Contribution Title

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Abstract. Despite the remarkable success of deep reinforcement learning (RL) across various domains, its deployment in the real world remains limited due to safety concerns. To address this challenge, constrained RL has been extensively studied as an approach for learning safe policies while maintaining performance. However, since constrained RL enforces constraints in the form of cumulative costs, it cannot guarantee state-wise safety. In this paper, we extend the Lagrangian based approach, a representative method in constrained RL, by introducing state-dependent Lagrange multipliers so that the policy is trained to account for state-wise safety. Our results show that the proposed method enables more fine-grained specification of the constraints and allows the policy to satisfy them more effectively by employing state-dependent Lagrange multipliers instead of a single scalar multiplier.

1 INTRODUCTION

Reinforcement learning (RL) learns a policy that maximizes rewards through trial and error. Learning in this manner may look very simple and straightforward, yet it is highly effective. This effectiveness has been clearly demonstrated in practice. Over the past few years, RL has demonstrated impressive achievements in diverse applications [15], [4], [12], [10]. Nevertheless, deploying RL in physical real-world environments remains a major challenge. To deploy RL-trained agents in real-world environments, two requirements must be satisfied: first, the agent must be able to successfully accomplish the given tasks, and second, it must ensure safety and reliability. In standard RL, such requirements are learned exclusively from reward signals, thereby necessitating careful reward engineering. However, reward engineering is inherently difficult, and even with carefully designed rewards, the learned policy may exploit unintended loopholes (reward hacking) or remain hard to interpret, making it difficult to ensure safety and reliability [3]. These limitations highlight the necessity of an explicit constraint function, separated from the reward, which has motivated research on constrained RL.

Constrained RL is a method that learns a policy to maximize rewards while satisfying constraints, thus enabling agents to behave safely while successfully performing tasks. A common formulation of constrained RL specifies constraints in terms of the expected cumulative cost [5]. For instance, in a driving scenario,

a constraint may be to minimize unintended lane intrusions (i.e., crossing into an adjacent lane), in which case the cumulative cost can be defined as the total number of lane intrusions. In this way, the agent can satisfy the constraint in expectation and keep the total number of intrusions within an acceptable limit. However, even a single violation at the level of an individual state—such as an unintended lane intrusion near another vehicle at a critical moment—may nevertheless lead to accidents or catastrophic failures.

This limitation highlights the need to consider state-wise constraints, which explicitly enforces constraints at the level of individual states. Accordingly, we introduce state-wise Lagrange multipliers to encourage the policy to satisfy safety requirements at every timestep. Our contributions are as follows:

- In contrast to most existing methods that utilize a single Lagrange multiplier for cumulative cost constraints, our approach addresses state-wise constraints, which inherently demand state-wise multipliers. To this end, neural networks are utilized to approximate these multipliers, enabling adaptive enforcement of safety at the level of individual states.
- The proposed method allows for finer specification of constraints and achieves better constraint satisfaction compared to existing constrained RL approaches under the same training budget.
- The learned Lagrange multiplier network can be utilized during deployment to assess state-wise safety, providing interpretability and insight into the agent's behavior.

2 Related Work

In the context of constrained RL, Liu et al. [9] discuss various formulations in the CMDP setting and introduce corresponding model-free approaches. Most existing works focus on enforcing constraints defined in terms of the discounted cumulative cost. Among representative methods is constrained policy optimization (CPO) [1], which uses surrogate functions to approximate both the objective and the constraints, and employs a projection step to enforce constraint satisfaction. This procedure requires a backtracking line search, which makes the algorithm computationally expensive. Another line of work includes Proximal Policy Optimization (PPO) Lagrangian and Trust Region Policy Optimization (TRPO) Lagrangian [11], which extend the PPO [14] and TRPO [13] algorithms to the constrained RL setting through Lagrangian relaxation, reformulating the constrained policy optimization problem as an unconstrained max-min optimization problem. By adaptively adjusting the Lagrange multiplier, these methods encourage policy updates toward satisfying the constraints. However, since constraint violations are necessary for learning a safe policy, the constraints are typically not satisfied during training. In interior-point policy optimization (IPO) [8], logarithmic barrier functions are added to the objective as penalty terms to account for the constraints. While this approach is easy to implement and can handle multiple constraints, it assumes feasible iterates, which can be problematic in situations such as random agent initialization where constraint violations

may occur. Stooke et al. [16] demonstrate that in Lagrangian-based methods, the Lagrange multiplier is updated only through integral control, resulting in oscillation and overshoot issues, and proposea a method that incorporates proportional and derivate terms to stabilize the updates.

