Electric Charges and fields

Electric Charges: Electric Charge is an intrinsic property of elementary particles of matter which gives rise to electric force between various objects.

it is represented by q. The SI unit of electric charge is Coulomb (C). A proton has positive charge (+e) and an electron has negative charge (−e) where $e = 1.6 \times 10^{-19}$ C.

- Like charges repel each other and unlike charges attract each other.
- The property which differentiates the two kinds of charge is called the "Polarity" of charge.
- Charge is a scalar quantity. When some electrons are removed from the atom, it acquires a positive charge and when some electrons are added to the atom, it acquires a negative charge.
- Charge does not depend on the velocity of the particle.

Properties of electric Charge

(1). **Additivity of Charges**: Charges are additive in nature. Total charge of a system is obtained simply by adding algebraically.

Question :→ What is the total charge of a system containing five charges +1, +2, −3, +4 and −5 in some arbitrary unit?

Answer : Total Charge = $+1+2-3+4-5 = -1$ in the same unit

(2). Charge is conserved

The total charge of an isolated System remain conserved (Constant). Charge can neither be created nor be destroyed. It can only transferred from one body to another body

(3). Quantization of Charge

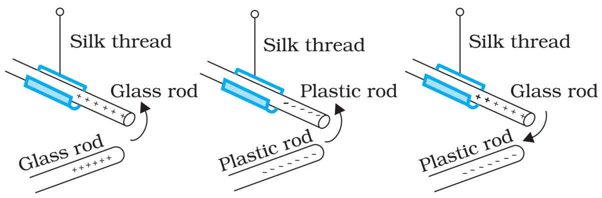
- The charge on any body will be some integral multiple of e. i.e $\boxed{q = \pm ne}$ $n = 1, 2, 3 ....$ e = quantum of charge.
- Charge on any body can never be $\left(\frac{1}{3}e\right)$, 1.5 etc.

CONDUCTORS: The substances through which electric charges can flow easily are called conductors. e.g. Metal, human body and animal bodies, graphite acids etc. are conductor.

Insulators: The substances through which electric charges cannot flow easily are called insulators. e.g Non- metal like glass, diamond, porcelain, plastic, nylon, wood, mica etc are insulators.

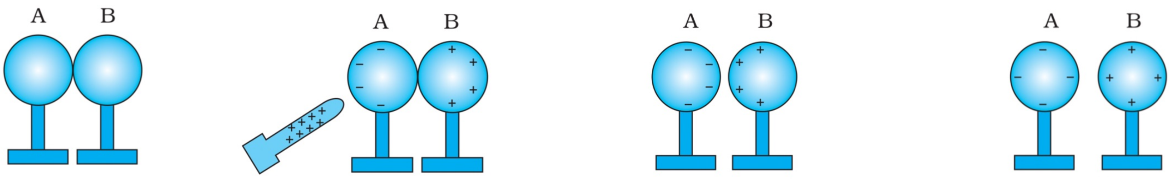
Methods of Charging

(1). Charging by rubbing: when two bodies rubbed together, due to friction some electrons from one body pass on to the other body. The body that donates the electrons become positively charged and body which gains electrons becomes negatively charged.

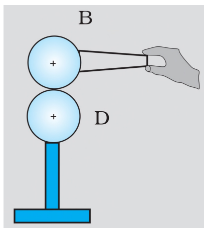


2). Charging by induction

The process of giving one object a net electric charge without touching the object to a second charged object is called "Charging by induction".



3). Charging by Contact or conduction: When a charged body is made to touch an uncharged body, some of the charge from the charged body is transferred to the other body. This is called Charging by Touch.



Q = Does in charging the mass of a body change?

Ans = Yes, as charging a body means addition and removal of electrons and electrons has a mass.

Difference between Charge and Mass

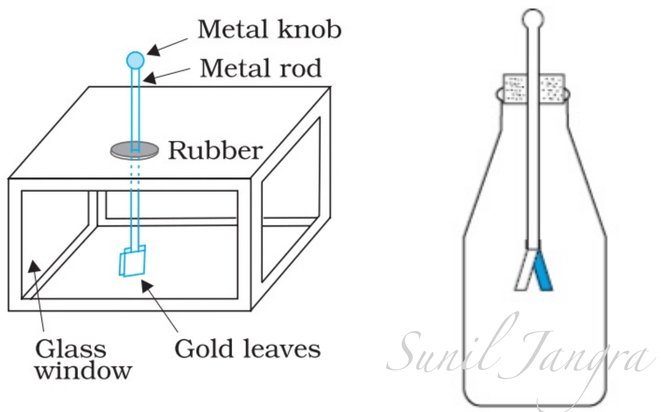
CHARGE	MASS
(1). Electric Charge can be +ve, -ve or zero.	1). Always a positive quantity.
(2). Charge on a body does not depend upon velocity of the body.	2). Mass of a body increases with its velocity as $m = \frac{m_0}{\sqrt{1-v^2/c^2}}$ C = Velocity of light in vacuum, m = mass of the body moving with velocity v and m_0 rest mass of the body.
3). Charge is quantized.	3). The quantization of mass is yet to be eastablish.
4). Electric charge is always conserved.	4). Mass is not conserved and it can be changed into energy and vice-versa.
5). Force b/w two point charges can be attractive, or repulsive.	5). The Gravitational force between two Masses is always attractive.

