Astrophysics

Lecture 16
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[Interstellar Dust]

- Silicates
  - The two main types of silicates in dust are pyroxene and olivine.

olivine	$Mg_{2x}Fe_{2-2x}SiO_4$	forsterite – Mg_2SiO_4	fayalite – Fe_2SiO_4
pyroxene	$Mg_xFe_{1-x}SiO_3$ $(x = 0 - 1)$	enstatite – $MgSiO_3$	ferrosilite – $FeSiO_3$

[Left] Olivine is the simplest silicate structure, which is composed of isolated tetrahedra bonded to iron and/or magnesium ions. No oxygen atom is shared to two tetrahedra.

[Middle] In pyroxene, silica tetrahedra are linked together in a single chain, where one oxygen ion from each tetrahedra is shared with the adjacent tetrahedron.

[Right] Other types are possible. In amphibole structures, two oxygen ions from each tetrahedra are shared with the adjacent tetrahedra.

In mica structures, the tetrahedra are arranged in continuous sheets, where each tetrahedron shares three oxygens with adjacent tetrahedra.

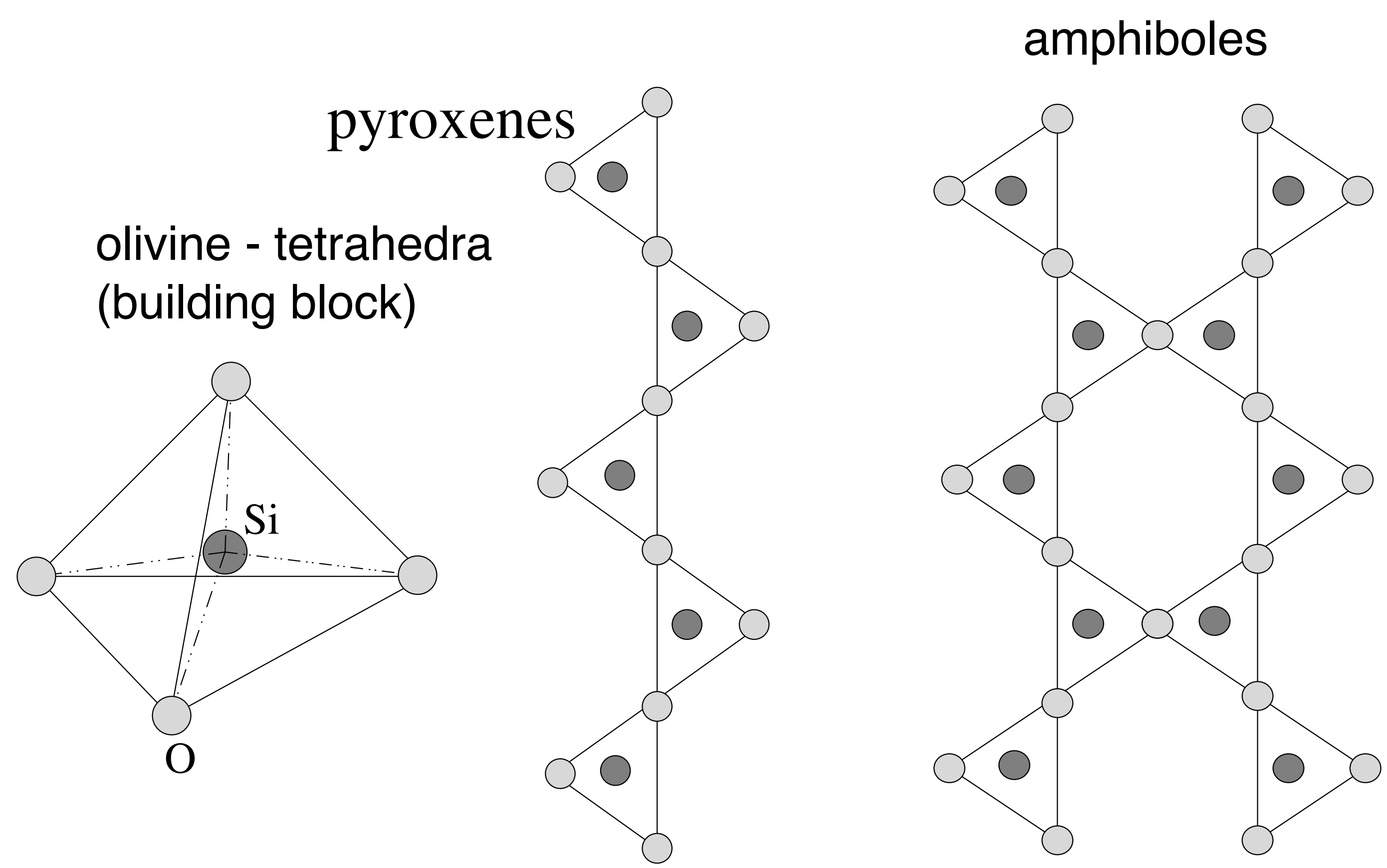
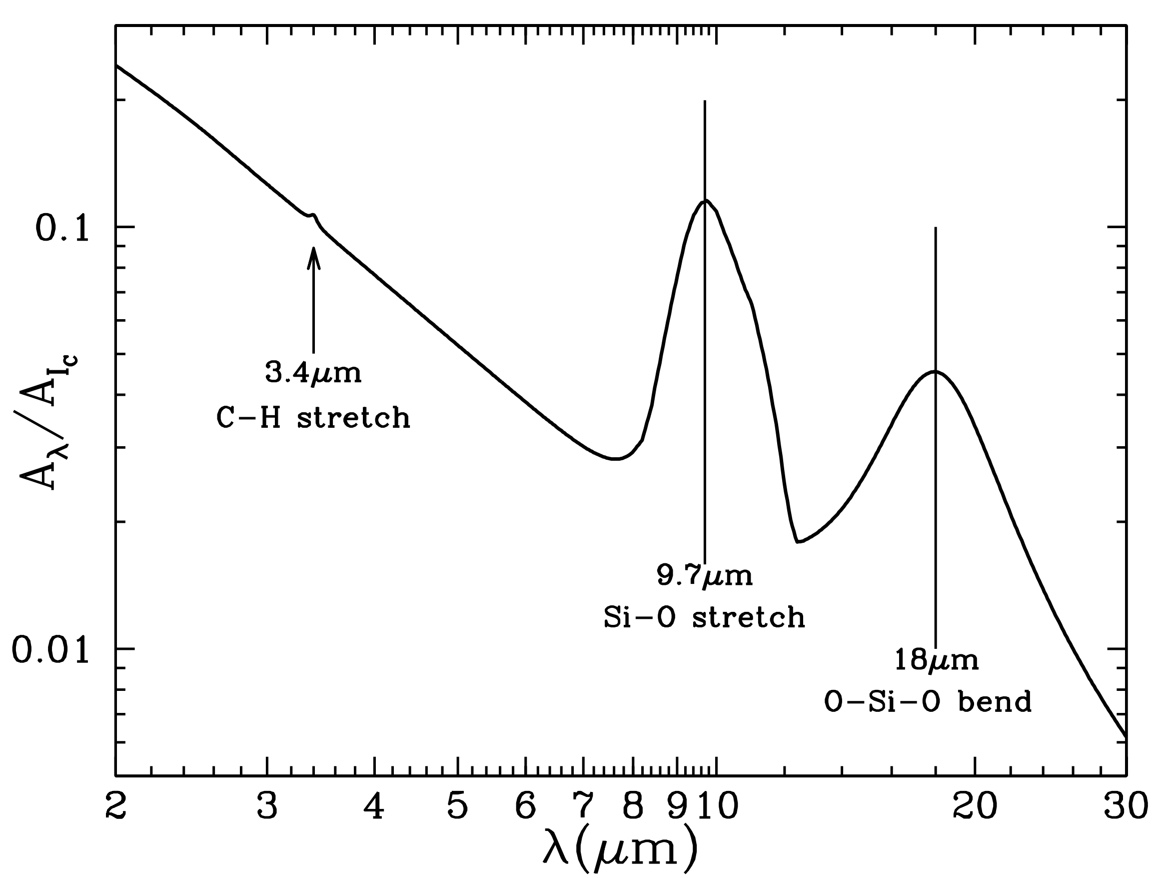


Fig 5.9 Krugel
[An Introduction to the Physics of Interstellar Dust]

Silicate Features

- Mid-Infrared Silicate Features:
  - There is a conspicuous IR absorption feature at 9.7µm. Silicate minerals generally have strong absorption responses due to the Si-O stretching mode near 10µm.
  - It seems virtually certain that the interstellar 9.7µm feature is due to silicates. This conclusion is strengthened by the fact that the 10µm emission feature is seen in the outflows from oxygen-rich stars (which would be expected to condense silicate dust) but not in the outflows from carbon-rich stars.
  - Near 18µm, interstellar dust shows another feature, attributable to the Si-O-Si bending mode in amorphous silicates.



The fact that the 9.7µm band is fairly featureless, unlike what is seen in laboratory silicate crystals, suggests that this “astrophysical” silicate is primarily amorphous rather than crystalline in nature.

IR extinction curve.
[Fig 23.2 Draine]

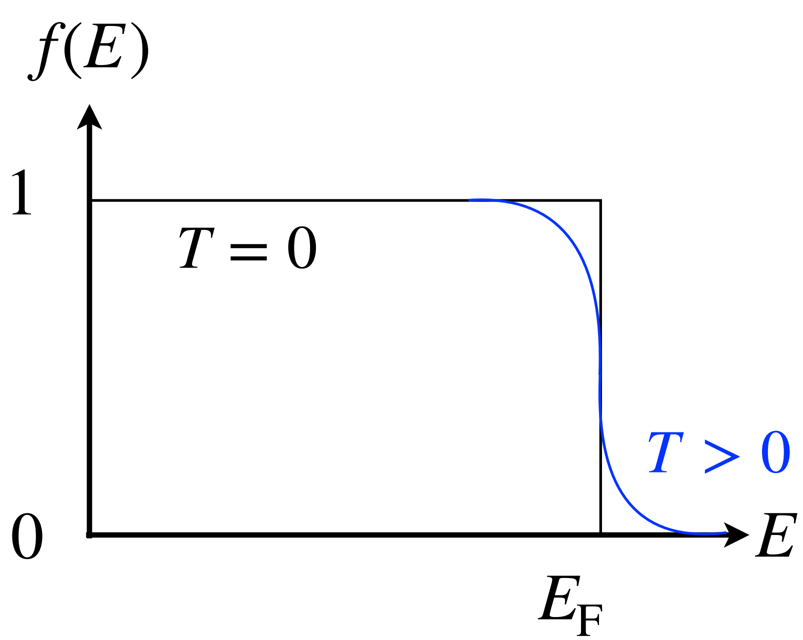
[The Fermi Level]

- The probability distribution of identical fermions, such as electrons in a solid, over the energy states of a system at temperature T, is given by the **Fermi-Dirac distribution**:

$$f(E) = \frac{1}{\exp\left(\frac{E-E_\mathrm{F}}{k_\mathrm{B}T}\right)+1}$$

Here E_F is the Fermi level, which is the energy required to add an electron to the system and an intrinsic quantity characterizing a solid.

