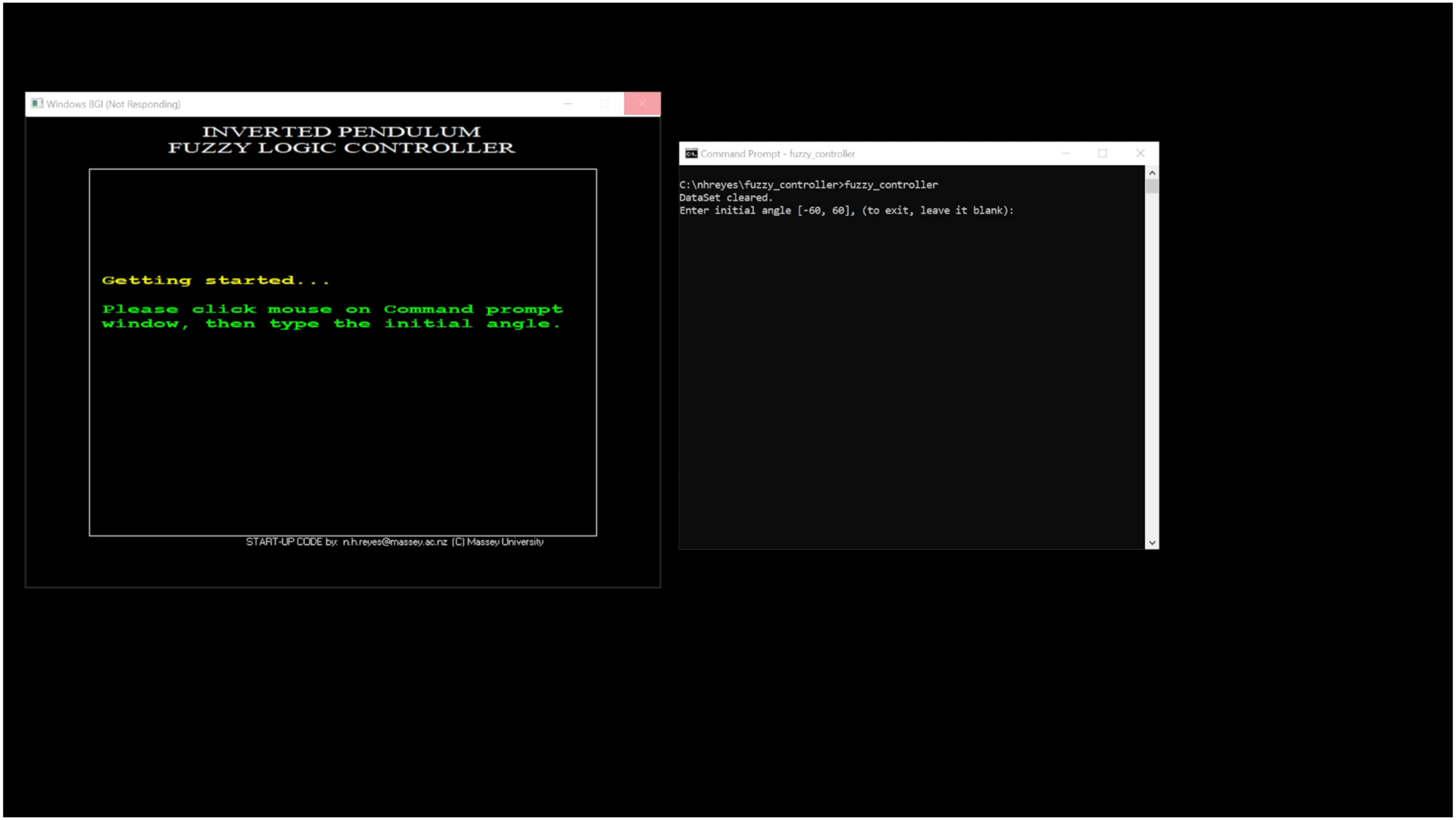
Demonstration of a well-calibrated Fuzzy System for controlling the Inverted Pendulum



Fuzzy System Design #1

Reference: (Book) Neural Network and Fuzzy Logic Applications in C/C++ (Wiley Professional Computing) by Stephen Welstead

Fuzzy System Design #2 **(best solution!)**

Reference: (Journal article), Takeshi Yamakawa, "A Fuzzy Inference Engine in Nonlinear Analog Mode and its Application to a Fuzzy Logic Control"

Fuzzy System Design #1

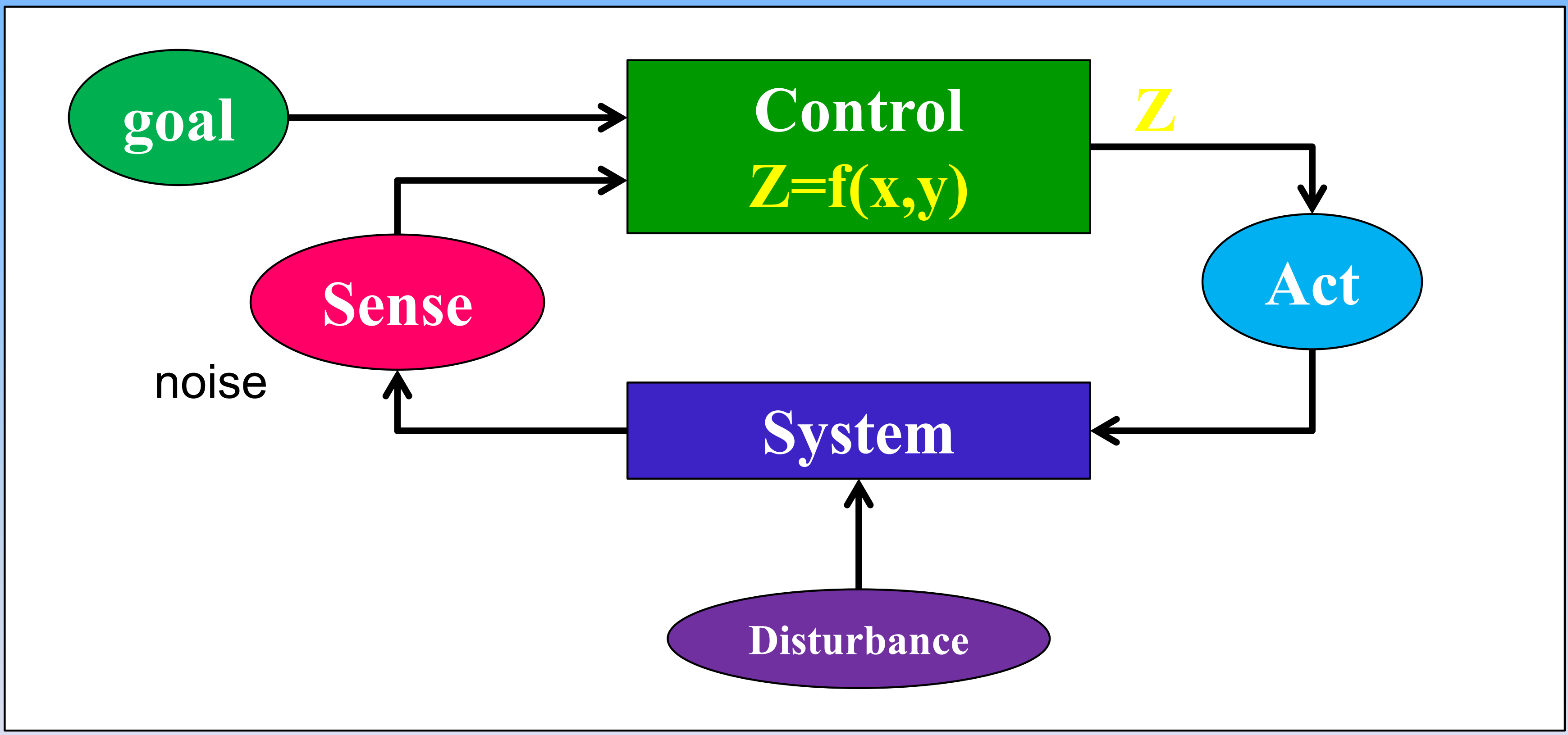
Inverted Pendulum Problem

Takagi-Sugeno Fuzzy Inference System

Reference: Neural Network and Fuzzy Logic Applications in C/C++ (Wiley Professional Computing) by Stephen Welstead

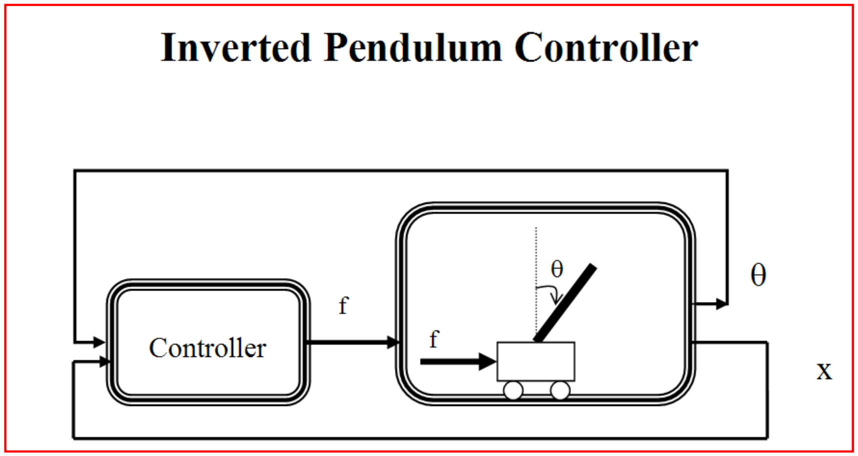
Feedback Control/Closed Loop Control

- Closed-loop control allows for **uncertainty** in the model as well as **noise** and **disturbances** in the system under control



***Controllers** are used in the industry to **regulate** temperature, pressure, flow rate, chemical composition, speed and practically every other variable for which a measurement exists.

