

Math Homework Week #4, Continuous Optimization

OSM Lab, Eun-Seok Lee

1. (6.1)

$$\begin{aligned} \min & -e^{-w^T x} \\ \text{s.t. } & -w^T x + w^T A w - w^T A y \leq -a \\ & y^T w - w^T x = b \end{aligned}$$

2. (6.5)

$$\begin{aligned} \min & -(0.07m + 0.05k) \\ \text{s.t. } & 4m + 3k = 240,000 \\ & 2m + k = 6,000 \end{aligned}$$

3. (6.6)

$$\begin{aligned} f_1(x, y) &= 6xy + 4y^2 + y = 0 \\ f_2(x, y) &= 3x^2 + 8xy + x = 0 \end{aligned}$$

$$\text{Hessian} = \begin{bmatrix} 6y & 6x + 8y + 1 \\ 6x + 8y + 1 & 8x \end{bmatrix}$$

- i) (0,0) – saddle point because Hessian is indefinite.
- ii) (0, -1/4) – saddle point because Hessian is indefinite.
- iii) (-1/3, 0) – saddle point because Hessian is indefinite.
- iv) (-1/9, -1/12) – local maximum because Hessian is negative definite.

4. (6.11)

$$\begin{aligned} x_1 &= x_0 - \frac{f'(x_0)}{f''(x_0)} \\ f'(x) &= 0 \\ x &= -\frac{b}{2a} \text{ is an unique maximizer.} \\ f''(x) &= 2a > 0 \text{ so, this is maximizer.} \end{aligned}$$

Now, $x_1 = x_0 - (2ax_0 + b)\frac{1}{2a} = -\frac{b}{2a}$
So, for any x_0 , x_1 is an unique maximizer.

5. (6.14)

I uploaded .py file on math/Week4.

References