Modern Astronomy

Part 1. Interstellar Medium (ISM)

Week 2

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updated 03/12, 14:20

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UST / KASI

Kirchhoff's Law in TE and in LTE

- In **(full) thermodynamic equilibrium** at temperature *T*, by definition, we know that

$$\frac{dI_\nu}{ds} = 0 \quad \text{and} \quad I_\nu = B_\nu(T)$$

We also note that
$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- Then, we can obtain ***the Kirchhoff's law for a system in TE***:

$$\boxed{\frac{j_\nu}{\alpha_\nu} = B_\nu(T), \qquad j_\nu = \alpha_\nu B_\nu(T)}$$

- ***Kirchhoff's law applies not only in TE but also in LTE***:
  - Recall that $B_\nu(T)$ ***is independent of the properties of the radiating /absorbing material.***
  - In contrast, both $j_\nu(T)$ ***and*** $\kappa_\nu(T)$ ***depend only on the materials in the cavity and on the temperature of that material***; they do not depend on the ambient radiation field or its spectrum.
  - Therefore, the Kirchhoff's law should be true even for the case of LTE.
  - ***In LTE, the ratio of the emission coefficient to the absorption coefficient is a function of temperature and frequency only.***

Blackbody radiation vs. Thermal radiation

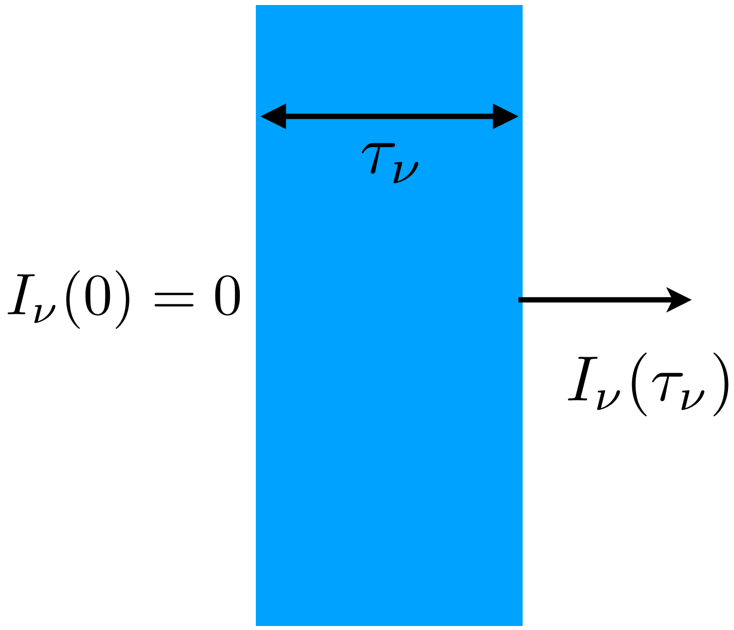
- ***Blackbody radiation*** means $I_\nu = B_\nu(T)$. An object for which the intensity is the Planck function is emitting blackbody radiation.

- ***Thermal radiation is defined to be radiation emitted by "matter" in LTE***. Thermal radiation means $S_\nu = B_\nu(T)$. An object for which the source function is the Planck function is emitting thermal radiation.

- ***Thermal radiation becomes blackbody radiation only for optically thick media.***

- To see the difference between thermal and blackbody radiation,
  - Consider a slab of material with optical depth τ_ν that is producing thermal radiation.
  - If no light is falling on the back side of the slab, the intensity that is measured on the front side of the slab is

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu\left(1 - e^{-\tau_\nu}\right)$$
$$= B_\nu\left(1 - e^{-\tau_\nu}\right)$$



  - If the slab is optical thick at frequency ν $(\tau_\nu \gg 1)$, then

$$I_\nu \approx B_\nu$$

  - If the slab is optically thin $(\tau_\nu \ll 1)$, then

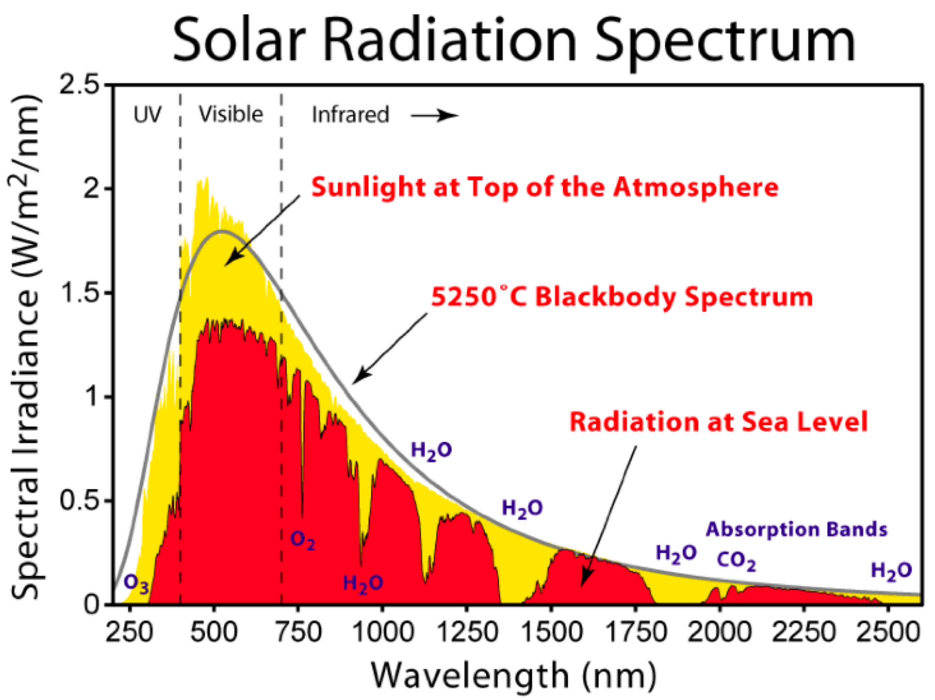
$$I_\nu \approx \tau_\nu B_\nu \ll B_\nu$$

This indicates that the radiation, although thermal, will not be blackbody.

Thermal radiation becomes blackbody radiation only for optical thick media.

Spectrum of Blackbody Radiation

- There is no perfect blackbody.
  - However, the cosmic microwave background comes quite close; stars can sometimes be usefully approximated as blackbodies.
  - By the end of the 19th century, the blackbody spectrum was fairly well known empirically, from laboratory studies. In 1900, Max Planck, using his idea of quantized energies, derived the blackbody spectrum.



https://pages.uoregon.edu/imamura/321/122/lecture-3/stellar_spectra.html

- The frequency dependence of blackbody radiation is given by the ***Planck function***:

$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp\left(h\nu/k_\mathrm{B}T\right) - 1} \quad \text{or} \quad B_\lambda(T) = \frac{2hc^2/\lambda^5}{\exp\left(hc/\lambda k_\mathrm{B}T\right) - 1}$$

$$h = 6.63 \times 10^{-27}\ \text{erg s (Planck's constant)}$$

$$k_\mathrm{B} = 1.38 \times 10^{-16}\ \text{erg K}^{-1}\ \text{(Boltzmann's constant)}$$

Stefan-Boltzmann Law

- Emergent flux is proportional to T^4.

$$F = \pi \int B_\nu(T) d\nu = \pi B(T) \quad \longleftarrow \quad B(T) = \int B_\nu(T) d\nu = \frac{ac}{4\pi} T^4 = \frac{\sigma}{\pi} T^4$$

$$F = \sigma T^4$$

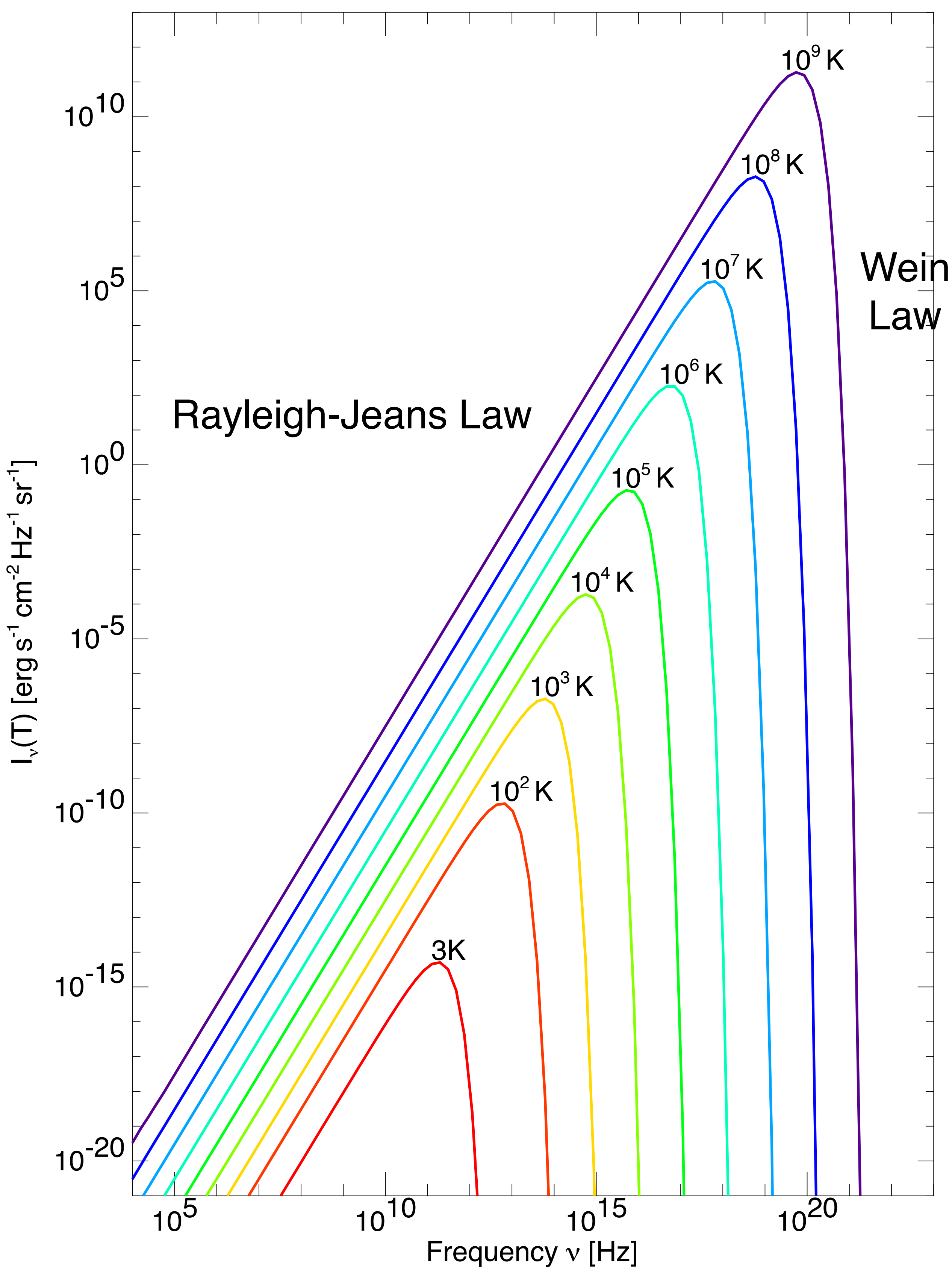
$$\text{Stephan} - \text{Boltzmann constant} : \sigma = \frac{2\pi^5 k_\mathrm{B}^4}{15 c^2 h^3} = 5.67 \times 10^{-5} \text{ erg cm}^2 \text{ s}^{-1} \text{ K}^{-4} \text{ sr}^{-1}$$

Rayleigh-Jeans Law & Wien Law

Rayleigh-Jeans Law (low-energy limit)

$$h\nu \ll k_{\rm B}T \quad (\nu \ll 2\times 10^{10}\,(T/1\,{\rm K})\,{\rm Hz})$$

$$I_\nu^{RJ}(T) = \frac{2\nu^2}{c^2} k_{\rm B} T$$



Wien Law (high-energy limit)

$$h\nu \gg k_{\rm B}T$$

$$I_\nu^{W}(T) = \frac{2h\nu^3}{c^2} \exp\left(-\frac{h\nu}{k_{\rm B}T}\right)$$

Characteristic Temperatures

- ***Brightness Temperature:***
  - The brightness temperature is defined to be the temperature such that a blackbody at that temperature would have specific intensity:

$$I_\nu = B_\nu(T_b) \quad \rightarrow \quad T_b(\nu) = \frac{h\nu/k_{\rm B}}{\ln\left[1 + 2h\nu^3/(c^2 I_\nu)\right]}$$

- ***Antenna Temperature:***
  - Radio astronomers are used to working at very low frequencies. They define the antenna temperature as being the brightness temperature in the Rayleigh-Jeans tail.

$$I_\nu = \frac{2\nu^2}{c^2} k_{\rm B} T_b \quad \rightarrow \quad T_A \equiv \frac{c^2}{2k_{\rm B}\nu^2} I_\nu$$

  - Radiative transfer equation in the RJ limit:
    - If the matter is in LTE and has its energy levels populated according to an excitation temperature $T_{\rm exc} \gg h\nu/k_{\rm B}$, then the source function is given by

$$S_\nu(T_{\rm exc}) = (2\nu^2/c^2) k_{\rm B} T_{\rm exc}$$

    - Then, RT equation becomes $\boxed{\dfrac{dT_A}{d\tau_\nu} = -T_A + T_{\rm exc} \quad \text{if } h\nu \ll k_{\rm B} T_{\rm exc}}$

$$T_A = T_A(0) e^{-\tau_\nu} + T_{\rm exc}(1 - e^{-\tau_\nu}) \quad \text{if } T_{\rm exc} \text{ is constant.}$$

- ***Color Temperature:***
  - By fitting the spectrum to a blackbody curve without regarding to vertical scale (absolute intensity scale), a color temperature T_c is obtained.
  - The color temperature correctly gives the temperature of a blackbody source of unknown absolute scale.
- ***Effective Temperature:***
  - The effective temperature of a source is obtained by equating the actual flux F to the flux of a blackbody at temperature $T_{\rm eff}$.

$$F = \int \int I_\nu \cos\theta d\nu d\Omega = \sigma T_{\rm eff}^4$$

- ***Excitation Temperature:***
  - The excitation temperature of level u relative to level ℓ is defined by

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} \exp\left(-\frac{E_{u\ell}}{k_{\rm B}T_{\rm exc}}\right) \quad \rightarrow \quad T_{\rm exc} \equiv \frac{E_{u\ell}/k_{\rm B}}{\ln\left(\frac{n_\ell/g_\ell}{n_u/g_u}\right)} \quad (E_{u\ell} \equiv E_u - E_\ell)$$

  - Radio astronomers studying the 21 cm line sometimes use the term "spin temperature" $T_{\rm spin}$ for excitation temperature.

Brief Introduction to Atomic Spectroscopy

[Reference]

Astronomical Spectroscopy:
An Introduction to the Atomic and Molecular Physics of Astronomical Spectra
author: Jonathan Tennyson, 2nd Edition

Quantum Numbers / H-atom

- Each bound state of the hydrogen atom is characterized by a set of four quantum numbers (n, l, m, m_s)
  - $n = 1, 2, 3, \cdots$: principal quantum number
  - $l = 0, 1, 2, \cdots, n-1$: ***orbital angular momentum*** quantum number
    - By convention, the values of l are usually designated by letters.

0	1	2	3	4	5	6	7	8	...
s	p	d	f	g	h	i	k	l	...

  - $m = -l, -l+1, \cdots, 0, \cdots, l-1, l$: magnetic quantum number.
    - It determines the behavior of the energy levels in the presence of a magnetic field.
    - This is the projection of the electron orbital angular momentum along the z-axis of the system.

- Spin
  - The electron possesses an intrinsic, ***spin angular momentum*** with the magnitude of $|s| = \frac{1}{2}$.
  - There are two states, $m_s = \pm\frac{1}{2}$, for the spin.

- Degeneracy for a given *n:* $2 \times \sum_{l=0}^{n-1} (2l+1) = 2n^2$

H-atom Spectra

- Spectral series of the H atom
  - The spectrum of H is divided into a number of series linking different upper levels n_2 with a single lower level n_1 value. ***Each series is denoted according to its n_1 value and is named after its discoverer.***
  - Within a given series, ***individual transitions are labelled by Greek letters.***

	$n_2 \longleftrightarrow n_1$		
n_1	Name	Symbol	Spectral region
1	Lyman	Ly	ultraviolet
2	Balmer	H	visible
3	Paschen	P	infrared
4	Brackett	Br	infrared
5	Pfund	Pf	infrared
6	Humphreys	Hu	infrared

$$\Delta n \equiv n_2 - n_1$$

$$\Delta n = 1 \text{ is } \alpha,$$

$$\Delta n = 2 \text{ is } \beta,$$

$$\Delta n = 3 \text{ is } \gamma,$$

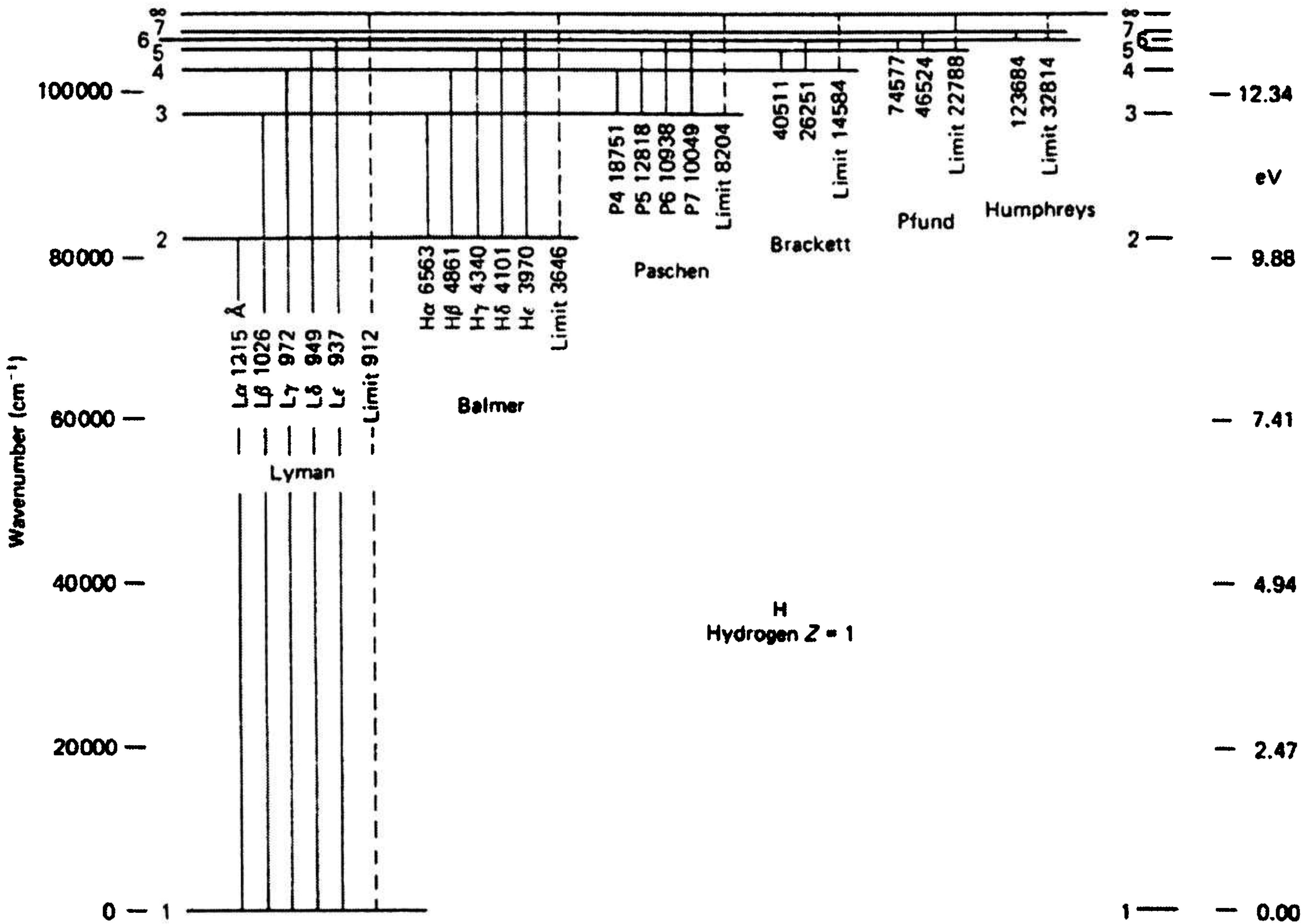
$$\Delta n = 4 \text{ is } \delta,$$

$$\Delta n = 5 \text{ is } \epsilon.$$

Lyman series : $\mathrm{Ly}\alpha, \mathrm{Ly}\beta, \mathrm{Ly}\gamma, \cdots$
Balmer series : $\mathrm{H}\alpha, \mathrm{H}\beta, \mathrm{H}\gamma, \cdots$
Paschen series: $\mathrm{P}\alpha, \mathrm{P}\beta, \mathrm{P}\gamma, \cdots$
Brackett series : $\mathrm{Br}\alpha, \mathrm{Br}\beta, \mathrm{Br}\gamma, \cdots$

Transitions with high Δn are labelled by the n_2. Thus, $H15$ is the Balmer series transition between $n_1 = 2$ and $n_2 = 15$.

