least_squares_simulation_with_synthetic_data

December 4, 2022

Least Square regression with simulated (synthetic data) reverse procedure

[1]:
```
import pandas as pd
import numpy as np
from numpy.random import seed
from numpy.random import normal
from sklearn.linear_model import LinearRegression
```

[2]:
```
#visualization libraries
import matplotlib.pyplot as plt
%matplotlib inline
import matplotlib.pyplot as plt
import seaborn as sns
```

We will be experimenting with synthetic data to develop the least square approximations on the reverse.
Our equation of the linear model:

$$y_i = \alpha + \beta x_i + \varepsilon_i (i = 1, ..., n)(1)$$

We are going to create synthetic data for a number of observations say n and we will simulate our experiment to find a and b.

We choose a constant term -a say $a = 10$ and a slope coefficient -b say, $b = 1$. We will generate n random disturbances $e_1, ...e_n$ from a normal distribution with mean zero and variance $\sigma^2 = 25$ so, $\sigma = 5$.

[3]:
```
#define our parameters a and b - sigma is the variance n is the number of␣
↪observations
a=10; b=1; sigma=5; n=20
```

[4]:
```
#define seed with 1 to make the experiment repeatable with same data
seed(1)
#Generate a sequence of n disturbances
e = normal(loc=0, scale=1, size=n)
e
```

[4]:
```
array([ 1.62434536, -0.61175641, -0.52817175, -1.07296862,  0.86540763,
       -2.3015387 ,  1.74481176, -0.7612069 ,  0.3190391 , -0.24937038,
```

```
 1.46210794, -2.06014071, -0.3224172 , -0.38405435,  1.13376944,
-1.09989127, -0.17242821, -0.87785842,  0.04221375,  0.58281521])
```

Apply a variance of 5 to disturbances We multiply the random normally distributed disturbances by variance sigma.

[5]:
```
EPS1=e*sigma
EPS1
```

[5]:
```
array([  8.12172682,  -3.05878207,  -2.64085876,  -5.36484311,
         4.32703815, -11.50769348,   8.72405882,  -3.8060345 ,
         1.59519548,  -1.24685188,   7.31053969, -10.30070355,
        -1.61208602,  -1.92027177,   5.66884721,  -5.49945634,
        -0.86214104,  -4.38929209,   0.21106873,   2.91407607])
```

[6]:
```
#Here we define our explained variable X of the equation 1
X=np.arange(1,n+1)

print(" The explanatory variable : " , X)
```

```
 The explanatory variable :  [ 1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17
18 19 20]
```

[7]:
```
#We are developing equation (1)  + x
YSYS=a+b*X
YSYS
```

[7]:
```
array([11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27,
       28, 29, 30])
```

Data Y, are generated with disturbances $\sim N(0, 25)$ by the equation $y_i = \alpha + \beta x_i + \varepsilon_i (i = 1, ..., n)$ with $n = 20, x_i = i$ with $i = 1, ...n, \alpha = 10$ and $\beta = 1$

