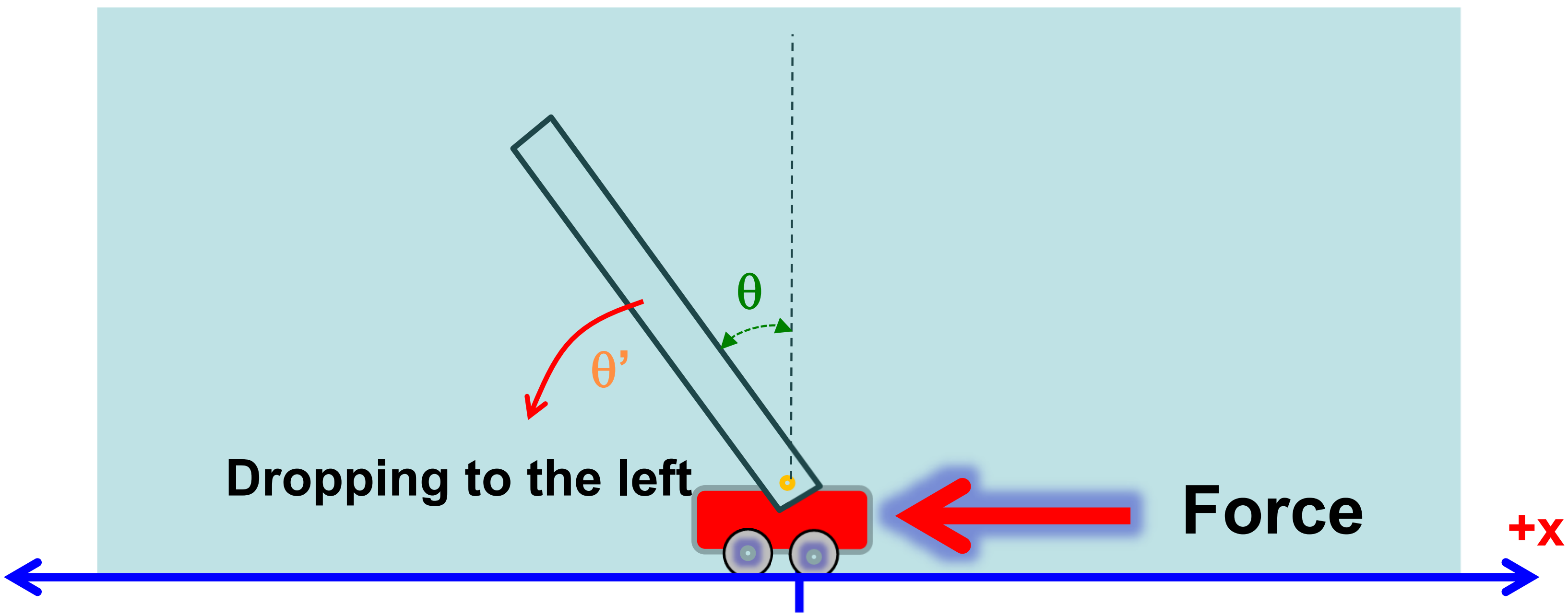
<u>Controller output:</u> **F** – force to be applied to the cart

Fuzzy Rule

e.g.

If (θ is **NEGATIVE** **and** θ' is **NEGATIVE**)

Then Apply a NEGATIVELY_LARGE_FORCE.



If (pole is leaning to the left AND pole is dropping to the left)
THEN largely push the cart to the left.

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Fuzzy Rule

e.g.

If (θ is **NEGATIVE** **and** θ' is **NEGATIVE**)

Then Apply a NEGATIVELY_LARGE_FORCE.

The linguistic term **NEGATIVE** associated with the input angle is represented as a **fuzzy set**, named $F_{\text{NEGATIVE }\theta}$.

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Fuzzy Rule

e.g.

If (θ is NEGATIVE **and** θ' is **NEGATIVE**)

Then NEGATIVELY_LARGE_FORCE.

The linguistic term **NEGATIVE** is represented as a **fuzzy set**, named $\mathbf{F}_{\mathbf{NEGATIVE}\ \theta}$.

On the other hand, the fuzzy set, $\mathbf{F}_{\mathbf{NEGATIVE}\ \theta}$, is implemented as a membership function, $\mathbf{F}_{\mathbf{NEGATIVE}}(\theta)$.

$\mathbf{F}_{\mathbf{NEGATIVE}}(\theta)$ = [0,1] computes for the degree of membership of any given input **angle** θ, in the fuzzy Set **NEGATIVE** θ

Fuzzy Rule

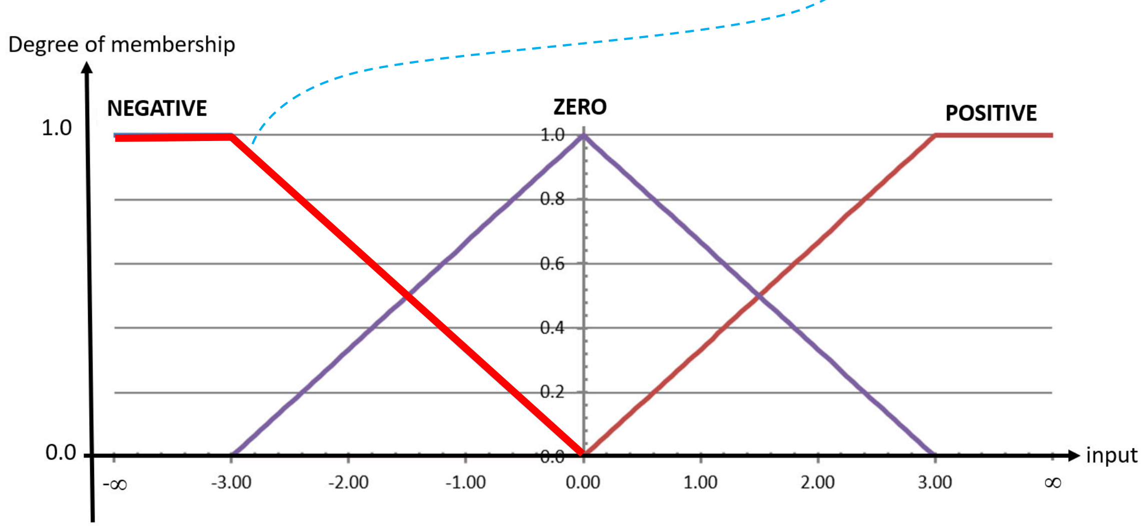
e.g.

If (θ is NEGATIVE **and** θ’ is **NEGATIVE**)

Then NEGATIVELY_LARGE_FORCE.

$F_{NEGATIVE}(\theta)$ = [0,1] computes for the degree of membership of any given input **angle** θ, in the fuzzy Set **NEGATIVE** θ

The shape of this membership function, $F_{NEGATIVE}(\theta)$, may take the form of a trapezoid opening to the left.



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Fuzzy Rule

e.g.

If (θ is NEGATIVE **and** θ' is **NEGATIVE**)

Then NEGATIVELY_LARGE_FORCE.

The linguistic term **NEGATIVE** associated with the input, angular velocity θ' is represented as a **fuzzy set**, named $\mathbf{F}_{\mathbf{NEGATIVE}\ \theta'}$.

On the other hand, the fuzzy set, $\mathbf{F}_{\mathbf{NEGATIVE}\ \theta'}$, is implemented as a membership function, $\mathbf{F}_{\mathbf{NEGATIVE}}(\theta')$.

$\mathbf{F}_{\mathbf{NEGATIVE}}(\theta')$ = [0,1] computes for the degree of membership of any given input **anglular velocity** θ', in the fuzzy Set **NEGATIVE θ'**

Fuzzy Rule

If (θ is NEGATIVE **and** θ' is **NEGATIVE**)

Then NEGATIVELY_LARGE_FORCE.

Could be a constant or another Membership Function

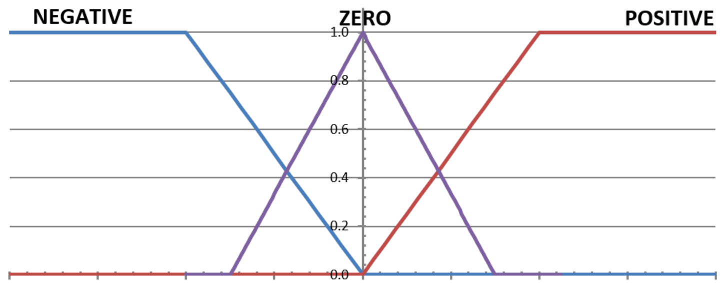
e.g. Negatively Large Force may be set to -100 N

Fuzzy Rule

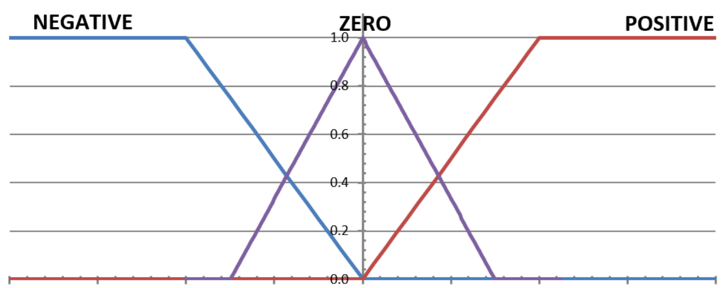
The linguistic terms should take into account the range of all possible values for each of the inputs.

Note: The range of input values and the shape of the membership functions are determined empirically

Input Angle θ: {N, ZE, P}={leaning to the left, centre, leaning to the right}



Input Angular velocity θ': {N, ZE, P}={dropping to the left, still, dropping to the right}



F: {PL, PS, ZE, NS, NL}={Large push to the right, small push to the right, don't push, small push to the left, Large push to the left}

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FUZZY LOGIC SYSTEM

Best solution
(Yamakawa's approach)

Reference journal article: Takeshi Yamakawa, "A Fuzzy Inference Engine in Nonlinear Analog Mode and its Application to a Fuzzy Logic Control"

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Inputs

We can reduce the number of rules further by combining related inputs together in a linear equation.

X = (**A** * **theta**) + (**B** * **theta_dot**)

Y = (**C***x**) + (**D** * **x_dot**)

Note: A, B, C & D are determined empirically.

This approach requires only a single FAMM to solve the control problem.

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Fuzzy System Design

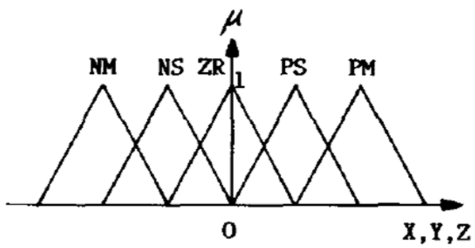
Yamakawa published his calibrated fuzzy rules and membership functions. Out of the 25 rules possible, he is only using 13 of them.

Fuzzy rules

$X = A\theta_t + B\dot{\theta}_t$

$Y = Cx + D\dot{x}$	NM	NS	ZR	PS	PM
PM	NS		PS		(PL)
PS		NS		PM	
ZR	NM		ZR		PM
NS		NM		PS	
NM	(NL)		NS		PS

(a)



(b)

Fig. 29. (a) A rule map including fuzzy rules for stabilizing a glass with wine. (b) Membership functions which characterize the fuzzy linguistic terms used in the rule map.

