New Zealand Mathematical Olympiad Committee

Maths Workshop

April 2022

Problems

1. Let $ABCD$ be a convex quadrilateral such that there is a circle which is tangent to all four sides. Prove that $AB + CD = AD + BC$.

2. Prove that the following equation has 4 distinct real solutions.

$$x^4 - 4x^3 + 8x + 1 = 0$$

3. What is the maximum number of times that 2 will divide into 50! exactly?

4. Josie has one more coin than Ross. Both players throw all of their coins simultaneously and observe the number that come up heads. Assuming all the coins are fair, what is the probability that Josie obtains strictly more heads than Ross?

5. Let $f(1), f(2), f(3), \ldots$ be an infinite sequence of positive integers, satisfying

$$f(n+3) = (f(n) + f(n+1))f(n+2)$$

   for all n. Find $f(7)$ given that $f(6) = 144$.

6. Is $\cos 1^\circ$ a rational number?