1.

A probability of bit error for fiber optic cable = $P_{b,vk} = 10^{-12}$.
For DPSK modulation = $P_{b,DPSK} = 1/2\bar{\gamma}$. [1],(6.60)

If $P_{b,vk} = P_{b,DPSK}$, => $10^{-12} = 1/2\bar{\gamma}$ => average SNR required to achieve the same $\bar{\gamma} = 1/(2\times10^{-12}) = 5\times10^{11}$ = 117 dB.

Because of this extremely high required SNR, wireless channels typically have P_b much larger that 10^{-12}.

- Fading coherence time ≈ symbol time ($T_c \approx T_s$)

  => we can assume that $\mathcal{H}$ is roughly constant over a symbol time

- Rayleigh fading

- $\bar{P}_b = \frac{1}{2(1+\bar{\mathcal{H}}_b)} \approx \frac{1}{2\bar{\mathcal{H}}_b}$ if $\bar{\mathcal{H}}_b$ is large

2.

Two-Ray Model [1] is used when a single ground reflection dominates the multipath effect:

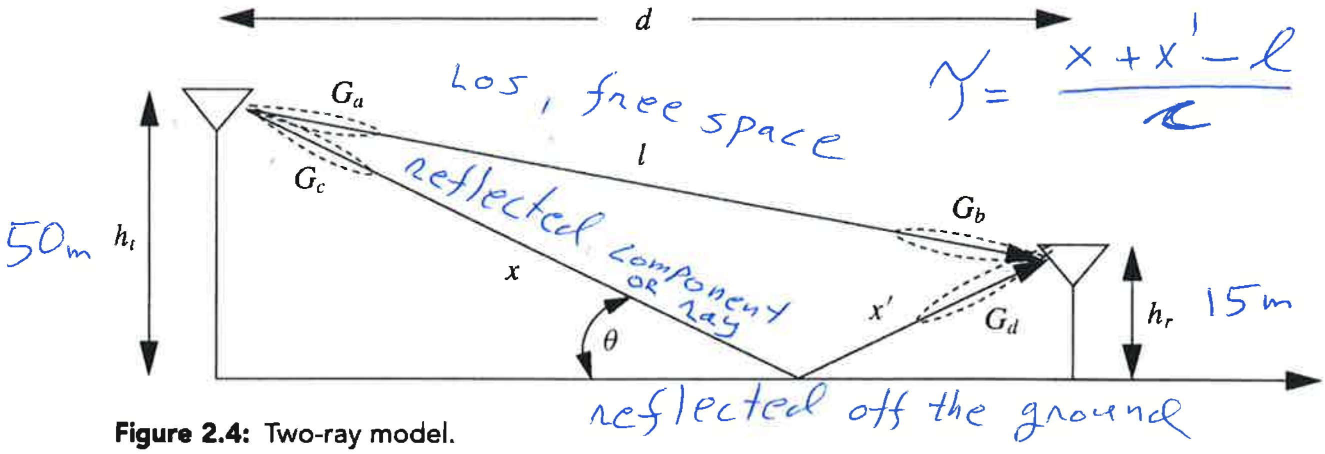


Figure 2.4: Two-ray model.

Matlab (WCI_E1_attachment1) equation [1], (2.12)

$$P_r = P_t \left[\frac{\lambda}{4\pi}\right]^2 \left| \frac{\sqrt{G_l}}{l} + \frac{R\sqrt{G_r}e^{-j\Delta\phi}}{x + x'} \right|^2,$$

· Is true, if the transmitted signal is narrowband relative to the delay spread ($\tau \ll \frac{1}{B_u}$)

$\Rightarrow u(t) \approx u(t-\tau)$

· $\tau \triangleq T_m \ll T_s = \frac{1}{B}$

where

P_r = received power
P_t = transmitted power
λ = wavelength (= c/f_c, where c = the speed of the light and f_c = carrier frequency) = 0,33 m
$G_l = G_a G_b$ = is the product of the transmit and receive antenna field radiation patterns in the LOS direction

$G_r = G_c G_d$ = is the product of the transmit and receive antenna field radiation patterns corresponding to the rays of length x and x', respectively = 1, and so on
R = ground reflection coefficient = –1
$\Delta\phi = 2\pi(x + x' - l)/\lambda$ = is the phase difference between the two received signal components.

