Vanderbilt University – College of Engineering
CS 2212, SPRING 2023 - Sections 01/02/03/04
Homework Assignment #1 (maximum 50 points)

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Section: CS 2212-02/Daniel Arena

Honor Statement:
By submitting this homework under your personal Gradescope account, you attest that you have neither given nor received unauthorized aid concerning this homework. You further acknowledge the instructors are the copyright owners of this HW. Any posting/uploading of HW questions for distribution (e.g., GitHub, Chegg) will be considered an honor code violation (even after finishing this class) and submitted to the honor council.

HW Philosophy:
The objective of HW in CS2212 is to expand on lecture topics and examine them in more detail and depth. For this reason, we typically give a couple of weeks to complete the HW. Keep in mind that this is not supposed to be an easy task. Expect HWs to take time, require thinking, and challenge you. The TAs and instructors are not here to provide you with an answer or tell you whether your answer is correct while you are working on the HW. We are here to help you think more critically and at a higher level than when you started this class.

Clear, Correct, and Concise:
One of the critical aspects of this course is learning to communicate your ideas effectively. This skill is not only necessary for this course (or CS in general) but also a skill that employers highly value. Answers on all HWs, exams, and other activities in this class are graded on correctness, clarity, and conciseness. Keeping this in mind, here are a couple of pieces of advice:

- **Know Your Audience** - For this class, you should imagine your audience are your fellow students. Keeping this in mind ensures you provide enough details to prove to the grader that you understand your solution while keeping your answer concise enough to avoid unnecessary information.
- **Reread, Rewrite and Refine** - As with any good piece of writing, a well-written proof or answer often goes through several revisions. Reread, rewrite, refine, and take the time to get it right.

You must type your answers (or you will lose 50% on the exercise). Each exercise should begin on a new page (you can put multiple parts to the same exercises on the same page). Save your file as a PDF and upload the PDF document in an electronic format to Gradescope (https://www.gradescope.com/).

You can use different colors to markup your solution but **avoid using a red font.** When submitting your work, designate the corresponding page(s) of your submission for the appropriate question (or risk losing 5 points). See the following video at the 0:46 mark: https://www.gradescope.com/get_started#student-submission

Questions:
Ask your instructor before/after class or your instructor or TAs during office hours.

Due Date:
Your completed homework, including all the questions and answers, must be uploaded as a PDF to Gradescope no later than **Wednesday, February 1st, @ 9 AM CT (Nashville Time).**

Exercise 1: Equivalently Equivalent [10 points] - ARENA

Skills: understanding propositional logic, compound logical expressions, and equivalences.

Consider the proposition: WFF = $((\neg A) \wedge D \wedge C \wedge B) \vee (A \wedge B \wedge C \wedge D) \vee \neg((\neg B) \vee C \vee (\neg D))$. Using **only equivalence rules**, prove that the WFF $\equiv$ $(B \wedge D)$. Show all the steps; for each step, be sure to mention the equivalence you are using as a justification. You may not need all the rows below. **Tip:** You might find the distributive property helpful.

ANSWER:

#	Statement	Justification (equivalence rule)
1	$((\neg A) \wedge D \wedge C \wedge B) \vee (A \wedge B \wedge C \wedge D) \vee \neg((\neg B) \vee C \vee (\neg D))$	Premise (Given)
2	$((\neg A) \wedge D \wedge C \wedge B) \vee (A \wedge B \wedge C \wedge D) \vee ((\neg\neg B) \wedge \neg C \wedge (\neg\neg D))$	**De Morgan's; 1**
3	$(\neg A \wedge D \wedge C \wedge B) \vee (A \wedge B \wedge C \wedge D) \vee (B \wedge \neg C \wedge D)$	**Double Negation; 2**
4	$(B \wedge D) \wedge ((\neg A \wedge C) \vee (A \wedge C) \vee (\neg C))$	**Distributivity; 3**
5	$(B \wedge D) \wedge ((C \wedge ((\neg A) \vee (A))) \vee (\neg C))$	**Distributivity; 4**
6	$(B \wedge D) \wedge ((C \wedge \text{True}) \vee (\neg C))$	**Complement; 5**
7	$(B \wedge D) \wedge (\text{True} \wedge C \vee \neg C)$	**Associative; 6**
8	$(B \wedge D) \wedge (\text{True} \wedge \text{True})$	**Complement; 7**
9	$(B \wedge D) \wedge (\text{True})$	**Idempotence; 8**
10	$(B \wedge D)$	**Idempotence; 9**
11	**QED**	
12		

