Efficient Globally Optimal Consensus Maximisation with Tree Search

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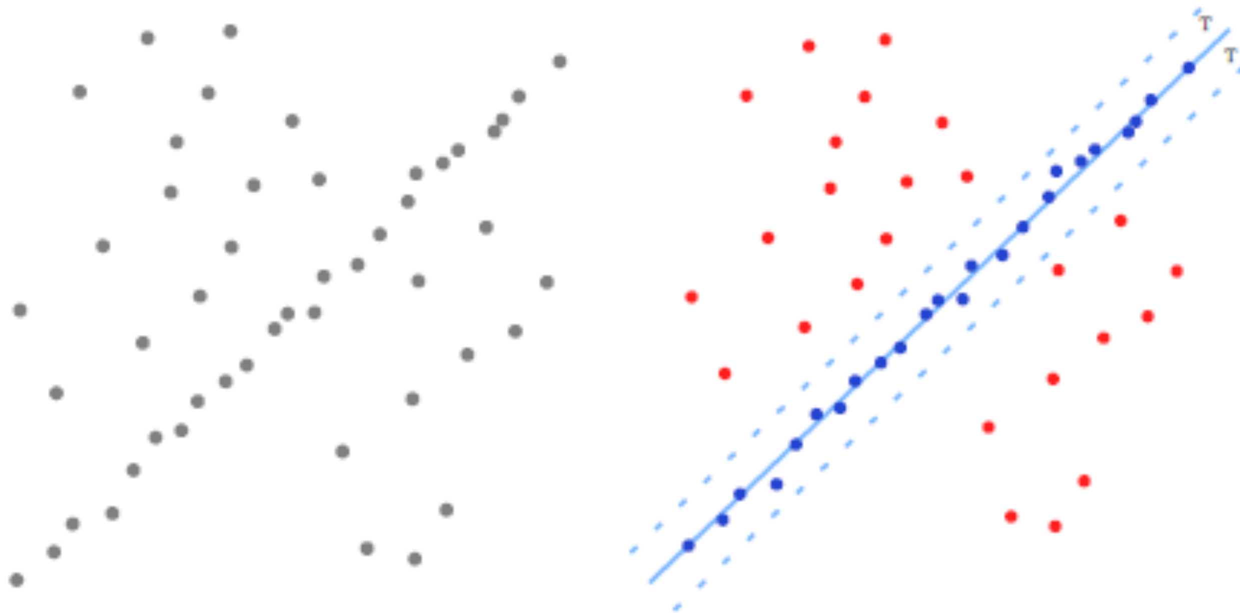
Maximum consensus

$$
\begin{aligned}
\max_{\theta,\, \mathcal{I} \subseteq \mathcal{X}} \quad & |\mathcal{I}| \\
\text{subject to} \quad & r_i(\theta) \leq \epsilon \;\; \forall \mathbf{x}_i \in \mathcal{I}
\end{aligned}
$$

Example 1: Line fitting



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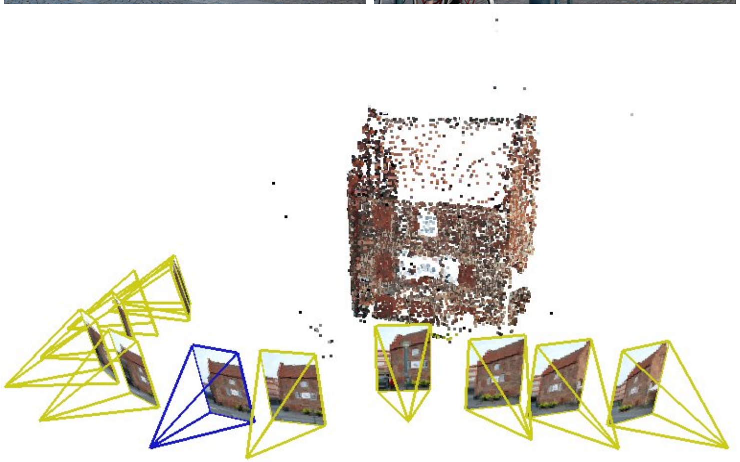
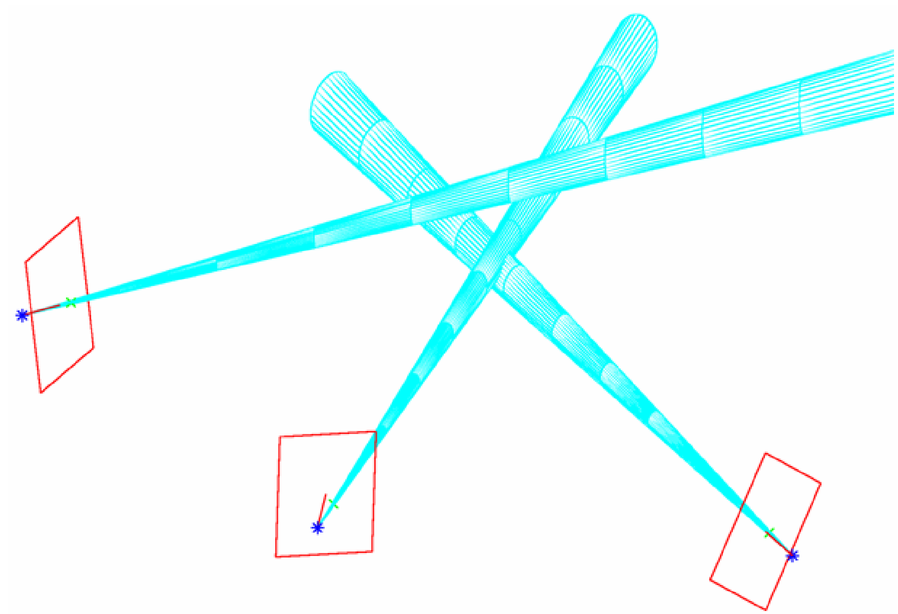
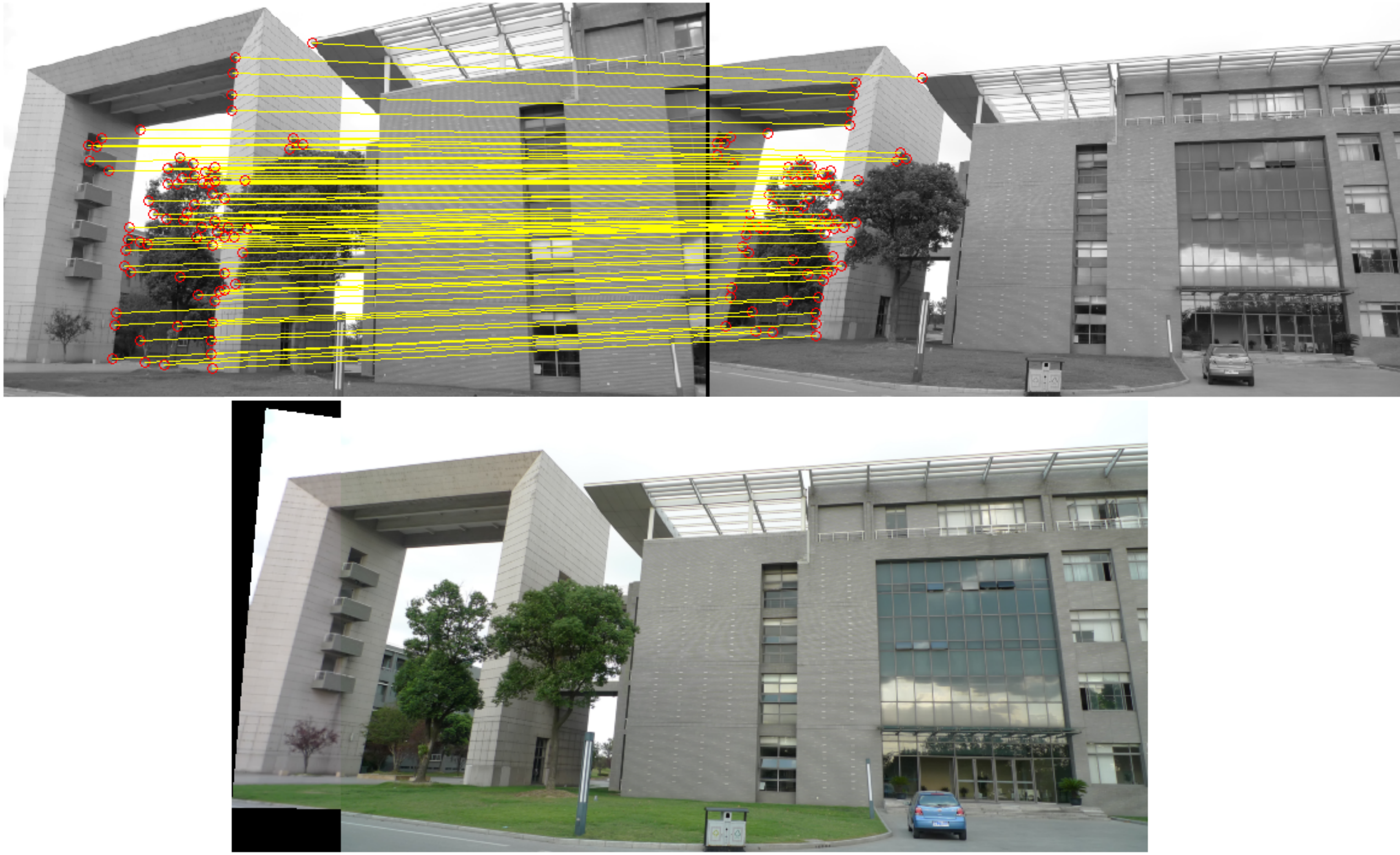
Example 2: Triangulation





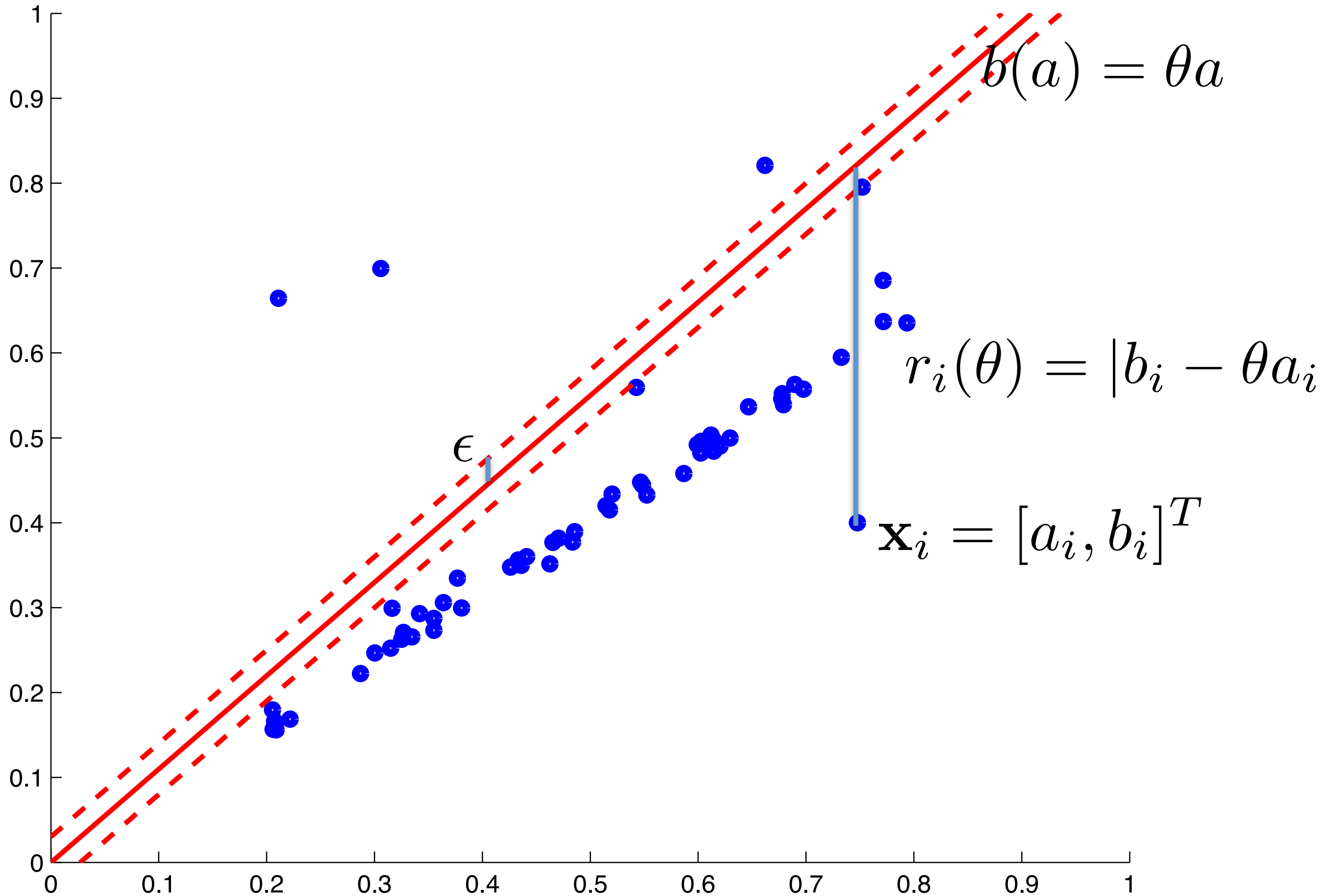
Olsson et al., Efficient optimization for L-infty problems using pseudoconvexity, ICCV 2007.
Hartley and Kahl, Optimal algorithms in multiview geometry, ACCV 2007.

Example 3: Homography fitting

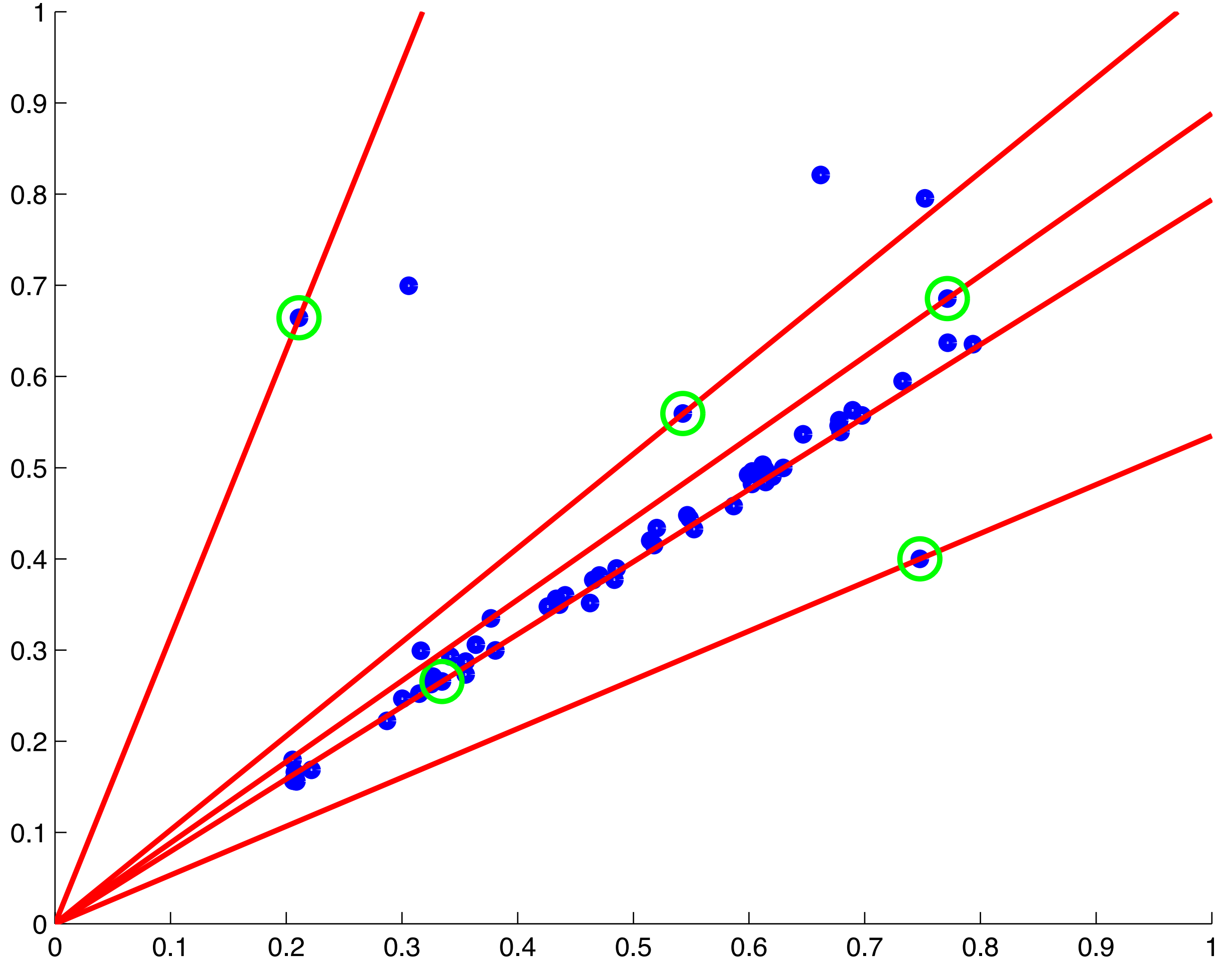


by T.-J. Chin Figure from http://sse.tongji.edu.cn/linzhang/CV14/Projects/panorama.htm.

