



## Problems

1. What is the remainder when  $2021^{2022^{2023}}$  is divided by 2022?
2. For the integers  $a, b, c$  and  $d$ : the difference between  $a$  and  $b$  is 2, the difference between  $b$  and  $c$  is 3, and the difference between  $c$  and  $d$  is 4. Which of the following values cannot be the difference between  $a$  and  $d$ ?

(a) 1      (b) 3      (c) 5      (d) 7      (e) 9

3. Given are two poles in a field, one having height 3m and the other 4m. Let  $A$  and  $B$  be the tips of the poles. There is a point,  $C$ , on the line segment joining the bases of the two poles, such that triangle  $ABC$  is equilateral. Determine the area of  $ABC$ .
4. Let  $\otimes$  be a binary operator, with the following three properties:
  - $a \otimes a = a + 2$ ;
  - $a \otimes b = b \otimes a$ ; and
  - $\frac{a \otimes (a+b)}{a \otimes b} = \frac{a+b}{b}$ .

Determine the value of  $8 \otimes 5$ .

5. Let  $m$  and  $n$  be given integers greater than 1. Consider an  $m \times n$  rectangular grid of points in the plane. Some  $k$  of these points are coloured red in such a way that no three red points are the vertices of a right-angled triangle with two sides parallel to the sides of the grid. Determine the greatest possible value of  $k$ .
6. Find all primes  $p$  such that  $\frac{2^{p-1}-1}{p}$  is a perfect square.