

# DeepWalk: Online Learning of Social Representations

Bryan Perozzi, Rami Al-Rfou, Steven Skiena  
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## 목차

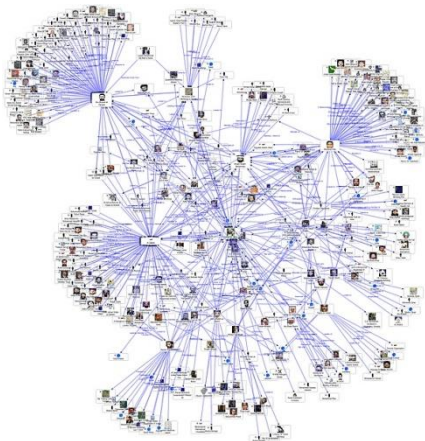
- 01 Introduction**
- 02 Problem Definition**
- 03 Background**
- 04 Method**
- 05 Experiments**
- 06 Code**
- 07 Conclusion**

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# 01 Introduction

## Graph

There are many graphs used in real world. We may want to execute tasks in Machine Learning/Deep Learning with these graph data.

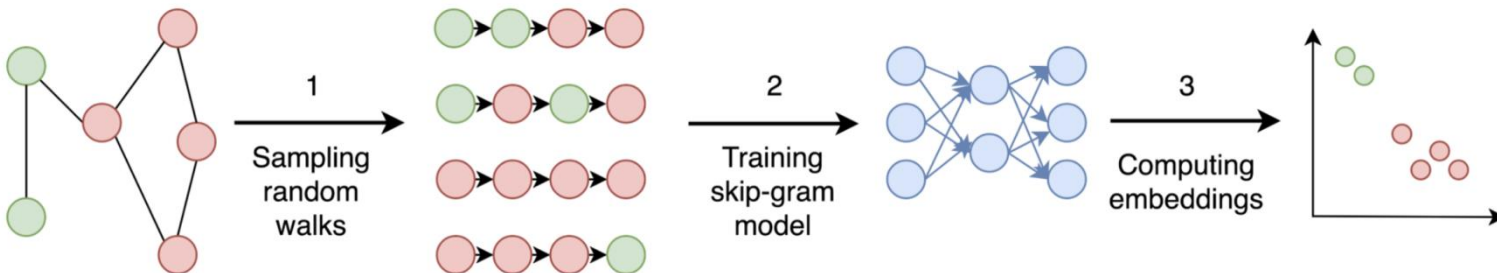


- network classification
- content recommendation
- anomaly detection
- ...

# 01 Introduction

## DeepWalk

- Deep walk is a graph embedding method.
- It uses method called “Random Walk” to make node sequence and use it on logics of language model to express the node in vector type.



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# 01 Introduction

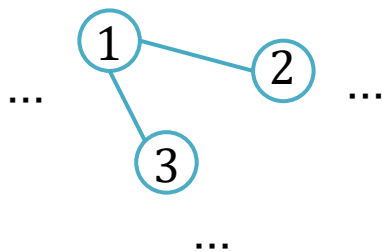
## DeepWalk's contribution

- learns the graph structure with short random walks.
- it's representation outperform its competitors with even less data.
- is parallelizable and can represent large web-scale graphs.

## 02 Problem Definition

### Goal

The goal of DeepWalk is to map nodes into an embedding space that has less dimension than the number of nodes in the graph.



Adjacency Matrix

1	0	1	1	
2	1	0	0	...
3	1	0	0	
				⋮

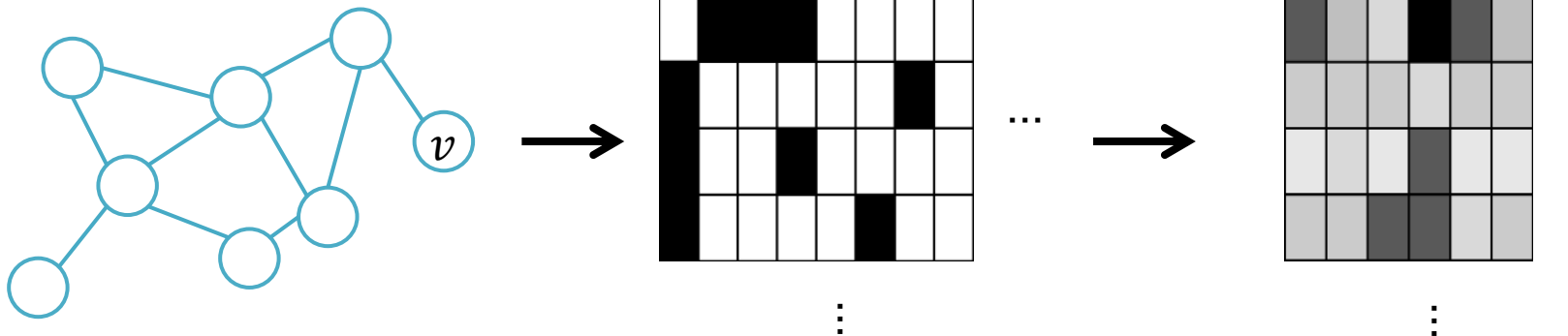
## 02 Problem Definition

### Goal

The goal of DeepWalk is to map nodes into an embedding space that has less dimension than the number of nodes in the graph.

**Input:** Graph = (V, E, X, Y) and HyperParameters (window size, embedding size ..)

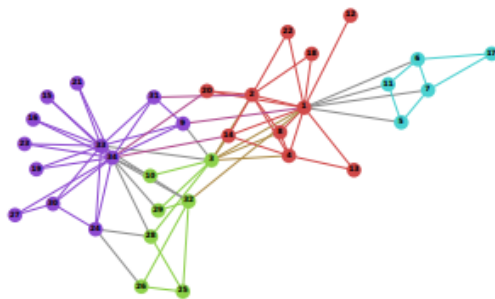
**Output:**  $\Phi: v \in V \rightarrow R^{|V|*d}$



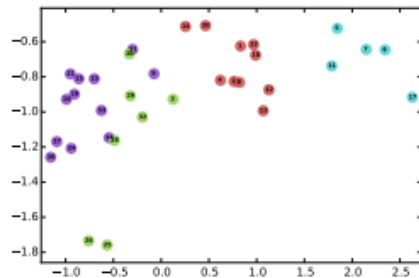
## 02 Problem Definition

### Goal

The similarity of a pair of nodes in the graph should correspond to their similarity in the embedding space as well.



