Q-LEARNING (AGAIN!)

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Metrowest Developers Machine Learning Group
11/28/2018

REFERENCES

The material for this talk is primarily drawn from the slides, notes and lectures of these courses:

University of California, Berkeley CS188:

- *CS188 – Introduction to Artificial Intelligence*, Profs. Dan Klein, Pieter Abbeel, et al. http://ai.berkeley.edu/home.html

David Silver, DeepMind:

- Introduction to Reinforcement Learning http://www0.cs.ucl.ac.uk/staff/d.silver/web/Teaching.html

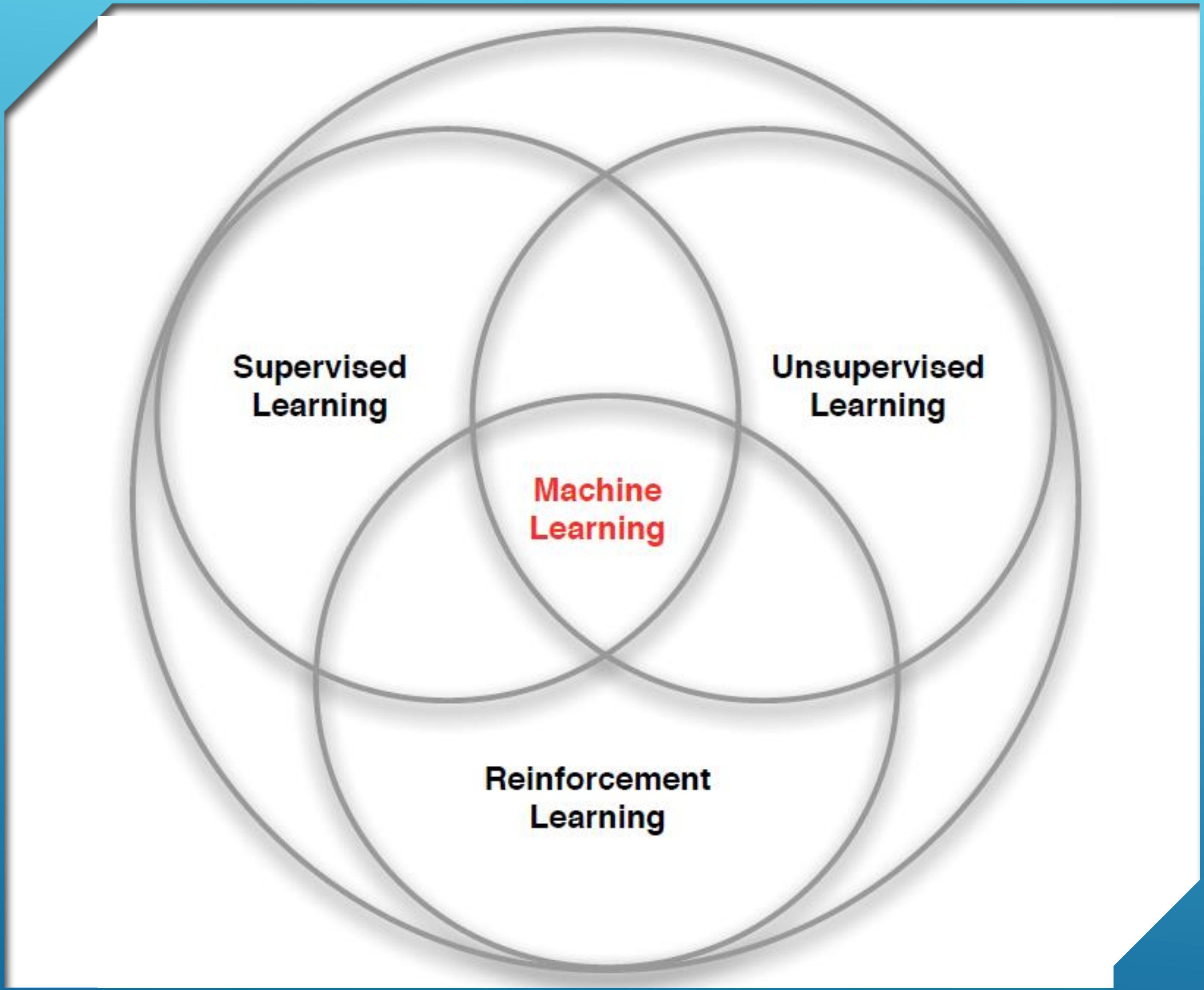
CS181 course at Harvard University:

- *CS181 Intelligent Machines: Perception, Learning and Uncertainty*, Sarah Finney, Spring 2009
- *CS181 Intelligent Machines: Perception, Learning and Uncertainty*, Prof. David C Brooks, Spring 2011
- *CS181 – Machine Learning*, Prof. Ryan P. Adams, Spring 2014. https://github.com/wihl/cs181-spring2014
- *CS181 – Machine Learning*, Prof. David Parkes, Spring 2017. https://harvard-ml-courses.github.io/cs181-web-2017/

Q-LEARNING: TALK OUTLINE

- Reinforcement Learning
- Model-based vs. Model-free RL
  - Model-based RL
  - Model-free RL: Q-Learning
- Q-Learning Demos
- Approximate Q-Learning

REINFORCEMENT LEARNING



BRANCHES OF MACHINE LEARNING

Credit: adapted from lecture slides David Silver, DeepMind, "Introduction to Reinforcement Learning

CHARACTERISTICS OF REINFORCEMENT LEARNING

- There is no supervisor, only a *reward* signal
- Feedback is delayed, not instantaneous
- Time really matters (sequential, data is not i.i.d. – independent and identically distributed.)
- Agent's actions affect the subsequent data it receives.

Credit: adapted from lecture slides David Silver, DeepMind, "Introduction to Reinforcement Learning

EXAMPLES OF REINFORCEMENT LEARNING

- Fly stunt maneuvers in a helicopter
- Defeat the world champion at Backgammon
- Manage an investment portfolio
- Control a power station
- Make a humanoid robot walk
- Play Atari games better than humans

Credit: adapted from lecture slides David Silver, DeepMind, "Introduction to Reinforcement Learning

EXAMPLES OF REINFORCEMENT LEARNING

RL Course by David Silver – Lecture1: Introduction to Reinforcement Learning

https://www.youtube.com/watch?v=2pWv7GOvuf0

12:25 – 22.00

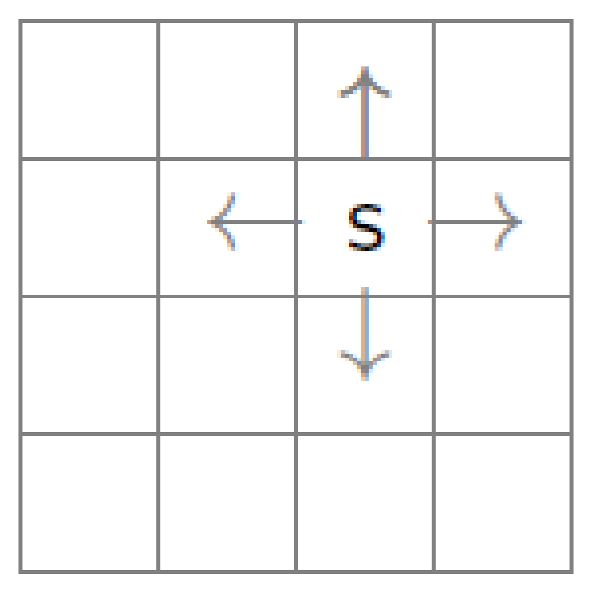
Credit: adapted from lecture slides David Silver, DeepMind, "Introduction to Reinforcement Learning

REINFORCEMENT LEARNING: THE BASIC IDEA

- Select an action.
- If action leads to reward, reinforce that action.
- If action leads to punishment, avoid that action.
- Basically, a computational form of Behaviorism (Pavlov, B. F. Skinner).

THE LEARNING FRAMEWORK

- Learning is performed **online**, learn as we interact with the world



- In contrast with supervised learning, there are no training or test sets. The reward is accumulated over interactions with the environment.
- Data is not fixed, more information is acquired as you go.
- The training distribution can be influenced by action decisions.

