Quantifying the Fiscal Backing for Monetary Policy

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Abstract

Successful inflation targeting requires fiscal policy to adjust the present value of the primary surplus path to meet changes in the market value of government debt due to monetary policy shocks. In this paper, I estimate the response of primary surpluses to a monetary policy shock and examine whether such a response is present in data, as suggested by the theory of monetary-fiscal policy interaction. The U.S. data estimates capture a 2% increase in primary surpluses total sum against a monetary contraction that raises the interest rate by 25 basis points. The result suggests that monetary contraction must be followed by fiscal contraction, mainly because of the dominant discount rate effect due to monetary policy shocks. I show that the expected future inflation path is the key to the monetary and fiscal policy interaction. Consequently, more aggressive inflation targeting monetary policy lessens the fiscal consequence.

Keywords: Fiscal Backing, Monetary Policy, Bayesian Estimation

JEL Codes: C11, E31, E63

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1 Introduction

Research on monetary-fiscal policy interaction has delivered a key insight: the existence of unique rational expectation equilibrium requires particular policy regimes which jointly regulate monetary and fiscal policies' behaviors. One such regime is well known for the Taylor rule monetary policy aggressively responding to inflation. In this regime, fiscal policy must provide fiscal solvency and backing for monetary policy. Fiscal backing refers to adjusting current and future primary surpluses against changes in the real market value of government debt due to monetary policy shocks, which is necessary for the equilibrium price level determination.

The monetary and fiscal policy interaction is imperative because both policies build on nominal bonds as their main tool.[1] The intrinsic value of the nominal bonds comes from a promise that the government will back them with real resources, the future taxation. It results in the government debt valuation in which the market value of outstanding government debt is equal to the present value of primary surpluses in equilibrium. So a monetary policy shock incurs a fiscal consequence when it changes the market value of debt.

To describe the necessity of fiscal backing concretely, suppose a monetary contraction raises the market value of privately-held government debt. But suppose further the primary surplus path does not respond to the contraction. Then it results in the market value of government debt being higher than its cash flow, the present value of primary surplus. The overpricing of the government bond creates a positive wealth effect that substitutes consumption for government bonds and stimulates the aggregate demand. This positive wealth effect is contrary to what the monetary contraction intends and incurs the price level indeterminacy. Hence the objective of fiscal backing is to adjust the primary surplus path to neutralize the wealth effect induced by monetary policy shocks on the government debt valuation. Cochrane (2011), Sims (2004, 2013, 2016), Caramp and Silva (2021) have made this point in a variety of theoretical settings. Missing from their works is a quantitative assessment determining whether how much fiscal backing is present in data.

In this paper, I estimate the response of primary surpluses to an exogenous monetary policy shock with the U.S. data through government debt valuation. I employ an estimated Dynamic Stochastic General Equilibrium(DSGE) model to make precise the objects I seek to measure

[1] The U.S. treasury has been issuing the inflation-indexed government debt(TIPS) since 1997. However, its average share in total outstanding debts from its introduction to 2020 is 7%, from the data of the Securities Industry and Financial Markets Association.

in data. The DSGE model extends the canonical medium-scale New Keynesian model with detailed fiscal parts. I set the priors of the policy parameters on the conventional regime in which monetary policy follows Taylor rule and fiscal policy stabilizes debt sufficiently to meet the equilibrium condition.

I also examine whether data can reveal the presence of fiscal backing without the restrictions on the policy regime. To this end, I estimate a Bayesian Vector Autoregression(VAR) model that shares the same dataset and takes an agnostic view on policy regime implementation. The identification strategy is to impose the same monetary policy rule in the DSGE model. Since the data is identical, imposing the same rule recovers the same error terms from the DSGE model.

Measuring how much fiscal backing monetary policy needs is a timely task given the unprecedentedly high level of debt and zero lower bound for the interest rate confronted by many economies. Under the distortionary tax system, there exists an upper bound for the present value of primary surpluses. Then sizable fiscal backing may not be affordable, restraining the central bank from anchoring the current price level. For instance, when the European Central Bank even introduced the negative interest rate in 2014, fiscal policies in the Euro area were under consolidation. Therefore estimating how much fiscal backing is required and exploring which economic environment amplifies its demand is crucial for policy implication.

The quantitative assessment must deal with several monetary policy transmissions to the government debt valuation. First, the market value of outstanding government debt depends on the nominal bond price and aggregate price level, which respond differently to monetary policy shocks. A monetary contraction that raises the nominal interest rate devalues the bond price but lowers the aggregate price level at the same time. It leads to the net effect on the real market value being quantitatively indeterminate. On the other hand, the discount factors on the primary surpluses become higher against the monetary contraction, deteriorating the present value of primary surpluses. Thus the estimation must take into account these monetary effects collectively for the quantitative aspect of fiscal backing.

The estimates without equilibrium restriction report a 0.38% devaluation of the market value of government debt to exogenous monetary policy shock that raises the interest rate by 25 basis points. However, this monetary contraction leads to a 2.25% increase in the primary surplus path since the discount rate depreciates the present value of primary surplus by 2.63%. It suggests that compensating for the severe discounting effect accounts for the fiscal backing motive mostly. Because of the dominant discount rate effect, the empirical fiscal backing pattern corresponds with the theoretical prescription: monetary contraction must be followed by fiscal contraction.

Next, I elaborate on the underlying economics of fiscal backing by decomposing the linearized government debt valuation. I show that the strong discount rate effect of the estimates comes from the persistent response of the expected nominal interest rate to monetary policy shock. Imposing a simple monetary policy rule and its shock process indicates that controlling the expected inflation path is the key to reducing the fiscal consequence of monetary policy shock. Consequently, the more hawkish monetary policy requires less fiscal backing.

I run counterfactual exercises on the estimated models to highlight the role of fiscal backing for monetary policy. While the estimates claim severe price rigidity, fiscal backing nature drastically changes with less rigidity as the market value of debt increases with monetary contraction. It boosts the fiscal backing demand, in line with Caramp and Silva (2021). With the estimated VAR, I compare the inflation responses to exogenous monetary contractions conditional on different fiscal backing scenarios. The exercise renders prominent differences in inflation responses at earlier periods.

To the best of my knowledge, I am the first to provide empirical evidence for the existence of fiscal backing explicitly to monetary-fiscal policy interaction literature. To all appearances, the institutional design seems to let monetary policy be exclusively responsible for inflation targeting and leave debt stabilization to the fiscal policy. The theoretical literature has pointed out that such manifestation is misleading since the primary surplus path must back the monetary policy by honoring equilibrium government debt valuation. My result discloses the theory's validity by revealing that the estimated model favors the environment that generates significant fiscal backing motive.

I also establish that the dominant discount rate effect is at the core of the fiscal backing from the empirical perspective. Theories have demonstrated the necessity of fiscal backing with rather parsimonious settings for comprehension. Contrastingly, an inclusive decomposition of government debt valuation is imperative, given that the estimates report the fall of the market value of debt from a monetary contraction.

Lastly, I shed light on that the fiscal consequence of monetary policy depends on the price rigidity and monetary policy rule that governs the expected inflation path. It can provide future policy implications that controlling the inflation expectation is central to affording fiscal backing with the unconventional policy environment.

Fiscal Backing in Literature Discussion of fiscal backing starts in earnest from the seminal works of Leeper (1991), Sims (1994), Woodford (1995), Cochrane (1999), namely the Fiscal Theory of Price Level(FTPL). Leeper (1991) denotes the policy regime of interest in this paper as active monetary and passive fiscal policy.[2] By active, it refers to monetary policy targets inflation uninterrupted by the consolidated government budget constraint. By passive, fiscal policy endogenously adjusts the primary surplus path to the real value of debt for government solvency. Since then, the theory has brought up the implicit assumptions on fiscal policy for inflation targeting monetary policy to the surface.[3]

This paper provides an empirical inquiry to the literature that studies the theoretical aspects of insufficient fiscal backing causing indeterminacy and identification issues in Taylor rule monetary policy. Particularly, Sims (2013) unveils that the essence of fiscal backing is the endogenous response of the primary surplus to the price level.[4] He also elicits the expectation aspects of fiscal backing. He points out that what matters for fiscal backing is whether the market perceives its existence, not the size of fiscal backing in equilibrium.[5] Dominguez and Gomis-Porqueras (2019) offer an interesting channel between the government debt and inflation by introducing secondary markets trading public debts in the DSGE model. They find that the conventional policy regime is more likely to rule out real and dynamic indeterminacies in this environment.

This paper complements the recent work of Caramp and Silva (2021), which analytically formalizes the wealth effect induced by monetary policy shocks and accompanied fiscal consequence. They decompose the monetary policy transmission channel into the substitution effect and wealth effect and then relate the latter one to fiscal policy. My work provides empirical investigation that shares many critical points with them. The first is about the wealth effect governing the initial response of inflation. Second, the maturity structure determines the sign of wealth effect of the monetary policy shock. The third is about the real effect of the monetary policy from the nominal rigidity matters for the fiscal consequence. In this context, my

[2] FTPL has explored the feasible policy mixtures that deliver unique and stable rational expectation equilibrium. While the FTPL literature aims to extend the analysis of inflation dynamics to other regimes, I limit my study to ask whether the fiscal policy has been passive enough to support the Taylor rule monetary policy.

[3] Leeper and Leith (2016), Cochrane (2019a) offer excellent surveys on the literature.

[4] Debt stabilizing fiscal policy may afford fiscal backing by responding to real debts. Bai and Leeper (2017) elucidate this subtle line between a passive fiscal policy and a debt stabilizing policy.

[5] Cochrane (2011) interprets this with off-equilibrium framework.

marginal contribution is to address the discount rate effect that turns out to be the dominant component of the debt valuation equation.

Cochrane (2021a) is methodologically the main reference for objectifying fiscal backing in empirics. I derive and decompose the linearized government debt valuation based on his method. Cochrane (2021a) reports the strong discount effect in the decomposition before me. However, his results are about unexpected shocks on the observables like inflation shock, surplus shock, other than monetary policy shock per se.[6]

Bianchi and Ilut (2017) is closely relevant to this paper in that their main contribution is to provide empirical evidence for joint monetary-fiscal inflation dynamics. They estimate a policy regime-switching DSGE model to evaluate the effect of agent's belief on future regime-switching to the macroeconomy. They conclude insufficient fiscal backing as a temporary event, interpreting it as a power game between policy authorities. In contrast, I do not preclude the possibility of chronic shortages of fiscal backing. It calls for a VAR estimation to incorporate with the viewpoint outside the equilibrium concept.

The subjects of interest are different from this paper to theirs too. Bianchi and Ilut (2017) explore how the propagations of each structural shock change with the agent's belief and information on regime-switchings. However, what I measure is the actual responses of primary surpluses to monetary policy shocks only. So the counterfactuals are about changing the realization of primary surpluses path.

Eusepi and Preston (2018) model the imperfect knowledge and learning about policies that break the Ricardian Equivalence and incur wealth effects. Accordingly, they show that passive fiscal policy always affects inflation under this bounded rationality. They also remark that the Taylor rule monetary policy must be more aggressive with higher debt levels. This paper goes along with the outcome but under the circumstance without imperfect knowledge and learning.

In terms of the subject of interest, Jiang et al. (2019, 2021) also quantify the market value of government debt and the present discounted value of primary surplus. They report that the market value of debt and debt-implied forecast on future primary surpluses have consistently exceeded the realized primary surplus path and interpret this mismatch as treasury investors' optimism. On the other hand, I search for the primary surplus path conditional on monetary

[6] Cochrane (2019b) finds half of the variation in the market value of debt to GDP ratio corresponds to varying forecasts of future primary surpluses, and a half to varying discount rates. His result is about the total variation, not conditional on monetary policy shocks.

policy shocks, and importantly, assume the debt valuation equation always holds.[7]

Hilscher et al. (2021) introduce a new method to measure the impact of inflation on the real value of public debt. As one of the implications, they offer a way to measure the changes in the present value of primary surpluses from the changes in the market value of debt and inflation from monetary policy shocks. However, It is different from my measure since I decompose the present value of primary surpluses into the primary surplus path and its discount factor because it is the primary surplus path that the fiscal policy governs entirely. In addition, they set the goal for their empirics only in a positive manner. However, I try to discuss 'what could have been done better' for monetary policy shocks for inflation targeting.

So far as my observation goes, the literature has shed light on various aspects of fiscal backing. However, the direct question-can the data show that the actual fiscal backing for conventional monetary policy has been sufficient?-is not yet answered.

2 Why Is Fiscal Backing Necessary For Inflation Targeting Monetary Policy?

In this section, I first derive the generic government debt valuation equation and depict how it can affect the price level determination in terms of the wealth effect of government bondholders. Then I apply an example that describes the necessity of fiscal backing under the active monetary and passive fiscal policy regime. Finally, I enumerate empirical factors that are critical in measuring fiscal backing.

2.1 Generic Government Debt Valuation Equation

Suppose that a government issues a full set of nominal zero-coupon bonds $B_{t,t+j}$ at period t with maturity j in a discrete time horizon. Then $Q_{t,t+j}$ denotes its price at t. Then government budget constraint at t can be written as

$$B_{t-1,t} = \sum_{j=1}^{\infty} Q_{t,t+j}(B_{t,t+j} - B_{t-1,t+j}) + P_t s_t \tag{1}$$

where P_t is aggregate price level and s_t is a real primary surplus. The equation (1) implies that the government must finance its matured short-term debt by adjusting its portfolio or additional primary surpluses. Let's denote B_t as a nominal market value of outstanding debt

[7] This is in line with the recent series of works Cochrane (2019b, 2021a).

portfolio at t,

$$B_t = \sum_{j=0}^{\infty} Q_{t,t+j} B_{t-1,t+j} \tag{2}$$

with $Q_{t,t} = 1$. Now let $\tilde{B}_t$ is the end-of-period market value of debt,

$$\tilde{B}_t = \sum_{j=0}^{\infty} Q_{t,t+j} B_{t,t+j} \tag{3}$$

Following the appendix in Cochrane (2021a), nominal gross return from the debt portfolio at t becomes

$$R_t^B = \frac{B_t}{\tilde{B}_{t-1}} \tag{4}$$

Plugging (2) and (4) into the government budget constraint (1) gives its simplified version,

$$R_t^B \tilde{B}_{t-1} = \tilde{B}_t + P_t s_t \tag{5}$$

I can transform this into a debt valuation equation showing the market value of debt equals to the expected present value of primary surplus path. Let $\tilde{b}_t = \tilde{B}_t / P_t$ and $r_t^b = R_t^B / \pi_t$ with inflation $\pi_t = P_t / P_{t-1}$. Then iterating the above equation forward with expectation at t and transversality condition yields the debt valuation equation.[8]

$$r_t^b \tilde{b}_{t-1} = \sum_{j=0}^{\infty} \mathbb{E}_t [m_{t+j} s_{t+j}]$$

where $m_{t+j} = \prod_{s=1}^{j} \frac{1}{r_{t+s}^b}$ with $m_t = 1$ is a real stochastic discount factor of government debt. Since $r_t^b \tilde{b}_{t-1} = B_t / P_t$,

$$\frac{B_t}{P_t} = \sum_{j=0}^{\infty} \mathbb{E}_t [m_{t+j} s_{t+j}] \tag{6}$$

[8] Bohn (1998), Canzoneri et al. (2001) present equilibria in which the real government debt can grow without bound. Chung et al. (2007) point out the lump-sum tax assumption is behind the cases and provide a counterexample. Because I do not specify the complete model environment, here I simply assume no lump-sum tax so that debt cannot grow without bounds.

The equation represents an asset pricing rule for the outstanding debt; the market value equals its cash flow, the expected present value of primary surpluses. It also tells why the nominal government bond can have intrinsic value as an asset; the government pays back the bondholders with real resources, the (real) primary surplus. Note that an implicit but essential fact here is that the government is the only taxation authority.[9]

In this generic setup, I can describe the effect of fiscal actions on the price level as a wealth effect. Suppose that the government issues more nominal debt today($B_t \uparrow$) with a commitment to raise extra surpluses($s_{t+j} \uparrow$) equal the debt increasement in real term. Then, bondholders have more assets today but also expect their tax increase equal to the asset gains.[10] Thus a bond issuance with an equal amount of expected tax hike in present value does not incur a wealth effect.

But what if there is no such commitment(or it is not credible)? Bondholders may bask in a positive wealth effect because while the market appreciates their assets more, they do not expect to pay more taxes in the future. This positive wealth effect pushes the demand of bondholders and ends up with a higher price level in equilibrium. The price level must rise to depreciate the nominal debt because it must meet its backing value in the equilibrium. In sum, as long as the update in expectation on future primary surpluses exactly meets the changes in the outstanding debt value, the price must take action to fill the gap.

2.2 Fiscal Backing in Active Monetary Passive Fiscal Policy Regime The previous debt valuation equation applies to most models as one of the equilibrium conditions. However, it abstracts away from how monetary and fiscal policy jointly determine the price level. The following example establishes the fiscal consequence of monetary policy shock and price level determination under the active monetary and passive fiscal policy regime. I bring it from Leeper and Leith (2016) and derive some of its solution sets. It shows the role of fiscal backing in price level determination and the unique and stable equilibrium of active monetary and passive fiscal policy regime analytically.

[9]FTPL explains the intrinsic value of fiat currency with this same rationale by simply adding the fiat currency holdings in the government budget constraint. By backing its nominal liabilities with real resources, a government can control inflations. See the introductions of Leeper and Leith (2016), Cochrane (2019a).

[10]This story may sound like the Ricardian Equivalance. However, it is the case only if the debt issuance is considered as a fiscal policy. Cochrane (2021b) sets apart the debt issuance into two contexts. Only the fiscal policy can raise the real revenue and thereby the real liabilities by debt issuances. Contrarily, debt issuances by monetary policy affect the inflation but revenue. See Del Negro and Sims (2015), Benigno (2020) for the different view on the financing by policy authorities with segregated budget constraints.

2.2.1 Example Model And Its Solution Suppose an infinite-horizon, constant endowment economy. The government has its bond portfolio of zero-coupon bonds B_t that declines at the fixed rate $\rho \in [0,1]$ each period. The geometric structure conveniently represents the maturity structure with a single parameter ρ. Then the government budget constraint becomes

$$\frac{(1+\rho Q_t)B_{t-1}}{P_t} = s_t + \frac{Q_t B_t}{P_t} \tag{7}$$

with Q_t is the bond portfolio price at t. Note that taking $\rho = 0$ boils down the maturity structure to short-term bonds only and $\rho = 1$ means all bonds are consols.

A representative household maximizes her utility under her budget constraint,

$$\max \mathbb{E}_t \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$
$$\text{s.t. } c_t + \frac{Q_t B_t}{P_t} + s_t = y + \frac{(1+\rho Q_t)B_{t-1}}{P_t}$$

s_t is a lump-sum tax, equal to primary surplus in this simple setup. The household optimization and the trivial constant endowment goods market clearing condition adds up to the intertemporal Euler equation,

$$\mathbb{E}_t \left[\beta \frac{P_t}{P_{t+1}} \frac{1+\rho Q_{t+1}}{Q_t} \right] = 1 \tag{8}$$

Using these two equations, let's derive the government debt valuation equation analogous to (6). Multiply $Q_t B_t / P_t$ on the each side of (8),

$$\frac{Q_t B_t}{P_t} = \mathbb{E}_t \left[\beta \frac{(1+\rho Q_{t+1})B_t}{P_{t+1}} \right] = \beta \mathbb{E}_t \left[s_{t+1} + \frac{Q_{t+1} B_{t+1}}{P_{t+1}} \right]$$

Imposing the boundedness of the real debt value leads to the debt valuation equation. Next, iterating the above equation forward with the government budget constraint (7) becomes,

$$\frac{(1+\rho Q_t)B_{t-1}}{P_t} = \frac{1+\rho Q_t}{Q_{t-1}} \frac{1}{\pi_t} \frac{Q_{t-1} B_{t-1}}{P_{t-1}} = \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t s_{t+j} \tag{9}$$

The real market value of debt consists of its par-value with holding period return $\frac{1+\rho Q_t}{Q_{t-1}}$ and the aggregate price level $\frac{1}{\pi_t}$, both subject to change at period t and the predetermined market value at the previous period, $\frac{Q_{t-1}B_{t-1}}{P_{t-1}}$. The present value of current and future primary surpluses is equal to its market value in equilibrium. In this context, the equation (9) intertwines the monetary and fiscal policy to jointly determine the price level. The goal is to study the the fiscal consequence of monetary policy shock on the real market value of outstanding debt and address the role of fiscal backing in equilibrium price level determination under the active monetary and passive fiscal policy.

To implement the policy regime, let's introduce a nominal short-term interest rate R_t and the term structure of interest rates in the form of no-arbitrage condition[11],

$$R_t = \mathbb{E}_t[R^B_{t+1}] = \mathbb{E}_t\left[\frac{1+\rho Q_{t+1}}{Q_t}\right] \tag{10}$$

where R^B_t denotes a holding period return of government bond portfolio at period t.

The monetary policy adjusts the gross nominal interest rate on the short-term bond against the deviation of inflation from its target level,

$$\frac{1}{R_t} = \frac{1}{R^*} + \alpha\left(\frac{1}{\pi_t} - \frac{1}{\pi^*}\right) - \varepsilon_t \tag{11}$$

the starred values represent the steady state or the target of each variable. The unconventional specification for the policy is to get closed-form solutions. The term ε_t is an exogenous monetary policy shock process. The negative sign on ε_t denotes that the monetary policy shock is contractionary.

By plugging the monetary policy rule (11) and term structure (10) into the Euler equation (8) results in the inflation dynamics equation,

$$\mathbb{E}_t\left[\frac{1}{\pi_{t+1}} - \frac{1}{\pi^*}\right] = \frac{\alpha}{\beta}\left(\frac{1}{\pi_t} - \frac{1}{\pi^*}\right) - \frac{1}{\beta}\varepsilon_t \tag{12}$$

How to solve for inflation depends on whther the coefficient α/β is inside or outside of unit circle. The Taylor rule in this model corresponds to $\alpha/\beta > 1$. It leads to solve for the inflation in a forward-looking manner,

$$\frac{1}{\pi_t} = \frac{1}{\pi^*} + \frac{1}{\alpha}\sum_{j=0}^{\infty}\left(\frac{\beta}{\alpha}\right)^j \mathbb{E}_t\varepsilon_{t+j} \tag{13}$$

[11]Technically, this condition abstracts away from the covariance term.

A contractionary monetary policy shock lowers inflation. Substituting the above solution into the monetary policy rule directly solves for the interest rate too.[12]

$$\frac{1}{R_t} = \frac{\beta}{\pi^*} + \sum_{j=1}^{\infty} \left(\frac{\beta}{\alpha}\right)^j \mathbb{E}_t \varepsilon_{t+j} \tag{14}$$

For the bond price Q_t, iterate the term structure (10) forward with respect to Q_t,

$$Q_t = \frac{1}{R_t} + \rho \mathbb{E}_t \left[\frac{Q_{t+1}}{R_t}\right] = \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t \left[\prod_{s=0}^{j} \frac{1}{R_{t+s}}\right] = \beta \sum_{j=1}^{\infty} (\beta\rho)^{j-1} \mathbb{E}_t \left[\prod_{s=1}^{j} \frac{1}{\pi_{t+s}}\right] \tag{15}$$

It links the current bond price to the whole path of expected inflations with maturity structure parameter ρ. Existence of long-term bond creates the trade-off between the current and future expected inflation.

Since the solution for the inflation and nominal rates seems to come entirely from the inflation dynamics (12), it seems that the model decouples into two independent parts. However, it is misleading because the real debt dynamics that involves the price level need to be solved. For this, we need to specify the systematic fiscal policy as well. The fiscal policy endogenously determines the primary surplus based on the real market value of debt $b_t = \dfrac{Q_t B_t}{P_t}$ of the previous period,

$$s_t = s^* + \gamma(b_{t-1} - b^*) \tag{16}$$

The fiscal policy parameter γ represents how much the government commits to stabilize debt to its target level. Plugging this fiscal policy rule into the government budget constraint (7) and rearraging it turns into the debt dynamics equation,

$$b_t - b^* = \left(\frac{1}{\pi_t} \frac{1 + \rho Q_t}{Q_{t-1}} - \gamma\right) b_{t-1} - (\beta^{-1} - \gamma) b^* \tag{17}$$

pushing the above equation one period forward and taking expectation at t brings a neater

[12] The response of the interest rate to a contractionary monetary policy shock is negative, even though the negative sign in ε_t clearly has a direct effect of raising R_t in the monetary policy rule (11). The same shock ε_t lowers inflation than its target level so the systemic rule part $\alpha(\pi_t^{-1} - \pi^{*-1})$ drags down R_t, because the model lacks intertemporal substitution effects on consumption choices.

version of the debt dynamics,

$$\mathbb{E}_t[b_{t+1} - b^*] = (\beta^{-1} - \gamma)(b_t - b^*) \tag{18}$$

Consequently, I can fully characterize the model economy with (12) and (18) as a linear bivariate system of the inverse of inflation and real market value of debt. The policy mixture emerges for the unique and stable rational expectation equilibrium of the system.[13] The active monetary and passive fiscal policy regime in this system amounts to the following policy parameter restrictions(excluding the oscillating case),

$$\frac{\alpha}{\beta} > 1, \quad r^* < \gamma \tag{19}$$

where $r^* = \beta^{-1} - 1$ is the steady state real interest rate. The first inequality restriction describes the Taylor principle for monetary policy. The second one tells that the fiscal policy must commit to adjusting primary surpluses more than the steady state interest rate. It means that the fiscal policy must retire outstanding debts and acquire debt stabilization as well.

