Math Homework Week #6, Linear Constrained Optimization

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1. (8.1)
   From vertices, $x = (14/5, 16/5)$ is optimizer.

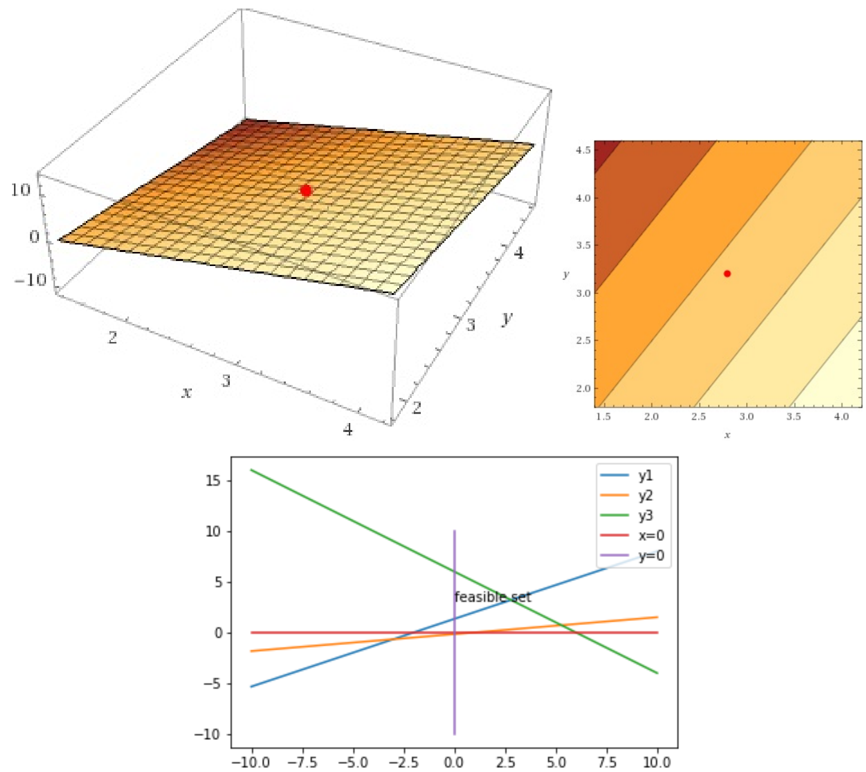


Figure 1

2. (8.2)
   (i)
   From vertices, $x = (6, 2)$ is optimizer.
   The optimal value is 20.
   Figures are below.

   (ii)
   From vertices, $(x, y) = (15, 12)$ is optimizer.
   The optimal value is 132.
   Figures are below.

