A·C NOTES

NEW NOTES 2022-23

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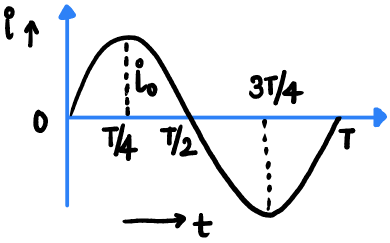
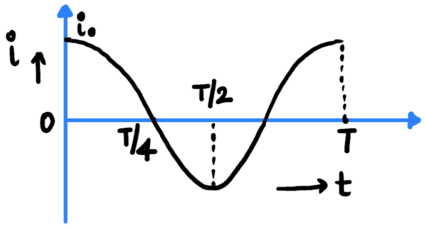
Alternating Current and Alternating EMF

An alternating current is one whose magnitude changes continuously with time between zero and a maximum value and whose direction reverses periodically.





The simplest type of alternating current is one which varies with time simple harmonically.
It is represented by $i = i_0 \sin\omega t$ or $i = i_0 \cos\omega t$

where i = instantaneous value of current at time t
i_0 = maximum (or peak) value of the current and is called "Current Amplitude".

- **Angular Frequency** (ω)

$$\boxed{\omega = \frac{2\pi}{T} = 2\pi f}$$

where T = time period
f = frequency.

- **Alternating emf**

The e.m.f (or voltage) whose magnitude changes continuously with time between zero and a maximum value and whose direction reverses periodically, is known as alternating emf.

The instantaneous value of alternating e.m.f may be represented by

$E = E_0 \sin\omega t$ or $E = E_0 \cos\omega t$

Note: The graphical representation of E as sine and cosine functions of t are of the same form as those of i.

Amplitude

The alternating current varies in magnitude and reverses in direction periodically. The maximum value of the current in either direction is called the 'peak value' or the 'amplitude' of the current. it is represented by i_0.

Periodic Time

The time taken by the alternating current to complete one cycle of variation is called the 'periodic-time' of the current. The periodic time T of the alternating current is given by

$$\boxed{T = \frac{2\pi}{\omega}}$$

Frequency

The number of cycles completed by an alternating current in one second is called the frequency of the current.

frequency $f = \frac{1}{T}$ or $\boxed{f = \frac{\omega}{2\pi}}$

Unit → cycles/second or Hertz (Hz)

Note: The frequency of the domestic alternating current is 50 cycles/second.

Mean (or Average Value)

An alternating current flows during one half-cycle in one-direction and during the other half-cycle in opposite direction. Hence, for one complete cycle, the mean value of alternating current is zero. However, the mean value of alternating current over half a cycle is finite quantity and in fact, it is the quantity which is defined as the 'mean value' of alternating current. It is given by

$$\boxed{i_{mean} = \frac{1}{T/2}\int_0^{T/2} i\,dt}$$

where i is the instantaneous value of the current.

$i = i_0 \sin\omega t$ $\quad$ i_0 = peak value

$T = 2\pi/\omega$

$$i_{mean} = \frac{\omega}{\pi}\int_0^{\pi/\omega} i_0 \sin\omega t\,dt \Rightarrow i_{mean} = \frac{\omega}{\pi}\frac{i_0}{\omega}(-\cos\omega t)_0^{\pi/\omega} \Rightarrow i_{mean} = -\frac{i_0}{\pi}[\cos\pi - \cos 0]$$

$$i_{mean} = -\frac{i_0}{\pi}[-1-1] \Rightarrow \boxed{i_{mean} = \frac{2i_0}{\pi}} \text{ or } \boxed{i_{mean} = 0.637\, i_0}$$

Root-mean square value

It is defined as that value of a direct current which produces the same amount of heating effect in a given resistor as is produced by the given alternating current when passed for the same time during a complete cycle.

It is also called virtual value or effective value of A.C.

