203/334 60.7%

Principles of Mathematical Analysis (3rd Edition)

**1. The Real and Complex Number Systems**

\* Introduction

\* Ordered Sets

\* Fields

\* The Real Field

\* The Extended Real Number System

\* The Complex Field

\* Euclidean Spaces

\* Appendix

**2. Basic Topology**

\* Finite, Countable, and Uncountable Sets

\* Metric Spaces

\* Compact Sets

\* Perfect Sets

\* Connected Sets

**3. Numerical Sequences and Series**

\* Convergent Sequences

\* Subsequences

\* Cauchy Sequences

\* Upper and Lower Limits

\* Some Special Sequences

\* Series

\* Series of Nonnegative Terms

\* The Number *e*

\* The Root and Ratio Tests

\* Power Series

\* Summation by Parts

\* Absolute Convergence

\* Addition and Multiplication of Series

\* Rearrangements

**4. Continuity**

\* Limits of Functions

\* Continuous Functions

\* Continuity and Compactness

\* Continuity and Connectedness

\* Discontinuities

\* Monotonic Functions

\* Infinite Limits and Limits at Infinity

**5. Differentiation**

\* The Derivative of a Real Function

\* Mean Value Theorems

\* The Continuity of Derivatives

\* L'Hospital's Rule

\* Derivatives of Higher Order

\* Taylor's Theorem

\* Differentiation of Vector-Valued Functions

**6. The Riemann-Stieltjes Integral**

\* Definition and Existence of the Integral

\* Properties of the Integral

\* Integration and Differentiation

\* Integration of Vector-Valued Functions

\* Rectifiable Curves

**7. Sequences and Series of Functions**

\* Discussion of Main Problem

\* Uniform Convergence

\* Uniform Convergence and Continuity

\* Uniform Convergence and Integration

\* Uniform Convergence and Differentiation

\* Equicontinuous Families of Functions

\* The Stone-Weierstrass Theorem

**8. Some Special Functions**

\* Power Series

\* The Exponential and Logarithmic Functions

\* The Trigonometric Functions

\* The Algebraic Completeness of the Complex Field

\* Fourier Series

\* The Gamma Function

(Finished Chapter 8 and exercises)

29/536 (5.4%)

Partial Differential Equations of Mathematical Physics and Integral Equations (Dover Edition)

**1. Elementary Modeling**

1.1 Introduction

1.2 Small Vibrations of an Elastic String

1.3 Heat Conduction

1.4 Diffusion-Dispersion Phenomena

1.5 Saturated Flows Underground

1.6 Telegrapher's System

1.7 Flow of Ideal Gases

1.8 Well-Posed and Ill-Posed Problems

**2. Partial Differential Equations of the First Order**

2.1 The Method of Characteristics for Quasilinear Equations

2.2 Justification of the Method of Characteristics

2.3 Immiscible Displacement

(Immiscible Displacement - Complete)

117/599 (19.5%)

Theoretical Numerical Analysis: A Functional Analysis Framework (3rd Edition)

**1. Linear Spaces** 1.1 Linear Spaces

1.2 Normed Spaces

1.2.1 Convergence

1.2.2 Banach Spaces

1.2.3 Completion of Normed Spaces

1.3 Inner Product Spaces

1.3.1 Hilbert Spaces

1.3.2 Orthogonality

1.4 Spaces of Continuously Differentiable Functions

1.4.1 Hölder Spaces

1.5 Lp Spaces

1.6 Compact Sets

**2. Linear Operators on Normed Spaces**

2.1 Operators

2.2 Continuous Linear Operators

2.2.1 L(V,W) as a Banach Space

2.3 The Geometric Series Theorem and its Variants

2.3.1 A Generalization

2.3.2 A Perturbation Result

2.4 Some More Results on Linear Operators

2.4.1 An Extension Theorem

2.4.2 Open Mapping Theorem

2.4.3 Principle of Uniform Boundedness

2.4.4 Convergence of Numerical Quadratures

2.5 Linear Functionals

2.5.1 An Extension Theorem for Linear Functionals

2.5.2 The Riesz Representation Theorem

2.6 Adjoint Operators

2.7 Weak Convergence and Weak Compactness

2.8 Compact Linear Operators

2.81. Compact Integral Operators on C(D)

2.8.2 Properties of Compact Operators

2.8.3 Integral Operators on L2(a,b)

2.8.4 The Fredholm Alternative Theorem

2.8.5 Additional Results on Fredholm Integral Equations

2.9 The Resolvent Operator

2.9.1 R() as a Holomorphic Function

**3. Approximation Theory**

3.1 Approximation of Continuous Functions by Polynomials

(Approximation of Continuous F.... - Complete)

100% done

Scientific Computing: An Introductory Survey (2nd Edition)