Schematic energy levels of the hydrogen atom with various spectral series identified.
The vertical numbers are wavelengths in Å.



Complex Atoms : Electron Configuration

- **The *configuration*** is the distribution of electrons of an atom in atomic **orbitals**.
  - The configuration of an atomic system is defined by specifying the nl values of all the electron orbitals: nl^x means x electrons in the orbital defined by n and l.
  - Each orbital labelled nl actually consists of orbitals with $2l+1$ different m values, each with two possible values of m_s. Thus the nl orbital can hold a maximum $2(2l+1)$ electrons.

$$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} \cdots$$

- **shells, subshells:**
  - ***Principal quantum number = shell***: Shells correspond with the principal quantum numbers (1, 2, 3, ...). They are labeled alphabetically with letters used in the X-ray notation (K, L, M, ...).
  - ***Orbital angular momentum quantum number = subshell***: Each shell is composed of one or more subshells. The first (K) shell has one subshell, called “1s”; The second (L) shell has two subshells, called “2s” and “2p”.

Angular Momentum Coupling

- Atoms contain several sources of angular momentum.
  - ***electron orbital*** angular momentum L
  - ***electron spin*** angular momentum S
  - ***nuclear spin*** angular momentum I
  - The nuclear spin arises from the spins of nucleons. Protons and neutrons both have an intrinsic spin of a half.

- As in classical mechanics, ***only the total angular momentum is a conserved quantity***.
  - It is therefore necessary to combine angular momenta together.

- Addition of two angular momenta:
  - The orbital and spin angular momenta are added vectorially as $\mathbf{J} = \mathbf{L} + \mathbf{S}$. This gives the total electron angular momentum.
  - One then combines the total electron and nuclear spin angular momenta to give the final angular momentum $\mathbf{F} = \mathbf{J} + \mathbf{I}$.

Lifting Degeneracy in Configuration: Angular Momentum Coupling, Terms

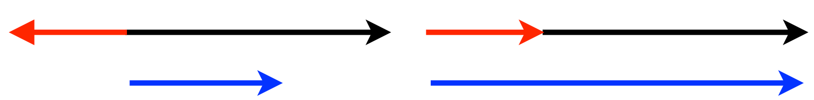
- ***L-S coupling (Russell-Saunders coupling):***
  - The orbital and spin angular momenta are added separately to give the total orbital angular momentum $\boldsymbol{L}$ and the total spin angular momentum $\boldsymbol{S}$. These are then added to give $\boldsymbol{J}$.

$$\boldsymbol{L} = \sum_i \boldsymbol{l}_i,\ \ \boldsymbol{S} = \sum_i \boldsymbol{s}_i \ \ \rightarrow \ \ \boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$$

  - The configurations split into **terms** with particular values of L and S.

- ***Adding two Angular Momenta***
  - Adding vector $\boldsymbol{a}$ and vector $\boldsymbol{b}$ gives a vector $\boldsymbol{c}$, whose length lies in the range

$|a-b| \le c \le a+b$ Here, $a,\ b,\ c$ are the lengths of their respective vectors.



$c = |a-b|$ $\quad\quad$ $c = a+b$

  - In quantum mechanics, a similar rule applies except that the results are quantized. The allowed values of the quantized angular momentum, c, span the range from the sum to the difference of a and b in steps of one:

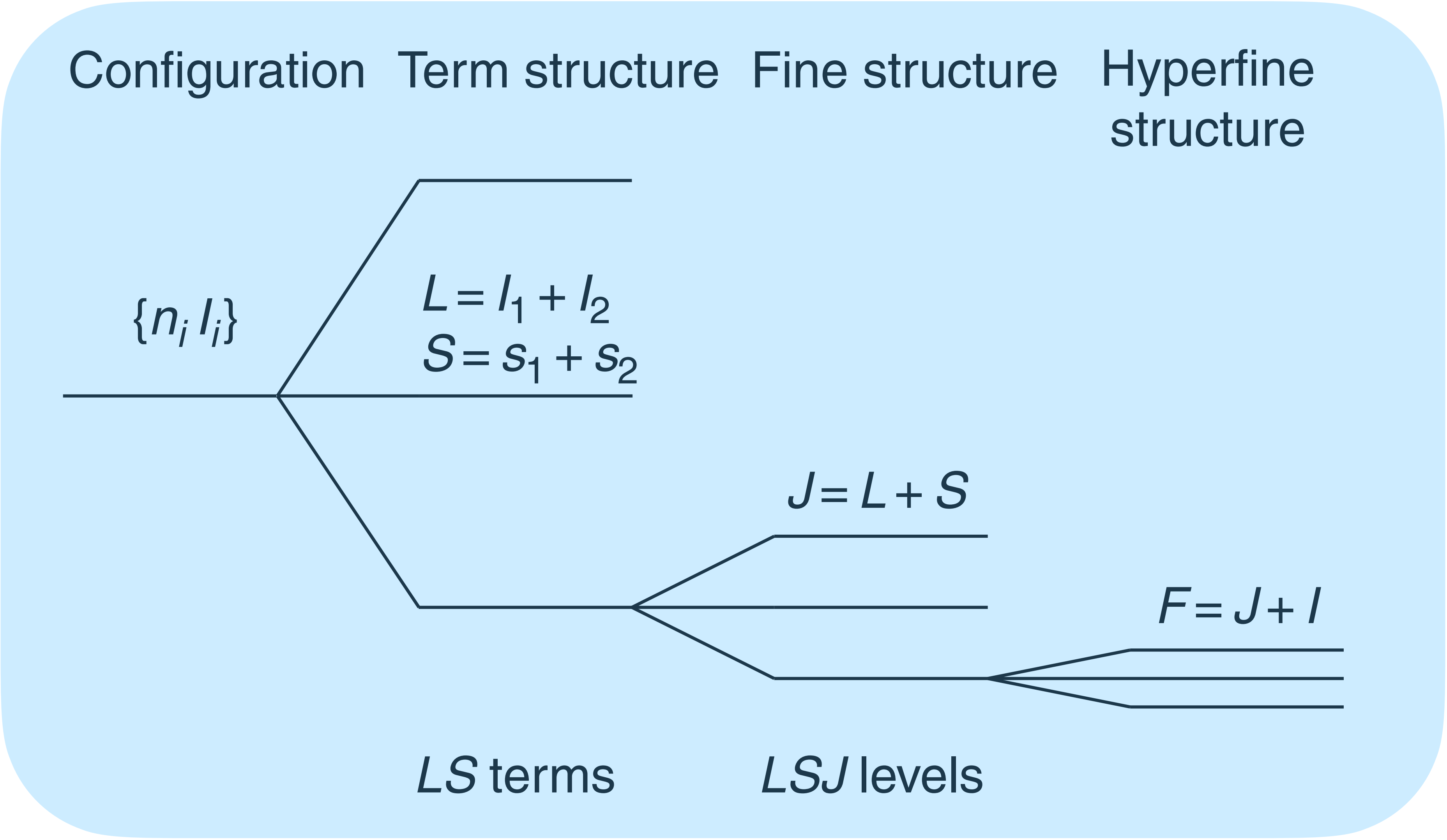
$$c = |a-b|, |a-b|+1, \cdots, a+b-1, a+b$$

  - For example, add the two angular momenta $L_1 = 2$ and $L_2 = 3$ together to give $\mathbf{L} = \mathbf{L}_1 + \mathbf{L}_2$. The result is

$$L = 1, 2, 3, 4, 5.$$

Energy Level Splitting

- Electronic configuration and energy level splitting
  - Configurations ⇒ Terms ⇒ Fine Structure (Spin-Orbit Interaction) ⇒ Hyperfine Structure (Interaction with Nuclear Spin)



[Pradhan & Nahar] Atomic Astrophysics and Spectroscopy

The Fine Structure of Hydrogen

- So far the discussion on H-atom levels has assumed that all states with the same principal quantum number, n, have the same energy.
  - However, this is not correct: inclusion of relativistic (or magnetic) effects split these levels according to the total angular momentum quantum number J. ***The splitting is called fine structure.***

- For hydrogen, $S = \dfrac{1}{2} \quad \rightarrow \quad J = L \pm \dfrac{1}{2}$

- Spectroscopic notation: $^{(2S+1)}L_J$

configuration	L	S	J	term	level
ns	0	$1/2$	$1/2$	^2S	$^2S_{1/2}$
np	1	$1/2$	$1/2,\ 3/2$	$^2P^o$	$^2P^o_{1/2},\ ^2P^o_{3/2}$
nd	2	$1/2$	$3/2,\ 5/2$	^2D	$^2D_{3/2},\ ^2D_{5/2}$
nf	3	$1/2$	$5/2,\ 7/2$	$^2F^o$	$^2F^o_{5/2},\ ^2D^o_{7/2}$

Note that the levels are called to be

singlet if 2S+1 = 1 $\quad S = 0, \quad J = L$

doublet if 2S+1 = 2 $\quad S = 1/2, \quad J = L \pm 1/2$

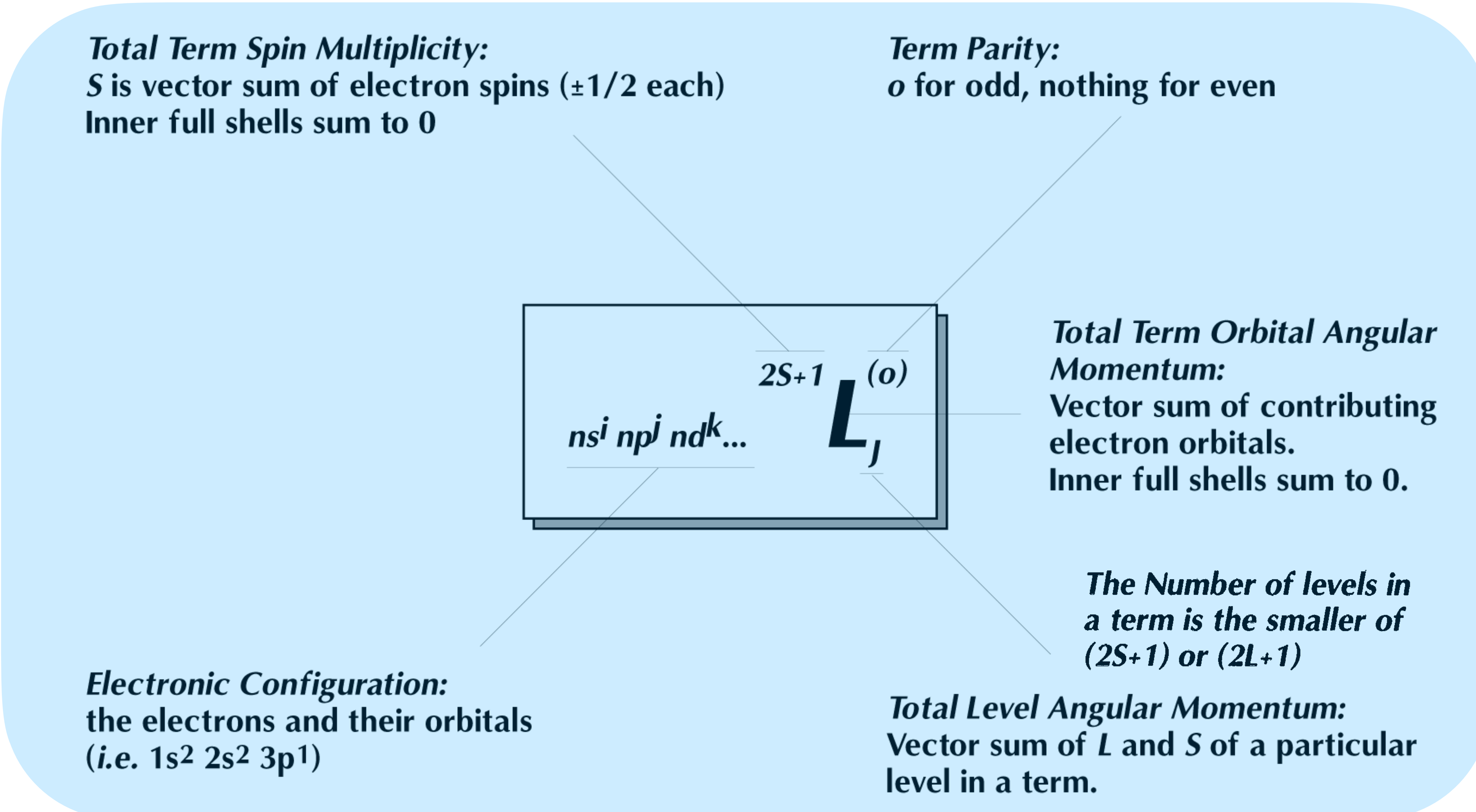
triplet if 2S+1 = 3 $\quad S = 1, \quad J = L-1, L, L+1$

(when L > 0)

  - The above table shows the fine structure levels of the H atom.
  - Note that the states with principal quantum number $n = 2$ give rise to three fine-structure levels. In spectroscopic notation, these levels are $2^2S_{1/2}$, $2^2P^o_{1/2}$ and $2^2P^o_{3/2}$.

Spectroscopic Notation

- **Spectroscopic Notation**



- A state with S = 0 is a 'singlet' as 2S+1 = 1.
  - $J = L$ (singlet)
- A state with S = 1/2 is a 'doublet' as 2S+1 = 2
  - $J = L - 1/2, L + 1/2$ (doublet if $L \geq 1$)
- One with S = 1 is a 'triplet' as 2S+1 = 3
  - $J = L - 1, L, L + 1$ (triplet $L \geq 1$)

$$n = 1, 2, 3, 4, 5 \cdots \quad \rightarrow \quad K, L, M, N, O, \cdots$$
$$\ell = 0, 1, 2, 3, 4 \cdots \quad \rightarrow \quad s, p, d, f, g, \cdots$$
$$L = 0, 1, 2, 3, 4 \cdots \quad \rightarrow \quad S, P, D, F, G, \cdots$$

sharp, principal, diffuse, fundamental,…

Selection Rules

- **Selection Rules**

(1) one electron jumps
(2) Δn any
(3) $\Delta l = \pm 1$
(4) parity change
→ selection rule for configuration

(5) $\Delta S = 0$ → ***intercombination*** line if only this rule is violated.

(6) $\Delta L = 0, \ \pm 1$ (except $L = 0 - 0$)
(7) $\Delta J = 0, \ \pm 1$ (except $J = 0 - 0$)
(8) $\Delta F = 0, \ \pm 1$ (except $F = 0 - 0$) → It is only rarely necessary to consider this.

- ***Allowed = Electric Dipole*** : Transitions which satisfy all the above selection rules are referred to as ***allowed transitions***. These transitions are strong and have **a typical lifetime of** $\sim 10^{-8}$ **s**. Allowed transitions are denoted without square brackets.

$$\text{e.g., C IV } 1548, 1550 \text{ Å}$$

- Photons do not change spin, so transitions usually occur between terms with the same spin state ($\Delta S = 0$). However, relativistic effects mix spin states, particularly for high *Z* atoms and ions. As a result, one can get (weak) spin changing transitions. These are called ***intercombination (semi-forbidden or intersystem) transitions*** or lines. They have **a typical lifetime of** $\sim 10^{-3}$ **s**. An intercombination transition is denoted with a single right bracket.

$$\text{C III] } 2s^2 \ {}^1S - 2s2p \ {}^3P^o \text{ at } 1908.7 \text{ Å}. \quad (\Delta S = 1)$$

- If any one of the rules 1-4, 6-8 are violated, they are called ***forbidden transitions*** or lines. They have **a typical lifetime of** $\sim 1 - 10^3$ **s**. A forbidden transition is denoted with two square brackets.

$$1906.7 \text{ Å [C III] } 2s^2 \ {}^1S_0 - 2s2p \ {}^3P^o_2, \quad (\Delta S = 1, \Delta J = 2)$$

- ***Resonance line*** denotes a dipole-allowed transition arising from the ground state of a particular atom or ion.

Forbidden Lines

- Forbidden lines are often difficult to study in the laboratory as collision-free conditions are needed to observe metastable states.
  - In this context, it must be remembered that laboratory ultrahigh vacuums are significantly denser than so-called dense interstellar molecular clouds.
  - ***Even in the best vacuum on Earth, frequent collisions knock the electrons out of these orbits (metastable states) before they have a chance to emit the forbidden lines.***
  - In astrophysics, low density environments are common. In these environments, the time between collisions is very long and an atom in an excited state has enough time to radiate even when it is metastable.
  - Forbidden lines of nitrogen ([N II] at 654.8 and 658.4 nm), sulfur ([S II] at 671.6 and 673.1 nm), and oxygen ([O II] at 372.7 nm, and [O III] at 495.9 and 500.7 nm) are commonly observed in astrophysical plasmas.
  - ***The forbidden 21-cm hydrogen line is particularly important for radio astronomy as it allows very cold neutral hydrogen gas to be seen.***
  - Since metastable states are rather common, forbidden transitions account for a significant percentage of the photons emitted by the ultra-low density gas in Universe.
  - ***Forbidden lines can account for up to 90% of the total visual brightness of objects such as emission nebulae.***

Notations

- Notations for Spectral Emission Lines and for Ions
  - There is a considerable confusion about the difference between these two ways of referring to a spectrum or ion, for example, C III or C^{+2}. These have very definite different physical meanings. However, in many cases, they are used interchangeably.
  - C^{+2} is a baryon and C III is a set of photons.
  - **C^{+2} refers to carbon with two electrons removed**, so that is doubly ionized, with a net charge of +2.
  - **C III is the spectrum produced by carbon with two electrons removed.** The C III spectrum will be produced by impact excitation of C^{+2} or by recombination of C^{+3}. So, depending on how the spectrum is formed. C III may be emitted by C^{+2} or C^{+3}.

    collisional excitation: $C^{+2} + e^- \rightarrow C^{+2*} + e^- \rightarrow C^{+2} + e^- + h\nu$

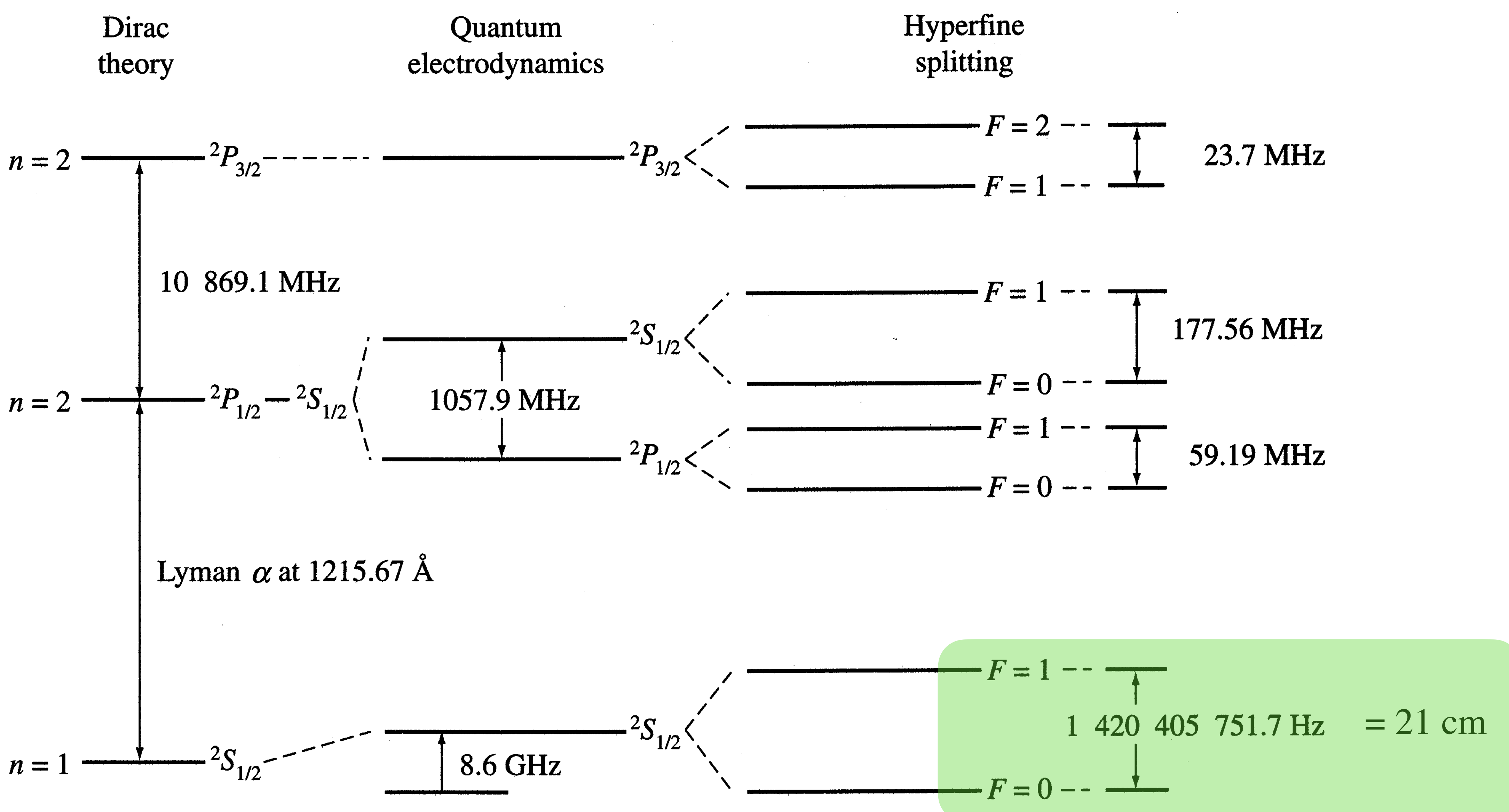
    recombination: $C^{+3} + e^- \rightarrow C^{+2} + h\nu$

  - There is no ambiguity in absorption line studies - only C^{+2} can produce a C III absorption line. This had caused many people to think that C III refers to the matter rather than the spectrum.
  - But this notation is ambiguous in the case of emission lines.

