MEGN 502
Advanced Engineering Analysis
Lab Assignment No. 7

27 October 2015

Note: This is an **INDIVIDUAL** assignment. You may discuss concepts with your fellow students, but you must turn in **INDIVIDUAL** work. This is not a group assignment.

For each question in this assignment provide an executive summary of your results (*typed, not handwritten*) and include an appendix with all supporting documentation and code you developed for each problem.

1. In class we discussed the general linear least squares problem one obtains for fitting data to an assumed polynomial. Say we are given a set of data $(x_i, y_i), \quad i = 1, 2, \ldots, M$ and we wish to follow the linear least squares procedure to fit a linear function to this data, $y = c_0 + c_1 x$. If we use the normal equations for solving the linear least squares problem, the values of the coefficients c_0 and c_1 are given by

$$c_0 = \frac{1}{D}\left(\sum_{i=1}^{M} x_i^2 \sum_{i=1}^{M} y_i - \sum_{i=1}^{M} x_i \sum_{i=1}^{M} x_i y_i\right)$$

$$c_1 = \frac{1}{D}\left(-\sum_{i=1}^{M} x_i \sum_{i=1}^{M} y_i + M \sum_{i=1}^{M} x_i y_i\right)$$

where

$$D = M \sum_{i=1}^{M} x_i^2 - \sum_{i=1}^{M} x_i \sum_{i=1}^{M} x_i$$

For this problem, answer the following:

(a) Derive the expressions given above

(b) Write a program for performing a least-squares fit to a given set of (x, y) data using an M^{th} order polynomial, $y(x) = \sum_{k=0}^{K} A_k x^k$

(c) Go to http://www.timeanddate.com/worldclock/astronomy.html?n=18 which provides sunrise data for Anchorage, Alaska. For the month of October, fit a polynomial to the sunrise data (use a decimal form for the time) as a function of the day of the month. What order of polynomial provides the best fit to the data?

2. Solve

$$\frac{d^2u}{dx^2} - e^u = 0, \quad u(0) = u(1) = 0$$

by assuming an approximate solution of the form

$$u(x) = \sum_{j=0}^{5} a_j x^j$$

Use the following weighted residual techniques:

(a) Deterministic point collocation (same number of collocation points as unknown constants).

(b) Deterministic subdomain collocation (same number of subdomains as unknown constants).

(c) Galerkin

3. For the remainder of this assignment you will investigate the use of the Method of Fundamental Solutions (MFS) for the solution of Laplace's equation. Consider the following problem:

$$\nabla^2 u = 0 \quad 0 \le x \le 1, \ 0 \le y \le 1$$

with boundary conditions

$$q(0, y) = -20, \ q(1, y) = 45, \ u(x, 0) = 10, \ u(x, 1) = -15.0$$

where $q = \frac{\partial u}{\partial n}$. For the Laplace operator,

$$G(\mathbf{x}; \mathbf{x}') = -\frac{1}{2\pi} \log r, \qquad \frac{\partial G}{\partial n}(\mathbf{x}; \mathbf{x}') = -\frac{1}{2\pi r^2} \left((x - x')n_x \ + \ (y - y')n_y\right)$$

where $r = \sqrt{(x - x')^2 + (y - y')^2}$ and (n_x, n_y) are the components of the local normal vector where $\frac{\partial G}{\partial n}$ is being computed.

4. For the problem given above, use the MFS with *fixed* source locations. Take 8 sources, two on each side of the domain. Take the locations of the sources as:

Source	x'	y'
1	1/3	-1/4
2	2/3	-1/4
3	5/4	1/3
4	5/4	2/3
5	2/3	5/4
6	1/3	5/4
7	-1/4	2/3
8	-1/4	1/3

(a) Write out the approximating function you are using. Does it satisfy the differential equation? Does it satisfy the boundary conditions?

(b) Compute the source strengths using a deterministic approach (8 equations, 8 unknowns). Plot the solution $u(x, y)$ as a function of y at $x = 0.5$. What can you say about the quality of your solution (and your confidence in it!)?

(c) Compute the source strengths again, but now use an overdeterministic, least-squares approach. Take a sufficient number of boundary collocation points for good convergence. Typically we would take 10 times as many boundary collocation points as we have unknown coefficients, so here that suggests taking 80 collocation points. You are free to solve the linear-least squares problem any way you wish, but remember the normal equations are inherently ill-conditioned. I suggest you use a QR or SVD approach (as we discussed in class), or simply the *backslash* command in Matlab. Repeat the plot you did in part (b). What can you conclude?

(d) Investigate the error in the boundary condition along all four boundaries by plotting the difference in your approximate solution and the theoretical boundary condition. What can you conclude?

(e) Repeat part (c), but move the sources further away from the boundary (from being located $\frac{1}{4}$ off the boundary to $\frac{5}{4}$ off the boundary). Does your solution improve or get worse?