(a) Input: Karate Graph



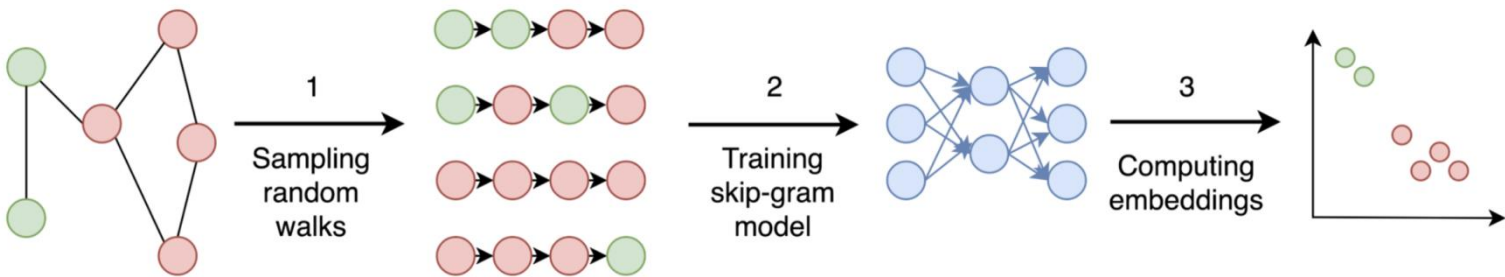
(b) Output: Representation



## 03 Background

### Learning Social Representation

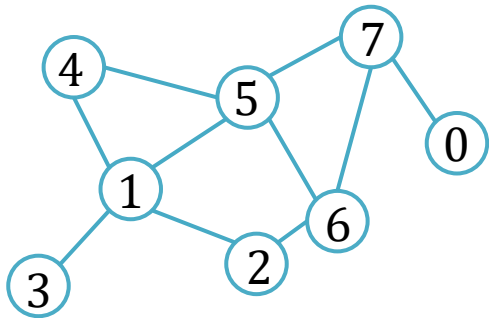
- RandomWalk
- Skip Gram
- Language Modeling



## 03 Background

### Random walk

- Random walk is a series of nodes, where next node is chosen randomly from the adjacent nodes.
- We can express random walk with  $Wv_i$  meaning a random walk starting at  $v_i$ .



if walk length is 5 then..

[1, 2, 6, 5, 4]

[5, 4, 1, 2, 6]

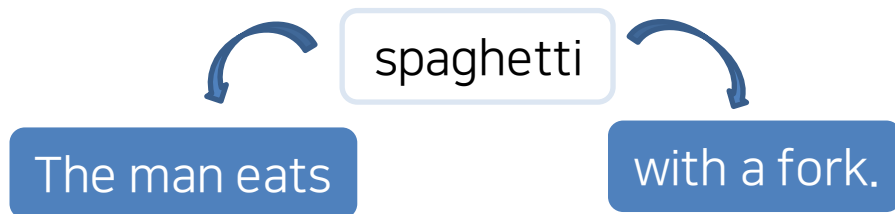
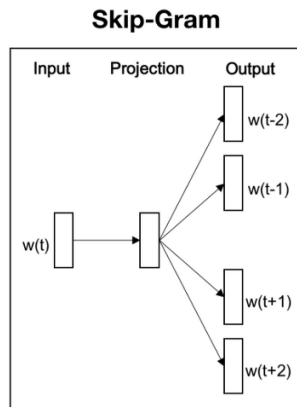
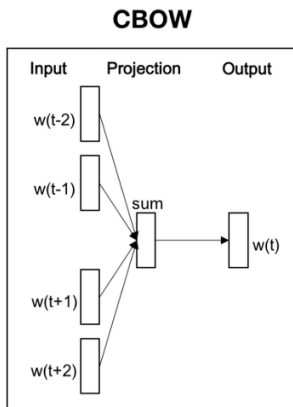
[7, 6, 5, 1, 3]

...

## 03 Background

### Skip-gram

- Word2Vec language model has two embedding algorithm methods.
- **CBOW** method tries to predict the center word based on the source of the context words.
- On the other hand, **Skip-gram** predicts the context words with the center word.



## 03 Background

### Skip-gram

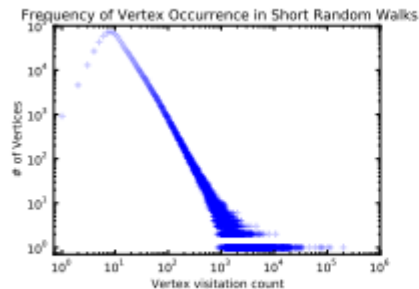
Language Model : Word – Sentence – Corpus

ex. ['The', 'man', 'eats', ... 'fork']

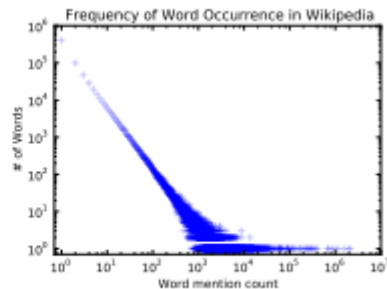
DeepWalk : Vertex – Random walk – Random walks

ex. [1, 3, 5, 4, ... Vt]

The language corpus word frequency follows power law and vertex frequency also follows the power law.



