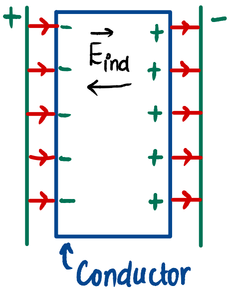
Capacitor and Capacitance

Behaviour of Conductors in Electrostatics Field.

(I). Net electrostatics Field is zero in the interior of a Conductor.



Net Field $\vec{E} = \vec{E}_{ext} - \vec{E}_{ind}$

Here $\vec{E}_{ext} = \vec{E}_{ind}$

So $\boxed{\vec{E} = 0}$

(II). At the Surface of a charged Conductor, electrostatic field must be normal to the surface at every point.

(III). The net Charge in the interior of a Conductor is zero and any excess charge resides at its surface.

(IV) Potential is constant within and on the surface of a Conductor.

we know that $E = -\frac{dV}{dr}$ inside a conductor $E = 0$ so $\frac{dV}{dr} = 0$ or $V =$ Constant

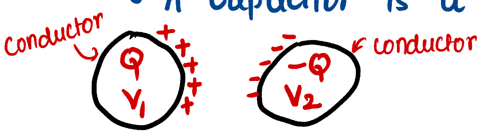
(V) Electric Field at the Surface of a Charged Conductor $\vec{E} = \frac{\sigma}{\epsilon_0}\hat{n}$ σ = Surface Charge density

Electrostatic Shielding

The phenomenon of making a region free from any electric field is Called electrostatic shielding. It is based on the fact that electric field vanishes inside the cavity of a hollow conductor.

Capacitor

- A Capacitor is a system of two Conductors separated by an insulator.



- +ve Charge is Called as the charge of the Capacitor.
- $V = V_1 - V_2$, Potential difference between the conductor.
- Capacitor is used to store charge in a small space.

Symbol of Capacitor

Capacitance / Capacity

Capacity of a Conductor is the ratio of Charge given to the conductor to the rise in its potential. it is denoted by C.

Capacity / Capacitance = $\frac{\text{Charge}}{\text{Potential}}$ i.e $\boxed{C = \frac{Q}{V}}$

Si unit of Capacitance is Farad, and farad is a very big unit.

- Capacitance depends upon the following factors

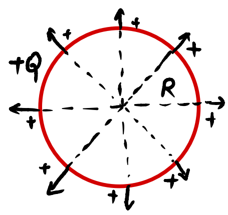
(1) Size and Shape of Conductor.

(2) Nature of the surrounding medium

(3) Presence of the other conductors in its neighbourhood.

Capacitance of an isolated Spherical Capacitor.

Consider an isolated spherical conductor of Radius. The Charge Q is uniformly distributed over its entire surface, assumed to be concentrated at the Centre of the Sphere.



- The potential at any point on the surface of the spherical conductor will be

$V = \frac{Q}{4\pi\epsilon_0 R}$ and Capacitance $C = \frac{Q}{V}$ $\Rightarrow C = 4\pi\epsilon_0 R$

i.e $C \propto R$ Radius of Conductor

• Capacitance of Earth

$C = 4\pi\varepsilon_0(\text{Radius of Earth})$ i.e $\boxed{C \simeq 711\,\mu F}$

Dielectric

Dielectric are insulating material that can produce electric effect without conduction.

Types (1). **Polar dielectric** 2). **Non-Polar dielectric**

1). Polar dielectric : The polar dielectric are made up of polar atoms/molecule, in which the centre of positive charge does not coincide with the centre of negative charge of the atom.
e.g H_2O, CO_2, NH_3 etc

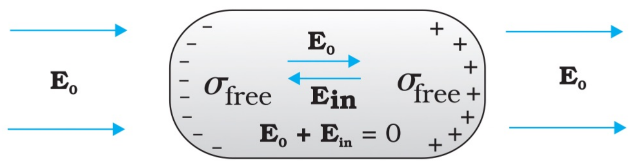
2). Non-Polar dielectric :→ The non-polar dielectric (like N_2, O_2, benzene, methane etc) are made up of non-polar atom/molecule, in which centre of positive charge coincides with the centre of negative charge of the atom molecule. e.g N_2, O_2, benzene, methane.