Moreover

$$l = \sqrt{(h_t - h_r)^2 + d^2} \quad \text{ja} \quad x + x' = r = \sqrt{(h_t + h_r)^2 + d^2}.$$

$\cdot$ If $d = 1 \Rightarrow l \approx 35,\ x + x' \approx 65 \Rightarrow \Delta\phi = \frac{2\pi(65-35)}{0,333}$
(or small)
$= 2\pi(\cdot 90)$ #

But remember $R = -1$

Lets mark the term after P_t at $K(d)$, and use decibels. We get

Not depending on d

$$10\log_{10} P_r = 10\log_{10} P_t + 10\log_{10} K(d).$$

$\log 1 = 0$ (normalisation)

$\Rightarrow 10\log_{10} P_r(d=1) = 10\log_{10} P_t + 10\log_{10} K(d=1)$,

We want to normalize the plots to start at approximately 0 dB, there for we have to subtract $10\log_{10} P_r(d=1)$ from both side on equation, so

$$10\log_{10} P_r - 10\log_{10} P_r(d=1) =$$
$$\cancel{10\log_{10} P_t} + 10\log_{10} K(d) - \cancel{10\log_{10} P_t} - 10\log_{10} K(d=1)$$
$$\Rightarrow \underbrace{10\log_{10} P_r - 10\log_{10} P_r(d=1)}_{=0\,dB,\ \text{if } d=1} = 10\log_{10} K(d) - 10\log_{10} K(d=1).$$

$10\log_{10} h_t = 10\log_{10} 50$
$= 17$

So we don't need P_t.

From the plots it can be seen that as G_r (gain of reflected path) is decreased, the asymptotic behavior of P_r tends toward d^{-2} from d^{-4}, which makes sense since the effect of reflected path is reduced, and it is more like having only a LOS path.

Also, the variation of power before and around d_c is reduced because the strength of the reflected path decreases as G_r decreases. Also note that the received power actually increases with distance up to some point. This is because for very small distances (i.e. d = 1), the reflected path is approximately two times the LOS path, making the phase difference very small. #
(360° or 0°)

Since R = −1, this causes the two paths to nearly cancel each other out. When the phase difference becomes 180 degrees (= π), the first local maximum is achieved. Additionally, the lengths of both paths are initially dominated by the difference between the antenna heights (which is 35 meters). Thus, the powers of both paths are roughly constant for small values of d, and the dominant factor is the phase difference between the paths.

Final maximum

The critical distance d_c, can be obtained by setting $\Delta\phi = \pi$ in the equation

$$\Delta\phi = \frac{2\pi(x + x' - l)}{\lambda} \approx \frac{4\pi h_t h_r}{\lambda d}, \qquad [1],(2.14)$$