Running example: Linear regression



RANSAC, minimal subset size = p



Minmax problem

- Minimise the maximum residual:

$$\min_{\theta} \max_{i} |b_i - \theta a_i|$$

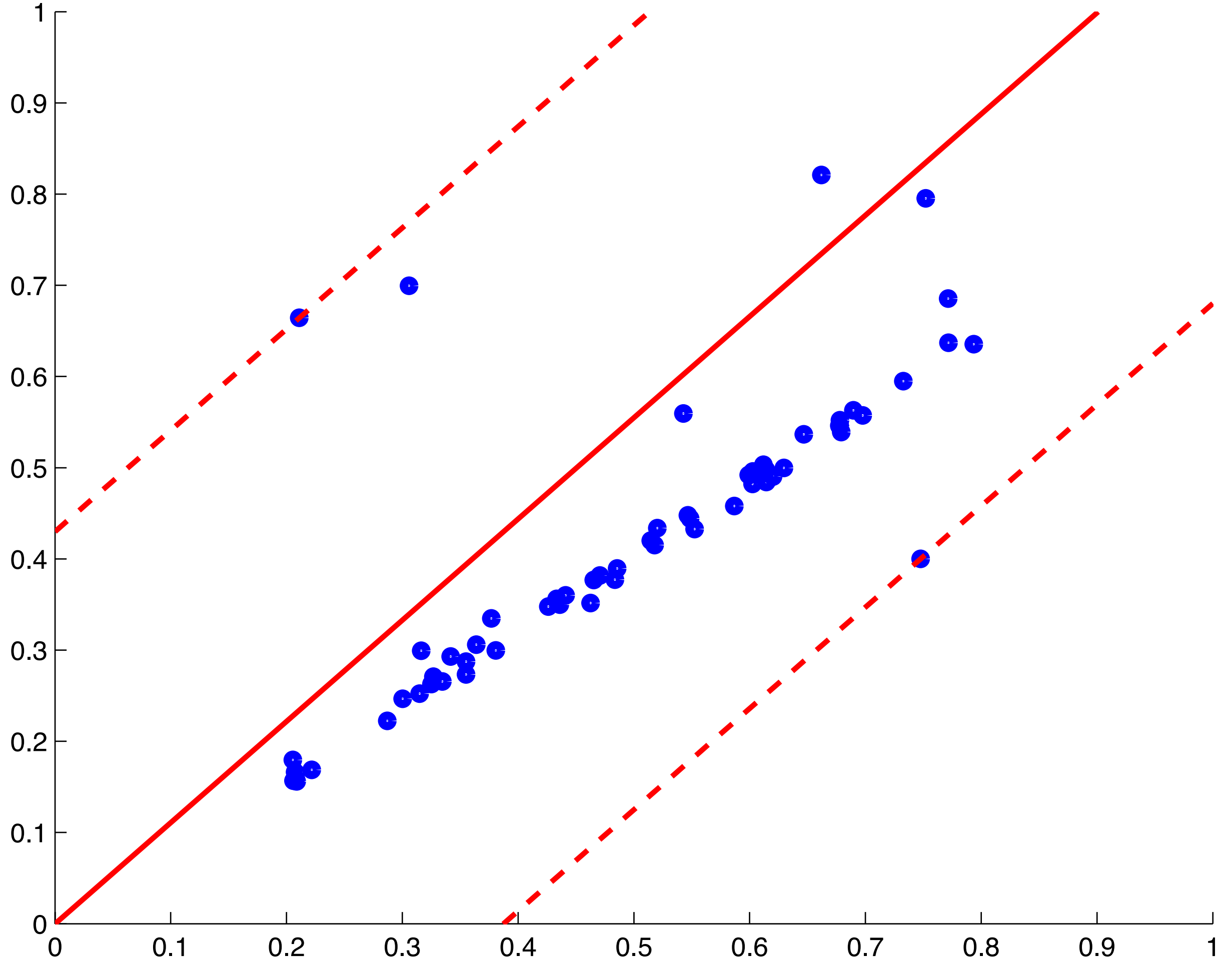
- Same as L-infinity minimisation[1,2]:

$$\min_{\theta} \left\| \begin{bmatrix} |b_1 - \theta a_1| \\ |b_2 - \theta a_2| \\ \vdots \\ |b_N - \theta a_N| \end{bmatrix} \right\|_{\infty}$$

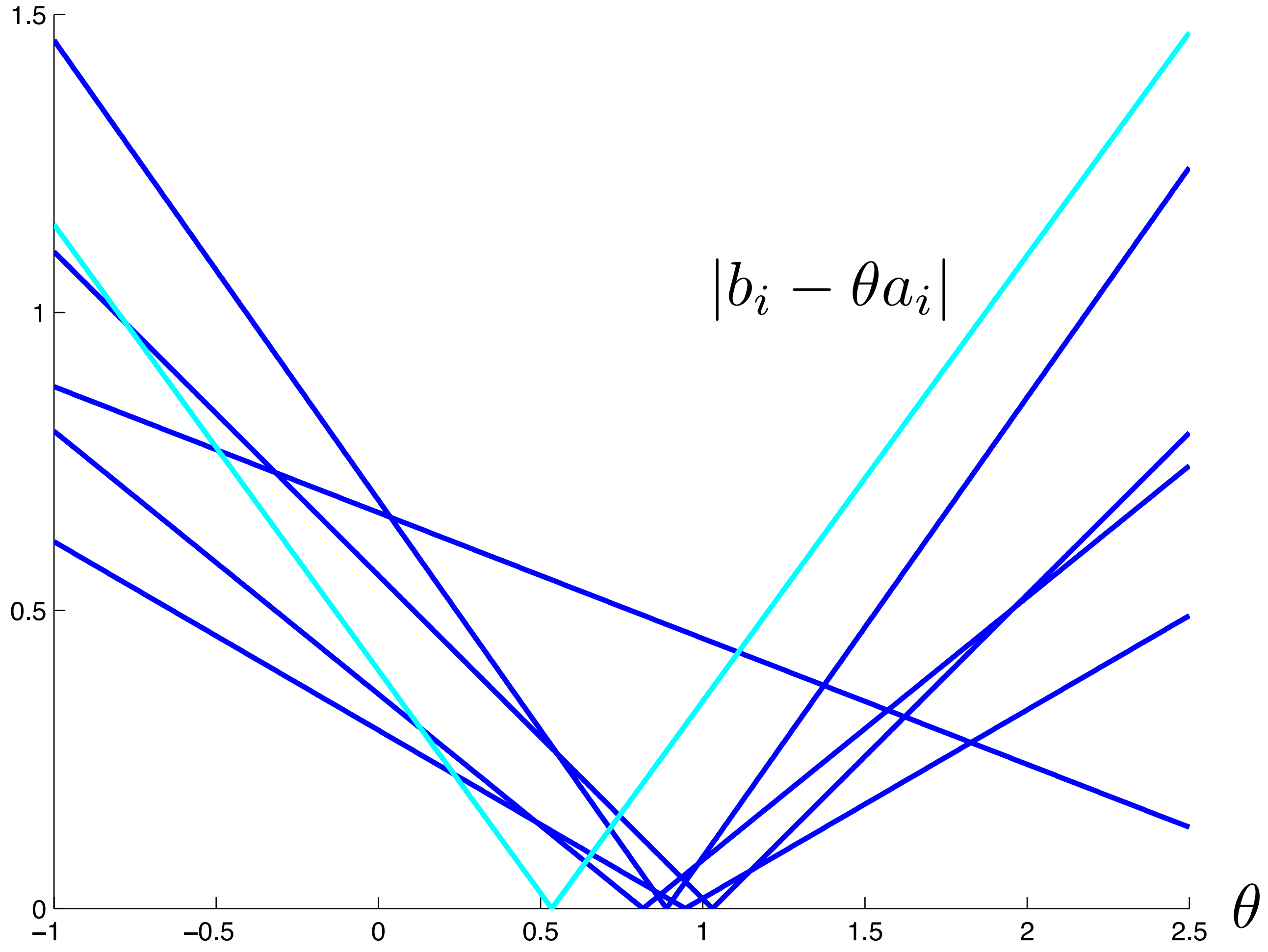
- A.k.a. Chebyshev approximation/regression.

[1]Hartley and Schaffalitzky, L-infty minimization in geometric reconstruction problems, CVPR 2004.
[2]Kahl and Hartley, Multiple-view geometry under the L-infty-norm, PAMI 2008.

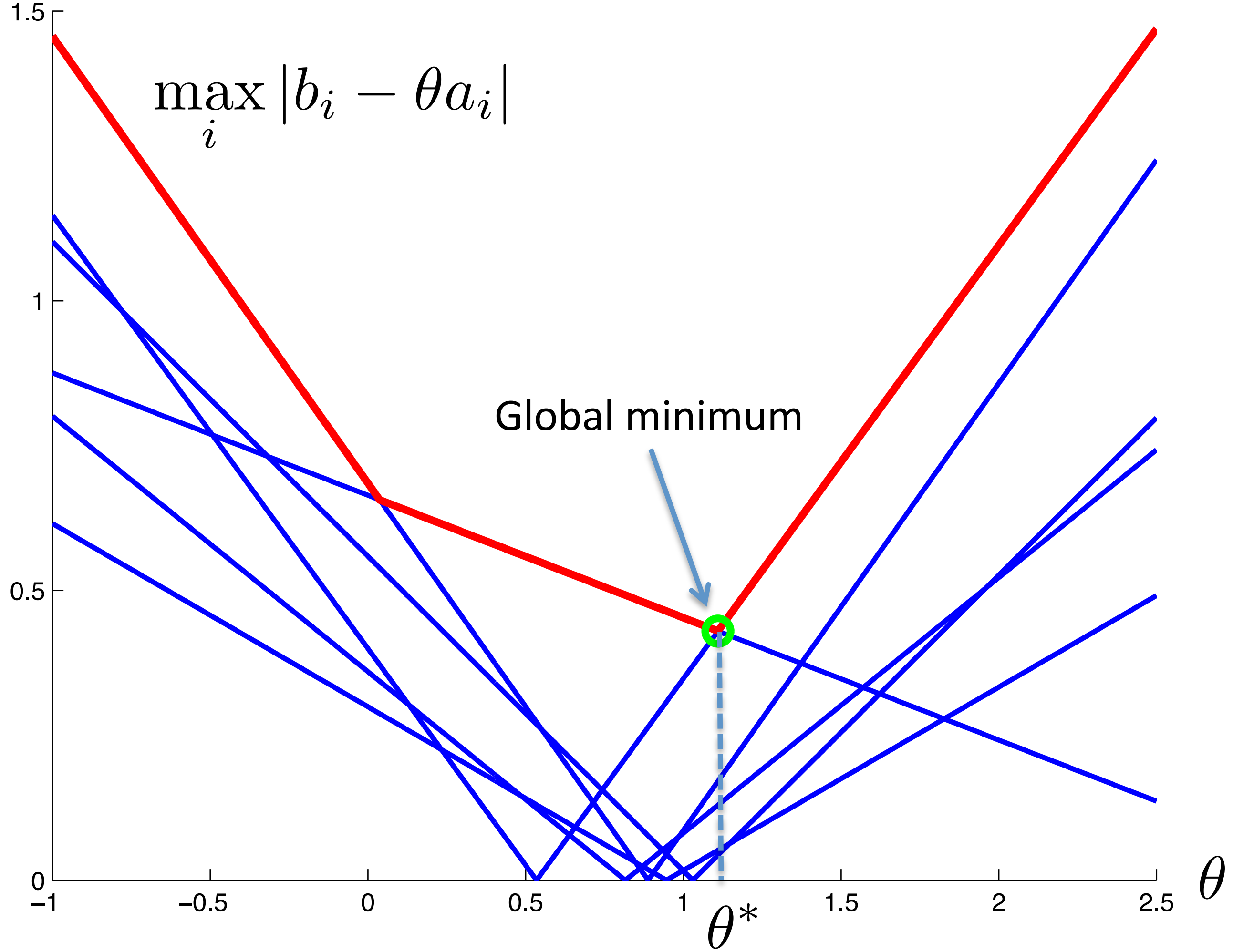
Minmax problem



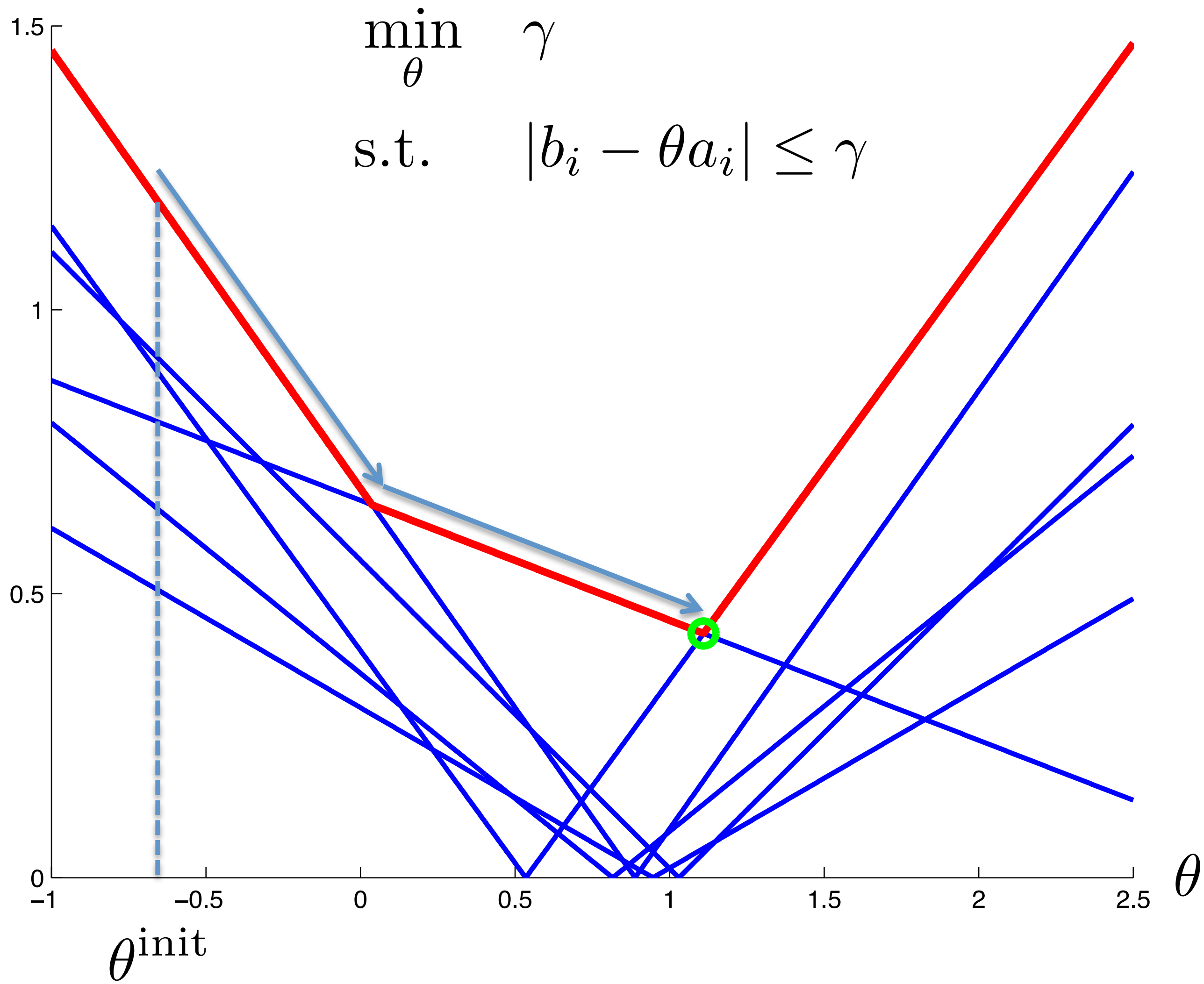
Minmax problem



Minmax problem



Simplex algorithm



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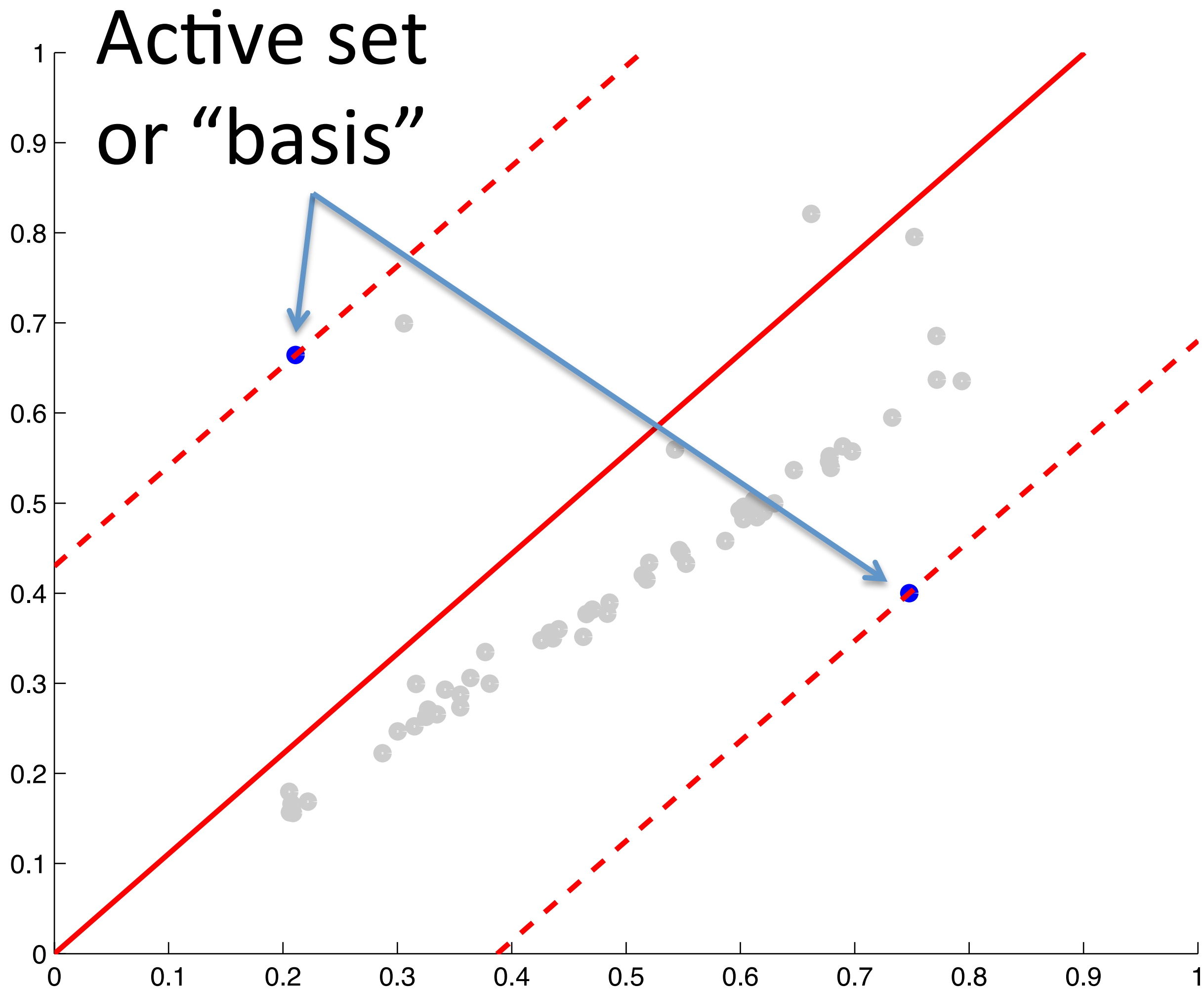
Combinatorial dimension = p+1