3 Methodology

3.1 Preliminaries

Problem Formulations

Markov Decision Processes In RL, problems are typically formulated as Markov decision process (MDP) [17]. An MDP is defined as a tuple $\langle S, A, P, R, \gamma \rangle$, where S and A denote the state and action spaces, P is the transition probability, R is the reward function, and $\gamma \in [0, 1)$ is the discount factor. The objective of RL is to find an optimal policy π^* that maximizes the cumulative reward, defined as:

$$\begin{aligned} J_R(\pi) &= \mathbb{E}_{\tau \sim \pi} \left[\sum_{t=0}^{\infty} \gamma^t R(s_t, a_t) \right] \\ \pi^* &= \arg\max J_R(\pi) \end{aligned} \tag{1}$$

Here, $\tau = (s_0, a_0, s_1, a_1, \ldots)$ denotes a trajectory generated by following policy π.

Constrained Markov Decision Processes In contrast, constrained RL is typically formulated as a constrained Markov decision process (CMDP) [2]. A CMDP extends an MDP by introducing a set of cost functions $C_1, \ldots, C_m$, which are separate from the reward function, together with corresponding limits $d_1, \ldots, d_m$. Formally, a CMDP is defined as a tuple $\langle S, A, P, R, C, d, \gamma \rangle$. In a CMDP, the set of feasible policies $\mathit{\Pi}_C$ is defined as:

$$\mathit{\Pi}_C = \{\pi \in \mathit{\Pi} : \forall i, J_{C_i}(\pi) \leq d_i\}, \tag{2}$$

Specifically, the cost-based return for constraint i is defined as:

$$J_{C_i}(\pi) = \mathbb{E}_{\tau \sim \pi} \left[\sum_{t=0}^{\infty} \gamma^t C_i(s_t, a_t) \right], \quad \forall i \in \{1, \ldots, m\}. \tag{3}$$

This definition parallels the expected return $J_R(\pi)$ in standard RL (Eq. 1). In standard RL, the objective is to sove an optimization problem that maximizes the expected return, as defined in Eq. (1). Whereas constrained RL solves the following constrained optimization problem:

$$\pi^* = \arg\max_{\pi} J_R(\pi) \text{ s. t. } J_{C_i}(\pi) \leq d_i, \ \forall i \in 1, \ldots, m. \tag{4}$$

State-wise Constrained Markov Decision Process The CMDP framework can be extended to use various types of cost-based constraints. One such extension is the state-wise constrained Markov decision process (SCMDP) [18], which imposes state-wise constraints to bound the expected cost at each state by a specified threshold. In a SCMDP, the set of feasible policies Π_{SC} is defined as:

$$\Pi_{SC} = \{\pi \in \Pi : \forall (s_t, a_t, s_{t+1}) \sim \tau, \\ \forall i, C_i(s_t, a_t, s_{t+1}) \leq w_i\} \tag{5}$$

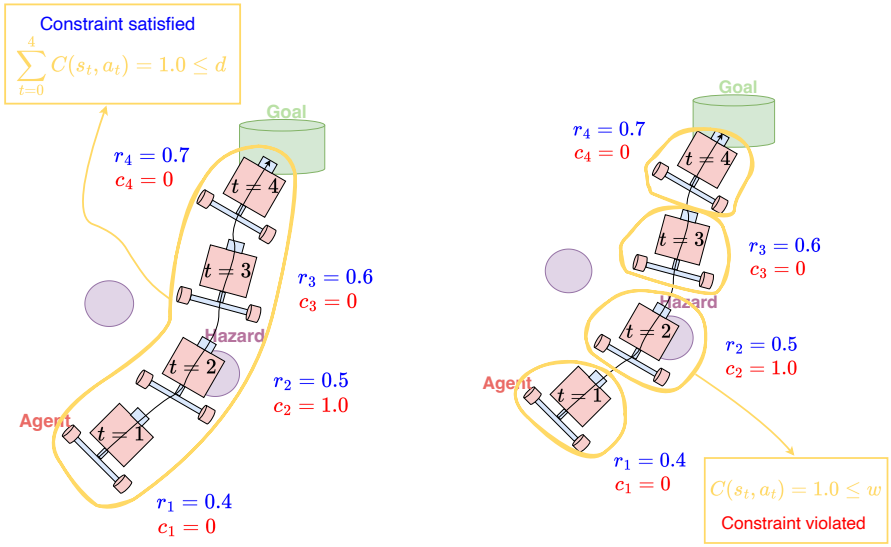
where $C_i(s_t, a_t, s_{t+1})$ is the cost incurred at state s_t after taking action a_t and transitioning to state s_{t+1}, and w_i is the corresponding limit. Similar to CMDP, the optimization problem in SCMDP can be formulated as