[8]:
```
Y=YSYS+EPS1
Y #this is our data explanatory variable
```

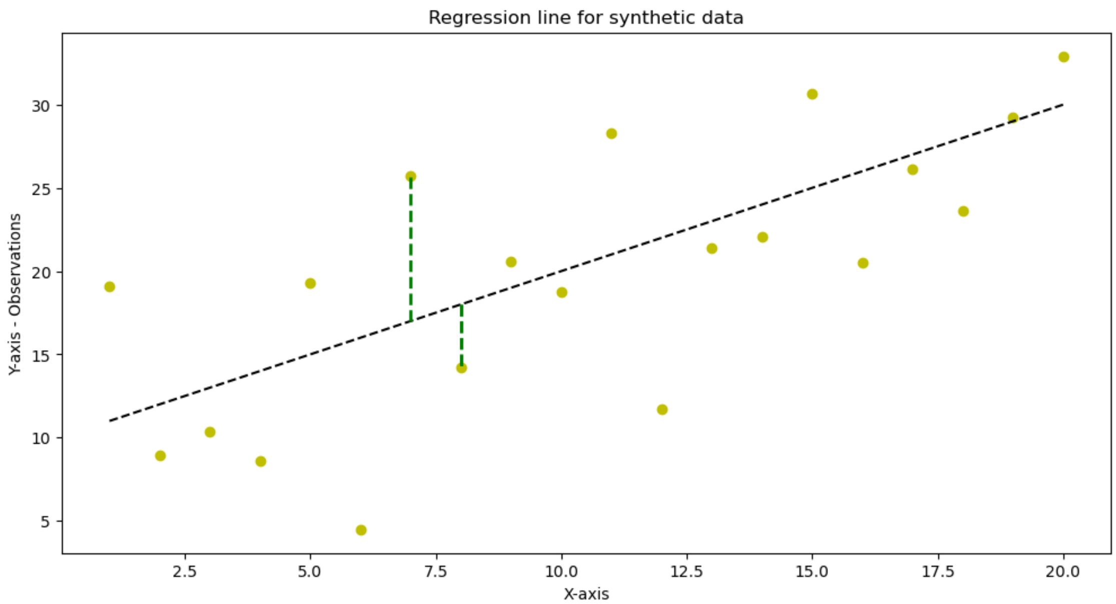
[8]:
```
array([19.12172682,  8.94121793, 10.35914124,  8.63515689, 19.32703815,
        4.49230652, 25.72405882, 14.1939655 , 20.59519548, 18.75314812,
       28.31053969, 11.69929645, 21.38791398, 22.07972823, 30.66884721,
       20.50054366, 26.13785896, 23.61070791, 29.21106873, 32.91407607])
```

[9]:
```
Y[7]

X[7]
```

[9]: 8

Now we have finished with the data generation process. Let's now display the "fitted" line and the data.

```
[10]: fig = plt.figure(figsize=(12,6))  #setting the figure size
      plt.title('Regression line for synthetic data')
      plt.xlabel('X-axis')
      plt.ylabel('Y-axis - Observations')
      plt.plot(X, Y, 'yo', X, b*X+a, '--k')
      plt.plot([X[7],X[7]], [18, Y[7]],color='green',linestyle='dashed',linewidth=2)
      plt.plot([X[7],X[7]], [18, Y[7]],color='green',linestyle='dashed',linewidth=2,␣
       ↪label="ei")
      plt.plot([X[6],X[6]], [17, Y[6]],color='green',linestyle='dashed',linewidth=2,␣
       ↪label="ei")
      plt.show()
```



Above: Scatter diagram with observed (synthetic) data (x_i, y_i), regression line $(y_i = \alpha + \beta x_i)$, and residual (e_i).

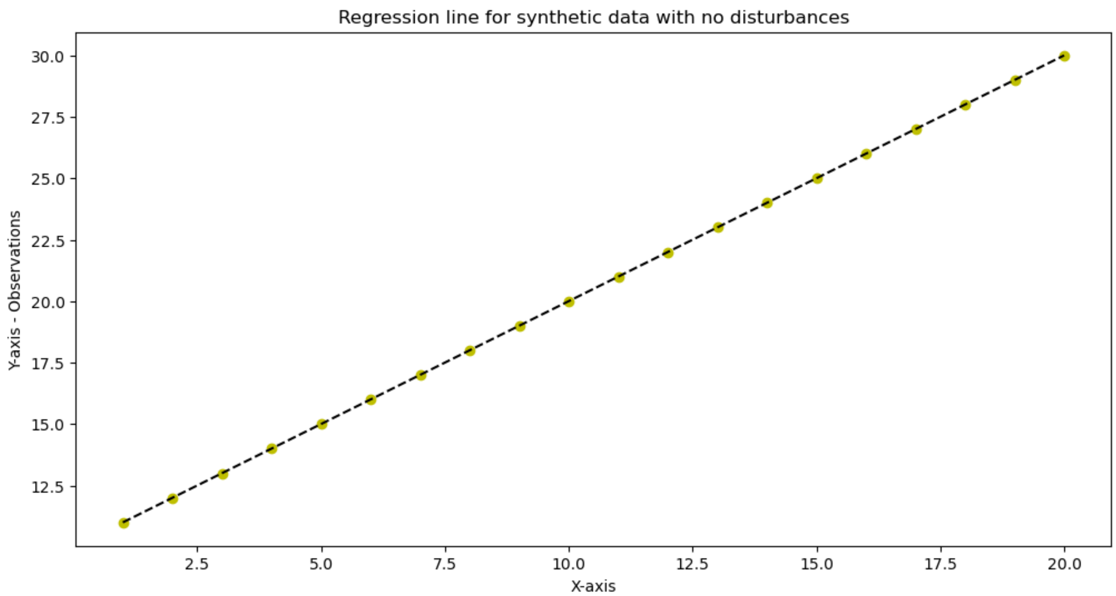
```
[11]: t=a+b*X
```

```
[12]: t
```

```
[12]: array([11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27,
             28, 29, 30])
```

```
[13]: fig = plt.figure(figsize=(12,6))  #setting the figure size
      plt.title('Regression line for synthetic data with no disturbances')
      plt.xlabel('X-axis')
      plt.ylabel('Y-axis - Observations')
      plt.plot(X, t, 'yo', X, b*X+a, '--k')
```