Hydrogen Atom : Fine & Hyperfine Structures

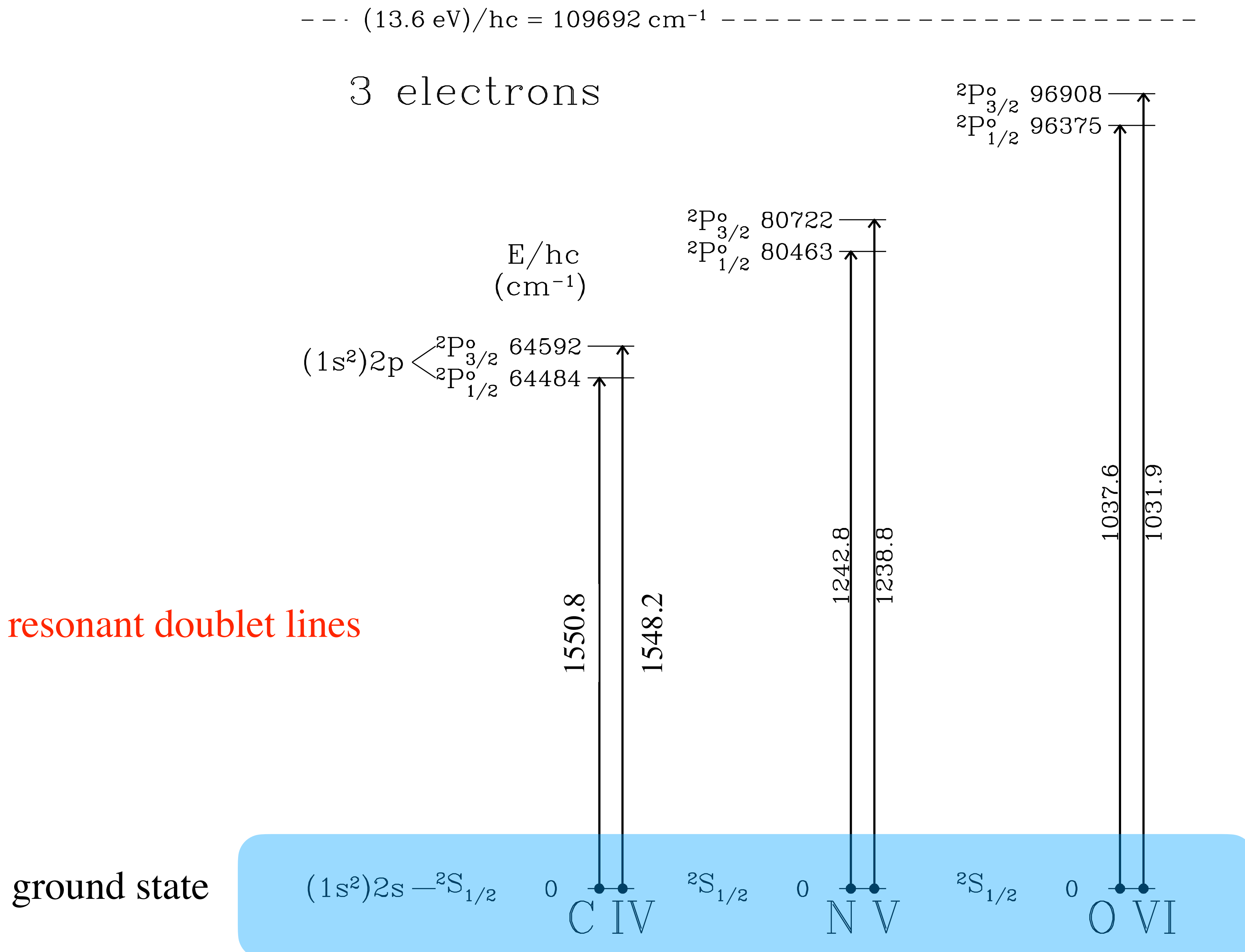
- ***Hyperfine Structure in the H atom***
  - Coupling the nuclear spin I to the total electron angular momentum J gives the final angular momentum F. For hydrogen this means

$$F = J + I = J \pm \frac{1}{2}$$



[Kwok] Physics and Chemistry of the ISM, [Bernath] Spectra of atoms and Molecules

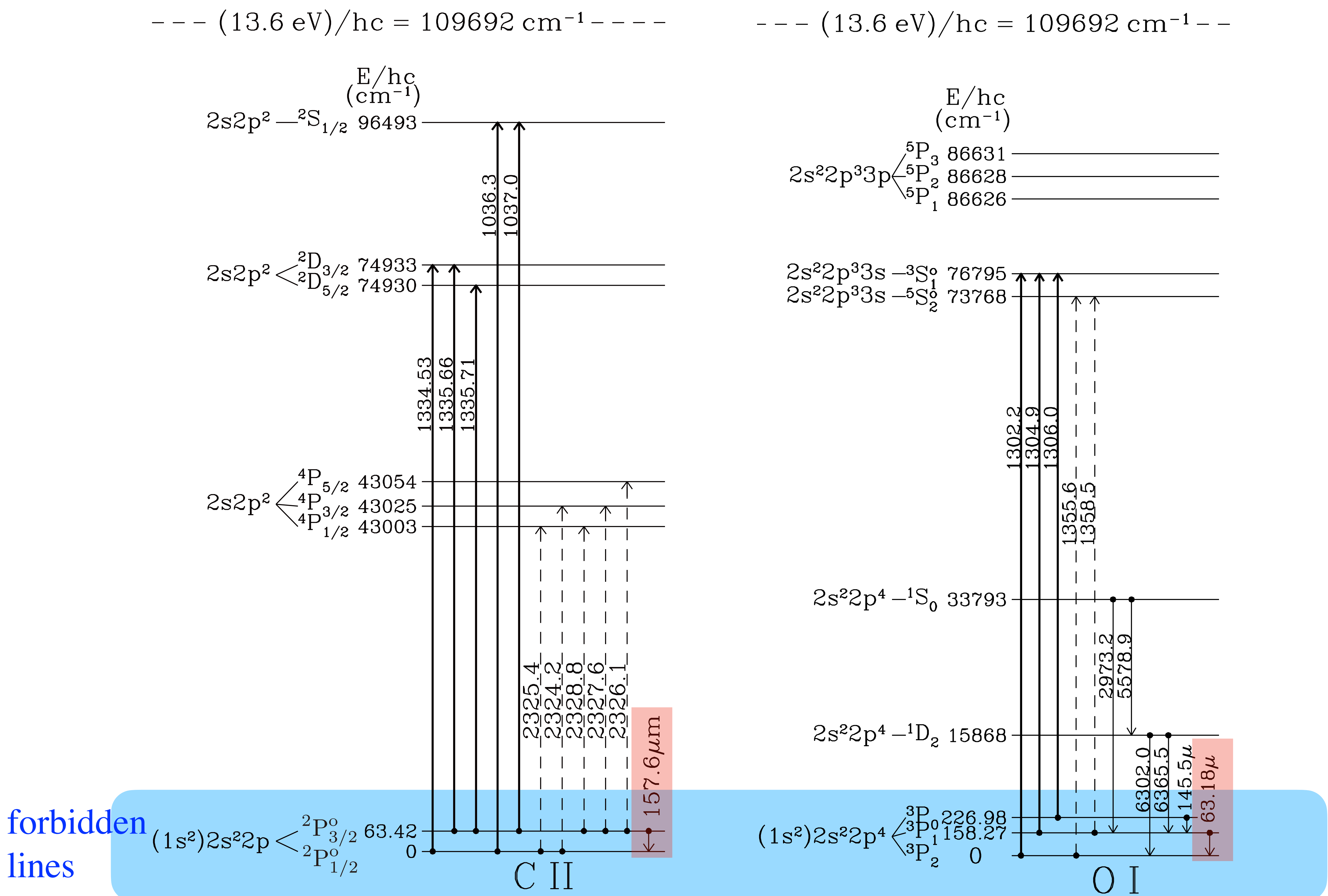
- 3 electrons (Lithium-like ions)



[Draine] Physics of the Interstellar and Intergalactic Medium

- 5 & 8 electrons

Upward heavy: allowed, Upward Dashed: intercombination, Downward solid: forbidden



[Draine] Physics of the Interstellar and Intergalactic Medium

Multiphase ISM

Five Phases of the ISM

Molecular clouds

- H_2 is the dominant form of molecules.
- **Number density ~ 10^6 cm^{-3} in the molecular cloud cores**, which are self-gravitating and form stars. (Note that 10^6 cm^{-3} is comparable to the density in the most effective cryo-pumped vacuum chambers in laboratories.)
- How to observe: for instance, 2.6, 1.3 and 0.9 mm (115, 230 and 345 GHz) emission lines from CO.

Cold neutral medium (CNM) ($T \sim 10^2$ K)

- The dominant form of CNM is H I (atomic hydrogen).
- The CNM is distributed in sheets and filaments occupying ~1% of the ISM volume.
- How to observe: UV and optical absorption lines in the spectra of background stars and quasars.

Warm neutral medium (WNM) ($T \sim 5 \times 10^3$ K)

- Its dominant form is H I (atomic hydrogen).
- A leading method of observing the WNM is using 21 cm emission.

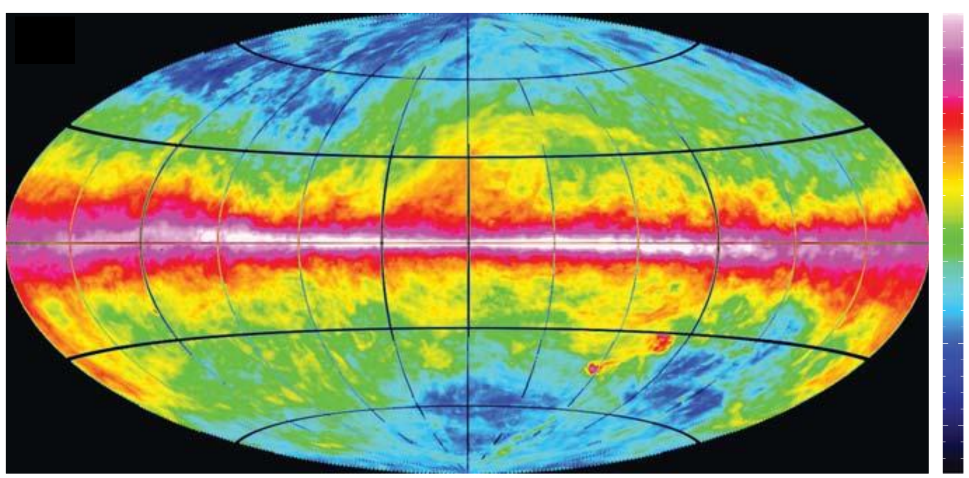
Warm ionized medium (WIM) or Diffuse ionized gas (DIG) ($T \sim 10^4$ K)

- The dominant form is H II (ionized hydrogen or proton).
- The WIM is primarily photoionized by O- and B- type stars.
- Observed using Balmer emission lines (Hα).

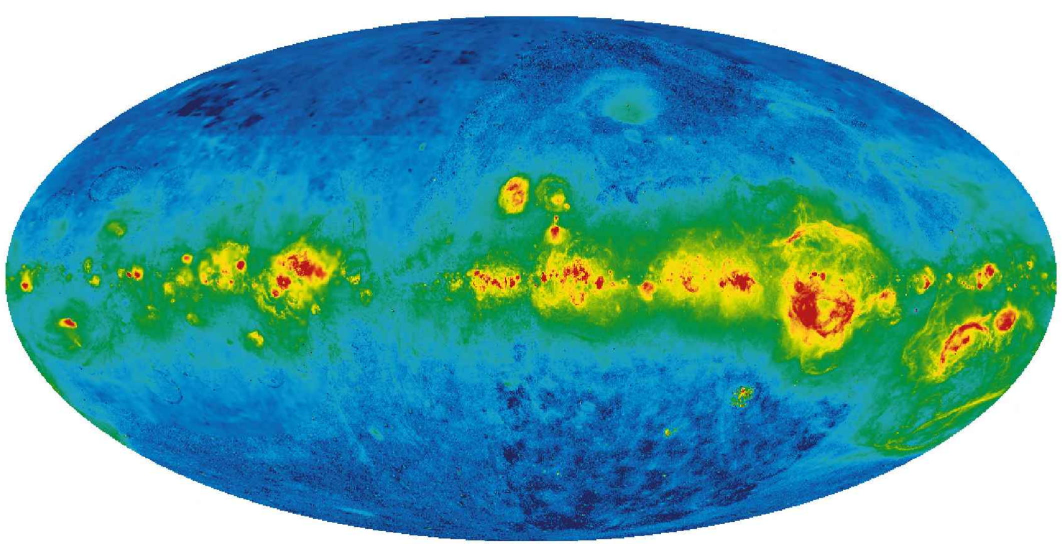
Hot ionized medium (HIM) or coronal gas ($T \gtrsim 10^{5.5}$ K)

- The HIM is primarily heated by supernovae.
- HIM occupies ~ half of the ISM volume, but provides only 0.2% of the ISM mass.
- soft X-ray emission. O VI, N V, and C IV emission or absorption lines in the spectra of background stars.

Name	T (K)	$n_H (cm^{-3})$	Mass fraction	Volume fraction
Molecular Clouds	20	> 100	35%	0.1%
Cold Neutral Medium	100	30	35%	1%
Warm Neutral Medium	5000	0.6	25%	40%
Warm Ionized Medium	10^4	0.3	3%	10%
Hot Ionized Medium	10^6	0.004	0.2%	50%



CNM + WNM: All-sky 21 cm map

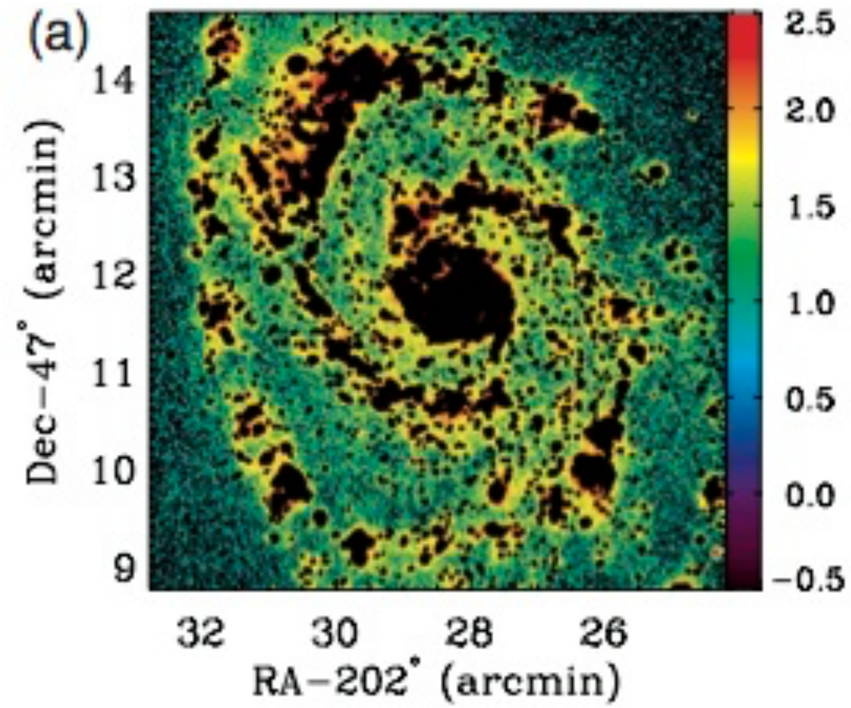


WIM: All-sky map of Hα (6563Å)

M51 (NGC5195)
Plate 1 [Lequeux]

B band - blue
V band - green
Hα - red (DIG)

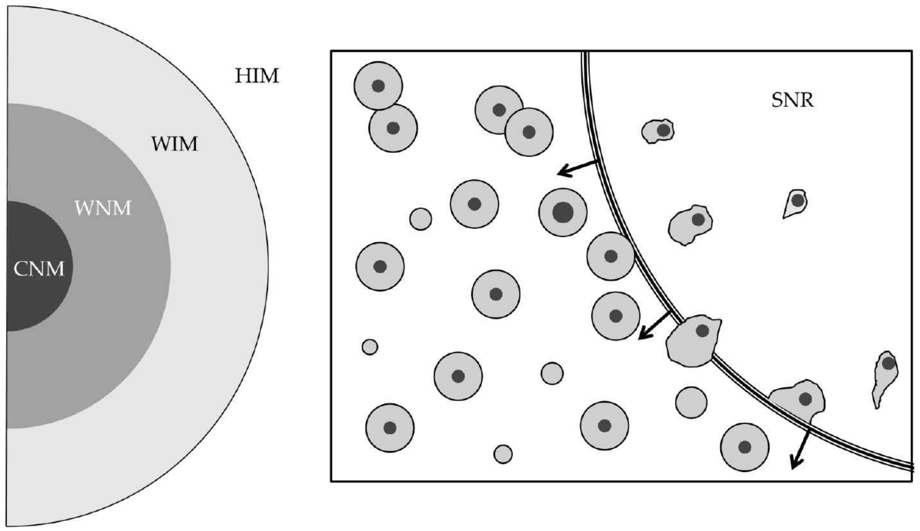
DIG: M 51 (Seon 2009)



Pressure Equilibrium

- All five phases of the ISM have a pressure $P \sim 4 \times 10^{-19}$ atm, equivalent to a thermal energy density $(3/2)nkT \sim 0.4$ eV cm^{-3}.
  - Thus, it is tempting to assume that the phases are in pressure equilibrium, with

$$n_1 k T_1 = n_2 k T_2 = 4 \times 10^{-19} \text{ atm}$$
$$n_1 T_1 = n_2 T_2 = 2{,}935 \text{ cm}^{-3}\,\text{K}$$
$$(1 \text{ atm} = 1.013 \times 10^6 \,\text{dyn}\,\text{cm}^{-2})$$



  - Earlier views of the ISM did assume the pressure equilibrium. Denser, cooler ***"clouds"*** in a tenuous, hotter ***"intercloud medium."***
  - However, current studies of the ISM have had reject this simple picture. The ISM has indeed tendencies toward pressure equilibrium, but something always happens to throw things out of equilibrium.
    - ✦ The ubiquity of free electrons indicates that the ISM is coupled to the interstellar magnetic field. The turbulent energy density is not negligibly small. Thus, they have to be taken into account.
    - ✦ Supernova explosions are going off in the ISM, increasing the temperature *T*.
    - ✦ Hot young stars are pouring ionizing radiation into the ISM, splitting up atoms and increasing *n*.

Heating and Cooling in the ISM

- The temperature of the ISM is also determined by a balance between heating and cooling.
  - Each phase has a temperature where the balance is a stable one.

- Definitions
  - heating gain per atom G, cooling loss per atom L in units of erg s^{-1}.
  - volumetric heating rate $g = nG$, volumetric cooling rate $\ell = nL$ in units of erg cm^{-3} s^{-1}.
  - cooling function Λ in units of erg cm^3 s^{-1}, which is useful for two-body interactions.
  - $\ell = nL = n^2\Lambda$, where n is the total number density of gas particles.
  - Even when only one type of particle is losing energy, the energy loss is shared among all the gas particles due to the relatively short thermalization time scale in the ISM.

Reference: Collisional time scale in the CNM

- $t_{\rm coll}(\rm HH) \sim 2.2\,\rm yr$ for atom-atom collisions
- $t_{\rm coll}(e\rm H) \sim 120\,\rm yr$ for atom-electron collisions
- $t_{\rm coll}(ee) \sim 1.2\,\rm hr$ for electron-electron collision

Mean free path in the CNM

- $\lambda_{\rm mfp}(\rm HH) \sim 0.74\,\rm AU$ for atom-atom collisions
- $\lambda_{\rm mfp}(e\rm H) \sim 1700\,\rm AU$ for atom-electron collisions
- $\lambda_{\rm mfp}(ee) \sim 1.9 \times 10^{-3}\,\rm AU$ for electron-electron collision

- Heating -

- Heating processes
  - The primary heating mechanisms of the ISM involve providing free electrons with high energies. Through collisions, the fast free electrons share their kinetic energy with other particles, and through further collisions, the distribution of velocities approaches a Maxwellian distribution.

  - **Source of free electrons**
    - ✦ Ionization by cosmic rays
    - ✦ **Photoionization of dust grains by starlight UV - the most important one.**
    - ✦ Photoionization of atoms (H, He, C, Mg, Si, Fe, etc) by X-rays or starlight UV.

