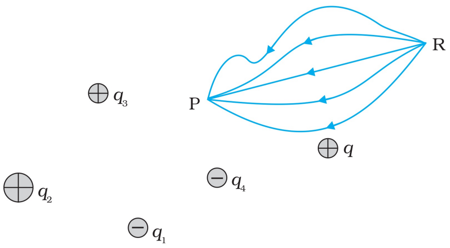
Electrostatic Potential

Electric Potential difference : Potential difference between two points in an electric field may be defined as the amount of work done in moving a unit positive charge from one point to the other against the electrostatic forces.



if V_P and V_R be the electric potential at point P and R respectively, then $\Delta V = V_P - V_R$

or $\Delta V = \frac{W_{PR}}{q}$ and $\boxed{V_P - V_R = \frac{W_{PR}}{q}}$

W_{PR} = external work done

$W_{PR} = -W_{E.F}$

SI unit of Potential difference $\frac{J}{C} = JC^{-1} = 1$ Volt.

Note:→ • Test charge is so small that it does not disturb the distribution of the Source Charge
• we just apply so much external force on the test charge that it just balances the repulsive electric force on it and hence does not produce any acceleration in it. (i.e Potential difference is path independent).

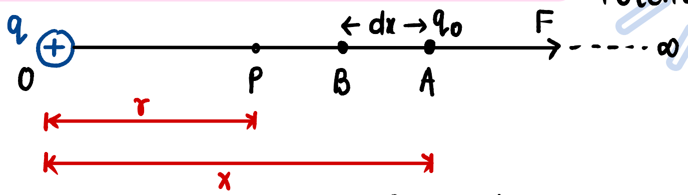
Electric Potential

Electric Potential at any point in an electric field is the amount of work done in moving a unit positive charge from infinity to that point against the electrostatic forces.

if the point R lies at infinity, then $V_R = 0$ so that $V = V_P = \frac{W}{q}$ or $V = \frac{W}{q}$

- Electric Potential always decrease in the direction of electric field.
- Electric Potential is a scalar quantity.

Potential due to a Point Charge



Consider a +ve point charge q placed at the origin O. we have to ...e the potential at point P. Let a test ch... placed at point A.

Acc. to Coulomb Law, Force acting on charge q_0 is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{x^2}$$

Work done in moving the test charge q_0 from A to B through small displacement $d\vec{x}$ against the electrostatic force is $dW = \vec{F}\cdot d\vec{x} = dW = Fdx\cos180° \Rightarrow dW = -Fdx$

The total work done in moving the charges q_0 from infinity to that point p will be

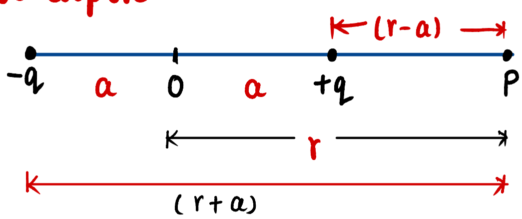
$$W = \int dW = -\int_\infty^r F\,dx = -\int_\infty^r \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{x^2}dx \qquad W = -\frac{qq_0}{4\pi\varepsilon_0}\int_\infty^r x^{-2}dx \qquad W = -\frac{qq_0}{4\pi\varepsilon_0}\left[-\frac{1}{x}\right]_\infty^r$$

$$W = \frac{qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}$$

Hence Electric Potential $V = \frac{W}{q_0} = \frac{qq_0}{4\pi\varepsilon_0 r} \times \frac{1}{q_0} \Rightarrow \boxed{V = \frac{q}{4\pi\varepsilon_0 r}}$

Potential due to dipole

Potential due to dipole at a point on axial line.



Potential due to +q & −q charge at point P is

$$V_+ = \frac{q}{4\pi\varepsilon_0(r-a)} \quad \text{—①} \qquad V_- = \frac{-q}{4\pi\varepsilon_0(r+a)} \quad \text{—②}$$

Total Potential at point P. is

$$V = V_+ + V_-$$

$$V = \frac{q}{4\pi\varepsilon_0(r-a)} - \frac{q}{4\pi\varepsilon_0(r+a)} \Rightarrow V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)} - \frac{1}{(r+a)}\right] \Rightarrow V = \frac{q}{4\pi\varepsilon_0}\left[\frac{r+a-r+a}{(r^2-a^2)}\right] \Rightarrow V = \frac{q\,2a}{4\pi\varepsilon_0(r^2-a^2)} \Rightarrow V = \frac{KP}{(r^2-a^2)}$$