MODEL-BASED VS. ***MODEL-FREE*** REINFORCEMENT LEARNING

RL MODEL ≅ (PO)MDP

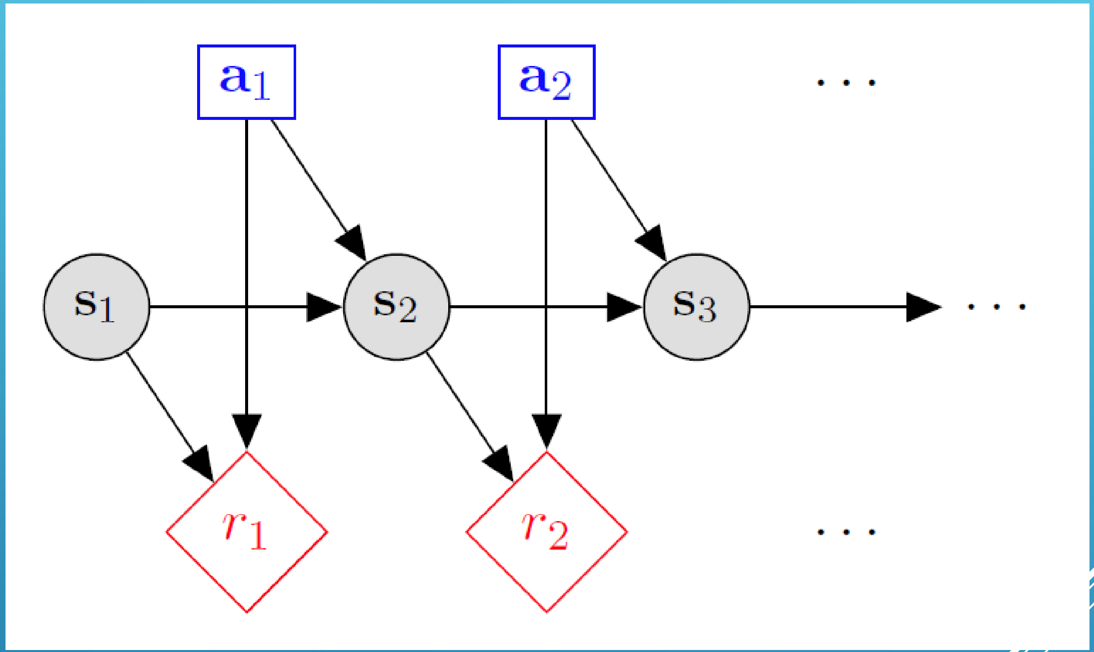
- "Reinforcement learning model" usually implies a ***Markov Decision Process*** (**MDP**) or a ***Partially-Observable MDP*** (**POMDP**).
- The Markov Decision Process provides a mathematical framework for reinforcement learning.
- MDP is used to model decision making in situations where outcomes are partly random and partly under the control of a decision maker.

MARKOV DECISION PROCESSES

- **States:** $s_1, \ldots, s_n$
- **Actions:** $a_1, \ldots, a_m$
- **Reward Function:**

  $r(s, a, s') \in R$
- **Transition model:**

  $T(s,a,s') = P(s'|s, a)$
- **Discount factor:** $\gamma \in [0, 1]$



<https://en.wikipedia.org/w/index.php?title=Markov_decision_process&oldid=855934986> [accessed 11 September 2018]

MARKOV DECISION PROCESSES

- The initial analysis of MDPs assumes **complete knowledge** of states, actions, rewards, transitions, and discounts.
- When MDPs are applied to reinforcement learning, it is assumed that the ***transition model*** and the ***rewards model*** are <u>unknown</u>.

<https://en.wikipedia.org/w/index.php?title=Markov_decision_process&oldid=855934986> [accessed 11 September 2018]

MODEL-BASED VS. MODEL-FREE

- Model-Based RL
  - Agent learns the transition and reward models for the environment a'la MDP.
  - Agent can predict:
    * how likely it is that it will transition from one state to another given the action it takes.
    * the likely reward it will get for taking an action in a given state.

- Model-Free RL
  - Agent directly learns a policy that optimizes its reward.
  - Agent CANNOT make predictions about transitions.

MODEL-BASED RL PROS AND CONS

- **Pros:**
  - Makes maximal use of experience.
  - Solves model optimally, given enough experience.
- **Cons:**
  - Requires computationally expensive solution procedure.
  - Requires the model to be small enough to solve

MODEL-FREE RL PROS AND CONS

- **Pros:**
  - Solution procedure is relatively more efficient.
  - Can handle much larger models.

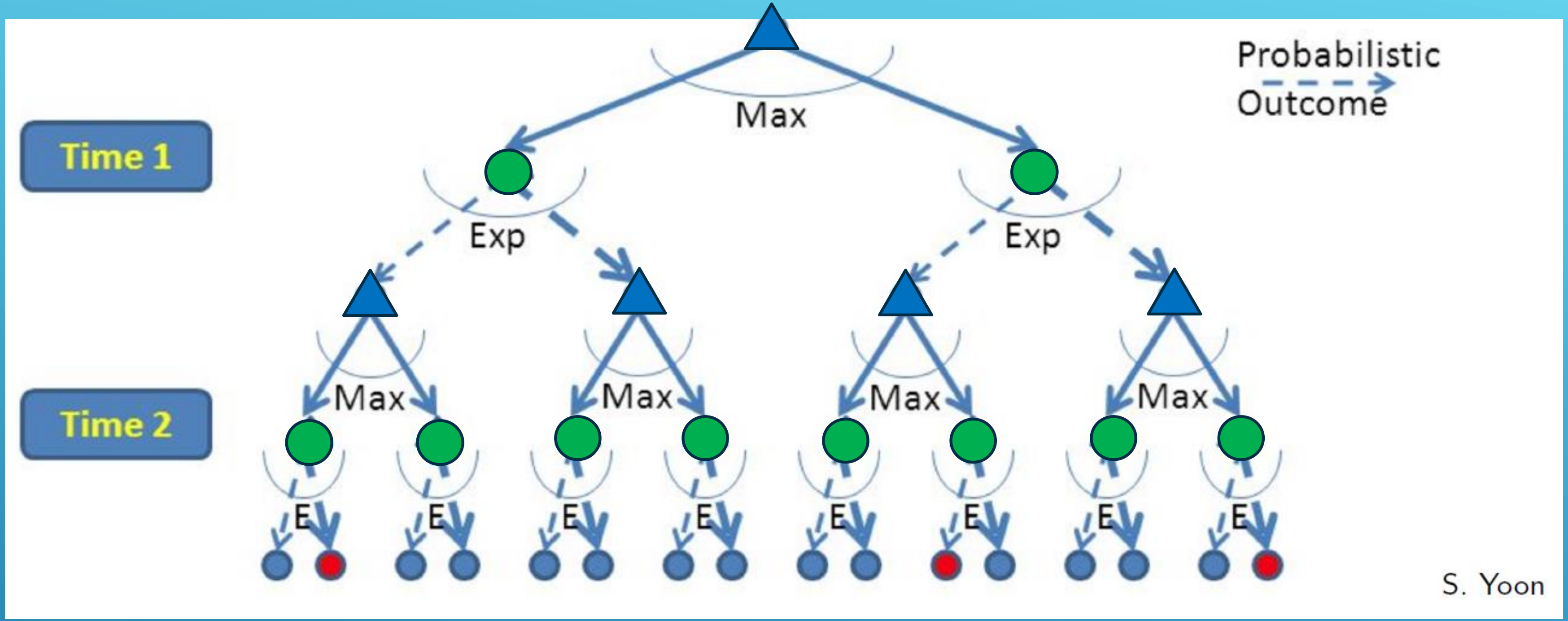
- **Cons:**
  - Learns more slowly. Does not learn as much model-based RL in a single training episode. ("Leaves information on the table").
  - Unable to make predictions about transitions in the environment.

MODEL-FREE REINFORCEMENT LEARNING: Q-LEARNING

Q-LEARNING

- Don’t learn a model, learn the Q function directly.
  - (But actually, this is a just a different kind of model.)
- Appropriate when model is too large to store, solve or learn
  - Model-based/Value Iteration transition cost: $O(|S^2|)$
  - Model-based/Value Iteration cost: $O(|A||S^2|)$
  - Model-free / Q-Learning: size of Q function $O(|A||S|)$

RL IS LIKE A GAME AGAINST NATURE



- Reinforcement learning is like a game-playing algorithm.
- Nodes where you move are called **states**: S (▲)
- Nodes where nature moves are called **Q-states**: <S,A> (●)

OPTIMAL QUANTITIES

- The value (utility) of a **state s**:

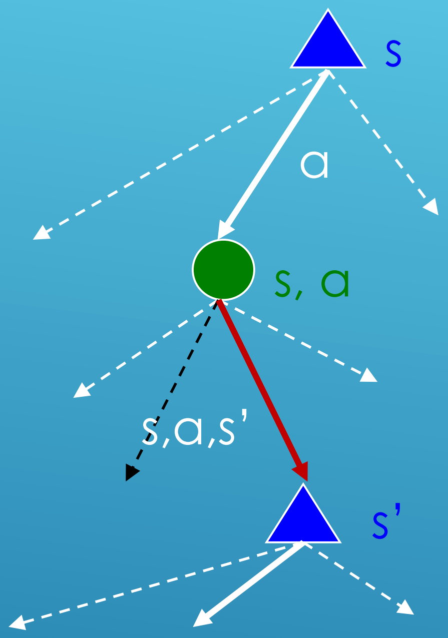
  $V^*(s)$ = expected utility starting in s and acting optimally

- The value (utility) of a **q-state (s,a)**:

  $Q^*(s,a)$ = expected utility starting out having taken action a from state s and (thereafter) acting optimally

- The optimal policy:

  $\pi^*(s)$ = optimal action from state s



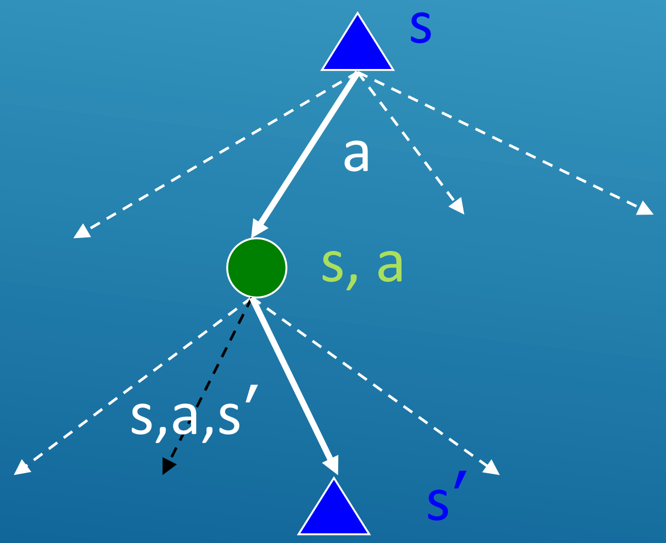
s is a *state*

(s, a) is a *q-state*

(s,a,s') is a *transition*

RECALL THE BELLMAN EQUATIONS

- There is one equation $V^*(s)$ for each state *s*.
- There is one equation $Q^*(s,a)$ for each state *s* and action *a*.
- These are equations, not assignments. They define a relationship, which when satisfied guarantees that $V^*(s)$ and $Q^*(s,a)$ are optimal for each state and action.
- This in turn guarantees that the policy π^* is optimal.



$$V^*(s) = \max_a Q^*(s,a)$$

$$Q^*(s,a) = \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma V^*(s')\right]$$

$$V^*(s) = \max_a \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma V^*(s')\right]$$

Credit: CS188 – Intro to AI, UC Berkeley, ai.berkely.edu
Credit: CS181 – Machine Learning, D. Parkes, Harvard Univ, Spring 2017

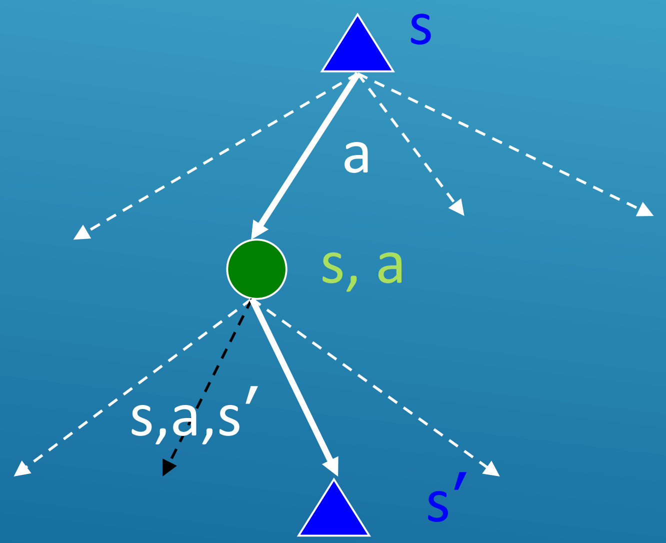
THE OPTIMAL VALUE UTILITY EQUATION V*

V* is rewritten as a recurrence relationship by substituting equation [2] into equation [1]

$$V^*(s) = \max_a Q^*(s,a) \quad \textbf{[1]}$$

$$Q^*(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s') \right] \quad \textbf{[2]}$$

$$V^*(s) = \max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')]$$



Credit: CS188 – Intro to AI, UC Berkeley, ai.berkely.edu
Credit: CS181 – Machine Learning, D. Parkes, Harvard Univ, Spring 2017

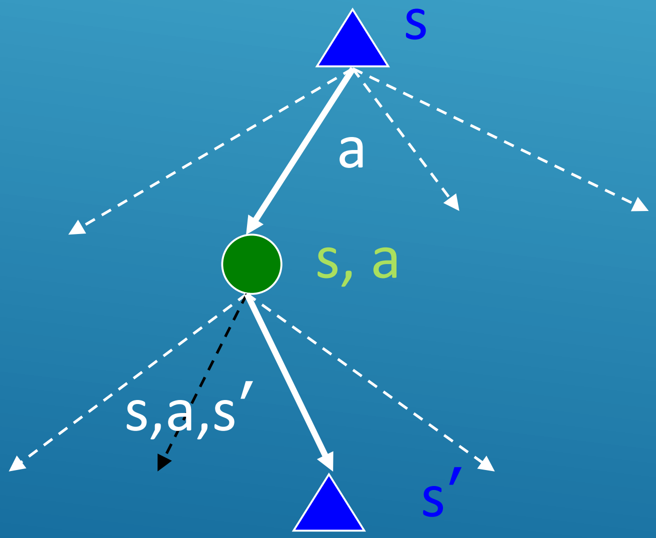
THE OPTIMAL VALUE UTILITY EQUATION Q*

Q* is rewritten as a recurrence relationship by substituting equation [1] into equation [2]

$$V^*(s) = \max_a Q^*(s,a) \qquad [1]$$

$$Q^*(s,a) = \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma V^*(s')\right] \qquad [2]$$

$$Q^*(s,a) = \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma \max_{a'} Q^*(s',a')\right]$$



Credit: CS188 – Intro to AI, UC Berkeley, ai.berkely.edu
Credit: CS181 – Machine Learning, D. Parkes, Harvard Univ, Spring 2017

FROM VALUE ITERATION TO Q-VALUE ITERATION

- Value iteration: find successive (depth-limited) values
  - Start with $V_0(s) = 0$, which we know is right
  - Given V_k, calculate the depth k+1 values for all states:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V_k(s')]$$

- But Q-values are more useful, so compute them instead
  - Start with $Q_0(s,a) = 0$, which we know is right
  - Given Q_k, calculate the depth k+1 q-values for all q-states:

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')\right]$$

Q-LEARNING

- What to do about *T(s,a,s')* and *R(s,a,s'), since we don't have these functions?*

$$\boxed{Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')\right]}$$

- Use sample-based Q-value iterantion
- Learn Q(s,a) values as you go
  - Receive a sample transition (s,a,r,s')
  - Consider your old estimate: $\boxed{Q(s,a)}$
  - Consider your new sample estimate:

$$\boxed{Q(s,a) = r + \gamma \max_{a'} Q_k(s',a')}$$

  - Incorporate the new estimate into a running average:

$$\boxed{Q(s,a) \leftarrow (1-\alpha)Q(s,a) + \alpha\left[r + \gamma \max_{a'} Q_k(s',a')\right]}$$

Q-LEARNING UPDATE RULE

- On transitioning from state *s* to state *s'* on action *a*, and receiving reward *r*, update:

$$\boxed{Q(s,a) \leftarrow (1-\alpha)Q(s,a) + \alpha\left[r + \gamma \max_{a'} Q_k(s',a')\right]}$$

- α is the **learning rate**
  - a large α results in quicker learning but may not converge.
  - α is often decreased as learning goes on.
- γ is the **discount rate** i.e., discounts future rewards

Q-LEARNING ALGORITHM

For each state *s* and action *a*:

$\boxed{Q(s,a) \leftarrow 0}$

Begin in state s:

Repeat:

For all actions associated with state *s*,
→ **CHOOSE ACTION *a*** ← based on the Q values for state *s*

Receive reward *r* and transition to s'

$$\boxed{Q(s,a) \leftarrow (1-\alpha)Q(s,a) + \alpha\left[r + \gamma \max_{a'} Q_k(s',a')\right]}$$

$\boxed{\mathrm{s} \leftarrow s'}$

CHOOSING THE ACTION

- Learned Q function determines the policy
  - in state *s*, choose action with largest *Q(s,a)*
- Still must worry about exploration vs. exploitation.

CHOOSING AN ACTION: EXPLORATION VS EXPLOITATION

- **Exploit:** use your current model to maximize the expected utility now.
- **Explore:** choose an action that will help you improve your model.
- **How to Exploit?** use the current policy.
- **How to Explore?**
  - choose an action randomly
  - choose an action you haven't chosen yet
  - choose an action that will take you to an unexplored state.

EXPLORATION STRATEGY: ϵ-GREEDY

- Explore with probability ϵ. Exploit with probability $1 - \epsilon$.
- Weaknesses:
  - Does not exploit when learning has converged.
- Uses:
  - appropriate if the world is changing.