(a) YouTube Social Graph



(b) Wikipedia Article Text

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## 03 Background

### Language Model

Language Model : *maximize*  $\Pr(w_n | w_0, w_1, \dots, w_{n-1})$

DeepWalk : *maximize*  $\Pr(v_i | \Phi(v_1), \Phi(v_2), \dots, \Phi(v_{i-1}))$

However, when there are lots of vertex, the computational cost grow exponentially. Therefore, the function changes to calculating probability of context words of  $v_i$ .

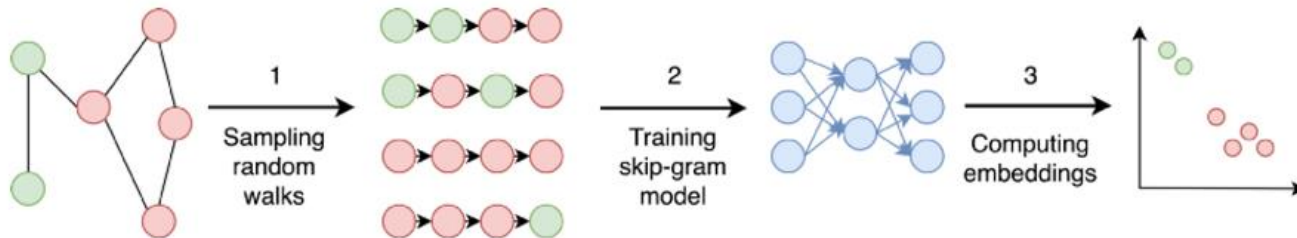
$$\text{minimize } \Phi - \log \Pr(\{v_{i-w}, \dots, v_{i-1}, v_{i+1}, \dots, v_{i+w}\} | \Phi(v_i))$$

## 04 Method

### DeepWalk Algorithm

DeepWalk consists of two parts :

1. Random walk generator
2. Skip-gram



## 04 Method

### DeepWalk Algorithm

Input:  $Graph = (V, E, X, Y)$  and HyperParameters

- window size  $w$
- embedding size  $d$
- walks per vertex  $\gamma$
- walk length  $t$

Output:  $\Phi: v \in V \rightarrow \mathbb{R}^{|V| \times d}$

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**Algorithm 1** DEEPWALK( $G, w, d, \gamma, t$ )

---

**Input:** graph  $G(V, E)$

    window size  $w$

    embedding size  $d$

    walks per vertex  $\gamma$

    walk length  $t$

**Output:** matrix of vertex representations  $\Phi \in \mathbb{R}^{|V| \times d}$

1: Initialization: Sample  $\Phi$  from  $\mathcal{U}^{|V| \times d}$

2: Build a binary Tree  $T$  from  $V$

3: **for**  $i = 0$  to  $\gamma$  **do**

4:    $\mathcal{O} = \text{Shuffle}(V)$

5:   **for each**  $v_i \in \mathcal{O}$  **do**

6:      $\mathcal{W}_{v_i} = \text{RandomWalk}(G, v_i, t)$

7:      $\text{SkipGram}(\Phi, \mathcal{W}_{v_i}, w)$

8:   **end for**

9: **end for**

---

## 04 Method

### Hierarchical Softmax

- utilizes a multi-layer binary tree where the probability of a word is calculated on whether it goes to left or right edge
- an alternative to *softmax* but faster to evaluate
- $O(\log n)$  time compared to  $O(n)$  for *softmax*
- can use Huffman coding to reduce the access time of frequent elements in the tree

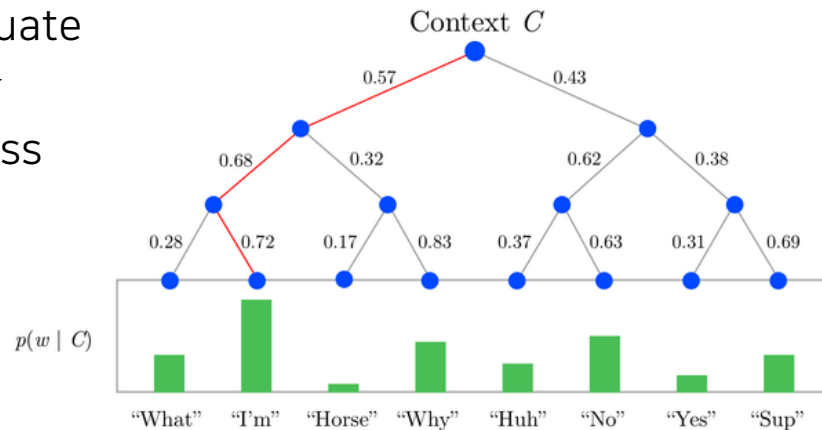
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**Algorithm 1** DEEPWALK( $G, w, d, \gamma, t$ )

---

**Input:** graph  $G(V, E)$ window size  $w$ embedding size  $d$ walks per vertex  $\gamma$ walk length  $t$ **Output:** matrix of vertex representations  $\Phi \in \mathbb{R}^{|V| \times d}$ 1: Initialization: Sample  $\Phi$  from  $\mathcal{U}^{|V| \times d}$ 2: Build a binary Tree  $T$  from  $V$ 3: **for**  $i = 0$  to  $\gamma$  **do**4:    $\mathcal{O} = \text{Shuffle}(V)$ 5:   **for each**  $v_i \in \mathcal{O}$  **do**6:      $\mathcal{W}_{v_i} = \text{RandomWalk}(G, v_i, t)$ 7:     SkipGram( $\Phi, \mathcal{W}_{v_i}, w$ )8:   **end for**9: **end for**

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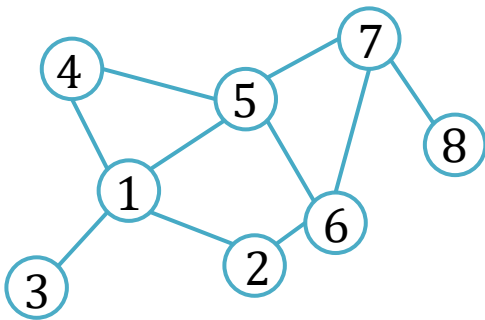




## 04 Method

### Outer loop with $\gamma$

- Outer loop depends on walk per vertex  $\gamma$
- Shuffle the node order of original graph



[1, 5, 7, ...]