Inverted Pendulum Problem



A Classic test case in embedded control

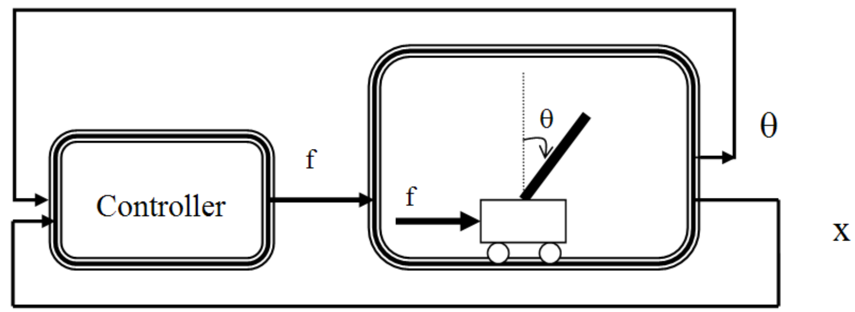
A pole with a weight on top is mounted on a motor-driven cart. The pole can swing freely, and the cart must move back and forth to keep it vertical.

A controller monitors the angle and motion of the pole and directs the cart to execute the necessary balancing movements.

A Glimpse at History: International Conference in Tokyo (1987) **Takeshi Yamakawa** demonstrated the use of fuzzy control, through a set of simple dedicated fuzzy logic chips, in an "**inverted pendulum**" experiment. (Later experiments: mounted a wine glass containing water or even a live mouse to the top of the pendulum).

Inverted Pendulum Problem

Inverted Pendulum Controller



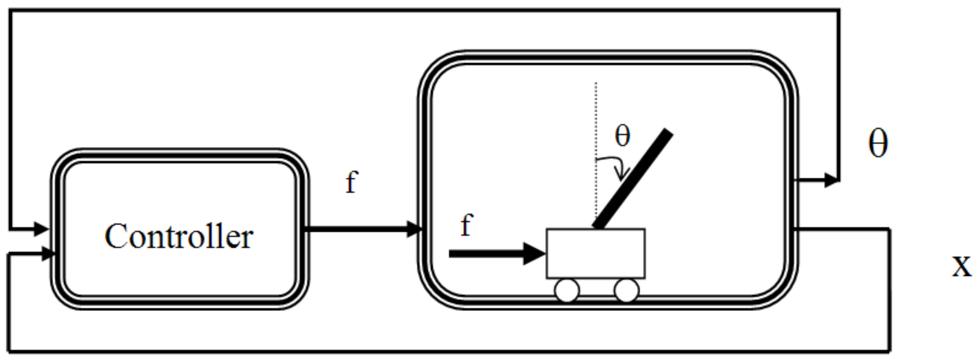
Conventional mathematical solution

The solution uses a second-order differential equation that describes cart motion as a function of pole position and velocity:

$$\left[m \frac{\partial^2}{\partial t^2}(x + l\sin\theta) \right] l\cos\theta - \left[m \frac{\partial^2}{\partial t^2}(l\cos\theta) \right] l\sin\theta = mgl\sin\theta$$

Inverted Pendulum Problem

Inverted Pendulum Controller



<u>Sensed values:</u>

X – position of object with respect to the horizontal axis

θ - angle of pole relative to the vertical axis

<u>Derived values:</u>

X' - Velocity along the x-axis

θ' - Angular velocity

<u>Input variables:</u> sensed and derived values

<u>Controller output:</u> **F** – force to be applied to the cart

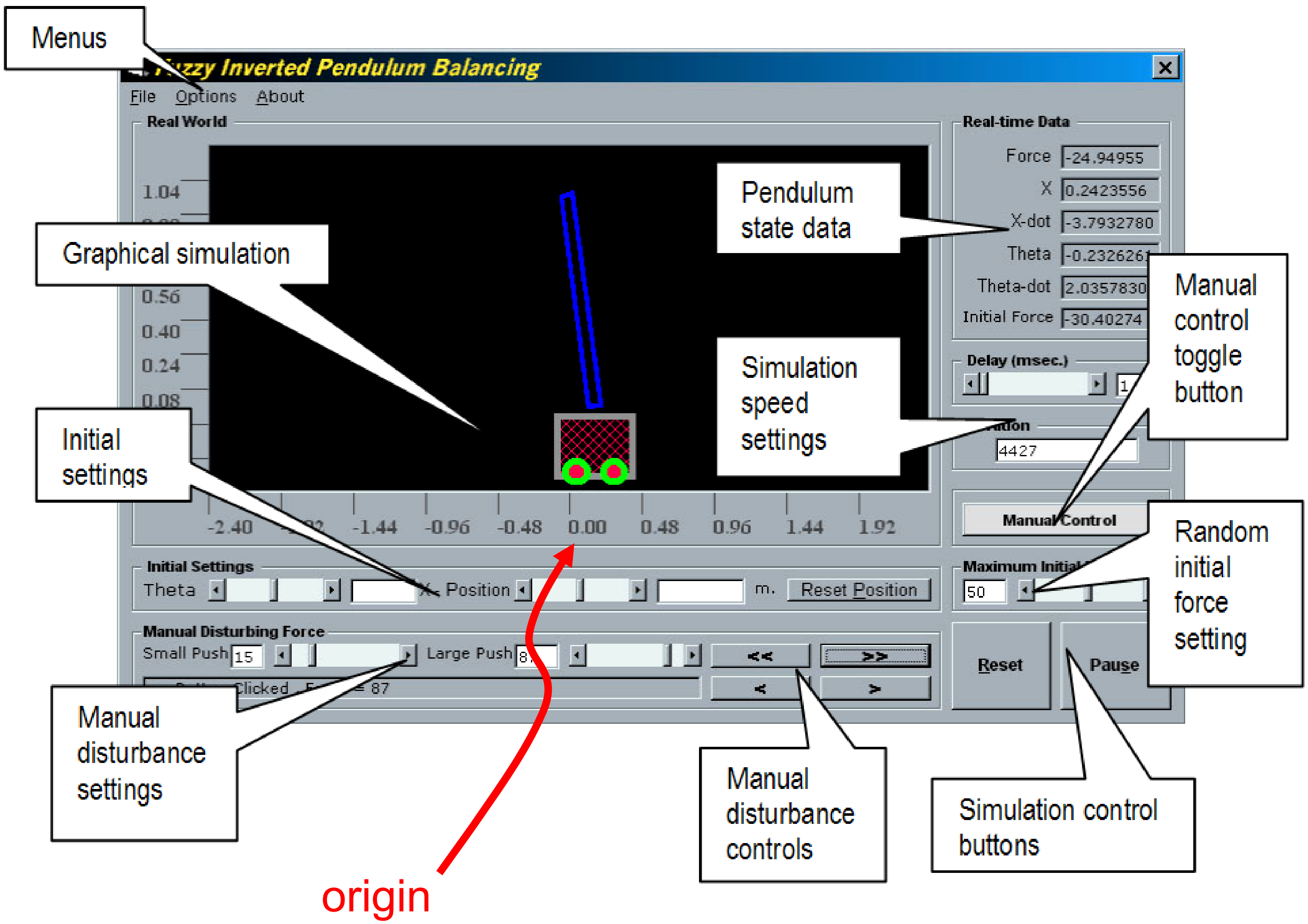
Derived Input Values

We can derive new input values for our Fuzzy Control System using Physics equations.

