Q:1 [10-points]

a) True
b) III (0)
c) II $(-y/x)$
d) I $(\alpha\beta)^{-1}$
e) II (∞) -2
f) IV $\left(\frac{-16}{3125}\right)$
g) III $\left(\int_{-\sqrt{2}}^{\sqrt{2}}(8-2x^4)\,dx\right)$
h) IV $\left(\frac{1}{24}\right)$
i) III $\left[x\sin^{-1}(3x)+\sqrt{1-9x^2}+C\right]$
j) I $\left[A=\lim_{n\to\infty}\sum_{k=1}^{\infty 3} f(x_k^*)\,\Delta x\right]$

$(\cos y-1)y'=y-1$
$y'\cos y-y'=y-1$
$1+y'\cos y=y+y'$

Q # 2 (05)

Sol:- $x+\sin y=xy$

$1+\cos y\frac{dy}{dx}=x\frac{dy}{dx}+y(1)$

$\cos y\frac{dy}{dx}-x\frac{dy}{dx}=y-1$

$\frac{dy}{dx}(\cos y-x)=y-1$

$\boxed{\frac{dy}{dx}=\frac{y-1}{\cos y-x}}$ (2.5) $\cos y-x$ (1)

Now, $\frac{d^2y}{dx^2}=\frac{(\cos y-x)\left(\frac{dy}{dx}\right)-(y-1)\left(-\sin y\frac{dy}{dx}-1\right)}{(\cos y-x)^2}$ 3/10

$\frac{d^2y}{dx^2}=\frac{\cancel{(\cos y-x)}\frac{(y-1)}{\cancel{(\cos y-x)}}+\sin y(y-1)\frac{dy}{dx}+y-1}{(\cos y-x)^2}$

$$\frac{d^2y}{dx^2} = \frac{(y-1) + \sin y \frac{(y-1)^2}{(\cos y - x)} + (y-1)}{(\cos y - x)^2}$$

$$= \frac{(\cos y - x)(y-1) + \sin y (y-1)^2 + (y-1)(\cos y - x)}{(\cos y - x)^3}$$

$$= (y-1)\left[\cos y - x + y\sin y - \sin y + \cos y - x\right]$$

$$2\cos y + 2x + \sin y\,(y-1)$$

+2.5

Q, $f(x) = \cot\left(\frac{\csc 2x}{x^3+5}\right)$ +3

$$f'(x) = -\csc^2\left(\frac{\csc 2x}{x^3+5}\right) \cdot \frac{d}{dx}\left(\frac{\csc 2x}{x^3+5}\right)$$

$$= -\csc^2\left(\frac{\csc 2x}{x^3+5}\right) \cdot \left[\frac{(x^3+5)(-\csc 2x \cdot \cot 2x) \cdot 2 - \csc 2x (3x^2)}{(x^3+5)^2}\right]$$

$$= -\csc^2\left(\frac{\csc 2x}{x^3+5}\right)\left[\frac{-2(x^3+5)\csc 2x \cdot \cot 2x - 3x^2 \csc 2x}{(x^3+5)^2}\right]$$

+2

Easy →

Q # 3

ⓐ $\int \frac{dx}{2+\cos x}$

Sol:- Let $t = \tan\left(\frac{x}{2}\right)$ (1 point)

$dt = \sec^2\left(\frac{x}{2}\right) \cdot \frac{1}{2} dx$

$2dt = \sec^2\left(\frac{x}{2}\right) dx$

$dx = \frac{2}{1+t^2} dt$

$\because \cos x = \frac{1-t^2}{1+t^2}$

put all in given eq (3 point Integral)

$\int \frac{1}{2+\frac{1-t^2}{1+t^2}} \cdot \frac{2}{1+t^2} dt$

$= \int \frac{2}{3+t^2} dt$ (1

$= \frac{2}{3} \int \frac{1}{1+\left(\frac{t}{\sqrt{3}}\right)^2} dt$

$\because \int \frac{1}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

$= \frac{2}{\sqrt{3}} \tan^{-1}\left[\frac{1}{\sqrt{3}} \tan\left(\frac{x}{2}\right)\right] + C$ (1 point)

