Open Source Macroeconomics Laboratory Boot Camp Homework Assignements

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DSGE

Exercise 1. I guess that $k_{t+1} = \alpha\beta e^{z_t} k_t^{\alpha}$ by Week 3, SMM problem. I will check with FOC condition.

$A = \alpha\beta$

$V(k_t, z_t) = max \log(e^{z_t} k_t^{\alpha} - k_{t+1}) + \beta E_t\{V(K_{t+1}, z_{t+1})\}$

Then, we can check with Euler equation.

$\frac{1}{e^{z_t} k_t^{\alpha} - k_{t+1}} = \beta E_t[\frac{\alpha e^{z_{t+1}} k_{t+1}^{(alpha-1)}}{e^{z_{t+1}} k_{t+1}^{\alpha} - k_{t+2}}]$

$\frac{1}{(1-\alpha\beta) e^{z_t} k_t^{\alpha}} = \beta E_t[\frac{\alpha e^{z_{t+1}} k_{t+1}^{(alpha-1)}}{e^{z_{t+1}} k_{t+1}^{\alpha} - k_{t+2}}]$

$\frac{1}{e^{z_t} k_t^{\alpha} - k_{t+1}} = \beta E_t[\frac{\alpha e^{z_{t+1}} k_{t+1}^{(alpha-1)}}{e^{z_{t+1}} k_{t+1}^{\alpha} - k_{t+2}}]$

$\frac{1}{(1-\alpha\beta) e^{z_t} k_t^{\alpha}} = \beta E_t[\frac{\alpha e^{z_{t+1}} k_{t+1}^{(alpha-1)}}{e^{z_{t+1}} k_{t+1}^{\alpha} - \alpha\beta e^{z_{t+1}} k_{t+1}^{\alpha}}]$

$\frac{1}{e^{z_t} k_t^{\alpha} - k_{t+1}} = \frac{1}{e^{z_t} k_t^{\alpha} - k_{t+1}}$