---

**Algorithm 1** DEEPWALK( $G, w, d, \gamma, t$ )

---

**Input:** graph  $G(V, E)$

    window size  $w$

    embedding size  $d$

    walks per vertex  $\gamma$

    walk length  $t$

**Output:** matrix of vertex representations  $\Phi \in \mathbb{R}^{|V| \times d}$

1: Initialization: Sample  $\Phi$  from  $\mathcal{U}^{|V| \times d}$

2: Build a binary Tree  $T$  from  $V$

3: **for**  $i = 0$  to  $\gamma$  **do**

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5:   **for each**  $v_i \in \mathcal{O}$  **do**

6:      $\mathcal{W}_{v_i} = \text{RandomWalk}(G, v_i, t)$

7:     SkipGram( $\Phi, \mathcal{W}_{v_i}, w$ )

8:   **end for**

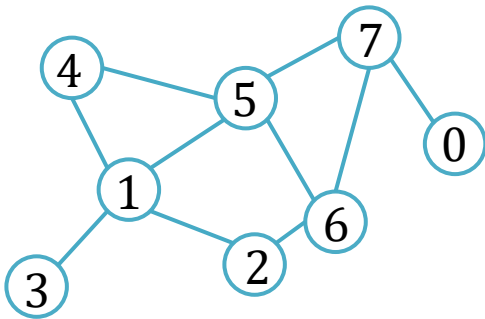
9: **end for**

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## 04 Method

### Inner loop with RandomWalk

- Inner loop depends on the number of node
- It makes a random walk with length  $t$  for  $v_i$



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**Algorithm 1** DEEPWALK( $G, w, d, \gamma, t$ )

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**Input:** graph  $G(V, E)$ window size  $w$ embedding size  $d$ walks per vertex  $\gamma$ walk length  $t$ **Output:** matrix of vertex representations  $\Phi \in \mathbb{R}^{|V| \times d}$ 1: Initialization: Sample  $\Phi$  from  $\mathcal{U}^{|V| \times d}$ 2: Build a binary Tree  $T$  from  $V$ 3: **for**  $i = 0$  to  $\gamma$  **do**4:    $\mathcal{O} = \text{Shuffle}(V)$ 5:   **for each**  $v_i \in \mathcal{O}$  **do**6:      $\mathcal{W}_{v_i} = \text{RandomWalk}(G, v_i, t)$ 7:     SkipGram( $\Phi, \mathcal{W}_{v_i}, w$ )8:   **end for**9: **end for**

---

if walk length is 5 then..

[1, 2, 6, 5, 4]

[5, 4, 1, 2, 6]

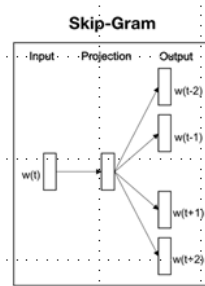
[7, 6, 5, 1, 3]

...

## 04 Method

### SkipGram

- analyze a single vertex to maximize the probability of the surrounding nodes
- In general, it has to modify all node vectors in the vocabulary but if the vocabulary size is huge, it will take a long time to run
- Therefore, we can use methods like Hierarchical Softmax or negative sampling.



---

#### Algorithm 1 DEEPWALK( $G, w, d, \gamma, t$ )

---

**Input:** graph  $G(V, E)$   
window size  $w$   
embedding size  $d$   
walks per vertex  $\gamma$   
walk length  $t$

**Output:** matrix of vertex representations  $\Phi \in \mathbb{R}^{|V| \times d}$

```
1: Initialization: Sample  $\Phi$  from  $\mathcal{U}^{|V| \times d}$ 
2: Build a binary Tree  $T$  from  $V$ 
3: for  $i = 0$  to  $\gamma$  do
4:    $\mathcal{O} = \text{Shuffle}(V)$ 
5:   for each  $v_i \in \mathcal{O}$  do
6:      $\mathcal{W}_{v_i} = \text{RandomWalk}(G, v_i, t)$ 
7:     SkipGram( $\Phi, \mathcal{W}_{v_i}, w$ )
8:   end for
9: end for
```

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#### Algorithm 2 SkipGram( $\Phi, \mathcal{W}_{v_i}, w$ )

---

```
1: for each  $v_j \in \mathcal{W}_{v_i}$  do
2:   for each  $u_k \in \mathcal{W}_{v_i}[j - w : j + w]$  do
3:      $J(\Phi) = -\log \Pr(u_k | \Phi(v_j))$ 
4:      $\Phi = \Phi - \alpha * \frac{\partial J}{\partial \Phi}$ 
5:   end for
6: end for
```

---

## 04 Method

### Parallelizability

- Researcher found out that DeepWalk is parallelizable

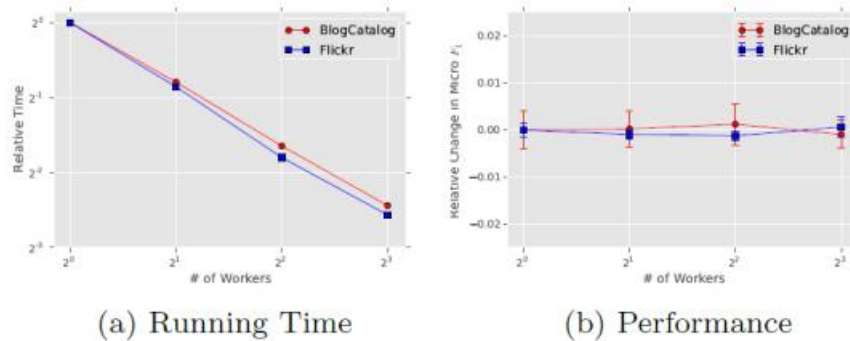


Figure 4: Effects of parallelizing DEEPWALK

## 05 Experiments

### Task “Node Classification”

- a task to predict unlabeled nodes.
- Datasets are **BlogCatalog**, **Flickr**, **YouTube**.
- For baseline models, they used
  - SpectralClustering
  - MaxModularity
  - EdgeCluster(K-means)
  - weighted vote Relational Neighbor(wvRN)
  - Majority

Name	BLOGCATALOG	FLICKR	YOUTUBE
$ V $	10,312	80,513	1,138,499
$ E $	333,983	5,899,882	2,990,443
$ Y $	39	195	47
Labels	Interests	Groups	Groups

Table 1: Graphs used in our experiments.

