New Zealand Mathematical Olympiad Committee

Maths Workshop October 2025

mathsolympiad.org.nz/workshops/

Problems

1. Prove that the fraction $\dfrac{9n+4}{2n+1}$ is in lowest terms for all positive integers n.

2. Consider the sequence $x_1, x_2, x_3, \ldots, x_{100}$ defined by: $x_1 = 1$ and $x_2 = 3$ and
$$x_{n+2} = 4x_{n+1} - 3x_n$$
for all positive integers n. Determine the units digit of x_{100}.

3. In how many ways can one arrange the numbers 21, 31, 41, 51, 61, 71 and 81 such that the sum of any four consecutive terms is a multiple of 3.

4. Let ABC be a triangle with $AB = 7$, $AC = 8$ and $BC = 5$. Let the incircle of triangle ABC be tangent to sides AB and AC at points P and Q respectively. Determine length PQ.

5. Find all integers n, such that both $n+3$ and n^2+3 are perfect cubes.

6. In triangle ABC, let M be the midpoint of BC. Let P and Q be the intersection of the angle bisector of $\angle ABC$ with AM and AC respectively. Prove that if $\angle BPM = 90°$ then $CQ = 2QA$.