

# Math Homework Week #6, Linear Contrained Optimization

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1. (8.1)

From vertices,  $x = (14/5, 16/5)$  is optimizer.

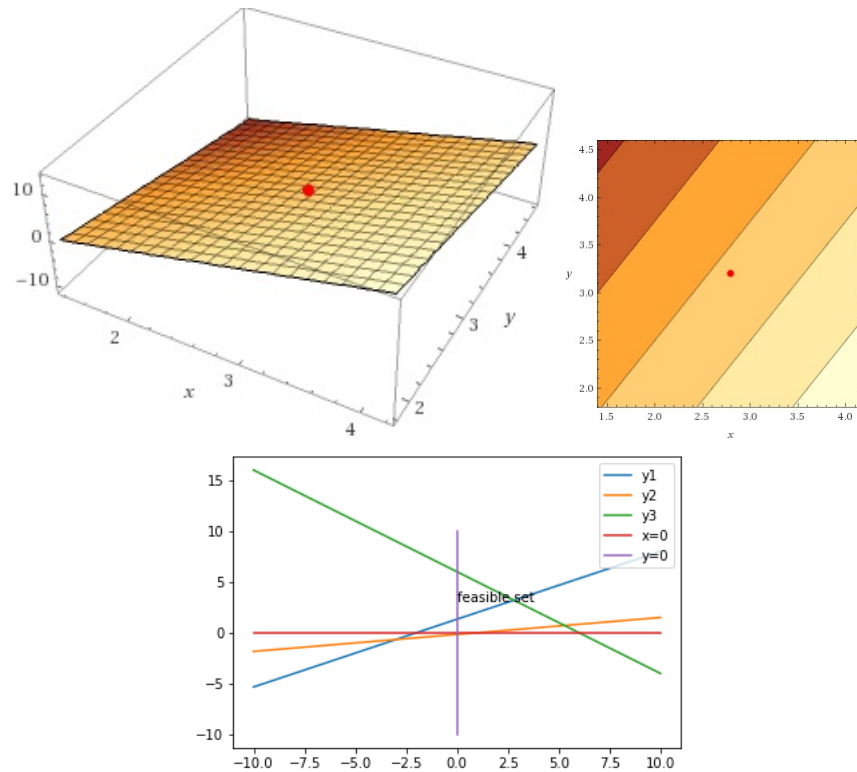


Figure 1

2. (8.2)

(i)

From vertices,  $x = (6, 2)$  is optimizer.

The optimal value is 20.

Figures are below.

(ii)

From vertices,  $(x, y) = (15, 12)$  is optimizer.

The optimal value is 132.

Figures are below.

