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Optimized adiabatic passage with dephasing

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We study adiabatic population transfer with dephasing in two-level models driven by a chirped driving field. We show that the population transfer is maximized when the dynamics follows specific ellipses as trajectories in the parameter space. We determine the optimal parameters and estimate the losses in a closed form. These estimates show a similar robustness as for the standard lossless adiabatic processes with respect to variations of the parameters.

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I. INTRODUCTION

Dynamics by adiabatic passage is now widely used, mainly to take advantage of its robustness with respect to fluctuations of the external control fields as well as to the imperfect knowledge of the model [1–7]. Adiabatic processes are defined as a limit of an infinitely slow dynamics. The long duration of the processes can be expected to be detrimental due to dissipation.

The decoherence produced by a pure dephasing effect, of rate Γ, induces the randomization of the relative phases of the coherent superpositions of states [8]. For a process of characteristic duration T, the loss is of order $T\Gamma$. Such a loss is detrimental for a process of complete population transfer, since the dynamics goes necessarily through a transient superposition of states. *Dephasing effects appear thus only transiently during a process of complete population transfer.*

The effects of dephasing can be reduced using shorter processes, but this short duration implies the apparition of nonadiabatic effects. The evolution times required to minimize the two type of losses, nonadiabatic and dephasing, are not compatible, and an optimal compromise has to be found. This is the goal of this paper.

We show specific *robust* trajectories in the adiabatic parameter space that minimize the total losses in a class of models for which the trajectories are defined as ellipses and calculate an estimate of the losses.

We study the problem of adiabatic passage with the Lindblad formalism [8,9], taking into account pure dephasing decoherence effects.

In Sec. II we recall some results for the nondissipative regime. We present the dissipative model in Sec. III. Section IV is devoted to the formulation of adiabatic passage for a dissipative Hamiltonian. We present the optimization of adiabatic passage in Sec. V, before concluding in Sec. VI.

II. NONDISSIPATIVE MODEL

In this section we define the lossless model and the class of the time-dependent parameters that will be used. We also recall the notion of optimal adiabatic passage.

We can generically describe population transfer by adiabatic passage in a lossless two-state model (with real parameters) by the Hamiltonian

$$H(s) = \frac{1}{2}\begin{bmatrix} -\Delta(s) & \Omega(s) \\ \Omega(s) & \Delta(s) \end{bmatrix}, \tag{1}$$

with a pulse-shaped coupling (e.g., a laser pulse) of the form

$$\Omega(s) = \Omega_0\Lambda(s), \quad 0 < \Lambda(s) < 1, \quad \Lambda(\pm\infty) = 0, \quad \Lambda(0) = 1 \tag{2}$$

and a the time-dependent monotonic detuning

$$\Delta(s) = \Delta_0 f(s), \quad 0 < |f(s)| < 1, \quad f(\pm\infty) = \pm 1, \quad f(0) = 0, \tag{3}$$

using without loss of generality positive amplitudes $\Omega_0, \Delta_0 > 0$ and a normalized time $s = t/T$, where T is the characteristic pulse length.

We will consider a class of models with

$$\Lambda^2(s) + f^2(s) = 1, \tag{4}$$

which define ellipses of semiaxes Ω_0 and Δ_0 as trajectories in the parameter space (Ω, Δ).

We diagonalize the Hamiltonian (1),

$$R^\dagger H R = \begin{bmatrix} \lambda_+ & 0 \\ 0 & \lambda_- \end{bmatrix}, \tag{5}$$

using the transformation

$$R = \begin{bmatrix} \cos\theta/2 & -\sin\theta/2 \\ \sin\theta/2 & \cos\theta/2 \end{bmatrix}, \quad \tan\theta = -\frac{\Omega}{\Delta}, \quad \theta \in [0, \pi[\tag{6}$$

associated with the instantaneous eigenvectors

$$\psi_+ = \begin{bmatrix} \cos\theta/2 \\ \sin\theta/2 \end{bmatrix}, \quad \psi_- = \begin{bmatrix} -\sin\theta/2 \\ \cos\theta/2 \end{bmatrix} \tag{7}$$

and the eigenvalues

$$\lambda_\pm = \pm\frac{1}{2}\lambda, \quad \lambda = \sqrt{\Omega^2 + \Delta^2}. \tag{8}$$

The Schrödinger equation ($\hbar = 1$) $i\frac{\partial}{\partial s}\phi = TH\phi$, with the scaled time $s = t/T$, expressed in terms of the adiabatic states $\tilde{\phi} = R^\dagger\phi$, becomes