At $T = 0$ K, it gives a probability to electrons to occupy energy levels $E \leq E_\mathrm{F}$, whereas zero probability to energy levels $E > E_\mathrm{F}$

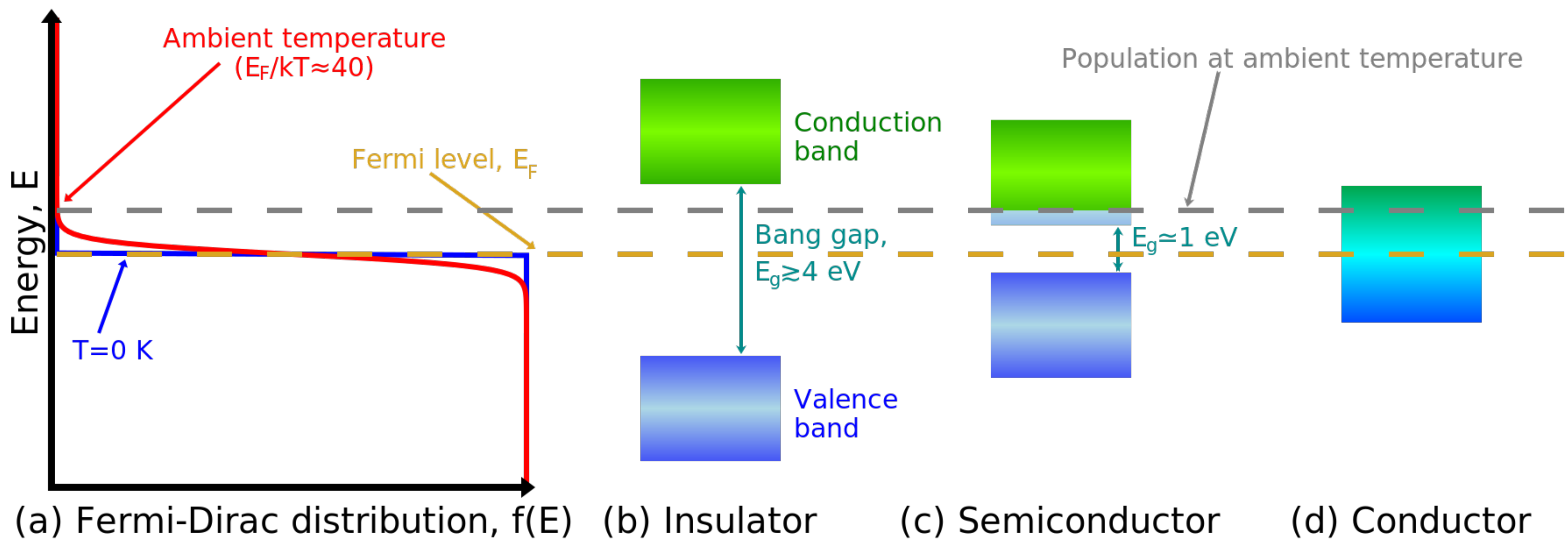


at $T = 0$ K

$$f(E > E_\mathrm{F}) = \frac{1}{\exp(+\infty)+1} = 0$$

$$f(E < E_\mathrm{F}) = \frac{1}{\exp(-\infty)+1} = 1$$

[Types of Solid]



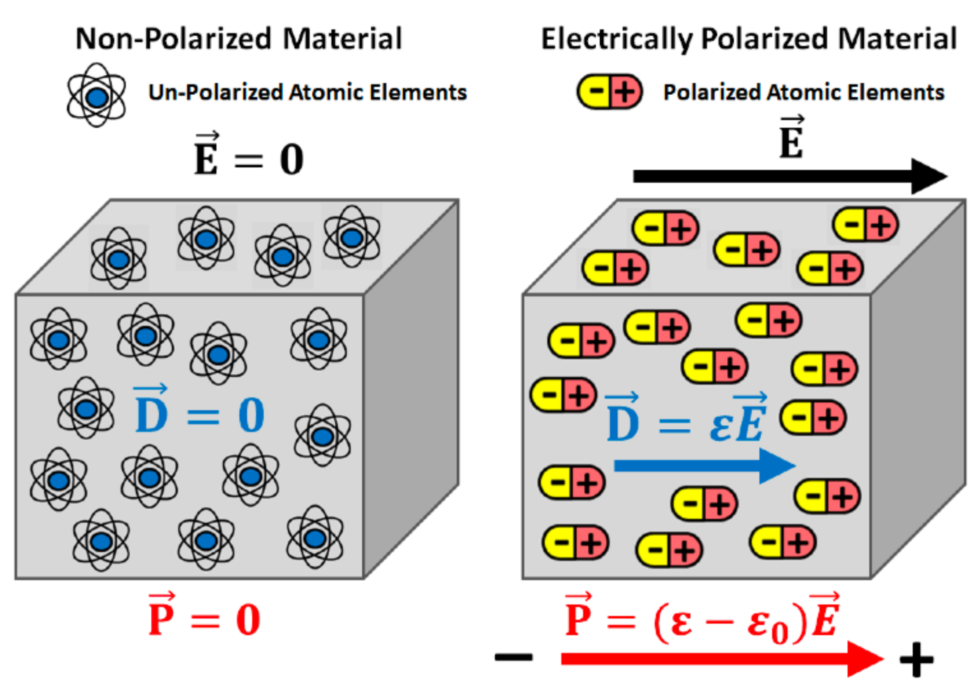
- **Insulators** have their valence and conduction bands widely spread apart. At ambient temperature, no electron will populate the conduction band.
- **Semiconductors** have their valence and conduction bands close to each other. They are insulators at $T = 0$ K, but their conduction band can be populated at ambient temperature (kT gets close to E_F).
- **Conductors** are solids for which have valence and conduction bands are the same. The Fermi level is within the band. In other words, the valence electrons are free to move through the lattice at any temperature.

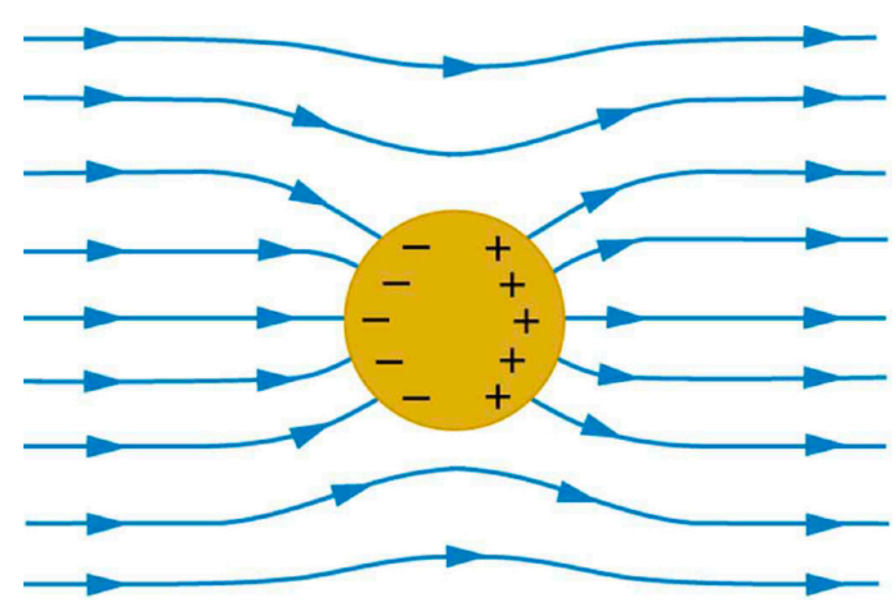
The Fermi level and the different types of solid.

In the figure, the left plot shows the rotated Fermi-Dirac distribution, for two values of the temperature, $T = 0$ K and $T \simeq 300$ K.
The three diagrams on the right show the valence and conduction bands relative to the Fermi level, E_F, for insulators, semiconductors and conductors. For conductors, the valence band is also the conduction band.

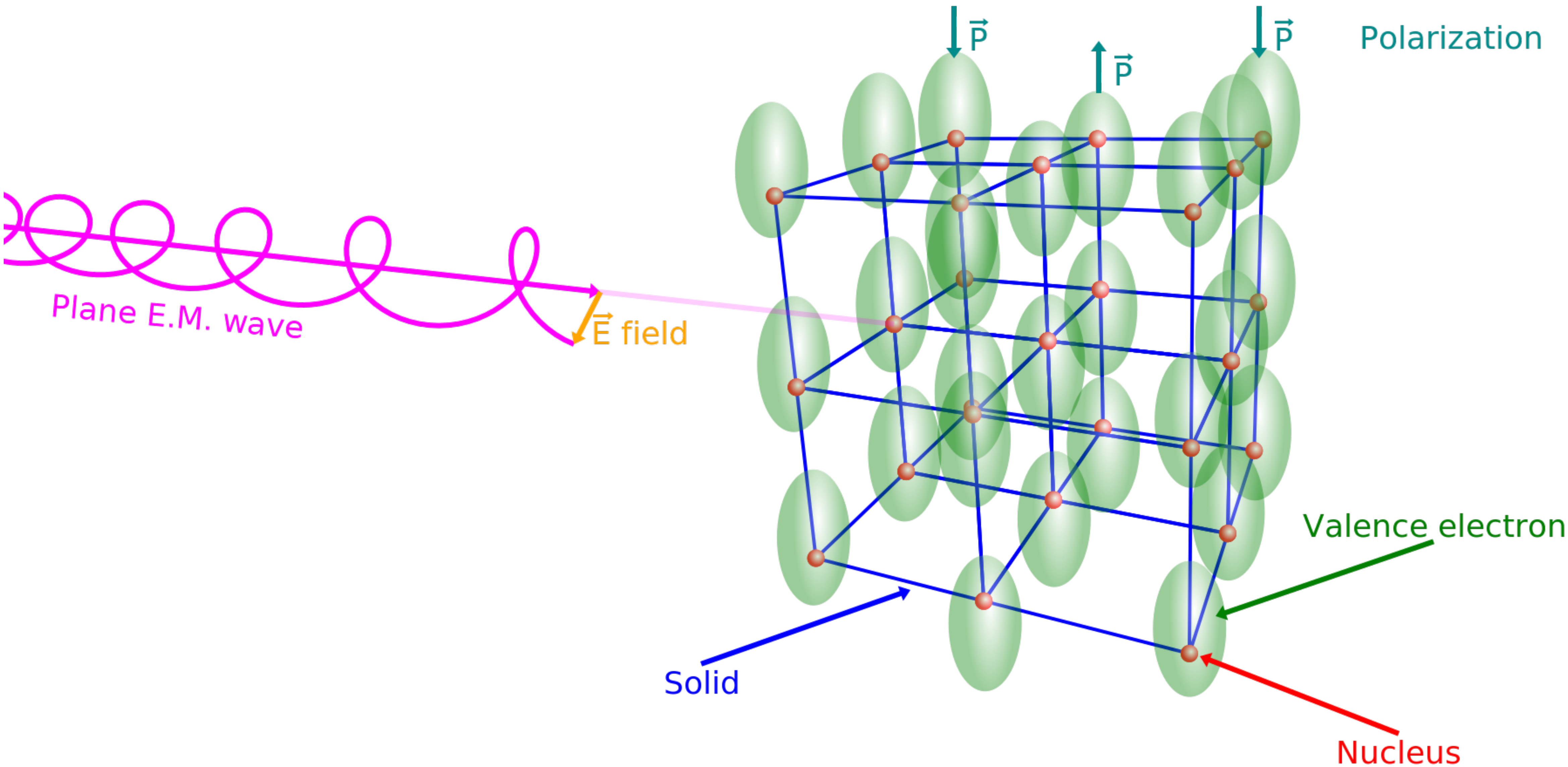
[Dielectrics and Conductors]