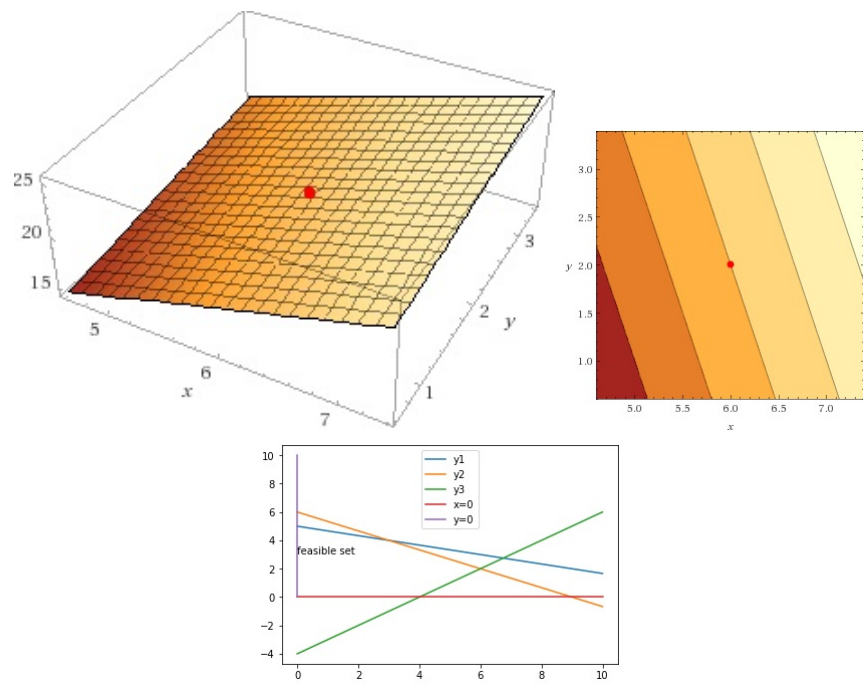


Figure 2

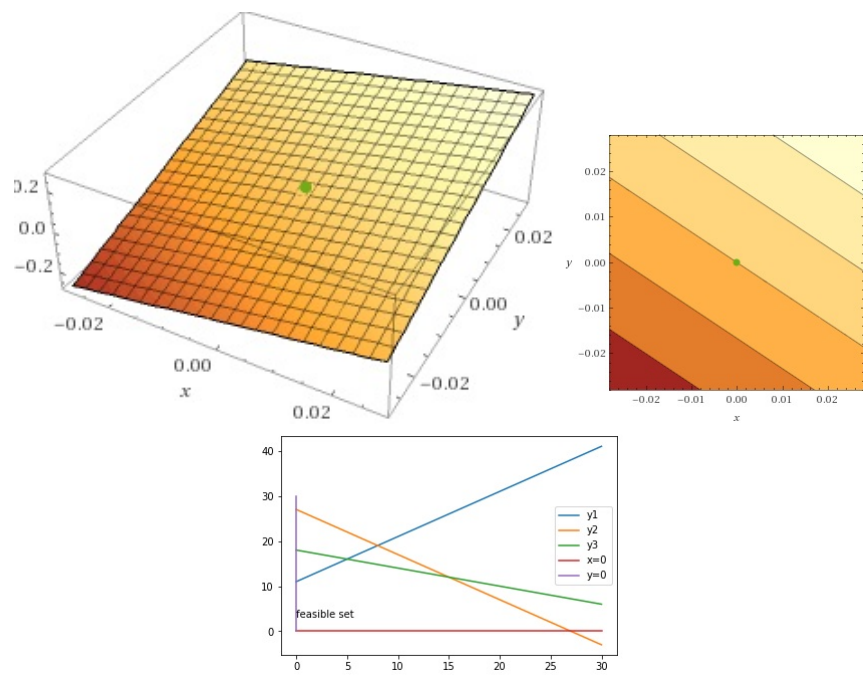


Figure 3

3. (8.5)

(i) "By Hand,"

1)  $f = 3x + y$

$w_1 = 15 - x - 3y$

$w_2 = 18 - 2x - 3y$

$w_3 = 4 - x + y$

2)  $f = 4y + 12 - 3w_3$

$w_1 = 11 - 2y + w_3$

$w_2 = 10 - 5y + 2w_3$

$x = 4 + y - w_3$

3)  $f = 4(2 + \frac{2}{5}w_3 - \frac{1}{5}w_2) + 12 - 3w_3$

$w_1 = 11 - 2(2 + \frac{2}{5}w_3 - \frac{1}{5}w_2) + w_3$

$y = (2 + \frac{2}{5}w_3 - \frac{1}{5}w_2)$

$x = 4 + (2 + \frac{2}{5}w_3 - \frac{1}{5}w_2) - w_3$

The optimizer is (6, 2), and it is exactly same to (8.2).

(ii) I used the simplex solver, and its optimizer is (15, 12) and it is exactly same to (8.2). The dictionaries are below.