# 05 Experiments

## BlogCatalog

- In this experiment, they varied the labeled nodes from 10~90%.
- DeepWalk showed better performance where there were few labeled nodes.

	% Labeled Nodes	10%	20%	30%	40%	50%	60%	70%	80%	90%
Micro-F1(%)	DEEPWALK	36.00	38.20	39.60	40.30	41.00	41.30	41.50	41.50	42.00
	SpectralClustering	31.06	34.95	37.27	38.93	39.97	40.99	41.66	42.42	42.62
	EdgeCluster	27.94	30.76	31.85	32.99	34.12	35.00	34.63	35.99	36.29
	Modularity	27.35	30.74	31.77	32.97	34.09	36.13	36.08	37.23	38.18
	wvRN	19.51	24.34	25.62	28.82	30.37	31.81	32.19	33.33	34.28
	Majority	16.51	16.66	16.61	16.70	16.91	16.99	16.92	16.49	17.26
Macro-F1(%)	DEEPWALK	21.30	23.80	25.30	26.30	27.30	27.60	27.90	28.20	28.90
	SpectralClustering	19.14	23.57	25.97	27.46	28.31	29.46	30.13	31.38	31.78
	EdgeCluster	16.16	19.16	20.48	22.00	23.00	23.64	23.82	24.61	24.92
	Modularity	17.36	20.00	20.80	21.85	22.65	23.41	23.89	24.20	24.97
	wvRN	6.25	10.13	11.64	14.24	15.86	17.18	17.98	18.86	19.57
	Majority	2.52	2.55	2.52	2.58	2.58	2.63	2.61	2.48	2.62

Table 2: Multi-label classification results in BLOGCATALOG

# 05 Experiments

## Flickr

- In this experiment, they varied the labeled nodes from 1~10%.
- DeepWalk showed better performance on cases on Flickr.

	% Labeled Nodes	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
Micro-F1(%)	DEEPWALK	<b>32.4</b>	<b>34.6</b>	<b>35.9</b>	<b>36.7</b>	<b>37.2</b>	<b>37.7</b>	<b>38.1</b>	<b>38.3</b>	<b>38.5</b>	<b>38.7</b>
	SpectralClustering	27.43	30.11	31.63	32.69	33.31	33.95	34.46	34.81	35.14	35.41
	EdgeCluster	25.75	28.53	29.14	30.31	30.85	31.53	31.75	31.76	32.19	32.84
	Modularity	22.75	25.29	27.3	27.6	28.05	29.33	29.43	28.89	29.17	29.2
	wvRN	17.7	14.43	15.72	20.97	19.83	19.42	19.22	21.25	22.51	22.73
	Majority	16.34	16.31	16.34	16.46	16.65	16.44	16.38	16.62	16.67	16.71
Macro-F1(%)	DEEPWALK	<b>14.0</b>	<b>17.3</b>	<b>19.6</b>	<b>21.1</b>	<b>22.1</b>	<b>22.9</b>	<b>23.6</b>	<b>24.1</b>	<b>24.6</b>	<b>25.0</b>
	SpectralClustering	13.84	<b>17.49</b>	19.44	20.75	21.60	22.36	23.01	23.36	23.82	24.05
	EdgeCluster	10.52	14.10	15.91	16.72	18.01	18.54	19.54	20.18	20.78	20.85
	Modularity	10.21	13.37	15.24	15.11	16.14	16.64	17.02	17.1	17.14	17.12
	wvRN	1.53	2.46	2.91	3.47	4.95	5.56	5.82	6.59	8.00	7.26
	Majority	0.45	0.44	0.45	0.46	0.47	0.44	0.45	0.47	0.47	0.47

Table 3: Multi-label classification results in FLICKR

# 05 Experiments

## YouTube

- YouTube dataset is close to a real world graph.
- It is so huge that SpectralClustering and Modularity couldn't train on it
- Result shows that DeepWalk can scale to large graphs and perform exceedingly well in sparsely labeled graph

	% Labeled Nodes	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
Micro-F1(%)	DEEPWALK	37.95	39.28	40.08	40.78	41.32	41.72	42.12	42.48	42.78	43.05
	SpectralClustering	—	—	—	—	—	—	—	—	—	—
	EdgeCluster	23.90	31.68	35.53	36.76	37.81	38.63	38.94	39.46	39.92	40.07
	Modularity	—	—	—	—	—	—	—	—	—	—
	wvRN	26.79	29.18	33.1	32.88	35.76	37.38	38.21	37.75	38.68	39.42
	Majority	24.90	24.84	25.25	25.23	25.22	25.33	25.31	25.34	25.38	25.38
Macro-F1(%)	DEEPWALK	29.22	31.83	33.06	33.90	34.35	34.66	34.96	35.22	35.42	35.67
	SpectralClustering	—	—	—	—	—	—	—	—	—	—
	EdgeCluster	19.48	25.01	28.15	29.17	29.82	30.65	30.75	31.23	31.45	31.54
	Modularity	—	—	—	—	—	—	—	—	—	—
	wvRN	13.15	15.78	19.66	20.9	23.31	25.43	27.08	26.48	28.33	28.89
	Majority	6.12	5.86	6.21	6.1	6.07	6.19	6.17	6.16	6.18	6.19

