New Zealand Mathematical Olympiad Committee

Maths Workshop

February 2022

Problems

1. Can you prove that $3^{44} + 4^{29}$ is composite?

2. Prove that the three altitudes of a triangle are concurrent.
   (an altitude of a triangle is a line through a vertex perpendicular to the opposite side)

3. Prove that there exists an integer n such that
$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} > 2022.$$

4. A domain contains 3 red particles, 4 green particles and 5 blue particles. Whenever two particles of different colours collide, they merge into a single particle of the third colour. Is it possible that the final colour will be blue?
   (available online: https://www.gustygames.co.nz/apps/challenges/Particles.php)

5. How many increasing sequences $0 < a < b < c < d < e < 14$ are there such that b and d are even, while a, c and e are odd.

6. Find all integer solutions of $y^2 = x^3 - 432$.