Modern Astronomy

Part 1. Interstellar Medium (ISM)

Lecture 2
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< The magnitude scale >

- For historical reasons, fluxes in the optical and infrared are measured in magnitudes.

- On the basis of naked eye observations, the Greek astronomer Hipparchus (190-120 BC) classified **all the stars into six classes** according to their apparent brightness.
    - **The brighter ones belong to the first magnitude class.** The faintest ones belong to the sixth magnitude class.

- Pogson (1856) noted that **the faintest stars visible to the naked eye are about 100 times fainter compared to the brightest stars.**
    - The brightest and faintest stars differ by five magnitude classes.
    - Therefore, stars in two successive classes should differ in apparent brightness by a factor $100^{1/5}$.

- Note that the human eye is more sensitive to a geometric progression (I_0, $2I_0$, $4I_0$, $8I_0$, $\cdots$) of intensity rather than an arithmetic progression (I_0, $2I_0$, $3I_0$, $4I_0$, $\cdots$). In other words, ***the apparent magnitude as perceived by the human eye scales roughly logarithmically with the radiation flux.***

- Suppose two stars have apparent brightnesses F_1 and F_2 and their magnitude classes are m_1 and m_2.

$$\frac{F_2}{F_1} = (100)^{\frac{1}{5}(m_1 - m_2)}.$$

- Then, on taking the logarithm of this, we find

$$m_1 - m_2 = 2.5 \log_{10}\left(\frac{F_2}{F_1}\right).$$

- This is the definition of ***apparent magnitude*** denoted by m, which is a measure of the apparent brightness of an object in the sky.
  - Note that the magnitude scale is defined in such a fashion that ***a fainter object has a higher value of magnitude.***

magnitudes of extinction and optical depth

The observed flux is reduced by the optical depth exponential absorption factor. The level of this ISM absorption can also be characterized in terms of the number of magnitudes of extinction (A).

$$\boxed{F_{\rm obs} = F_0 e^{-\tau}} \qquad A \equiv m_{\rm obs} - m_0 = 2.5 \log_{10}\left(\frac{F_0}{F_{\rm obs}}\right) = 2.5\tau \log_{10} e \quad \Rightarrow \boxed{A \approx 1.086\tau}$$

- We need a measure that quantifies the luminosity or intrinsic brightness of an object.
- The ***absolute magnitude*** of a celestial object is ***the magnitude it would have if it were placed at a distance of 10 pc.***
  - If the object is at a distance d pc, then $(10\,\mathrm{pc}/d)^2$ is the ratio between its apparent brightness and the brightness it would have if it were at a distance of 10 pc.

$$\frac{F(d)}{F(10\,\mathrm{pc})} = \left(\frac{10\,\mathrm{pc}}{d}\right)^2$$

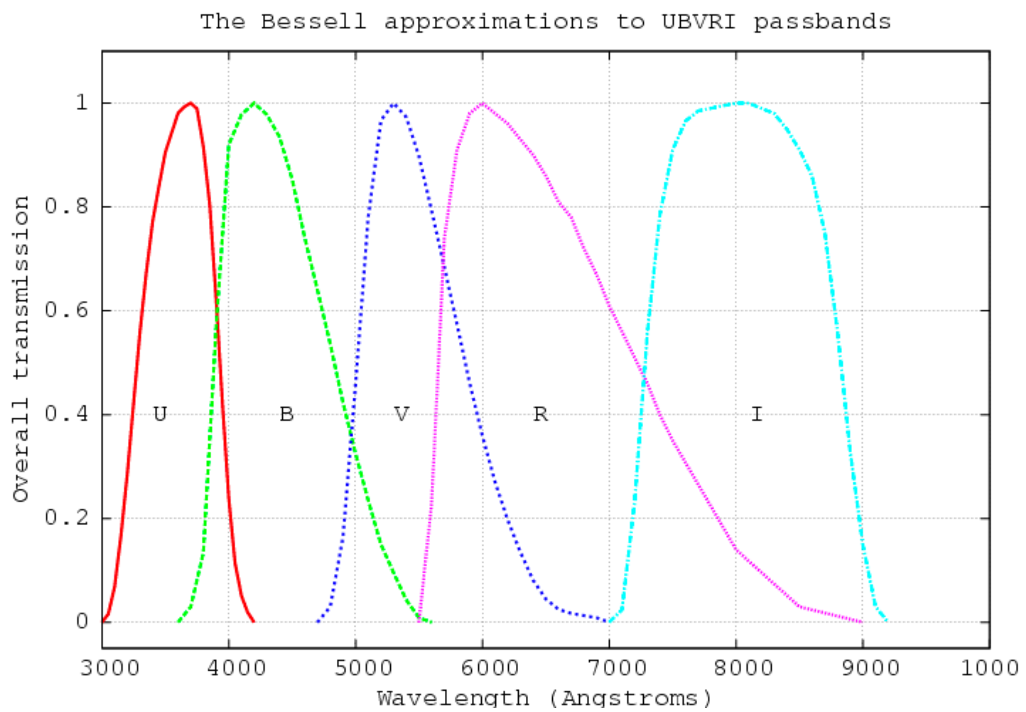
  - Then, the relation between apparent magnitude m and absolute magnitude M is

$$m - M = 2.5\log_{10}\left(\frac{d}{10\,\mathrm{pc}}\right)^2 = 5\log_{10}\left(\frac{d}{10\,\mathrm{pc}}\right)$$

  - The difference $m - M$ is called the ***distance modulus***.

Filters and Wavebands

- Common bandpasses



Johnson system (or Johnson-Morgan system)

U (ultraviolet)	365 nm
B (blue)	440 nm
V (visible)	550 nm
R (red)	641 nm
I (near-infrared)	0.896 μm
J	0.900 μm
H	1.22 μm
K	2.19 μm

  - These are the central wavelengths of each band, which extend ~10% in wavelength to either side.
  - Magnitude at each bandpass is denoted by m_U, m_B, m_V, m_R, m_K, etc.

- Zero-points in the Vega magnitude system
  - Note that the magnitude scale has been relatively defined.
  - ***The zero-points are defined such that the magnitude of a standard star (Vega) is 'zero in all wavebands.'***

Vega magnitude system

- In the Vega system, Vega's magnitude is always 0 in every passband:

$$U_{\rm Vega} = 0,\ B_{\rm Vega} = 0,\ V_{\rm Vega} = 0,\ R_{\rm Vega} = 0,\ I_{\rm Vega} = 0$$

Vega does not have a flat spectral energy distribution so it doesn't make much sense to force its magnitudes to be flat.

This becomes even more problematic for UV and IR surveys (surveys outside of the optical), where Vega deviates substantially from a flat SED.

A solution is to calibrate the system using the absolute physical flux. This system is the AB system.

AB magnitude

- Oke & Gunn (1983) defined the AB magnitude system.

