Math Homework Week #4, Spectral Theory

OSM Lab, Eun-Seok Lee

1. (6.1)
   $min - e^{-w^T x}$
   $$s.t \; -w^T x \leq -w^T Aw + w^T Ay - a$$
   $$y^T w = w^T x + b$$

2. (6.5)
   $min - (0.07m + 0.05k)$
   $$s.t \; 4m + 3k = 240,000$$
   $$2m + k = 6,000$$

3. (6.6)
   $f_1(x,y) = 6xy + 4y^2 + y = 0$
   $f_2(x,y) = 3x^2 + 8xy + x = 0$

   $Hessian = \begin{bmatrix} 6y & 6x+8y+1 \\ 6x+8y+1 & 8x \end{bmatrix}$
   i) (0,0) – saddle point because Hessian is indefinite.
   ii) (0, -1/4) – saddle point because Hessian is indefinite.
   iii) (-1/3, 0) – saddle point because Hessian is indefinite.
   iv) (-1/9, -1/12) – local maximum because Hessian is negative definite.

4. (6.11)
   $x_1 = x_0 - \frac{f'(x_0)}{f''(x_0)}$
   $f'(x) = 0$
   $x = -\frac{b}{2a}$ is an unique maxizer.
   $f''(x) = 2a > 0$ so, this is maximizer.

   Now, $x_1 = x_0 - (2ax_0 + b)\frac{1}{2a} = -\frac{b}{2a}$
   So, for any x_0, x_1 is an unique maxizer.

5. (6.14)
   I uploaded .py file on math/Week4.

References