  - **Other heating sources:**
    - ✦ **Heating by shock waves and other MHD phenomena.**

Heating: Photoelectric Heating by Dust

- UV and X-ray photons can knock electrons free from dust grains. The ejected electrons carry kinetic energy, which can be effective at heating the surrounding gas.
- ***Photoelectrons emitted by dust grains dominate the heating of the diffuse neutral ISM (CNM and WNM) in the Milky Way.***
- The work function, analogous to the ionization energy of an atom, for graphite is $4.50 \pm 0.05\,\mathrm{eV}$. Therefore, UV photons with $h\nu \gtrsim 5\,\mathrm{eV}$ can kick out photoelectrons from dust grains. The photoelectric heating by dust is dominated by photons with $h\nu \gtrsim 8\,\mathrm{eV}$.

$$G_{\rm pe} \approx 1.4 \times 10^{-26} \frac{n_{\rm ph}(8-13.6\,\mathrm{eV})}{3\times 10^{-3}\,\mathrm{cm^{-3}}} \frac{\left\langle \sigma_{\rm abs} \right\rangle}{10^{-21}\,\mathrm{cm^2}} \frac{\langle Y \rangle}{0.1} \frac{\left\langle E_{\rm pe} \right\rangle - \left\langle E_c \right\rangle}{1\,\mathrm{eV}} \ \ \mathrm{erg\ s^{-1}}$$

The gain is independent of temperature.

Here,

$n_{\rm ph}(8-13.6\,\mathrm{eV})$ = number density of $8 < h\nu < 13.6\,\mathrm{eV}$ photons
$\left\langle \sigma_{\rm abs} \right\rangle$ = total dust photo absorption cross section per H nucleon, averaged over the photon spectrum.
$\langle Y \rangle$ = photoelectric yield averaged over the spectrum of 8 to 13.6 eV photons absorbed by the interstellar grain mixture.
$\left\langle E_{\rm pe} \right\rangle$ = mean kinetic energy of escaping photoelectrons.
$\left\langle E_{\rm c} \right\rangle$ = mean kinetic energy of electrons captured from the plasma by grains.

- ***Photoelectric heating from dust may be an order of magnitude larger than the cosmic ray heating rate.***

- Cooling -

- Decreasing the average kinetic energy of particles in the ISM is usually done by ***radiative cooling***.
    - In the CNM, cooling is performed by infrared photons emitted by carbon and oxygen.
        - ✦ Oxygen is nearly all in the form of neutral O I. (the ionization energy = 13.26 eV)
        - ✦ Carbon will be nearly always in the form of singly ionized C II. (ionization energy = 11.26 eV) The background starlight in our galaxy has enough photons in the relevant energy range $11.26\,\mathrm{eV} < h\nu < 13.60\,\mathrm{eV}$ to keep the C atoms ionized.

- [C II] 158μm (collisionally excited line emission)
    - The electronic ground state of C II is split into two fine levels, separated by an energy $E_{u\ell} = 7.86 \times 10^{-3}\,\mathrm{eV}$, which corresponds to $\lambda = 158\,\mu\mathrm{m}$ and $T = E_{u\ell}/k = 91.2\,\mathrm{K}$.
    - The upper level is populated by collisions with hydrogen atoms and free electrons.
    - If C II is excited by collisions with free electrons, the cooling function is given by, for a C abundance $n_\mathrm{C}/n_\mathrm{H} = 3 \times 10^{-4}$,

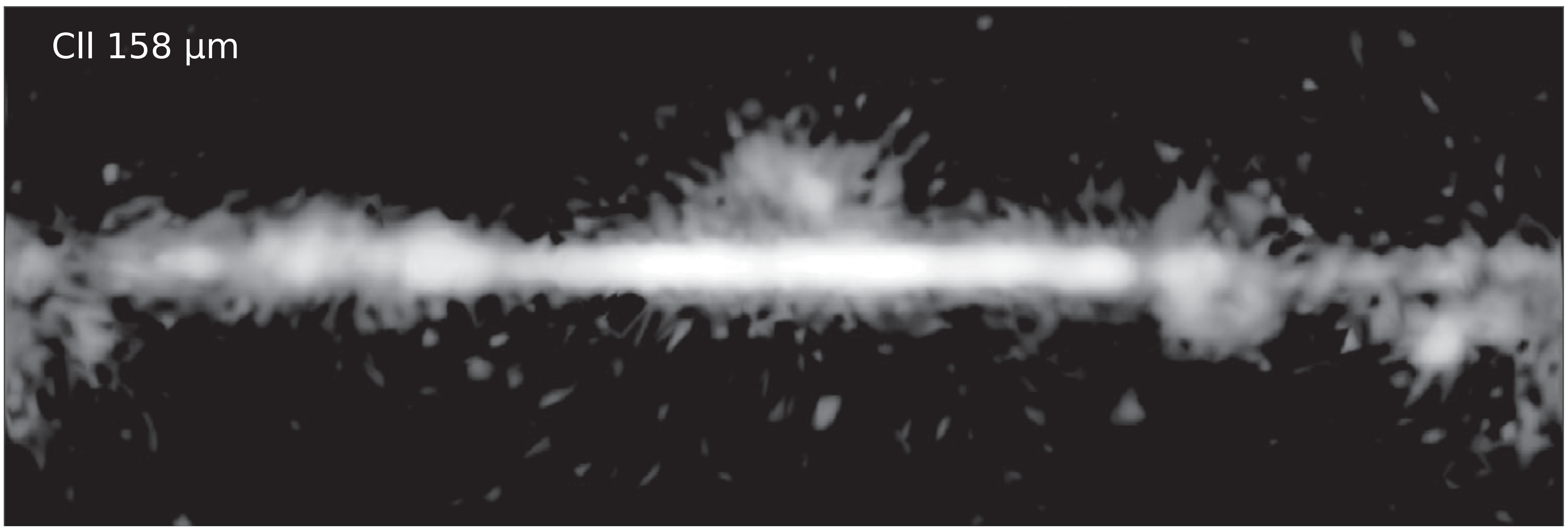
$$\frac{\Lambda^{e}_{[\mathrm{CII}]}}{10^{-25}\,\mathrm{erg\ cm^3\ s^{-1}}} \approx 0.03 \left(\frac{x}{10^{-3}}\right) \left(\frac{T}{100\,\mathrm{K}}\right)^{-1/2} \exp\left(-\frac{91.2\,\mathrm{K}}{T}\right)$$

Here, $x = n_e/n$ is the ionization fraction.

- If the C II is excited by collisions with hydrogen atoms, the cooling function is

$$\frac{\Lambda^{\rm H}_{\rm [CII]}}{10^{-25}\,{\rm erg\ cm^3\ s^{-1}}} \approx 0.06 \left(\frac{T}{100\,{\rm K}}\right)^{0.13} \exp\left(-\frac{91.2\,{\rm K}}{T}\right)$$

- In the CNM, both contribute significantly to the excitation of C II.



C II 158 **µ**m line emission in the Galaxy. The map size is -180º to 180º in Galactic longitude and -60º and 60º in Galactic latitude. The data is from all-sky maps created by the Cosmic Microwave Background Explorer.

[Fig. 5.5. Introduction to the Interstellar Medium, J. P. Williams]

- [O I] 63.2μm (collisionally excited emission line)
  - The electronic ground state of O I has a fine splitting of $E_{u\ell}/k = 228\,\mathrm{K}$.
  - The upper level is populated primarily by collisions with hydrogen atoms.
  - The resulting cooling function due to the emission of 63.2μm is, for an abundance of $n_\mathrm{O}/n_\mathrm{H} = 5.4\times10^{-4}$,

$$\frac{\Lambda^{\mathrm{H}}_{[\mathrm{OI}]}}{10^{-25}\,\mathrm{erg\ cm^3\ s^{-1}}} \approx 0.04\left(\frac{T}{100\,\mathrm{K}}\right)^{0.42}\exp\left(-\frac{228\,\mathrm{K}}{T}\right)$$

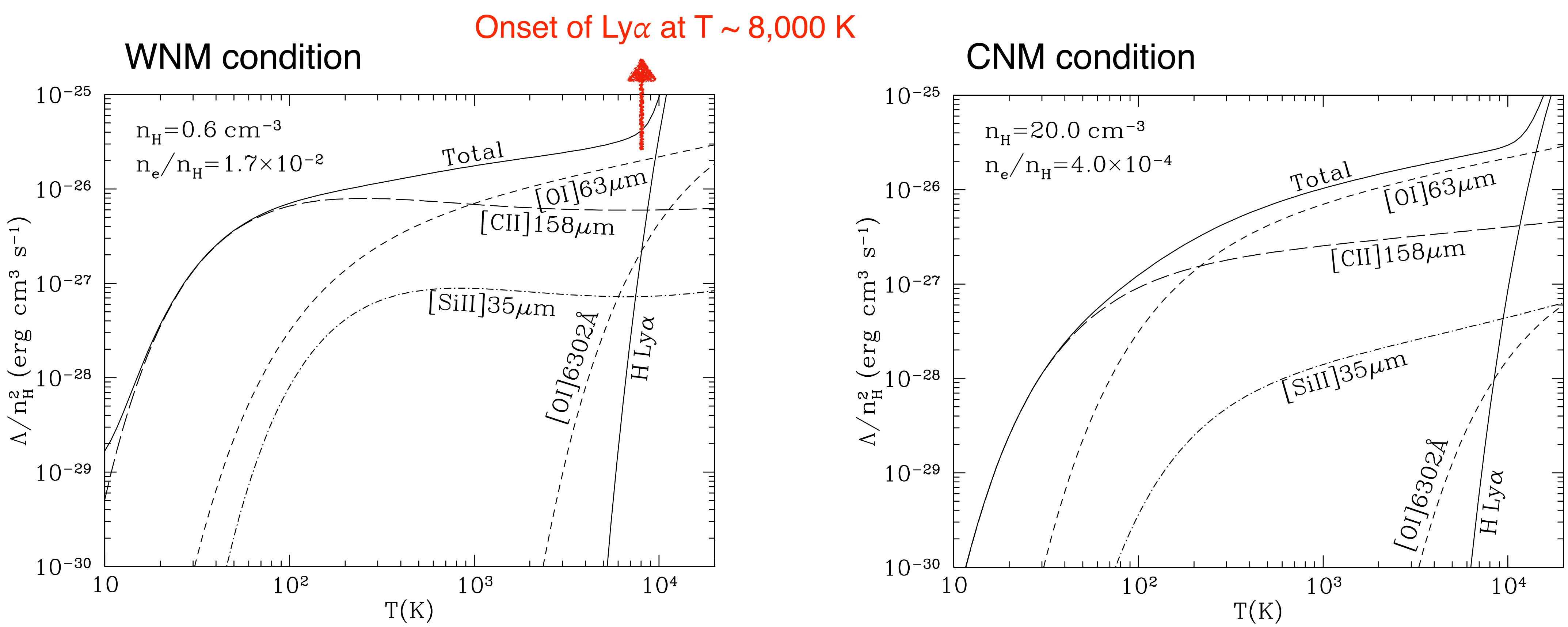
- Note:
  - [C II] and [O I] are the dominant form of cooling in molecular clouds and the CNM.
  - Molecular clouds can also cool by emission from the vibrational and rotational transitions of molecules.

- Lyα 1216Å
  - The first excited level of atomic hydrogen is $E_{21} = 10.20\,\mathrm{eV}$ above the ground state.
  - Although the first excited level will not be highly populated by collisions until the temperature reaches $T \sim E_{21}/k = 118{,}000\,\mathrm{K}$, hydrogen is extremely abundant. Thus the cooling by Lyα can compete with cooling by IR fine-structure lines at temperature as low as T ~ 8000 K.
  - The cooling function for H excited by collisions with free electrons is

$$\frac{\Lambda^{\mathrm{e}}_{[\mathrm{Ly}\alpha]}}{10^{-25}\,\mathrm{erg\ cm^3\ s^{-1}}} \approx 7000 \left(\frac{x}{10^{-3}}\right)\left(\frac{T}{100\,\mathrm{K}}\right)^{-0.5} \exp\left(-\frac{118{,}000\,\mathrm{K}}{T}\right)$$

- When T > 15,000 K,
  - atomic hydrogen can be collisionally ionized, followed by radiative recombination to a high energy level, and followed by a cascade down to the ground state.
  - The recombination lines are an important cooling mechanism in the WNM and WIM.
  - These phases are also cooled by line emission from more highly ionized atoms such as O III, C IV, and O VI.

- Thermal Bremsstrahlung
  - In the HIM at T > 10^6 K, the "braking radiation" emitted by electrons when they are accelerated by other charged particles can be a significant cooling mechanism.
  - The cooling function is $\Lambda \propto T^{1/2}$.
- Cooling Function



  - For 10 < T < 10^4 K, [C II] 158μm line is a major coolant. The [O I] 63μm line is important for T > 100 K. Lyα cooling dominates only at T > 10^4 K.

Stable & Unstable Equilibrium

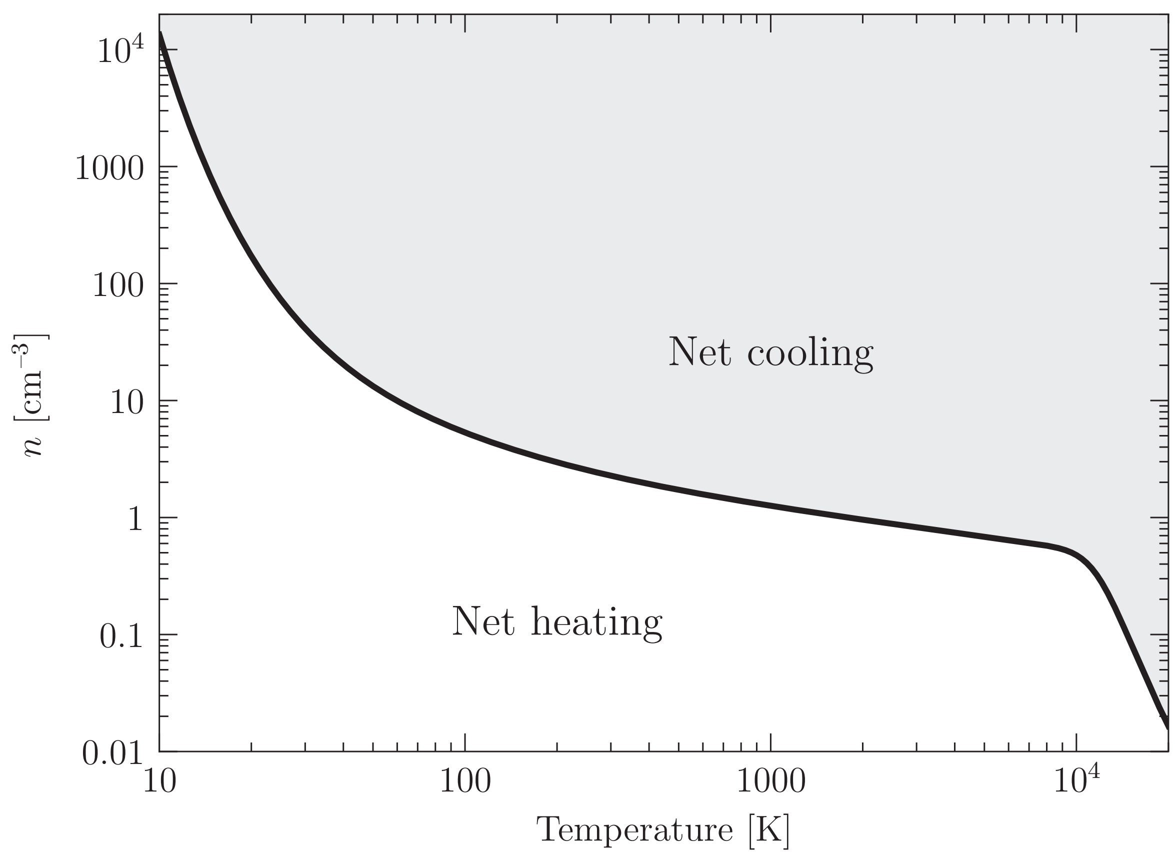
- A ***thermal equilibrium*** must have heating and cooling balanced: $g = \ell$.
  - We assume ***photoelectric heating by dust*** and ***cooling by [C II], [O I], and Lyα***. Then, the equilibrium density is obtained by

$$n_{\rm eq} G = n_{\rm eq}^2 \Lambda \quad \rightarrow \quad n_{\rm eq}(T) = \frac{G}{\Lambda(T)}$$

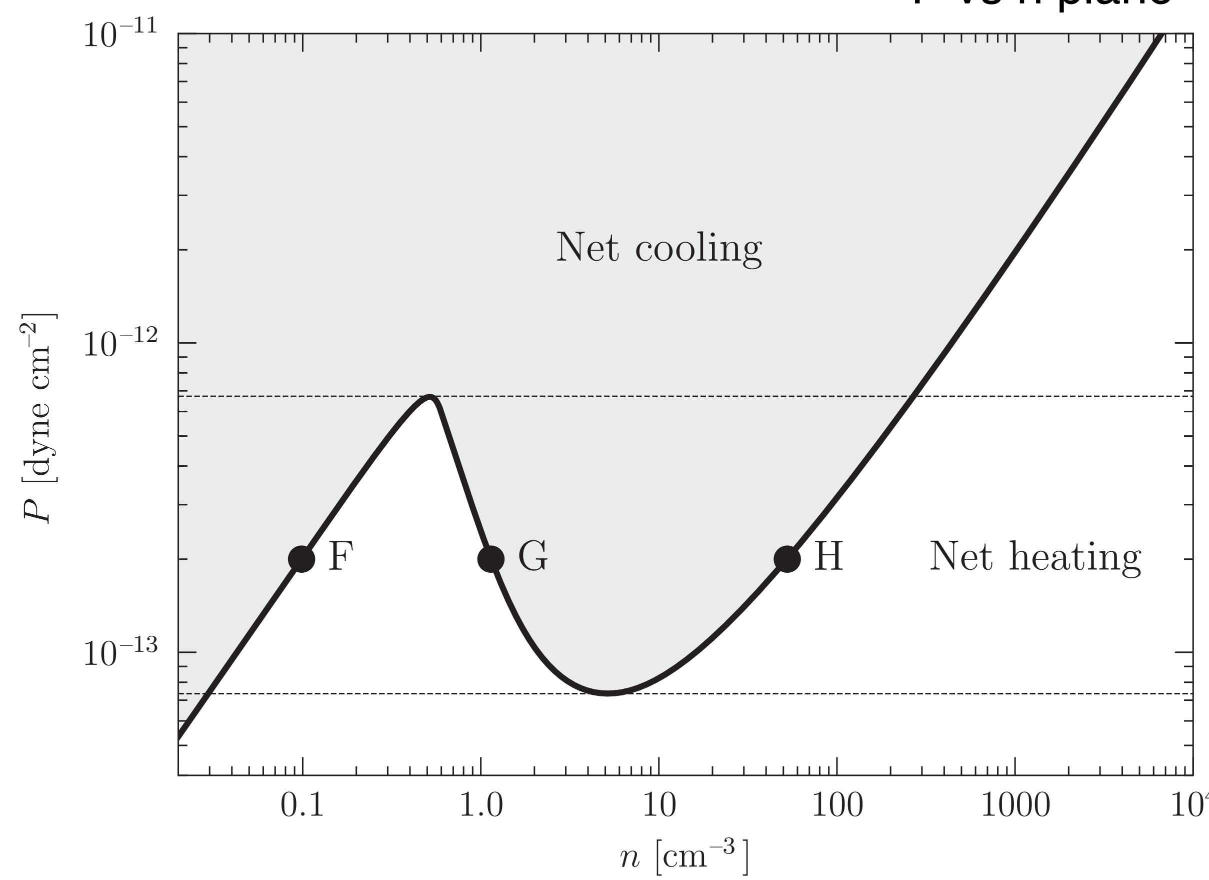
Note that G is a (nearly) constant.

$P = n k_{\rm B} T$

n vs T plane



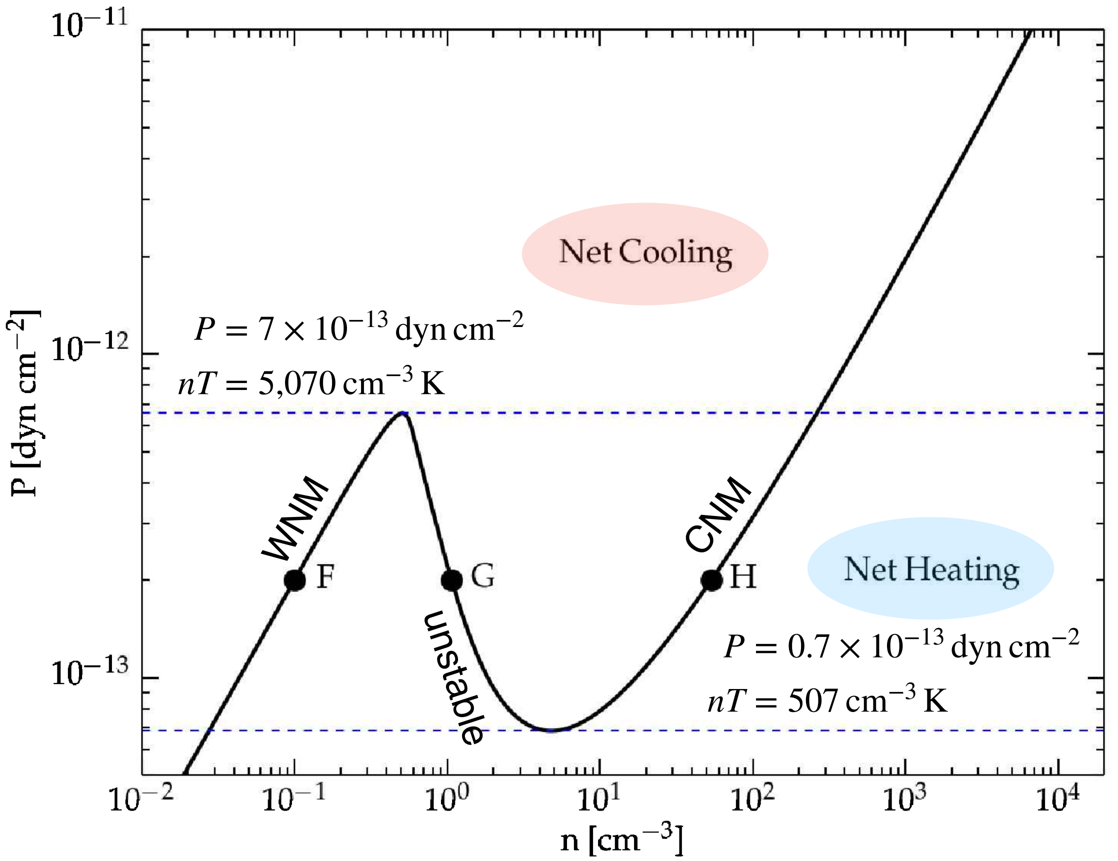
P vs n plane



  - If every point along the above equilibrium line represented a stable equilibrium, then there could be a continuous distribution of temperatures, and thus of number densities.
  - However, it's not the case. Not every equilibrium point is a stable equilibrium.