2.2.2 FISCAL CONSEQUENCE OF MONETARY POLICY

To study the fiscal consequence of monetary policy shock on the market value of the debt side, suppose the monetary policy shock follows a stationary AR(1) process decaying with a parameter φ. Then the effect of monetary policy shock on the real market value of debt in (9) becomes,

$$\begin{aligned}\frac{\partial}{\partial \varepsilon_t}\left(\frac{(1+\rho Q_t)B_{t-1}}{P_t}\right) &= \frac{Q_{t-1}B_{t-1}}{P_{t-1}}\Bigg\{\underbrace{\frac{\rho}{Q_{t-1}\pi_t}\frac{\partial Q_t}{\partial \varepsilon_t}}_{\text{Bond price revaluation}} + \underbrace{\frac{1+\rho Q_t}{Q_{t-1}}\frac{\partial}{\partial \varepsilon_t}\left(\frac{1}{\pi_t}\right)}_{\text{Inflation adjustment}}\Bigg\} \\ &= \frac{B_{t-1}}{P_t}\frac{\rho\beta}{\alpha - \beta\varphi}\sum_{j=1}^{\infty}(\beta\rho)^{j-1}\mathbb{E}_t\left[\prod_{s=1}^{j}\frac{1}{\pi_{t+s}}\sum_{s=1}^{j}\varphi^s \pi_{t+s}\right] + \frac{(1+\rho Q_t)B_{t-1}}{P_{t-1}}\frac{1}{\alpha - \beta\varphi}\end{aligned} \tag{20}$$

More aggressive monetary policy reduces the fiscal consequence. Dependence on the previous real market debt level $\frac{Q_{t-1}B_{t-1}}{P_{t-1}}$ makes the fiscal consequence of a monetary policy shock state-dependent.[14] Because higher stakes get more leverage effect from a monetary policy shock.

[13] Inflation is non-predetermined while debt is. Hence one of the two eigenvalues of the system, $\frac{\alpha}{\beta}$ and $\beta^{-1} - \gamma$, should be outside while the other should be inside of the unit circle. See Leeper (1991), Sims (1994) for detailed establishments regarding to the policy regime.

[14] Eusepi and Preston (2018) address the state dependency with imperfect knowledge framework.

The fiscal consequence on the market value of debt divides into the bond price revaluation and inflation adjustment. The bond price revaluation effect refers to bond portfolio price changes due to a monetary policy shock. It is in effect only if there exist long-term government liabilities and the monetary shock is not transitory. It gets larger with more longer maturity ρ and more persistent monetary shock φ.[15] The other factor of changes in the market value of debt is the inflation adjustment. It simply shows that the lower inflation from the monetary contraction increases the debt value in the real term. Although it shares a similar comparative analysis with the bond price revaluation, it occurs even with the short-term debt only and/or transitory monetary shock cases. The fact that the inflation adjustment effect is always present in changes in the market value of debt demonstrates that the primary surplus must respond to inflation to align the debt valuation equation(Sims, 2013).[16]

2.2.3 The Role of Fiscal Backing: Eliminating Wealth Effect of Monetary Policy

The previous exercise verifies that a monetary policy shock changes the real market value of debt in (20). Since the debt valuation (9) is one of the equilibrium conditions, the present value of primary surpluses must respond to meet this change from the other side of the equation in equilibrium. Suppose that the expected primary surplus path does not respond to a monetary contraction at all. It ends up with the real market value of debt being higher than its actual backing value in (9). Then the government bond-holders would regard this difference as a positive wealth effect.

Now we can see more vividly why the primary surplus path is necessary to respond to a monetary policy shock. The equilibrium restriction $\beta^{-1} - \gamma < 1$ on the real debt dynamics (18) assures the deviation of the debt from its target diminishes each period. It implies that the primary surplus responds to the extent that the increased debt from a monetary policy shock can be retired. In fact, the coefficient in (17) relates fiscal backing to the wealth effect in a coarse but straightforward fashion.

$$\mathbb{E}_{t-1}\left(\frac{1}{\pi_t}\frac{1+\rho Q_t}{Q_{t-1}} - \gamma\right) < 1 \tag{21}$$

[15] Optimal maturity structure policy is a compelling topic. See for example Lustig et al. (2008), Bhandari et al. (2019).

[16] It also indicates why the fiscal policy stabilizing the real market value of debt to some extent or more can achieve the same goal. It is in line with the point from Bai and Leeper (2017) that the debt stabilizing fiscal policy is not passive unless it responds to the price level.

This condition prevents the real debt dynamics from taking an explosive path. In this model, the potential wealth effect induced by a monetary policy shock solely comes from the term $\frac{1}{\pi_t}\frac{1+\rho Q_t}{Q_{t-1}}$. So this inequality restriction in the real debt dynamics embodies the role of fiscal backing for joint determination for equilibrium price level; under the rational expectation, it shrinks and then eliminates the wealth effect of the government debt induced by monetary policy shocks over time. Put it differently, fiscal backing renders policy coordinations of fiscal contraction following a monetary contraction and fiscal expansion following monetary expansion.

2.3 Empirical Aspects Of Quantifying Fiscal Backing The previous example model provides some analytics to understand the role of fiscal backing, thanks to simplified model setups. Departing from this parsimonious model environment entails several challenges to recover the fiscal backing in data.

First, introducing consumption and production choices changes the behavior and adds new channels of the wealth effect induced by monetary policy shocks. Unlike the previous example, a higher nominal rate lowers inflation in the New Keynesian monetary policy. It hinges on the relation that the real interest rate follows the nominal rates so that the households substitute their current consumptions to savings and the inflation falls. Therefore monetary contractions lead to higher nominal rates and lower bond portfolio prices. It puts the bond price revaluation and inflation adjustment effect in (20) into opposing direction.

Second, endogenous discounting factor m_{t+j} counts in as an additional channel of the wealth effect in (6). Because it revises the expected present value of future primary surpluses and fiscal backing is no longer a sole determinant of its present value. The problem is that the discount rate effect would likely move in the opposite direction to the fiscal backing against monetary policy shocks. For example, a monetary contraction raising the nominal interest rate would lower the expected inflation and higher future bond returns. Then it calls for even larger primary surpluses to compensate for this higher discounting effect.[17]

Consequently, there are now three determinants of the wealth effect induced by monetary policy shocks that fiscal backing must deal with; (i) inflation adjustment, (ii) bond price revaluation, and (iii) discount rate effect. These three effects convolute the need for fiscal backing not only quantitatively but also qualitatively. For instance, (i) and (iii) require the fiscal back-

[17] Cochrane (2021a) reports a significant role of discount rate in inflation variation.

ing to be contractionary against a monetary contraction by raising the market value relative to its backing value. However, (ii) depreciates the market value at the same time. Hence if (ii) outweighs the others, then the wealth effect becomes negative from a monetary contraction. It may change the nature of fiscal backing from a fiscal contraction to expansion following a monetary contraction, motivating a quantitative assessment for the fiscal backing and wealth effect.

Third, a distortionary tax system can have substantial impacts on fiscal backing at high debt states. Unlike a lump-sum tax case that sufficient fiscal backing is always affordable, distortionary taxation may create an upper bound of tex revenue, a fiscal limit. This possibility would pressure the other fiscal policies than taxes like government spending and transfers to arrange the resource for fiscal backing.[18] Such a fiscal limit can deteriorate the expectations of future primary surpluses and immediately affect the current price level.

Lastly, the transmission of monetary policy shocks on output is also crucial for bond pricing.[19] The real effect of monetary policy shock on the output that links to the business cycle will be converted into the unexpected bond price shock. The endogenous discount rate is also dependent on the real effect of the monetary policy shock. These channels are hard to be captured analytically. Instead, I take a quantitative approach which leaves the door open for such complexities.

2.4 Decomposing the Linearized Debt Valuation Equation

In this section, I use a linearized government budget constraint to decompose the debt valuation eqaution into aforementioned factors following Cochrane (2021a). It allows me to measure fiscal backing and the potential wealth effect explicitly for the linear macro model in general. Log-linearizng the government budget constraint (7) I get,

$$\beta\hat{b}_t + (1-\beta)\hat{s}_t = \hat{R}_t^B - \hat{\pi}_t + \hat{b}_{t-1} \tag{22}$$

[18]It is foreseeable that the immense bill of use for public expenditures concerning agings is inevitable. It could cause imminent troubles for the current market value of debt in terms of its forward-looking nature.

[19]In the previous model, the households value the government bond only for the consumption smoothing motive. Introducing default risks and liquidity services would also amplify the monetary policy transmission on bond pricing. See Krishnamurthy and Vissing-Jorgensen (2012).

with $\hat{R}_t^B$ the linearized ex-post return of government bond portfolio. Iterating forward its ex-ante version becomes the linearized debt valuation,

$$\hat{R}_t^B - \hat{\pi}_t + \hat{b}_{t-1} = (1-\beta)\sum_{j=0}^{\infty}\beta^j\mathbb{E}_t[\hat{s}_{t+j}] - \sum_{j=1}^{\infty}\beta^j\mathbb{E}_t[\hat{R}_{t+j}^B - \hat{\pi}_{t+j}] \tag{23}$$

Next, impose the no arbitrage condition between the government bond return and nominal interest rate $\hat{R}_t = \mathbb{E}_t\hat{R}_{t+1}^B$ and rearrange the equation,

$$\hat{R}_t^B - \hat{\pi}_t + \hat{b}_{t-1} = (1-\beta)\sum_{j=0}^{\infty}\beta^j\mathbb{E}_t[\hat{s}_{t+j}] - \beta\hat{R}_t - \sum_{j=1}^{\infty}\beta^j\mathbb{E}_t[\beta\hat{R}_{t+j} - \hat{\pi}_{t+j}] \tag{24}$$

Denote a monetary policy shock ε_t and take the derivative of the debt valuation,

$$\underbrace{\frac{\partial\hat{R}_t^B}{\partial\varepsilon_t}}_{\substack{\text{Bond price}\\ \text{revaluation}}} - \underbrace{\frac{\partial\hat{\pi}_t}{\partial\varepsilon_t}}_{\substack{\text{Inflation}\\ \text{adjustment}}} = \underbrace{(1-\beta)\sum_{j=0}^{\infty}\beta^j\left[\frac{\partial\mathbb{E}_t\hat{s}_{t+j}}{\partial\varepsilon_t}\right]}_{\substack{\text{Fiscal backing}\\ \text{=Future primary surplus path}}} - \underbrace{\frac{\partial\beta\hat{R}_t}{\partial\varepsilon_t} - \sum_{j=1}^{\infty}\beta^j\left[\frac{\partial\mathbb{E}_t\beta\hat{R}_{t+j}}{\partial\varepsilon_t} - \frac{\partial\mathbb{E}_t\hat{\pi}_{t+j}}{\partial\varepsilon_t}\right]}_{\text{Discount rate effect}} \tag{25}$$

The left-hand-side represents the impulse response of the market value of debt which divides into the bond price revaluation and inflation adjustment as in (20). The right-hand side casts the impulse response of the present value of primary surpluses. Now fiscal backing is no longer the only determinant of the present value of primary surpluses.

Rewrite the equation with respect to fiscal backing links the fiscal backing and the wealth effect,

$$\underbrace{(1-\beta)\sum_{j=0}^{\infty}\beta^j\left[\frac{\partial\mathbb{E}_t\hat{s}_{t+j}}{\partial\varepsilon_t}\right]}_{\text{Fiscal backing}} = \underbrace{\frac{\partial\hat{R}_t^B}{\partial\varepsilon_t} + \sum_{j=0}^{\infty}\beta^j\left[\frac{\partial\beta\mathbb{E}_t\hat{R}_{t+j}}{\partial\varepsilon_t} - \frac{\partial\mathbb{E}_t\hat{\pi}_{t+j}}{\partial\varepsilon_t}\right]}_{\substack{\text{Potential wealth effect}\\ \text{induced by monetary policy}}} \tag{26}$$

The right-hand side amounts to the potential wealth effect induced by a monetary policy shock. It shows that the role of fiscal backing is to eliminate the wealth effect. Note that not only the size but also the sign of fiscal backing is conditional on the difference between the bond price revaluation and discount rate effect. Suppose a monetary contraction devalues bond price far more than the discount rate depreciates the present value of primary surpluses. Then raising extra primary surpluses is no longer necessary to lower the price level because the wealth effect is negative from this monetary contraction. It infers that fiscal backing is equivalent to a fiscal contraction following monetary contraction, only if monetary contraction generates a positive wealth effect. Hence measuring the components in (25) respectively and recovering the potential wealth effect is essential to assess fiscal backing from the empirical perspective.

The equation (26) suggests that the responses of expected path of the interest rate and inflation are the key determinants of fiscal backing. More persistent responses requires more fiscal backing as inflation would response negatively to the monetary contraction shock ε_t.

To see how monetary policy rule affects the fiscal consequence, let's impose a simple monetary policy rule $\hat{R}_t = \alpha\hat{\pi}_t + u_t$, where $u_t = \varphi u_{t-1} + \varepsilon_t$ follows an AR(1) process. Then the fiscal backing computation boils down to

$$(1-\beta)\sum_{j=0}^{\infty}\beta^j\frac{\partial\mathbb{E}_t\hat{s}_{t+j}}{\partial\varepsilon_t} = \frac{\partial\hat{R}_t^B}{\partial\varepsilon_t} + (\alpha\beta-1)\sum_{j=0}^{\infty}\beta^j\frac{\partial\mathbb{E}_t\hat{\pi}_{t+j}}{\partial\varepsilon_t} + \frac{1}{1-\beta\varphi} \tag{27}$$

Interestingly, the condition $\alpha > \beta$ appears as a sign determinant of the expected inflation responses. Given that the expected inflation responds negatively to monetary contraction, more aggressive monetary policy($\alpha\uparrow$) lessens fiscal backing demand. At the same time, conditional on the monetary policy rule α, the equation states that fiscal backing depends on the expected inflation path response. Lastly, more persistent monetary policy shock($\varphi\uparrow$) enlarges fiscal backing as it devalues the present value of primary surpluses critically.

Note that deriving this outcome requires the government budget constraint, transversality, and no-arbitrage condition between short-term and long-term bonds. These are equilibrium conditions essentially for every macro model which encompasses fiscal solvency. Thus a linearized DSGE model and/or VAR model in which a government takes part can always derive this equation (25).

The main strategy of the empirical exercise is to evaluate how the fiscal policy supports a monetary policy shock in the price level determination through (25). It turns into measuring the expected future primary surplus path through a DSGE and VAR model collectively. A structural estimation requires its model to have a unique and stable equilibrium in general. It inevitably precludes the shortage or absence of fiscal backing. A VAR is an atheoretical counterpart of a DSGE model that is free from this equilibrium regime restrictions and does not force the existence of perfect fiscal backing. Therefore I must accompany a structural estimation with a VAR part to answer whether fiscal backing in data has been sufficient.

The computation for the decomposition in (25) is readily implementable to generic state space models including DSGEs and VARs. They admit the following transition equation for a vector X_t,

$$X_t = AX_{t-1} + B\varepsilon_t \tag{28}$$

then equation (25) is equivalent to

$$(\mathbf{c}'_{R^B} - \mathbf{c}'_{\pi})B\mathbf{c}_{\varepsilon^M} = (1-\beta)\mathbf{c}'_s[I-\beta A]^{-1}B\mathbf{c}_{\varepsilon^M} - (\mathbf{c}'_{R^B} - \mathbf{c}'_{\pi})[I-\beta A]^{-1}B\mathbf{c}_{\varepsilon^M} + (\mathbf{c}'_{R^B} - \mathbf{c}'_{\pi})B\mathbf{c}_{\varepsilon^M} \tag{29}$$

where $\mathbf{c}_z$ is a selection vector for a variable z_t, *i.e.* $z_t = \mathbf{c}'_z X_t$. Note that the difference equation (29) holds for any structural shock in ε_t, I can apply it to study whether the monetary policy and fiscal backing are more important in the market value of debt variations than the other shocks.

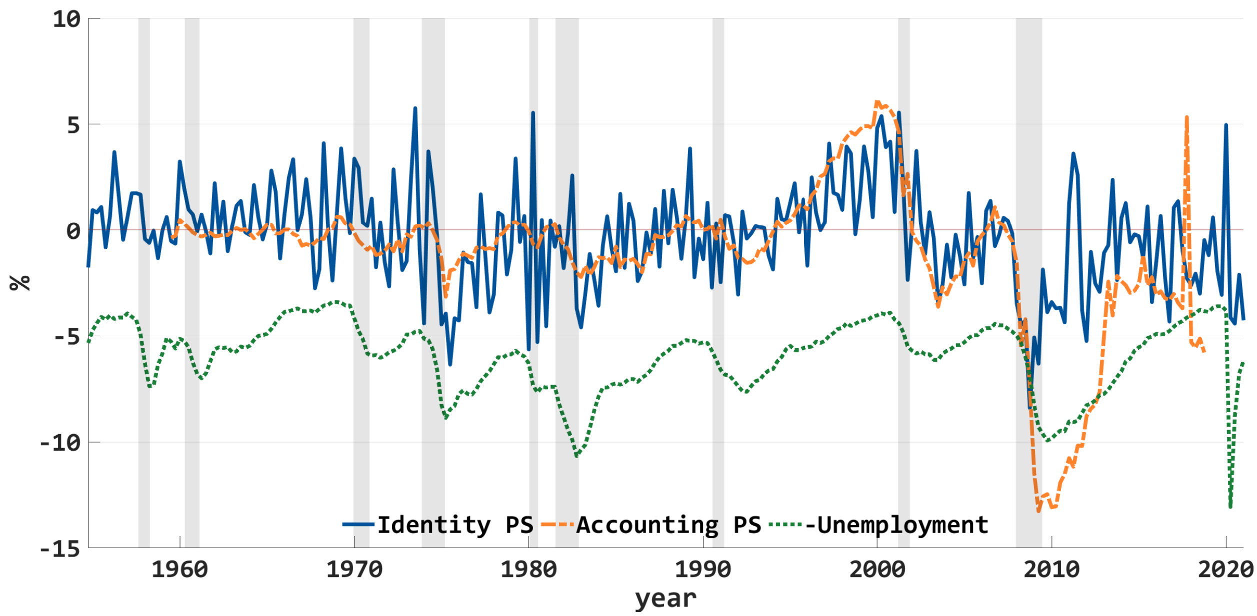
3 Data

Both the DSGE and VAR models for estimation share the same observable set of nine variables. The benchmark selections are output y_t, investment x_t, real wage w_t, labor n_t, market value of debt b_t, primary surplus s_t, inflation π_t, short-term nominal interest rate R_t, and ex-post nominal return on government bond portfolio R^B_t. I choose the observable set with standard ones from the medium-scale DSGE estimations, plus the fiscal observables essential for my main interest. Hall et al. (2018) provides data for the market value of government debt and its holding return.

The primary surplus data is certainly the target of interest, as the goal is to pick up the response of the primary surplus conditional on monetary policy shocks. Figure 1 shows two available options for primary surplus data. The identity plot is the primary surplus computed to hold the government budget identity from using the rest of the components. I follow the method of Hall and Sargent (2011) to derive the one labeled as accounting. The series is a federal government's total receipt net of its expenditure. Because it is primary but not gross surplus, the interest and pension payments are off from the expenditure. I also plot the unemployment rate to highlight the impact of tax revenue cyclicality on primary surpluses. It implies that the effect of monetary policy on the business cycle would be important to the fiscal consequences.

The figure tells that the two primary surplus data are quite different in that the accounting plot has much fewer spikes in high frequencies. It points to that the difference is more than just measurement errors. I suspect the bond price revaluation in each period mainly causes this difference. For the benchmark case, I use the primary surplus from the identity to ensure the market value of debt aligns exactly with the measured surplus. However, as shown in Figure 1, the primary surplus from the identity suffers from a severe seasonality due to the not-seasonally-adjusted market value of debt. I compromise to filter the primary surplus variable although it could mean that I mess with information sets. I add a measurement error term to the primary surplus in the measurement equation of the state space modeling for the DSGE model.

Figure 1: Primary Surplus Data Comparison



Identity(Solid): Primary surplus computed based on (22), Accounting(Dash-dot): Computed from components of NIPA table 3 following Hall and Sargent (2011), -Unemployment(dotted): Negative unemployment rate. Each data is quarterly. Grey shaded areas depict the NBER recessions.

The frequency of the data is quarterly and the full sample ranges from 1960 to 2006. However, many papers including Sims and Zha (2006), Davig et al. (2006), Bianchi and Ilut (2017) document the regime-switching scenarios in the U.S. and warn that the active monetary and passive fiscal policy regime was not prevailing in the pre-Volcker era, even with some temporary periods of conflicting policies. I do not include the Great Recession era because fiscal backing mechanism hinges on the future fiscal policy expectation, and the unconventional policies are questionable in this aspect.

4 Medium Scale New Keynesian Model Estimation

4.1 Model Description I use a DSGE model for estimation closely resembles that of Leeper et al. (2017). It extends the prototypical Smets and Wouters (2007) with two extra fiscal ingredients; a geometric maturity structure of government bonds and detailed fiscal policy responding to the real market value of debt and output.

4.1.1 Labor Packers A competitive labor packer producers composite labor input with differentiated household labors from a production technology, $n_t = \left(\int_0^1 n_{i,t}^{\frac{1}{1+\eta_t^w}} \right)^{1+\eta_t^w}$ where η_t^w is a time-varying exogenous markup process. The labor packer maximizes its profit subject to the labor cost,

$$\max_{n_{i,t}} W_t n_t - \int_0^1 W_{i,t} n_{i,t} di$$

where $W_{i,t}$ and $n_{i,t}$ are indivdual nominal wage and labor supply. W_t is the aggregate nominal wage. Solving the maximization problem yields the individual labor demand.

4.1.2 Household The continuum of households indexed with $i \in [0,1]$ chooses consumption c_t, investment x_t, labor supply $n_{i,t}$, wage $w_{i,t}$, utilized captial k_t, its utilization v_t, short-term and long-term government bond B_t^s, B_t to maximize lifetime utility. The long-term bond has a geometric maturity structure decaying at a constant rate ρ, which leads to the maturity duration of $(1-\beta\rho)^{-1}$.[20] To set up the maximizing problem, I assign the real detrended Lagrange multiplier $\frac{\lambda_t}{P_t \Gamma_t}$ to the budget constraint and $\frac{\lambda_t q_t}{\Gamma_t}$ to the law of motion of capital in which Γ_t is the permanent technology growth of the economy augmented as labor productivity and q_t is

[20] Hilscher et al. (2021) claims the exponential distribution of the maturity structure is not appropriate approximation of reality. On the other hand, Bhandari et al. (2019) prescribes a fixed geometric maturity structure as optimal debt management regarding asset pricing implications.

the Tobin's Q. The household solves the following maximizing problem,

$$
\begin{aligned}
V_t = & \max_{\substack{c_t, x_t, n_{i,t}, W_{i,t} \\ v_t, \bar{k}_t, B_t^s, B_t}} u_t^U \left\{ \ln(c_t - \eta c_{t-1}) - \vartheta \frac{n_{i,t}^{1+\chi^{-1}}}{1+\chi^{-1}} \right\} + \beta \iota_c \mathbb{E}_t[V_{t+1}] \\
& + \frac{\lambda_t}{P_t \Gamma_t} \{ (1-\tau^n) W_{i,t} n_{i,t} + (1-\tau^k) R_t^k v_t \bar{k}_{t-1} + \Upsilon_{i,t} + B_{t-1}^s + (1+\rho Q_t) B_{t-1} + P_t z_t \\
& - (1+\tau^c) P_t c_t - P_t x_t - \psi(v_t) \bar{k}_{t-1} - R_t^{-1} B_t^s - Q_t B_t \} \\
& + \frac{\lambda_t q_t}{\Gamma_t} \left[u_t^x \left\{ 1 - \mathcal{S}\left(\frac{x_t}{x_{t-1}} \right) \right\} x_t + (1-\delta) \bar{k}_{t-1} - \bar{k}_t \right]
\end{aligned}
$$

The total capital $\bar{k}_t$ is utilized as $k_t = v_t \bar{k}_{t-1}$. Each household pays taxes for consumption, labor and captial rental respectively. She receives profit $\Upsilon_{i,t}$ of the intermediate producer as an owner and lump-sum transfer z_t from the government.[21] Wage, captial rental rate, short-term and long-term bond prices in the budget constraints are nominal. η is a habit formation parameter. Augmenting long-term debts tends to produce prolonged responses of endogenous variables. So I parameterize the habit's internalization level with $\iota_c \in [0, 1]$. If $\iota_c = 1$ the model nests internal habit and if $\iota_c = 0$ the model becomes 'Catching up with the Joneses'. ϑ is a leisure preference term that matches the steady-state hours of work value. χ is the Frisch elasticity of labor supply, β is a time discount rate. ψ is an utilization cost function that $\psi(1) = 0$. A parameter $\psi \in [0, 1)$ represents its required property as $\frac{\psi''(1)}{\psi'(1)} = \frac{\psi}{1-\psi}$.