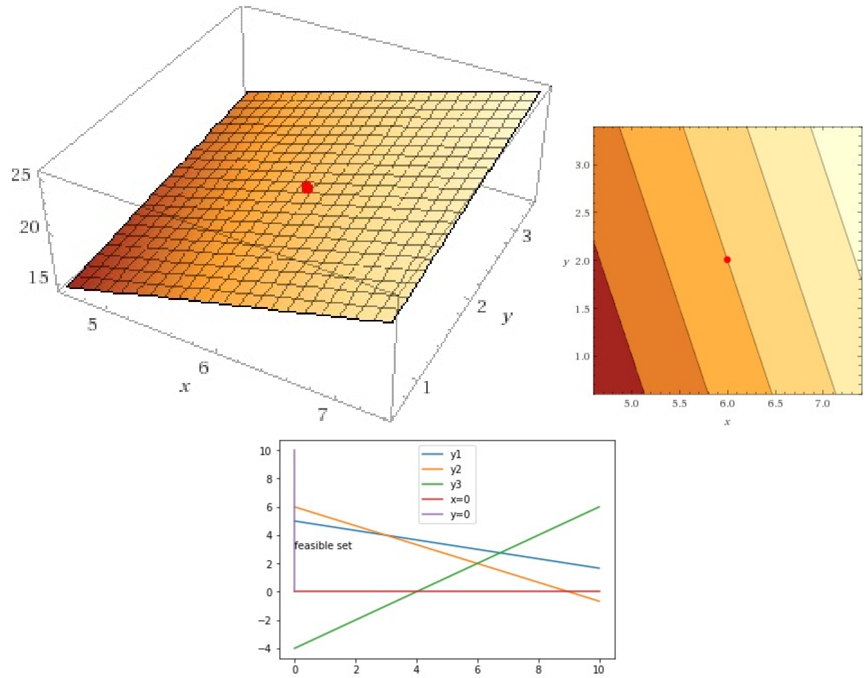


Figure 2

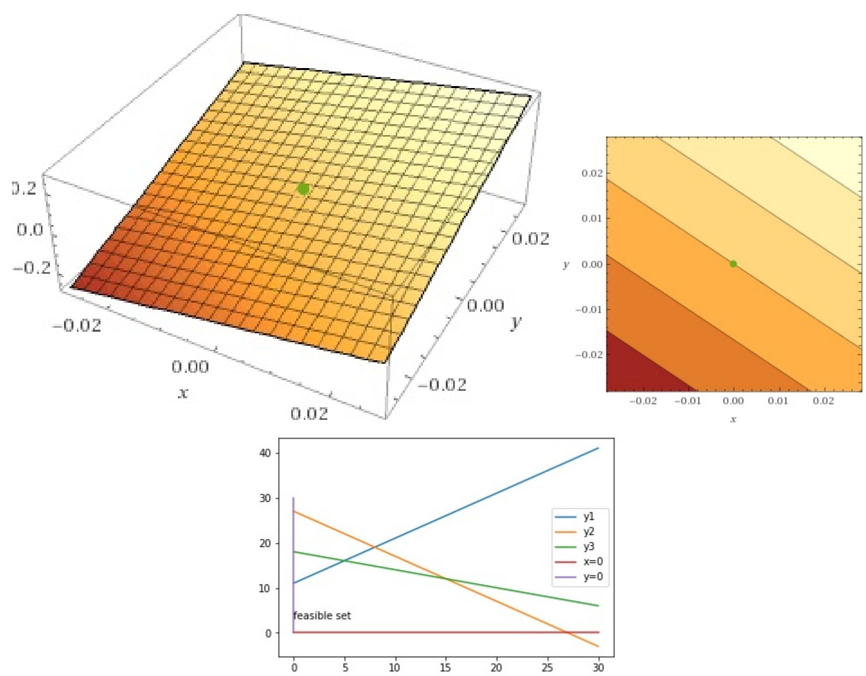


Figure 3

3. (8.5)

(i) "By Hand,"

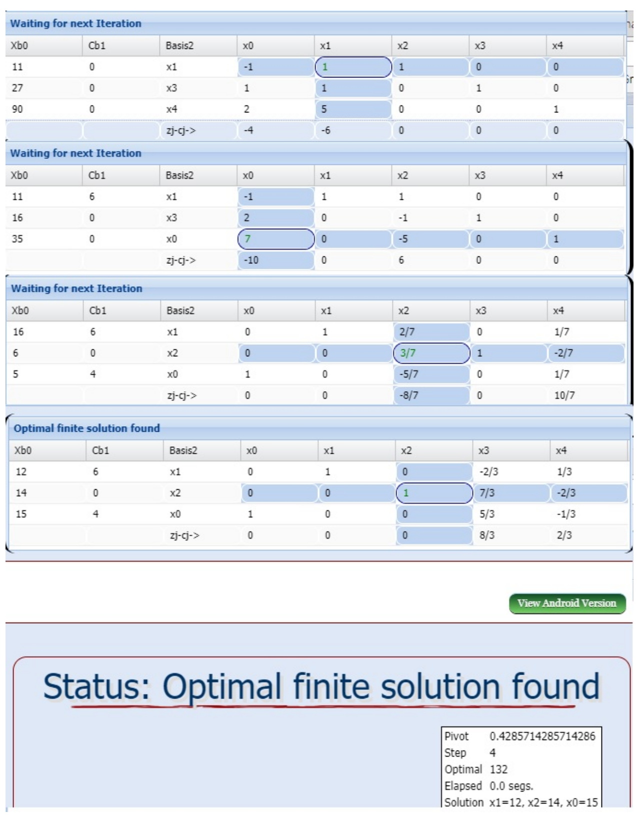
1) $f = 3x + y$
$w_1 = 15 - x - 3y$
$w_2 = 18 - 2x - 3y$
$w_3 = 4 - x + y$