- The **monochromatic AB magnitude** is defined as follows:

$$m_{\mathrm{AB}} = -2.5\log_{10} f_\nu(\mathrm{Jy}) + 8.90 \approx -2.5\log_{10}\left(\frac{f_\nu}{3631\,\mathrm{Jy}}\right)$$

$$m_{\mathrm{AB}} = -2.5\log_{10} f_\nu(\mathrm{erg\,s^{-1}\,cm^{-2}Hz^{-1}}) - 48.60$$

Here, $\mathrm{Jy} = 10^{-23}\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}}$

- The **bandpass AB magnitude** is defined such that the zero point corresponds to a ‘constant’ spectral flux density of 3631 Jy at all frequencies.

$$m_{\mathrm{AB}} = -2.5\log_{10}\left[\frac{\int (f_\nu/h\nu)\, e_\nu d\nu}{\int (3631\,\mathrm{Jy}/h\nu)\; e_\nu d\nu}\right] = -2.5\log_{10}\left[\frac{\int f_\lambda\, \lambda e_\lambda d\lambda}{\int (c/\lambda) e_\lambda d\lambda}\right] - 48.60$$

Here, e_ν is the “equal-energy” filter response function, expressed in terms of per unit energy. Modern systems of passbands, such as the SDSS ugriz filter system are on the AB magnitude system.

$$e_\lambda \equiv e_\nu(\nu = c/\lambda),\ f_\nu d\nu = f_\lambda d\lambda,\ (3631\,\mathrm{Jy})d\nu = (3631\,\mathrm{Jy})(c/\lambda^2)d\lambda$$

< Thermodynamic equilibrium >

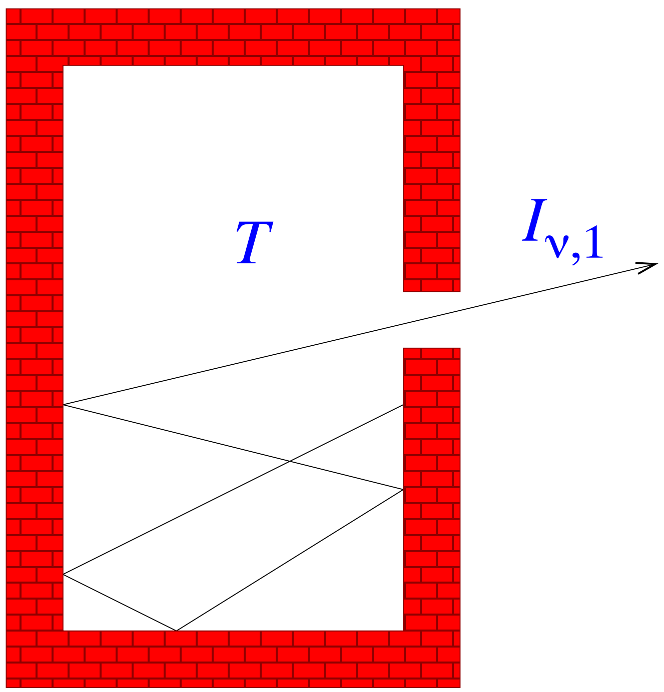
- In general, equilibrium means a state of balance.
- Thermal Equilibrium
  - ***Thermal equilibrium* refers to steady states of temperature, which defines the average energy of material or photons.**

    (for ideal gas, $E_{\mathrm{avg}} = \frac{3}{2} k_{\mathrm{B}} T$)
  - In a state of (complete) ***thermodynamic equilibrium (TE)***, no net flows of matter or of energy, no phase changes, and no unbalanced potentials (or driving forces), within the system. ***In TE, matter and radiation are in equilibrium at the same temperature T.***
  - When the material is ‘locally’ in thermodynamic equilibrium, and only the radiation field is allowed to depart from its TE, we refer to the state of the system as being in ***local thermodynamic equilibrium (LTE)***
  - In other words, if the material is (locally) in thermodynamic equilibrium at a well-defined temperature T, ***it is said to be in local thermodynamic equilibrium (LTE) even if it is not in equilibrium with the radiation field.***

Blackbody

- A blackbody is an idealized physical body that absorbs all incident radiation regardless of frequency or angle of incidence (i.e., perfect absorber). The above cavity can be regarded to be a blackbody.
- Radiation from a blackbody in thermal equilibrium is called the blackbody radiation.
- **The blackbody radiation spectrum is the universal function.**

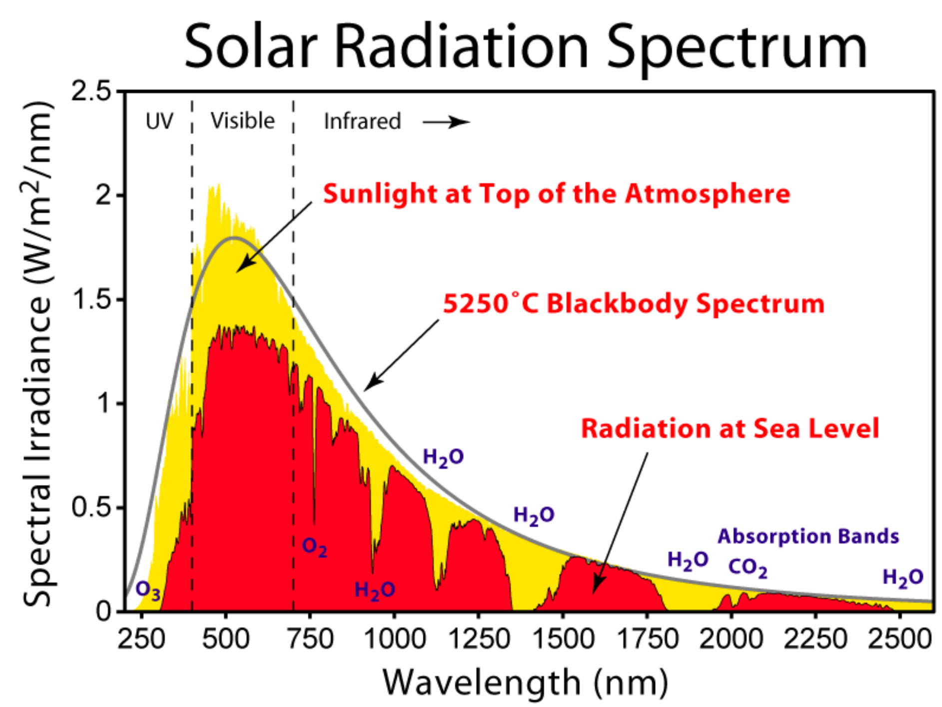
- Imagine a container bounded by opaque walls with a very small hole.
  - ***Photons will be scattered and absorbed many times, (and eventually trapped and completely absorbed in the box).*** Under such conditions, the particles and photons continually share their kinetic energies. In perfect thermal equilibrium, ***the average particle kinetic energy will equal to the average photon energy, and a unique temperature T can be defined.***



box 1

Spectrum of Blackbody Radiation

- In reality, there is no perfect blackbody.
  - However, the cosmic microwave background comes quite close; stars can sometimes be usefully approximated as blackbodies.
  - By the end of the 19th century, the blackbody spectrum was fairly well known empirically, from laboratory studies. In 1900, Max Planck, using his idea of quantized energies, derived the blackbody spectrum.



https://pages.uoregon.edu/imamura/321/122/lecture-3/stellar_spectra.html

- The frequency dependence of blackbody radiation is given by the ***Planck function***:

$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp\left(h\nu/k_\mathrm{B}T\right) - 1} \quad \text{or} \quad B_\lambda(T) = \frac{2hc^2/\lambda^5}{\exp\left(hc/\lambda k_\mathrm{B}T\right) - 1}$$

$$h = 6.63 \times 10^{-27}\ \text{erg s (Planck's constant)}$$

$$k_\mathrm{B} = 1.38 \times 10^{-16}\ \text{erg K}^{-1}\ \text{(Boltzmann's constant)}$$

