New Zealand Mathematical Olympiad Committee

Maths Workshop August 2025

mathsolympiad.org.nz/workshops/

Problems

1. Find all integers n such that $n^2 + 1$ is a multiple of $n + 1$.

2. How many ordered triples (a, b, c) of odd integers exist such that $a + b + c = 99$?

3. Triangle ABC has sidelengths 5, 7 and 8. Find the radius of its incircle[1].

4. Show that there are infinitely many integer solutions to the following equation.

$$4xy + x + y = z^2.$$

5. Let $\mathbb{Z}^+$ denote the set of positive integers. Find all functions $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ which has the following properties:
   - For all x and y, $f(xy) = f(x)f(y)$,
   - $f(30) = 1$, and
   - For any n whose last digit is 7, $f(n) = 1$.

6. Do there exist positive integers n and m such that

$$m < n\sqrt{103} < m + \frac{1}{n\sqrt{103}} \ ?$$

[1] An incircle is the largest circle inside the triangle; the circle which is tangent to all three sides of the triangle