- Pressure Equilibrium

- Let's assume that the interstellar gas is in pressure equilibrium.
  - For pressures in the range $0.7 \times 10^{-13}\,\mathrm{dyn\,cm^{-2}} < P < 7 \times 10^{-13}\,\mathrm{dyn\,cm^{-2}}$, bounded by the dashed lines, there are three possible values of $n_{\rm eq}$ at a fixed pressure.



- Consider what happens at a point, for instance F, if you slightly change the temperature while keeping the pressure fixed.
  - If T increases, n must decrease, and you must move left from point F. This moves you into the net cooling portion, and T consequently decreases.
  - If T decrease, n must increase, and this moves you rightward into the net heating portion, and T consequently increases.
  - Thus, a negative feedback restores the original temperature.
- A similar negative feedback maintains temperature stability at point H.
- However, now consider what happens at G.
  - If T increases, n must decrease, and you must move left from point G. This moves you into the net heating portion, and T increases further, until you reach F.
  - If T decrease, n must increase, and this moves you rightward into the net cooling portion, and T decrease further, until you reach H.
  - Thus, a positive feedback makes the point unstable.
- **Consequently, we have two stable equilibrium points (F and H). F = WNM, H = CNM**

Two-Phase Model & Three-Phase Model

- As a result of their analysis, Field, Goldsmith, and Habing (1969) created a two-phase model of the ISM, consisting of Cold Neutral Clouds, with n ~ 10 cm^{-3} and T ~ 100 K, embedded within a Warm Intercloud Medium, with n ~ 0.1 cm^{-3} and T ~ 10,000 K.

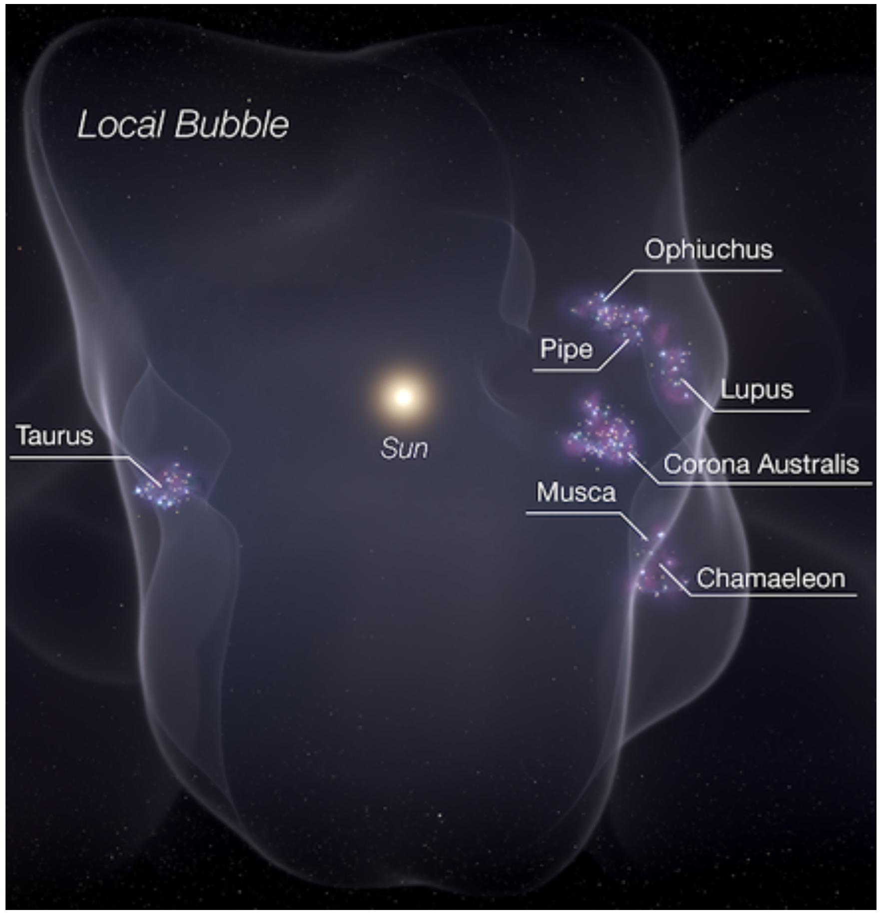
  ✦ They were unaware of the role played by dust in heating the ISM, assumed that ***collisional ionization by cosmic rays provided the bulk of the heating***.

  ✦ FGH (1969) advocated a two-phase model. However, they also speculated "an existence of a third stable phase at T > 10^6 K, with bremsstrahlung the chief cooling process."

- In the 1970s, detection of a diffuse soft X-ray background and of emission lines such as O VI 1032, 1038Å hinted at the existence of interstellar gas with T ~ 10^6 K. In fact, the Sun resides in a "***Local Bubble***" of hot gas, with T ~ 10^6 K and n ~ 0.004 cm^{-3}.

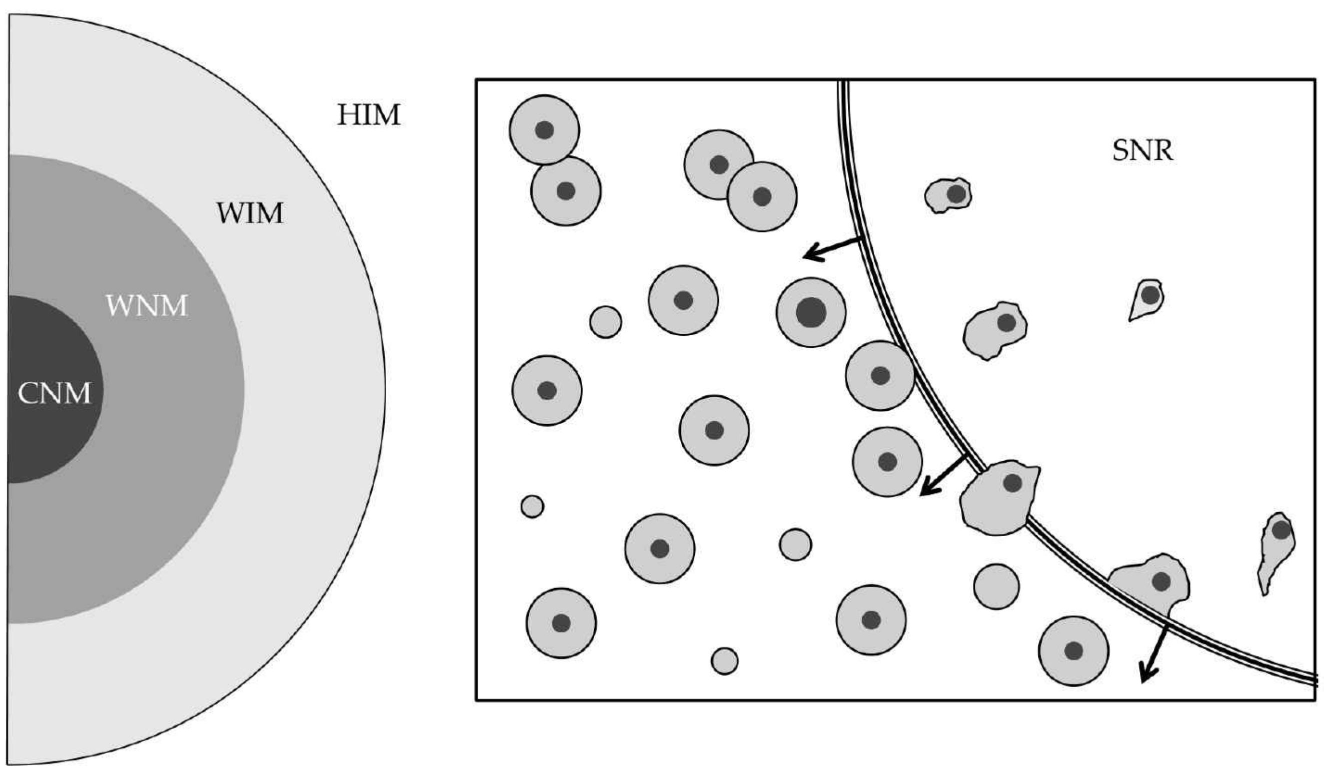
- Cox & Smith (1974) suggested that supernova remnants could produce a bubbly hot phase, and that the bubbles blown by supernovae would occupy a large volume fraction of the ISM.

- A superbubble or supershell is a cavity which is ~100 pc across and is populated with hot (10^6 K) gas atoms, less dense than the surrounding ISM, blown against that medium and carved out by multiple supernovae and stellar winds.



McKee & Ostriker’s Three-Phase Model

- McKee & Ostriker (1977)
  - They made a more elaborate argument for three phases within the ISM.
  - ***Cold Neutral Medium***, with $T \sim 80\,\mathrm{K}$, $n \sim 40\,\mathrm{cm}^{-3}$, and a low fractional ionization $x = n_e/n \lesssim 0.001$.
  - ***Warm Medium***, containing both ionized and neutral components, $T \sim 8000\,\mathrm{K}$ and $n \sim 0.3\,\mathrm{cm}^{-3}$, the ionization fraction ranging from $x \sim 0.02$ in the neutral component (WNM) to $x \sim 0.15$ in the ionized component (WIM).
  - ***Hot Ionized Medium***, consisting of the overlapping supernova bubbles, with $T \sim 10^6\,\mathrm{K}$ and $n \sim 0.002\,\mathrm{cm}^{-3}$, and $x \sim 1$ (nearly complete ionization).



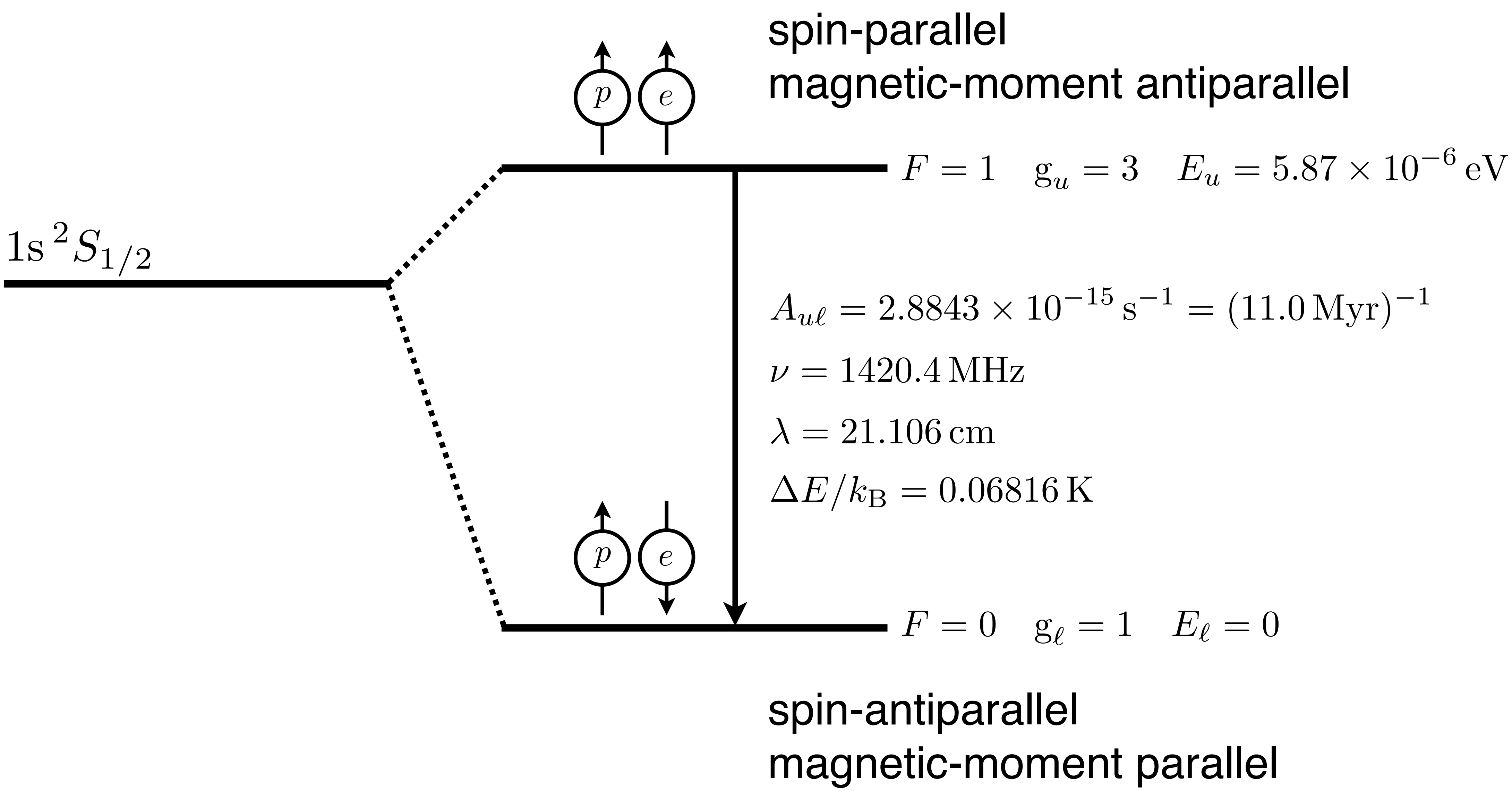
- However, in many ways, the ISM is a dynamic, turbulent, dusty, magnetized place.

Atomic Gas / Hydrogen Gas

Hydrogen Gas - 21 cm hyperfine line

- The CNM and WNM, taken together, provide over half the mass of the ISM.
  - H is the most abundant element in the universe. In the CNM and WNM, most of the hydrogen is in the form of neutral atoms.
  - The Lyα line of H provides a useful probe of the properties of the CNM and WNM. However, at its wavelength the Earth's atmosphere is highly opaque, and thus observing Lyα absorption requires orbiting UV satellites. In addition, Lyα can be seen in absorption only along those lines of sight toward sources with a high UV flux.
  - To do a global survey of atomic hydrogen in the galaxy, we need some way of easily detecting radiation from hydrogen, regardless of its kinetic temperature or number density.
  - Such a way was first found in 1944, by Henk van de Hulst.
    - He attempted to find emission lines at the wavelengths ~ 1 cm to 20 m, at which the Earth's atmosphere is transparent. He then realized that the hyperfine structure line resulting from a flip of the electron spin within a hydrogen atom should have a wavelength of 21 cm.
    - This was confirmed by Harold Ewen and Edward Purcell in 1951, when they first detected 21 cm emission from the Milky Way.

Hyperfine splitting of the 1s ground state of atomic H



Note that the magnetic moment is proportional to the charge, so the electron and proton have opposite directions of the magnetic moments.

Difference between Lyα and 21 cm transitions

- The excitation energy for Lyα (E = 10.2 eV, E/k = 118,000 K) is much higher than the kinetic temperature of the neutral ISM.

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} \exp\left(-\frac{118,000\,\mathrm{K}}{T}\right) = 1.7 \times 10^{-51} \text{ at } T = 1000\,\mathrm{K} \qquad (g_u/g_\ell = 3)$$

  - Collisional excitation is unimportant, and most hydrogen atoms are in the lower level of the Lyα transition.
  - The Lyα has a higher energy by a factor of $1.7\mathrm{x}10^6$ than the 21 cm.

- The excitation energy for 21 cm is ~ 5.9 μeV, and its equivalent temperature E/k = 0.068 K is much lower than the temperature of the cosmic microwave background.
  - Even the CMB is able to populate the upper level.
  - If collisions are frequent, then **the spin temperature will be solely determined by collisions**, and thus will be a good tracer of the gas kinetic temperature.
  - Thus, there is ample opportunity to populate the upper energy level of the 21 cm hyperfine transition. The level populations for the 21 cm levels, since $T_{\mathrm{exc}} \gg 0.068\,\mathrm{K}$ in all circumstances of the ISM.

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} e^{-h\nu/kT_{\mathrm{exc}}} = 3\,e^{-0.068\,\mathrm{K}/T_{\mathrm{exc}}} \simeq 3 \longrightarrow \boxed{n_u \simeq \frac{3}{4} n_{\mathrm{H}}, \; n_\ell \simeq \frac{1}{4} n_{\mathrm{H}}}$$

  - However, in many cases (in particular in WNM), the hyperfine levels may not be in excitation equilibrium. Radio astronomers use the term ***spin temperature*** for 21 cm rather than the “excitation temperature.”

Typical Optical Depths of the 21-cm line

- Typical optical depths of the 21-cm line:

$$\tau_\nu = \kappa_\nu \Delta s = \frac{3}{32\pi} A_{u\ell} \frac{hc\,\lambda_{u\ell}}{kT_{\rm spin}} (n_{\rm H} \Delta s)\, \phi_\nu$$

$$\tau_0 = 0.311 \left(\frac{N_{\rm H\,I}}{10^{21}\ {\rm cm}^{-2}}\right) \left(\frac{100\,{\rm K}}{T_{\rm spin}}\right) \left(\frac{10\,{\rm km\,s}^{-1}}{b}\right) \longleftarrow$$

column density:

$$N_{\rm HI} = n_{\rm HI} \Delta s \quad [{\rm cm}^{-2}]$$

- In the CNM, a typical spin temperature is $T_{\rm spin} \approx 50 - 100\,K$:

$$\tau_0^{\rm CNM} \approx 0.3 - 0.6$$
$$e^{-\tau_0} \approx 0.55 - 0.74$$

The CNM is in general optically thin, but show significant absorption.

- In the WNM, a typical spin temperature is $T_{\rm spin} \approx 5000 - 8000\,K$:

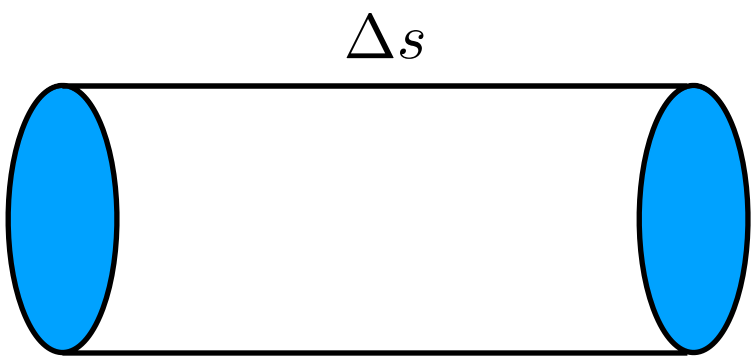
$$\tau_0^{\rm WNM} \approx 0.004 - 0.006$$
$$e^{-\tau_0} \approx 0.995$$

The 21-cm absorption is negligible in the WNM.