In the second constraint, capital depreciates with δ and investment adjustment function $\mathcal{S}\left(\frac{x_t}{x_{t-1}}\right)$. It requires convexity and is normalized to zero in the steady state, *i.e.* $\mathcal{S}(\gamma) = 0$, $\mathcal{S}(\gamma)' = 0$ and $\mathcal{S}''(\gamma) = \varsigma > 0$ where γ is the steady state growth rate of the economy.

There are two exogenous shocks on the household side. u_t^U is the intertemporal preference shock and u_t^x is a marginal investment efficiency shock.

The remainder of the household problem is about maximizing her present value of lifetime utility under the Calvo type wage rigidity. Each household updates her wage freely with probability $1 - \zeta_w$. Otherwise, she indexes her wage to the lagged inflation π_{t-1} with grwoth rate of the economy $\gamma_t = \frac{\Gamma_t}{\Gamma_{t-1}}$ at rate ι_w and the steady state of growth and inflation $\gamma\pi$ at rate $1 - \iota_w$

[21] The final goods market is competitive, so its profit is zero.

with probability ζ_w, that is,

$$W_{i,t} = \begin{cases} W^{\#}_{i,t} & \text{with probability } 1-\zeta_w \\ (\gamma_{t-1}\pi_{t-1})^{\iota_w}(\gamma\pi)^{1-\iota_w} W_{i,t-1} & \text{with probability } \zeta_w \end{cases}$$

This implies that the household whose wage has been optimized at t and sticking up to $t+s$ becomes $W_{i,t+s|t} = \pi^{\mathsf{w}}{}_{t+s} W^{\#}_{i,t}$, where $\pi^{\mathsf{w}}{}_{t+s} = \prod_{k=1}^{s} \left\{ (\gamma_{t+k-1}\pi_{t+k-1})^{\iota_w}(\gamma\pi)^{1-\iota_w} \right\}$ describes the indexation rule. Under the Calvo rigidity at an arbitrary period $T \geq t$, the household i chooses her reset wage $W^{\#}_{i,t}$ to maximize her value function associated with wage and labor choice,

$$V^w_T = \max_{W^{\#}_{i,t}} \left\{ -u^U_T \vartheta \frac{n_{i,T|t}^{1+\chi^{-1}}}{1+\chi^{-1}} + \lambda_T (P_T \Gamma_T)^{-1} \{(1-\tau^n_T) W_{i,T|t} n_{i,T|t}\} + \beta\zeta_w \mathbb{E}_T[V^w_{T+1}] \right\}$$
$$\text{subject to } n_{i,T|t} = n_T \left(\frac{W_{i,T|t}}{W_T} \right)^{-\eta^w_T}$$

4.1.3 Final Good Producers Analogous to the labor packer, a competitive final good producer combines the intermediate goods to maximize her profit,

$$\max_{y_{j,t}} P_t \left(\int_0^1 y_{j,t}^{\frac{1}{1+\eta^p_t}} dj \right)^{1+\eta^p_t} - \int_0^1 P_{j,t} y_{j,t} dj$$

which also yields the demand for intermediate good j and the aggregate price index with zero profit.

4.1.4 Intermediate Good Producers Each intermediate good producer j in the monopolistic competitive market minimizes her cost subject to Cobb-Douglas production technology meeting the demand,

$$\mathcal{L} = \max_{\tilde{k}_{j,t}, n_{j,t}} -(W_t n_{j,t} + R^k_t k_{j,t}) + \Psi_{j,t} \left(k^{\alpha}_{j,t} (\Gamma_t n_{j,t})^{1-\alpha} - \Gamma_t \Omega - \left(\frac{P_{j,t}}{P_t} \right)^{-\eta^p_t} y_t \right)$$

Ω is a fixed cost parameter which ensures no entry by setting the zero profit at the steady state. α is the usual parameter which measures the share of capital in production.

Next is to examine the intertemporal optimization for each intermediate producer. Analogous to the wage setting process, each producer is constrained by the price rigidity denoted as

ζ_p,

$$P_{j,t} = \begin{cases} P^{\#}_{j,t} & \text{with probability} 1-\zeta_p \\ \pi_{t-1}^{\iota_p}\pi^{1-\iota_p}P_{j,t-1} & \text{with probability} \zeta_p \end{cases}$$

Similar to the sticky wage case, the indexed price becomes $P_{j,t+s|t} = \pi^{\mathsf{p}}{}_{t+s}P^{\#}_{j,t}$ with indexation rule $\pi^{\mathsf{p}}{}_{t+s} = \prod_{k=1}^{s}\left(\pi_{t+k-1}^{\iota_p}\pi^{1-\iota_p}\right)$. Note that the profit of intermediate firm at t is

$$\begin{aligned} \Upsilon_{j,t} &= P_{j,t}y_{j,t} - W_t n_{j,t} - R^k_t k_{j,t} \\ &= P_{j,t}y_{j,t} - \Psi_t k^{\alpha}_{j,t}(\Gamma_t n_{j,t})^{1-\alpha} \\ &= P_{j,t}y_{j,t} - \Psi_t y_{j,t} - \Psi_t\Gamma_t\Omega \end{aligned}$$

subject to its demand for $y_{j,t}$. Conditional on the period t, the producer j chooses its reset price $P^{\#}_{j,t}$ to maximize its profit through the value function of the household(as an owner) at an arbitrary period $T \geq t$,

$$V^p_T = \max_{P^{\#}_{j,t}} \left\{\lambda_T(P_T\Gamma_T)^{-1}\Upsilon_{j,T|t} + \beta\zeta_p\mathbb{E}_T[V^p_{T+1}]\right\}$$

4.1.5 Monetary and Fiscal Policy

The policy interaction of the model follows the conventional New Keynesian model with a spectrum of interaction between monetary and fiscal policy. Here I pick a certain policy interaction which brings the unique stable equilibrium of the model, the active monetary and passive fiscal policy regime. The monetary policy follows the rule,

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R}\left\{\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\left(\frac{y_t}{y_t^{target}}\right)^{\phi_y}\right\}^{1-\rho_R}u^M_t \tag{30}$$

where y_t^{target} the policy target for output. If $y_t^{target} = y$, then the monetary policy simply responds to the output deviation from its steady state. If $y_t^{target} = y^f_t$ where y^f_t is the output without both nominal rigidities and markup shocks, then the monetary policy responds to the output gap.

The fiscal authority operates its debt stabilizing policy based on the following budget identity,

$$Q_tB_t + P_t\tau_t = (1+\rho Q_t)B_{t-1} + P_tg_t + P_tz_t \tag{31}$$

where $P_t\tau_t$ is the total nominal tax revenue,

$$P_t\tau_t = \tau^cP_tc_t + \tau^nW_tn_t + \tau^kR_t^ku_tk_t \tag{32}$$

the short-term bond is off the government budget constraint because of the implicit assumption of zero net supply at the equilibrium. The fiscal authority controls the government spending g_t and lump-sum transfer z_t to stabilize the real market value of debt, $b_t = \frac{Q_tB_t}{P_t}$. It also responds to output gap and its own lagged term. Note that the fiscal authority observes b_{t-1} and then stabilizes it in next period.

$$\frac{g_t/\Gamma_t}{g} = \left(\frac{g_{t-1}/\Gamma_{t-1}}{g}\right)^{\rho_g}\left\{\left(\frac{y_t}{y_t^{target}}\right)^{\varphi_x}\left(\frac{b_{t-1}}{b}\right)^{-\varphi_b}\right\}^{1-\rho_g}u_t^g \tag{33}$$

$$\frac{z_t/\Gamma_t}{z} = \left(\frac{z_{t-1}/\Gamma_{t-1}}{z}\right)^{\rho_z}\left\{\left(\frac{y_t}{y_t^{target}}\right)^{\psi_x}\left(\frac{b_{t-1}}{b}\right)^{-\psi_b}\right\}^{1-\rho_z}u_t^z \tag{34}$$

A remark here is the sign of fiscal expenditure parameters φ_x and ψ_x responding to output. While countercyclical behavior of tax revenue is uncontroversial, that of government expenditure is quite ambiguous. Still the primary surplus overall should be debt-responsible. So one way to reflect this feature is to specify the expenditure is neutral($\varphi_x = \psi_x = 0$). Alternative way is to make the persistency of expenditure very large, so that the process becomes more exogenous.

4.1.6 Aggregation

The factor market clearing conditions are straightforward.

$$n_t = \int_0^1 n_{j,t}dj \tag{35}$$

$$k_t = \int_0^1 k_{j,t}dj \tag{36}$$

To derive the goods market clearing condition substitute the total nominal profit from the household budget constraint. The total profit is the sum of profits from intermediate market because the final good market has zero profit by construction.

$$\Upsilon_t = \int_0^1 \Upsilon_{j,t} dj = \int_0^1 P_{j,t} y_{j,t} dj - W_t \int_0^1 n_{j,t} dj - R_t^k \int_0^1 k_{j,t} dj = P_t y_t - W_t n_t - R_t^k k_t$$

thus the integrated household budget constraint becomes

$$\begin{aligned}
&(1+\tau^c) P_t c_t + P_t x_t + R_t^{-1} B_{t+1}^s + Q_t B_{t+1} \\
&= -\tau^n W_t n_t - \tau^k R_t^k v_t \bar{k}_{t-1} - \psi(v_t)\bar{k}_{t-1} + P_t y_t + B_t^s + (1+\rho_L Q_t) B_t + P_t z_t
\end{aligned}$$

and then imposing the government budget constraint with zero net supply of short-term bond brings the goods market clearing condition,

$$P_t c_t + P_t x_t + P_t g_t + \psi(v_t)\bar{k}_{t-1} = P_t y_t \tag{37}$$

4.1.7 Exogenous Shocks

Each shock of the model follows normal distribution,

$$\varepsilon_t^s \sim \mathbb{N}(0, \sigma_s^2) \quad s \in \{\gamma, U, x, w, p, M, g, z\}$$

and follows the typical AR(1) process

$$\begin{aligned}
\gamma_t &= \gamma^{1-\varrho_\gamma} \gamma_{t-1}^{\varrho_\gamma} \exp(\varepsilon_t^\gamma) \\
u_t^s &= u^{s\,1-\varrho_s} u_{t-1}^{s\ \ \varrho_s} \exp(\varepsilon_t^s) \quad s \in \{U, x, M, g, z\} \\
\eta_t^s &= \eta^{s\,1-\varrho_s} \eta_{t-1}^{s\ \ \varrho_s} \exp(\varepsilon_t^s) \quad s \in \{w, p\}
\end{aligned}$$

4.2 DSGE Estimation

I use the Sequential Monte Carlo(SMC) algorithm from Herbst and Schorfheide (2015) as a main estimation method. It has several advantages over the traditional Markov Chain Monte Carlo(MCMC) algorithm like Metropolis-Hastings. First, it fits to deal with the global identification issue. Although I restrict the policy parameters within the conventional regime, I cannot preclude the possibilities that the Markov chain reaches restrained regions and gets stuck. SMC can bypass this issue with multiple chains in terms of particles. Second, SMC makes use of parallel computation that save a lot of time resources. Third, setting up the covariance matrix for Markov chain is not troublesome in SMC. The method can have an adaptation for covariance matrix by computing the covariance of the particles through each stage. For the estimation, I use 10,000 particles and 500 stages, a single mutation step but with 5 random blocks for 37 parameters.

I calibrate some parameters due to the identification issue mostly. Table 1 lists the calibration. The maturity structure parameter targets the average maturity duration of 5 years. Shortening the duration from 5 to 3 years does not affect the estimation result. The high persistence of the lump-sum transfer part is from the estimation of federal current transfer payments following an AR(1) process. It also reflects the strong exogeneity of the social security programs related to aging. Tax rates and steady state fiscal values are computed based on the U.S. fiscal data.

Table 1: Parameter Calibration

Parameters	Value	Description
β	0.99	Time discount rate
δ	0.025	Capital depreciation rate
α	1/3	Production share of capital
μ_w	0.14	Steady state wage markup
μ_p	0.14	Steady state price markup
ρ	0.9596	Decaying rate of long-term government bond
ρ_z	0.98	Persistence of lump-sum transfer
ϱ_z	0.14	Persistence of lump-sum transfer shock
τ_n	0.186	(Steady state) Labor tax rate
τ_k	0.218	(Steady state) Capital tax rate
τ_c	0.023	(Steady state) Consumption tax rate
s_b	0.80	Debt to GDP ratio target level
s_g	0.08	Steady state government spending to GDP ratio

4.3 ESTIMATION RESULT

Table 2 reports SMC sampler statistics of the estimated DSGE model. The estimates give tight posteriors for most of the parameters and are similar to the results of former studies(Smets and Wouters, 2007, Leeper et al., 2017). As reported in many papers, the estimates show severe price rigidity indicating a strong real effect of the monetary policy.

The posterior estimates are indeterminate about the cyclicality of the systemic reactions of the government spending and transfer against output. The behavior of the spending could vary depending on the different model environment, such as the government spending turning to a public good.[22] The transfer response to debt is higher than that of the spending. Given the tax rate fixed, it implies that fiscal backing in this model relies more on the transfer adjustment than spending adjustment.

[22] Leeper et al. (2017) take the model open to the public goods being complementary or substitution of private consumption goods.

Table 2: DSGE Posterior Estimates

	Parameter	Prior				Posterior		
		Dist	Mean	SD	90 percent CS	Mean	SD	90 percent CS
100γ	SS tech growth	N	0.4	0.05	[0.32, 0.48]	0.33	0.03	[0.28, 0.38]
χ_n	inverse Frisch	G	2	0.5	[1.26, 2.87]	1.89	0.22	[1.37, 2.17]
η	habit formation	B	0.5	0.2	[0.17, 0.82]	0.72	0.02	[0.70, 0.75]
ν	capital util	B	0.6	0.15	[0.34, 0.84]	0.31	0.04	[0.26, 0.38]
ς	inv adst cost	N	6	1.5	[3.59, 8.54]	7.83	0.76	[6.53, 9.07]
ζ_p	price calvo	B	0.5	0.2	[0.17, 0.82]	0.91	0.02	[0.87, 0.92]
ζ_w	wage calvo	B	0.5	0.2	[0.17, 0.82]	0.63	0.03	[0.56, 0.67]
ι_p	price idx	B	0.5	0.2	[0.17, 0.82]	0.46	0.07	[0.37, 0.62]
ι_w	wage idx	B	0.5	0.2	[0.17, 0.82]	0.09	0.04	[0.05, 0.17]
ρ_R	MP persistence	B	0.5	0.2	[0.17, 0.82]	0.41	0.07	[0.35, 0.58]
ϕ_π	MP inflation	U	2.04	0.6	[1.11, 2.98]	1.03	0.06	[1.00, 1.09]
ϕ_x	MP output	N	0.15	0.2	[-0.18, 0.48]	0.28	0.03	[0.23, 0.33]
ρ_g	FP spnd persistence	B	0.5	0.2	[0.17, 0.82]	0.92	0.05	[0.82, 0.95]
φ_x	FP spnd output	N	0.15	0.5	[-0.68, 0.97]	0.18	0.34	[-0.37, 0.73]
φ_b	FP spnd debt	N	1.2	0.5	[0.38, 2.01]	0.12	0.09	[0.07, 0.23]
ψ_x	FP trans output	N	0.15	0.5	[-0.68, 0.97]	0.15	0.34	[-0.42, 0.71]
ψ_b	FP trans debt	N	1.2	0.5	[0.38, 2.01]	0.89	0.27	[0.63, 1.56]
ϱ_γ	tech growth	B	0.5	0.2	[0.17, 0.82]	0.12	0.06	[0.06, 0.24]
ϱ_U	preference	B	0.5	0.2	[0.17, 0.82]	0.99	0.01	[0.98, 0.99]
ϱ_i	invst	B	0.5	0.2	[0.17, 0.82]	0.65	0.04	[0.60, 0.70]
ϱ_w	wage markup	B	0.5	0.2	[0.17, 0.82]	0.54	0.03	[0.49, 0.58]
ϱ_p	price markup	B	0.5	0.2	[0.17, 0.82]	0.45	0.04	[0.39, 0.53]
ϱ_g	govt spending	B	0.5	0.2	[0.17, 0.82]	0.26	0.13	[0.15, 0.60]
ϱ_M	MP	B	0.5	0.2	[0.17, 0.82]	0.85	0.07	[0.68, 0.90]
$100\sigma_\gamma$	tech growth	IG	0.1	1	[0.02, 0.28]	1.10	0.05	[1.03, 1.18]
$100\sigma_U$	preference	IG	0.1	1	[0.02, 0.28]	0.98	0.42	[0.73, 2.09]
$100\sigma_i$	invst	IG	0.1	1	[0.02, 0.28]	0.49	0.03	[0.45, 0.54]
$100\sigma_w$	wage markup	IG	0.1	1	[0.02, 0.28]	0.22	0.02	[0.20, 0.27]
$100\sigma_p$	price markup	IG	0.1	1	[0.02, 0.28]	0.11	0.01	[0.10, 0.12]
$100\sigma_g$	govt spending	IG	0.1	1	[0.02, 0.28]	5.96	0.57	[5.48, 7.06]
$100\sigma_z$	lumpsum transfer	IG	0.1	1	[0.02, 0.28]	24.03	1.05	[22.46, 25.68]
$100\sigma_M$	MP	IG	0.1	1	[-1.63, 1.65]	0.28	0.01	[0.26, 0.31]
$\bar{n}$	SS labor	N	0	1	[-1.70, 1.63]	0.29	0.51	[-0.56, 1.15]
$\bar{\pi}$	SS π	N	0.79	0.25	[0.38, 1.20]	0.52	0.16	[0.26, 0.79]
$\bar{s}$	SS primary surplus	N	0.21	0.5	[-0.62, 1.03]	-0.04	0.06	[-0.12, 0.05]
$100\sigma_s^{ME}$	ME primary surplus	IG	0.1	1	[0.02, 0.28]	0.90	0.08	[0.79, 1.03]

The third column marks the prior distribution. B(Beta), G(Gamma), IG(Inverse-Gamma), N(Normal), U(Uniform)

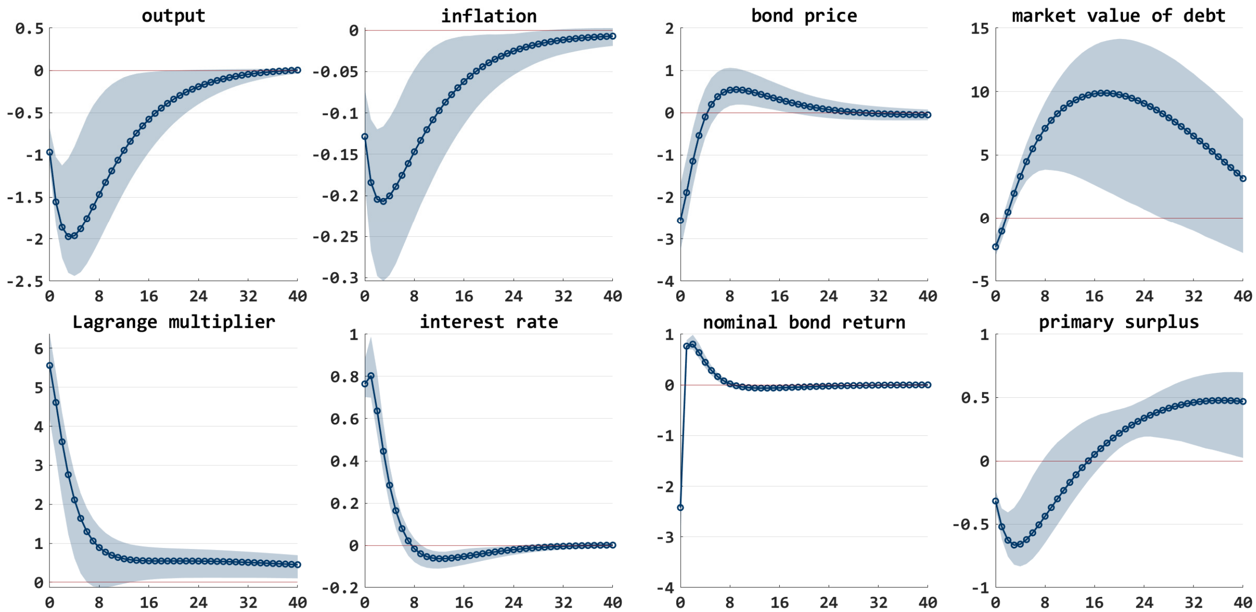
Figure 2 draws the 90 percent set of posterior impulse response on some variables of the estimated model to a monetary contraction of one standard deviation of exogenous monetary policy shock. The overall responses are consistent with the conventional monetary policy theory qualitatively. The responses of the output, inflation, and interest rate are very similar to those of Smets and Wouters (2007), implying the result follows the New Keynesian monetary policy story in general.

The response of the government debt shows a very long persistence. Because the fiscal policies target the debt status, the response of the primary surplus also follows the same pattern. It is in line with Leeper et al. (2017), although their result is about the government spending shock. Yet, the persistence comes from the matrix A in (28), so it can underpin that the estimates resemble theirs.

The bond price falls at the impact of unanticipated contraction. However, it starts to rise after acknowledging the autoregressive monetary shock and gradually goes down quite a while. The nominal bond return that marks the market value of the bond portfolio is tightly related to the bond price in (**??**). The huge negative impact response of return contributes to the devaluation of the real market value of government debt. On the other hand, the few big positive responses after the impact boost their associated discount rates. It results in the discount rate effect dominating the following market value of debt analysis, along with the Lagrange multiplier of the household. The reversing movement of the bond return suggests the ambiguous net effect on the wealth effect induced by the monetary shock through the bond holdings of the private sector.

The fiscal backing shapes the long-run characteristics of the model responses. A contractionary monetary policy dampens output at the early stages. Due to the distorting tax base, huge primary deficits arise. It may seem odd because the fiscal backing should imply fiscal austerity following the contractionary monetary policy. However, it is the present value of the primary surplus that backs the market value of debt. In this context, the reverting primary surplus path response amounts to the surplus in terms of the present value discounting. It also explains the large accumulation of the market value of debt despite the decreased bond price. Since it takes relatively many periods to arrange the required fiscal backing, bond selling is inevitable to fill in the government balance. Such adjustments would take more time with more debts outstanding indicating the state dependency of the fiscal backing.

Figure 2: Posterior Impulse Response of DSGE Model to Monetary Policy Shock

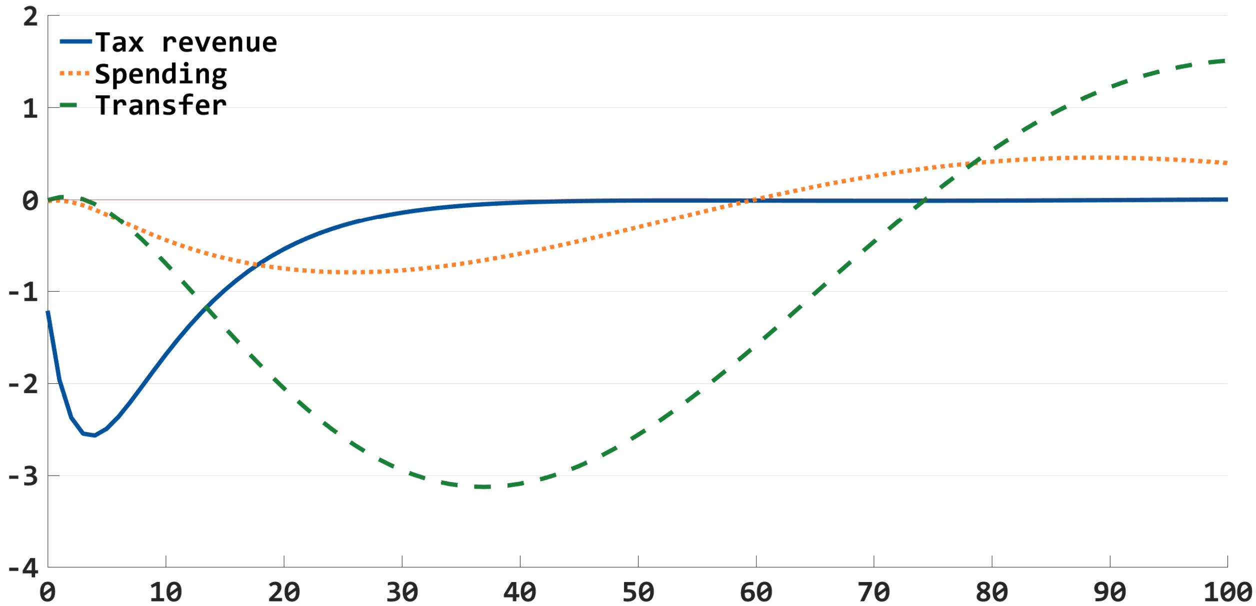


Responses over 40 quarters to an exogenous monetary contraction of a one standard deviation of exogenous monetary policy shock and the sample period 1960Q1 to 2006Q4. Solid line depicts the posterior mean and the shaded areas are the 90 percent credible intervals.