- ***Dielectrics (Insulators)***: Dielectrics are substances which do not contain free charge carriers. They are isolators and no constant current can be sustained within them. Nevertheless, alternating currents produced by a time-variable electric field are possible. In these currents, the charges do no travel far from their equilibrium positions.
- ***Conductors (Metals)***: The substances having free charge carriers are called the conductors. When a piece of metal is connected at its ends to the poles of a battery, a steady current flows under the influence of an electric field. When this piece of metal is placed in a static electric field, the charges accumulate at its surface and arrange themselves in such a way that the electric field inside vanishes and then there is no internal current. However, time-varying electric fields and currents are possible.
- In the interstellar medium, one finds both dielectric and metallic particles, but the latter are far from being perfect conductors.





[credit: Frédéric Galliano]

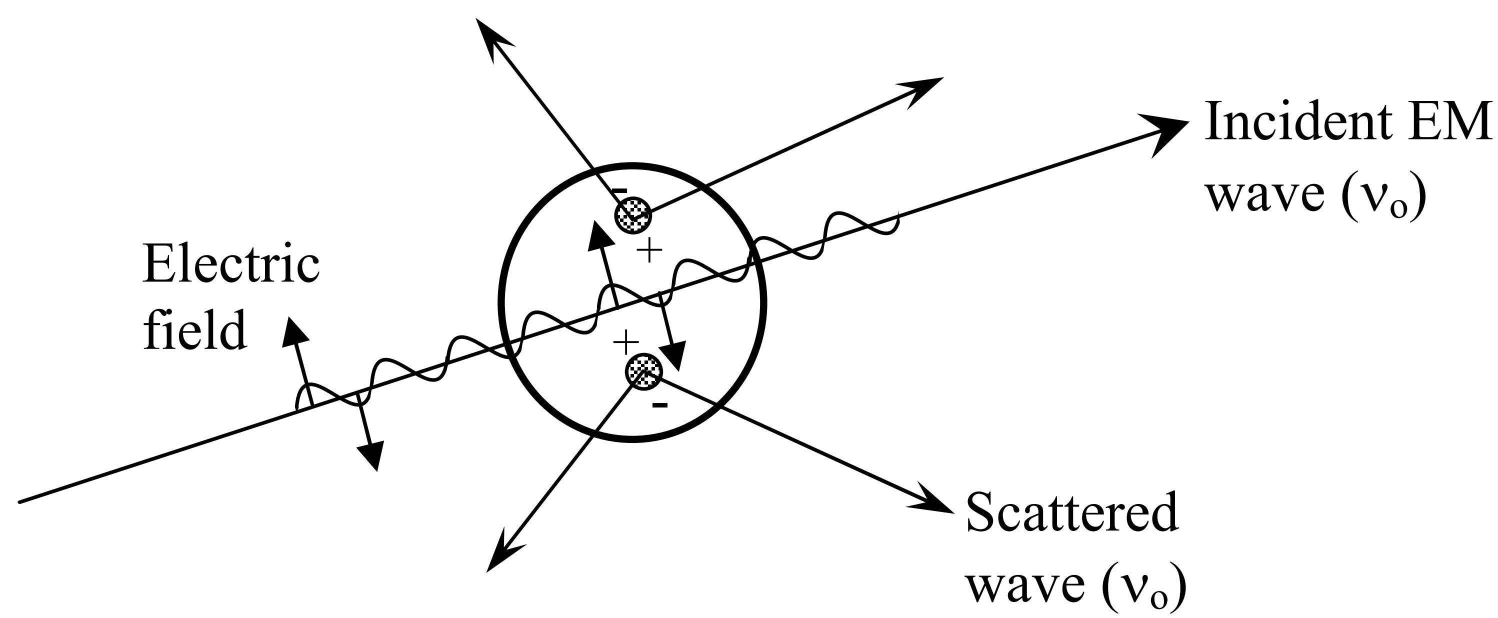


Effect of an electromagnetic wave on a dielectric. An incoming, circularly polarized, electromagnetic wave is figured in magenta. The cube on the right represents a solid. The nuclei, assumed to be fixed, are the the red spheres. The valence electrons are the green ellipsoids. They are displaced out of their equilibrium positions by the electromagnetic wave, inducing a time-dependent polarization.

Optical Properties of Grains

- Physical Basis for Scattering and Absorption
  - If an obstacle (which could be a single electron, an atom or molecule, a solid or liquid particle) is illuminated by an electromagnetic wave, electric charges in the obstacle are set into oscillatory motion with ***the same frequency*** as the electric field of the incident wave.
  - We consider the dielectric material to be made up of an infinite number of infinitely small electric and magnetic dipoles whose dipole strengths are proportional to the imposed field strengths. The induced dipoles create their own field or wave in return. The dust particles emits its own field or waves in reaction to the imposed field of waves.

scattering =
excitation + reradiating



  - Accelerated electric charges radiate electromagnetic energy in all directions; it is this secondary radiation that is called ***the radiation scattered*** by the obstacle:

- In addition to reradiating electromagnetic energy, the excited elementary charges may transform part of the incident electromagnetic energy into other forms (thermal energy, for example), a process called ***absorption***.

    - ***Rayleigh scattering*** (Lord Rayleigh), applicable to small, dielectric (non-absorbing), spherical particles. ==> simple

$$|m|\frac{2\pi a}{\lambda} \ll 1 \quad (m = \text{the refractive index}, \; a = \text{radius of the spherical particle})$$

    - ***Mie scattering*** (Gustave Mie), the general solution for (absorbing or non-absorbing) spherical particles without a particular bound on particle size. ==> complex
    - Geometric optics regime: The particle is much larger than the wavelength, so that it can be regarded in the geometric optics regime. This does not mean that its scattering is simple. Reflection on the surface and refraction in the interior can still be quite complex (e.g., light passing through a rain drop), but it can be calculated using ray-tracing through the particle and off the particle's surface.

[Dust Theory: cross section and efficiency factors]

- ***Cross Sections***:
  - A dust grain has wavelength-dependent cross sections for absorption and scattering. Extinction is the sum of absorption and scattering processes.

$$C_{\rm ext}(\lambda) = C_{\rm abs}(\lambda) + C_{\rm sca}(\lambda)$$

  - For a population of dust grains with number density $n_{\rm d}$, the extinction cross section is related to the extinction coefficient and the dust optical depth by:

$$\begin{aligned} \kappa_\lambda &= n_{\rm d} C_{\rm ext}(\lambda) \\ \tau_\lambda &= n_{\rm d} C_{\rm ext}(\lambda) L \qquad L = \text{pathlength} \\ &= 1.086 A_\lambda \end{aligned}$$

- ***Efficiency Factors***:
  - The cross section is often expressed in terms of efficiency factors, normalized to the geometric cross section of an equal-solid-volume sphere:

$$Q_{\rm ext}(\lambda) = \frac{C_{\rm ext}(\lambda)}{\pi a^2}, \quad Q_{\rm abs}(\lambda) = \frac{C_{\rm abs}(\lambda)}{\pi a^2}, \quad Q_{\rm sca}(\lambda) = \frac{C_{\rm sca}(\lambda)}{\pi a^2}$$

$$V = \frac{4\pi}{3} a^3 \qquad a = \text{the radius of an equal-volume sphere}$$

- Albedo and Scattering phase function
  - The ***albedo*** is defined by

$$\omega(\lambda) = \frac{C_{\rm sca}(\lambda)}{C_{\rm ext}(\lambda)}$$

  - Scattering is a function of the scattering angle and thus expressed in terms of the differential scattering cross section:

$$C_{\rm sca}(\lambda) = \int_0^{2\pi}\int_0^{\pi} \frac{d\sigma_{\rm sca}(\theta,\phi;\lambda)}{d\Omega}\sin\theta d\theta d\phi$$

  - The ***scattering asymmetry factor*** is defined by:

$$\mathrm{g} \equiv \langle\cos\theta\rangle = \frac{1}{\sigma_{\rm sca}}\int_0^{2\pi}\int_0^{\pi}\cos\theta\frac{d\sigma_{\rm sca}}{d\Omega}\sin\theta d\theta d\phi$$

  - The scattering phase function can be described by the Rayleigh function (long wavelengths) or Henyey-Greenstein function (in short wavelengths):

$$\mathcal{P}(\theta) \equiv \frac{1}{\sigma_{\rm sca}}\int_0^{\pi}\frac{d\sigma_{\rm sca}}{d\Omega}d\phi \longrightarrow$$

$$\mathcal{P}_{\rm Ray}(\theta) = \frac{1}{2}\left(1+\cos^2\theta\right) \quad \text{for } \frac{2\pi a}{\lambda} \ll 1 \longrightarrow \langle\cos\theta\rangle = 0$$

$$\mathcal{P}_{\rm HG}(\theta) = \frac{1}{2}\frac{1-\mathrm{g}^2}{\left(1+\mathrm{g}^2-2\mathrm{g}\cos\theta\right)^{3/2}} \quad \text{for } \frac{2\pi a}{\lambda} \gg 1 \longrightarrow \langle\cos\theta\rangle = \mathrm{g}$$

[How to calculate the cross-sections] - Maxwell's equations

- **Maxwell's eqs.** (in macroscopic forms) relates fields to charge and current densities.