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$$i\frac{\partial}{\partial s}\tilde{\phi}=\begin{bmatrix} T\lambda_+ & i\dot{\theta}/2 \\ -i\dot{\theta}/2 & T\lambda_- \end{bmatrix}\tilde{\phi}, \tag{9}$$

with the off-diagonal nonadiabatic coupling

$$\frac{\dot{\theta}}{2}=\frac{1}{2}\frac{\Omega\dot{\Delta}-\dot{\Omega}\Delta}{\Delta^2+\Omega^2}. \tag{10}$$

The overdot denotes derivatives with respect to the scaled time s. In the adiabatic limit, defined as $T\to\infty$, the nonadiabatic coupling can be neglected and the dynamics follows the adiabatic state(s) connected to the initial state.

For a process of finite duration, the condition of *local* adiabaticity $|\dot{\theta}|\ll|\lambda_+-\lambda_-|$ reads

$$|\Omega\dot{\Delta}-\dot{\Omega}\Delta|\ll T\lambda^3. \tag{11}$$

One can derive a rough condition for adiabaticity around the avoided crossing, occurring at $s=0$, by linearizing the detuning $\Delta(s)\sim\Delta_0 s$, considering the peak amplitude Ω_0 for the field and assuming the derivatives $\dot{\Omega}\sim\Omega_0/T$, $\dot{\Delta}\sim\Delta_0/T$, which leads to the condition

$$T\Delta_0\ll(T\Omega_0)^2. \tag{12}$$

Efficient population transfer by adiabatic passage occurs when $\Delta_0\sim\Omega_0$, leading to the well-known condition $T\Omega_0\gg 1$. The resonant case $\Delta_0\ll\Omega_0$ leads to a lifting of quasidegeneracy [23], and the far off-resonant case $\Delta_0\gg\Omega_0$ is dominated by nonadiabatic effects.

Optimal adiabatic passage has been defined as the cancellation of the nonadiabatic losses at the end of the process in the Dykhne-Davis-Pechukas (DDP) formula [10]. Such a formulation allows one to determine the *global* (i.e., at the end of the process) nonadiabatic losses in the adiabatic limit, when the couplings are analytic. In this case, these losses are exponentially small—i.e., beyond any power of T—consistent with the superadiabatic analysis [11,12]. A discontinuity of the nth derivative of the couplings corresponds to global nonadiabatic losses of the order $O(T^{-n})$. Optimal adiabatic passage corresponds to the use of *parallel* eigenvalues for the dynamics, since they allow the cancellation of the global nonadiabatic losses of the DDP formula in the adiabatic limit $T\to+\infty$. For the two-state problem, parallel eigenvalues correspond to circular trajectories of radius Ω_0 in the parameter space: $\Omega^2+\Delta^2=\Omega_0^2$. For a process of finite duration T, the global nonadiabatic loss depends on the choice of the level line. It has been numerically shown that already a modest $T\Omega_0$ yields a remarkable efficiency of the adiabatic passage [13].

III. DISSIPATIVE MODEL

The dissipative two-state model is described in terms of the density matrix, which can be represented in the basis $\{\mathbf{I}_2,\boldsymbol{\sigma}_x,\boldsymbol{\sigma}_y,\boldsymbol{\sigma}_z\}$ as $\boldsymbol{\rho}(t)=(\mathbf{I}_2+\rho_x\boldsymbol{\sigma}_x+\rho_y\boldsymbol{\sigma}_y+\rho_z\boldsymbol{\sigma}_z)/2$, with ρ_x, ρ_y, and ρ_z the real coordinates of the vector $\varrho(t)$ on the Bloch sphere. These coefficients can be expressed in terms of the elements ρ_{ij} of the density matrix as $\rho_x=\rho_{+-}+\rho_{-+}$, $\rho_y=i(\rho_{+-}-\rho_{-+})$, and $\rho_z=\rho_{++}-\rho_{--}$. The total population is conserved during the process $\rho_{++}+\rho_{--}=1$, and the population inversion is given by ρ_z. In this basis, the dephasing operator reads $\boldsymbol{\Gamma}_d=\sqrt{\Gamma/2}\boldsymbol{\sigma}_z$ with Γ the dephasing rate, and the Lindblad equation [9] takes the form of the well-known dissipative Bloch equation [14]

$$|\dot{\varrho}(t)\rangle=\mathcal{L}(t)|\varrho(t)\rangle, \tag{13}$$

with

$$\mathcal{L}(t)=\begin{bmatrix} -\Gamma & -\Delta(t) & 0 \\ \Delta(t) & -\Gamma & -\Omega(t) \\ 0 & \Omega(t) & 0 \end{bmatrix},\quad |\varrho\rangle=\begin{bmatrix}\rho_x\\ \rho_y\\ \rho_z\end{bmatrix}. \tag{14}$$

