



$$T_{BR}(0, 0, -D), T_{BL}(0, 0, D), T_{RB}(0, 0, D), T_{LB}(0, 0, -D)$$

$$A_{RB} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -D & 0 & 1 \end{bmatrix} \quad v_R = A_{RB} v_B.$$

$$\Rightarrow \begin{bmatrix} \dot{\theta} \\ v_{xR} \\ v_{yR} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix} \Rightarrow \begin{bmatrix} \dot{\theta} \\ v_{xR} \\ v_{yR} \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ -D\dot{\theta} + v_x \\ v_y \end{bmatrix}$$

Similarly for the left wheel:

$$\Rightarrow \begin{bmatrix} \dot{\theta} \\ v_{xL} \\ v_{yL} \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ D\dot{\theta} + v_x \\ v_y \end{bmatrix}$$

For a conventional wheel,

$$v_{xL} = r\dot{\phi}_L, v_{xR} = r\dot{\phi}_R, y_{xL} = 0, y_{xR} = 0, \text{ (where } r \text{ is the radius of the wheel)}$$

$$\Rightarrow \begin{bmatrix} \dot{\theta} \\ r\dot{\phi}_L \\ v_x \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ -D\dot{\theta} + v_x \\ v_y \end{bmatrix}, \quad \begin{bmatrix} \dot{\theta} \\ r\dot{\phi}_R \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ D\dot{\theta} + v_x \\ v_y \end{bmatrix}$$

Rearranging to get equations for $\dot{\phi}_L$ & $\dot{\phi}_R$

$$\begin{bmatrix} \dot{\phi}_L \\ \dot{\phi}_R \end{bmatrix} = \gamma \begin{bmatrix} -D & 1 & 0 \\ D & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix}$$

$$U = H\gamma \Rightarrow \gamma = H^T U$$

$$H^T = (H^T H)^{-1} H^T$$

$$= \left(\gamma_2 \begin{bmatrix} -D & D \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -D & 1 & 0 \\ D & 1 & 0 \end{bmatrix} \right)^{-1} \gamma \begin{bmatrix} -D & D \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \left(\gamma_2 \begin{bmatrix} 2D^2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right)^{-1} \gamma \begin{bmatrix} -D & D \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\gamma^2}{2D^2} & 0 & 0 \\ 0 & \frac{\gamma^2}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \gamma \begin{bmatrix} -D & -D \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$H^T = \begin{bmatrix} -\frac{\gamma}{2D} & \frac{\gamma}{2D} \\ \frac{\gamma}{2} & \frac{\gamma}{2} \\ 0 & 0 \end{bmatrix}$$