PAGE No
DATE

* **Set** : A set is a collection of meaningful objects and things.
These objects and thing are known as members of set.

In a set the order of element as well as repetition of elements is also immaterial.
$A = \{1,2,3,4\}$; $B = \{4,3,2,1\}$, $C = \{4,4,3,2,2,1,4,1\}$
A, B, C are same

→ Null set (ϕ): contains no elements

→ Universal set : contains everything in the given frame of reference

→ Equal set : they have same elements

→ Subset : A set is said to be subset of B, if every element of set A is also an element of set B.
$x \in A$ then $x \in B$
$\therefore A \subseteq B$

$\therefore$ Two sets are equal if $A \subseteq B$ & $B \subseteq A$,
$x \in A \Leftrightarrow x \in B$

→ Venn diagram : is a geometric representation of sets and their relation.

→ Cardinality of set : The not of elements in set A, $|A|$

PAGE No			
DATE			

→ Power set: is the set of all subsets of set A. P(A).

A = {1,2,3}

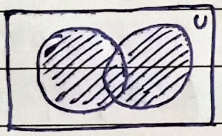
P(A) = {{1},{2},{3},{1,2},{2,3},{1,3},{1,2,3},{ϕ}}

→ a set of n elements will have 2^n elements in power set of that set.

* Operations on sets:

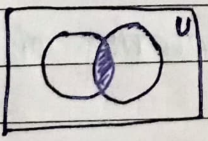
① Union : let A & B be two sets, then A∪B is the set which contains all the elements present in A or B.

$A \cup B = \{x : x \in A \text{ or } x \in B\}$



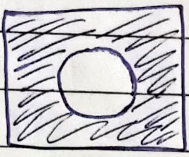
② Intersection : let A & B be two sets, then their intersection A∩B is the set which contains all elements present in A and B.

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

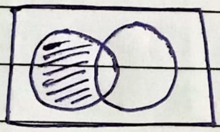


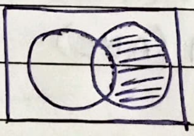
(3) complement : is the set of all those elements which are not in A.

$\bar{A}$ or A^c



(4) Difference : is a set of all elements of A which are not in B for A - B, and vice versa for B - A.



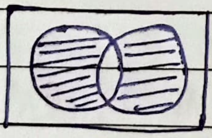


$A - B = \{x : x \in A \text{ and } x \notin B\}$

$B - A = \{x : x \in B \text{ and } x \notin A\}$

(5) Symmetric difference : $A \oplus B$

set of A-B and $\cup$ B-A



PAGE No

DATE

* Properties of set operation :

1) Commutative law :

$A \cup B = B \cup A$

$A \cap B = B \cap A$

2) Associative law :

$A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

3) Distributive law :

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

4) Idempotent law :

$A \cup A = A$

$A \cap A = A$

5) Complement properties –

$(\bar{\bar{A}}) = A$

$A \cup \bar{A} = U$

$A \cap \bar{A} = \phi$

$\bar{U} = \phi$

$\bar{\phi} = U$

6) De Morgan's Law –

$\overline{A \cup B} = \bar{A} \cap \bar{B}$

$\overline{A \cap B} = \bar{A} \cup \bar{B}$

→ proved $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$

let,

$$x \in \overline{(A \cup B)} = x \notin (A \cup B)$$
$$= x \notin A \text{ and } x \notin B$$
$$= x \in \bar{A} \text{ and } x \in \bar{B}$$
$$= x \in (\bar{A} \cap \bar{B})$$

$\therefore \overline{(A \cup B)} \subseteq \bar{A} \cap \bar{B}$ —— (I)

conversely,

$$x \in \bar{A} \cap \bar{B} \Rightarrow x \in \bar{A} \text{ and } x \in \bar{B}$$
$$= x \notin A \text{ and } x \notin B$$
$$\Rightarrow x \notin (A \cup B)$$
$$\Rightarrow x \in \overline{(A \cup B)}$$

$\therefore \bar{A} \cap \bar{B} \subseteq \overline{A \cup B}$ —— (II)

from (I) & (II)

$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$

* **Principle of inclusion & exclusion** –

Let A & B be two disjoint sets (nothing common) then
$|A \cup B| = |A| + |B|$

If A & B are two non-disjoint sets then,
$|A \cup B| = |A| + |B| - |A \cap B|$

If A, B & C are three non-empty sets which are not mutually disjoint then,

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

example – Total 260 students,

64 – maths, 94 – CS, 58 – Business

28 – maths & Bus, 26 – maths & CS, 22 – CS & Bus.

