New Zealand Mathematical Olympiad Committee

Maths Workshop December 2024

mathsolympiad.org.nz/workshops/

Problems

1. The difference between a number and its reciprocal is $\frac{9}{20}$. What is the sum of the number and it's reciprocal?

2. The number 6095184273 is called *altripan* because it contains all ten digits exactly once each, and for every string of four consecutive digits, the sum of the first three and the last three differ by 1. What are the possible values of the units digit of an altripan number?

3. Let ABC be any acute angled triangle. Points D, E and F are the feet of the perpendiculars from A, B and C to sides BC, AC and AB respectively. Prove that line AD bisects angle $\angle EDF$.

4. Find all positive integers a, b and c such that

$$45a + 63b + 91c = 1197.$$

5. King Ross has 500 barrels of wine, but one of them is poisoned. Anyone drinking the poisoned wine will die within 12 hours. The king has four prisoners whom he is willing to sacrifice in order to find the poisoned barrel. Can this be done within 48 hours?

6. A lattice point is any point in the xy-plane such that both x and y are integers. What is the minimum area of a pentagon whose vertices are all lattice points? What is the minimum area of a *convex* pentagon whose vertices are all lattice points?