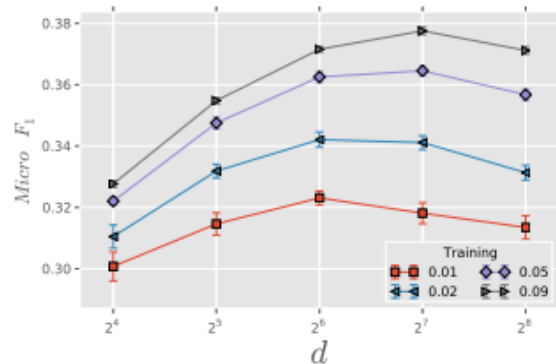
Table 4: Multi-label classification results in YOUTUBE



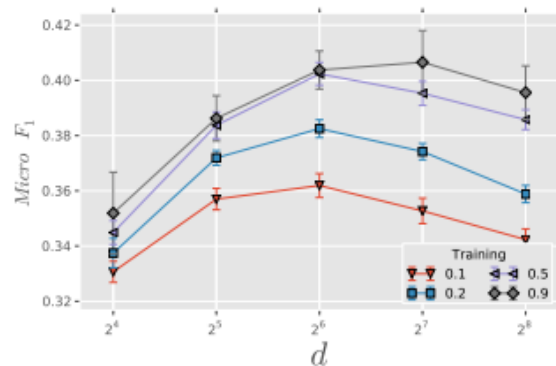
## 05 Experiments

### Parameter sensitivity

- Test on parameters (number of walk per vertex  $\gamma$ , amount of training data  $T_R$  and dimensions  $d$  on x-axis)
- More data, the better



(a1) FLICKR,  $\gamma = 30$

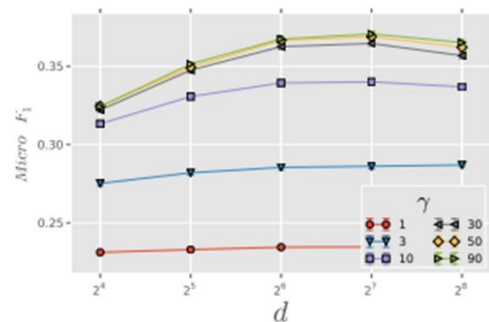


(a3) BLOGCATALOG,  $\gamma = 30$

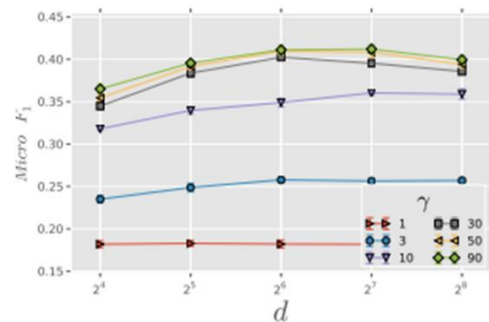
# 05 Experiments

## Parameter sensitivity

- In Flickr and BlogCatalog dataset, when  $\gamma$  was 30, it shows most efficient performance
- When the value  $\gamma$  is fixed, the performance between different dimensions is relatively stable.



(a2) FLICKR,  $T_R = 0.05$

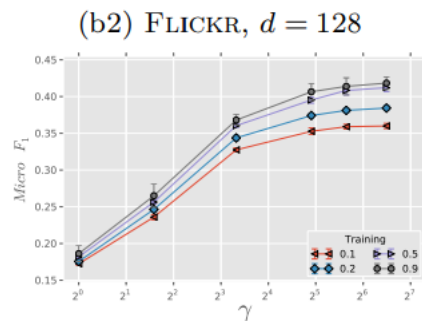
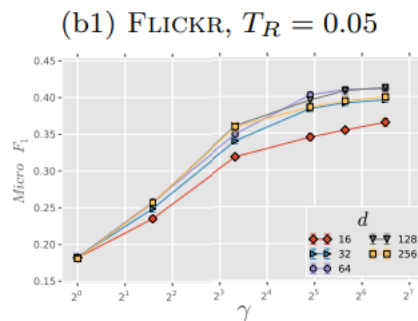
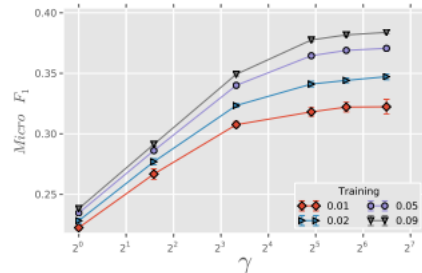
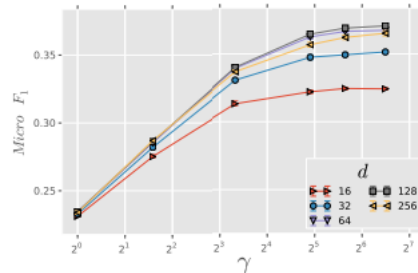


(a4) BLOGCATALOG,  $T_R = 0.5$

# 05 Experiments

## Sampling frequency

- Increasing  $\gamma$  has effects but quickly slows down when  $\gamma$  is over 10.



(b3) BLOGCATALOG,  $T_R = 0.5$       (b4) BLOGCATALOG,  $d = 128$

(a) Stability over number of walks,  $\gamma$

## 06 Code

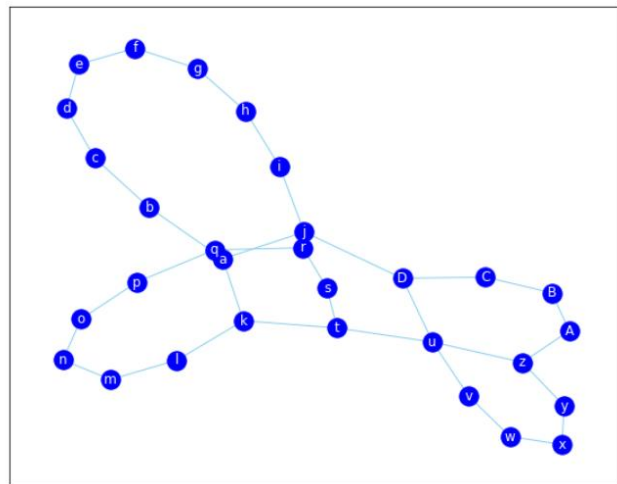
### (1) init

Input:  $Graph = (V, E, X, Y)$  and HyperParameters

- window size  $w$
- embedding size  $d$
- walks per vertex  $\gamma$
- walk length  $t$