READ [1], p. 36-37

so, we get

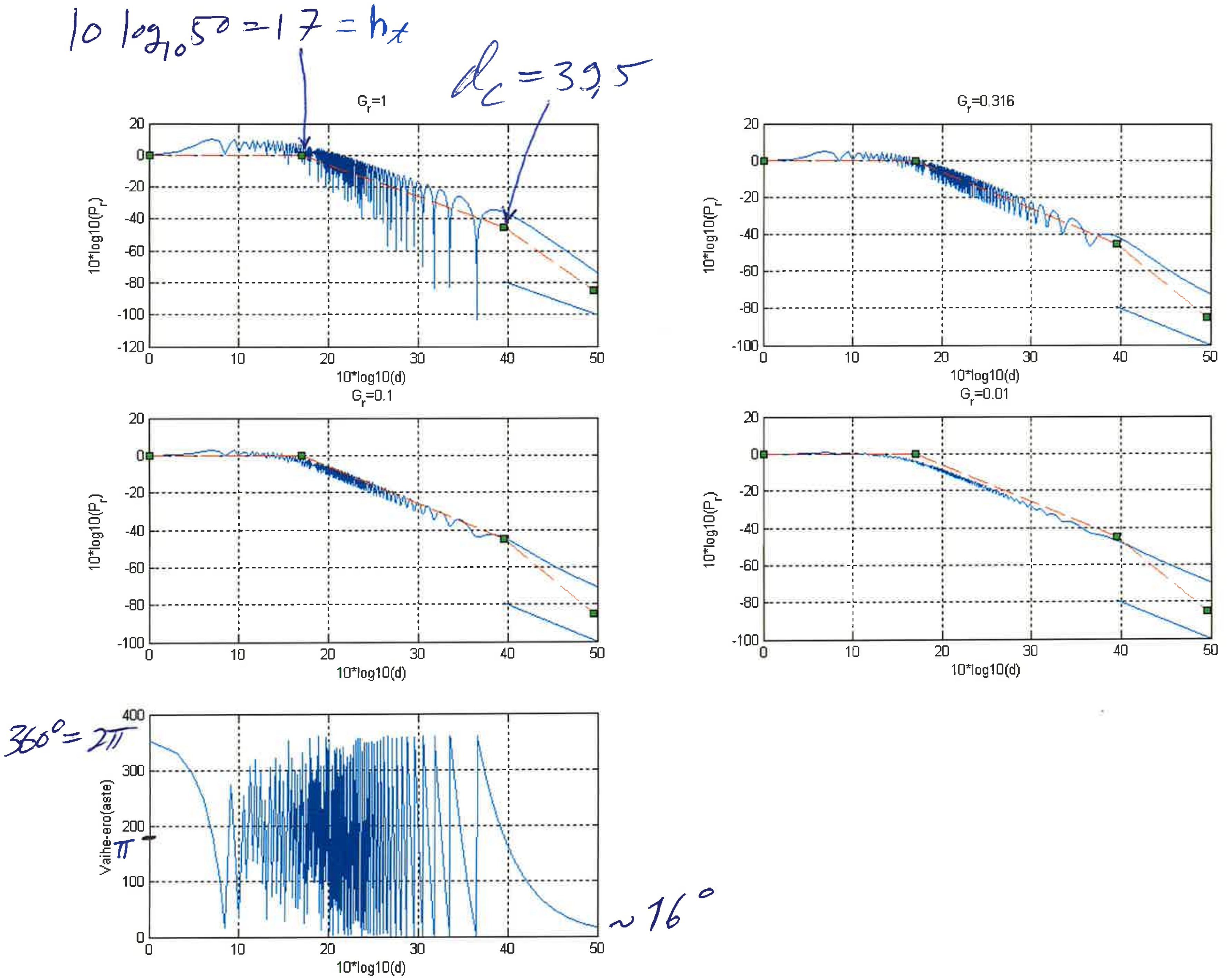
$$d_c = \frac{4h_t h_r}{\lambda} = \frac{4 \cdot 50 \cdot 15}{3 \cdot 10^8 / 900 \cdot 10^6} = 9000\ \text{m} \triangleq 39{,}5.$$

Note that at large distances it becomes increasingly difficult to have $G_r \ll G_l$ since it requires extremely precise angular directivity in the antennas.