- 2 inputs, 5 fuzzy sets per input, 5x5=25 possible rules
- 3 inputs, 5 fuzzy sets per input, 5x5x5=125 possible rules
- 4 inputs, 5 fuzzy sets per input, 5x5x5x5=625 possible rules

X = (A * theta) + (B * theta_dot)

Y = (C*x) + (D * x_dot)

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Fuzzy System Design

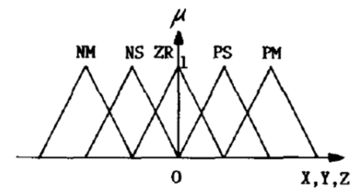
Here are some graphical interpretations of some of the rules.

Rule: If (X is NM) and (Y is NM) THEN F = NL.

$X = A\theta + B\dot{\theta}$

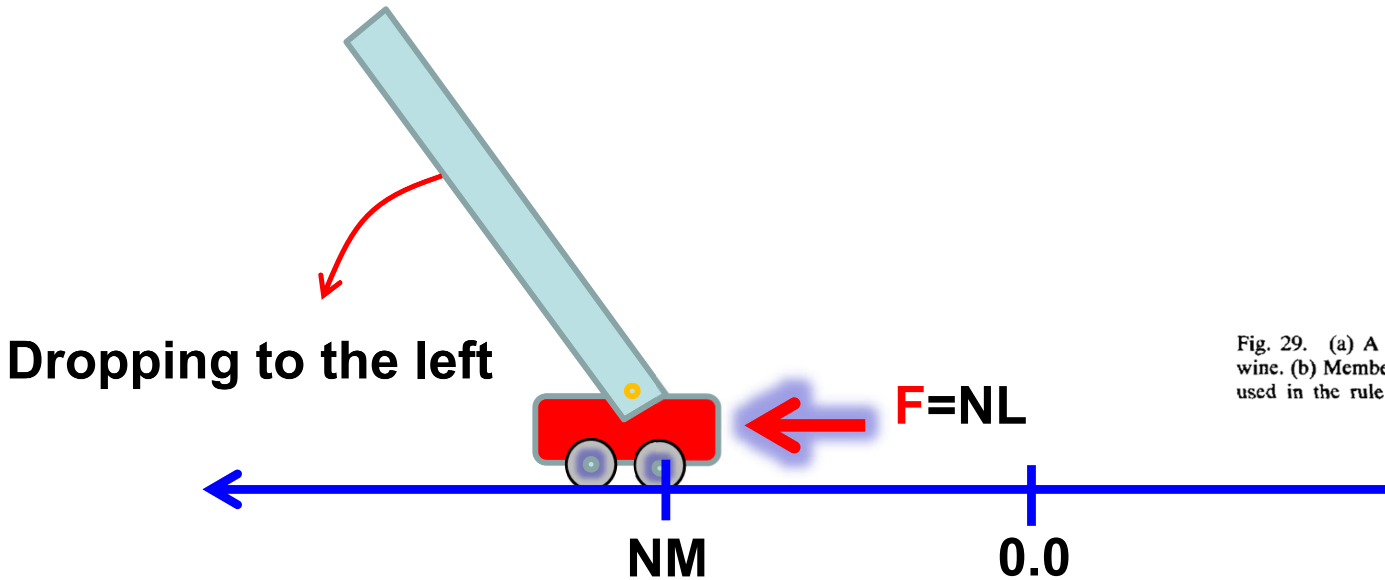
$Y = Cx + D\dot{x}$	NM	NS	ZR	PS	PM
PM	NS		PS		(PL)
PS		NS		PM	
ZR	NM		ZR		PM
NS		NM		PS	
NM	$\boxed{\text{(NL)}}$		NS		PS

(a)



(b)

Fig. 29. (a) A rule map including fuzzy rules for stabilizing a glass with wine. (b) Membership functions which characterize the fuzzy linguistic terms used in the rule map.



X = (**A** * **theta**) + (**B** * **theta_dot**)

Y = (**C***x**) + (**D** * **x_dot**)

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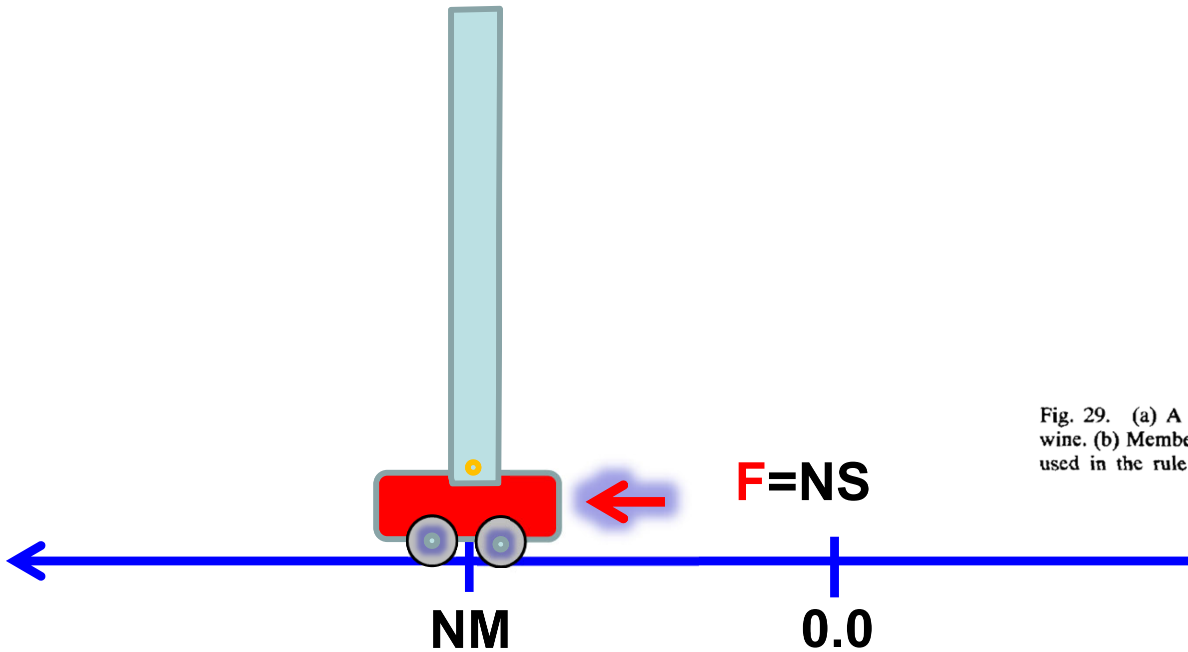
Fuzzy System Design

Here are some graphical interpretations of some of the rules.

Rule: If (X is ZE) and (Y is NM) THEN F = NS.



$X = (A * theta) + (B * theta\_dot)$

$Y = (C*x) + (D * x\_dot)$

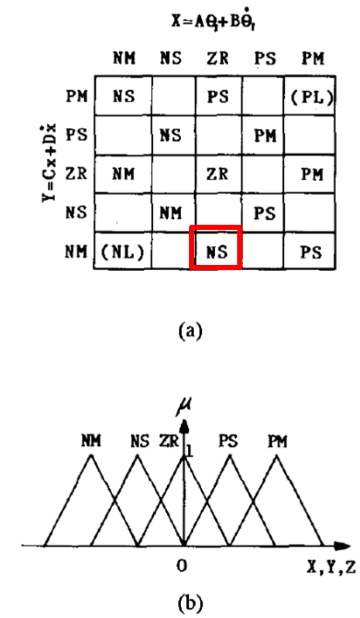


Fig. 29. (a) A rule map including fuzzy rules for stabilizing a glass with wine. (b) Membership functions which characterize the fuzzy linguistic terms used in the rule map.

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Fuzzy System Design

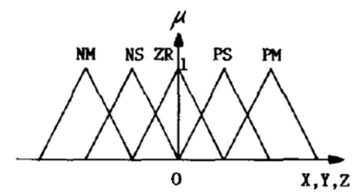
Here are some graphical interpretations of some of the rules.

Rule: If (X is PM) and (Y is NM) THEN F = PS.

$X = A\theta + B\dot{\theta}$

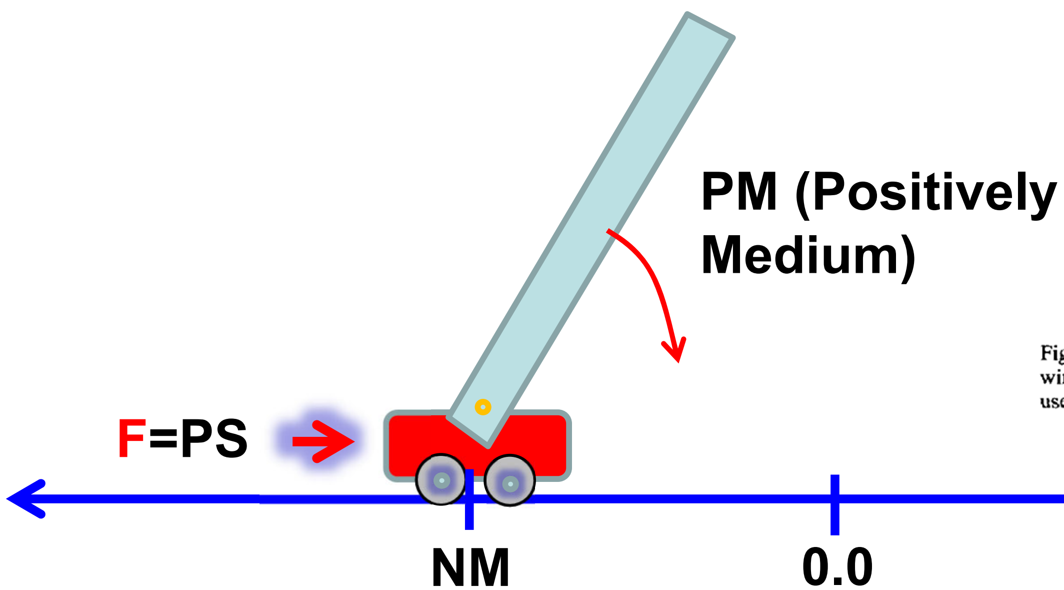
$Y = Cx + D\dot{x}$	NM	NS	ZR	PS	PM
PM	NS		PS		(PL)
PS		NS		PM	
ZR	NM		ZR		PM
NS		NM		PS	
NM	(NL)		NS		$\boxed{\text{PS}}$

(a)



(b)

Fig. 29. (a) A rule map including fuzzy rules for stabilizing a glass with wine. (b) Membership functions which characterize the fuzzy linguistic terms used in the rule map.



X = (**A** * **theta**) + (**B** * **theta_dot**)

Y = (**C***x) + (**D** * **x_dot**)

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Derivation of Formulas and Hand Simulation of a Complete Fuzzy Inference System

Part 3b

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Fuzzification

Fuzzy Control

Different stages of Fuzzy control

1. Fuzzification

Input variables are assigned degrees of membership in various classes

e.g. A temperature input might be graded according to its degree of coldness, coolness, warmth or heat.

The purpose of **fuzzification** is to **map the inputs** from a set of sensors (or features of those sensors) **to values from 0 to 1** using a set of input membership functions.