**1. Scientific Computing**

1.1 Introduction

1.1.1 Computational Problems

1.1.2 General Strategy

1.2 Approximations in Scientific Computing

1.2.1 Sources of Approximation

1.2.2 Absolute Error and Relative Error

1.2.3 Data Error and Computational Error

1.2.4 Truncation Error and Rounding Error

1.2.5 Forward Error and Backward Error

1.2.6 Sensitivity and Conditioning

1.2.7 Stability and Accuracy

1.3 Computer Arithmetic

1.3.1 Floating-Point Numbers

1.3.2 Normalization

1.3.3 Properties of Floating-Point Systems

1.3.4 Rounding

1.3.5 Machine Precision

1.3.6 Subnormals and Gradual Underflow

1.3.7 Exceptional Values

1.3.8 Floating-Point Arithmetic

1.3.9 Cancellation

1.3.10 Other Arithmetic Systems

1.3.11 Complex Arithmetic

1.4 Mathematical Software

1.4.1 Mathematical Software Libraries

1.4.2 Scientific Computing Environments

1.4.3 Extended Arithmetic Packages

1.4.4 Practical Advice on Software

1.5 Historical Notes and Further Reading

**2. Systems of Linear Equations**

2.1 Linear Systems

2.2 Existence and Uniqueness

2.3 Sensitivity and Conditioning

2.3.1 Vector Norms

2.3.2 Matrix Norms

2.3.3 Matrix Condition Number

2.3.4 Error Bounds

2.3.5 Residual

2.4 Solving Linear Systems

2.4.1 Problem Transformations

2.4.2 Triangular Linear Systems

2.4.3 Elementary Elimination Matrices

2.4.4 Gaussian Elimination and LU Factorization

2.4.5 Pivoting

2.4.6 Implementation of Gaussian Elimination

2.4.7 Complexity of Solving Linear Systems

2.48 Gauss-Jordan Elimination

2.49 Solving Modified Problems

2.4.10 Improving Accuracy

2.5 Special Types of Linear Systems

2.5.1 Symmetric Positive Definite Systems

2.5.2 Symmetric Indefinite Systems

2.5.3 Banded Systems

2.6 Iterative Methods for Linear Systems

2.7 Software for Linear Systems

2.7.1 LINPACK and LAPACK

2.7.2 Basic Linear Algebra Subprograms

2.8 Historical Notes and Further Reading

**3. Linear Least Squares**

3.1 Linear Least Squares Problems

3.2 Existence and Uniqueness

3.2.1 Normal Equations

3.2.2 Orthogonality and Orthogonal Projectors

3.3 Sensitivity and Conditioning

3.4 Problem Transformations

3.4.1 Normal Equations

3.4.2 Augmented System

3.4.3 Orthogonal Transformations

3.4.4 Triangular Least Squares Problems

3.4.5 QR Factorization

3.5 Orthogonalization Methods

3.5.1 Householder Transformations

3.5.2 Givens Rotations

3.5.3 Gram-Schmidt Orthogonalization

3.5.4 Rank Deficiency

3.6 Singular Value Decomposition

3.6.1 Other Applications of SVD

3.7 Comparison of Methods

3.8 Software for Linear Least Squares

3.9 Historical Notes and Further Reading

**4. Eigenvalue Problems**

4.1 Eigenvalues and Eigenvectors

4.2 Existence and Uniqueness

4.2.1 Characteristic Polynomial

4.2.2 Multiplicity and Diagonalizability

4.2.3 Eigenspaces and Invariant Subspaces

4.2.4 Properties of Matrices and Eigenvalue Problems

4.2.5 Localizing Eigenvalues

4.3 Sensitivity and Conditioning

4.4 Problem Transformations

4.4.1 Diagonal, Triangular, and Block Triangular Forms

4.5 Computing Eigenvalues and Eigenvectors

4.5.1 Power Iteration

4.5.2 Inverse Iteration

4.5.3 Rayleigh Quotient Iteration

4.5.4 Deflation

4.5.5 Simultaneous Iteration

4.5.6 QR Iteration

4.5.7 Krylov Subspace Methods

4.5.8 Jacobi Method

4.5.9 Bisection or Spectrum-Slicing

4.5.10 Divide-and-Conquer

4.5.11 Relatively Robust Representation

4.5.12 Comparison of Methods

4.6 Generalized Eigenvalue Problems

4.7 Computing the Singular Value Decomposition

4.8 Software for Eigenvalue Problems

4.9 Historical Notes and Further Reading

**5. Nonlinear Equations**

5.1 Nonlinear Equations

5.2 Existence and Uniqueness

5.3 Sensitivity and Conditioning

5.4 Convergence Rates and Stopping Criteria

5.5 Nonlinear Equations in One Dimension

5.5.1 Interval Bisection

5.5.2 Fixed-Point Iteration

5.5.3 Newton's Method

5.5.4 Secant Method

5.5.5 Inverse Interpolation

5.5.6 Linear Fractional Interpolation

5.5.7 Safeguarded Methods

5.5.8 Zeros of Polynomials

5.6 Systems of Nonlinear Equations

5.6.1 Fixed-Point Iteration

5.6.2 Newton's Method

5.6.3 Secant Updating Methods

5.6.4 Robust Newton-Like Methods

5.7 Software for Nonlinear Equations

5.8 Historical Notes and Further Reading

**6. Optimization**

6.1 Optimization Problems

6.2 Existence and Uniqueness

6.2.1 Convexity

6.2.2 Unconstrained Optimality Conditions

6.2.3 Constrained Optimality Conditions