The model defines the primary surplus as a difference between the tax revenue and public expenditure divided into spending and transfer. Figure 3 splits the primary surplus response into each component. As noted in Table 2, the transfer response is larger than spending's. The figure demonstrates the negative impact response of primary surplus to monetary contraction; the output fall leads to the drop in the tax revenue at the impact while the adjustments from the expenditure side are insignificant.[23] The tax revenue affected by changes in the output provides an additional fiscal backing motive other than the reasons discussed previously. This indirect output channel is crucial for fiscal backing from the quantitative perspective. The primary surplus reacts explicitly to the output from its specification in terms of the spending and transfer. The distortionary tax system implicitly affects the primary surplus as well and leads the tax revenue to be procyclical. It implies the nominal rigidities which represent the level of the real effect of the monetary policy is a critical factor in estimating the fiscal backing.

[23]Note that the debt stabilizing mechanism operates against b_{t-1} not b_t. It naturally generates the mean-reverting primary surplus response. Cochrane (2021a,b) contends it as a distinctive evidence for debt financing. He interprets this pattern as fiscal policy being responsible to debt stabilization.

Figure 3: Posterior Fiscal Impulse Response of DSGE Model to Monetary Policy Shock



Responses of posterior means over 100 quarters to a monetary contraction of one standard deviation of exogenous monetary policy shock and the sample period 1960Q1 to 2006Q4. The fiscal responses are those of tax revenue(solid), government spending(dotted), and lump-sum transfer(dashed).

Table 3 reports the estimates of components in (25), the main interest of this paper. The equilibrium model always satisfies the debt valuation equation, so the fiscal backing is equivalent to the potential wealth effect. It allows a counterfactual exercise in which the present value of primary surplus does not change and amounts to compute the potential wealth effect induced by the monetary shocks.

The second and third column shows the prior predictive estimates. A prior predictive analysis is useful to examine the range of values that the model estimates can generate. Because DSGE models often involve many restrictions, they may conclude the estimates and prevent them from being updated by data.

The prior predictives in Table 3 show that the DSGE model regulates the signs of inflation at the impact, discount rate effect, and fiscal backing. However, the theory alone cannot confirm the bond price revaluation qualitatively or quantitatively. The revaluation can have any value depending on the variety of monetary shock transmission channels. In this context, the extensive support of the prior predictive is suitable, ready to hear what data tells. On the other hand, the prior predictive restricts the sign of the present value of primary surplus elements coherent to the prescribed fiscal backing mechanism. As a result, the credible set of the prior predictive of fiscal backing leaves the role of data in estimation. Therefore the prior predictive practices in the table support the prior choice that largely resembles those of in Leeper et al. (2017) in which they use diffusing priors to examine the various arguments on fiscal multipliers.

Table 3: Prior and Posterior Debt Value Decomposition of DSGE Model

Debt value components	Prior				Posteior			
	Median	Mean	5th quantile	95th quantile	Median	Mean	5th quantile	95th quantile
Inflation adjustment	-0.25	-0.34	-1.49	-0.04	-0.13	-0.13	-0.18	-0.08
Bond price revaluation	-1.05	-0.89	-2.59	3.16	-2.43	-2.42	-3.00	-1.78
Discount rate effect	-2.16	-2.41	-6.73	-1.08	-5.13	-5.12	-5.64	-4.38
Fiscal backing	1.32	1.85	0.25	9.80	2.82	2.82	1.63	3.78

Each row is the response of (25) to a monetary contraction of a one standard deviation of exogenous monetary policy shock.

The posterior result suggests that the discount rate effect is the dominant factor in the debt valuation, which is in line with the main results of Cochrane (2021a).[24] In particular, the absolute value of the discount rate effect size is always larger than that of bond price revaluation that turns out to be negative in the whole 90 percent posterior intervals. It results in that the real market value of government debt is higher than the present value of primary surpluses and generates a positive wealth effect, even when the market value falls. Consequently, the future primary surplus path must rise to soak up this positive wealth effect to honor the equilibrium condition. The dominant discount factor effect is direct evidence of positive wealth effect from a monetary contraction. At the same time, it is indirect evidence for strong fiscal backing.

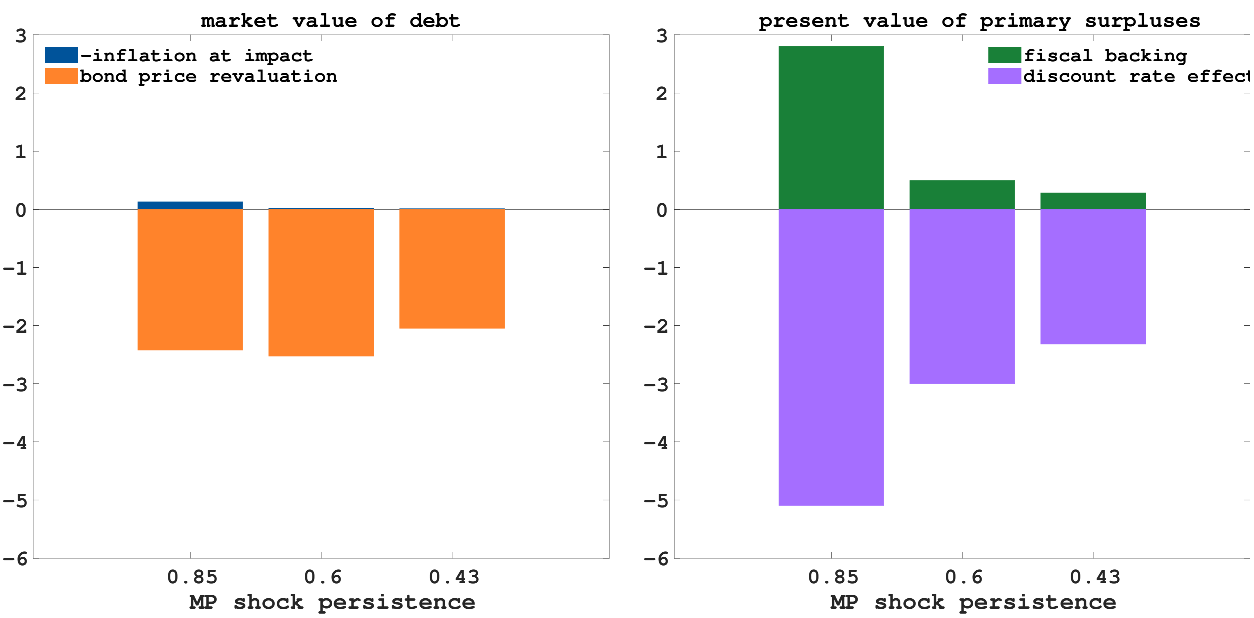
Fiscal Consequence of Monetary Policy Revisit The posterior estimates of the debt valuation decomposition assert that even though a monetary contraction reduces the market value of government debt, fiscal backing must be positive to compensate for the severe discount rate effect. Based on (26), it infers the persistent responses of the expected interest rate and inflation paths are the main causes. Given the high level of price rigidity and monetary policy

[24] His results are not about the identified monetary policy shocks. But he still finds the discount rate effect is dominant in most of the identified shocks. Cochrane (2019b) finds half of the variations in the market value of debt to GDP ratio come from the varying discount rate.

shock persistence, the nominal interest rate path seems to amplify the need for fiscal backing. In order to see this is the case, I run counterfactual exercises that suggest how the model quantifies fiscal backing.

Figure 4 plots the mean of posterior and counterfactuals with respect to the monetary policy shock persistence parameter ϱ_M. Less persistent shock greatly alleviates the discount rate effect. It results in drastic decrease of fiscal backing, along with the bond devaluation.

Figure 4: Mean Response of Debt Valuation Counterfactuals on Monetary Policy Shock Persistence

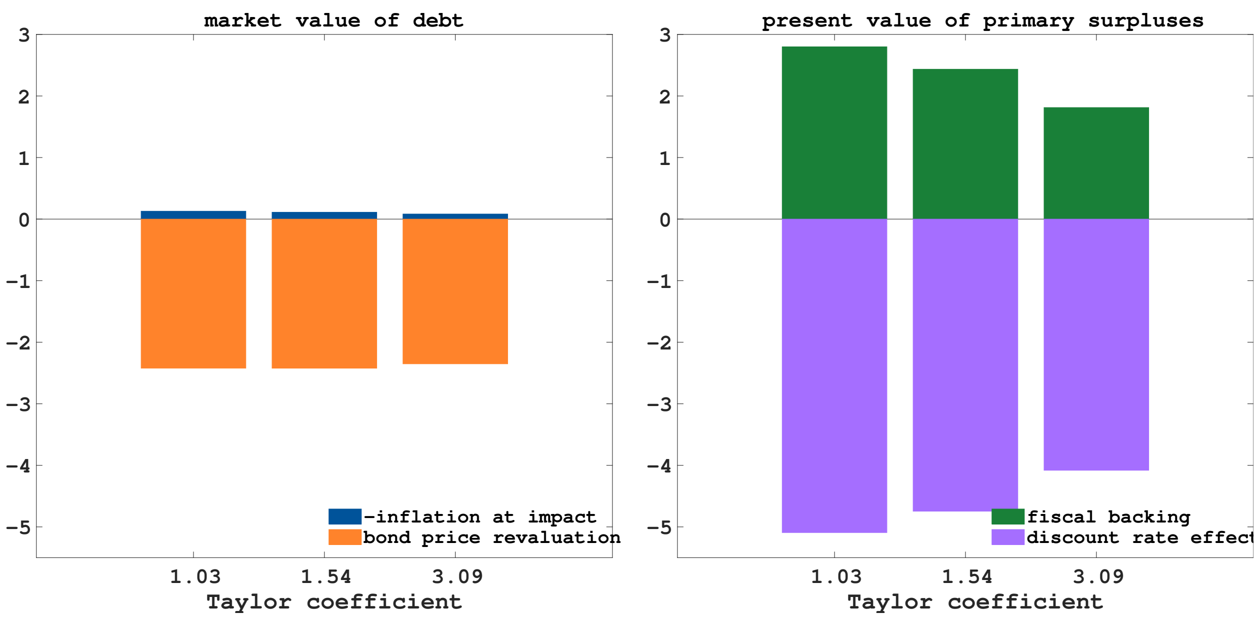


Posterior and counterfactual means. The government debt valuation holds with -inflation at impact(blue) + bond price revaluation(orange) = fiscal backing(green) + discount rate effect(purple)

The counterfactual exercise on shock persistence generates a significant difference, but it is not implementable as a policy advice. In this sense, a counterfactual on monetary policy rule may provide pragmatic implications. I examine how the fiscal consequence depends on different monetary policy rule in terms of the Taylor coefficient ϕ_π.

Figure 5 repeats the counterfactual responses of debt valuation with the level of aggressiveness of monetary policy rule. The more hawkish monetary policy is effectively reducing the size of fiscal backing as it does on the discount rate effect. It indicates that this aggressive systemic rule stabilizes the expected inflation path and thereby the future interest path as well. It is noteworthy for the potential stimulative policies against current recessions with the high level of debts. For the expansionary monetary policy, it is crucial to elicit the expectation of the private sector that the central bank will stick to the Taylor principle, so that the required fiscal backing is affordable even with the given level of debt.

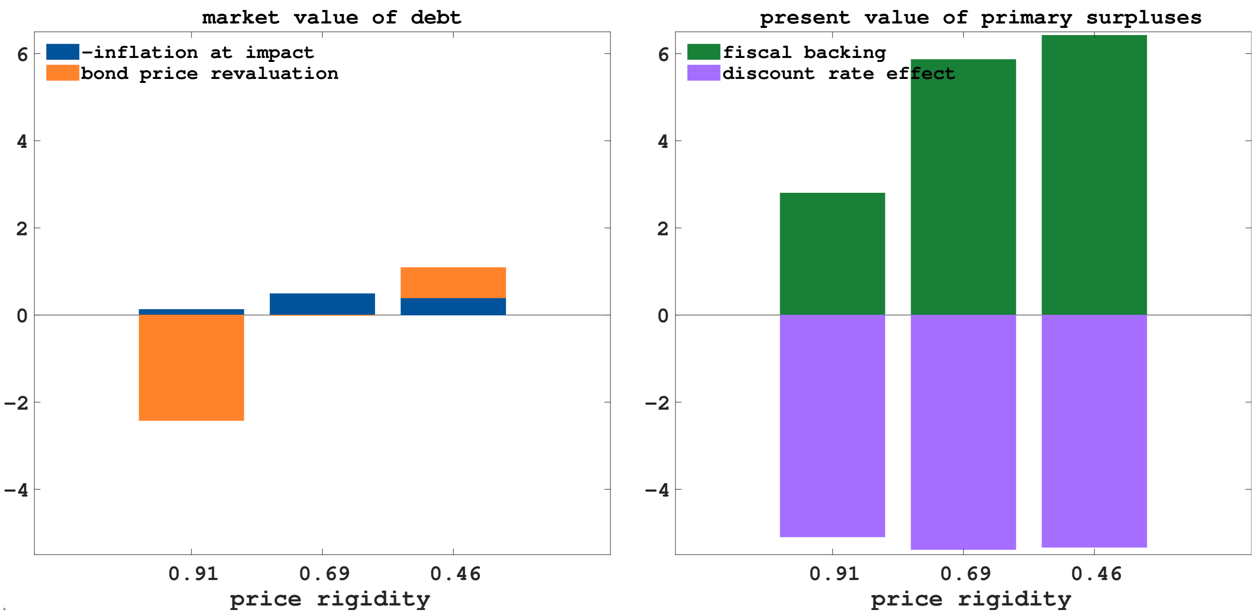
Figure 5: Mean Response of Debt Valuation Counterfactuals on Monetary Policy Rule



Posterior and counterfactual means. The government debt valuation holds with -inflation at impact(blue) + bond price revaluation(orange) = fiscal backing(green) + discount rate effect(purple)

Lastly, I excute a counterfactual with respect to price rigidity, which is suspected to be a major source of the prevailing discount rate effect. Figure 6 plots the counterfactuals on the Calvo price rigidity ζ_p. As the pricing gets more flexible, the nature of the market value of debt changes drastically. With 69% of not having a chance to update prices, bond price revaluation is almost neutralized. With 46%, the bond price actually gets an appreciation from a monetary contraction. It amplifies the potential wealth effect through the government debt valuation so that fiscal backing becomes larger. This is in line with Caramp and Silva (2021), in that the policy interaction becomes more crucial in the environment in which the price is more flexible.

Figure 6: Mean Response of Debt Valuation Counterfactuals on Price Rigidity



Posterior and counterfactual means. The government debt valuation holds with -inflation at impact(blue) + bond price revaluation(orange) = fiscal backing(green) + discount rate effect(purple)

5 Partially Identified Bayesian VAR Model Estimation

The structural estimation in the previous section captures the existence of fiscal backing by construction. It was possible because the model restriction imposes debt valuation as one of the equilibrium conditions. The active monetary and passive fiscal policy regime enforces the fiscal policy to respond to the real debt so that the model can be solved and estimated. Such restricted estimates alone cannot answer whether fiscal backing is coming from the data or the model restriction.

To this end, I introduce a Bayesian VAR model as the second stage of fiscal backing estimation. Using the same dataset, I can interpret it as a less restricted version of the previous linearized DSGE model from the equilibrium condition perspective. Many popular priors of the VAR are suitable for listening to the data and free from regime restriction. In turn, it can take an agnostic view on fiscal backing and seek to the data response to the monetary policy shock. Some diffusing priors can incorporate with the impact response of inflation in any scale and direction. The impulse responses of other variables can also have wide supports compared to those of the DSGE estimates.

Another relevant feature of the VAR is that it can establish the identification of the monetary policy in a variety of flexible manners. The main idea is to impose the same monetary policy rule estimated in the DSGE model. Because the data is identical, it recovers the identical error term(Leeper et al., 1996). Treating this error term as the exogenous monetary policy shock ensures that the DSGE and VAR model considers the same monetary policy specification, including the realized shocks. The benchmark method I take is to treat the estimated monetary policy shock from the DSGE model as a proxy variable for the Instrumental Variable(IV) identification. I apply Mertens and Ravn (2013), Caldara and Herbst (2019) for the proxy external IV VAR estimation whose methods are based on the classical and Bayesian estimation respectively.[25]

The partially identified Structural VAR(SVAR) is another way to utilize the shared dataset and model specification. The point is to restrict the equation associated with the interest rate variable in the VAR to the identical monetary policy rule in the DSGE model. The procedure recovers the identical error terms ε_t^M in (30) without extra assumptions for the other structural shocks. I implement this idea with the tool developed by Arias et al. (2018), which incorporates both zero and sign restrictions in the SVAR model through orthogonal reduced-form parameterization.

5.1 Econometric Framework

A reduced form VAR(p) model is represented as

$$y_t = c + \sum_{\ell=1}^{p} A_\ell y_{t-\ell} + u_t, \quad u_t \sim \mathbb{N}[0, \Sigma] \tag{38}$$

where y_t is a $n \times 1$ vector of observables, c is a constant vector and A_ℓ is a coefficient matrix with respect to each ℓ lagged vector $y_{t-\ell}$. The reduced form error term u_t is a forecast error. Next, suppose that the VAR model has the following structural representation,

$$B_0 y_t = B_0 c + \sum_{\ell=1}^{p} B_\ell y_{t-\ell} + \varepsilon_t, \quad \varepsilon_t \sim \mathbb{N}[0, I_{n\times n}] \tag{39}$$

[25] Caldara and Herbst (2019) provide an econometric framework that allows treating measurement errors on the proxy formally. I apply their method and report the results in subsection D.2.

with $B_\ell = B_0 A_\ell$ for all $\ell = 1, \ldots, p$. Then the relationship between the forecast error u_t and the structural innovation ε_t becomes

$$u_t = B_0^{-1}\varepsilon_t = B\varepsilon_t \tag{40}$$

Each column of $B = B_0^{-1}$ corresponds to the impact response for each structural innovation ε_{it} respectively.

The covariance matrix of the reduced form (38) is

$$\Sigma = \mathbb{E}[u_t u_t'] = \mathbb{E}[B\varepsilon_t \varepsilon_t' B'] = BB' \tag{41}$$

Identification problem arises because one can only estimate Σ with observable in (38) but B.

Given that the interest of this paper is limited to a monetary policy shock, without loss of generality, I can divide the structural innovation into two parts, $\varepsilon_t = [\varepsilon_{1,t}' \varepsilon_{2,t}']'$ where $\varepsilon_{1,t}$ is a monetary policy shock and $\varepsilon_{2,t}$ represents the rest of orthogonal shocks. Subsequently, it leads to a partial identification of interest, $B = [B_1 B_2]$ where B_1 is a column vector that represents the impact response of each observable in y_t to monetary policy shock and B_2 is a $n \times (n-1)$ matrix for other shocks. Therefore the goal is to estimate B_1 instead of the whole B matrix.

To recover the structural column B_1, a proxy variable Z_t can serve as an external instrumental variable with the following conditions,

$$\mathbb{E}[Z_t \varepsilon_{1,t}'] = \phi \tag{42}$$

$$\mathbb{E}[Z_t \varepsilon_{2,t}'] = 0 \tag{43}$$

The first condition (42) is a relevance condition and ϕ represents the degree of the instrumental variable's proximity to the shock of interest $\varepsilon_{1,t}$. The second condition (43) is a exclusion condition that the instrumental variable is not correlated to other orthogonal shocks.

The benchmark identification strategy of this paper is to take the estimated monetary policy shock in (30) from the previous DSGE model as an instrumental variable. Since I use a single instrumental variable and focus on one structural shock, I can estimate B in the following method. Let's start with by multiplying the instrumental variable Z_t to the transposed version of (40),

$$Z_t u_t' = Z_t \varepsilon_t' B'$$

and then taking expectation conditional on the matrix B,

$$\begin{aligned}\mathbb{E}[Z_t u_t'] &= \mathbb{E}[Z_t(B_1\varepsilon_{1,t} + B_2\varepsilon_{2,t})'] \\ &= \mathbb{E}[Z_t\varepsilon_{1,t}']B_1' + \mathbb{E}[Z_t\varepsilon_{2,t}']B_2' \\ &= \phi B_1'\end{aligned} \tag{44}$$

because of the instrumental conditions (42) and (43).

Next, using the inverse of (41) and multiplying the above term to it front and back,

$$\mathbb{E}[Z_t u_t']\Sigma^{-1}[\mathbb{E}[Z_t u_t']]' = \phi B_1'(BB')^{-1}B_1\phi = \phi^2 \tag{45}$$

since $B^{-1}B_1 = [1, 0, \ldots, 0]'$. Therefore, combining (44) and (45) results in B_1 only as,

$$[\mathbb{E}[Z_t u_t']]'\left[\mathbb{E}[Z_t u_t']\Sigma^{-1}[\mathbb{E}[Z_t u_t']]'\right]^{-\frac{1}{2}} = B_1$$

which has the following sample analogue,

$$\frac{1}{T}\sum_{t=1}^{T} Z_t \hat{u}_t' \left(\frac{1}{T}\sum_{t=1}^{T} Z_t \hat{u}_t' \hat{\Sigma}^{-1} \frac{1}{T}\sum_{t=1}^{T} \hat{u}_t Z_t'\right)^{-\frac{1}{2}} = \hat{B}_1 \tag{46}$$

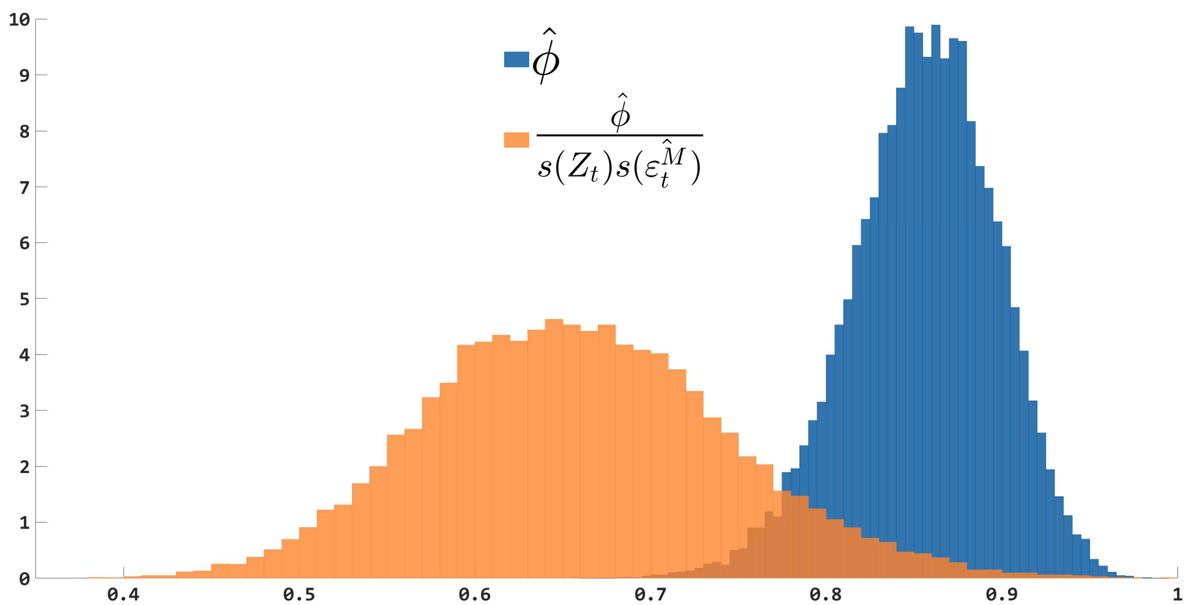
where T is the sample size associated with the instrumental variable Z_t.

5.2 Bayesian Estimation Given the same dataset of the counterpart DSGE model and identification strategy established above, I estimate the reduced form VAR model (38) with the hierarchical Bayesian modeling approach proposed by Giannone et al. (2015) for commonly used priors for reduced form VAR models. The basic idea is to replace subjective choices on hyperparameters with hyperprior distributions. By selecting a conjugate hyperprior distribution, the conventional MCMC estimation is applicable with a closed-form marginal likelihood. The method can incorporate with the Minnesota prior, including the sum-of-coefficients and single-unit-root restrictions. The hierarchical modeling runs MCMC with hyperpriors through maximizing one-step-ahead forecasting of the reduced form model.

5.2.1 Estimation Result I estimate the model with 200,000 MCMC draws and then discard the first half. On top of that, I resample the second half systemically with every 10th samples to address the potential autocorrelation of the MCMC sampler. These process results in the final 20,000 posterior draws.