Membership Function: $F_{Set}(x) = [0, 1]$ = *assigns a degree of membership of x, in the fuzzy set defined.*

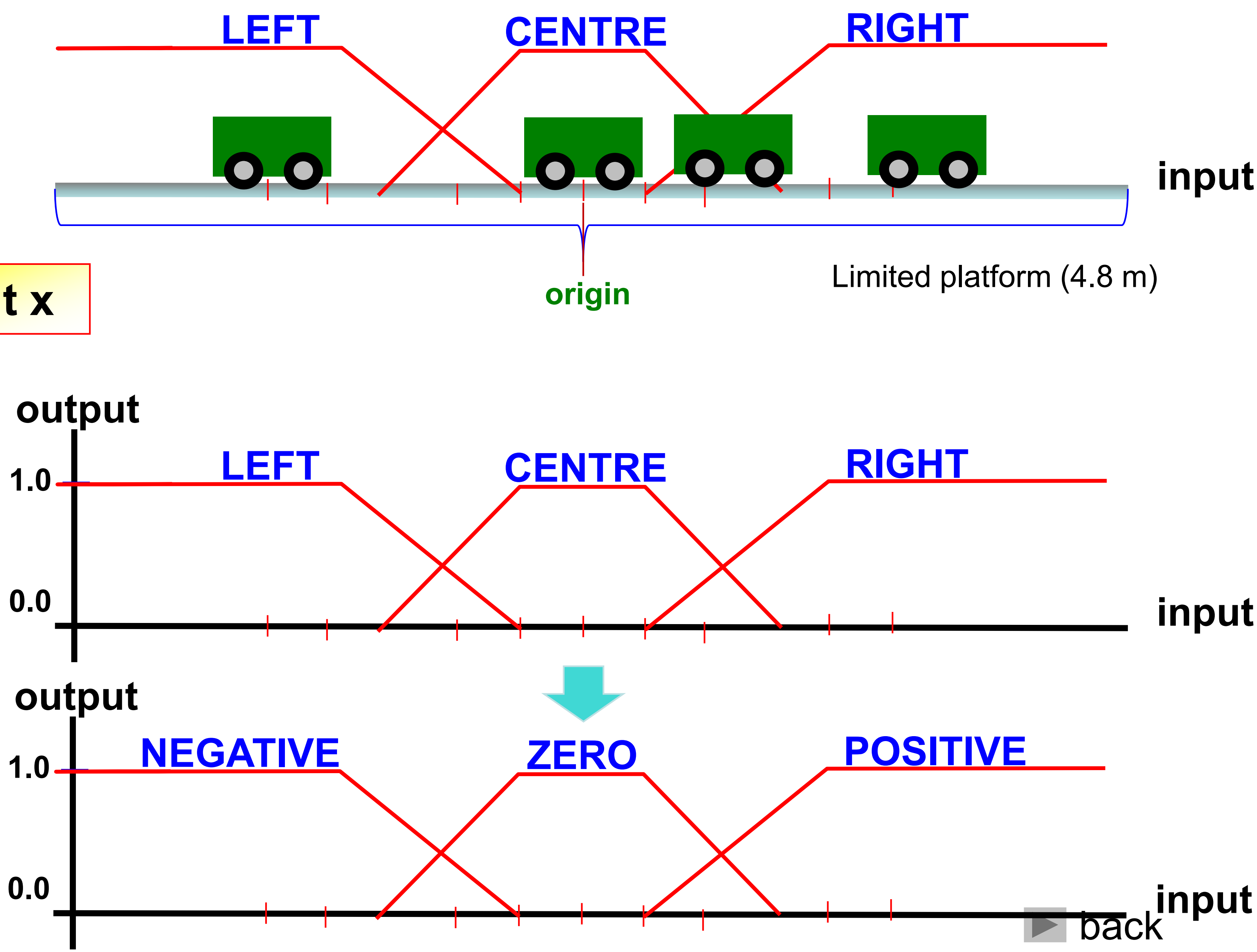
e.g. $F_{NEAR}(x=0.2) = [0, 1]$

We will see a complete example of the steps involved later.

Fuzzification

Let's consider one of the inputs, x (position of cart):

Input x



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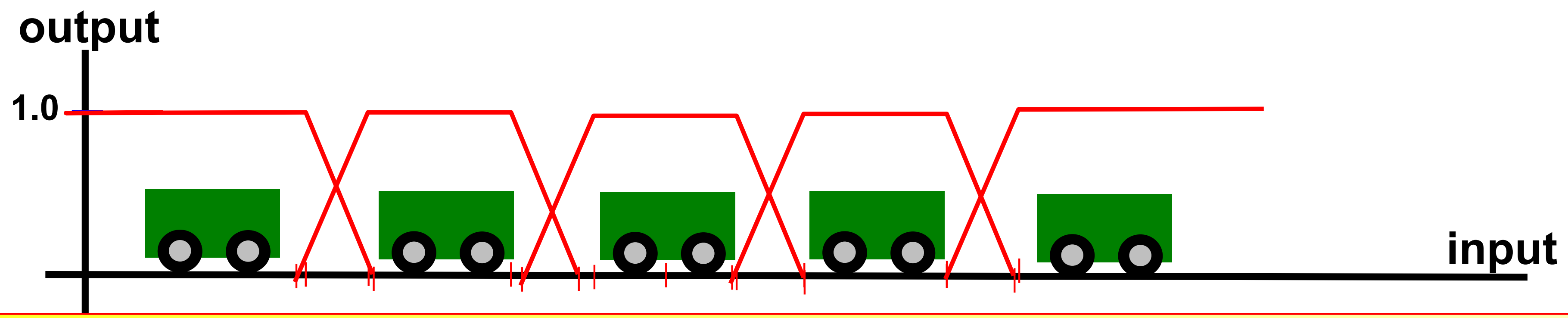
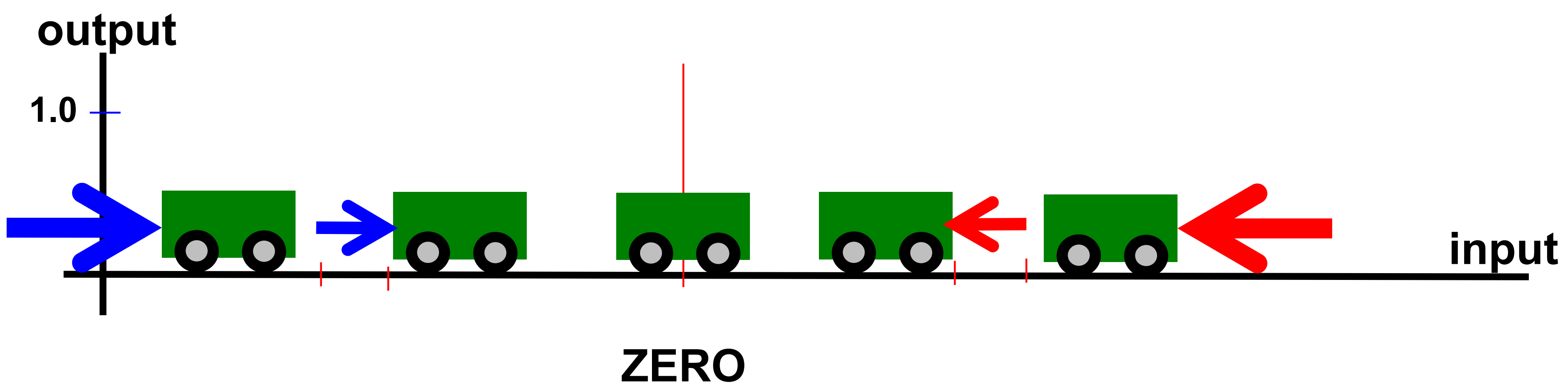
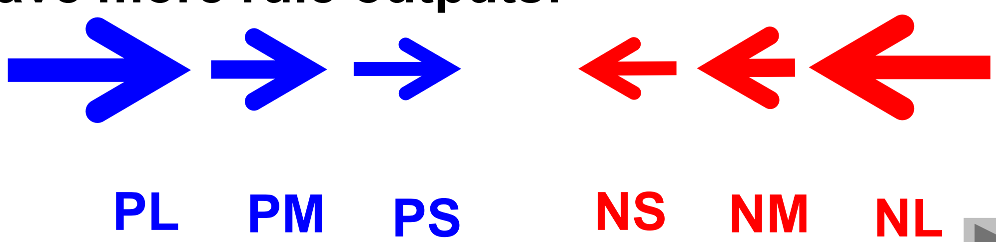
Fuzzification



By defining more fuzzy sets, more accurate control rules can be defined.



As desired, we can have more rule outputs:



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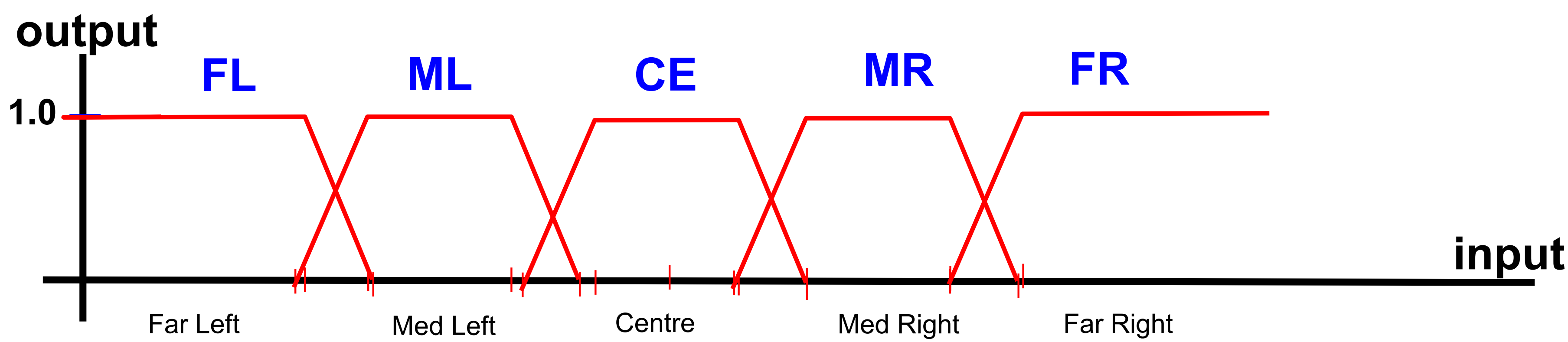
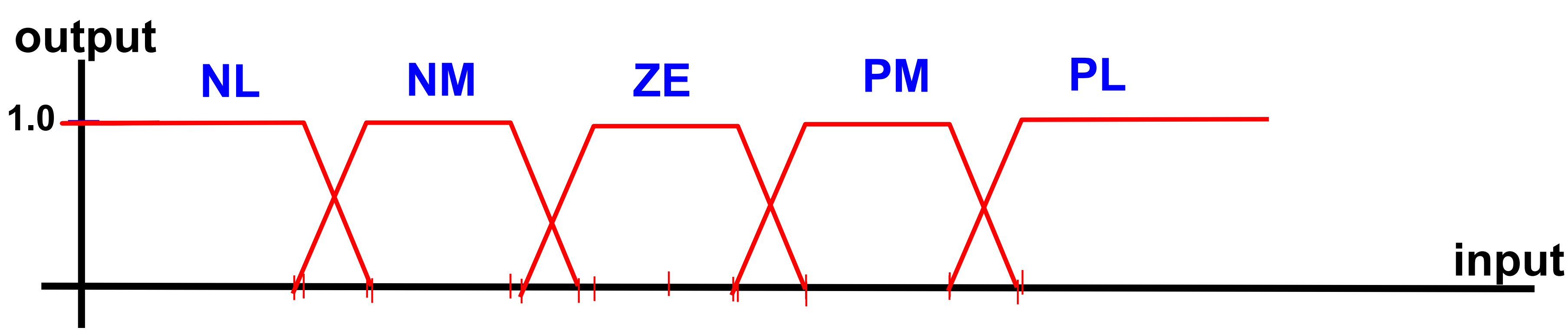
Fuzzification

We can describe the position of the cart using the following 5 fuzzy sets.





