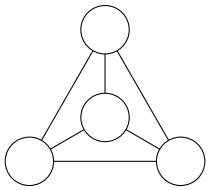
New Zealand Mathematical Olympiad Committee

Maths Workshop

March 2023

Problems

1. In the diagram below we write a number in each circle. We then label each line segment with the sum of the two numbers in the circles on the end of the line segment. For which quadruples of numbers can we obtain the numbers $0, 1, 2, 3, 4, 5$ (in some order) on the line segments?



2. Given the following eight symbols, your task is to construct a large number.

   $$1 \quad 2 \quad 3 \quad 4 \quad (\quad) \quad . \quad -$$

   Each digit may only be used once, the other symbols may be used zero or more times. For example, a teenager in Florida came up with $4^3 - 12 = 52$. What is the largest number you can achieve?

3. The *reverse* of any two-digit positive integer can be obtained by interchanging it's digits. I have a pair of 2-digit positive integers. When I add my integers I obtain S. If I add the reverses of my two integers, I obtain $4S$. Determine all possible pairs of 2-digit positive integers that I might have.

4. Let ABC be a triangle with sidelengths: $AB = 4$, $AC = 10$ and $BC = 7$. Let D be the point on side BC (between B and C) such that AD bisects angle $\angle BAC$. What is the length of BD?

5. In how many ways can the letters of MISSISSIPPI be rearranged so that no two S's are adjacent?

6. Let a, m, and n be positive integers, with $a > 1$, and m odd. What is the greatest common divisor of $a^m - 1$ and $a^n + 1$?