Waiting for next Iteration							
x <sub>b0</sub>	C <sub>b1</sub>	Basis2	x <sub>0</sub>	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>
11	0	x <sub>1</sub>	-1	1	1	0	0
27	0	x <sub>3</sub>	1	1	0	1	0
90	0	x <sub>4</sub>	2	5	0	0	1
		z <sub>j</sub> -c <sub>j</sub> ->	-4	-6	0	0	0

Waiting for next Iteration							
x <sub>b0</sub>	C <sub>b1</sub>	Basis2	x <sub>0</sub>	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>
11	6	x <sub>1</sub>	-1	1	1	0	0
16	0	x <sub>3</sub>	2	0	-1	1	0
35	0	x <sub>0</sub>	7	0	-5	0	1
		z <sub>j</sub> -c <sub>j</sub> ->	-10	0	6	0	0

Waiting for next Iteration							
x <sub>b0</sub>	C <sub>b1</sub>	Basis2	x <sub>0</sub>	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>
16	6	x <sub>1</sub>	0	1	2/7	0	1/7
6	0	x <sub>2</sub>	0	0	3/7	1	-2/7
5	4	x <sub>0</sub>	1	0	-5/7	0	1/7
		z <sub>j</sub> -c <sub>j</sub> ->	0	0	-8/7	0	10/7

Optimal finite solution found							
x <sub>b0</sub>	C <sub>b1</sub>	Basis2	x <sub>0</sub>	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>
12	6	x <sub>1</sub>	0	1	0	-2/3	1/3
14	0	x <sub>2</sub>	0	0	1	7/3	-2/3
15	4	x <sub>0</sub>	1	0	0	5/3	-1/3
		z <sub>j</sub> -c <sub>j</sub> ->	0	0	0	8/3	2/3

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Status: Optimal finite solution found

Pivot 0.4285714285714286  
Step 4  
Optimal 132  
Elapsed 0.0 segs.  
Solution x1=12, x2=14, x0=15

Figure 4

4. (8.7)
- (i) The optimizer is (3, 4) and optimal value is 11.
  - (ii) It is NOT feasible.
  - (iii) The optimizer is (0, 2) and optimal value is 2.

Each dictionary is below.

Waiting for next iteration							
x <sub>b</sub> 0	C <sub>b</sub> 1	Basis2	x <sub>0</sub>	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>
-8	0	x <sub>2</sub>	-4	-2	1	0	0
6	0	x <sub>1</sub>	-2	3	0	1	0
3	0	x <sub>4</sub>	1	0	0	0	1
zj-cj->			-1	-2	0	0	0
Waiting for next iteration							
x <sub>b</sub> 0	C <sub>b</sub> 1	Basis2	x <sub>0</sub>	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>
-4	0	x <sub>0</sub>	-16/3	0	1	2/3	0
2	2	x <sub>1</sub>	-2/3	1	0	1/3	0
3	0	x <sub>4</sub>	1	0	0	0	1
zj-cj->			-7/3	0	0	2/3	0
Waiting for next iteration							
x <sub>b</sub> 0	C <sub>b</sub> 1	Basis2	x <sub>0</sub>	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>
3/4	1	x <sub>0</sub>	1	0	-3/16	-1/8	0
5/2	2	x <sub>1</sub>	0	1	-1/8	1/4	0
9/4	0	x <sub>2</sub>	0	0	3/16	1/8	1
zj-cj->			0	0	-7/16	3/8	0
Optimal finite solution found							
x <sub>b</sub> 0	C <sub>b</sub> 1	Basis2	x <sub>0</sub>	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>
3	1	x <sub>0</sub>	1	0	0	0	1
4	2	x <sub>1</sub>	0	1	0	1/3	2/3
12	0	x <sub>2</sub>	0	0	1	2/3	16/3
zj-cj->			0	0	0	2/3	7/3

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**Status: Optimal finite solution found**

Pivot	0.1875
Step	4
Optimal	11
Elapsed	0.001 secs.
Solution	x <sub>0</sub> =3, x <sub>1</sub> =4, x <sub>2</sub> =12

Figure 5: (i)

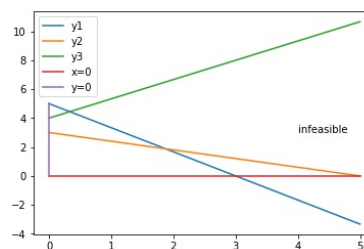


Figure 6: (ii)