Objects acquiring two kinds of charges on rubbing

Positive Charge	Negative Charge
1. Glass rod	1). Silk Cloth
2). Fur or woolen cloth	2). Ebonite, Amber, Rubber rod
3). Woolen Coat	3). Plastic Seat
4). Woolen Carpet	4). Rubber shoes
5). Nylon or Acetate	5). Cloth
6). Dry hair.	6). Comb.

Gold Leaf Electroscope

A simple apparatus to detect charge on a body and it's polarity is the Gold leaf Electroscope. it consist of a vertical metal rod housed in a box, with two thin gold leaves attached to its bottom end. When a charged object touches the metal knob at the top of the rod, charge flow on to the leaves and they diverge. The degree of divergance is an indicator of the amount of charge.



Note: Charges were named as positive and negative by the American scientist Benjamin franklin.

Coulomb's Law: → It states that "the force of interaction between any two point charges is directly proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between them.

q_1 •———————• q_2
r

i.e $F = k\frac{|q_1 q_2|}{r^2}$ where k = Constant of proportionality i.e $k = \frac{1}{4\pi\varepsilon_0 K}$ for vacuum K=1

K = dielectric Constant also known as permittivity.

ε_0 = absolute permittivity = $8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$ $k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2/C^2$

for medium $F_m = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{r^2}$ → for medium

$\frac{F_m}{F} = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{r^2} \times \frac{r^2\, 4\pi\varepsilon_0}{q_1 q_2}$ i.e $\frac{F_m}{F} = \frac{1}{K}$ or $F_m = \frac{F}{K}$

so we can say that force decreases K time on introducing medium b/w the charges.

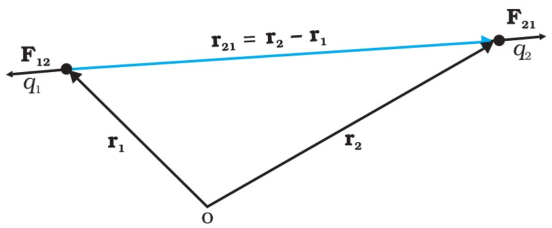
F_m = force when any medium is Introduced between the charges

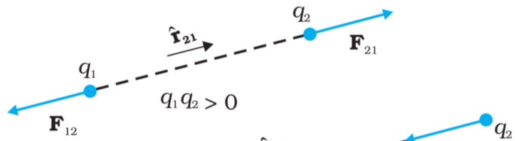
F = " when vacuum / air between the charges.

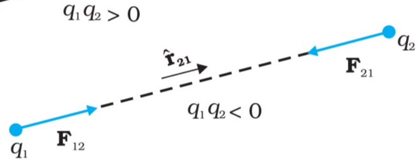
Dielectric Constant or Relative Permittivity $\boxed{K = \frac{\varepsilon}{\varepsilon_0}}$ or $\boxed{\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}}$ $= \frac{\text{Permittivity of any medium}}{\text{Permittivity of free Space.}}$

Coulomb's law in Vector form:

$\vec{F}_{12}$ = force on q_1 due to q_2
$\vec{F}_{21}$ = force on q_2 due to q_1







Acc. to Coulomb's Law

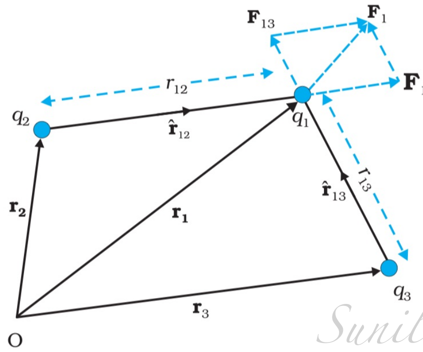
$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

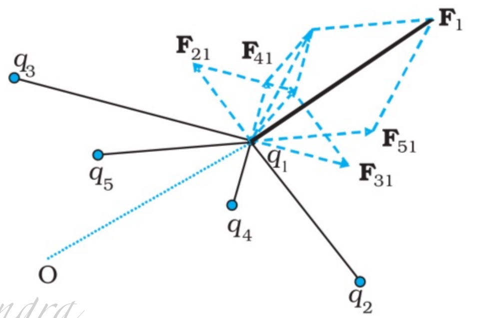
similarly $\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21}$ where $\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|}$ & $\hat{r}_{21} = \frac{\vec{r}_{21}}{|\vec{r}_{21}|}$ $\hat{r}_{12}$ & $\hat{r}_{21}$ are unit vectors.