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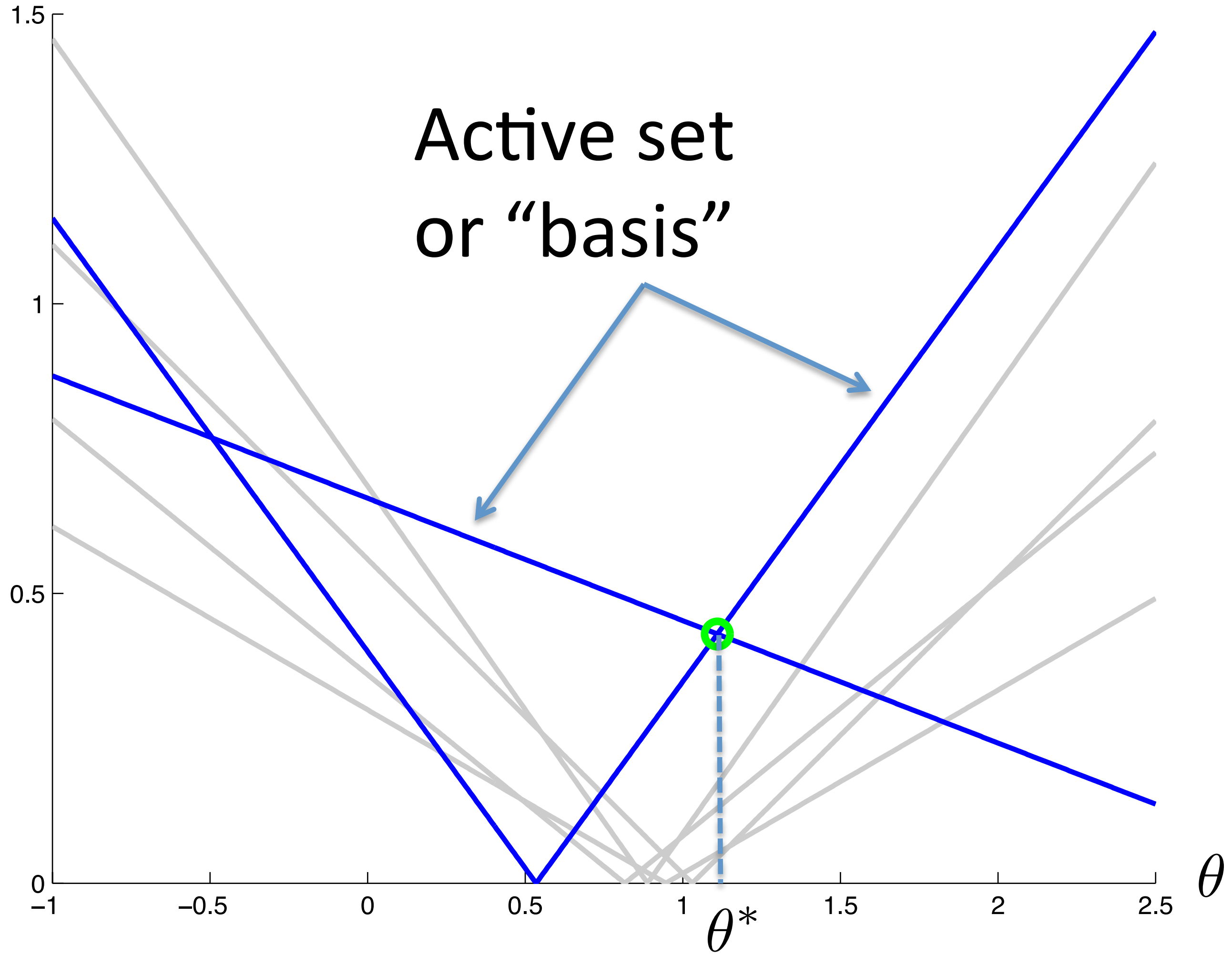

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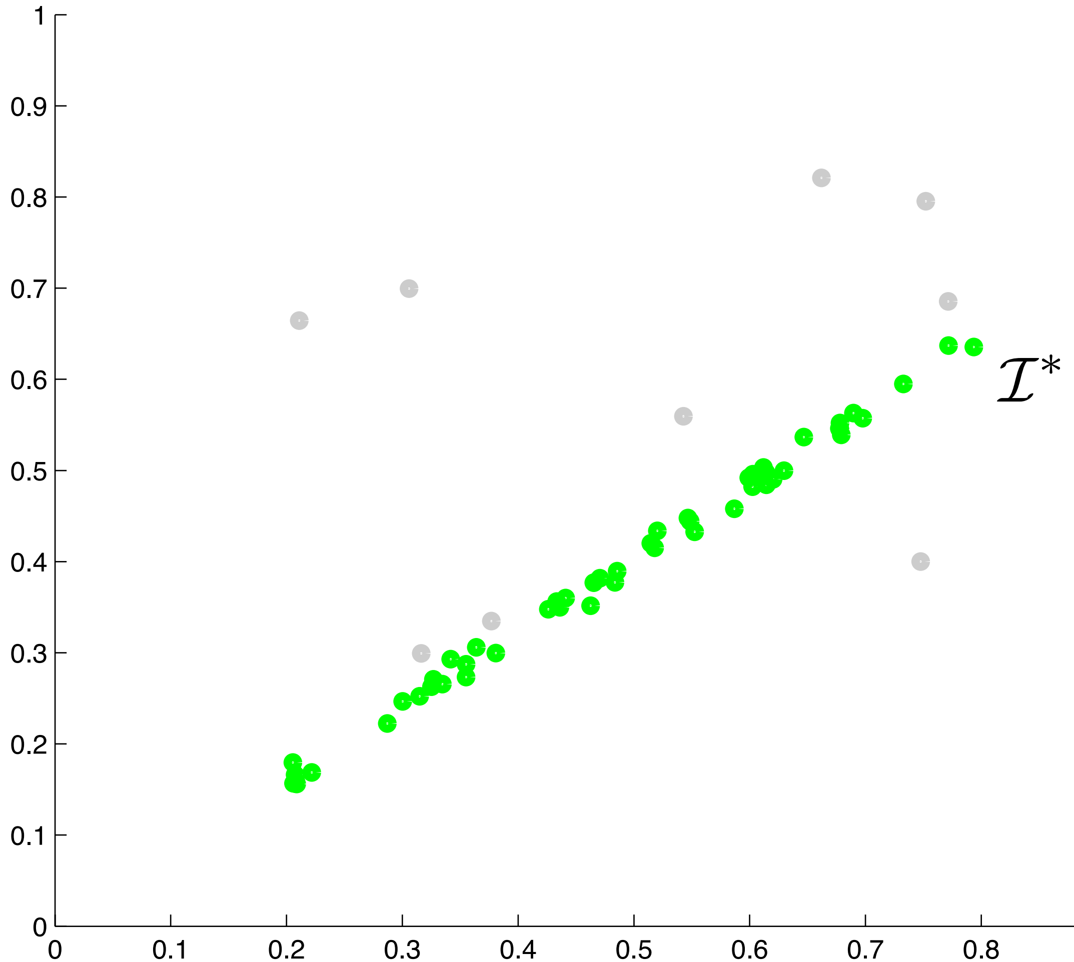
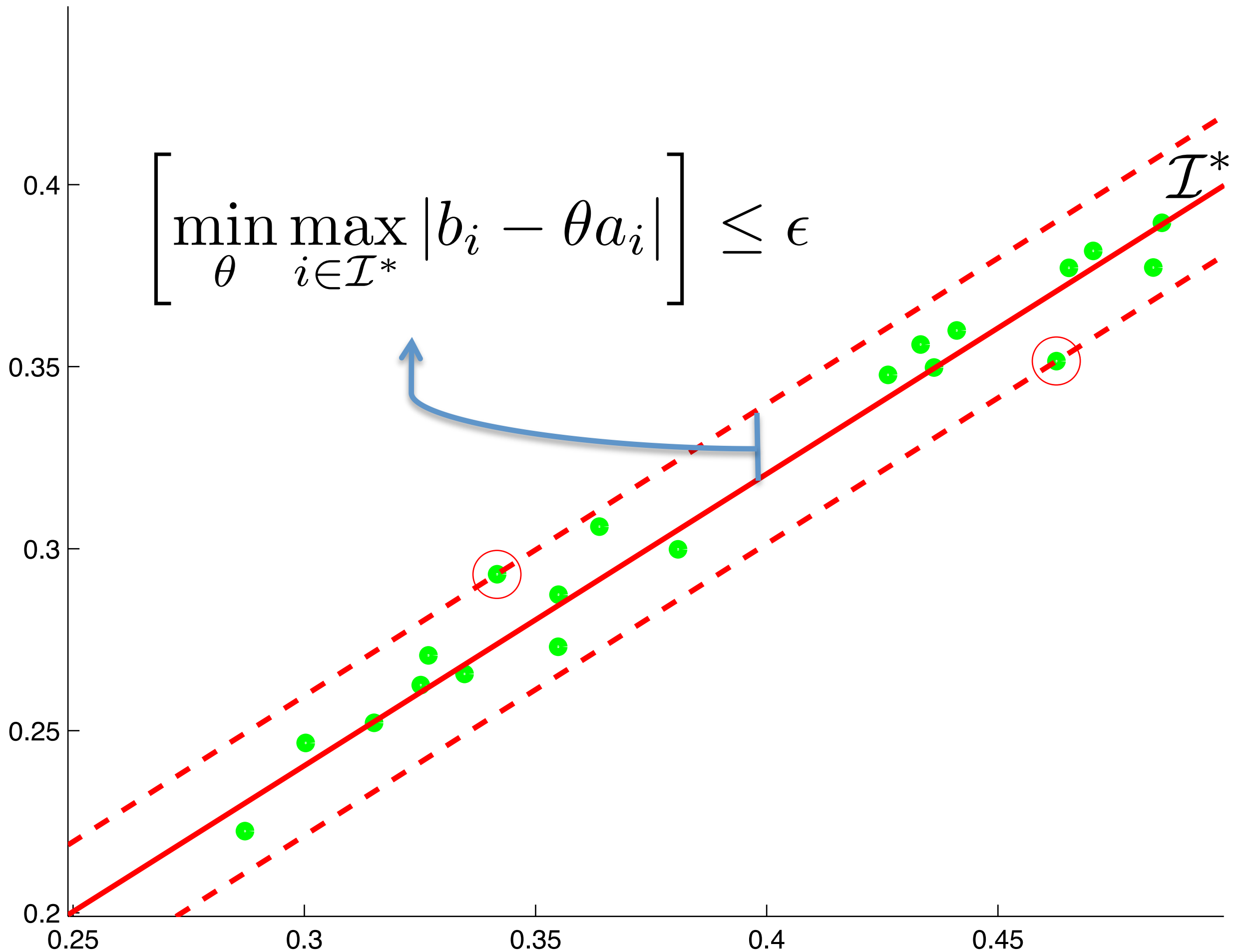
Maximum consensus



Maximum consensus



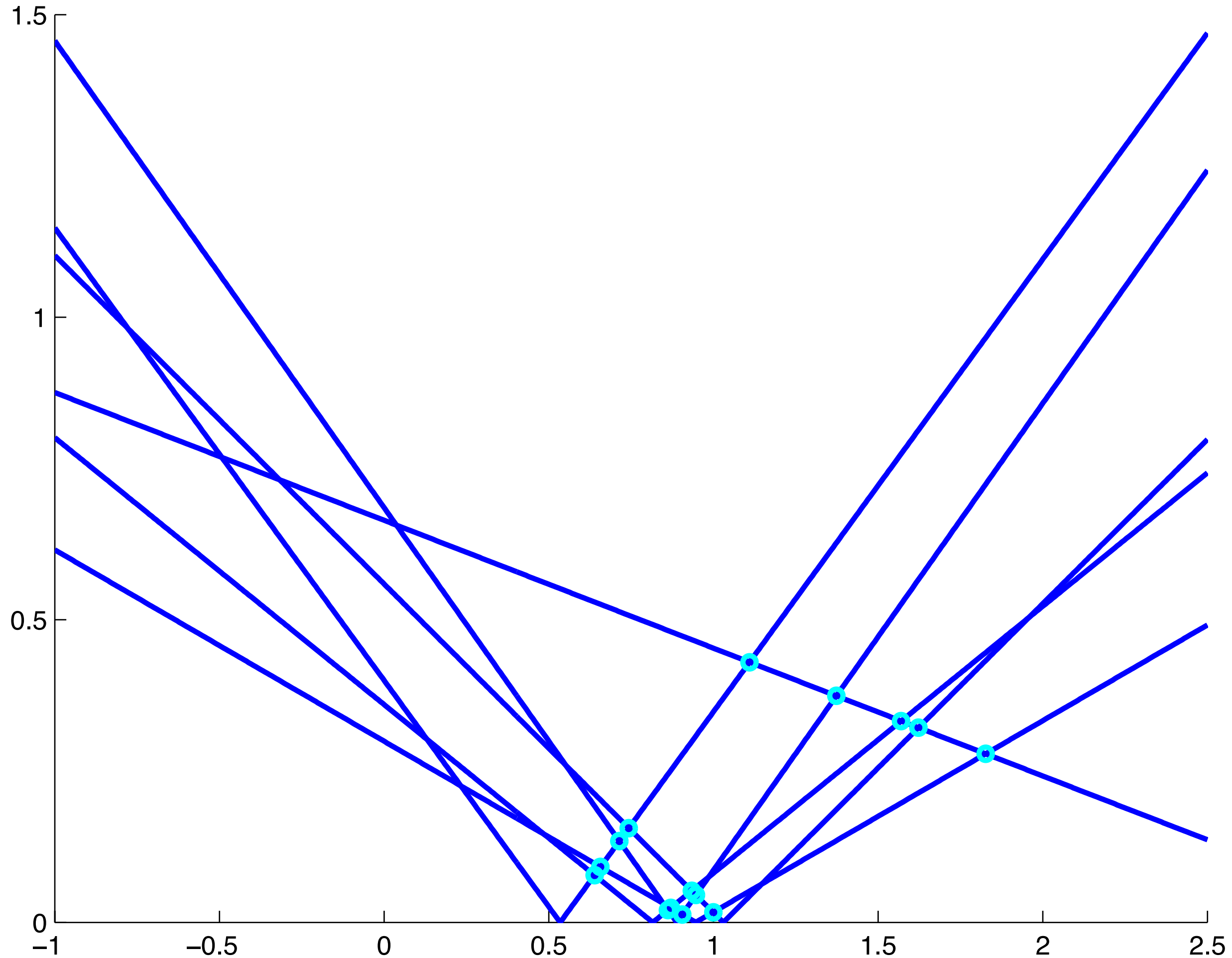
An algorithm

- For all subsets $\mathcal{B} \subset \mathcal{X}$ of size (p+1);
  - Solve minmax problem on $\mathcal{B}$.
  - If maximum residual of $\mathcal{B}$ is $\leq \epsilon$;
    - If the coverage of $\mathcal{B}$ is greater than the current largest;
      - Set $\mathcal{I}^*$ as the coverage of $\mathcal{B}$.