The proxy for the IV method seems to have a strong relevance to the latent monetary policy shock. Figure 7 collects the histograms of two measures for the proximity of the IV method. The two histograms are asymptotically identical because both $s(Z_t)$ and $s(\varepsilon_t^M)$ are assumed to be one. However, they are estimated differently as the standard deviation estimates of recovered latent monetary policy shock in the VAR model are slightly greater than one. Nevertheless, these numbers lessen the concerns about the weak instrumental variables.

Figure 7: Histogram of Estimated Proxy Relevance in (42)



$\hat{\phi}$ is the estimated covariance between the proxy and the latent variable in (42). $s(\cdot)$ is a sample standard deviation.

Figure 8 depicts the estimated impulse response of the VAR model to an exogenous monetary contraction raising the interest rate 25 basis points. I also put the posterior mean of DSGE model estimates for comparison. The result shows that the monetary contraction is expansionary to output, wage, and labor at the earlier periods.[26] Yet both models have the similar response of investment, market value of debt, interest rate, and bond return on the

[26] Ramey (2016) indicates this output puzzle is prevalent in the instrumental variable identification with the narrative approach of Romer and Romer (2004) as well as with recursive zero restrictions. In subsection D.3, I apply the SVAR estimation approach to deal with this issue in the spirit of Arias et al. (2019). I find that the result is robust on this output puzzle.

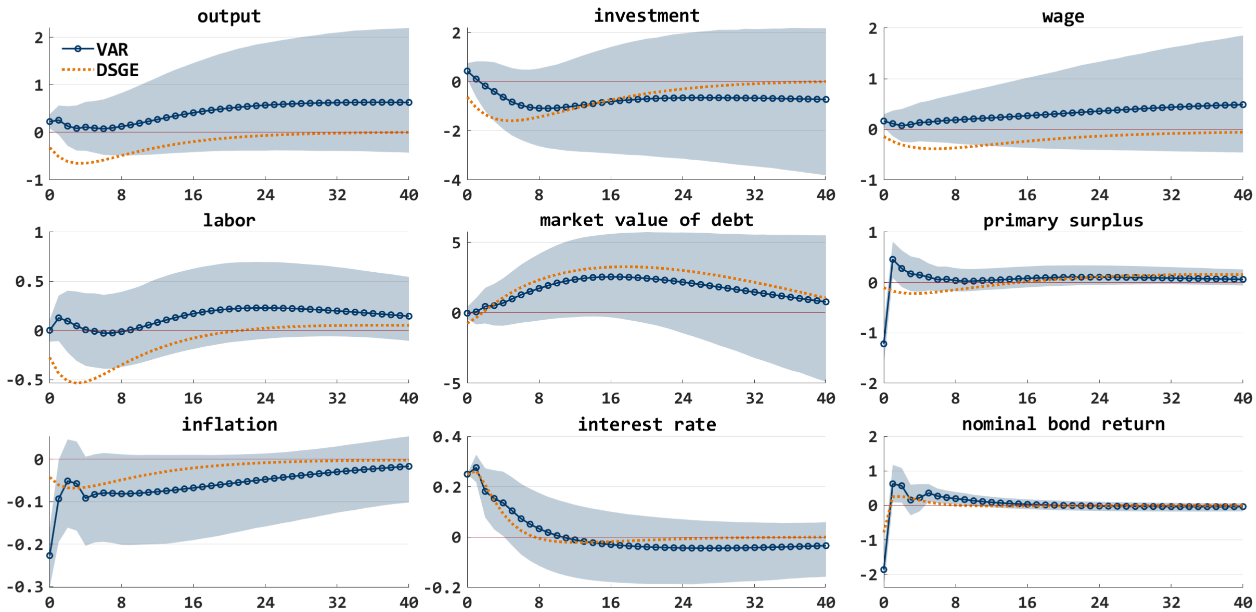
other hand. Inflation response is similar too, but VAR shows a stronger impact response than that of DSGE.

The primary surplus shows negative response at the impact, but with different scales. Although the output response is expansionary at the impact, I interpret the deficit at the impact as the falling of the tax revenue.[27]

However, both models indicate the path of primary surplus ends up with positive financing. The primary surplus path in the DSGE model has a continuously reverting pattern that sums up to the positive value as noted in Table 3. The fiscal policy of the DSGE model runs the primary surplus based on an explicit debt level targeting, so it moves closer to how debt responds to the shock. The same path in the VAR model only reverts once right after the negative impact and keeps collecting surplus afterward. So it suffices the mechanism of fiscal backing in that it arranges primary surpluses against the monetary contraction. Importantly, the similar responses of nominal bond return and inflation infer that the discount rate effect is also large in VAR estimation. Therefore fiscal backing must compensate the discount rate effect on the present value of primary surpluses.

[27] Cochrane (2021a) points out the strong correlation between the primary surplus and the unemployment rate, due to procyclical tax revenues and discretionary counter-cyclical stimulus spending.

Figure 8: Posterior Fiscal Impulse Response of VAR Model to Monetary Policy Shock



Responses of posterior means over 40 quarters to an exogenous monetary contraction raising the interest rate 25 BPS and the sample period 1960Q1 to 2006Q4. The solid line with the circled mark is a posterior mean response of the benchmark VAR model and the shaded areas are the 90 percent credible intervals. The dotted line is the posterior mean response from the counterpart DSGE model.

The estimation result suggests the presence of fiscal backing in data without imposing the active monetary and passive fiscal policy. Table 4 presents the debt valuation decomposition of the VAR estimation against a monetary contraction raising the interest rate 25 basis points. The discount rate effect dominates the bond devaluation effect, similar to the DSGE estimation result. Therefore it reveals the necessity of fiscal backing without the equilibrium restrictions. Fiscal backing should take the form of fiscal contraction to compensate for the strong discounting and meet the present value of primary surpluses to the market value of debt.[28]

Figure 9 visualizes the estimated posterior distribution in Table 4 and compares the estimates from the DSGE model. The posterior samplers of the DSGE model have shorter intervals than those of the VAR model and the equilibrium theory takes the intervals away from the zero line, following the fiscal consequence of monetary policy. The estimates of the VAR also follow those of the DSGE model qualitatively in that their mean values are in line with the

[28]Since I filtered the seasonality of the primary surplus data, the computed wealth effect does not exactly eqaul the estimated fiscal backing. The last row in Table 4 shows that the potential wealth effect is slightly greater than the fiscal backing by 27 basis points.

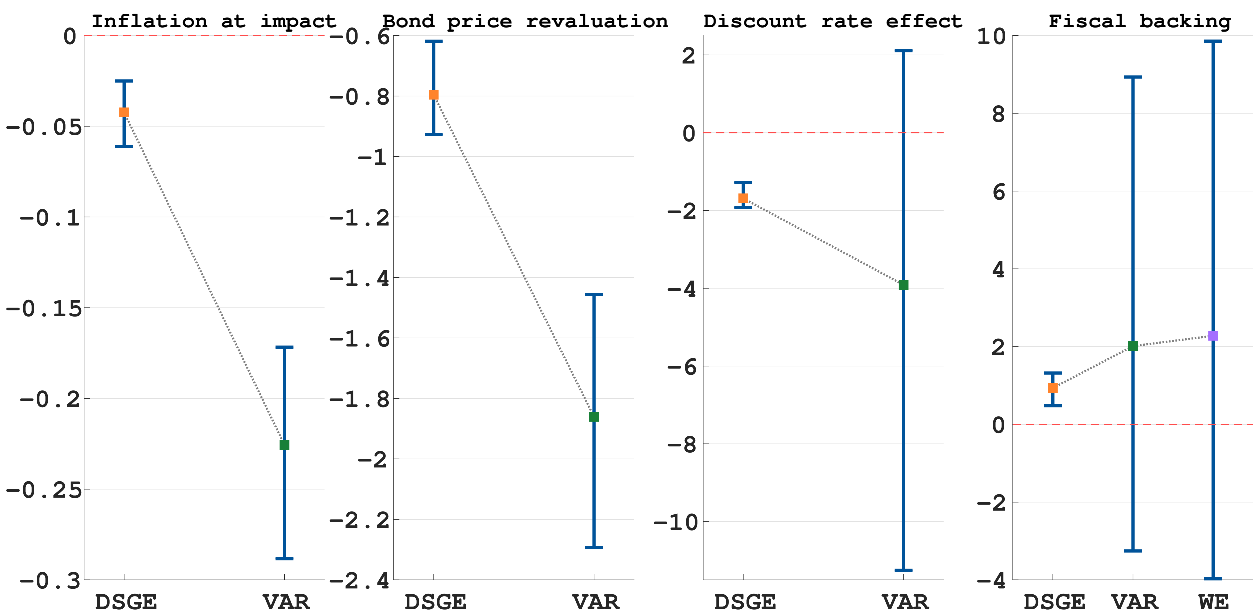
DSGE counterparts only with larger scales. Discount rate effect and fiscal backing on the other hand have much longer intervals due to severe high-frequency noises from their observable.

Table 4: Posterior Debt Value Decomposition of VAR Model

Debt value components	Median	Mean	5th quantile	95th quantile
Inflation adjustment	-0.22	-0.23	-0.29	-0.17
Bond price revaluation	-1.86	-1.86	-2.29	-1.46
Discount rate effect	-3.75	-3.91	-11.25	2.11
Fiscal backing	1.81	2.01	-3.26	8.93
Wealth effect	2.12	2.28	-3.97	9.85

Each row is the response of component in (25) to an exogenous monetary contraction raising the interest rate 25bps.

Figure 9: Posterior Debt Value Decomposition of VAR Model



Each error bar represents 90 percent posterior interval and the squared mark is its mean value. DSGE and VAR are estimates of each model. WE is a wealth effect computed from VAR estimates based on the debt valuation relation (26)

5.2.2 Counterfactual Analysis

I conduct a counterfactual analysis with the estimated VAR model to ask how would the VAR responses change in different fiscal backings. First, the estimated fiscal backing in total is defined from (29) as

$$\widehat{FB} = (1-\beta)\mathbf{c}_s'[I-\beta\hat{A}]^{-1}\hat{B}_1$$

where $\mathbf{c}_s$ is a selection vector for the primary surplus, $\hat{A}$ is the estimated coefficient matrix for reduced form VAR and $\hat{B}_1$ is the estimated monetary policy shock at the impact from (46). I changed the estimated reduced form VAR equation associated with the primary surplus to follow an exogenous autoregressive process,

$$s_t = \beta s_{t-1}$$

next, I replace the estimated impact response of the primary surplus $\hat{B}_{s,1}$-the element of the column vector $\hat{B}_1$ associated to the primary surplus-with counterfactual value FB^f. Then the counterfactual impulse response of the primary surplus to the identified monetary policy shock $\varepsilon_{1,t}$ is

$$\frac{\partial s_{t+h}}{\varepsilon_{1,t}} = \beta^h(1-\beta)FB^f$$

so that the sum of the primary surplus path becomes

$$\sum_{h=0}^{\infty}\frac{\partial s_{t+h}}{\varepsilon_{1,t}} = \frac{(1-\beta)FB^f}{1-\beta} = FB^f$$

Thus if $FB^f = \widehat{FB}$, then it preserves the amount of expected primary surplus path response.

I impose the debt valuation equation to compute counterfactual inflation at the impact, $B_{\pi,1}$, the impact response of inflation as

$$B^f_{\pi,1} = \hat{B}_{\pi,1} + \widehat{FB} - FB^f$$

So a counterfactual fiscal backing larger than the estimate will lower the price level at the impact and vice versa. With the different values of FB^f, the counterfactual VAR impulse response addresses the effect of fiscal backing for a monetary contraction to the VAR model.

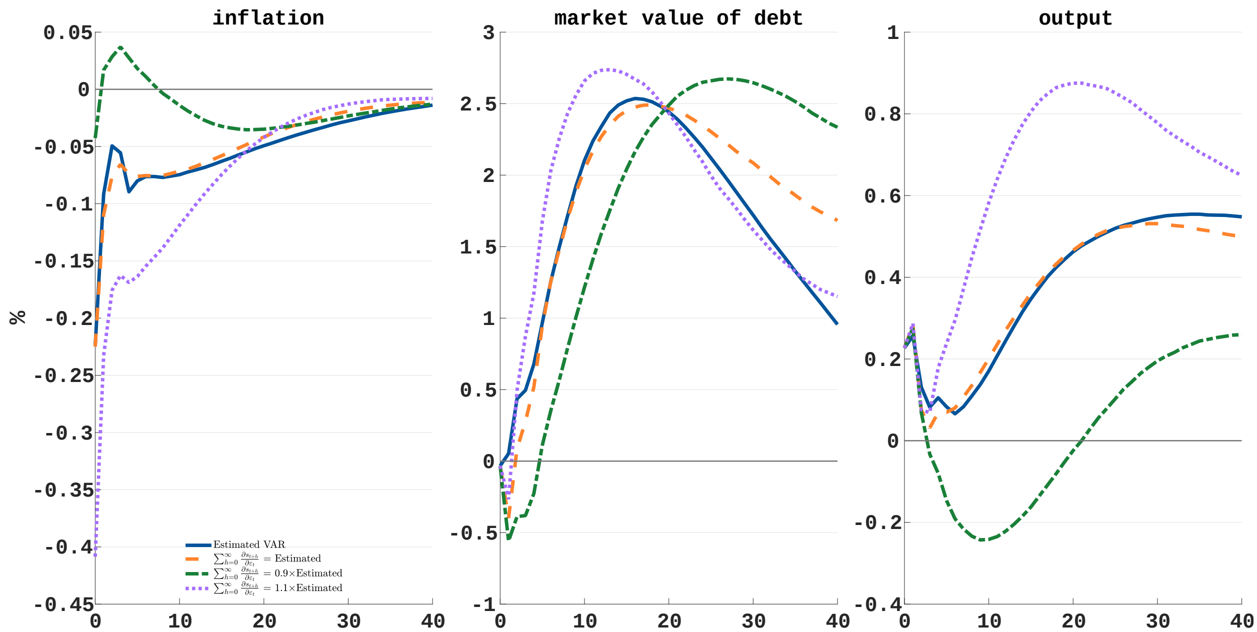
Figure 10 delivers the counterfactual exercises with different fiscal backing value. The dashed line posits the exogenous fiscal backing has the same total value as the estimated fiscal backing($FB^f = \widehat{FB}$). It shows a similar move to the original estimated impulse response, the solid line except for the response of market value of debt after the 18th quarter. It stems from the different primary surplus paths, although their sums are equal. The estimated VAR infers the financing of fiscal backing is concentrated in the earlier periods while the counterfactual path is almost constant with a relatively high discount rate $\beta = .99$. So the estimated primary surplus path retires the outstanding debt faster than the exogenous version does.

The dash-dotted line draws impulse responses with a counterfactual fiscal backing smaller than the estimated by a factor of 0.1($FB^f = .9\widehat{FB}$). The inflation response in the earlier periods highlights the shortage of fiscal backing for monetary policy. Along with the slow financing resulting from the exogenous primary surplus, higher inflation also elevates the market value of debt response.

Lastly, the dotted line depicts impulse responses with a counterfactual fiscal backing larger than the estimated by a factor of 0.1($FB^f = 1.1\widehat{FB}$). The inflation response gets relatively large drops in earlier quarters, particularly at the impact. Other than that, the responses are mirrored to the smaller fiscal backing case mostly. It is in line with the threshold mechanism of passive fiscal policy found in the DSGE counterfactual.

The different output responses are quite unexpected. Fiscal backing seems to have a lot to do with the output recovery from the monetary contraction.

Figure 10: Counterfactual Fiscal Backing in the Estimated VAR



Each line draws the mean impulse response of estimated VAR(solid), counterfactual exogenous primary surplus path equal to the estimated(dashed), counterfactual exogenous primary surplus path equal to the 90 percent of the estimated(dash-dotted), and counterfactual exogenous primary surplus path equal to the 110 percent of the estimated(dotted) respectively.

5.2.3 How Much Does Fiscal Backing Take in for Fiscal Policy?

Fiscal backing refers to conditional responses of primary surpluses on monetary policy shocks. It implies that such responses are not coming from the original fiscal motives.

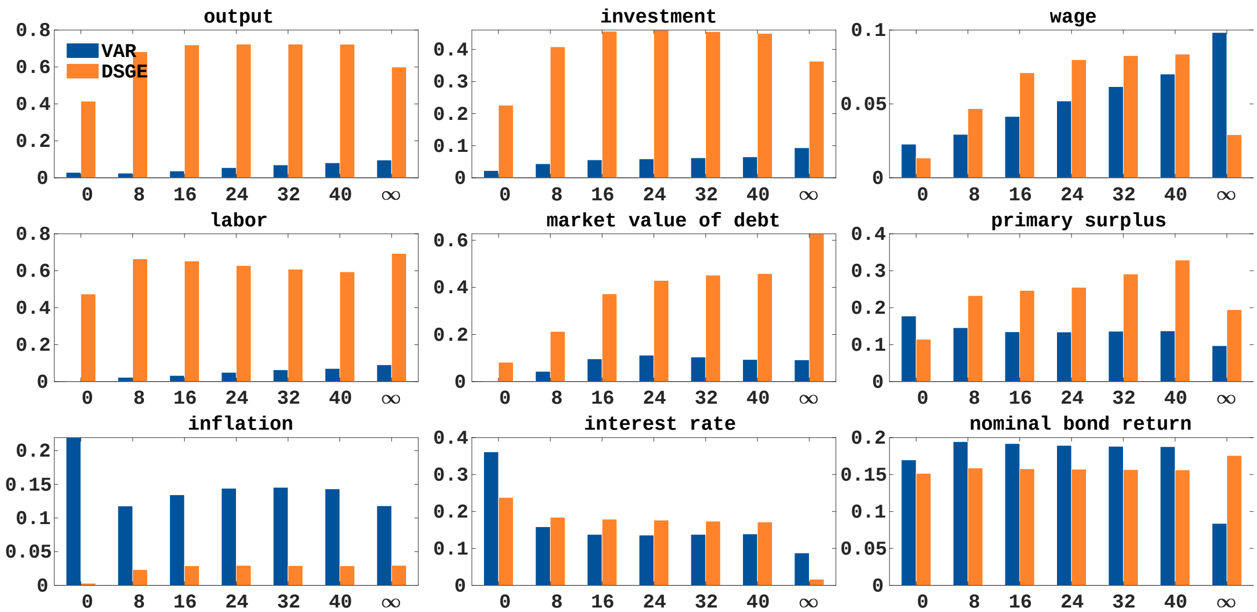
Let $s_t|_{\varepsilon_t^M}$ be the path of the primary surplus due to fiscal backing of monetary policy. Then the stance of fiscal policy is more accurately given by

$$\tilde{s}_t = s_t - s_t|_{\varepsilon_t^M} \tag{47}$$

This adjustment is akin to "cyclical-adjusted" primary surpluses. Since the proxy IV method of VAR estimation recovers the underlying monetary policy shock ε_t^M, I can compute this this filtered primary surplus $\tilde{s}_t$ through historical decomposition on the primary surplus data. Vaporizing the historical attribution of ε_t^M to the primary sruplus will get $\tilde{s}_t$.

Figure 11 reports the variance decomposition analysis for monetary policy shock for both DSGE and VAR models. Results in the DSGE model tend to be higher than those in VAR except the inflation. Notably, the models predict the share of monetary shocks for the total variability takes between 10 to 20 percent for the primary surplus. It infers there are some chances that the primary surplus observable may not reflect the actual fiscal stance.

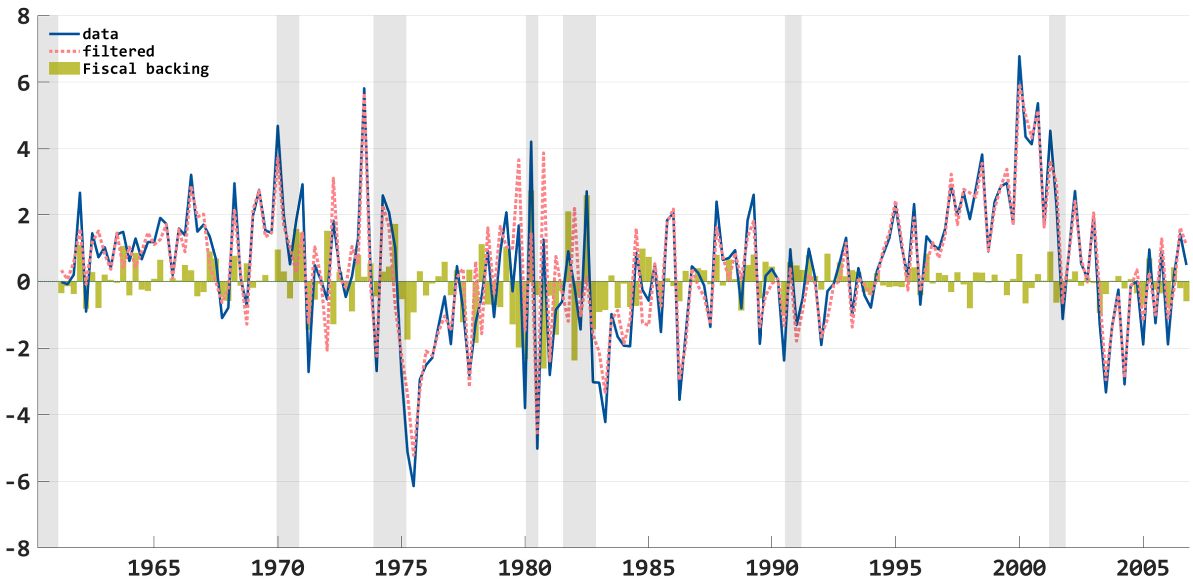
Figure 11: Variance Decomposition of Monetary Policy Shock in DSGE and VAR



Each bar depicts the variance share of monetary shock in the total forecast error at every 2 years horizon. The last bar in each panel shows the long run variance decomposition.

Figure 12 combines the primary surplus data and filtered primary surplus along with their difference, the monetary policy historical decomposition. The filterization does not bring about significant changes to the fiscal stance, particularly after the Great Moderation. Still, the pronounced monetary policy shocks around 1980 seems to attribute to the relatively larger share of fiscal backing in the total variability of the primary surplus.

Figure 12: Monetary Policy Shock Historical Decomposition of Primary Surplus from VAR Estimate



The figure plots the primary surplus data s_t (solid), filtered primary surplus $\tilde{s}_t$ (dotted) and the monetary policy shock historical decomposition (bar). Grey shaded area depicts the NBER recessions.

6 Conclusion

The monetary and fiscal policy interaction literature has established fiscal backing-responses of primary surpluses to monetary policy shocks-for inflation-targeting monetary policy. This theory describes the role of fiscal backing as to adjust the present value of primary surpluses to meet the changes in the market value of government debt due to monetary policy shocks. It amounts to neutralizing the potential wealth effect that may arise if the market value of government debt differs from its backing value, the present value of primary surpluses.

This paper provides a quantitative assessment of how much the monetary policy requires fiscal backing for inflation targeting. I apply both the DSGE and VAR models to evaluate the role of policy regime equilibrium conditions in quantifying fiscal backing and the potential wealth effect. The estimation with U.S. data favors the presence of fiscal backing as the fiscal contraction follows monetary contraction. More importantly, the main result stems from that the discount rate on the future primary surplus path gets more effects from monetary policy shocks than other components of the government debt valuation. This dominant discount rate effect motivates fiscal backing to be always present in as the prescription from the theory; fiscal backing behind a monetary contraction's decrease in inflation is a fiscal contraction that returns real government debt to target.

Through decomposing the government debt valaution, I document the responses of the expected interest rate and inflation are critical for the fiscal consequence of monetary policy. In particular, given the strong rigidity and persistence of the exogenous shock process, more aggressive monetary policy rule can mitigate the demand for fiscal backing. It highlights the importance of the policy interaction with the unprecedentedly high level of debt that many economies are facing today. Additionally, this paper points out that controlling the expected inflation path is key for the fiscal backing management.

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A Data

The followings are the benchmark dataset I use for both DSGE and VAR estimation.

- GDP: Gross Domestic Product, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate
- FPI: Fixed Private Investment, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate
- CPHNBS: Nonfarm Business Sector: Compensation Per Hour, Index 2012=100, Quarterly, Seasonally Adjusted
- NNBS = PRS85006023$\times$CE16OV: Nonfarm Business Sector: Average Weekly Hours, Index 2012=100, Quarterly, Seasonally Adjusted times Employment Level, Thousands of Persons, Quarterly(Averaging Monthly data), Seasonally Adjusted
- MVDP: Market value of debt held by private from Hall et al. (2018)
- PSGBC: Primary surplus from government budget constraint
- GDPDEF: Gross Domestic Product: Implicit Price Deflator, Index 2012=100, Quarterly, Seasonally Adjusted
- FEDFUNDS: Effective Federal Funds Rate, Percent, Annualize and average Monthly data to quarterly, Not Seasonally Adjusted
- NHPR: Nominal holding period return of government debt portfolio from Hall et al. (2018)

The acronyms are the variable names from the FRED Economic Data, except for MVDP, PSGBC and NHPR.