14 – maths, bus., CS.

① How many are doing No courses ?

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

$= 64 + 94 + 58 - 28 - 26 - 22 + 14$

$= 154$

$\therefore$ No. of none $= |\overline{A \cup B \cup C}| = U - |A \cup B \cup C|$

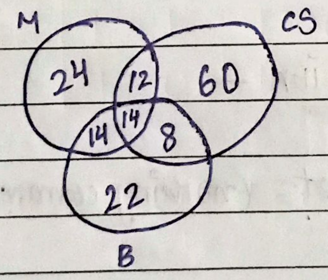
$= 260 - 154$

$= 106$

② How many only CS ?

$94 - 14 - 8 - 12 = \underline{60}$

③ Create a Venn diagram.



PAGE No
DATE

* A matrix is said to a boolean matrix if it has only two types of elements - 1 or 0

There are following 3 types of operations on a boolean matrix -

① **Join** - let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two boolean matrix then join of A & B denoted as $A \vee B$, is defined as a matrix, $C = [c_{ij}]$ where

$$c_{ij} = \begin{cases} 1, & \text{if } a_{ij} = 1 \text{ or } b_{ij} = 1 \\ 0, & \text{if } a_{ij} = 0 \text{ and } b_{ij} = 0 \end{cases}$$

② **Meet** - let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two boolean matrix then the meet of A & B denoted as $A \wedge B$, is defined as matrix, $D = [d_{ij}]$ where

$$d_{ij} = \begin{cases} 1, & \text{if } a_{ij} = 1 \text{ and } b_{ij} = 1 \\ 0, & \text{otherwise} \end{cases}$$

③ **Boolean product** - let $A = [a_{ij}]$ is $m \times p$ boolean matrix & $B = [b_{ij}]$ is $p \times n$ boolea matrix, Then boolean product of A & B is denoted as $A \odot B$, is defined as a matrix $C = [c_{ij}]$ of order $m \times n$, when

$$c_{ij} = \begin{cases} 1, & \text{if } a_{ik} = 1 \text{ and } b_{jk} = 1, \text{ for some } 1 \le k \le p \\ 0, & \text{otherwise} \end{cases}$$

→ example: $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$A \vee B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ $A \wedge B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

→ example: $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

$A \odot B = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

Note: $A \odot B \neq B \odot A$

PAGE No
DATE

→ **properties :**

① Commutative : $A \vee B = B \vee A$
$A \wedge B = B \wedge A$

② Associative : $A \vee (B \vee C) = (A \vee B) \vee C$
$A \wedge (B \wedge C) = (A \wedge B) \wedge C$

③ Distributive : $A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$
$A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$

* Binary operation : An operation is said to be a binary operation if for that operation we need two members to operate.

* Unary operation : An operation is said to be unary if for that operation we require just one member or operand.

We sometimes denote binary operation by $\square$, $\triangledown$ symbol and unary operation by $\mathring{o}$ symbol.

* Closure property : A mathematical str is said to have a closure property for a binary operation, if that binary operation produces another member of the given set.

PAGE No
DATE

→ An element e is called identity element for the binary operation if,

$$x \,\square\, e = e \,\square\, x = x$$

If for an element x : element y of structure such that $x \,\square\, y = y \,\square\, x = e$
then y is said to be inverse of x.

Let $\square$, ∇, $\circ$ these operations be defined for the set $\{0,1\}$ by the following tables –

$\square$	0	1
0	0	1
1	1	0

∇	0	1
0	0	0
1	0	1

x°	x
0	1
1	0

→ $\square$ is commutative
→ ∇ is associative
→ check de Morgan's law –

PAGE No
DATE

* Proposition / statement :

→ A statement or proposition is a declarative sentence which is either true or false but not both,

→ A proposition is denoted by p, q, r, s, t etc these are proposition variables.

* Conjunction — let p & q be two statements then compound statements - p and q, denoted as $p \wedge q$.
The conjunction is true when both p & q are true.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

* Disjunction — let p & q be two statements then the disjunction of p & q is denoted as $p \vee q$, compound statements - p or q
The disjunction is true when either of p or q is true otherwise false

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

PAGE No
DATE

* **Negation** — The negation of a statement p is denoted by ~p and it's compound statement is not p.

p	~p
T	F
F	T

* **Conditional Statements** —

→ If p & q are two statements then the compound statement "If p then q" is known as conditional statement or an Implication.
→ It is denoted as $p \Rightarrow q$
→ p is known as hypothesis or antecedent
→ q is conclusion
→ conditional statement if both are true or ~~false~~ p is false.