Exercise 2: Logically Illogical [15 points] - ARENA

Skills: logical reasoning, quantifiers

There are (3) parts to this exercise (A, B, C)

Using the domain of natural numbers N+ (i.e., 1, 2, 3, ….), let's define D(x, y) to be true when y divides x and false otherwise. For example, if x=6 and y=2, then D(x, y) is true because 2 divides 6. On the other hand, if x=5 and y=2, then D(x, y) is false since 2 does not divide 5 over the domain of natural numbers.

Now, let's define P(x) over N+ such that P(x) is true if x is a prime number and false otherwise.

A. **[5 points]** Using quantifiers, logical operators, and D(x, y) as previously defined, write the symbolic representation to express P(x). **Hint:** We say that any natural number x is prime if the only numbers that divide it are 1 and x. Note that 1 is not a prime number.

i. **[3 points]** Write a symbolic representation for a WFF that accurately defines P(x).

$P(x) = (x \neq 1) \Lambda (\neg\exists y\ (x \neq y\ \Lambda\ y \neq 1\ \Lambda\ D(x, y)))$

ii. **[2 points]** To demonstrate you understand the representation from part A(i), using no more than three sentences, explain why your answer correctly describes P(x).

My representation from part A(i) answers this question because it takes care of this circumstance when x is 1 (not a prime number, and therefore evaluates the expression to false). Furthermore, if x is another number, to determine if it is prime, it depends on the value of y. The evaluation of the formula within the second pair of parantheses yields true when y divides x, x = y, and y = 1, which is the criteria for a prime number.

B. **[5 points]** Consider the factual statement, "There are an infinite number of natural numbers that are divisible by 5." Using quantifiers, logical operators, and our previously defined D(x, y), express this statement as a WFF (well-formed formula). You may continue to assume a domain of N+.

i. **[3 points]** Write a symbolic representation for a WFF that accurately defines the statement, "There are an infinite amount of natural numbers that are divisible by 5."

$WFF = \forall x \exists y\ (D(x, 5) \rightarrow D(y, 5)\ \Lambda\ (y > x))$

ii. **[2 points]** To demonstrate you understand the representation from part B(i), using no more than three sentences, explain why your answer correctly describes the statement.

For all natural numbers x that are divisible by 5, there exists some natural number y that is divisible by 5 such that y is greater than x. This statement proves that from the subset of all natural numbers, there is an infinite group of numbers (multiples of 5) that is divisible by 5.

C. **[5 points]** Consider a world in which the following is true -- "There are not an infinite number of prime numbers." Using quantifiers, logical operators, and P(x) as previously defined, express this statement as a WFF (well-formed formula). You may continue to assume a domain of N+.

i. **[3 points]** Write a symbolic representation for a WFF that accurately defines the statement, “There are not an infinite number of prime numbers.”

WFF = $\exists x \forall y\ (P(x) \land P(y) \rightarrow (x > y))$

There exists a prime number that is greater than all other prime numbers

ii. **[2 points]** To demonstrate you understand the representation from part C(i), using no more than three sentences, explain why your answer correctly describes the statement.

The statement that there is not an infinite number of prime numbers means that there exists some prime number that is greater than all the other prime numbers, which puts an upper bound to the number of prime numbers there are. To symbolically express this, I used the existential quantifier to illustrate that for some natural number x to be prime and all natural numbers y to be prime, this singular natural number x must be greater than all the other natural prime numbers.