Here $\vec{F}_{12} = -\vec{F}_{21}$ i.e $\vec{F}_{12} + \vec{F}_{21} = 0$ Coulomb's law agrees with the newton's third law.

Forces between Multiple Charges / Superposition Principle

Acc. to the principle of superposition
" total force on a given charge due to number of charges is the vector sum of individual forces acting on that charge due to the presence of other charges.





i.e $\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13}$

or

$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \_\_\_\_\_ + \vec{F}_{1n}$

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}^2}\hat{r}_{12} + \frac{q_1q_3}{r_{13}^2}\hat{r}_{13} + \_\_\_\_ + \frac{q_1q_n}{r_{1n}^2}\hat{r}_{1n}\right]$$ i.e $$\vec{F}_1 = \frac{q_1}{4\pi\varepsilon_0}\sum\frac{q_i}{r_{1i}^2}\hat{r}_{1i}$$

Note:→ The Vector sum is obtained as usual by the Parallelogram law of addition of vectors

Unit of Charge: from Coulomb's Law $F = k\frac{q_1q_2}{r^2}$ if $q_1 = q_2 = 1C$ and $r = 1m$

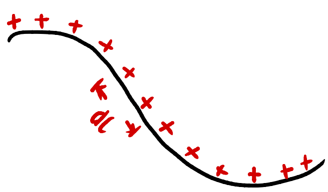
then $F = 9\times10^9$ N

Continuous Charge Distribution

The region in which charges are closely spaced is said to have continuous distribution of charge.
it is of three types.

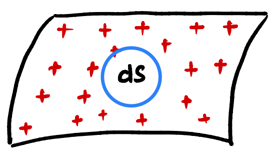
1). Linear Charge distribution
The distribution in which charge is distributed along the length of a body or object.
linear charge density is defined as the charge per unit length. it is denoted by λ.

$\lambda = \frac{dq}{dl}$ SI unit $= \frac{C}{m} = Cm^{-1}$



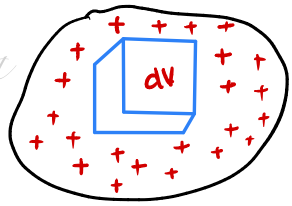
2). Surface Charge distribution
when the Charge is distributed or spread over a two dimensional surface S is Called Surface Charge distribution.
Surface Charge density is defined as the Charge per unit area. It is denoted by σ.

$\sigma = \frac{dq}{ds}$ SI unit $= \frac{C}{m^2} \Rightarrow Cm^{-2}$



3). Volume Charge Distribution
When the charge is distributed over a three dimensional volume or region V, is Called as Volume Charge distribution.
Volume Charge distribution is defined as the charge per unit volume. it is denoted by ρ.

$\rho = \frac{dq}{dV}$ SI unit $= Cm^{-3}$



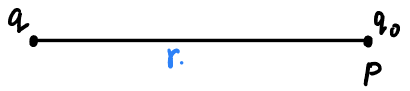
Electric field:→ The space in the surrounding of any charge in which its influence can be experienced by other charges is called Electric Field. It is denoted by E.

Electric Field Intensity: The electrostatic force acting per unit positive (test) charge in electric field is called electric field Intensity at that point. It is denoted by $\vec{E}$.

i.e $\vec{E} = \dfrac{\vec{F}}{q_0}$ force experienced by test charge q_0.   SI unit of $\vec{E}$ is N/C (Newton/Coulomb)

it is a vector quantity and its direction is in the direction of electrostatic force acting on positive charge.

Electric Field due to a Point Charge



The force between two charges is given by coulomb's law.

