①

Task 2.2: Senthil Arumugam Ramasamy

1565724

Q) Given a random variable process $x(\xi, t)$ with an Ensemble of three functions: $x(\xi_0, t)$, $x(\xi_1, t)$, $x(\xi_2, t)$ occuring with probabilies $P(\xi_0) = 1/2$ $P(\xi_1) = 1/3$ $P(\xi_2) = 1/6$

determine $s_{xx}(t_1, t_2) = ?$

Answer: There are three possible realizations given ξ_0, ξ_1, ξ_2 from the graph:

$x(\xi_0, t_1) = -1$ $\quad x(\xi_1, t_1) = 3$ $\quad x(\xi_2, t_1) = 1$

$x(\xi_0, t_2) = 2$ $\quad x(\xi_2, t_2) = 1$ $\quad x(\xi_2, t_2) = -1$

$$s_{xx}(t_1, t_2) = \sum_{i=1}^{3} \sum_{i=1}^{3} \left[x_i(\xi_1, t_1)\, x_j(\xi, t_2) \right]$$

$$= x(\xi_0, t_1) \cdot x(\xi_0 \cdot t_2) .\, P\{\xi_0\}$$

$$+ x(\xi_1, t_1) \cdot x(\xi_1, t_2)\, P(\xi_1) +$$

$$x(\xi_2, t_1) \cdot x(\xi_2, t_2)\, P\{\xi_2\}$$

$$= (-1)(\cancel{2})(1/\cancel{2}) + \cancel{3}(1)\left(\frac{1}{\cancel{3}}\right) + 1(-1)(1/6) = -1/6$$

Task: 2.3

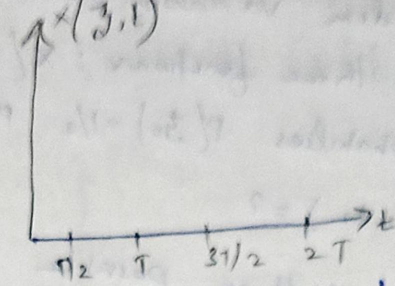
* Random process $x(\xi, t) = A(\xi) \sin\left(\frac{t}{T}\pi + \varphi(\xi)\right)$

with $T > 0$ $\quad f_y(y) = p\delta(y) + 1 - p\delta(\varphi + \pi/2)$ with

Probability $1 - P_1 = y = -\pi/2$

Case 1:

$A(\xi_0)=0 \quad y=0$ & $y=\pi/2$ probabily $=1/2$

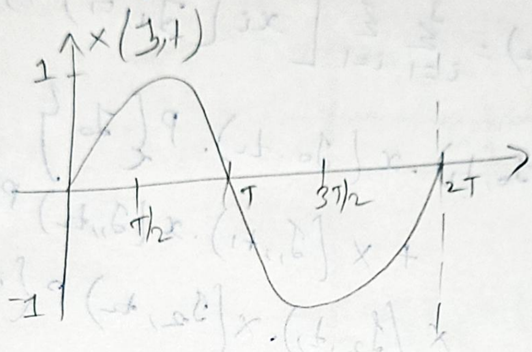


Signal is zero for all t

Case 2:

$\varphi=0 \quad A(\xi)=1$ Probabity $=1/2 \times P$

$x(\xi,t)=\sin(t/T\,\pi)$



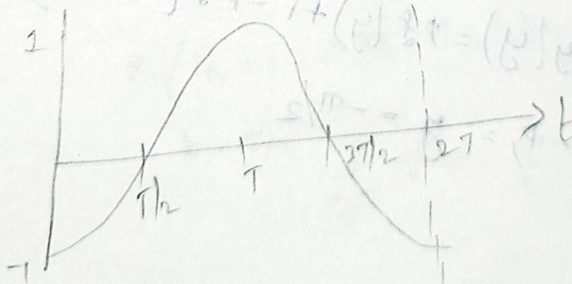
t	$x(\xi,t)$
0	0
$T/2$	1
T	0
$3T/2$	-1
$2T$	0