$$\pi^* = \arg\max_{\pi} J_R(\pi) \text{ s.t. } J_{SC_i}(\pi) \leq w, \ \forall i \in 1, \ldots, m \tag{6}$$

In this formulation, the state-wise constraint is defined as:

$$J_{SC_i}(\pi) = \mathbb{E}_{(s_t, a_t, s_{t+1}) \sim \tau, \tau \sim \pi}\big[C_i(s_t, a_t, s_{t+1})\big], \quad \forall i \in \{1, \ldots, m\}. \tag{7}$$

This formulation differs from the CMDP, where constraints are imposed only on the cumulative cost. In contrast, the SCMDP enforces constraints directly on the state-wise cost, thereby ensuring per-state safety guarantees.



(a) Constrained RL (b) State-wise constrained RL

Fig. 1: Comparison of constrained RL and state-wise constrained RL. In constrained RL, the agent is feasible if the cumulative cost is below the limit, while in state-wise constrained RL, a violation occurs if the cost at any single state exceeds the limit.

Lagrangian Relaxation for Constrained Policy Optimization A common approach to solve constrained optimization problems is to use Lagrangian relaxation. In this approach, the constrained optimization problem (4) is reformulated as an unconstrained optimization problem by introducing Lagrange multiplier $\lambda \geq 0$ that penalizes constraint violations. The resulting Lagrangian can be written as:

$$L(\theta, \lambda) = J_R(\pi_\theta) - \lambda^\top (J_C(\pi_\theta) - d), \tag{8}$$

where θ denotes the parameter of the policy. The objective is then to find a saddle point (θ^*, λ^*) that satisfies:

$$L(\theta^*, \lambda) \geq L(\theta^*, \lambda^*) \geq L(\theta, \lambda^*). \tag{9}$$

Since finding a global saddle point is often computationally intractable, in practice one aims to find a locally optimal solution using iterative updates of the policy parameters and the Lagrange multiplier. A common approach is to apply gradient-based updates of the form

$$\theta_{n+1} = \theta_n + \eta_\theta \nabla_\theta \Big(J_R(\pi_\theta) - \lambda_n^\top J_C(\pi_\theta) \Big), \tag{10}$$

$$\lambda_{n+1} = \Big[\lambda_n + \eta_\lambda \big(J_C(\pi_\theta) - d \big) \Big]_+, \tag{11}$$

where $\eta_\theta, \eta_\lambda > 0$ are step sizes, and $[\cdot]_+$ denotes the projection onto the nonnegative reals to ensure $\lambda \geq 0$.

3.2 Formulations of PPO under MDP, CMDP, and SCMDP

PPO Among policy gradient methods, PPO is one of the most widely used algorithms, originally proposed to solve the optimization problem in Eq. 1. To improve stability, PPO introduces a clipping mechanism that prevents large policy updates. The objective function of PPO is defined as:

$$J_R^{\text{PPO}}(\theta) = \mathbb{E}_{\pi_{\theta_{\text{old}}}} \left[\min \left(r(\theta) A^{\pi_{\theta_{\text{old}}}}(s,a), \text{clip}(r(\theta), 1-\epsilon, 1+\epsilon) A^{\pi_{\theta_{\text{old}}}}(s,a) \right) \right] \tag{12}$$

where $r(\theta) = \frac{\pi_\theta(a|s)}{\pi_{\theta_{\text{old}}}(a|s)}$ denotes the probability ratio between the current and the old policies, and $A^{\pi_{\theta_{\text{old}}}}(s,a) = Q^{\pi_{\theta_{\text{old}}}}(s,a) - V^{\pi_{\theta_{\text{old}}}}(s)$ represents the advantage function under the old policy.