• Polarization in dielectric and Polarisation Vector.

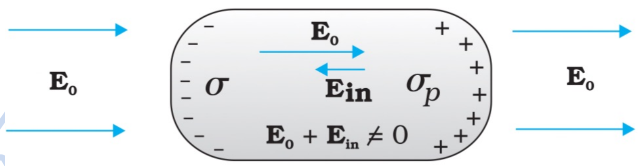
↳ When a dielectric material is placed in an electric field, dipole moment appears in any volume in it. This fact is known as Polarisation of the material. $P = \chi_e E$

↳ It is defined as the dipole moment per unit volume, and its magnitude is referred to as the polarisation.
Polarisation vector $= \dfrac{\text{Dipole moment}}{\text{Volume}}$

• Behaviour of a conductor and a dielectric in an external electric field.



Conductor



Dielectric

Here E_{in} = induced electric field.
E_0 = External electric field.
and $\vec{E_0} = -\vec{E_i}$

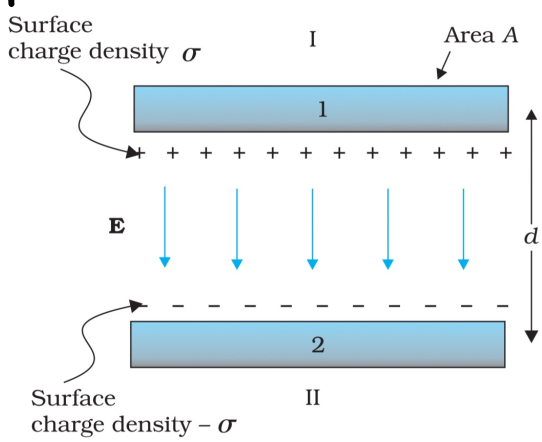
Here $\vec{E_0} \neq -\vec{E_i}$
i.e Net Electric Field $\vec{E} = \dfrac{\vec{E_0}}{K}$

Dielectric Constant (K)

The ratio of strength of the applied electric field to the strength of the reduced value of electric field on placing the dielectric between the plates of a capacitor is the dielectric constant, denoted by K. $K = \dfrac{E_0}{E}$

The Parallel Plate Capacitor

The parallel plate capacitor consists of two metal plates parallel to each other and separated by a distance d.



The field due to the two charged plates (in inner region)

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} \Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

for uniform electric field, Potential difference is

$$V = Ed \Rightarrow V = \frac{\sigma d}{\varepsilon_0} \quad \& \quad \sigma = \frac{Q}{A}$$

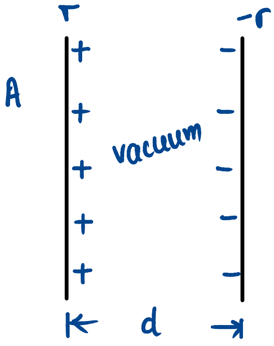
Q = Charge on Capacitor.

So $\dfrac{Q}{V} = \dfrac{\varepsilon_0 A}{d}$ i.e $\boxed{C = \dfrac{A\varepsilon_0}{d}}$

Note :→ Farad is a very big unit.

In practical the area of the plates needed to have C = 1F for a separation of 1cm then $A = \dfrac{Cd}{\varepsilon_0}$ $A = 10^9\ m^2$
which is a plate about 30 km in length and breadth.

