CS M51A Homework 2

Thursday, October 8, 2020 7:42 PM

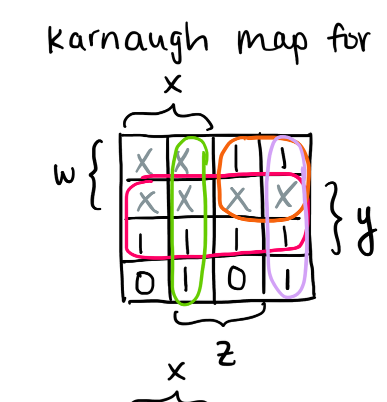
I completed this assignment entirely on my own except for discussion with Janice Tsai

1.

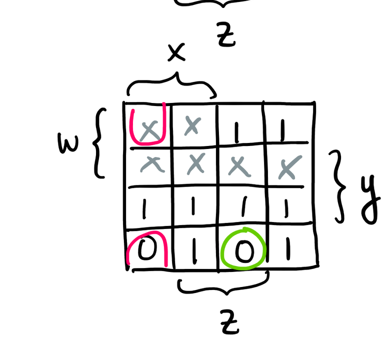
w	x	y	z	a	b	c	d	e	f	g
0	0	0	0	1	1	1	0	1	1	1
0	0	0	1	0	0	1	0	0	1	0
0	0	1	0	1	0	1	1	1	0	1
0	0	1	1	1	0	1	1	0	1	1
0	1	0	0	0	1	1	1	0	1	0
0	1	0	1	1	1	0	1	0	1	1
0	1	1	0	1	1	0	1	1	1	1
0	1	1	1	1	0	1	0	0	1	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	0

$\underbrace{w\ x\ y\ z}_{\text{BCD digit}}$

Karnaugh map for 'a':

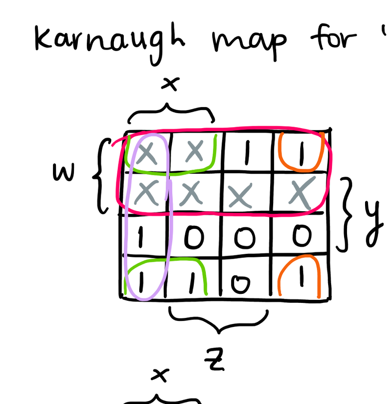


sum of products: $y + w \cdot x' + x \cdot z + x' \cdot z'$

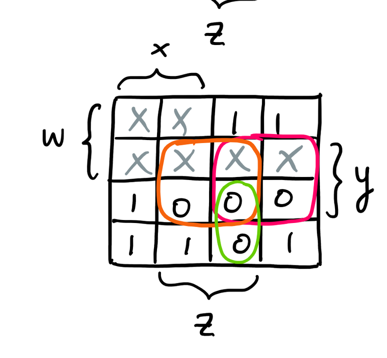


product of sums: $(x' + y + z) \cdot (w + x + y + z')$

Karnaugh map for 'b':

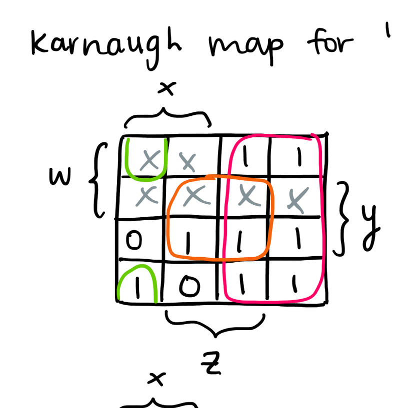


sum of products: $w + x' \cdot y' \cdot z' + x \cdot y' + x \cdot z'$

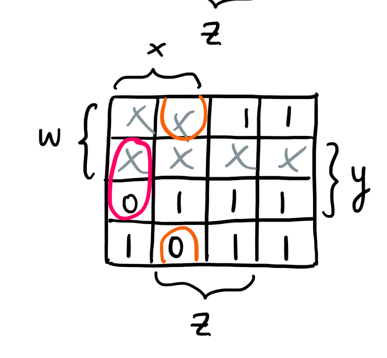


product of sums: $(x + y') \cdot (y' + z') \cdot (w + x + z')$

Karnaugh map for 'c':

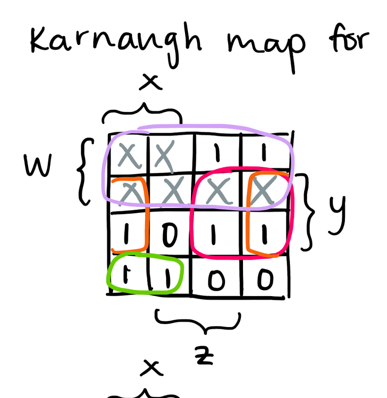


sum of products: $x' + y \cdot z + x \cdot y' \cdot z'$

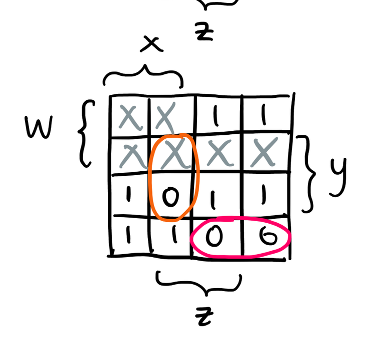


product of sums: $(x' + y' + z) \cdot (x' + y + z')$

Karnaugh map for 'd':