The DSGE model for estimation has a state space model representation,

$$\text{Transiton Equation: } X_t = C + GX_{t-1} + M\varepsilon_t \tag{A.1}$$

$$\text{Measurement Equation: } Y_t = D + HX_t + Wu_t \tag{A.2}$$

I transform the raw dataset into observable variables Y_t and put it into the measurement equation (A.2) as

$$\begin{bmatrix} 100*\Delta\ln(GDP_t/GDPDEF_t/POP_t) \\ 100*\Delta\ln(FPI_t/GDPDEF_t/POP_t) \\ 100*\Delta\ln(CPHNBS_t/GDPDEF_t) \\ 100*\Delta\ln(NNBS_t/POP_t) \\ 100*\Delta\ln(MVDP_t/GDPDEF_t/POP_t) \\ \ln(PSGBC_t) \\ 100*\Delta\ln(GDPDEF_t) \\ FEDFUNDS_t \\ NHPR_t \end{bmatrix} = \begin{bmatrix} 100\gamma \\ 100\gamma \\ 100\gamma \\ \bar{n} \\ 100\gamma \\ \bar{s}-100\gamma+\bar{R} \\ \bar{\pi} \\ \bar{\pi}+\bar{R} \\ \bar{\pi}+\bar{R} \end{bmatrix} + 100 \begin{bmatrix} y_t - y_{t-1} + \gamma_t \\ x_t - x_{t-1} + \gamma_t \\ w_t - w_{t-1} + \gamma_t \\ n_t \\ b_t - b_{t-1} + \gamma_t \\ s_t + (\beta - 1)b_t \\ \pi_t \\ R_t \\ R_t^B \end{bmatrix}$$

The constants with bar are measurement means with the following priors, $\bar{n} \sim \mathbb{N}(0,1)$, $\bar{\pi} \sim \mathbb{N}(.79,.25)$, $\bar{s} \sim \mathbb{N}(.21,.5)$, and $\bar{R} = 100(\gamma\beta^{-1} - 1)$. POP is a popluation measure that I use the population data CNP16OV(Population Level, Thousands of Persons, Monthly averaged to quarterly, Not Seasonally Adjusted).

Figure 13: Benchmark Observable for Estimation

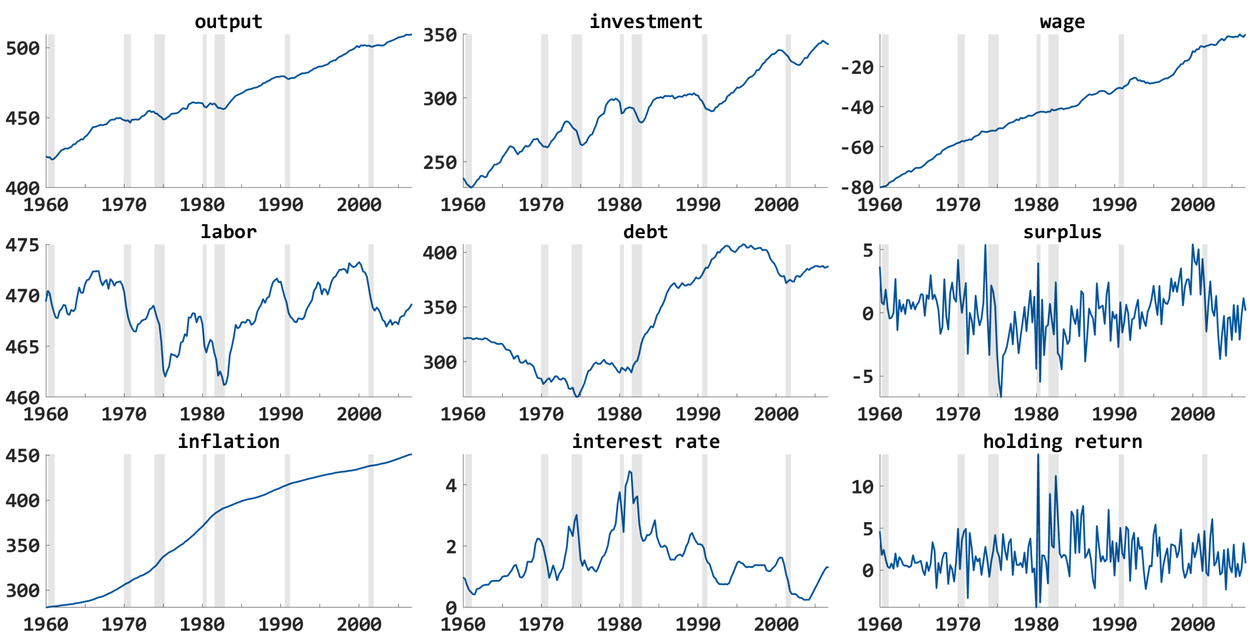

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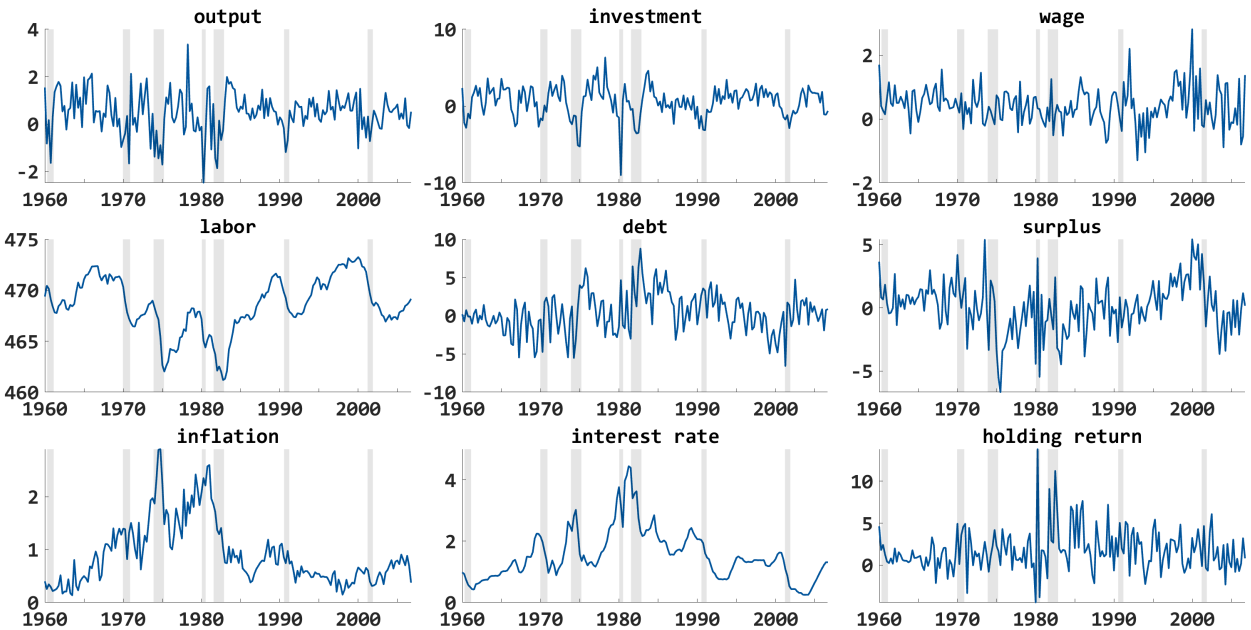
Figure 14: Benchmark Observable for Estimation: Differenced



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B Medium Scale New Keynesian Model

B.1 Detrended Model Equilibrium The DSGE model is nonstationary in some variables with growth Γ_t. Detrending such variables is necessary for solving the model by approximating around the steady state. The variables with trends are $c_t, x_t, \bar{k}_t, w_t, w_t^{\#}, y_t, y_t^{\#}, k_t, g_t, z_t, \tau_t$ and b_t. I detrend those variables by dividing them with Γ repectively.(*i.e.* $\tilde{X}_t = X_t\Gamma_t^{-1}$) After detrending and laying out all variables in real terms, the model equilibrium conditions become as followed.

Household FOC for consumption

$$\frac{u_t^U}{\tilde{c}_t - \eta\gamma_t^{-1}\tilde{c}_{t-1}} - \beta\iota_c\eta\mathbb{E}_t\left[\gamma_{t+1}^{-1}\frac{u_{t+1}^U}{\tilde{c}_{t+1} - \eta\gamma_{t+1}^{-1}\tilde{c}_t}\right] = \lambda_t(1+\tau^c) \tag{B.1}$$

Household FOC for investment

$$\begin{aligned}&\mathbb{E}_t\left[\beta\gamma_{t+1}^{-1}\lambda_{t+1}q_{t+1}u_{t+1}^x\mathcal{S}'\left(\gamma_{t+1}\frac{\tilde{x}_{t+1}}{\tilde{x}_t}\right)\left(\gamma_{t+1}\frac{\tilde{x}_{t+1}}{\tilde{x}_t}\right)^2\right]\\&+\lambda_t q_t u_t^x\left\{1-\mathcal{S}'\left(\gamma_t\frac{\tilde{x}_t}{\tilde{x}_{t-1}}\right)\gamma_t\frac{\tilde{x}_t}{\tilde{x}_{t-1}} - \mathcal{S}\left(\gamma_t\frac{\tilde{x}_t}{\tilde{x}_{t-1}}\right)\right\} = \lambda_t\end{aligned} \tag{B.2}$$

Household FOC for capital utilization

$$(1-\tau^k)r_t^k = \psi'(\upsilon_t) \tag{B.3}$$

Household FOC for physical capital stock

$$q_t = \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\gamma_{t+1}^{-1}\{(1-\tau^k)r_{t+1}^k\upsilon_{t+1} + (1-\delta)q_{t+1} - \psi(\upsilon_{t+1})\}\right] \tag{B.4}$$

Household FOC for short-term and long-term bond

$$\mathbb{E}_t[\beta\lambda_{t+1}\gamma_{t+1}^{-1}\pi_{t+1}^{-1}R_t] = \lambda_t \tag{B.5}$$

$$\mathbb{E}_t[\beta\lambda_{t+1}\gamma_{t+1}^{-1}\pi_{t+1}^{-1}(1+\rho Q_{t+1})Q_t^{-1}] = \lambda_t \tag{B.6}$$

Household FOC for reset wage and the wage indexation rule

$$\mathbb{E}_t\left[\sum_{s=0}^{\infty}(\beta\zeta_w)^s\eta_{t+s}^w u_{t+s}^U\vartheta {n_{t+s}^{\#}}^{1+\chi^{-1}}\right] = \tilde{w}_t^{\#}\mathbb{E}_t\left[\sum_{s=0}^{\infty}(\beta\varphi_w)^s(\eta_{t+s}^w-1)\lambda_{t+s}(1-\tau^n)\bar{\pi}_{t+s}^{\mathsf{w}}n_{t+s}^{\#}\right] \tag{B.7}$$

$$\bar{\pi}_{t+s}^{\mathsf{w}} = \prod_{k=1}^{s}\left\{\left(\frac{\gamma_{t+k-1}}{\gamma}\frac{\pi_{t+k-1}}{\pi}\right)^{\iota_w}\left(\frac{\gamma}{\gamma_{t+s}}\frac{\pi}{\pi_{t+s}}\right)\right\} \tag{B.8}$$

Individual labor demand

$$n_{t+s}^{\#} = n_{t+s}\left(\bar{\pi}_{t+s}^{\mathsf{w}}\frac{\tilde{w}_t^{\#}}{\tilde{w}_{t+s}}\right)^{-\eta_{t+s}^w} \tag{B.9}$$

Law of motion for wage

$$\tilde{w}_t^{1-\eta_t^w} = (1-\zeta_w){\tilde{w}_t^{\#}}^{1-\eta_t^w} + \zeta_w\left\{\left(\frac{\gamma_{t-1}}{\gamma}\frac{\pi_{t-1}}{\pi}\right)^{\iota_w}\frac{\gamma}{\gamma_t}\frac{\pi}{\pi_t}\right\}^{1-\eta_t^w}\tilde{w}_{t-1}^{1-\eta_t^w} \tag{B.10}$$

Law of motion for physical capital

$$\tilde{\bar{k}}_t = \upsilon_t^x\left\{1-\mathsf{S}\left(\gamma_t\frac{\tilde{x}_t}{\tilde{x}_{t-1}}\right)\right\}\tilde{x}_t + (1-\delta)\gamma_t^{-1}\tilde{\bar{k}}_{t-1} \tag{B.11}$$

Capital utilization

$$\tilde{k}_t = \upsilon_t\gamma_t^{-1}\tilde{\bar{k}}_{t-1} \tag{B.12}$$

Real marginal cost for production

$$\psi_t = \frac{\tilde{w}_t}{(1-\alpha)\tilde{k}_t^{\alpha} n_t^{-\alpha}} \tag{B.13}$$

Marginal rate of substitution between production inputs

$$\frac{\tilde{w}_t}{r_t^k} = \frac{1-\alpha}{\alpha}\frac{\tilde{k}_t}{n_t} \tag{B.14}$$

Firm FOC for reset price and the price indexation rule

$$\frac{\pi_t^{\#}}{\pi_t}\mathbb{E}_t\left[\sum_{s=0}^{\infty}(\beta\zeta_p)^s\lambda_{t+s}\tilde{y}_{t+s}^{\#}(\eta_{t+s}^p-1)\bar{\pi}_{t+s}^{\mathsf{p}}\right] = \mathbb{E}_t\left[\sum_{s=0}^{\infty}(\beta\zeta_p)^s\lambda_{t+s}\tilde{y}_{t+s}^{\#}\eta_{t+s}^p\psi_{t+s}\right] \tag{B.15}$$

$$\bar{\pi}_{t+s}^{\mathsf{p}} = \prod_{k=1}^{s}\left(\left(\frac{\pi_{t+k-1}}{\pi}\right)^{\iota_p}\frac{\pi}{\pi_{t+k}}\right) \tag{B.16}$$

Intermediate good demand

$$y_{t+s}^{\#} = y_{t+s}\left(\bar{\pi}_{t+s}^{\mathsf{p}}\frac{\pi_t^{\#}}{\pi_t}\right)^{-\eta_{t+s}^p} \tag{B.17}$$

Law of motion for inflation

$$\left(\frac{\pi_t}{\pi}\right)^{1-\eta_t^p} = (1-\zeta_p)\left(\frac{\pi_t^{\#}}{\pi}\right)^{1-\eta_t^p} + \zeta_p\left(\frac{\pi_{t-1}}{\pi}\right)^{\iota_p(1-\eta_t^p)} \tag{B.18}$$

Aggregate production function

$$\tilde{k}_t^{\alpha} n_t^{1-\alpha} - \Omega = \tilde{y}_t\nu_t^p \tag{B.19}$$

Price dispersion

$$\nu_t^p = (1-\zeta_p)\pi_t^{\#^{-\eta_t^p}}\pi_t^{\eta_t^p} + \zeta_p\pi_{t-1}^{-\iota_p\eta_t^p}\pi_t^{\eta_t^p}\nu_{t-1}^p \tag{B.20}$$

B.2 Steady State First, some policy targets are exogenous. Steady state inflation π is labeled as target inflation rate, set by the monetary authority. Likewise, fiscal authority equates steady state debt target b to its target value.[29] The geometric maturity structure is also computed based on the average duration,

$$\text{Average duration} = 1 + \beta\rho + (\beta\rho)^2 + \cdots = \frac{1}{1-\beta\rho}$$
$$\Rightarrow \rho = \beta^{-1}(1 - \text{Average duration}^{-1})$$

Steady state tax rates $\{\tau^c, \tau^n, \tau^k\}$ are also calibrated outside of the model. The share of government spending to output $g = s_g y$ is also calibrated as well. The full utilization of capital at steady state, $v = 1$ is a natural assumption to take. The steady state trend growth γ should also be given outside of the model. The leisure preference parameter ϑ will be calibrated to match the steady state labor(hours of work) to one third. Then using the detrended equilibrium conditions above, each steady state value along with some parameters can be computed sequentially,

$$M = \frac{\beta}{\gamma\pi} \tag{B.21}$$

$$R = M^{-1} \tag{B.22}$$

$$Q = \frac{1}{R-\rho} \tag{B.23}$$

$$\pi^{\#} = \pi \tag{B.24}$$

$$\nu^p = \frac{1-\zeta_p}{1-\zeta_p \pi^{\eta^p(1-\iota_p)}} \tag{B.25}$$

$$\psi = \frac{\eta^p - 1}{\eta^p} \tag{B.26}$$

$$q = 1 \tag{B.27}$$

$$r^k = \frac{q}{1-\tau^k} \tag{B.28}$$

$$\frac{k}{n} = \left(\frac{\alpha\psi}{r^k}\right)^{\frac{1}{1-\alpha}} \tag{B.29}$$

$$w = \frac{1-\alpha}{\alpha}\frac{k}{n}r^k \tag{B.30}$$

[29]Since the transfer should be weakly positive, steady state debt ratio must be restricted as

$$b \leq \frac{\tau - g}{(\beta^{-1}-1)y}$$

which stems from the model steady state derivation.

$$w^{\#} = w \tag{B.31}$$

$$\frac{x}{n} = (\gamma - 1 + \delta)\frac{k}{n} \tag{B.32}$$

$$\frac{y}{n} = w + r^k \frac{k}{n} \tag{B.33}$$

$$\frac{c}{n} = (1 - s_g)\frac{y}{n} - \frac{x}{n} \tag{B.34}$$

$$n = \left\{ \frac{w^{\#}}{\vartheta} \frac{\eta^w - 1}{\eta^w} \frac{1 - \beta\iota_c\eta\gamma^{-1}}{1 - \eta\gamma^{-1}} \frac{1 - \tau^n}{1 + \tau^c} \frac{n}{c} \right\}^{\frac{1}{1+\chi^{-1}}} \tag{B.35}$$

$$\bar{k} = \gamma k \tag{B.36}$$

$$\lambda = \frac{1}{(1 + \tau^c)c} \frac{1 - \beta\iota_c\eta\gamma^{-1}}{1 - \eta\gamma^{-1}} \tag{B.37}$$

$$\Omega = \left(\frac{k}{n}\right)^{\alpha} n - y\nu^p \tag{B.38}$$

$$\tau = \tau^c c + \tau^n wn + \tau^k r^k k \tag{B.39}$$

$$z = (1 - \beta^{-1})\ell + \tau - g \tag{B.40}$$

B.3 Log Linearization

The log linearization follows the convetion, $\hat{X}_t = \frac{X_t - X}{X} = \ln X_t - \ln X$. I skip the hat notation for brevity.

$$U_{c_t} - u_t^U + \frac{\gamma}{\gamma - \eta} c_t + \frac{\eta}{\gamma - \eta}\gamma_t = \frac{\eta}{\gamma - \eta} c_{t-1} \tag{B.41}$$

$$\frac{1}{1 - \kappa_c} U_{c_t} + \kappa_c \mathbb{E}_t \gamma_{t+1} - \kappa_c \mathbb{E} U_{c_{t+1}} - \lambda_t = 0 \tag{B.42}$$

$$x_t + \frac{1}{1+\beta}\gamma_t - \kappa_x^{-1} q_t - \frac{\beta}{1+\beta}\mathbb{E}_t x_{t+1} - \frac{\beta}{1+\beta}\mathbb{E}_t \gamma_{t+1} - \tilde{u}_t^x = \frac{1}{1+\beta} x_{t-1} \tag{B.43}$$

$$r_t^k = \frac{\nu}{1 - \nu} v_t = 0 \tag{B.44}$$

$$-q_t + \mathbb{E}_t \lambda_{t+1} - \lambda_t - \mathbb{E}_t \gamma_{t+1} + \frac{\beta}{\gamma}\left(\frac{\gamma}{\beta} - 1 + \delta\right)\mathbb{E} r_{t+1}^k + \frac{\beta}{\gamma}(1 - \delta)\mathbb{E}_t q_{t+1} = 0 \tag{B.45}$$

$$\mathbb{E}_t \lambda_{t+1} - \mathbb{E}_t \gamma_{t+1} - \mathbb{E}_t \pi_{t+1} - \lambda_t + R_t = 0 \tag{B.46}$$

$$R_t^B - \rho\beta\gamma^{-1} Q_t = Q_{t-1} \tag{B.47}$$

$$R_t - \mathbb{E}_t R_{t+1}^B = 0 \tag{B.48}$$

$$-\psi_t^w + w_t - \chi_n n_t - u_t^U + \lambda_t = 0 \tag{B.49}$$

$$\begin{aligned} w_t - \frac{\beta}{1+\beta}\mathbb{E} w_{t+1} - \frac{\beta}{1+\beta}\mathbb{E}_t \pi_{t+1} - \frac{\beta}{1+\beta}\mathbb{E}_t \gamma_{t+1} + \frac{1+\beta\iota_w}{1+\beta}\pi_t + \frac{1+\beta\iota_w}{1+\beta}\gamma_t + \kappa_w \psi_t^w - u_t^w \\ = \frac{1}{1+\beta} w_{t-1} + \frac{\iota_w}{1+\beta}\gamma_{t-1} + \frac{\iota_w}{1+\beta}\pi_{t-1} \end{aligned} \tag{B.50}$$

$$-\gamma \bar{k}_t - (1 - \delta)\gamma_t + (\gamma - 1 + \delta)x_t + (\gamma - 1 + \delta)\kappa_x \tilde{u}_t^x = -(1 - \delta)\bar{k}_{t-1} \tag{B.51}$$

$$k_t + \gamma_t - v_t = \bar{k}_{t-1} \tag{B.52}$$

$$w_t + n_t - r_t^k - k_t = 0 \tag{B.53}$$

$$-\psi_t^p + \alpha r_t^k + (1-\alpha) w_t = 0 \tag{B.54}$$

$$-\pi_t + \frac{\beta}{1+\beta\iota_p}\mathbb{E}_t \pi_{t+1} + \kappa_p \psi_t^p + u_t^p = -\frac{\iota_p}{1+\beta\iota_p}\pi_{t-1} \tag{B.55}$$

$$-y y_t + c c_t + x x_t + g g_t + (\gamma\beta^{-1} - 1 + \delta) k v_t = 0 \tag{B.56}$$

$$-\frac{y}{y+\Omega} y_t + \alpha k_t + (1-\alpha) n_t = 0 \tag{B.57}$$

$$-R_t + (1-\rho_R)\phi_\pi \pi_t + (1-\rho_R)\phi_x y_t + u_t^M = -\rho_R R_{t-1} \tag{B.58}$$

$$-\tau\tau_t + \tau^c c c_t + \tau^n w n (w_t + n_t) + \tau^k r^k k (r_t^k + k_t) = 0 \tag{B.59}$$

$$-s s_t + \tau\tau_t - g g_t - z z_t = 0 \tag{B.60}$$

$$-\beta b_t - (1-\beta) s_t + R_t^B - \gamma_t - \pi_t = -b_{t-1} \tag{B.61}$$

$$g_t - (1-\rho_g)\varphi_x y_t - u_t^g = \rho_g g_{t-1} - (1-\rho_g)\varphi_b b_{t-1} \tag{B.62}$$

$$z_t - (1-\rho_z)\psi_x y_t - u_t^z = \rho_z z_{t-1} - (1-\rho_z)\psi_b b_{t-1} \tag{B.63}$$

$$u_t^s = \varrho_s u_{t-1}^s + \varepsilon_t^s, \quad \text{for } s \in \{\gamma, U, x, w, p, g, z, M\} \tag{B.64}$$

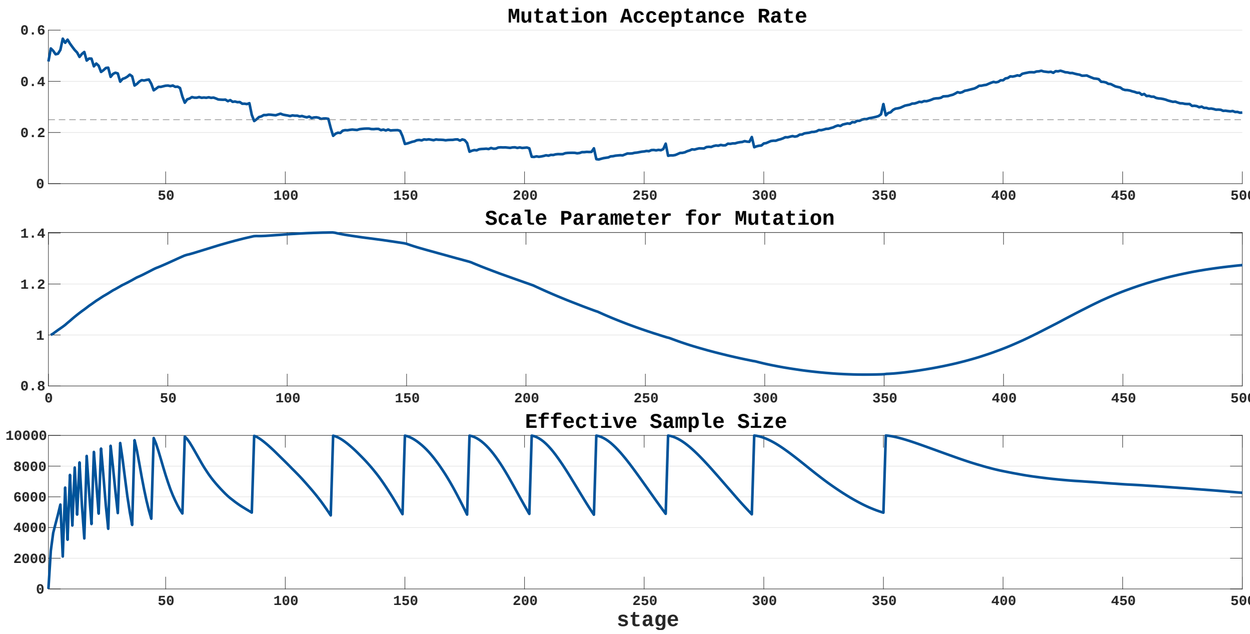
C Sequential Monte Carlo Estimation

The SMC algorithm following Herbst and Schorfheide (2015) uses likelihood tempering with the tempering scheudle ϕ_n defined as

$$\phi_n = \left(\frac{n}{N_\phi}\right)^\lambda$$

for each stage n and total stage number N_ϕ. I set $\lambda = 2.1$ following Cai et al. (2021), as the likelihood may change drastically even in the earlier stages. I also use stratified resampling in the selection step, as it is more efficient than the multinominal repsampling method. Figure 15 plots the adaptations of the hyperparamters in SMC estimation. The figure indicates that the hyperparameters work accordingly to target the mutation acceptance rate to 25% as desired.