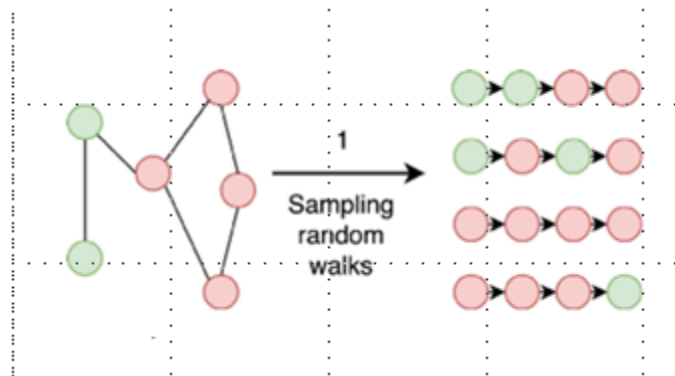
Output:  $\Phi: v \in V \rightarrow R^{|V|*d}$

```
class DeepWalk(torch.nn.Module):  
    def __init__(self, G, w=10, d=2, gamma=20, t=6):  
        super(DeepWalk, self).__init__()  
        self.G = G  
        self.w = w    #window size  
        self.d = d    #embedding size  
        self.gamma = gamma    # walks per vertex  
        self.t = t    # walk length  
        self.embedding = None
```



## 06 Code

### (2) randomWalk



```
def randomWalk(self, st_v):  
    one_walk = []  
    current_node = st_v  
    one_walk.append(str(st_v))  
    for j in range(self.t - 1): # self.t = walk_length  
        neighbors = list(self.G.edges([st_v]))  
        if (len(neighbors) > 0):  
            random_edge = random.choice(neighbors)  
            if (random_edge[0] == current_node):  
                current_node = random_edge[1]  
            else :  
                current_node = random_edge[0]  
            one_walk.append(str(current_node))  
    return one_walk
```

## 06 Code

### (3) train

---

**Algorithm 1** DEEPWALK( $G, w, d, \gamma, t$ )

---

**Input:** graph  $G(V, E)$

    window size  $w$

    embedding size  $d$

    walks per vertex  $\gamma$

    walk length  $t$

**Output:** matrix of vertex representations  $\Phi \in \mathbb{R}^{|V| \times d}$

```
1: Initialization: Sample  $\Phi$  from  $\mathcal{U}^{|V| \times d}$ 
2: Build a binary Tree  $T$  from  $V$ 
3: for  $i = 0$  to  $\gamma$  do
4:    $\mathcal{O} = \text{Shuffle}(V)$ 
5:   for each  $v_i \in \mathcal{O}$  do
6:      $\mathcal{W}_{v_i} = \text{RandomWalk}(G, v_i, t)$ 
7:     SkipGram( $\Phi, \mathcal{W}_{v_i}, w$ )
8:   end for
9: end for
```

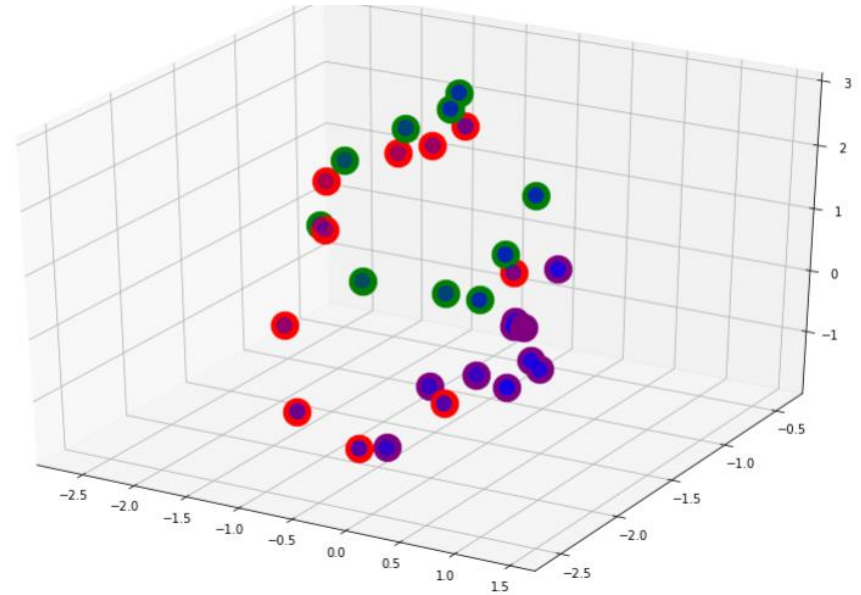
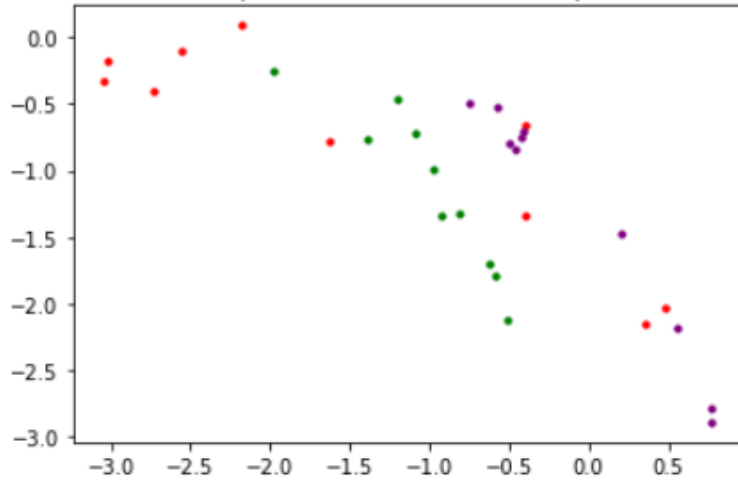
---

```
def train(self):
    walks = []
    nodes = list(self.G.nodes())
    print('starting', len(self.G.nodes()))
    for i in range(self.gamma):
        random.shuffle(nodes)
        for node in nodes:
            walks.append(self.randomWalk(node))

    skipgrammodelresult = Word2Vec(walks, size=self.d, window=self.w, sg=1, hs=1)
    self.embedding = skipgrammodelresult
    return self.embedding
```

## 06 Code

### (4) plot



---

## 06 Code

### (5) dataset

- Facebook Large Page-Page Network
- Each node represent official Facebook pages while the link are mutual likes between sites.
- The nodes are divided into 4 categories which are defined by Facebook: politicians, governmental organizations, television shows, and companies.