Instantaneous value of alternating current

$$I = I_0 \sin\omega t$$

if dH is small amount of heat produced in time dt in Resistor R, then

$dH = i^2 R\,dt$ (in one complete cycle)

then total heat produced is

$$\int_0^H dH = \int_0^T i^2 R\,dt \Rightarrow H = \int_0^T i_0^2 \sin^2\omega t\, R\,dt \Rightarrow H = i_0^2 R\int_0^T \sin^2\omega t\,dt$$

$$H = i_0^2 R\int_0^T\left(\frac{1-\cos 2\omega t}{2}\right)dt \Rightarrow H = \frac{i_0^2 R}{2}\left[\int_0^T dt - \int_0^T \cos 2\omega t\,dt\right]$$

$$H = \frac{i_0^2 R}{2}\left[(T-0) - \left(\frac{\sin 2\omega t}{2\omega}\right)_0^T\right] \Rightarrow H = \frac{i_0^2 R}{2}\left[T - \frac{1}{2\omega}(\sin 2\omega T - \sin 2\omega \times 0)\right]$$

Here $T = \frac{2\pi}{\omega}$

$$H = \frac{i_0^2 R}{2}\left[T - \frac{1}{2\omega}(\sin 4\pi - \sin 0)\right] \quad \text{i.e.} \quad \boxed{H = \frac{i_0^2 RT}{2}} \quad -(i)$$

if I_{rms} is rms value of alternating current and H is the heat produced by rms current, then

$$H = I_{rms}^2 RT \quad -(ii)$$

From eq (i) and (ii)

$$I_{rms}^2 \cancel{RT} = \frac{I_0^2 \cancel{RT}}{2}$$

$$I_{rms}^2 = \frac{I_0^2}{2} \Rightarrow \boxed{I_{rms} = \frac{I_0}{\sqrt{2}}}$$

$I_{rms} = 0.707 I_0 \Rightarrow I_{rms} = 70.7\%$ of I_0

rms value of alternating emf

$\boxed{E_0 = I_0 R}$

$$E_{rms} = I_{rms} R$$

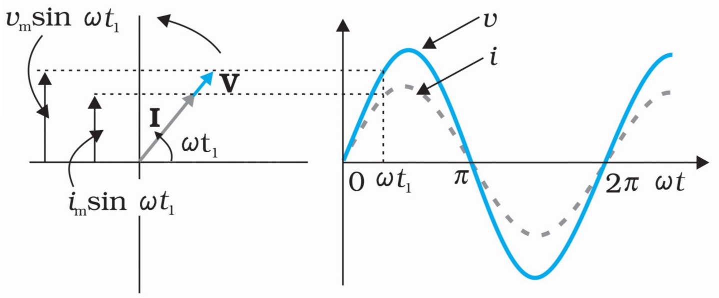
$$E_{rms} = \frac{I_0}{\sqrt{2}} R$$

$$E_{rms} = \frac{E_0}{\sqrt{2}} = 0.707 E_0 \Rightarrow 70.7\% \text{ of } E_0$$

Phasors

A phasor is a vector which rotates about the origin with angular speed ω.

The vertical components of phasors V and I represent the sinusoidally varying quantities v and i.



A.C Voltage applied to a Resistor

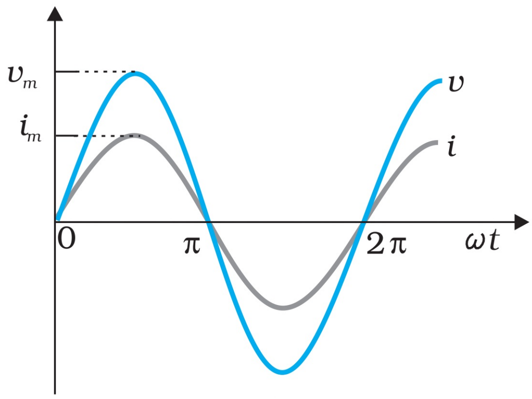
we consider a source which produces sinusoidally varying potential difference across its terminals. This potential difference also called as ac voltage, given by