A sample calculation of some of the derived values: angular velocity (theta’)

theta	time	theta'
2	1	
10	2	8
30	3	20
40	4	10
47	5	7
32	6	-15
28	7	-4
19	8	-9

Inverted Pendulum Problem



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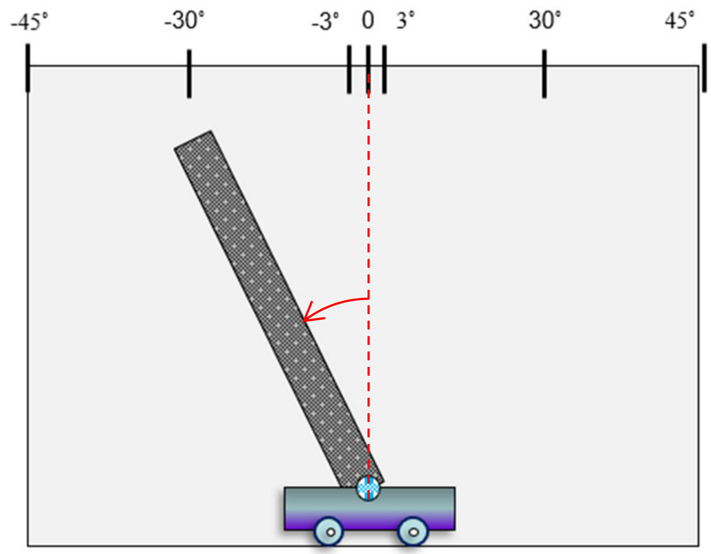
Parameters for a Fuzzy System

Once you have determined the appropriate inputs and outputs for your application, there are three steps to designing the parameters for a fuzzy system:

1. specify the **fuzzy sets** to be associated with each variable.
2. decide on what the **fuzzy rules** are going to be.
3. specify the **shape of the membership functions**.

Fuzzy Sets

Input **angle** = the pole angle measured relative to the vertical axis.



Note: the figure is not drawn to scale

We can begin designing a fuzzy system by subdividing the two input variables (pole angle and angular velocity) into membership sets.

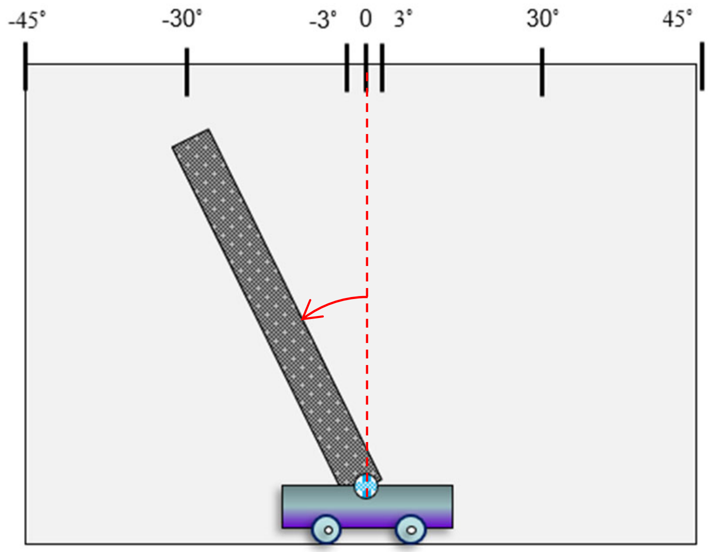
The **angle** could be described as:

1. Inclined to the Left (Negative).
2. Vertical (Zero).
3. Inclined to the Right (Positive).

Linguistic terms for describing an angle measurement

Fuzzy Sets

Input **angle** = the pole angle measured relative to the vertical axis.



Note: the figure is not drawn to scale

Fuzzy logic provides the mathematical facility for transforming vague linguistic terms into their equivalent mathematical function representation.

Linguistic terms	**Fuzzy sets**
1. Inclined to the Left (Negative).	$F_{negative}(angle)$
2. Vertical (Zero).	$F_{zero}(angle)$
3. Inclined to the Right (Positive).	$F_{positive}(angle)$

Implemented as **membership functions**

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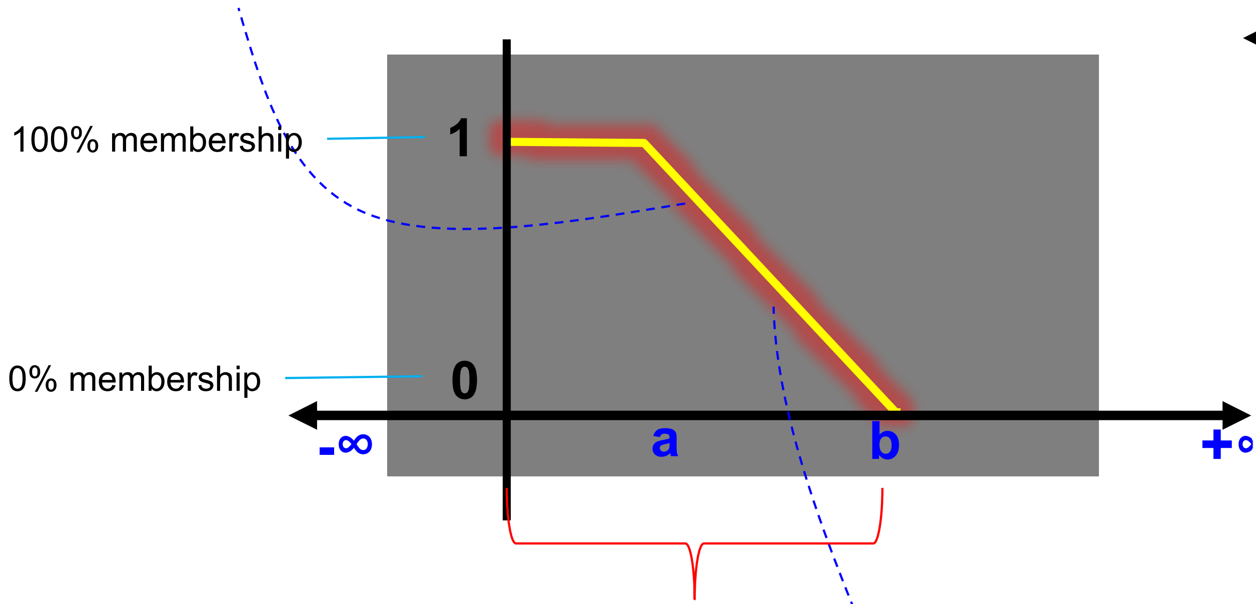
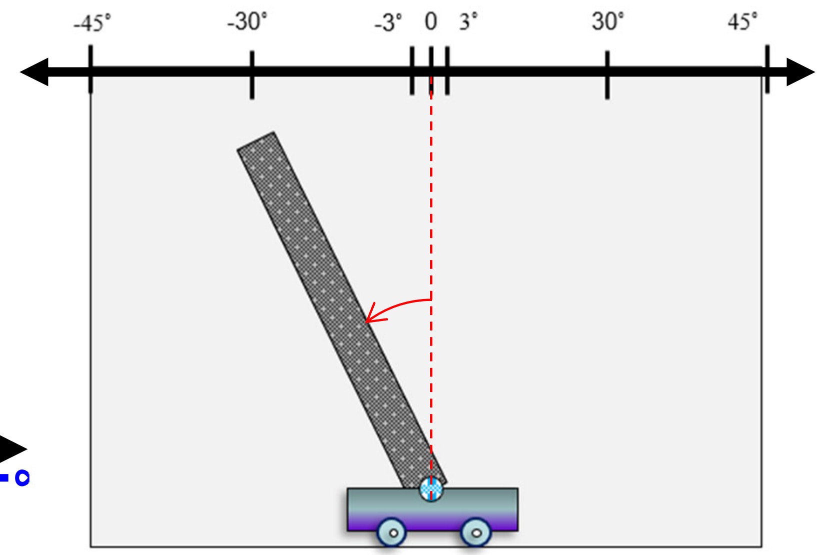
Membership Functions

Membership Functions for the Pole Angle: Inclined to the Left (Negative)

Inclined to the Left (also referred to as Negative angle)





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Range of values associated with the term “Inclined to the Left”

Linguistic term **Fuzzy set**

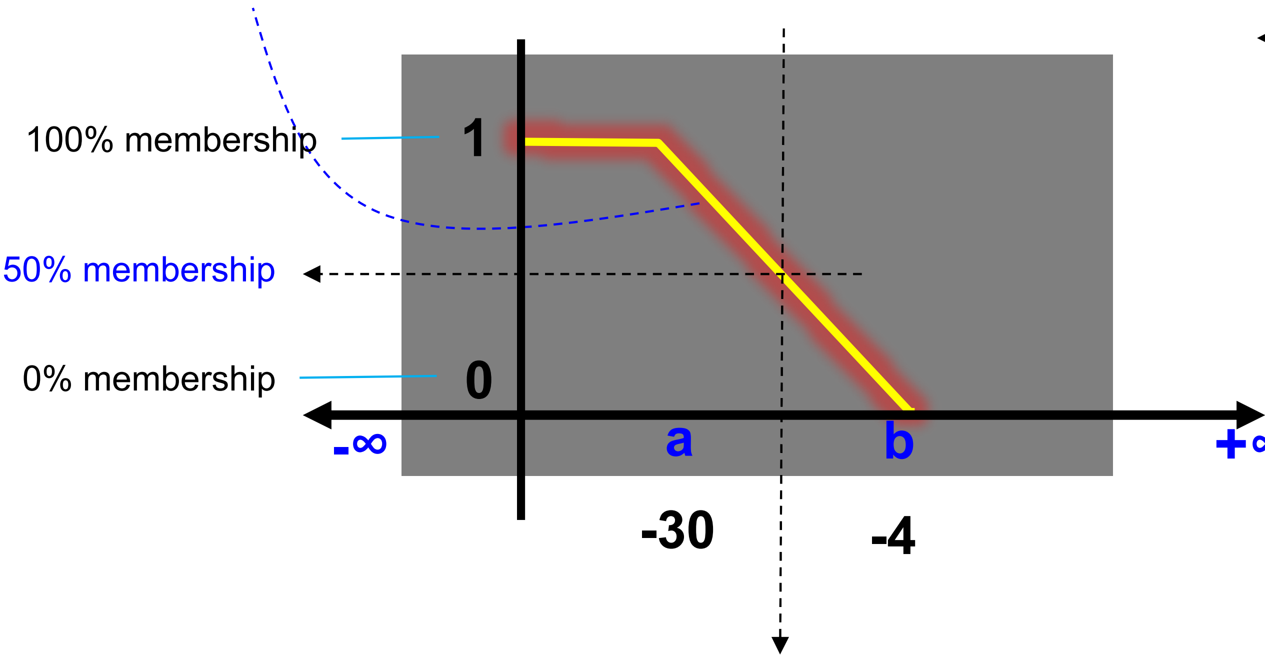
1. Inclined to the Left (Negative). ⇨ $F_{negative}$(angle) = [0, 1]

