Assignment One

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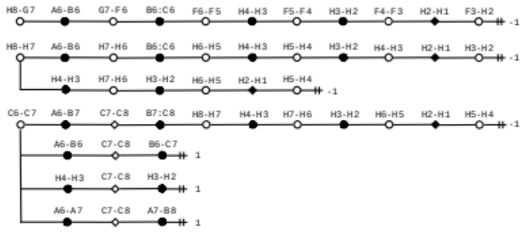
Question One

Below is a "smart" search tree for the game provided.



The "smart" search tree was contstructed with the following assumption in mind: The game is over if the target is destroyed and the piece could not be caputered by the opponenent on the next turn (the following opponents move was shown to illustrate this). If a move destroyed the target, it is denoted on the tree with a diamond rather than a circle.

This tree is very condensed from the full search tree and even a full "smart" search. However, the tree does illustrate the edge cases and the reader can draw conclusions from those to see that any case inbetween will result in the same outcome.

For instance, if the black King persues the white pawn, and the white King moves diagonally, to try and support the pawn, it does not make it in time (first branch). It also shows that if the white King directly persues the black Pawn, the black King can capture the white Pawn and still leave time for the black Pawn to advance and destroy the target before the white King can intercept. From this, we can easily infer the white King could advance along the diagonal, and still fall short of reaching the black Pawn before it destroys the target (even though the number of moves did not change).
Black appears to have the advantage in this game. The only way white may win is if it's Pawn persues the target first and the black King does not persue the Pawn on the diagonal towards the target.

Question Two

Part A

The general reachability formula for the Queen is as follows.

$$\begin{aligned} R_Q(x, y) = {} & (x = (x_1, x_2) \wedge (1 \leq x_1 \leq 8) \wedge (1 \leq x_2 \leq 8)) \wedge \\ & (y = (y_1, y_2) \wedge (1 \leq y_1 \leq 8) \wedge (1 \leq y_2 \leq 8)) \wedge \\ & (|y_1 - x_1| \leq 8 \wedge |y_2 - x_2| \leq 8) \end{aligned} \tag{1}$$

x $\in$ Z $\times$ Z, y $\in$ Z $\times$ Z

Part B

The reachability formulas for the King in a three dimensional space is as follows:

$$
\begin{aligned}
R_K(x, y, z) = {} & (x = (x_1, x_2) \wedge (1 \leq x_1 \leq 8) \wedge (1 \leq x_2 \leq 8)) \wedge \\
& (y = (y_1, y_2) \wedge (1 \leq y_1 \leq 8) \wedge (1 \leq y_2 \leq 8)) \wedge \\
& (z = (z_1, z_2) \wedge (1 \leq z_1 \leq 8) \wedge (1 \leq z_2 \leq 8)) \wedge \\
& (|y_1 - x_1 - z_1| \leq 1 \wedge |y_2 - x_2 - z_2| \leq 1)
\end{aligned}
\tag{2}
$$

The reachability formulas for the Knight in a three dimensional space is as follows:

$$
\begin{aligned}
R_K(x, y, z) = {} & (x = (x_1, x_2) \wedge (1 \leq x_1 \leq 8) \wedge (1 \leq x_2 \leq 8)) \wedge \\
& (y = (y_1, y_2) \wedge (1 \leq y_1 \leq 8) \wedge (1 \leq y_2 \leq 8)) \wedge \\
& (z = (z_1, z_2) \wedge (1 \leq z_1 \leq 8) \wedge (1 \leq z_2 \leq 8)) \wedge \\
& (|y_1 - x_1 - z_1| = 1 \wedge |y_2 - x_2 - z_2| = 2) \vee \\
& (|y_1 - x_1 - z_1| = 2 \wedge |y_2 - x_2 - z_2| = 1)
\end{aligned}
\tag{3}
$$

Assuming the following both of the equations:

$$
x \in Z \times Z, y \in Z \times Z, z \in Z \times Z \tag{4}
$$