Case 3:

$\phi=-\pi/2 \quad A(\xi)=1$ Probabity $=1/2(1-P)$

$$x(\xi,t)=\sin\left(\frac{t}{T}\pi-\pi/2\right)$$

$$=-\cos\left(\frac{t}{T}\pi\right)$$

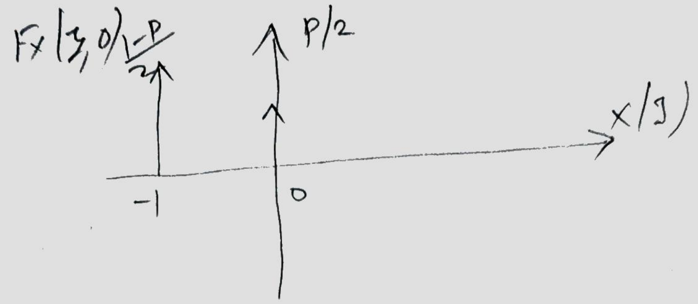


t	$x(\xi,t)$
0	-1
$T/2$	0
T	1
$3T/2$	0
$2T$	-1

(b) Sketch CDF $F_X(x, t)$ at $t=0$ and $t = T/4$

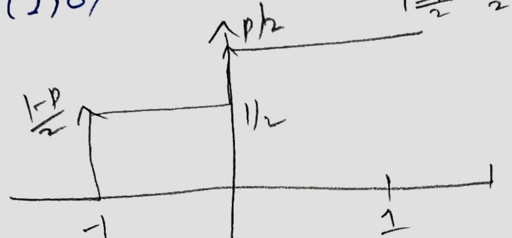
$x(\xi, 0) = A(\xi) \sin(0 + \varphi(\xi))$

$x(\xi, 0) = A(\xi) \sin(\varphi(\xi))$



A	φ	$x(0)$	p
			0.5
0	-	0	0.5×p
1	0	0	
1	$-\pi/2$	-1	$\frac{1-p}{2}$

$F_X(\xi, 0)$



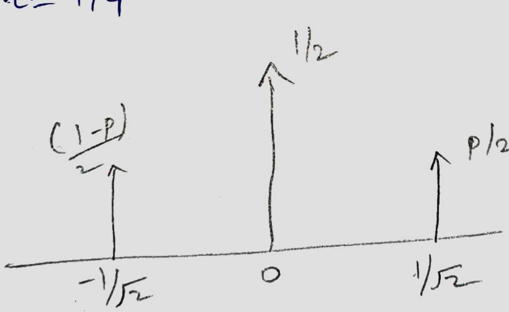
At $x < -1$: CDF $= 0$

At $x = 1$ CDF $\Rightarrow \frac{1-p}{2}$

At $x = 0$ CDF $\Rightarrow \frac{1}{2} + \frac{p}{2} + \frac{1-p}{2}$

$F_X(0^+) = \frac{1-p}{2} + \frac{p}{2} + \frac{1}{2} = 1$

At $t = T/4$



$A = 1,\ \varphi = 0$	$\sin \pi/4$
$A = 1\ \ \varphi = -\pi/2$	$\sin(-\pi/4)$
$A = 0$	0

$x(\xi, t) = A(\xi) \sin(t/T\, \pi + \varphi(\xi))$

$t = T/4$ $\quad \frac{t}{T} \cdot \pi \Rightarrow \frac{T}{4} \cdot \frac{\pi}{T} = \pi/4$

$$x(J,t) = A(J)\sin(\pi/4 + \varphi(J))$$ (4)