EXPLORATION STRATEGY: BOLTZMANN

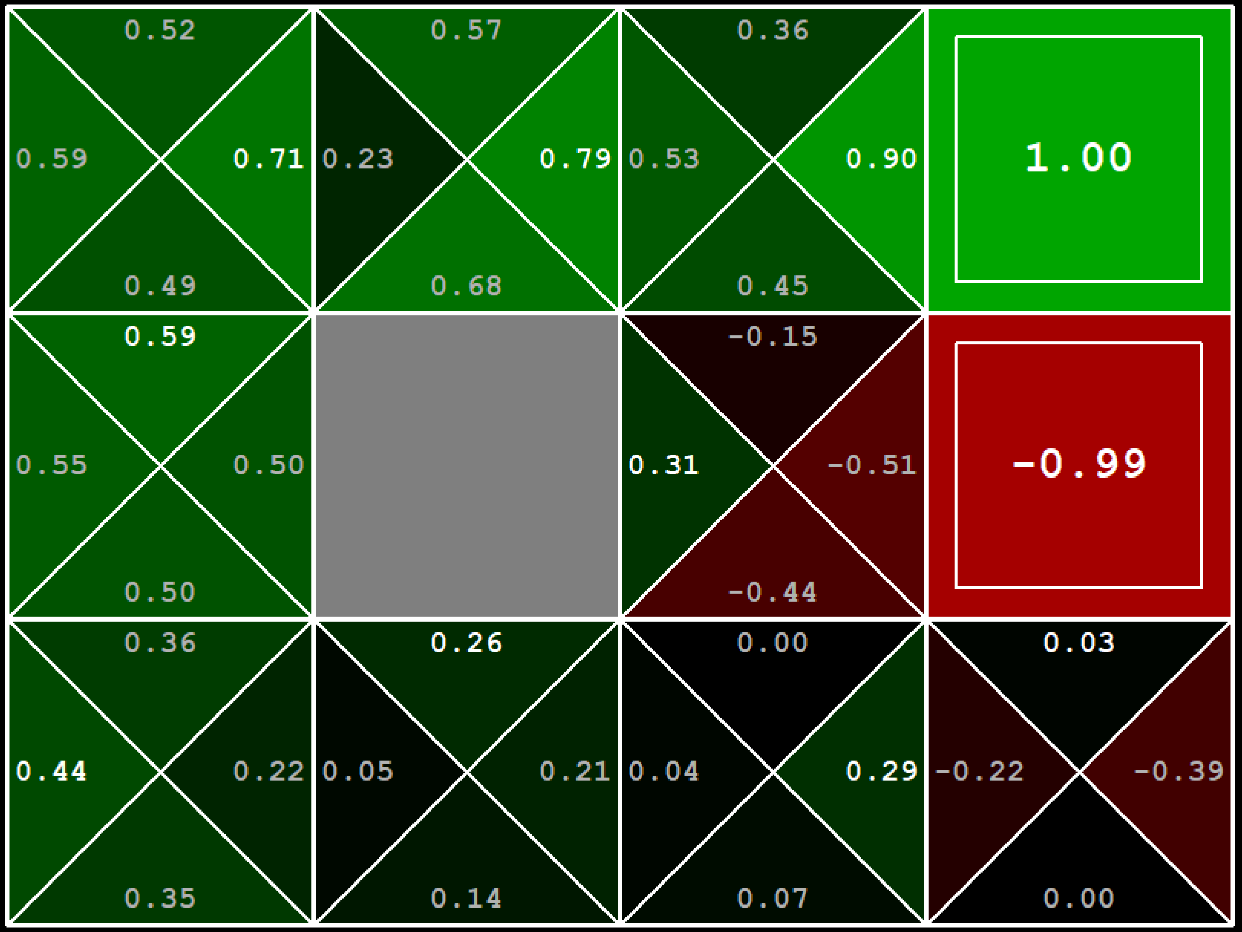
- In state s, choose action a with probability p:

$$p = \frac{e^{\frac{Q(s,a)}{t}}}{\sum_{a'} e^{\frac{Q(s,a')}{t}}}$$

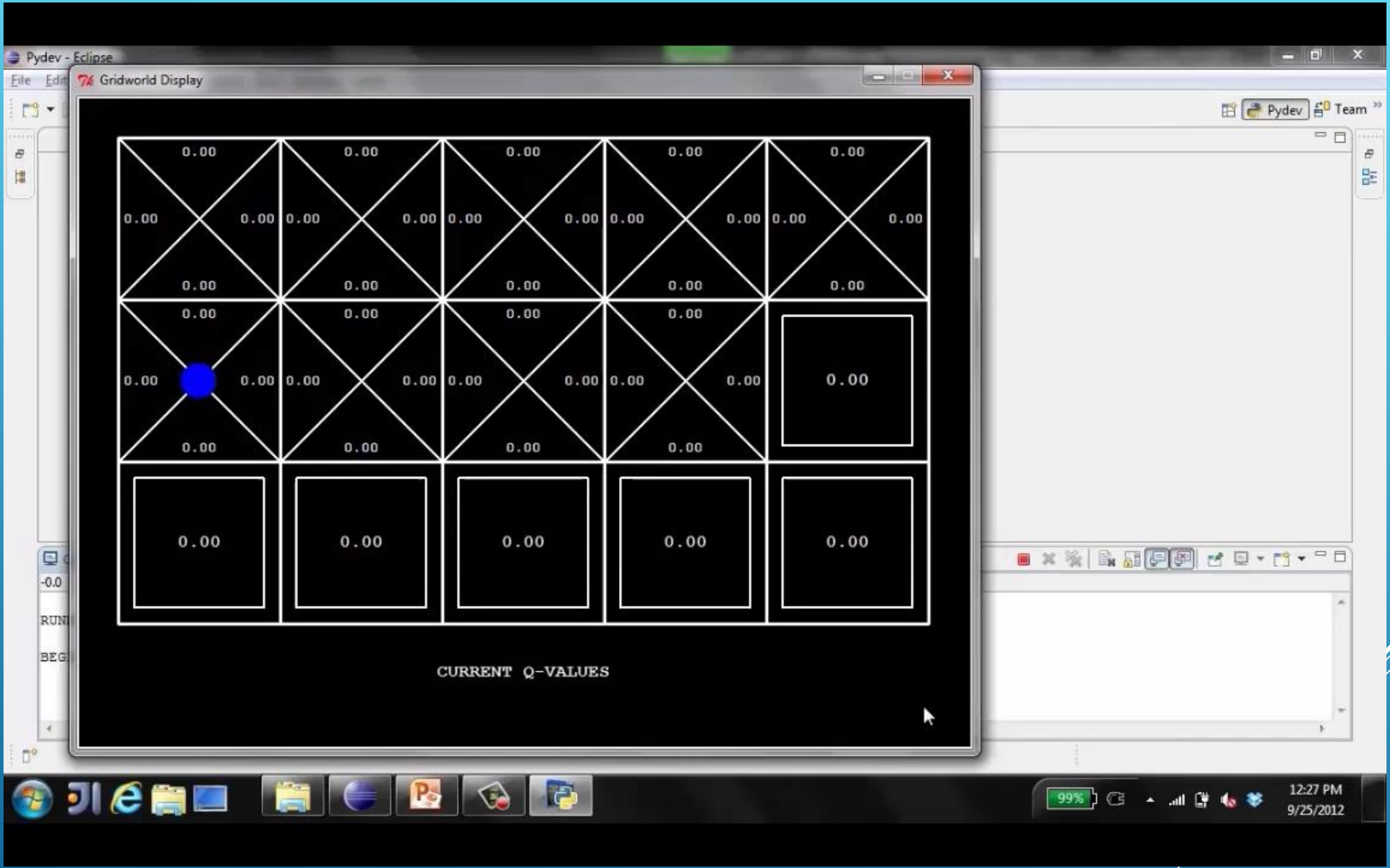
- Simulated annealing: *t* is a "temperature"
- High temperature means more exploration
- Over time, t cools, reducing exploration
- Sensitive to cooling schedule.