The Lindblad equation is the most general master equation in the Markov approximation that describes the dynamics of the density matrix of the system (i.e., after tracing out the environment variables). It generalizes the von Neumann–Liouville equation to the dissipative case and the Bloch equation to more general dissipative terms [8]. The mathematical structure of Eq. (13) has the same form as a Schrödinger equation with a non-Hermitian "Hamiltonian" $\mathcal{H}(t)=i\mathcal{L}(t)$ acting on the three-dimensional space $\mathbb{C}^3$. We will use this analogy to analyze the dynamics.

Adiabatic passage for non-Hermitian operators is analyzed in [15–17]. We consider here the regime of "weak non-Hermiticity" of Ref. [15]—i.e., $T\Gamma\lesssim 1$—where a generalization of the adiabatic theorem is possible for nondegenerate eigenvalues.

Adiabatic passage is described for dephasing in [18,19] through the calculation of approximate instantaneous eigenvectors and eigenvalues of $i\mathcal{L}(t)$. It has been shown in [18] that the local adiabatic conditions are (i) the one of the lossless model—i.e., Eqs. (11) and (12)—and (ii)

$$\lambda\equiv\sqrt{\Omega^2+\Delta^2}\gg\Gamma. \tag{15}$$

Below we address two important questions: (i) We determine for the dissipative model the conditions of optimal global adiabatic passage—i.e., that minimize the global *nonadiabatic* losses—and (ii) we determine the conditions that minimize the *total* losses, due to both nonadiabatic and decoherence effects. The first question is of fundamental interest, while the latter is of practical interest to reduce the total loss in a process of population transfer.

Processes of finite duration T, like, e.g., the interaction of atoms with laser pulses, depend on the pulse coupling $\Omega(t)$ [14,20]. If the pulsed coupling is of peak amplitude Ω_0, a robust adiabatic process will allow a complete population transfer when $T\Omega_0\gg 1$. This complete population transfer is also achieved by a resonant half Rabi oscillation with the area of the coupling $\int dt\Omega(t)=\pi$ ("π-pulse" transfer). We conclude that in both cases, the peak amplitude Ω_0 can be increased to shorten duration of the dynamics and thus to reduce the decoherence losses. Since the coupling amplitude is usually limited by other undesirable processes, such as ionization, the efficiency of adiabatic passage needs to be studied for given values of the ratio Ω_0/Γ. A way of analysis leading to similar conclusions consists in arguing that the minimum duration of the process is technically limited (due to the bandwidth of the pulse for instance) and that efficiency of the passage has to be studied for given values of $T\Gamma$.

A pure dephasing effect is more detrimental when the detuning is closer to zero, due to the occurrence of superpositions of states. This suggests that the detuning should be swept fast through the resonance to minimize this effect and that the sweep speed should be faster for larger Γ. The preceding requirements impose roughly $\Delta_0 \gg \Omega_0, \Gamma$. This intuitive picture is made more precise below.

IV. ADIABATIC PASSAGE AND NONADIABATIC CORRECTIONS FOR A DISSIPATIVE MODEL

A. Adiabatic passage for the dephasing Lindblad equation

The Bloch equation has the same mathematical structure as a Schrödinger equation with a non-Hermitian "Hamiltonian" $\mathcal{H}=i\mathcal{L}$, Eqs. (14) (but with non-normalized dynamical state vectors since the vectors on the Bloch sphere are not normalized for nonpure states). This analogy is of interest, since it shares some properties with different problems featuring losses, such as systems involving a continuum. In this case, the non-Hermitian effective Hamiltonians are obtained by perturbation theory on a complex-scaled Hamiltonian.

We define the right (left) eigenvectors $|\psi_n^R\rangle$ ($|\psi_m^L\rangle$) of the Lindbladian $\mathcal{H}$ ($\mathcal{H}^\dagger$), respectively, by

$$\mathcal{H}|\psi_n^R\rangle = \lambda_n|\psi_n^R\rangle, \tag{16a}$$

$$\mathcal{H}^\dagger|\psi_m^L\rangle = \lambda_m^*|\psi_m^L\rangle \quad \text{or } \langle\psi_m^L|\mathcal{H} = \lambda_m\langle\psi_m^L|, \tag{16b}$$

with $\dagger$ denoting the adjoint and $*$ the complex conjugate. We consider for simplicity a nondegenerate spectrum: $\lambda_m \neq \lambda_n$ for $m\neq n$. This allows one to write $\langle\psi_m^L|\mathcal{H}|\psi_n^R\rangle=\lambda_n\langle\psi_m^L|\psi_n^R\rangle=\lambda_m\langle\psi_m^L|\psi_n^R\rangle$—i.e., $(\lambda_n-\lambda_m)\langle\psi_m^L|\psi_n^R\rangle=0$—and thus to define the scalar product as

$$\langle\psi_m^L|\psi_n^R\rangle = \delta_{m,n}, \tag{17}$$

with the appropriate normalization.

