

## Min-Max Deque

This is the hard version of the problem. Here, there are updates to the array, and the constraints on  $Q$  are  $1 \leq Q \leq 2 \cdot 10^5$ .

Consider an array  $A$  of length  $N$ .  $N$  is odd.

Alice and Bob will play the following game on it:

- Let  $B$  be an initially empty array.
- Alice and Bob take turns playing, with Alice going first.
- On the  $i$ -th turn, the current player will choose element  $A_i$ , and insert it to either the beginning or the end of  $B$ .  
So, Alice will place  $A_1$ , then Bob will place  $A_2$ , then Alice will place  $A_3$ , and so on.
- This continues till all  $N$  elements have been placed in  $B$ .

Once the game has finished, Alice's *score* is defined to be  $\min(B_1, B_N)$ .

Alice's objective is to maximize her score, while Bob's objective is to minimize Alice's score.  
Define  $f(A)$  to be the final score if both players play optimally, for the starting array  $A$ .

You are given an array  $A$  of length  $N$ , where  $N$  is odd.

There are  $Q$  point updates to the array. Each update gives you two integers  $p$  and  $X$ , and you are required to set  $A_p = X$ .

Report the value of  $f(A)$  before any updates are performed, and also after each update.  
Updates to  $A$  are permanent.

## Input Format

- The first line of input will contain a single integer  $T$ , denoting the number of test cases.
- Each test case consists of multiple lines of input.
  - The first line of each test case contains two space-separated integers  $N$  and  $Q$  — the length of  $A$  and the number of updates, respectively.
  - The second line contains  $N$  space-separated integers  $A_1, A_2, \dots, A_N$ .
  - The next  $Q$  lines describe the updates. Each of them contains two space-separated integers  $p$  and  $X$ , denoting a point update requiring you to set  $A_p := X$ .

## Output Format

For each test case, output one line containing  $Q + 1$  space-separated integers: the first integer should be the value of  $f(A)$  before any updates, and the  $i$ -th of the next  $Q$  integers should be the value of  $f(A)$  after the first  $i$  updates.

## Constraints

- $1 \leq T \leq 10^4$
- $3 \leq N \leq 2 \cdot 10^5 - 1$
- $N$  is odd.
- $1 \leq Q \leq 2 \cdot 10^5$