Q: $\int_0^5 \frac{w}{w-2}\,dw$ ©

This is improper integral: (1 point for ...)

$$= \int_0^2 \frac{w}{w-2}\,dw + \int_2^5 \frac{w}{w-2}\,dw \quad \text{— (A)}$$

$$\because \int_0^2 \frac{w}{w-2}\,dw = \lim_{b\to 2^-}\int_0^b \frac{w}{w-2}\,dw$$

$$= \lim_{b\to 2^-}\int_0^b \frac{w-2+2}{w-2}\,dw$$

$$= \lim_{b\to 2^-}\int_0^b \left[1 + \frac{2}{w-2}\right]dw$$

$$= \lim_{b\to 2^-}\Big|w + 2\ln(w-2)\Big|_0^b \quad \text{2 point}$$

$$= \lim_{b\to 2^-}\left[b + 2\ln(b-2) - \{0 + 2\ln(2)\}\right] \quad \text{1 point}$$

$$= \infty \quad \text{1 point}$$

so the integral is divergent.

(c) $\int \frac{5}{x^3+2x^2+5x}\,dx$

Sol:-

Consider

$$\frac{5}{x(x^2+2x+5)} = \frac{A}{x} + \frac{Bx+C}{(x^2+2x+5)}$$

2 points

$A+B=0$

$2A+C=0$

$5=5A$

$\boxed{A=1}$, $\boxed{B=-1}$, $\boxed{C=-2}$

$$\int \frac{5}{x(x^2+2x+5)}dx = \int\left[\frac{1}{x} + \frac{-1x-2}{(x^2+2x+5)}\right]dx$$

$$= \int\left(\frac{1}{x} - \frac{x+2}{x^2+2x+5}\right)dx$$

$$= \ln x - \int \frac{x+2}{(x+1)^2+4}dx$$

Let $y = x+1 \Rightarrow x = y-1$

$dy = dx$

$$= \ln x - \int \frac{y+1}{y^2+4}dy$$

2 points

$$= \ln x - \left[\int \frac{y}{y^2+4}dy + \int \frac{1}{y^2+4}dy\right]$$

$$= \ln x - \left[\frac{1}{2}\ln(y^2+4) + \frac{1}{2}\tan^{-1}\left(\frac{y}{2}\right)\right] + C$$

$$= \ln x - \frac{1}{2}\ln[(x+1)^2+4] - \frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + C$$

(d) $\int \frac{1}{2x^2+4x+7}\,dx$

$= \frac{1}{2}\int \frac{1}{x^2+2x+\frac{7}{2}}\,dx$

$= \frac{1}{2}\int \frac{1}{(x^2+2x+1)+\frac{7}{2}-1}\,dx$

$= \frac{1}{2}\int \frac{1}{(x+1)^2+\frac{5}{2}}\,dx \longrightarrow$ 1 point

Let $u = x+1$, $du = dx$ $\longrightarrow$ 1 point

$= \frac{1}{2}\int \frac{du}{u^2+\left(\frac{\sqrt{5}}{\sqrt{2}}\right)^2}$

$\because \int \frac{du}{u^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)$

$= \frac{1}{2}\times\frac{1}{\sqrt{\frac{5}{2}}}\tan^{-1}\left(\frac{u}{\sqrt{\frac{5}{2}}}\right)+C$ 2 points

$= \frac{\sqrt{2}}{2\sqrt{5}}\tan^{-1}\left(\frac{\sqrt{2}\,u}{\sqrt{5}}\right)+C$

$= \frac{\sqrt{2}}{2\sqrt{5}}\tan^{-1}\left(\frac{\sqrt{2}\,(x+1)}{\sqrt{5}}\right)+C$

