

Excercise Set1 for Economics

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1. (Please see py file) After computing both successive approximation and matrix algebra, the results were very similar. I set tolerance rate as "1e-7" but if I take higher tolerance rate, it would become more similar. The successive approximation result is $[-0.89552246, 13.34328364, 45.64179143]^T$ and matrix algebra result is $[-0.89552239, 13.34328358, 45.64179104]^T$.

2. Claim: by Banach's fixed point theorem, it has an unique solution.
Metric Space is complete, so all I have to do is to prove T is a contraction mapping.

$$\begin{aligned}\rho(Tx, Ty) &= |\beta \sum \max(w_k, x)p_k - \beta \sum \max(w_k, y)p_k| \\ &\leq \beta \sum |\max(w_k, x)p_k - \max(w_k, y)p_k| \\ &\leq \beta |x - y| \\ &= \beta \rho(x, y)\end{aligned}$$

$$\therefore \rho(Tx, Ty) \leq \beta \rho(x, y)$$

If this problem satisfies fixed point property, I can use successive approximation.
In the next problem, I used it, and got a solution for reservation wage.

3. (Please see py file) The reservation wage increases, and this coincides with my intuition; as compensation increases, the incentive for work may decrease.