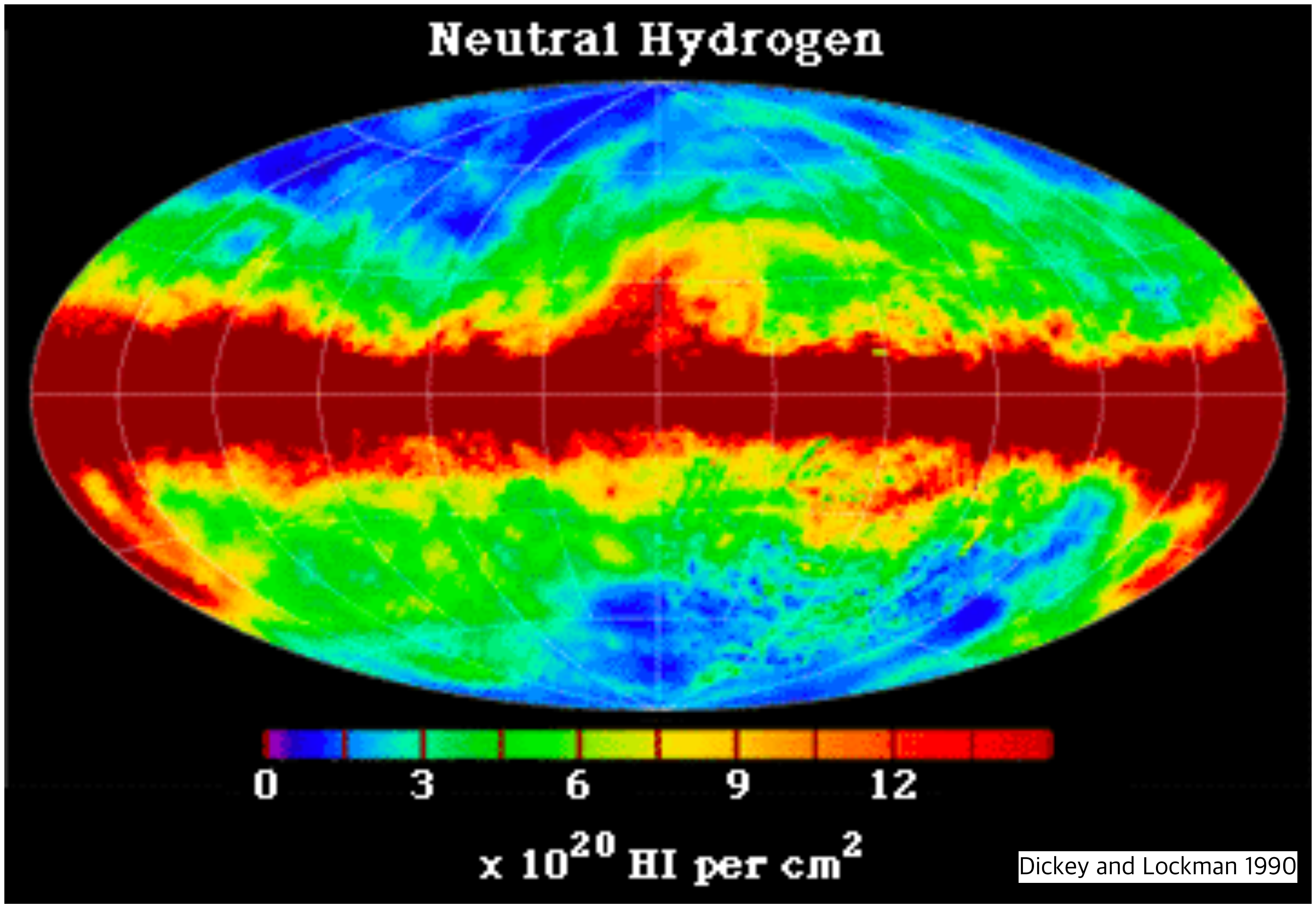
Typical H I column density in our Galaxy

- Column density

$$N_{\rm HI} = n_{\rm HI}\Delta s \quad [{\rm cm}^{-2}] \qquad \Leftarrow \qquad N_{\rm HI} = \frac{n_{\rm HI} V}{\Delta A}$$



$$\Delta A = 1\,{\rm cm}^2$$



$$N_{\rm HI} = 10^{20} - 10^{22}\,{\rm cm}^2$$

in our Galaxy, except for the Lockman hole

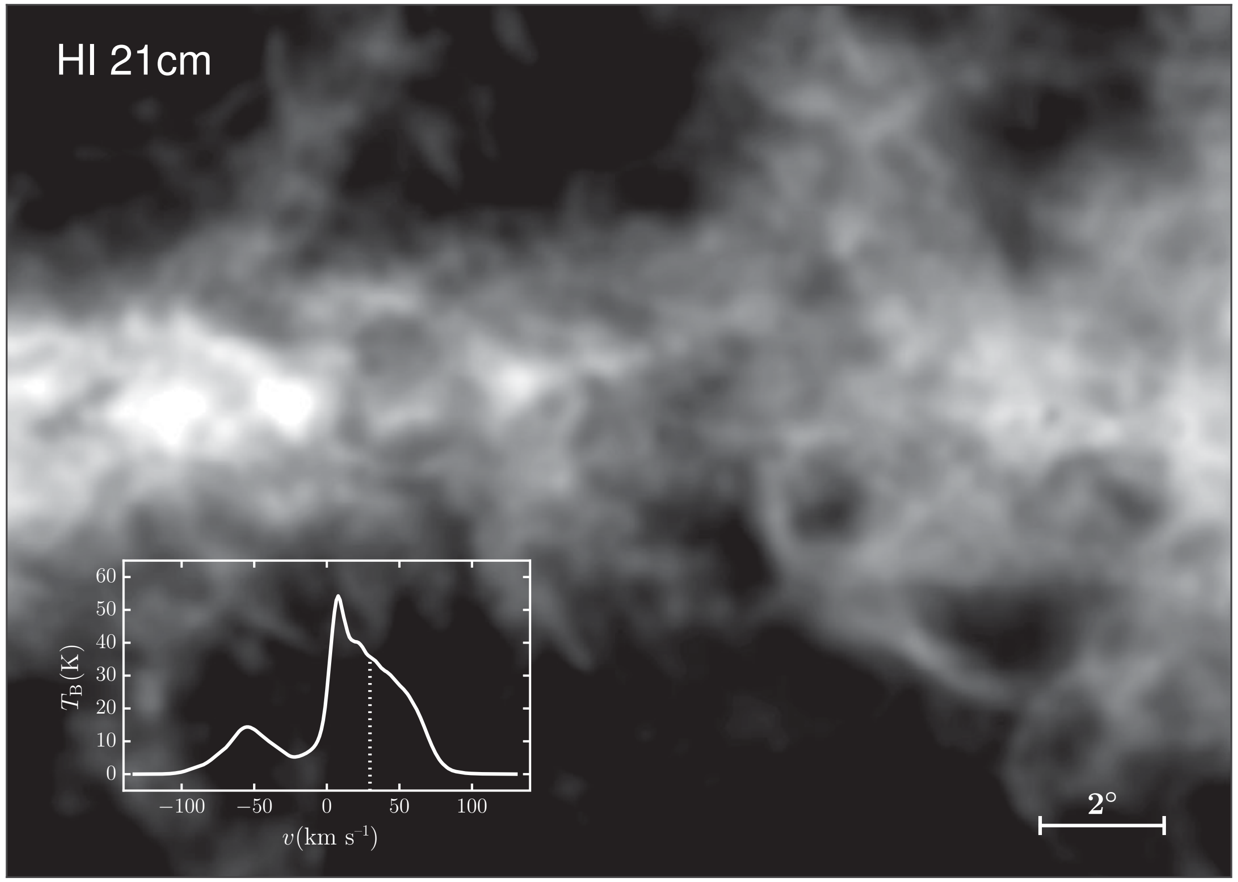
The Lockman hole is an area of the sky in which minimal amounts of neutral hydrogen gas are observed.

Column density in Lockman hole

$$N_{\rm HI} \approx 5 \times 10^{19}\ {\rm cm}^2$$

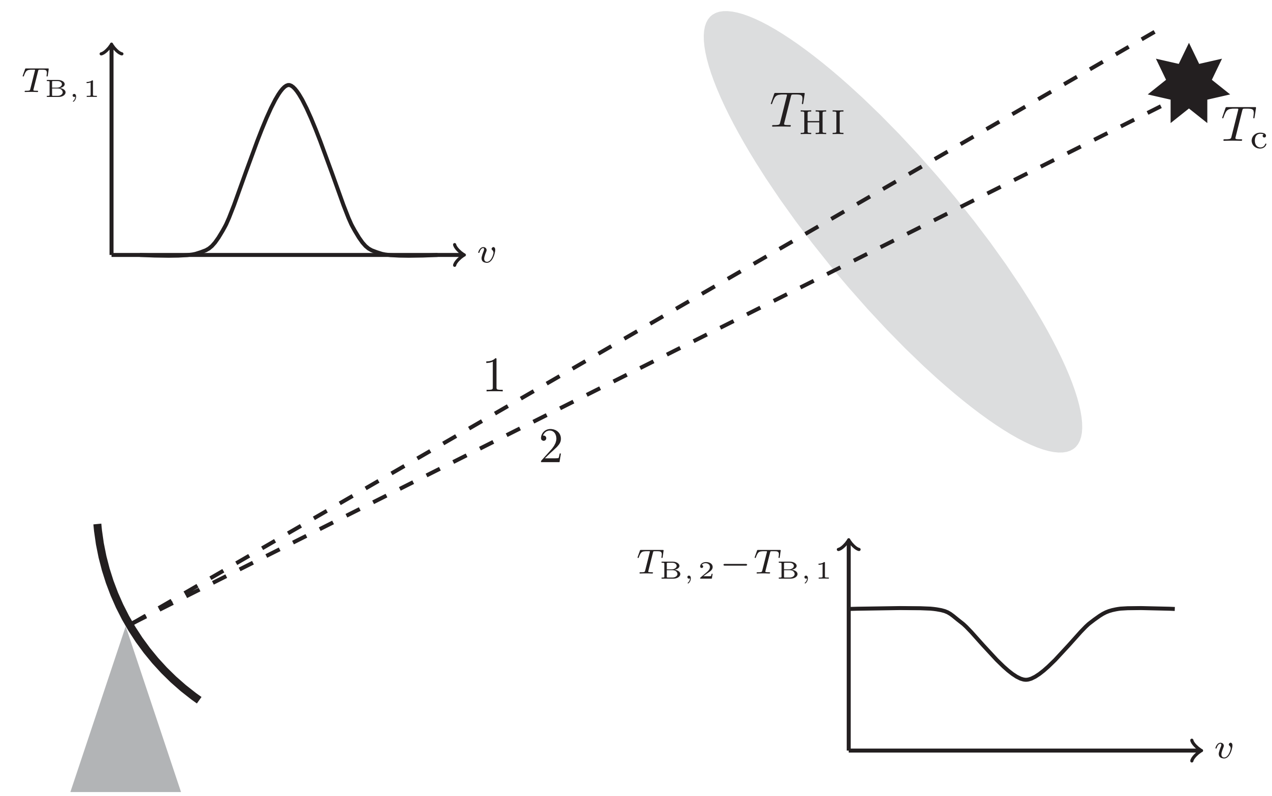
Coordinates:
(*l*, *b*) = (149.77, 52.03)
(RA, dec) = (10h45m, +58deg)
Size: ~ 15 square degrees

21 cm emission & absorption lines



Galactic 21 cm H I line emission from the HI4PI survey.

The inset shows the mean spectrum of this region in units of brightness temperature versus velocity.
The vertical dotted line shows the velocity slice shown in the image.



Schematic of an absorption experiment to measure the temperature of an atomic cloud.

The first line of sight measures the emission from the cloud only.
The second line of sight contains a background source with continuum (brightness) temperature T_c.
The difference between the two lines of sight reveals a dip in the continuum emission due to absorption by the cloud.

[Introduction to Interstellar Medium, J.P. Williams]

Ionized Gas - Ionization Equilibrium

- ***Photoionization Equilibrium***:
  - Balance between photo-ionization and the process of recombination.

- ***Collisional Ionization Equilibrium (CIE)*** or coronal equilibrium
  - Balance at a given temperature between collisional ionization from the ground states of the various atoms and ions, and the process of recombination from the higher ionization stages.
  - In this equilibrium, effectively, all ions are in their ground state.

H II regions: Photoionization and Recombination of Hydrogen

- Interstellar medium (ISM), which is primarily composed of hydrogen, is transparent to $h\nu$ < 13.6 eV photons, but is very opaque to ionizing photons with $h\nu$ > 13.6 eV.

  - Ionized atomic hydrogen regions, broadly termed "H II regions", are composed of gas ionized by photons with energies above the hydrogen ionization energy of 13.6 eV.

$$\mathrm{H} + h\nu \;\rightarrow\; \mathrm{H}^+ + e^- + \Delta E$$

  - Sources of ionizing photons include massive, hot young stars, hot white dwarfs, and supernova remnant shocks.

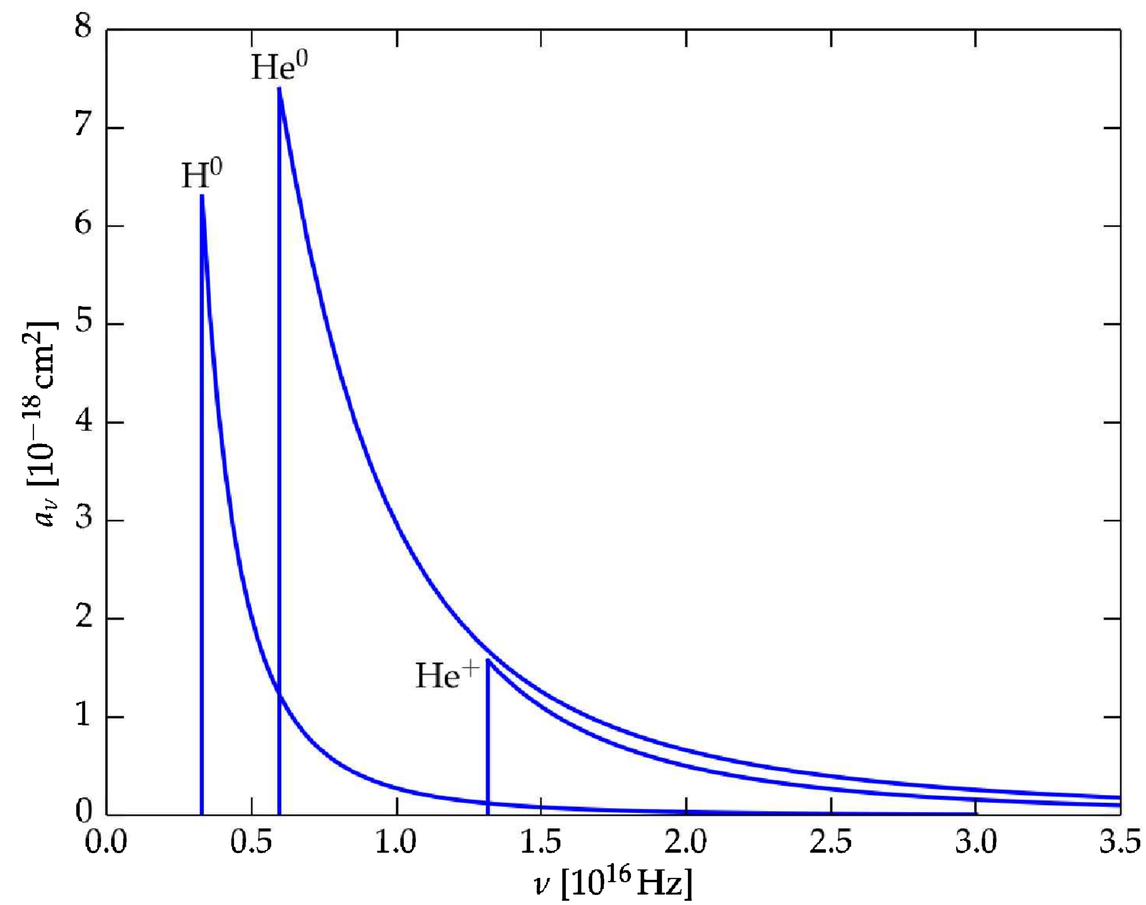
- The photoionization cross-section of H depends strongly on frequency and is reasonably approximated by a power-law:

$$\sigma_{\rm pi}(\nu) \approx \sigma_0 \left(\frac{h\nu}{I_{\rm H}}\right)^{-3} \quad \text{for } I_{\rm H} \lesssim h\nu \lesssim 100 I_{\rm H}$$

$$(I_{\rm H} = 13.6\,{\rm eV})$$

$$\sigma_0 \equiv \frac{2^9\pi}{3{\rm e}^4}\alpha\pi a_0^2 = 6.304 \times 10^{-18}\,{\rm cm}^{-2}$$

fine-structure constant
$(\alpha \equiv e^2/hc = 1/137.04,\ {\rm e} = 2.71828...)$



Photoionization cross section for hydrogen(H^0), hydrogenic helium (He$^+$), and neutral helium (He^0). [Fig. 4.1 in Ryden]

- The recombination cross-section (recombination to levels $n \geq 2$) is given in an approximate power law form,

$$\alpha_{\rm B}(T) \equiv \sum_{n=2}^{\infty}\sum_{\ell=0}^{n-1}\alpha_{n\ell}(T)$$

$$\approx 2.59 \times 10^{-13}\, T_4^{-0.833-0.034\ln T_4}\ [{\rm cm}^3\,{\rm s}^{-1}] \quad \text{for } 0.3 \lesssim T_4 = T/10^4\,{\rm K} \lesssim 3$$

The Strömgren Sphere: A Uniform, Pure Hydrogen Nebula

- ***Strömgren Sphere***:
  - Following Strömgren (1939), we consider the simple idealized problem of a fully ionized, spherical region of uniform medium plus a central source of ionizing photons.
  - The ionization is assumed to be maintained by absorption of the ionizing photons radiated by a central hot star. The central source produces ionizing photons, with energy $\nu > \nu_0 = I_{\rm H}/h$ at a constant rate $Q_0 \equiv \dot{N}_{\rm ionize}$ [photons s^{-1}].
- ***O star*** temperatures, masses, and ionizing photon rates
  - The emission of the most massive (O-type) stars can be approximated as a blackbody with a peak energy $E \sim 5hc/kT \approx 12.9$ eV, very close to the hydrogen ionization potential.
  - O stars produce enough ionizing photons to create large ionized, or H II, regions.

SpT	T_*(K)	$M_*(M_\odot)$	$\log_{10}(L_*/L_\odot)$	$\log_{10}(\dot{N}_{\rm ionize})$
O3	44600	58.3	5.83	49.63
O4	43400	46.2	5.68	49.47
O5	41500	37.3	5.51	49.26
O6	38200	31.7	5.30	48.96
O7	35500	26.5	5.10	48.63
O8	33380	22.0	4.90	48.29
O9	31500	18.0	4.72	47.90

- Ionization and Recombination rates:
  - Ionization rate per neutral hydrogen atom at a radius r = 1 pc from an O star is

$$R_{\rm ionize} = \frac{\dot{N}_{\rm ionize}}{4\pi r^2}\sigma_0 \approx 3\times 10^{-7}\,{\rm s}^{-1}$$

  - This is much greater than the recombination rate per hydrogen ion,

$$R_{\rm recombine} = \alpha_{\rm B} n_e \approx 3\times 10^{-12}\left(\frac{n_e}{10\,{\rm cm}^{-3}}\right)\ {\rm s}^{-1}\quad {\rm at}\ T = 10^4\,{\rm K}$$

  - This implies a neutral hydrogen atom near the center of the nebula would survive a few months before being ionized but then, once ionized, would have to wait about ten thousand years before recombining with a free electron. We conclude that, at 1 pc, the nebula is almost fully ionized.
  - As we move outwards in the nebular away from the O star the ionization rate decreases as $1/r^2$ and eventually will match the recombination rate. This define the boundary of the H II region.

- The transition from ionized to neutral will occur at a length scale of the mean free path of photoionization in the neutral gas.

$$\lambda_{\rm mfp} = \frac{1}{n_{\rm H}\sigma_{\rm pi}} \sim 5 \times 10^{-4}\,{\rm pc} \left(\frac{n_{\rm H}}{10^2\,{\rm cm}^{-2}}\right)^{-1} \left(\frac{\sigma_{\rm pi}}{6.304 \times 10^{-18}\,{\rm cm}^{-2}}\right)^{-1}$$

  - This length scale is over a thousand times smaller than the observed size of the nebula.
  - Therefore, the gas is either fully ionized or neutral.
  - This tells us that the transition from ionized gas to neutral gas at the boundary of the H II region will occur over a distance that is very small compared to the Strömgren radius.