Q-LEARNING DEMOS

Q-LEARNING EXAMPLE: GRIDWORLD

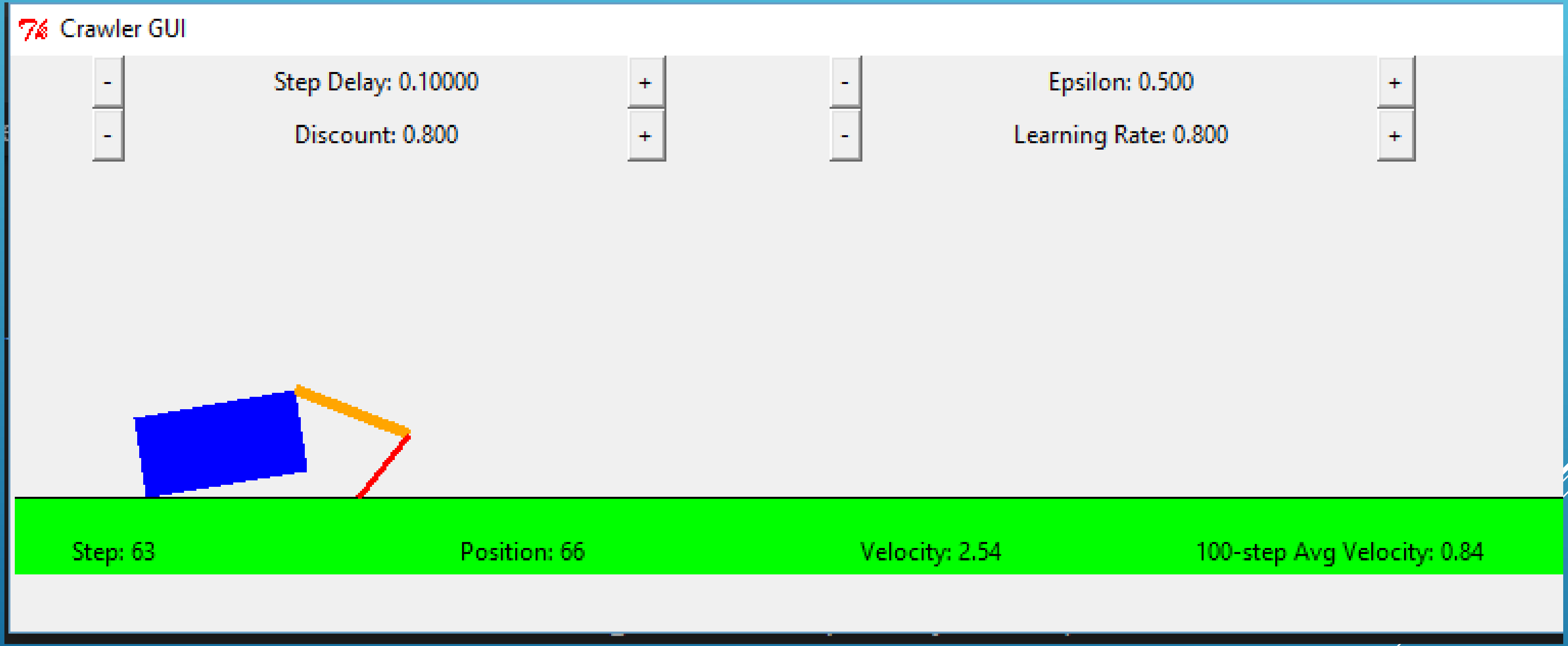


Q-VALUES AFTER 40 EPISODES



VIDEO OF Q-LEARNING DEMO -- GRIDWORLD

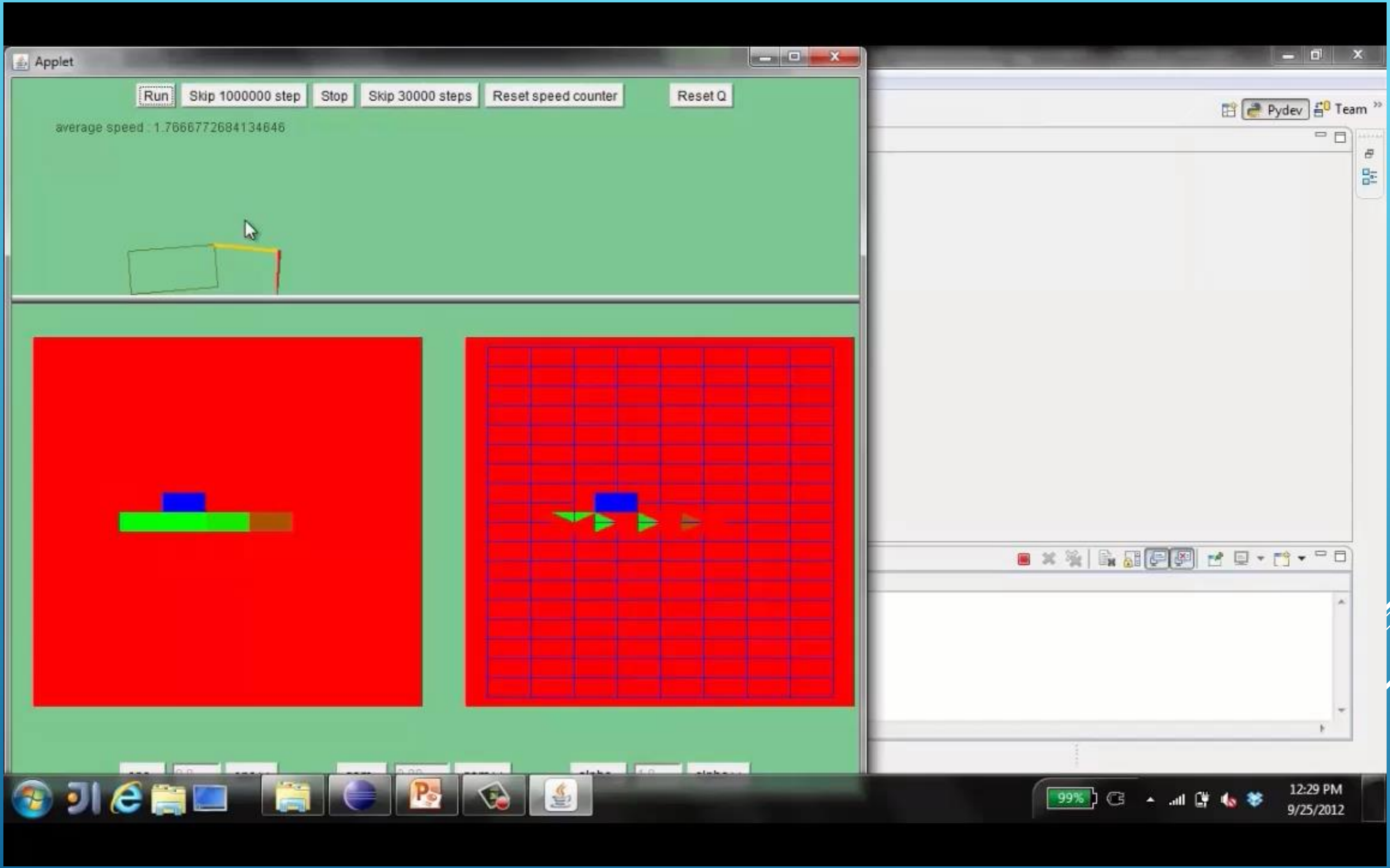
Q-LEARNING EXAMPLE: CRAWLER ROBOT



Credit: CS188 – Intro to AI, UC Berkeley, ai.berkely.edu

VIDEO 1 OF Q-LEARNING DEMO -- CRAWLER

Credit: CS188 – Intro to AI, UC Berkeley, ai.berkely.edu



VIDEO 2 OF Q-LEARNING DEMO -- CRAWLER

Credit: CS188 – Intro to AI, UC Berkeley, ai.berkely.edu

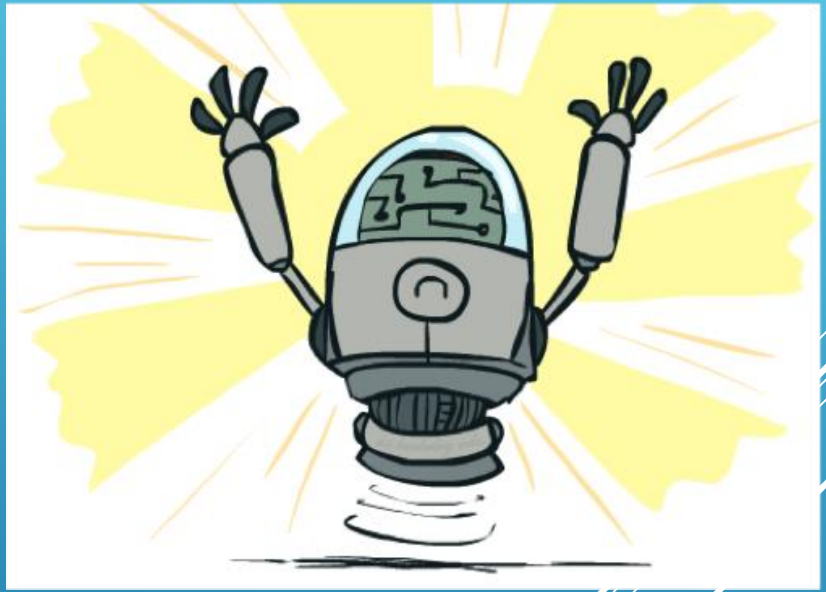
Q-LEARNING EXAMPLE: DISCOUNT EFFECT

Update rule: $$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)\left[r + \gamma \max_{a'} Q(s',a')\right]$$

Description	Training Steps	Discount (γ)	Learning Rate (α)	Avg Velocity
Default	~100K	**0.8**	0.8	~1.73
Low γ	~100K	**0.5**	0.8	0.0
High γ	~100K	**0.919**	0.8	~3.33
Low α	~100K	0.8	0.2	~1.73
High α	~100K	0.8	0.9	~1.73
High γ, Low α	~100K	0.919	0.2	~3.33

Q-LEARNING PROPERTIES

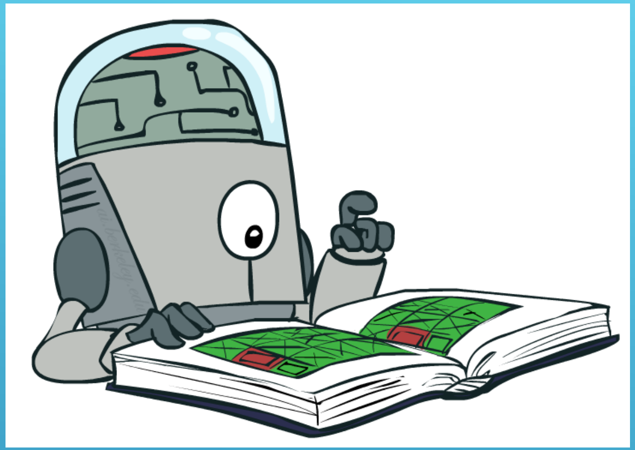
- Amazing result: Q-learning converges to optimal policy -- even if you're acting sub-optimally!
- This is called ***off-policy learning***
- Caveats:
  - You have to explore enough
  - You have to eventually make the learning rate small enough
  - … but not decrease it too quickly
  - Basically, in the limit, it doesn't matter how you select actions (!)