$\nabla \cdot \mathbf{D} = 4\pi\rho_f$	Gauss's law
$\nabla \cdot \mathbf{B} = 0$	Gauss's law for magnetism (no magnetic monopoles)
$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$	Maxwell-Faraday equation
$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J}_f + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$	Ampere-Maxwell equation

Here, f denotes the free charge or free current.

$\mathbf{D}$,$\mathbf{H}$: macroscopic fields

$\mathbf{B}$,$\mathbf{E}$: microscopic fields

$\mathbf{D} = \epsilon \mathbf{E}$

$\mathbf{B} = \mu \mathbf{H}$

ϵ : dielectric constant

μ : magnetic permeability

Dielectric material (절연체): an electrical insulator that can be polarized by an applied electric field. Electric charges do not flow through the material as they do in a conductor, but only slightly shift from their average equilibrium positions causing dielectric polarization.

Permeability (투자율): the degree of magnetization of a material in response to a magnetic field.

Note $\epsilon = \mu = 1$ in the absence of dielectric or permeability media.

D and **E** / **H** and **B**

- Griffiths, Introduction to Electrodynamics, 3rd

- **D** allows us to write Gauss's law in terms of the free charge alone.

  The electric displacement provide a particularly useful way to express Gauss's law, in the context of dielectrics, because it makes reference only to free charges, and free charge is the stuff we control. Bound charge comes along for the ride: when we put the free charge in place, a certain polarization automatically ensues, and this polarization produces the bound charge.

- **H** plays a role in magnetostatics analogous to **D** in electrostatics:

  H permits us to express Ampere's law in terms of the free current alone - and free current is what we control directly. Bound current, like bound charge, comes along for the ride - the material gets magnetized, and this results in bound currents; we cannot turn term on or off independently, as we can free currents.

  Many authors call **H**, not **B**, the "magnetic field." Then they have to invent a new word for **B**: the "flux density," or magnetic "induction" (an absurd choice, since that term already has at least two other meanings in electrodynamics). Anyway, **B** is indisputably the fundamental quantity, so it would better to call it the "magnetic field," as everyone does in the spoken language. **H** has no sensible name: just call it "**H**".

Waves in a medium

In order to calculate scattering and absorption of electromagnetic waves by dust grains, we need to characterize the response of the target material to the local oscillating electric fields.

$\mathbf{D}$ electric displacement

$\mathbf{E}$ electric field

$\mathbf{B}$ magnetic flux density (magnetic induction)

$\mathbf{H}$ magnetic field strength (magnetic field)

Macroscopic Maxwell's equations

Gauss
$$\nabla \cdot \mathbf{D} = 4\pi\rho_f$$
$$\nabla \cdot \mathbf{B} = 0$$

Faraday
$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

Ampere
$$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J}_f + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$$

Constitutive Relations

$$\mathbf{D} = \epsilon \mathbf{E}$$
$$\mathbf{B} = \mu \mathbf{H}$$
$$\mathbf{J} = \sigma \mathbf{E}$$

$\epsilon =$ permitivity (유전율)

$\mu =$ permeability (투자율)

$\sigma =$ conductivity (전도율)

Continuity Equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0$$

$$\nabla \cdot \left(\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} \right)$$
$$0 = 4\pi \nabla \cdot \mathbf{J} + \frac{\partial}{\partial t}\nabla \cdot \mathbf{D}$$
$$0 = \nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t}$$

Assume a space and time variation of all quantities of the form $\exp\left[i\left(\mathbf{k}\cdot\mathbf{r} - \omega t\right)\right]$

$$i\mathbf{k} \cdot \mathbf{D} = 4\pi\rho$$
$$i\mathbf{k} \cdot \mathbf{B} = 0$$
$$i\mathbf{k} \times \mathbf{E} = i\frac{\omega}{c}\mathbf{B}$$
$$i\mathbf{k} \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J} - i\frac{\omega}{c}\mathbf{D}$$
$$-i\omega\rho + i\mathbf{k}\cdot\mathbf{J} = 0$$

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) = \frac{\omega}{c}\mathbf{k} \times \mathbf{B}$$
$$\mathbf{k}(\mathbf{k}\cdot\mathbf{E}) - \mathbf{E}(\mathbf{k}\cdot\mathbf{k}) = \frac{\omega\mu}{c}\mathbf{k} \times \mathbf{H}$$
$$\boxed{k^2\mathbf{E} = \frac{\omega^2\epsilon\mu}{c^2}\left(1 + i\frac{4\pi\sigma}{\omega\epsilon}\right)\mathbf{E}}$$

$$\mathbf{k} \times \mathbf{H} = -i\frac{4\pi}{c}\mathbf{J} - \frac{\omega}{c}\mathbf{D}$$
$$= -\frac{\omega\epsilon}{c}\left(1 + i\frac{4\pi\sigma}{\omega\epsilon}\right)\mathbf{E}$$

Dispersion Relation

$$k^2 = \frac{\omega^2}{c^2}m^2$$
$$m^2 = \mu\left(\epsilon + i\frac{4\pi\sigma}{\omega}\right)$$

m **= complex index of refraction**

m is sometimes referred to as the **optical constants**, or simply the "n and k".

- $\mathbf{k}$ is property of the wave, however, $\epsilon\mu$ is a property of the medium. **Here, we will ignore the magnetic field, i.e., $\mu = 1$ and consider a dielectric material (σ = 0).**
  - These enter into the theory through the ***complex index of refraction***, $m = n_r + in_i$, where the real and imaginary part are functions of the wavelength.
  - Alternatively, the optical properties of a material can be expressed in terms of the ***dielectric function*** (or dielectric constant) $\epsilon = \epsilon_1 + i\epsilon_2$. The dielectric function and the complex index of refraction are related through.

$$m = n_r + in_i \quad (\text{or } m = n + ik)$$

$\epsilon = \epsilon_1 + i\epsilon_2$	$\longrightarrow$	$\epsilon_1 = n_r^2 - n_i^2$
$\epsilon = m^2$		$\epsilon_2 = 2n_r n_i$

For a conductor, the electrical conductivity σ, if any, can be absorbed within the imaginary part of the dielectric function.

$$\mathbf{J} = \sigma\mathbf{E} \qquad \epsilon \to \epsilon + \frac{4\pi i\sigma}{\omega}$$

  - The refractive index is often referred to as optical constants, or simply the "*n* and *k*".
- Consider a plane wave traveling in the z direction represented by

$$E = E_0 \exp\left[i(kz - \omega t)\right]$$

  - In free space, the wave vector is given by

$$k = \omega/c = 2\pi/\lambda \ (\lambda = \text{wavelength in vacuum})$$

- In a material with the index of refraction m, the wave vector is:

$$k = m\omega / c$$

The electric field becomes:

$$E = E_0 \exp\left(-\frac{n_i \omega}{c} z\right) \exp\left[-i\omega\left(t - \frac{n_r z}{c}\right)\right]$$

Thus, ***the real part of the index of refraction introduces a phase shift*** while the ***imaginary part results in damping.*** The power of electromagnetic wave will decrease as it propagates through the material, with

$$|E|^2 \propto e^{-2n_i \omega z / c}$$

The attenuation coefficient will be

$$\kappa = 2n_i \frac{\omega}{c} = \frac{4\pi n_i}{\lambda}$$

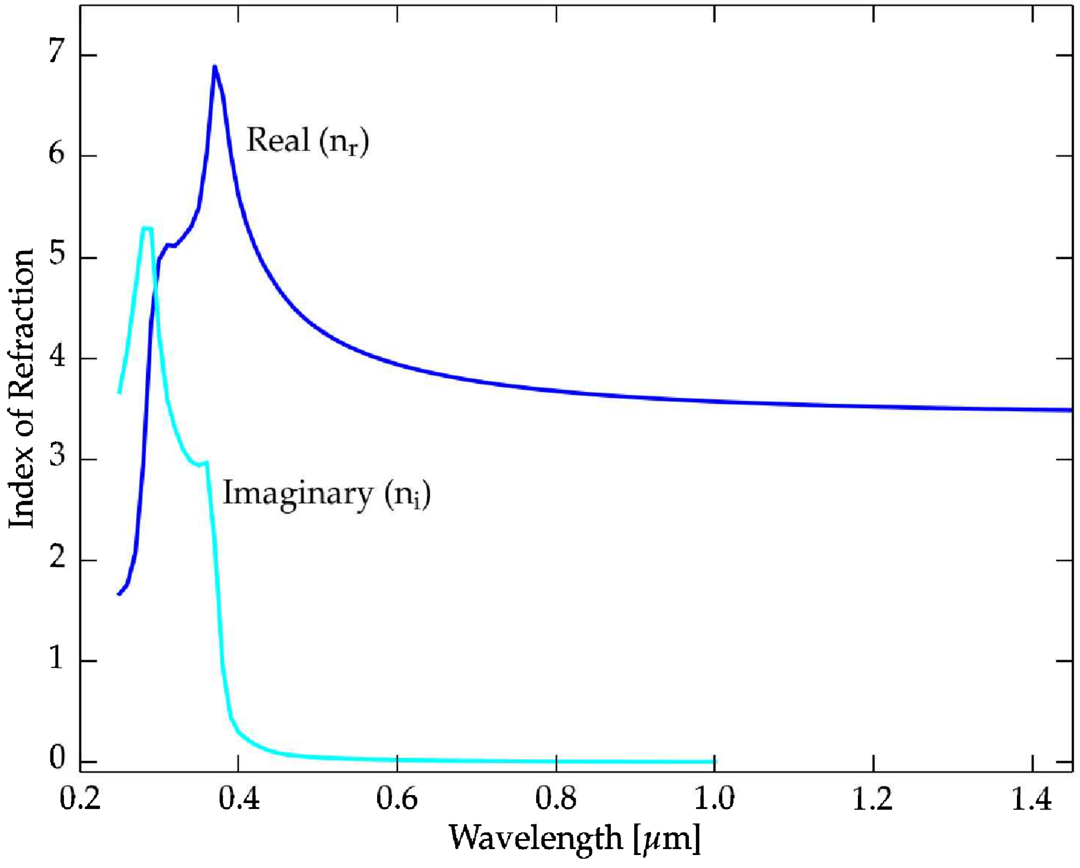
- Examples:
  - For transparent substances, the imaginary part of the index of refraction is much smaller than one.

$$m = 1.31 + i(3.1 \times 10^{-9})$$ pure water ice, $\lambda = 5500\text{Å}$

  - For highly reflective substances, the imaginary part of the index of refraction is comparable to or greater than one.

$$m = 0.36 + i2.69$$ gold, $\lambda = 5500\text{Å}$

  - The index of refraction can be strongly dependent on wavelength.
  - Silicon goes from being opaque in the UV to being transparent in the near IR.



