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CS 161 HW 5

1. $P \Rightarrow \neg Q$, $Q \Rightarrow \neg P$
   $\neg P \vee \neg Q$ $\neg Q \vee \neg P$

P	Q	¬P	¬Q	P ⇒ ¬Q	Q ⇒ ¬P
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

$P \Rightarrow \neg Q$ and $Q \Rightarrow \neg P$ are equivalent

$P \Leftrightarrow \neg Q$: $P \Rightarrow \neg Q, \neg Q \Rightarrow P, ((P \wedge \neg Q) \vee (\neg P \wedge Q))$
$(\neg P \vee \neg Q) \wedge (Q \vee P)$

P	Q	¬P	¬Q	P ⇔ ¬Q	((P ∧ ¬Q) ∨ (¬P ∧ Q))
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	F

$P \Leftrightarrow \neg Q$ and $((P \wedge \neg Q) \vee (\neg P \wedge Q))$ are equivalent

2. $(smoke \Rightarrow fire) \Rightarrow (\neg smoke \Rightarrow \neg fire)$
   $(\neg smoke \vee fire) \Rightarrow (smoke \vee \neg fire)$
   $\neg(\neg smoke \vee fire) \vee (smoke \vee \neg fire)$
   $(smoke \wedge \neg fire) \vee (smoke \vee \neg fire)$

smoke	fire	smoke ∧ ¬fire	smoke ∨ ¬fire	(smoke ∧ ¬fire) ∨ (smoke ∨ ¬fire)
T	T	F	T	T
T	F	T	T	T
F	T	F	F	F
F	F	F	T	T

since $(smoke \Rightarrow fire) \Rightarrow (\neg smoke \Rightarrow \neg fire)$ is satisfied in 3 of the 4 words, it is neither valid nor unsatisfied.

$(smoke \Rightarrow fire) \Rightarrow ((smoke \vee heat) \Rightarrow fire)$
$(\neg smoke \vee fire) \Rightarrow (\neg(smoke \vee heat) \vee fire)$
$\neg(\neg smoke \vee fire) \vee ((\neg smoke \wedge \neg heat) \vee fire)$
$(smoke \wedge \neg fire) \vee ((\neg smoke \wedge \neg heat) \vee fire)$

smoke	fire	heat	smoke ∧ ¬fire	(¬smoke ∧ ¬heat) ∨ fire	(smoke ∧ ¬fire) ∨ ((¬smoke ∧ ¬heat) ∨ fire)
T	T	T	F	T	T
T	T	F	F	T	T

T	F	T	T	F	T
T	F	F	T	F	T
F	T	T	F	T	T
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

since (smoke ⇒ fire) ⇒ ((smoke ∨ heat) ⇒ fire) is satisfied in 7 of the 8 worlds, it is neither valid nor unsatisfied.

((smoke ∧ heat) ⇒ fire) ⟺ ((smoke ⇒ fire) ∨ (heat ⇒ fire))
(¬(smoke ∧ heat) ∨ fire) ⟺ ((¬smoke ∨ fire) ∨ (¬heat ∨ fire))
((¬smoke ∨ ¬heat) ∨ fire) ⟺ ((¬smoke ∨ fire) ∨ (¬heat ∨ fire))

{ ((¬smoke ∨ ¬heat) ∨ fire) ⟹ ((¬smoke ∨ fire) ∨ (¬heat ∨ fire))
{ ((¬smoke ∨ fire) ∨ (¬heat ∨ fire)) ⟹ ((¬smoke ∨ ¬heat) ∨ fire)

{ ¬((¬smoke ∨ ¬heat) ∨ fire) ∨ ((¬smoke ∨ fire) ∨ (¬heat ∨ fire))
{ ¬((¬smoke ∨ fire) ∨ (¬heat ∨ fire)) ∨ ((¬smoke ∨ ¬heat) ∨ fire))

{ (¬(¬smoke ∨ ¬heat) ∧ ¬fire) ∨ (¬smoke ∨ fire ∨ ¬heat)
{ (¬(¬smoke ∨ fire) ∧ ¬(¬heat ∨ fire)) ∨ (¬smoke ∨ ¬heat ∨ fire)

