



Problems

1. When the number 5^{2022} is divided by 11, what is the remainder?
2. The integers from 1 to 2022 are written on the blackboard. Two randomly chosen numbers are erased and replaced by their difference giving a sequence with one less number. This process is repeated until there is only one number remaining. Is the remaining number even or odd?
3. Let $ABCD$ be any convex quadrilateral. Let M , N , P and Q be the midpoints of AB , BC , CD and DA respectively. Prove that $MNPQ$ is a parallelogram.
4. Find all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ (functions that map real numbers to real numbers) such that

$$f(x - f(y)) = 1 - x - y$$

for all real numbers x and y .

5. Let A , B and C be the angles of an acute triangle. Show that

$$\cos A + \cos B + \cos C > 1.$$

6. Show that for $n > 0$, the binomial coefficient $\binom{2n}{n}$ is divisible by both $(n+1)$ and $(4n-2)$.