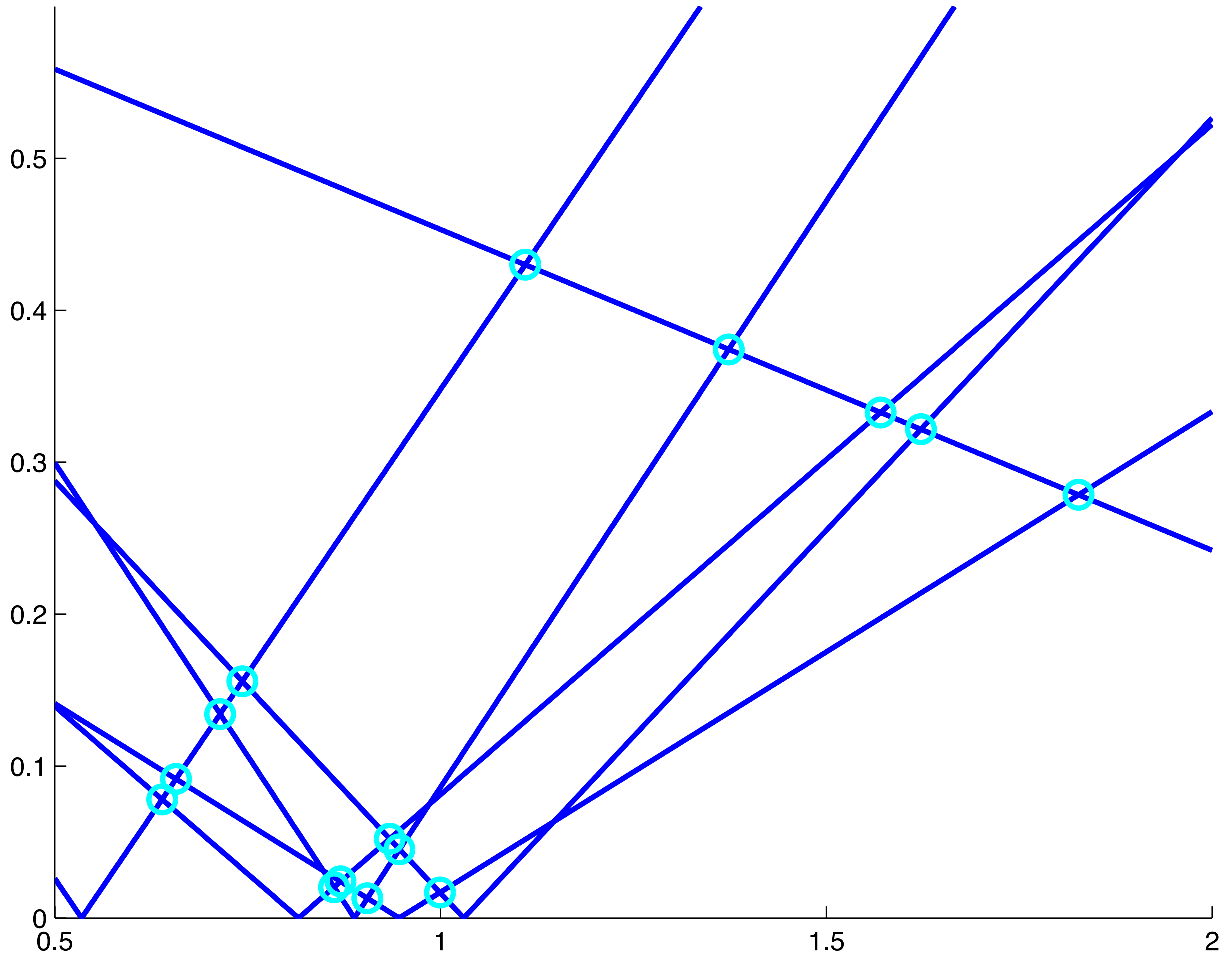
$$\binom{N}{p+1} = \frac{N!}{(p+1)!(N-p-1)!}$$

$$= \frac{1}{(p+1)!} N(N-1)\dots(N-p) \equiv \mathcal{O}(N^{p+1})$$

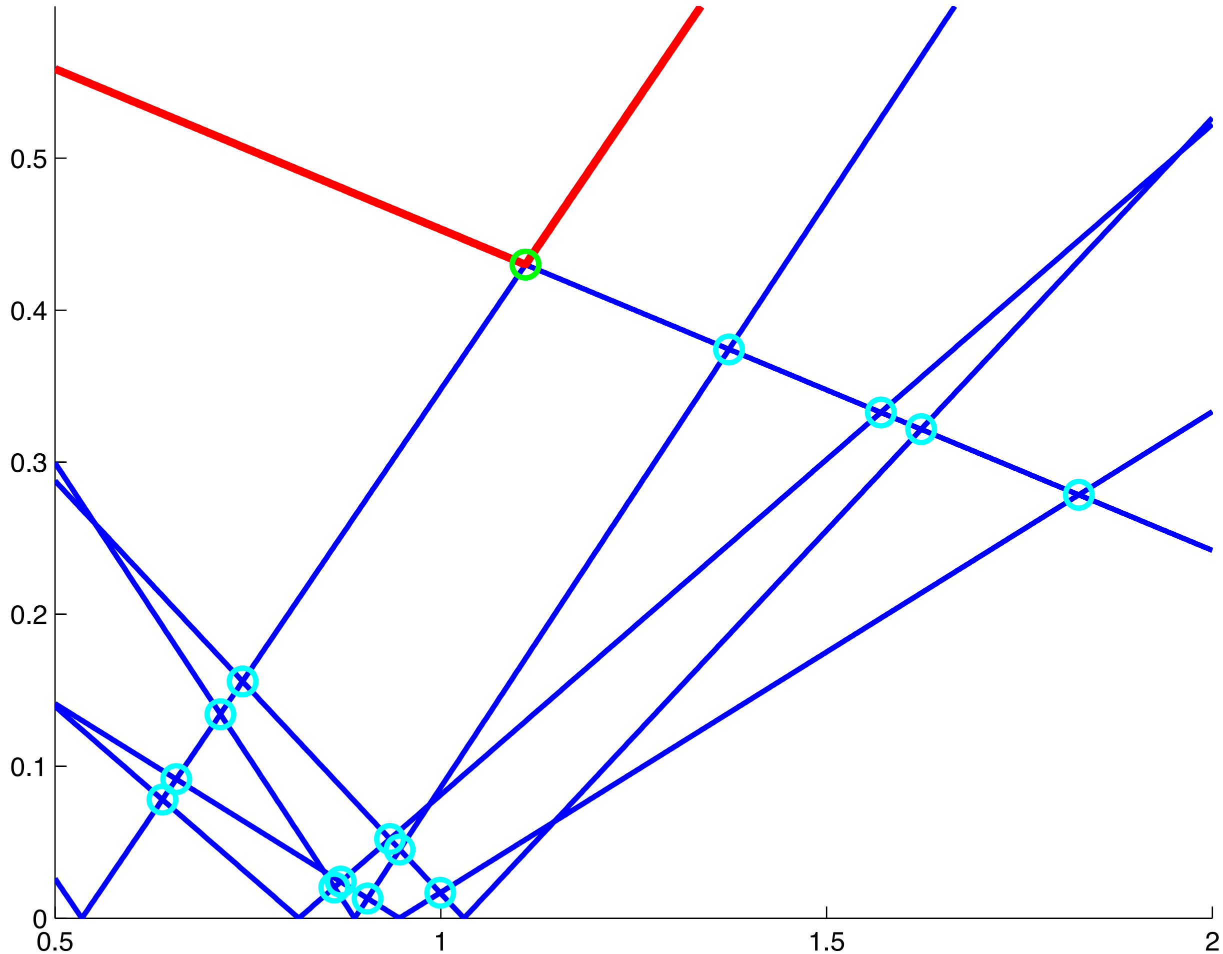
Minmax problem



Minmax problem



Recursive minmax



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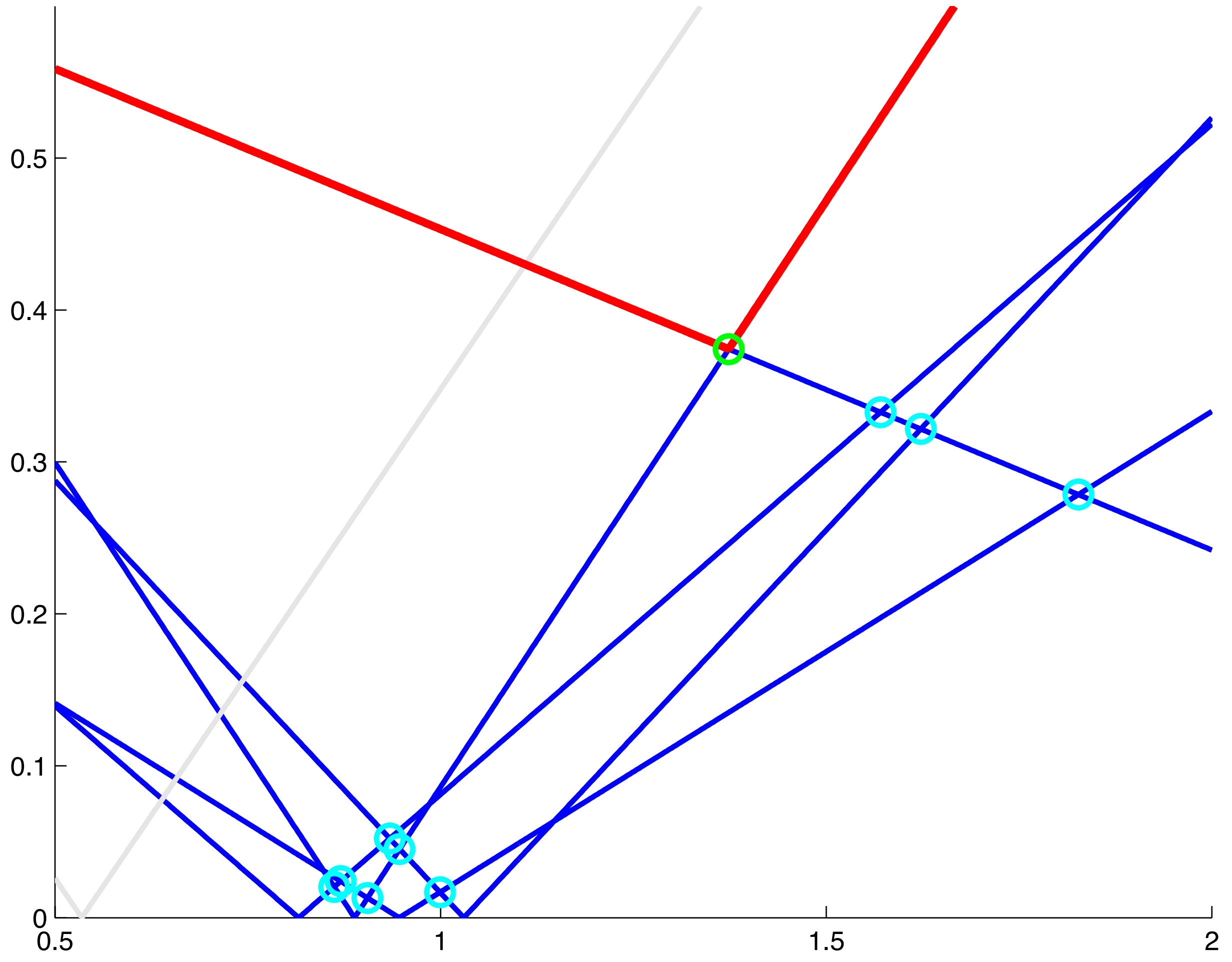
Recursive minmax



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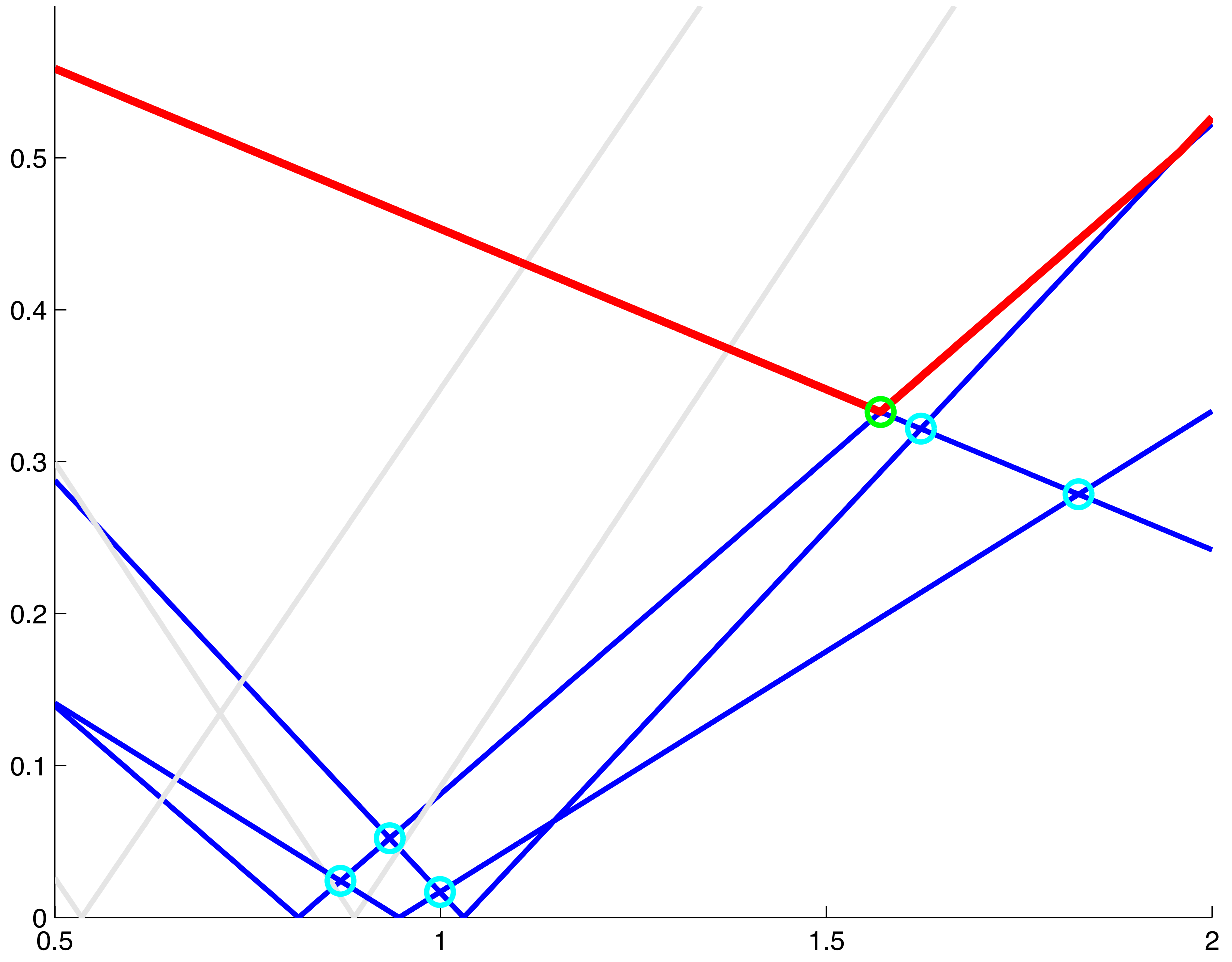
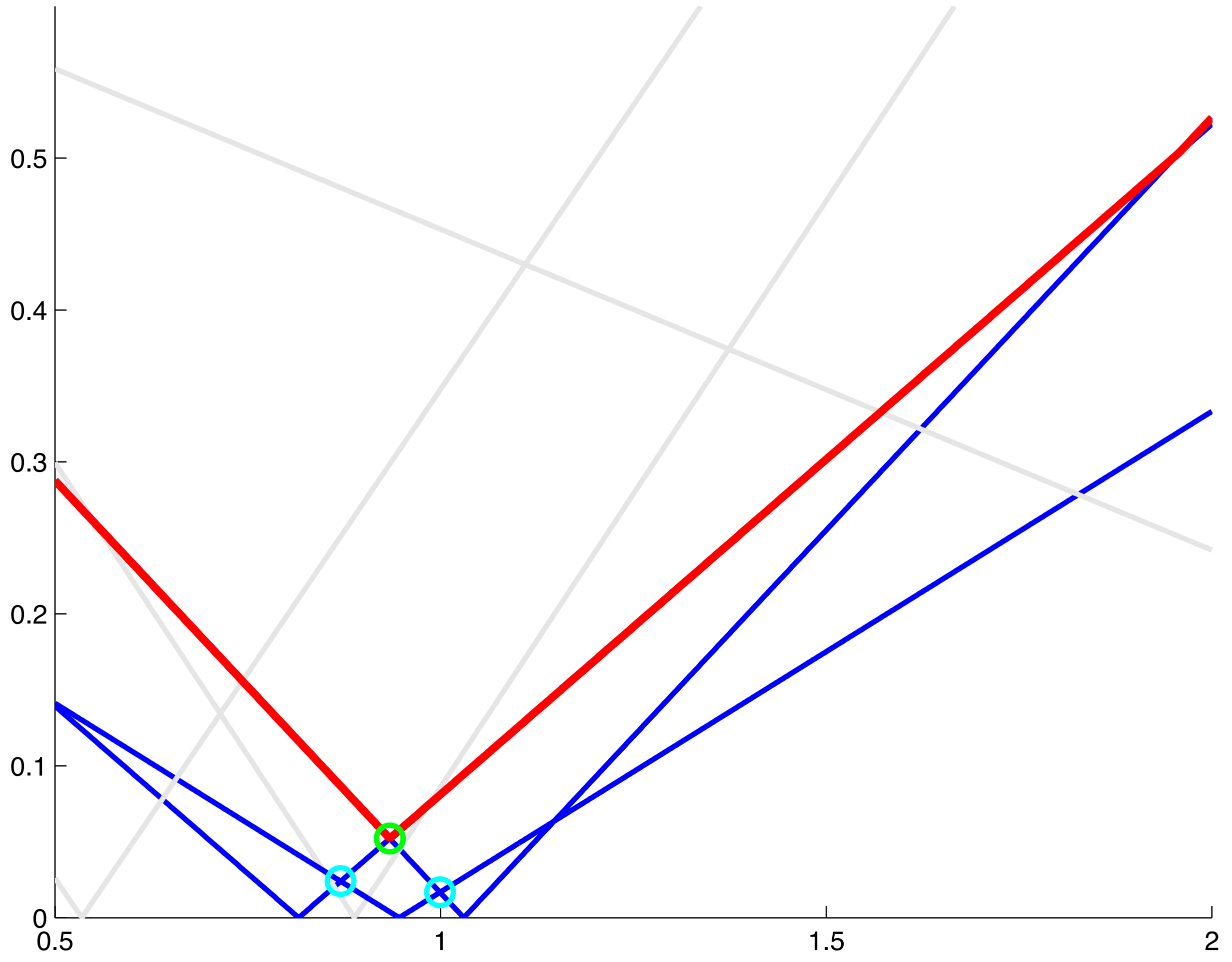
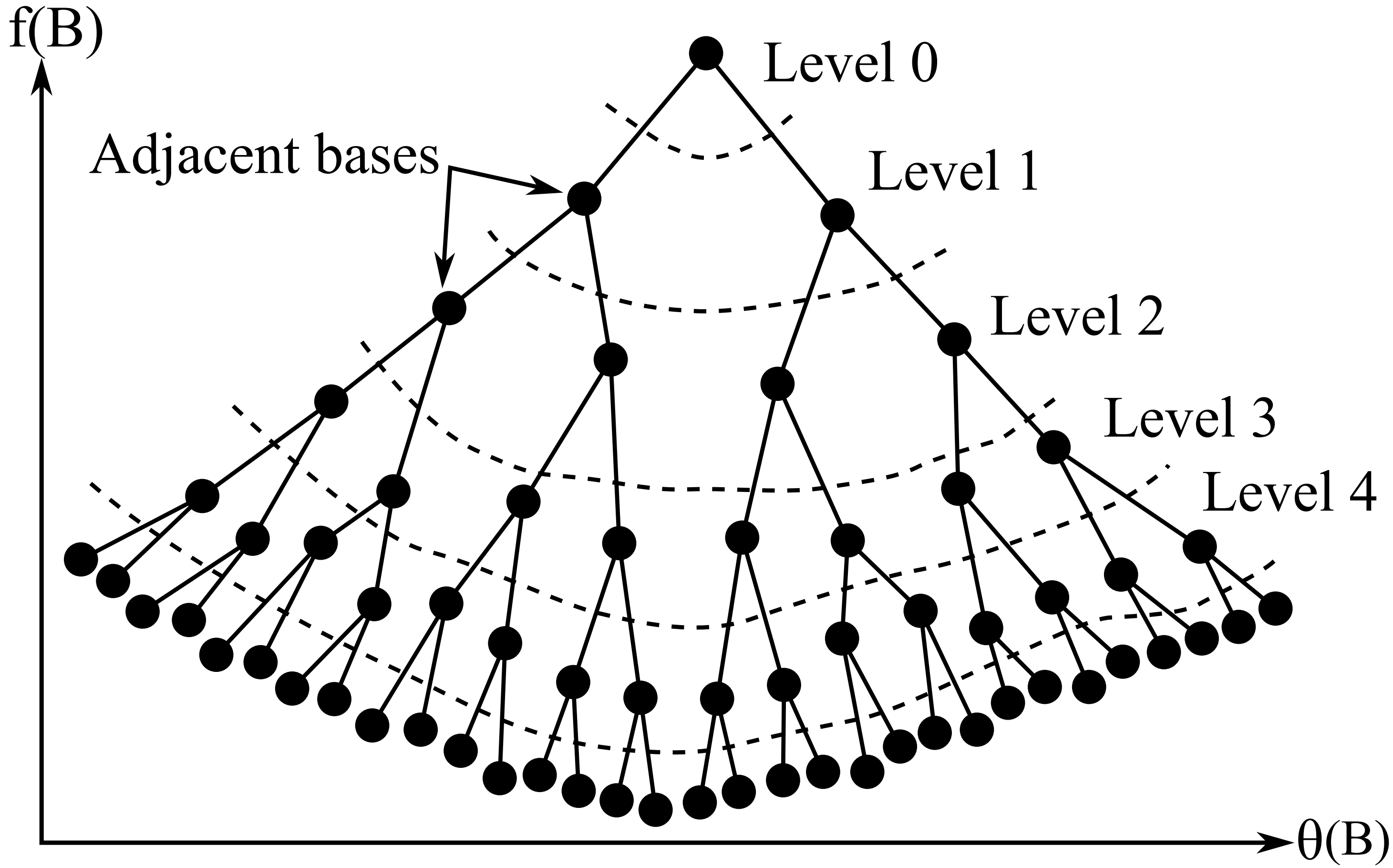
Recursive minmax