The real (blue) and imaginary (cyan) components of the index of refraction for silicon at $T = 300$ K.

[Mie Theory]

- The derivation of the equations is somewhat elaborate.
  - See Chapter 4 of Bohren & Huffman [Absorption and Scattering of Light by Small Particles]
- Summary of the Results:
  - The interaction of an incident wave with a sphere of radius a causes the sphere to radiate electromagnetic waves. This outgoing wave can be written in terms of vector spherical harmonics. Like with spherical harmonics, this involves Legendre polynomials and Bessel functions.
  - The ***extinction and scattering cross sections*** can be written in terms of the scattering coefficients a_n and b_n :

$$Q_{\rm ext} = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)\,\mathrm{Re}\{a_n + b_n\}$$

$$Q_{\rm sca} = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)\left(|a_n|^2 + |b_n|^2\right)$$

Here, x is the size parameter and a is the dust radius:

$$x = \frac{2\pi a}{\lambda}$$ = ratio of the size of the particle over the wavelength

  - The ***asymmetry factor*** is given by:

$$\mathrm{g} = \frac{4}{x^2 Q_{\rm sca}}\sum_{n=1}^{\infty}\left[\frac{n(n+2)}{n+1}\mathrm{Re}\{a_n^* a_{n+1} + b_n^* b_{n+1}\} + \frac{2n+1}{n(n+1)}\mathrm{Re}\{a_n^* b_n\}\right]$$

The scattering coefficients are expressed in terms of Riccati-Bessel functions ψ and ξ.

$$a_n = \frac{m\psi_n(mx)\psi_n'(x) - \psi_n(x)\psi_n'(mx)}{m\psi_n(mx)\xi_n'(x) - \xi_n(x)\psi_n'(mx)}$$
$$b_n = \frac{\psi_n(mx)\psi_n'(x) - m\psi_n(x)\psi_n'(mx)}{\psi_n(mx)\xi_n'(x) - m\xi_n(x)\psi_n'(mx)}$$

Recurrence relations:

$$\psi_n(x) = xj_n(x)$$
$$\psi_n'(x) = xj_{n-1}(x) - nj_n(x)$$
$$\xi_n(x) = x\left[j_n(x) + iy_n(x)\right]$$
$$\xi_n'(x) = x\left[j_{n-1}(x) + iy_{n-1}(x)\right] - n\left[j_n(x) + iy_n(x)\right]$$

The spherical Bessel functions satisfy the recurrence relation:

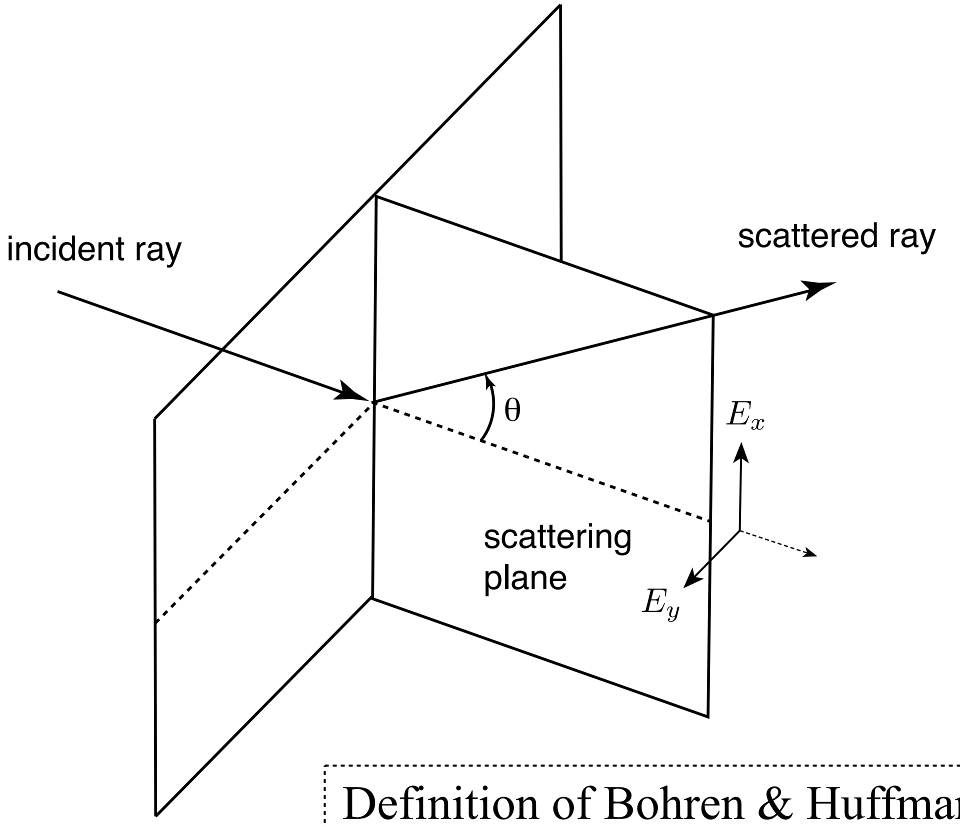
$$j_n(x) = -j_{n-2}(x) + \frac{2n-1}{x}j_{n-1}(x) \qquad j_0(x) = \frac{\sin x}{x} \qquad y_0(x) = -\frac{\cos x}{x}$$
$$y_n(x) = -y_{n-2}(x) + \frac{2n-1}{x}y_{n-1}(x) \qquad j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x} \qquad y_1(x) = -\frac{\cos x}{x^2} - \frac{\sin x}{x}$$

The larger the particle is compared to the wavelength, the more terms have to be included in the sum. A good Mie code is BHMIE of Bohren & Huffman, a version of which can be downloaded from the website of Bruce Draine (http://www.astro.princeton.edu/~draine/scattering.html).

Draine & Lee (1984) developed the first self-consistent, physically motivated dielectric functions for interstellar dust, which have been cited more than 3500 times.

Scattering Phase Function

Let's define the scattering geometry as follows:



Definition of Bohren & Huffman

$E_x = E_{\parallel}^{\mathrm{BH}}$ parallel to the scattering plane

$E_y = -E_{\perp}^{\mathrm{BH}}$ perpendicular to the scattering plane

Then, the scattered electric fields are given by:

$$\begin{pmatrix} E'_{\parallel} \\ E'_{\perp} \end{pmatrix} = \frac{e^{ik(r-z)}}{-kr} \begin{pmatrix} S_2 & 0 \\ 0 & S_1 \end{pmatrix} \begin{pmatrix} E_{\parallel} \\ E_{\perp} \end{pmatrix}$$

Here, the elements of the amplitude scattering matrix are

$$S_1 = \sum_n \frac{2n-1}{n(n+1)} (a_n \pi_n + b_n \tau_n)$$

$$S_2 = \sum_n \frac{2n-1}{n(n+1)} (a_n \tau_n + b_n \pi_n)$$

The functions π_n and τ_n are defined by

$$\pi_n(\cos\theta) = \frac{P_n^1(\cos\theta)}{\sin\theta}$$

$$\tau_n(\cos\theta) = \frac{dP_n^1}{d\theta}$$

Recurrence relations:

$$\pi_n(\mu) = \frac{2n-1}{n-1}\mu\pi_{n-1} - \frac{n}{n-1}\pi_{n-2}$$

$$\tau_n(\mu) = n\mu\pi_n - (n+1)\pi_{n-1}$$

$$\pi_0 = 0 \quad \text{and} \quad \pi_1 = 1$$

$$\mu = \cos\theta$$

For an unpolarized incident light $(|E_{\parallel}| = |E_{\perp}|)$, the intensities of the incident and scattered radiation into the direction θ are related by

$$I \equiv |E_{\parallel}|^2 + |E_{\perp}|^2$$
$$I' \equiv |E'_{\parallel}|^2 + |E'_{\perp}|^2$$

$\longrightarrow$

$$I'(\theta) = S_{11} I \qquad \text{where} \quad S_{11} = \frac{1}{2}\left(|S_1|^2 + |S_2|^2\right)$$

$S_{11}(\cos\theta)$ is the scattering phase function, after a proper normalization.

When integrated over all directions, the S_{11} is related to the scattering efficiency:

$$\int_0^\pi S_{11}(\cos\theta)\sin\theta d\theta = \frac{1}{2}x^2 Q_{\mathrm{sca}}$$

Then, the normalized phase function is given by

$$\mathcal{P}(\cos\theta) = \frac{2}{x^2 Q_{\mathrm{sca}}} S_{11}(\cos\theta) \qquad \int_0^\pi \mathcal{P}(\cos\theta)\sin\theta d\theta = 1$$

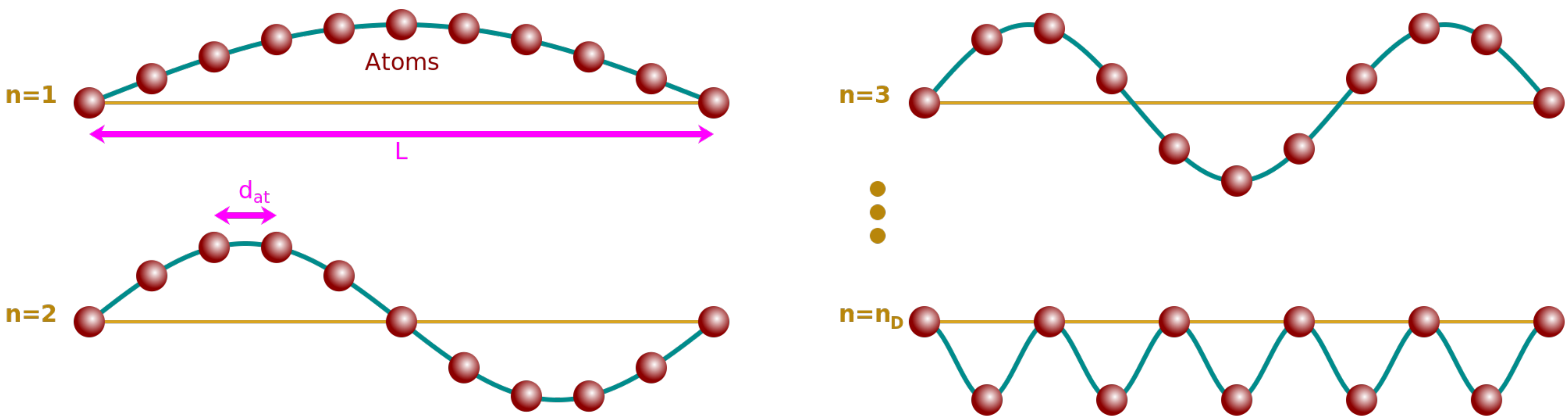
[Temperatures of Interstellar Grains]

- The "temperature" of a dust grain is a measure of the internal energy present in vibrational modes and possibly also in low-lying electronic excitations.

