Multilevel Analysis 1(level1:individual)

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Description of dataset

log.radon	basement	uranium	county	county.name
0.7885	1	-0.6890	1	AITKIN
0.7885	0	-0.6890	1	AITKIN
1.0647	0	-0.6890	1	AITKIN
0.0000	0	-0.6890	1	AITKIN
1.1314	0	-0.8473	2	ANOKA

Dataset: Radon data

- **House variables:**
  - **log.radon**:Numeric. Logarithm of indoor radon measurement values
  - **basement**:Binary. Measurement position indication (0=basement/1=1st floor)
- **County variables:**
  - **county**:Factor. ID of each county with 85 levels
  - **county.name**:Factor. Name of each county
  - **uranium**:Numeric. Average uranium in the soil of each county

Analysis method and model specification

Random intercept model:

Level 1: $Y_{ij} = \beta_{0j} + \beta_{1j}\text{Basement}_{ij} + R_{ij} \quad R_{ij} \sim \mathcal{N}(0, \sigma^2)$
Level 2: $\beta_{0j} = \gamma_1 + U_j \quad U_j \sim \mathcal{N}(0, \tau_0^2)$
Combined: $Y_{ij} = \gamma_1 + \beta_{1j}\text{Basement}_{ij} + U_j + R_{ij}$
$Y_{ij} = \text{log.radon}_{ij}$

Parameters:

- γ_1: Fixed effect. Mean intercept across all counties, controlling for Basement.
- β_{1j}: Fixed effect. Mean Basement slope across all counties.
- σ^2: Random effect. Variance of county intercepts around γ_1, controlling for Basement.
- τ_0^2: Random effect. Variance of the houses around their county mean, controlling for Basement.

Analysis method and model specification

```r
library(lme4)
Basement.fixed <- lmer(
  log.radon ~ 1 + basement + (1 | county),
  data = radon)
```

```
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  county   (Intercept) 0.1077   0.3282
##  Residual             0.5709   0.7556
## Number of obs: 919, groups:  county, 85
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  1.46160    0.05158  28.339
## basement    -0.69299    0.07043  -9.839
```

```
## # Intraclass Correlation Coefficient
##
##     Adjusted ICC: 0.159
##   Unadjusted ICC: 0.145
```

Results

Random intercept model:

By default, we use REML to estimate parameters:

$$\text{log.radon}_{ij} = 1.462 - 0.693\text{Basement}_{ij} + U_j + R_{ij}$$
$$U_j \sim \mathcal{N}(0, 0.108)$$
$$R_{ij} \sim \mathcal{N}(0, 0.571)$$

ICC

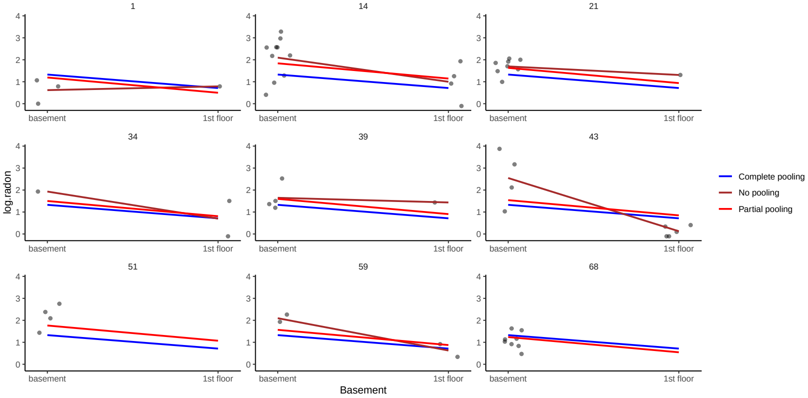
Adjusted ICC: 0.159

Unadjusted ICC: 0.145

Interpretation:

- Adjusted:
  County grouping accounts for 15.9% of the total variance of the log.radon under the control of Basement.
- Unadjusted:
  County grouping accounts for 14.5% of the total variance of the log.radon

Pooling



- Complete pooling assumes no variance between counties, but it's often larger.
- No pooling assumes larger variance between counties(independence), but it's often smaller.
- Partial pooling is in between.

