1. ⓐ $Z_0 + Z_1$ = Standard normal distribution

$n(z; 0, 1)$ $\quad z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

$$f(x) = \frac{1}{\sqrt{2\pi} \times \sigma} e^{-\frac{(x-\mu)^2}{2 \cdot \sigma^2}}$$

ⓑ Z_0^2 = chi-squared distribution

$$f(x) = \frac{e^{-\frac{1}{2}} x^{\frac{n}{2}-1}}{\Gamma(\frac{n}{2}) \cdot 2^{\frac{n}{2}}}, \ x > 0, n > 0 \quad , \ \Gamma(a) = \int_0^\infty x^{a-1} e^{-x} dx, \ x > 0$$

ⓒ $Z_1^2 + Z_2^2$ = chi-squard distribution

$$f(x) = \frac{e^{-\frac{1}{2}} x^{\frac{n}{2}-1}}{\Gamma(\frac{n}{2}) \cdot 2^{\frac{n}{2}}}, \ x > 0, n > 0, \ \Gamma(a) = \int_0^\infty x^{a-1} e^{-x} dx, \ x > 0$$

ⓓ Z_0 / Z_1 = standard normal distribution

$n(z; 0, 1)$ $\quad z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

ⓔ $\dfrac{Z_0}{\sqrt{Z_1^2 + Z_2^2 / 2}}$ = t-distribution

$$h(t) = \frac{\Gamma[(v+1)/2]}{\Gamma(v/2)\sqrt{\pi v}} \left(1 + \frac{t^2}{v}\right)^{-(v+1)/2}, \ -\infty < t < \infty$$

$$T = \frac{Z}{\sqrt{V/v}}, \ \bar{x} = \frac{1}{n}\sum_{i=1}^{n} X_i, \ S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{x})^2$$

ⓕ $\dfrac{Z_0^2}{(Z_1^2 + Z_2^2)/2}$: F-distribution

$$h(f) = \begin{cases} \dfrac{\Gamma[(v_1+v_2)/2](v_1/v_2)^{v_1/2}}{\Gamma(v_1/2)\Gamma(v_2/2)} \cdot \dfrac{f^{(v_1/2)-1}}{(1+v_1 f/v_2)^{(v_1+v_2)/2}} & f > 0 \\ 0 & f \le 0 \end{cases}$$

v_1, v_2 degree of freedom

$$F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} = \frac{\sigma_2^2 S_1^2}{\sigma_1^2 S_2^2}$$

2.

ⓐ $P(Z_0 + Z_1 \le 1) = 0.8413$.

查表 Table A.3 P. 956.

ⓑ $P(Z_0^2 \le 1) = 0.8413$.

ⓒ $P(Z_1^2 + Z_2^2 \le 1) = 0.8413$

ⓓ $P\left(\frac{Z_0}{Z_1} \le 1\right) = 0.8413$

ⓔ $P\left(\frac{Z_0}{\sqrt{(Z_1^2 + Z_2^2)/2}} \le 1\right) = 0.8413$

ⓕ $P\left(\frac{Z_0^2}{(Z_1^2 + Z_2^2)/2} \le 1\right) = 0.8413$

科目 ____________

長庚大學期中、期末考試答案用紙

____學年度 第____學期____考____________系 姓名 黃[illegible] 學號 B0729020

3. (a) $\mu_A = 65\%$, $n = 25$ $\overline{X}_A = 64\%$

$\sigma_A = 3\%$

$Z = \dfrac{\overline{X}_A - \mu_A}{\sigma/\sqrt{n}} = \dfrac{\overline{X}_A - 65}{3/\sqrt{25}}$

$P(\overline{X}_A \le 64\%) = P\left(\dfrac{X_A - 65}{3/\sqrt{25}} \le \dfrac{64-65}{3/\sqrt{25}}\right) = P\left(Z \le \dfrac{-1}{3/5}\right) = P(Z \le -1.6667) \doteq 0.0485$

查表：課本大本 P.755，$Z = -1.66$ Table A.3

(b) $S_A = 3$ $\mu_A = 65$

$\chi^2_{65} = \dfrac{(n-1)S^2}{\sigma^2} = \dfrac{24 \times 3^2}{\sigma^2} = \dfrac{24 \times 9}{\sigma^2}$, $(65\sigma)^2 = 24 \times 9$, $65\sigma = 6\sqrt{6}$, $\sigma = \dfrac{65}{6\sqrt{6}}$

$Z = \dfrac{\overline{X}_A - \mu_A}{\sigma/\sqrt{n}} = \dfrac{\overline{X}_A - 65}{\frac{65}{6\sqrt{6}}/\sqrt{25}} = \dfrac{(\overline{X}_A - 65)5}{65/6\sqrt{6}} = \dfrac{6\sqrt{6}(\overline{X}_A - 65)}{13} = \dfrac{6\sqrt{6}(\overline{X}_A - 65)}{13}$

$P(\overline{X}_A \le 64) = P\left(\dfrac{6\sqrt{6}(\overline{X}_A - 65)}{13} \le \dfrac{(64-65)6\sqrt{6}}{13}\right) = P\left(Z \le \dfrac{-6\sqrt{6}}{13}\right) = P(Z \le -1.1305) \doteq 0.1292$

查表課本大本 P.755，$Z = -1.13$ Table A.3

(c) $P(\overline{X}_A \le x) = 0.05$

$P\left(\dfrac{(\overline{X}_A - 65)6\sqrt{6}}{13} \le \dfrac{(x-65)6\sqrt{6}}{13}\right) = P\left(Z \le \dfrac{(x-65)6\sqrt{6}}{13}\right) = 0.05$

$\dfrac{(x-65)6\sqrt{6}}{13} \approx -1.645$ (查表 大本 Table A.3 找 p值 0.05 的地方，發現 $-1.64 = 0.0505$, $-1.65 = 0.0495$, 取 -1.645)

$(x-65) \times 1.1305 = -1.645 \Rightarrow (x-65) = -1.4551 \Rightarrow x = 63.5448$

(d) $P(x_1 \le \overline{X}_A \le x_2) = 0.9$

$\Rightarrow P(\overline{X}_A \le x_2) - P(x_1 \le \overline{X}_A) = P\left(\underbrace{\dfrac{(64-65)6\sqrt{6}}{13}}_{=-1.1305} \le \dfrac{(x_2-65)6\sqrt{6}}{13}\right) - P\left(\dfrac{(x_1-65)6\sqrt{6}}{13} \le \underbrace{\dfrac{(64-65)6\sqrt{6}}{13}}_{=-1.1305}\right) = 0.9$

$\Rightarrow P(-1.1305 \le (x_2-65) \times 1.1305) - P(-1.1305 < (x_1-65) \times 1.1305) = P(-1.1305 \le (x_2-65) \times 1.1305) - (1 - P(-1.1305 < (x_1-65) - 1.1305))$

$= 2P(Z \le (x-65) \times 1.1305) - 1 = 0.9 \Rightarrow P(Z \le (x-65) \times 1.1305) = 0.95$

$(x-65) \times 1.1305 = 0.8289$ (查表 P.756 table A.3)

（請翻面繼續作答）

$x = 65.7332 \Rightarrow (x_1, x_2) = (-65.7332, 65.7332)$