Here $p = q\,2a$

For Short dipole $r \gg a$ $\boxed{V_{axial} = \frac{kp}{r^2}}$

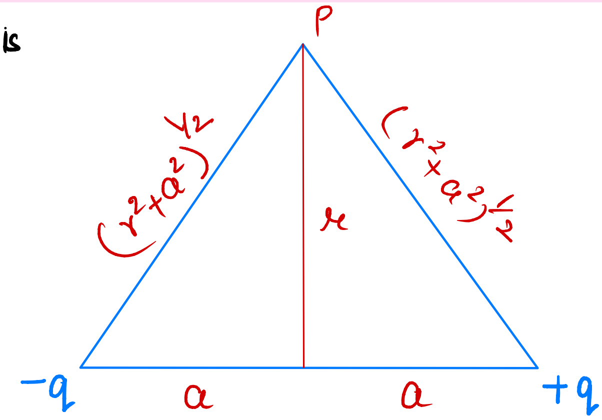
• Potential due to dipole at a point on equatorial line

Potential due to positive charge & Negative charge at point P is

$V_+ = \frac{q}{4\pi\varepsilon_0 (r^2+a^2)^{1/2}}$ & $V_- = \frac{-q}{4\pi\varepsilon_0 (r^2+a^2)^{1/2}}$

So Total Potential at Point P is

$\boxed{V_{eq} = 0}$



Potential due to dipole at any point

Potential due to +q and −q at point P are respectively

$V_+ = \frac{q}{4\pi\varepsilon_0 r_1}$ & $V_- = \frac{-q}{4\pi\varepsilon_0 r_2}$

Net Potential at point P is

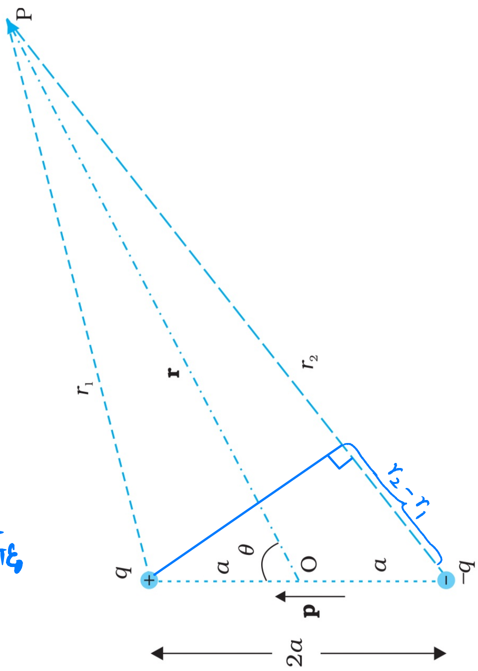
$V = V_+ + V_- \Rightarrow V = \frac{q}{4\pi\varepsilon_0 r_1} - \frac{q}{4\pi\varepsilon_0 r_2} \Rightarrow V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{r_2 - r_1}{r_1 r_2}\right]$

if the point P lies far away from the dipole, then

$r_1 - r_2 = 2a\cos\theta$ and $r_1 r_2 \simeq r^2$

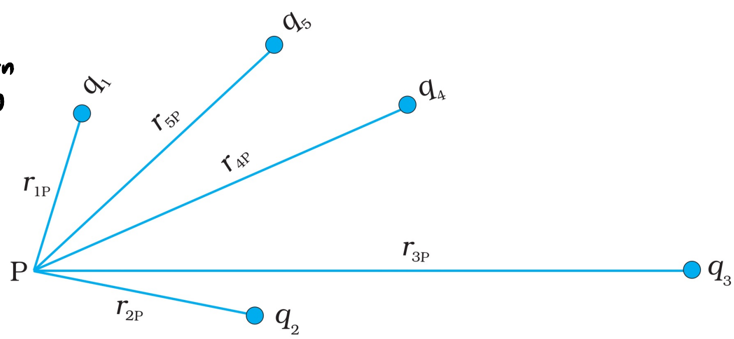
$\therefore$ $V = \frac{q}{4\pi\varepsilon_0} \times \frac{2a\cos\theta}{r^2} \Rightarrow \boxed{V = \frac{KP\cos\theta}{r^2}}$ $\because$ $P = q \times 2a$ & $K = \frac{1}{4\pi\varepsilon_0}$



Potential due to a System of Charges

Let there be n number of charges $q_1\ q_2\ q_3 \ldots q_n$ at distances $r_1\ r_2\ r_3\ r_4 \ldots r_n$ respectively from the point P.

Potential due to q_1 & q_2 charge at point P is given by $V_1 = \frac{q}{4\pi\varepsilon_0 r_{1P}}$ $V_2 = \frac{q}{4\pi\varepsilon_0 r_{2P}}$



Similarly

$V_n = \frac{q_n}{4\pi\varepsilon_0 r_{nP}}$

The total Potential at point P due individual Charges is given by.

