MULTI-ARMED BANDITS

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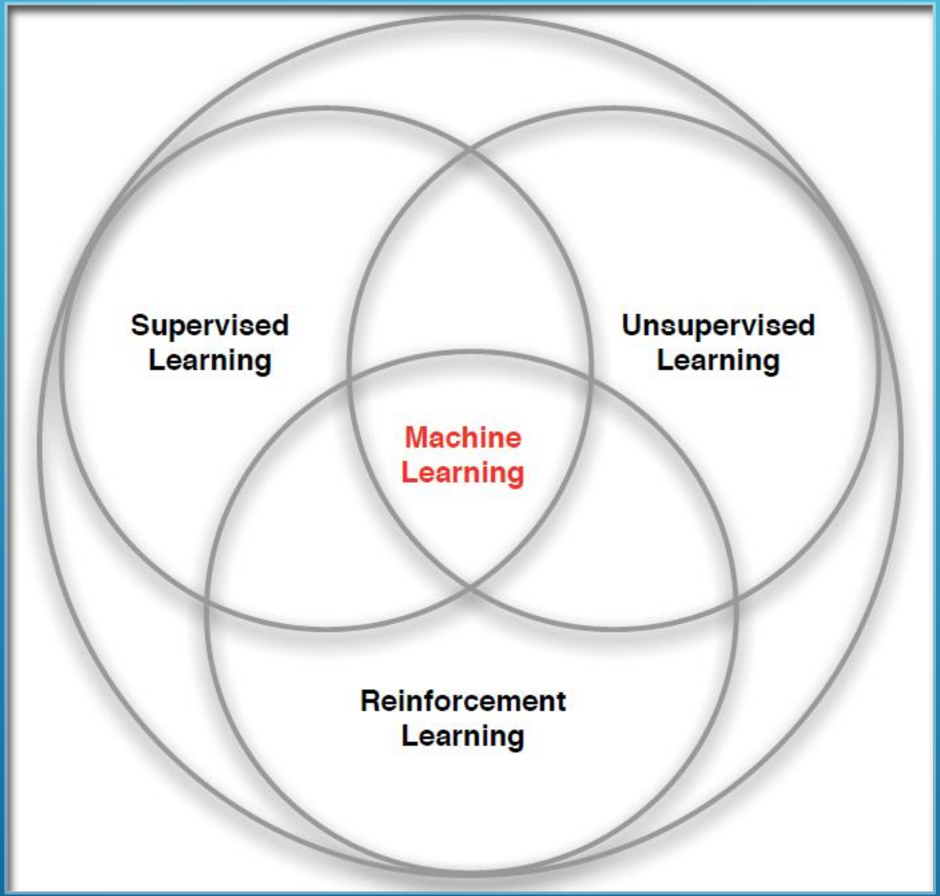
02/12/2020

3 TYPES OF MACHINE LEARNING



Supervised Learning – Learn a function from labeled data that maps input attributes to an output label e.g., linear regression, decision trees, SVMs.

Unsupervised Learning – Learn patterns in unlabeled data e.g., principle component analysis or clustering algorithms such as K-means, HAC, or Gaussian mixture models.

Reinforcement Learning – An agent learns to maximize rewards while acting in an uncertain environment.

Credit: image borrowed from lecture slides David Silver, DeepMind, "Introduction to Reinforcement Learning

SOME CHARACTERISTICS OF REINFORCEMENT LEARNING

- Learning happens as the agent interact with the world.
- There are no training or test sets. Training is guided by rewards and punishments obtained by acting in an environment.
- The amount of data an agent receives is not fixed. More information is acquired as you go.
- Actions are not always rewarded or punished immediately. "Delayed gratification" is possible.
- Agent actions can affect the subsequent data it receives. E.g., closing a door that can't be opened again.

MARKOV DECISION PROCESSES

- The ***Markov Decision Process*** (MDP) provides a mathematical framework for reinforcement learning.
- An MDP is used to model <u>optimal decision-making</u> in situations where outcomes are <u>uncertain</u>.

<https://en.wikipedia.org/w/index.php?title=Markov_decision_process&oldid=855934986> [accessed 11 September 2018]

THE MDP DECISION FRAMEWORK

Markov decision processes model uncertainty

Use probability to model uncertainty about the domain.

Markov decision processes model an agent's objectives

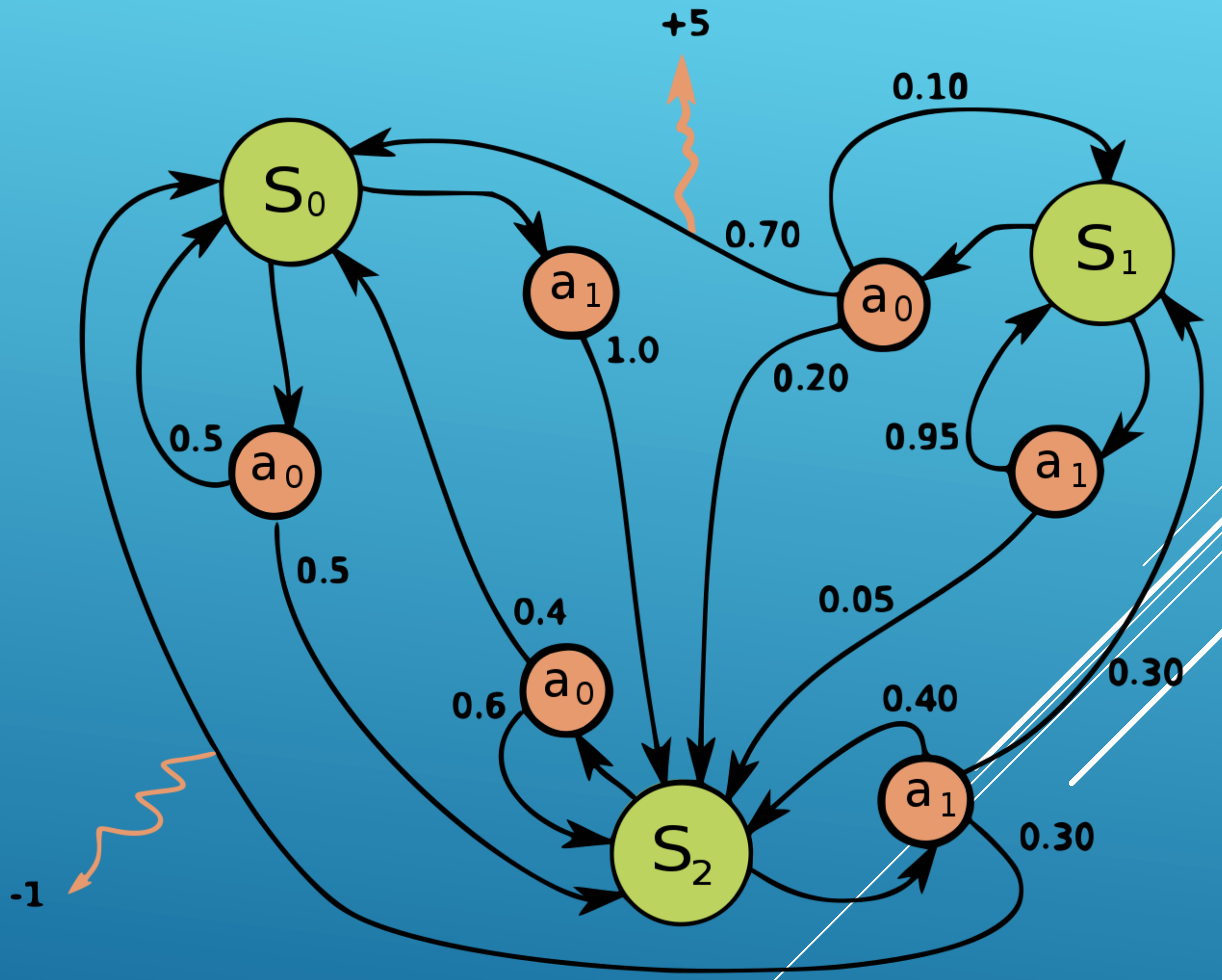
Use utility to model an agent's objectives. The higher the utility, the "happier" your agent is.

Markov decision processes find an optimal decision policy

The goal is to discover an optimal decision policy π specifying how the agent should act in all possible states in order to maximize its expected utility.

MARKOV DECISION PROCESSES

- **States**: $s_0, \dots, s_n$
- **Actions**: $a_0, \dots, a_m$
- **Reward Function**:
- **Transition model**:
- **Discount factor**: $\gamma \in [0, 1]$



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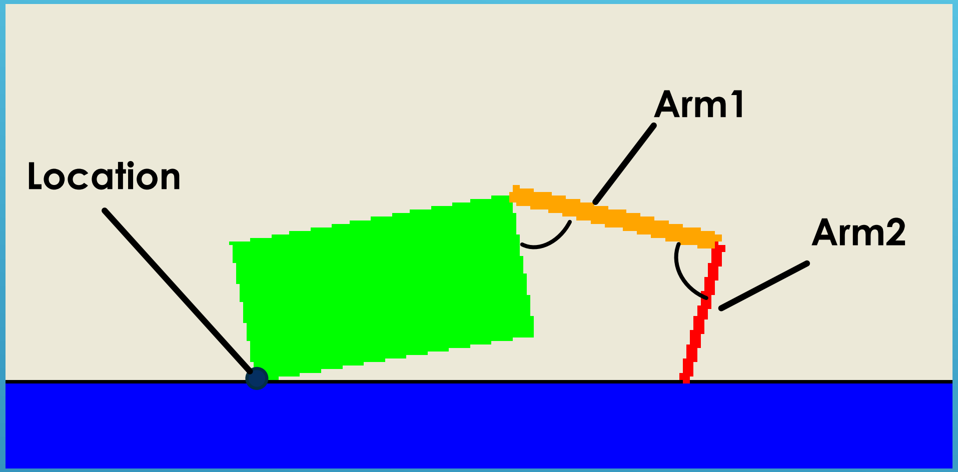
APPLICATION: CRAWLER ROBOT



- **States:** <Location, Arm1 angle, Arm2 angle>
- **Actions:** increase Arm1 angle, decrease Arm1 angle,
  increase Arm2 angle, decrease Arm2 angle.
- **Reward Function:** +1 if robot moves right, -1 if robot moves left.
- **Transition model:** model of box movement caused by arm movements.

