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Q1

Let assume there is a positive integer between 0 and 1, and X is a set such that $X = \{n \in Z^{+} | 0 < n < 1\}$. Since we assume there is a positive integer between 0 and 1, X is a non-empty set. Therefore by Well Ordering Property, X has a smallest element.
Lets call this smallest element y. Therefore $0 < y < 1$. Multiply this equation with y.
And we get $0 < y^2 < y$. 2 is an element of positive integers, and so does y, according to our assumption. Therefore y^2 is a positive integer too. And it is seen by our equation that $y^2 < y$.
But y was the smallest element of the set X. We reach a contradiction.
Hence X is an empty set. There is no positive integer between 0 and 1. Obviously 0 is not a positive number, and any real number less than 0 is a negative real number.
Therefore any number less than 1 can not be a positive integer, and 1 is the smallest positive integer.

Q2

Q3

a. With 3 dots, we can construct 1 triangle.
With 6 dots, we can construct 6 triangle.(5 difference)
With 10 dots, we can construct 15 triangle.(9 difference)
With 15 dots, we can construct 28 triangle.(13 difference)

It is observed from the dots schema that in every new level, the difference from the previous one is, the difference of the previous one from the 2 previous one +4. Actually this is caused because in each new level, a new square is introduced different than the previous incrementations. 4 new triangles can be constructed from that square, therefore difference is increases by 4 in every level.

With 21 dots, we can construct 45 triangle.(17 difference)
With 28 dots, we can construct 66 triangle.(21 difference)
With 36 dots, we can construct 91 triangle.(25 difference)

The given grid has 36 dots, and 91 triangles can be constructed.

b. For function to be onto from a set with 6 elements to a set with 4 elements, every element in the range must be filled with at least 1 element from the domain. There are 2 case to fulfill this requirement.

First option

3 elements in the domain goes to same element in the range, and other 3 element in the domain goes to unique leftover element in the range.
$\binom{4}{1}$ For selecting the element in the range which is filled by 3 element from domain.
$\binom{6}{3}$ For selecting 3 element from the domain which is goes to same element in the range.
3! For the leftover 3 elements in the domain to goes unique 3 leftover element in the range.
$\binom{4}{1} * \binom{6}{3} * 3! = 480$ cases for the first option.

Second option

3 elements in the domain goes to same element in the range, and other 3 element in the domain goes to unique leftover element in the range.
$\binom{4}{2}$ For selecting two elements in the range which will be filled by 2 element from domain.
$\binom{6}{2}$ For selecting 2 element from the domain which is goes to same element in the range.
$\binom{4}{2}$ For selecting 2 element from the domain (Since now left 4 element in the domain) which is goes to same element in the range.
2! For the leftover 2 elements in the domain to goes unique 2 leftover element in the range.
$\binom{4}{2} * \binom{6}{2} * 42 * 2! = 1080$ cases for the second option.

Since these 2 options are disjoint events ad covers all the posibilities from a set with 6 elements to a set with 4 elements to be a onto function, We just add them, Hence There are $480 + 1080 = 1560$ onto functions from a set with 6 elements to a set with 4 elements.

Q4

a. Let me present number of ternary strings of length n that contain two consecutive symbols that are the same as a(n).
There are 2 cases to inspect, ternary strings start with two consequtive same symbols, and those are not.

First Option

There are 3 sub-options.
Our string can start with 2 consequtive 0's, 1's and 2's. For each of them, any string of length n-2 added to them are ternary strings start with two consequtive same symbols.
Therefore $3 * 3^{n-2} = 3^{n-1}$ different cases for the first option.

Second Option

Again, there are 3 suboptions.

A ternary string can start with 0, and followed by a string of length n-1 with two consecutive same symbols.
There are a(n-1) such that string. But 1/3 of them starts with 0, and if it starts with zero, our overall string would start with 00, but we already covered that case.
Therefore 2/3 *a(n-1) different cases for strings starting with 0, but does not start with 2 consecutive symbols.

A ternary string can start with 1, and followed by a string of length n-1 with two consecutive same symbols.
There are a(n-1) such that string. But 1/3 of them starts with 1, and if it starts with 1, our overall string would start with 11, but we already covered that case.
Therefore 2/3 *a(n-1) different cases for strings starting with 1, but does not start with 2 consecutive symbols.

A ternary string can start with 2, and followed by a string of length n-1 with two consecutive same symbols.
There are a(n-1) such that string. But 1/3 of them starts with 2, and if it starts with zero, our overall string would start with 22, but we already covered that case.
Therefore 2/3 *a(n-1) different cases for strings starting with 2, but does not start with 2 consecutive symbols.

Hence 2/3 *3 a(n-1) = 2a(n-1) different cases for this option. This 2 options are disjoint events and cover all the cases for ternary strings of length n which has 2 consecutive symbols.
Sum up together, 2a(n-1)+3^{n-1} is our recurrence relation

b. a(1) = 0. Since we can not have a string has 2 consecutive symbols with length 1.
This is the initial condition.

c. $a_n = a_n^h + a_n^p$
We must solve both homogeneous and particular equations.

For the homogeneous solution: a(n) = 2*a(n-1)
The characteristic equation is : $x^2 - 2x = 0$, and $x_1 = 0, x_2 = 0$
Hence a_n^h is $A \cdot 2^n$ where A is constant.

For the non-homogeneous solution, since particular part 3^{n-1} is in the y^n form where y is a constant, our particular solution will be in the form $B \cdot 3^n$.
We substitute $B \cdot 3^n$ in the recurrence relation formula,
$B \cdot 3^n = 2B \cdot 3^{n-1} + 3^{n-1}$, and divide both sides by 3^{n-1}:
$3B = 2B + 1$, Hence B is equal to 1.
Therefore $a_n^p = 3^n$

$a_n = A \cdot 2^n + 3^n$, Since $a_n = a_n^h + a_n^p$
We know from the initial condition that a(1)=0. Substitite the formula, and we get
0 = 2A+3
Therefore A = -3/2

$a_n = -3 \cdot 2^{n-1} + 3^n$, $n \geq 1$