$F = k\dfrac{q q_0}{r^2}$ where q = Source charge and q_0 = Test charge

Electric field intensity due to q at point P is $\vec{E} = \dfrac{\vec{F}}{q_0}$ or $E = \dfrac{F}{q_0}$ ⇒ $E = k\dfrac{q q_0}{r^2} \times \dfrac{1}{q_0}$

$\boxed{E = \dfrac{kq}{r^2}}$

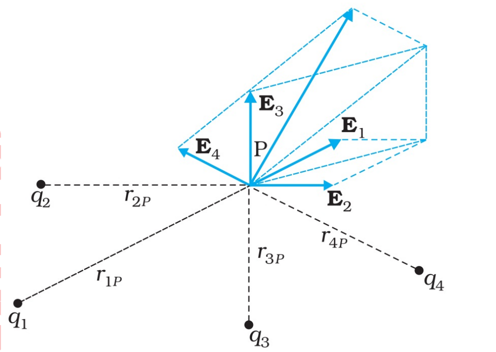
- if q > 0 i.e positive charge $\vec{E}$ is directed away from source
- if q < 0 i.e negative charge $\vec{E}$ is directed towards the source charge.

Electric Field due to System of Charges

The electric field at point P due to the system of n charges is $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \_\_\_\_ + \vec{E}_n$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2P}^2}\hat{r}_{2P} + \_\_\_\_ + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_{nP}^2}\hat{r}_{nP}$$

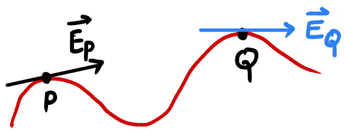
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{iP}^2}\hat{r}_{iP}$$

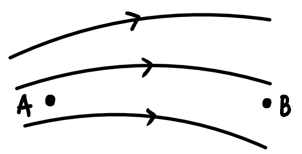


$\vec{E}$ is a vector quantity that varies from one point to another point in space and is determined from the positions of the source charges.

Electric Field lines

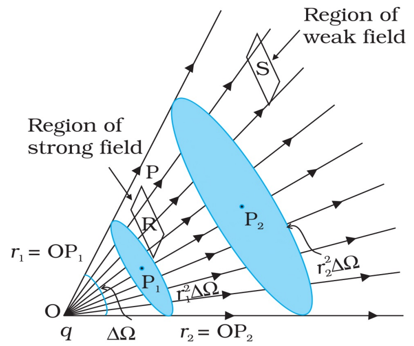
An electric field line is an imaginary line or curve drawn through a region of space so that its tangent at any point is in the direction of the electric field vector at that point.





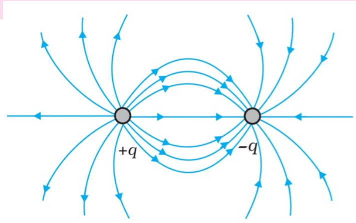
$|E_A| > |E_B|$

, The relative closeness of the lines at some place give an idea about the intensity of electric field at that point.



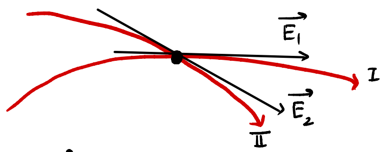
Properties of Electric field lines

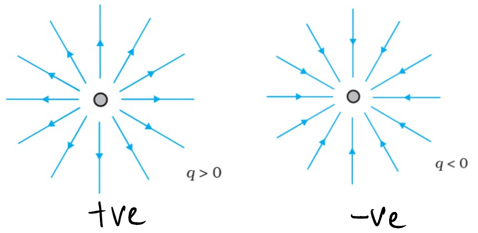
(1). Electric field lines start from positive charge and ends at negative charges. if there is a single charge, they may start or end at infinity.



2). In a charge free region, electric field lines can be taken to be continuous curves without any breaks.

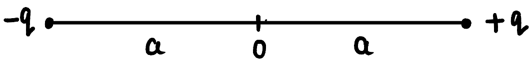
3). Two field lines can never cross each other because at the point of intersection the Electric field Intensity will have two direction which is not possible.





4) Electric field lines do not form any closed loops, due to conservative nature of electric field.

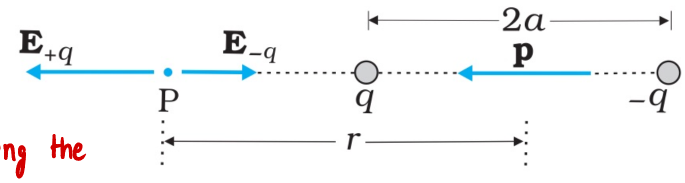
Electric Dipole: It is a System of two equal and opposite charges separated by a Certain distance (Dipole length 2a).



Electric dipole moment: Electric Dipole moment is a vector $\vec{P}$ directed from negative to positive Charge. The magnitude of dipole moment is $P = q(2a)$ and vector form $\vec{P} = q\,2\vec{a}$.

THE FIELD OF AN ELECTRIC DIPOLE

(i) For points on the axis



Let the point P be at distance r from the Centre of dipole on the side of the charge q, as shown in fig.