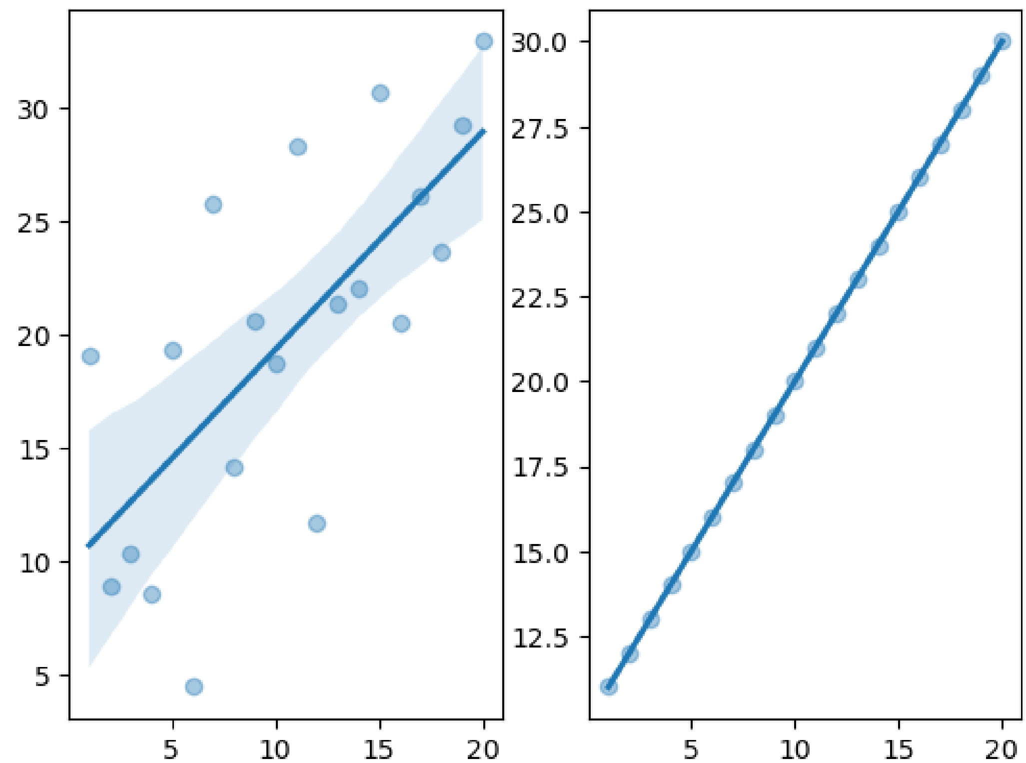
```python
plt.show()
```



[14]:
```python
fig, ax = plt.subplots(1, 2)
sns.regplot(X, Y, ax=ax[0], scatter_kws={'alpha': 0.4})
sns.regplot(X, t, ax=ax[1], scatter_kws={'alpha': 0.4})
```

```
C:\Users\ippok\conda3\lib\site-packages\seaborn\_decorators.py:36:
FutureWarning: Pass the following variables as keyword args: x, y. From version
0.12, the only valid positional argument will be `data`, and passing other
arguments without an explicit keyword will result in an error or
misinterpretation.
  warnings.warn(
```

[14]:
```
<AxesSubplot:>
```



The Reverse procedure

> Now we will be taking the reverse procedure to verify and confirm the parameters a and b. Suppose we are given the data X and Y above and we were asked to calculate the parameters a and b and display the fitted line.