{ (smoke ∧ heat ∧ ¬fire) ∨ (¬smoke ∨ fire ∨ ¬heat)
{ ((smoke ∧ ¬fire) ∧ (heat ∧ ¬fire)) ∨ (¬smoke ∨ ¬heat ∨ fire)

smoke	heat	fire	(smoke ∧ heat ∧ ¬fire) ∨ (¬smoke ∨ fire ∨ ¬heat)	((smoke ∧ ¬fire) ∧ (heat ∧ ¬fire)) ∨ (¬smoke ∨ ¬heat ∨ fire)
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

((smoke ∧ heat) ⇒ fire) ⟺ ((smoke ⇒ fire) ∨ (heat ⇒ fire))
is T in all worlds so it is valid.

3. a) mythical ⇒ immortal
¬mythical ⇒ ¬immortal ∧ mammal
immortal ∨ mammal ⇒ horned
horned ⇒ magical

M ⇒ I	abbreviated
¬M ⇒ ¬I ∧ A	M = mythical
I ∨ A ⇒ H	I = immortal
H ⇒ G	A = mammal
	H = horned
	G = magical

b) ¬M ∨ I
M ∨ (¬I ∧ A) distribute to (M ∨ ¬I) ∧ (M ∨ A)
¬(I ∨ A) ∨ H = (¬I ∧ ¬A) ∨ H distribute to (¬I ∨ ¬A) ∧ (¬A ∨ H)
¬H ∨ G
Δ: {¬M ∨ I, (M ∨ ¬I) ∧ (M ∨ A), (¬I ∨ ¬A) ∧ (¬A ∨ H), ¬H ∨ G}
CNF: (¬M ∨ I) ∧ (M ∨ ¬I) ∧ (M ∨ A) ∧ (¬I ∨ ¬A) ∧ (¬A ∨ H) ∧ (¬H ∨ G)

c) M = mythical
I = immortal
A = mammal
H = horned
G = magical
$\Delta : \{\neg M \vee I, (M \vee \neg I) \wedge (M \vee A), (\neg I \vee \neg A) \wedge (\neg A \vee H), \neg H \vee G\}$

* Is unicorn mythical?
$\alpha = M$
KB: $\Delta \wedge \neg\alpha$

0.	$\neg M \vee I$	Δ
1.	$M \vee \neg I$	Δ
2.	$M \vee A$	Δ
3.	$\neg I \vee \neg A$	Δ
4.	$\neg A \vee H$	Δ
5.	$\neg H \vee G$	Δ
6.	$\neg M \vee A$	Δ
7.	$\neg M$	$\neg\alpha$
8.	A	2, 7
9.	$\neg I$	2, 8
10.	H	4, 8
11.	G	5, 10

no contradiction is found, so $\Delta \nvdash \alpha$ and unicorn cannot be proven mythical.

* Is unicorn magical?
$\alpha = G$
KB: $\Delta \wedge \neg\alpha$

0.	$\neg M \vee I$	Δ
1.	$M \vee \neg I$	Δ
2.	$M \vee A$	Δ
3.	$\neg I \vee \neg A$	Δ
4.	$\neg A \vee H$	Δ
5.	$\neg H \vee G$	Δ
6.	$\neg M \vee A$	Δ
7.	$\neg G$	$\neg\alpha$
8.	$\neg H$	5, 7
9.	$\neg A$	4, 8
10.	(M)	2, 9
11.	I	0, 10
12.	($\neg$M)	6, 9

contradiction, so $\Delta \vdash \alpha$
unicorn is proven magical

* Is unicorn horned?
$\alpha = H$
KB: $\Delta \wedge \neg\alpha$

0.	$\neg M \vee I$	Δ
1.	$M \vee \neg I$	Δ
2.	$M \vee A$	Δ
3.	$\neg I \vee \neg A$	Δ

4. ¬A ∨ H
5. ¬H ∨ G
6. ¬M ∨ A
7. ¬H } α
8. ¬A 4, 7
9. M 2, 8
10. ⊥ 0, 9
11. ¬M 6, 8

contradiction, so $\Delta \models \alpha$
unicorn is proven horned

4.