Rayleigh-Jeans Law & Wien Law

Rayleigh-Jeans Law (low-energy limit)

$$h\nu \ll k_{\rm B}T \quad \left(\nu \ll 2\times 10^{10}\,(T/1\,{\rm K})\,{\rm Hz}\right)$$

$$I_\nu^{RJ}(T) = \frac{2\nu^2}{c^2}k_{\rm B}T$$

Wien Law (high-energy limit)

$$h\nu \gg k_{\rm B}T$$

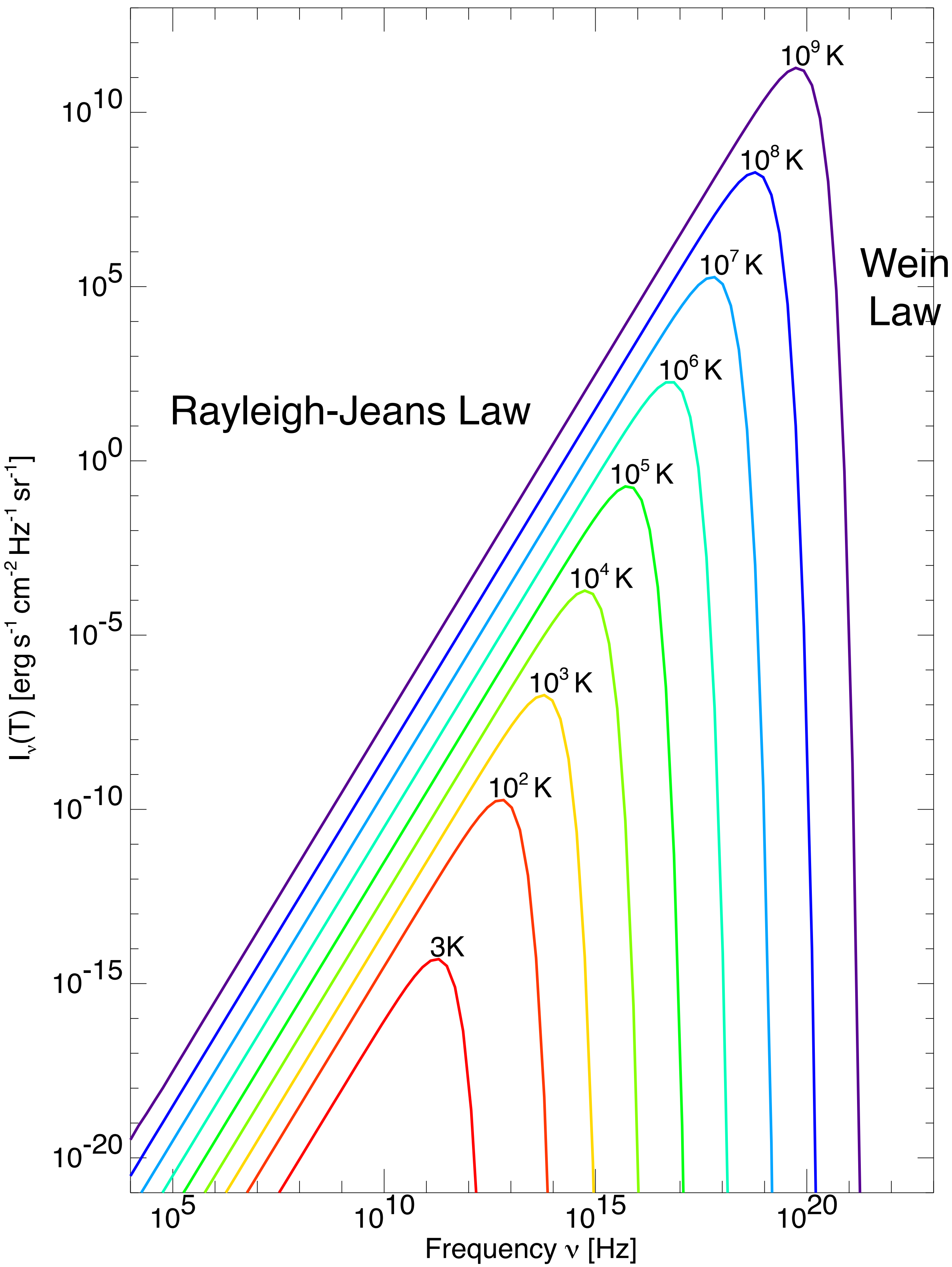
$$I_\nu^{W}(T) = \frac{2h\nu^3}{c^2}\exp\left(-\frac{h\nu}{k_{\rm B}T}\right)$$

Stefan-Boltzmann Law (flux at the surface)

$$F = \pi\int B_\nu(T)d\nu = \sigma_{\rm SB}T^4$$

Stephan – Boltzmann constant :

$$\sigma_{\rm SB} = \frac{2\pi^5 k_{\rm B}^4}{15c^2h^3} = 5.67\times 10^{-5}\ {\rm erg\ cm^2\ s^{-1}\ K^{-4}\ sr^{-1}}$$



Kirchhoff's Law in TE and in LTE

- In **(full) thermodynamic equilibrium** at temperature *T*, by definition, we know that

$$\frac{dI_\nu}{ds} = 0 \quad \text{and} \quad I_\nu = B_\nu(T)$$

We also note that $$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- Then, we can obtain ***the Kirchhoff's law for a system in TE***:

$$\boxed{\frac{j_\nu}{\alpha_\nu} = B_\nu(T), \qquad j_\nu = \alpha_\nu B_\nu(T)}$$

- ***Kirchhoff's law applies not only in TE but also in LTE***:
  - Recall that $B_\nu(T)$ ***is independent of the properties of the radiating /absorbing material.***
  - In contrast, both $j_\nu(T)$ ***and*** $\alpha_\nu(T)$ ***depend only on the materials in the cavity and on the temperature of that material***; they do not depend on the ambient radiation field or its spectrum.
  - Therefore, the Kirchhoff's law should be true even for the case of LTE.
  - ***In LTE, the ratio of the emission coefficient to the absorption coefficient is a function of temperature and frequency only.***

Blackbody radiation vs. Thermal radiation

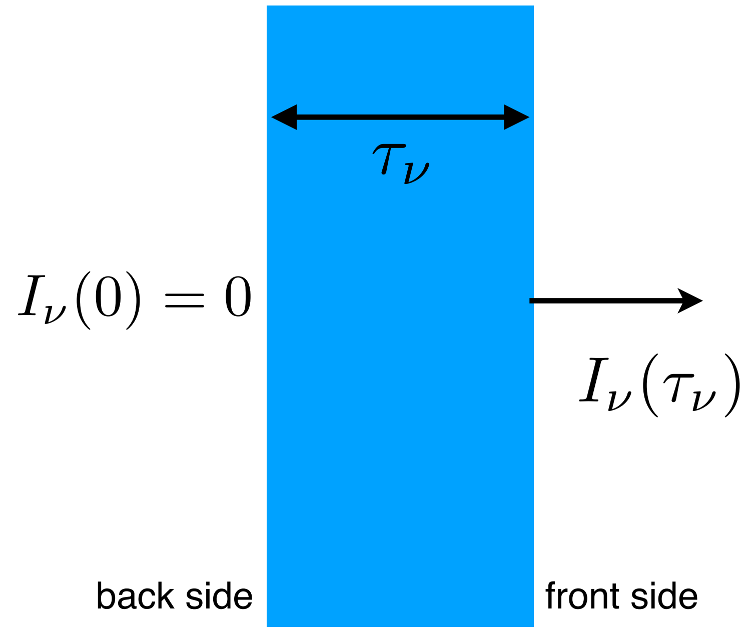
- ***Blackbody radiation*** means $I_\nu = B_\nu(T)$. An object for which the intensity is the Planck function is emitting blackbody radiation.