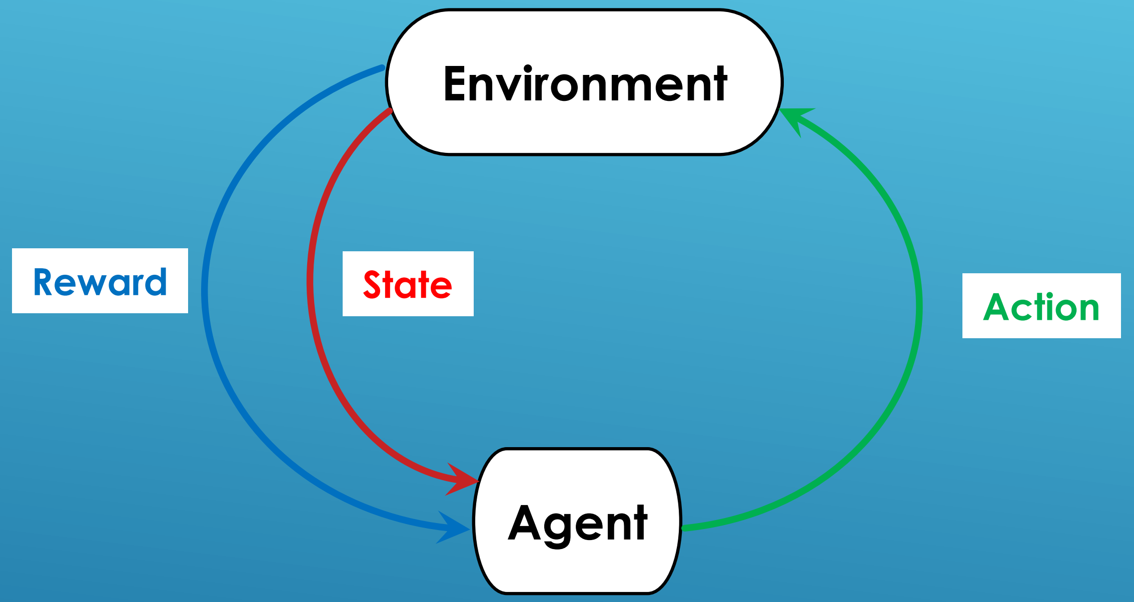
APPLICATION: THE MULTI-ARMED BANDIT

A Simple Reinforcement Learning Problem

- **States:** only 1 state
- **Actions:** pull one of *k* arms
- **Reward Function**: receive reward based on a probability distribution associated with each arm.
- **Transition model:** no transitions

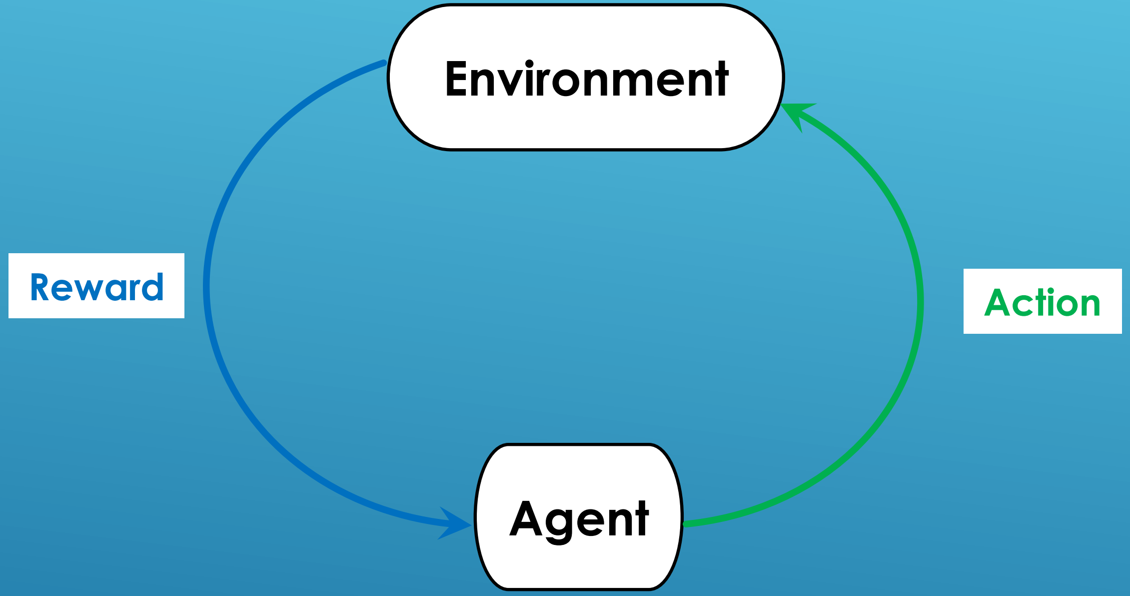
<https://en.wikipedia.org/w/index.php?title=Multi-armed_bandit&oldid=935049159> [accessed 12 February 2020]

THE REINFORCEMENT LEARNING PROBLEM



$$s_0, a_0, r_0, s_1, a_1, r_1, s_2 \ldots, s_{n-1}, a_{n-1}, r_{n-1}, s_n$$

THE MULTI-ARM BANDIT LEARNING PROBLEM



$$a_0, r_0, a_1, r_1, a_2, r_2, \ldots, a_n, r_n$$

WHY STUDY MULTI-ARMED BANDITS?

- Multi-armed bandits can be used to create agents to solve real-world problems.
- A simple model can make it easier to understand important issues that hard to address using a more complex model.
- Here, we will use multi-armed bandits to approach the exploration vs. exploitation trade-off in reinforcement learning.

MULTI-ARMED BANDIT APPLICATIONS

- Medical treatment in clinical trials
- Online ad placement
- Webpage personalization
- Network packet routing

CHOOSING AN ACTION: EXPLORATION VS EXPLOITATION

- How should an agent choose an action? An obvious answer is simply to follow the current policy. However, this is often not the best way to improve your model.
- **Exploit:** use your current model to maximize the expected utility now.
- **Explore:** choose an action that will help you improve your model.

HOW/WHEN SHOULD WE EXPLORE / EXPLOIT ?

- **How to Exploit?** use the current policy.

- **How to Explore?**
  - choose an action randomly
  - be optimistic about the rewards you will receive.
  - choose an action you have used less often

- **When to exploit? When to explore? How much time in exploration vs time in exploitation?**

5 MULTI-ARMED BANDIT ALGORITHMS

1. Greedy
2. ϵ-Greedy
3. Greedy with optimistic initial values
4. ϵ-Greedy with decreasing ϵ.
5. Upper-confidence bound (UCB) action selection

GREEDY METHOD

- At time *t*, estimate a value for each action:

$$Q_t(a) = \frac{\text{Sum of rewards when a taken prior to t}}{\text{Number of times a taken prior to t}}$$

- Select the action with the maximum value.

$$A_t = argmax\ Q_t(a)$$

GREEDY METHOD WEAKNESSES

- Always exploits current knowledge, no exploration.
- Can get stuck with a suboptimal action.

Ε-GREEDY METHOD

- At time *t*, estimate a value for each action:

$$Q_t(a) = \frac{\text{Sum of rewards when a taken prior to t}}{\text{Number of times a taken prior to t}}$$

- With probability 1 - ϵ, select the action with the maximum value.

$$A_t = argmax\ Q_t(a)$$

- With probability 1 - ϵ, randomly select an action from all the actions with equal probability.

EXPERIMENTS

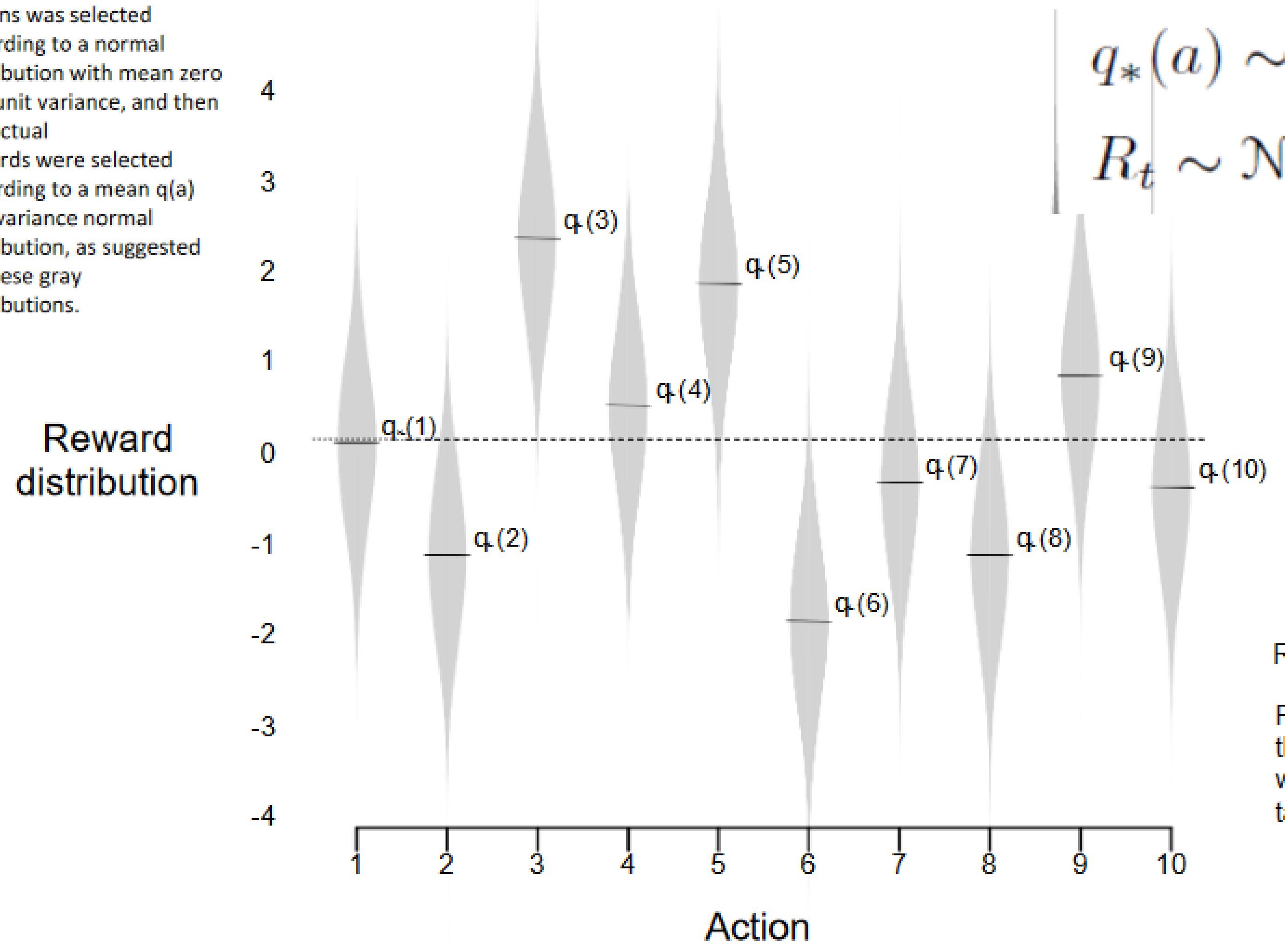
- Setup: (one run)
  - 10-armed bandits
  - Draw u_i from $Gaussian(0,1), i = 1, \dots, 10$
    - the expectation/mean of rewards for action i.
  - Rewards of action *I* at time *t*: $x_i(t)$
    - $x_i(t) \sim Gaussian(u_i, 1)$
  - Play 2000 rounds/steps
  - Average return at each time step
- Average over 1000 runs

The 10-armed Testbed

Figure 2.1: An example bandit problem from the 10-armed testbed. The true value q(a) of each of the ten actions was selected according to a normal distribution with mean zero and unit variance, and then the actual rewards were selected according to a mean q(a) unit variance normal distribution, as suggested by these gray distributions.