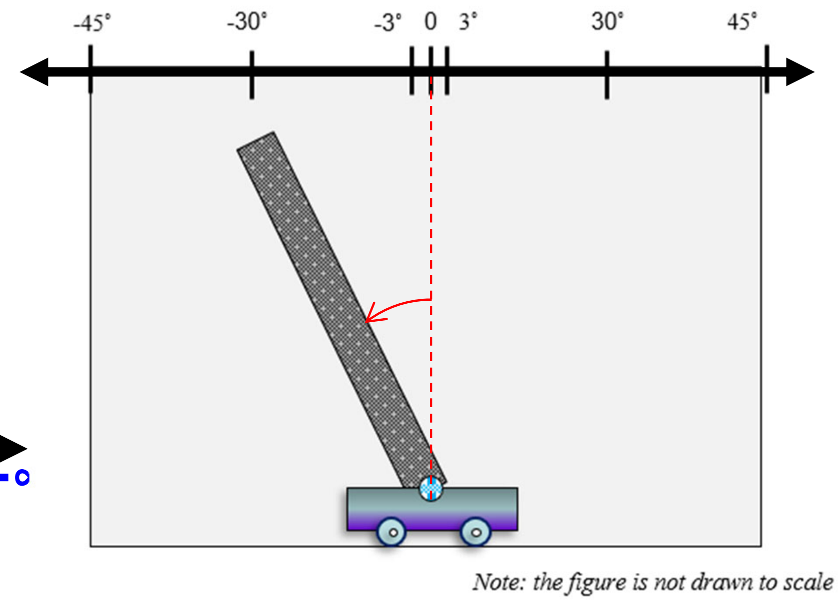
input

Membership Functions

Membership Functions for the Pole Angle: Inclined to the Left (Negative)

Inclined to the Left (also referred to as Negative angle)





Given an **input angle** = **-13**

Fuzzy set

$F_{negative}$(**angle**) = ?

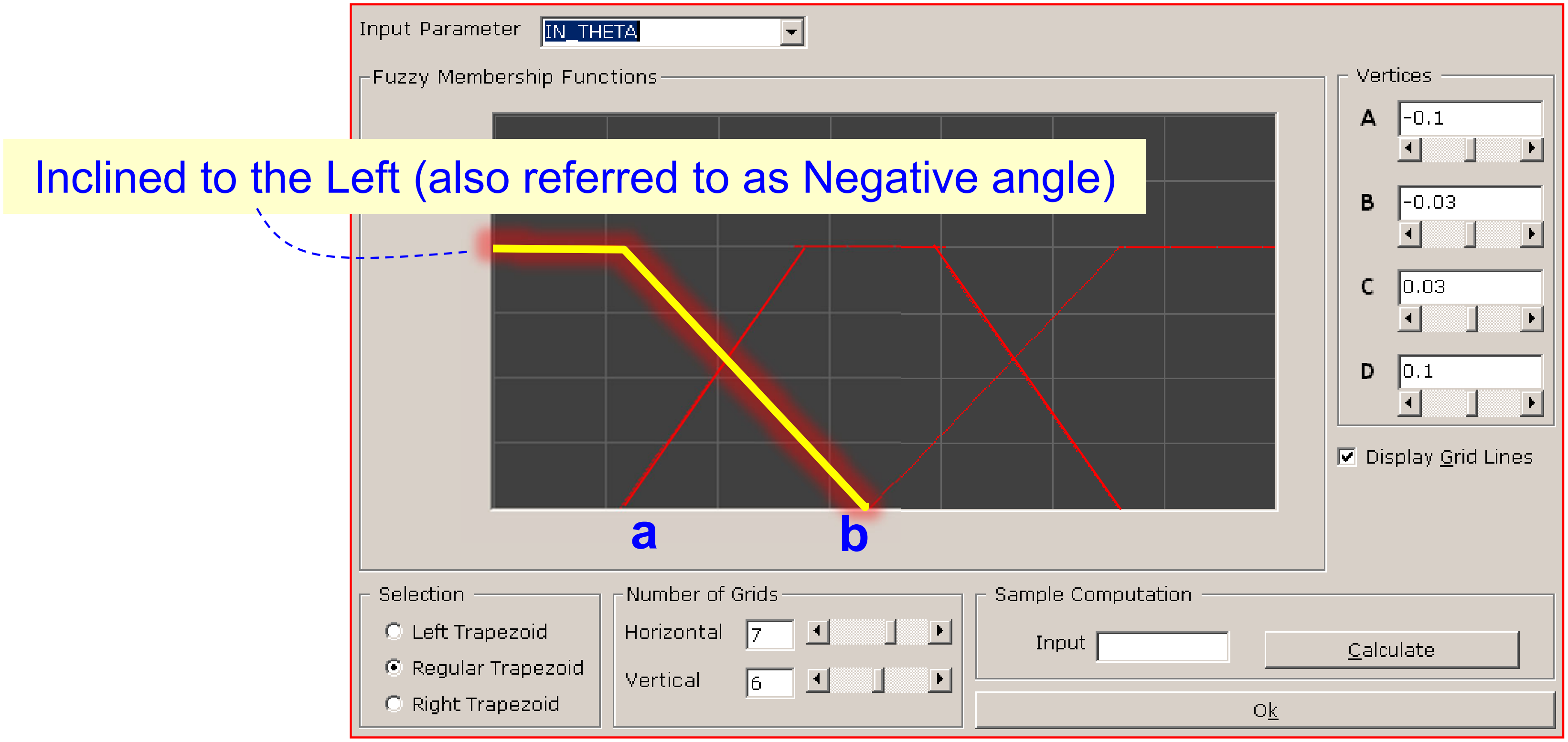
input

Degree of membership of the input angle = **-13** to the fuzzy set $F_{negative}$

$F_{negative}$(**-13**) ~ **0.5**

Membership Functions

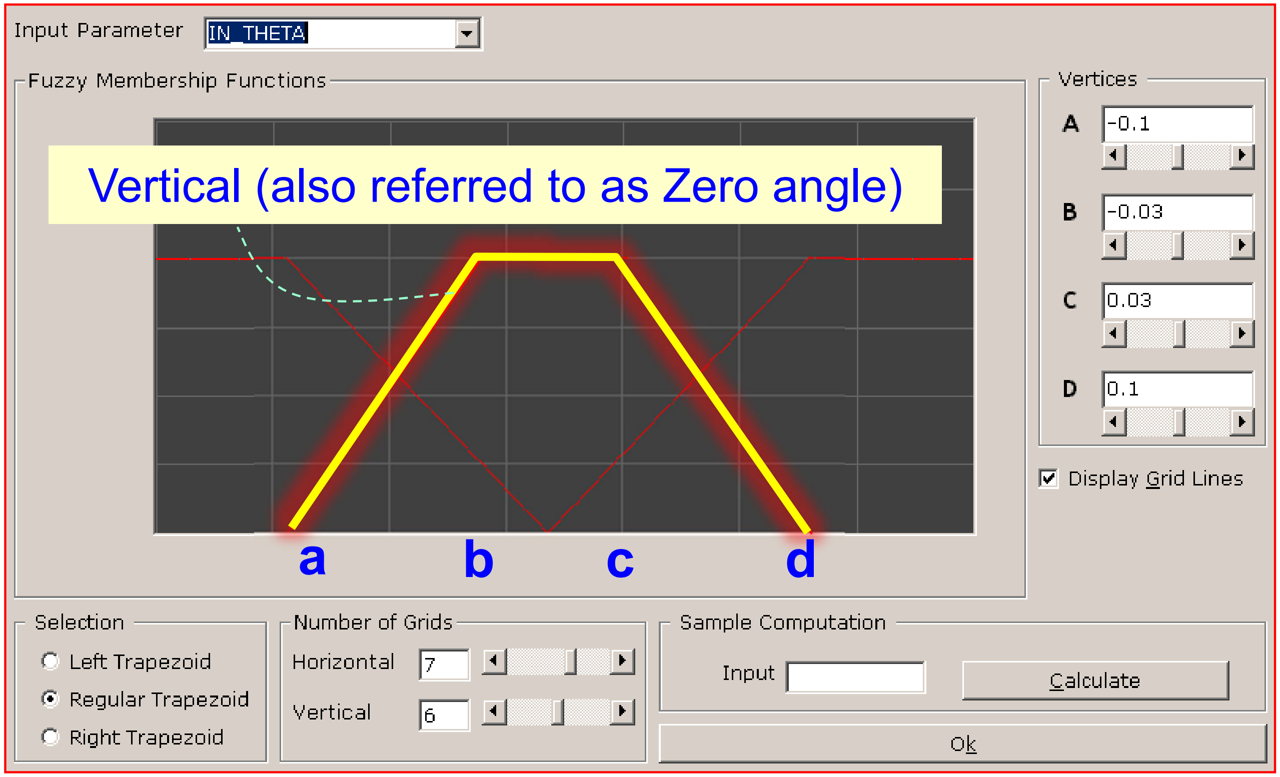
Membership Functions for the Pole Angle: Inclined to the Left (Negative)