6.3 Sensitivity and Conditioning

6.4 Optimization in One Dimension

6.4.1 Golden Section Search

6.4.2 Successive Parabolic Interpolation

6.4.3 Newton's Method

6.4.4 Safeguarded Methods

6.5 Unconstrained Optimization

6.5.1 Direct Search

6.5.2 Steepest Descent

6.5.3 Newton's Method

6.5.4 Quasi-Newton Methods

6.5.5 Secant Updating Methods

6.5.6 Conjugate Gradient Method

6.5.7 Truncated or Inexact Newton Methods

6.6 Nonlinear Least Squares

6.6.1 Gauss-Newton Method

6.6.2 Levenberg-Marquardt Method

6.7 Constrained Optimization

6.7.1 Sequential Quadratic Programming

6.7.2 Penalty and Barrier Methods

6.7.3 Linear Programming

6.8 Software for Optimization

6.9 Historical Notes and Further Reading

**7. Interpolation**

7.1 Interpolation

7.2 Existence, Uniqueness, and Conditioning

7.3 Polynomial Interpolation

7.3.1 Monomial Basis

7.3.2 Lagrange Interpolation

7.3.3 Newton Interpolation

7.3.4 Orthogonal Polynomials

7.3.5 Interpolating Continuous Functions

7.4 Piecewise Polynomial Interpolation

7.4.1 Hermite Cubic Interpolation

7.4.2 Cubic Spline Interpolation

7.4.3 B-splines

7.5 Software for Interpolation

7.5.1 Software for Special Functions

7.6 Historical Notes and Further Reading

**8. Numerical Integration and Differentiation**

8.1 Integration

8.2 Existence, Uniqueness, and Conditioning

8.3 Numerical Quadrature

8.3.1 Newton-Cotes Quadrature

8.3.2 Clenshaw-Curtis Quadrature

8.3.3 Gaussian Quadrature

8.3.4 Progressive Gaussian Quadrature

8.3.5 Composite Quadrature

8.3.6 Adaptive Quadrature

8.4 Other Integration Problems

8.4.1 Tabular Data

8.4.2 Improper Integrals

8.4.3 Double Integrals

8.4.4 Multiple Integrals

8.5 Integral Equations

8.6 Numerical Differentiation

8.6.1 Finite Difference Approximations

8.6.2 Automatic Differentiation

8.7 Richardson Extrapolation

8.8 Software for Integration and Differentiation

8.9 Historical Notes and Further Reading

**9. Initial Value Problems for Ordinary Differential Equations**

9.1 Ordinary Differential Equations

9.2 Existence, Uniqueness, and Conditioning

9.3 Numerical Solution of ODEs

9.3.1 Euler's Method

9.3.2 Accuracy and Stability

9.3.3 Implicit Methods

9.3.4 Stiffness

9.3.5 Taylor Series Methods

9.3.6 Runge-Kutta Methods

9.3.7 Extrapolation Methods

9.3.8 Multistep Methods

9.3.9 Multivalue Methods

9.4 Software for ODE Initial Value Problems

9.5 Historical Notes and Further Reading

**10. Boundary Value Problems for Ordinary Differential Equations**

10.1 Boundary Value Problems

10.2 Existence, Uniqueness, and Conditioning

10.3 Shooting Method

10.4 Finite Difference Method

10.5 Collocation Method

10.6 Galerkin Method

10.7 Eigenvalue Problems

10.8 Software for ODE Boundary Value Problems

10.9 Historical Notes and Further Reading

**11. Partial Differential Equations**

11.1 Partial Differential Equations

11.2 Time-Dependent Problems

11.2.1 Semidiscrete Methods

11.2.2 Fully Discrete Methods

11.3 Time-Independent Problems

11.3.1 Finite Difference Methods

11.3.2 Finite Element Methods

11.4 Direct Methods for Sparse Linear Systems

11.4.1 Sparse Factorization Methods

11.4.2 Fast Direct Methods

11.5 Iterative Methods for Linear Systems

11.5.1 Stationary Iterative Methods

11.5.2 Jacobi Method

11.5.3 Gauss-Seidel Method

11.5.4 Successive Over-Relaxation

11.5.5 Conjugate Gradient Method

11.5.6 Rate of Convergence

11.5.7 Multigrid Methods

11.6 Comparison of Methods

11.7 Software for Partial Differential Equations

11.7.1 Software for Initial Value Problems

11.7.2 Software for Boundary Value Problems

11.7.3 Software for Sparse Linear Systems

11.8 Historical Notes and Further Reading

**12. Fast Fourier Transform**

12.1 Trigonometric Interpolation

12.1.1 Discrete Fourier Transform

12.2 FFT Algorithm

12.2.1 Limitations of FFT

12.3 Applications of DFT

12.3.1 Fast Polynomial Multiplication

12.4 Wavelets

12.5 Software for FFT

12.6 Historical Notes and Further Reading

**13. Random Numbers and Stochastic Simulation**

13.1 Stochastic Simulation

13.2 Randomness and Random Numbers

13.3 Random Number Generators

13.3.1 Congruential Generators

13.3.2 Fibonacci Generators

13.3.3 Nonuniform Distributions

13.4 Quasi-Random Sequences

13.5 Software for Generating Random Numbers

13.6 Historical Notes and Further Reading

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100 % done

Numerical Linear Algebra (1st Edition)