$$q_*(a) \sim \mathcal{N}(0, 1)$$

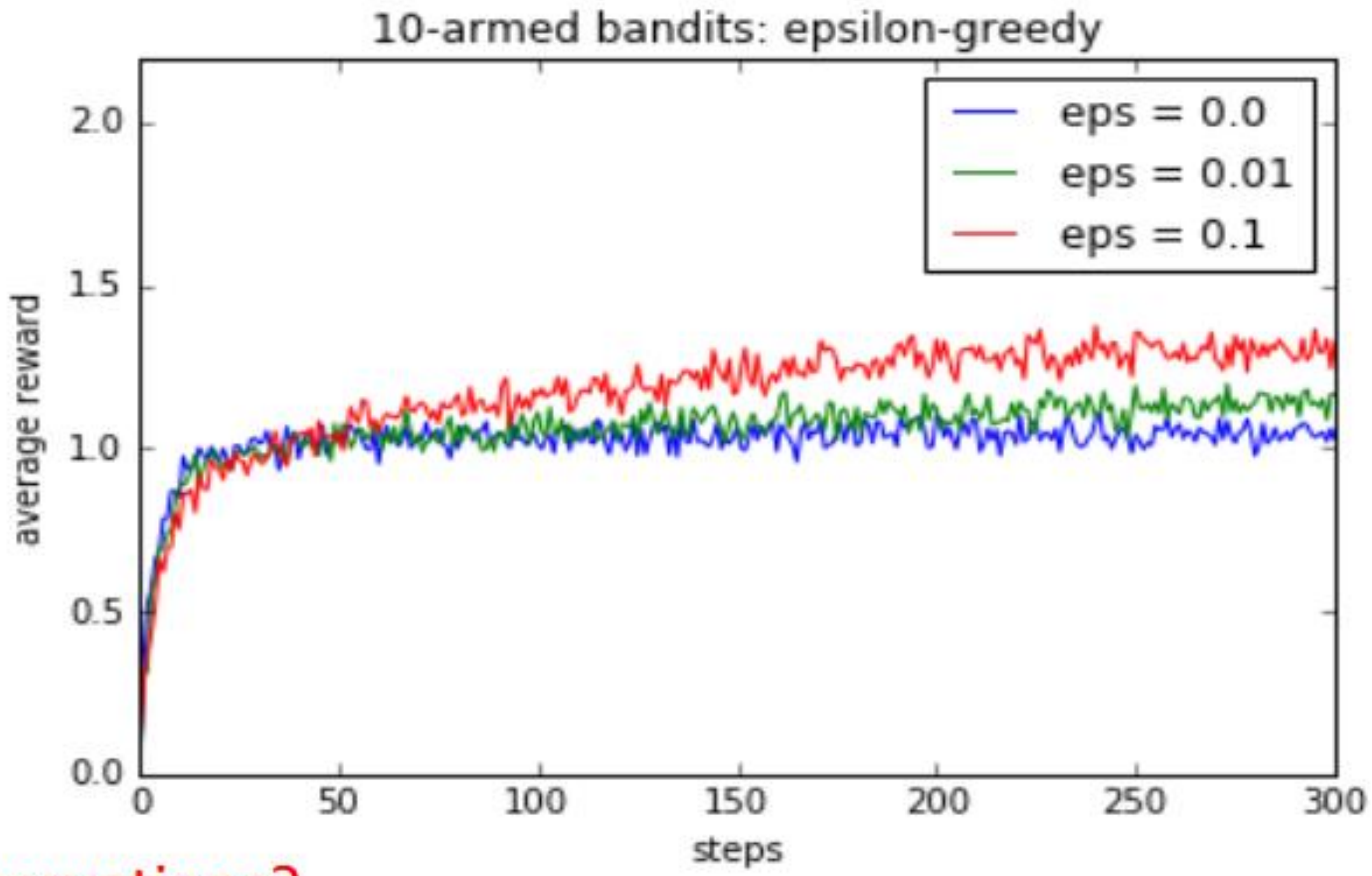
$$R_t \sim \mathcal{N}(q_*(a), 1)$$



Run for 1000 steps

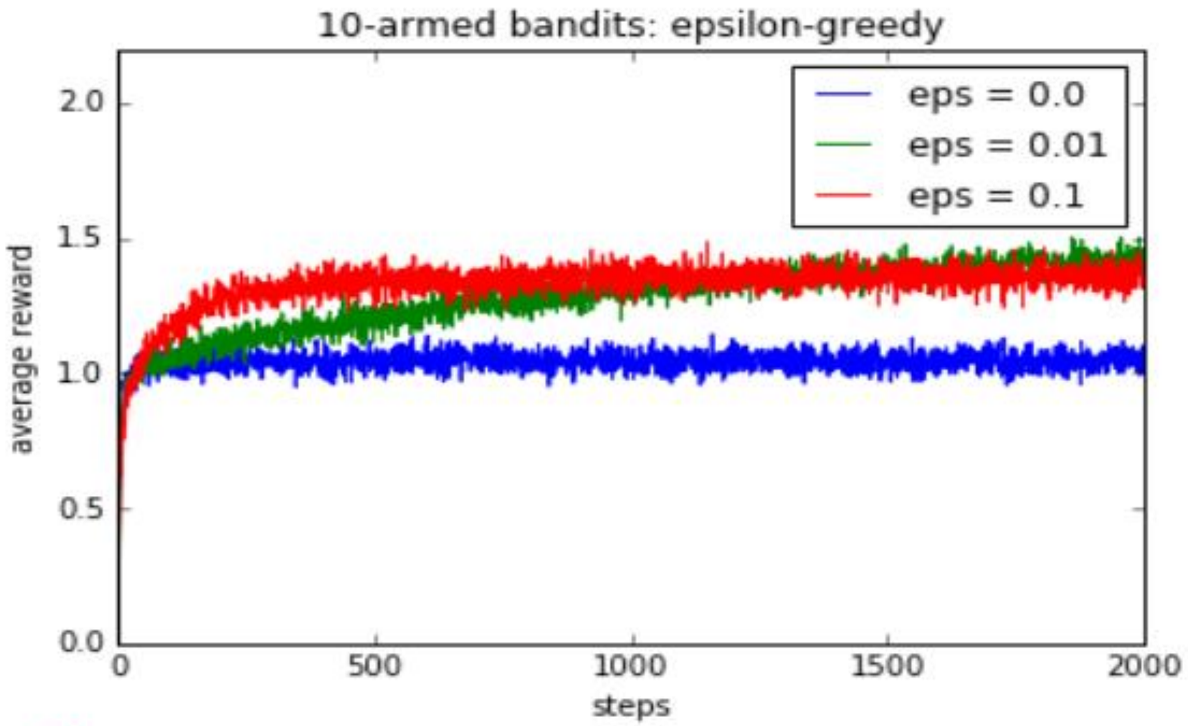
Repeat the whole thing 2000 times with different bandit tasks

Experimental results: average over 1000 runs



Observations?

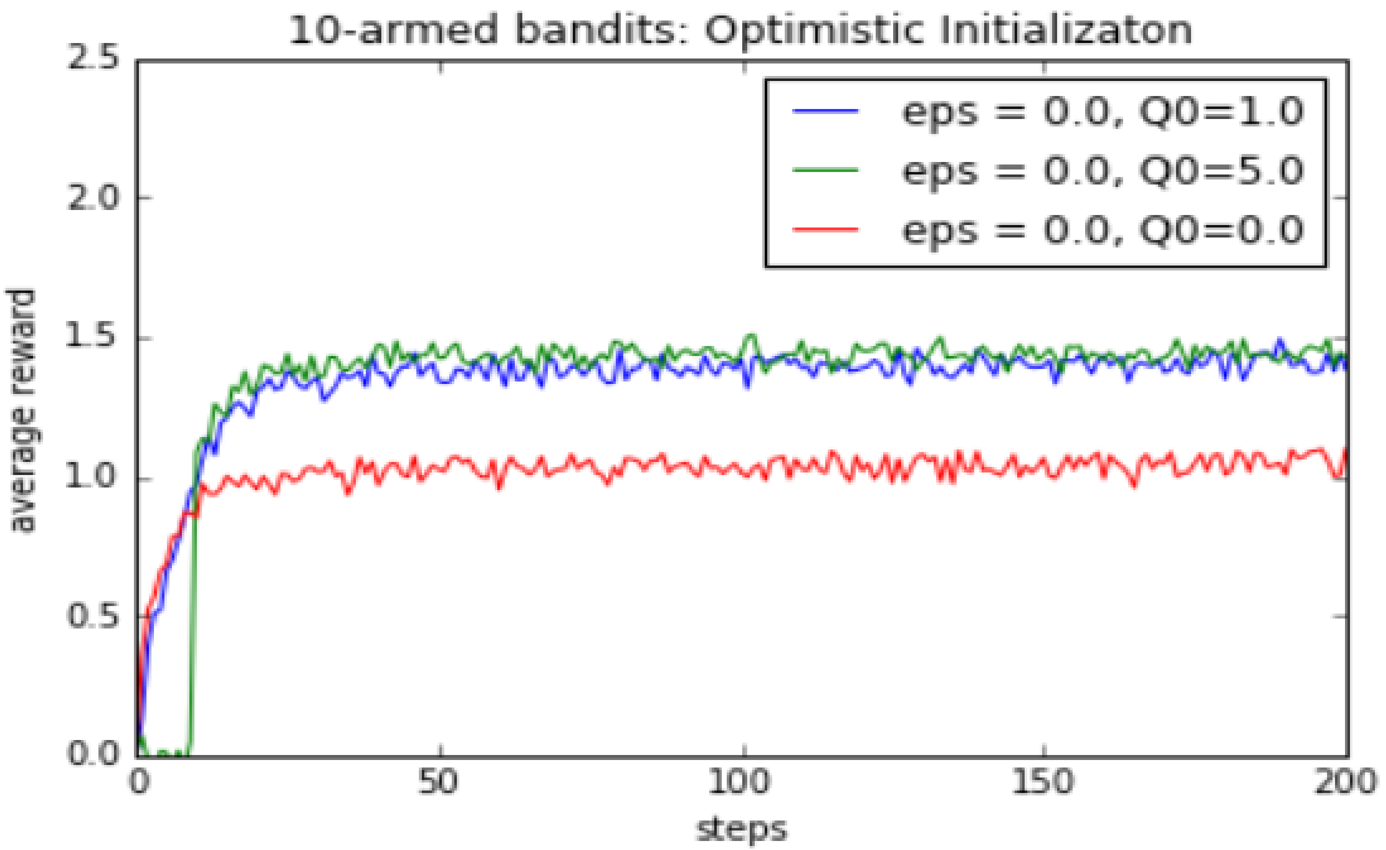
What will happen if we run more steps?