2) $f = 4y + 12 - 3w_3$
$w_1 = 11 - 2y + w_3$
$w_2 = 10 - 5y + 2w_3$
$x = 4 + y - w_3$

3) $f = 4(2 + \frac{2}{5}w_3 - \frac{1}{5}w_2) + 12 - 3w_3$
$w1 = 11 - 2(2 + \frac{2}{5}w_3 - \frac{1}{5}w_2) + w_3$
$y = (2 + \frac{2}{5}w_3 - \frac{1}{5}w_2)$
$x = 4 + (2 + \frac{2}{5}w_3 - \frac{1}{5}w_2) - w_3$

The optimizer is (6, 2), and it is exactly same to (8.2).

(ii) I used the simplex solver, and its optimizer is (15, 12) and it is exactly same to (8.2). The dictionaries are below.



Waiting for next Iteration

Xb0	Cb1	Basis2	x0	x1	x2	x3	x4
11	0	x1	-1	1	1	0	0
27	0	x3	1	1	0	1	0
90	0	x4	2	5	0	0	1
		zj-cj->	-4	-6	0	0	0

Waiting for next Iteration

Xb0	Cb1	Basis2	x0	x1	x2	x3	x4
11	6	x1	-1	1	1	0	0
16	0	x3	2	0	-1	1	0
35	0	x0	7	0	-5	0	1
		zj-cj->	-10	0	6	0	0

Waiting for next Iteration

Xb0	Cb1	Basis2	x0	x1	x2	x3	x4
16	6	x1	0	1	2/7	0	1/7
6	0	x2	0	0	3/7	1	-2/7
5	4	x0	1	0	-5/7	0	1/7
		zj-cj->	0	0	-8/7	0	10/7

Optimal finite solution found

Xb0	Cb1	Basis2	x0	x1	x2	x3	x4
12	6	x1	0	1	0	-2/3	1/3
14	0	x2	0	0	1	7/3	-2/3
15	4	x0	1	0	0	5/3	-1/3
		zj-cj->	0	0	0	8/3	2/3

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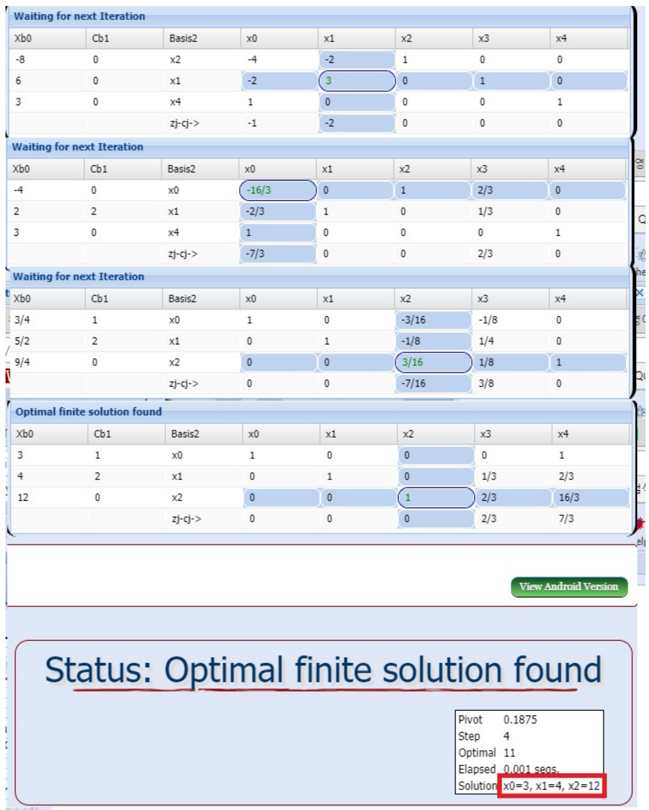
Status: Optimal finite solution found

Pivot	0.4285714285714286
Step	4
Optimal	132
Elapsed	0.0 segs.
Solution	x1=12, x2=14, x0=15

Figure 4

4. (8.7)
   (i) The optimizer is (3, 4) and optimal value is 11.
   (ii) It is NOT feasible.
   (iii) The optimizer is (0, 2) and optimal value is 2.