These terms are more commonly used in the literature.

Derivation of Fuzzification Formulas

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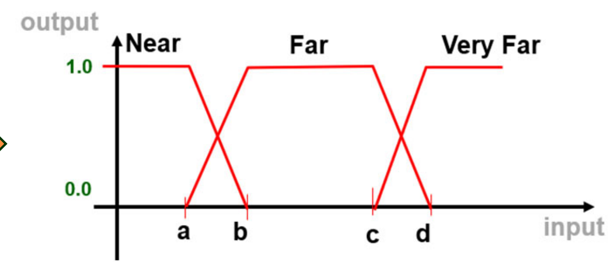
From linguistic terms to precise valuations

Linguistic terms → **Fuzzy Sets** →

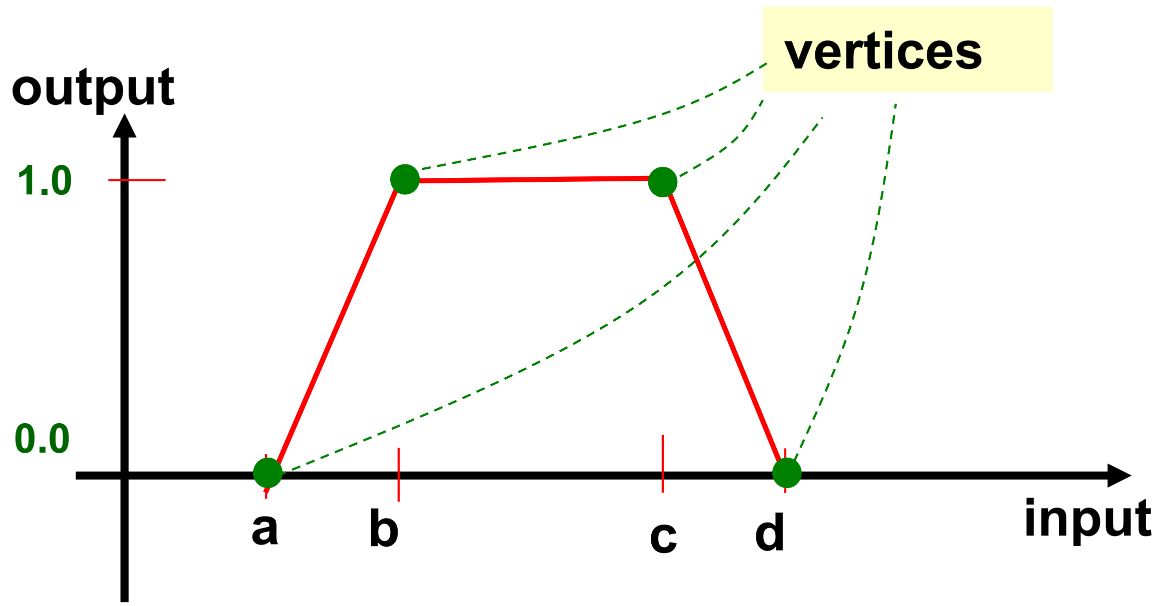


Membership Functions

e.g. Distance is “Near”	Fuzzy set “Near”	e.g. $F_{NEAR}(\mathbf{x}) = [0, 1]$
Distance is “Far”	Fuzzy set “Far”	$F_{Far}(\mathbf{x}) = [0, 1]$
Distance is “Very Far”	Fuzzy set “Very Far”	$F_{VERY\ FAR}(\mathbf{x}) = [0, 1]$

Computing for the degree of membership of x in a fuzzy set

Given a crisp input x, find the degree of membership of x in a fuzzy set.



Trapezoidal Fuzzy Set

Given input x, find y.

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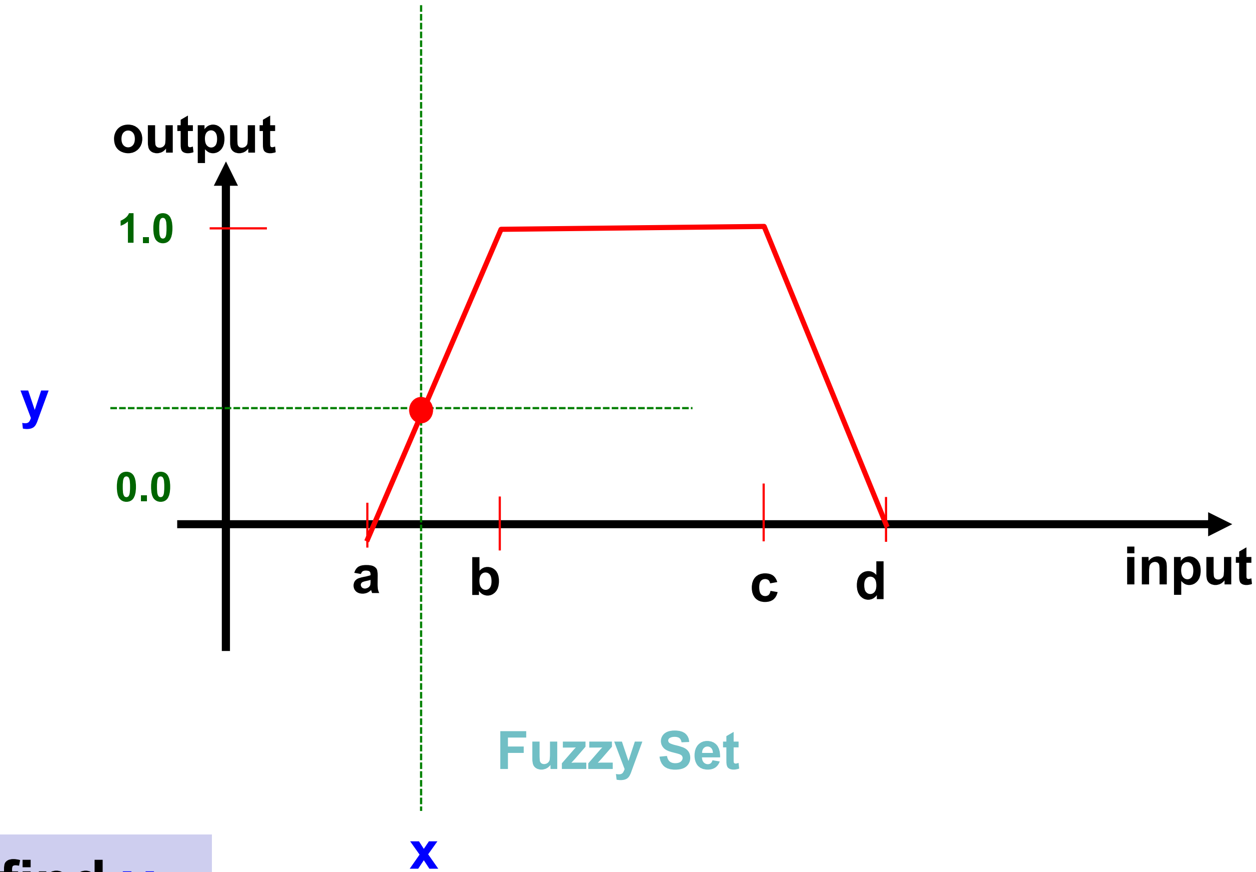
Computing for the degree of membership of x in a fuzzy set

Given a crisp input x, find the degree of membership of x in a fuzzy set.

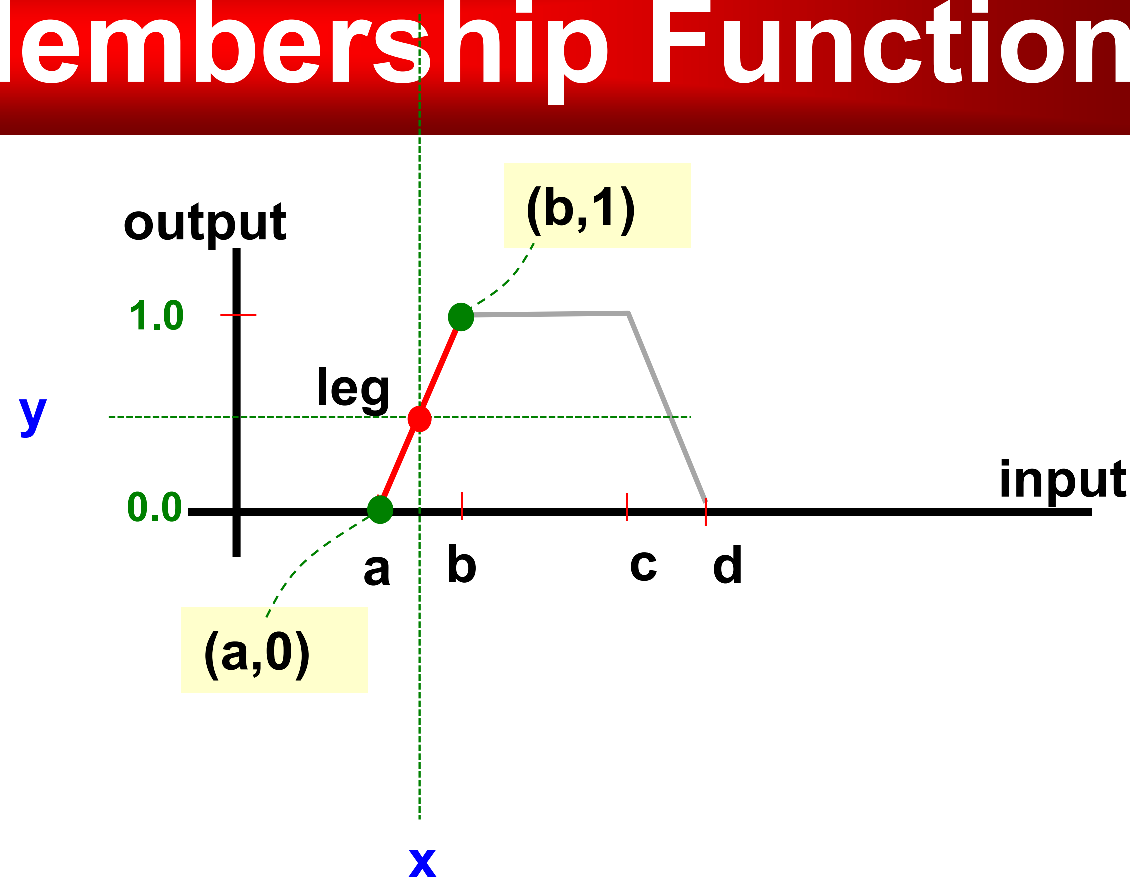


Given input x, find y.