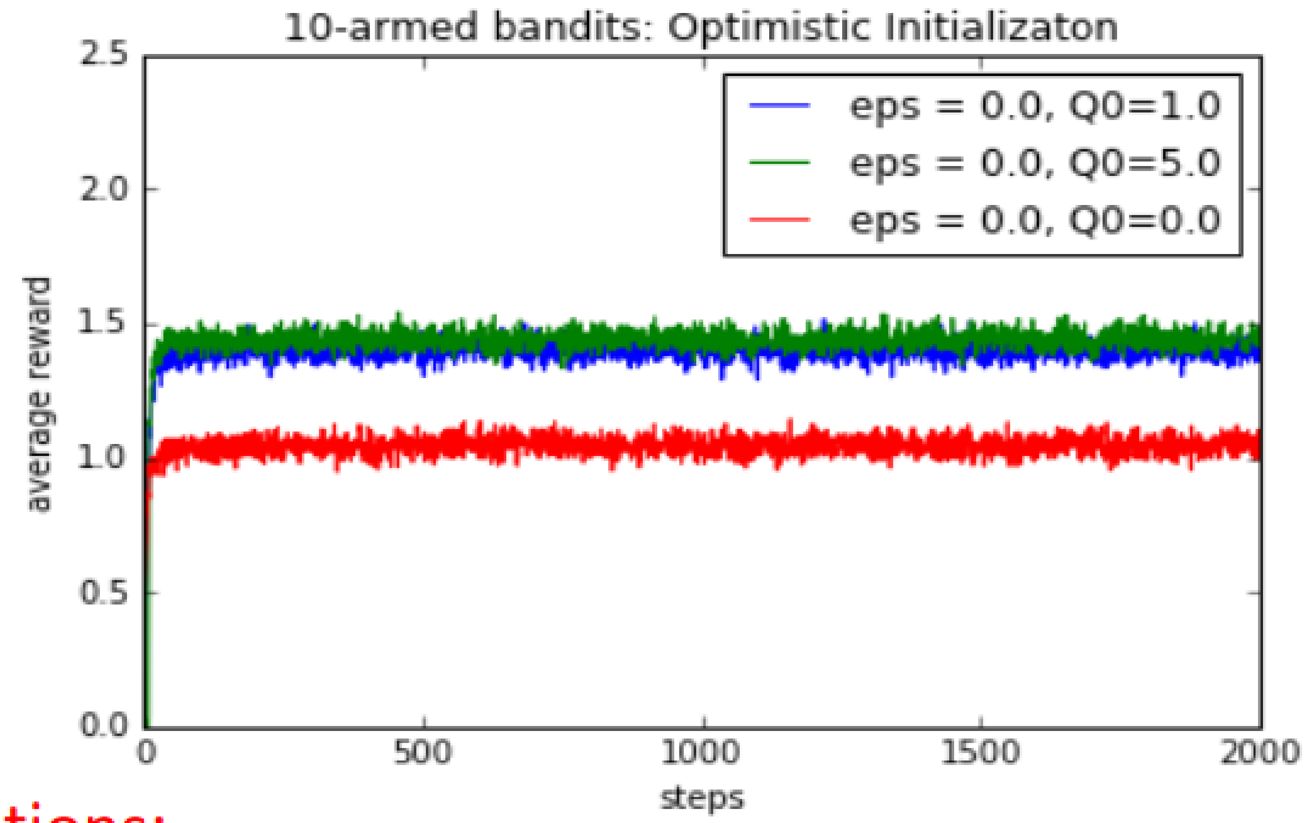
Observations:

- Greedy method improved faster at the very beginning, but level off at a lower level.
- ε- Greedy methods continue to Explore and eventually perform better.
- The$\varepsilon = 0.01$ method improves slowly, but eventually performs better than the $\varepsilon = 0.1$ method.

Improve the Greedy method with optimistic initialization



Greedy with optimistic initialization

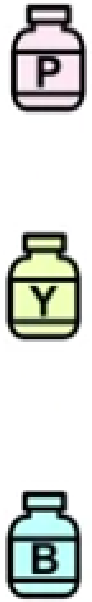


Observations:

- Big initial Q values force the Greedy method to explore more in the beginning.
- No exploration afterwards.

Example: Clinical Trials

A reward of 1 if the treatment succeeds otherwise 0



Example: Clinical Trials

A reward of 1 if the treatment succeeds otherwise 0

$$Q_1(\text{P}) = 0.0$$

$$Q_1(\text{Y}) = 0.0$$

$$Q_1(\text{B}) = 0.0$$

Example: Clinical Trials

A reward of 1 if the treatment succeeds otherwise 0

$$Q_1(\boxed{\text{P}}) = 2.0$$

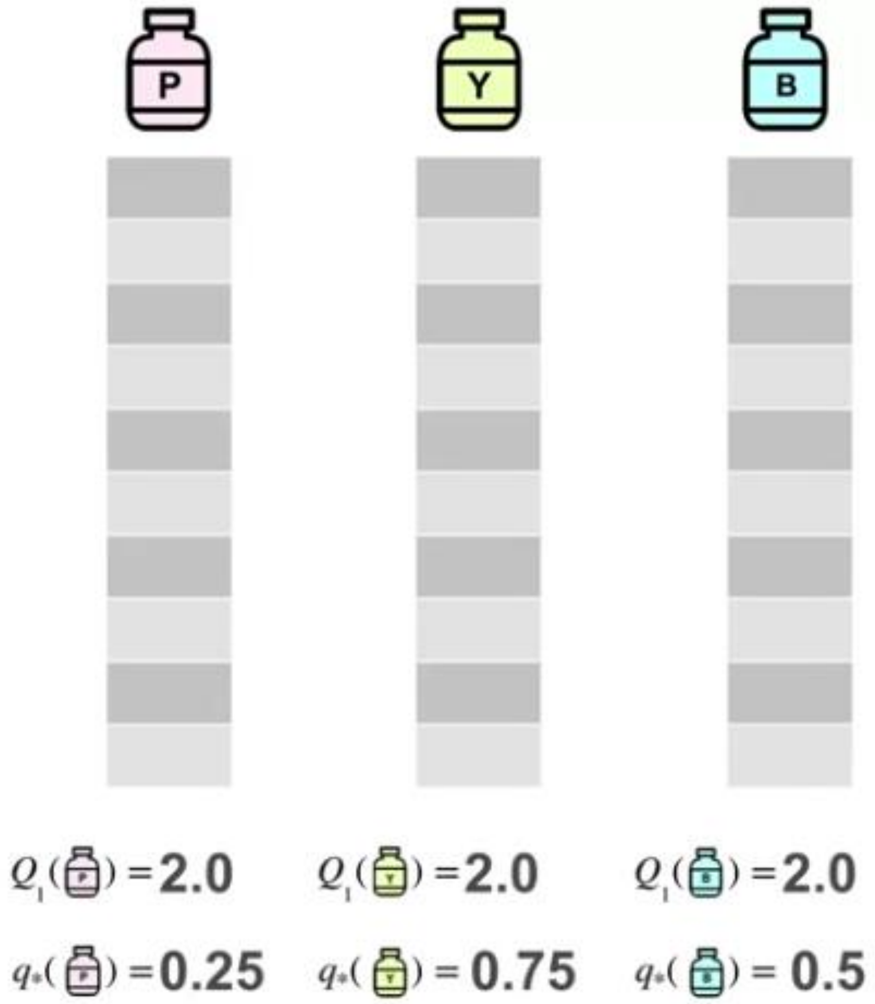
$$Q_1(\boxed{\text{Y}}) = 2.0$$

$$Q_1(\boxed{\text{B}}) = 2.0$$

A reward of 1 if the treatment succeeds otherwise 0

$$Q_{n+1} \leftarrow Q_n + \alpha\left(R_n - Q_n\right)$$

Let $\alpha = 0.5$



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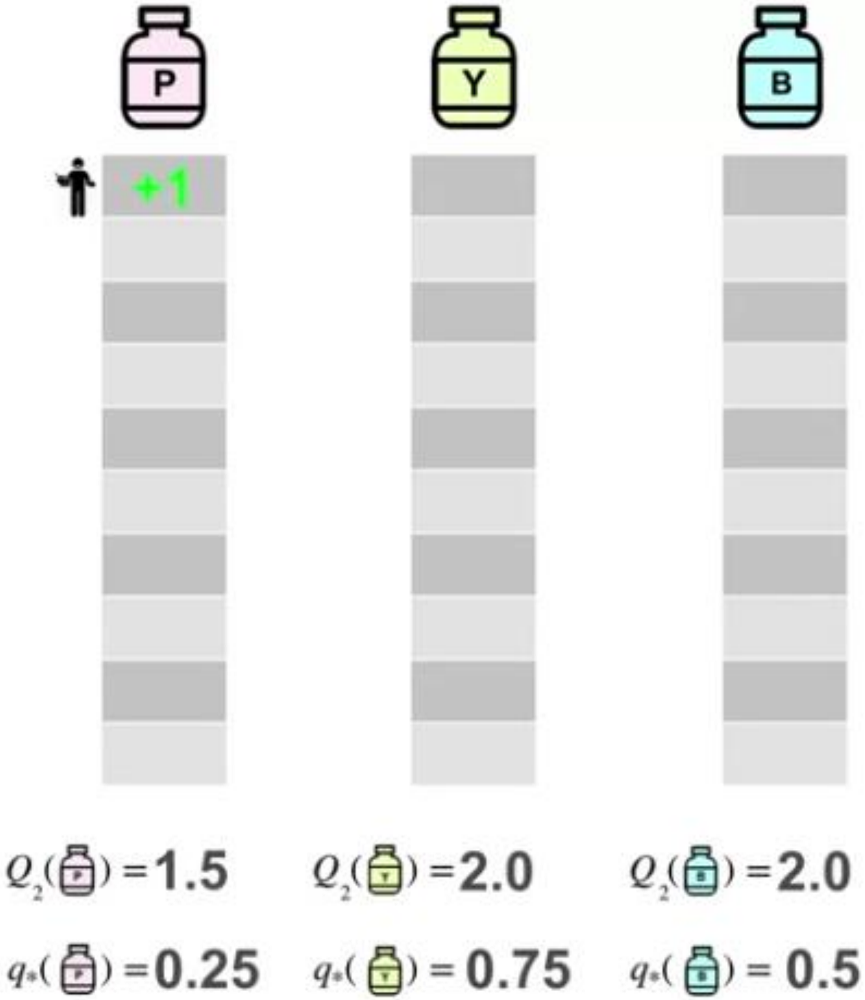