- ***Thermal radiation is defined to be radiation emitted by "matter" in LTE***. Thermal radiation means $S_\nu = B_\nu(T)$. An object for which the source function is the Planck function is emitting thermal radiation.

- ***Thermal radiation becomes blackbody radiation only for optically thick media.***

- To see the difference between thermal and blackbody radiation,
  - Consider a slab of material with optical depth τ_ν that is producing thermal radiation.
  - If no light is falling on the back side of the slab, the intensity that is measured on the front side of the slab is

$$\begin{array}{l} I_\nu(0) = 0 \\ S_\nu = B_\nu \end{array} \longrightarrow \quad \begin{aligned} I_\nu(\tau_\nu) &= I_\nu(0)e^{-\tau_\nu} + S_\nu\left(1 - e^{-\tau_\nu}\right) \\ &= B_\nu\left(1 - e^{-\tau_\nu}\right) \end{aligned}$$



  - If the slab is optical thick at frequency ν $(\tau_\nu \gg 1)$, then

$$\boxed{I_\nu = B_\nu \quad \text{as } \tau_\nu \to \infty}$$

  - If the slab is optically thin $(\tau_\nu \ll 1)$, then

$$\boxed{I_\nu \approx \tau_\nu B_\nu \ll B_\nu \quad \text{as } \tau_\nu \ll 1}$$

This indicates that the radiation, although it is thermal, will not be blackbody radiation.

Thermal radiation becomes blackbody radiation only for optical thick media.

The state of LTE

- Macroscopically, LTE is characterized by the following three equilibrium distributions:
  - **Maxwellian velocity distribution** of particles, written here in terms of distribution for the absolute values of velocity,

$$f(v)dv = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) 4\pi v^2 dv$$

    where m is the particle mass and k the Boltzmann constant.
  - **Boltzmann excitation equation**,

$$\frac{n_i}{N_I} = \frac{g_i}{U_I} e^{-E_i/kT}$$

    where n_i is the population of level i, g_i is its statistical weight, and E_i is the level energy, measured from the ground state; N_I and U_I are the total number density and the partition function of the ionization state I to which level i belongs, respectively.
  - **Saha ionization equation**,

$$\frac{N_I}{N_{I+1}} = n_e \frac{U_I}{U_{I+1}} \left(\frac{h^2}{2\pi m_e kT}\right)^{3/2} e^{\chi_I/kT}$$

    where χ_I is the ionization potential of ion I.

- Microscopically, LTE holds if all atomic processes are in detailed balance, i.e., if the number of processes $A \rightarrow B$ is exactly balanced by the number of inverse processes $B \rightarrow A$.

Characteristic Temperatures

- ***Brightness Temperature:***
  - The brightness temperature is defined to be the temperature such that a blackbody at that temperature would have specific intensity:

$$I_\nu = B_\nu(T_b) \quad \rightarrow \quad T_b(\nu) = \frac{h\nu/k_\mathrm{B}}{\ln\left[1 + 2h\nu^3/(c^2 I_\nu)\right]}$$

- ***Antenna Temperature:***
  - Radio astronomers are used to working at very low frequencies. They define the antenna temperature as being the brightness temperature in the Rayleigh-Jeans tail.

$$I_\nu = \frac{2\nu^2}{c^2} k_\mathrm{B} T_b \quad \rightarrow \quad T_A \equiv \frac{c^2}{2k_\mathrm{B}\nu^2} I_\nu$$

  - Radiative transfer equation in the RJ limit:
    - If the matter is in LTE and has its energy levels populated according to an excitation temperature $T_\mathrm{exc} \gg h\nu/k_\mathrm{B}$, then the source function is given by

$S_\nu(T_\mathrm{exc}) = (2\nu^2/c^2) k_\mathrm{B} T_\mathrm{exc}$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

    - Then, RT equation becomes $$\frac{dT_A}{d\tau_\nu} = -T_A + T_\mathrm{exc} \quad \text{if } h\nu \ll k_\mathrm{B} T_\mathrm{exc}$$

$$T_A = T_A(0)e^{-\tau_\nu} + T_\mathrm{exc}(1 - e^{-\tau_\nu}) \quad \text{if } T_\mathrm{exc} \text{ is constant.}$$

- ***Color Temperature:***
  - By fitting the spectrum to a blackbody curve without regarding to vertical scale (absolute intensity scale), a color temperature T_c is obtained.
  - The color temperature correctly gives the temperature of a blackbody source of unknown absolute scale.
- ***Effective Temperature:***
  - The effective temperature of a source is obtained by equating the actual flux F to the flux of a blackbody at temperature $T_{\rm eff}$.

$$F = \iint I_\nu \cos\theta d\nu d\Omega = \sigma_{\rm SB} T_{\rm eff}^4$$

cf. Stefan-Boltzmann law

- ***Excitation Temperature:***
  - The excitation temperature of level u relative to level ℓ is defined by

cf. Boltzmann distribution

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} \exp\left(-\frac{E_{u\ell}}{k_{\rm B} T_{\rm exc}}\right) \quad \rightarrow \quad T_{\rm exc} \equiv \frac{E_{u\ell}/k_{\rm B}}{\ln\left(\frac{n_\ell/g_\ell}{n_u/g_u}\right)} \quad (E_{u\ell} \equiv E_u - E_\ell)$$

  - Radio astronomers studying the 21 cm line sometimes use the term "spin temperature" $T_{\rm spin}$ for excitation temperature.

Hydrogen Atom / Atomic Spectroscopy

[Reference]