When the Lindbladian is Hermitian, one recovers $|\psi_n^R\rangle=|\psi_n^L\rangle$ from Eq. (16). When the Lindbladian satisfies $\mathcal{H}^T=\mathcal{H}$, we have $|\psi_n^L\rangle=|\psi_n^R\rangle^*$.

In the present case the Lindbladian satisfies

$$\mathcal{H}^* = -\mathcal{H}, \tag{18}$$

which implies that one eigenvalue, say λ_2, is purely imaginary, its associated eigenvector $|\psi_2^R\rangle$ (and $|\psi_2^L\rangle$) is real, and that the two others satisfy $\lambda_3=-\lambda_1^*$ (i.e., same imaginary parts and opposite real parts) and $|\psi_1^R\rangle=|\psi_3^R\rangle^*$ ($|\psi_1^L\rangle=|\psi_3^L\rangle^*$).

The adiabatic theorem can be thus formulated for the solution of the Schrödinger equation in the limit $T\to\infty$ as

$$|\phi^R(t)\rangle = \langle\psi_n^L(t_i)|\phi^R(t_i)\rangle\exp\left(-i\int_{t_i}^t ds\lambda_n(s)\right)|\psi_n^R(t)\rangle + |\varepsilon^R(t)\rangle, \tag{19}$$

if we consider that the state is exactly connected to the single eigenstate $|\psi_n^R(t_i)\rangle$ initially—i.e., $\langle\psi_m^L(t_i)|\phi^R(t_i)\rangle=0$, $\forall\ m\neq n$, where $|\varepsilon^R(t)\rangle$ is the time-dependent nonadiabatic correction, associated with the adiabatic following along a right eigenvector, which is of order $O(1/T)$. It is well known that, for Hermitian Hamiltonians with analytic time-dependent parameters, the global nonadiabatic correction (between times for which the coupling is off) is of order $e^{-\text{const}\times T}$—i.e., beyond any power of $1/T$—as given by superadiabatic analysis and the DDP formula. We expect such a result to hold also under some conditions for non-Hermitian Hamiltonians (see, for instance, the Landau-Zener model with a dissipative Hamiltonian [21] and the recent study of adiabatic evolution for non-Hermitian Hamiltonians [22]).

The additional geometric phase has been chosen as 0 by imposing the condition of parallel transport $\langle\psi_n^L(t)|\frac{d}{dt}|\psi_n^R(t)\rangle=0$.

A similar construction can be formulated for the dual vectors $\langle\phi^L(t_i)|\psi_m^R(t_i)\rangle=0$, $\forall\ m\neq n$; then, we have

$$\langle\phi^L(t)| = \langle\phi^L(t_i)|\psi_n^R(t_i)\rangle\exp\left(i\int_{t_i}^t ds\lambda_n^*(s)\right)\langle\psi_n^L(t)| + \langle\varepsilon^L(t)|, \tag{20}$$

with $\langle\varepsilon^L(t)|$ the time-dependent nonadiabatic correction, associated with the adiabatic following along a left eigenvector, of order $O(1/T)$, which leads to

$$\langle\phi^L(t)|\phi^R(t)\rangle = \langle\psi_n^L(t_i)|\phi^R(t_i)\rangle\langle\phi^L(t_i)|\psi_n^R(t_i)\rangle \times e^{i\int_{t_i}^t ds[\lambda_n^*(s)-\lambda_n(s)]} + O(1/T). \tag{21}$$

For our problem, we have $|\phi^R\rangle\equiv|\varrho^R\rangle\equiv|\varrho\rangle$. If we consider that initially $\rho_{--}(t_i)=1$—i.e., $\rho_z(t_i)=-1$—then the connection is such that $\langle\psi_2^L(t_i)|\phi^R(t_i)\rangle=1=\langle\phi^L(t_i)|\psi_2^R(t_i)\rangle$ (with an appropriate choice of the initial phase of the eigenvectors; see below), implying a dynamics along the eigenstate associated to the purely imaginary eigenvalue $\lambda_2(t)$. This leads to

$$\langle\phi(t)|\phi(t)\rangle = \exp\left(2\int_{t_i}^t ds\ \mathrm{Im}\ \lambda_2(s)\right) + O(1/T). \tag{22}$$

The dynamical nonadiabatic losses are featured by the z component of $|\varepsilon^R(t)\rangle$, which is given by the DDP formula when $\Gamma=0$ as $\rho_{--}(t_f)=-\frac{1}{2}\langle\sigma_z|\varepsilon^R(t_f)\rangle$.