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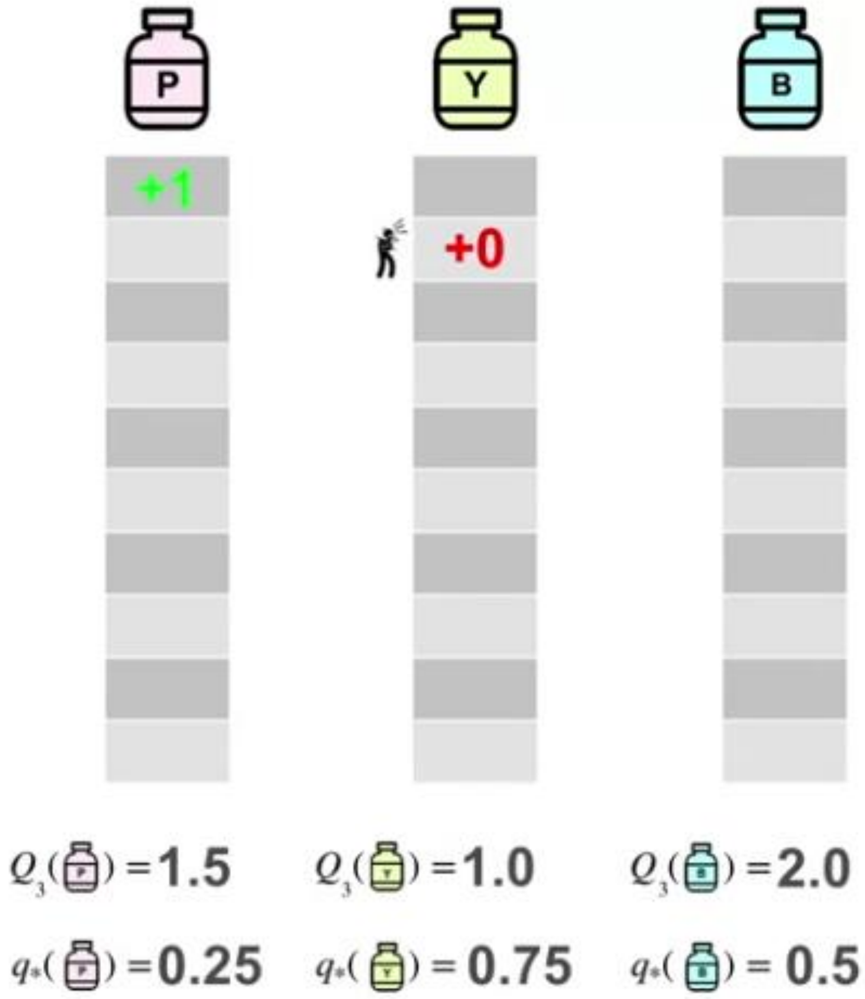

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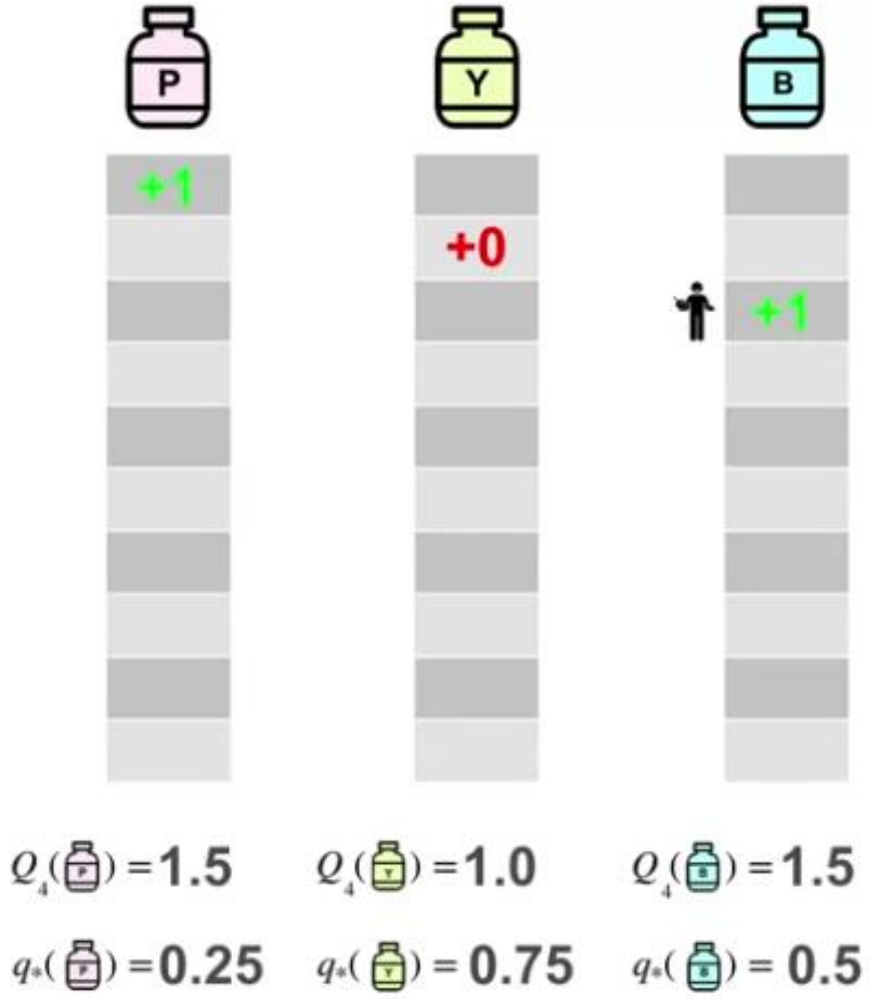

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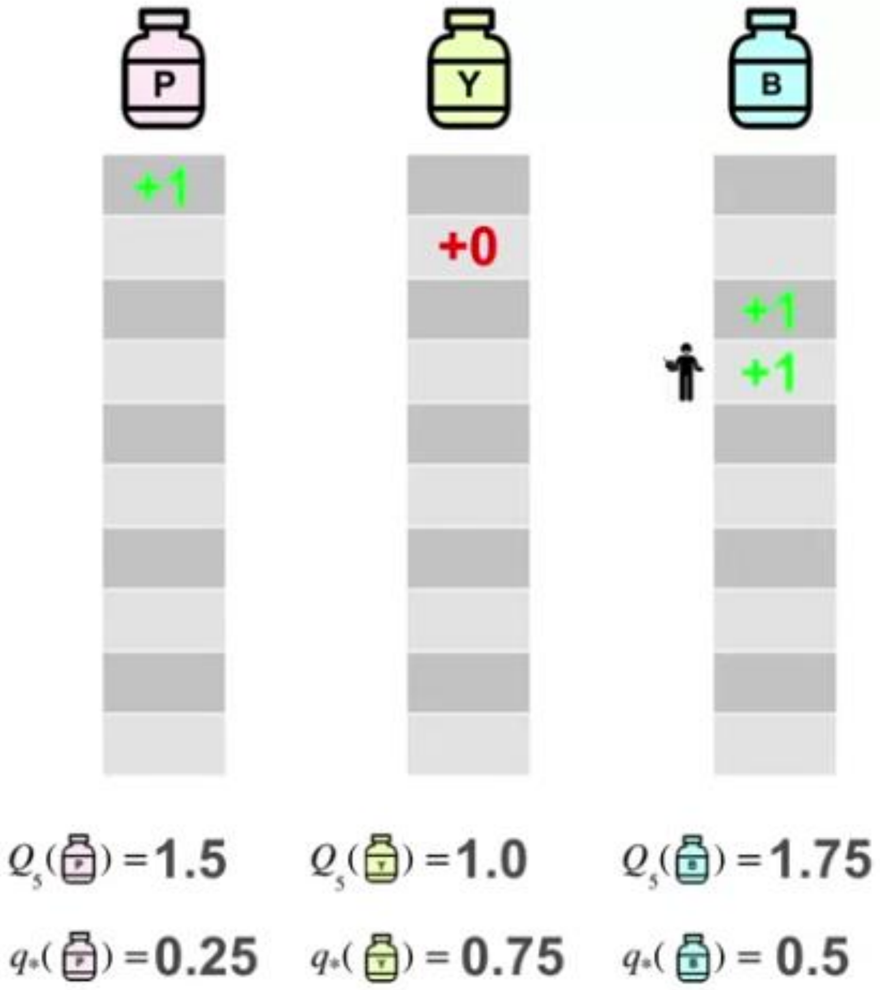

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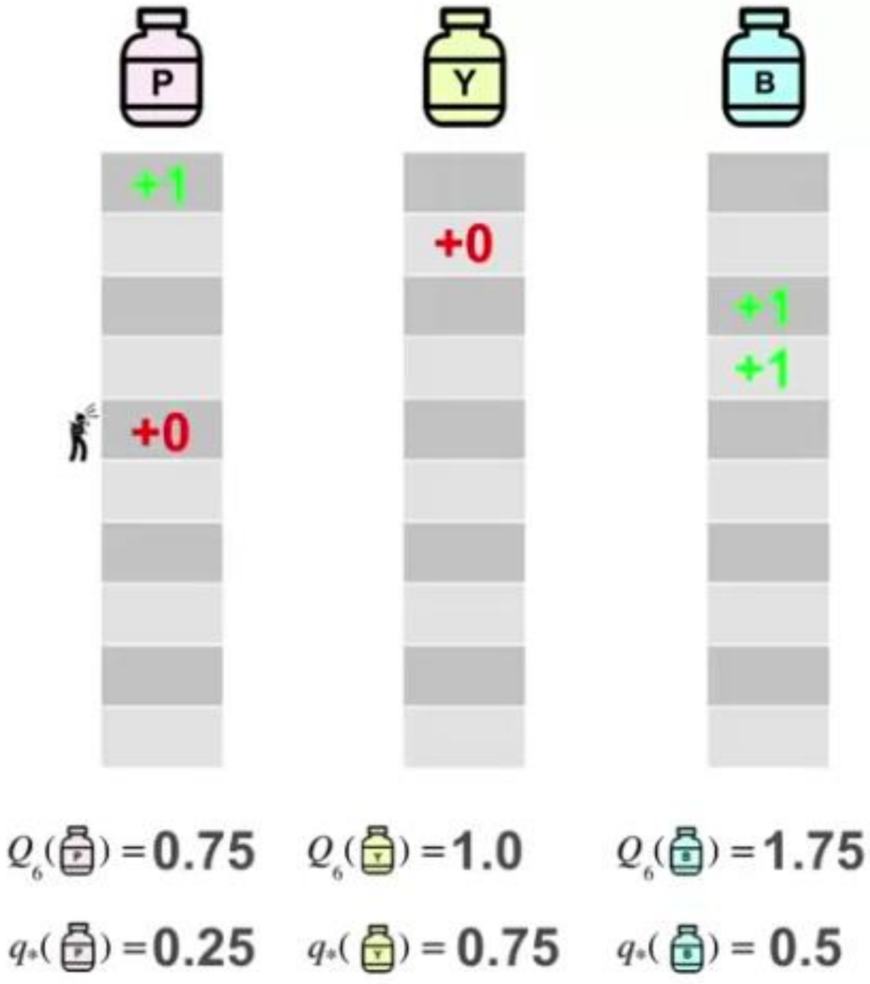