**I. Fundamentals**

1. Matrix-Vector Multiplication

\* Familiar Definitions

\* A Matrix Times a Vector

\* A Matrix Times a Matrix

\* Range and Nullspace

\* Rank

\* Inverse

\* A Matrix Inverse Times a Vector

\* A Note on *m* and *n*

2. Orthogonal Vectors and Matrices

\* Adjoint

\* Inner Product

\* Orthogonal Vectors

\* Components of a Vector

\* Unitary Matrices

\* Multiplication by a Unitary Matrix

3. Norms

\* Vector Norms

\* Matrix Norms Induced by Vector Norms

\* Examples

\* Cauchy-Schwarz and Hölder Inequalities

\* Bounding ||AB|| in an Induced Matrix Norm

\* General Matrix Norms

\* Invariance under Unitary Multiplication

4. The Singular Value Decomposition

\* A Geometric Observation

\* Reduced SVD

\* Full SVD

\* Formal Definition

\* Existence and Uniqueness

5. More on the SVD

\* A Change of Bases

\* SVD vs. Eigenvalue Decomposition

\* Matrix Properties via the SVD

\* Low-Rank Approximations

\* Computation of the SVD

**II. QR Factorization and Least Squares**

6. Projectors

\* Projectors

\* Complementary Projectors

\* Orthogonal Projectors

\* Projection with an Orthonormal Basis

\* Projection with an Arbitrary Basis

7. QR Factorization

\* Reduced QR Factorization

\* Full QR Factorization

\* Gram-Schmidt Orthogonalization

\* Existence and Uniqueness

\* When Vectors Become Continuous Functions

\* Solution of *Ax = b*by QR Factorization

8. Gram-Schmidt Orthogonalization

\* Gram-Schmidt Projections

\* Modified Gram-Schmidt Algorithm

\* Operation Count

\* Counting Operations Geometrically

\* Gram-Schmidt as Triangular Orthogonalization

9. MATLAB

\* MATLAB

\* Experiment 1: Discrete Legendre Polynomials

\* Experiment 2: Classical vs. Modified Gram-Schmidt

\* Experiment 3: Numerical Loss of Orthogonality

10. Householder Triangularization

\* Householder and Gram-Schmidt

\* Triangularizing by Introducing Zeros

\* Householder Reflectors

\* The Better of Two Reflectors

\* The Algorithm

\* Applying or Forming *Q*

\* Operation Count

11. Least Squares Problems

\* The Problem

\* Example: Polynomial Data-Fitting

\* Orthogonal Projection and the Normal Equations

\* Pseudoinverse

\* Normal Equations

\* QR Factorization

\* SVD

\* Comparison of Algorithms

**III. Conditioning and Stability**

12. Conditioning and Condition Numbers

\* Condition of a Problem

\* Absolute Condition Number

\* Relative Condition Number

\* Examples

\* Condition of Matrix-Vector Multiplication

\* Condition Number of a Matrix

\* Condition of a System of Equations

13. Floating Point Arithmetic

\* Limitations of Digital Representations

\* Floating Point Numbers

\* Machine Epsilon

\* Floating Point Arithmetic

\* Machine Epsilon, Again

\* Complex Floating Point Arithmetic

14. Stability

\* Algorithms

\* Accuracy

\* Stability

\* Backward Stability

\* The Meaning of *O*(machine)

\* Dependence on *m* and *n*, not *A* and *b*

\* Independence of Norm

15. More on Stability

\* Stability of Floating Point Arithmetic

\* Further Examples

\* An Unstable Algorithm

\* Accuracy of a Backward Stable Algorithm

\* Backward Error Analysis

16. Stability of Householder Triangularization

\* Experiment

\* Theorem

\* Analyzing an Algorithm to Solve *Ax* = *b*

17. Stability of Back Substitution

\* Triangular Systems

\* Backward Stability Theorem

\* *m* = 1

\* *m* = 2

\* *m* = 3

\* General *m*

\* Remarks

18. Conditioning of Least Squares Problems

\* Four Conditioning Problems

\* Theorem

\* Transformation to a Diagonal Matrix

\* Sensitivity of *y* to Perturbations in *b*

\* Sensitivity of *x* to Perturbations in *b*

\* Tilting the Range of *A*

\* Sensitivity of *y* to Perturbations in *A*

\* Sensitivity of *x* to Perturbations in *A*

19. Stability of Least Squares Algorithms

\* Example

\* Householder Triangularization

\* Gram-Schmidt Orthogonalization

\* Normal Equations

\* SVD

\* Rank-Deficient Least Squares Problems

**IV. Systems of Equations**

20. Gaussian Elimination

\* LU Factorization

\* Example

\* General Formulas and Two Strokes of Luck

\* Operation Count

\* Solution of *Ax = b* by LU Factorization

\* Instability of Gaussian Elimination Without Pivoting

21. Pivoting

\* Pivots

\* Partial Pivoting

\* Example

\* *PA = LU* Factorization and a Third Stroke of Luck

\* Complete Pivoting

22. Stability of Gaussian Elimination

\* Stability and the Size of *L* and *U*

\* Growth Factors

\* Worst-Case Instability

\* Stability in Practice

\* Explanation

23. Cholesky Factorization

\* Hermitian Positive Definite Matrices

\* Symmetric Gaussian Elimination

\* Cholesky Factorization

\* The Algorithm

\* Operation Count

\* Stability

\* Solution of *Ax = b*

**V. Eigenvalues**

24. Eigenvalue Problems

\* Eigenvalues and Eigenvectors

\* Eigenvalue Decomposition

\* Geometric Multiplicity

\* Characteristic Polynomial

\* Algebraic Multiplicity

\* Similarity Transformations

\* Defective Eigenvalues and Matrices

\* Diagonalizability

\* Determinant and Trace

\* Unitary Diagonalization

\* Schur Factorization

\* Eigenvalue-Revealing Factorizations

25. Overview of Eigenvalue Algorithms

\* Shortcomings of Obvious Algorithms

\* A Fundamental Difficulty

\* Schur Factorization and Diagonalization

\* Two Phases of Eigenvalue Computations

26. Reduction to Hessenberg or Tridiagonal Form

\* A Bad Idea

\* A Good Idea

\* Operation Count

\* The Hermitian Case: Reduction to Tridiagonal Form

\* Stability

27. Rayleigh Quotient, Inverse Iteration

\* Restriction to Real Symmetric Matrices

\* Rayleigh Quotient

\* Power Iteration

\* Inverse Iteration

\* Rayleigh Quotient Iteration

\* Operation Counts

28. QR Algorithm Without Shifts

\* The QR Algorithm

\* Unnormalized Simultaneous Iteration  
 \* Simultaneous Iteration

\* Simultaneous Iteration ⇔ QR Algorithm

\* Convergence of the QR Algorithm

29. QR Algorithm With Shifts

\* Connection with Inverse Iteration

\* Connection with Shifted Inverse Iteration

\* Connection with Rayleigh Quotient Iteration

\* Wilkinson Shift

\* Stability and Accuracy

30. Other Eigenvalue Algorithms

\* Jacobi

\* Bisection

\* Divide-and-Conquer

31. Computing the SVD

\* SVD of A and Eigenvalues of A\*A

\* A Different Reduction to an Eigenvalue Problem

\* Two Phases

\* Golub-Kahan Bidiagonalization

\* Faster Methods for Phase 1

\* Phase 2

**VI. Iterative Methods**

32. Overview of Iterative Methods

\* Why Iterate?