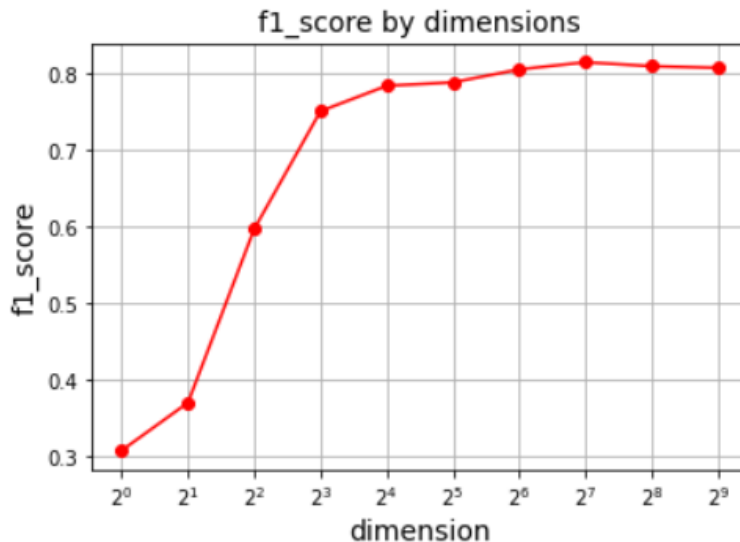
	Facebook
Nodes	22,470
Edges	171,002
Density	0.001
Transitivity	0.232



## 06 Code

### (6) Experiment on dimensions

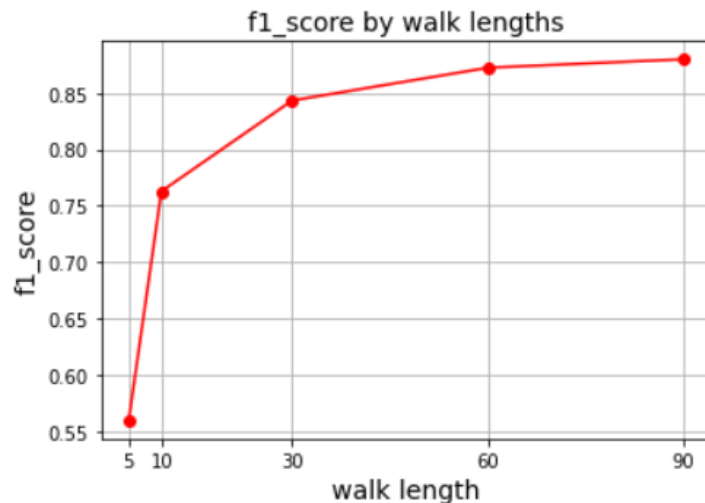
- window\_size = 5
- gamma = 5
- walk\_length = 10
- dimension = [1, 2, 4, 8, 16, 32, 64, 128, 256, 512]
- Result : In this dataset, 128 dimensions worked best.
- After 8-dimension, the f1 score did not change dramatically.



## 06 Code

### (6) Experiment on walk lengths

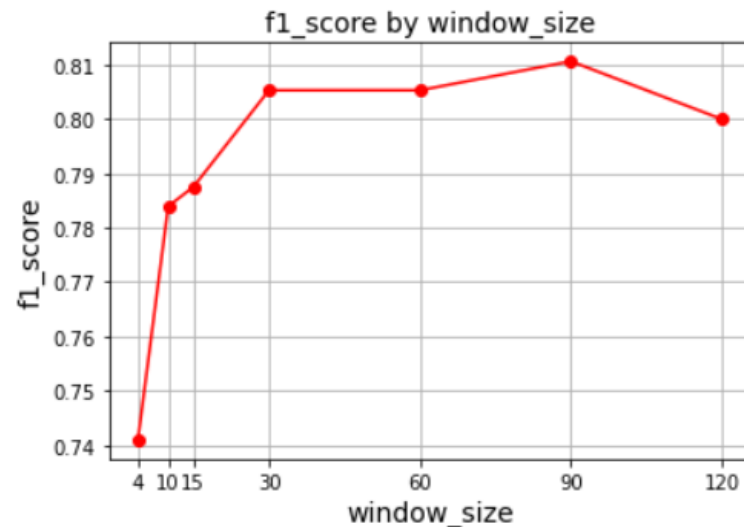
- window\_size = 5
- dimension = 8
- gamma = 5
- walk length = [5, 10, 30, 60, 90]
- Result : In this dataset, if the walking length exceeds 30, it is effective.
- Walk length had the greatest effect on calculation time.



## 06 Code

### (7) Experiment on window size

- dimension = 8
- gamma = 5
- walk length = 10
- window\_size = [4, 10, 15, 30, 60, 90, 120]
- Result : In this dataset, the best value was 90.



---

## 07 Conclusions

### Conclusion

- Deepwalk is appealing generalization of language modeling.
- It outperforms other methods on creating meaningful representations with large graphs.
- The approach is parallelizable, allowing workers to update different parts of the model concurrently.

---

## 07 Conclusions

### Future works

Since it is unweighted random walk, the frequency or importance does not influence embeddings.

Node2Vec address some of the limitation of DeepWalk.

---

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