HW01_Financial_Mathematics_sy3420

September 28, 2022

Homework 1: How good are financial models?

[2]:
```
import pandas as pd
import numpy  as np
import matplotlib.pyplot as plt
import scipy.stats as stats
from datetime import datetime

import copy as cp

DATA_PATH = "./HW/FinancialMath/data/"
```

[3]:
```
HW01_SPX = pd.read_csv("HW01_SPX.csv")
SPX_put = pd.read_csv("SPX_put_X20210716.csv")
```

1 Getting comfortable with some financial math

In this home work, you will get some experience working with some common financial mathematics and then have a chance to see if you can use this to do some financial data mining.

(Note that for purposes of this home work, we are making some simplifying assumptions about the data (e.g., that we can ignore weekends and holidays; that we don't need write extra code to make our functions fail gracefully, such as code to check to see if a date is out of range; etc.)

1.1 Return math

1.1.1 Calculate returns and log returns for the following price series. Compare the two. What do you notice?

DOW	Closing Price
Mon	\$1.00
Tue	\$1.50
Wed	\$1.00
Thr	\$0.50

DOW	ret	log ret
Mon	-	
Tue	50%	40.5%
Wed	-30%	-40.5%
Thr	-50%	-40.5%

[4]:

```
#return
return_t = (1.5-1)/1
return_w = (1-1.5)/1.5
return_th = (0.5-1)/1

print("Return for Tuesday is $",return_t)
print("Return for Wednesday is $",return_w)
print("Return for Thursday is $",return_th)
```

```
Return for Tuesday is $ 0.5
Return for Wednesday is $ -0.3333333333333333
Return for Thursday is $ -0.5
```

Return for Tuesday is $ 0.5

Return for Wednesday is $ -0.3333333333333333

Return for Thursday is $ -0.5

[5]:

```
#log return
log_t = np.log(1.5/1)*100
log_w = np.log(1/1.5)*100
log_th = np.log(0.5/1)*100

print("Log Return for Tuesday is", log_t,"%")
print("Log Return for Wednesday is", log_w,"%")
print("Log Return for Thursday is", log_th,"%")
```

```
Log Return for Tuesday is 40.546510810816436 %
Log Return for Wednesday is -40.54651081081644 %
Log Return for Thursday is -69.31471805599453 %
```

Read in the file `HW01_SPX.csv`.

1.1.2 Calculate returns and log returns for the closing price and plot the distribution of the log returns. What do you notice? (Hint: Use a large bin number like 41 or 51.) Add two new columns to your S&P data frame: `ret` and `log_ret`.

[41]:

```
HW01_SPX
```

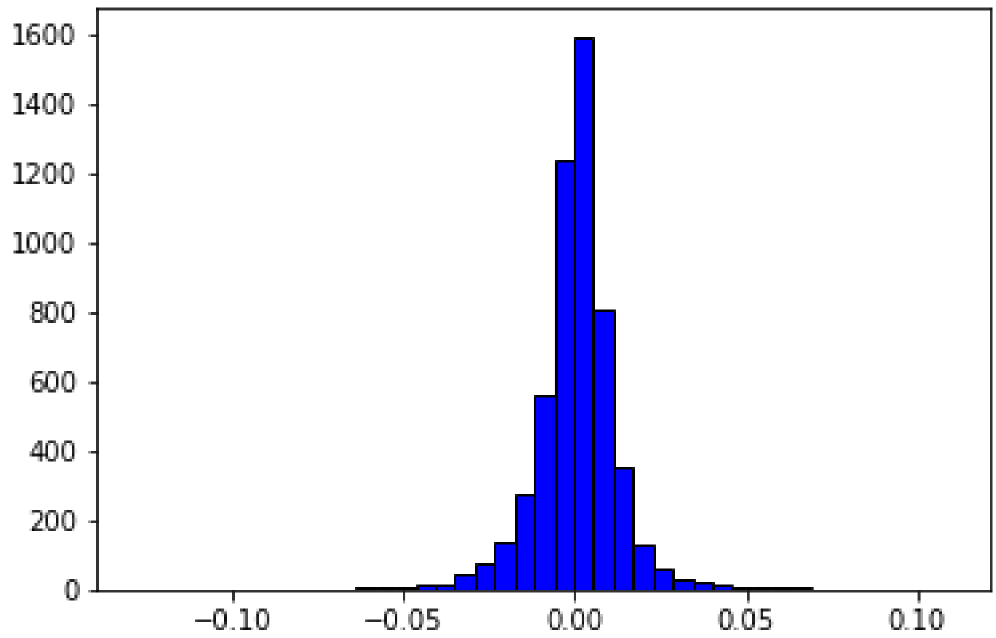
[41]:
```
            Date     Open     High      Low    Close    Return  Log_Return
0     2000-01-03  1455.22  1478.00  1438.36  1455.22       NaN         NaN
1     2000-01-04  1399.42  1455.22  1397.43  1399.42 -0.038345   -0.039099
2     2000-01-05  1402.11  1413.27  1377.68  1402.11  0.001922    0.001920
3     2000-01-06  1403.45  1411.90  1392.02  1403.45  0.000956    0.000955
4     2000-01-07  1441.47  1441.47  1400.53  1441.47  0.027090    0.026730
…            …       …      …       …       …       …        …
5414  2021-07-12  4372.41  4386.68  4364.03  4384.63  0.003451    0.003445
5415  2021-07-13  4381.07  4392.37  4366.92  4369.21 -0.003517   -0.003523
5416  2021-07-14  4380.11  4393.68  4362.36  4374.30  0.001165    0.001164
5417  2021-07-15  4369.02  4369.02  4340.70  4360.03 -0.003262   -0.003268
5418  2021-07-16  4367.43  4375.09  4322.53  4327.16 -0.007539   -0.007568

[5419 rows x 7 columns]
```