\* Structure, Sparsity, and Black Boxes

\* Projection into Krylov Subspaces

\* Number of Steps, Work per Step, and Preconditioning

\* Exact vs. Approximate Solutions

\* Direct Methods that Beat O(m3)

33. The Arnoldi Iteration

\* The Arnoldi/Gram-Schmidt Analogy

\* Mechanics of the Arnoldi Iteration

\* QR Factorization of a Krylov Matrix

\* Projection onto Krylov Subspaces

34. How Arnoldi Locates Eigenvalues

\* Computing Eigenvalues by the Arnoldi Iteration

\* A Note of Caution: Nonnormality

\* Arnoldi and Polynomial Approximation

\* Invariance Properties

\* How Arnoldi Locates Eigenvalues

\* Arnoldi Lemniscates

\* Geometric Convergence

35. GMRES

\* Residual Minimization in Kn

\* Mechanics of GMRES

\* GMRES and Polynomial Approximation

\* Convergence of GMRES

\* Polynomials Small on the Spectrum

36. The Lanczos Iteration

\* Three-Term Recurrence

\* The Lanczos Iteration

\* Lanczos and Electric Charge Distribution

\* Example

\* Rounding Errors and "Ghost" Eigenvalues

37. From Lanczos to Gauss Quadrature

\* Orthogonal Polynomials

\* Jacobi Matrices

\* The Characteristic Polynomial

\* Quadrature Formulas

\* Gauss Quadrature

\* Gauss Quadrature via Jacobi Matrices

\* Example

38. Conjugate Gradients

\* Minimizing the 2-Norm of the Residual

\* Minimizing the *A*-Norm of the Residual

\* The Conjugate Gradient Iteration

\* Optimality of CG

\* CG as an Optimization Algorithm

\* CG and Polynomial Approximation

\* Rate of Convergence

\* Example

39. Biorthogonalization Methods

\* Where We Stand

\* CGN = CG Applied to the Normal Equations

\* Tridiagonal Biorthogonalization

\* BCG = Biconjugate Gradients

\* Example

\* QMR and Other Variants

40. Preconditioning

\* Preconditioners for *Ax = b*

\* Left, Right, and Hermitian Preconditioners

\* Example

\* Survey of Preconditioners for *Ax = b*

\* Preconditioners for Eigenvalue Problems

\* A Closing Note

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100 % done

Introduction to Computational PDEs: Course Notes for AMATH 442/CM 452 (1st Edition)