For $\varphi(J) = 0$

$$x(J, T/4) = A(J)\sin(\pi/4)$$

For $\varphi(J) = \pi/2$

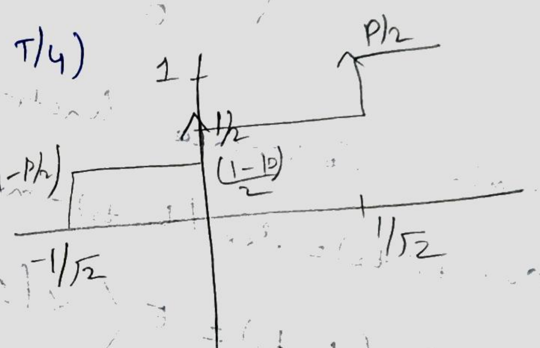
$$x(J, T/4) = A(J)\sin(-\pi/4)$$

For $\varphi(J) = \pi/2$

$$x(J, T/4) = A(J)\sin(-\pi/4)$$

For $A = 0$ $\quad x(J, T/4) = 0$

For $(J, T/4)$



(c) Calculate $m_x^{(1)}, t$

$$m_x^{(1)}(t) = E[x(J,t)]$$

$$x(J,t) = A(J)\cdot\sin\left(\frac{t}{T}\pi + \varphi(J)\right)$$

Case 1: $A = 0$

$$x(J,t) = 0$$

$$p = 1/2 \Rightarrow 1/2 \times 0 = 0 \quad -(1)$$

Case 2 : $A = 1 \quad \varphi = 0$

$$x(s, t) = \sin(t/T \, \pi)$$

$$P = \frac{1}{2} \cdot P = P/2 \Rightarrow \frac{P}{2} \cdot \sin\left(\frac{t}{T}\pi\right) \quad -(2)$$

Case 3: $\quad A = 1 \quad \varphi = -\pi/2$

$$x(s, t) = \sin\left(\frac{t}{T}\pi - \pi/2\right)$$

Probability $= 1/2 \cdot (1-P)$

$$\therefore \frac{1-P}{2} \cdot \sin\left(\frac{t}{T}\pi - \pi/2\right) \quad -(3)$$

Putting (1) (2) & (3) together

$$m_x^{(1)}(t) = \begin{cases} \frac{P}{2} \cdot \sin\left(\frac{t}{T}\pi\right) + \frac{1-P}{2} \cdot \sin\left(\frac{t}{T}\pi - \pi/2\right) & \text{if } A = 1 \\ 0 & \text{if } A = 0 \end{cases}$$

(d) Determine Co-Varience function $C_{xx}(t_1, t_2)$

$$C_{xx}(t_1, t_2) = S_{xx}(t_1, t_2) - m_x^{(1)}(t_1) - m_x^{(1)}(t_2)$$

$$S_{xx}(t_1, t_2) = E\left\{ x(s_1, t_1)\, x(s_1, t_2) \right\}$$

$$= \frac{P}{2} \sin\left(\frac{t_1}{T}\pi\right) \cdot \sin\left(\frac{t_2}{T}\pi\right) + \frac{1-P}{2}\left(-\cos\left(\frac{t_1}{T}\pi\right) \times -\cos\frac{t_1}{T}\pi\right)$$

$$= 0.5P \sin\left(\frac{t_1}{T}\pi\right) \cdot \sin\left(\frac{t_2}{T} \cdot \pi\right) + s(1-P) \cos\left(\frac{t_1}{T} \cdot \pi\right) \cdot \cos\left(\frac{t_2}{T} \cdot \pi\right)$$

$$\therefore C_{xx}(t_1, t_2) = 0.5P \left[\sin\left(\frac{t_1}{T}\pi\right) \cdot \sin\left(\frac{t_2 \pi}{T}\right)\right] + 0.5(1-P)\left[\cos\frac{t_1}{T}\pi\right) \cos\left(\frac{t_2}{T}\pi\right)\right] -$$

$$\left[0.5P \sin\left(\frac{t_1}{T}\pi\right) + 0.5(1-P)\cos\left(\frac{t_1}{T}\pi\right) - 0.5P\sin\left(\frac{t_2}{T}\pi\right) + 0.5(1-P)\cos\left(\frac{t_2 \pi}{T}\right) \right]$$

(e) A Process if Ergodic if time average = Ensemble average, Here time average depends on realisations (i.e) different outcomes gives different time averages, but the Ensemble average is constant across them.