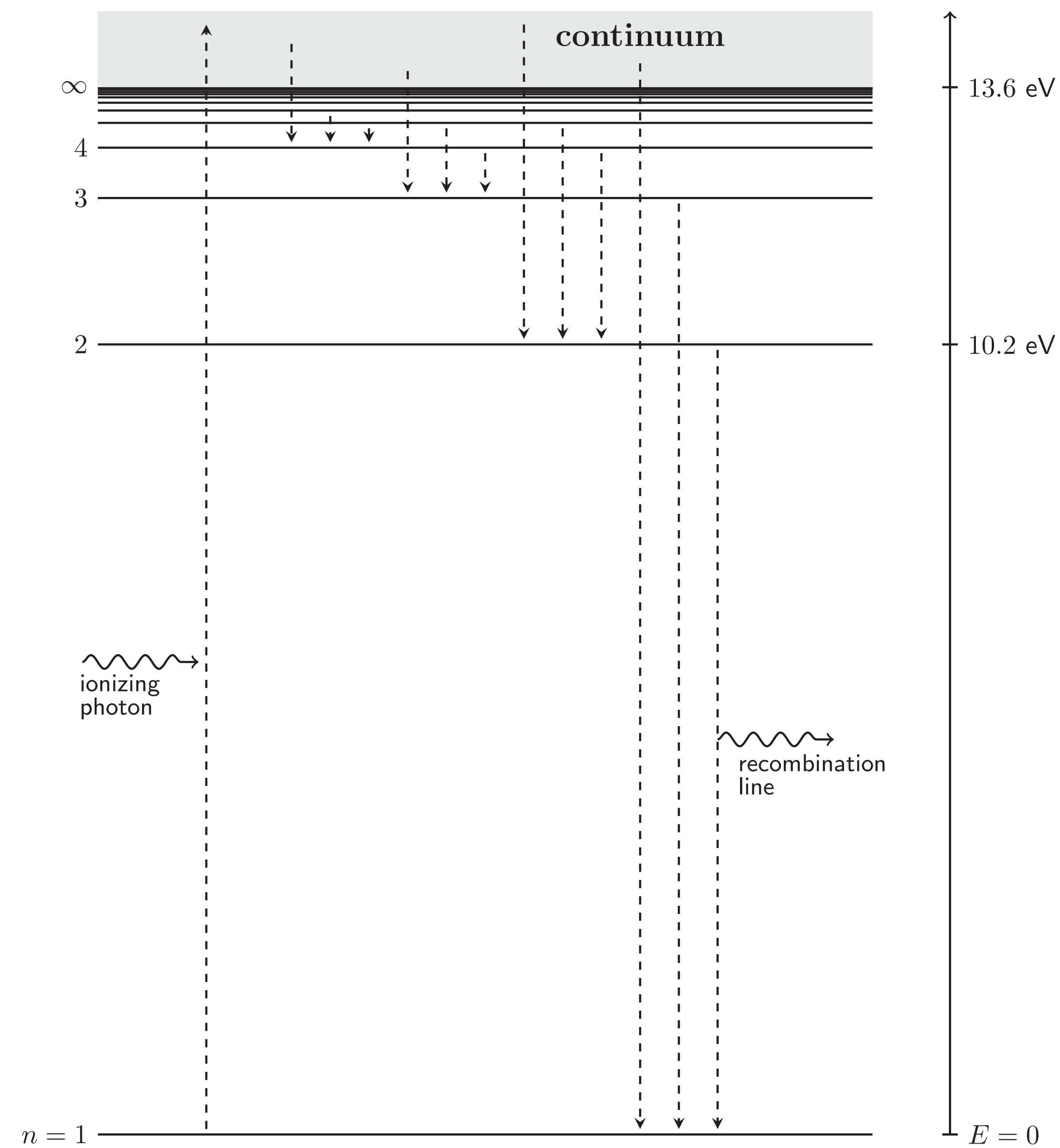
Astronomical Spectroscopy:
An Introduction to the Atomic and Molecular Physics of Astronomical Spectra
author: Jonathan Tennyson, 2nd Edition

< The Hydrogen Atom >

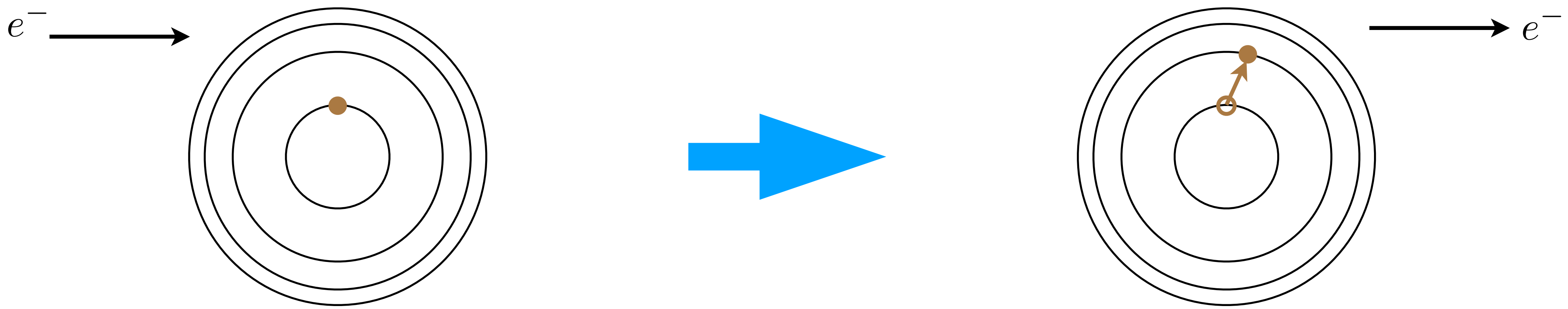
- The most common element in the Universe!

- Electronic Energy Levels
  - Ionization potential $E_{\rm IP} = 13.6$ eV, which is equivalent to $\lambda = hc/E_{\rm IP} = 912\text{Å}$, or high-speed collisions $(2E_{\rm IP}/m)^{1/2} \sim 50\,{\rm km\,s^{-1}}$.
  - A neutral hydrogen atom has bound levels labeled by quantum number, $n = 1,2,3\cdots$, which are well described by the Bohr model,

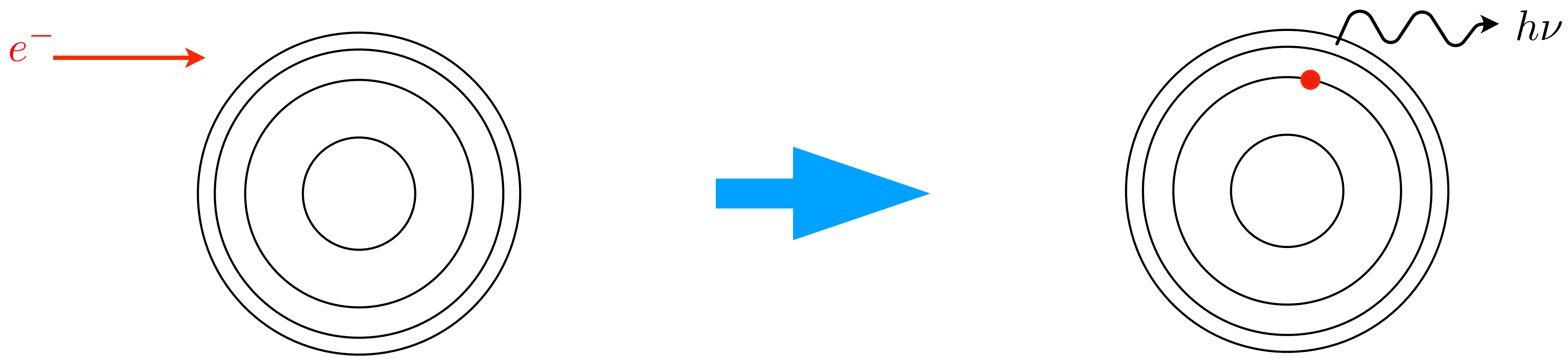
$$E_n = E_{\rm IP}\left(1 - \frac{1}{n^2}\right).$$



- **Collisional Excitation**
  - The electron orbital energy of the first excited level ($n = 2$), $E_2 - E_1 = 10.2$ eV, is much greater than the average kinetic energy found in most parts of the ISM ($\Delta E/k = 1.2 \times 10^5\,\mathrm{K}$).
  - Collisional excitation of a neutral hydrogen cloud is therefore rare. Consequently, almost all the atoms lie in the ground state ($n = 1$).
  - A hydrogen in the ISM will neither absorb nor emit optical and infrared photons as these have energies of a few tenths to a few eV.



