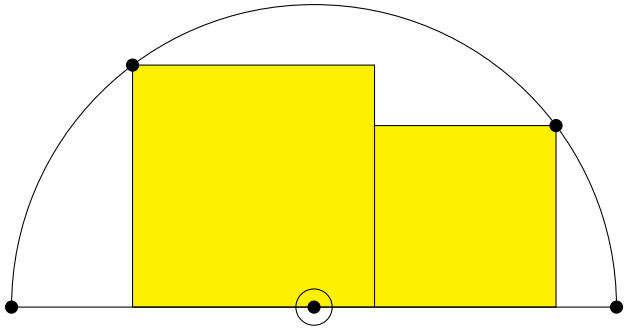
New Zealand Mathematical Olympiad Committee

Maths Workshop

October 2021

Problems

1. Two squares are inscribed in a semicircle of diameter 10 as shown. What is the shaded area?



2. Replace each letter in the word CLEVER with a digit or elementary arithmetic symbol ($+$, $-$, $\times$, $\div$, $=$, 0,1,2,3,4,5,6,7,8 and 9) so that the word becomes a balanced equation. The same letter must represent the same symbol. Different letters must represent different symbols.

3. Find all real numbers x such that

$$\sqrt{x+2\sqrt{x-1}}+\sqrt{x-2\sqrt{x-1}}=2.$$

4. A bag contains 10 balls numbered 1 to 10. Four balls are chosen uniformly at random. On average, what will the maximum value of the 4 chosen balls be?

5. Prove that for any three positive real numbers a, b, c we have

$$\frac{a}{a+2b}+\frac{b}{b+2c}+\frac{c}{c+2a}\geq 1.$$

6. Show that all the divisors of $2\times 10^{23}-1=1999\ldots999$ have units digit 1 or 9.