**1. Overview of PDEs**

1.1 Linear Second Order PDEs with Two Independent Variables

1.1.1 A Note About Leibniz and Subscript Notation

1.1.2 Classification of Linear Second-Order PDEs

1.1.3 Derivation of the Heat Equation

1.2 Hyperbolic PDEs with Two Independent Variables

1.2.1 The Linear Advection Equation

1.2.2 The Wave Equation

1.2.3 d'Alembert's Solution for the Wave Equation IVP

1.2.4 Domain of Influence and Domain of Dependence

1.2.5 Existence and Uniqueness for the IVBVP

1.3 Elliptic PDEs with Two Independent Variables

1.3.1 The Dirac Delta

1.3.2 Domain of Influence

1.3.3 Discontinuous Boundary Conditions

1.3.4 Existence and Uniqueness

1.4 Parabolic PDEs with Two Independent Variables

1.4.1 Domain of Influence and Domain of Dependence

1.4.2 Discontinuous Initial Conditions

1.5 Linear Second Order PDEs with Three Independent Variables

1.5.1 A Note About Vector Calculus Notation

**2. Finite Difference Methods**

2.1 Finite Difference Methods for Elliptic PDEs

2.1.1 1D Elliptic Model Problem

\* Matrix Form of the BVP

\* Actual Error and Convergence

2.1.2 2D Elliptic Model Problem

\* Matrix Form of the BVP

2.1.3 Convergence Theory

\* The Error Equation

\* Lax Convergence Theorem for Elliptic PDEs

\* 2-Norm Convergence for 1D Elliptic Problems

2.2 FD Methods for Hyperbolic PDEs

2.2.1 FD Methods for the 1D Linear Advection Equation

2.2.2 Stability

\* Example: The Forward Upwind Scheme

\* Graphical Techniques for Demonstrating Stability

\* Example: The Backward Central Scheme

\* Discussion

\* Link With the Discrete Fourier Transform

2.2.3 Dissipation and Dispersion

\* Dissipation and Dispersion for PDEs

\* Dissipation and Dispersion for Difference Formulas

2.2.4 Finite Difference Methods for the Wave Equation

\* Physical Interpretation of CFL Condition for Explicit Methods

2.2.5 Finite Difference Methods in 2D and 3D

2.3 Finite Difference Methods for Parabolic PDEs

\* Stability

2.4 Finite Difference Convergence Theory for Time-Dependent Problems

2.4.1 Actual Error, Truncation Error, and Consistency

2.4.2 Stability and Convergence: Lax Convergence Theorem

2.4.3 2-Norm Convergence

**3. Finite Volume Methods for Nonlinear Hyperbolic Conservation Laws**

3.1 Characteristic Curves

3.2 1D Conservation Laws and the Burgers' Equation

3.2.1 Integral Forms of Conservation Laws

3.2.2 Characteristic Curves of the Burgers' Equation

3.2.3 Shock Speed: The Rankine-Hugoniot Relation

3.3 Problems with FD Methods for Hyperbolic Conservation Laws

3.3.1 Problem 1: Oscillations When Solution is Discontinuous

3.3.2 Problem 2: Standard FD Methods Can Give the Wrong Shock

Speeds

3.4 Finite Volume Methods

3.4.1 The Finite Volume Principle

3.4.2 The Local Lax-Friedrichs Method in 1D

\* Stability, Accuracy, and Consistency

3.4.3 Numerical Conservation

3.4.4 FV Methods and the Linear Advection Equation

3.5 Conservation Laws in Higher Dimensions

3.5.1 Gauss' Divergence Theorem

3.5.2 Conservation Laws in Higher Dimension

3.5.3 Finite Volume Methods in 2D

3.6 Systems of Conservation Laws

**4. Finite Element Methods for Elliptic Problems**

4.1 An Introductory Example

4.2 The 1D Model Problem

4.2.1 Weighted Residual Form and Weak Form

\* The Weighted Residual Form of the ODE

\* The Weak Form of the ODE

\* The Difference Between the Forms of the ODE

4.2.2 Discrete Weak Form

\* Matrix Form of the Discrete ODE Problem

4.2.3 Choice of Basis Functions

\* The Tent Functions as Basis Functions

4.3 The 2D Model Problem

4.3.1 Weighted Residual Form and Weak Form

4.3.2 Discrete Weak Form

\* Matrix Form of the Discrete PDE Problem

4.3.3 Simple Finite Elements in 2D

\* Constructing the Global Stiffness Matrix and Load Vector

\* Pseudo-Code

4.4 Neumann Boundary Conditions

4.4.1 Compatibility Between *h* and *f*

4.4.2 Weighted Residual Form and Weak Form

4.4.3 Discrete Weak Form

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124/351 (35.3%)

Understanding and Implementing the Finite Element Method (1st Edition)

**I. The Basic Framework for Stationary Problems**

1. Some Model PDEs

1.1 Laplace's Equation; Elliptic BVPs

1.1.1 Physical Experiments Modeled by Laplace's Equation

\* Steady-State Heat Flow

\* Units and Physical Parameters

\* Small Vertical Deflections of a Membrane

1.2 Other Elliptic BVPs

1.2.1 The Equations of Isotropic Elasticity

1.2.2 General Linear Elasticity

2. The Weak Form of a BVP

2.1 Review of Vector Calculus

2.1.1 The Divergence Theorem

\* The Compatibility Condition for the Neumann Problem

2.1.2 Green's Identity

2.1.3 Other Forms of the Divergence Theorem and Green's Identity

2.2 The Weak Form of a BVP

2.2.1 Minimization of Energy

2.2.2 Relaxing the PDE

\* Sobolev Spaces

2.2.3 A Few Details About Sobolev Spaces

2.3 The Weak Form for Other Boundary Conditions and PDEs

2.3.1 Neumann Conditions and the Weak Form

2.3.2 Mixed Boundary Conditions

2.3.3 Inhomogeneous Boundary Conditions

\* Inhomogeneous Neumann Conditions

\* Inhomogeneous Dirichlet Conditions

2.3.4 Other Elliptic BVP's

2.4 Existence and Uniqueness Theory for the Weak Form of a BVP

2.4.1 Vector Spaces and Inner Products

2.4.2 Hilbert Spaces

2.4.3 Linear Functionals

2.4.4 The Riesz Representation Theorem

2.4.5 Variational Problems and the Riesz Representation Theorem

2.5 Examples of Ellipticity

2.5.1 The Model Problem

2.5.2 The Equations of Isotropic Elasticity

2.6 Variational Formulation of Nonsymmetric Problems

3. The Galerkin Method

3.1 The Projection Theorem

3.2 The Galerkin Method for a Variational Problem

3.2.1 Another Interpretation of the Galerkin Method

3.2.2 The Galerkin Method for a Nonsymmetric Problem

4. Piecewise Polynomials and the Finite Element Method

4.1 Piecewise Linear Functions Defined on a Triangular Mesh

4.1.1 Using Piecewise Linear Functions in Galerkin's Method

\* Inhomogeneous Dirichlet Conditions

4.1.2 The Sparsity of the Stiffness Matrix

4.2 Quadratic Lagrange Triangles

4.2.1 Continuous Piecewise Quadratic Functions

4.2.2 The Finite Element Method with Quadratic Lagrange

Triangles

4.3 Cubic Lagrange Triangles

4.3.1 Continuous Piecewise Cubic Functions

4.3.2 The Finite Element Method with Cubic Lagrange

Triangles

4.4 Lagrange Triangles of Arbitrary Degree

4.4.1 Hierarchical Bases for Finite Element Spaces

4.5 Other Finite Elements: Rectangles and Quadrilaterals

4.5.1 Rectangular Elements

4.5.2 General Quadrilaterals

4.6 Using a Reference Triangle in Finite Element Calculations

4.7 Isoparametric Finite Element Methods

4.7.1 Isoparametric Quadratic Triangles

4.7.2 Isoparametric Triangles of Higher Degree

5. Convergence of the Finite Element Method

5.1 Approximating Smooth Functions by Continuous Piecewise Linear

Functions

5.1.1 The Standard Refinement of a Triangulation

5.1.2 Nondegenerate Families of Triangulations

5.1.3 Approximation by Piecewise Linear Functions

5.2 Approximation by Higher-Order Piecewise Polynomials

5.3 Convergence in the Energy Norm

5.4 Convergence in the L2-Norm

5.5 Variational Crimes

5.5.1 Numerical Integration

5.5.2 Outline of the Analysis of the Effect of Quadrature

5.5.3 Isoparametric Finite Elements

(Section 5 - Convergence.... completely done)

68/158 (43.0%)

Discontinuous Galerkin Methods for Solving Elliptic and Parabolic Equations: Theory and Implementation (1st Edition)