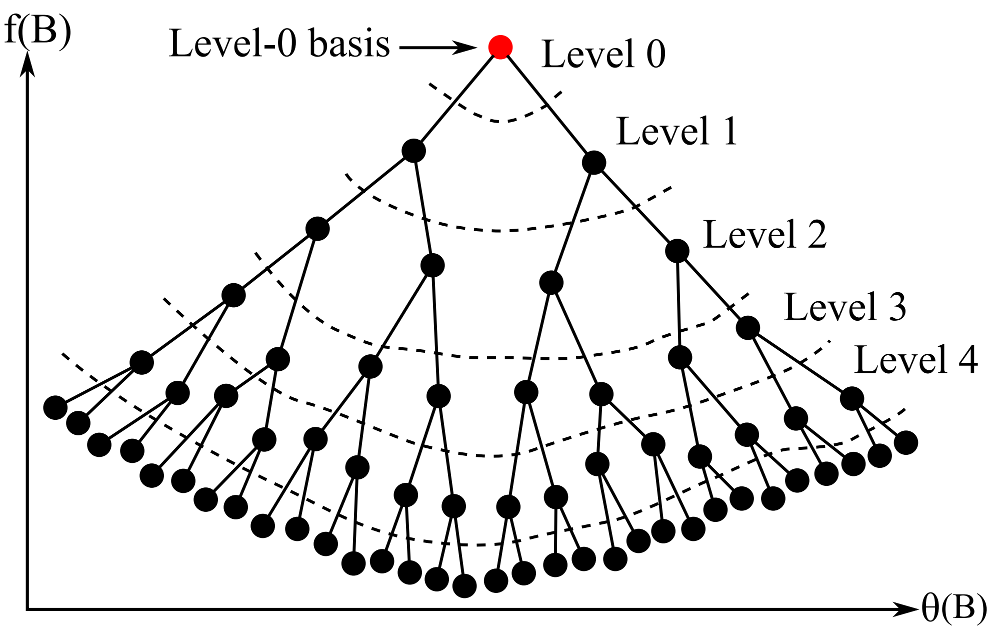
Recursive minmax

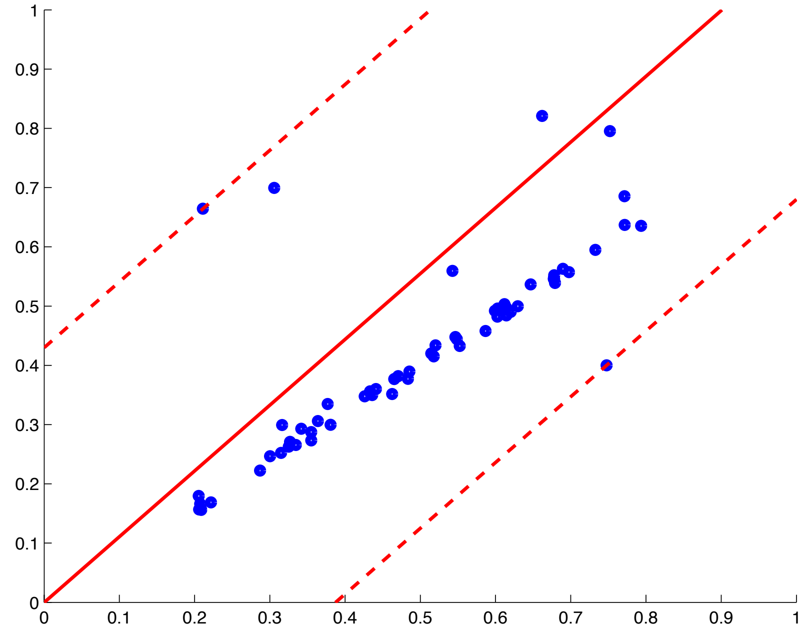


It's a tree!

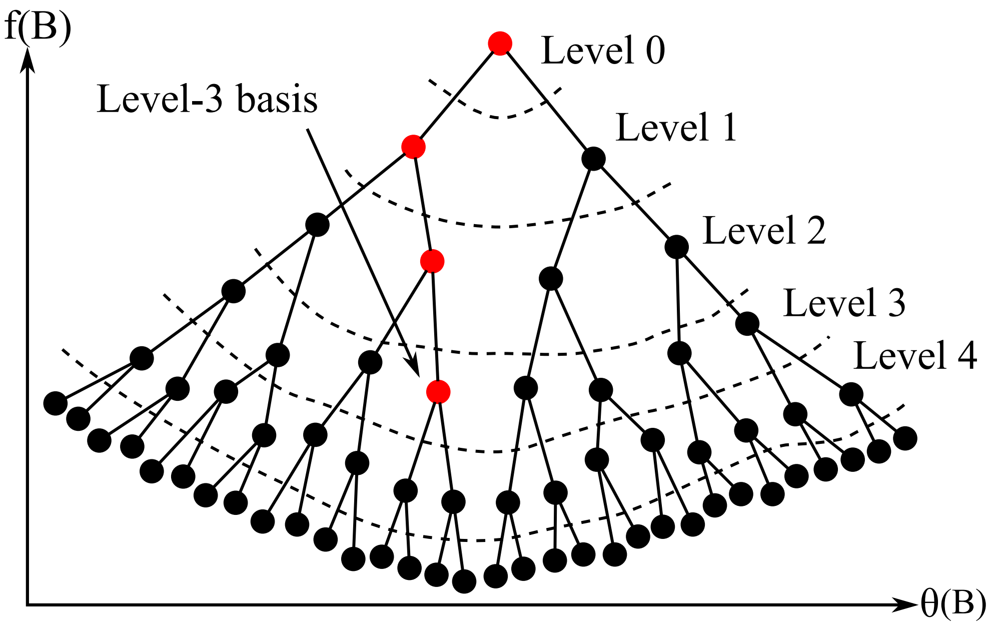


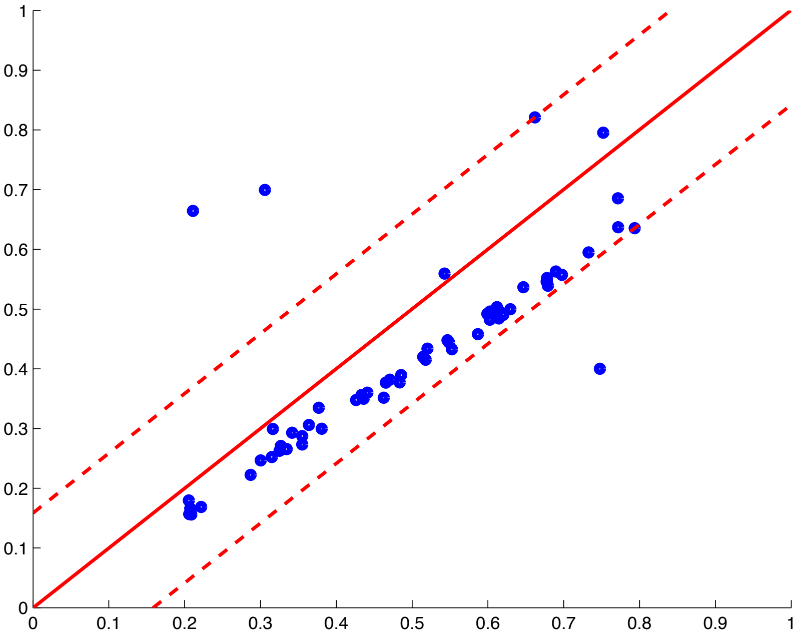
Tree depth or level





Tree depth or level





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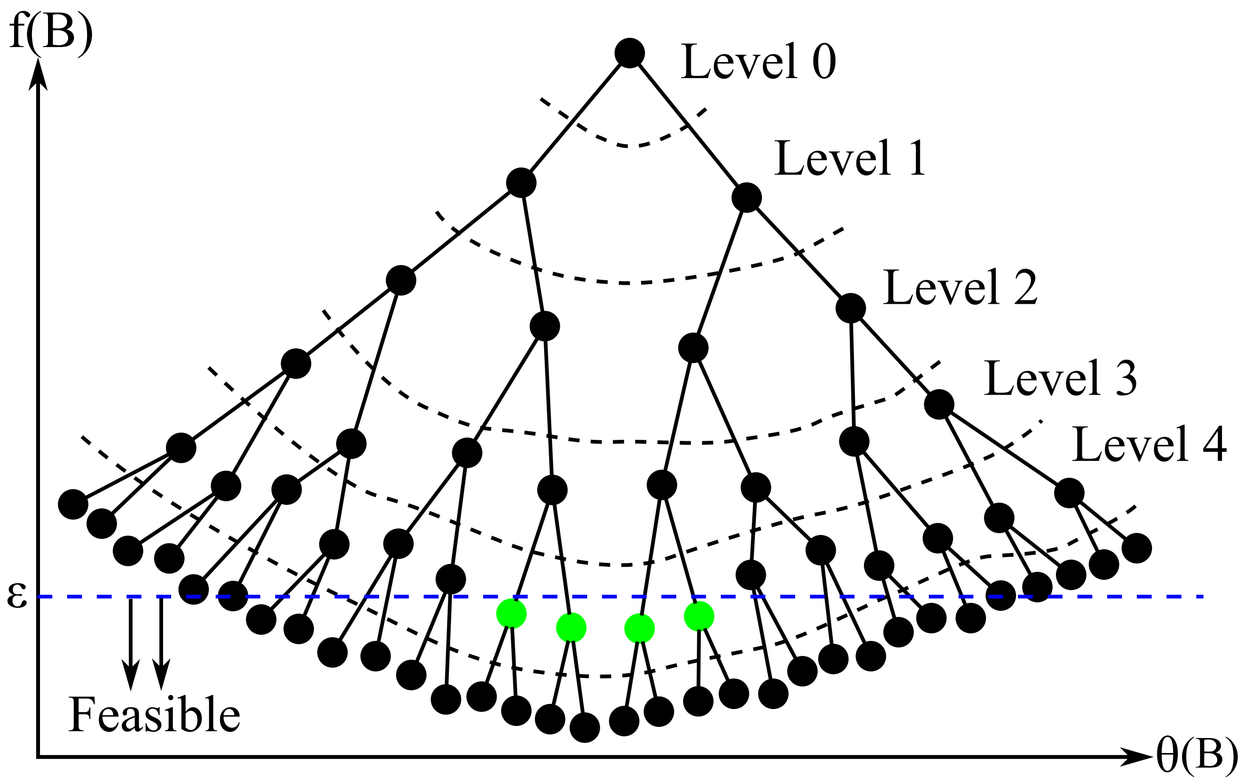
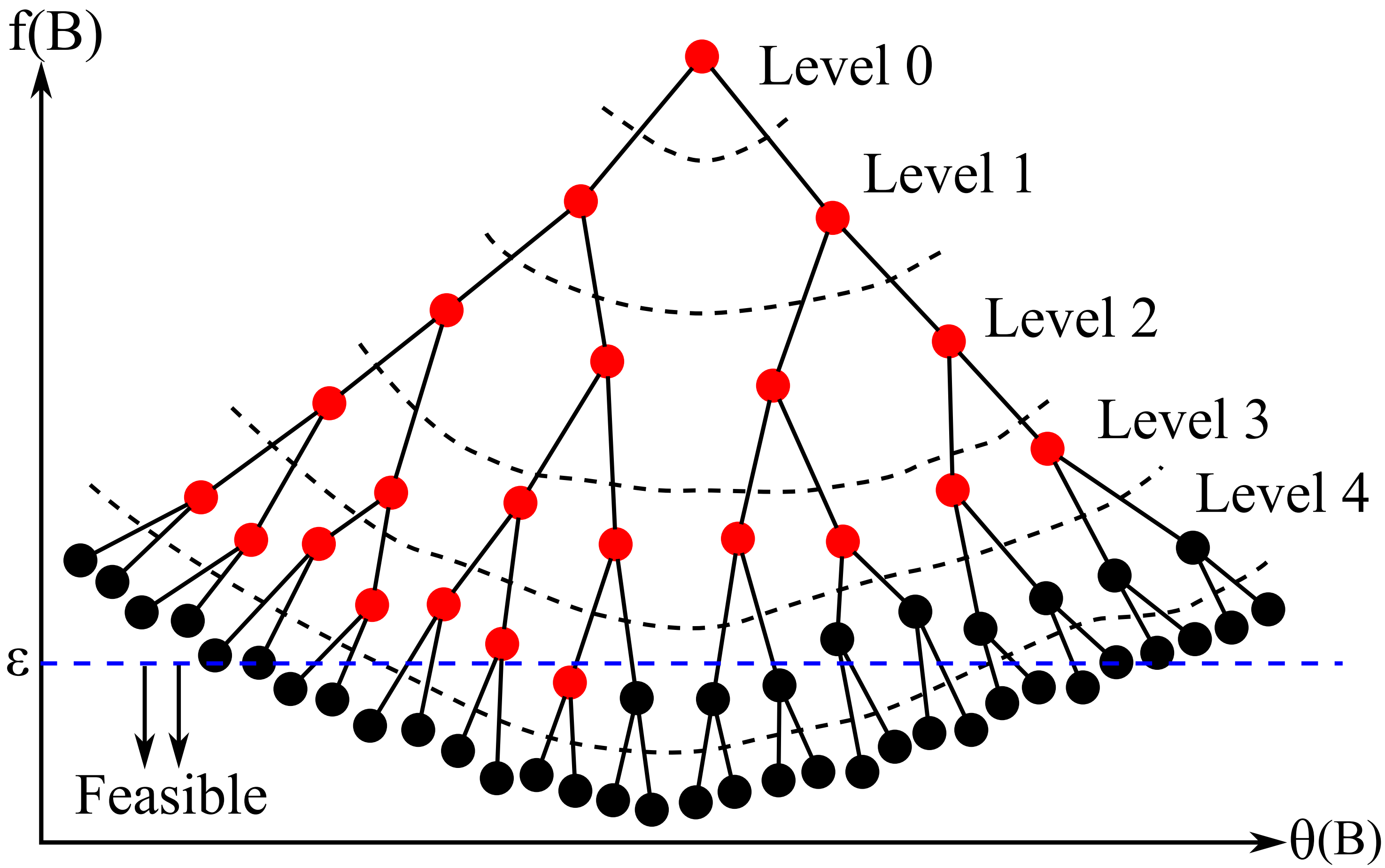
Maximum consensus II

$$\min_{\mathcal{B}} \; l(\mathcal{B}) \quad \text{s.t.} \;\; f(\mathcal{B}) \leq \epsilon.$$