- Grain Heating
  - In diffuse regions, where ample starlight is present, grain heating is dominated by absorption of starlight photons.
  - In dense dark clouds, grain heating can be dominated by inelastic collisions with atoms or molecules from the gas (grain-grain collisions are too infrequent).

- When an optical or UV photon is absorbed by a grain, an electron is raised into an excited electronic state; three cases can occur.
  - If the electron is sufficiently energetic, it may be able to escape from the solid as a "**photoelectron**."
  - In most solids or large molecules, however, the electronically excited state will deexcite nonradiatively, with the energy going into ***many vibrational modes - i.e., heat***.

- The atoms in a solid may oscillate along the chain (longitudinal wave) or perpendicular to it (transverse wave). These collective vibrational modes are sound waves.



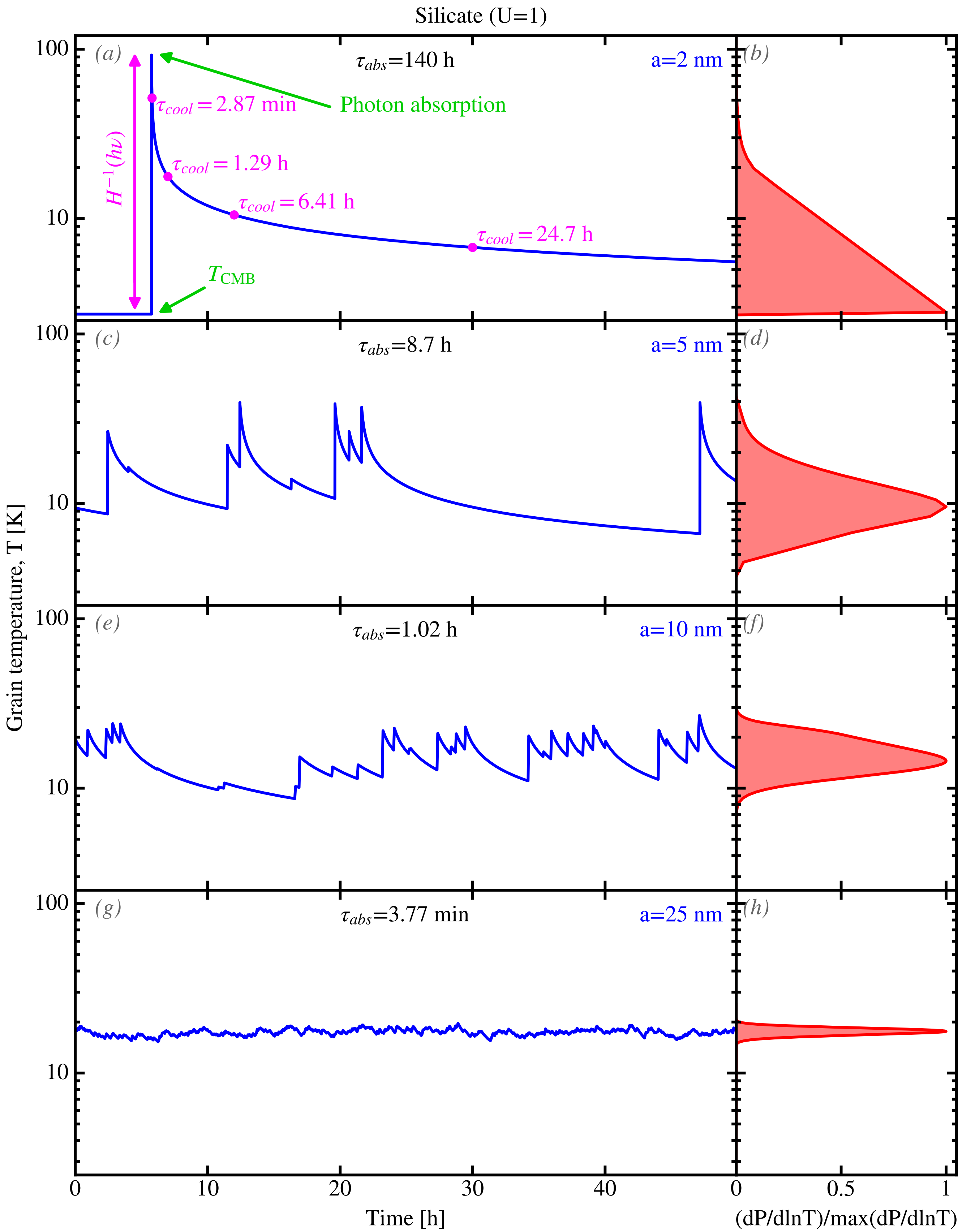
Phonon modes. We represent the simplest case of a string of atoms (red spheres). The total length of the solid is materialized by the yellow horizontal line. The two atoms at each end of this line are fixed. The modes are thus quantified. The shortest possible wavelength is $2d_{\mathrm{at}}$, corresponding to the $n = n_{\mathrm{D}}$ mode.

Temperature of Large Grains and Small Grains

- Large Grains
  - Grains with radii $a \gtrsim 0.03\,\mu\text{m}$, can be considered "classical." These grains are macroscopic - absorption or emission of single quanta do not appreciably change the total energy in vibrational or electronic excitations.
  - The temperature of a large dust grain can be obtained by equating the heating rate to the cooling rate.

- Very Small Grains
  - For ultra-small particles, ranging down to large molecules, quantum effects are important (this include the "spinning" dust grains responsible for microwave emission).
  - When a dust particle is very small, its temperature will fluctuate. This happens because whenever an energetic photon is absorbed, the grain temperature jumps up by some not negligible amount and subsequently declines as a result of cooling.

  - To compute their emission, we need their optical and thermal properties.
    - The optical behavior depends in a sophisticated way on the the complex index of refraction and on the particle shape.
    - The thermal behavior is determined more simply from the specific heat.
  - We need to calculate the distribution function of temperature.



Temperature fluctuations of grains with different radii.

The left panels show the time variation of the temperature of silicate grains (Draine 2003b,c), exposed to the Mathis et al. (1983) interstellar radiation field with $U = 1$. The radius of the grain a increases downward.

The right panels show the corresponding probability distribution of the temperature.

The simulation were performed using the Draine & Anderson (1985) Monte-Carlo method.

See Draine (2003a) for a similar simulation with graphite.

[credit: Frédéric Galliano]

Nuclear Burning

Chapter 2
Astrophysics Processes: The Physics of Astronomical Phenomena
(H. Bradt)

Chapter 6
An Introduction to Stellar Astrophysics (F. LeBlanc)

[Introduction]

- The power source for most stars is the burning of hydrogen in the core of the star.

  The pressures and temperatures there are sufficient to allow the hydrogen nuclei to undergo fusion reactions that lead to helium. (The core temperature of the sun is 1.6×10^6 K.)

  Such reactions are exothermic. They release energy in the form of kinetic energy of the reaction productions. The result is that the star remains in a fairly stable state for much of its active life — some 10^{10} yr in the case of the sun.

- Stable equilibrium (is maintained by a negative feedback mechanism.)

  It the star is perturbed to smaller size, the densities and temperature increase owing to the greater gravitational force. This leads to more nuclear reactions. The increased energy output into the core causes the star to expand, thus returning it to its original state.

  If the star is perturbed to a larger size, the reduced densities and temperatures diminish the energy output, and the star will shrink back to its original stable state.

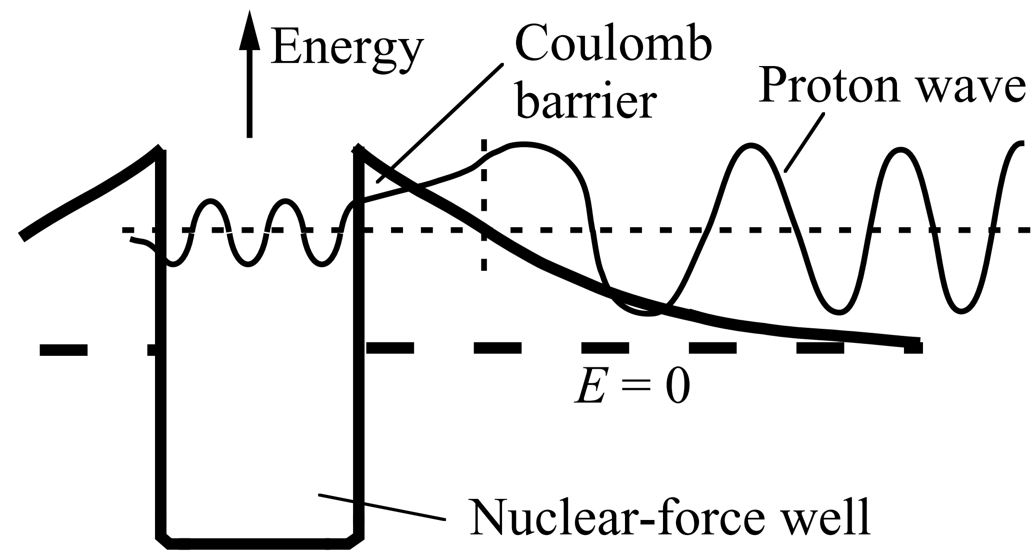
Quantum Tunneling

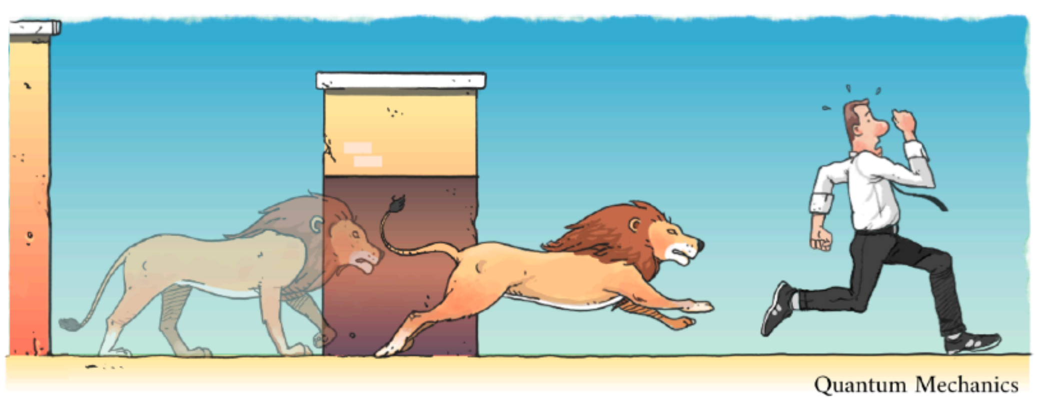
- Coulomb Barrier & Tunneling

The dominant element in the sun is hydrogen, and it is completely ionized throughout most of the solar volume.

