New Zealand Mathematical Olympiad Committee

Maths Workshop

April 2021

Problems

1. Find all pairs of positive integers x and y such that

$$x^2 + y^2 = 2048.$$

2. How many ways can a 7-letter word be constructed from the English alphabet if every letter must be distinct? (it doesn't matter if the word is not in the dictionary)

3. Let $ABCD$ be a convex quadrilateral such that lengths AB, BC, CD, DA and BD are equal to 15, 12, 16, 25 and 20 respectively. What is AC?

4. Evaluate

$$1 \times 2 \times 3 + 3 \times 4 \times 5 + 5 \times 6 \times 7 + \cdots + 99 \times 100 \times 101.$$

5. Let p_0 be any positive integer. Define the sequence $p_0, p_1, p_2, \ldots$ recursively by

$$p_{n+1} = 2p_n + 1$$

for all integers $n > 0$. Must this sequence contain a composite number?

6. What is the value of $\sqrt{4 + \sqrt{4 - \sqrt{4 + \sqrt{4 - \cdots}}}}$