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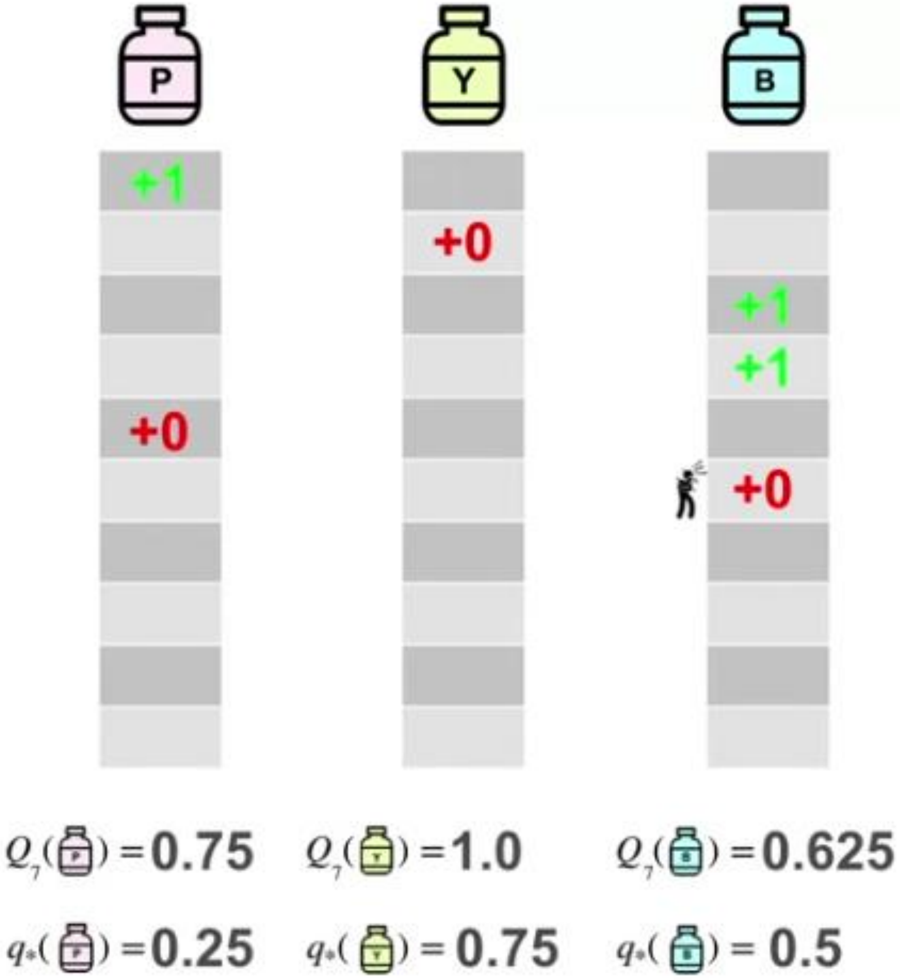

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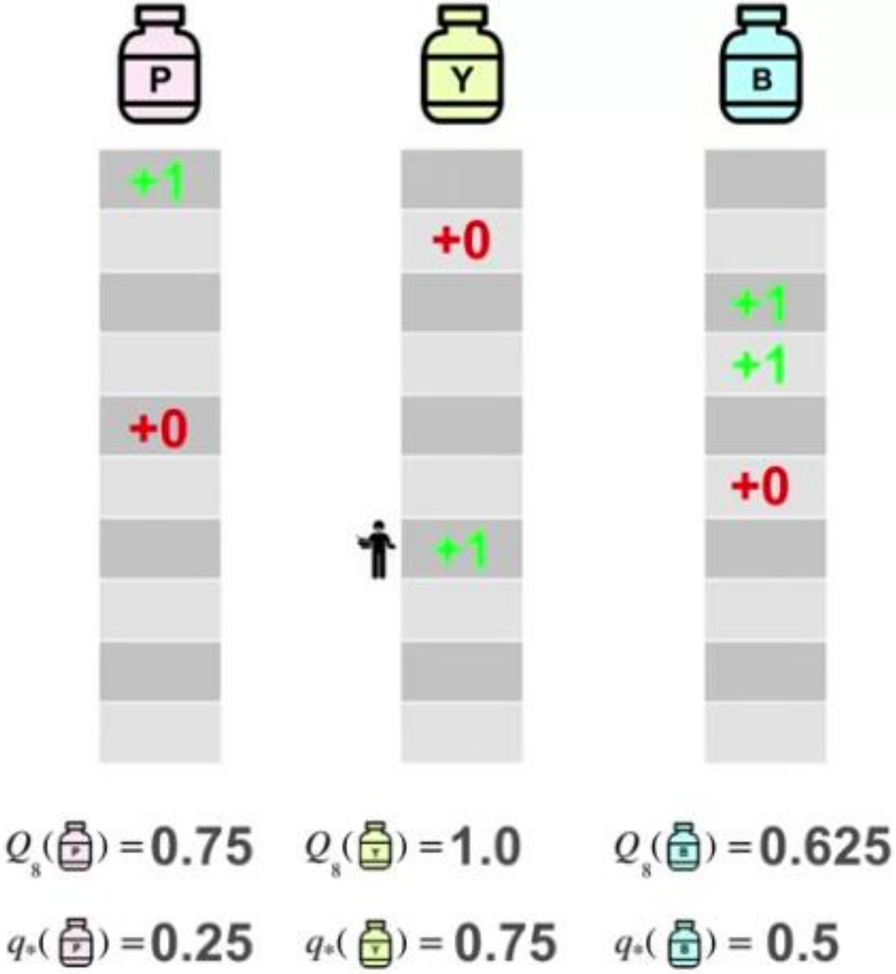

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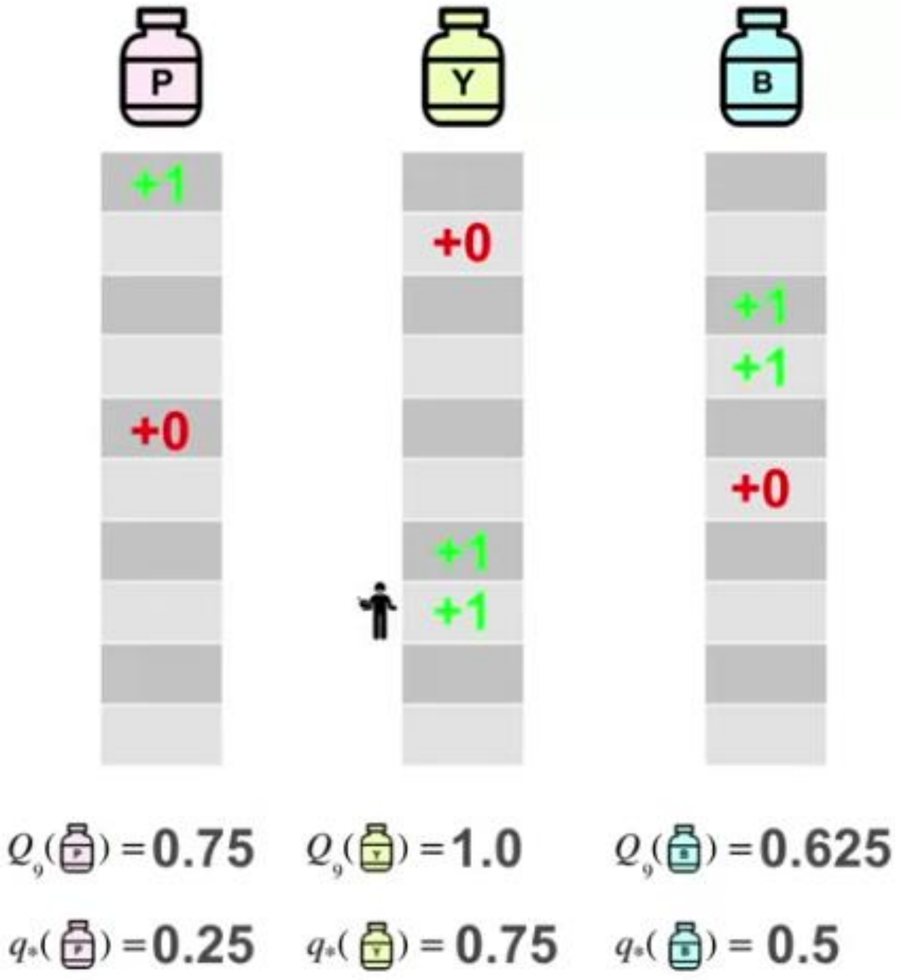

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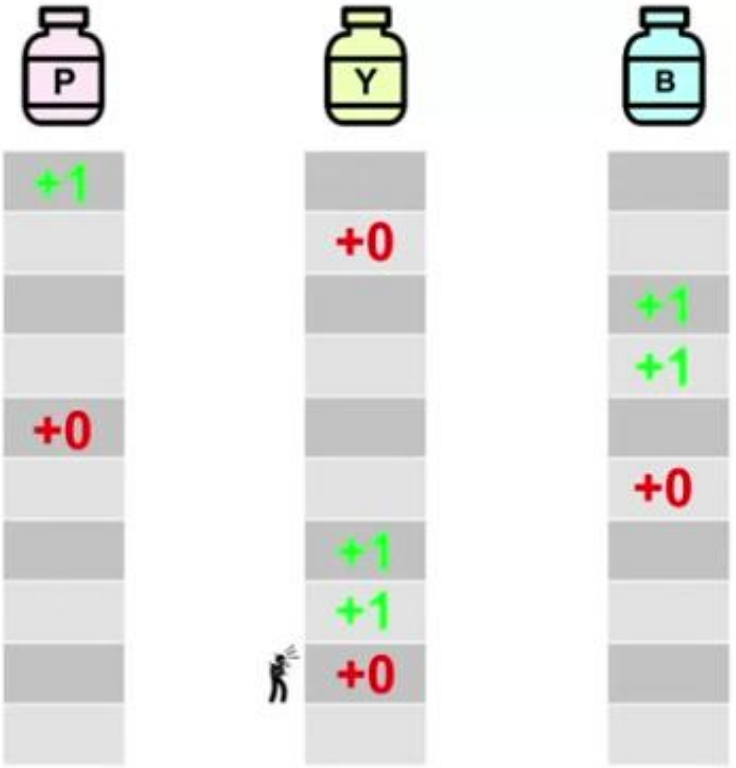


