New Zealand Mathematical Olympiad Committee

Maths Workshop

June 2023

Problems

1. Find six different pairs of positive integers (x, y) that satisfy the equation
$$\frac{3}{x} + \frac{4}{y} = 1.$$

2. Consider an 8×8 chessboard with the top-left and bottom-right squares removed. Initially there is a knight on each of the 62 squares of the chessboard. Is it possible to move all 62 knights simultaneously (according to the usual chess knight moves) such that no two knights land on the same square?

3. Each term in the sequence $a_0, a_1, a_2, a_3, \ldots$ is either 0 or 1. No two consecutive terms sum to equal the sum of the next two consecutive terms.[1] Similarly no three consecutive terms sum to equal the sum of the next three consecutive terms. Given that $a_0 = 0$, determine the value of a_{2022}.

4. Let ABC be a triangle, let D be a point on side AB between A and B, and let E be a point on side AC between A and C. Let F be the intersection of BE and CD. Consider the four areas: quadrilateral $ADFE$, triangle BDF, triangle BFC and triangle CEF. Is it possible that these four areas are equal?

5. (a) How many ways can 8 be written as an ordered sum of 4 positive integers?

   (b) How many ways can 8 be written as an ordered sum of 4 non-negative integers?

6. Let $f(x) = ax^2 + bx + c$ with $a \neq 0$, be a quadratic such that there exists an integer s with
$$f(s) = f(3s - 1).$$
Prove that there does not exist an integer t such that
$$f(t) = f(2 - 3t)$$

[1] The next consecutive pair after (a_i, a_{i+1}) is the pair (a_{i+2}, a_{i+3}). The next consecutive triple after (a_j, a_{j+1}, a_{j+2}) is the triple $(a_{j+3}, a_{j+4}, a_{j+5})$.