$= \frac{\sqrt{10}}{10}\tan^{-1}\left(\frac{10}{5}(x+1)\right)+C$ 1 point

Q#4

$R(x) = (x-1)^2 e^{3x}$ (a)

Sol:-

$R'(x) = (x-1)^2 \cdot 3e^{3x} + 2(x-1) \cdot e^{3x}$

$= 3(x-1)^2 e^{3x} + 2e^{3x}(x-1)$

$= e^{3x}(x-1)\left[3(x-1)+2\right]$

$= e^{3x}(x-1)(3x-3+2)$ 1 point

$= e^{3x}(x-1)(3x-1)$

For Critical Point:-

$R'(x) = 0$

$e^{3x}(x-1)(3x-1) = 0$

$e^{3x} = 0$ Not possible

$x - 1 = 0$, $\boxed{x = 1}$

$3x - 1 = 0$, $\boxed{x = \frac{1}{3}}$

1.5 points

$-\infty$		$1/3$		1		$+\infty$
	+ve		-ve		+ve	

Increasing: $(-\infty, \frac{1}{3}) \cup (1, +\infty)$

Decreasing: $(\frac{1}{3}, 1)$

2.5 points

$$R'(x) = \underset{u}{e^{3x}}\,\underset{v}{(x-1)(3x-1)}$$

$$R''(x) = e^{3x}\left[(x-1)(3) + (3x-1)(1)\right] + (x-1)(3x-1)\cdot 3e^{3x}$$

$$= e^{3x}\left[3x-3+3x-1+3(x-1)(3x-1)\right]$$

$$= e^{3x}\left[3x-3+3x-1+(3x-3)(3x-1)\right]$$

$$= e^{3x}\left[\cancel{3x}-\cancel{3}+\cancel{3x}-1+9x^2-\cancel{3x}-9x+\cancel{3}\right]$$

$$= e^{3x}\left[9x^2-6x-1\right]$$

1 point

Inflection Point:-

$$f''(x) = 0 \quad , \quad f''(x) = \text{undefined}$$

(Not possible)

$$e^{3x}(9x^2-6x-1) = 0$$

$e^{3x} = 0$ — Not exist

$$9x^2-6x-1 = 0$$

$$x = \frac{1+\sqrt{2}}{3} = 0.8$$

$$x = \frac{1-\sqrt{2}}{3} = -0.138$$

(1.5 points

+ve | -ve | +ve

$-\infty$ | -0.13 | 0.8 | $+\infty$

Concave up:- $(-\infty, -0.13) \cup (0.8, +\infty)$ 2.5 points

Concave down:- ~~(0.8, +∞)~~ $(-0.13, 0.8)$

$3e^{3t}$

4 b

$f(t) = 2t + e^{-2t}$ is continuous fun on $[-2, 3]$
& diff at $(-2, 3)$.

$f(t) = 2t + e^{-2t}$
$f(-2) = -4 + e^{4}$
$f(3) = 6 + e^{-6}$

2 points

By Mean Value Theorem

$\therefore f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$2 - 2e^{-2c} = \dfrac{6 + e^{-6} + 4 - e^{4}}{3 + 2}$

$5(2 - 2e^{-2c}) = 10 + e^{-6} - e^{4}$

$\cancel{10} - 10e^{-2c} = \cancel{10} + e^{-6} - e^{4}$

$e^{-2c} = \dfrac{e^{4} - e^{-6}}{10}$

$\boxed{c = -0.8486}$

3 points

4c $\lim_{x\to 0^+}\left(\frac{1}{x^2}-\frac{1}{\tan x}\right)$; $\infty-\infty$

$= \lim_{x\to 0^+}\left(\frac{\tan x - x^2}{x^2\tan x}\right)$; $\left(\frac{0}{0}\right)$ 1 point

