科目 ______

長庚大學期中、期末考試答案用紙

_____學年度 第____學期____考 資工 系 姓名 黃俞嘉 學號 B0729020.

3. ⓐ 迴積分定理，主要幫助 laplace 積分反轉換之進行

ⓑ $L^{-1}\{F(s)G(s)\} = \int_0^t f(\tau)g(t-\tau)d\tau$ 或 $L\{\int_0^t f(\tau)g(t-\tau)d\tau\} = F(s)G(s)$

其中 $F(s) = L\{f(t)\}$ ， $G(s) = L\{g(t)\}$.

$$L\{\int_0^t f(\tau)g(t-\tau)d\tau\} = \int_{\tau=0}^{\tau=\infty}\int_{T=0}^{T=\infty} f(\tau)g(T)e^{-s(T+\tau)}\,dT\,d\tau$$

$$= \int_{\tau=0}^{\tau=\infty}\int_{T=0}^{T=\infty} f(\tau)g(T)\,e^{-sT}\,e^{-s\tau}\,dT\,d\tau$$

$$= \int_{\tau=0}^{\tau=\infty}\left[\int_{T=0}^{T=\infty} f(\tau)g(T)e^{-sT}e^{-s\tau}dT\right]d\tau$$

$$= \int_{\tau=0}^{\tau=\infty}\left[f(\tau)e^{-s\tau}\int_{T=0}^{T=\infty} g(T)e^{-sT}dT\right]d\tau$$

$$= \left[\int_{T=0}^{T=\infty} g(T)\,e^{-sT}dT\right]\left[\int_{\tau=0}^{\tau=\infty} f(\tau)\,e^{-s\tau}d\tau\right] = G(s)F(s)$$

1. ⓐ $X_k = \sum_{n=0}^{7} x_n \cdot e^{\frac{-2\pi i}{8}\cdot kn}$

$X_0 = \sum_{n=0}^{7} x_n \cdot e^{\frac{-2\pi i}{8}\cdot 0n} = \sum_{n=0}^{7} x_n \cdot 1 = 1$

$X_1 = \sum_{n=0}^{7} x_n e^{\frac{-2\pi i}{8}\cdot n} = 1\cdot e^{\frac{-2\pi i}{8}\cdot 0} + 0\cdot e^{\frac{-2\pi i}{8}} + 0\cdot e^{\frac{-2\pi i}{8}\cdot 2} + \dots + 0\cdot e^{\frac{-2\pi i}{8}\cdot 7} = 1$

$X_2 = \sum_{n=0}^{7} x_n e^{\frac{-2\pi i}{8}\cdot 2n} = 1\cdot e^{\frac{-2\pi i}{8}\cdot 2\cdot 0} = 1$

⋮

↳ $X_1[k] = [1,1,1,1,1,1,1,1]$

ⓑ $X_k = \sum_{n=0}^{7} x_n e^{\frac{-2\pi i}{8}\cdot kn}$

$X_0 = 1\cdot e^{\frac{-2\pi i}{8}\cdot 0\cdot 0} + 1\cdot e^{\frac{-2\pi i}{8}\cdot 0\cdot 1} + \dots = 8$

```
ys=[1,1,1,1,1,1,1,1]
dft(ys)
```

out array([8.00000000e+00+0.00000000e+00j, -5.55111512e-16+2.22044605e-16j, -4.28626380e-16-4.44089210e-16j, -2.22044605e-16+8.88178420e-16j, 0.00000000e+00-4.89858720e-16j, -2.10942375e-15-1.22124533e-15j, -2.93296830e-15-6.66133815e-16j, 3.55271368e-15+1.11022302e-15j])

ⓒ
```
ys=[1,-1,1,-1,1,-1,1,-1]
dft(ys)
```

out array([0, 1.11e-16-1.11-16j, 9.55e-17-1.11-16j, 8.88e-16-1.55-15j, 8.00e+00+3.42e-15j, -2.66e-15+1.11-16j, 2.93e-15-6.66e-16j, -5.21e+15-2.46-15j])

(請翻面繼續作答)

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d. $x_4[n] = [3, 0, 2, 0, 2, 0, 2, 0] = ys4$

```
dft(ys4)
out array([9.+0.00e+00j, 1.-4.44e-16j, 1.-4.89e-16j, 1.-6.66e-16j, 9.+2.93e-15j,
       1.-1.33e-15j, 1.-1.4e-15j, 1.-1.77e-15j])
```

e. 相反，反過來

f. $x_6[n]$

a.
2、$h[n] = (f * g)[n] = \sum_{m=-\infty}^{\infty} f[m]\,g[n-m] = \sum_{m=0}^{2} f[m]\,g[n-m] = [f_0g_0,\ f_0g_1+f_1g_0,\ f_0g_2+f_1g_1+f_2g_0]$

b. $y[n] = (x * w)[n] = \sum_{m=-\infty}^{\infty} x[m]\,w[n-m] = \sum_{m=0}^{4} x[m]\,w[n-m] = [1\cdot 1,\ 1\cdot 1+1\cdot 1,\ 0+1\cdot 1+1\cdot 1+0,$
$1\cdot 0+1\cdot 0+1\cdot 1+1\cdot 1] = [1, 2, 2, 2].$

c. $X[k] = [4, 0, 0, 0]$
$W[k] = [2, 0]$
$Y[k] = [7, -1, -1, -1]$

[1]
g.
```
a=[1, 1, 0, 0]
ys= idft(a)
ys
out array([0.5+0.j, 0.25+0.25j, 0.+0.j, 0.25-0.25j])
b=[1, 1, 0, 0, 0, 0, 0, 0]
ys= idft(b)
ys
out array([0.25, 0.213+0.088j, 0.125+0.125j, 0.037+0.088j, 0, 0.037-0.088j, 0.125-0.125j,
       0.213-0.088j])
```

1個⊕ 1個⊖

差異：x_8 比較長且有相反的關係 ex: 如上面畫的線

(請翻面繼續作答)