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

* With every conditional statement there are associated 3 other condition statements — $(p \Rightarrow q)$
① converse $(q \Rightarrow p)$
② Inverse $(\sim p \Rightarrow \sim q)$
③ contrapositive $(\sim q \Rightarrow \sim p)$

PAGE No | DATE

example : If I run fast then I will win the race

→ CONVERSE : If I have to win the race then I will have to run fast

→ INVERSE : If I don't run fast then I will not win the race

→ CONTRAPOSITIVE : If I don't win the race then I will not run fast.

* NOTE : A contrapositive statement is always the converse of inverse statement.
A contrapositive statement is equivalent to original statement.

* Bi-conditional statement — If p & q are two statements then the compound statement "p if and only if q" denoted as $p \Leftrightarrow q$ is known as biconditional statement or an equivalence.

→ Bi-cond. statement are true when p & q both have same truth values otherwise it is false.

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

PAGE No		
DATE		

→ $p \vee q \Leftrightarrow q \vee p$

$p \vee q$	$q \vee p$	$p \vee q \Leftrightarrow q \vee p$
T	T	T
T	T	T
T	T	T
F	F	T

★ Tautology — a statement is said to be a tautology if the last column of its truth table shows all true values, irrational of truth values of preceeding columns.

★ Contradiction — a statement is said to be a contradiction if last column of its truth table shows all false values, irrational of truth values of preceeding columns.

★ Contingency — a statement is said to be a contingency if the last column of its truth table shows some true values & some false values.

Unlike the numbers, we can't say that two statements are equal, insted we say that two statements p & q are logically equivalent.

equivalence, ($\cong$)

→ example: check if $p \Rightarrow q \cong \sim q \Rightarrow \sim p$ for Tautology

p	q	$\sim p$	$\sim q$	$p \Rightarrow q$	$\sim q \Rightarrow \sim p$	$p \Rightarrow q \Leftrightarrow \sim q \Rightarrow \sim p$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

∴ Tautology

* NOTE: a conditional statement is always equivalent to its contrapositive statement.

→ example: Write equivalnt statement for that –

"If I run fast then I will win the race"

"If I do not win the race then I will not run fast"

(contrapositive statement).

→ check $\sim(p \wedge q) \vee (\sim p \vee \sim q)$

p	q	$\sim p$	$\sim q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$	$\sim(p \wedge q) \vee (\sim p \vee \sim q)$
T	T	F	F	F	F	F
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

∴ contingency

PAGE No

DATE

Q - If $P \Rightarrow q$ = False, then det value of

$\sim(p \wedge q) \Rightarrow q$ ——— (1)

for

$p \Rightarrow q$ = False

p = True

q = False, putting in (1)

$\sim(T \wedge F) \Rightarrow F$

$T \Rightarrow F$

= False

Q - If $p \Rightarrow q$ = True, then can we determine the truth values of statement : $(p \wedge q) \Rightarrow \sim q$

for

$p \Rightarrow q$ to be True

case (1) : p = True, q = True

then $(T \wedge T) \Rightarrow F$

$T \Rightarrow F$

F

case (2): p = False, q = True

then $(F \wedge T) \Rightarrow F$

$F \Rightarrow F$

T

case (3): p = False, q = False

then $(F \wedge F) \Rightarrow T$

$F \Rightarrow T$

T

PAGE No	
DATE	

* properties of join, meet and negation —

1) $(p \wedge q) \cong (q \wedge p)$
   $(p \vee q) \cong (q \vee p)$ } commutative

2) $p \wedge (q \wedge r) \cong (p \wedge q) \wedge r$
   $p \vee (q \vee r) \cong (p \vee q) \vee r$ } associative

3) $p \vee (q \wedge r) \cong (p \vee q) \wedge (p \vee r)$
   $p \wedge (q \vee r) \cong (p \wedge q) \vee (p \wedge r)$ } distributeve

4) $p \wedge p \cong p$
   $p \vee p \cong p$ } Idempotent

5) $\sim(\sim p) \cong p$

6) De morgan's law
   $\sim(p \wedge q) \cong \sim p \vee \sim q$
   $\sim(p \vee q) \cong \sim p \wedge \sim q$

→ example : write negation of following statement —

(i) It is hot today and yesterday it was raining.
It is not hot today or yesterday it was not raining.

