FINAL PROJECT E

NO. 1

$$K_v = \sum_u \omega_{uv}$$

weighted degree of v = sum of all weights of edges adjacent to v.

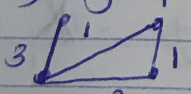
3 + 2 + 1 = 6

NO. 2

Sum of all edges weights in the graph.
It is multiplied by half to avoid double counting since each edge is counted twice.
For example $w(a,b)$ and $w(b,a)$

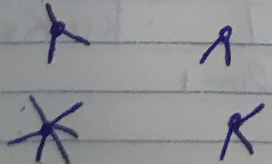
NO. 3

Using graph in question 1



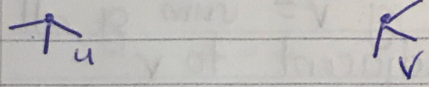
number of stubs = 2m

= 14



$P[\text{connecting one stub with another}] = 1/_{2m-1}$

If we want to choose 2 stubs between 2 nodes with $k_u$ and $k_v$ stubs



$k_u$ stubs $\quad$ $k_v$ stubs

$m_{uv}$ = total edges connecting u and v after randomizing.

$\mathbb{E}[m_{uv}] = ?$

Define $X_{ij} = \begin{cases} 1 & \text{if stub } i \text{ from } u \text{ connects to stub } j \text{ from } j \\ 0 & \text{else} \end{cases}$

$$m_{uv} = X_{11} + X_{12} + \dots + X_{1k_v} + \dots + X_{k_u k_v}$$

$$m_{uv} = \sum_{i=1}^{k_u} \sum_{j=0}^{k_v} X_{ij}$$

$$E[m_{uv}] = E[X_{11} + X_{12} + \dots + \cancel{X}_{1k_v} + \dots + X_{k_u k_v}]$$

$$E[X_{ij}] = 1/_{2m-1}$$

$$\cancel{E[X_{ij}]} = E[m_{uv}] = \frac{1}{2m-1} \cdot k_u k_v =$$

$$\frac{k_u k_v}{2m-1}$$

No. 4

Modularity Q, compares our given network to a given network with the same weighted degrees, but in which all edges are rewired to random.

$$Q = \frac{1}{2m}\sum_{k=1}^{l}\sum_{i,j\in C_k}\left[w_{ij} - \frac{k_i k_j}{2m}\right]$$

This ~~tells~~ helps us determine the structure of the community.

~~$C_k$ is the partitioning of the nodes of network~~

The modularity Q above of has been normalized so that $-1 < Q < 1$

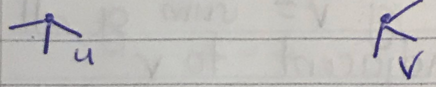
~~What this t~~

This means that we minimise the number of edges between communities and maximise the number of ~~communities~~ edges within a community.

$C_k$ = partitioning of the nodes of network,

$P[\text{connecting one stub with another}] = 1/_{2m-1}$

If we want to choose 2 stubs between 2 nodes with $k_u$ and $k_v$ stubs.

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$$E[X_{ij}] = 1/_{2m-1}$$

$$\cancel{E[X_{ij}]} = E[m_{uv}] = \frac{1}{2m-1} \cdot k_u k_v =$$

$$\frac{k_u k_v}{2m-1}$$