Model comparison

Different fixed part(with level 2 predictor)

Model		Equation			Deviance	df
$\mathcal{M}_0$	$Y_{ij} =$	$\gamma_1 + \beta_{1j}\text{Basement}_{ij}$	$+$	$U_j + R_{ij}$	$D_0 = 2163.7$	$v_0 = 4$
$\mathcal{M}_1$	$Y_{ij} =$	$\gamma_1 + \beta_{1j}\text{Basement}_{ij} + \gamma_2\text{Uranium}_j$	$+$	$U_j + R_{ij}$	$D_1 = 2122.8$	$v_1 = 5$

Since models only have difference of fixed part, thus we **must** use ML to estimate parameters:

```
Basement.fixed         <- lmer(log.radon ~ 1 + basement +           (1 | county), data = radon)
Uranium.Basement.fixed <- lmer(log.radon ~ 1 + basement + uranium + (1 | county), data = radon)
anova(Basement.fixed,Uranium.Basement.fixed,
      refit=TRUE)
```

```
## Data: radon
## Models:
## Basement.fixed: log.radon ~ 1 + basement + (1 | county)
## Uranium.Basement.fixed: log.radon ~ 1 + basement + uranium + (1 | county)
##                        npar    AIC    BIC  logLik -2*log(L)  Chisq Df
## Basement.fixed            4 2171.7 2190.9 -1081.8    2163.7
## Uranium.Basement.fixed    5 2132.8 2156.9 -1061.4    2122.8 40.834  1
##                        Pr(>Chisq)
## Basement.fixed
## Uranium.Basement.fixed  1.658e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Model comparison

Different fixed part(with level 2 predictor)

Model		Equation			Deviance	df
$\mathcal{M}_0$	$Y_{ij} =$	$\gamma_1 + \beta_{1j}\text{Basement}_{ij}$	$+$	$U_j + R_{ij}$	$D_0 = 2163.7$	$v_0 = 4$
$\mathcal{M}_1$	$Y_{ij} =$	$\gamma_1 + \beta_{1j}\text{Basement}_{ij} + \gamma_2\text{Uranium}_j$	$+$	$U_j + R_{ij}$	$D_1 = 2122.8$	$v_1 = 5$

- Test statistic: $D_0 - D_1 \sim \mathcal{X}_1^2$
- $D_0 - D_1$=2163.7-2122.8=40.834
- p=P[$D_0 - D_1$>40.834]<.001

Conclusion

At $\alpha = 5\%$, we reject $\mathcal{M}_0$.

Adding Uranium significantly imporves the model fit.

Model comparison

Different random part(with random slope)

Model	Equation	Deviance	df
$\mathcal{M}_0$	$Y_{ij} = \gamma_1 + \gamma_2 \text{Uranium}_j + \beta_{1j} \text{Basement}_{ij} + U_{0j} + R_{ij}$	$D_0 = 2134.2$	$v_0 = 5$
$\mathcal{M}_1$	$Y_{ij} = \gamma_1 + \gamma_2 \text{Uranium}_j + \beta_{1j} \text{Basement}_{ij} + U_{0j} + U_{1j} \text{Basement}_{ij} + R_{ij}$	$D_1 = 2128.6$	$v_1 = 7$

Since models only have difference of random part, thus we **should** use REML to estimate parameters:

```
Uranium.Basement.fixed  <- lmer(log.radon ~ 1 + basement + uranium + (1 | county), data = radon)
Uranium.Basement.random <- lmer(log.radon ~ 1 + basement + uranium + (1 + basement|county), data = radon)
anova(Uranium.Basement.fixed,Uranium.Basement.random,
      refit=FALSE)
```

```
## Data: radon
## Models:
## Uranium.Basement.fixed: log.radon ~ 1 + basement + uranium + (1 | county)
## Uranium.Basement.random: log.radon ~ 1 + basement + uranium + (1 + basement | county)
##                         npar    AIC    BIC  logLik -2*log(L)  Chisq Df
## Uranium.Basement.fixed     5 2144.2 2168.3 -1067.1    2134.2
## Uranium.Basement.random    7 2142.6 2176.4 -1064.3    2128.6 5.5459  2
##                         Pr(>Chisq)
## Uranium.Basement.fixed
## Uranium.Basement.random    0.06248 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Model comparison

Different random part(with random slope)

Model	Equation	Deviance	df
$\mathcal{M}_0$	$Y_{ij} = \gamma_1 + \gamma_2\text{Uranium}_j + \beta_{1j}\text{Basement}_{ij} + U_{0j} + R_{ij}$	$D_0 = 2134.2$	$v_0 = 5$
$\mathcal{M}_1$	$Y_{ij} = \gamma_1 + \gamma_2\text{Uranium}_j + \beta_{1j}\text{Basement}_{ij} + U_{0j} + U_{1j}\text{Basement}_{ij} + R_{ij}$	$D_1 = 2128.6$	$v_1 = 7$

- Test statistic: $D_0 - D_1 \sim \mathcal{X}_2^2$
- $D_0 - D_1$=2134.2-2128.6=5.5459
- p=P[$D_0 - D_1$>5.5459]=0.0628

Conclusion

At $\alpha = 5\%$, we fail to reject $\mathcal{M}_0$.
There is not enough evidence that adding random slopes for Basement imporves model fit.
We decude to retain $\mathcal{M}_0$.

All codes and dataset are avaliable in github