Then $\vec{E}_{-q} = -\dfrac{q}{4\pi\varepsilon_0 (r+a)^2}\hat{P}$ — ① ($\hat{P}$ is the unit vector along the dipole axis (from −q to q).

also $\vec{E}_{+q} = \dfrac{q}{4\pi\varepsilon_0 (r-a)^2}\hat{P}$ — ②

The total Electric Field at point P is $\vec{E} = \vec{E}_{+q} + \vec{E}_{-q}$

we get $\vec{E} = \dfrac{q}{4\pi\varepsilon_0}\dfrac{4ar}{(r^2-a^2)^2}\hat{P}$ and magnitude $E = \dfrac{P\,2r}{4\pi\varepsilon_0 (r^2-a^2)^2}$ $(P = q2a)$

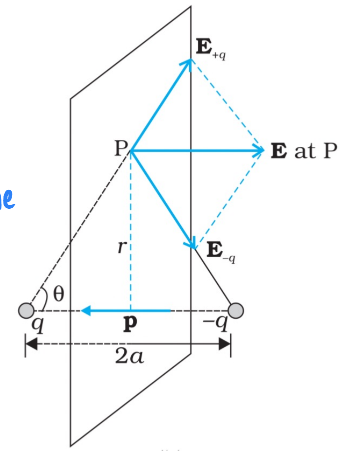
For r>>a (for short dipole)

$E = \dfrac{2kP}{r^3}$ where $k = \dfrac{1}{4\pi\varepsilon_0}$

NOTE: Total Electric Field & Dipole moment both are in same direction.

ii) For Points on the equatorial plane :→ The magnitudes of the electric fields due to the two Charges +q and −q are given by

$E_{+q} = \dfrac{q}{4\pi\varepsilon_0}\dfrac{1}{r^2+a^2}$ —① & $E_{-q} = \dfrac{q}{4\pi\varepsilon_0}\dfrac{1}{(r^2+a^2)}$ —(2). are equal



The direction of $\vec{E}_{+q}$ and $\vec{E}_{-q}$ are shown in figure. Clearly the Components along the dipole axis cancel away. The Components along the dipole axis add up. The total electric field is opposite to $\hat{P}$. $\vec{E} = -(E_{+q} + E_{-q})\cos\theta\,\hat{P}$

$\vec{E} = -\dfrac{2qa}{4\pi\varepsilon_0 (r^2+a^2)^{3/2}}\hat{P}$

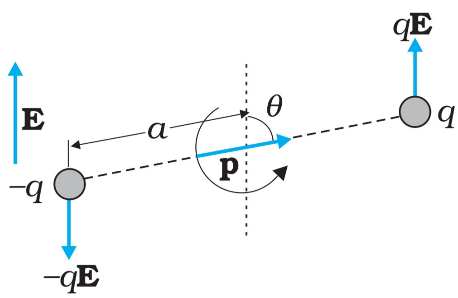
For short Dipole i.e r>>a

$\vec{E} = -\dfrac{2qa}{4\pi\varepsilon_0 r^3}\hat{P}$ and Magnitude $\boxed{E = \dfrac{kP}{r^3}}$

Electric field due to dipole at any point from centre of Dipole is

$E = \dfrac{kP}{r^3}\sqrt{3\cos^2\theta + 1}$

Dipole in a Uniform External Field



consider a permanent dipole of dipole moment $\vec{P}$ in a uniform external field $\vec{E}$, as shown in fig. There a force $q\vec{E}$ on q and $-q\vec{E}$ on −q. The net force on dipole is zero, since $\vec{E}$ is uniform.

When the net force is zero, the torque is independent of the origin.

Torque = Magnitude of either force x perpendicular distance between the two antiparallel forces.

$\tau = qE \times 2a\sin\theta \Rightarrow \tau = PE\sin\theta$ i.e $\vec{\tau} = \vec{P}\times\vec{E}$

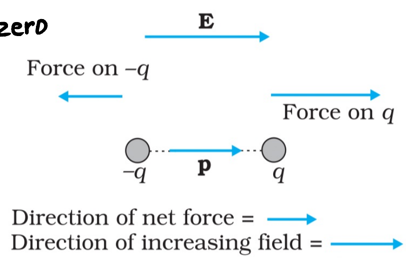
τ is a vector quantity and its direction is normal to the paper, coming out of it.

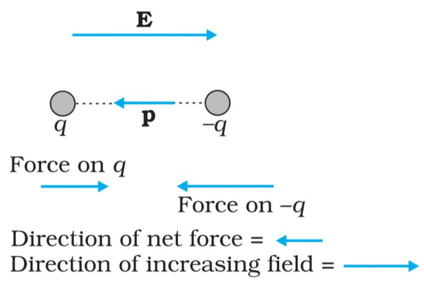
This torque will tend to align the dipole with the field $\vec{E}$.

when $\vec{P}$ is alinged to $\vec{E}$, the torque is zero i.e $\tau = PE\sin 0^\circ = 0$

Dipole in a Non-Uniform External field

In that Case net force on dipole is non-zero. Hence dipole will experience both force and torque when placed in non-uniform electric field.