**I. Elliptic Problems**

1. One-Dimensional Problem

1.1 Model Problem

1.2 A Class of DG Methods

1.3 Existence and Uniqueness of the DG Solution

1.4 Linear System

1.4.1 Computing the Matrix *A*

1.4.2 Computing the Right-hand Side *b*

1.4.3 Imposing Boundary Conditions Strongly

1.5 Convergence of the DG Method

1.6 Numerical Experiments

1.7Bibliographical Remarks

2. Higher Dimensional Problem

2.1 Preliminaries

2.1.1 Vector Notation

2.1.2 Sobolev Spaces

2.1.3 Trace Theorems

2.1.4 Approximation Properties

2.1.5 Green's Theorem

2.1.6 Cauchy-Schwarz's and Young's Inequalities

2.2 Model Problem

2.2.1 Weak Solution

2.2.2 Numerical Solution

2.3 Broken Sobolev Spaces

2.3.1 Jumps and Averages

2.4 Variational Formulation

2.4.1 Consistency

2.5 Finite Element Spaces

2.5.1 Reference Elements Versus Physical Elements

2.5.2 Basis Functions

2.5.3 Numerical Quadrature

2.6 DG Scheme

2.7 Properties

2.7.1 Coercivity of Bilinear Forms

2.7.2 Continuity of Bilinear Form

2.7.3 Local Mass Conservation

2.7.4 Existence and Uniqueness of DG Solution

2.8 Error Analysis

2.8.1 Error Estimates in the Energy Norm

2.8.2 Error Estimates in the *L2* norm

2.9 Implementing the DG Method

2.9.1 Data Structure

2.9.2 Local Matrices and Right-Hand Sides

2.9.3 Global Matrix and Right-Hand Side

2.10 Numerical Experiments

2.10.1 Smooth Solution

2.10.2 Singular Solution

2.10.3 Condition Number

2.11 The Local Discontinuous Galerkin Method

2.11.1 Definition of the Mixed DG Method

2.11.2 Existence and Uniqueness of the Solution

2.11.3 A Priori Error Estimates

2.12 DG Versus Classical Finite Element Method

2.13 Bibliographical Remarks

(Chapter 2 plus exercises= Done)

39/513 (7.6%)

Iterative Methods for Sparse Linear Systems (2nd Edition)

**1. Background in Linear Algebra**

1.1 Matrices

1.2 Square Matrices and Eigenvalues

1.3 Types of Matrices

1.4 Vector Inner Products and Norms

1.5 Matrix Norms

1.6 Subspaces, Range, and Kernel

1.7 Orthogonal Vectors and Subspaces

1.8 Canonical Forms of Matrices

1.8.1 Reduction to the Diagonal Form

1.8.2 The Jordan Canonical Form

1.8.3 The Schur Canonical Form

1.8.4 Application to Powers of Matrices

1.9 Normal and Hermitian Matrices

1.9.1 Normal Matrices

1.9.2 Hermitian Matrices

1.10 Nonnegative Matrices, M-Matrices

1.11 Positive-Definite Matrices

1.12 Projection Operators

1.12.1 Range and Null Space of a Projector

1.12.2 Matrix Representations

1.12.3 Orthogonal and Oblique Projectors

1.12.4 Properties of Orthogonal Projectors

(Finished 1.12.4)

96/396 (24.2 %)

The Least-Squares Finite Element Method: Theory and Applications in Computational Fluid Dynamics and Electromagnetics (1st Edition)

**I. Basic Concepts of LSFEM**

**1. Introduction**

1.1 Why Finite Elements?

1.2 Why Least-Squares?

**2. First-Order Scalar Equation in One Dimension**

2.1 A Model Problem

2.2 Function Spaces Hm(Ω)

2.3 The Classic Galerkin Method - Global Approximation

2.4 The Least-Squares Method - Global Approximation

2.5 One-Dimensional Finite Elements

2.6 The Classic Galerkin Finite Element Method

2.7 The Least-Squares Finite Element Method

2.7.1 The Least-Squares Formulation

2.7.2 The Euler-Lagrange Equation

2.7.3 Error Estimates

2.7.4 Condition Number

2.7.5 A Numerical Example

2.8 Concluding Remarks

**3. First-Order System in One Dimension**

3.1 A Model Problem

3.2 The Rayleigh-Ritz Method

3.3 The Mixed Galerkin Method

3.4 The Least-Squares Finite Element Method

3.4.1 The Least-Squares Formulation

3.4.2 Stability Estimate

3.4.3 Error Analysis

3.4.4 Numerical Results

3.5 Concluding Remarks

**II. Fundamentals of LSFEM**

**4. Basis of LSFEM**

4.1 Function Spaces

4.2 Linear Operators

4.3 The Bounded Inverse Theorem

4.4 The Friedrichs Inequality

4.5 The Poincaré Inequality

4.6 Finite Element Spaces

4.6.1 Regularity Requirements

4.6.2 Linear Triangular Element

4.6.3 Interpolation Errors

4.7 First-Order System

4.8 General Formulation of LSFEM

4.9 The Euler-Lagrange Equation

4.10 Error Estimates for LSFEM

4.10.1 General Problems

4.10.2 Elliptic Problems

4.11 Implementation of LSFEM

4.11.1 The Least-Squares Solution to Linear Algebraic Equations

4.11.2 The Least-Squares Finite Element Collocation Method

4.11.3 Importance of the Order of Gaussian Quadrature

4.12 Concluding Remarks

**5. Div-Curl-System**

5.1 Basic Theorems

5.2 Determinacy and Ellipticity

5.3 The Div-Curl Method

5.4 The Least-Squares Method

5.5 The Euler-Lagrange Equation

5.6 The Friedrichs Second Div-Curl Inequality

5.7 Concluding Remarks

(Section 5 - Complete)

100% done

Div, Grad, Curl, and All That: An Informal Text on Vector Calculus (1st Edition)