For proton-proton interactions to take place, the protons must come within the short range of the nuclear forces, and their kinetic energies should be great enough to overcome the huge Coulomb repulsion force at these short distances. However, the average kinetic energy of protons at the core of the sun is ~ 1000 less than required.

This problem is surmounted by the wave nature of particles that allows them to penetrate some distance into potential barriers. If the barrier is sufficiently narrow, a particle can leak through it into the nuclear potential well. There are sufficient numbers of particles in the high-energy tail of the Maxwell-Boltzmann distribution at 10^7 K to provide the required leakage into the nuclear well and hence nuclear reactions. The reaction rates are highly temperature sensitive. A modest temperature rise markedly increase the rate of nuclear interactions.





credit: LibreTexts, Chemistry

- Nuclear Warmer

  Only a tiny fraction of the stellar thermal energy of a star is radiated away from the stellar surface — only $\sim 5 \times 10^{-8}$ for the sun. The nuclear energy that must be supplied to compensate this loss is thus only a very small fraction of the total thermal energy of the sun.

  Therefore, the sun is not like a raging nuclear furnace but a huge ball of hot gas with a low-powered nuclear "warmer." In other words, the sun is a very big house with high thermal content.

  A basic model of a normal star can thus treat the star simply as a gravitationally bound, stable ball of hot gas.

Proton-proton (pp) chain

- The dominant chain of nuclear interactions in the sun is the proton-proton (pp) chain.

  The pp chain can take place at temperatures above 5×10^6 K.

  The series of reactions in this chain coverts four protons to a helium nucleus (two protons + two neutrons). The latter is referred to as an ***alpha particle***.

PPI chain

$$
\begin{aligned}
&{}^1\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^2\mathrm{H} + e^+ + \nu_e \\
&{}^1\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^2\mathrm{H} + e^+ + \nu_e \\
&{}^1\mathrm{H} + {}^2\mathrm{H} \rightarrow {}^3\mathrm{He} + \gamma \\
&{}^1\mathrm{H} + {}^2\mathrm{H} \rightarrow {}^3\mathrm{He} + \gamma \\
&{}^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2\,{}^1\mathrm{H}
\end{aligned}
$$

$$
\begin{aligned}
&{}^1\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^2\mathrm{H} + e^+ + \nu_e \\
&{}^1\mathrm{H} + {}^2\mathrm{H} \rightarrow {}^3\mathrm{He} + \gamma
\end{aligned}
$$

69 %: ${}^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2\,{}^1\mathrm{H}$ (PPI)

31 %: ${}^3\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^7\mathrm{Be} + \gamma$

99.7 %:

$$
\begin{aligned}
&{}^7\mathrm{Be} + e^- \rightarrow {}^7\mathrm{Li} + \nu_e \\
&{}^7\mathrm{Li} + {}^1\mathrm{H} \rightarrow 2\,{}^4\mathrm{He}
\end{aligned}
$$

(PPII)

0.3 %:

$$
\begin{aligned}
&{}^7\mathrm{Be} + {}^1\mathrm{H} \rightarrow {}^8\mathrm{B} + \gamma \\
&{}^8\mathrm{B} \rightarrow 2\,{}^4\mathrm{He} + e^+ + \nu_e
\end{aligned}
$$

(PPIII)

CNO Cycles

- The CNO cycles are made up of reactions in which protons are fused with C, N and O nuclei to produce helium.
- The CNO cycles dominate energy generation in main-sequence stars only for masses larger than 1.5 solar mass.

CNOI		CNOII		CNOIII
${}^{1}H + {}^{12}C \rightarrow {}^{13}N + \gamma$				
${}^{13}N \rightarrow {}^{13}C + e^{+} + \nu_e$				
${}^{1}H + {}^{13}C \rightarrow {}^{14}N + \gamma$				
${}^{1}H + {}^{14}N \rightarrow {}^{15}O + \gamma$	$\longrightarrow$	${}^{1}H + {}^{14}N \rightarrow {}^{15}O + \gamma$		
${}^{15}O \rightarrow {}^{15}N + e^{+} + \nu_e$	$\longrightarrow$	${}^{15}O \rightarrow {}^{15}N + e^{+} + \nu_e$		
${}^{1}H + {}^{15}N \rightarrow {}^{12}C + {}^{4}He$		${}^{1}H + {}^{15}N \rightarrow {}^{16}O + \gamma$	$\longrightarrow$	${}^{1}H + {}^{15}N \rightarrow {}^{16}O + \gamma$
		${}^{1}H + {}^{16}O \rightarrow {}^{17}F + \gamma$	$\longrightarrow$	${}^{1}H + {}^{16}O \rightarrow {}^{17}F + \gamma$
		${}^{17}F \rightarrow {}^{17}O + e^{+} + \nu_e$	$\longrightarrow$	${}^{17}F \rightarrow {}^{17}O + e^{+} + \nu_e$
		${}^{1}H + {}^{17}O \rightarrow {}^{14}N + {}^{4}He$		${}^{1}H + {}^{17}O \rightarrow {}^{18}F + \gamma$
				${}^{18}F \rightarrow {}^{18}O + e^{+} + \nu_e$
				${}^{1}H + {}^{18}O \rightarrow {}^{15}N + {}^{4}He$

Helium-Burning Phase - Triple-α process

- During the hydrogen-burning phase, 4 protons are transformed into ^{4}He nuclei and therefore the composition of the core gradually changes.

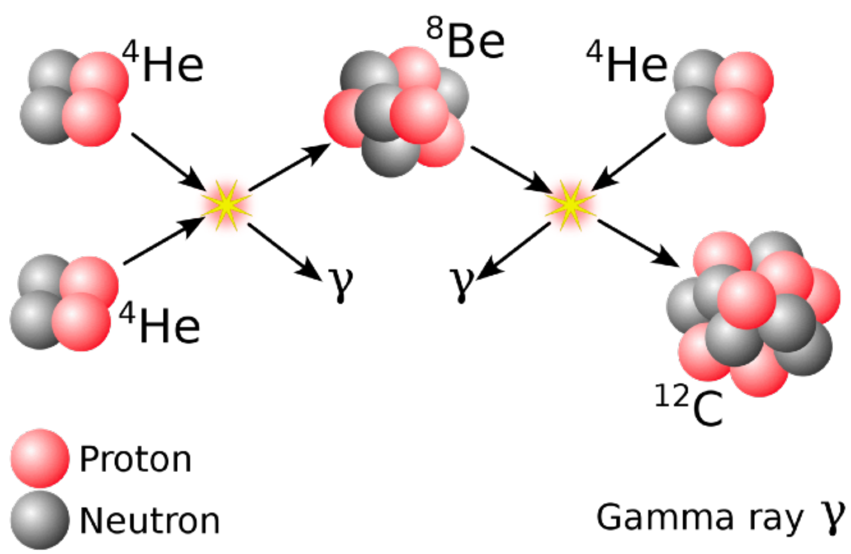
  This process leads to an increase to the mean molecular weight in the stellar core.

  An increase of the mean molecular weight leads to a decrease of gas pressure. The core progressively contracts during the core hydrogen-burning phase thereby increasing the density and temperature (and pressure).

  If the mass of the star is larger than 0.5 solar mass, the core will, following its contraction, attain the critical temperature ($\approx 10^8$ K) needed for the fusion of helium.

  Helium in the core of the evolved star can burn via the following chain of reactions:

$$
\begin{aligned}
&^4\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^8\mathrm{Be} + \gamma \\
&^8\mathrm{Be} + {}^4\mathrm{He} \rightarrow {}^{12}\mathrm{C}^* + \gamma \\
&^{12}\mathrm{C}^* \rightarrow {}^{12}\mathrm{C} + \gamma
\end{aligned}
$$



wikipedia

This chain is commonly called the triple-α reaction since the three α particles fuse to create a carbon nucleus.

Alpha process elements

- Alpha process elements (or alpha elements) are so-called since their most abundant isotopes are integer multiples of four - the mass of the helium nucleus (the alpha particle).

  The stable alpha elements are C, O, Ne, Mg, Si, and S.

  The alpha process generally occurs only if the star is sufficiently massive ($\gtrsim 10M_{\odot}$). These stars contract as they age, increasing core temperature and density to high enough levels to enable the alpha process.

  Requirements increase with atomic mass, especially in later stages — sometimes referred to as silicon burning — and thus most commonly occur in supernovae.

  Type II supernovae mainly synthesize oxygen and the alpha-elements (Ne, Mg, Si, S, Ar, Ca, and Ti) while Type Ia supernovae mainly produce elements of the iron peak (Ti, V, Cr, Mn, Fe, Co, and Ni). An iron peak element is an element with an atomic number in the vicinity of iron's (26).

  The abundance of total alpha elements in stars is usually expressed in terms of logarithms:

$$\left[\frac{\alpha}{\mathrm{Fe}}\right] \equiv \log_{10}\left(\frac{N_{\mathrm{E}\alpha}}{N_{\mathrm{Fe}}}\right)_{\mathsf{Star}} - \log_{10}\left(\frac{N_{\mathrm{E}\alpha}}{N_{\mathrm{Fe}}}\right)_{\mathsf{Sun}},$$

  The relative abundances of the two groups reveal information of past supernovae and star formation history.

Burning Phases

- Carbon burning leads to the creation of oxygen, neon and magnesium.

  $^{12}\mathrm{C} + {}^{4}\mathrm{He} \rightarrow {}^{16}\mathrm{O} + \gamma$

  $^{12}\mathrm{C} + {}^{12}\mathrm{C} \rightarrow {}^{20}\mathrm{Ne} + {}^{4}\mathrm{He}$

  $^{12}\mathrm{C} + {}^{12}\mathrm{C} \rightarrow {}^{24}\mathrm{Mg} + \gamma$

- Neon burning:

  It would be natural that the next phase of nuclear burning would be fusion reactions related to oxygen. However, the critical temperature for neon fusion with α particles is lower than for oxygen burning. At the high temperatures, a sufficient number of energetic photons are present that can lead to photodisintegration of a portion of the neon nuclei

  $^{20}\mathrm{Ne} + \gamma \rightarrow {}^{16}\mathrm{O} + {}^{4}\mathrm{He}$

  This photodisintegration reactions create α particles that can then react with the remaining neon via the reactions

  $^{20}\mathrm{Ne} + {}^{4}\mathrm{He} \rightarrow {}^{24}\mathrm{Mg} + \gamma$

  $^{24}\mathrm{Mg} + {}^{4}\mathrm{He} \rightarrow {}^{28}\mathrm{Si} + \gamma$

  $^{20}\mathrm{Ne} + {}^{20}\mathrm{Ne} \rightarrow {}^{16}\mathrm{O} + {}^{24}\mathrm{Mg}$

- Oxygen-Burning:

  $^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow {}^{30}\mathrm{Si} + 2\,{}^{1}\mathrm{H}$

  $^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow {}^{28}\mathrm{Si} + {}^{4}\mathrm{He}$

  $^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow {}^{31}\mathrm{P} + {}^{1}\mathrm{H}$

  $^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow {}^{31}\mathrm{S} + \mathrm{n}$

  The last reaction emits a free neutron that can participate in the formation of the heavier elements via the s process.

