Trigonometric Formulae

Fundamental Identities

1. $\sin^2 x + \cos^2 x = 1$
2. $1 + \tan^2 x = \sec^2 x$
3. $1 + \cot^2 x = \cos ec^2 x$

Trigonometric Functions of Sum and Difference

4. $\sin(x + y) = \sin x \cos y + \cos x \sin y$
5. $\sin(x - y) = \sin x \cos y - \cos x \sin y$
6. $\cos(x + y) = \cos x \cos y - \sin x \sin y$
7. $\cos(x - y) = \cos x \cos y + \sin x \sin y$
8. $\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$
9. $\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

Trigonometric Functions of Multiple Angles

10. $\sin 2x = 2 \sin x \cos x = \dfrac{2 \tan x}{1 + \tan^2 x}$
11. $\cos 2x = \cos^2 x - \sin^2 x = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$
12. $\tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x}$
13. $\sin 3x = 3 \sin x - 4 \sin^3 x$
14. $\cos 3x = 4 \cos^3 x - 3 \cos x$
15. $\tan 3x = \dfrac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$

Formulae for Conversion of Product into Sum and Difference

16. $\sin x \cos y = \dfrac{1}{2}[\sin(x - y) + \sin(x + y)]$
17. $\cos x \cos y = \dfrac{1}{2}[\cos(x - y) + \cos(x + y)]$
18. $\sin x \sin y = \dfrac{1}{2}[\cos(x - y) - \cos(x + y)]$

Formulae for Conversion of Powers into Sum and Difference

19. $\sin^2 x = \dfrac{1 - \cos 2x}{2}$
20. $\cos^2 x = \dfrac{1 + \cos 2x}{2}$
21. $\sin^3 x = \dfrac{3 \sin x - \sin 3x}{4}$
22. $\cos^3 x = \dfrac{3 \cos x + \cos 3x}{4}$

Hyperbolic Functions

Hyperbolic Functions

1. $\sinh x = \dfrac{e^x - e^{-x}}{2}$

2. $\cosh x = \dfrac{e^x + e^{-x}}{2}$

3. $\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

Hyperbolic Functions

4. $\sinh^{-1} x = \log(x + \sqrt{x^2 + 1})$

5. $\cosh^{-1} x = \log(x + \sqrt{x^2 - 1})$

6. $\tanh^{-1} x = \dfrac{1}{2}\log\left(\dfrac{1+x}{1-x}\right)$

Fundamental Identities

7. $\cosh^2 x - \sinh^2 x = 1$

8. $1 - \tanh^2 x = \sec h^2 x$

9. $\coth^2 x - \cos ech^2 x = 1$

Differentiation Formulae

1. $\dfrac{d}{dx}(k) = 0$ where 'k' is any constant

2. $\dfrac{d}{dx}(x^n) = nx^{n-1}$

3. $\dfrac{d}{dx}(\sqrt{x}) = \dfrac{1}{2\sqrt{x}}$

4. $\dfrac{d}{dx}(\log x) = \dfrac{1}{x}$

5. $\dfrac{d}{dx}(e^x) = e^x$

6. $\dfrac{d}{dx}(a^x) = a^x \log a,$ if a > 0

7. $\dfrac{d}{dx}(\sin x) = \cos x$

8. $\dfrac{d}{dx}(\cos x) = -\sin x$

9. $\dfrac{d}{dx}(\tan x) = \sec^2 x$

10. $\dfrac{d}{dx}(\cot x) = -\cos ec^2 x$

11. $\dfrac{d}{dx}(\sec x) = \sec x \tan x$

12. $\dfrac{d}{dx}(\cos ecx) = -\cos ecx \cot x$

13. $\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$

14. $\dfrac{d}{dx}(\cot^{-1} x) = -\dfrac{1}{1+x^2}$

15. $\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}$

16. $\dfrac{d}{dx}(\cos^{-1} x) = -\dfrac{1}{\sqrt{1-x^2}}$

17. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

18. $\frac{d}{dx}(\cos ec^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$

19. $\frac{d}{dx}(\sinh x) = \cosh x$

20. $\frac{d}{dx}(\cosh x) = \sinh x$

21. $\frac{d}{dx}(\tanh x) = \sec h^2 x$

22. $\frac{d}{dx}(\sec hx) = \sec hx \tanh x$

23. $\frac{d}{dx}(\cos echx) = -\cos echx \coth x$

24. $\frac{d}{dx}(\coth x) = -\cos ech^2 x$

25. $\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$

26. $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$

27. $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}, \quad |x| < 1$

28. $\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}, \quad |x| > 1$

29. $\frac{d}{dx}(\cos ech^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$

30. $\frac{d}{dx}(\sec h^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$

Rules of Diffrentiation:

If u & v are differentiable functions of x then

1. $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$

2. $\frac{d}{dx}(\frac{u}{v}) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

3.Chain Rule: If y is a differentiable functions of u *where* u is differentiable functions of x then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

By using this rule derivatives of composite functions can be obtained directly as follows

1. $\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1} f'(x)$ where dash denotes derivative w. r. t. ' x '

2. $\frac{d}{dx}\sqrt{f(x)} = \frac{1}{2\sqrt{f(x)}} f'(x)$

3. $\frac{d}{dx}[\log f(x)] = \frac{1}{f(x)} f'(x)$

4. $\frac{d}{dx} e^{f(x)} = e^{f(x)} f'(x)$

5. $\frac{d}{dx} a^{f(x)} = a^{f(x)} f'(x) \log a, \quad$ if a > 0

6. $\frac{d}{dx}\sin[f(x)] = \cos[f(x)] f'(x)$

7. $\frac{d}{dx}\cos[f(x)] = -\sin[f(x)] f'(x)$

And so on.

Integration Formulae

1. $\int k\,dx = kx + c$
2. $\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad \text{if n} \neq -1$
3. $\int \frac{1}{x} dx = \log x + c$
4. $\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c$
5. $\int e^x dx = e^x + c$
6. $\int a^x dx = \frac{a^x}{\log a} + c, \quad \text{if a} > 0$
7. $\int \sin x dx = -\cos x + c$
8. $\int \cos x dx = \sin x + c$
9. $\int \tan x dx = \log \sec x + c$
10. $\int \cot x dx = \log \sin x + c$
11. $\int \sec x dx = \log(\sec x + \tan x) + c \ = \log \tan(\frac{x}{2} + \frac{\pi}{4}) + c$
12. $\int \cos ec x dx = \log(\cos ec x - \cot x) + c = \log \tan \frac{x}{2} + c$
13. $\int \sec^2 x dx = \tan x + c$
14. $\int \cos ec^2 x dx = -\cot x + c$
15. $\int \sec x \tan x dx = \sec x + c$
16. $\int \cos ec x \cot x dx = -\cos ec x + c$
17. $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1}(\frac{x}{a}) + c$
18. $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log(\frac{a+x}{a-x}) + c = \frac{1}{a} \tanh^{-1}(\frac{x}{a}) + c \ , \quad \text{x}^2 < \text{a}^2$
19. $\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log(\frac{x-a}{x+a}) + c = -\frac{1}{a} \coth^{-1}(\frac{x}{a}) + c , \quad \text{x}^2 > \text{a}^{2.}$
20. $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}(\frac{x}{a}) + c$
21. $\int \frac{1}{\sqrt{x^2 - a^2}} dx = \log(x + \sqrt{x^2 - a^2}) + c = \cosh^{-1}(\frac{x}{a}) + c$
22. $\int \frac{1}{\sqrt{x^2 + a^2}} dx = \log(x + \sqrt{x^2 + a^2}) + c = \sinh^{-1}(\frac{x}{a}) + c$
23. $\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2}) + \frac{a^2}{2} \sin^{-1}(\frac{x}{a}) + c$
24. $\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2}) - \frac{a^2}{2} \log(x + \sqrt{x^2 - a^2}) + c$
25. $\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2}) + \frac{a^2}{2} \log(x + \sqrt{x^2 + a^2}) + c$
26. $\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + c = \frac{e^{ax}}{\sqrt{a^2 + b^2}} \sin(bx - \tan^{-1} \frac{b}{a}) + c$
27. $\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + c = \frac{e^{ax}}{\sqrt{a^2 + b^2}} \cos(bx - \tan^{-1} \frac{b}{a}) + c$

28. $\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + c, \quad \text{if n} \neq -1$

29. $\int \frac{f'(x)}{f(x)} dx = \log f(x) + c$

30. $\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c$

31. $\int e^{f(x)} f'(x)dx = e^{f(x)} + c$

32. $\int a^{f(x)} f'(x)dx = \frac{a^{f(x)}}{\log a} + c, \quad \text{if a} > 0$

33. $\int [\sin f(x)] f'(x)dx = -\cos f(x) + c$

34. $\int [\cos f(x)] f'(x)dx = \sin f(x) + c$

35. $\int [\tan f(x)] f'(x)dx = \log \sec f(x) + c$

36. $\int [\cot f(x)] f'(x)dx = \log \sin f(x) + c$

37. $\int [\sec^2 f(x)] f'(x)dx = \tan f(x) + c$

38. $\int [\cos ec^2 f(x)] f'(x)dx = -\cot f(x) + c$

39. $\int e^x [f(x) + f'(x)]dx = e^x f(x) + c$

40. General Rule of Integration by Parts

$$\int uvdx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + ...$$

where dashes denotes the order of derivative & lower suffixes denotes the order of integration.