$V = V_1 + V_2 + V_3 + \ldots + V_n$

$V = \frac{q_1}{4\pi\varepsilon_0 r_{1P}} + \frac{q_2}{4\pi\varepsilon_0 r_{2P}} + \ldots + \frac{q_n}{4\pi\varepsilon_0 r_{nP}}$

So $\boxed{V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{iP}}}$

Equipotential Surfaces

- Any Surface that has same electric potential at every point on it is Called "Equipotential Surface". Equipotential Surface is an imaginary Surface.

Properties of Equipotential Surface

- No work is done in moving a test charge over an equipotential Surface.

$\because$ $W = q_0 (V_A - V_B)$ on equipotential Surface $V_A - V_B = 0$ So $W = 0$

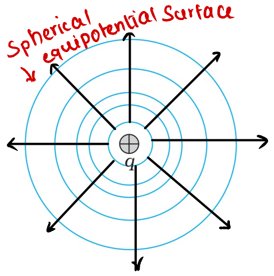
- Electric field is always perpendicular (Normal) to the equipotential Surface at every point.

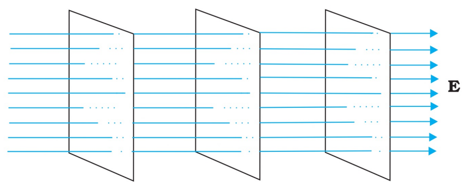
If the field were not Normal to the equipotential Surface, it would have a non zero component along the surface. so to move a test charge against this Component, a work would have to be done.

- Equipotential Surfaces are closer together in the region of strong field and farther apart in the regions of weak fields.

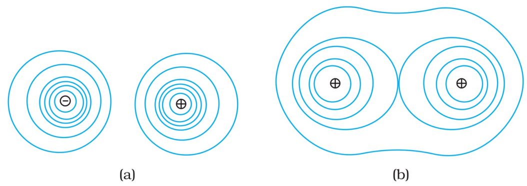
Because $E = -\frac{dV}{dr}$ or $dr = -\frac{dV}{E}$ on equipotential surface $dV =$ Constant then $r \propto \frac{1}{E}$

- No two equipotential Surfaces can intersect each other. because at the point of intersection there will be two values of electric potential at the point of intersection, which is impossible.





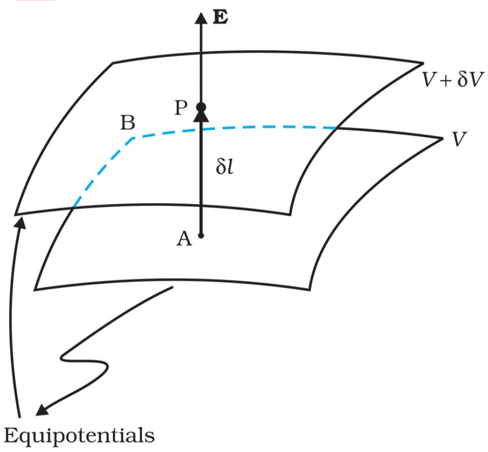
Equipotential surfaces for a uniform electric field.



Some equipotential surfaces for (a) a dipole, (b) two identical positive charges.

Relation between field and Potential

Let P be a point on the surface B. δl is the perpendicular distance of the Surface A from P. Imagine that a unit positive charge is moved along this perpendicular from the Surface B to surface A against the electric field. The work done in this process is



$\delta W = \vec{F}_{ef} \cdot \vec{\delta l}$ here $\vec{F}_{ef} = -\vec{F}_{ext}$ (External force is against electrostatic Field force)

i.e $\delta W = -\vec{F}_{ext} \cdot \vec{\delta l}$

$\delta W = -q_0 \vec{E} \cdot \vec{\delta l}$

$\frac{\delta W}{q_0} = -\vec{E} \cdot \vec{\delta l}$

$\boxed{\delta V = -\vec{E} \cdot \vec{\delta l}}$ for uniform electric field.

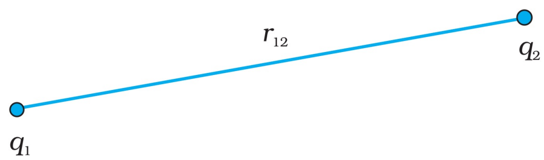
$-ve$ showing that potential always decreases in the direction of electric field.

For Non- Uniform E.F

$$V = -\int_B^A \vec{E} \cdot \vec{dl} \qquad \text{or} \qquad E = -\frac{dV}{dr}$$

Potential Energy of a System of Charges

Note :→ There is no external field against which work needs to be done.



Work done in bringing a charge from infinity to the point $\vec{r_1}$.