$V = V_m \sin\omega t$ applying KCl 2nd law we get

$V_m \sin\omega t = iR \Rightarrow i = \frac{V_m \sin\omega t}{R}$

$\frac{V_m}{R} = i_m$ So $\boxed{i = i_m \sin\omega t}$

Graphical Representation



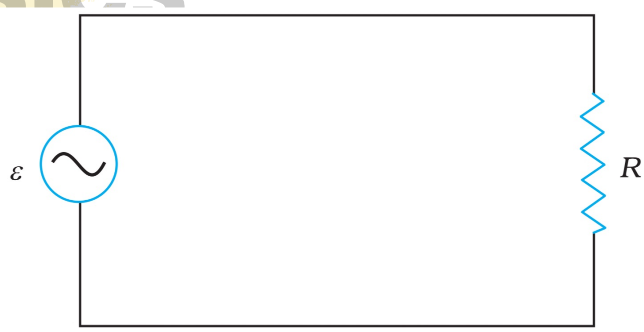
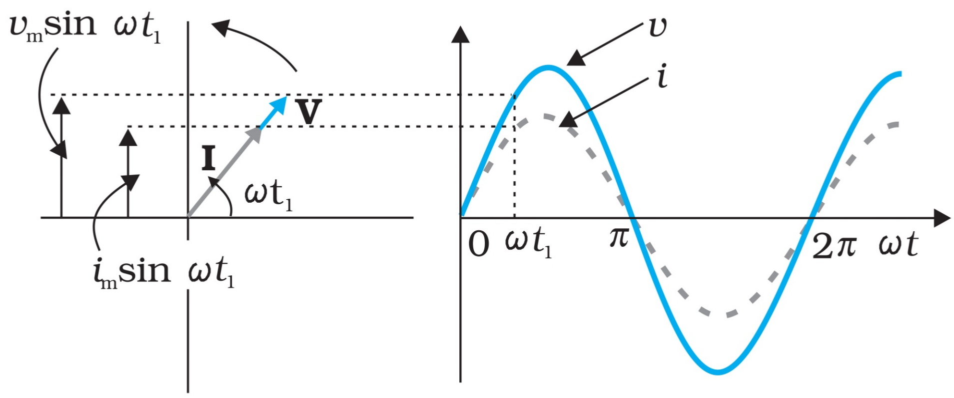


FIGURE 7.1 AC voltage applied to a resistor.

NOTE: Voltage and Currents are in phase with each other.

Phasor :→ A phasor is a vector which rotates about the origin with angular speed ω.

Phasor diagram



A.C Voltage applied to a inductor

An a.c source connected to an inductor. Let the voltage across the source be $V = V_m \sin\omega t$

Apply KCL 2, we get $V - L\frac{di}{dt} = 0 \Rightarrow V = L\frac{di}{dt}$

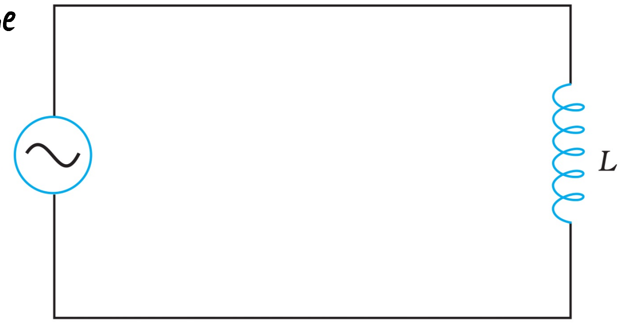


FIGURE An ac source connected to an inductor.

$Ldi = V dt \Rightarrow Ldi = V_m \sin\omega t\, dt$

Integrate above equation both Side

$\int L di = \int V_m \sin\omega t\, dt \Rightarrow Li = -V_m \frac{\cos\omega t}{\omega} \Rightarrow i = -\frac{V_m \cos\omega t}{L\omega}$

where $X_L = \omega L$ $\quad i = \frac{V_m}{X_L}\sin(\omega t - \pi/2)$ $\quad \because -\cos\omega t = \sin(\omega t - \pi/2)$

and $\boxed{i = i_m \sin(\omega t - \pi/2)}$ where $i_m = \frac{V_m}{X_L}$

$X_L = L\omega$ = Inductive Reactance [Resistance due to inductor]

unit of X_L is ohm (Ω).

NOTE: Current lags the voltage by $\pi/2$.

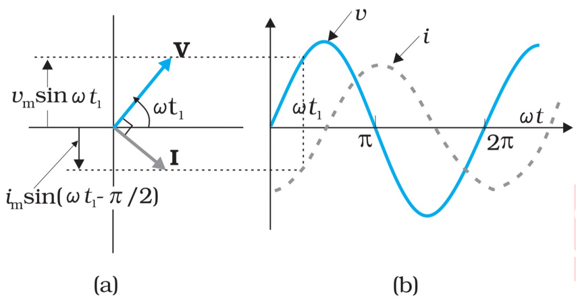


FIGURE (a) A Phasor diagram for the circuit in
(b) Graph of v and i versus ωt.

The average power supplied to an inductor over one complete cycle is ZERO.