   Each dictionary is below.



Waiting for next Iteration

Xb0	Cb1	Basis2	x0	x1	x2	x3	x4
-8	0	x2	-4	-2	1	0	0
6	0	x1	-2	3	0	1	0
3	0	x4	1	0	0	0	1
		zj-cj->	-1	-2	0	0	0

Waiting for next Iteration

Xb0	Cb1	Basis2	x0	x1	x2	x3	x4
-4	0	x0	-16/3	0	1	2/3	0
2	2	x1	-2/3	1	0	1/3	0
3	0	x4	1	0	0	0	1
		zj-cj->	-7/3	0	0	2/3	0

Waiting for next Iteration

Xb0	Cb1	Basis2	x0	x1	x2	x3	x4
3/4	1	x0	1	0	-3/16	-1/8	0
5/2	2	x1	0	1	-1/8	1/4	0
9/4	0	x2	0	0	3/16	1/8	1
		zj-cj->	0	0	-7/16	3/8	0

Optimal finite solution found

Xb0	Cb1	Basis2	x0	x1	x2	x3	x4
3	1	x0	1	0	0	0	1
4	2	x1	0	1	0	1/3	2/3
12	0	x2	0	0	1	2/3	16/3
		zj-cj->	0	0	0	2/3	7/3

Figure 5: (i)

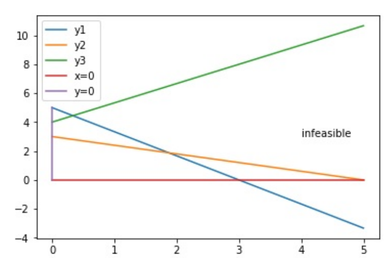


Figure 6: (ii)

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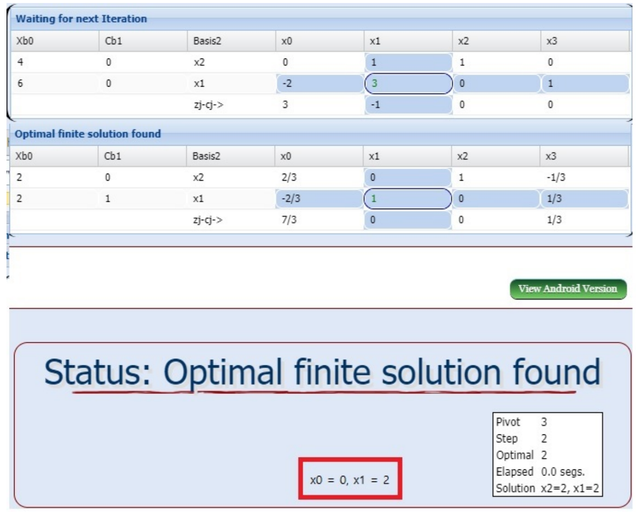


Figure 7: (iii)

5. (8.13)
   If each constraint is zero, it means that only feasible set is $(0, \cdots, 0)$.
   If $c^T x$ is defined at $x = (0, \cdots, 0)$, then it is optimum.
   However, if it is not defined, it is unbounded by definition.

6. (8.17) From dual problem, I will restore primal problem by definition.

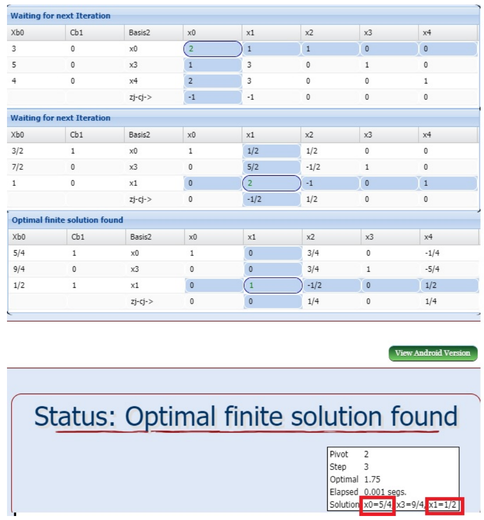
   min $b^T y$
   $s.t. A^T y \geq c$
   $y \geq 0$

   By the definition of duality,
   max $c^T y$
   $s.t. (A^T)^T y \leq b$
   $y \geq 0$

   Then, y can be changed to x because it is not determined variables.

   max $c^T x$
   $s.t. (A^T)^T x \leq b$
   $x \geq 0$
   This is exactly same to the primal problem.