[7]:
```python
HW01_SPX["Return"] = (HW01_SPX["Close"].pct_change())
```

[8]:
```python
HW01_SPX["Log_Return"] = (np.log(HW01_SPX["Close"]/HW01_SPX["Close"].shift(1)))
```

[9]:
```python
plt.hist(HW01_SPX["Log_Return"], color = 'blue', edgecolor = 'black', bins = 41)
```

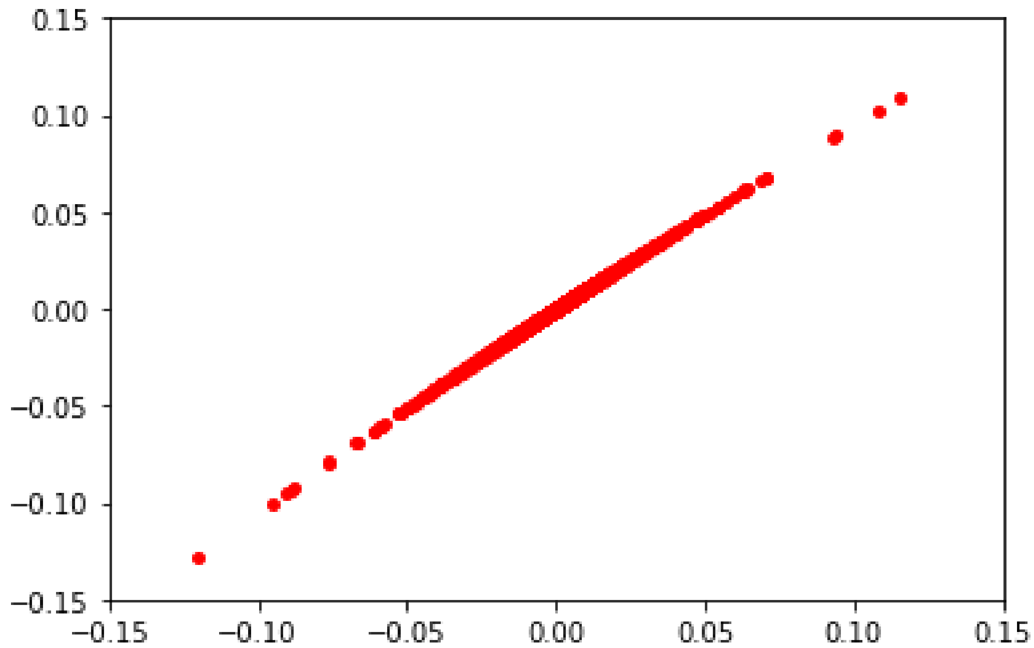
[9]:
```
(array([1.000e+00, 0.000e+00, 0.000e+00, 0.000e+00, 1.000e+00, 2.000e+00,
        1.000e+00, 0.000e+00, 2.000e+00, 0.000e+00, 2.000e+00, 5.000e+00,
        4.000e+00, 8.000e+00, 1.300e+01, 1.600e+01, 4.600e+01, 7.900e+01,
        1.380e+02, 2.780e+02, 5.570e+02, 1.238e+03, 1.592e+03, 8.090e+02,
        3.530e+02, 1.280e+02, 5.900e+01, 3.100e+01, 2.200e+01, 1.000e+01,
        8.000e+00, 3.000e+00, 5.000e+00, 3.000e+00, 0.000e+00, 0.000e+00,
        0.000e+00, 2.000e+00, 0.000e+00, 1.000e+00, 1.000e+00]),
 array([-0.12765214, -0.12186619, -0.11608023, -0.11029428, -0.10450833,
        -0.09872237, -0.09293642, -0.08715047, -0.08136451, -0.07557856,
        -0.0697926 , -0.06400665, -0.0582207 , -0.05243474, -0.04664879,
        -0.04086284, -0.03507688, -0.02929093, -0.02350498, -0.01771902,
        -0.01193307, -0.00614711, -0.00036116,  0.00542479,  0.01121075,
         0.0169967 ,  0.02278265,  0.02856861,  0.03435456,  0.04014052,
         0.04592647,  0.05171242,  0.05749838,  0.06328433,  0.06907028,
         0.07485624,  0.08064219,  0.08642814,  0.0922141 ,  0.09800005,
         0.10378601,  0.10957196]),
 <a list of 41 Patch objects>)
```



The log return distribution somewhat represents a normal distribution.