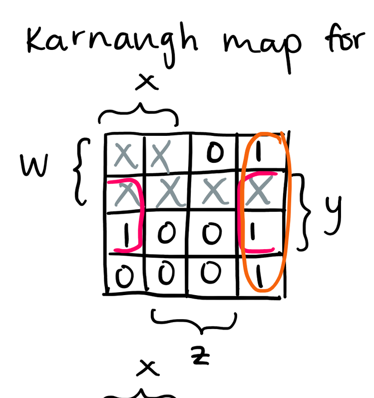


sum of products: $x' \cdot y + y \cdot z' + w' \cdot x \cdot y' + w$

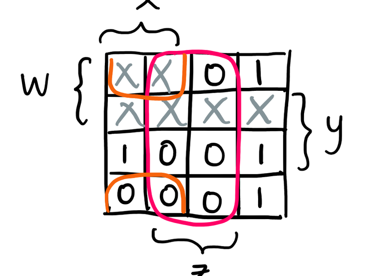


product of sums: $(w + x + y) \cdot (x' + y' + z')$

Karnaugh map for 'e':

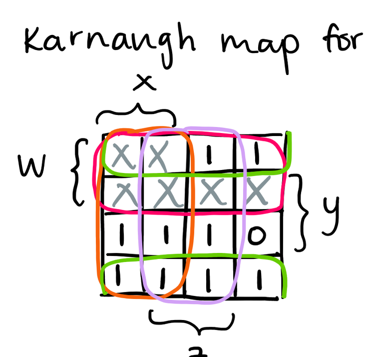


sum of products: $y \cdot z' + x' \cdot z'$

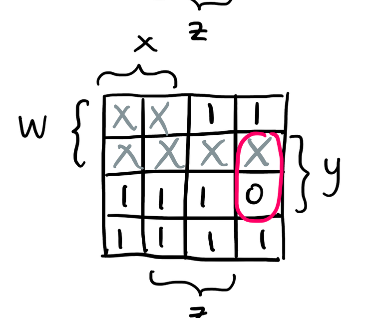


product of sums: $z' \cdot (x' + y)$

Karnaugh map for 'f':

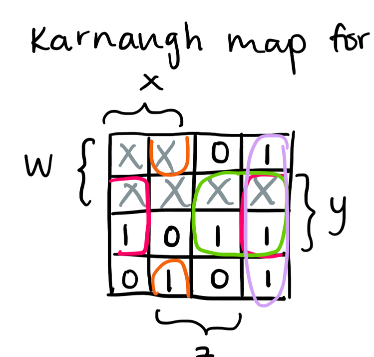


sum of products: $w + x + y' + z$

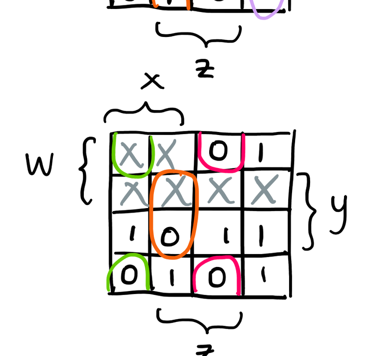


product of sums: $x + y' + z$

Karnaugh map for 'g':



sum of products: $y \cdot z' + x \cdot y' \cdot z + x' \cdot y + x' \cdot z'$



product of sums: $(x + y + z') \cdot (x' + y' + z') \cdot (x' + y + z)$

2. Already know : NAND, NOR

my intuition:
- can't be solely based on x, or solely based on y
- can't be all true, or all false

		F	AND		x		y		OR	NOR				NOT		NAND	T
x	y	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}	f_{16}
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

in discussion we talked about AND and OR needing NOT as they are monotonously increasing, so it can't be AND or NOT
In same way, NOT is monotonously decreasing so it needs AND or OR.

Possibly universal: $f_3, f_5, f_7, f_{10}, f_{11}, f_{12}, f_{14}$

★ f_3: $x \cdot y'$

→ NOT: $x' = 1\ f_3\ x$
$= 1 \cdot x'$
$= x'$

→ OR: x OR y
$(1\ f_3\ x)\ f_3\ (1\ f_3\ y)$
$((1 \cdot x') \cdot (1 \cdot y'))'$

x	y	x OR y	$(1 f_3 y)\ f_3\ (y f_3 x)$
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	1

