

科目 _____

長庚大學期中、期末考試答案用紙

學年度 第 _____ 學期 考 ^比 _良 工 系 姓名 ^甘 _國 ¹ ₁₀ 學號 B0729020

1. ① $\mu=0 \quad \sigma=1$
 $f_z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2\pi^2}(x-0)^2}, -\infty < x < \infty = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}, -\infty < x < \infty$

② $z = \frac{x-\mu}{\sigma} = \frac{x-0}{1} = x$

$P(-1 \leq z \leq 1) = \int_{-1}^1 \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} = 0.8413 - 0.1587 = 0.6826$

③ $z = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} = 0.95 \Rightarrow z_x = \frac{x-0}{1} = x \Rightarrow P(z \leq x) = 0.95 \Rightarrow P(z \leq -x) = 0.05$
 $x = 1.96$

自由度 (degree of freedom) 為 1 的 χ^2

④ $f_{\chi^2}(x) = f(x^2; v) = \begin{cases} \frac{1}{2^{\frac{1}{2}} \Gamma(\frac{1}{2})} x^{\frac{1}{2}} e^{-\frac{x}{2}}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$

① $E(X) = V = 1$

② $sd = \sqrt{2V} = \sqrt{2}$

③ $P(X \leq 1) = 1 - e^{-\lambda x} \quad (\lambda = \frac{1}{\beta} = \frac{1}{2}) = 1 - e^{-\frac{1}{2}x} = 1 - e^{-\frac{1}{2}} \approx 0.3934$

④ $f_T(t) = \begin{cases} \frac{1}{\beta} e^{-\frac{t}{\beta}}, & t > 0 \\ 0, & \text{elsewhere} \end{cases} = \begin{cases} e^{-t}, & t > 0 \\ 0, & \text{elsewhere} \end{cases}$

⑤ $E(T) = \beta = 1$

⑥ $sd[T] = \beta^2 = 1$

⑦ $P(T > 1) = e^{-\lambda x}, \lambda = \frac{1}{\beta} = 1 \Rightarrow e^{-1}$

⑧ $f_{\Gamma}(t) = \begin{cases} \frac{1}{\beta^a \Gamma(a)} x^{a-1} e^{-\frac{x}{\beta}}, & x > 0 \\ 0, & \text{elsewhere} \end{cases} = \begin{cases} \frac{1}{2!} x^2 e^{-x}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$

$a=3 \quad \beta=1$

⑨ $E[T_3] = a\beta = 3 \times 1 = 3$

⑩ $sd[T_3] = a\beta^2 = 3 \times 1^2 = 3$

⑪ $P(T_3 > 3) = e^{-3} \approx 0.04978$

⑫ $P(T_3 > 7) = e^{-7} \approx 0.0009$

不能: e^{-7} 太小, 表示概率不太可能。

(請翻面繼續作答)