Exercise 2. ** Budget Constraint

$c_t = (1-\tau)[w_t l_t + (r_t - \delta)k_t] + k_t + T_t - k_{t+1}$

* Intertemporal Euler Equation

$\frac{1}{c_t} = \beta E_t[\frac{(r_{t+1} - \delta)(1-\tau) + 1}{c_{t+1}}]$

* Consumption-leisure Euler Equation

$\frac{1}{1-l_t} = \frac{w_t(1-\tau)}{c_t}$

*Firm FOC

$R_t = \alpha e^{z_t} K_t^{\alpha-1} L_t^{1-\alpha}$

$W_t = (1-\alpha) e^{z_t} K_t^{\alpha} L_t^{-\alpha}$

* Gov

$\tau[w_t l_t + (r_t - \delta)k_t] = T_t$

* Adding-up and Market Clearing

$l_t = L_t$

$k_t = K_t$

$w_t = W_t$

$r_t = R_t$

* Exogeneous Laws of Motion

$z_t = (1-\rho_z)\bar{z} + \rho z_{t-1} + \epsilon_t^z$

$\epsilon_t^z \sim iid(0, \sigma_z^2)$

We could not solve the way in number 1 because of δ

Exercise 3. ** Budget Constraint

$c_t = (1-\tau)[w_t l_t + (r_t - \delta)k_t] + k_t + T_t - k_{t+1}$

* Intertemporal Euler Equation

$c_t^{-\gamma} = \beta E_t[\frac{(r_{t+1}-\delta)(1-\tau)+1}{c_{t+1}^{\gamma}}]$

* Consumption-leisure Euler Equation

$\frac{1}{1-l_t} = \frac{w_t(1-\tau)}{c_t^{\gamma}}$

*Firm FOC

$R_t = \alpha e^{z_t} K_t^{\alpha-1} L_t^{1-\alpha}$

$W_t = (1-\alpha) e^{z_t} K_t^{\alpha} L_t^{-\alpha}$

* Gov

$\tau[w_t l_t + (r_t - \delta)k_t] = T_t$

* Adding-up and Market Clearing

$l_t = L_t$

$k_t = K_t$

$w_t = W_t$

$r_t = R_t$

* Exogeneous Laws of Motion

$z_t = (1-\rho_z)\bar{z} + \rho z_{t-1} + \epsilon_t^z$

$\epsilon_t^z \sim iid(0, \sigma_z^2)$

Exercise 4. ** Budget Constraint

$c_t = (1-\tau)[w_t l_t + (r_t - \delta)k_t] + k_t + T_t - k_{t+1}$

* Intertemporal Euler Equation

$c_t^{-\gamma} = \beta E_t[\frac{(r_{t+1}-\delta)(1-\tau)+1}{c_{t+1}^{\gamma}}]$

* Consumption-leisure Euler Equation

$\frac{1}{1-l_t} = \frac{w_t(1-\tau)}{c_t^{\gamma}}$

*Firm FOC

$R_t = e^{z_t}[\alpha K_t^{\eta} + (1-\alpha)L_t^{\eta}]^{\frac{1}{\eta}-1}\alpha K_t^{\eta-1}$

$W_t = e^{z_t}[\alpha K_t^{\eta} + (1-\alpha)L_t^{\eta}]^{\frac{1}{\eta}-1}(1-\alpha)L_t^{\eta-1}$

* Gov

$\tau[w_t l_t + (r_t - \delta)k_t] = T_t$

* Adding-up and Market Clearing

$l_t = L_t$

$k_t = K_t$

$w_t = W_t$

$r_t = R_t$

* Exogeneous Laws of Motion

$z_t = (1-\rho_z)\bar{z} + \rho z_{t-1} + \epsilon_t^z$

$\epsilon_t^z \sim iid(0, \sigma_z^2)$

Exercise 5. ** Budget Constraint

$c_t = (1-\tau)[w_t l_t + (r_t - \delta)k_t] + k_t + T_t - k_{t+1}$

* Intertemporal Euler Equation

$c_t^{-\gamma} = \beta E_t[\frac{(r_{t+1}-\delta)(1-\tau)+1}{c_{t+1}^{\gamma}}]$

* Consumption-leisure Euler Equation

It is fixed.