- **Recombination**
  - In ionized gas, an electron that recombines with a proton may for a hydrogen atom in an excited state, $n > 1$.
  - The electron will then cascade down the energy ladder, producing a series of recombination lines, $n_2 \rightarrow n_1$.
  - These processes produce radio or infrared emission at large n_1, to the near-infrared for $n_1 = 3$ (Paschen series), optical for $n_2 = 2$ (Balmer series), and ultraviolet for n_1 (Lyman series).



H-atom Spectra

- Spectral series of the H atom
  - The spectrum of H is divided into a number of series linking different upper levels n_2 with a single lower level n_1 value. ***Each series is denoted according to its n_1 value and is named after its discoverer.***
  - Within a given series, ***individual transitions are labelled by Greek letters.***

$n_2 \longrightarrow n_1$

n_1	Name	Symbol	Spectral region
1	Lyman	Ly	ultraviolet
2	Balmer	H	visible
3	Paschen	P	infrared
4	Brackett	Br	infrared
5	Pfund	Pf	infrared
6	Humphreys	Hu	infrared

$\Delta n \equiv n_2 - n_1$

$\Delta n = 1$ is α,

$\Delta n = 2$ is β,

$\Delta n = 3$ is γ,

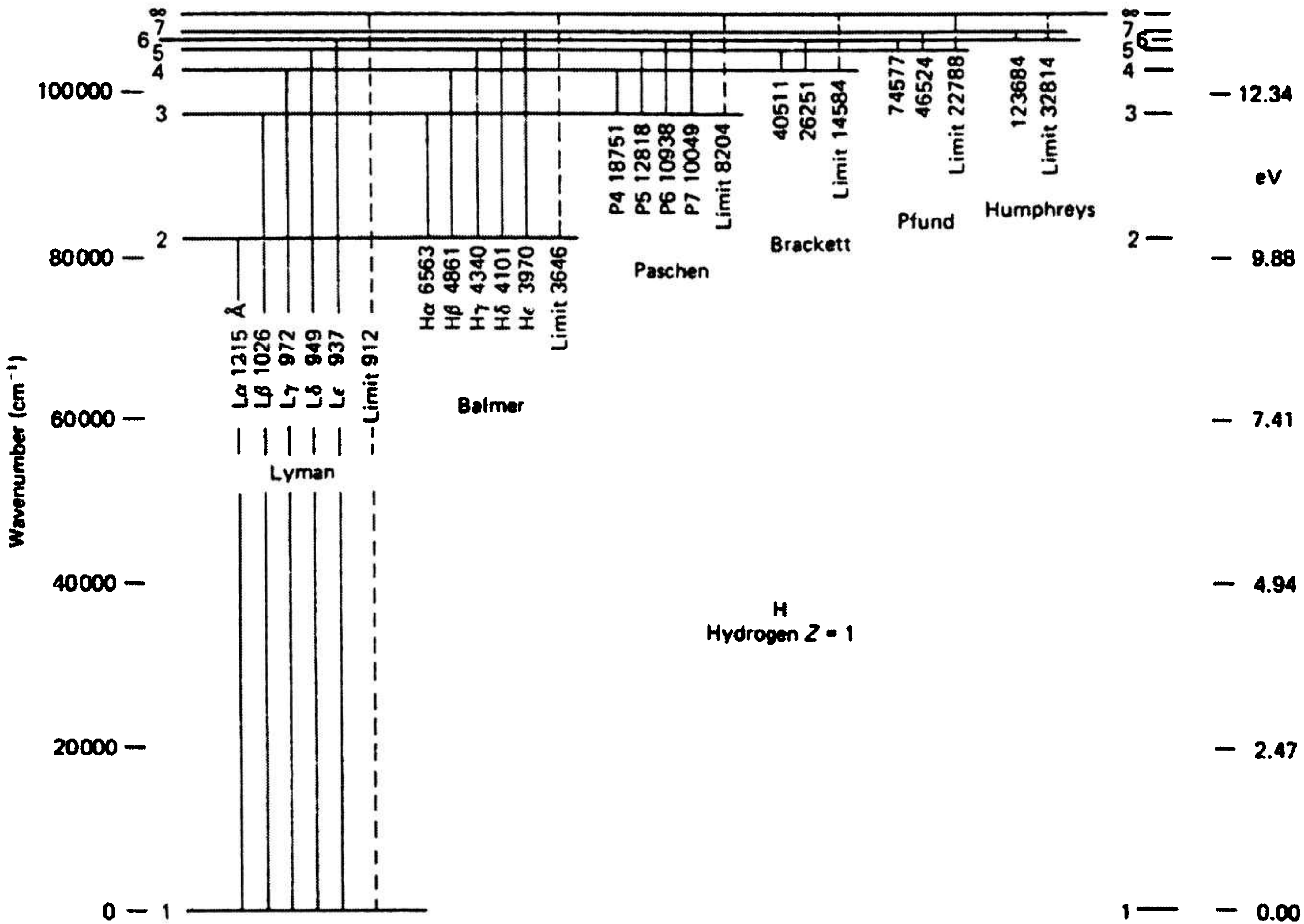
$\Delta n = 4$ is δ,

$\Delta n = 5$ is ϵ.

Lyman series : $\mathrm{Ly}\alpha, \mathrm{Ly}\beta, \mathrm{Ly}\gamma, \cdots$
Balmer series : $\mathrm{H}\alpha, \mathrm{H}\beta, \mathrm{H}\gamma, \cdots$
Paschen series: $\mathrm{P}\alpha, \mathrm{P}\beta, \mathrm{P}\gamma, \cdots$
Brackett series : $\mathrm{Br}\alpha, \mathrm{Br}\beta, \mathrm{Br}\gamma, \cdots$

Transitions with high Δn are labelled by the n_2. Thus, $H15$ is the Balmer series transition between $n_1 = 2$ and $n_2 = 15$.

Schematic energy levels of the hydrogen atom with various spectral series identified.
The vertical numbers are wavelengths in Å.



< Quantum Mechanics > — Hydrogen Atom

- Each bound state of the hydrogen atom is characterized by a set of four quantum numbers (n, l, m, m_s)
  - $n = 1, 2, 3, \cdots$: **[1] principal quantum number** (shell)
  - $l = 0, 1, 2, \cdots, n-1$: **[2] orbital angular momentum quantum number** (subshell)
    - By convention, the values of l are usually designated by small letters.

0	1	2	3	4	5	6	7	8	...
s	p	d	f	g	h	i	k	l	...

  - $m = -l, -l+1, \cdots, 0, \cdots, l-1, l$: **[3] magnetic quantum number**
    - It determines the behavior of the energy levels in the presence of a magnetic field.
    - This is the projection of the electron orbital angular momentum along the z-axis of the system.

- Spin
  - The electron possesses an intrinsic, ***spin angular momentum*** with the magnitude of $|s| = \frac{1}{2}$.
  - There are **[4] two spin states**, $m_s = \pm\frac{1}{2}$, for the spin.

- Degeneracy for a given *n:* $2 \times \sum_{l=0}^{n-1} (2l+1) = 2n^2$

Complex Atoms : Electron Configuration

- **The *configuration* is the distribution of electrons of an atom in atomic orbitals.**
  - The configuration of an atomic system is defined by specifying the nl values of all the electron orbitals: nl^x means x electrons in the orbital defined by n and l.
  - Each orbital labelled nl actually consists of orbitals with $2l+1$ different m values, each with two possible values of m_s. Thus the nl orbital can hold a maximum $2(2l+1)$ electrons.

$$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} \cdots$$

- **shells, subshells:**
  - ***Principal quantum number = shell***: Shells correspond with the principal quantum numbers (1, 2, 3, ...). They are labeled alphabetically with letters used in the X-ray notation (K, L, M, ...).
  - ***Orbital angular momentum quantum number = subshell***: Each shell is composed of one or more subshells. The first (K) shell has one subshell, called “1s”; The second (L) shell has two subshells, called “2s” and “2p”.

