New Zealand Mathematical Olympiad Committee

Maths Workshop

March 2021

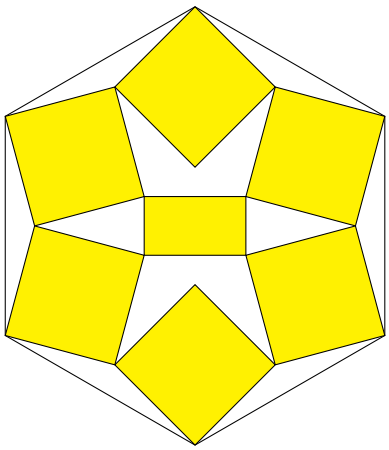
Problems

1. Find all positive integers x and y such that

$$xy = 5(x + y).$$

   What about negative integers?

2. Six congruent squares and a rectangle are fitted into a regular hexagon as shown. What fraction of the hexagon do they cover?



3. In how many ways can 6 married couples be seated in a circle so that every husband is next to his wife?

4. Is there a way to label the faces of a cube with a positive real number so that the number on each face equals the product of the numbers on the adjacent faces (i.e. the faces with which it shares an edge), other than the labeling with only ones?

5. If a, b and c are the side lengths of a triangle, prove that

$$\frac{3}{2} \leq \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \leq 2.$$

6. There are six towns in a flat plane. The smallest distance between any two of six towns is m kilometers. The largest distance between any two of the towns is M kilometers. Show that M/m is greater than or equal to $\sqrt{3}$.