- Assuming that ***the ionization is nearly complete (*** $n_p = n_e = n_{\rm H}$ ***) within*** R_s , and nearly zero ($n_{\rm H^0} = n_{\rm H},\ n_e = 0$) outside R_s , we obtain the size of the ionized region by simply equating the rate of ionization to the rate of recombination (over the whole volume of the ionized region):

$$Q_0 = \left(n_{\rm H}^2 \alpha_{\rm B}\right) \frac{4\pi}{3} R_s^3$$

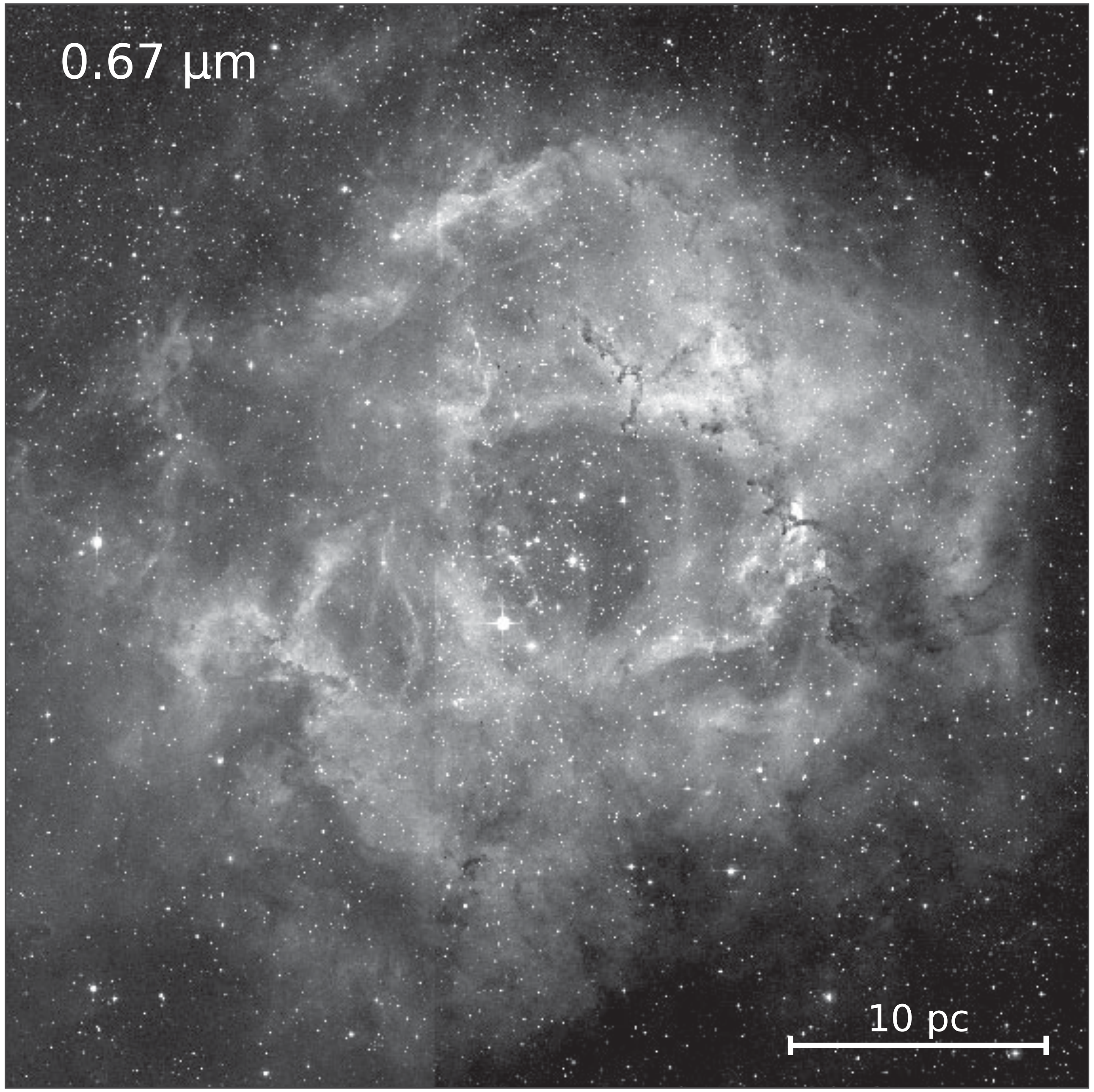
- This gives the Stromgren radius:

$$\begin{aligned} R_s &= \left(\frac{3}{4\pi} \frac{Q_0}{\alpha_{\rm B} n_{\rm H}^2}\right)^{1/3} \\ &= 3.17 \left(\frac{Q_0}{10^{49}\,\text{photons s}^{-1}}\right)^{1/3} \left(\frac{n_{\rm H}}{10^2\,\text{cm}^{-3}}\right)^{-2/3} \left(\frac{T}{10^4\,\text{K}}\right)^{0.28} \quad [\text{pc}] \end{aligned}$$

The physical meaning of this is that ***the total number of ionizing photons emitted by the star balances the total number of recombinations within the ionized volume*** $(4\pi/3)R_s^3$, often called the Strömgren sphere. It's radius R_s is called the Strömgren radius.

H II region - Rosette nebula

- For the Rosette, we find R_S = 40 pc which is about a factor of 2 larger than its observed extent. The discrepancy is because some of he ionizing photons are absorbed by dust.

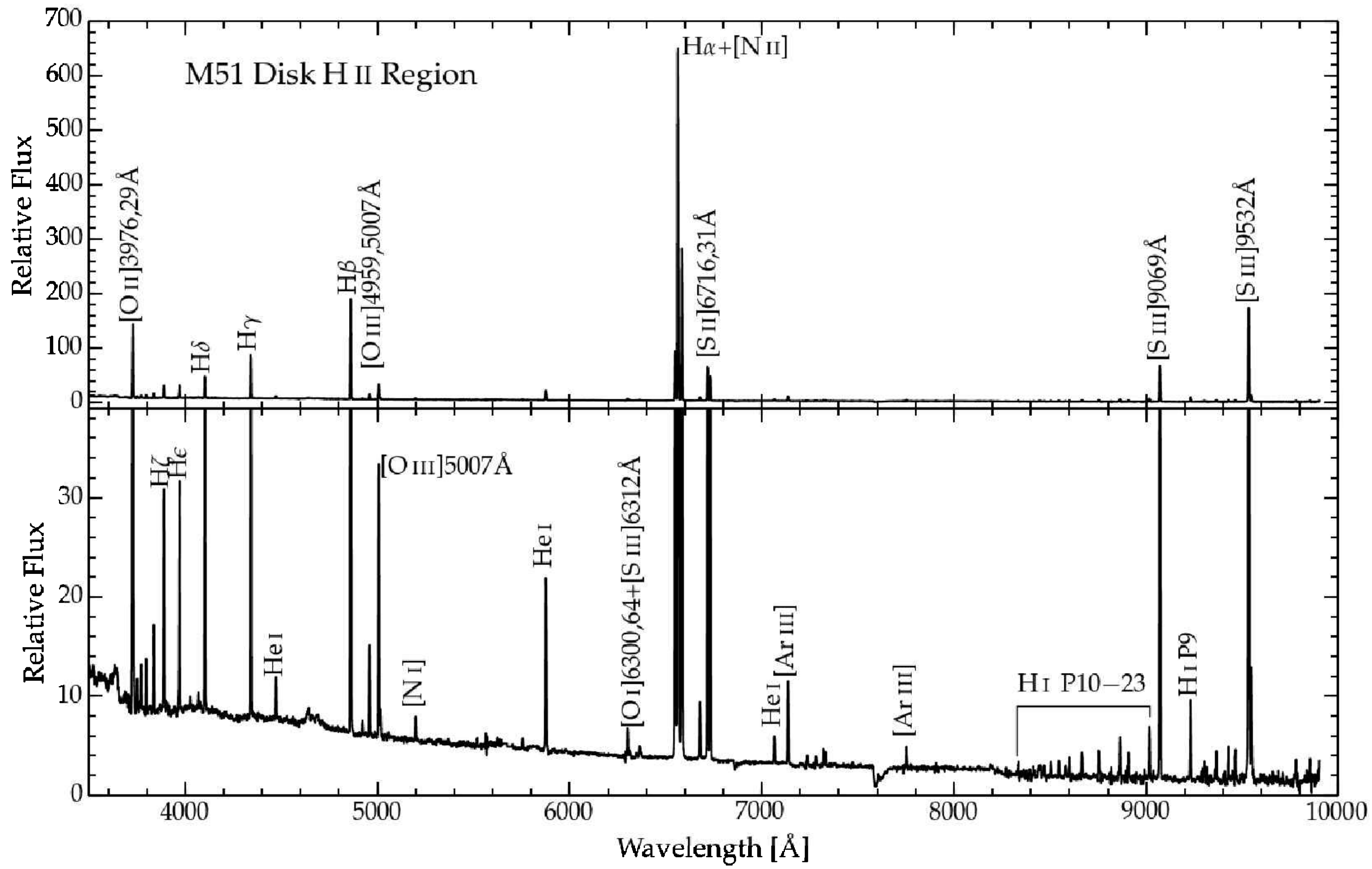


Optical (DSS2 Red) image of the Rosette nebula.

[Fig 6.1, J. P. Williams]

Nebular Emission Lines

- In the figure, the continuum is a mixture of free-bound continuum (from radiative recombination), free-free emission (thermal bremsstrahlung), and two-photon emission.
- The ***collisionally excited emission lines*** are much stronger than the continuum spectrum. The collisional emission lines are utilized to measure the temperature and density of the medium.



Spectrum of a disk HII region in the Whirlpool galaxy (M51).

(top) bright lines
(bottom) scaled to show faint lines.

Figure 4.6 [Ryden]

Case A and B (Radiative Recombination of Hydrogen)

- ***On-the-spot approximation:***
  - In optically thick regions, it is assumed that every photon produced by radiative recombination to the ground state of hydrogen is immediately, then and there, destroyed in photoionizing other hydrogen atom.
  - In the on-the-spot approximation, recombination to the ground state has no net effect on the ionization state of the hydrogen gas.
- Baker & Menzel (1938) proposed two limiting cases:
  - ***Case A:*** ***Optically thin*** to ionizing radiation, so that every ionizing photon emitted during the recombination process escapes. For this case, we sum the radiative capture rate coefficient $\alpha_{n\ell}$ over all levels $n\ell$.
  - ***Case B:*** ***Optically thick*** to radiation just above $I_{\mathrm{H}} = 13.60\,\mathrm{eV}$, so that ionizing photons emitted during recombination are immediately reabsorbed, creating another ion and free electron by photoionization. In this case, the recombinations directly to $n = 1$ do not reduce the ionization of the gas: ***only recombinations to*** $n \geq 2$ ***act to reduce the ionization.***

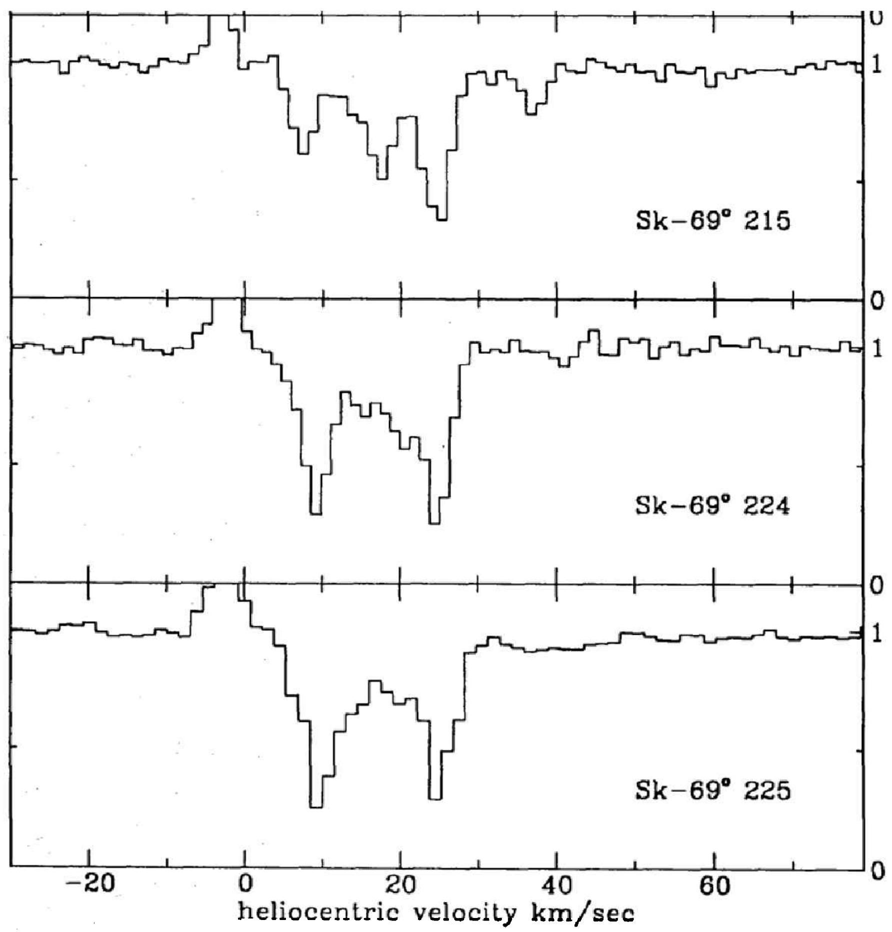
  - **Case B in photoionized gas**: Photoionized nebulae around OB stars (H II regions) usually have large enough densities of neutral H. For this situation, case B is an excellent approximation.
  - **Case A in collisionally ionized gas**: Regions where the hydrogen is collisional ionized are typically very hot ($T > 10^6$ K) and contain a very small density of neutral hydrogen. For these shock-heated regions, case A is an excellent approximation.

Properties of Ionized Hydrogen Regions

- Ionized atomic hydrogen regions, broadly termed "H II regions", are composed of gas ionized by photons with energies above the hydrogen ionization energy of 13.6 eV.
  - ***Ionization Bounded:*** These objects include "***classical H II regions***" ionized by hot O or B stars (or clusters of such stars) and associated with regions of recent massive-star formation, and "planetary nebulae", the ejected outer envelopes of AGB stars photoionized by the hot remnant stellar core.
  - ***Density Bounded: Warm Ionized Medium*** / Diffuse Ionized Gas: Ionized Gas in the diffuse ISM, far away from OB associations.
  - The UV, visible and IR spectra of H II regions are very rich in emission lines, primarily collisional excited lines of metal ions and recombination lines of hydrogen and helium. H II regions are also observed at radio wavelengths, emitting radio free-free emission from thermalized electrons and radio recombination lines from highly excited states of H, He, and some metals (e.g., H109α and C lines).
- Three processes govern the physics of H II regions:
  - ***Photoionization Equilibrium***: the balance between photoionization and recombination. This determines the spatial distribution of ionic states of the elements in the ionized zone.
  - ***Thermal Balance*** between heating and cooling. Heating is dominated by photoelectrons ejected from hydrogen and helium with thermal energies of a few eV. Cooling is mostly dominated by electron-ion impact excitation of metal ion followed by emission of "forbidden" lines from low-lying fine structure levels. It is these cooling lines that give H II regions their characteristic spectra.
  - ***Hydrodynamics***, including shocks, ionization and photodissociation fronts, and outflows and winds from the embedded stars.

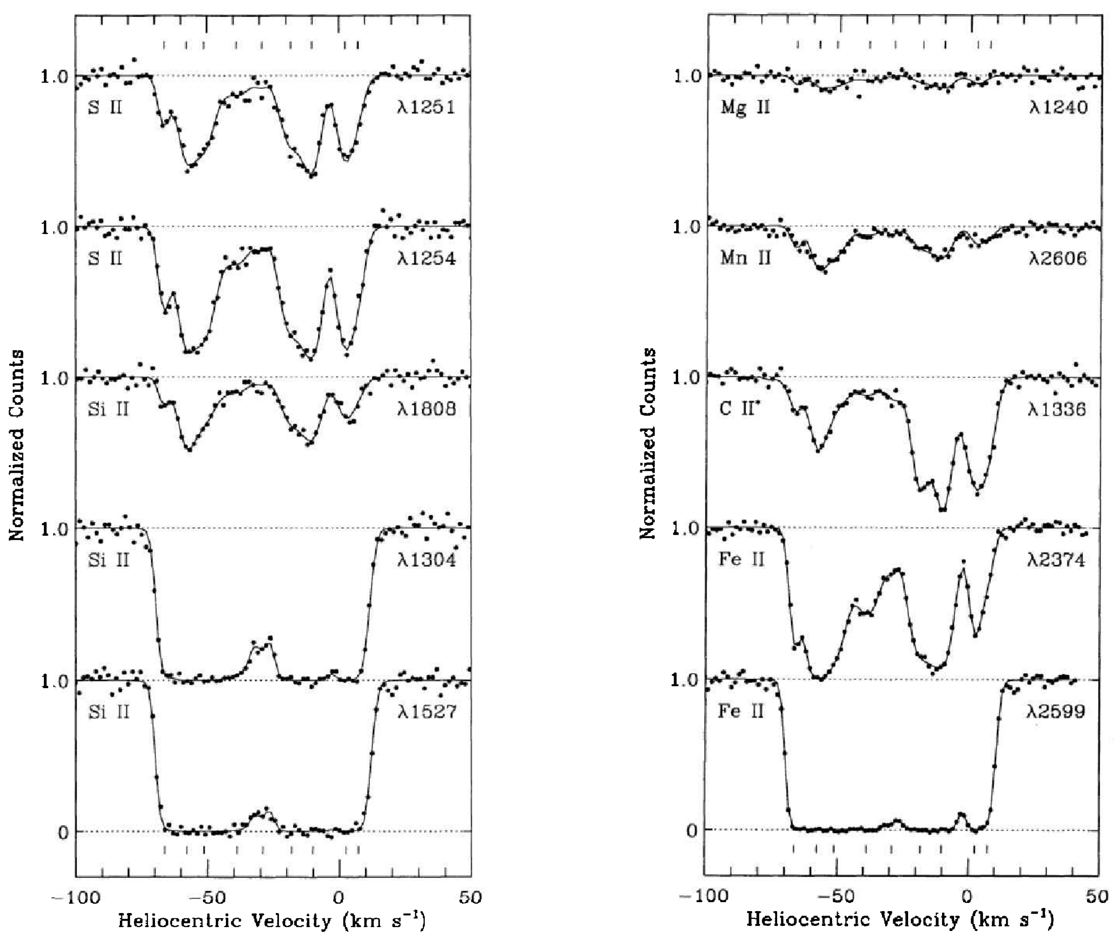
Observations of Metallic Absorption Lines Toward the CNM

- The CNM gives rise to a number of absorption features in the spectra of hot background stars.
  - The most prominent absorption lines at visible wavelengths are Ca II K and H lines at $\lambda = 3933, 3968\ \text{Å}$, and Na I D_1 and D_2 doublet lines at $\lambda = 5890, 5896\ \text{Å}$.



Na I D_2 interstellar absorption line seen along 3 lines of sight to stars in LMC (Molaro et al. 1993)

[Note] The cold gas is ~100 pc away from Earth.



UV interstellar absorption lines toward an O-type star HD93521. (Spitzer & Fitzpatrick 1993)

[Note] (1) multiple velocity components and (2) line saturation on Si II and Fe II.
The multiple velocities are due primarily to the differential rotation of our galaxy. (clouds at different distances)

- The composition and excitation of interstellar gas can be studied using absorption lines that appear in the spectra of background stars (or other sources).
- The interstellar lines are typically narrow compared to spectral features produced by absorption in stellar photospheres, and in practice can be readily distinguished.
- It is normally possible to detect absorption only by the ground state (and perhaps the excited fine-structure levels of the ground electronic state) - the populations in the excited electronic states are too small to be detected in absorption.

- The widths of absorption lines are usually determined by Doppler broadening, with line widths of a few km s^{-1} (or $\Delta\lambda/\lambda \approx 10^{-5}$) - often observed in cool clouds.

- Absorption lines (and emission lines) contains a lots of information about number density, temperature, chemical abundances, ionization states, and excitation states.
- However, interpreting the information requires understanding the ways in which light interacts with baryonic matter, radiative transfer.
- **We need to know the line profile to analyze absorption lines.**

Absorption & Emission Line Profile

- ***In the classical / quantum theory of spectral lines,***

we obtain a Lorentzian line profile:

$$\sigma_\nu = f_{nn'} \frac{\pi e^2}{m_e c} \frac{\gamma/4\pi^2}{(\nu-\nu_0)^2 + (\gamma/4\pi)^2}$$

$$\int_0^\infty \sigma_\nu d\nu = f_{nn'} \frac{\pi e^2}{m_e c}$$

$m_e =$ electron mass
$e =$ electric charge

where $f_{nn'}$ is called the ***oscillator strength*** or ***f-value*** for the transition between states n and n'.

$\gamma = A$ is the ***damping constant (or Einstein A-coefficient)***.

