New Zealand Mathematical Olympiad Committee

Maths Workshop

February 2023

Problems

1. Determine the number of positive divisors of $10^6 - 1 = 999999$.
2. Two squares of sidelength 1 have a common center. Show that the area of their intersection is greater than $\frac{3}{4} = 0.75$.
3. Find all real numbes x and y such that
$$x^3 + y^3 = 1 \qquad \text{and} \qquad x^4 + y^4 = 1.$$
4. Ross has a rectangular table. It is possible to place 100 identical coins on it (centers must be on the table) such that none of the coins overlap, and it is impossible to place any more coins on the table without causing an overlap.
Prove that using 400 coins and allowing overlaps, we can cover Ross' table completely.
5. Positive integers are written on all the faces of a cube, one on each. At each corner (vertex) of the cube, the product of the numbers on the faces that meet at the corner is written. The sum of the numbers written at all the corners is 2022. If T denotes the sum of the numbers on all the faces, find all the possible values of T.
6. Let n be a three digit number. It is known that exactly $\frac{1}{k}$ of the numbers: $\{1, 2, 3, \ldots, n\}$ begin with the digit 3. Detemine all possible values of n.