Figure 15: Hyperparameter Adaptations for SMC



Dashed line in the top panel depicts the target acceptance rate.

D Estimation Results

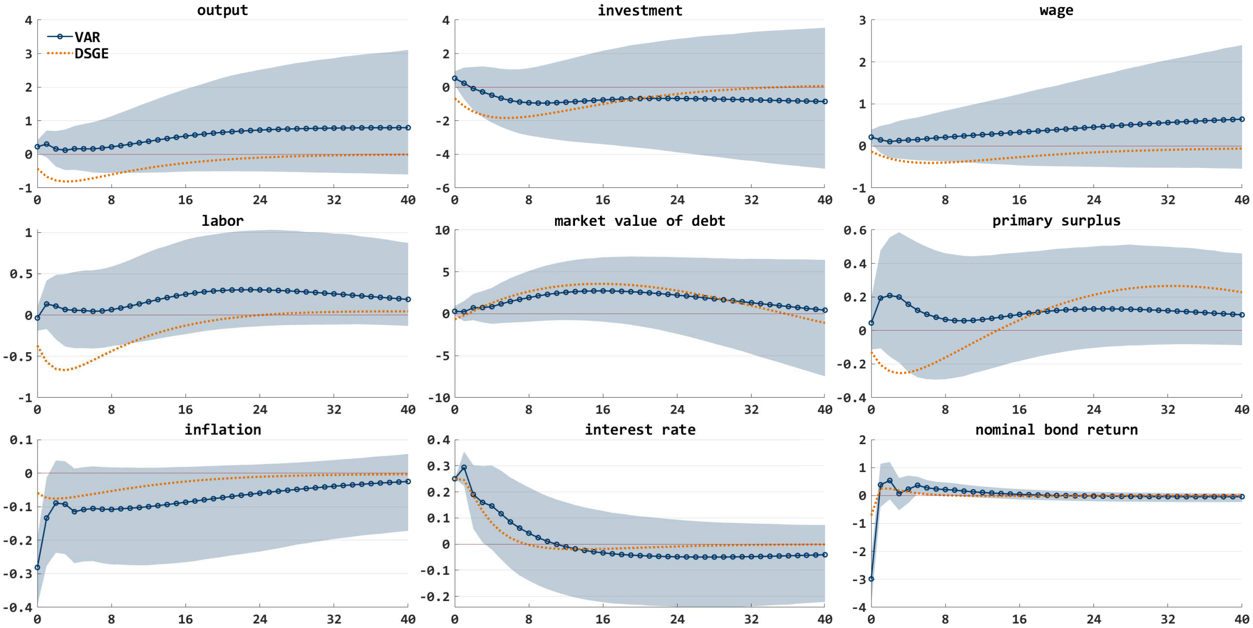
D.1 Estimation With Accounting Primary Surplus Data Replace primary surplus data with the one from accounting method.

Table 5: Posterior Debt Valuation Decomposition of DSGE model

Debt value components	Median	Mean	5th quantile	95th quantile
Inflation adjustment	-0.16	-0.16	-0.22	-0.11
Bond price revaluation	-2.03	-2.01	-2.57	-1.33
Discount rate effect	-4.64	-4.65	-5.13	-4.22
Fiscal backing	2.77	2.79	2.03	3.73

Each row is the response of (25) to a monetary contraction of a one standard deviation of exogenous monetary policy shock.

Figure 16: Posterior Fiscal Impulse Response of VAR Model to Monetary Policy Shock



Responses of posterior means over 40 quarters to an exogenous monetary contraction raising the interest rate 25 BPS and the sample period 1960Q1 to 2006Q4. The solid line with circled mark is a posterior mean response of the benchmark VAR model and the shaded areas are the 90 perecent credible intervals. The dotted line is the posterior mean response from the counterpart DSGE model.

Table 6: Posterior Debt Value Decomposition of VAR Model

Debt value components	Median	Mean	5th quantile	95th quantile
Inflation adjustment	-0.28	-0.28	-0.37	-0.21
Bond price revaluation	-2.96	-2.97	-3.74	-2.35
Discount rate effect	-4.06	-4.28	-13.14	2.60
Fiscal backing	3.47	3.82	-7.71	17.86
Wealth effect	1.39	1.59	-5.86	11.00

Each row is the response of component in (25) to an exogenous monetary contraction raising the interest rate 25bps.

Figure 17: Posterior Debt Value Decomposition of VAR Model: Primary Surplus Accounting Method

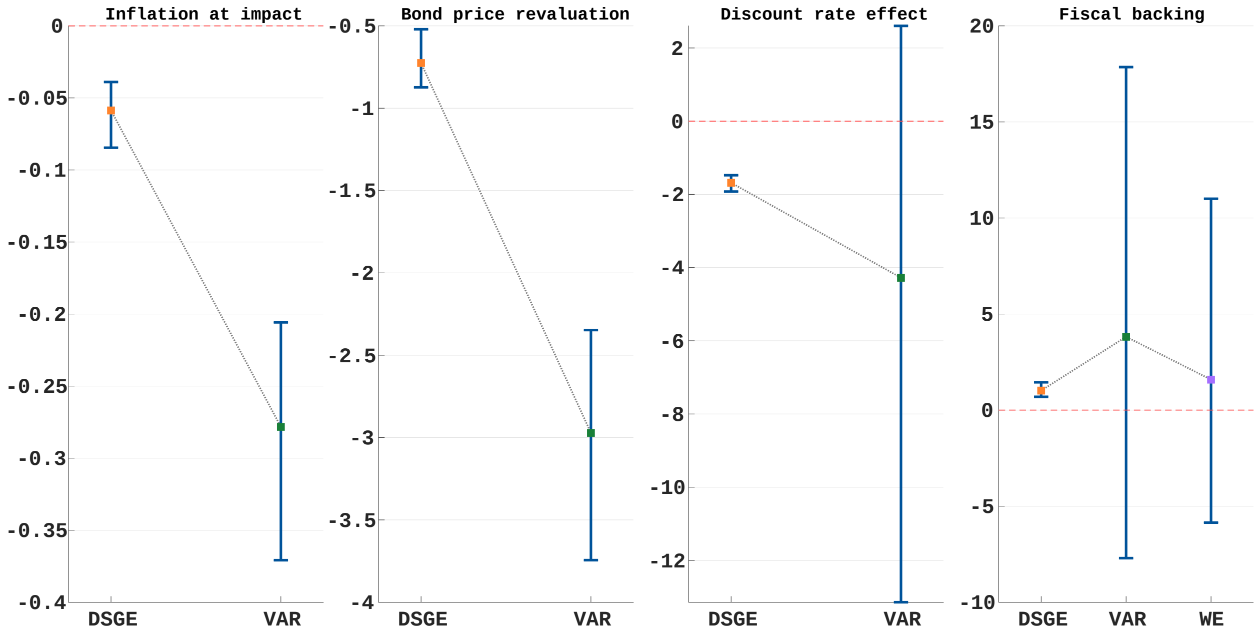
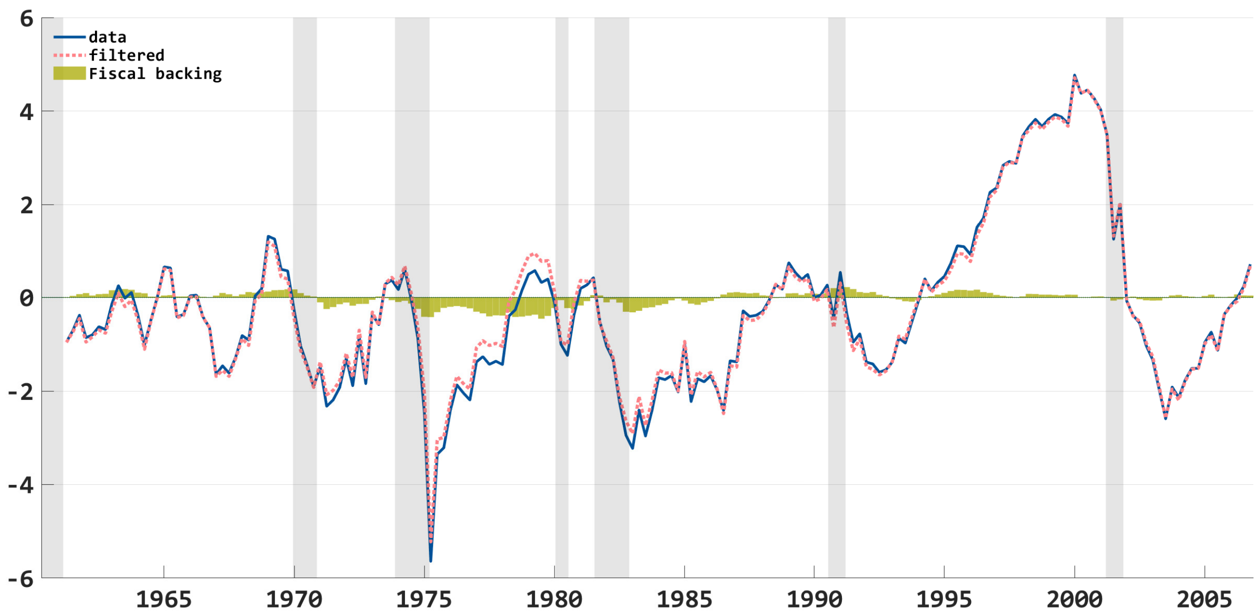


Figure 18: Monetary Policy Shock Historical Decomposition of Primary Surplus from VAR Estimate



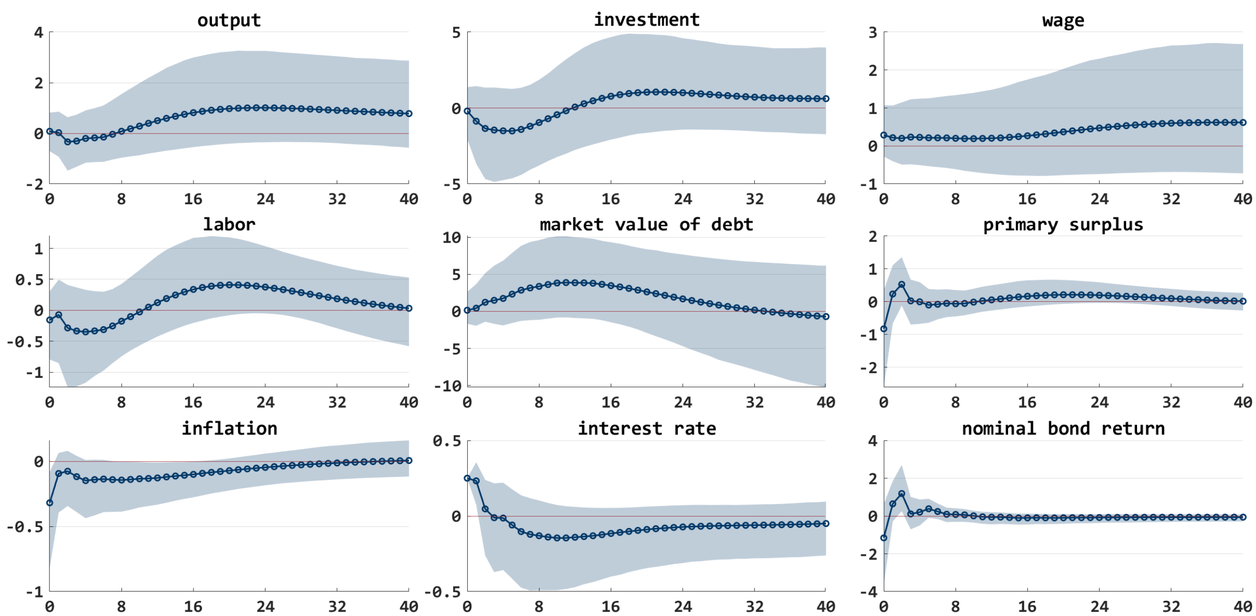
The figure plots the primary surplus data s_t (solid), filtered primary surplus $\tilde{s}_t$ (dotted) and the monetary policy shock historical decomposition (bar). Grey shaded area depicts the NBER recessions.

D.2 Fully Bayesian Proxy IV VAR Model Estimation Caldara and Herbst (2019) augment the following measurement equation between the proxy and latent structural shock of interest to the SVAR model.

$$Z_t = \beta \varepsilon_t^M + \sigma_n u \nu_t \tag{D.1}$$

where ν_t following the standard normal is assumed as an iid measurement error, independent to the latent monetary policy shock.[30] This setup allows to implement the SVAR estimation fully Bayesian, inclduing the parameters in the measurement equation. Figure 19 and Table 7 exhibit very similar results to the benchmark VAR estimation.

Figure 19: Posterior Impulse Response Bayesian Proxy SVAR Model with Monetary Policy Rule (30)



[30]The relevance restriction in (42) is equal to

$$corr(Z_t, \varepsilon_t^M)^2 = \frac{\beta^2}{\beta^2 + \sigma_\nu^2} = \phi^2$$

since the population variances of Z_t and ε_t^M are one.

Table 7: Posterior Debt Value Decomposition of BPSVAR Model

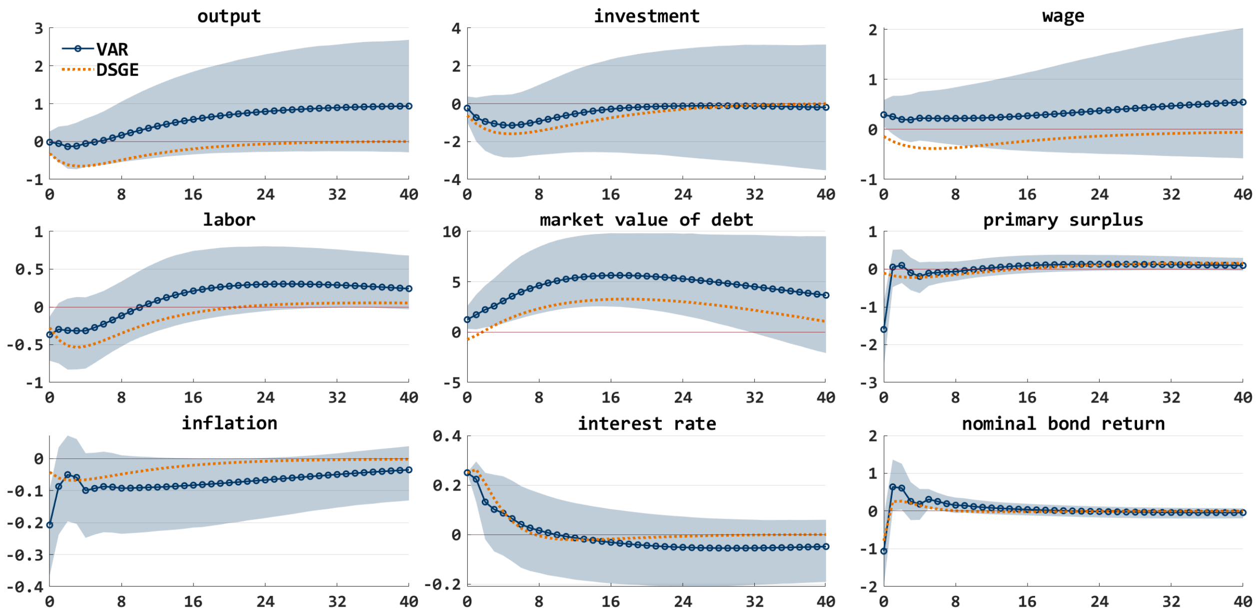
Debt value components	Median	Mean	5th quantile	95th quantile
Inflation adjustment	-0.23	-0.26	-0.66	-0.10
Bond price revaluation	-1.22	-1.19	-3.09	0.32
Discount rate effect	-2.88	-2.94	-10.06	3.75
Fiscal backing	1.45	1.50	-3.36	7.01
Wealth effect	1.95	2.01	-6.28	10.28

Each row is the response of component in (25) to an exogenous monetary contraction raising the interest rate 25bps.

D.3 Output Puzzle and SVAR Estimation The impulse response estimates in Figure 8 reveal the expansionary output response in the earlier periods against a monetary contraction. It may stem from the policy regime assumption of the DSGE model that treats the pre-Volcker era as the active monetary and passive fiscal policy regime. But several studies do not share this view(Sims and Zha, 2006, Davig et al., 2006, Bianchi and Ilut, 2017).

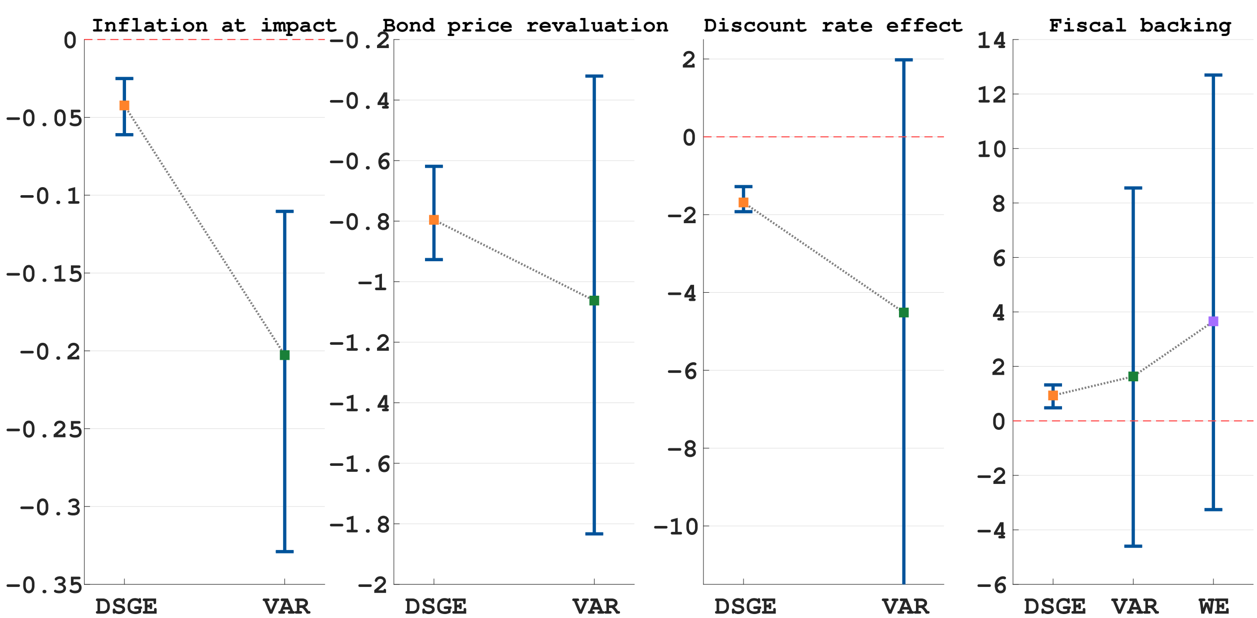
To see whether this argument is sensible, I estimate the VAR with the subsample of the monetary policy shock proxy after 1980 to exclude the pre-Volcker periods. Figure 20 collects the same graph analogous to the benchmark VAR with a full sample proxy for monetary policy shock. The output puzzle seems to be alleviated in that the output does not show the expansionary response at the impact. Although the later part of the impulse response seems to suffer the issue, the mean response of DSGE might be telling that only the level of response is problematic. The other responses and the debt value decomposition error bands give qualitatively similar results.

Figure 20: Posterior Impulse Response and Debt Value Decomposition of VAR Model with Proxy Excluding Pre-Volcker Era



The figure draws the same objective of Figure 8 with subsample of monetary policy shock proxy from the estimated DSGE model excluding the pre-Volcker era(up to 1980).

Figure 21: Posterior Impulse Response and Debt Value Decomposition of VAR Model with Proxy Excluding Pre-Volcker Era



The figure draws the same objective of Figure 9 with subsample of monetary policy shock proxy from the estimated DSGE model excluding the pre-Volcker era(up to 1980).

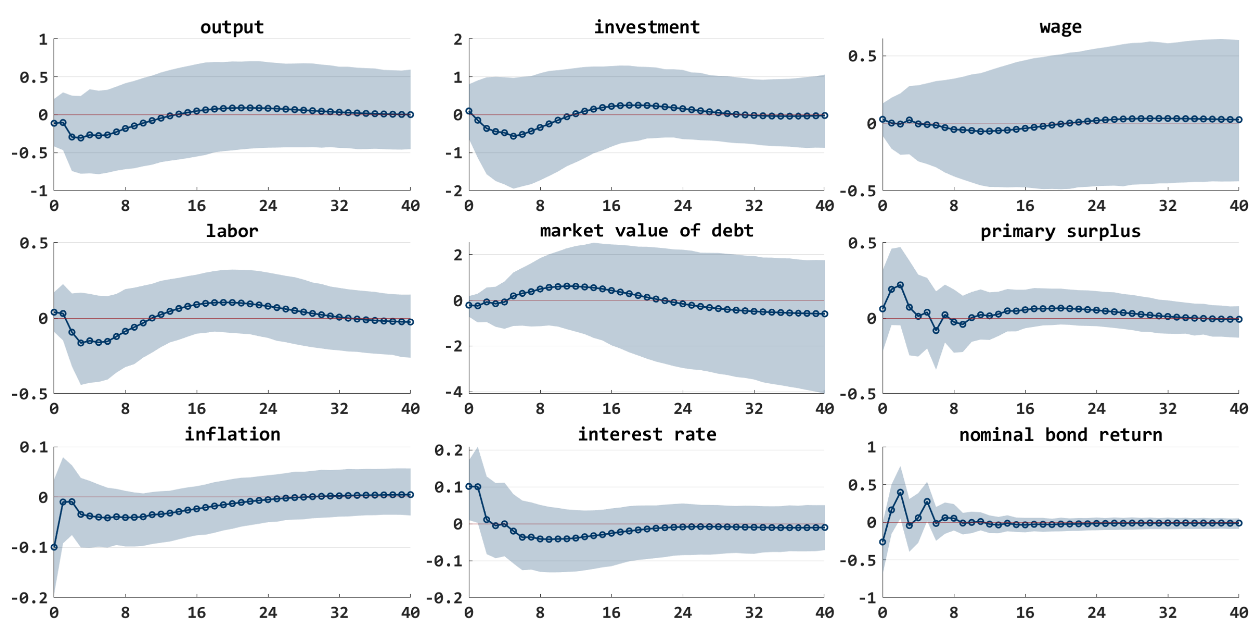
Arias et al. (2018) propose a computation algorithm based on the importance sampling to estimate a SVAR model involving both zero and sign restrictions. Arias et al. (2019) apply this method to recover monetary policy shocks by imposing the following monetary policy rule on the eqaution associated to interest rate in the reduced form VAR,

$$R_t = \psi_p p_t + \psi_y y_t + \varepsilon_t^M$$

with $\psi_p, \psi_y > 0$. The sign restrictions are on the B_0 matrix in (39) which governs the contemporaneous relationship within the variables of the VAR model. It also restricts the other variable's coefficient in the B_0 to be zero, which constitute the identification of monetary policy rule. I apply this method to estimate the SVAR model with the monetary policy rule specified in the DSGE model as (30).

The posterior output response in Figure 22 is relatively free from the output puzzle, though insignificant. Unlike the benchmark case and the DSGE estimates, fiscal policy shows primary surplus not deficit at the impact. The bond return response seems to suffer from the high frequency noise of the data.

Figure 22: Posterior Impulse Response SVAR Model with Monetary Policy Rule (30)



Although the estimates of the SVAR are somewhat different, its debt valuation decomposition arrives at similar conclusion, in that the discount rate effect has larger size than the bond price revaluation in Table 8.