Exercise 3: Who Teaches What? [10 points] - ARENA

Skills: logical reasoning, quantifiers

There are (4) parts to this exercise (A, B, C, D)

Define the following predicates over the domain of Vanderbilt courses and professors.

P(x): x is a professor. C(y): y is a course. T(x, y): professor x teaches course y.	Also, define the following symbols: d = “CS 2212 discrete structures” a = “Professor Dan Arena” h = “Professor Kamrul Hasan”

Using the above predicates, symbols, and appropriate quantifiers, express the following statements symbolically. Here's a quick list of symbols for logic that you can easily copy/paste as needed in your HW instead of using the equation editor:

Symbol	**Meaning**
∀	FOR ALL
∃	THERE EXISTS
⟶	IMPLIES
⇔	IF AND ONLY IF
v	OR
Λ	AND
¬	NOT

A. **[2 points]** Some courses are taught by more than one professor.

$\exists y \exists x \exists z(T(x, y) \land T(z, y) \land P(x) \land P(z) \land C(y) \land \neg(x=z))$

B. **[3 points]** CS 2212 (Discrete Structures) is taught by exactly two professors: Dan Arena and Kamrul Hasan.

Rewrite: Dan Arena and Kamrul Hasan teach CS 2212, and there does not exist a professor such that teaches the course and the professor is not hasan and the professors it not arena

$T(h, d) \land T(a, d) \land P(a) \land P(h) \land C(d) \land (\neg\exists x(T(x, d)) \land P(x) \land \neg(x=a) \land \neg(x=h))$

C. **[2 points]** Some courses are not offered (i.e., some courses are taught by no one).

$\exists y \forall x((C(y)) \land (P(x) \longrightarrow \neg T(x, y)))$

D. **[3 points]** No professor teaches more than two courses.

$\forall x\ \forall g \forall e \forall f\ (\neg(T(x, g) \land T(x, e) \land T(x, f)) \land P(x) \land C(g) \land C(e) \land C(f))$

Exercise 4: LOVE ME [15 points] - HASAN

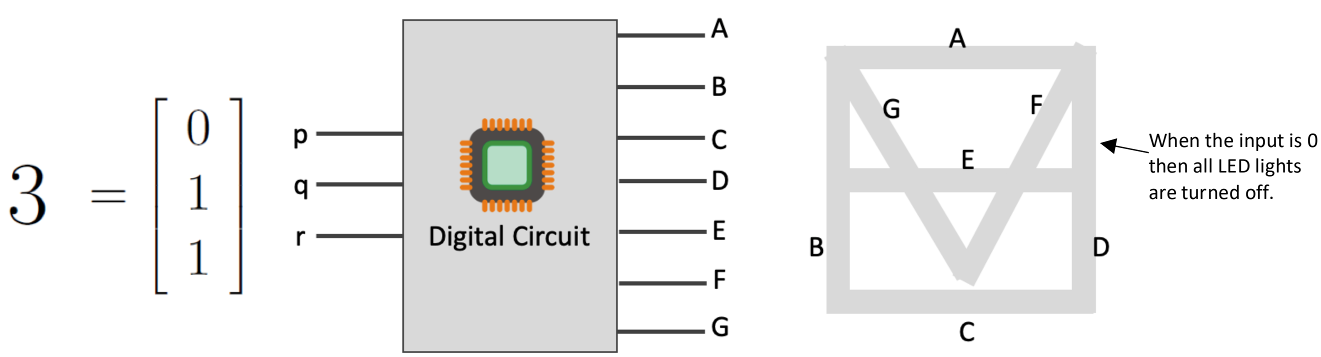
Skills: critical thinking and logical reasoning

[15 points] Nadha Skolar wants to impress his girlfriend for Valentine's Day by designing a logic circuit that when supplied the correct values will light up a particular letter. Given the correct sequence, Nadha hopes to spell out "Love Me." Unfortunately, he's running into some trouble designing his circuit.