Potential Energy of Dipole

When an electric dipole is placed in an electric field $\vec{E}$, a torque $\vec{\tau} = \vec{P} \times \vec{E}$ acts on it. if we rotate the dipole through a small angle $d\theta$ opposite to the torque is

$dW = \tau d\theta \Rightarrow dW = -PE$ The change in electric Potential energy of the dipole is there $dU = -dW$

so $dU = PE\sin\theta \, d\theta$

Integrating $\int_0^U dU = \int_{90^\circ}^{\theta} PE\sin\theta \, d\theta \Rightarrow U = PE[-\cos\theta]_{90}^{\theta} \Rightarrow U = -PE\cos\theta \Rightarrow U = -\vec{P}\cdot\vec{E}$

Equilibrium of Dipole

$U = -PE\cos\theta$

(i) if $\theta = 0^\circ$

$U = -PE$ (minimum)

$\vec{F}_{Net} = 0 \quad \vec{\tau} = 0$

Stable equilibrium position.

(ii) if $\theta = 180^\circ$

$U = +PE$

$\vec{F}_{Net} = 0 \quad \vec{\tau} = 0$

• unstable equilibrium position.

Electric Flux

Electric flux over an area is equal to the total number of electric field line crossing this area

Electric flux $\Delta\phi$ through an area element ΔS is given by

$\Delta\phi = \vec{E}\cdot\vec{\Delta S} = E\Delta S\cos\theta$

ΔS area vector

$\vec{E}$ Electric field Intensity.

SI unit = Nm^2/C

if $\theta = 0^\circ$

then $\Delta\phi = E\Delta S\cos 0^\circ$

$\Delta\phi = E\Delta S$

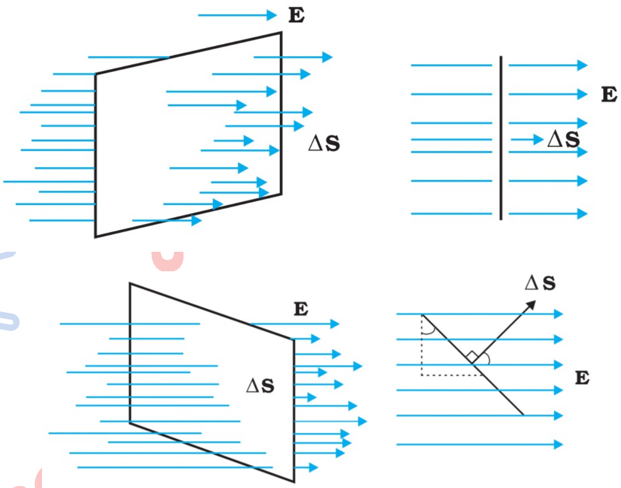
Maximum flux.

if $\theta = 90^\circ$

then $\Delta\phi = E\Delta S\cos 90^\circ$

$\Delta\phi = 0$

$\because \cos 90^\circ = 0$.

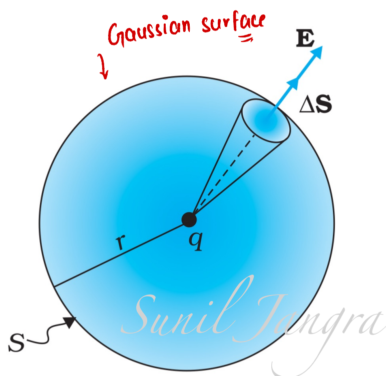


Gauss' Theorem

The electric flux over any closed surface is $\frac{1}{\varepsilon_0}$ times the total charge enclosed by that surface. i.e

$$\phi_E = \oint \vec{E}\cdot\vec{ds} = \frac{1}{\varepsilon_0}\Sigma q$$

q = enclosed charge.



if a charge q is placed at the centre of a cube then total electric flux linked with the whole cube $= \frac{q}{\varepsilon_0}$

& electric flux with one face of the cube $= \frac{q}{6\varepsilon_0}$

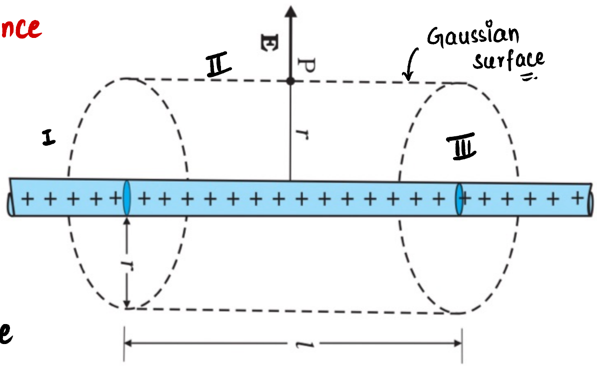
Application-I : Field due to an infinitely long straight uniformly charged wire.