Angular Momentum Coupling

- Atoms contain several sources of angular momentum.
  - ***electron orbital*** angular momentum L
  - ***electron spin*** angular momentum S
  - ***nuclear spin*** angular momentum I
  - The nuclear spin arises from the spins of nucleons. Protons and neutrons both have an intrinsic spin of a half.

- As in classical mechanics, ***only the total angular momentum is a conserved quantity***.
  - It is therefore necessary to combine angular momenta together.

- Addition of two angular momenta:
  - The orbital and spin angular momenta are added vectorially as $\mathbf{J} = \mathbf{L} + \mathbf{S}$. This gives the total electron angular momentum.
  - One then combines the total electron and nuclear spin angular momenta to give the final angular momentum $\mathbf{F} = \mathbf{J} + \mathbf{I}$.

Lifting Degeneracy in Configuration: Angular Momentum Coupling, Terms

- ***L-S coupling (Russell-Saunders coupling):***
  - The orbital and spin angular momenta are added separately to give the total orbital angular momentum $\boldsymbol{L}$ and the total spin angular momentum $\boldsymbol{S}$. These are then added to give $\boldsymbol{J}$.

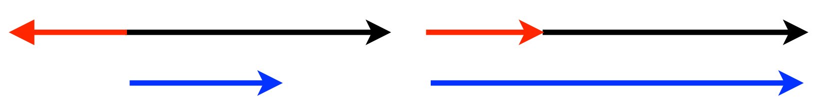
$$\boldsymbol{L} = \sum_i \boldsymbol{l}_i, \ \ \boldsymbol{S} = \sum_i \boldsymbol{s}_i \quad \rightarrow \quad \boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$$

  - The configurations split into **terms** with particular values of L and S.

- ***Adding two Angular Momenta***
  - Adding vector $\boldsymbol{a}$ and vector $\boldsymbol{b}$ gives a vector $\boldsymbol{c}$, whose length lies in the range

$$|a - b| \leq c \leq a + b$$

Here, a, b, c are the lengths of their respective vectors.



$$c = |a - b| \qquad c = a + b$$

  - In quantum mechanics, a similar rule applies except that the results are quantized. The allowed values of the quantized angular momentum, c, span the range from the sum to the difference of a and b in steps of one:

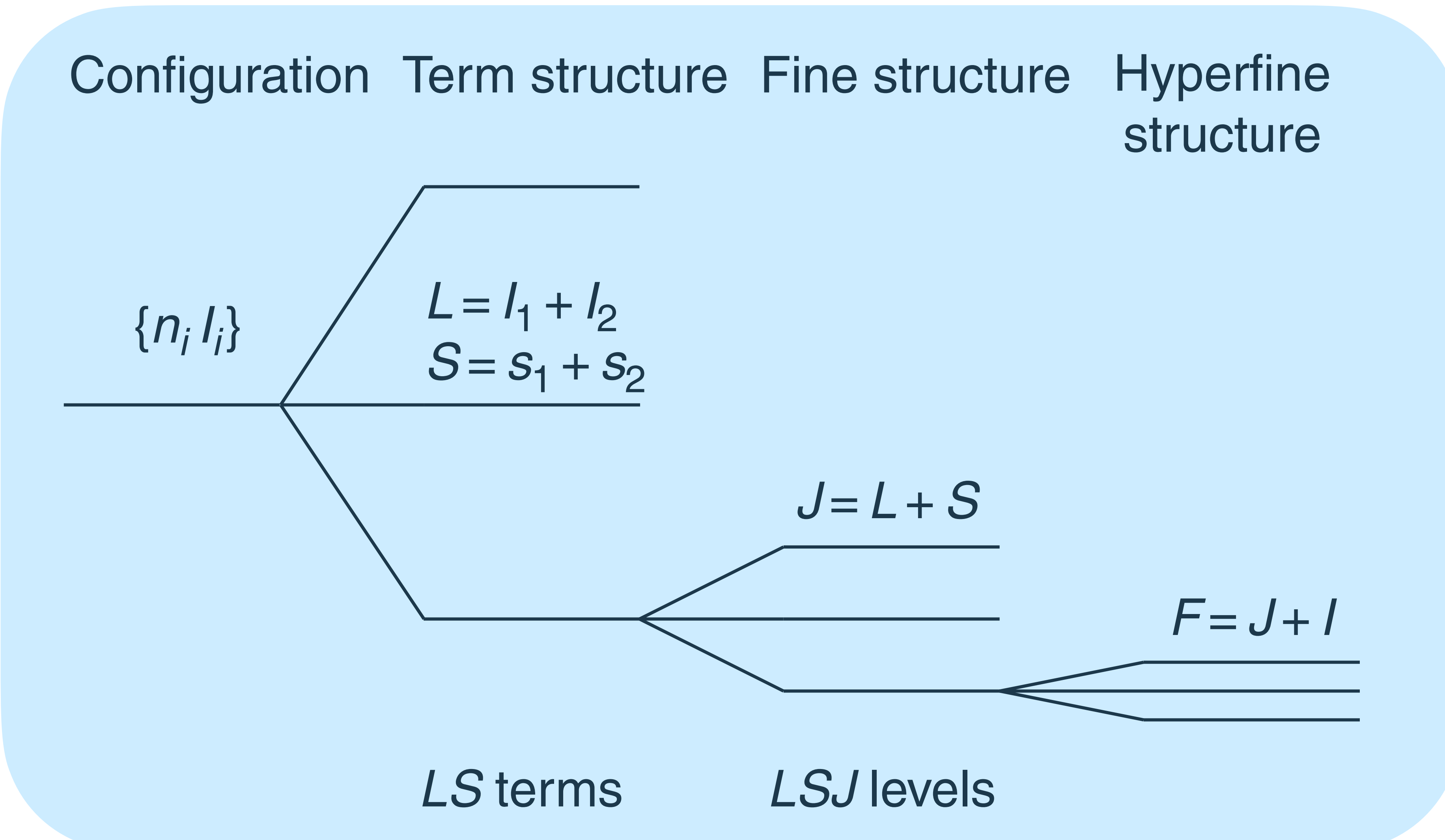
$$c = |a-b|, |a-b|+1, \cdots, a+b-1, a+b$$

  - For example, add the two angular momenta $L_1 = 2$ and $L_2 = 3$ together to give $\mathbf{L} = \mathbf{L}_1 + \mathbf{L}_2$. The result is

$$L = 1, 2, 3, 4, 5.$$

Energy Level Splitting

- Electronic configuration and energy level splitting
  - Configurations ⇒ Terms ⇒ Fine Structure (Spin-Orbit Interaction) ⇒ Hyperfine Structure (Interaction with Nuclear Spin)



[Pradhan & Nahar] Atomic Astrophysics and Spectroscopy

The Fine Structure of Hydrogen

- So far the discussion on H-atom levels has assumed that all states with the same principal quantum number, n, have the same energy.
  - However, this is not correct: inclusion of relativistic (or magnetic) effects split these levels according to the total angular momentum quantum number J. ***The splitting is called fine structure.***

