New Zealand Mathematical Olympiad Committee

Maths Workshop November 2025

mathsolympiad.org.nz/workshops/

Problems

1. Find all triangular numbers which are sums of squares of two consecutive integers.

2. Can you divide an 11×11 square into five rectangles such that the set of sidelengths, of these five rectangles is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$?

3. Explain why $2^{2^{100}} - 1$ has at least 100 different prime divisors.

4. Let S be a subset of $\{1, 2, 3, \ldots, 99\}$. Suppose that no two elements of S have difference equal to 4 or 7. How many elements can S have?

5. Let ABC be a triangle with $\angle BAC = 90°$. Let D be the point on AC such that BD is the angle bisector of $\angle ABC$. If $AD = 2$ and $DC = 3$ then find the length BD.

6. Let number $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. For example

   $$\left\lfloor \sqrt{2} \right\rfloor = 1 \quad \text{and} \quad \left\lfloor \sqrt{8} \right\rfloor = 2 \quad \text{and} \quad \left\lfloor \pi \right\rfloor = 3 \quad \text{and} \quad \left\lfloor 5 \right\rfloor = 5.$$

   Let A be the set of all numbers of the form $\lfloor n\sqrt{2} \rfloor$ where n is a positive integer. In other words:

   $$\begin{aligned} A &= \left\{ \left\lfloor \sqrt{2} \right\rfloor, \left\lfloor 2\sqrt{2} \right\rfloor, \left\lfloor 3\sqrt{2} \right\rfloor, \left\lfloor 4\sqrt{2} \right\rfloor, \ldots \right\} \\ &= \{1, 2, 4, 5, \ldots\} \end{aligned}$$

   Let B be the set of all numbers of the form $\lfloor n(2 + \sqrt{2}) \rfloor$ where n is a positive integer. In other words:

   $$\begin{aligned} B &= \left\{ \left\lfloor 2 + \sqrt{2} \right\rfloor, \left\lfloor 2(2 + \sqrt{2}) \right\rfloor, \left\lfloor 3(2 + \sqrt{2}) \right\rfloor, \ldots \right\} \\ &= \{3, 6, 10, \ldots\} \end{aligned}$$

   - Does there exist a number which is in both A and B?
   - Which positive integers are in neither A nor B?