Breadth-first search (BFS)



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A* algorithm

- Basis expansion is prioritised by

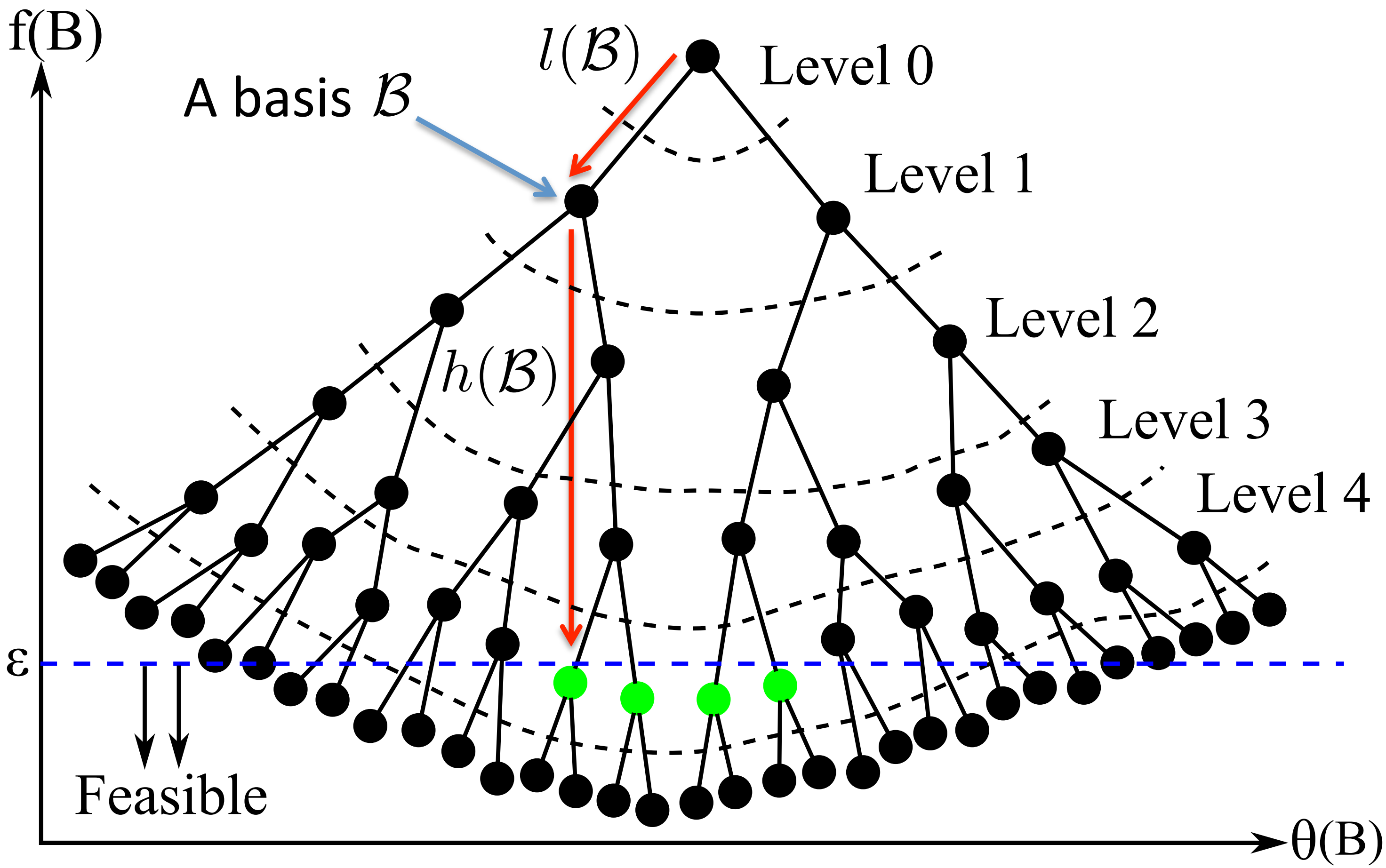
$$e(\mathcal{B}) = l(\mathcal{B}) + h(\mathcal{B})$$

$l(\mathcal{B})$: Level of basis $\mathcal{B}$.

$h(\mathcal{B})$: An estimate of the number of steps remaining to feasibility.

Hart et al., A formal basis for the heuristic determination of minimum cost paths, IEEE Trans. on Systems Science and Cybernetics, 4(2):100–107, 1968.

A* algorithm



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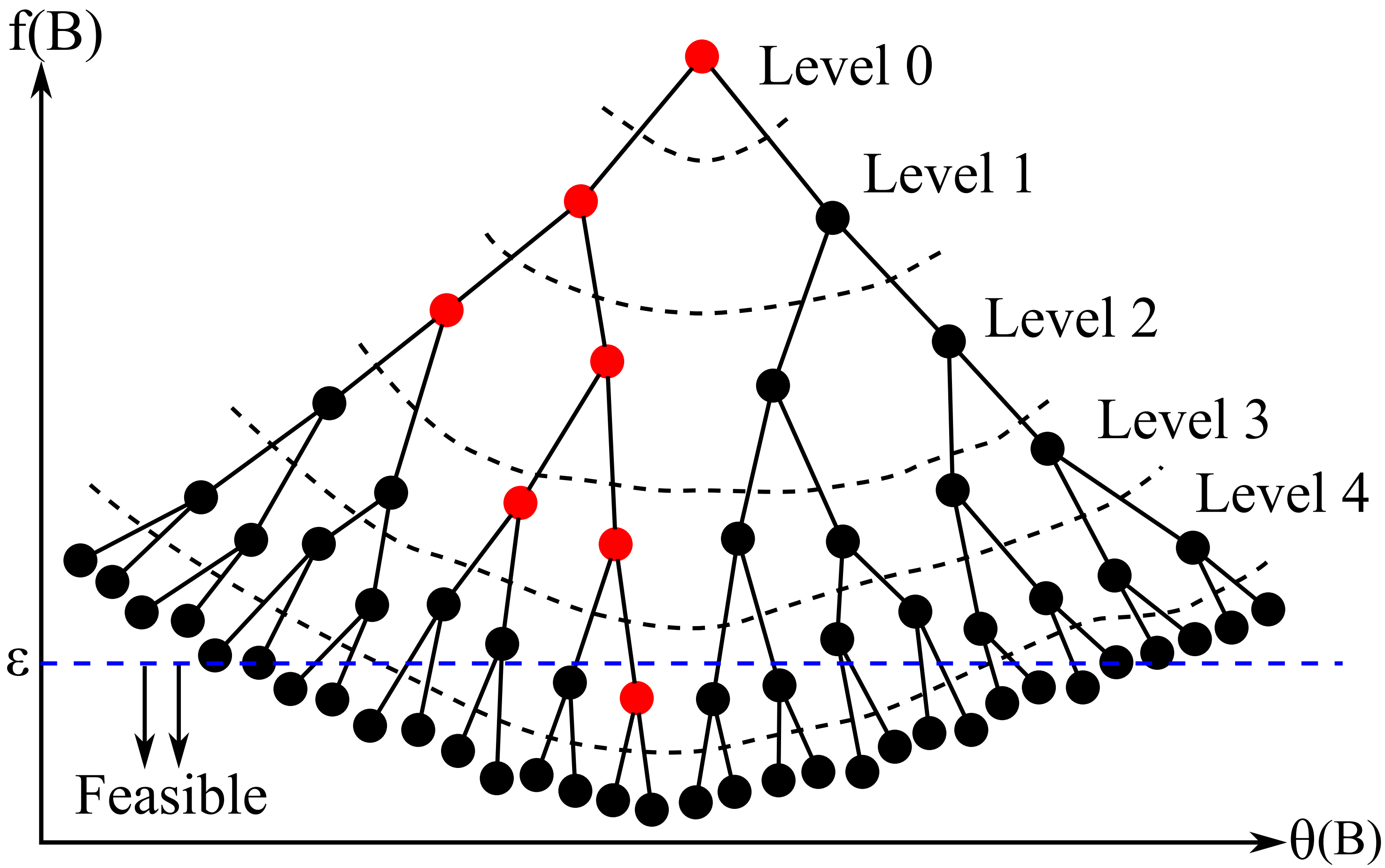
A* algorithm



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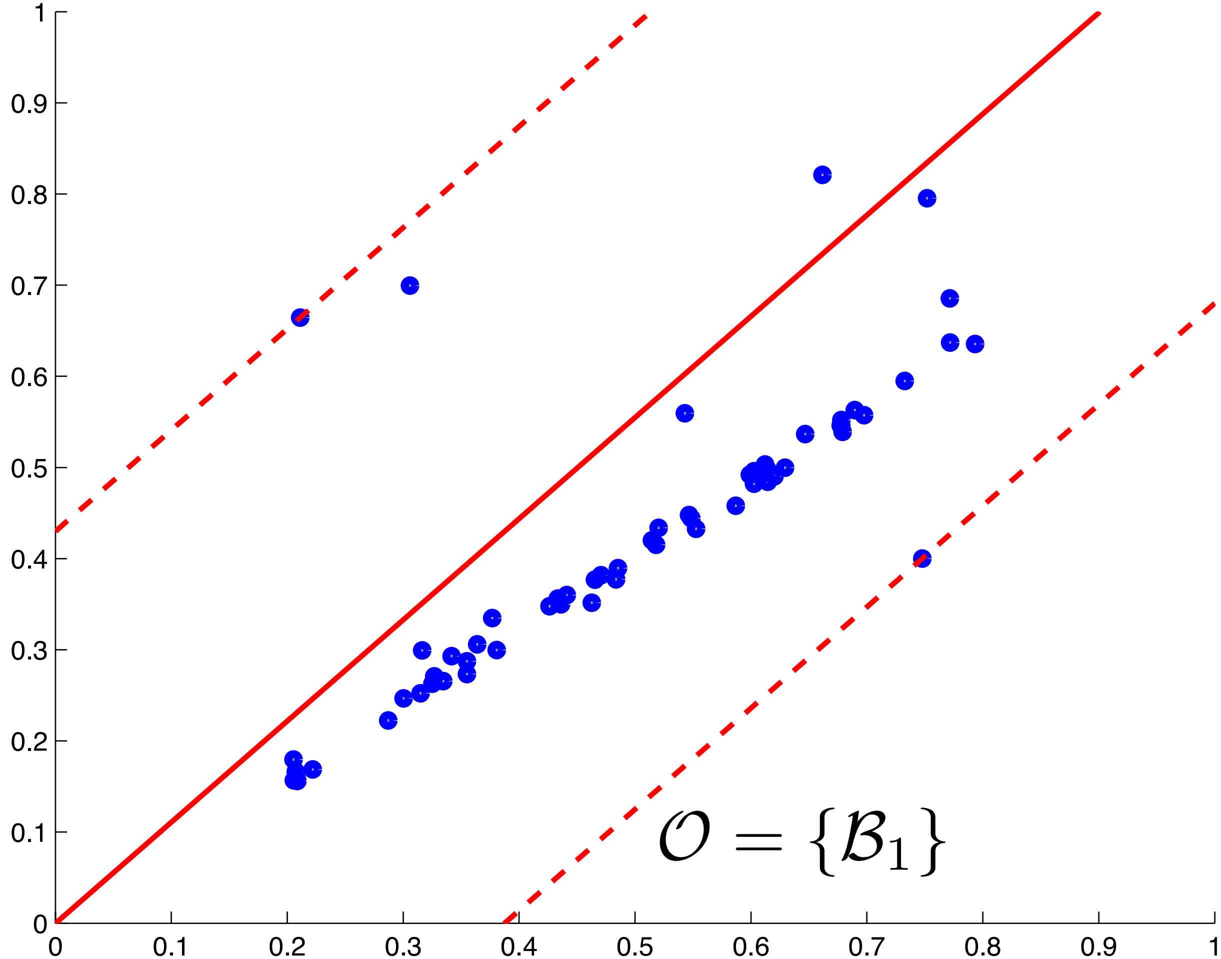
Heuristic function



