New Zealand Mathematical Olympiad Committee

Maths Workshop March 2024

mathsolympiad.org.nz/workshops/

Problems

1. When the number 4^{4^5} is expressed in the form n^n, what is the value of n?

2. Consider a rectangle $ABCD$ with $BC = 2 \cdot AB$. Let ω be the circle that touches the sides AB, BC, and AD. A tangent drawn from point C to the circle ω intersects the segment AD at point K. Determine the ratio $\frac{AK}{KD}$.

3. Ross has an analog clock with only hour and minute hands. Determine the total length of time per day where the angle between the clock hands is less than 1 degree. Both clock hands move continuously and at a constant speed.

4. You roll a 20-sided die repeatedly. On which roll are you most likely to first see a number you've already seen before?

5. Evaluate
$$\sum_{n=2}^{\infty} \big(\log(n^3+1) - \log(n^3-1)\big)$$

6. Josie is thinking of a positive integer $n \leqslant 100$, and your task is to guess this number. You can choose two positive integers $a, b \leqslant 100$ and ask for $\gcd(a+n, b)$. Show that you can determine n with at most seven questions.