Exercise 1.

$$\alpha = e^{i\varphi_\alpha} \cos\left(\frac{\theta}{2}\right)$$

$$\beta = e^{i\varphi_\beta} \sin\left(\frac{\beta}{2}\right)$$

$$|\alpha|^2 = |e^{i\varphi_\alpha}|^2 \cdot \cos^2\left(\frac{\theta}{2}\right) = \cos^2\left(\frac{\theta}{2}\right)$$

$$Re(\beta) = \frac{e^{i\varphi_\beta} + e^{-i\varphi_\beta}}{2} \sin\left(\frac{\theta}{2}\right)$$

$$Im(\beta) = \frac{e^{i\varphi_\beta} - e^{-i\varphi_\beta}}{2i} \sin\left(\frac{\theta}{2}\right)$$

$$|Re(\beta)|^2 = \frac{1}{4}\left| e^{i\varphi_\beta} + e^{-i\varphi_\beta}\right|^2 \sin^2\left(\frac{\theta}{2}\right)$$

$$= \frac{1}{4}\left(\left|e^{i\varphi_\beta}\right|^2 + 2\left|e^{i\varphi_\beta}e^{-i\varphi_\beta}\right| + \left|e^{-i\varphi_\beta}\right|^2\right) \sin^2\left(\frac{\theta}{2}\right)$$

$$= \frac{1}{4}(1 + 2 + 1) \sin^2\left(\frac{\theta}{2}\right)$$

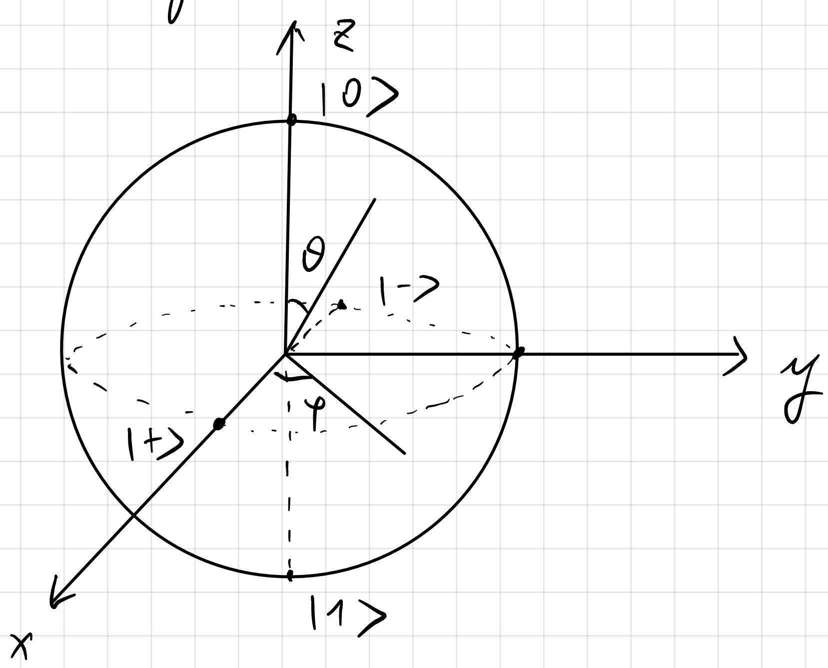
$$= \sin^2\left(\frac{\theta}{2}\right)$$

$$|Im(\beta)|^2 = \frac{1}{4}\left(\left|e^{i\varphi_\beta}\right|^2 - 2\left|e^{i\varphi_\beta}e^{-i\varphi_\beta}\right| + \left|e^{-i\varphi_\beta}\right|^2\right) \sin^2\left(\frac{\theta}{2}\right)$$

$$= \frac{1}{4}(1 - 2 + 1) \cdot \sin^2\left(\frac{\theta}{2}\right)$$

$$|\alpha|^2 + |Re(\beta)|^2 + |Im(\beta)|^2 = \cos^2\left(\frac{\theta}{2}\right) + \sin^2\left(\frac{\theta}{2}\right) + 0 = 1$$

Bloch sphere



(ϑ, φ)

$|0\rangle = (0, 0)$

$|1\rangle = (\pi, 0)$

$|+\rangle = (\frac{\pi}{2}, 0)$

$|-\rangle = (\frac{\pi}{2}, \pi)$

Ex 2.1. Hadamard

$$H|0\rangle = |+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$H\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$H|1\rangle = |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$H\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\Rightarrow \quad H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Ex 4.1 CNOT

$$CNOT\,(a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)$$
$$= a|00\rangle + b|01\rangle + c|11\rangle + d|10\rangle$$

$$CNOT\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} a \\ b \\ d \\ c \end{bmatrix}$$

$$\Rightarrow \quad CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$