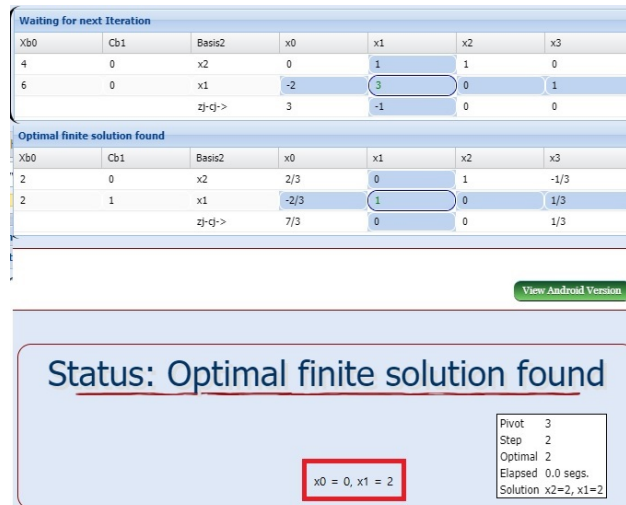


Figure 7: (iii)

5. (8.13)

If each constraint is zero, it means that only feasible set is  $(0, \dots, 0)$ .

If  $c^T x$  is defined at  $x = (0, \dots, 0)$ , then it is optimum.

However, if it is not defined, it is unbounded by definition.

6. (8.17) From dual problem, I will restore primal problem by definition.

$$\begin{aligned} \min \quad & b^T y \\ \text{s.t.} \quad & A^T y \geq c \\ & y \geq 0 \end{aligned}$$

By the definition of duality,

$$\begin{aligned} \max \quad & c^T x \\ \text{s.t.} \quad & (A^T)^T y \leq b \\ & y \geq 0 \end{aligned}$$

Then,  $y$  can be changed to  $x$  because it is not determined variables.

$$\begin{aligned} \max \quad & c^T x \\ \text{s.t.} \quad & (A^T)^T x \leq b \\ & x \geq 0 \end{aligned}$$

This is exactly same to the primal problem.

7. (8.18)

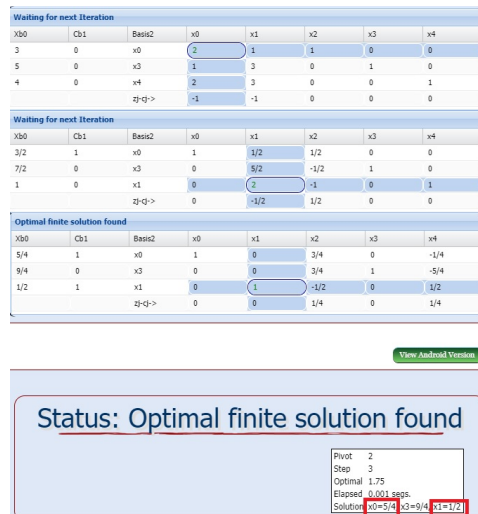


Figure 8: Primal Problem, and the optimal value is 1.75.

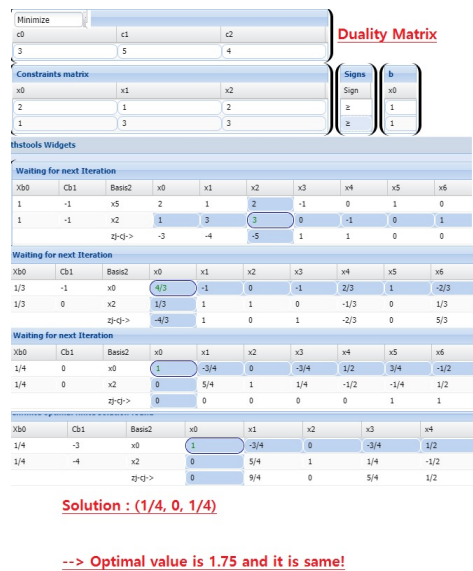


Figure 9: Dual Problem

## References