7. (8.18)



Waiting for next Iteration

Xb0	Cb1	Basis2	x0	x1	x2	x3	x4
3	0	x0	2	1	1	0	0
5	0	x3	1	3	0	1	0
4	0	x4	2	3	0	0	1
		zj-cj->	-1	-1	0	0	0

Waiting for next Iteration

Xb0	Cb1	Basis2	x0	x1	x2	x3	x4
3/2	1	x0	1	1/2	1/2	0	0
7/2	0	x3	0	5/2	-1/2	1	0
1	0	x1	0	2	-1	0	1
		zj-cj->	0	-1/2	1/2	0	0

Optimal finite solution found

Xb0	Cb1	Basis2	x0	x1	x2	x3	x4
5/4	1	x0	1	0	3/4	0	-1/4
9/4	0	x3	0	0	3/4	1	-5/4
1/2	1	x1	0	1	-1/2	0	1/2
		zj-cj->	0	0	1/4	0	1/4

View Android Version

Status: Optimal finite solution found

Pivot	2
Step	3
Optimal	1.75
Elapsed	0.001 segs.
Solution	x0=5/4, x3=9/4, x1=1/2

Figure 8: Primal Problem, and the optimal value is 1.75.

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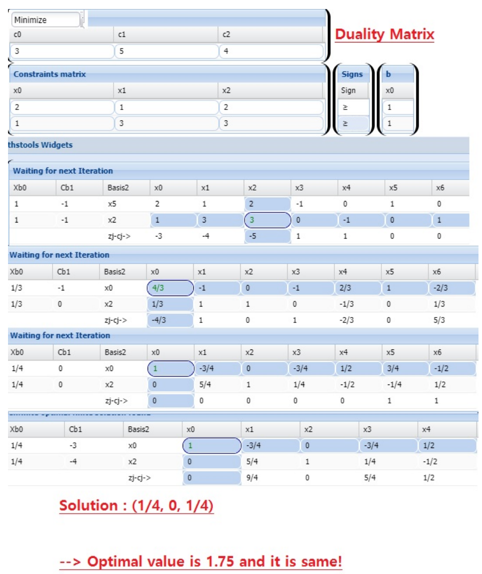

Minimize

c0	c1	c2
3	5	4

Duality Matrix

Constraints matrix / **Signs** / **b**

x0	x1	x2	Sign	x0
2	1	2	≥	1
1	3	3	≥	1

thstools Widgets

Waiting for next Iteration

Xb0	Cb1	Basis2	x0	x1	x2	x3	x4	x5	x6
1	-1	x5	2	1	2	-1	0	1	0
1	-1	x2	1	3	3	0	-1	0	1
		zj-cj->	-3	-4	-5	1	1	0	0

Waiting for next Iteration

Xb0	Cb1	Basis2	x0	x1	x2	x3	x4	x5	x6
1/3	-1	x0	4/3	-1	0	-1	2/3	1	-2/3
1/3	0	x2	1/3	1	1	0	-1/3	0	1/3
		zj-cj->	-4/3	1	0	1	-2/3	0	5/3

Waiting for next Iteration

Xb0	Cb1	Basis2	x0	x1	x2	x3	x4	x5	x6
1/4	0	x0	1	-3/4	0	-3/4	1/2	3/4	-1/2
1/4	0	x2	0	5/4	1	1/4	-1/2	-1/4	1/2
		zj-cj->	0	0	0	0	0	1	1

Xb0	Cb1	Basis2	x0	x1	x2	x3	x4
1/4	-3	x0	1	-3/4	0	-3/4	1/2
1/4	-4	x2	0	5/4	1	1/4	-1/2
		zj-cj->	0	9/4	0	5/4	1/2

Solution : (1/4, 0, 1/4)

--> Optimal value is 1.75 and it is same!

Figure 9: Dual Problem

References