New Zealand Mathematical Olympiad Committee

Maths Workshop April 2024

mathsolympiad.org.nz/workshops/

Problems

1. Arrange 5 sheep and 3 wolves into a 5×5 grid such that no row, column nor diagonal contains both a sheep and a wolf. Each animal must be placed within a single cell in the grid and no two animals can occupy the same cell.

2. Ross likes every number which is equal to 13 times the sum of it's digits. How many positive integers does Ross like?

3. Find x such that $x^{x^3} = 36$.

4. A convex quadrilateral is inscribed in a rectangle with exactly one quadrilateral's vertex on each side of the rectangle. Prove that the area of the rectangle is twice the area of the quadrilateral if and only if a diagonal of the quadrilateral is parallel to two parallel sides of the rectangle.

5. Find all solutions of $3^x + 4^y = 5^z$, for integers x, y, z.

6. Prove that there is no function $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ such that

$$f(f(n)) = n + 1987$$

for all positive integers n.