So, our purpose is to find a and b that minimize the square difference

$$S(a,b) = \sum (y_i - a - bx_i^2)$$

We derive the partial derivatives with respect to a and b and equate to zero

$$\partial S/\partial \alpha = -2\sum (y_i - a - bx_i) = 0$$

$$\partial S/\partial \beta = -2\sum x_i(y_i - a - bx_i) = 0$$

Dividing by 2n finally we find

$$a = \hat{y} - b\hat{x}$$

$$b = \frac{\sum (x_i - \hat{x})(y_i - \hat{y})}{\sum (x_i - \hat{x})^2}$$

We can further simplify our fraction for b as folows:

$$\sum (x_i - \hat{x})(y_i - \hat{y}) = \sum (x_i y_i) - \frac{1}{n}\sum x_i \sum y_i$$

$$\sum (x_i - \hat{x})^2 = \sum x_i^2 - \frac{1}{n}\left(\sum x_i\right)^2$$

[15]:
```
#calculate the mean for x and y
x_mean=np.mean(X)
y_mean=np.mean(Y)
```

[16]:
```
#Calculate Σxi and Σyi
Sx=np.sum(X)
Sy=np.sum(Y)
#Square X and Y
X_sq=X**2
Y_sq=Y**2
#Calculate Σxiyi
Sxy=np.dot(X, Y.T)
```

[17]:
```
X_sq
```

[17]:
```
array([  1,   4,   9,  16,  25,  36,  49,  64,  81, 100, 121, 144, 169,
       196, 225, 256, 289, 324, 361, 400], dtype=int32)
```

[18]:
```
Y_sq
```

[18]:
```
array([ 365.64043651,   79.9453781 ,  107.3118072 ,   74.5659345 ,
        373.53440352,   20.18081783,  661.72720223,  201.46865649,
        424.16207687,  351.68056451,  801.48665727,  136.87353748,
        457.44286441,  487.51439856,  940.57818929,  420.2722905 ,
        683.18767113,  557.46552803,  853.28653656, 1083.33640345])
```

[19]:
```
X_sum_sq=np.sum(X_sq)
Y_sum_sq=np.sum(Y_sq)
```

[20]:
```
print (X_sum_sq)
print (Y_sum_sq)
```

```
2870
9081.661354455558
```

[21]:
```
Sy
```

[21]:
```
396.6635363539271
```

[22]:
```
#Our nominator is
nom=Sxy-((Sx*Sy)/20)
print (nom)
```

```
637.388638050521
```

[23]:
```
#Our dennominator is
den=X_sum_sq-(1/n)*(Sx)**2
print (den)
```

```
665.0
```

[24]:
```
#And we finally find b
b_c=nom/den
print ("parameter b is", b_c)
```

```
parameter b is 0.9584791549631895
```

[25]:
```
#And we find a as well
alpha=np.mean(Y)-b*np.mean(X)
print ("parameter a is", alpha)
```