B. Eigenvalues and eigenvectors of the Lindbladian

The Lindbladian $\mathcal{H}(t)=i\mathcal{L}(t)$ admits the three following instantaneous eigenvalues:

$$\lambda_1 = -\frac{2i\Gamma}{3} - \frac{i}{2}(C-D) + \frac{\sqrt{3}}{2}(C+D), \tag{23a}$$

$$\lambda_3 = -\frac{2i\Gamma}{3} - \frac{i}{2}(C-D) - \frac{\sqrt{3}}{2}(C+D), \tag{23b}$$

$$\lambda_2 = -\frac{2i\Gamma}{3} + i(C-D), \tag{23c}$$

with the real coefficients

$$C = \frac{1}{3}\sqrt[3]{\frac{\beta-\alpha}{2}}, \tag{24a}$$

$$D = \frac{1}{3}\sqrt[3]{\frac{\beta+\alpha}{2}}, \tag{24b}$$

$$\alpha = -2\Gamma^3 - 18\Gamma\Delta^2 + 9\Gamma\Omega^2, \tag{24c}$$

$$\beta = \sqrt{4(3\Delta^2 + 3\Omega^2 - \Gamma^2)^3 + \alpha^2}. \tag{24d}$$

The instantaneous right and left eigenvectors followed during the dynamics are associated with the imaginary eigenvalue λ_2 and can be written as

$$|\Psi_2^R\rangle = \frac{1}{\sqrt{N}}\begin{bmatrix} \Omega \\ -\frac{\Omega}{\Delta}\left(\frac{\Gamma}{3} + C - D\right) \\ \Delta\left(1 + \frac{(\Gamma/3 + C - D)^2}{\Delta^2}\right) \end{bmatrix}, \tag{25a}$$

$$\langle\Psi_2^L| = \frac{1}{\sqrt{N}}\begin{bmatrix} \Omega \\ \frac{\Omega}{\Delta}\left(\frac{\Gamma}{3} + C - D\right) \\ \Delta\left(1 + \frac{(\Gamma/3 + C - D)^2}{\Delta^2}\right) \end{bmatrix}^T, \tag{25b}$$

where the real coefficient N provides the normalization $\langle\Psi_2^L|\Psi_2^R\rangle = 1$. One can show that the definition (25) of the instantaneous eigenvectors satisfies the condition of parallel transport:

$$\langle\Psi_2^L|\frac{d}{dt}|\Psi_2^R\rangle = 0. \tag{26}$$

These eigenvectors satisfy $|\Psi_2^R(t_i)\rangle = \{0\ 0\ \mathrm{sgn}(\Delta(t_i))\}^T$ and $\langle\Psi_2^L(t_i)| = \{0\ 0\ \mathrm{sgn}(\Delta(t_i))\}$ and thus the initial connection $\langle\Psi_2^L(t_i)|\rho^R(t_i)\rangle = 1$ since $\mathrm{sgn}[\Delta(t_i)] = -1$ and $|\rho^R\rangle = [0\ 0\ -1]^T$.

The population inversion at the end of the interaction is given by the z component of (19) and can be written as

$$\rho_z(t_f) = e^{-\eta} + \langle\sigma_z|\varepsilon^R(t_f)\rangle, \tag{27}$$

with the damping term

$$\eta = i\int_{t_i}^{t_f}\lambda_2(s)ds = \int_{t_i}^{t_f}\Omega_0\left(\frac{\Omega^2(s)}{\Omega^2(s) + \Delta^2(s)}\frac{\Gamma}{\Omega_0} + O((\Gamma/\Omega_0)^3)\right)ds. \tag{28}$$

At first order in Γ/Ω_0 and in the adiabatic limit, the final population inversion (27) is identical to formula (26) of Ref. [19]:

$$\rho_z(t_f) \simeq \rho_z^0(t_f)\exp\left(-\Gamma\int_{t_i}^{t_f}\frac{\Omega^2}{\Omega^2 + \Delta^2}dt\right), \tag{29}$$

where $\rho_z^0(t_f)$ is the final nondissipative population inversion. This formula allows one to write the population inversion approximately as a product of a term $\rho_z^0(t_f)$ involving purely nonadiabatic effects with a purely dissipative term. We have $\rho_z^0(t_f) = 1$ in the adiabatic limit.