Consider a long line charge with a linear charge density λ.

we have to calculate the electric field at a point, a distance r from the line charge.

Acc. to Gauss Theorem

$\phi = \frac{q}{\varepsilon_0} \Rightarrow \int \vec{E}\cdot\vec{ds} = \frac{q}{\varepsilon_0} \Rightarrow \oint_{I} \vec{E}\cdot\vec{ds} + \oint_{II} \vec{E}\cdot\vec{ds} + \oint_{III} \vec{E}\cdot\vec{ds} = \frac{q}{\varepsilon_0}$



for I & III faces $\vec{E}$ and $\vec{ds}$ perpendicular to each other

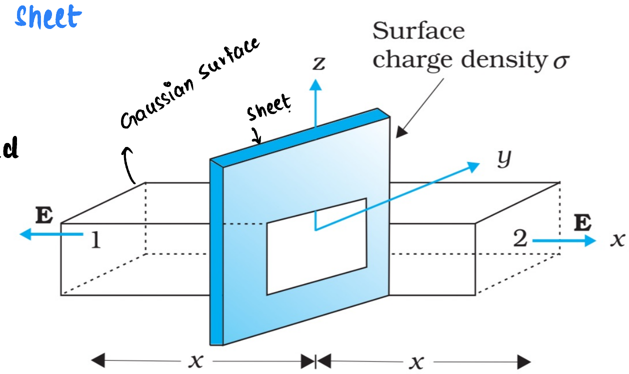
i.e $\theta = 90^\circ$ & $\cos 90^\circ = 0$. so we will Consider only II Phase because $\vec{E}$ & $\vec{ds}$ are parallel i.e $\theta = 0^\circ$.

$$\oint_{II} E\,ds\cos 0^\circ = \frac{q}{\varepsilon_0} \Rightarrow \oint E\,ds = \frac{q}{\varepsilon_0} \Rightarrow E\oint ds = \frac{q}{\varepsilon_0} \Rightarrow E \times 2\pi r l = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{2\pi r l \varepsilon_0} \Rightarrow \boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}} \quad \text{i.e } E \propto \frac{1}{r}$$

(Note $\int ds$ = Curved surface area ($2\pi r l$) & $\lambda = q/l$

Application : 2 → Field due to a uniformly charged infinite plane sheet

Let σ be the uniform surface charge density of an infinite plane sheet
Plane of the sheet is Normal to the x-axis.
A = cross-sectional area of gaussian surface.
From figure we can say that only face 1 & 2 will considered here. Acc. to Gauss's Theorem



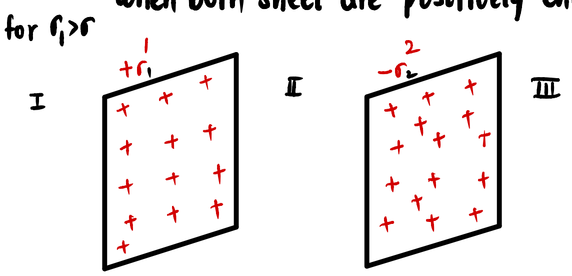
$$\oint_1 \vec{E}\cdot d\vec{s} + \oint_2 \vec{E}\cdot d\vec{s} = \frac{q}{\varepsilon_0} \Rightarrow EA + EA = \frac{q}{\varepsilon_0} \Rightarrow 2EA = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{2A\varepsilon_0} \quad \text{and } \sigma = \frac{q}{A} \quad E = \frac{\sigma}{2\varepsilon_0}$$

(Note $\int ds = A$)