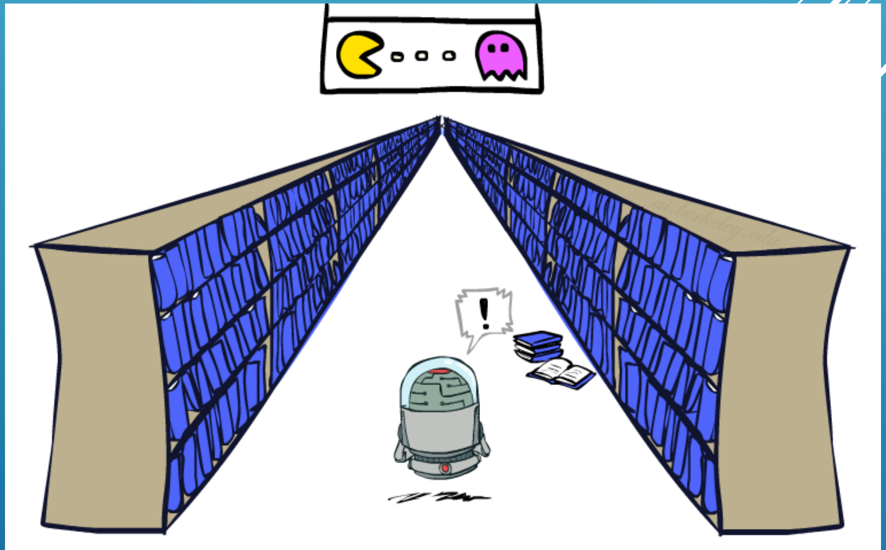


Credit: CS188 – Intro to AI, UC Berkeley, ai.berkely.edu

GENERALIZING ACROSS STATES

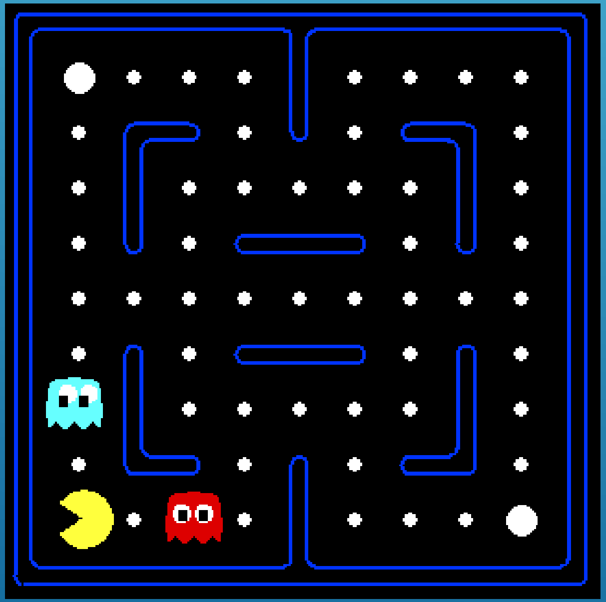
- Basic Q-Learning keeps a table of all q-values
- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory
- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar situations



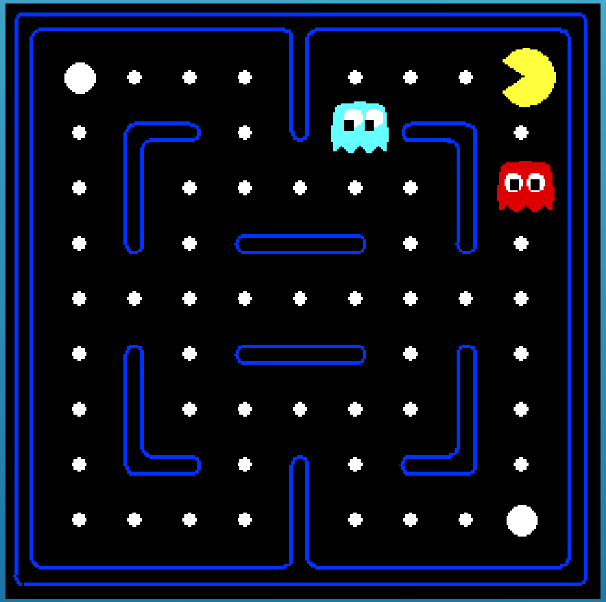


EXAMPLE: PACMAN

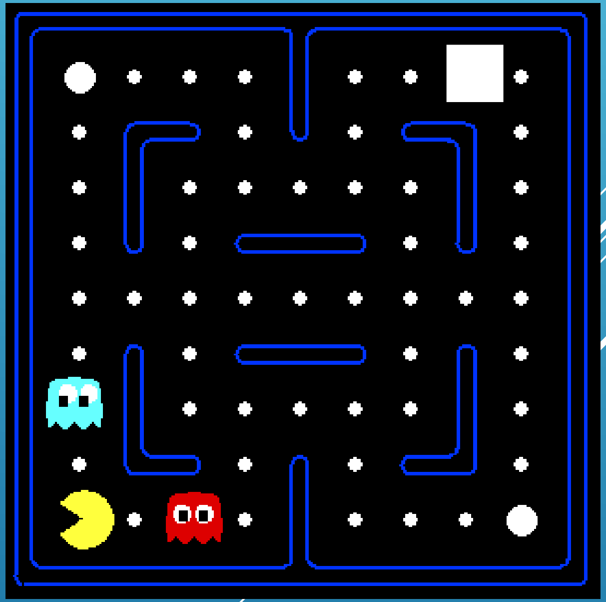
Let's say we discover through experience that this state is bad:



In naïve q-learning, we know nothing about this state:



Or even this one!



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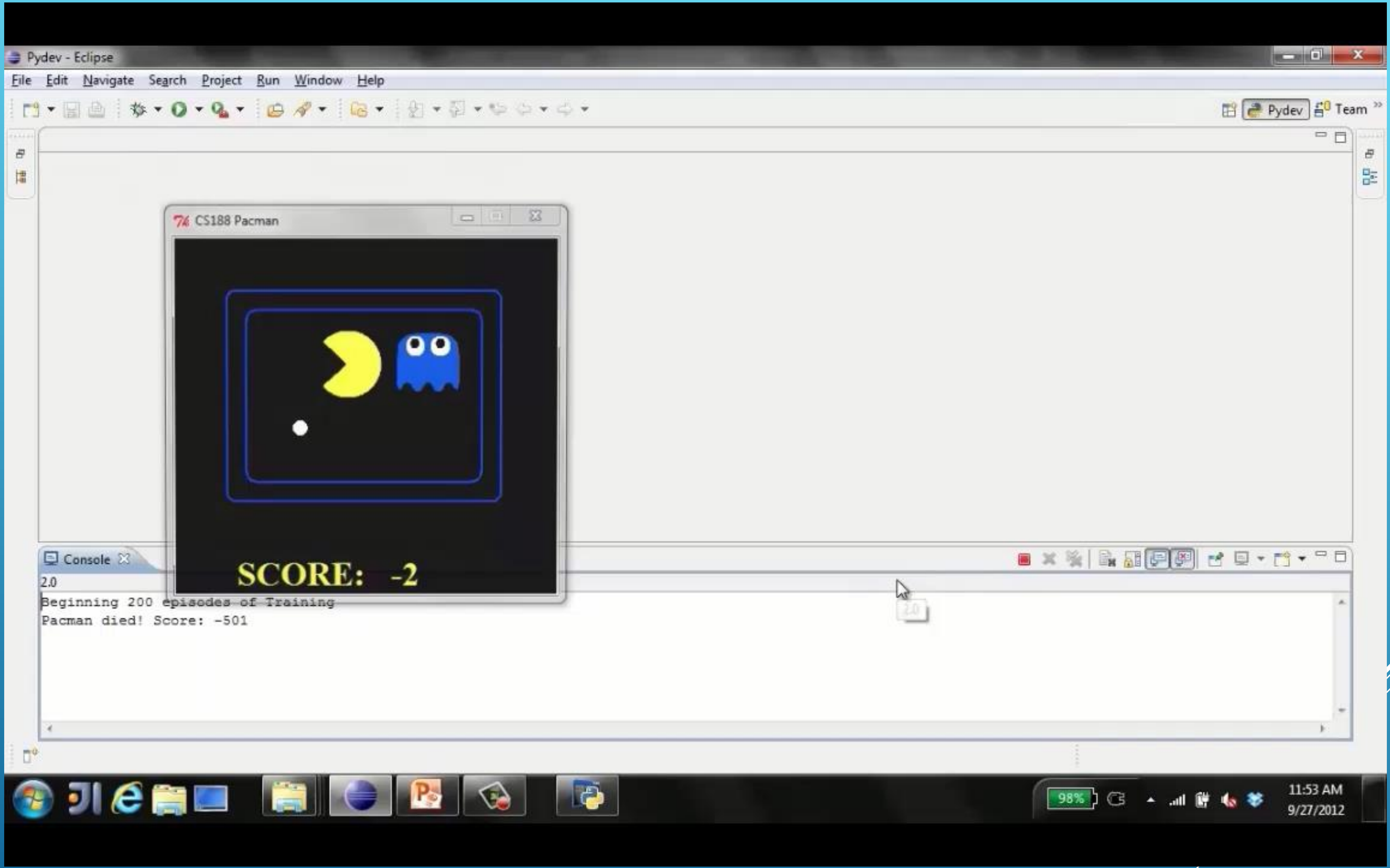
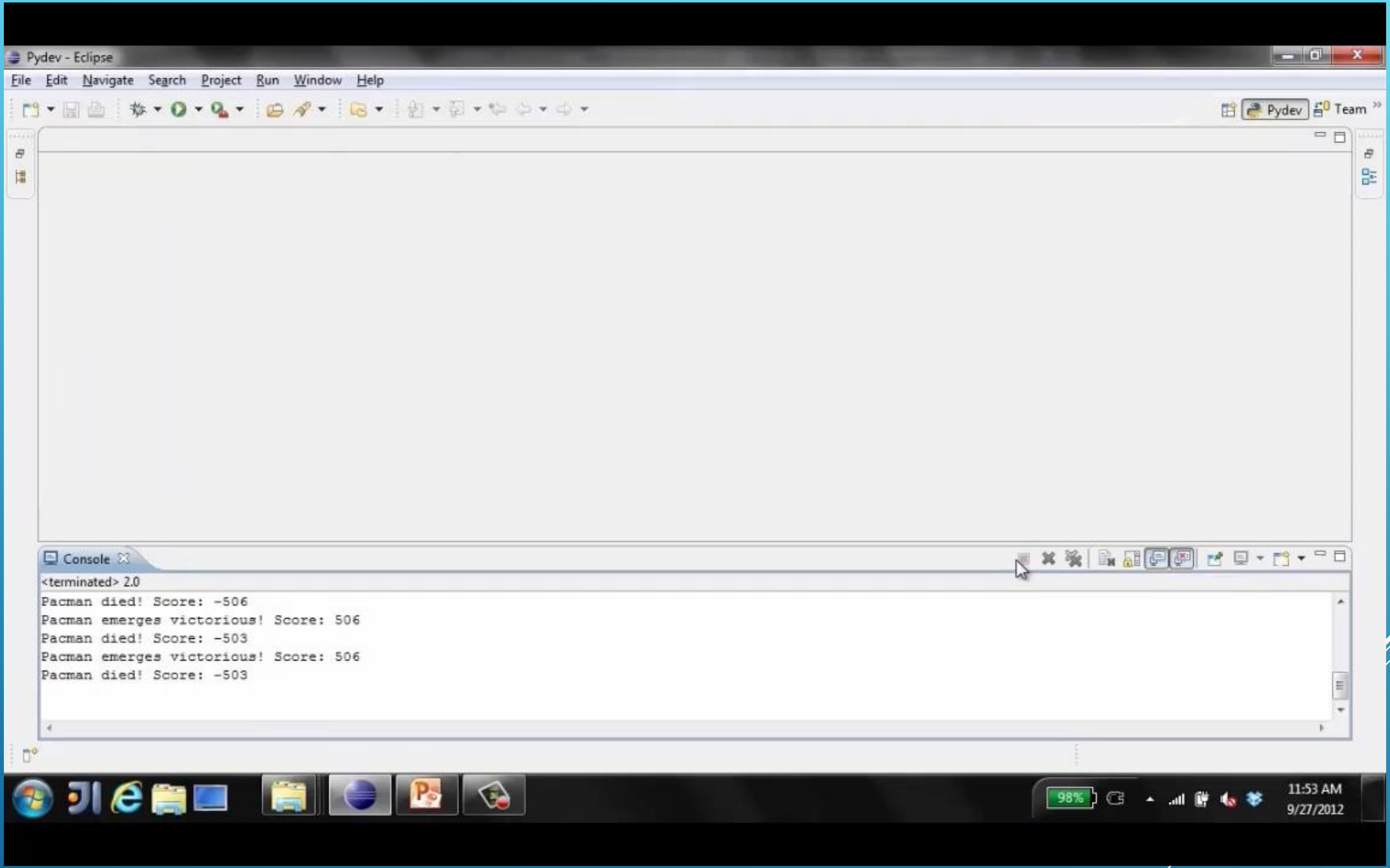