Effect of Dielectric on Capacitance



when there is a vacuum between the plates then

$$E_0 = \frac{\sigma}{\varepsilon_0}$$

When there is a dielectric between the plates then Net Electric field E is given by

$$E = E_0 - E_{in} \qquad \text{Here } E = \frac{E_0}{K}$$

$$\frac{E_0}{K} = E_0 - E_{in}$$

$$\frac{\sigma_0}{\varepsilon_0 K} = \frac{\sigma_0}{\varepsilon_0} - \frac{\sigma_i}{\varepsilon_0} \quad \text{i.e} \quad \frac{\sigma_0}{K} = \sigma_0 - \sigma_i \quad \text{where } K = \text{dielectric Constant.}$$

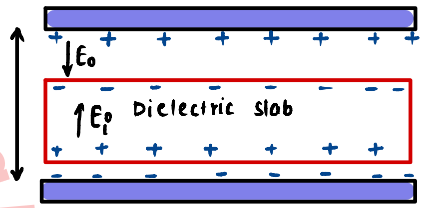
Now Potential difference $V = \frac{\sigma_0 d}{K\varepsilon_0} \Rightarrow V = \frac{Qd}{A\varepsilon_0 K} \Rightarrow \frac{Q}{V} = \frac{A\varepsilon_0 K}{d}$

$$C = \frac{Q}{V} \Rightarrow \boxed{C = C_0 K} \qquad C_0 = \frac{A\varepsilon_0}{d} \text{ [for Vacuum]}$$

↳ Capacitance increases K times.

Capacitance of a Parallel Plate Capacitor with a dielectric Slab

Partially filled.



Capacitance $C_0 = \frac{\varepsilon_0 A}{d}$ (for vacuum) & Electric field $E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$ (Vacuum)

When dielectric slab is placed partially between the plates of the Capacitor. Hence the potential difference between the Capacitor plates is

$$V = E_0(d-t) + Et \Rightarrow V = E_0(d-t) + \frac{E_0}{K}t \qquad \text{where } E = E_0/K$$

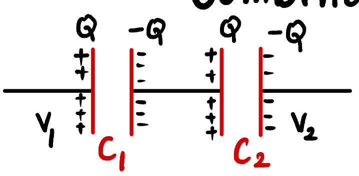
$$V = E_0\left[(d-t) + \frac{t}{K}\right] \Rightarrow V = \frac{Q}{A\varepsilon_0}\left[d - t + \frac{t}{K}\right] \Rightarrow \frac{Q}{V} = \frac{A\varepsilon_0}{d - t + t/K} \Rightarrow C = \frac{Q}{V} \Rightarrow \boxed{C = \frac{A\varepsilon_0}{d - t + \frac{t}{K}}}$$

- if the dielectric fills the entire space between the plates then $t = d$ $\quad C = K\frac{A\varepsilon_0}{d}$

if Conductor is placed between the plates of Capacitor then $C = \frac{A\varepsilon_0}{d-t}$ $\because$ $K = \infty$ for conductor

Combination of Capacitors

SERIES



In Series Charge remain Same and Potential divided

The total Potential drop V across the combination is

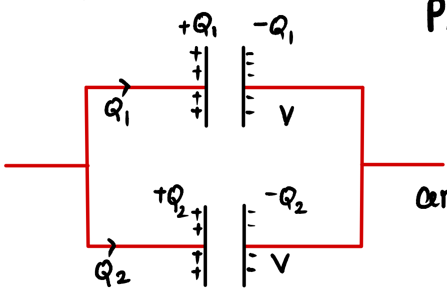
$$V = V_1 + V_2 \qquad \text{Here } V = \frac{Q}{C}$$

$$\text{and } V_1 = \frac{Q}{C_1} \quad \& \quad V_2 = \frac{Q}{C_2}$$

$$V = V_1 + V_2 \Rightarrow \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

PARALLEL



In Parallel Combination Voltage remain same and charge divided.

Total Charge $Q = Q_1 + Q_2$

$$Q = CV \qquad Q_1 = C_1V \qquad Q_2 = C_2V$$

$$CV = C_1V + C_2V$$

$$CV = (C_1 + C_2)V$$

$$\boxed{C = C_1 + C_2}$$

Energy Stored in a Capacitor

The work done in charging the Capacitor is stored as its electrical Potential energy. This energy is supplied by the battery. In charging of the Capacitor electric charges transferred from one plate to another plate.