Output y = degree of membership of x in the fuzzy set = [0, 1]

Trapezoidal Membership Function

Crisp Input: x
Membership Function: F(x)



If x is between a and b:

$$y - y_1 = m(x - x_1)$$

Let $x_1 = a$ and $y_1 = 0$,
$x_2 = b$ and $y_2 = 1$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

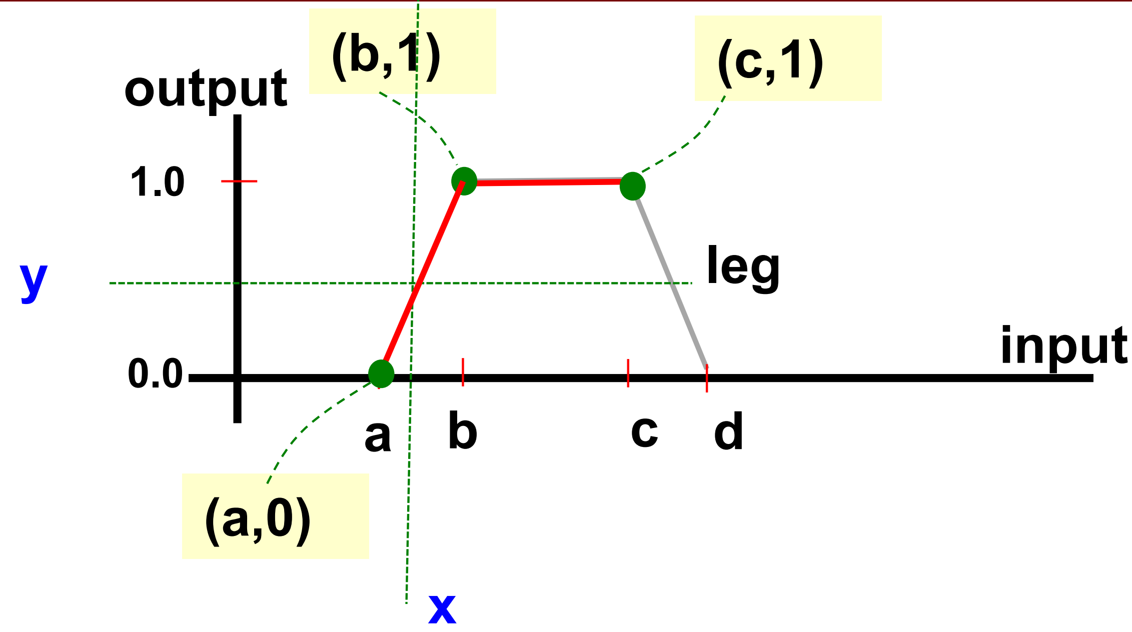
$$y - \mathbf{0} = m(x - \boldsymbol{a})$$

$$m = \frac{\mathbf{1} - \mathbf{0}}{\boldsymbol{b} - \boldsymbol{a}}$$

$$\boldsymbol{y} = \frac{\boldsymbol{x} - \boldsymbol{a}}{\boldsymbol{b} - \boldsymbol{a}}$$

Trapezoidal Membership Function

Crisp Input: x
Membership Function: F(x)



If x is between a and b:

$$y = \frac{x - a}{b - a}$$

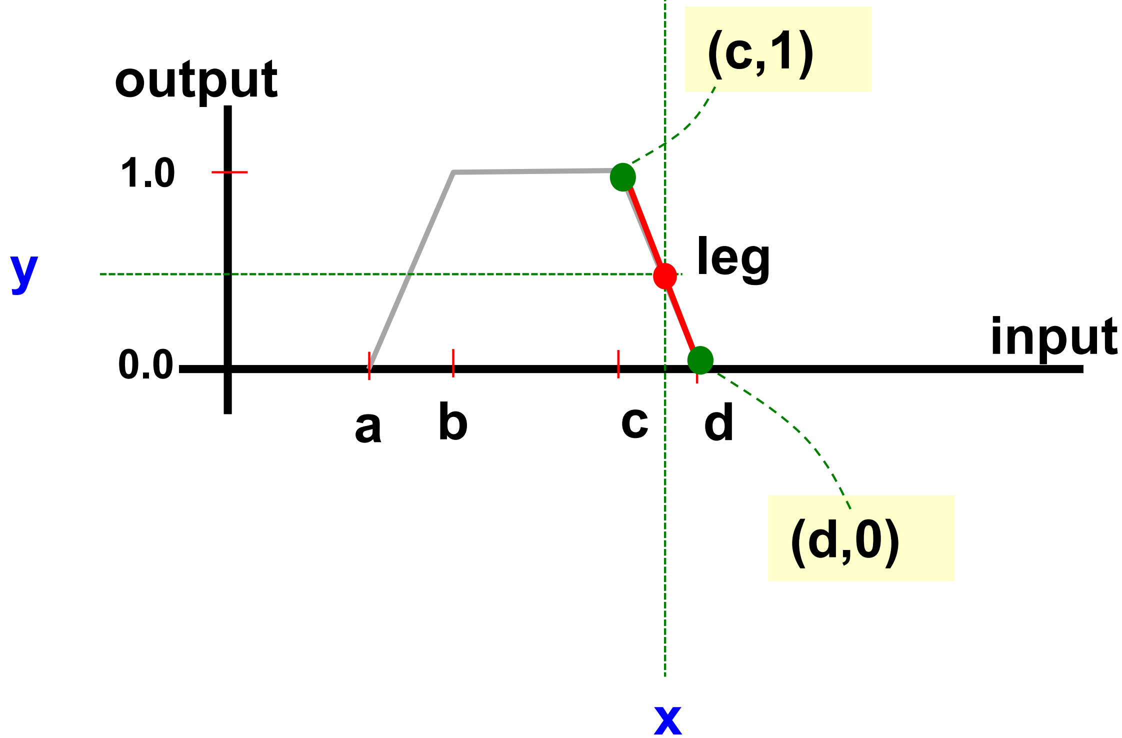
If x is between b and c:

$$y = 1$$

$$F_{right_{trapezoid}}(x) = \underbrace{\max\left(\overbrace{\min\left(1, \frac{x-a}{b-a}\right)}^{\text{Value cannot exceed 1}}, 0\right)}_{\text{Value cannot be less than 0}} = [\mathbf{0}, \mathbf{1}]$$

Trapezoidal Membership Function

Crisp Input: x
Membership Function: F(x)



If x is between c and d:

$$y - y_1 = m(x - x_1)$$

Let $x_1 = d$ and $y_1 = 0$,
$x_2 = c$ and $y_2 = 1$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y - \mathbf{0} = m(x - \boldsymbol{d})$$

$$m = \frac{\mathbf{1 - 0}}{\boldsymbol{c - d}}$$

$$\boldsymbol{y} = \frac{\boldsymbol{x-d}}{\boldsymbol{c-d}} = \frac{\boldsymbol{d-x}}{\boldsymbol{d-c}}$$

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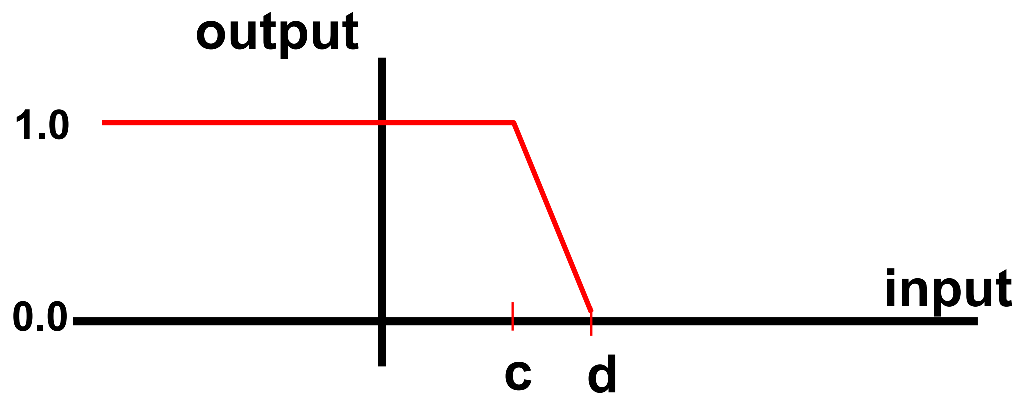
Trapezoidal Membership Function

Crisp Input: x
Membership Function: F(x)



$$F_{left\_trapezoid}(x) = \max\left(\min\left(1, \frac{d-x}{d-c}\right), 0\right)$$

Given input

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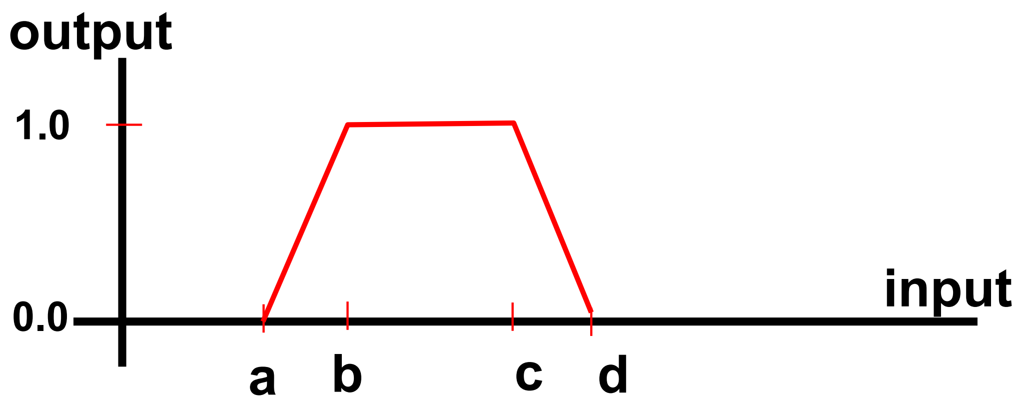
Trapezoidal Membership Function

Crisp Input: x
Membership Function: F(x)



$$F_{regular\_trapezoid}(x) = \max\left(\min\left(\underbrace{\frac{x-a}{b-a}}, 1, \underbrace{\frac{d-x}{d-c}}\right), 0\right)$$

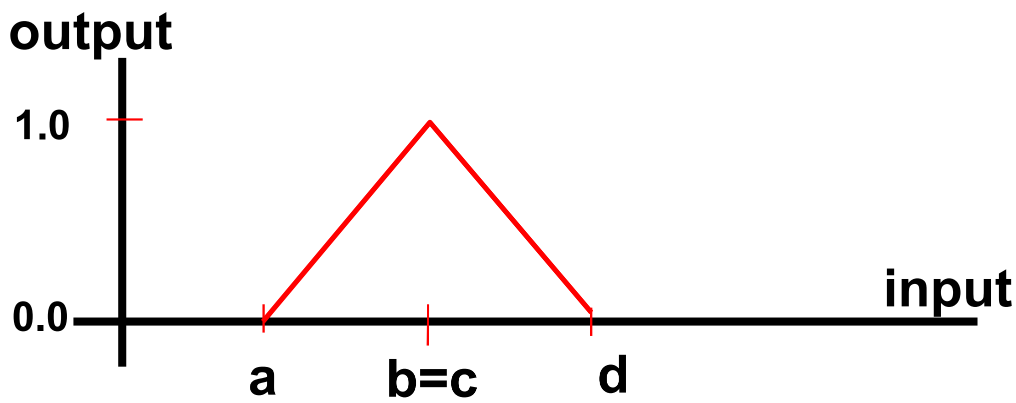
Given input

If x is between a and b:

If x is between c and d:

Triangular Membership Function

Crisp Input: x
Membership Function: F(x)



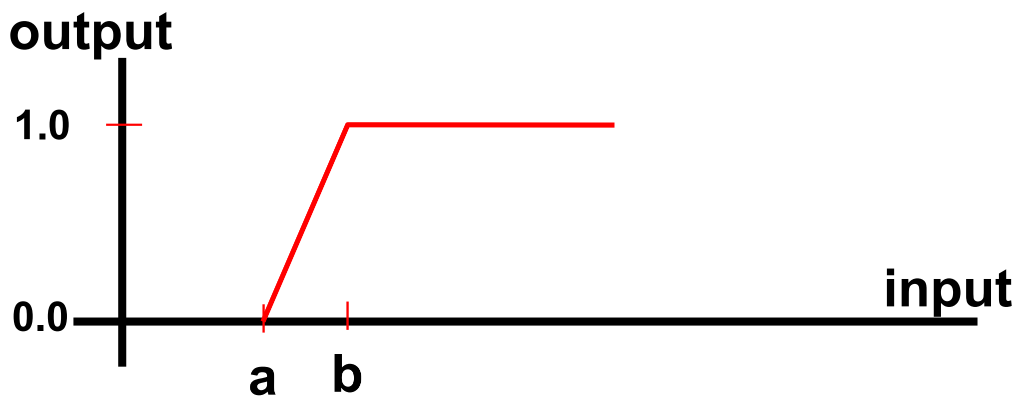
$$F_{regular\_trapezoid}(x) = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right)$$

If b is equal to c:

$$F_{triangular}(x) = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-b}\right), 0\right)$$

Trapezoidal Membership Function

Crisp Input: x
Membership Function: F(x)



$$F_{right\_trapezoid}(x) = \max\left(\min\left(1, \frac{x-a}{b-a}\right), 0\right)$$

Given input

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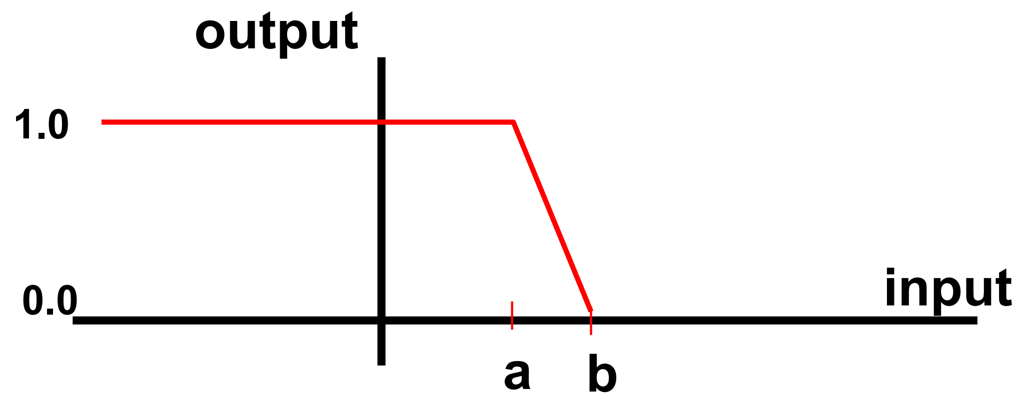
Trapezoidal Membership Function

Note: In the Fuzzy Logic engine codes, parameters c and d were replaced by a and b

Crisp Input: x
Membership Function: F(x)

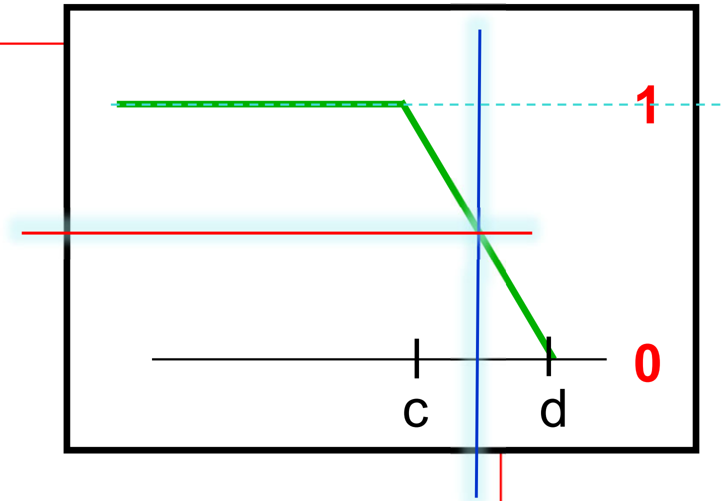


$$F_{left\_trapezoid}(x) = \max\left(\min\left(1, \frac{\boldsymbol{b} - x}{\boldsymbol{b} - \boldsymbol{a}}\right), 0\right)$$

Given input

Trapezoidal Membership Functions

Alternative Code Implementation



LeftTrapezoid

Left_Slope = 0
Right_Slope = 1 / (c - d)

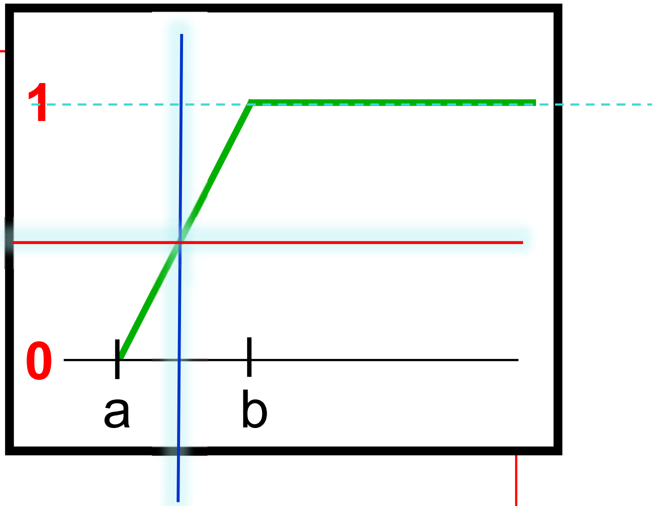
CASE 1: X <= c
Membership Value = 1

CASE 2: X >= d
Membership Value = 0

CASE 3: c < x < d
Membership Value = Right_Slope * (X - d)

Trapezoidal Membership Functions

Alternative Code Implementation



RightTrapezoid

Left_Slope = 1 / (b - a)
Right_Slope = 0

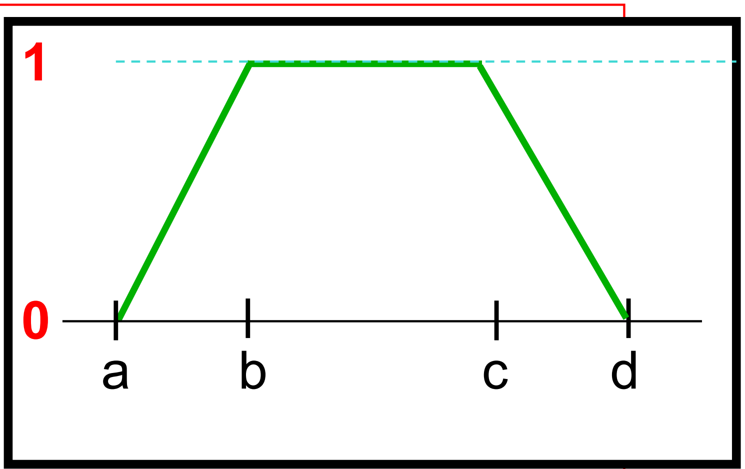
CASE 1: X <= a
Membership Value = 0

CASE 2: X >= b
Membership Value = 1

CASE 3: a < x < b
Membership Value = Left_Slope * (X - a)

Trapezoidal Membership Functions

Alternative Code Implementation



Regular Trapezoid

Left_Slope = 1 / (b - a)
Right_Slope = 1 / (c - d)

CASE 1: X <= a Or X >= d
Membership Value = 0

CASE 2: X >= b And X <= c
Membership Value = 1

CASE 3: X >= a And X <= b
Membership Value = Left_Slope * (X - a)

CASE 4: (X >= c) And (X <= d)
Membership Value = Right_Slope * (X - d)

Hand Simulation

Complete Fuzzy Inference System

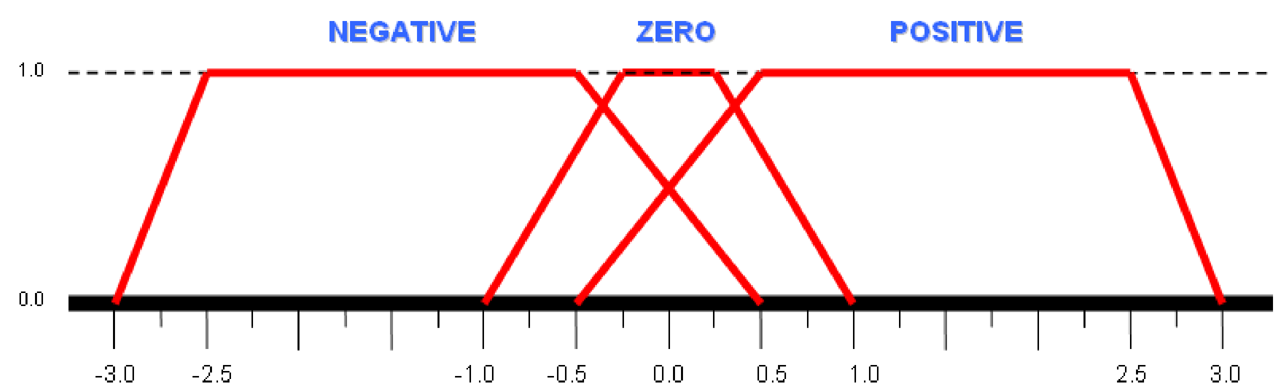
Sample Problem

Given the following FAMM, membership functions and rule outputs, compute for the final crisp output of the fuzzy system when input **x**=0.25 and **y**=-0.25.