PPO Lagrangian PPO Lagrangian is proposed as an extension of PPO to the constrained RL setting, enabling the algorithm to handle explicit constraints. The original PPO algorithm enhances stability by limiting policy updates through clipping, as defined in Eq. 12, but it does not directly handle constraints. To address constraints defined in terms of the cumulative cost in Eq. 3, PPO Lagrangian applies the Lagrangian relaxation technique, as discussed in Section 3.1,

thereby converting the constrained optimization problem in Eq. 4 into an unconstrained one.

$$J^{\text{PPO-Lag}}(\theta) = \mathbb{E}_{\pi_{\theta_{\text{old}}}}\Big[\min\big(r(\theta)A^{\pi_{\theta_{\text{old}}}}(s,a), \text{clip}(r(\theta), 1-\epsilon, 1+\epsilon)A^{\pi_{\theta_{\text{old}}}}(s,a)\big) - \lambda r(\theta)A_c^{\pi_{\theta_{\text{old}}}}\Big] \tag{13}$$

where $\lambda \geq 0$ is the Lagrange multiplier that imposes a penalty on constraint violations.

3.3 Proposed Method

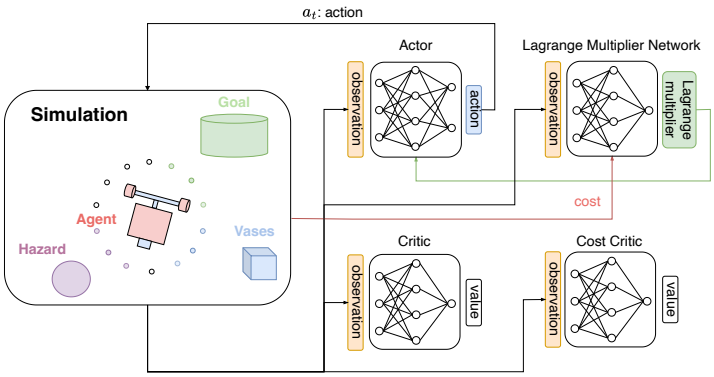


Fig. 2: Overview of the proposed method. Unlike standard PPO-Lagrangian, which employs a scalar Lagrange multiplier due to the constraint being defined on cumulative cost, our approach imposes constraints in a state-wise manner, requiring state-varying multipliers. To estimate these multipliers, we introduce an additional neural network, termed the Lagrange Multiplier Network.

In this paper, we propose a method that extends PPO Lagrangian to the state-wise constrained RL setting by estimating state-wise Lagrange multipliers. Similar to PPO Lagrangian in Eq. 13, which employs a scalar Lagrange multiplier to handle constraints on the cumulative cost, addressing the state-wise constraint defined in Eq. 7 requires a state-varying multiplier. To this end, we introduce an additional neural network, referred to as the Lagrange multiplier network, that takes the state as input and outputs the corresponding Lagrange multiplier, as illustrated in Fig. 2. The Lagrange multiplier network is parameterized by ξ and takes the state s as input, outputting the corresponding Lagrange multiplier $\lambda_\xi(s)$. This allows the application of Lagrangian relaxation to the constrained policy optimization defined in Eq. 6, analogous to the way

PPO-Lagrangian handles cumulative cost constraints. The objective function of the proposed method can thus be expressed as:

$$J^{\text{PPO-Lagnet}}(\theta) = \mathbb{E}_{\pi_{\theta_{\text{old}}}}\Big[\min\big(r(\theta)A^{\pi_{\theta_{\text{old}}}}(s,a), \text{clip}(r(\theta),1-\epsilon,1+\epsilon)A^{\pi_{\theta_{\text{old}}}}(s,a)\big) - \lambda_\xi(s)r(\theta)A_c^{\pi_{\theta_{\text{old}}}}\Big] \tag{14}$$

The parameters of the Lagrange multiplier network are updated using the following objective:

$$J_\lambda(\xi) = \mathbb{E}_{(s_t,a_t,s_{t+1})\sim\tau,\tau\sim\pi_{\theta_{\text{old}}}}[\lambda_\xi(s_t)(C(s_t,a_t,s_{t+1})-w)] \tag{15}$$