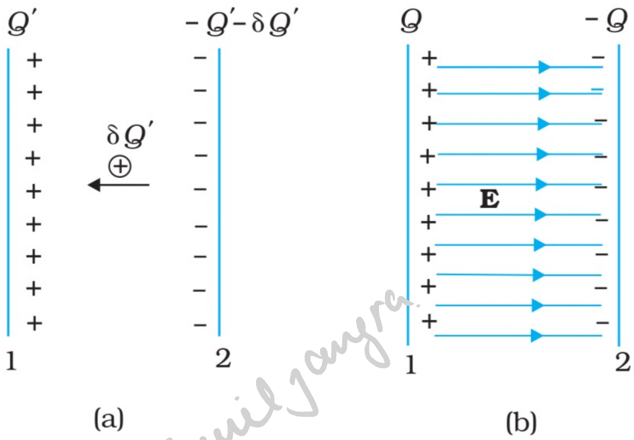


FIGURE (a) Work done in a small step of building charge on conductor 1 from Q' to $Q'+\delta Q'$. (b) Total work done in charging the capacitor may be viewed as stored in the energy of electric field between the plates.

Suppose now a small additional charge dQ' be transferred from plate 2 to plate 1. The work done will be

$$dW = V'dQ' \Rightarrow dW = \frac{Q'}{C}\cdot\delta Q'$$

Total work done in charging the Capacitor

$$W = \int dW = \int_0^Q \frac{Q'}{C}\,\delta Q' \qquad W = \left[\frac{Q'^2}{2C}\right]_0^Q$$

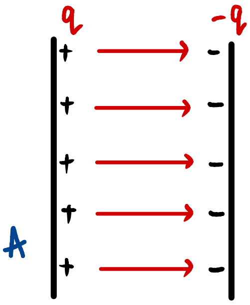
$$W = \frac{1}{2C}\left[Q^2 - 0^2\right]$$

$$W = \frac{1}{2C}Q^2$$

This work done is stored as electrical potential energy U.

$$\boxed{U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV \quad \because Q = CV}$$

Force between the plates of a Capacitor



Electric field on the negative plate due to positive charge is

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{q}{2A\varepsilon_0}$$

The magnitude of force on the charge in -ve plate is

$$F = qE \Rightarrow F = \frac{q^2}{2A\varepsilon_0}$$

Electric Susceptibility (χ_e)

It describes the electrical behaviour of a dielectric. It is a dimensionless constant. for vacuum $\chi_e = 0$

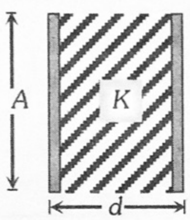
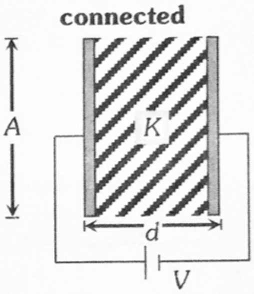
Relation between dielectric constant & electric Susceptibility can be given as

$$\boxed{K = 1 + \chi_e}$$

Energy density (u) between the plates

$$u = \frac{\text{Energy Stored}}{\text{Volume of Capacitor}} \Rightarrow u = \frac{U}{Ad} \Rightarrow u = \frac{1}{2}\varepsilon_0 E^2$$

Table Variation of different variable (Q, C, V, E and U) of parallel plate capacitor when dielectric is introduced

Quantity	Battery is Removed	Battery Remains connected
		
Capacity	$C' = KC$	$C' = KC$
Charge	$Q' = Q$	$Q' = KQ$
Potential	$V' = V/K$	$V' = V$
Intensity	$E' = E/K$	$E' = E$
Energy	$U' = U/K$	$U'' = KU$