Plot the returns against the log returns using the same axis limits on the x and y axes. Write down your observations.

[10]:
```
plt.plot(HW01_SPX["Return"], HW01_SPX["Log_Return"], 'ro', markersize = 4)
plt.axis([-0.15, 0.15, -0.15,  0.15])
plt.show()
```



The observation plot represents a somewhat straight line. This suggests that the returns and log returns have little difference.

1.1.3 Calculate the volatility of the equity series over the most recent 30, 90, 120 and 360 days. For now, you may assume that calendar days are equivalent to trading days (i.e., that there are no days on which trading is not done, and that the last x observations in the data frame represent the last x days). Has the market become more or less volatile in the past 120 days?

[11]:
```
np.std(HW01_SPX["Return"][-30:], ddof=1 )
```

[11]: 0.005724051108350765

[12]:
```
np.std(HW01_SPX["Return"][-90:],  ddof=1 )
```

[12]: 0.006948492055594805

[13]:
```
np.std(HW01_SPX["Return"][-120:],  ddof=1 )
```

[13]: 0.00847155600783947

[14]:
```
np.std(HW01_SPX["Return"][-360:],  ddof=1 )
```

[14]: 0.018766654064227898

We can see that volatility increases as the days increase. Therefore, this sugguests that market is becoming more volatile in the past 120 days

1.1.4 Repeat this again, but this time for the (end) date 2021-06-21.

[15]:
```
#30 days
np.std(HW01_SPX["Return"][5370:5401],  ddof=1 )
```

[15]: 0.007965966105401581

[16]:
```
#90 days
np.std(HW01_SPX["Return"][5310:5401],  ddof=1 )
```

[16]: 0.008397351028491025

[17]:
```
#120 days
np.std(HW01_SPX["Return"][5280:5401],  ddof=1 )
```

[17]: 0.008711173331491202

[18]:
```
np.std(HW01_SPX["Return"][5040:5401],  ddof=1 )
```

[18]: 0.01880583659956322

We can see that volatility increases as the days increase. Therefore, this sugguests that market is becoming more volatile.

2 Valuing options

Recall that we may calculate the value of a European put, $P(\cdot)$, under the Black-Scholes-Merton pricing framework as:

$$P = \; N(-d_2)Ke^{-rt} - N(-d_1)S, \text{ where} \tag{1}$$

$$d_1 = \frac{1}{\sigma\sqrt{t}}\left[\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}t\right)\right], \tag{2}$$

$$\begin{aligned} d_2 &= \frac{1}{\sigma\sqrt{t}}\left[\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)\right] \\ &= d_1 - \sigma t, \quad \text{and} \end{aligned}$$

$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}z^2}dz \;; \tag{3}$$

and where:

$$
\begin{aligned}
t &\equiv \text{the time (in years) between today and the expiration of the option, i.e., } T - t_0\text{, where, } T \text{ is the closin} \\
S &\equiv \text{the price of the stock today;} \\
K &\equiv \text{the price at which the option holder may sell the stock at time } t; \\
\sigma &\equiv \text{the property-specific price volatility (e.g., as measured by the volatility of the AVM estimates);} \\
r &\equiv \text{the risk free rate (e.g., on U.S. government securites with a maturity date of today+} t\text{);} \\
N(x) &\equiv \text{the standard normal cdf evaluated at } x; \text{ and} \\
P &= P(S, K, \sigma, r, t) \equiv \text{the value of a European put option allowing the holder to sell one share of the unde} \\
& \quad S \text{ today at price } K \text{ at time } t\text{, given an annualized volatility of } \sigma \text{ and a risk-free rate } r\ .
\end{aligned}
$$

Note that $N(x)$ is the standard normal CDF (i.e., mean $\mu = 0$ and variance $\sigma^2 = 1$), which we can approximate numerically using the `scipy.stats` function `norm.cdf(x, 0, 1)`.