Inductive Reactance (X_L):→ Opposition offered by inductive circuit / inductor to the flow of Current.

$X_L = L\omega = 2\pi\nu L \quad \nu_{dc} = 0$ for d.c $X_L = 0$

$\nu_{ac} = 50$ Hz $\quad X_L$ = very large value, so inductor passes d.c only.

AC Voltage Applied to a Capacitor

An ac Source ε generating ac voltage $V = V_m \sin\omega t$ connected to a Capacitor only, a purely Capacitive ac circuit.

Let q be the charge on the capacitor at any time t. The instantaneous voltage v across the Capacitor is

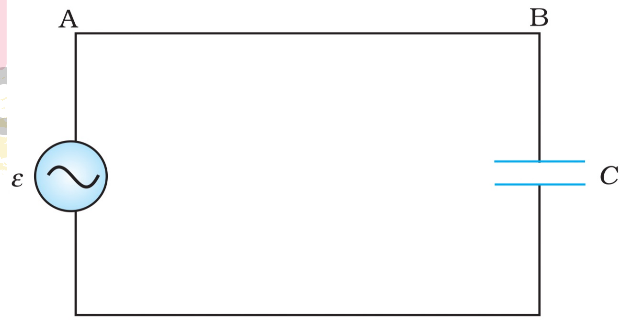


FIGURE An ac source connected to a capacitor.

$V = \frac{q}{C}$ Applying KCl 2nd Law $V_m \sin\omega t = \frac{q}{C} \Rightarrow q = CV_m \sin\omega t$

To find Current, we use $i = \frac{dq}{dt} \Rightarrow i = \frac{d}{dt}(CV_m \sin\omega t)$

$i = CV_m \cos\omega t\, \omega \Rightarrow i = \frac{V_m}{1/C\omega}\cos\omega t \Rightarrow i = \frac{V_m}{X_C}\cos\omega t$

where X_C = Capacitive Reactance [Resistance due to Capacitor] and $X_C = 1/C\omega$ $\quad \boxed{\omega = 2\pi\nu}$

and $i = i_m \cos\omega t \Rightarrow i = i_m \sin(\omega t + \pi/2)$ NOTE: Current leads the voltage by $\pi/2$.

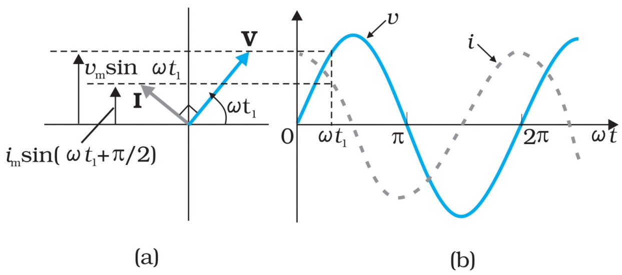


FIGURE (a) A Phasor diagram for the circuit in Fig. 7.8. (b) Graph of v and i versus ωt.

NOTE:→ The average power supplied to an capacitor over one complete cycle is ZERO.

• Capacitive Reactance (X_C) = Opposition offered by capacitive circuit $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$ for dc $X_C = \infty$

So d.c can't pass through capacitor.

Admittance (Y) = Reciprocal of impedance

i.e $Y = \frac{1}{Z}$

Susceptance (S): Reciprocal of reactance is defined as susceptance. it is of two Types.

(i) inductive susceptance $S_L = 1/X_L$

(ii) Capacitive Susceptance $S_C = \frac{1}{X_C}$

AC Voltage Applied to a Series LCR Circuit

Figure shows a series LCR circuit connected to an ac source ε. The voltage of the source to be $\boxed{V = V_m \sin\omega t}$

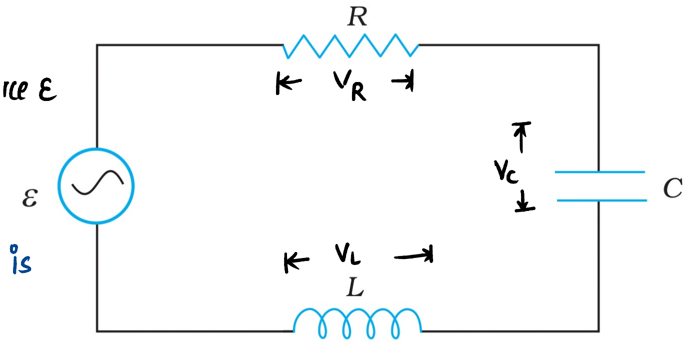
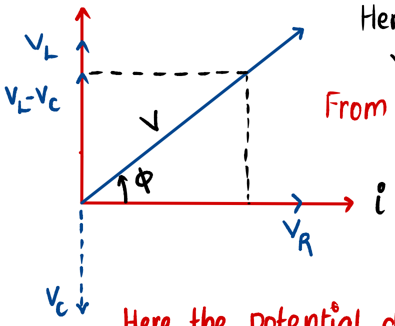


FIGURE A series *LCR* circuit connected to an ac source.