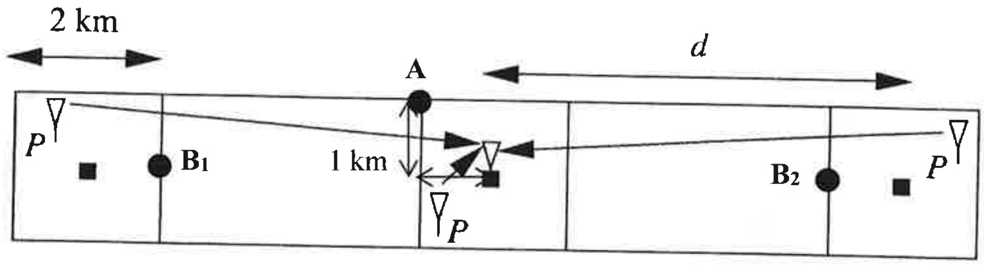
Large d, $x + x' \simeq d \simeq l$, $\theta \simeq 0 \Rightarrow G_l \simeq G_r$

$P_r \simeq \left[\frac{\sqrt{G_l}\, h_t h_r}{d^2}\right]^2 P_t \Rightarrow P_r\,dB_m = P_t\,dB_m + 10\log_{10}(G_l) + 20\log_{10}(h_t h_r) - 40\log_{10}(d)$

- Distances $h_t \to d_c$ wave experiences constructive and destructive interference of the two rays $\Rightarrow$ sequence of maxima and minima [Constructive / Destructive] $\triangleq$ SMALL-SCALE or multipath fading


$10 \log_{10} 50 = 17 = h_t$
$d_c = 39,5$
G_r=1
G_r=0.316
G_r=0.1
G_r=0.01
10*log10(P_r)
10*log10(d)
Vaihe-ero(aste)
$360° = 2\pi$
π
$\sim 16°$

3



■ Base Station/Cell Center

d = reuse distance
P = mobile transmit power (all P_t are equal)
A = master user
B1 = **B2** = interferers (use the same frequency as master user mobile A).

SIR (signal-to-interference power ratio) from mobiles to the base station must be 20 dB $\triangleq 10^{20/10} = 100$ (or greater).

Min. SIR will result when main user is at **A** and Interferers are at **B**

Distance between A and base station is

$d_A = \sqrt{2}$ (km).

Distance between B and base station is

$d_{B_i} = d - 1$ (km), mark as d_B.

a) Propagation for both signal and interference follow a free-space model

$$\frac{P_r}{P_t} = \left[\frac{\sqrt{G}\lambda}{4\pi d}\right]^2. \qquad [1],(2.7)$$

$\sqrt{G}$ is the product of the transmit and receive antenna field radiation patterns in LOS direction. $P_r = P_t\, []^2 = P[]^2$

So we can find $d_{\min}$

$$\mathrm{SIR}_{\min} = \left(\frac{S}{I}\right)_{\min} = 100 = \frac{P\left(\frac{\sqrt{G}\lambda}{4\pi d_A}\right)^2}{2P\left(\frac{\sqrt{G}\lambda}{4\pi d_B}\right)^2} = \frac{d_B^2}{2d_A^2} = \frac{(d_{\min}-1)^2}{4}$$

$$\Rightarrow \quad d_{\min} = 21\,\text{km}.$$

2 interference sources

Since integer number of cells (2 km) should be accommodated in distance, we get $d_{\min}$ = 22 km.

---

b) Propagation for both signal and interference follow the simplified path loss model

$$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma}, \qquad [1],(2.39)$$

$d_0 = \frac{2 d_a^{\,2}}{\lambda}$

d_a is the diameter of the antenna

where

> K is unit less constant that depends on the antenna characteristics and average channel attenuation, and now = 1

d_0 is a reference distance for the antenna far field = 100 m

γ is the path-loss exponent = 3.