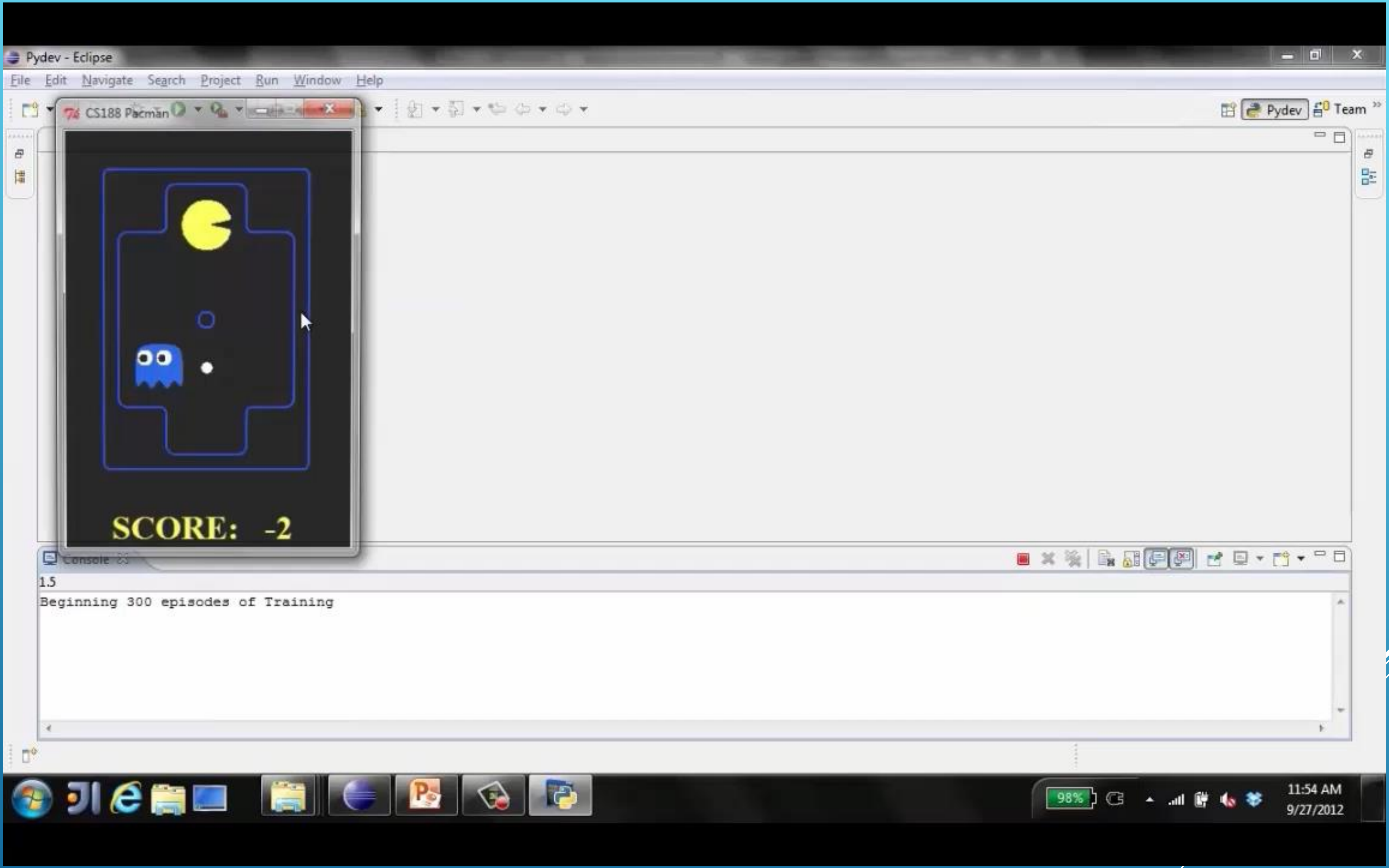
TINY PACMAN DEMO 1

Credit: CS188 – Intro to AI, UC Berkeley, ai.berkely.edu



TINY PACMAN DEMO 2

Credit: CS188 – Intro to AI, UC Berkeley, ai.berkely.edu

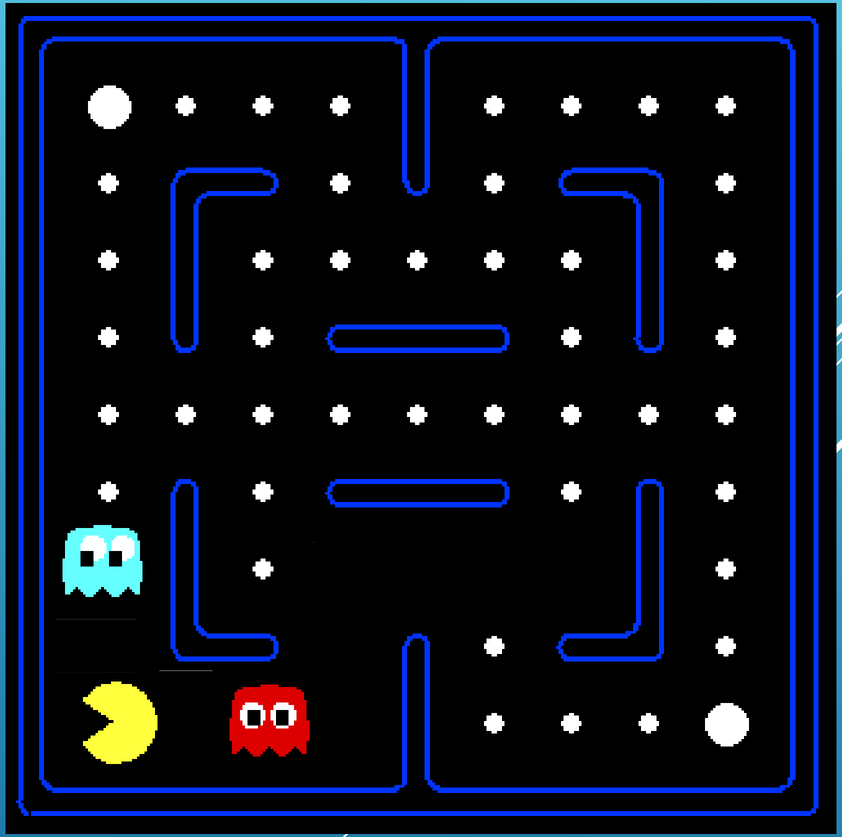


TINY PACMAN DEMO 3

Credit: CS188 – Intro to AI, UC Berkeley, ai.berkely.edu

FEATURE-BASED REPRESENTATIONS

- Solution: describe a state using a vector of features (properties)
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - 1 / (dist to dot)^2
    - Is Pacman in a tunnel? (0/1)
    - …… etc.
    - Is it the exact state on this slide?
  - Can also describe a q-state (s, a) with features (e.g. action moves closer to food)



Credit: CS188 – Intro to AI, UC Berkeley, ai.berkely.edu

LINEAR VALUE FUNCTIONS

- Using a feature representation, we can write a q function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- Advantage: our experience is summed up in a few powerful numbers
- Disadvantage: states may share features but actually be very different in value!