Rule Outputs

NL=-5
NS=-2.5
ZE=0
PS=2.5
PL=5.0

y \ x	N	ZE	P
N	NL	NS	NS
ZE	NS	ZE	PS
P	PS	PS	PL



Assume that the logical operator used is the fuzzy-AND

Crisp Inputs: **x = 0.25, y = -0.25**

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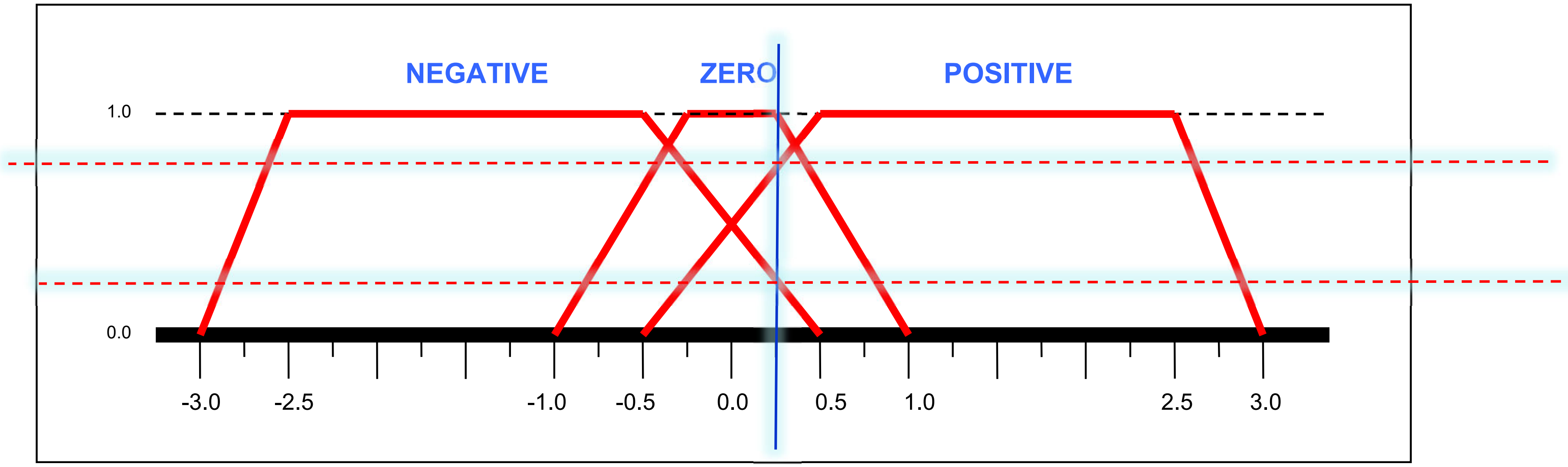
Fuzzification

Fuzzification Example

Fuzzy Sets = { Negative, Zero, Positive }



Assuming that we are using the same trapezoidal membership functions for both input variables, x and y.

Crisp Inputs: **x = 0.25, y = -0.25**

What is the degree of membership of x and y in each of the Fuzzy Sets?

Sample Calculations

Crisp Input: $x = 0.25$

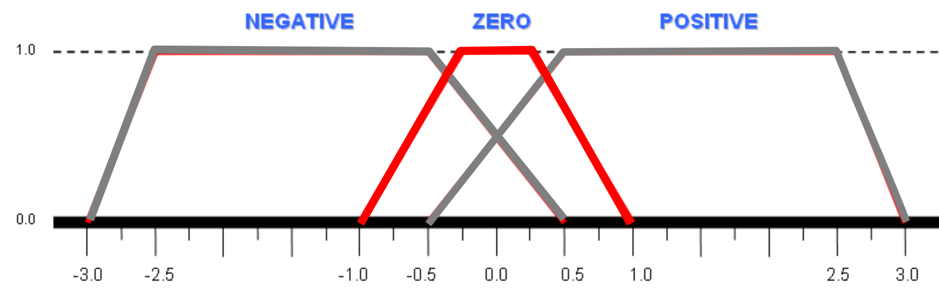
$F_{zero}(0.25)$

Given input

$$F_{ZE}(0.25) = \max\left(\min\left(\frac{x-a}{b-a},1,\frac{d-x}{d-c}\right),0\right)$$

$$= \max\left(\min\left(\frac{0.25-(-1)}{-0.25-(-1)},1,\frac{1-0.25}{1-0.25}\right),0\right)$$

$$= \underbrace{\max(\underbrace{\min(1.67,1,1)}_{\mathbf{1}},0)}_{\mathbf{1}} = 1$$



F_{zero}

- **a=-1**
- **b=-0.25**
- **c=0.25**
- **d=1.0**

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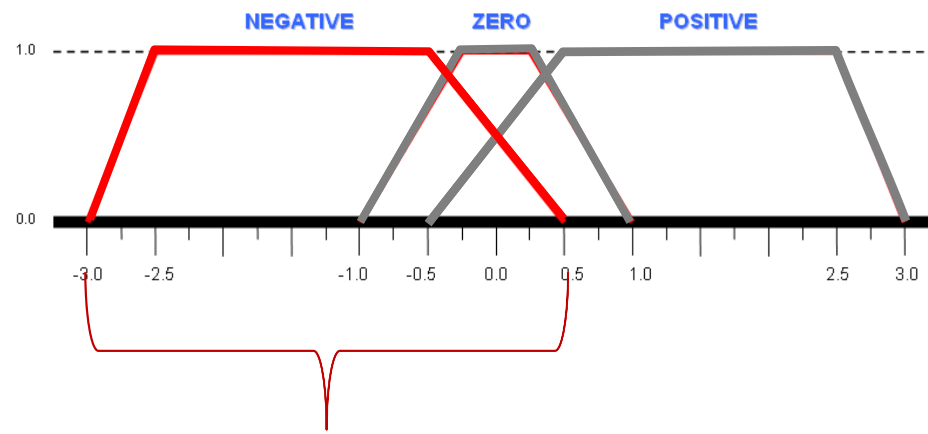
Sample Calculations

Crisp Input: $x = 0.25$



$F_{zero}(0.25)$

F_{zero}: a=-1,b=-0.25, c=0.25, d=1.0

Given input

$$F_{ZE}(0.25) = \max\left(\min\left(\frac{x-a}{b-a},1,\frac{d-x}{d-c}\right),0\right)$$

$$= \max\left(\min\left(\frac{0.25-(-1)}{-0.25-(-1)},1,\frac{1-0.25}{1-0.25}\right),0\right)$$

$$= \max\left(\min\left(1.67,1,1\right),0\right)$$

$$= 1$$

$F_{negative}$:

- **a=-3.0**
- **b=-2.5**
- **c=-0.5**
- **d=0.5**

$F_{positive}(0.25)$

$F_{positive}$: a=-0.5,b=0.5, c=2.5, d=3.0

$$F_P(0.25) = \max\left(\min\left(\frac{0.25-(-0.5)}{0.5-(-0.5)},1,\frac{3-0.25}{3-2.5}\right),0\right)$$

$$= \max\left(\min\left(0.75,1,5.5\right),0\right)$$

$$= 0.75$$

$F_{negative}(0.25)$

$F_{negative}$: a=-3.0,b=-2.5, c=-0.5, d=0.5

$$F_N(0.25) = \max\left(\min\left(\frac{0.25-(-3)}{-2.5-(-3)},1,\frac{0.5-0.25}{0.5-(-0.5)}\right),0\right)$$

$$= \max\left(\min\left(6.5,1,0.25\right),0\right)$$

$$= 0.25$$

Sample Calculations

Crisp Input: $y = -0.25$

$$= \max\left(\min\left(\frac{x-a}{b-a},1,\frac{d-x}{d-c}\right),0\right)$$

F_{zero}(-0.25)

F_{zero}: a=-1,b=-0.25, c=0.25, d=1.0

Given input

$$F_{ZE}(-0.25) = \max\left(\min\left(\frac{-0.25-(-1)}{-0.25-(-1)},1,\frac{1-(-0.25)}{1-0.25}\right),0\right)$$

$$= \max(\min(1,1,1.67),0)$$

$$= 1$$

$F_{positive}$(-0.25)

$$F_p(-0.25) = \max\left(\min\left(\frac{-0.25-(-0.5)}{0.5-(-0.5)},1,\frac{3.0-(-0.25)}{3.0-2.5}\right),0\right)$$

$$= \max(\min(0.25,1,6.5),0)$$

$$= 0.25$$

$F_{positive}$: a=-0.5,b=0.5, c=2.5, d=3.0

$F_{negative}$(-0.25)

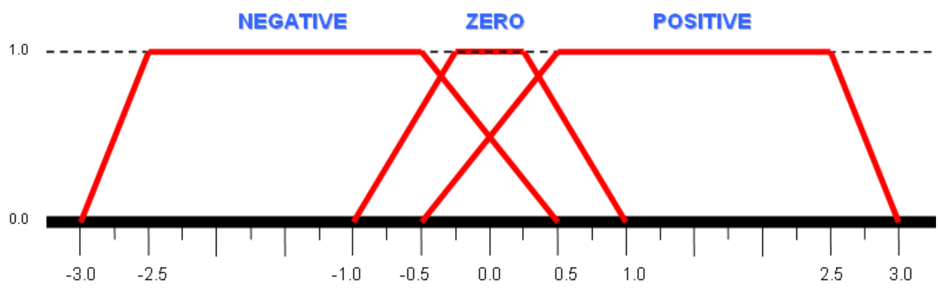
$$F_N(-0.25) = \max\left(\min\left(\frac{-0.25-(-3)}{-2.5-(-3)},1,\frac{0.5-(-0.25)}{0.5-(-0.5)}\right),0\right)$$

$$= \max(\min(5.5,1,0.75),0)$$

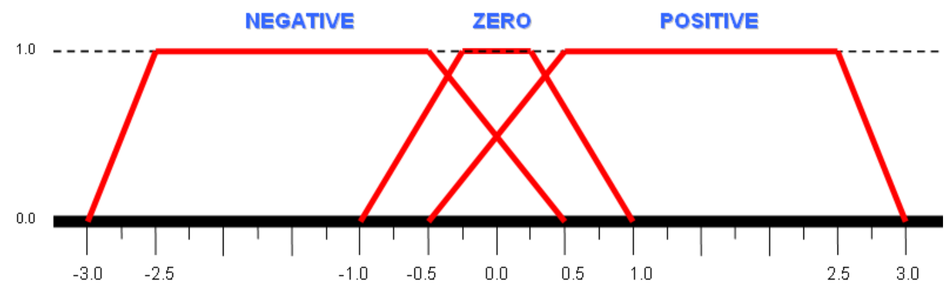
$$= 0.75$$

$F_{negative}$: a=-3.0,b=-2.5, c=-0.5, d=0.5

Fuzzification results



Crisp Input: $x = 0.25$



Crisp Input: $y = -0.25$

$F_{zero}(0.25)=1$

F_{zero}: a=-1,b=-0.25, c=0.25, d=1.0

$F_{positive}(0.25)=0.75$

$F_{positive}$: a=-0.5,b=0.5, c=2.5, d=3.0

$F_{negative}(0.25) = 0.25$

$F_{negative}$: a=-3.0,b=-2.5, c=-0.5, d=0.5

$F_{zero}(-0.25) = 1$

F_{zero}: a=-1,b=-0.25, c=0.25, d=1.0

$F_{positive}(-0.25) = 0.25$

$F_{positive}$: a=-0.5,b=0.5, c=2.5, d=3.0

$F_{negative}(-0.25) = 0.75$

$F_{negative}$: a=-3.0,b=-2.5, c=-0.5, d=0.5

Rule Evaluation

Fuzzy Control

Different stages of Fuzzy control

2. Rule Evaluation

Inputs are applied to a set of **if/then** fuzzy control rules, to compute for the degree of firing of the rules.

e.g. **IF** temperature is very hot, **THEN** set fan speed very high.

antecedent

consequent

Fuzzy set

Fuzzy set or a constant

Fuzzy Control

Different stages of Fuzzy control

Fuzzy logical connectives can also be used to define the antecedent of a rule.