So $d_{\min}$ can solve for equation

$$\mathrm{SIR}_{\min} = \left(\frac{S}{I}\right)_{\min} = 100 = \frac{PK\left(\dfrac{d_0}{d_A}\right)^{\gamma}}{2PK\left(\dfrac{d_0}{d_B}\right)^{\gamma}} = \frac{1}{2}\left(\frac{d_B}{d_A}\right)^{\gamma} = \frac{1}{2}\left(\frac{d_{\min}-1}{\sqrt{2}}\right)^{\gamma}$$

$$= \frac{1}{2}\left(\frac{d_{\min}-1}{\sqrt{2}}\right)^{3}$$

$$\Rightarrow \quad d_{\min} = 9{,}27\ \text{km} \quad \Rightarrow \quad d_{\min} = 10\ \text{km} \quad \text{(divisible by 2).}$$

---

c) The same model as b), but now $\gamma_A = 2$ and $\gamma_B = 4$. So $d_{\min}$ is

$$\mathrm{SIR}_{\min} = \left(\frac{S}{I}\right)_{\min} = 100 = \frac{PK\left(\dfrac{d_0}{d_A}\right)^{\gamma_A}}{2PK\left(\dfrac{d_0}{d_B}\right)^{\gamma_B}} = \frac{d_B^{\gamma_B}}{2d_A^{\gamma_A}} d_0^{\gamma_A-\gamma_B}$$

$$= \frac{(d_{\min}-1)^4}{0{,}04}$$

$$\Rightarrow \quad d_{\min} = 2{,}41\ \text{km} \quad \Rightarrow \quad d_{\min} = 4\ \text{km} \quad \text{(divisible by 2).}$$

4 Large scale fading

We define **outage probability** under path loss and shadowing to be the probability that the received power at a given distance d, $P_r(d)$, falls below $P_{\min}$:

$$P_{\text{out}}(P_{\min}, d) = p(P_r(d) < P_{\min}).$$

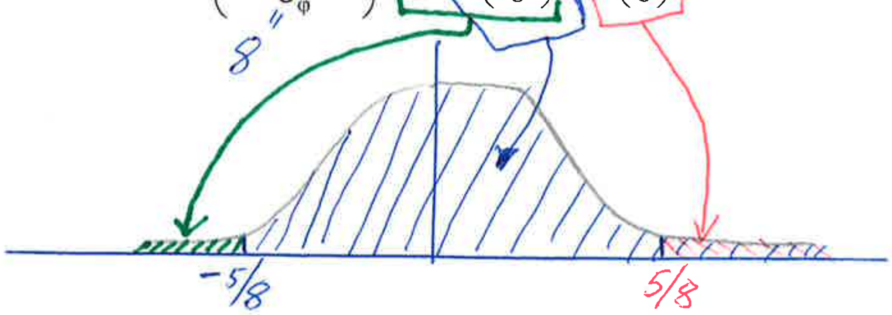
Now $P_{\min}$ = 10 dBm and the mean of the received signal power is μ_φ dBm and standard deviation is σ_φ dBm.

a) Now μ_φ = 15 dBm and σ_φ = 8 dBm. If we take dB-value from log-normal variable, we get Gauss-distributed variable. So we get (compare to [1],(2.52) and B.18–B.20)

$Q(0{,}625) \approx 0{,}26$

$0{,}625 = 5/8$

$$P_{\text{out}} = 1 - Q\left(\frac{\overset{10-15}{P_{\min} - \mu_\varphi}}{\underset{=8}{\sigma_\varphi}}\right) = 1 - Q\left(\frac{-5}{8}\right) = Q\left(\frac{5}{8}\right) = 26\,\%.$$



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b) Now σ_φ = 4 dBm and outage probability must be below 1 %.

$Q(x) = 0{,}01$
$\Rightarrow x = 2{,}33$

So, we can find μ_φ from equation

$$P_{\text{out}} = 1 - Q\left(\frac{P_{\min} - \mu_\varphi}{\sigma_\varphi}\right) \le 0{,}01 \Rightarrow Q\left(\frac{P_{\min} - \mu_\varphi}{\sigma_\varphi}\right) > 0{,}99$$

$$Q\left(\frac{P_{\min}-\mu_{\varphi}}{\sigma_{\varphi}}\right)>99\,\% \quad \Rightarrow \quad \frac{10-\mu_{\varphi}}{4}<-2{,}33 \quad \Rightarrow \quad \mu_{\varphi}>19{,}3\ \text{dBm}\,.$$