APPROXIMATE Q-LEARNING

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

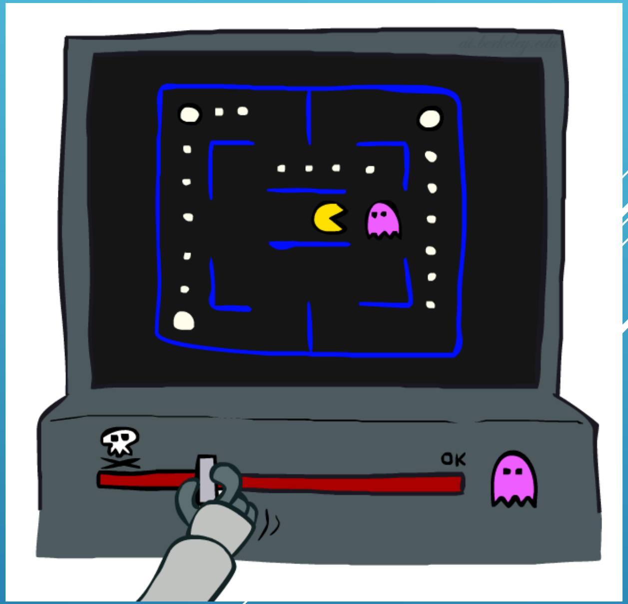
- Q-learning with linear Q-functions:

$$\text{transition} = (s, a, r, s')$$

$$\text{difference} = \left[r + \gamma \max_{a'} Q(s', a')\right] - Q(s,a)$$

$$Q(s,a) \leftarrow Q(s,a) + \alpha\,[\text{difference}]$$ Exact Q's

$$w_i \leftarrow w_i + \alpha\,[\text{difference}]\, f_i(s,a)$$ Approximate Q's

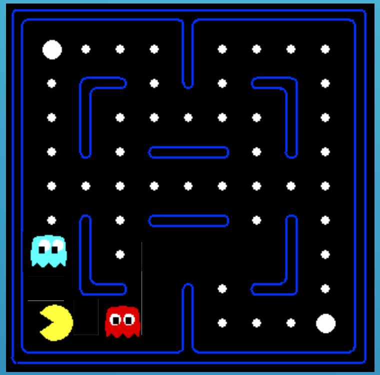


- Intuitive interpretation:
  - Adjust weights of active features
  - E.g., if something unexpectedly bad happens, blame the features that were on i.e., "dis-prefer" all states with that state's features
- Formal justification: online least squares

EXAMPLE: Q-PACMAN

$$Q(s,a) = 4.0 f_{DOT}(s,a) - 1.0 f_{GST}(s,a)$$

s

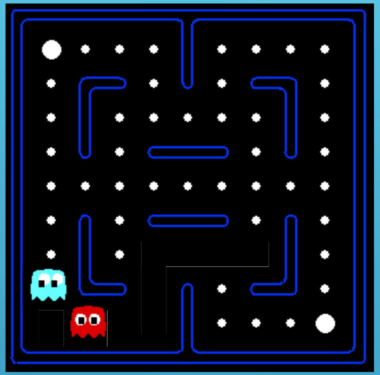


$f_{DOT}(s, \text{NORTH}) = 0.5$

$f_{GST}(s, \text{NORTH}) = 1.0$

$a = \text{NORTH}$
$r = -500$

s'



$Q(s, \text{NORTH}) = +1$

$r + \gamma \max_{a'} Q(s', a') = -500 + 0$

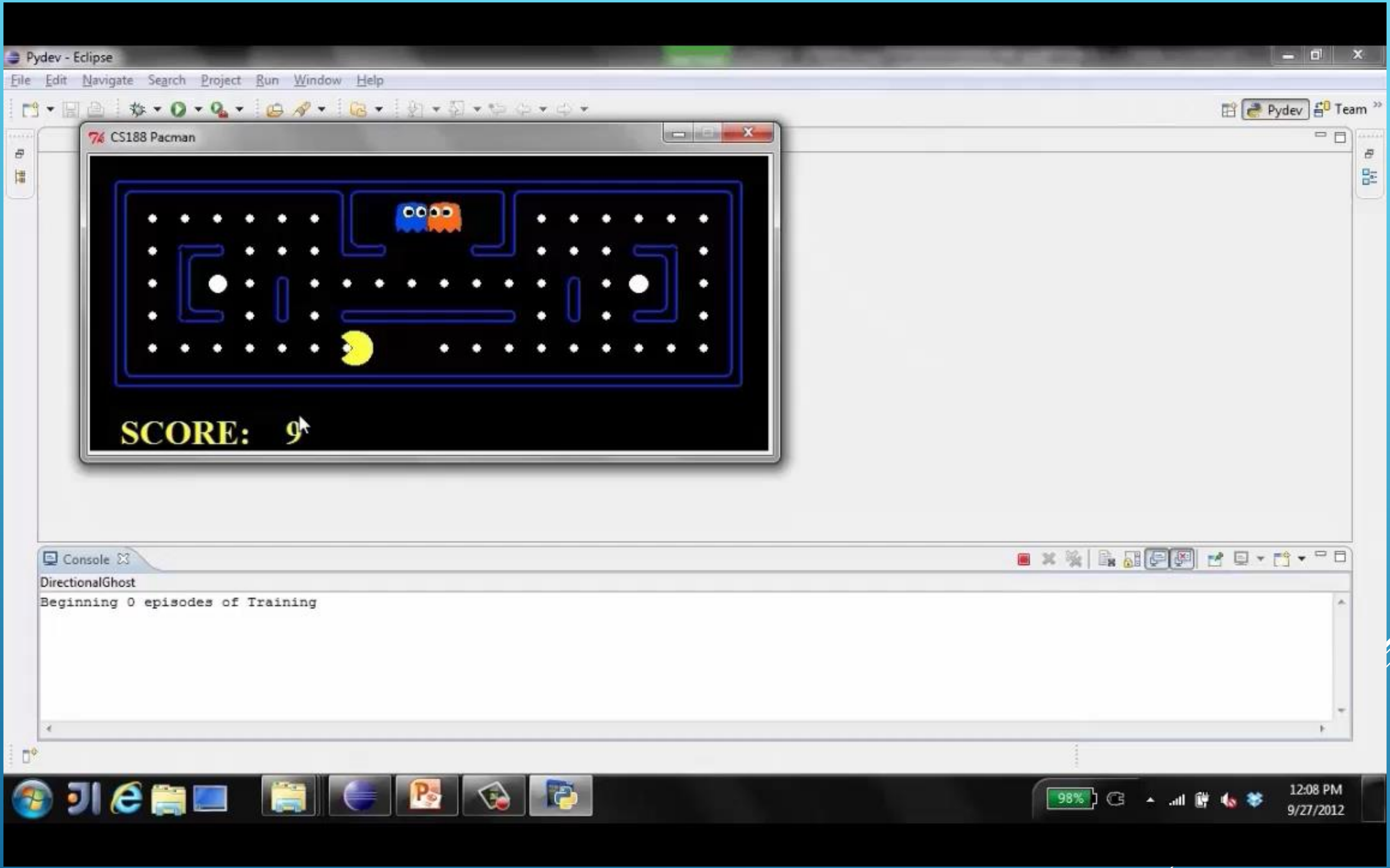
$Q(s', \cdot) = 0$

difference $= -501$

$w_{DOT} \leftarrow 4.0 + \alpha \left[-501\right] 0.5$

$w_{GST} \leftarrow -1.0 + \alpha \left[-501\right] 1.0$

$$Q(s,a) = 3.0 f_{DOT}(s,a) - 3.0 f_{GST}(s,a)$$



APPROXIMATE Q-LEARNING DEMO -- PACMAN

Credit: CS188 – Intro to AI, UC Berkeley, ai.berkely.edu

NEXT TIME: MORE RL

Possible Topics:

- Credit Assignment
- Online Learning vs Offline learning
  - SARSA vs Q-Learning
- Hybrid Approaches:
  - Temporal Difference (TD) Value Learning
  - Dyna-Q
- Partially Observable Markov Decision Processes (POMDPs)
- Deep Q-Learning