2.0.1 Write a python function `EuroPut(S,K, sigma, r, t)` to price a European put option

[19]:
```python
from scipy.stats import norm
import math
```

[20]:
```python
N = norm.cdf
def EuroPut(S, K, sigma, r, t):
    d1 = ( (1/(sigma*np.sqrt(t))) * ( np.log(S/K) + (r + (np.power(sigma,2))/
 ↪2)*t  ))
    #d2 = ((np.log(S/K) + (r - ((np.power(sigma, 2))/2))))/sigma*(np.sqrt(t))
    d2 = d1 - sigma*np.sqrt(t)
    P = N(-d2, 0, 1)*K*math.exp(-r*t) - N(-d1, 0, 1)*S

    return P
```

2.0.2 Use the function you wrote to price an option on a stock with the following characteristics:

price today	\$20.00
exercise price	\$35.00
time to expiration (yrs)	1.00
volatility	0.25
risk-free rate	0.05

[21]:
```
EuroPut(20, 35, 0.25, 0.05, 1.0)
```

[21]:
```
13.342106213937623
```

Plot (a) the price of the put option for volatility values of `[0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.4]`; and (b) price the put option for time to expiration values of `[0.25, 0.50, 0.75, 1.0, 1.25, 1.5, 2.0]`.

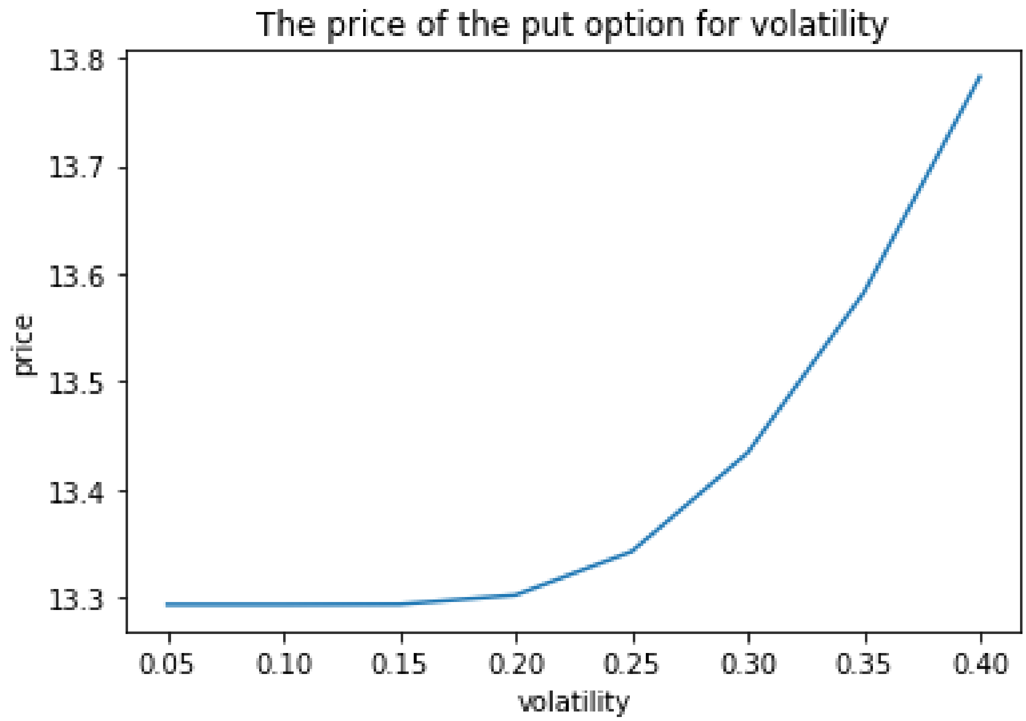
(Two seperate plots)

[22]:
```
sig = [0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.4]
time = [0.25, 0.50, 0.75, 1.0, 1.25, 1.5, 2.0]
```

[23]:
```
#the price of the put option for volatility values
print(EuroPut(20, 35, 0.05, 0.05, 1.0))
print(EuroPut(20, 35, 0.1, 0.05, 1.0))
print(EuroPut(20, 35, 0.15, 0.05, 1.0))
print(EuroPut(20, 35, 0.2, 0.05, 1.0))
print(EuroPut(20, 35, 0.25, 0.05, 1.0))
print(EuroPut(20, 35, 0.3, 0.05, 1.0))
print(EuroPut(20, 35, 0.35, 0.05, 1.0))
print(EuroPut(20, 35, 0.4, 0.05, 1.0))
```

```
13.293029857524992
13.293029939451085
13.293367798740647
13.301895472341705
13.342106213937623
13.43384914500923
13.582201279981447
13.782727769860102
```