(ii) I want to go to field or it will be hot today.
I do not want to go to field and it will not be hot today.

→ Important –

① $(p \Rightarrow q) \cong (\sim p \vee q)$

② $\sim(p \Rightarrow q) \cong (p \wedge \sim q)$

③ $(p \Rightarrow q) \cong \sim q \Rightarrow \sim p$

→ check whether $(p \Rightarrow q) \cong (\sim p \vee q)$

p	q	$\sim p$	$p \Rightarrow q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Hence shown, $(p \Rightarrow q) \cong (\sim p \vee q)$

④ $(p \Leftrightarrow q) \cong (p \Rightarrow q) \wedge (q \Rightarrow p)$

⑤ $\sim(p \Leftrightarrow q) \cong (p \wedge \sim q) \vee (q \wedge \sim p)$ → using ①

→ Write negation of statement:

I will win football match if and only if I study hard

I will win football match and not study hard or I will study hard and not win football match.

PAGE No
DATE

* **Arguments** : an arguments is a series of statements where all but last are known as hypothesis & the last statement is known as conclusion.

→ If it is valid the argument is known as valid argument otherwise it is know as fallacy.

→ Critical rows are those rows in which hypothesis is all true.

→ For an argument to be valid all the critical rows, conclusion must also be True.

→ If there is atleast 1 critical row for which conclusion is false then the argument will be a fallacy.

eg- $\left.\begin{array}{l} p \Rightarrow q \\ q \Rightarrow r \end{array}\right\}$ hypothesis

$\therefore p \Rightarrow r$ → conclusion , check validity

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$		$p \Rightarrow r$
T	T	T	T	T	→	T
T	T	F	T	F		F
T	F	T	F	T		T
T	F	F	F	T		F
F	T	T	T	T	→	T
F	T	F	T	F		T
F	F	T	T	T	→	T
F	F	F	T	T	→	T

All critical rows conclusion is True

∴ valid argument

PAGE No			
DATE			

* Methods of proof :

① Through Truth Table

② Rules of Inferences :-

(i) Modus penons :- rule of affirmation

$$\begin{array}{l} p \Rightarrow q \\ \underline{p} \\ \therefore q \end{array}$$

(ii) Modus Tollens :- rule of negation

$$\begin{array}{l} p \Rightarrow q \\ \underline{\sim q} \\ \therefore \sim p \end{array}$$

(iii) Hypothetical syllogism :-

$$\begin{array}{l} p \Rightarrow q \\ \underline{q \Rightarrow r} \\ \therefore p \Rightarrow r \end{array}$$

(iv) Disjunctive syllogism :-

$$\begin{array}{l} p \vee q \\ \underline{\sim q} \\ \therefore p \end{array}$$

PAGE No
DATE

→ Check validity of argument –

① $p \Rightarrow \sim q$ (i)
$r \Rightarrow q$ (ii)
r (iii)
$\therefore \sim p$

→ M.P. on ii & iii
$p \Rightarrow \sim q$
q
→ M.T. → $\therefore \sim p$

Valid argument

② $p \Rightarrow q$ (i)
$q \Rightarrow r$ (ii)
$r \Rightarrow s$ (iii)
$\sim s$ (iv)
$p \vee t$ (v)
$\therefore t$

→ H.S. on i, ii, iii
$p \Rightarrow s$ (i)
$\sim s$ (iv)
$p \vee t$ (v)
$\therefore t$

→ M.T. on i, iv
$p \vee t$ (V)
$\sim p$ (vi)
→ D.S. on V, Vi → $\therefore t$

Valid argument

③ Check validity –

→ If I graduated this semester (p) then I will have passed the physics course (q)

→ If I do not study physics for 10 hours a week (~r) then I will not pass physics (~q)

→ If study physics for 10 hours a week (r) the I cannot play volleyball. (s)

∴ If I play volleyball (~s) then I will not graduate this semester. (~p)

PAGE No	
DATE	

$p \Rightarrow q$
$\sim r \Rightarrow \sim q$
$r \Rightarrow s$
$\therefore \sim s \Rightarrow \sim p$

We know, contrapositive statement is equivalent to original statement

$p \Rightarrow q$	$\rightarrow p \Rightarrow q$
$r \Rightarrow q$	$\rightarrow q \Rightarrow r$
$r \Rightarrow s$	$\rightarrow r \Rightarrow s$
$\therefore s \Rightarrow p$	$\rightarrow \therefore p \Rightarrow s$

using, hypothetical syllogism on i, ii, iii

$\therefore p \Rightarrow s$

Valid argument

④ check validity

→ If my plumbing plans do not meet the construction code then I can't build my house

→ If I hire a licensed contractor then my plumbing plans will meet construction code.