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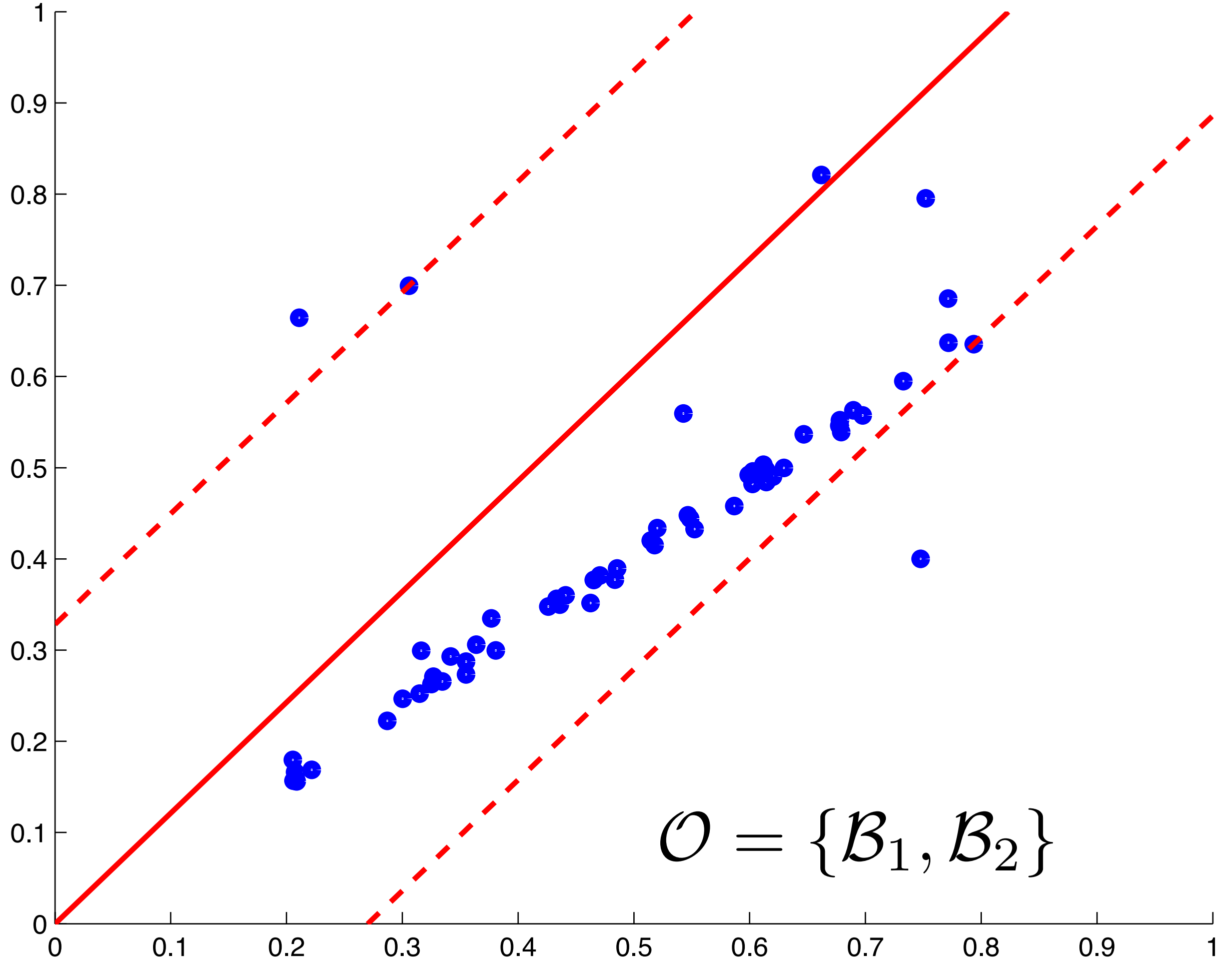
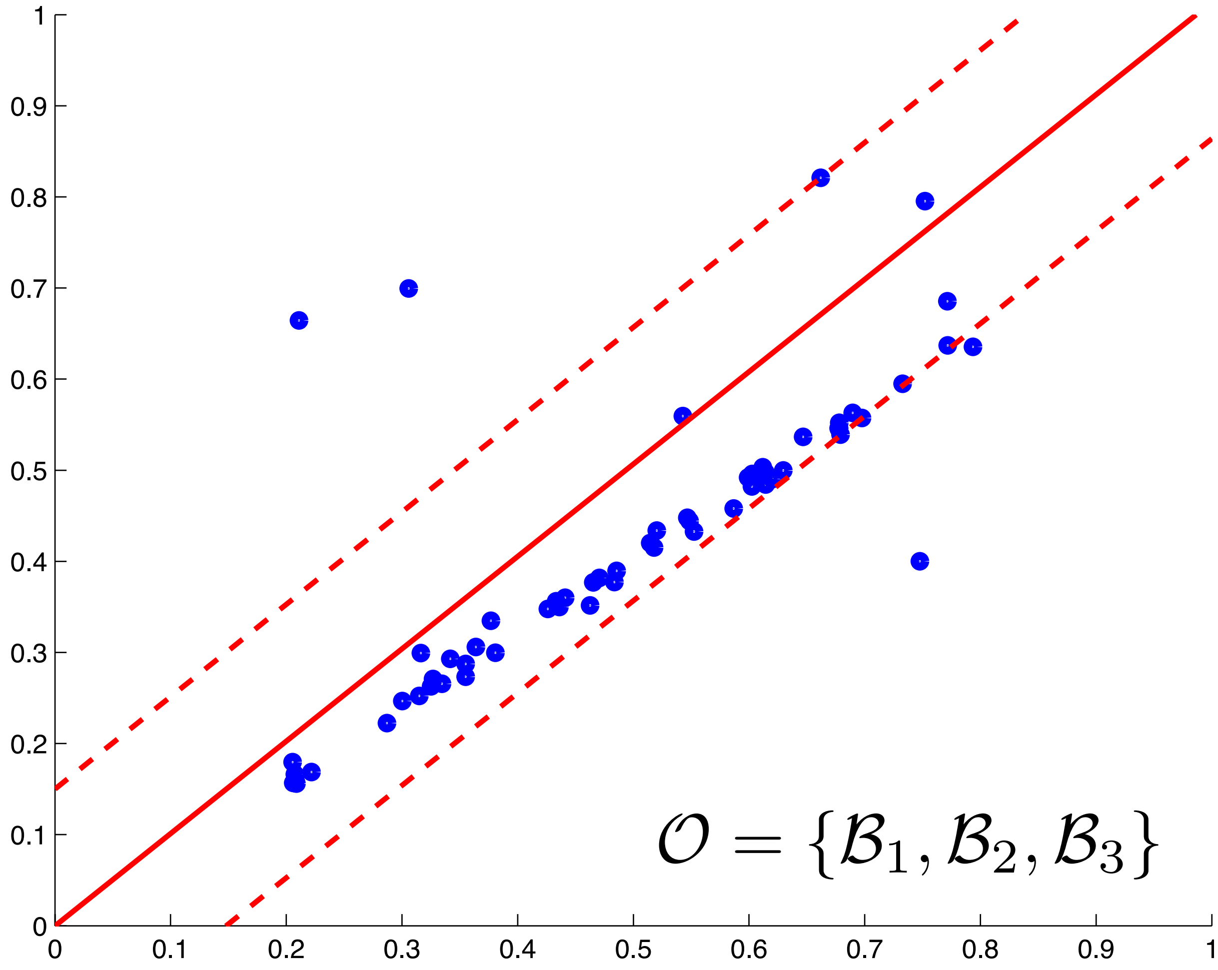
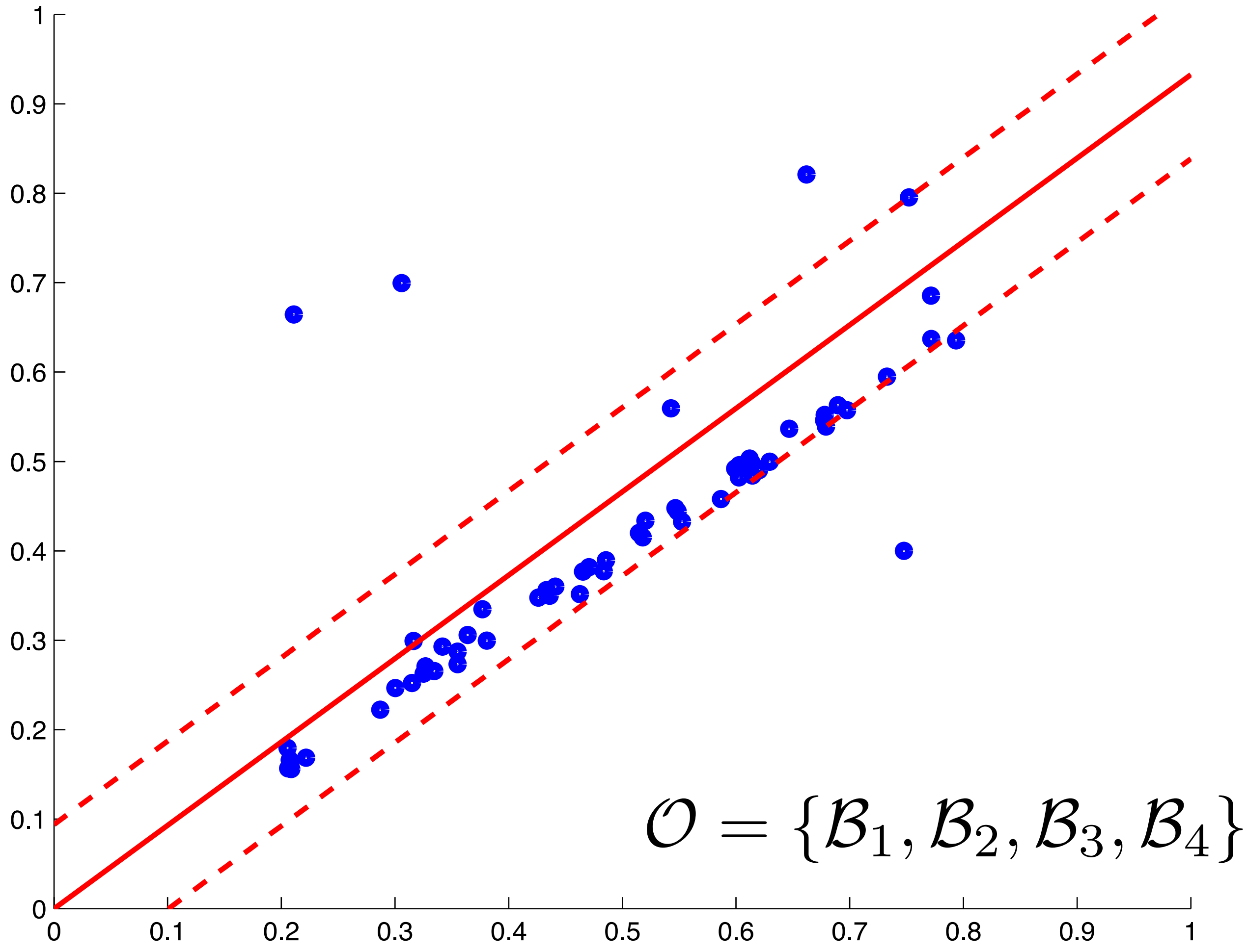


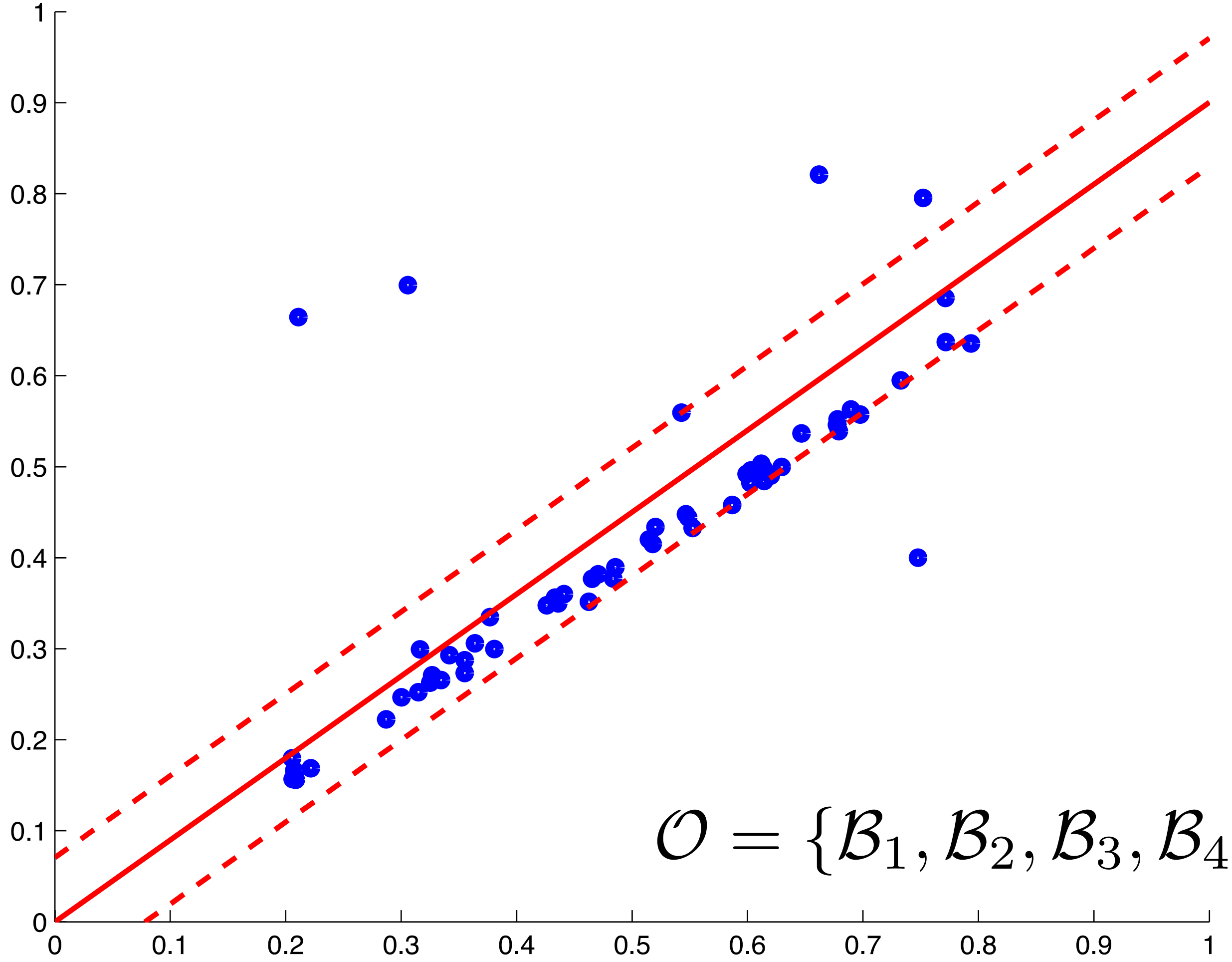
Heuristic function



Heuristic function

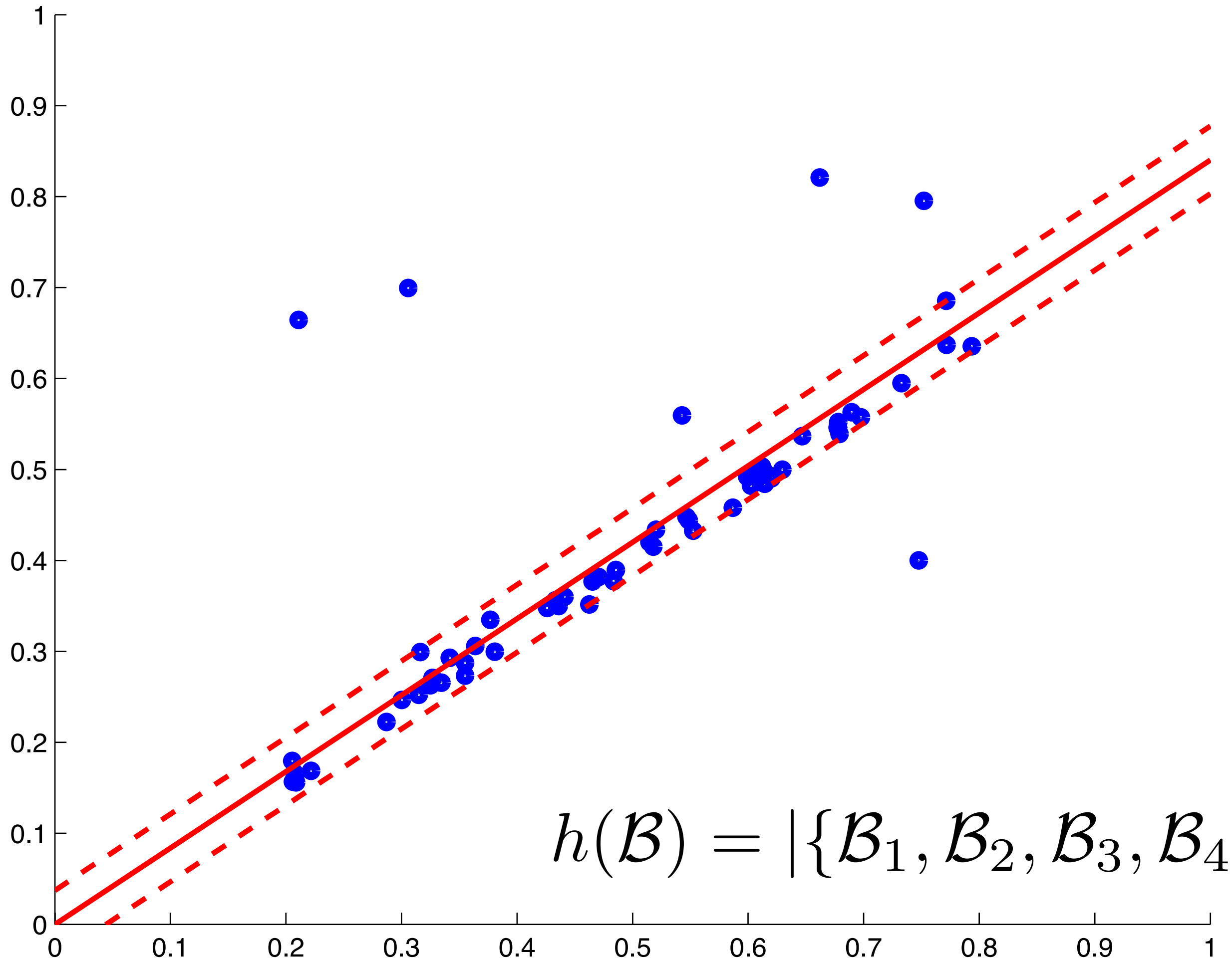


Heuristic function



$$\mathcal{O} = \{\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4, \mathcal{B}_5\}$$

Heuristic function



$$h(\mathcal{B}) = |\{\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4, \mathcal{B}_5\}| = 5$$

Definition (Admissibility):

A heuristic is admissible if it satisfies

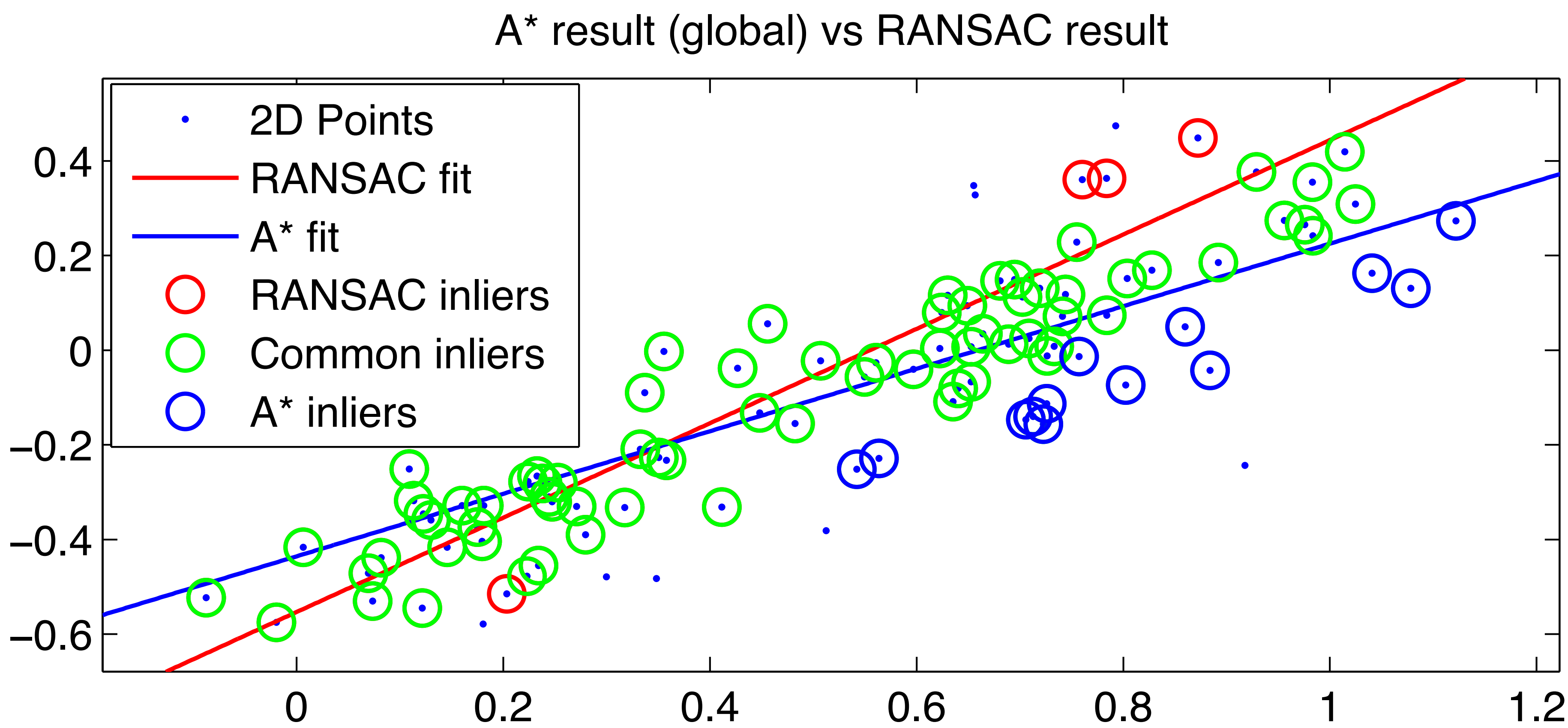
$$h(\mathcal{B}) \geq 0 \quad \text{and} \quad h(\mathcal{B}) \leq h^*(\mathcal{B})$$

where $h^*(\mathcal{B})$ is the true remaining cost from $\mathcal{B}$ to feasibility.