[24]:
```
price_v = [(EuroPut(20, 35, 0.05, 0.05, 1.0)), (EuroPut(20, 35, 0.1, 0.05, 1.
↪0)), (EuroPut(20, 35, 0.15, 0.05, 1.0)),
(EuroPut(20, 35, 0.2, 0.05, 1.0)), (EuroPut(20, 35, 0.25, 0.05, 1.0)),␣
↪(EuroPut(20, 35, 0.3, 0.05, 1.0)),
(EuroPut(20, 35, 0.35, 0.05, 1.0)), (EuroPut(20, 35, 0.4, 0.05, 1.0))]
volatility=[0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.4]

plt.plot(volatility, price_v)
plt.title("The price of the put option for volatility")
plt.xlabel("volatility")
plt.ylabel("price")
plt.show()
```

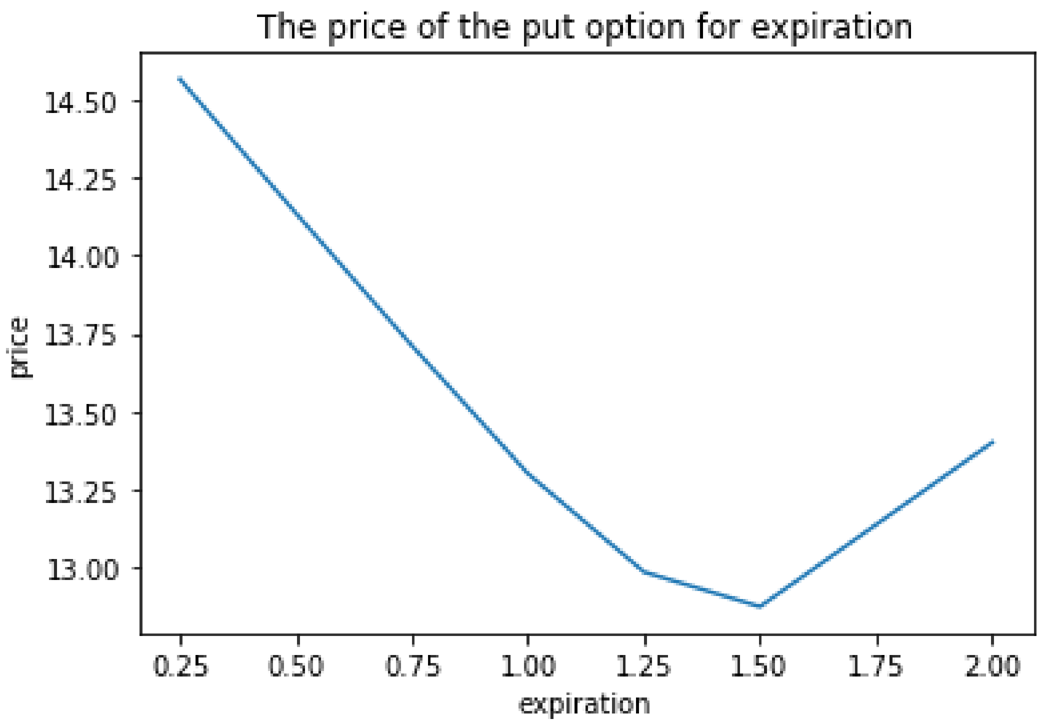


[25]:
```
#price the put option for time to expiration values
print(EuroPut(20, 35, 0.05, 0.05, 0.25))
print(EuroPut(20, 35, 0.1, 0.05, 0.5))
print(EuroPut(20, 35, 0.15, 0.05, 0.75))
print(EuroPut(20, 35, 0.2, 0.05, 1))
print(EuroPut(20, 35, 0.25, 0.05, 1.25))
print(EuroPut(20, 35, 0.3, 0.05, 1.5))
print(EuroPut(20, 35, 0.35, 0.05, 1.2))
```

```
14.565223017285852
14.135846920991646
13.711826704498435
13.301895472341705
12.986639208457351
12.87565066898544
13.401376698250676
```

[26]:
```
price_e = [(EuroPut(20, 35, 0.05, 0.05, 0.25)), (EuroPut(20, 35, 0.1, 0.05, 0.
 ↪5)), (EuroPut(20, 35, 0.15, 0.05, 0.75)),
(EuroPut(20, 35, 0.2, 0.05, 1)), (EuroPut(20, 35, 0.25, 0.05, 1.25)),␣
 ↪(EuroPut(20, 35, 0.3, 0.05, 1.5)), (EuroPut(20, 35, 0.35, 0.05, 1.2))]
```

```
expiration=[0.25, 0.50, 0.75, 1.0, 1.25, 1.5, 2.0]

plt.plot(expiration, price_e)
plt.title("The price of the put option for expiration")
plt.xlabel("expiration")
plt.ylabel("price")
plt.show()
```



2.0.3 Imagine that the actual closing price of the option today is $14.25. What value of volatility is implied given the price in the marketplace? (Hint, use your `EropeanPut` function to search for a value of `sigma` that makes the price equal to $\$14.75 \pm \$0.005$ or less by trying different values of `sigma`.)