Phasor diagram Solution

We know that current and voltage remain in same phase in case of Resistor. And in case of inductor and Capacitor there is a phase difference of π/2 in voltage and current. Then



Here $V_R = iR$

$V_L = iX_L$ and $V_C = iX_C$

From phasor diagram

$$V^2 = V_R^2 + (V_L - V_C)^2 \Rightarrow V^2 = (iR)^2 + (iX_L - iX_C)^2 \Rightarrow \frac{V}{i} = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\frac{V}{i} = Z \text{ (Impendance)} \quad \text{i.e} \quad Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \text{and} \quad Z = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

Here the potential difference leads the current by an angle.

So $V = V_m \sin(\omega t + \phi)$ in case of Series LCR Circuit

Direction of Resultant voltage is given by $\tan\phi = \left|\frac{V_L - V_C}{V_R}\right| \Rightarrow \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

and $\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$

Resonance Condition of a Series LCR Circuit

A Series LCR circuit is said to be in the resonance condition when the current through it has its maximum value.

The frequency at which the current amplitude I_0 attains a peak value is called Natural or resonant frequency.

Condition for Resonance is $X_L = X_C$

i.e $\omega L = \frac{1}{\omega C} \Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow \omega_r = \frac{1}{\sqrt{LC}}$ $\quad \omega_r = 2\pi f_r$

So $2\pi f_r = \frac{1}{\sqrt{LC}} \Rightarrow \boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$

f_r = Natural frequency.

ω_r = Angular frequency. (Natural)

The Current amplitude at resonance frequency will be

$I_0 = \frac{E_0}{R}$ Due to $X_L = X_C$ $\quad Z = R$ i.e Z is minimum and Current is maximum.

NOTE :→ Series Resonant circuit is also called an "acceptor circuit".

→ Resonance occurs only in Series LCR Circuit, not in LR and LC Circuit.

SHARPNESS OF RESONANCE : Q-Factor

The sharpness of resonance is measured by a coefficient called the quality or Q-factor. The resonance frequency is independent of R, but sharpness of peak depends on R.

Because $i_0 = \frac{E_0}{R}$

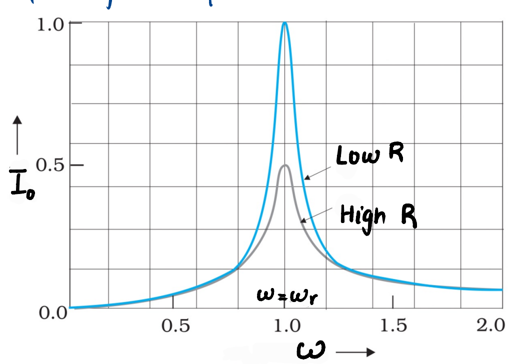


FIGURE Variation of i_m with ω for two

Q-factor: The Q-factor of a series LCR-Circuit may be defined as the ratio of the voltage drop across the inductance (or capacitance) at resonance to the applied voltage.

$$Q = \frac{\text{Voltage drop across L (or C)}}{\text{Applied Voltage}} \Rightarrow Q = \frac{\omega_r L\, I_{rms}}{R\, I_{rms}}$$

$Q = \frac{\omega_r L}{R}$ we know that $\omega_r = \frac{1}{\sqrt{LC}}$ so we get $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

Power in A·C Circuit

The rate of dissipation of energy in an electrical circuit is called the power. It is equal to the product of voltage and current. The power of an alternating-current circuit depends upon the phase difference between the voltage and current.