---

c) Like in b), but now σ_{φ} = 12 dBm. We get

$$Q\left(\frac{P_{\min}-\mu_{\varphi}}{\sigma_{\varphi}}\right)>99\,\% \quad \Rightarrow \quad \frac{10-\mu_{\varphi}}{12}<-2{,}33 \quad \Rightarrow \quad \mu_{\varphi}>38\ \text{dBm}\,.$$

---

d) For mitigating the effect of shadowing, we can use macroscopic diversity. The idea in macroscopic diversity is to send the message from different base stations to achieve uncorrelated shadowing. In this way the probability of power outage will be less because both base stations are unlikely to experience an outage at the same time, if they are uncorrelated.

5

Wideband channel characterized by the autocorrelation function of channel impulse response $c(\tau,t)$ gives the average output power associated with the channel as a function of the multipath delay $\tau = \tau_1 = \tau_2$ and the difference Δt in observation time.

$$A_c(\tau,\Delta t) = \begin{cases} \operatorname{sin}c(W\Delta t) & 0 \le \tau \le 10\mu s \\ 0 & \text{otherwise,} \end{cases}$$

where $W = 100$ Hz and $\operatorname{sinc}(x) = \sin(\pi x)/(\pi x)$.

In WSSUS channel model (WSS = *Wide-Sense Stationary*, where US = *Uncorrelated Scattering)*

$$A_c(\tau_1,\tau_2;\Delta t) = \mathbf{E}\left[c^*(\tau_1;t)c(\tau_2;t+\Delta t)\right] = A_c(\tau_1;\Delta t)\delta(\tau_1-\tau_2) \triangleq A_c(\tau;\Delta t),$$
[1],(3.52)

The process is stationary if PDF (Probability Density Function) and moments are not depending on the specific time instants t_1 and t_2, but, instead, it depends on the time difference $t_1 - t_2 = \Delta t$.

If the attenuation and phase shift of the channel associated with path delay τ_1 is uncorrelated with the attenuation and phase shift associated with path delay τ_2, then it is called uncorrelated scattering.

a) Delay spread (or Multipath delay spread, or Time delay spread), T_m of the channel is $10\mu s$, so the difference between length of first and last path is

$$d = ct = 3\cdot 10^5 \text{km/s}\cdot 10^{-5}\text{s} = 3\text{ km}.$$

It is outdoor channel.

b) A Fourier transform of the channel autocorrelation function is the channel scattering function, where ρ is Doppler frequency, so

$$S_c(\tau,\rho) = \mathrm{F}_{\Delta t}\left[A_c(\tau,\Delta t)\right] = \int_{-\infty}^{\infty} A_c(\tau,\Delta t) e^{-j2\pi\rho\Delta t} d\Delta t \quad [1],(3.53)$$

The channel scattering function characterizes the average output power associated with the channel as a function of the multipath delay τ and Dopper ρ.

$\operatorname{sinc}(W\Delta t) = \frac{\sin \pi W \Delta t}{\pi W \Delta t}$

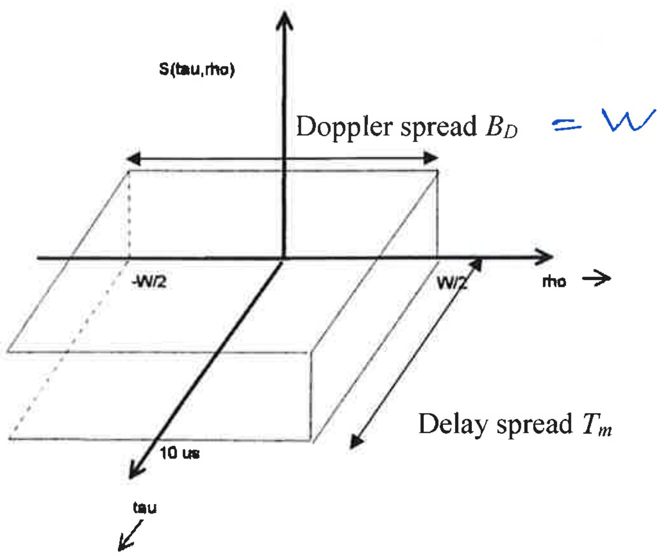
$\sin u = \frac{1}{2j}\left(e^{+ju} - e^{-ju}\right)$