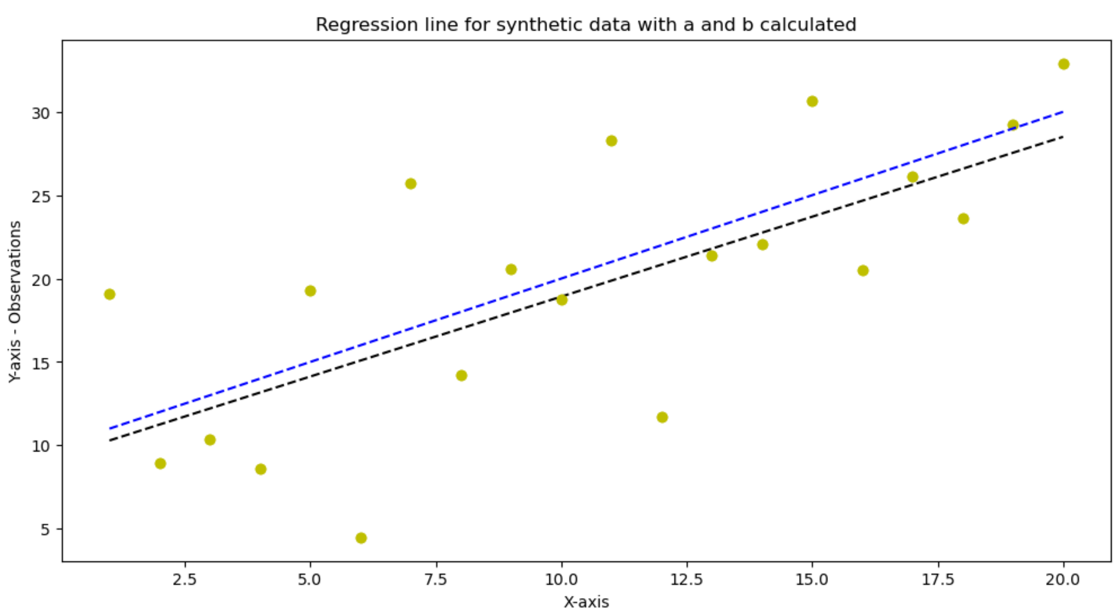
```
parameter a is 9.333176817696355
```

So, we calculated manually the regression line parameters and found them as;

$$a + \beta x = 9.333176817696355 + 0.9584791549631895x$$

Which is very close to our initial given values of $a = 10$ and $\beta = 1$

[26]:
```
fig = plt.figure(figsize=(12,6))  #setting the figure size
plt.title('Regression line for synthetic data with a and b calculated')
plt.xlabel('X-axis')
plt.ylabel('Y-axis - Observations')
plt.plot(X, Y, 'yo', X, b_c*X+alpha, '--k')
plt.plot(X, Y, 'yo', X, b*X+a, '--b')
plt.show()
```



[27]:
```python
import statsmodels.api as sm
X = sm.add_constant(X)
```

[28]:
```python
#A different way obtaining LS regression through the package
model = sm.OLS(Y,X)
results = model.fit()
results.params
```