For two parallel Charged sheet

when both sheet are positively charged

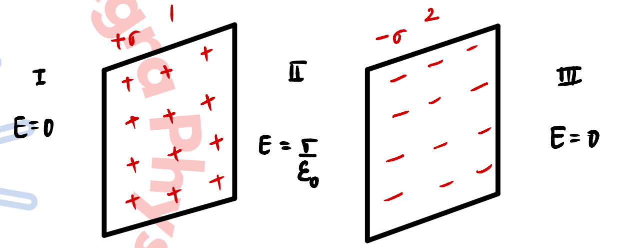


For region I $E = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0}$

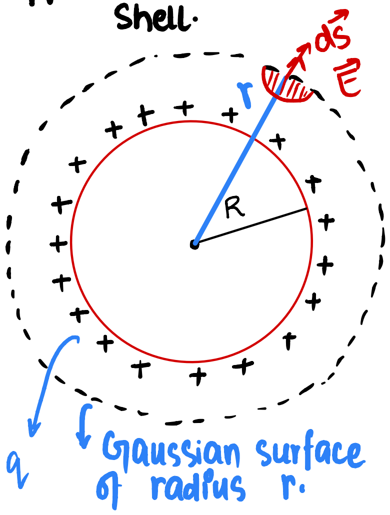
for region II $E = \frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0}$

For III region $E = \frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0}$

when both sheet are oppositively charged.



Application - III Field due to uniformly charged thin Spherical Shell.



Let σ be the uniform surface charge density of an uniformly charged thin spherical shell.

Case I when $r > R$ {outside the sphere}
Acc. to Gauss law

$$\oint \vec{E}\cdot d\vec{s} = \frac{q}{\varepsilon_0} \Rightarrow \int E\,ds\cos\theta = \frac{q}{\varepsilon_0}$$

Here $\theta = 0^\circ$ $\vec{E}$ & $d\vec{s}$ are in same direction.
$\cos 0^\circ = 1$

ie $\oint E\,ds = \frac{q}{\varepsilon_0}$ & $q = \sigma 4\pi R^2$

$$E\int ds = \frac{\sigma 4\pi R^2}{\varepsilon_0} \quad \& \int ds = 4\pi r^2$$

$$E \times \cancel{4\pi} r^2 = \frac{\sigma}{\varepsilon_0}\cancel{4\pi} R^2 \qquad \boxed{E = \frac{\sigma R^2}{\varepsilon_0 r^2}}$$

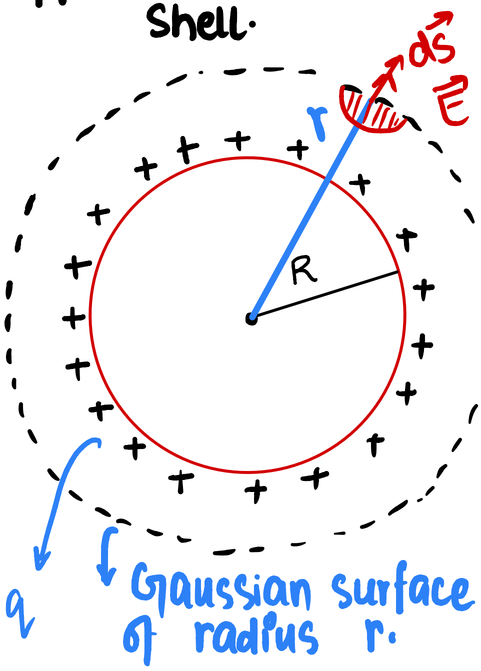
Case-II ($r=R$) on the sphere

$$E = \frac{\sigma \cancel{R^2}}{\varepsilon_0 \cancel{R^2}} \qquad E = \frac{\sigma}{\varepsilon_0}$$

Case-III ($r<R$) inside the sphere.

$$\boxed{E=0}$$

Application – III Field due to uniformly charged thin spherical Shell.



Let σ be the uniform surface charge density of an uniformly charged thin spherical shell.

Case I when $r > R$ {outside the sphere}

Acc. to Gauss law

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Here $\theta = 0^\circ$ $\vec{E}$ & $\vec{ds}$ are in same direction.

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ie $\oint E\,ds = \frac{q}{\varepsilon_0}$ & $q = \sigma 4\pi R^2$

$$E\int ds = \frac{\sigma 4\pi R^2}{\varepsilon_0}$$

& $\int ds = 4\pi r^2$

$$E \times \cancel{4\pi} r^2 = \frac{\sigma}{\varepsilon_0}\cancel{4\pi} R^2$$

$$\boxed{E = \frac{\sigma R^2}{\varepsilon_0 r^2}}$$

Case-II ($r = R$) on the sphere

$$E = \frac{\sigma \cancel{R^2}}{\varepsilon_0 \cancel{R^2}} \qquad E = \frac{\sigma}{\varepsilon_0}$$

Case-III ($r < R$) inside the Sphere.

$$\boxed{E = 0}$$

IMPORTANT TOPIC OF THE CHAPTER

- Gauss's Theorem
- it's Application
- Electric flux

- Electric Field due to Dipole → Equatorial line; ↓ Axial line

- Torque on a dipole
- Equilibrium of Dipole
- Coulomb's Law
- Vector form of Coulomb's Law.