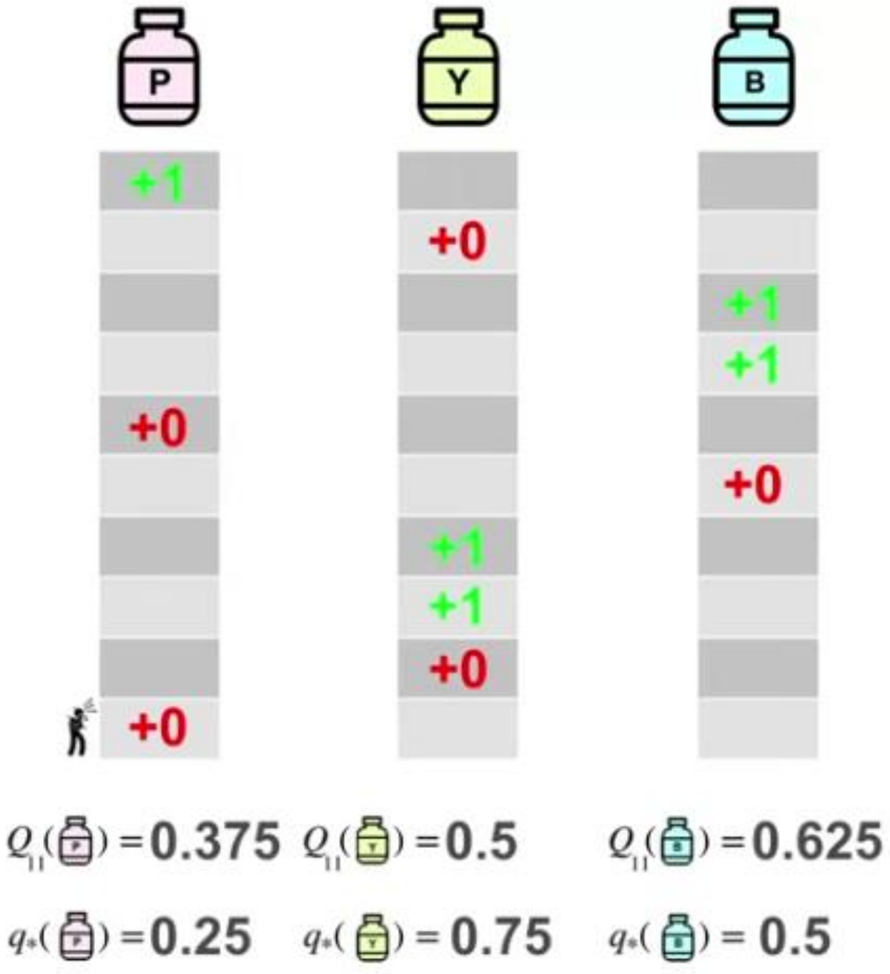
$Q_{10}(\text{P}) = 0.75$ $Q_{10}(\text{Y}) = 0.5$ $Q_{10}(\text{B}) = 0.625$

$q_*(\text{P}) = 0.25$ $q_*(\text{Y}) = 0.75$ $q_*(\text{B}) = 0.5$

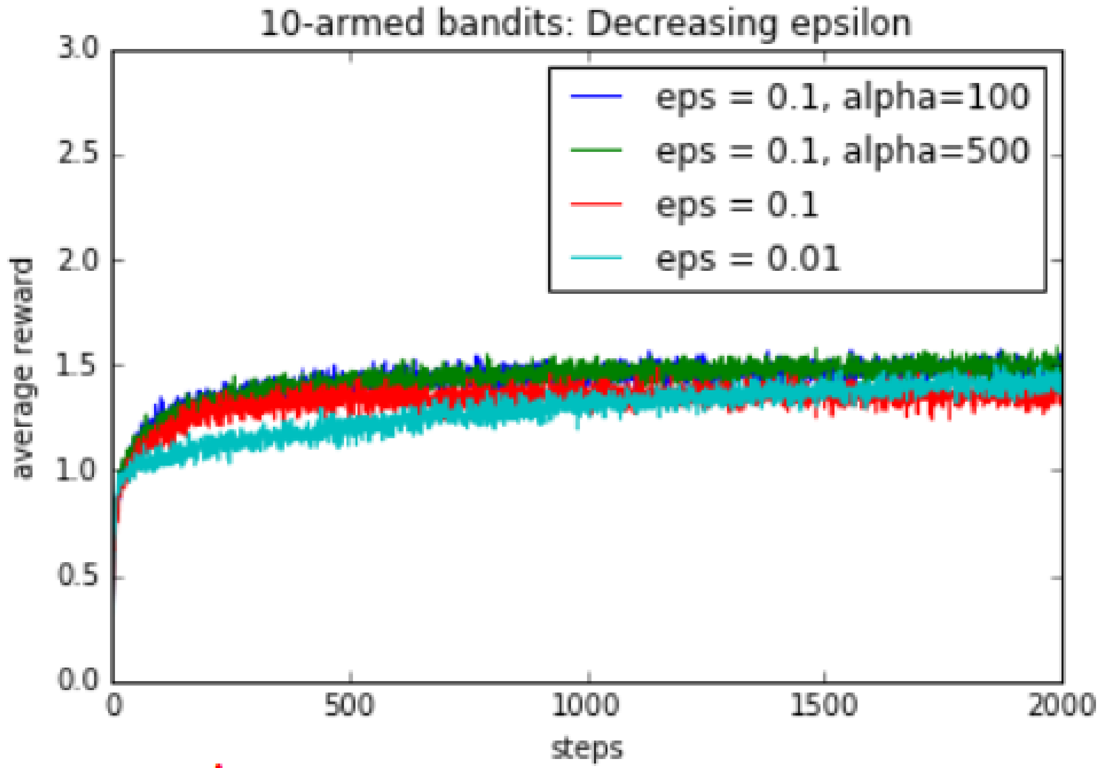
A reward of 1 if the treatment succeeds otherwise 0

$$Q_{n+1} \leftarrow Q_n + \alpha\left(R_n - Q_n\right)$$

Let $\alpha = 0.5$



Improve ε-greedy with decreasing ε over time



Decreasing over time:

- $\varepsilon_t = \varepsilon_t * (\alpha)/(t + \alpha)$
- Improves faster in the beginning, also outperforms fixed ε-greedy methods in the long run.

ε-GREEDY METHOD WEAKNESSES

- Randomly selects an action to explore, does not explore more “promising” actions.
- Does not consider prior experience with an action. If an action has been taken many times, there less need to explore it.

UPPER CONFIDENCE BOUND (UCB) ALGORITHM

- Take each action once.
- At any time *t* > *K*,

  - $A_t = \text{argmax}_a\left(Q_t(a) + c\sqrt{\frac{\ln t}{N_a(t)}}\right)$, where

  $\boldsymbol{Q_t(a)}$ is the average reward obtained from action a.
  $\boldsymbol{N_a(t)}$ is the number of times action a has been taken.
  c is a user-specified parameter that controls the amount exploration that takes place.

UPPER CONFIDENCE BOUND (UCB) ALGORITHM

$$A_t = \underset{a}{\operatorname{argmax}}\left(Q_t(a) + c\sqrt{\frac{\ln t}{N_a(t)}}\right),$$

Exploitation

Exploration

Upper-Confidence Bound (UCB) Action Selection

$$c\sqrt{\frac{ln\ t}{N_t(a)}} \rightarrow c\sqrt{\frac{ln \text{ timesteps}}{\text{times action } a \text{ taken}}}$$

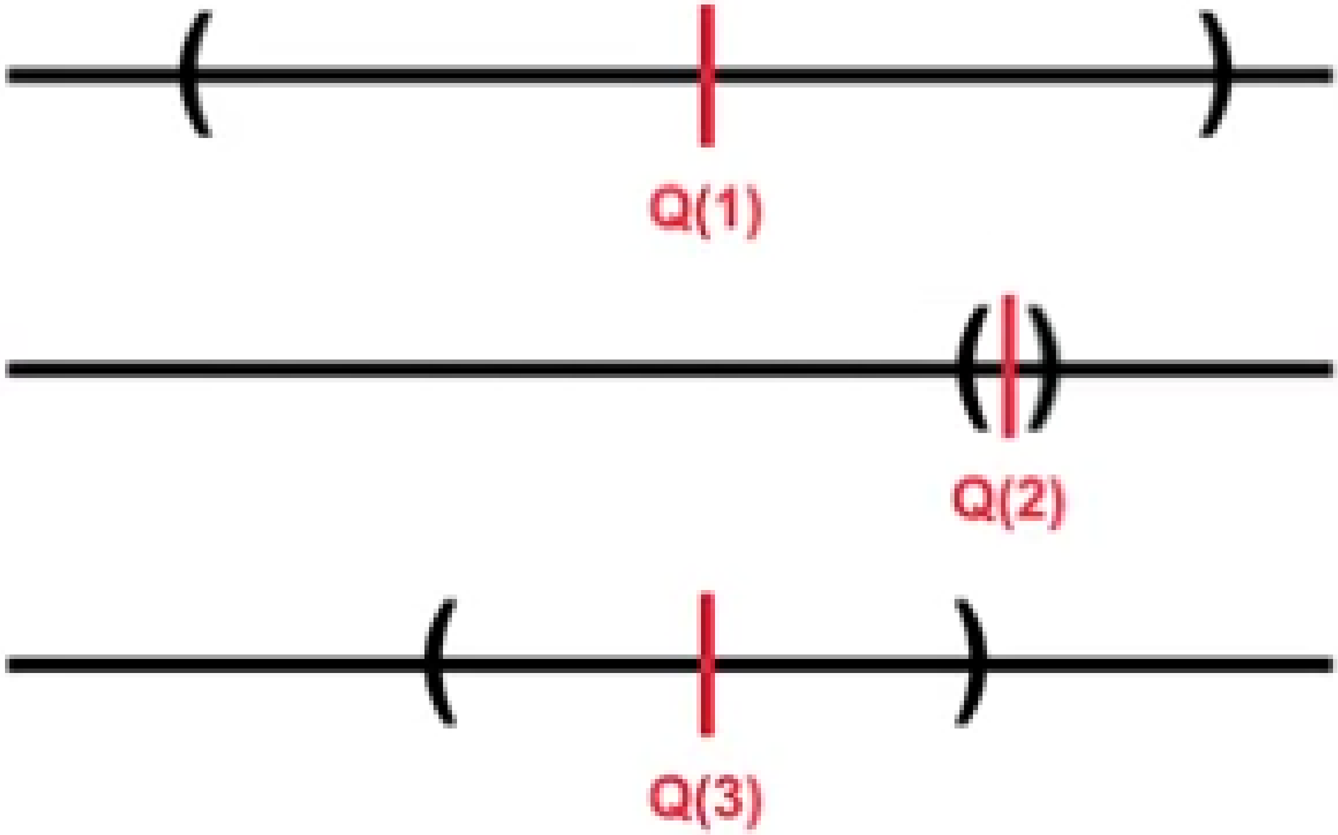
$$c\sqrt{\frac{ln\ 10000}{5000}} \rightarrow 0.043c$$

