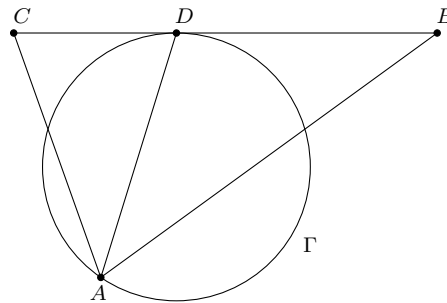## Maths Workshop
September 2021

Problems

1. Show that in a filled Sudoku grid the entries in the 16 cells around the central 3×3 box are exactly the entries in the four 2×2-blocks in the corners of the grid.

2. The bisector of angle A in triangle ABC intersects the side BC at D. A circle, Γ through A is tangent to BC at D. Prove that Γ is tangent to the circumcircle of ABC at A.

3. Find all real numbers x such that

$$x^2 - 2x = \frac{2}{x} - \frac{1}{x^2}.$$

4. Find all positive integers p, such that $\{p, 4p^2 + 1, 6p^2 + 1\}$ are all prime numbers.

5. Is the following number prime or composite?

$$\frac{2^{58} + 1}{5}$$

6. A fair coin is tossed n times and the outcome of each toss is recorded. Find the probability that in the resulting sequence of tosses a head immediately follows a head exactly h times and a tail immediately follows a tail exactly t times. Express your answer in terms of h and t.
(For example, for the sequence HHHTTHTHH, we have n = 9, h = 3, and t = 1.)