Table 8: Posterior Debt Value Decomposition of SVAR Model

Debt value components	Median	Mean	5th quantile	95th quantile
Inflation adjustment	-0.23	-0.34	-2.41	0.03
Bond price revaluation	-0.60	-0.70	-2.83	-0.08
Discount rate effect	-2.20	-2.29	-10.47	4.34
Fiscal backing	1.97	2.32	-1.31	10.65
Wealth effect	1.79	1.93	-5.07	11.52

Each row is the response of component in (25) to an exogenous monetary contraction raising the interest rate 25bps.

D.4 DSGE Model Estimation

Since the DSGE model consists of fully identified structural shocks, variance decomposition is applicable.

Figure 23: Variance Decomposition of DSGE Model

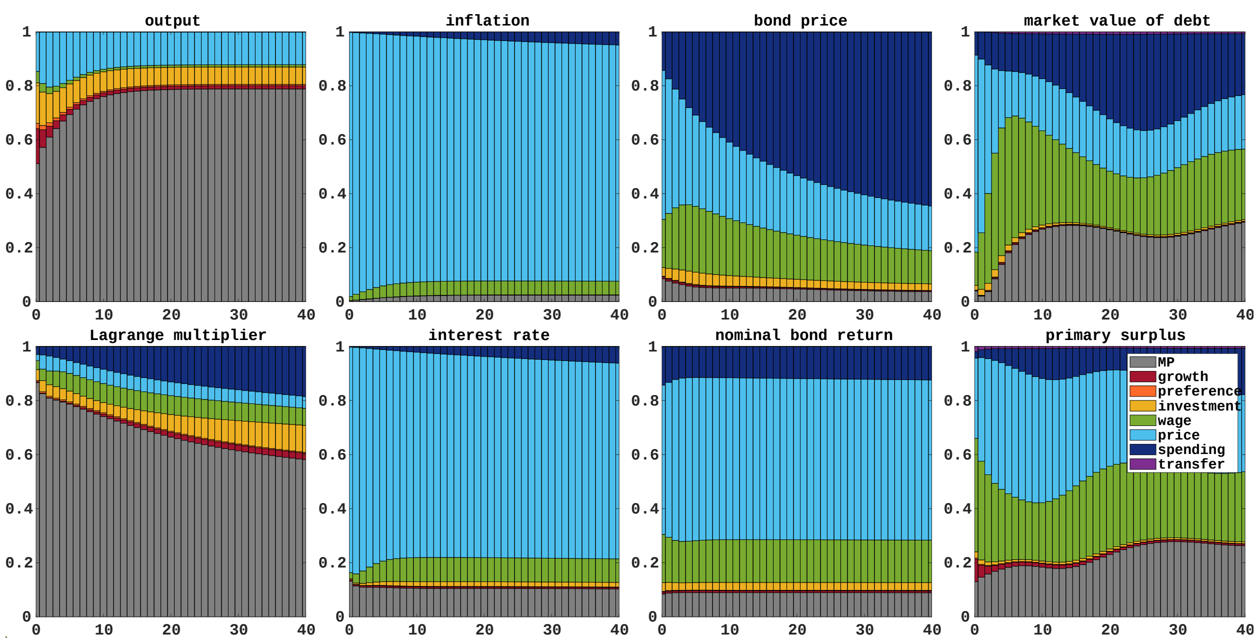
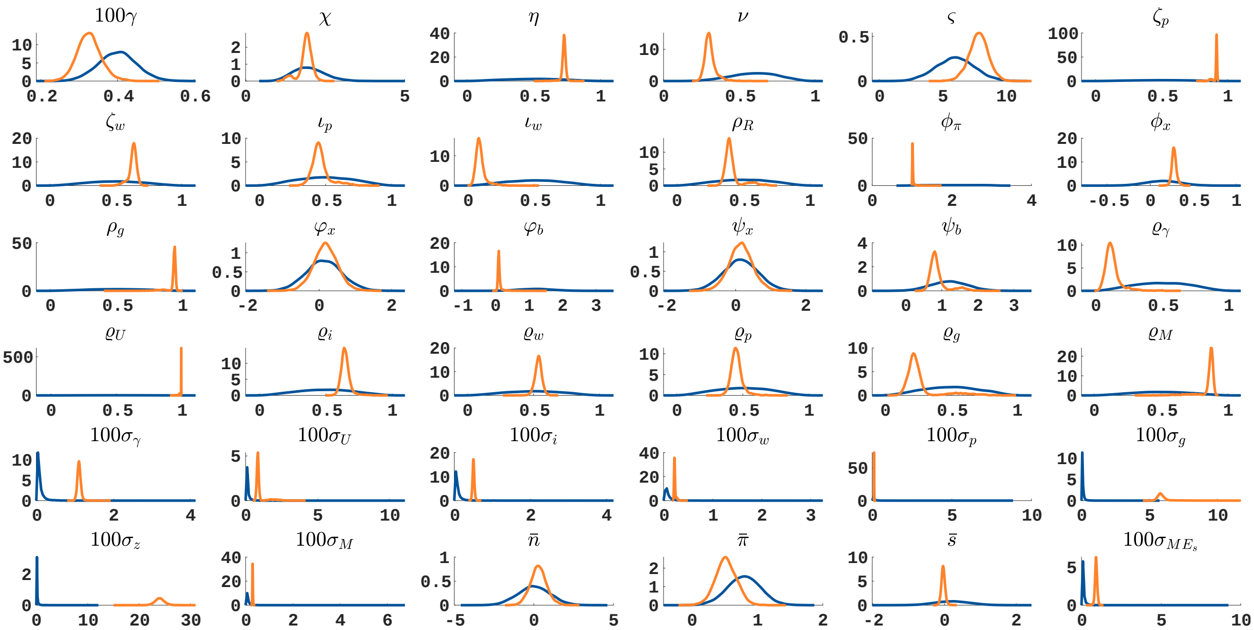
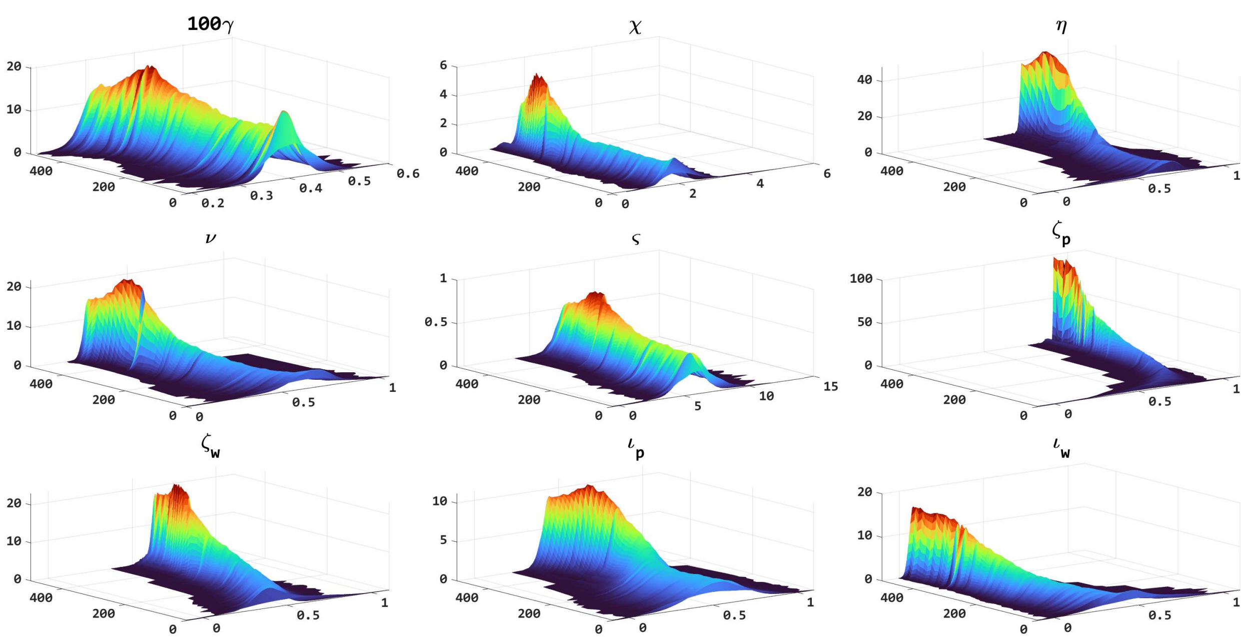


Figure 24: Parameter Estimation Adaptation of Particle Sampler in SMC: Sampler Kernel Density



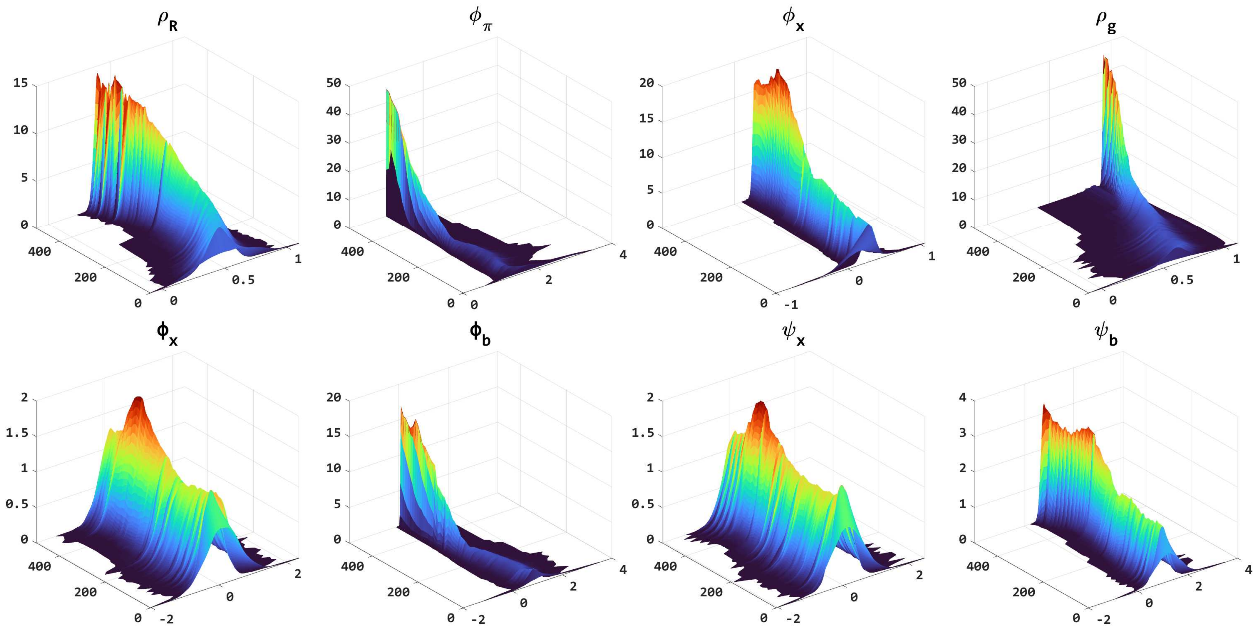
Darker line is the kernel density of prior, and the light colored line is the kernel density of posterior.

Figure 25: Parameter Estimation Adaptation of Particle Sampler in SMC: Model Parameters



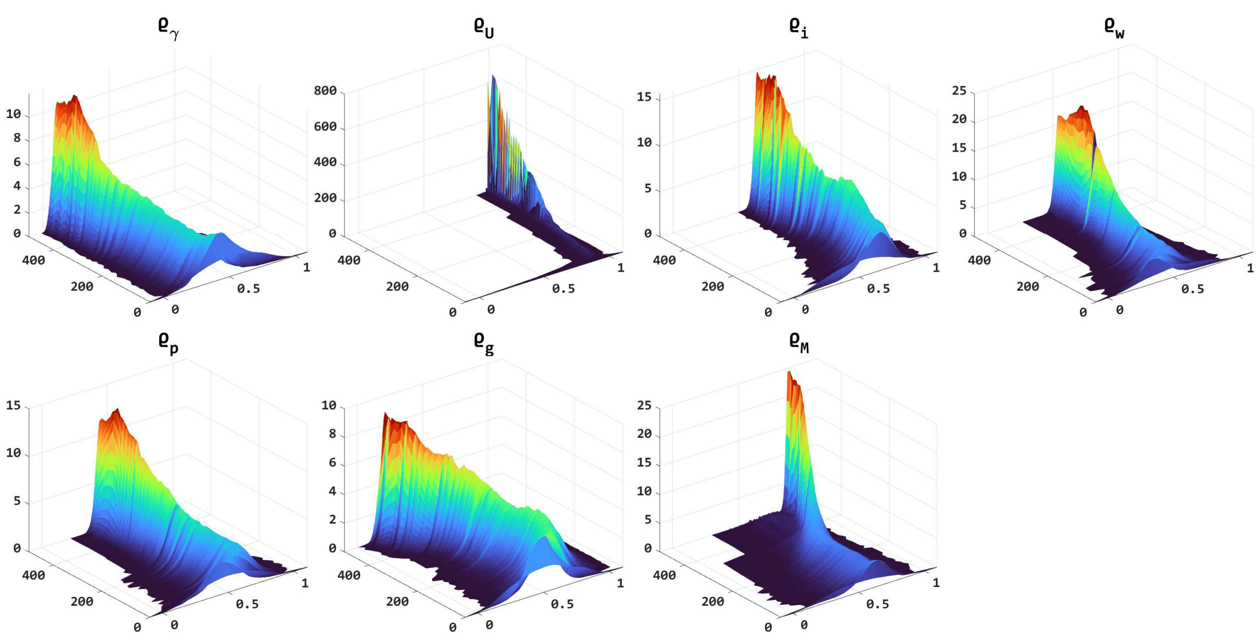
Left axis(stages in SMC), right axis(support of posterior sampler of each parameter), vertical axis(kernel density)

Figure 26: Parameter Estimation Adaptation of Particle Sampler in SMC: Policy Parameters



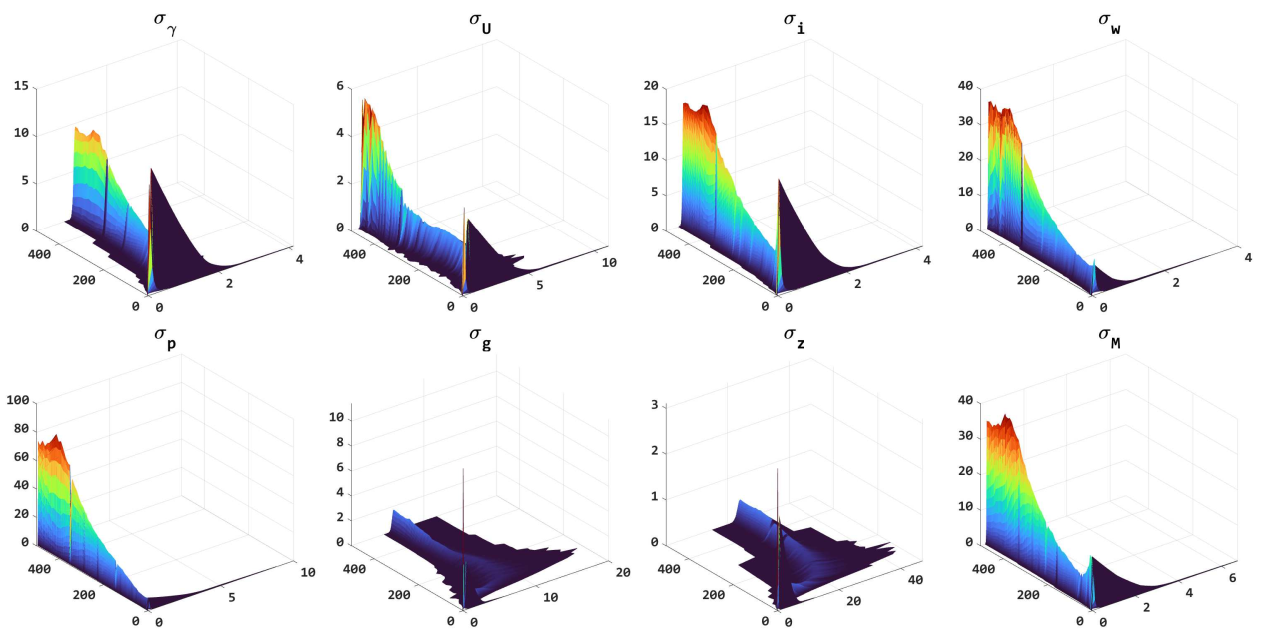
Left axis(stages in SMC), right axis(support of posterior sampler of each parameter), vertical axis(kernel density)

Figure 27: Parameter Estimation Adaptation of Particle Sampler in SMC: Autoregression Coefficient for Structural Shock Process



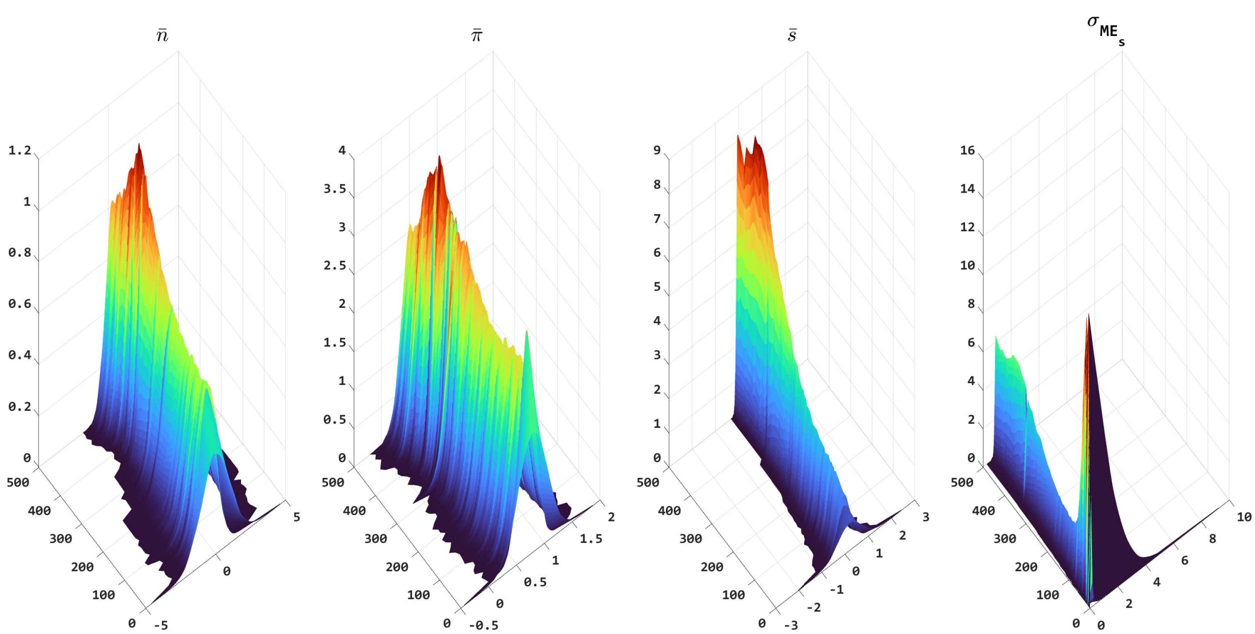
Left axis(stages in SMC), right axis(support of posterior sampler of each parameter), vertical axis(kernel density)

Figure 28: Parameter Estimation Adaptation of Particle Sampler in SMC: Standard Deviation for Structural Shock Process



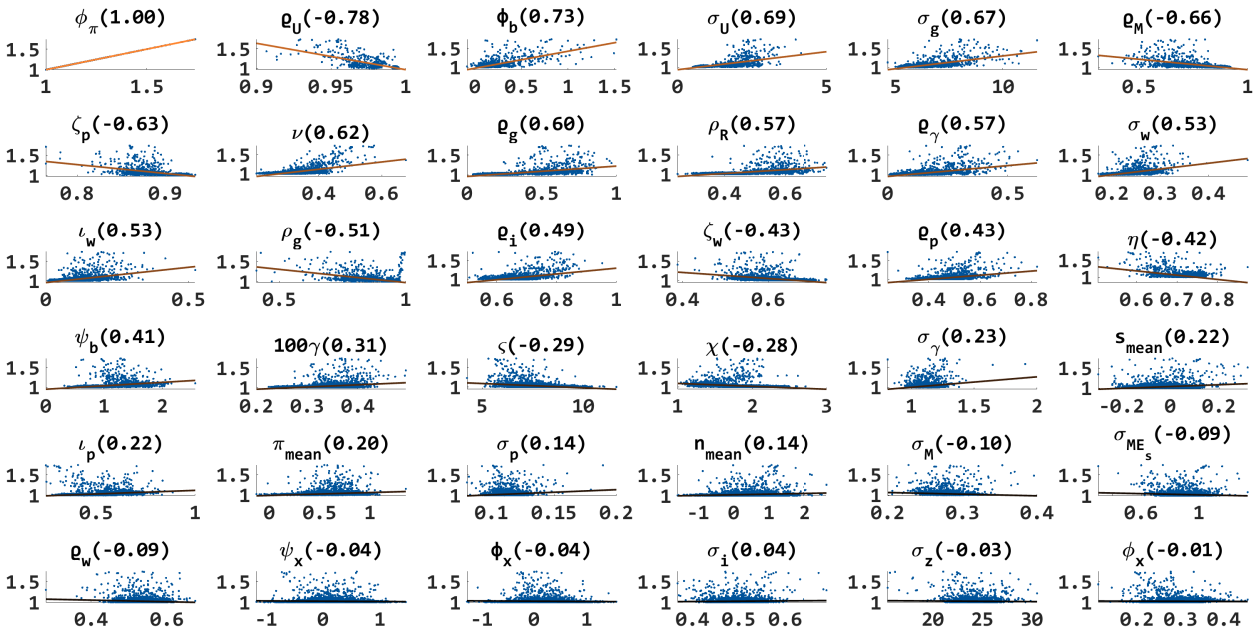
Left axis(stages in SMC), right axis(support of posterior sampler of each parameter), vertical axis(kernel density)

Figure 29: Parameter Estimation Adaptation of Particle Sampler in SMC: Measurement Equation Parameter



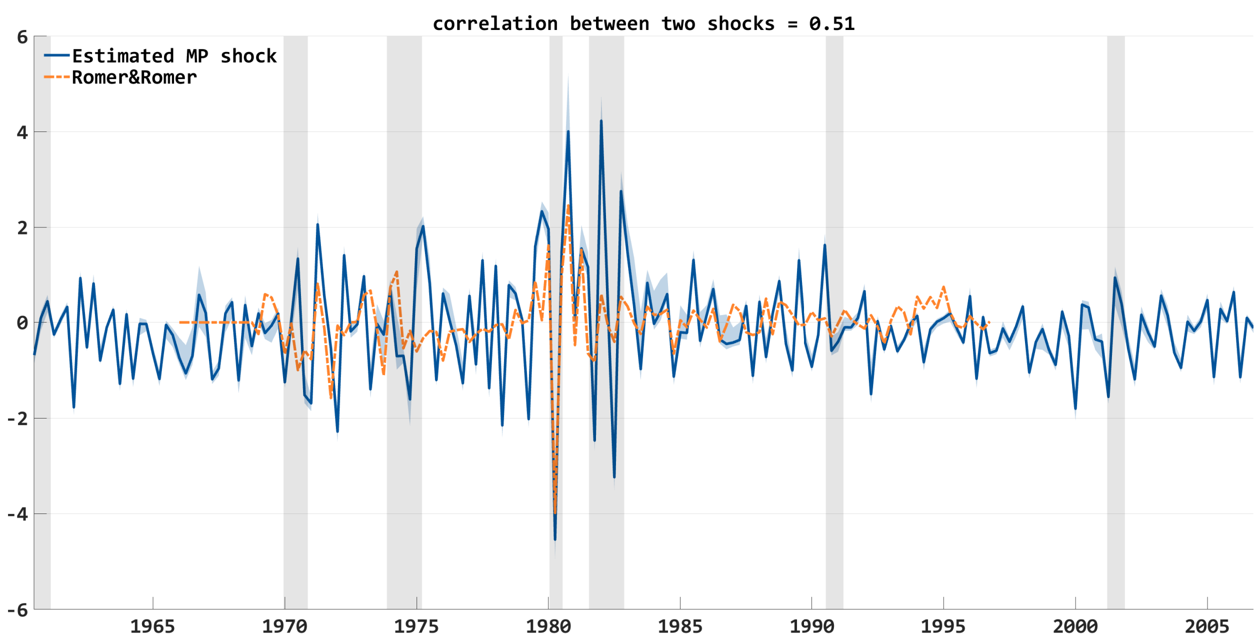
Left axis(stages in SMC), right axis(support of posterior sampler of each parameter), vertical axis(kernel density)

Figure 30: Posterior Sampler Correlation of DSGE Model



Each Panel depicts the scatter plot with the Taylor rule coefficient ϕ_π.

Figure 31: Estimated Monetary Policy Shock Comparison



Median of estimated monetary policy shock from the benchmark DSGE model with 90 percent credible set(solid) and narrative shock from Romer and Romer (2004).