MEGN 517A Inelastic Constitutive Relations

Homework 2

Due: 3/1/2016

---

1. Tissue in the skin of animals consists of a protein called Collagen. This protein has a fibrous structure and provides the structural strength to the skin. There is an effort going on to understand the mechanical properties of collagen- one of the motivations being the creation of artificial skin. Hydrolyzed collagen (essentially collagen that is exposed to moisture, like in skin) shows a non-linear elastic (hyperelastic) material response. Figure below shows a typical engineering stress-strain curve obtained experimentally for hydrolyzed collagen. The curve is for simple uniaxial tension.

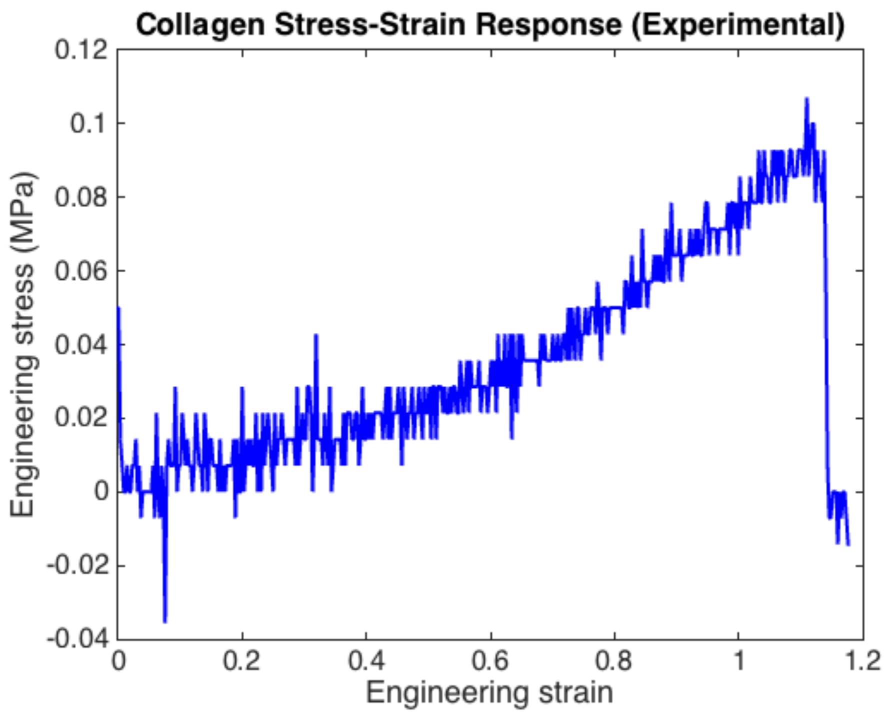
   Using Matlab or other software of your choice, determine which of the following hyperelastic material models gives the best fit to the observed response. The second column in the table gives the constitutive law in terms of stretch and engineering stress for each of the material models. Third column gives the material constants that you need to fit to the data.

   a. Clearly list the assumptions you made when fitting the parameters and making plots.
   b. List the values of the material constants for each model that give the best fit.
   c. Overall, which model is the most appropriate for modeling the observed response? State the reasons.

	Model	Constitutive law	Material parameters
1	Linear elastic	$P = G(\lambda - 1)$	G
2	Neo-Hookean	$P = G(\lambda - {}^{1}/_{\lambda^2})$	G
3	Mooney-Rivlin	$P = \mu_1 \left(\lambda - {}^{1}/_{\lambda^2}\right) + \mu_2 \left(1 - {}^{1}/_{\lambda^3}\right)$	μ_1, μ_2

Note: Data in the plot is available as Excel/MAT file on Blackboard. It is not necessary to derive the constitutive laws. But see question 2 below. The curve is extremely noisy, partly because of the very low stiffness. It is your choice to smooth the data. The sudden unloading at the end is because of failure. It is not necessary to capture that response. Mooney-Rivlin is another model for non-linear elastic material response, much like the Neo-Hookean model. We did not discuss the

Mooney-Rivlin model in the class, but its derivation is similar to that of the Neo-Hookean model.



2. The strain energy density for the Mooney-Rivlin model is given by: $W = \mu_1(I_1 - 3) + \mu_2(I_2 - 3)$. Here I_1 and I_2 are the invariants of the deformation tensor $\boldsymbol{B}$. Assuming incompressible behavior and uniaxial tension, derive the constitutive law for the Mooney-Rivlin model. You can follow the same procedure we followed in the class – Write the invariants in terms of the stretch ratios. assume a specific form of the stretch ratios for uniaxial tension and incompressible behavior. Find the stress by differentiating. You can calculate the engineering stress (First Piola-Kirchhoff stress) from the strain energy density simply as: $F = {}^{\partial W}/_{\partial \lambda}$.