New Zealand Mathematical Olympiad Committee

Maths Workshop February 2024

mathsolympiad.org.nz/workshops/

Problems

1. Does there exist a perfect square which is the product of 4 consecutive odd numbers?

2. Let $ABCD$ be a convex quadrilateral with sidelengths AB, BC, CD, DA equal to $2, 3, 2, 4$ (in that order). Given that diagonal AC has length 4, determine the length of diagonal BD.

3. Given is a 4×4 grid. What is the maximum number of coins that can be placed on the centres of the squares of the grid so that all of the distances between the centres of the coins are different?

4. Ross invests some positive amount of money at some positive interest rate and Josie invests some other positive amount of money at twice Ross' interest rate. At the end of each year Ross gives Josie all the interest from his account and Josie invests it (at her higher interest rate) and Josie gives Ross all the interest from her account and he invest it (at his low interest rate). In the long run what happens to the ratio of Josie's wealth to Ross'?

5. Find all pairs of positive integers (x, y), such that $x^2 + 3y$ and $y^2 + 3x$ are both perfect squares.

6. Given any convex quadrilateral, trisect each of the sides. Join up the corresponding points on opposite sides with straight lines (like a skewed tic-tac-toe board).

   Prove that the central of the nine sub-quadrilaterals has an area one ninth of the area of the original.