$= \lim_{x\to 0^+}\frac{\sec^2 x - 2x}{x^2\sec^2 x + 2x\tan x}$ 2.5

$= \infty$ 2.5 points

Q # 5

a

2 points

$\frac{d\theta}{dt} = ?$ $y = 200$

$\frac{dy}{dt} = -15$

As we know that:

$\tan\theta = \frac{y}{500}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{dy}{dt}\frac{1}{500}$ 2points

$$\frac{d\theta}{d\cancel{t}} = \frac{dy}{dt}\,\frac{\cos^2\theta}{500}$$

$$= \frac{(-15)}{500}\cos^2(0.38051)$$

$$\boxed{\frac{d\theta}{dt} = -0.02586}$$

1 point

$\tan\theta = \frac{200}{500}$

$\theta = \tan^{-1}\left(\frac{2}{5}\right)$

$\theta = 0.38051$

5b $y = x^4 + \ln(x+10)$, $y = x^3 + \ln(x+10)$

Sol:-

$x^4 + \cancel{\ln(x+10)} = x^3 + \cancel{\ln(x+10)}$

$x^4 - x^3 = 0$

$x^3(x-1) = 0$

$x = 0, 1$

1 point

$0 \quad \frac{x^3 + \ln(x+10)}{x^4 + \ln(x+10)} \quad 1$

1 point

$$A = \int_0^1 \left[x^3 + \cancel{\ln(x+10)} - \left(x^4 + \cancel{\ln(x+10)}\right)\right] dx$$