*Firm FOC

$R_t = \alpha K_t^{\alpha-1}(L_t e^{z_t})^{1-\alpha}$

$W_t = (1-\alpha)K_t^{\alpha} L_t^{-\alpha}(e^{z_t})^{(1-\alpha)}$

* Gov

$\tau[w_t l_t + (r_t - \delta)k_t] = T_t$

* Adding-up and Market Clearing

$l_t = L_t$

$k_t = K_t$

$w_t = W_t$

$r_t = R_t$

* Exogeneous Laws of Motion

$z_t = (1-\rho_z)\bar{z} + \rho z_{t-1} + \epsilon_t^z$

$\epsilon_t^z \sim iid(0, \sigma_z^2)$

** S-S version

$\bar{c} = (1-\tau)[\bar{w}\bar{l} + (\bar{r} - \delta)\bar{k}] + \bar{T}$

$1 = \beta[(\bar{r} - \delta)(1-\tau) + 1]$

$\bar{r} = \alpha \bar{k}^{\alpha-1}(e^{\bar{z}})^{1-\alpha}$

$\bar{w} = (1-\alpha)\bar{k}^{\alpha}(e^{\bar{z}})^{(1-\alpha)}$

$\tau[\bar{w} + (\bar{r} - \delta)\bar{k}] = \bar{T}$

$\bar{k} = [\bar{r}\frac{1}{\alpha e^{\bar{z}(1-\alpha)}}]^{\frac{1}{\alpha-1}}$

$\bar{r} = (\frac{1}{\beta} - 1)(\frac{1}{1-\tau}) + \delta$

Exercise 6. ** Budget Constraint

$c_t = (1-\tau)[w_t l_t + (r_t - \delta)k_t] + k_t + T_t - k_{t+1}$

* Intertemporal Euler Equation

$c_t^{-\gamma} = \beta E_t[\frac{(r_{t+1}-\delta)(1-\tau)+1}{c_{t+1}^{\gamma}}]$

* Consumption-leisure Euler Equation

$a(1-l_t)^{-\xi} = c_t^{-\gamma} w_t (1-\tau)$

*Firm FOC

$R_t = \alpha K_t^{\alpha-1}(L_t e^{z_t})^{1-\alpha}$

$W_t = (1-\alpha)K_t^{\alpha} L_t^{-\alpha}(e^{z_t})^{(1-\alpha)}$

* Gov

$\tau[w_t l_t + (r_t - \delta)k_t] = T_t$

* Adding-up and Market Clearing

$l_t = L_t$

$k_t = K_t$

$w_t = W_t$

$r_t = R_t$

* Exogeneous Laws of Motion

$z_t = (1-\rho_z)\bar{z} + \rho z_{t-1} + \epsilon_t^z$

$\epsilon_t^z \sim iid(0, \sigma_z^2)$

** S-S version

$\bar{c} = (1-\tau)[\bar{w}\bar{l} + (\bar{r}-\delta)\bar{k}] + \bar{T}$

$1 = \beta[(\bar{r}-\delta)(1-\tau)+1]$

$a(1-\bar{l})^{-\xi} = \bar{c}^{-\gamma}\bar{w}(1-\tau)$

$\bar{r} = \alpha\bar{k}^{\alpha-1}(\bar{l}e^{\bar{z}})^{1-\alpha}$

$\bar{w} = (1-\alpha)\bar{k}^{\alpha}\bar{l}^{-\alpha}(e^{\bar{z}})^{(1-\alpha)}$

$\tau[\bar{w} + (\bar{r}-\delta)\bar{k}] = \bar{T}$

$\bar{k} = [\bar{r}\frac{1}{\alpha e^{\bar{z}(1-\alpha)}}]^{\frac{1}{\alpha-1}}$

$\bar{r} = (\frac{1}{\beta}-1)(\frac{1}{1-\tau}) + \delta$

Exercise 7. No Homework

Exercise 8. Please see my py file.

Linearization

Exercise 9. Please see my py file.

Exercise 10. Please see my py file.

Exercise 11. $F(P\tilde{X}_t + Q\tilde{Z}_{t+1}) + G(P\tilde{X}_{t-1} + Q\tilde{Z}_t) + H\tilde{X}_{t-1} + L(N\tilde{Z}_t + \epsilon_{t+1}) + M\tilde{Z}_t$

$= FP(P\tilde{X}_{t-1} + Q\tilde{Z}_t) + FQ(N\tilde{Z}_t + \epsilon_{t+1} + G(P\tilde{X}_{t-1} + Q\tilde{Z}_t) + H\tilde{X}_{t-1} + L(N\tilde{Z}_t + \epsilon_{t+1}) + M\tilde{Z}_t$

$= [(FP+G)P+H]\tilde{X}_{t-1} + [FQN + LN + FPQ + GQ + M]\tilde{Z}_t + (FQ+L)\epsilon_{t+1}$

After Expectation, it goes to (8)

Exercise 12. Please see my py file.

Exercise 13. Please see my py file.

Exercise 14. Please see my py file.

Exercise 15. Please see my py file.

Exercise 16. Please see my py file.

Exercise 17. Please see my py file.

Exercise 18. Please see my py file.

Exercise 19. Please see my py file.

Perturbation

Exercise 20. $F_{xxx}[x_u^3] + F_{xxu}[3x_u^2] + F_{xuu}[3x_u] + F_{xx}[3x_u x_{uu}] + F_{xu}[3x_{uu}] + F_{uuu} + F_x x_{uuu} = 0$
$\therefore x_{uuu} = -\frac{F_{xxx}[x_u^3] + F_{xxu}[3x_u^2] + F_{xuu}[3x_u] + F_{xx}[3x_u x_{uu}] + F_{xu}[3x_{uu}] + F_{uuu}}{F_x}$

Exercise 21. Please see my py file.

Exercise 22. Please see my py file.

Exercise 23. Please see my py file.

Exercise 24. Please see my py file.

Filtering

Exercise 25. Please see my py file.

Exercise 26. Please see my py file.

Exercise 27. Please see my py file.

Exercise 28. Please see my py file.

References