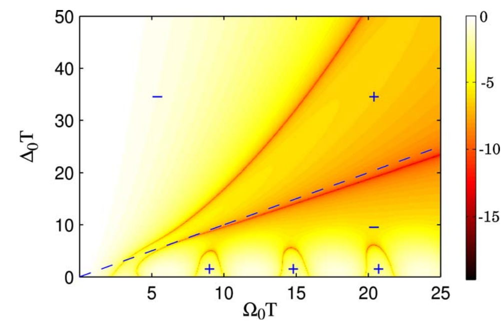

FIG. 1. (Color online) Contour plot of the natural logarithm of the absolute value of the global nonadiabatic losses $\langle\sigma_z|\varepsilon^R(t_f)\rangle$ for $T\Gamma = 0.5$, when the dynamics follows ellipses in the parameter space, as a function of $T\Delta_0$ and $T\Omega_0$. The black lines indicate losses equal to zero; signs + and − indicate the zones where the losses are, respectively, positive and negative. The dashed line indicates the parameters leading to circular trajectories. Note that the complete "π-pulse" transfer (i.e., for $\Delta_0 = 0$) of the lossless model ($\Gamma = 0$) would occur for $T\Omega_0 = 2\sqrt{\pi\ln 2}(2p+1) \approx 3(2p+1)$ with p an integer.

C. Minimization of the global nonadiabatic losses

The difficulty to derive analytically regions that minimize the global nonadiabatic losses is that there is no known formula to describe the nonadiabatic term $\langle\sigma_z|\varepsilon^R(t_f)\rangle$, like the DDP formula in the nondissipative case. However, one can expect that at first order Eq. (29) can give us some information. Indeed, the nondissipative population inversion ρ_z^0 contains nonadiabatic effects given by the DDP formula

$$\rho_z^0 = 1 - 2e^{-2\,\mathrm{Im}[\mathcal{D}(t_c)]}, \tag{30}$$

where

$$\mathcal{D}(t_c) = 2\int_0^{t_c}\sqrt{\Omega^2(t) + \Delta^2(t)}dt \tag{31}$$

and with t_c the imaginary time where the eigenvalues (8) cross—i.e., $\lambda(t_c) = 0$. This leads to the approximate expression of the global nonadiabatic losses in the dissipative case:

$$\langle\sigma_z|\varepsilon^R(t_f)\rangle \simeq -2e^{-2\,\mathrm{Im}[\mathcal{D}(t_c)] - \eta}. \tag{32}$$

The numerical calculation of the global nonadiabatic losses is plotted in Fig. 1 for a dephasing term $T\Gamma = 0.5$ when the dynamics follows ellipses,

$$\left(\frac{\Omega(t)}{\Omega_0}\right)^2 + \left(\frac{\Delta(t)}{\Delta_0}\right)^2 = 1, \tag{33}$$

defined by Ω_0 and Δ_0 in the parameter space, with the parametrization

$$\Omega(t) = \Omega_0 e^{-4\ln 2(t/T)^2}, \tag{34a}$$

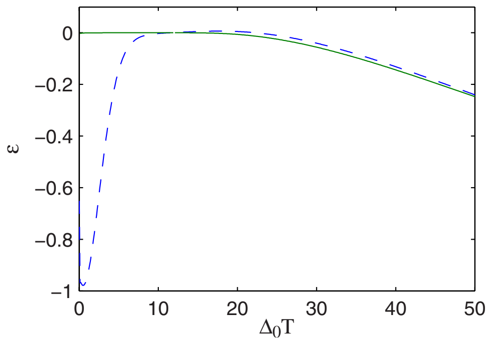

FIG. 2. (Color online) Comparison of the global nonadiabatic losses obtained by numerical simulation (dashed line) and formula (32) (solid line) for a dephasing rate $T\Gamma=0.5$ and a coupling $T\Omega_0=12$.

$$\Delta(t) = \mathrm{sgn}(t)\Delta_0\sqrt{1-\Omega(t)^2/\Omega_0^2}, \tag{34b}$$

where T is the full width at half maximum of the Rabi frequency.