Theorem:

A* algorithm is optimal if $h(\mathcal{B})$ is admissible.

Results



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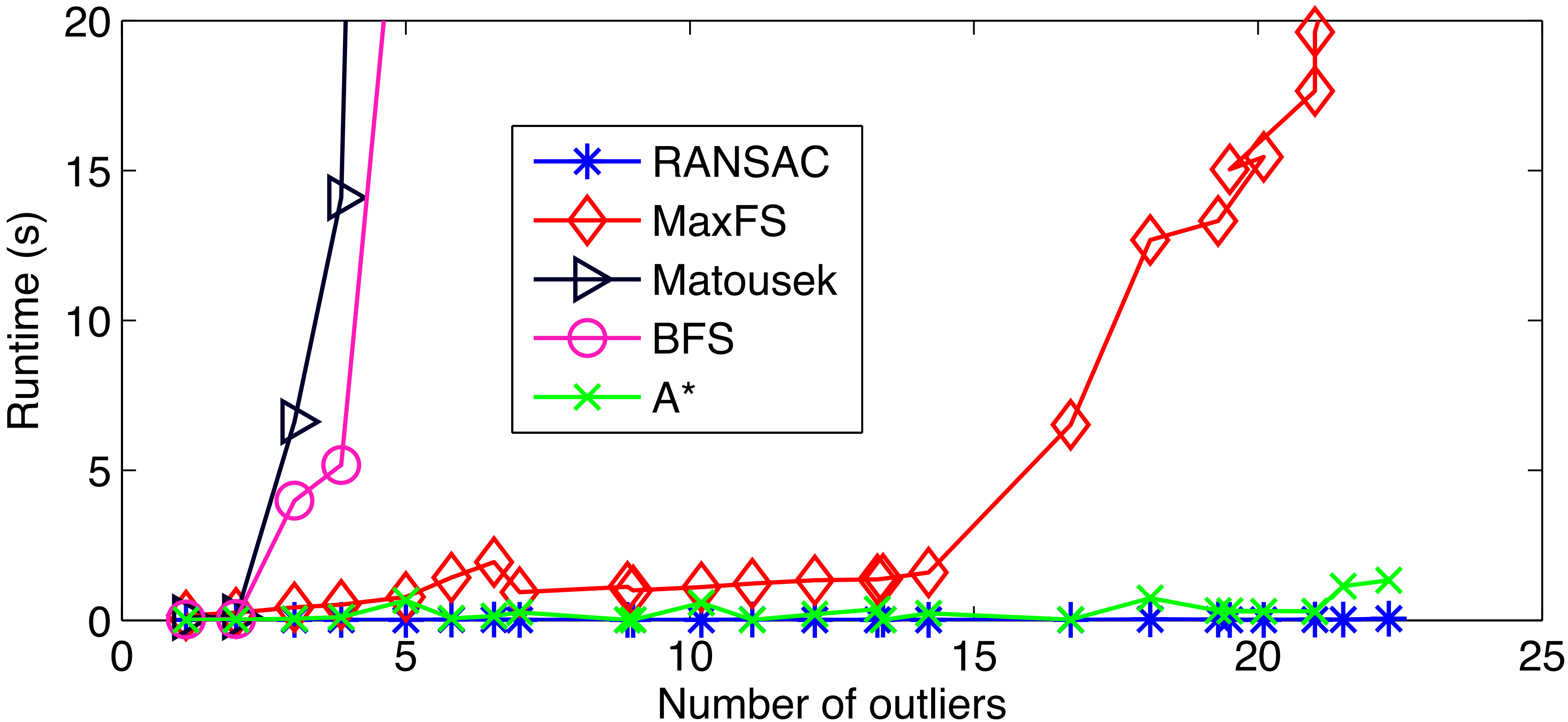
Results



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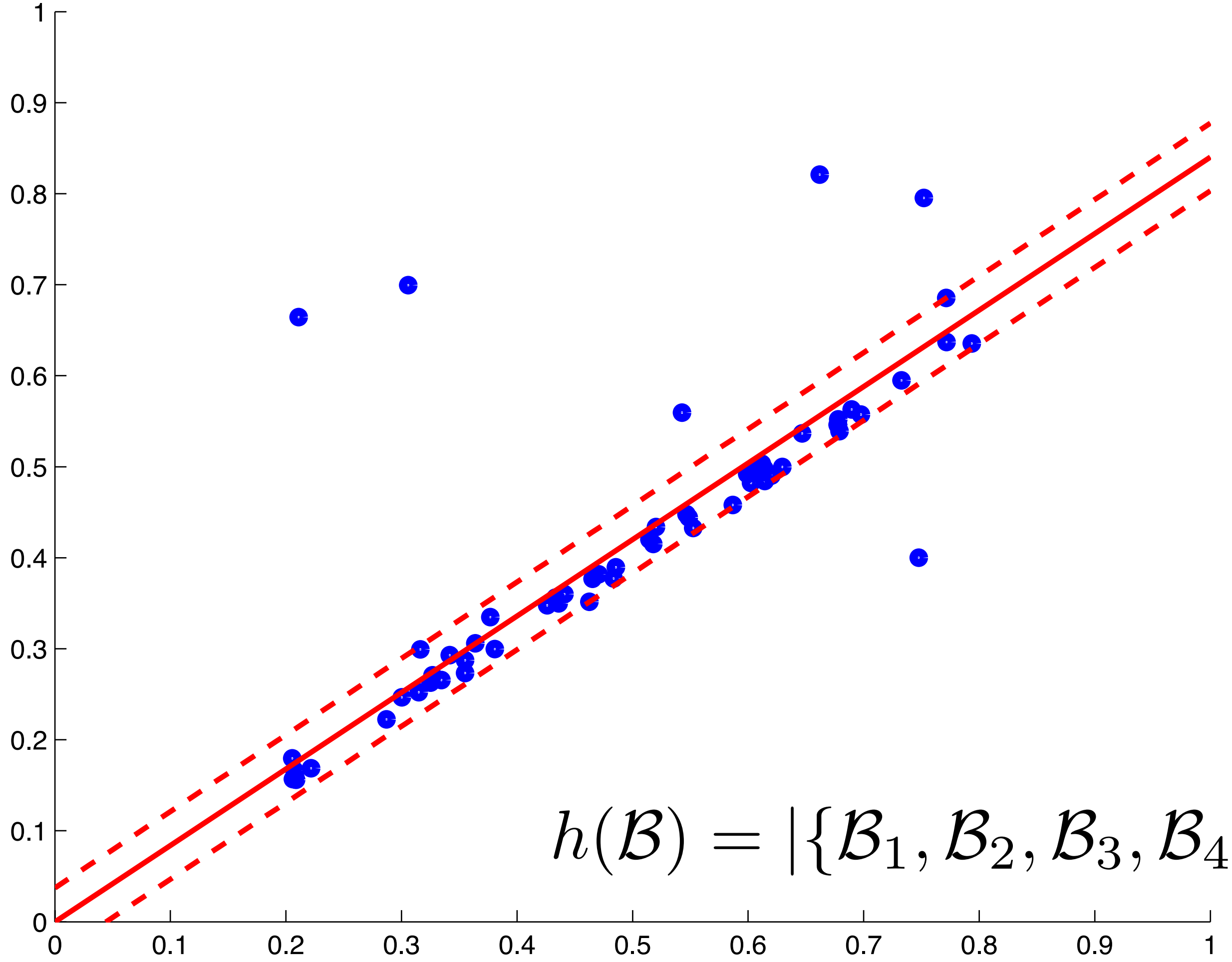
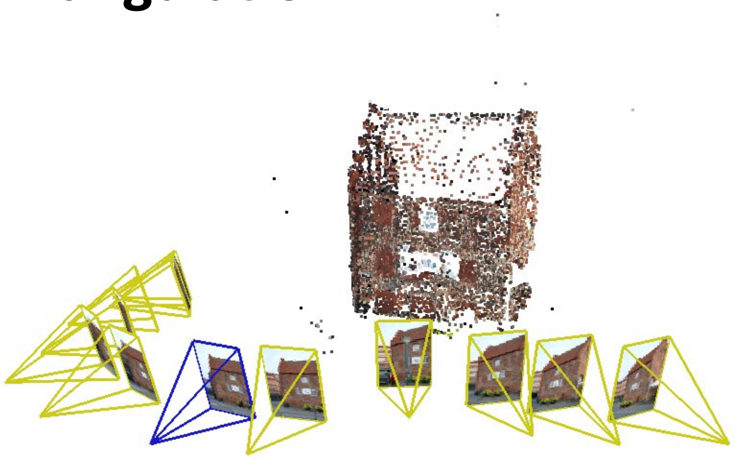
Limitation: Outlier ratio $\leq \frac{1}{p+1}$



$$h(\mathcal{B}) = |\{\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4, \mathcal{B}_5\}| = 5$$

Other residual functions?

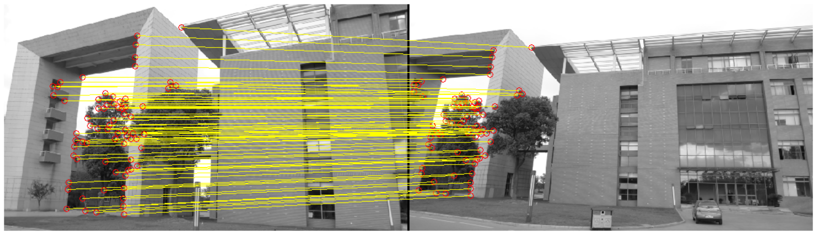
Triangulation



Reprojection error:

$$r_i(\theta) = \frac{\|(\mathbf{P}_{i,1:2} - \mathbf{x}_i \mathbf{P}_{i,3})\tilde{\theta}\|}{\mathbf{P}_{i,3}\tilde{\theta}}$$

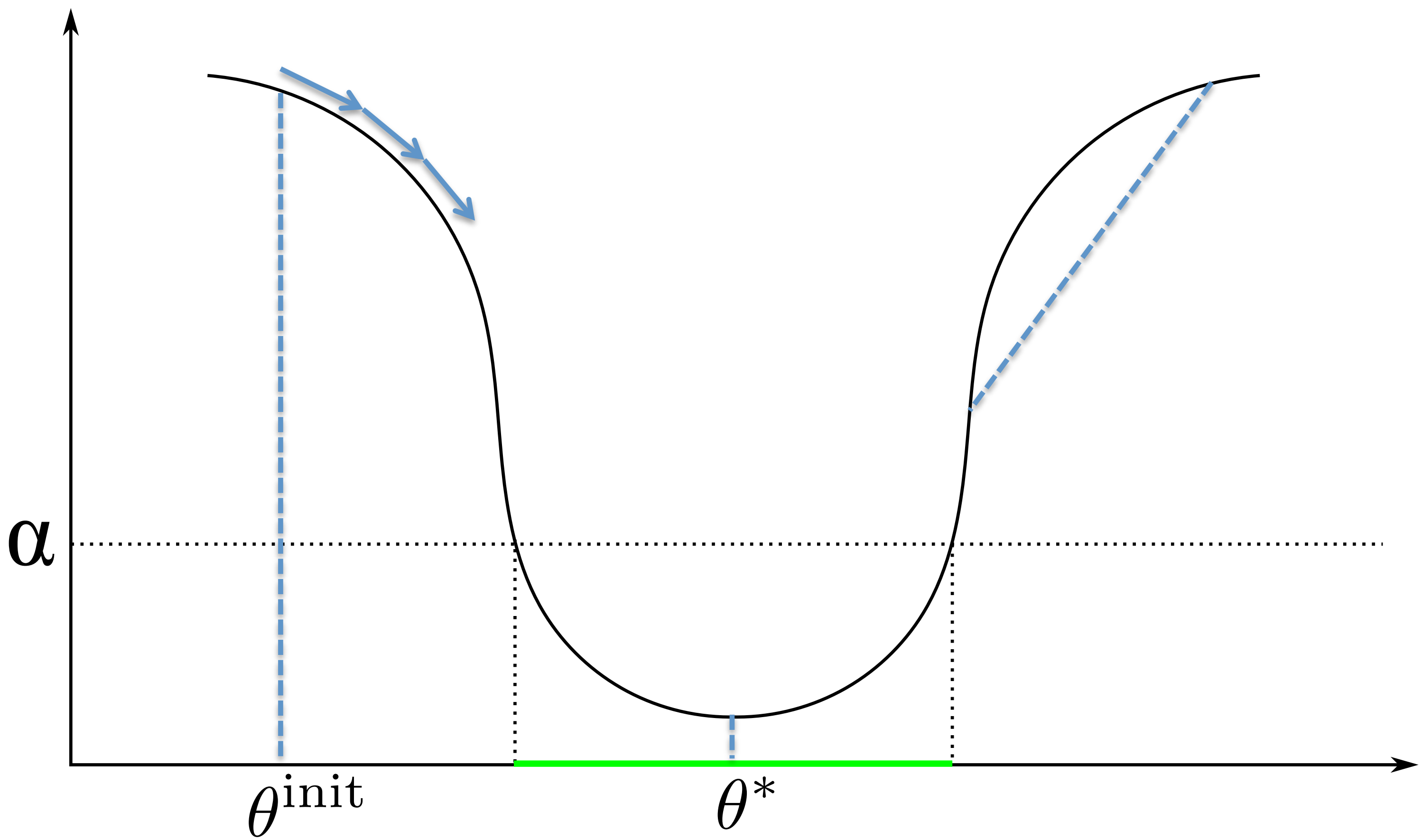
Homography fitting



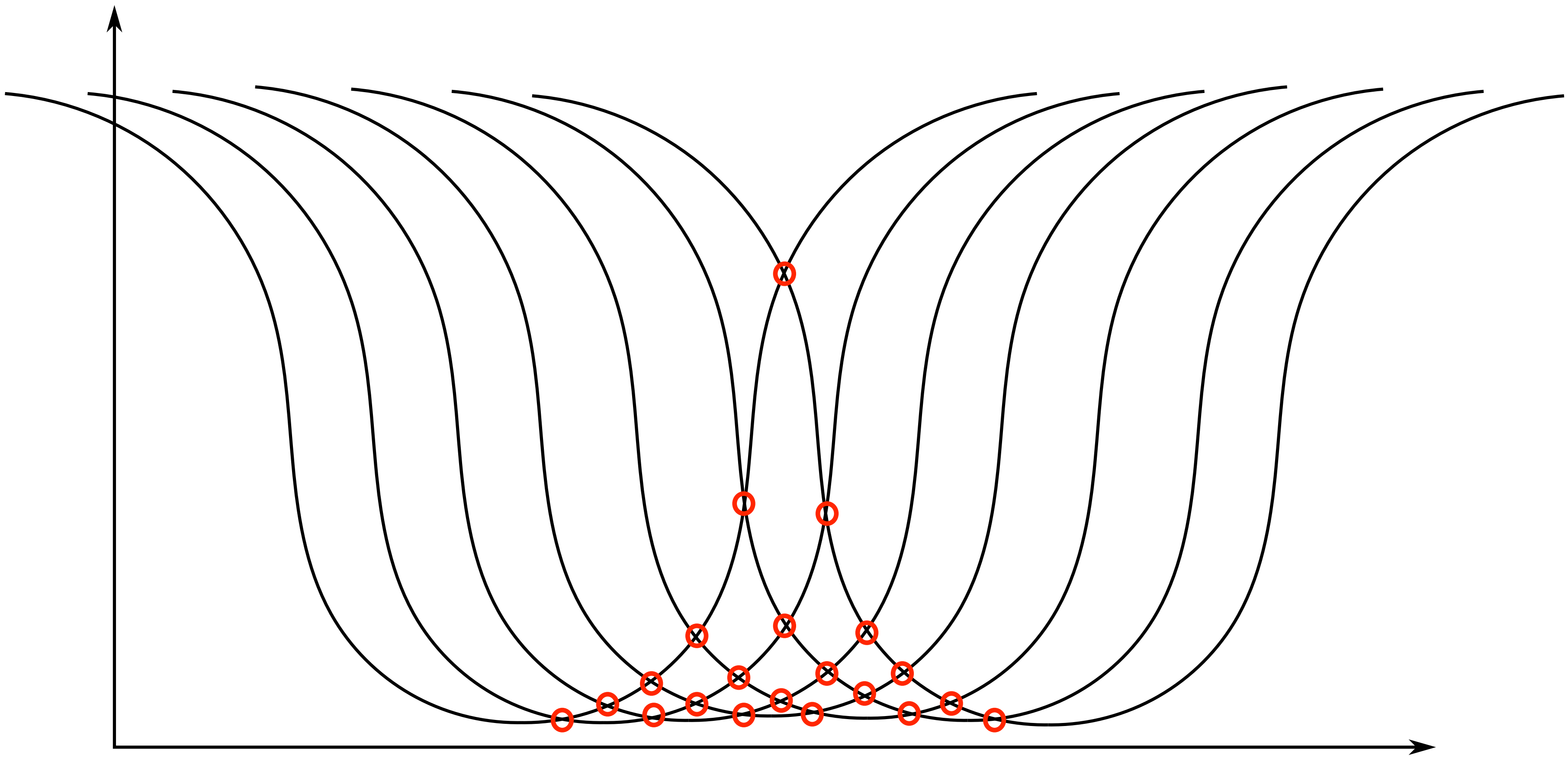
Transfer error:

$$r_i(\theta) = \frac{\|(\theta_{1:2} - \mathbf{u}'_i \theta_3)\tilde{\mathbf{u}}_i\|}{\theta_3 \tilde{\mathbf{u}}_i}$$

Pseudoconvex residual



Combinatorial dimension = p+1



Thank you!

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