[27]:
```
p = EuroPut(20,35,0.49,.05,1)
p
```

[27]: 14.24996615668878

sigma = 0.49 would be it. We can simply look at the graph and expiration = 0.49 seems to be approrpriate.

3 How well does Black-Scholes predict?

Read in the file `SPX_put_X20210716.csv` which contains several months of history for an option on the S&P500 index.

The terms of the option are:

current_price	in `underlying_last`
strike price	$3685
expiration date	2021-07-16
risk-free rate	0.0025 *

* the risk-free rate varies daily, but for convenience in this example we have set it to a constant value.

3.0.1 Use your `EuroPut` function to price the option as of the first date in our data set (2021-06-21). Use volatilities you calculated from that date back 30 days and 120 days.

[28]:

```
SPX_put.head()
```

[28]:

```
   Unnamed: 0  quote_unixtime     quote_readtime   quote_date  \
0      108508      1624305600   2021-06-21 16:00   2021-06-21
1      116717      1624392000   2021-06-22 16:00   2021-06-22
2      125088      1624478400   2021-06-23 16:00   2021-06-23
3      133233      1624564800   2021-06-24 16:00   2021-06-24
4      141537      1624651200   2021-06-25 16:00   2021-06-25

   quote_time_hours  underlying_last  expire_date  expire_unix  strike  p_bid  \
0                16          4224.66   2021-07-16   1626465600    3685   5.09
1                16          4246.38   2021-07-16   1626465600    3685   3.90
2                16          4241.83   2021-07-16   1626465600    3685   3.40
3                16          4266.39   2021-07-16   1626465600    3685   2.85
4                16          4280.68   2021-07-16   1626465600    3685   2.21

   p_ask       p_size  p_last p_volume  strike_distance  strike_distance_pct
0   5.30    923 x 811     0.0                     539.7                0.128
1   4.09   1389 x 100     0.0                     561.4                0.132
2   3.61   886 x 1621     0.0                     556.8                0.131
3   3.00   106 x 1310     0.0                     581.4                0.136
4   2.35    109 x 401     0.0                     595.7                0.139
```

[29]:

```
#30 days
sigma = np.std(HW01_SPX["Return"][5370:5400],  ddof=1 ) * np.sqrt(365)

start = datetime(2021, 6, 21)
```

```
end = datetime(2021, 7, 16)
diff = end - start
t = (diff.days + diff.seconds/86400)/365
S = SPX_put["underlying_last"][0]
K = SPX_put["strike"][0]
r = 0.0025

EuroPut(S, K, sigma, r, t)
```

[29]: 0.006762959166845217

[30]:
```
#90 days

sigma = np.std(HW01_SPX["Return"][5280:5401],  ddof=1 ) * np.sqrt(365)

start = datetime(2021, 6, 21)
end = datetime(2021, 7, 16)
diff = end - start
t = (diff.days + diff.seconds/86400)/365
S = SPX_put["underlying_last"][0]
K = SPX_put["strike"][0]
r = 0.0025

EuroPut(S, K, sigma, r, t)
```

[30]: 0.03944987844840542

3.0.2 Now write a short function to calculate the implied volatility for all of the options prices in the data set (you may assume that the data conventions are the same in both the SPX and SPX put option files):

[39]:
```
def IV(S,K,r,t,target_price,bound=.005):
    vol = 0
    call = EuroPut(S,K,vol,r,t)
    step_size = 0.01
    if call > target_price + bound:
        return
    else:
        while True:
            next_price = EuroPut(S,K,step_size,r,t)
            if next_price < target_price:
                vol = vol + step_size
            else:
                high_vol = vol + step_size
                vol_low = vol
```

```
        while abs(target_price - call) > bound:
            vol = (vol_up-vol_low)/2
            p = EuroPut(S,K,vol,r,t)
            if target_price - call > 0:
                vol_low = vol
            else:
                vol_up = vol
    return vol
```

[40]:

```
implied_vol = []

K = 3685
r = .0025
end_date = datetime(2021,7,16)

for i in range(len(SPX_put['p_last'])-1):
    price = SPX_put['p_last'][i]
    S = SPX_put['underlying_last'][i]

    t_str = SPX_put['quote_date'][i]
    split = t_str.split('-')
    split1 = list(map(int, split))
    start_date = datetime(split1[0],split1[1],split1[2])
    difference  = end_date - start_date
    t = (difference.days + difference.seconds/86400)/365 #have to adjust for␣
 ↪year
    implied_vol.append(IV(S,K,r,t,price,bound=.005))
print(implied_vol)
```

```
[1e-08, 1e-08, 1e-08, 1e-08, 1e-08, 1e-08, 1e-08, 0.33203126, 0.352734385,
0.3796875099999999, 0.44453125999999993, 0.4773437599999999, 0.4734375099999999,
0.45312500999999994, 0.6093750099999999, 0.69062501, 0.8531250099999999,
1.1812500100000003]
```