In the nondissipative case, when the dynamics follows a circle in the parameter space, the global nonadiabatic losses become very small. In the dissipative case, the global nonadiabatic losses $\langle\sigma_z|\varepsilon^R(t_f)\rangle$ [Eq. (27)] are plotted on Fig. 1 and show several cases. When $\Delta_0\approx 0$, there is a lifting of quasidegeneracy and the population oscillates as a function of Ω_0 [23]. When the ellipses with $\Delta_0<\Omega_0$ approach circles—e.g., by fixing Ω_0 and increasing Δ_0—we see that the global nonadiabatic losses go through zero and become positive. When $\Delta_0>\Omega_0$, there is another curve where $\langle\sigma_z|\varepsilon^R(t_f)\rangle$ goes again through zero as Δ_0 increases and then becomes negative.

Figure 1 shows surprisingly that in regions above the circular trajectories the global nonadiabatic losses $\langle\sigma_z|\varepsilon^R(t_f)\rangle$ can be positive—i.e., improving the population transfer with respect to an ideal adiabatic regime. The region+bounded by the two black lines corresponds thus to a region of interest with respect to nonadiabatic losses, and it extends the circular (level line) trajectories of the nondissipative system. This effect is not described by the approximate formula (32), which gives only negative losses. However, this formula is still a good approximation at first order, since the positive losses are very small, as shown in Fig. 2.

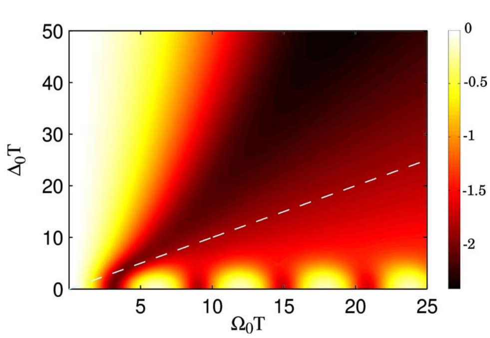

FIG. 3. (Color online) Contour plot of the natural logarithm of the total losses $\rho_{--}(t_f)\equiv[1-\rho_z(t_f)]/2$. for a dephasing rate $T\Gamma=0.5$ as a function of $T\Delta_0$ and $T\Omega_0$.

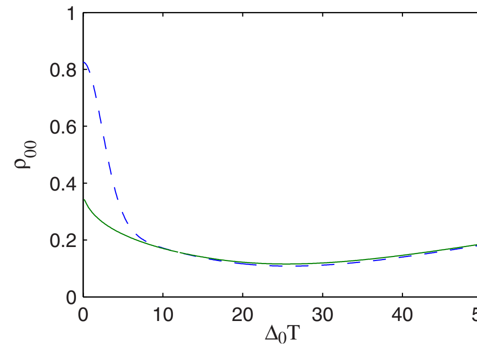

FIG. 4. (Color online) Comparison of the total losses obtained by numerical simulation (dashed line) and formula (32) (solid line) for a dephasing rate $T\Gamma=0.5$ and a coupling $T\Omega_0=12$.

V. MINIMIZATION OF THE TOTAL LOSSES

The total losses are defined as the population left in the ground state at the end of the process: $\rho_{--}(t_f)\equiv[1-\rho_z(t_f)]/2$. From Eq. (29) we see that the losses due to dephasing effects are reduced when $\Delta_0\gg\Omega_0$. Since we showed in the preceding section that the global nonadiabatic losses are optimized in a quite wide region above the circular trajectories, we expect a minimization of the total losses (nonadiabatic and dissipative) occurring as a compromise between this region and $\Delta_0\gg\Omega_0$.

We find below specific elliptic trajectories in the parameter space that yield this compromise.

The numerical simulation of the total losses is plotted on Fig. 3 for a dephasing rate $T\Gamma=0.5$. The domain below the circular trajectories shows Rabi oscillations for $\Delta_0=0$ and their attenuation when Δ_0 grows. We see that the minimum of losses is obtained in a quite wide domain above the circular trajectories, providing some robustness for the parameters of the ellipses. For given Ω and Γ, the approximate formula (29) can be used to determined which parameters Δ minimize the losses when the dynamics follows ellipses.

Figure 4 and Table I show a good agreement between the

TABLE I. Comparison of the numerical data given from numerical simulation and formula (29) for some dephasing rate $T\Gamma$ and coupling $T\Omega_0$.

