New Zealand Mathematical Olympiad Committee

Maths Workshop June 2025

mathsolympiad.org.nz/workshops/

Problems

1. Let p be a prime number and let q be the next prime number after p. Define the sequence $(x_1, x_2, x_3, \ldots)$ by $x_1 = p$, and for all positive integers n:

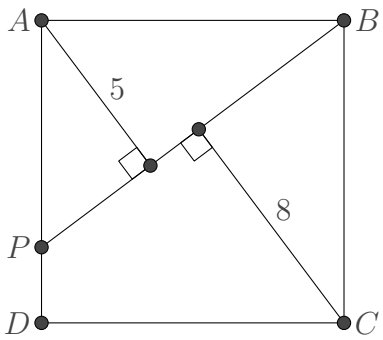
   - If n divides x_n, then $x_{n+1} = x_n + 1$.
   - If n does not divide x_n then $x_{n+1} = x_n$.

   Prove or disprove that $x_q = q$.

2. What is the next number in the sequence? $1, 2, 4, 6, 10, 12, 16, 18, 22, 28, \ldots$

3. A spider has one shoe and one sock for each of its eight legs. In how many different orders can the spider put on its socks and shoes? (assume that on each leg, the sock must be put on before the shoe)

4. Point P lies on side AD of square $ABCD$. Given that the perpendicular distances from A and C to BP are 5 and 8 respectively, determine the area of the square $ABCD$.



5. One deals out a deck of 52 cards (face up) into a 4×13 array. Then one tries to select 13 cards, one from each column, in such a way as to get one card of each denomination (but not necessarily of the same suit). Must this always be possible?

6. A circle ω is tangent to two parallel lines l_1 and l_2. A second circle ω_1 is tangent to l_1 at A and to ω externally at C. A third circle ω_2 is tangent to l_2 at B and to ω externally at D and to ω_1 at E. Let Q be the intersection of AD and BC. Prove that Q is the circumcentre of triangle CDE.