New Zealand Mathematical Olympiad Committee

Maths Workshop September 2025

mathsolympiad.org.nz/workshops/

Problems

1. Find all integers n such that $2^n + 1$ is a perfect square.

2. Let $ABCD$ be a parallelogram. Point P lies on side AD (between A and D) such that PB is the angle bisector of $\angle ABC$ and PC is the angle bisector of $\angle BCD$. Determine the ratio of the sides of the parallelogram $AB : BC$.

3. Ross is thinking of 16 different positive integers all less than 60. Is it guaranteed that Ross is thinking of 4 different numbers: a, b, c, d such that $a + b = c + d$?

4. Find all positive integers a, b, c such that
$$a^3 + b^3 + c^3 = 3abc + 4.$$

5. The candy store sells chocolates in the flavours: white, milk and dark. You can buy them in three types of coloured boxes. The three boxes have the following contents:
   - Gold: 3 white, 3 milk, 1 dark.
   - Silver: 1 white, 2 milk, 4 dark.
   - Bronze: 5 white, 1 milk, 2 dark.

   Ross buys some boxes of chocolates (at least one) and when he gets home, it turns out he has exactly the same number of chocolates of each flavour. What is the minimum number of boxes that Ross could have bought?

6. Given an initial number $n_0 > 1$, two players $\mathcal{A}$ and $\mathcal{B}$ choose integers $n_1, n_2, n_3, \ldots$ alternately according to the following rules.
   - Knowing n_{2k}, $\mathcal{A}$ chooses any integer n_{2k+1} such that $n_{2k} \leqslant n_{2k+1} \leqslant n_{2k}^2$.
   - Knowing n_{2k+1}, $\mathcal{B}$ chooses any integer n_{2k+2} such that $\frac{n_{2k+1}}{n_{2k+2}} = p^r$ where p is a prime and $r \geqslant 1$ is an integer.

   Player $\mathcal{A}$ wins the game by choosing the number 1990; player $\mathcal{B}$ wins the game by choosing the number 1. For each value n_0 determine if: player $\mathcal{A}$, player $\mathcal{B}$ or neither player have a winning strategy?