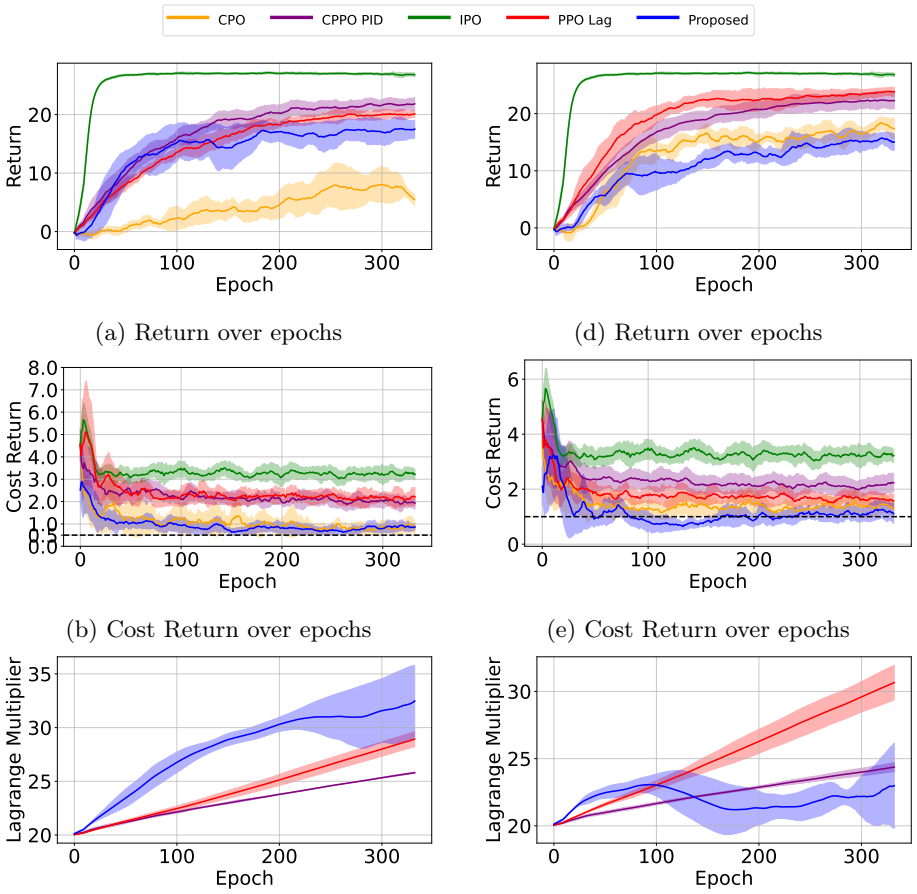
where w denotes the state-wise constraint limit. In this formulation, the objective $J_\lambda(\xi)$ drives the multiplier $\lambda_\xi(s)$ to increase when the observed cost $C(s_t,a_t,s_{t+1})$ exceeds the limit w, and to decrease when it falls below w.

4 Experiments

We evaluate our approach in the Safety Gymnasium [6] environment, focusing on the `PointGoal` task under different cost limits. Safety Gymnasium provides two cost definitions: a binary indicator and object-specific values. We adopt the object-specific cost (`constrain_indicator=False`) in all experiments. We evaluate our approach in terms of constraint satisfaction performance by comparing it with CPO [1], PPO Lagrangian [11], IPO [8], and CPPO [16], using the implementations provided by Omnisafe [7].

4.1 Results

Figures. 3 and 4 present the comparison on the `PointGoal` task under different cost limits. Each figure shows how return, cost return, and the Lagrange multiplier evolve during training, highlighting how varying cost limits affect the agent's performance and constraint satisfaction. Since all methods except ours define the constraint in terms of cumulative cost, the threshold for our method was set to 1/1000 of theirs (as each episode consists of 1,000 steps). As shown in Fig. 3 (b), (e), and Fig. 4 (b), (e), our proposed method consistently satisfies the constraints across different constraint settings, unlike the other methods. However, as the constraints are enforced more strictly, the reward performance is lower compared to some of the other methods, as shown in Fig. 3 (d). This is a common trade-off in constrained RL. In addition, as shown in Fig. 3 (f) and Fig. 4 (c) and (f), when comparing the values of the Lagrange multiplier, our proposed method satisfies the constraints, resulting in bounded values or even decreasing trends. An exception occurs when the cost limit is set to 0.5, which is particularly strict and difficult to satisfy, leading to continuously increasing values.



