New Zealand Mathematical Olympiad Committee

NZMO Round One 2025 — Instructions

Submissions due date: 15^{th} *July*

The **New Zealand Mathematical Olympiad** (NZMO) consists of two rounds:

Round One (the NZMO1): A take home exam (the following set of 8 problems). Solutions are to be submitted by 15^{th} July (21:00 NZST).

Round Two (the NZMO2): A three hour supervised exam on the 5^{th} of September.

Awards for the NZMO (Gold/Silver/Bronze/Honourable Mention) will be made based on the results of the NZMO2. Scores will not be announced for either round. Participants in the NZMO1 will only receive an indication of whether they have been invited to participate in the NZMO2; participants in the NZMO2 will just be told the level of their award, if any.

In addition, the results of both rounds of the NZMO will be used to select about 25 students to participate in the training camp in January 2026, and around 20 female or non-binary students to attend the training camp later in 2026. Only students who are New Zealand citizens or permanent residents, and who will still be enrolled in school in 2026, are eligible for the January training camp. Only students who attend this camp are eligible for selection to represent New Zealand at the 2026 International Mathematical Olympiad, (IMO).

General instructions:

- Participation in the NZMO is open to any student enrolled in the New Zealand education system, at secondary school level or below.
- The NZMO is an individual competition. Participants must work on the problems entirely on their own, without assistance from anyone else or any kind of calculator/computer (other than purely for word processing purposes). This includes but is not limited to using calculations, graphing functions, or using tools such as GeoGebra.
- Electronic devices may not be used to assist in solving the problems. This includes but is not limited to calculators, computers, tablets, smart phones and smart watches.
- The internet may not be used, except as a reference for looking up definitions.
- Although some problems may seem to require only a numerical answer, in order to receive full credit for the problem a complete justification must be provided.
- Submitting rough working and/or partial solutions cannot negatively affect the score awarded. We encourage you to submit any rough work or partial solutions.
- It is forbidden to discuss the problems until the official solutions are posted on the NZMOC website. Typically this will be 3 days after the submission deadline.
- Please address **all** queries to Kevin Shen, info@mathsolympiad.org.nz

The NZMO1 submission url is: https://www.mathsolympiad.org.nz/nzmo1_submission. If you are having trouble loading the submission form, please try logging out of your google account, or opening the form in an incognito window. All your solutions and partial solutions should be submitted as a **single document** in PDF format. Please complete the submission form **carefully**, especially your contact details.

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NZMO Round One 2025 — Problems

1. Let a and b be positive integers with no common factor greater than 1. What are the possible values for the greatest common divisor of $(a+b)$ and $(a-b)$?
2. Let ABC be a right-angled triangle with $\angle BAC = 90°$, $\angle ABC = 70°$, and $AB = 1$. Let M be the midpoint of BC. Let D be the point on the extension of AM beyond M such that $\angle CDA = 110°$. Find the length of CD.
3. Let $P(x) = x^3 + ax^2 + bx - 8$ be a polynomial with 3 real roots. Show that $a^2 \geq 2b + 12$.
4. Find the largest integer k such that any string of 2025 letters consisting only of A's and B's contains a palindromic substring of length k or longer.
   A palindromic substring is a string of consecutive letters which reads the same backwards as forwards.
5. Alice plays a game with the Mad Hatter. The Mad Hatter will write two rows of numbers on a blackboard, each a distinct permutation of $\{1, 2, \ldots, n\}$. On each move, Alice is allowed to swap the positions of the numbers a and $a+1$ in the first row, for some $1 \leq a < n$. What is the minimum number of moves Alice needs in order to guarantee that she can turn the first row of numbers into the second, regardless of the permutations the Mad Hatter writes?
6. Determine the largest real number M such that for each infinite sequence $x_0, x_1, x_2, \ldots$ of real numbers satisfying $x_0 = 1$, $x_1 = 3$ and
   $$x_0 + x_1 + \cdots + x_{n-1} \geq 3x_n - x_{n+1} \quad \text{for all } n \geq 1,$$
   the inequality
   $$\frac{x_{n+1}}{x_n} > M$$
   holds for all $n \geq 0$.
7. Let ABC be a triangle and let D be a point inside the triangle ABC such that AD bisects $\angle BAC$. Let line BD meet side AC at E. Let line CD meet side AB at F. Let T be the intersection of the (internal) angle bisectors of $\angle AED$ and $\angle AFD$. Prove that if T lies on segment AD, then triangle ABC is isosceles.
8. Show that there are infinitely many triples (a, b, c) of positive integers such that
   $$a^2 + b^2 + c^2 + (a+b+c)^2 = abc.$$