Middle East Technical University Department of Computer Engineering

Discrete Computational Structures
Take Home Exam 1

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Question 1 (10 pts)

a) Prove that the compound proposition

$$\neg(p \land q) \leftrightarrow (\neg q \to p)$$

is logically equivalent to

$$(p \lor q) \land (\neg p \lor \neg q)$$

1) By the defination of double implication

$$(\neg(p \land q) \leftrightarrow (\neg q \to p) \equiv \neg(p \land q) \to ((\neg q \to p)) \land ((\neg q \to p) \to \neg(p \land q))$$

2) By the defination of implies x4

$$\equiv ((p \land q) \lor (q \lor p)) \land (\neg(q \lor p) \lor \neg(p \land q))$$

3) By using De Morgan's Law x2

$$\equiv ((p \land q) \lor (q \lor p)) \land ((\neg q \land \neg p) \lor (\neg p \lor \neg q))$$

4) Using Distribution Law x2

$$\equiv ((p \land q) \lor q) \lor (p \land q) \lor p)) \land ((\neg q \land \neg p) \lor \neg p) \lor ((\neg q \land \neg p) \lor \neg q))$$

5) Using Absorption Law x2

$$(q \lor p) \land (\neg p \lor \neg q)$$

Which is equivalent to

$$(p \lor q) \land (\neg p \lor \neg q)$$

-End of Proof-

Question 2 (30 pts)

Translate the following English sentences into compound predicate logic propositions using the predicates below.

I(x, y): x is an intern in faculty y.
E(x, y): x has employee id number y.
S(x, y): x is supervised by y.
A(x, y): x is admitted to job position y.
J(x, y): x is a job position in faculty y.

a. Two different interns in the same faculty cannot have the same employee id number.
b. There are some interns in all faculties who are supervised by no one but themselves.
c. At most two interns can be admitted to each job position in the medicine faculty.
a

a)

$\forall x, y, a, b, (x \neq z)[((I(x, b) \wedge I(y, b)) \rightarrow (\neg((E(x, a) \wedge E(z, a)))]$

b)

$\forall x, z \exists y, (y \neq z)[I(y, x) \rightarrow (\neg S(y, z) \wedge S(y, y))]$

c)

$\forall a, b, c, k \neg \exists x, y, z([I(x, a) \wedge (A(x, J(k, Medicine)))] \wedge [I(y, b) \wedge (A(x, J(k, Medicine)))] \wedge [I(z, c) \wedge (A(x, J(k, Medicine)))] \wedge x \neq y \wedge x \neq z \wedge y \neq z)$

Question 3a)

$$p \vee \neg q, p \vee r \vdash (r \to q) \to p$$

1. $p \vee \neg q$	premise
2. $p \vee r$	premise
3. $(r \to q)$	assumption
4. $\neg p$	assumption
5. p	assumption
6. $\perp$	4,5 $\neg e$
7. $\neg q$	6 $\perp e$
8. $\neg q$	assumption
9. $\neg q$	1,5-7,8 $\vee e$
10. p	assumption
11. $\perp$	4,10 $\neg e$
12. r	11 $\perp e$
13. r	assumption
14. r	2,10-12,13 $\vee e$
15. q	3,14 $\to e$
16. $\perp$	9,15 $\neg e$
17. $\neg\neg p$	4-16 $\neg i$
18. p	17 $\neg\neg e$
19. $(r \to q) \to p$	3-18 $\to i$

Question 3b)

$$\vdash ((q \to p) \to q) \to q$$

1.	$(q \to p) \to q$	assumption
2.	$\neg q$	assumption
3.	q	assumption
4.	$\neg p$	assumption
5.	q	3 Reiteration(copy)
6.	$\neg q$	2 Reiteration
7.	$\bot$	8 $\neg e$
8.	$\neg\neg p$	4-7 $\neg i$
9.	p	5,6 $\neg\neg e$
10.	$q \to p$	3-9 $\to i$
11.	$q \to p$	assumption
12.	$\neg q$	2 Reiteration
13.	$(q \to p) \to \neg q$	11-12 $\to i$
14.	$q \to p$	assumption
15.	q	1,14 $\to e$
16.	$\neg q$	13,14 $\to e$
17.	$\bot$	15,16 $\neg e$
18.	$\neg(q \to p)$	14-17 $\neg i$
19.	$\bot$	10,18 $\neg e$
20.	$\neg\neg q$	3-19 $\neg i$
21.	q	20 $\neg\neg e$
22.	$((q \to p) \to q) \to q$	1-21 $\to i$

Question 4a)

$$\neg\forall x(P(x) \to Q(x)) \vdash \exists x(P(x) \land \neg Q(x))$$

1. $\neg\forall x(P(x) \to Q(x))$	premise
2. $\neg\exists x(P(x) \land \neg Q(x))$	assumption
3. $freshname : a$	
4. $\neg(P(a) \to Q(a))$	assumption
5. $\neg(P(a) \land \neg Q(a))$	assumption
6. $P(a)$	assumption
7. $\neg Q(a)$	assumption
8. $P(a) \land \neg Q(a)$	6,7 $\land i$
9. $\exists x(P(x) \land \neg Q(x))$	8 $\exists i$
10. $\bot$	2,9 $\neg e$
11. $\neg\neg Q(a)$	7-10 $\neg i$
12. $Q(a)$	11 $\neg\neg e$
13. $P(a) \to Q(a)$	6-12 $\to i$
14. $\bot$	4,13 $\neg e$
15. $\neg\neg(P(a) \land \neg Q(a))$	5-14 $\neg i$
16. $P(a) \land \neg Q(a)$	15 $\neg\neg e$
17. $\exists x(P(x) \land \neg Q(x))$	16 $\exists i$
18. $\bot$	2,17 $\neg e$
19. $\neg\neg(P(a) \to Q(a))$	4-18 $\neg i$
20. $P(a) \to Q(a)$	19 $\neg\neg e$
21. $\forall x(P(x) \to Q(x))$	3-20 $\forall i$
22. $\bot$	1,21 $\neg e$
23. $\neg\neg\exists x(P(x) \land \neg Q(x))$	2-22 $\neg i$
24. $\exists x(P(x) \land \neg Q(x))$	23 $\neg\neg e$