1. **Conjunction: A AND B = min(A, B)**
2. **Disjunction: A OR B = max(A, B)**
3. **Negation: NOT A = 1 - A**

Fuzzy rules may be written in the following form:

If *(***input1** *is membership function1)* ***and/or***
*(***input2** *is membership function2)* ***and/or*** *….*

Then *(output is output membership function).*

For example, one could make up a rule that says:

if *(temperature is high* ***and*** *humidity is high)* ***then*** *room is hot.*

Fuzzy AND

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Fuzzy Control

Different stages of Fuzzy control

Rule Firing Strength

Inputs are applied to a set of **if/then** fuzzy control rules, taking into account the logical connectives that link the inputs, to compute the degree of firing of the rules.

In our solution, we will associate one variable for each rule firing strength.

e.g. **W1** is the **rule firing strength** of **Rule #1.**

Rule #1

Rule firing strength of Rule #1

Outputs
NL=-5
NS=-2.5
ZE=0
PS=2.5
PL=5.0

FAMM

y \ x	N	ZE	P
N	NL	NS	NS
ZE	NS	ZE	PS
P	PS	PS	PL

Rule Strength

W1	**W4**	**W7**
W2	**W5**	**W8**
W3	**W6**	**W9**

Fuzzy Control

Rule Evaluation Example

Given that we are using the **conjunction operator (Fuzzy AND)** in the antecedents of the rules, we calculate the **rule firing strength W_n**.

FAMM

		x N	ZE	P
y	N	NL	NS	NS
	ZE	NS	ZE	PS
	P	PS	PS	PL

Rule #1: IF (x is N **AND** y is N), **THEN** output=NL.

$x = 0.25$ $\quad y = -0.25$

Rule firing strength of Rule #1

Rule Strength

W1	**W4**	**W7**
W2	**W5**	**W8**
W3	**W6**	**W9**

Fuzzified inputs back menu

Fuzzy Control

Rule Evaluation Example

Given that we are using the **conjunction operator (Fuzzy AND)** in the antecedents of the rules, we calculate the **rule firing strength W_n**.

FAMM

y \ x	N	ZE	P
N	NL	NS	NS
ZE	NS	ZE	PS
P	PS	PS	PL

Rule #1: IF (x is N **AND** y is N), **THEN** output=NL.

$\underbrace{\text{x is N}}_{F_{negative}(x=0.25)}$ $\underbrace{\text{y is N}}_{F_{negative}(y=-0.25)}$

Rule Strength

W1	**W4**	**W7**
W2	**W5**	**W8**
W3	**W6**	**W9**

Rule firing strength of Rule #1

Fuzzified inputs back menu

Fuzzy Control

Rule Evaluation Example

Given that we are using the **conjunction operator** (**Fuzzy AND**) in the antecedents of the rules, we calculate the **rule firing strength** W_n.

FAMM

y \ x	N	ZE	P
N	NL	NS	NS
ZE	NS	ZE	PS
P	PS	PS	PL

Rule #1: IF (x is N **AND** y is N), **THEN** output=NL.

$\underbrace{\text{x is N}}_{F_{negative}(x=0.25)}$ $\underbrace{\text{y is N}}_{F_{negative}(y=-0.25)}$

$$W_1 = \mathbf{min}[F_N(0.25), F_N(-0.25)]$$

Rule Strength

W1	W4	W7
W2	W5	W8
W3	W6	W9

Fuzzified inputs back menu

Fuzzy Control

Rule Evaluation Example

Given that we are using the **conjunction operator (Fuzzy AND)** in the antecedents of the rules, we calculate the **rule firing strength W_n**.

FAMM

y \ x	N	ZE	P
N	NL	NS	NS
ZE	NS	ZE	PS
P	PS	PS	PL

Rule #1: IF (x is N **AND** y is N), **THEN** output=NL.

$F_{negative}(x=0.25)$

$F_{negative}(y=-0.25)$

Rule Strength

W1	W4	W7
W2	W5	W8
W3	W6	W9

$$W_1 = \mathbf{min}[F_N(0.25), F_N(-0.25)]$$

$$W_1 = \min[0.25, 0.75] = 0.25$$

Fuzzified inputs back menu

Fuzzy Control

Rule Evaluation Example

Given that we are using the **conjunction operator (Fuzzy AND)** in the antecedents of the rules, we calculate the **rule firing strength W_n**.

FAMM

y \ x	N	ZE	P
N	NL	NS	NS
ZE	NS	ZE	PS
P	PS	PS	PL

Rule Strength

W1	**W4**	**W7**
W2	**W5**	**W8**
W3	**W6**	**W9**

$$W_1 = \min\left[F_N(0.25), F_N(-0.25)\right] = \min\left[0.25, 0.75\right] = 0.25$$

$$W_2 = \min\left[F_N(0.25), F_{ZE}(-0.25)\right] = \min\left[0.25, 1\right] = 0.25$$

$$W_3 = \min\left[F_N(0.25), F_P(-0.25)\right] = \min\left[0.25, 0.25\right] = 0.25$$

$$W_4 = \min\left[F_{ZE}(0.25), F_N(-0.25)\right] = \min\left[1, 0.75\right] = 0.75$$

$$W_5 = \min\left[F_{ZE}(0.25), F_{ZE}(-0.25)\right] = \min\left[1, 1\right] = 1$$

$$W_6 = \min\left[F_{ZE}(0.25), F_P(-0.25)\right] = \min\left[1, 0.25\right] = 0.25$$

$$W_7 = \min\left[F_P(0.25), F_N(-0.25)\right] = \min\left[0.75, 0.75\right] = 0.75$$

$$W_8 = \min\left[F_P(0.25), F_{ZE}(-0.25)\right] = \min\left[0.75, 1\right] = 0.75$$

$$W_9 = \min\left[F_P(0.25), F_P(-0.25)\right] = \min\left[0.75, 0.25\right] = 0.25$$

Fuzzified inputs
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Does a FAMM need to be square?

Is it possible to use more than 2 input parameters for a FAMM?

FAMM

y \ x	N	ZE	P
N	NL	NS	NS
ZE	NS	ZE	PS
P	PS	PS	PL

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Defuzzification

Fuzzy Control

Different stages of Fuzzy control

3. Defuzzification

The contributions of all the rules are combined to generate an output distribution. Subsequently, this distribution is defuzzified to produce a single crisp output.

Fuzzy outputs are combined into discrete values needed to drive the control mechanism

(e.g. Exact steering angle for a robot)

We will see a complete example of the steps involved in the next slide.

Fuzzy Control

Defuzzification Example

Assuming that we are using the **center of mass** defuzzification method.

$$OUTPUT = \frac{\sum_i \boldsymbol{\mu}(x_i)x_i}{\sum_i \boldsymbol{\mu}(x_i)}$$

where $\boldsymbol{\mu}(x_i)$ is the membership value for point x_i in the universe of discourse.

Fuzzy Control

Defuzzification Example

Assuming that we are using the **center of mass** defuzzification method.

$$OUTPUT = \frac{\sum_i \boldsymbol{\mu}(x_i)x_i}{\sum_i \boldsymbol{\mu}(x_i)}$$

where $\boldsymbol{\mu}(x_i)$ is the membership value for point x_i in the universe of discourse.

Fuzzy Control

Defuzzification Example

Assuming that we are using the **center of mass** defuzzification method.

$$OUTPUT = \frac{\sum_i \boldsymbol{\mu}(x_i)x_i}{\sum_i \boldsymbol{\mu}(x_i)}$$

where $\boldsymbol{\mu}(x_i)$ is the membership value for point x_i in the universe of discourse.

$$\mathrm{OUTPUT} = \frac{(\mathrm{W_1 \cdot NL + W_2 \cdot NS + W_3 \cdot PS + W_4 \cdot NS + W_5 \cdot ZE + W_6 \cdot PS + W_7 \cdot NS + W_8 \cdot PS + W_9 \cdot PL})}{\sum_{i=1}^{9} \mathrm{W_i}}$$

Fuzzy Control

Defuzzification Example

Assuming that we are using the **center of mass** defuzzification method.

$$\text{OUTPUT} = \frac{(W_1 \cdot NL + W_2 \cdot NS + W_3 \cdot PS + W_4 \cdot NS + W_5 \cdot ZE + W_6 \cdot PS + W_7 \cdot NS + W_8 \cdot PS + W_9 \cdot PL)}{\sum_{i=1}^{9} W_i}$$

$$= \frac{\left(0.25 \cdot (-5) + 0.25 \cdot (-2.5) + 0.25 \cdot 2.5 + 0.75 \cdot (-2.5) + 1 \cdot 0 + 0.25 \cdot 2.5 + 0.75 \cdot 2.5 + 0.75 \cdot 2.5 + 0.25 \cdot 5\right)}{\left(0.25 + 0.25 + 0.25 + 0.75 + 1 + 0.25 + 0.75 + 0.75 + 0.25\right)}$$

<u>Outputs</u>

NL = **-5**
NS = -2.5
ZE = 0
PS = 2.5
PL = 5.0

W1	**W4**	**W7**
W2	**W5**	**W8**
W3	**W6**	**W9**

FAMM

y \ x	N	ZE	P
N	NL	NS	NS
ZE	NS	ZE	PS
P	PS	PS	PL

Fuzzified inputs back menu

Fuzzy Control

Defuzzification Example

Assuming that we are using the **center of mass** defuzzification method.

$$\text{OUTPUT} = \frac{(W_1 \cdot NL + W_2 \cdot NS + W_3 \cdot PS + W_4 \cdot NS + W_5 \cdot ZE + W_6 \cdot PS + W_7 \cdot NS + W_8 \cdot PS + W_9 \cdot PL)}{\sum_{i=1}^{9} W_i}$$

$$= \frac{(0.25 \cdot (-5) + 0.25 \cdot (-2.5) + 0.25 \cdot 2.5 + 0.75 \cdot (-2.5) + 1 \cdot 0 + 0.25 \cdot 2.5 + 0.75 \cdot 2.5 + 0.75 \cdot 2.5 + 0.25 \cdot 5)}{(0.25 + 0.25 + 0.25 + 0.75 + 1 + 0.25 + 0.75 + 0.75 + 0.25)}$$

$$\boxed{= -1.25/\ 4.5 \ = -0.278}$$

FAMM

<u>Outputs</u>
NL=-5
NS=-2.5
ZE=0
PS=2.5
PL=5.0

W1	**W4**	**W7**
W2	**W5**	**W8**
W3	**W6**	**W9**

		x		
		N	ZE	P
y	N	NL	NS	NS
	ZE	NS	ZE	PS
	P	PS	PS	PL

Fuzzified inputs

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Summary of Steps

To compute the output of the Fuzzy Inference System, given the inputs, one must follow six steps:

1. Determine a set of fuzzy rules.
2. Fuzzify the inputs using the input membership functions.
3. Combine the fuzzified inputs according to the logical connectives set in the fuzzy rules to calculate the firing strength of each rule.
4. Determine the consequence of each rule by combining the rule strength with the rule output.
5. Combine the consequences to obtain an output distribution.
6. Defuzzify the output distribution using the center of mass.