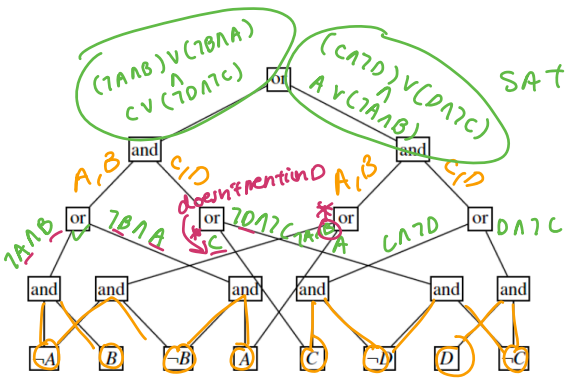

Figure 1

- decomposable ✓
- deterministic X
- not smooth *

Figure 1 is decomposable, but not deterministic nor smooth because the top OR gate has children that are SAT and some of the OR gates have children w/ a different set of variables.

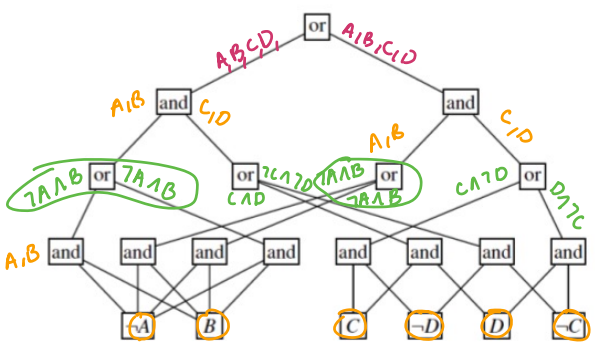

Figure 2

- decomposable ✓
- deterministic X
- smooth ✓

Figure 2 is decomposable and smooth because all the AND gates do not share variables and all OR gates have children that mention the same variables. It is NOT deterministic because there are OR gates w/ children that are SAT.

5. $w(A, \neg B, C) = w(A)\, w(\neg B)\, w(C)$

wmc: added weight of its satisfying assignments

$w(A) = 0.1$ $w(B) = 0.3$ $w(C) = 0.5$ $w(D) = 0.7$

$w(\neg A) = 0.9$ $w(\neg B) = 0.7$ $w(\neg C) = 0.5$ $w(\neg D) = 0.3$

a) $(\neg A \wedge B) \vee (\neg B \wedge A)$

A	B	$\neg A \wedge B$	$\neg B \wedge A$	$(\neg A \wedge B) \vee (\neg B \wedge A)$	weight
T	T	F	F	F	X
T	F	F	T	T	$(0.1)(0.7) = 0.07$
F	T	T	F	T	$(0.9)(0.3) = 0.27$
F	F	F	F	F	† X

wmc: 0.34

b)

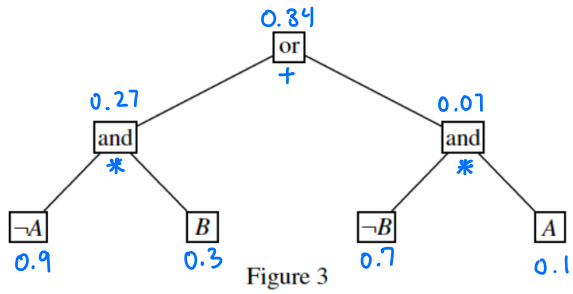

Figure 3

The count on the root is equal to the wmc.

c)

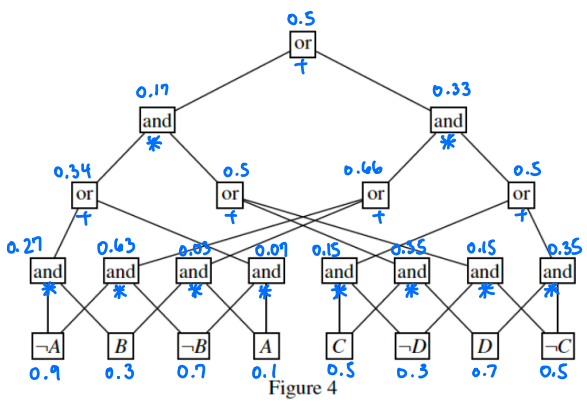

Figure 4

wmc = 0.5