Radiative Transfer Code

1. Coordinate System

Consider two reference frames, F_1 and F_2, and a vector $\boldsymbol{v}$ whose components are known in F_1, represented as $\{\boldsymbol{v}\}_1 = (v_{x_1}, v_{y_1}, v_{z_1})^T$. We want to determine the representation of the same vector in F_2, or $\{\boldsymbol{v}\}_2 = (v_{x_2}, v_{y_2}, v_{z_2})^T$.

The rotation sequence used in the Monte-Carlo simulation code is the 321, or $z \to y \to x$. Considering a rotation from F_1 to F_2 the first rotation is about z_1 through an angle ϕ_z which is positive according to the right hand rule about the z_1 axis. With two rotations to go, the resulting alignment in general is oriented with neither F_1 or F_2, but some intermediate reference frame (the first of two) denoted F'. Since the rotation was about z_1, z' is parallel to it but neither of the other two primed axes is. The next rotation is through an angle ϕ_y about the axis y' of the first intermediate reference frame to the second intermediate reference frame, F''. Note that $y'' = y'$, and neither y'' or z'' are necessarily axes of either F_1 or F_2. The final rotation is about x'' through angle ϕ_x and the final alignment is parallel to the axes of F_2.

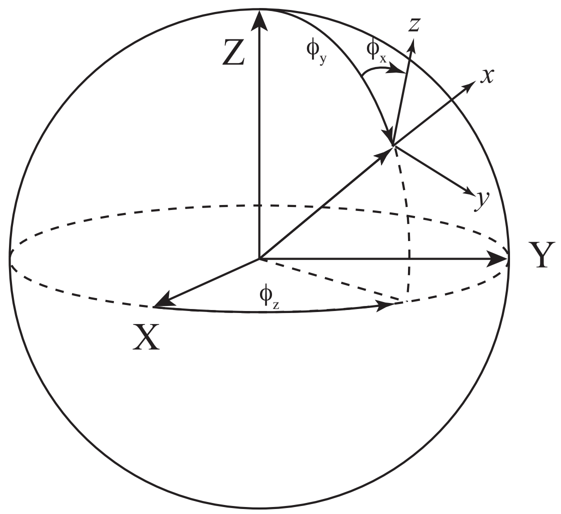

Figure 1: Coordinate system transfrom from (X, Y, Z) to (x, y, z).

Consider first the rotation about z_1, $\{\boldsymbol{v}\}' = T_{F'1}\{\boldsymbol{v}\}_1$ in which

$$T_{F'1} = \begin{pmatrix} \cos\phi_z & \sin\phi_z & 0 \\ -\sin\phi_z & \cos\phi_z & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

From the rotation about y' we get $\{\boldsymbol{v}\}'' = T_{F''F'}\{\boldsymbol{v}\}'$ in which

$$T_{F''F'} = \begin{pmatrix} \cos\phi_y & 0 & -\sin\phi_y \\ 0 & 1 & 0 \\ \sin\phi_y & 0 & \cos\phi_y \end{pmatrix}.$$

From the rotation about x'', finally, $\{\boldsymbol{v}\}_2 = T_{2F''}\{\boldsymbol{v}\}''$ with

$$T_{2F''} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_x & \sin\phi_x \\ 0 & -\sin\phi_x & \cos\phi_x \end{pmatrix}.$$

We now have $\{\boldsymbol{v}\}_2 = T_{21}\{\boldsymbol{v}\}_1 = T_{2F''}T_{F''F'}T_{F'1}\{\boldsymbol{v}\}_1$ in which

$$T_{21} = \begin{pmatrix} \cos\phi_y\cos\phi_z & \cos\phi_y\sin\phi_z & -\sin\phi_y \\ (\sin\phi_x\sin\phi_y\cos\phi_z - \cos\phi_x\sin\phi_z) & (\sin\phi_x\sin\phi_y\sin\phi_z + \cos\phi_x\cos\phi_z) & \sin\phi_x\cos\phi_y \\ (\cos\phi_x\sin\phi_y\cos\phi_z + \sin\phi_x\sin\phi_z) & (\cos\phi_x\sin\phi_y\sin\phi_z - \sin\phi_x\cos\phi_z) & \cos\phi_x\cos\phi_y \end{pmatrix}.$$

The inverse transform of T_{21} is

$$T_{12} = T_{21}^T = \begin{pmatrix} \cos\phi_y\cos\phi_z & (\sin\phi_x\sin\phi_y\cos\phi_z - \cos\phi_x\sin\phi_z) & (\cos\phi_x\sin\phi_y\cos\phi_z + \sin\phi_x\sin\phi_z) \\ \cos\phi_y\sin\phi_z & (\sin\phi_x\sin\phi_y\sin\phi_z + \cos\phi_x\cos\phi_z) & (\cos\phi_x\sin\phi_y\sin\phi_z - \sin\phi_x\cos\phi_z) \\ -\sin\phi_y & \sin\phi_x\cos\phi_y & \cos\phi_x\cos\phi_y \end{pmatrix}.$$

In the code, the coordinate system F_1 is the base coordinate system ((X, Y, Z) in Figure 1) to represent the dust density grid. The coordinate system F_2 is the coordinate system ((x, y, z) in Figure 1) of the observer who measure the photons from the dust cloud. We note here that ϕ_y and ϕ_x are closely related to the inclination angle of the cloud system and position angle, respectively. However, they differ from the conventionally-defined inclination and position angles by $\pi/2$. In most cases, only the rotation angle about y axis ϕ_y may be needed to vary.

The coordinate of the observer at a distance d, rotated by (ϕ_x, ϕ_y, ϕ_z) angles, is $(d, 0, 0)$ in the observer's coordinate system. Then the observer's coordinates in the dust cloud system are give by

$$\begin{aligned} x_{\rm obs} &= d\cos\phi_y\cos\phi_z \\ y_{\rm obs} &= d\cos\phi_y\sin\phi_z \\ z_{\rm obs} &= -d\sin\phi_y \end{aligned}$$

The detector plane coincides with the y_2z_2 plane of the observer's coordinate system. If the coordinates of the observer and photon are $(x_{\rm obs}, y_{\rm obs}, z_{\rm obs})$ and $(x_{\rm p}, y_{\rm p}, z_{\rm p})$, respectively, in the dust cloud system, the photon direction vector to the observer is $\{\boldsymbol{v}\}_1 = (x_{\rm obs} - x_{\rm p}, y_{\rm obs} - y_{\rm p}, z_{\rm obs} - z_{\rm p})^T/|\boldsymbol{v}|$ in the dust cloud system. Here, the normalization factor $|\boldsymbol{v}| = \left[(x_{\rm obs} - x_{\rm p})^2 + (y_{\rm obs} - y_{\rm p})^2 + (z_{\rm obs} - z_{\rm p})^2\right]^{1/2}$. The angular coordinates (α, δ) of the photon direction toward the observer in the y_2z_2 plane are

$$\begin{aligned} \alpha &= \mathrm{atan2}(-v_{y_2}, v_{x_2}), \\ \delta &= \mathrm{atan2}(-v_{z_2}, v_{x_2}). \end{aligned}$$

The array indices on the image plane with angular bins $(\Delta\alpha, \Delta\delta)$ are then given

$$\begin{aligned} i &= \mathrm{int}\left(\frac{\alpha}{\Delta\alpha} + \frac{N_i - 1}{2}\right), \\ j &= \mathrm{int}\left(\frac{\delta}{\Delta\delta} + \frac{N_j - 1}{2}\right). \end{aligned}$$