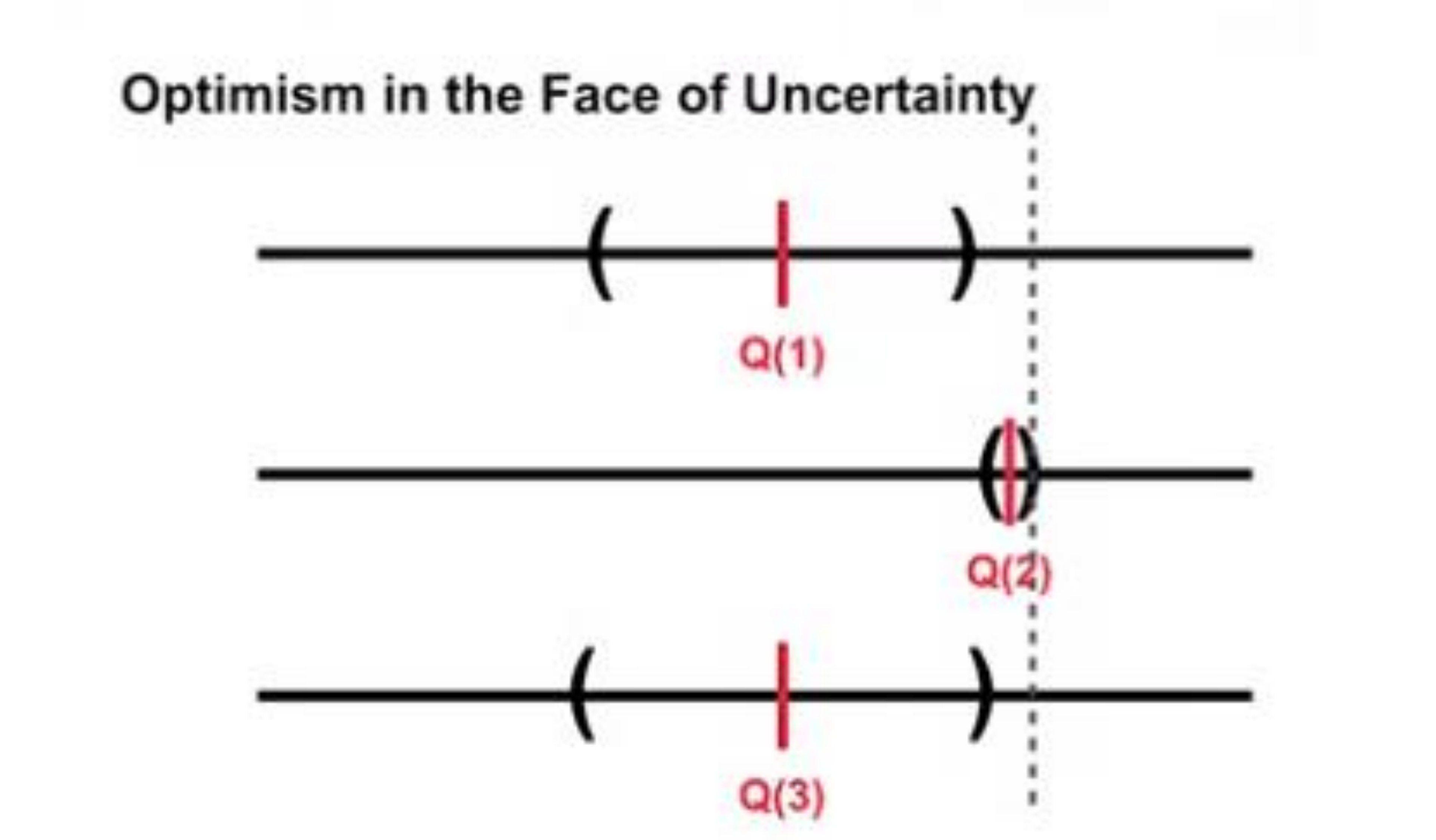
$$c\sqrt{\frac{ln\ 10000}{100}} \rightarrow 0.303c$$

UPPER CONFIDENCE BOUND (UCB) ALGORITHM

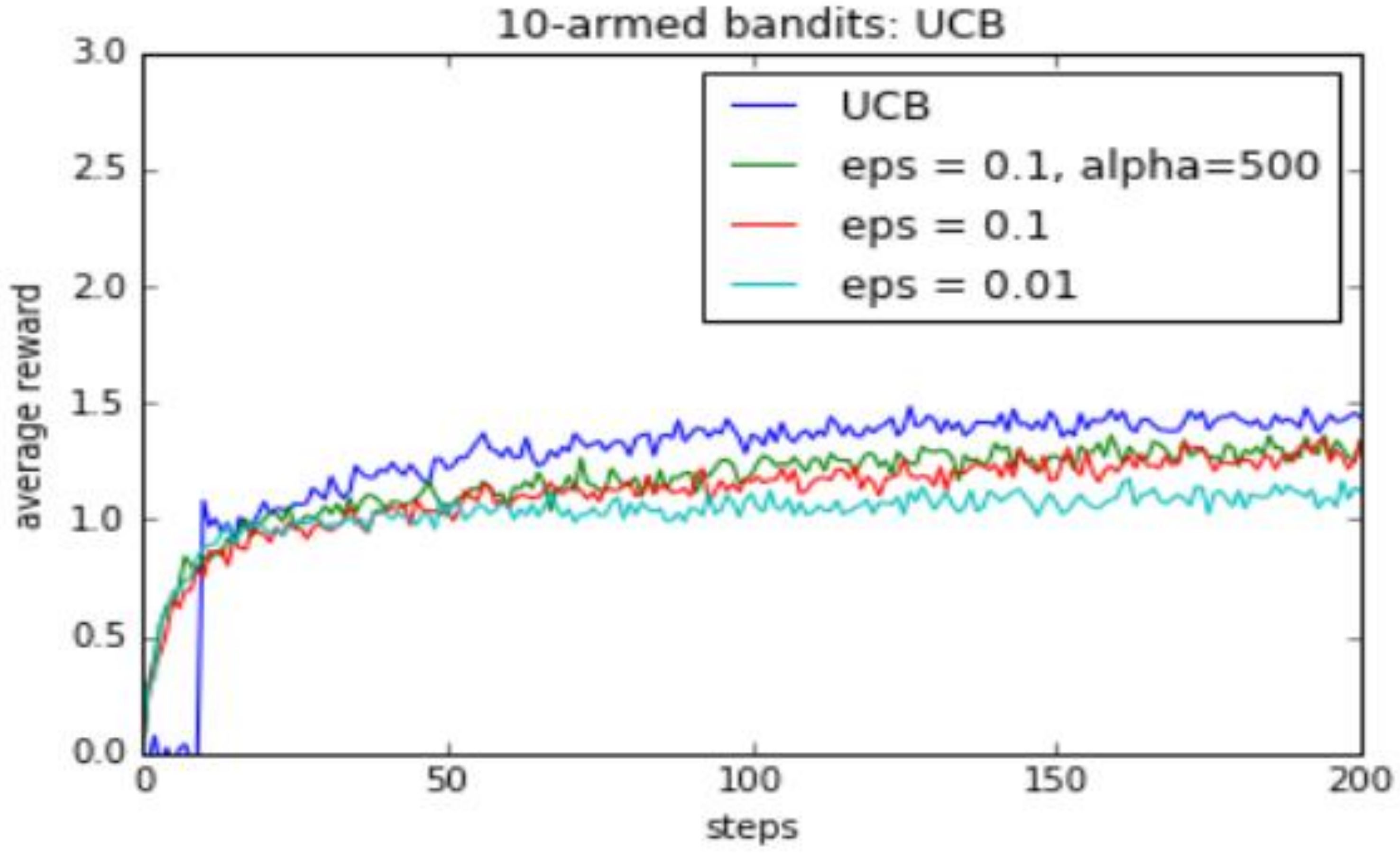
- $A_t = \underset{a}{\operatorname{argmax}}\left(Q_t(a) \; + c\sqrt{\frac{\ln t}{N_a(t)}}\right)$
- $c\sqrt{\frac{\ln t}{N_a(t)}}$ is the confidence interval for the average reward.
- Small $N_a(t)$ indicates a large confidence interval i.e., $Q_t(a)$ is more uncertain.
- Large $N_a(t)$ indicates a small confidence interval i.e., $Q_t(a)$ is more accurate.

Optimism in the Face of Uncertainty




Optimism in the Face of Uncertainty
Q(1)
Q(2)
Q(3)

How good is UCB? Experimental results



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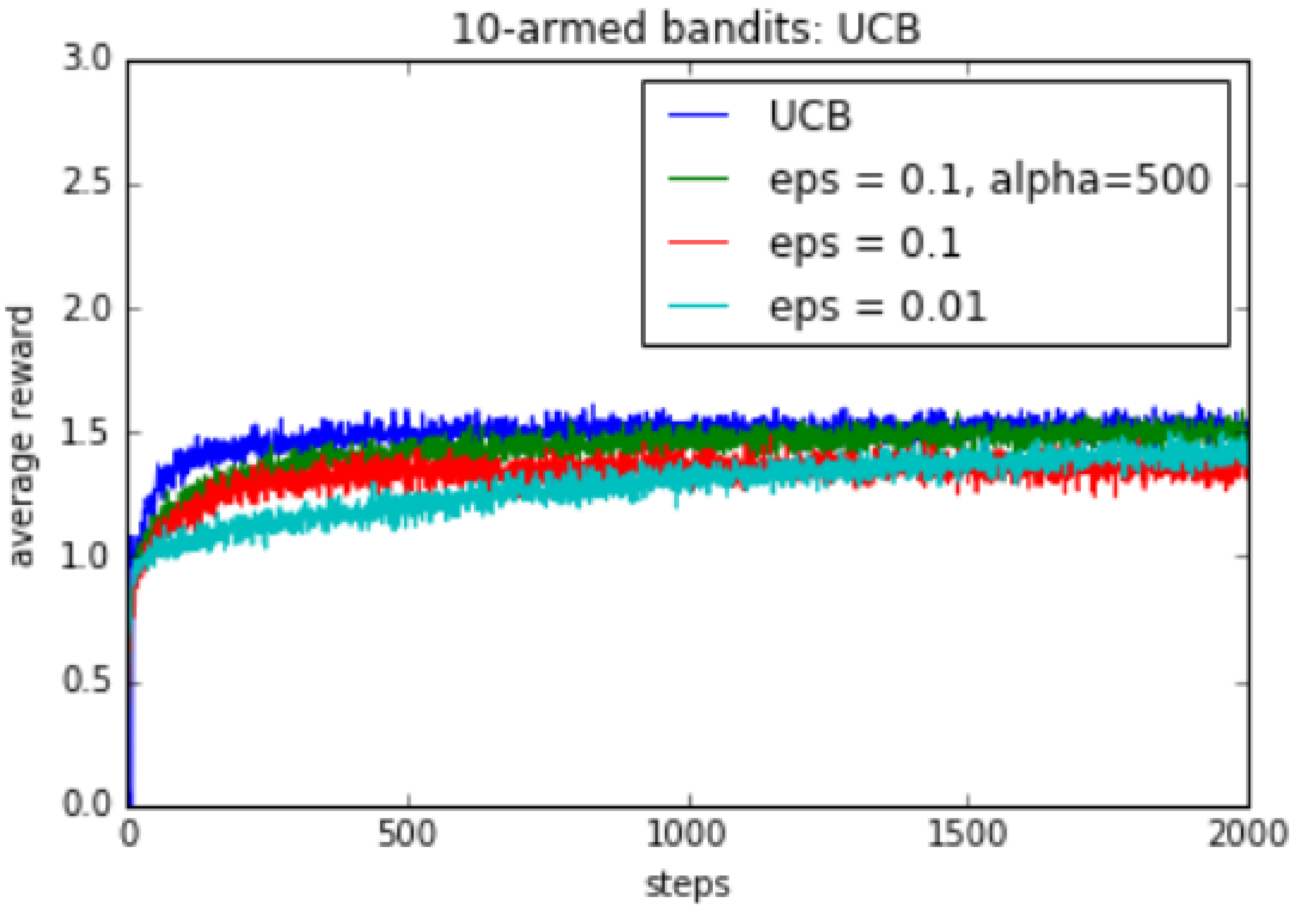

Observations: After initial 10 steps, improves quickly and outperforms ε-Greedy methods.

MULTI-ARMED BANDITS

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