marking $u = \Delta t$

$$\Rightarrow S_c(\tau,\rho) = \int_{-\infty}^{\infty} A_c(\tau,u) e^{-j2\pi\rho u} du = \int_{-\infty}^{\infty} \operatorname{sinc}(Wu) e^{-j2\pi\rho u} du$$

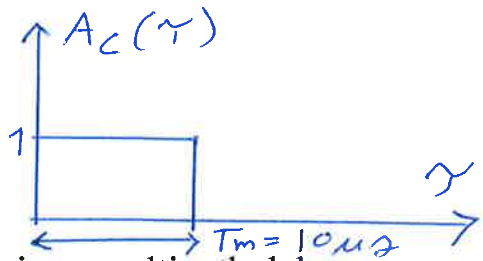
$$= \int_{-\infty}^{\infty} \frac{1}{2j} \frac{e^{j\pi W u} - e^{-j\pi W u}}{\pi W u} e^{-j2\pi\rho u} du$$

$$= \ldots = \frac{1}{W}\operatorname{rect}\left(\frac{\rho}{W}\right), \quad \textit{when } 0 \le \tau \le 10\,\mu\text{s}$$



---

c) Let's define first *delay power spectrum* a.k.a. *power delay profile* a.k.a. *multipath intensity profile)* $A_c(\tau)$, which is find from the autocorrelation function, when $\Delta t = 0$, so in this case



$$A_c(\tau) \triangleq A_c(\tau, \Delta t = 0) = \begin{cases} 1, & \text{when } 0 \le \tau \le 10\ \mu\text{s} \\ 0, & \text{otherwise} \end{cases}$$

Represents the average power associated with a given multipath delay.
since sinc(0) = 1. So we get for channel's average delay spread

$$\mu_{T_m} = \frac{\int_0^\infty \tau A_c(\tau)d\tau}{\int_0^\infty A_c(\tau)d\tau} = \frac{\int_0^{10\mu s} \tau d\tau}{\int_0^{10\mu s} d\tau} = \frac{\Big/_0^{10\mu s} \frac{1}{2}\tau^2}{\Big/_0^{10\mu s} \tau} = \frac{\frac{1}{2}(10\mu s)^2}{10\mu s} \qquad [1],(3.54)$$

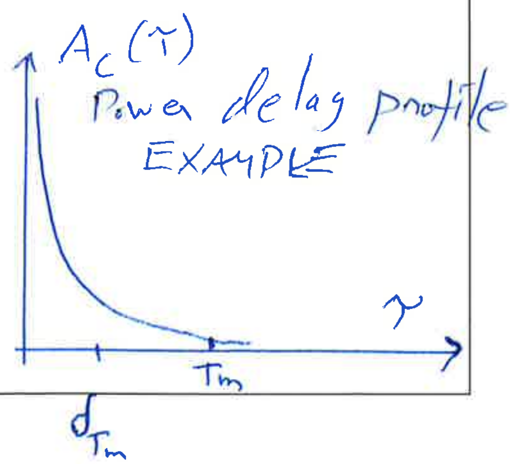
$$= 5\ \mu\text{s}$$

RMS (Root Mean Square) delay spread is

$$\sigma_{T_m} = \sqrt{\frac{\int_0^\infty (\tau - \mu_{T_m})^2 A_c(\tau)d\tau}{\int_0^\infty A_c(\tau)d\tau}} = \sqrt{\frac{\int_0^\infty (\tau-5)^2 d\tau}{10}}\ \mu\text{s} \qquad [1],(3.55)$$

$$= \sqrt{\frac{\frac{1}{3}\Big/_0^{10\mu s}(\tau-5)^3}{10}}\ \mu\text{s} = \sqrt{\tfrac{1}{30}\left[(10-5)^3 - (-5)^3\right]}\ \mu\text{s}$$

$$= \sqrt{\tfrac{1}{15}(5)^3}\ \mu\text{s} = \sqrt{\tfrac{1}{3}\cdot 25}\ \mu\text{s} \approx 2{,}89\ \mu\text{s}.$$