[28]:
```
array([9.76914569, 0.95847915])
```

[29]:
```python
#add the constant
X
```

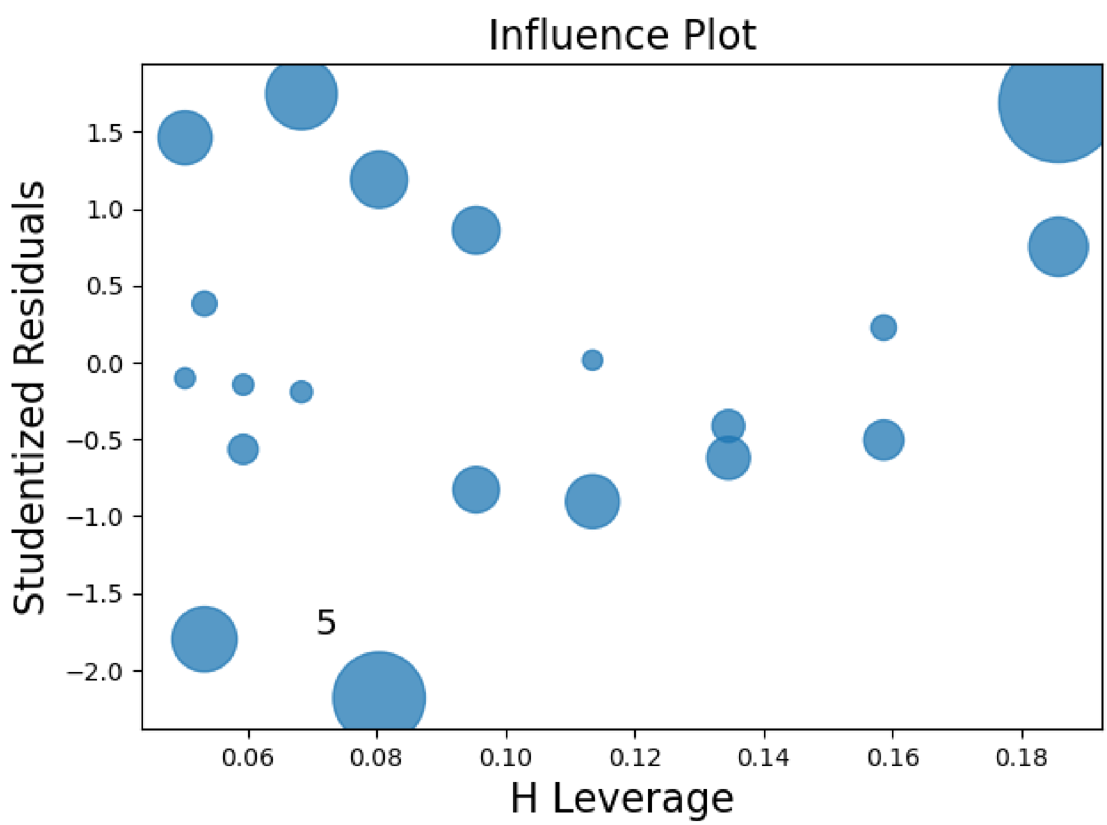
[29]:
```
array([[ 1.,  1.],
       [ 1.,  2.],
       [ 1.,  3.],
       [ 1.,  4.],
       [ 1.,  5.],
       [ 1.,  6.],
       [ 1.,  7.],
       [ 1.,  8.],
       [ 1.,  9.],
       [ 1., 10.],
       [ 1., 11.],
       [ 1., 12.],
       [ 1., 13.],
       [ 1., 14.],
       [ 1., 15.],
       [ 1., 16.],
       [ 1., 17.],
       [ 1., 18.],
       [ 1., 19.],
       [ 1., 20.]])
```

[30]:
```python
results.summary()
```

[30]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.503
Model:                            OLS   Adj. R-squared:                  0.475
Method:                 Least Squares   F-statistic:                     18.22
Date:                Sat, 03 Dec 2022   Prob (F-statistic):           0.000463
Time:                        23:55:17   Log-Likelihood:                -62.451
```

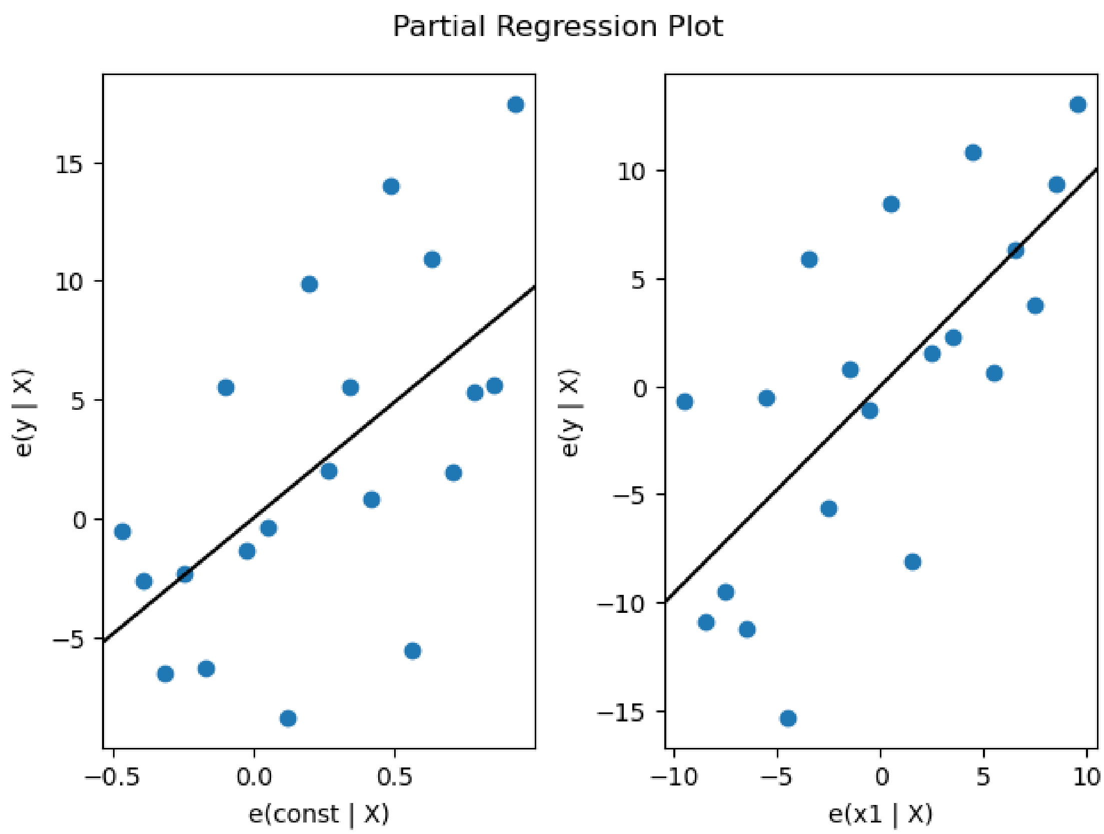
```
No. Observations:                  20   AIC:                             128.9
Df Residuals:                      18   BIC:                             130.9
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          9.7691      2.690      3.632      0.002       4.117      15.421
x1             0.9585      0.225      4.268      0.000       0.487       1.430
==============================================================================
Omnibus:                        0.100   Durbin-Watson:                   2.956
Prob(Omnibus):                  0.951   Jarque-Bera (JB):                0.322
Skew:                          -0.058   Prob(JB):                        0.851
Kurtosis:                       2.390   Cond. No.                         25.0
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
"""
```

[31]:
```
fig = sm.graphics.influence_plot(results, criterion="cooks")
fig.tight_layout(pad=1.0)
```



[32]:
```
fig = sm.graphics.plot_partregress_grid(results)
fig.tight_layout(pad=1.0)
```

