07/15/2016
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Linear Algebra
HW #2.
Lectures 9-16.

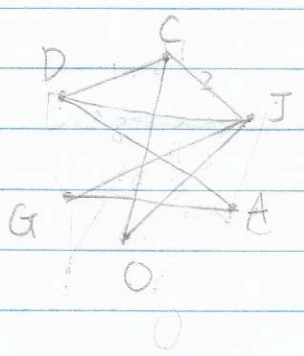
Part 1: Written Work.

① The basis of all 3x3 matrixes is 9. Therefore the dimension of M is 9.

② The basis of all symmetric 3x3 matrices is 6. therefore the dimension of them is also 6.

③. The dimension of intersection of matrices S and U is 15.

④



$n=6$
$m=8$.

$$\begin{array}{c} \\ D \\ C \\ J \\ A \\ O \\ G \end{array}\begin{array}{c} \begin{array}{cccccc} D & C & J & A & O & G \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

For the matrix and the graph to better demonstrate the different types of associations, I would assign different weights to different edges/relationships. This way I will also be able to demonstrate the strength of the nodes.

⑤ $x = (x_0, x_1, x_2, x_3, \dots x_n)$

$y = (y_0, y_1, y_2, y_3 \dots y_n)$

• $-x^T y = \begin{bmatrix} -x_1 \\ -x_2 \\ \vdots \\ -x_n \end{bmatrix} [y_1, y_2, y_3 \dots y_n] = -x_1y_1 - x_2y_2 \cdots - x_ny_n$

$= -(x_1y_1 + x_2y_2 \cdots + x_ny_n)$

$= -(x \cdot y)$

$= 0$

• $y^T x = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} [x_1, x_2, x_3 \dots x_n] = y_1x_1 + y_2x_2 \cdots + y_nx_n$

$= y \cdot x$

$$\boxed{\therefore -x^T y = y^T x = 0}$$

⑥ $* \ (A^T)^T = A$ $\quad \therefore (A^TA)^T = A^T(A^T)^T = A^TA$

$* \ (AB)^T = A^TB^T$

⑦ 1) $Q = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & & & \\ \vdots & & & \\ q_{n1} & & & \end{bmatrix} \times Q^T = \begin{bmatrix} q_{11} & q_{21} & \cdots & q_{n1} \\ q_{12} & & & \\ \vdots & & & \\ q_{1n} & & & \end{bmatrix} = I$

$Q^T = Q^{-1} \Rightarrow Q^TQ = I$

$\begin{pmatrix} q_{11}^2 + q_{21}^2 + \cdots q_{n1}^2 & \cdots \\ q_{12}^2 + q & \cdots \\ \vdots & \end{pmatrix} = \begin{bmatrix} 1 & 0 & \cdots \\ 0 & 1 & 0 & \cdots \\ 0 & 0 & 1 & \cdots \end{bmatrix}$

$\Rightarrow (q_{11}^2 + q_{21}^2 + \cdots q_{n1}^2) = 1$

$\|$

• $\|q\|^2 = \left(\sqrt{q_{11}^2 + q_{21}^2 \cdots}\right)^2 = 1$

$\therefore$

2) As shown on the answer of question #5 by the definition of orthogonal vectors, the multiplication of two distinct columns is 0 and therefore perpendicular.

$\theta = 0$.

3) $\begin{bmatrix} \cos\theta & \cos(\theta+\frac{\pi}{2}) \\ \sin\theta & \sin(\theta+\frac{\pi}{2}) \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = Q$.

$Q^TQ = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$.

$\therefore Q^T = Q^{-1}$

8) 1) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2) $A + A^{-1} = 2A$

$A^{-1} = 2A - A$

$A^{-1} = A \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

↳ identity matrix

4) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

if one row is all 0's then all determinants are 0.

5) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

satisfies #1 & 2.

9) $E \cdot M = A$

find the inverse of M

↳ multiply by A.

$E = A \cdot M^{-1}$

$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$