- Silicon-Burning:

  Photodisintegration can create α particles in stellar cores. These α particles can then fuse with heavier nuclei. Silicon may then burn via the reactions:

  $^{28}\mathrm{Si} + {}^{4}\mathrm{He} \rightarrow {}^{32}\mathrm{S} + \gamma$

  $^{32}\mathrm{S} + {}^{4}\mathrm{He} \rightarrow {}^{36}\mathrm{Ar} + \gamma, \quad {}^{36}\mathrm{Ar} + {}^{4}\mathrm{He} \rightarrow {}^{40}\mathrm{Ca} + \gamma, \quad {}^{40}\mathrm{Ca} + {}^{4}\mathrm{He} \rightarrow {}^{44}\mathrm{Ti} + \gamma$

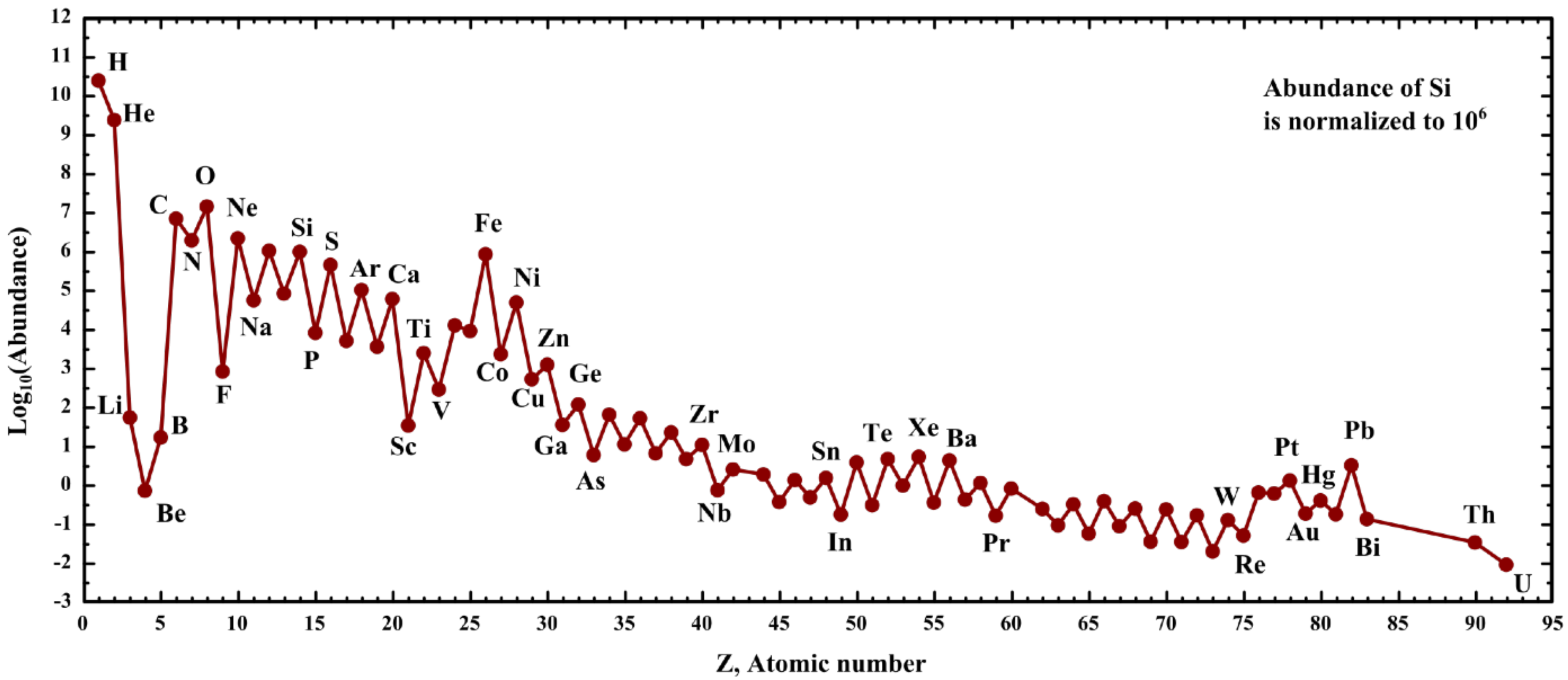
  $^{44}\mathrm{Ti} + {}^{4}\mathrm{He} \rightarrow {}^{48}\mathrm{Cr} + \gamma, \quad {}^{48}\mathrm{Cr} + {}^{4}\mathrm{He} \rightarrow {}^{52}\mathrm{Fe} + \gamma, \quad {}^{52}\mathrm{Fe} + {}^{4}\mathrm{He} \rightarrow {}^{56}\mathrm{Ni} + \gamma$

  Since $^{56}\mathrm{Ni}$ is unstable, it disintegrate via the reactions:

  $^{56}\mathrm{Ni} \rightarrow {}^{56}\mathrm{Co} + \mathrm{e}^{+} + \nu_{\mathrm{e}}, \quad {}^{56}\mathrm{Co} \rightarrow {}^{56}\mathrm{Fe} + \mathrm{e}^{+} + \nu_{\mathrm{e}}$

  $^{56}\mathrm{Fe}$ is one of the most stable nuclei found in nature. Energy generation via nuclear fusion essentially ends at the creation of iron (and other iron-peak elements).

- The iron peak is a local maximum in the vicinity of Fe (Cr, Mn, Fe, Co and Ni) on the plot of the abundances of the chemical elements.



wikipedia

Burning phase	Elements produced	Central temperature	Timescale
H	He	6.0×10^7 K	7×10^6 yr
He	C, O	2.0×10^8 K	5×10^5 yr
C	O, Ne, Mg	9.0×10^8 K	600 yr
Ne	O, Mg, Si	1.7×10^9 K	0.5 yr
O	Si, S	2.3×10^9 K	6 d
Si	Fe-peak	4.0×10^9 K	1 d

https://www.chem.ualberta.ca/~massspec/atomic_mass_abund.pdf

- The various thermonuclear reactions taking place in stars can therefore create most elements up to the iron-peak elements. However, heavier elements are found on Earth. Another physical process is therefore needed to explain the existence of the elements heavier than those of the iron peak.
- Most of the isotopes of these elements can be explained by the capture of neutrons by nuclei followed by the emission of an electron via the reaction (sometimes called a β^- decay.
- $n \rightarrow p + e^- + \bar{\nu}_e$
- This decay increases the number of protons in the nucleus and consequently produces a new elements. These processes are efficient since neutrons can generally interact more easily with nuclei compared to charged particles (other nuclei) because they do not need to overcome the Coulomb repulsion of the nucleus.

Burning Shells: Onion-like structure

- Above a certain mass ($M_* \gtrsim 10M_\odot$), stars are able to fuse elements up to iron. (The exact value of this mass is not known with precision.)

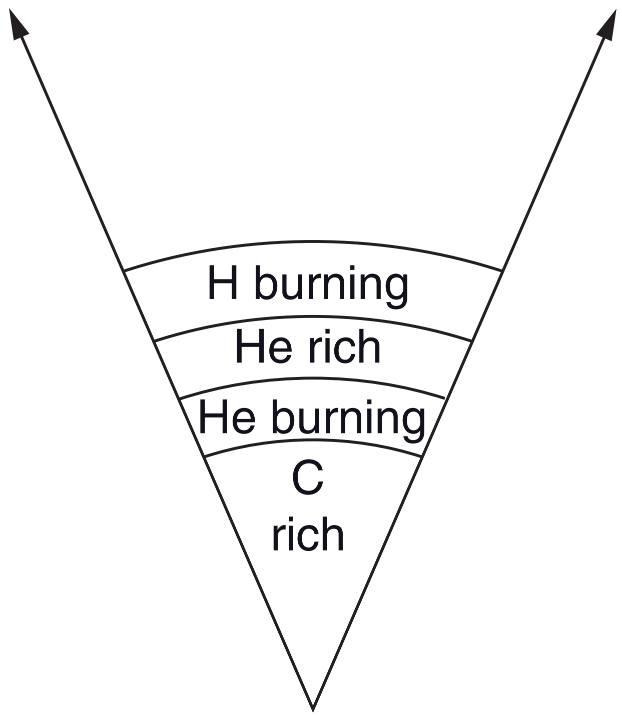
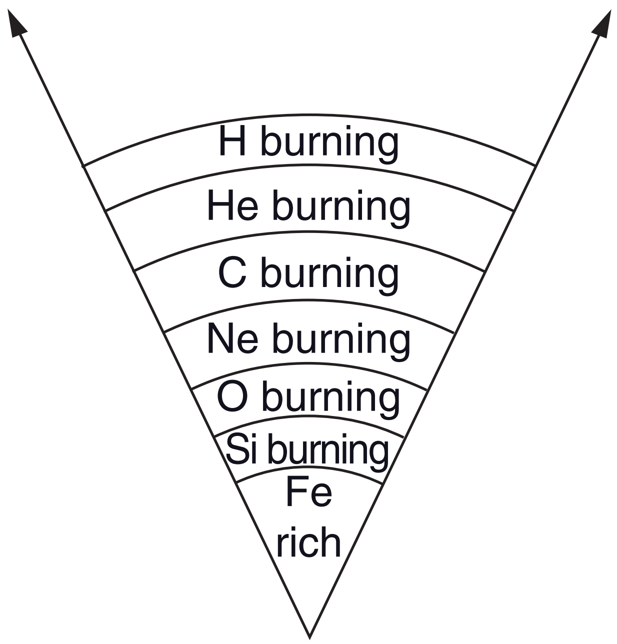


Illustration of the central region of 1 solar mass star near the end of its nuclear-burinig life.



Onion-like structure of a massive star ($M_* \gtrsim 10M_\odot$) near the end of its life.