This is the implied volatility results

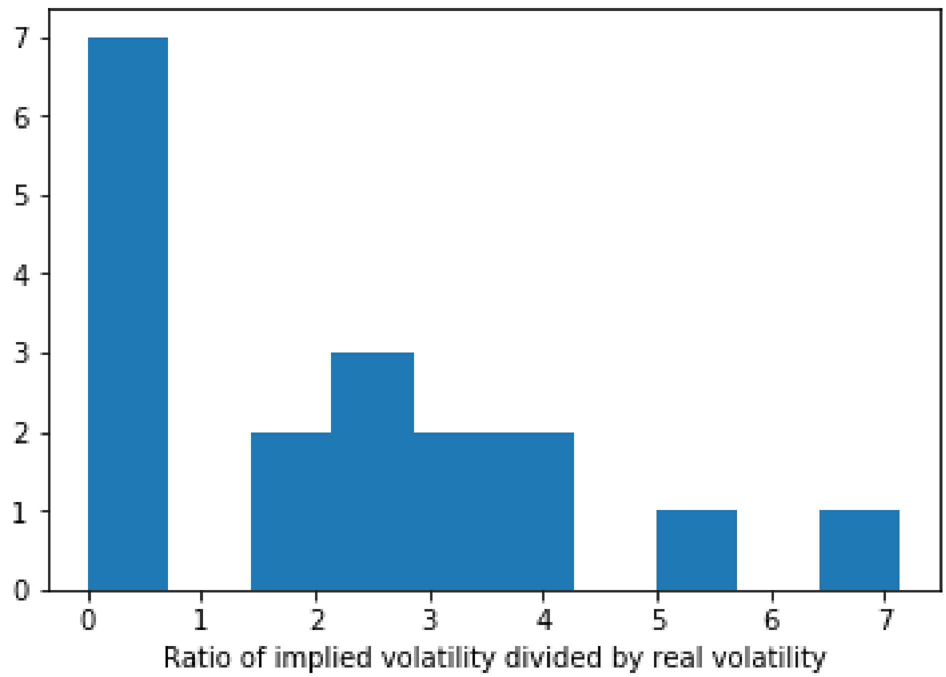
3.0.3 Next calculate the *ratio* of the implied volatility to the realized volatility at 120 days for all option prices in the options data and plot the distribution (don't worry about error trapping, etc.).

[180]:

```
for i in range(len(SPX_put['p_last'])-1):
    S = SPX_put['underlying_last'][5401:5426]
    price = SPX_put['p_last'][5401:5426]
    op_prices.append(price)

    date_ind = HW01_SPX["Date"][5401:5426]
```

```
    real_vol.append((np.std(HW01_SPX["Return"][5280:5400],  ddof=1 ))*np.
↪sqrt(365))
```

[362]:
```
ratio = []
for v in range(len(real_vol[1:19])):
    ratio.append(imp_vol[v]/real_vol[v])
plt.hist(ratio)
plt.xlabel('Ratio of implied volatility divided by real volatility')
plt.show()
```



4 Some data financial data mining

4.1 Now create a new data set for modeling (with lagged features)

1. The data set should contain R_t as well as all of the the original data from the the options data set *except* for`underlying_last_t1`.

2. Next create a new variable, `underlying_last_t1` that contains all of the values of `underlying_last` shifted forward one day.

3. Example: The row of the final data set for the date 2021-06-28, would contain all of the data for 2021-06-28 (except that day's value of `underlying_last`), and also and *tomorrow's* (2021-06-29) value of `underlying_last` which would be stored in the variable `underlying_last_t1`

in the row for 2021-06-28.

[215]:
```
SPX_put.head()
```

[215]:
```
   Unnamed: 0  quote_unixtime    quote_readtime   quote_date  \
0      108508      1624305600  2021-06-21 16:00   2021-06-21
1      116717      1624392000  2021-06-22 16:00   2021-06-22
2      125088      1624478400  2021-06-23 16:00   2021-06-23
3      133233      1624564800  2021-06-24 16:00   2021-06-24
4      141537      1624651200  2021-06-25 16:00   2021-06-25

   quote_time_hours  underlying_last  expire_date  expire_unix  strike  p_bid  \
0                16          4224.66   2021-07-16   1626465600    3685   5.09
1                16          4246.38   2021-07-16   1626465600    3685   3.90
2                16          4241.83   2021-07-16   1626465600    3685   3.40
3                16          4266.39   2021-07-16   1626465600    3685   2.85
4                16          4280.68   2021-07-16   1626465600    3685   2.21

   p_ask       p_size  p_last p_volume  strike_distance  strike_distance_pct  \
0   5.30    923 x 811     0.0                     539.7                0.128
1   4.09   1389 x 100     0.0                     561.4                0.132
2   3.61   886 x 1621     0.0                     556.8                0.131
3   3.00   106 x 1310     0.0                     581.4                0.136
4   2.35    109 x 401     0.0                     595.7                0.139

   underlying_last_t1
0                0.00
1             4224.66
2             4246.38
3             4241.83
4             4266.39
```