(a) Return over epochs

(d) Return over epochs

(b) Cost Return over epochs

(e) Cost Return over epochs

(c) Lagrange multipliers over epochs

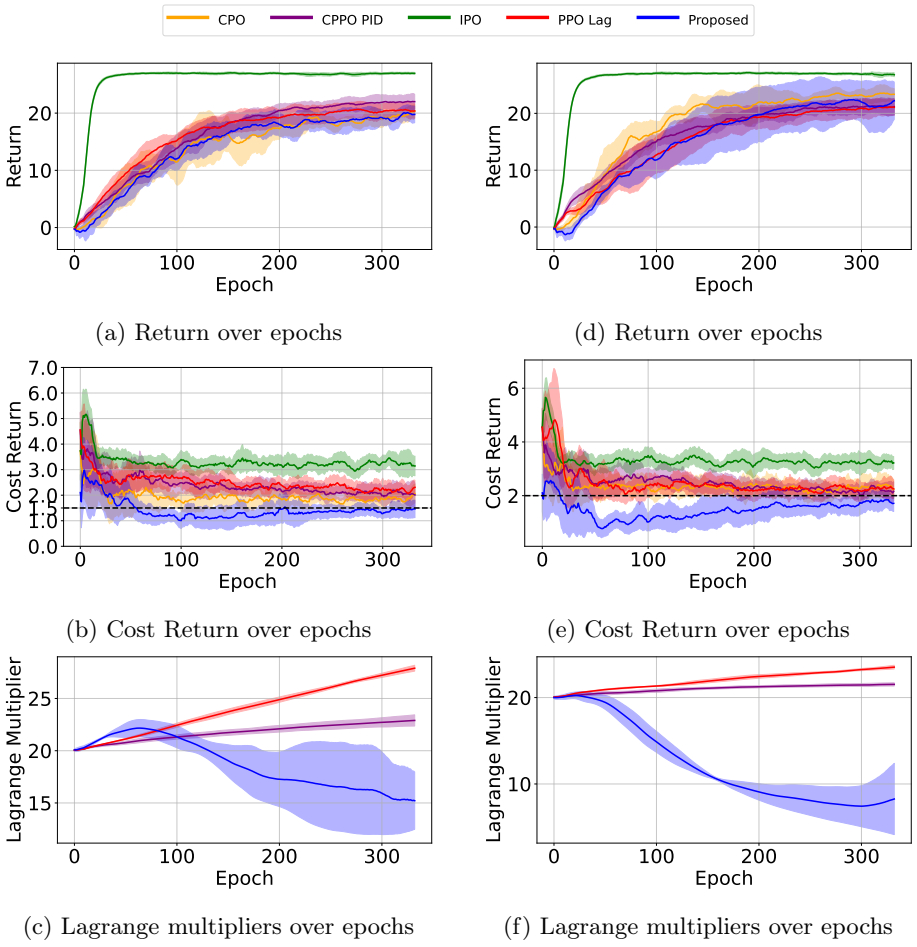
(f) Lagrange multipliers over epochs

Cost return limit: 0.5

Cost return limit: 1

Fig. 3: Learning curves. The left column, (a)–(c), illustrates the return, cost return, and Lagrange multiplier when the cost limit is set to 0.5. The right column, (d)–(f), presents the corresponding results under a cost limit of 1.0. Each row highlights a different metric, showing how the return (top), cost return (middle), and Lagrange multiplier (bottom) evolve over training epochs.

Here, the initial value of 20 was chosen empirically to accelerate convergence. Unlike PPO Lagrangian and CPPO PID, which impose constraints over cumulative costs across an episode and therefore requires a single scalar multiplier, our method enforces state-wise constraints and thus needs state-varying multipliers. For our method, the state-wise multiplier values were averaged over each epoch for visualization.



(a) Return over epochs

(d) Return over epochs

(b) Cost Return over epochs

(e) Cost Return over epochs

(c) Lagrange multipliers over epochs

(f) Lagrange multipliers over epochs

Cost return limit: 1.5

Cost return limit: 2

Fig. 4: Learning curves. The left column, (a)–(c), illustrates the return, cost return, and Lagrange multiplier when the cost limit is set to 1.5. The right column, (d)–(f), presents the corresponding results under a cost limit of 2.0. Each row highlights a different metric, showing how the return (top), cost return (middle), and Lagrange multiplier (bottom) evolve over training epochs.

4.2 Analysis on Lagrange Multiplier

Figures 5 and 6 present the evaluation results of the learned Lagrange multiplier network. Figure 5 shows the per-step reward, cost, and the output of the network throughout a single episode. The output of the Lagrange multiplier increases as the agent approaches obstacles and potential constraint violations, while decreasing toward zero when the agent remains in safe states, as observed

from the green curve in the bottom subplot of Fig. 5. Figure 6 shows simulation frames corresponding to the two starred timesteps in Fig. 5: the black star corresponds to subfigure (a), where the agent violates the constraint and the multiplier reaches its maximum value, while the white star corresponds to subfigure (b), where the agent remains in a safe state and the output stays close to zero.