Trapezoid Vertices		
Left Trapezoid	**Regular**	**Right**
A = -0.1 rad. = -5.73 deg.	A = -0.1 rad = -5.73 deg.	A = 0
B = 0	B = -0.03 = -1.72 deg.	B = 0.1 rad = 5.73 deg.
C = 0	C = 0.03 = 1.72 deg.	C = 0
D = 0	D = 0.1 rad = 5.73 deg.	D = 0

Membership Functions

Membership Functions for the Pole Angle: Inclined to the Left (Negative)



Trapezoid Vertices		
Left Trapezoid	**Regular**	**Right**
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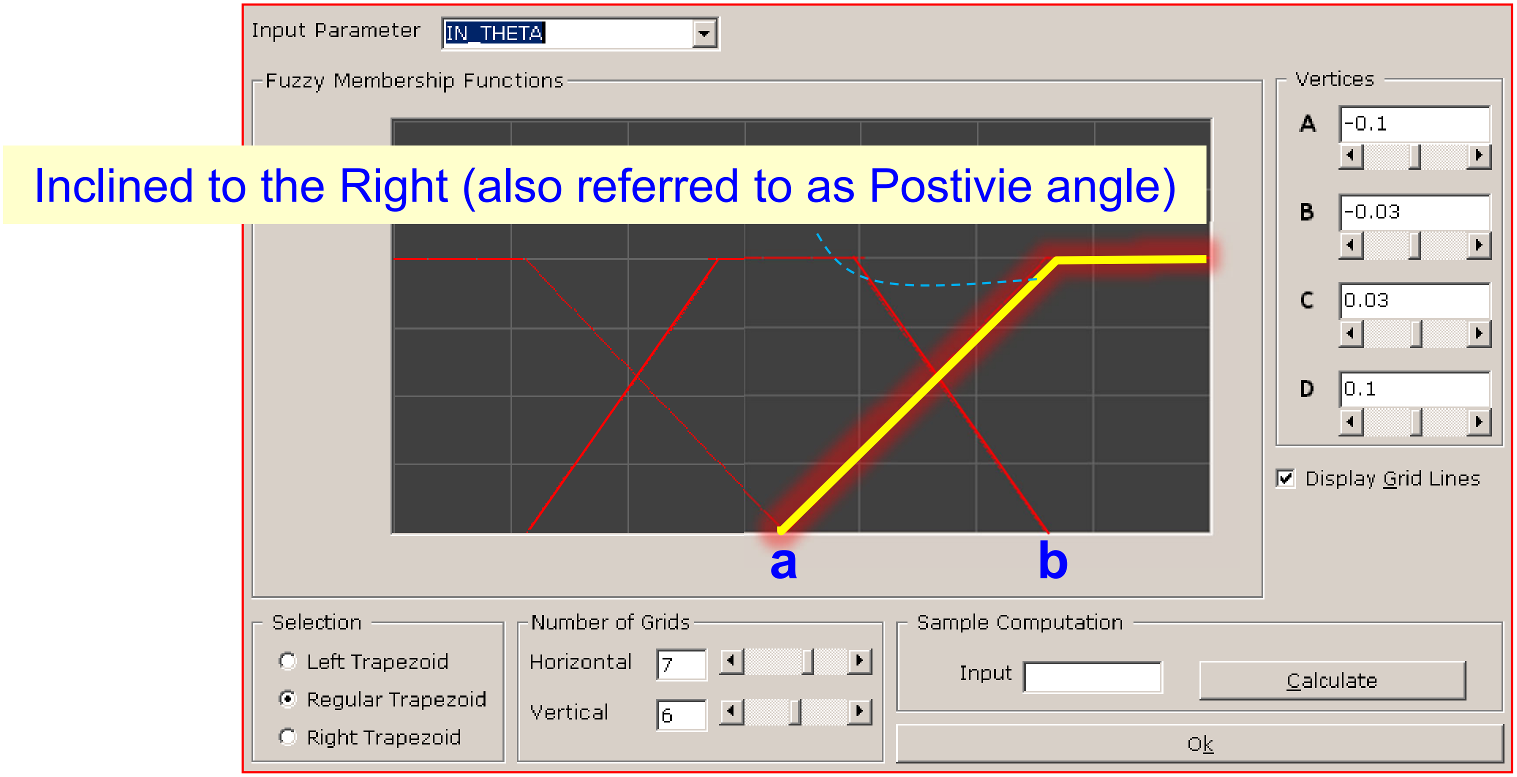
B = 0	B = -0.03 = -1.72 deg.	B = 0.1 rad = 5.73 deg.
C = 0	C = 0.03 = 1.72 deg.	C = 0
D = 0	D = 0.1 rad = 5.73 deg.	D = 0

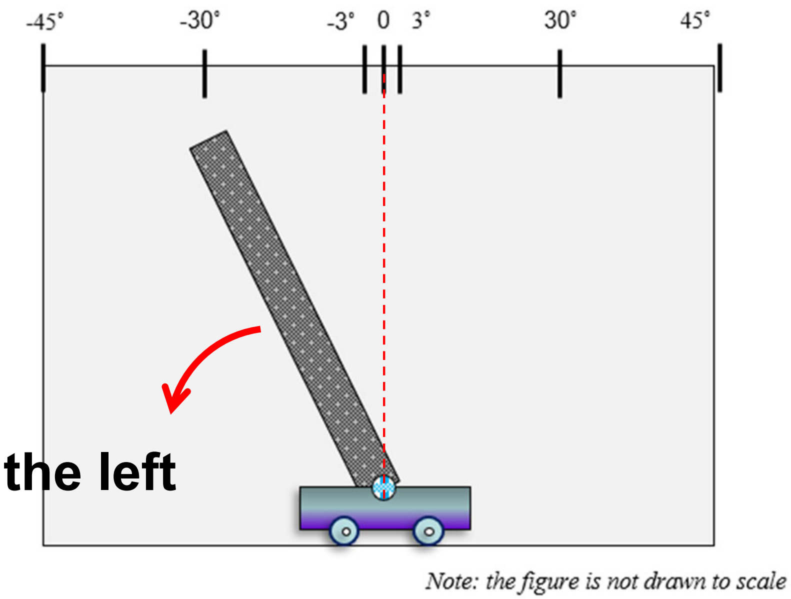
Membership Functions

Membership Functions for the Pole Angle: Inclined to the Left (Negative)



Trapezoid Vertices		
Left Trapezoid	**Regular**	**Right**
A = -0.1 rad. = -5.73 deg.	A = -0.1 rad = -5.73 deg.	A = 0
B = 0	B = -0.03 = -1.72 deg.	B = 0.1 rad = 5.73 deg.
C = 0	C = 0.03 = 1.72 deg.	C = 0
D = 0	D = 0.1 rad = 5.73 deg.	D = 0

Fuzzy Sets



Falling to the left

The **angular velocity** could be described as:

1. Falling to the Left (N).
2. Still (Zero).
3. Falling to the Right (P).

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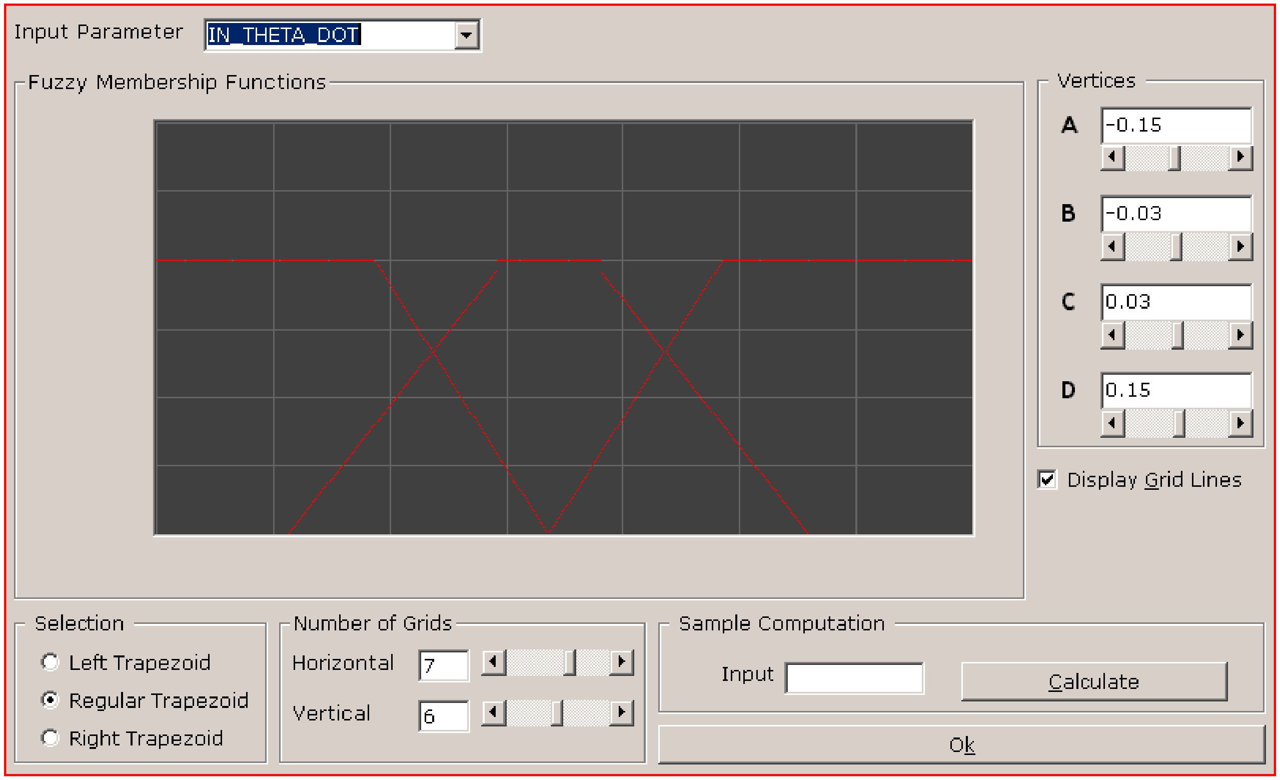
Membership Functions

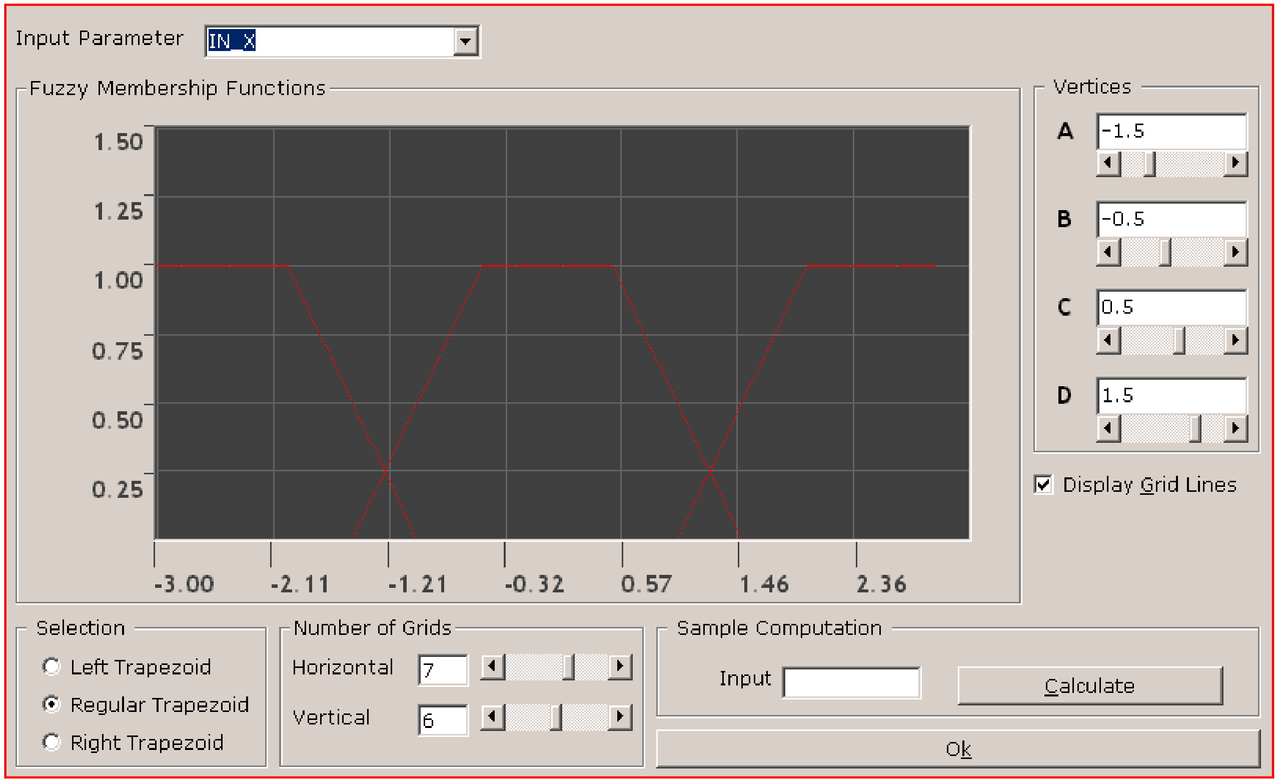
Membership Functions for the Pole Angular Velocity



Trapezoid Vertices		
Left Trapezoid	**Regular**	**Right**
A = -0.1=-5.73 deg/s.	A = -0.15 rad/s = -8.59 deg/s	A = 0
B = 0	B = -0.03 rad/s =-1.72 deg/s	B = 0.1 rad/s = 5.73 deg/s
C = 0	C = 0.03=1.72 deg/s	C = 0
D = 0	D = 0.15= 8.59 deg/s	D = 0

Membership Functions

Membership Functions for the Cart Position x

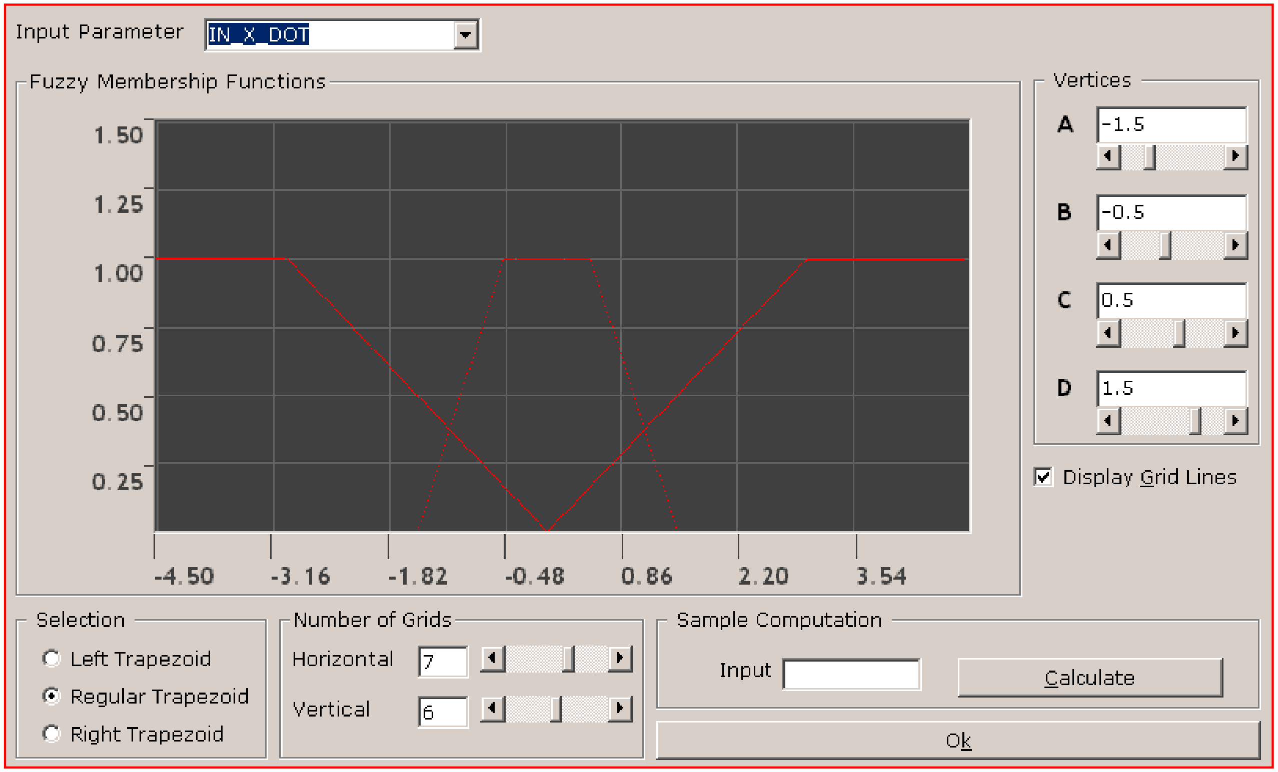


Take note of the position of the origin.