[186]:
```
SPX_put["underlying_last_t1"] = SPX_put["underlying_last"].shift(1)
```

[214]:
```
SPX_put['underlying_last_t1'] = SPX_put['underlying_last_t1'].fillna(0)
```

4.2 **Use your new data set: Build a (small!) decision tree classifier to predict whether the options price will rise or fall tomorrow, using any of the variables you like. You may also include the current value of estimated volatility, if you would like. Please show your results and discuss (1-2 paragraphs) your impressions. Does the implied volatility provide any useful power?**

[287]:
```
from sklearn.tree import DecisionTreeClassifier
from sklearn.linear_model          import LogisticRegression# Import Decision␣
 ↪Tree Classifier
from sklearn.model_selection import train_test_split # Import train_test_split␣
 ↪function
from sklearn import metrics
from sklearn import utils
```

[323]:
```
SPX_put.columns
```

[323]:
```
Index(['Unnamed: 0', 'quote_unixtime', 'quote_readtime', 'quote_date',
       'quote_time_hours', 'underlying_last', 'expire_date', 'expire_unix',
       'strike', 'p_bid', 'p_ask', 'p_size', 'p_last', 'p_volume',
       'strike_distance', 'strike_distance_pct', 'underlying_last_t1'],
      dtype='object')
```

[330]:
```
feature_cols = ['underlying_last',
       'strike', 'p_bid', 'p_size', 'p_last',
       'strike_distance', 'strike_distance_pct', 'underlying_last_t1']
X = SPX_put[feature_cols]
y = SPX_put.underlying_last
```

[332]:
```
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.3,␣
 ↪random_state=1)
```

[335]:
```
len(y_train)
```

[335]:
```
13
```

[343]:
```
from sklearn import preprocessing
label_encoder = preprocessing.LabelEncoder()
train_Y = label_encoder.fit_transform(y_train)
# train_X = label_encoder.fit_transform(X_train)
test_Y = label_encoder.fit_transform(y_test)
# test_X = label_encoder.fit_transform(X_test)
```

[344]:
```
# utils.multiclass.type_of_target(X_train)
```

[348]:
```python
clf = DecisionTreeClassifier()
# clf.fit(X_train.values.reshape(-1, 1), y_train)
clf.fit(train_X.reshape(-1, 1), train_Y)
print("Accuracy:", metrics.accuracy_score(test_Y, y_pred))
```

```
Accuracy: 0.16666666666666666
```

I first divided the data into 70 percent training and 30 percent testing. Not a very great decision tree. Implied volatility seems to have not helped in this case. I got an accuracy rate of 0.16666666666666666.

In the future, we could use larger datasets. Since this was a smaller dataset, there could be missing vital innformation which could be helpful to make a better decision tree with high accuracy.

5 Extra Credit* (not required)

5.1 Flexing with a larger data set

If you are feeling ambitious, or you are just intrigued by this problem, you can load the data set: `HW01_SPX_2021_eod.txt` (tab delimited) which contains just under 1MM records, in the same format as the data you have been using (but there is now an extra column `p_iv` which contains the pre-computed implied volatility. In this case, I have only included records for which there was actually volume traded in the option on the date indicated. (Why might this be a bad idea if we were trying to do this for real??) Can you do better than Black-Scholes? How would you test your model?

[521]:
```python
f = open("HW01_SPX_2021_eod.txt", "r")
print(f.readline())
print(f.readline())
```

```
quote_unixtime  quote_readtime  quote_date      quote_time_hours
underlying_last expire_date     expire_unix     strike  p_bid   p_ask   p_size
p_last  p_iv    p_volume        strike_distance strike_distance_pct

1609794000       2021-01-04 16:00        2021-01-04     16      3701.38
2021-02-19      1613768400      100     0       0.04    0 x 296         0
2.83668         3601.4  0.973
```

Implied volatility is just the market's forecast of a likely movement in security's price. It is used to outline future moves and supplies and demands. This provides insight into how the market thinnks about a stock movement. Since this volatility is forcasting, it's important to include records for volumes traded and not traded for possible comparison. It's basically leaving our vital information. In real life, there is so much uncertanity and chaos. Even working with finance math like this, investors don't even know how true the math is. But leaving out information such as no volume traded can cause bias and inaccurate.

Yes, we can do better than black-scholes. Black-scholes assumes that the volaility is constant, while heston model allows stochastic volatility which allows for more flexibility and can perform better with data!

```
[ ]:
```