National University of Computer & Emerging Sciences, Karachi
FAST School of Computing
Fall-2023, Final Exam
26th December 2023, 09:00 AM - 12:00 Noon

Course Code: MT-1003	**Course Name: Calculus and Analytical Geometry**
Instructors Name: Ms. Urooj / Ms. Alishba Tariq / Ms. Fareeha sultan / Mr. Nadeem Khan / Mr. Mairaj Ahmed	
Student Roll No: 23K-0727	**Section No:** 1J

Instructions:

- Attempt all questions. There are **06 Questions and 04 pages.**
- Solve the paper according to the sequence given and preform all the necessary steps.
- Graphical Calculator is not allowed.
- Return the question paper with the answer copy.

Time: 180 minutes **Max Marks:100**

Question 01: **[CLO-3]** **[10]**

a. **T/F:** The improper integral $\int_1^\infty \frac{1}{x^2}\,dx$ represents a finite area

b. The function $f(x) = x^{\frac{5}{11}}$ has a point of inflection with an x-coordinate of

I) $\frac{5}{11}$ II) $-\frac{5}{11}$ III) 0 IV) Does not exist

c. First derivative of $xy = 90$ is equal to.

I) $\frac{dy}{dx} = \frac{y}{x}$ II) $\frac{dy}{dx} = -\frac{y}{x}$ III) $\frac{dy}{dx} = xdy + ydx$ IV) $\frac{dy}{dx} = \frac{x}{y}$

d. If a is a constant, what is the derivative of $y = x^a$?

I) ax^{a-1} II) $(a-1)^x$ III) x^{a-1} IV) ax

e. Evaluate $\lim_{x\to 2^-} \frac{x^2-2x}{x^2-5x+6}$

I) $-\infty$ II) -2 III) $\frac{2}{5}$ IV) Does not exist

f. Given that $f(1) = 5$, $f'(1) = 4$ and $g(x) = [f(x)]^{-4}$ find $g'(1)$

I) 2 II) $\frac{93}{31}$ III) $\frac{-37}{4}$ IV) $\frac{-16}{3125}$

g. The curves $y = x^4 - 3$ and $y = -x^4 + 5$ enclosed an area. Set up a definite integral which calculates the area of this region.

I) $\int_{-\sqrt{2}}^{\sqrt{2}} 2\,dx$ II) $\int_{-1}^{1} 2\,dx$ III) $\int_{-\sqrt{2}}^{\sqrt{2}} (8-2x^4)dx$ IV) $\int_{-1}^{1} (8-2x^4)dx$

h. If $f(x) = \sqrt{1+\sqrt{1+x}}$ then $f'(8) =$?

I) $\frac{1}{12}$ II) $\frac{1}{8}$ III) $\frac{1}{9}$ IV) $\frac{1}{24}$

i. If $f(x) = sin^{-1}(3x)$ then $\int f(x)dx =$?

I) $xsin^{-1}(3x) + \frac{\sqrt{1-9x^2}}{9} + c$ II) $xcos^{-1}(3x) + \frac{\sqrt{1-9x^2}}{9} + c$

III) $xsin^{-1}(3x) + \frac{\sqrt{1-9x^2}}{3} + c$ IV) $xsin^{-1}(3x) - \frac{\sqrt{1-9x^2}}{9} + c$

j. If the function f is continuous on $[a, b]$ and if $f(x) \geq 0$ for all x in $[a, b]$, then the area A under the curve $y = f(x)$ over the interval $[a, b]$ is defined as ______________, with x_k^* as the right endpoint of each subinterval

I) $A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$ II) $A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_{k-1})\Delta x$

III) $A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$ V) $A = \lim_{n\to\infty} \sum_{k=1}^{n} f[\frac{1}{2}(x_{k-1} + x_k)]\Delta x$

Question 02: [CLO-3] [5+5=10]

a. Find $\frac{d^2y}{dx^2}$ by using implicit differentiation.

$$x + \sin y = xy$$

b. Find the derivative of

$$f(x) = \cot\left[\frac{\operatorname{cosec}2x}{x^3+5}\right]$$

Question 03: [CLO-3] [5+5+5+5=20]

Evaluate the integral of the following

a. $\displaystyle\int \frac{dx}{2+\cos x}$

b. $\displaystyle\int_0^5 \frac{w}{w-2}\,dw$

c. $\displaystyle\int \frac{5}{x^3+2x^2+5x}\,dx$

d. $\displaystyle\int \frac{1}{2x^2+4x+7}\,dx$

Question 04: [CLO-4] [10+5+5=20]

a. A study on optimizing revenue function R from a website is,

$$R(x) = (x-1)^2 e^{3x}$$

where x measures the proportion of the total bandwidth requested by a customer. Find intervals in which the $R(x)$ is decreasing, increasing, concave up and concave down.

b. Show that the function $f(t) = 2t + e^{-2t}$ satisfies the hypotheses of the Mean-Value Theorem over the interval $[-2, 3]$ and find all values of c in the interval $(-2, 3)$ at which the tangent line to the graph of $f(t)$ is parallel to the secant line joining the points $(-2, f(-2))$ and $(3, f(3))$.

c. Use L-Hopital's rule to compute the limit

$$\lim_{x\to 0^+}\left[\frac{1}{x^2} - \frac{1}{\tan x}\right]$$

Question 05: [CLO-4] [5+5+5=15]

a. The angle of elevation is the angle formed by a horizontal line and a line joining the observer's eye to an object above the horizontal line. A person is 500 feet way from the launch point of a hot air balloon. The hot air balloon is starting to come back down at a rate of 15 ft/sec. At what rate is the angle of elevation, θ, changing when the hot air balloon is 200 feet above the ground.

b. Find the area of the region bounded by the curves

$$y = x^4 + \ln(x+10) \text{ and } y = x^3 + \ln(x+10)$$