→ I hire a licensed contractor

→ ∴ I can build my house

PAGE No	
DATE	

* Mathematical Induction –

1) Basic step: $P(n_0)$ is true $\quad (\forall\, n \geq n_0,\ P(n))$
2) Induction step: $p(k) \Rightarrow p(k+1)$ is a tautology
   i.e. if $p(k)$ is true, then $p(k+1)$ is also true
   where $(n_0 < k \leq n)$

→ example – $1^2 + 3^2 + 5^2 + \ldots\ldots (2n-1)^2 = \dfrac{n(2n+1)(2n-1)}{3}$

Basic step,

$P(1) \Rightarrow$ LHS, $(2\times 1-1)^2 = 1$

RHS, $\dfrac{1\times 3\times 1}{3} = 1$

Induction step,

$$p(k) \Rightarrow 1^2 + 3^2 + 5^2 + \ldots + (2k-1)^2 = \frac{k(2k+1)(2k-1)}{3}$$

$$p(k+1) \Rightarrow 1^2+3^2+5^2+\ldots + (2k-1)^2 + (2k+1)^2 = \frac{k(2k+1)(2k-1)}{3} + (2k+1)^2$$

$$= \frac{k(2k+1)(2k-1) + 3(2k+1)^2}{3}$$

$$= \frac{(2k+1)}{3}\left[k(2k-1) + 3(2k+1)\right]$$

$$= \frac{(2k+1)}{3}(2k^2 - k + 6k + 3)$$

$$= \frac{(2k+1)}{3}(2k^2 + 5k + 3)$$

$$= \frac{(2k+1)}{3}(2k+3)(k+1)$$

Q - $1^2+2^2+3^2+4^2+\dots+n^2 = \dfrac{n(n+1)(2n+1)}{6}$

Basic step,

$P(1) \rightarrow 1^2 = 1$

$$\frac{1(2)(3)}{6} = 1$$

Induction step,

$$P(k) \rightarrow 1^2+2^2+3^2+\dots+k^2 = \frac{k(k+1)(2k+1)}{6}$$

$$p(k+1) \rightarrow 1^2+2^2+3^2+\dots+k^2+(k+1)^2 = \frac{k(k+1)(2k+1)+(k+1)^2}{6}$$

$$= \frac{(k+1)}{6}\left[k(2k+1)+6(k+1)\right]$$

$$= \frac{(k+1)}{6}\left[2k^2+k+6k+6\right]$$

$$= \frac{(k+1)}{6}(2k^2+7k+6)$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

Q - Prove that, $1 + 2^n < 3^n$, for $n \geq 2$

Basic step

$p(2) \rightarrow 1 + 2^2 < 3^2$

$5 < 9$

Induction step,

$p(k) \Rightarrow 1 + 2^k < 3^k$

$$
\begin{aligned}
p(k+1) \Rightarrow 1 + 2^{k+1} &= 1 + 2 \cdot 2^k \\
&= 1 + 2^k + 2^k < 3^k + 2^k \\
&< 3^k + 2 \cdot 3^k \\
&< \underline{\underline{3^{k+1}}}
\end{aligned}
$$

Q - Find the least n for which $(1+n^2) < 2^n$

Q - Show that, $1+2+3+\ldots+n < \frac{(2n+1)^2}{8}$

PAGE No
DATE

* **Product rule** - let a task T be completed in $T_1, T_2, T_3, \ldots T_k$ steps.
Suppose task T_1 can be done in n_1 ways, T_2 in n_2 ways and T_k in n_k ways, then the whole task can be done in -
$(n_1 \cdot n_2 \cdot n_3 \ldots\ldots n_k)$ ways

* **Permutation** -
The permutation is a kind of arrangement of things in a given order
Let there be 'n' distinct objects, then the permutation of n objects taken r at a time $(r \le n)$ is denoted as
$$^{n}P_{r} = \frac{n!}{(n-r)!}$$