```
eval_env: 1
eval_env: 1
```

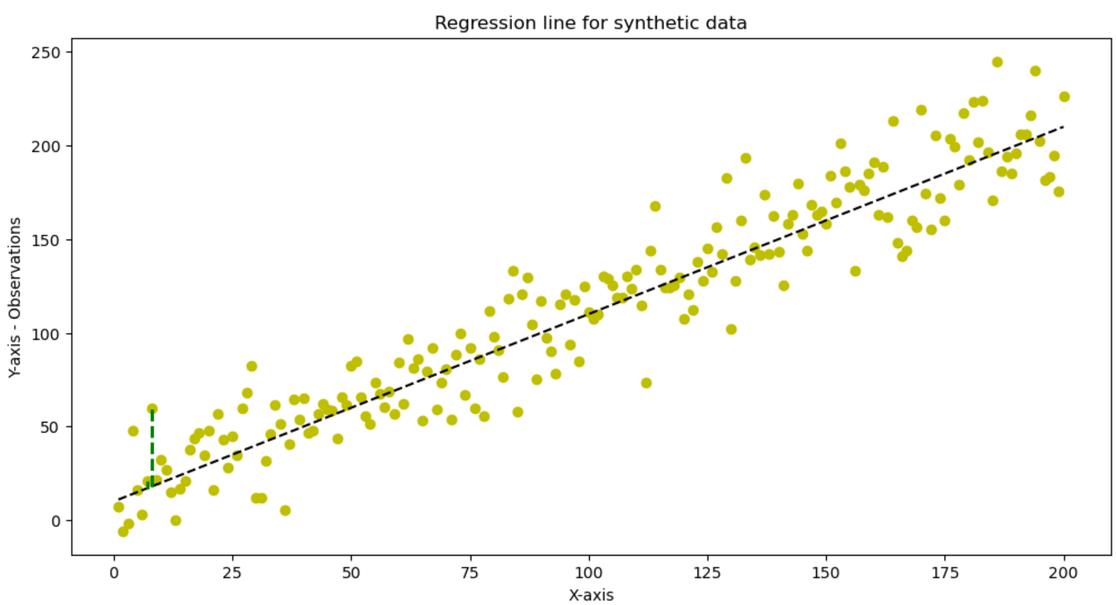


[33]:
```
def make_regression(var, obs, a, b):
    #define seed with 1 to make the experiment repeatable with same data
    #seed(1)
    #Generate a sequence of n disturbances
    e = normal(loc=0, scale=1, size=obs)
    EPS1=e*var
    X=np.arange(1,obs+1)
    #We are developing equation (1)  + x
    YSYS=a+b*X
    Y=YSYS+EPS1
    Y #this is our data explanatory variable
    return X, Y
```

