New Zealand Mathematical Olympiad Committee

Maths Workshop

March 2022

Problems

1. Let $N = 111\ldots11$ be the number consisting of 91 ones in a row. Prove that N is not a prime number.

2. Find real numbers a, b, c such that

$$6a + c = 7c + b = 8b + a = 2022.$$

3. Let ABC be any triangle such that $AB = 4$ and $AC = 6$. Let M be the midpoint of BC. Prove that $AM \leq 5$.

4. How many ways are there to rearrange the letters of the word "MISSISSIPPI"?

5. Triangle ABC satisfies $AB = AC$. The point M is situated so that C is the midpoint of AM. Let the perpendicular bisector of AM intersect AB at point P. Given that lines BC and MP are parallel, prove that the triangle APM is equilateral.

6. If the equation $x^3 - 3x + 1 = 0$ has roots a, b, c, show that $1/a + 1/b$ is a root of

$$x^3 - 6x^2 + 9x - 1 = 0.$$