The instantaneous values of the voltages and current in an A·C Circuit are given by

$$V = V_0 \sin(\omega t + \phi) \qquad i = i_0 \sin\omega t$$

ϕ = phase difference between the voltage and current

The instantaneous power in the Circuit is

$$P_{ins} = Vi$$
$$= V_0 \sin(\omega t + \phi) \times i_0 \sin\omega t$$
$$= V_0 i_0 \sin\omega t\,(\sin\omega t \cos\phi + \cos\omega t \sin\phi)$$
$$= V_0 i_0 [\sin^2\omega t \cos\phi + \sin\omega t \cos\omega t \sin\phi]$$

For one complete cycle, $\sin^2\omega t = \frac{1}{2}$ and $\sin 2\omega t = 0$

Therefore, the average power P in the circuit is given by

$$P_{avg} = \frac{V_0 i_0}{2}\cos\phi$$

$$P_{avg} = \frac{V_0}{\sqrt{2}}\frac{i_0}{\sqrt{2}}\cos\phi \quad \text{i.e} \quad \boxed{P_{avg} = V_{rms} I_{rms} \cos\phi}$$

Note: $\cos\phi$ is known as 'power factor' of the circuit and its value depends upon the nature of the circuit.

Special Cases

Case (i) *Resistive circuit*: If the circuit contains only pure *R*, it is called *resistive*. In that case $\phi = 0$, $\cos\phi = 1$. There is maximum power dissipation.

Case (ii) *Purely inductive* or *capacitive circuit*: If the circuit contains only an inductor or capacitor, we know that the phase difference between voltage and current is $\pi/2$. Therefore, $\cos\phi = 0$, and no power is dissipated even though a current is flowing in the circuit. This current is sometimes referred to as *wattless current*.

Case (iii) *LCR series circuit*: In an *LCR* series circuit, power dissipated is given by Eq. (7.38) where $\phi = \tan^{-1}(X_c - X_L)/R$. So, ϕ may be non-zero in a *RL* or *RC* or *RCL* circuit. Even in such cases, power is dissipated only in the resistor.

Case (iv) *Power dissipated at resonance in LCR circuit*: At resonance $X_c - X_L = 0$, and $\phi = 0$. Therefore, $\cos\phi = 1$ and $P = I^2 Z = I^2 R$. That is, maximum power is dissipated in a circuit (through *R*) at resonance.

Wattless Current: → The current in a.c circuit is said to be wattless if the average power consumed in the circuit is zero.

Power-factor: → defined as the ratio of the true power to the apparent power of an a.c circuit.

$$\text{Power-factor} = \frac{\text{True Power}}{\text{Apparent Power}}$$

Apparent Power = $V_{rms} \times i_{rms}$

True Power = Apparent Power $\times \cos\phi$.

also $\boxed{\cos\phi = \frac{R}{Z}}$

Transformer

It is a device which is either used to increase or decrease the voltage in A.C circuits through mutual induction. A transformer consists of two coils wound on the same core.

The coil connected to input is called primary while the other connected to output is called secondary coil.

An alternating current passing through the primary creates a continuosly changing flux through the core. This changing flux induces an alternating emf in the secondary. As magnetic field lines are closed curves, the flux per turn of primary must be equal to flux per turn of the secondary.

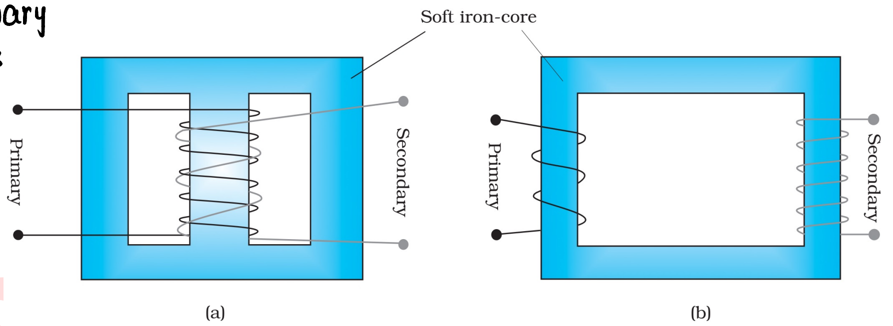


FIGURE Two arrangements for winding of primary and secondary coil in a transformer: (a) two coils on top of each other, (b) two coils on separate limbs of the core.