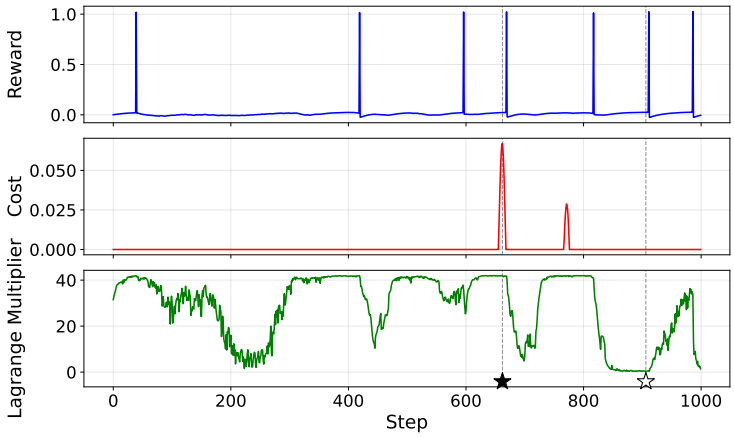
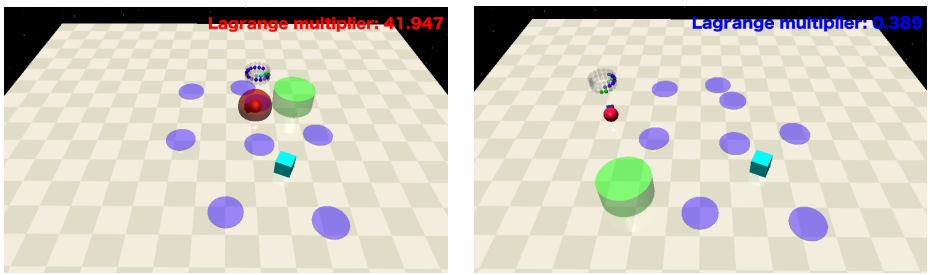


Fig. 5: Evaluation results of a single episode. The plot shows the per-step reward, cost, and the output of the learned Lagrange multiplier network. The output increases as the agent approaches obstacles and constraint violations, while it decreases toward zero when the agent remains in safe states.



(a) (b)

Fig. 6: Frames from the same episode at the timesteps marked with stars in Fig. 5. Subfigure (a) shows the agent violating the constraint, where the network outputs its maximum value, while subfigure (b) shows the agent in a safe situation, where the output remains close to zero.

5 CONCLUSIONS

In this work, we proposed a method that introduces state-wise Lagrange multipliers to learn policies that satisfy constraints defined in terms of state-wise costs. Unlike most existing approaches that address constraints based on cumulative costs, our method handles state-wise constraints, which inherently require state-dependent multipliers. To this end, neural networks are employed to approximate these multipliers, enabling the policy to more effectively satisfy constraints at the level of individual states. The proposed approach allows constraints to be specified more finely and demonstrates consistent satisfaction of them. Moreover, the learned Lagrange multiplier network can also be utilized at deployment time to assess state-wise safety, thereby providing interpretability and insight into the agent's behavior.

Building on the current findings, we identify several directions for future research. First, in our current approach, learning a safe policy inherently involves constraint violations during training, and even after convergence, deterministic satisfaction of the constraints cannot be strictly guaranteed. To provide deterministic safety guarantees, one possible direction is to leverage the learned Lagrange multiplier network to assess the current state and, when a risky state is detected, employ additional mechanisms such as safety filters or control barrier functions to block unsafe actions. Second, since PPO is an on-policy algorithm, its data efficiency is inherently lower than that of off-policy methods. This limitation becomes particularly critical in tasks where learning must be performed in real-world environments. A promising direction to address this issue is to explore extensions toward more sample-efficient approaches, such as adopting off-policy algorithms or incorporating model-based methods.

Despite these limitations, our findings highlight the potential of state-wise constrained RL and demonstrate that our approach provides a promising foundation for developing safer and more interpretable RL agents. We hope this work will inspire further research on bridging theoretical safety guarantees and practical RL applications.

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