→ AND: $x \cdot y = ((x \cdot y)')'$ De Morgan's Law
$= (x' + y')'$
$= (\underbrace{(1\ f_3\ x)}_{x}\ \text{OR}\ \underbrace{(1\ f_3\ y)}_{y})'$ substitute NOT, OR
$= ((1\ f_3 (1 f_3 x))\ f_3\ (1\ f_3 (1 f_3 y)))'$
$= 1\ f_3\ (((1 f_3 (1 f_3 x))\ f_3\ (1\ f_3 (1 f_3 y))))$

∴ $\{f_3\}$ is universal

★ Because of duality, let's check f_{14}: $x' + y$

→ NOT: $x' = x\ f_{14}\ 0$
$= x' + 0$
$= x'$

→ OR: $x + y = (x\ f_{14}\ 0)' + (y\ f_{14}\ 0)'$
$(x\ f_{14}\ 0)\ f_{14}\ ((y\ f_{14}\ 0)' + 0)$
$(x\ f_{14}\ 0)\ f_{14}\ ((y\ f_{14}\ 0)\ f_{14}\ 0)$

x	y	OR
0	0	0 + 0 = 0
0	1	1 + 0 = 1
1	0	0 + 1 = 1
1	1	1 + 1 = 1

→ AND: $x \cdot y = ((x \cdot y)')'$
$= (x' + y')'$ De Morgan's Law
$= (\underbrace{(x\ f_{14}\ 0)}_{x} + \underbrace{(y\ f_{14}\ 0)}_{y})'$ substitute NOT, OR
$= (((x\ f_{14}\ 0)\ f_{14}\ 0)\ f_{14}\ (((y\ f_{14}\ 0)\ f_{14}\ 0)\ f_{14}\ 0))'$
$= (((x\ f_{14}\ 0)\ f_{14}\ 0)\ f_{14}\ (((y\ f_{14}\ 0)\ f_{14}\ 0)\ f_{14}\ 0))\ f_{14}\ 0$

∴ $\{f_{14}\}$ is universal

★ f_5: $x' \cdot y$

→ NOT: $x' = x\ f_5\ 1$
$= x' \cdot 1$
$= x'$

→ AND: $x \cdot y = (x\ f_5\ 1)\ f_5\ ((y\ f_5\ 1)\ f_5\ 1)$
$= (x' \cdot 1)' \cdot ((y' \cdot 1)' \cdot 1)$

x	y	
0	0	$(1 \cdot 1)' \cdot ((1 \cdot 1)' \cdot 1) = 0 \cdot 0 = 0$ ✓
0	1	$(1 \cdot 1)' \cdot ((0 \cdot 1)' \cdot 1) = 0 \cdot 1 = 0$ ✓
1	0	$(0 \cdot 1)' \cdot ((1 \cdot 1)' \cdot 1) = 1 \cdot 0 = 0$ ✓
1	1	$(0 \cdot 1)' \cdot ((0 \cdot 1)' \cdot 1) = 1 \cdot 1 = 1$ ✓

→ OR: $x + y = ((x + y)')'$ De Morgan's Law
$= (x' \cdot y')'$ substitute NOT, OR
$= (\underbrace{(x\ f_5\ 1)}_{x} \cdot \underbrace{(y\ f_5\ 1)}_{y})'$
$= (((x\ f_5\ 1)\ f_5\ 1)\ f_5\ ((y\ f_5\ 1)\ f_5\ 1)\ f_5\ 1)'$
$= (((x\ f_5\ 1)\ f_5\ 1)\ f_5\ ((y\ f_5\ 1)\ f_5\ 1)\ f_5\ 1)\ f_5\ 1$

∴ $\{f_5\}$ is universal.

★ Because of duality, let's check f_{12}: $x + y'$

→ NOT: $x' = 0\ f_{12}\ x$
$= 0 + x'$
$= x'$

→ OR: $x + y = (0\ f_{12}\ (0\ f_{12}\ x))\ f_{12}\ (0\ f_{12}\ y)$

x	y	OR
0	0	0
0	1	1
1	0	1
1	1	1

$(0 + (0 + x')') + (0 + y')'$
$0 + 0 = 0$
$0 + 1 = 1$
$1 + 0 = 1$
$1 + 1 = 1$

→ AND: $x \cdot y = ((x \cdot y)')'$
$= (x' + y')'$ De Morgans Law
$= (\underbrace{(0\ f_{12}\ x)}_{x} + \underbrace{(0\ f_{12}\ y)}_{y})'$ substitute OR, NOT
$= ((0\ f_{12}\ (0\ f_{12}\ (0\ f_{12}\ x)))\ f_{12}\ (0\ f_{12}\ (0\ f_{12}\ y)))'$
$= 0\ f_{12}\ ((0\ f_{12}\ (0\ f_{12}\ (0\ f_{12}\ x)))\ f_{12}\ (0\ f_{12}\ (0\ f_{12}\ y)))$

∴ $\{f_{12}\}$ is universal

Individually universal boolean functions:

$\{f_3\}$, $f_3 = x \cdot y'$
$\{f_{14}\}$, $f_{14} = x' + y$
$\{f_5\}$, $f_5 = x' \cdot y$
$\{f_{12}\}$, $f_{12} = x + y'$