Selected Resonance Lines[a] with $\lambda < 3000$ Å

	Configurations	ℓ	u	$E_\ell/hc(\mathrm{cm}^{-1})$	$\lambda_{\mathrm{vac}}(\text{Å})$	$f_{\ell u}$
C IV	$1s^22s - 1s^22p$	$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^o_{1/2}$	0	1550.772	0.0962
		$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^o_{3/2}$	0	1548.202	0.190
N V	$1s^22s - 1s^22p$	$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^o_{1/2}$	0	1242.804	0.0780
		$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^o_{3/2}$	0	1242.821	0.156
O VI	$1s^22s - 1s^22p$	$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^o_{1/2}$	0	1037.613	0.066
		$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^o_{3/2}$	0	1037.921	0.133
C III	$2s^2 - 2s2p$	$^1\mathrm{S}_0$	$^1\mathrm{P}^o_1$	0	977.02	0.7586
C II	$2s^22p - 2s2p^2$	$^2\mathrm{P}^o_{1/2}$	$^2\mathrm{D}^o_{3/2}$	0	1334.532	0.127
		$^2\mathrm{P}^o_{3/2}$	$^2\mathrm{D}^o_{5/2}$	63.42	1335.708	0.114
N III	$2s^22p - 2s2p^2$	$^2\mathrm{P}^o_{1/2}$	$^2\mathrm{D}^o_{3/2}$	0	989.790	0.123
		$^2\mathrm{P}^o_{3/2}$	$^2\mathrm{D}^o_{5/2}$	174.4	991.577	0.110
C I	$2s^22p^2 - 2s^22p3s$	$^3\mathrm{P}_0$	$^3\mathrm{P}^o_1$	0	1656.928	0.140
		$^3\mathrm{P}_1$	$^3\mathrm{P}^o_2$	16.40	1656.267	0.0588
		$^3\mathrm{P}_2$	$^3\mathrm{P}^o_2$	43.40	1657.008	0.104
N II	$2s^22p^2 - 2s2p^3$	$^3\mathrm{P}_0$	$^3\mathrm{D}^o_1$	0	1083.990	0.115
		$^3\mathrm{P}_1$	$^3\mathrm{D}^o_2$	48.7	1084.580	0.0861
		$^3\mathrm{P}_2$	$^3\mathrm{D}^o_3$	130.8	1085.701	0.0957
N I	$2s^22p^3 - 2s^22p^23s$	$^4\mathrm{S}^o_{3/2}$	$^4\mathrm{P}_{5/2}$	0	1199.550	0.130
		$^4\mathrm{S}^o_{3/2}$	$^4\mathrm{P}_{3/2}$	0	1200.223	0.0862
O I	$2s^22p^4 - 2s^22p^33s$	$^3\mathrm{P}_2$	$^3\mathrm{S}^o_1$	0	1302.168	0.0520
		$^3\mathrm{P}_1$	$^3\mathrm{S}^o_1$	158.265	1304.858	0.0518
		$^3\mathrm{P}_0$	$^3\mathrm{S}^o_1$	226.977	1306.029	0.0519
Mg II	$2p^63s - 2p^63p$	$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^o_{1/2}$	0	2803.531	0.303
		$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^o_{3/2}$	0	2796.352	0.608
Al III	$2p^63s - 2p^63p$	$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^o_{1/2}$	0	1862.790	0.277
		$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^o_{3/2}$	0	1854.716	0.557

Table 9.4 in [Draine]
See also Table 9.3

Line Broadening Mechanisms

- ***Atomic levels are not infinitely sharp***, nor are the lines connecting them.
  - (1) Doppler (Thermal) Broadening
  - (2) Natural Broadening
  - (3) Collisional Broadening
  - (4) Thermal Doppler + Natural Broadening

- **Voigt profile : Thermal + Natural broadening**
  - Atoms shows both a Lorentz profile plus the Doppler effect.
  - In this case, we can write the profile as an average of the Lorentz profile over the various velocity states of the atom:
  - ***Voigt profile = convolution of a Lorentz function (natural broadening) and Gaussian function (thermal broadening).***

- The profile can be written using the Voigt function.

$$\phi(\nu) = \frac{1}{\Delta\nu_{\rm D}\sqrt{\pi}} H(u,a)$$

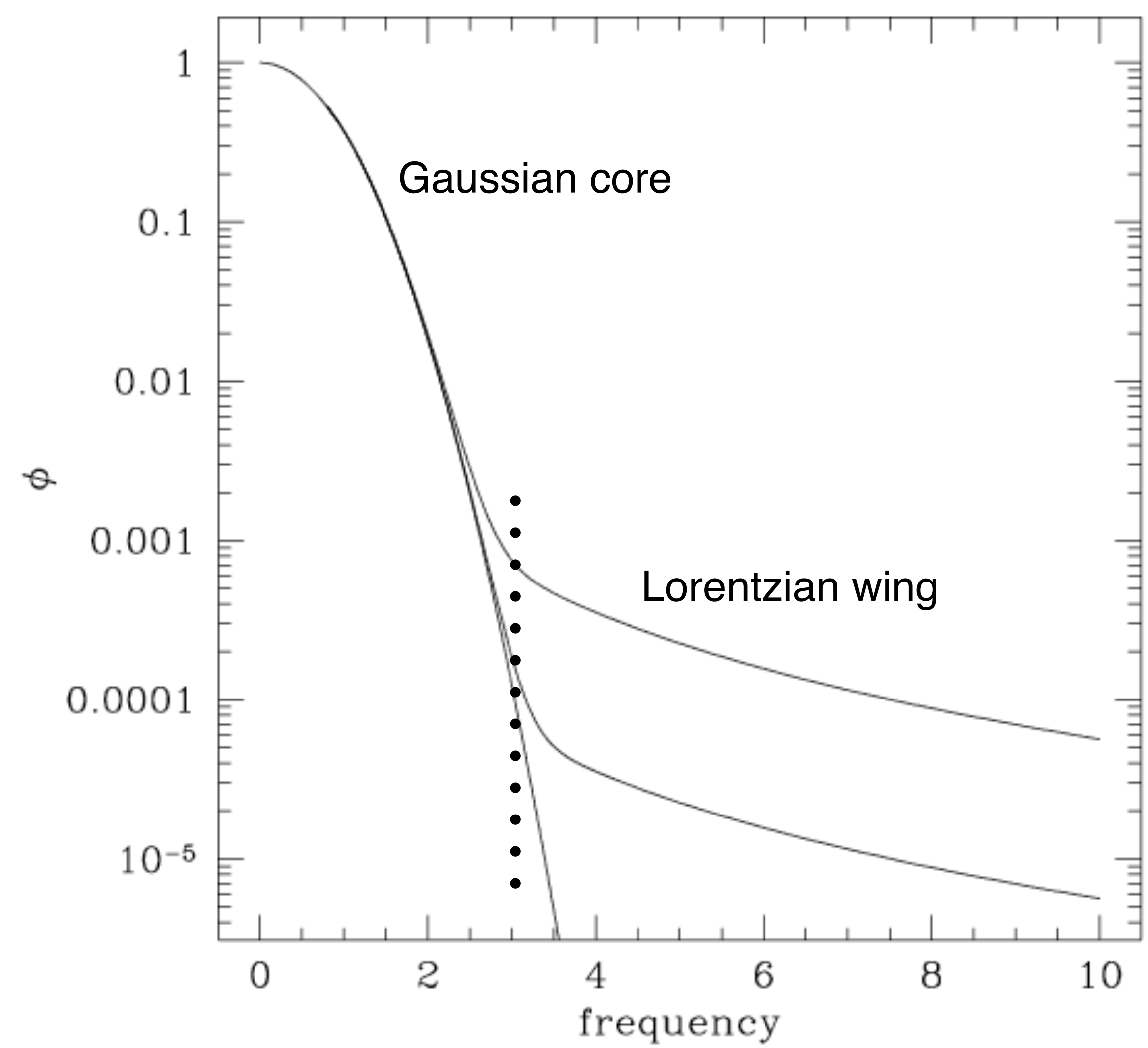
Voigt-Hjerting function:

$$H(u,a) \equiv \frac{a}{\pi}\int_{-\infty}^{\infty}\frac{e^{-y^2}dy}{(u-y)^2+a^2}$$

$$a \equiv \frac{\Gamma}{4\pi\Delta\nu_{\rm D}}$$

$$u \equiv \frac{\nu-\nu_0}{\Delta\nu_{\rm D}}$$

$$\Delta\nu_{\rm D} = \nu_0\frac{v_{\rm th}}{c} = \frac{\nu_0}{c}\sqrt{\frac{2kT}{m}}$$



Here, a is a ratio of the intrinsic broadening to the thermal broadening.

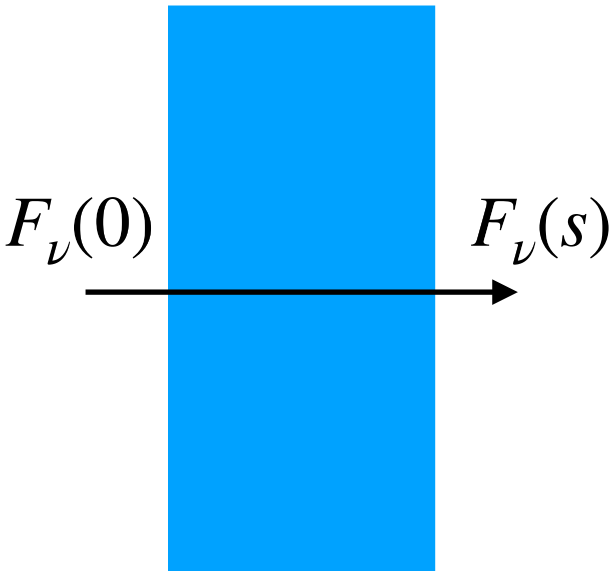
u is a measure of how far you are from the line center, in units of thermal broadening parameter.

Including the turbulent motion

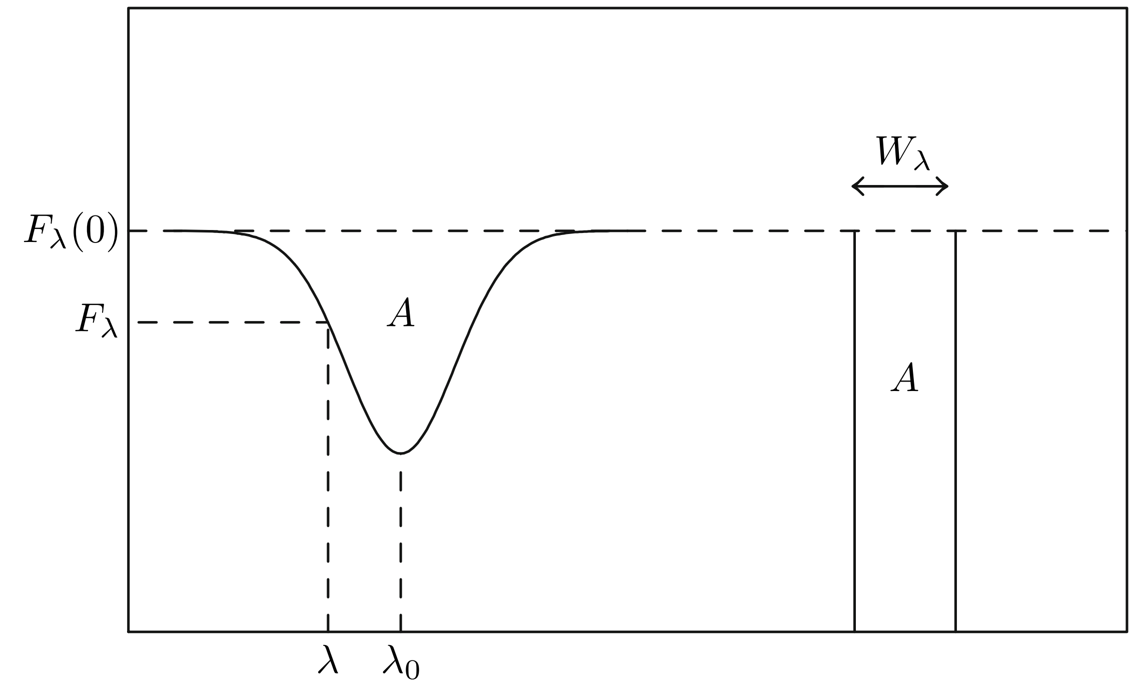
$$\Delta\nu_{\rm D} = \nu_0\frac{v_{\rm th}}{c} \;\rightarrow\; \Delta\nu_{\rm D} = \nu_0\frac{b}{c}$$

$$\text{where } b = \sqrt{v_{\rm th}^2 + v_{\rm turb}^2},\;\; v_{\rm th} = \sqrt{\frac{2kT}{m}}$$

Absorption Line & Equivalent Width



$$F_\nu = F_\nu(0) e^{-\tau_\nu}$$



$$\tau_\nu = \tau_0 H(u, a)$$

$$\tau_0 = \frac{\sqrt{\pi} e^2}{m_e c} f_{\ell u} \frac{\lambda_{\ell u}}{b} N_\ell$$

Here, τ_0 is the optical depth at the line center.
N_ℓ is the column density of the atoms in the lower (ground) level.

(wavelength) equivalent width

$$W_\lambda \equiv \int d\lambda \left[1 - \frac{F_\lambda}{F_\lambda(0)} \right] = \int d\lambda \left(1 - e^{-\tau_\lambda} \right)$$

- ***Equivalent width***
  - The spectrograph often lack the spectral resolution to resolve the profiles of narrow lines, but can measure the total amount of “missing power” resulting from a narrow absorption line.
  - The equivalent width is the width of a straight-sided, perfectly black absorption line that has the same integrated flux deficit as the actual absorption line.

Variation of Line Profiles & Curve of growth

- ***The absorption line profiles for*** $b = 10\ \mathrm{km\ s^{-1}}$
    - When $\tau_0 < 1$, $F_\nu / F_\nu(0) \approx 1 - \tau_\nu$ and thus the shape of an absorption line resembles and upside-down Voight function.
    - When $\tau_0 \gg 1$, the absorption line saturates at its center and becomes increasingly "box-shaped."

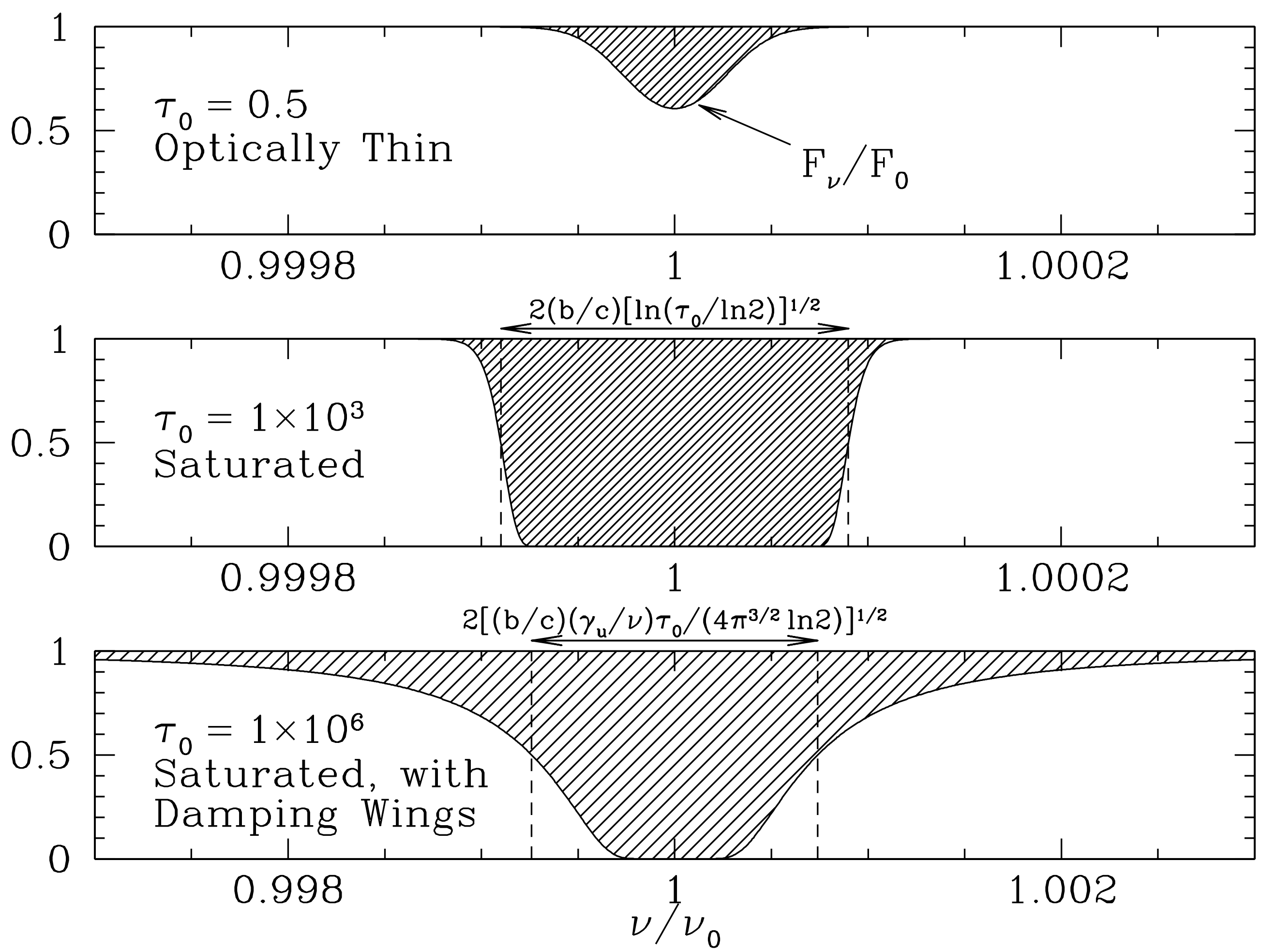
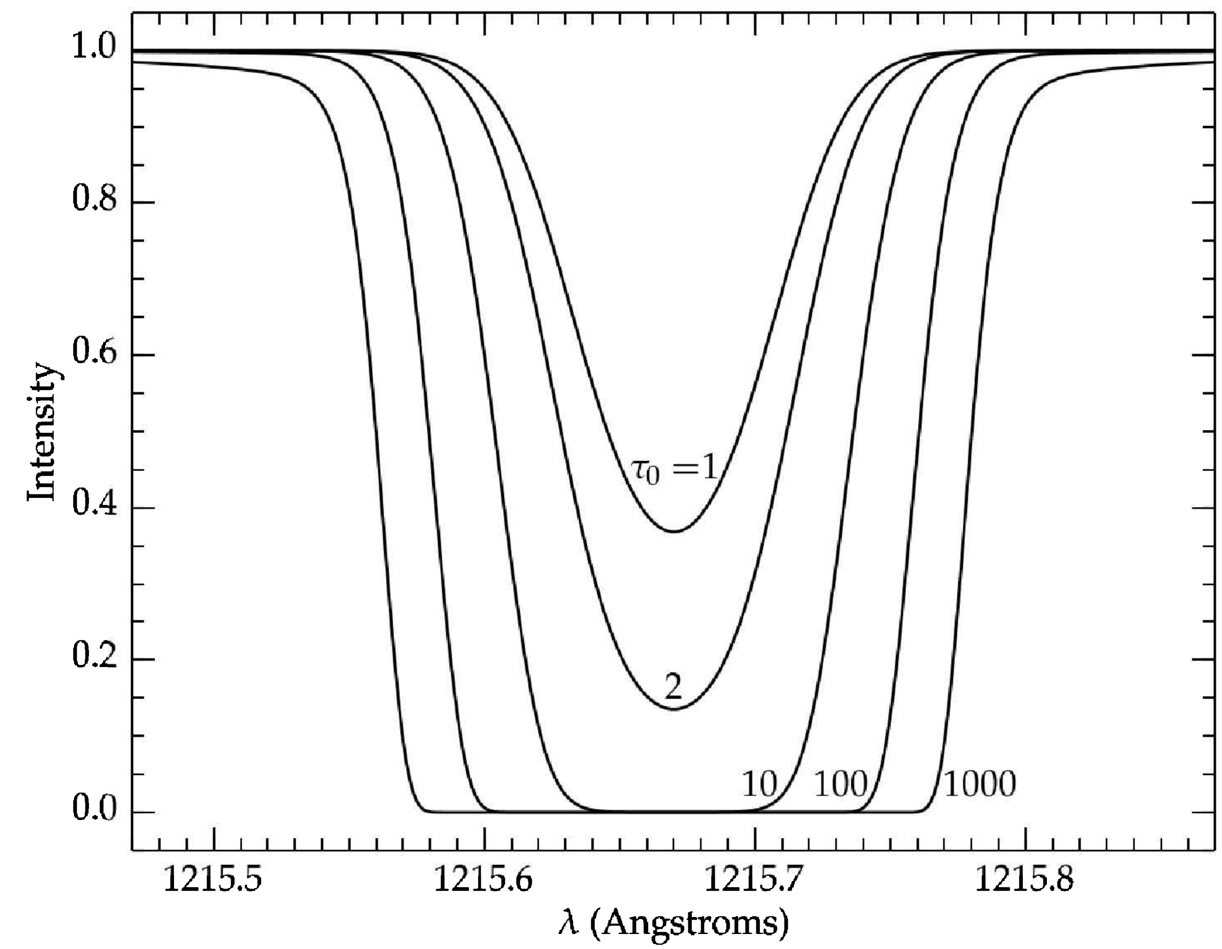


Figure 9.1 in [Draine]
Note the different abscissa in the lowest panel.



Lyman α absorption lines for $b = 10\ \mathrm{km\ s^{-1}}$.

Figure 2.6 in [Ryden]

- ***Curve of growth***
    - The curve of growth refers to the numerical relation between the observed equivalent width and the underlying optical depth (or the column density) of the absorber.

Homework (due date: 03/28)

[Q1] Consider an (isotropically emitting) star of uniform intensity $I_\nu = B$ at the surface, show that the flux at the surface is

$$F_\nu = \int I_\nu \cos\theta d\Omega = \pi B$$

[Q2] (a) The specific intensity of a star is, to first order, a blackbody. For a given effective temperature, $T_{\rm eff}$, and stellar radius, R, derive its bolometric luminosity.

(b) Look up values for these parameters and calculate this formula for the Sun.

- [Q3]
  - Suppose that we observe a radio-bright QSO and detect absorption lines from Milky Way gas in its spectra. The 21 cm line is seen in optically-thin absorption with a profile with FWHM(H I) = 10 km s^{-1}. We also have high-resolution observations of the Na I doublet lines referred to as D_1 (5898Å) and D_2 (5892Å) in absorption. The Na I D_2 5892Å line width is FWHM(Na I D_2) = 5 km s^{-1}. The line profiles are the result of a combination of thermal broadening plus turbulence with a Gaussian velocity distribution with one-dimensional velocity dispersion $\sigma_{v,\,\mathrm{turb}}$.

    You will want to employ the following theorem: If the turbulence has a Gaussian velocity distribution, the overall velocity distribution function of atoms of mass *M* will be Gaussian, with one-dimensional velocity dispersion:

$$v_{\mathrm{rms}}^2 = \sigma_v^2 = \sigma_{v,\,\mathrm{turb}}^2 + \frac{kT}{M}$$

  - If the Na I D_2 line is optically thin, estimate the kinetic temperature *T* and $\sigma_{v,\,\mathrm{turb}}$.

    Note hat for a Gaussian function, $\mathrm{FWHM} = 2\sqrt{2\ln 2}\sigma$.