New Zealand Mathematical Olympiad Committee

Maths Workshop

*February 2021*

# Problems

1. A cloth bag contains a pool ball, which is known to be a red ball. A second pool ball is chosen at random in such a way that it is equally likely to be a red or a blue ball. The ball is added to the bag, the bag is shaken, and a ball is drawn at random. This ball proves to be a red. What is the probability that the ball remaining in the bag is also a red?

2. A regular decagon $A_0A_1A_2\ldots A_9$ is given in the plane. Compute $\angle A_0A_3A_7$.

3. A triangle has angles in degrees that are all integers. One is a square, another is a cube and the third is a fourth power. What are angles?

4. Find real numbers $x, y, z$ such that

$$\begin{aligned} x^2 - yz &= 1 \\ y^2 - zx &= 2 \\ z^2 - xy &= 3. \end{aligned}$$

5. Find all positive integers $n$ such that $(x^7 - x)$ is a multiple of $n$ for all integers $x \geq 2$?

6. The sides of two squares (not necessarily of the same size) intersect in eight distinct points: $A$, $B$, $C$, $D$, $E$, $F$, $G$ and $H$. These eight points form an octagon. Join opposite pairs of vertices to form two non-adjacent diagonals. (For example, diagonals $AE$ and $CG$.) Show that these two diagonals are perpendicular.