$W_1 = 0$ $\because$ External field is zero.

and work in bringing a charge from infinity to the point $\vec{r_2}$:

$W_2 = V_1 q_2$ $V_1 =$ Potential due to q_1 charge at q_2 charge is

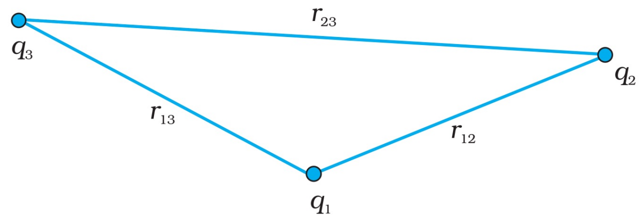
$W_2 = \frac{q_1}{4\pi\varepsilon_0 r_{12}} \times q_2$ $\because$ $V_1 = \frac{q_1}{4\pi\varepsilon_0 r_{12}}$

This work done gets stored in the form of Potential energy.

i.e $\boxed{U = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}}$

Potential energy of a system of three charges is given by $U = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$

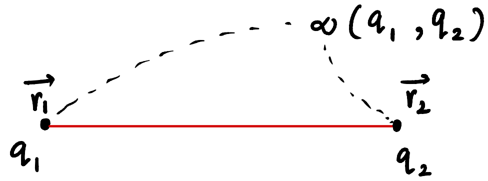
When there is no external electric field.



Potential energy in an External Electric Field

- For a System of two charges q_1 and q_2
- The work done in bringing a charge q_1 from infinity to the point $\vec{r_1}$ is

$W_1 = V(\vec{r_1})\, q_1$ $\qquad V(\vec{r_1})$ = Electric potential due to external field at position vector $\vec{r_1}$.



work done in bringing the charge q_2 from infinity to the point $\vec{r_2}$:

$W_2 = V(\vec{r_2})\, q_2 + V_1(\vec{r_{12}})\, q_2$ $\qquad$ { $V(\vec{r_2})$ = potential at r_2 due to external E.F).

Note :- $V_1(\vec{r_{12}})$ is the potential due to charge q_1 on charge q_2.

i.e $W_2 = V(\vec{r_2})q_2 + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$ $\qquad$ Hence $U = W_1 + W_2 \Rightarrow U = V(\vec{r_1})q_1 + V(\vec{r_2})q_2 + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$

Potential Energy for a Collection of More than Two Charges (i.e for n- charges)

$$U = \frac{K}{2}\sum_{\substack{i,j \\ i\neq j}}^{n} \frac{q_i q_j}{r_{ij}} \qquad \text{Here} \quad K = \frac{1}{4\pi\epsilon_0}$$

Question:→ (i). Calculate the potential at a point P due to a charge of 4×10^{-7} C Located 9cm away

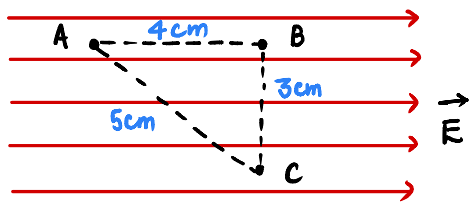
Solution: Potential at a point is given by $V = \frac{q}{4\pi\epsilon_0 r} \Rightarrow V = \frac{4\times10^{-7}\times100\times9\times10^{9}}{9}$ so $V = 4\times10^{4}$ V.

(ii). Hence obtain the work done in bringing a charge of 2×10^{-9} C from infinity to that point.

Solution: we know workdone $W = q_0 V$

$W = 2\times10^{-9}\times4\times10^{4} = 8\times10^{-5}$ Joule

Question:→ Three point A, B and C lie in a uniform electric field (E) of 5×10^{3} N/C as shown in the figure. Find the potential difference between A and C.



Solution:→ Potential at pt. B and C are Same

$V_B = V_C$

And Potential difference between A and C = Potential diff between A and B

i.e $\quad$ P.D $= -Edx = -5\times10^{3}\times4\times10^{-2} = -200$ Volt

Q= What is the angle between electric field and equipotential Surface?

a). 90° always (b) 0° always (c) 0° to 90° (d) 0° to 180° $\qquad$ Answer → (a).

Question: Find V_{ab} in an electric field $\vec{E} = (2\hat{i} + 3\hat{j} + 4\hat{k})$ N/C where $\vec{r_a} = (\hat{i} - 2\hat{j} + \hat{k})$m and $\vec{r_b} = (2\hat{i} + \hat{j} - 2\hat{k})$m

Solution: using $\quad dV = -\vec{E}\cdot d\vec{r}$ $\qquad$ Here $d\vec{r} = \vec{r_a} - \vec{r_b} \Rightarrow d\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) - (2\hat{i} + \hat{j} - 2\hat{k})$

$= -\hat{i} - 3\hat{j} + 3\hat{k}$

$V_a - V_b = -\vec{E}\cdot d\vec{r}$

$V_a - V_b = -(2\hat{i} + 3\hat{j} + 4\hat{k})\cdot(-\hat{i} - 3\hat{j} + 3\hat{k})$

$= -(-2 - 9 + 12) = -1$ Volt answer.