$(T\Omega_0;T\Gamma)$		Simulation	Formula (29)
(12; 0.5)	Optimal $\Delta_0 T$	25.90	25.45
	ρ_{++} (circle)	0.843	
	ρ_{++} (ellipse)	0.891	0.884
(20; 0.5)	Optimal $\Delta_0 T$	69	65
	ρ_{++} (circle)	0.843	
	ρ_{++} (ellipse)	0.916	0.912
(12; 1)	Optimal $\Delta_0 T$	30.96	31.07
	ρ_{++} (circle)	0.735	
	ρ_{++} (ellipse)	0.824	0.811
(20; 1)	Optimal $\Delta_0 T$	76	78
	ρ_{++} (circle)	0.735	
	ρ_{++} (ellipse)	0.852	0.854

total losses obtained by numerical simulation and formula (29), in the region above the circular trajectories. Table I indicates the improvement of the transition probability $p = \rho_{++}$ when the dynamics follows optimized elliptic trajectories instead of circular trajectories. The improvement is larger when the dephasing rate grows, 8.7% for $T\Gamma = 0.5$ with $T\Omega_0 = 20$ and 16% for $T\Gamma = 1$ with $T\Omega_0 = 20$.

VI. CONCLUSION AND DISCUSSION

In this paper we have considered the dephasing effects on a two-level system in the Lindblad formalism. We have seen that in the adiabatic limit, the population transfer is described by the evolution of an eigenstate of the Lindbladian, whose associated eigenvalue provides the losses by dephasing. We have seen that the total losses can be minimized when the dynamics follows specific ellipses in the parameter space and that the optimized parameters can be estimated at first order using formula (29), leading to an improvement of the transition probability.

There are different types of dissipative effects in quantum systems: (i) dephasing and spontaneous emission induced by an environment (dissipative effects referred to as decoherence) and (ii) losses of population due to coupling with a continuum (see, for instance [24], in the context of collisional process). In the first type, the total Hilbert space is the tensor product of the Hilbert spaces of the system and of the environment. The effective dynamics, after tracing out the environment variables, is described in the Markovian limit by a Lindblad equation. In the second type, the Hilbert space is the direct sum of the bound states and the continuum. The losses into the continuum can be treated effectively by a lossy Schrödinger equation for the bound states—i.e., with additional non-Hermitian terms, obtained, e.g., by adiabatic elimination (see, for instance, [25,26]). We remark that in some particular situations a system of the first type can be reduced to an effective description by a lossy Schrödinger equation. When this is possible, it is very convenient from the technical point of view—e.g., in a numerical simulation—since the dimension of the spaces needed is much smaller than for a Lindblad equation. This is the case, for instance, when one considers an N-level system with spontaneous emission into another level; the Lindblad equation of the complete system can be reduced to a lossy Schrödinger equation for the N-level system. This reduction is not applicable for the dephasing phenomenon we have analyzed in the present paper.

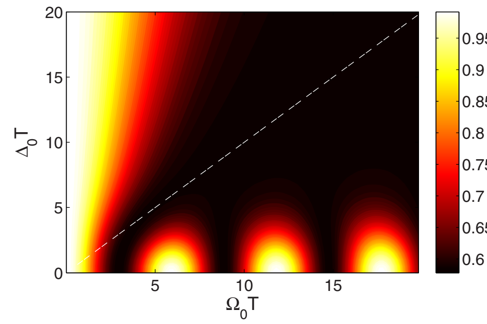

FIG. 5. (Color online) Contour plot of the total loss $1 - |\langle + | \phi(t_f)\rangle|^2$ for $T\gamma = 0.5$ as a function of $T\Delta_0$ and $T\Omega_0$ (to be compared with Fig. 3). The dashed line indicates the parameters leading to circular trajectories.

The case of a lossy Schrödinger equation is expected to lead to different results since, in this case, the losses are not transient as for the dephasing case, but continuous during the transfer *and* also after the transfer (if we consider a final lossy state). To illustrate this difference, we analyze the problem of population transfer considered in Sec. II, but with the Hamiltonian of the lossy upper state:

$$H(t) = \frac{1}{2}\begin{bmatrix} -\Delta(t) & \Omega(t) \\ \Omega(t) & \Delta(t) - i\gamma/2 \end{bmatrix}. \qquad (35)$$

We consider the pulse and the detuning (34) with a finite time interval $t \in [-\tau/2, \tau/2]$ of duration $\tau = 3.5T$.

Figure 5 shows the total loss $1 - |\langle + | \phi(t_f)\rangle|^2$ with $\phi(t)$ solution of the Schrödinger equation $i\frac{\partial}{\partial t}\phi(t) = H(t)\phi(t)$ for a moderate loss $T\gamma = 0.5$. It shows that the losses are minimized in a wide region that surrounds for $T\Omega_0 \gtrsim 3$ the circular trajectories (shown as a dashed line). No specific trajectories optimize the population transfer in this area.

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