0.5 pt

$$= \int_0^1 x^3 - x^4 \, dx$$

$$= \left(\frac{x^4}{4} - \frac{x^5}{5}\right)\Big|_0^1$$

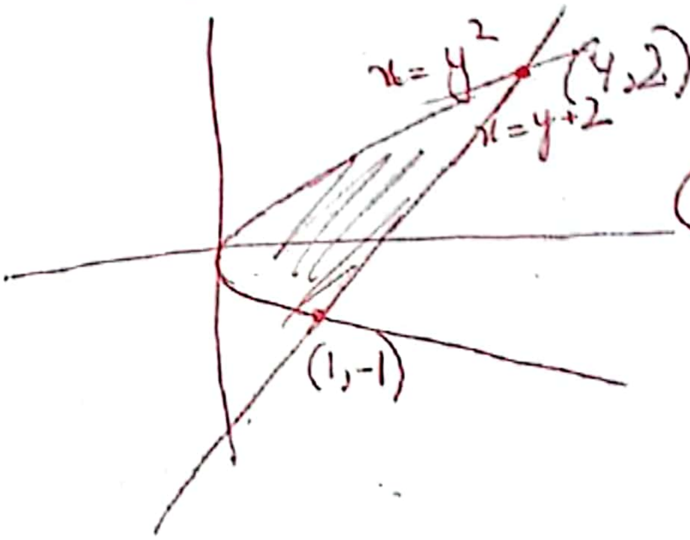
$$\boxed{A = \frac{1}{20}} = 0.05$$

2.5 points

5c

a

$x = y^2$ & $x = y + 2$



1 point

$\therefore\ y^2 = y + 2$

$y^2 - y - 2 = 0$

$y = 2, -1$

$x = 4 \quad x = 1$

1 point

$$V = \pi \int_{-1}^{2} \left[(y+2)^2 - y^4 \right] dy$$

0.5 point

$$= \int_{-1}^{2} \left(y^2 + 2y + 4 - y^4 \right) dy$$

$$= \left| \frac{y^3}{3} + y^2 + 4y - \frac{y^5}{5} \right|_{-1}^{2}$$

$$\boxed{V = \frac{72\pi}{5}}$$

14.4π

2.5 points

Q. 6

(a) $\frac{-4}{13}, \frac{4}{26}, \frac{-4}{39}, \frac{4}{52}, -\frac{4}{65}$

$$\boxed{a_n = \frac{(-1)^n 4}{13n}}$$ 2 points

For Convergent / Divergent

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{(-1)^n 4}{13n}$$

$$= \frac{4}{13} \lim_{n\to\infty} \frac{(-1)^n}{n}$$ 2 points

$$= \frac{4}{13}(0)$$

$$= 0 \text{ ; (Convergent)}$$ 1 point

(b) $$\sum_{k=1}^{\infty} \left[\frac{8}{6^{k+1}} + \frac{3}{4^{k+1}}\right]$$

$$= \sum_{k=1}^{\infty} \frac{8}{6^{k+1}} + \sum_{k=1}^{\infty} \frac{3}{4^{k+1}}$$

$$= \left\{\frac{8}{6^2} + \frac{8}{6^3} + \frac{8}{6^4} + \cdots\right\} + \left\{\frac{3}{4^2} + \frac{3}{4^3} + \frac{3}{4^4} + \cdots\right\}$$

$\frac{8}{6^3} \div \frac{8}{6^2}$

$\frac{8}{6^3} \times \frac{6^2}{8}$

$\frac{1}{6}$

$\because r = \frac{1}{6} < 1$ (Conver) 2 points $\because r = \frac{1}{4} < 1$ (Conver)

$$= \frac{a}{1-r} \quad \frac{8/36}{1-1/6} \quad \frac{8}{36} \div \left(\frac{5}{6}\right) = \frac{8}{36} \times \frac{6}{5} = \frac{8}{30} = \frac{4}{15} \qquad \frac{3/4^2}{1-1/4} \qquad \frac{3}{16} \div \frac{3}{4} \quad \frac{3}{16} \times \frac{4}{3}$$

$$= \frac{8}{\ } \quad \frac{4}{\ } \text{ 2 point} \quad \frac{3}{\ } \qquad = \frac{1}{4}$$

$\frac{4}{15} + \frac{1}{45n}$ $\frac{31}{60}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{12}$ 1 point (iii) $\frac{31}{60}$

2.0833

6 (i) $\sum_{k=1}^{\infty} \frac{\ln k}{k\sqrt{k}}$ 1 point (Integral test)

$\sum_{k=2}^{\infty} \frac{\ln k}{k\sqrt{k}}$

$\Rightarrow \int_2^{+\infty} \frac{\ln x}{x^{3/2}}\,dx$

$\Rightarrow \lim_{b\to\infty} \int_2^{b} \frac{\ln x}{x^{3/2}}\,dx$ 2 points

$\Rightarrow \lim_{b\to\infty} \int_2^{b} x^{-3/2} \ln x \,dx$

$\Rightarrow \lim_{b\to\infty} \left[\frac{-2\ln x}{x^{1/2}} - \frac{4}{x^{1/2}} \right]_2^b$

1 point

$\Rightarrow \sqrt{2}(\ln 2 + 2)$ (Convergent)

(iii) $\sum_{k=1}^{\infty} \left[\frac{(k+1)!}{5^{-k}\,k!}\right]$

Ratio test 1 point

$\therefore \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$

⋮ 2 point

$\frac{1}{5} < 1$ (Convergent) 1 point

$\frac{3}{4}\big|\frac{4,15}{4,5}$ 5 4

$\frac{16+15}{60}$

$\frac{31}{60}$

(ii) $\sum_{k=1}^{\infty} \left(\frac{k^{4/3}}{8k^2+5k+1}\right)$ a_n b_n

Comparison Test $\frac{k^{4/3}}{8k^2+5k+1} > \frac{k^{4/3}}{8k^2}$

1 point

$\frac{4}{3} - 2$

$\frac{4-6}{3} = -\frac{2}{3}$

$\frac{k^{4/3}}{8k^2+5k+1} > \frac{1}{8k^{2/3}}$ 2 points

$\therefore b_n$ is p-series $p = 2/3 < 1$ (divergent)

So the original series is also divergent

1 point