Therefore $\frac{\phi_P}{N_P} = \frac{\phi_S}{N_S}$ or $\frac{1}{N_P}\frac{d\phi_P}{dt} = \frac{1}{N_S}\frac{d\phi_S}{dt}$ $\left(\text{as } e \propto \frac{d\phi}{dt}\right)$ so $\boxed{\frac{e_s}{e_p} = \frac{N_s}{N_p}}$ —(1)

NOTE: → In an ideal transformer, there is no loss of power. Hence

Power input = Power Output ⇒ $e_s I_s = e_p I_p$ ⇒ $\frac{e_s}{e_p} = \frac{I_p}{I_s}$ from eq (1) $\boxed{\frac{e_s}{e_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}}$ Called as Transformation Ratio.

- In step-up transformer $N_s > N_p$. it increases voltage and reduces current.
- In step-up transformer $N_p > N_s$. it increases current and reduces voltage.
- It works only on A.C
- A transformer cannot increase (decrease) voltage and current simultaneously. As ei = constant.
- Some power is always lost due to eddy currents, hysteresis, etc.

- **Some energy losses in Transformer**

(i) *Flux Leakage*: There is always some flux leakage; that is, not all of the flux due to primary passes through the secondary due to poor design of the core or the air gaps in the core. It can be reduced by winding the primary and secondary coils one over the other.

(ii) *Resistance of the windings*: The wire used for the windings has some resistance and so, energy is lost due to heat produced in the wire (I^2R). In high current, low voltage windings, these are minimised by using thick wire.

(iii) *Eddy currents*: The alternating magnetic flux induces eddy currents in the iron core and causes heating. The effect is reduced by having a laminated core.

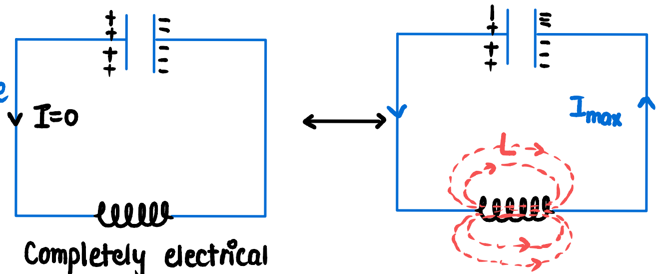
(iv) *Hysteresis*: The magnetisation of the core is repeatedly reversed by the alternating magnetic field. The resulting expenditure of energy in the core appears as heat and is kept to a minimum by using a magnetic material which has a low hysteresis loss.

- **Efficiency of transformer (η)** $= \frac{\text{Output Power}}{\text{Input Power}} = \frac{e_s I_s}{e_p I_p}$ ⇒ $\eta\% = \frac{e_s i_s}{e_p i_p} \times 100$

For ideal Transformer $\eta = 100\%$. But for Practical Transformer lies between 70% – 90%.

LC Oscillations

When a charged capacitor is allowed to discharge through a non-resistive, electrical oscillations of constant amplitude and frequency are produced. These oscillations are called LC oscillations.



Completely electrical energy

Completely Magnetic Energy

Working

When a capacitor is supplied with an AC current, it gets charged.

When this charged capacitor is connected with an inductor, current flows through inductor, giving rise to magnetic flux, hence induced emf is produced in the circuit.

Due to this, the charge (or energy) on the capacitor decreases and an equivalent amount of energy is stored in the inductor in the form of magnetic field when the discharging of the capacitor completes, current and magnetic flux linked with L starts decreasing.

Therefore, an induced emf is produced which recharges the capacitor in opposite direction.

This process of charging and discharging of capacitor is repeated and energy taken once from source keeps on oscillating between C and L. The equation of LC oscillation is given by

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0 \qquad \text{where } q = q_0\cos(\omega t + \phi)$$

and the charge oscillates with a frequency,

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

• Energy stored in Capacitor	• Energy stored in inductor
$U = \frac{1}{2}\frac{q^2}{C}$	$U = \frac{1}{2}LI^2$

The LC oscillations discussed above are not realistic for the two reasons.

i). Every inductor has some resistance. The effect of this resistance will introduce a damping effect on the charge and current in the circuit and oscillations finally die away.

ii). Even, if the resistance is zero, the total energy of the system would not remain constant. It is radiated away from the system in the form of electromagnetic waves. In fact, radio and TV transmitters depend on this radiation.

CHOKE COIL

Choke coil is a device having high inductance and negligible resistance. It is used in ac circuits for the purpose of adjusting current to any required value in such a way that power loss in a circuit can be minimised. It is used in fluroscent tubes.