```
[34]: (X, Y)=make_regression(5, 20, 10, 1)
Y
```

```
[34]: array([ 5.49690411, 17.72361855, 17.5079536 , 16.51247169, 19.50427975,
        12.5813607 , 16.38554887, 13.32115283, 17.6605596 , 22.65177733,
        17.54169624, 20.01623237, 19.5641365 , 19.77397179, 21.64376935,
        25.93667701, 21.41344826, 29.17207849, 37.29901089, 33.7102208 ])
```

```
[35]: (X, Y)=make_regression(20, 200, 10, 1)
fig = plt.figure(figsize=(12,6))  #setting the figure size
plt.title('Regression line for synthetic data')
plt.xlabel('X-axis')
plt.ylabel('Y-axis - Observations')
plt.plot(X, Y, 'yo', X, b*X+a, '--k')
plt.plot([X[7],X[7]], [18, Y[7]],color='green',linestyle='dashed',linewidth=2)
plt.plot([X[7],X[7]], [18, Y[7]],color='green',linestyle='dashed',linewidth=2,␣
 ↪label="ei")
plt.plot([X[6],X[6]], [17, Y[6]],color='green',linestyle='dashed',linewidth=2,␣
 ↪label="ei")
plt.show()
```




```
[65]: def find_reg_params(X, Y):
    #calculate the mean for x and y
    x_mean=np.mean(X)
    y_mean=np.mean(Y)
```

```
    #Calculate Σxi and Σyi
    Sx=np.sum(X)
    Sy=np.sum(Y)
    #Square X and Y
    X_sq=X**2
    Y_sq=Y**2
    #Calculate Σxiyi
    Sxy=np.dot(X, Y.T)
    X_sum_sq=np.sum(X_sq)
    Y_sum_sq=np.sum(Y_sq)
    #Our nominator is
    nom=Sxy-((Sx*Sy)/20)
    #print (nom)
    #Our dennominator is
    den=X_sum_sq-((1/n)*(Sx)**2)
    #print (den)
    #And we finally find b
    beta=nom/den
    #print ("calculated parameter b is", b_c)
    #And we find a as well
    alpha=np.mean(Y)-b*np.mean(X)
    #print ("calculated parameter a is", alpha)
    return alpha, beta
```

[66]:
```
(X_g, Y_g)=make_regression(5, 20, 10, 1)
(a_c,b_c)=find_reg_params(X_g, Y_g)
```

[67]:
```
a_c
```

[67]: 11.352733128353737

[68]:
```
b_c
```

[68]: 0.6931630139474644

[69]:
```
a_val=np.zeros(20001)
b_val=np.zeros(20001)
for i in range(20001):
    (X_gen,Y_gen)=make_regression(5, 20, 10, 1)
    (a_val[i],b_val[i]) = find_reg_params(X_gen, Y_gen)
```

[70]:
```
np.mean(b_val)
```

[70]: 0.9994615439817374

[71]:
```
np.mean(a_val)
```

[71]: 9.996208189089495

[72]:
```python
a200_val=np.zeros(20001)
b200_val=np.zeros(20001)
for i in range(20001):
    (X_gen200,Y_gen200)=make_regression(50, 200, 10, 1)
    (a200_val[i],b200_val[i]) = find_reg_params(X_gen200, Y_gen200)
```

[73]:
```python
print ("the mean value a found after 2000 simulations of 200 observations is␣
 ↪",np.mean(a200_val))
print ("the mean value b found after 2000 simulations of 200 observations is",␣
 ↪np.mean(b200_val))
```

```
the mean value a found after 2000 simulations of 200 observations is
9.990809897151705
the mean value b found after 2000 simulations of 200 observations is
1.103217771888285
```

[ ]:
```python

```