Trapezoid Vertices		
Left Trapezoid	**Regular**	**Right**
A = -2 m	A = -1.5 m	A = 1 m
B = -1 m	B = -0.5 m	B = 2 m
C = 0	C = 0.5 m	C = 0
D = 0	D = 1.5 m	D = 0

Membership Functions

Membership Functions for the Cart Velocity



Trapezoid Vertices		
Left Trapezoid	**Regular**	**Right**
A = -3 m/s	A = -1.5 m/s	A = 0
B = 0	B = -0.5 m/s	B = 3 m/s
C = 0	C = 0.5 m/s	C = 0
D = 0	D = 1.5 m/s	D = 0

FUZZY LOGIC SYSTEM

FAMM1: x vs. x'

Position vs. Velocity

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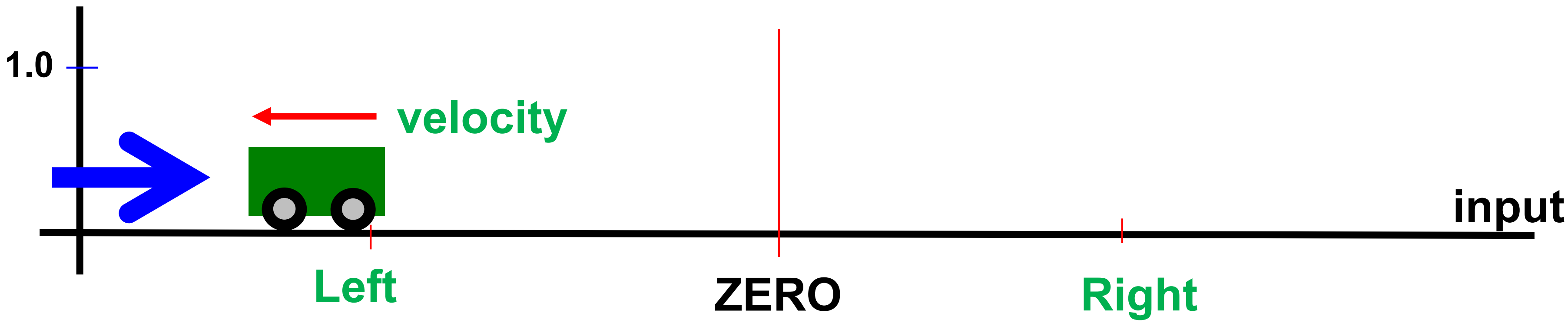
Fuzzy Rules (FAMM 1)

Fuzzy rule base and for the **position and velocity** vectors of the inverted pendulum-balancing problem



1. IF cart is on the left AND cart is going left THEN largely push cart to the right

Input X: {N, ZE, P} = {left, centre, right}

Input X': {N, ZE, P} = {going to the left, still, going to the right}

F: {PL, PS, ZE, NS, NL}={Large push to the right, small push to the right, don't push, small push to the left, Large push to the left}

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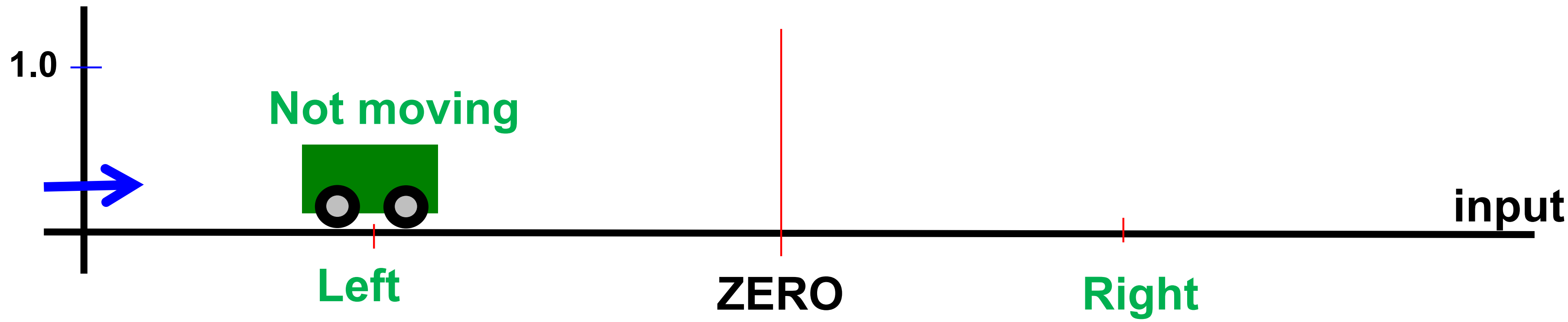
Fuzzy Rules (FAMM 1)

Fuzzy rule base and for the **position and velocity** vectors of the inverted pendulum-balancing problem



2. IF cart is on the left AND cart is not moving THEN slightly push cart to the right

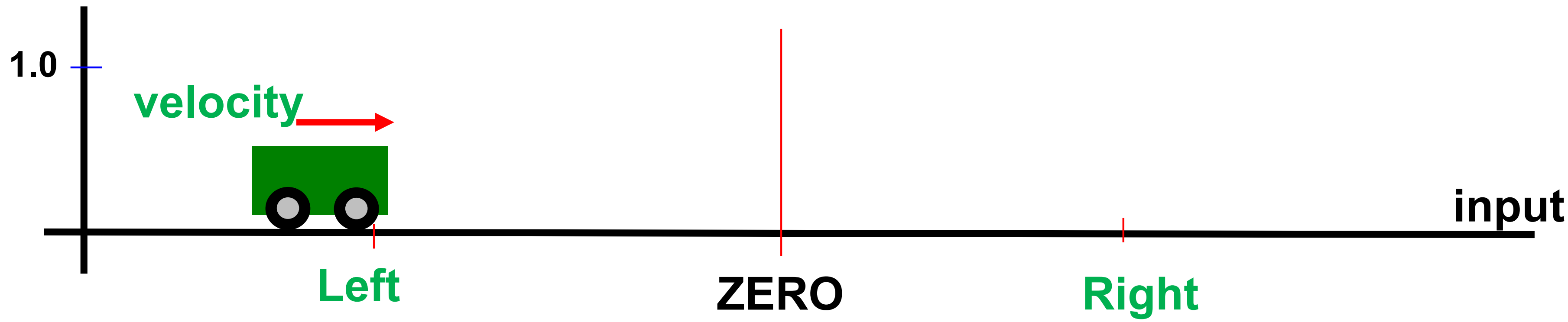
Input X: {N, ZE, P} = {left, centre, right}

Input X': {N, ZE, P} = {going to the left, still, going to the right}

F: {PL, PS, ZE, NS, NL}={Large push to the right, small push to the right, don't push, small push to the left, Large push to the left**}**

Fuzzy Rules (FAMM 1)

Fuzzy rule base and for the **position and velocity** vectors of the inverted pendulum-balancing problem



3. IF cart is on the left AND cart is going right THEN don’t push cart

Input X: {N, ZE, P} = {left, centre, right}

Input X’: {N, ZE, P} = {going to the left, still, going to the right}

F: {PL, PS, ZE, NS, NL}={Large push to the right, small push to the right, don’t push, small push to the left, Large push to the left**}**

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Fuzzy Rules (FAMM 1)

Fuzzy rule base and for the **position and velocity** vectors of the inverted pendulum-balancing problem

1. IF cart is on the left AND cart is going left THEN largely push cart to the right
2. IF cart is on the left AND cart is not moving THEN slightly push cart to the right
3. IF cart is on the left AND cart is going right THEN don't push cart
4. IF cart is centered AND cart is going left THEN slightly push cart to the right
5. IF cart is centered AND cart is not moving THEN don't push cart
6. IF cart is centered AND cart is going right THEN slightly push cart to the left
7. IF cart is on the right AND cart is going left THEN don't push cart
8. IF cart is on the right AND cart is not moving THEN push cart to the left
9. IF cart is on the right AND cart is going right THEN largely push cart to the left

Input X: {N, ZE, P} = {left, centre, right}

Input X': {N, ZE, P} = {going to the left, still, going to the right}

F: {PL, PS, ZE, NS, NL}={Large push to the right, small push to the right, don't push, small push to the left, Large push to the left**}**

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Position vs. Velocity (FAMM 1)

If the cart is too near the end of the path, then regardless of the state of the pole angle push the cart towards the other end.

