New Zealand Mathematical Olympiad Committee

Maths Workshop

June 2021

Problems

1. Triangle ABC has side lengths: 3, 4 and 5. On each side of triangle ABC we construct squares: $ABED$, $BCGF$ and $CAIH$ externally. What is the area of hexagon $DEFGHI$?

2. How many ways are there to travel from $(0, 0)$ to $(4, 4)$ moving in the positive x and y direction along the Cartesian gridlines such that the number of direction changes is odd?

3. What is the maximum possible value of

$$\frac{ab}{a^4 + b^2 + 1}$$

   Where a, b, c are nonzero real numbers?

4. The sum of two positive integers is 2310. Can their product be divisible by 2310?

5. A hat contains a number of colored balls, with equal numbers of each color. Adding 6 balls of a new color to the hat would not change the probability of drawing (without replacement) two balls of the same color. How many balls are in the hat? (Before the extra balls are added.)

6. Find all positive real solutions to the equations:

$$\begin{aligned} x + y^2 + z^3 &= 3 \\ y + z^2 + x^3 &= 3 \\ z + x^2 + y^3 &= 3. \end{aligned}$$