TU Delft
Faculty of Electrical Engineering, Mathematics, and Computer Science
Circuits and Systems Group

ET4350 Applied Convex Optimization

ASSIGNMENT

Compressed Sensing

1 Context

Compressed sensing (CS) is a popular signal processing tool to reconstruct signals and images from significantly fewer measurements than suggested by the traditionally used Nyquist sampling theory. In this assignment, we measure only a subset of frequency domain samples of a sparse time-domain signal. This results in an underdetermined system. Nonetheless, exploiting properties of the time-domain signal, such as sparsity and/or positivity, we may reconstruct the original signal.

This exercise consists of two parts: (a) formulate the CS-based signal reconstruction as a suitable convex optimization problem; and (b) implement the signal reconstruction algorithm. In a group of 2 students, make a short report (4-5 pages; pdf file) containing the required Matlab scripts, plots, and answers. Also, prepare a short presentation to explain your results and defend your choices.

Dataset explanation

Download `cs.mat` from the course webpage. The variable x contains the true sparse signal. In this assignment, we assume that we sample in frequency domain (e.g., similar to MRI scanners), but we use *random frequency domain sampling*. The variable `sampling_mask` is a $\times$4-fold subsampled random mask. This boils down to a $\times$4-fold reduction in the sensing time. The zero-filled frequency data is stored in the variable X_{us}, where $X_{us} = F_{us}x$ with F_{us} being the Fourier transform evaluated only at the subset of frequency domain samples. Can you use tools learnt from this course to reconstruct x

Vector = Square * Vector

from X_{us} by incorporating the structure (e.g, sparsity and positivity) of the signal?

2 Assignment

You will have to answer the following questions:

1. (2 pts) Formulate the CS-based image reconstruction as an optimization problem. Suggest a suitable convex approximation (i.e., derive a convex relaxed problem). Motivate the proposed formulation as well as the relaxation.

2. (2 pts) Implement the proposed convex optimization problem in your favorite off-the-shelf solver (e.g., `CVX`, `SeDuMi`, or `YALMIP`). How does this ready-made software solve your problem? Comment on the number of iterations, CPU time, and algorithm the ready-made solver uses.

3. (5 pts) Implement a low-complexity algorithm (e.g., projected (sub)gradient descent for the above problem, or provide a first-order algorithm to solve the primal and dual problems). Compare the obtained results with the solutions from the off-the-shelf solver. Comment on the number of iterations, CPU time, and convergence of your low-complexity algorithm.

4. (1 pt) Make a short presentation explaining your results clearly in 5 minutes.

3 Reference

R.G. Baraniuk . Compressive sensing [lecture notes]. IEEE signal processing magazine, vol. 24, pp 181-121, July 2007.

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