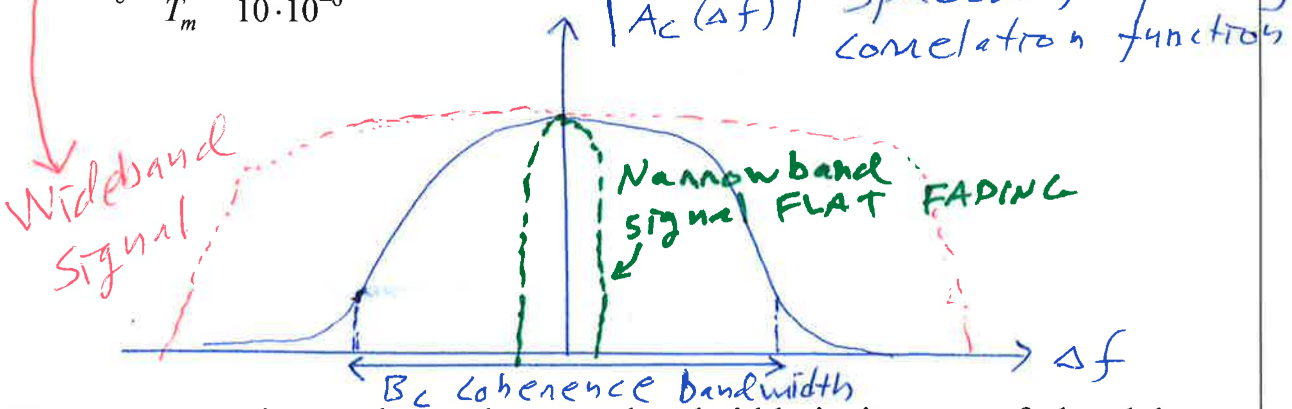
The maximum ρ-value for which $|S_C(\rho)|$ is greater than zero is called the Doppler spread. So

$$B_D = W = 100\text{ Hz}.$$

---

d) If the signal bandwidth (usually same as symbol rate) is greater than coherence bandwidth, then the channel amplitude values at frequencies separated by more than the coherence bandwidth are roughly independent. Thus, the channel amplitude varies widely across the signal bandwidth. In this case the channel or fading is called **frequency-selective**. => ISI

$$B_c \approx \frac{1}{T_m} = \frac{1}{10 \cdot 10^{-6}} = 10^5\text{ Hz} = 100\text{ kHz}.$$



We can approximate that coherence bandwidth is inverse of the delay spread (or average or RMS), so we exhibit frequency selective fading, if symbol rate is over 100 ksymb/s.

[If we are transmitting a narrowband signal with bandwidth $B \ll B_c$, then fading across the entire signal bandwidth is highly correlated => the fading is roughly equal across the entire signal bandwidth => fading is called frequency flat or non-selective or flat fading]

---

e) Rayleigh fading, since receiver power is evenly distributed relative to delay; no dominant LOS path

---

f) Average Fade Duration

$$\bar{t}_Z = \frac{e^{\rho^2} - 1}{\rho f_D \sqrt{2\pi}}, \qquad [1],(3.47)$$

where $\rho = \sqrt{P_0 / \bar{P}_r}$, Signal power level

where P_0 is the target power and $\bar{P}_r$ is average power. The $\rho = 1$. Doppler frequency is f_D = 50 Hz. So, we get

$$\bar{t}_Z = 0{,}0137\,\mathrm{s}$$