**1. Introduction, Vector Functions, and Electrostatics**

\* Introduction

\* Vector Functions

\* Electrostatics

**2. Surface Integrals and the Divergence**

\* Gauss' Law

\* The Unit Normal Vector

` \* Definition of Surface Integrals

\* Evaluating Surface Integrals

\* Flux

\* Using Gauss' Law to Find the Field

\* The Divergence

\* The Divergence in Cylindrical and Spherical Coordinates

\* The Del Notation

\* The Divergence Theorem

\* Two Simple Applications of the Divergence Theorem

**3. Line Integrals and the Curl**

\* Work and Line Integrals

\* Line Integrals Involving Vector Functions

\* Path Independence

\* The Curl

\* The Curl in Cylindrical and Spherical Coordinates

\* The Meaning of the Curl

\* Differential Form of the Circulation Law

\* Stokes' Theorem

\* Two Applications of Stokes' Theorem

\* Path Independence and the Curl

**4. The Gradient**

\* Line Integrals and the Gradient

\* Finding the Electrostatic Field

\* Using Laplace's Equation

\* Directional Derivatives and the Gradient

\* Geometric Significance of the Gradient

\* The Gradient in Cylindrical and Spherical Coordinates

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52/347 (14.9%)

Finite Elements: Theory, Fast Solvers, and Applications in Solid Mechanics (3rd Edition)

**1. Introduction**

1.1 Examples and Classification of PDE's

\* Examples

\* Classification of PDE's

\* Well-posed Problems

1.2 The Maximum Principle

\* Examples

\* Corollaries

1.3 Finite Difference Methods

\* Discretization

\* Discrete Maximum Principle

1.4 A Convergence Theory for Difference Methods

\* Consistency

\* Local and Global Error

\* Limits of the Convergence Theory

**2. Conforming Finite Elements**

2.1 Sobolev Spaces

\* Introduction to Sobolev Spaces

\* Friedrichs' Inequality

\* Possible Singularities of *H1* Functions

\* Compact Imbeddings

2.2 Variational Formulation of Elliptic Boundary-Value Problems of Second

Order

\* Variational Formulation

\* Reduction to Homogeneous Boundary Conditions

\* Existence of Solutions

\* Inhomogeneous Boundary Conditions

2.3 The Neumann Boundary-Value Problem. A Trace Theorem

\* Ellipticity in H1

\* Boundary-Value Problems with Natural Boundary Conditions

\* Neumann Boundary Conditions

\* Mixed Boundary Conditions

\* Proof of the Trace Theorem

\* Practical Consequences of the Trace Theorem

(Section 2.3 - Complete)

18/701 (2.56%)

The Finite Element Method in Electromagnetics (2nd Edition)

**1. Basic Electromagnetic Theory**

1.1 Brief Review of Vector Analysis

1.2 Maxwell's Equations

1.2.1 The General Integral Form

1.2.2 The General Differential Form

1.2.3 Electro- and Magnetostatic Fields

1.2.4 Time-Harmonic Fields

1.2.5 Constitutive Relations

1.3 Scalar and Vector Potentials

1.3.1 Scalar Potential for Electrostatic Field

1.3.2 Vector Potential for Magnetostatic Field

1.4 Wave Equations

1.4.1 Vector Wave Equations

1.4.2 Scalar Wave Equations

1.5 Boundary Conditions

1.5.1 At the Interface Between Two Media

1.5.2 At a Perfectly Conducting Surface

1.5.3 At an Imperfectly Conducting Surface

1.5.4 Across a Resistive and Conductive Sheet

1.6 Radiation Conditions

1.7 Fields in an Infinite Homogeneous Medium

1.8 Huygens's Principle

1.9 Radar Cross Sections

1.10 Summary

12/498 (2.4%)

Computer Simulation Using Particles (Paperback Edition)

**1. Computer Experiments Using Particle Models**

1.1 Introduction

1.2 The Computer Experiment

1.2.1 The Role of the Computer Experiment

1.2.2 Setting Up Computer Experiments

1.3 Length and Time Scales

1.4 Physical Systems

1.4.1Correlated Systems

(Finished 1.4.1)

58/653 (8.8%)

Fundamentals of Plasma Physics (3rd Edition)

**1. Introduction**

1. General Properties of Plasmas

1.1 Definition of a Plasma

1.2 Plasma as the Fourth State of Matter

1.3 Plasma Production

1.4 Particle Interactions and Collective Effects

1.5 Some Basic Plasma Phenomena

2. Criteria for the Definition of a Plasma

2.1 Macroscopic Neutrality

2.2 Debye Shielding

2.3 The Plasma Frequency

3. The Occurence of Plasmas in Nature

3.1 The Sun and Its Atmosphere

3.2 The Solar Wind

3.3 The Magnetosphere and the Van Allen Radiation Belts

3.4 The Ionosphere

3.5 Plasmas Beyond the Solar System

4. Applications of Plasma Physics

4.1 Controlled Thermonuclear Fusion

4.2 The Magnetohydrodynamic Generator

4.3 Plasma Propulsion

4.4 Other Plasma Devices

5. Theoretical Description of Plasma Phenomena

5.1 General Considerations on a Self-Consistent Formulation

5.2 Theoretical Approaches

**2. Charged Particle Motion in Constant and Uniform Electromagnetic Fields**

1. Introduction

2. Energy Conservation

3. Uniform Electrostatic Field

4. Uniform Magnetostatic Field

4.1 Formal Solution of the Equation of Motion

4.2 Solution in Cartesian Coordinates

4.3 Magnetic Moment

4.4 Magnetization Current

5. Uniform Electrostatic and Magnetostatic Fields

5.1 Formal Solution of the Equation of Motion

5.2 Solution in Cartesian Coordinates

6. Drift Due to an External Force

(Finished Chapter 2 and exercises)