You can think of a compound proposition as a logic circuit, which has only two possible outcomes, either 1 (true) or 0 (false). For instance, consider the expression:

$$A = (q \land \neg r) \lor (p \land \neg r)$$

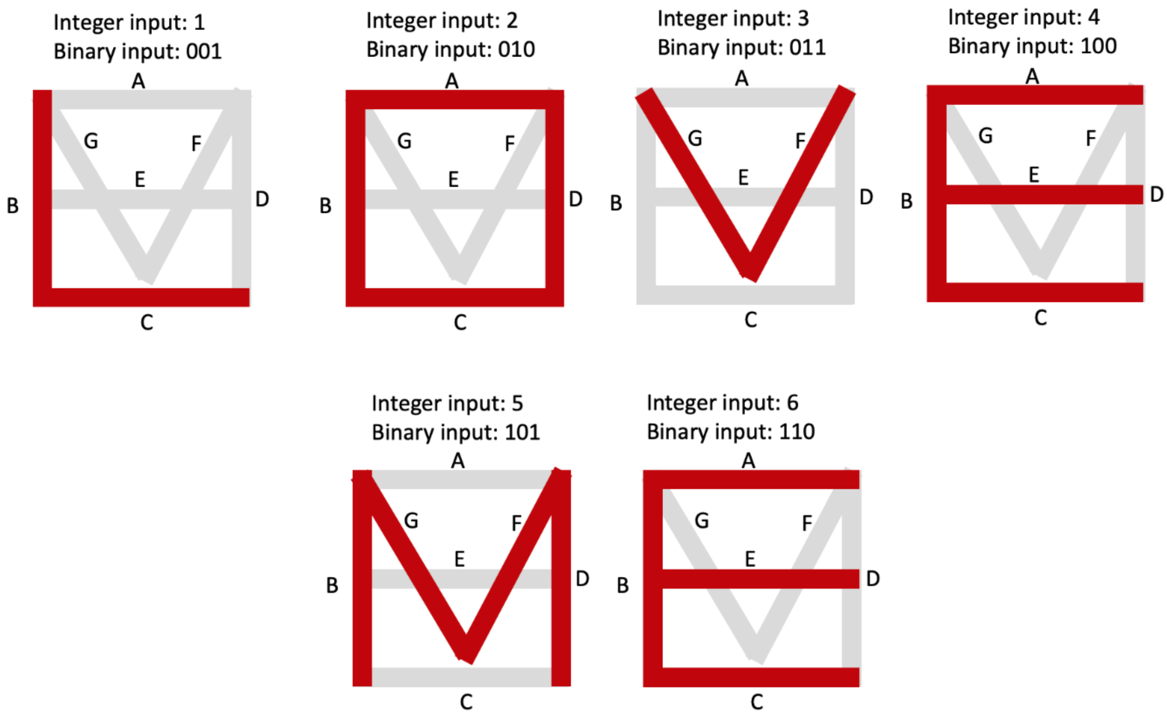
In terms of a digital circuit, A is the output, and p, q and r are input signals. If $p = 0$, $q = 1$, $r = 1$, the output signal $A = 0$. In signal A represented the horizontal line in the image below right, A would not light up. Now consider a 7-segment LED display consisting of 7 LED lights as shown in the below figure. The input consists of three 'signals' which correspond to the binary representation of a digit. For instance, if there is a digit 3, then it's binary representation is 0 1 1, and the value of three input signals are $p = 0$, $q = 1$ and $r = 1$.



You job is to design a digital circuit (compound proposition) for each of the seven LED lights (in other words, seven propositions). The goal is to have the LED light up according to display the following letters one at a time

“LOVE ME” (including the blank). Keep in mind the LED lights up when it gets a ‘1’ signal. For example, the logic circuit for LED light A could be: A = $(q \wedge \neg r) \vee (p \wedge \neg r)$. This would cause A to light up when given the binary value for 2 (100) and not light up when given the value 3 (011).