			X	
		N	ZE	P
	N	PL	PS	ZE
X’	ZE	PS	ZE	NS
	P	ZE	NS	NL

3 x 3 FAMM

Input X: {N, ZE, P} = {left, centre, right}

Input X’: {N, ZE, P} = {going to the left, still, going to the right}

F: {PL, PS, ZE, NS, NL}={Large push to the right, small push to the right, don’t push, small push to the left, Large push to the left**}**

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FUZZY LOGIC SYSTEM

FAMM2: θ vs. θ'

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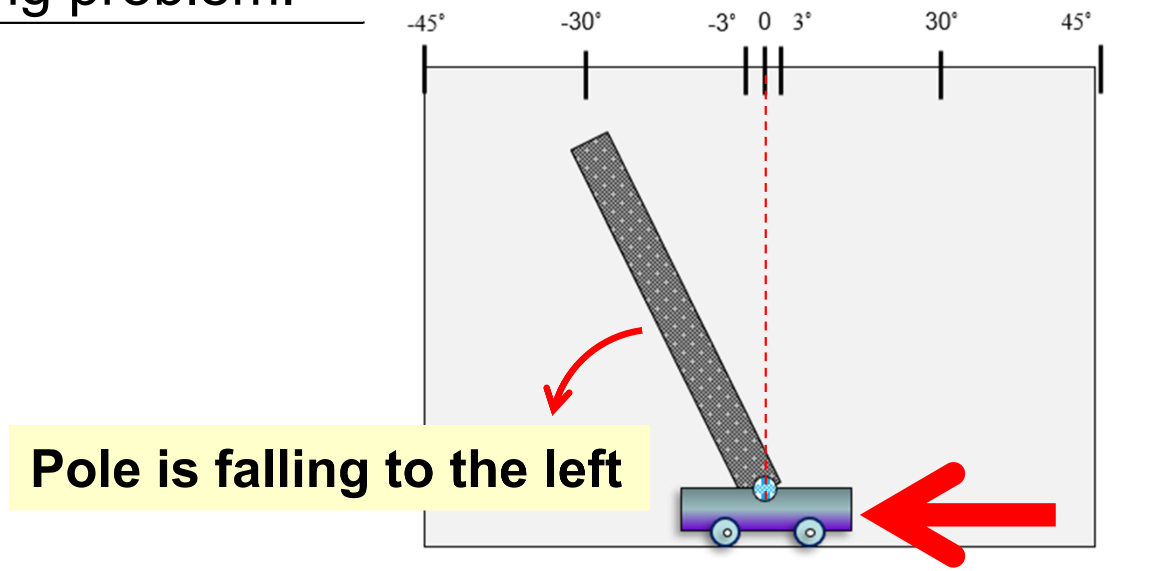
Fuzzy Rules (FAMM 2)

Fuzzy rule base for the **angle and angular velocity** vectors of the inverted pendulum-balancing problem.



Note: the figure is not drawn to scale

1. IF pole is leaning to the left AND pole is dropping to the left THEN largely push cart to the left

Input θ: {N, ZE, P}={leaning to the left, centre, leaning to the right}

Input θ': {N, ZE, P}={moving to the left, still, moving to the right}

F: {PL, PS, ZE, NS, NL}={Large push to the right, small push to the right, don't push, small push to the left, Large push to the left}

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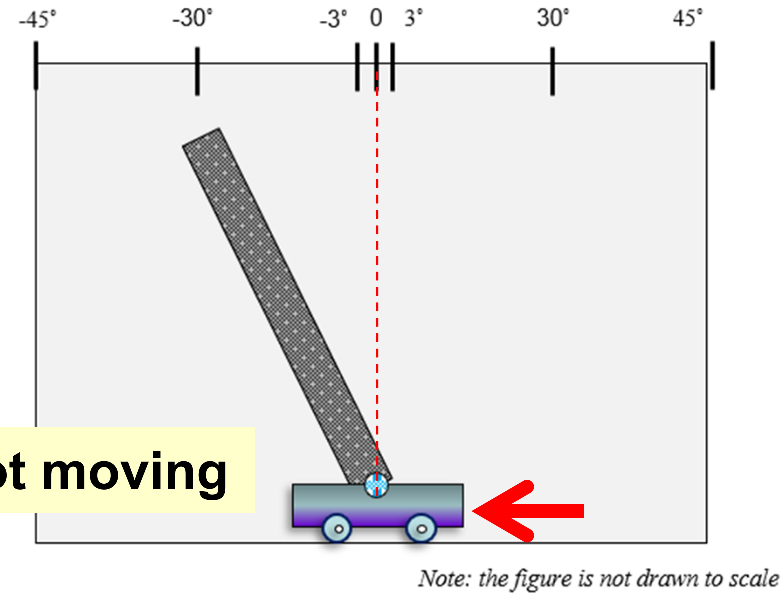
Fuzzy Rules (FAMM 2)

Fuzzy rule base for the **angle and angular velocity** vectors of the inverted pendulum-balancing problem.



2. IF pole is leaning to the left AND pole is not moving THEN slightly push cart to the left

Input θ: {N, ZE, P}={leaning to the left, centre, leaning to the right}

Input θ': {N, ZE, P}={moving to the left, still, moving to the right}

F: {PL, PS, ZE, NS, NL}={Large push to the right, small push to the right, don't push, small push to the left, Large push to the left}

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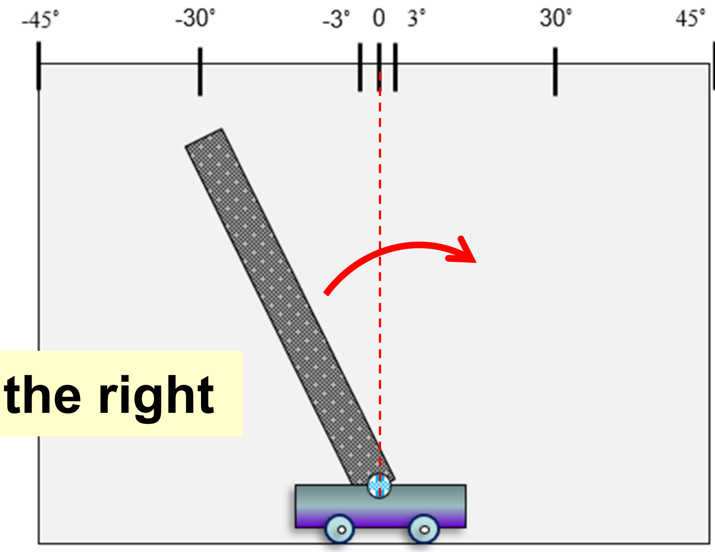
Fuzzy Rules (FAMM 2)

Fuzzy rule base for the **angle and angular velocity** vectors of the inverted pendulum-balancing problem.



Note: the figure is not drawn to scale

Pole is moving to the right

3. IF pole is leaning to the left AND pole is moving to the right THEN don't push the cart

Input θ: {N, ZE, P}={leaning to the left, centre, leaning to the right}

Input θ': {N, ZE, P}={moving to the left, still, moving to the right}

F: {PL, PS, ZE, NS, NL}={Large push to the right, small push to the right, don't push, small push to the left, Large push to the left**}**

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Fuzzy Rules (FAMM 2)

Fuzzy rule base and the corresponding FAMM for the **angle and angular velocity** vectors of the inverted pendulum-balancing problem

1. IF pole is leaning to the left AND pole is dropping to the left THEN largely push cart to the left

2. IF pole is leaning to the left AND pole is not moving THEN slightly push cart to the left

3. IF pole is leaning to the left AND pole is moving to the right THEN don't push the cart

…. and so on, and so forth

……..

Input θ: {N, ZE, P}={leaning to the left, centre, leaning to the right}

Input θ': {N, ZE, P}={moving to the left, still, moving to the right}

F: {PL, PS, ZE, NS, NL}={Large push to the right, small push to the right, don't push, small push to the left, Large push to the left**}**

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Angle vs. Angular Velocity (FAMM 2)

If the pole angle is too big or changing too quickly, then regardless of the location of the cart on the cart path, push the cart towards the direction it is leaning to.

3 x 3 FAMM

			θ	
		N	ZE	P
	N	NL	NM	ZE
θ'	ZE	NS	ZE	PS
	P	ZE	PM	PL

Input θ: {N, ZE, P}={leaning to the left, centre, leaning to the right}

Input θ': {N, ZE, P}={moving to the left, still, moving to the right}

F: {PL, PS, ZE, NS, NL}={Large push to the right, small push to the right, don't push, small push to the left, Large push to the left**}**

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Fuzzy Rule

If (θ is NEGATIVE **and** θ' is **NEGATIVE**)

Then NEGATIVELY_LARGE_FORCE.

$F_{SMALL}(\theta')$ = degree of membership of the given **ANGULAR VELOCITY** in the Fuzzy Set NEGATIVE

Could be a constant or another Membership Function

$F_{NEGATIVE}(\theta)$ = degree of membership of the given **ANGLE** in the Fuzzy Set NEGATIVE

Input θ: {N, ZE, P}={leaning to the left, centre, leaning to the right}

Input θ': {N, ZE, P}={dropping to the left, still, dropping to the right}

F: {PL, PS, ZE, NS, NL}={Large push to the right, small push to the right, don't push, small push to the left, Large push to the left**}**

(FAMM 1) and (FAMM 2)

The final output can be calculated by combining all the rule outputs from the two FAMMS, into one big centre of mass formula.

			X	
		N	ZE	P
	N	PL	PS	ZE
X’	ZE	PS	ZE	NS
	P	ZE	NS	NL

FAMM 1

			θ	
		N	ZE	P
	N	NL	NM	ZE
θ’	ZE	NS	ZE	PS
	P	ZE	PM	PL

FAMM 2

Final output

Force (magnitude and direction) for controlling the Inverted Pendulum