In addition to the propositions that represent the signals A through G, you must also provide the truth table that verifies your answer. For interger input 0 (binary: 000) the display shows nothing (no LED light will be turned on). As illustrated below, an integer input 1 (binary 001) will present the alphabet L, input 2 will show the letter “O” and so one. Also note that for digit 7, the output of lights could be anything, that is, we don’t care if it is 0 or 1.



A. **[5 POINTS]** List your propositions below for signals A through G and then complete the truth table demonstrating the correctness of your propositions

Digits	Inputs			Outputs						
	p	q	r	A	B	C	D	E	F	G
0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	1	1	0	0	0	0
2	0	1	0	1	1	1	1	0	0	0
3	0	1	1	0	0	0	0	0	1	1
4	1	0	0	1	1	1	0	1	0	0
5	1	0	1	0	1	0	1	0	1	1
6	1	1	0	1	1	1	0	1	0	0
7	1	1	1	x	x	x	x	x	x	x

Here x means, it could be anything (0 or 1).

B. **[10 POINTS]** Write the WFF for each LED light (B, C, D, F, and G) when you give an (integer) input value such as 0, 1, 2, 3, 4, 5, and 6. Note that each input is translated to a 3-bit binary value and assigned to p, q, and r.

B(ii) [2 POINTS] Write a WFF for B that outputs a binary value, either 0 (turn off the LED light B) or 1 (turn on the LED light B), based on the input values of p, q, and r.
Anwser: B = (p v ¬(q ⇔ r))

Truth Table

p	q	r	(q ⇔ r)	¬ (q ⇔ r)	p v ¬ (q ⇔ r)
0	0	0	1	0	0
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1	0	1	1
1	1	0	0	1	1

B(iii) [2 POINTS] Write a WFF for C that outputs a binary value, either 0 (turn off the LED light C) or 1 (turn on the LED light C), based on the input values of p, q, and r.
Anwser: C = (¬r ⇔ (p v q))

Truth Table

p	q	r	¬r	p v q	(¬r ⇔ (p v q))
0	0	0	1	0	0
0	0	1	0	0	1
0	1	0	1	0	1
0	1	1	0	0	0
1	0	0	1	0	1
1	0	1	0	0	0
1	1	0	1	1	1

B(iii) [2 POINTS] Write a WFF for D that outputs a binary value, either 0 (turn off the LED light D) or (turn on the LED light D), based on the input values of p, q, and r.
Anwser: D = (q Λ ¬p Λ -r) v (¬q Λ p Λ r)

Truth Table

p	¬p	q	¬q	r	-r	(q Λ ¬p Λ -r) v (¬q Λ p Λ r)
0	1	0	1	0	1	0
0	1	0	1	1	0	0
0	1	1	0	0	1	1
0	1	1	0	1	0	0
1	0	0	1	0	1	0
1	0	0	1	1	0	1
1	0	1	0	0	1	0

B(iv) [2 POINTS] Write a WFF for E that outputs a binary value, either 0 (turn off the LED light E) or 1 (turn on the LED light E), based on the input values of p, q, and r.
Anwser: E = p Λ (¬r v q)

Truth Table

p	q	r	p Λ (¬r v q)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1

B(v) [2 POINTS] Write a WFF for F and G that outputs a binary value, either 0 (turn off the LED light F and G) or 1 (turn on the LED light F and G), based on the input values of p, q, and r.
Anwser: F and G = (¬p ∧ q ∧ r) v (p ∧ ¬q ∧ r)

Truth Table

p	¬p	q	¬q	r	-r	(¬p ∧ q ∧ r) v (p ∧ ¬q ∧ r)
0	1	0	1	0	1	0
0	1	0	1	1	0	0
0	1	1	0	0	1	1
0	1	1	0	1	0	0
1	0	0	1	0	1	0
1	0	0	1	1	0	1
1	0	1	0	0	1	0