- For hydrogen, $S = \frac{1}{2} \quad \rightarrow \quad J = L \pm \frac{1}{2}$

- Spectroscopic notation: $^{(2S+1)}L_J$

configuration	L	S	J	term	level
ns	0	$1/2$	$1/2$	^2S	$^2S_{1/2}$
np	1	$1/2$	$1/2,\ 3/2$	$^2P^o$	$^2P^o_{1/2},\ ^2P^o_{3/2}$
nd	2	$1/2$	$3/2,\ 5/2$	^2D	$^2D_{3/2},\ ^2D_{5/2}$
nf	3	$1/2$	$5/2,\ 7/2$	$^2F^o$	$^2F^o_{5/2},\ ^2F^o_{7/2}$

Note that the levels are called to be

singlet if 2S+1 = 1 $\quad S = 0,\ \ J = L$

doublet if 2S+1 = 2 $\quad S = 1/2,\ \ J = L \pm 1/2$

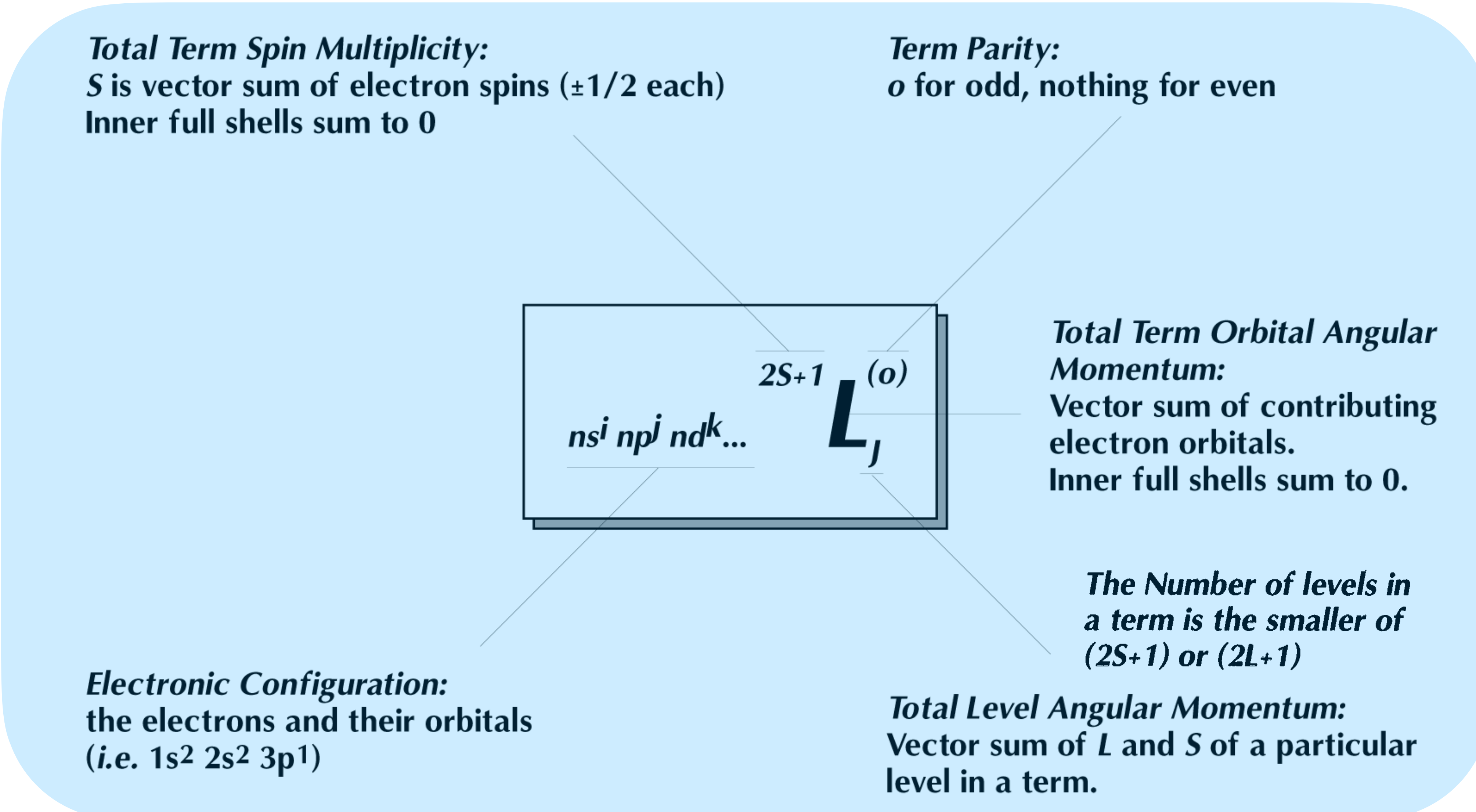
triplet if 2S+1 = 3 $\quad S = 1,\ \ J = L-1, L, L+1$

(when L > 0)

  - The above table shows the fine structure levels of the H atom.
  - Note that the states with principal quantum number $n = 2$ give rise to three fine-structure levels. In spectroscopic notation, these levels are $2^2S_{1/2}$, $2^2P^o_{1/2}$ and $2^2P^o_{3/2}$.

Spectroscopic Notation

- **Spectroscopic Notation**



- A state with S = 0 is a ‘singlet’ as 2S+1 = 1.
  - $J = L$ (singlet)
- A state with S = 1/2 is a ‘doublet’ as 2S+1 = 2
  - $J = L - 1/2, L + 1/2$ (doublet if $L \geq 1$)
- One with S = 1 is a ‘triplet’ as 2S+1 = 3
  - $J = L - 1, L, L + 1$ (triplet $L \geq 1$)

$$n = 1, 2, 3, 4, 5 \cdots \quad \rightarrow \quad K, L, M, N, O, \cdots$$
$$\ell = 0, 1, 2, 3, 4 \cdots \quad \rightarrow \quad s, p, d, f, g, \cdots$$
$$L = 0, 1, 2, 3, 4 \cdots \quad \rightarrow \quad S, P, D, F, G, \cdots$$

sharp, principal, diffuse, fundamental,…

Selection Rules

- **Selection Rules**

(1) one electron jumps
(2) Δn any
(3) $\Delta l = \pm 1$
(4) parity change

→ selection rule for configuration

(5) $\Delta S = 0$ → ***intercombination*** line if only this rule is violated.

(6) $\Delta L = 0, \ \pm 1$ (except $L = 0 - 0$)
(7) $\Delta J = 0, \ \pm 1$ (except $J = 0 - 0$)
(8) $\Delta F = 0, \ \pm 1$ (except $F = 0 - 0$) → It is only rarely necessary to consider this.

- ***Allowed = Electric Dipole*** : Transitions which satisfy all the above selection rules are referred to as ***allowed transitions***. These transitions are strong and have **a typical lifetime of** $\sim 10^{-8}$ **s**. Allowed transitions are denoted without square brackets.

$$\text{e.g., C IV } 1548, 1550 \text{ Å}$$

- Photons do not change spin, so transitions usually occur between terms with the same spin state ($\Delta S = 0$). However, relativistic effects mix spin states, particularly for high *Z* atoms and ions. As a result, one can get (weak) spin changing transitions. These are called ***intercombination (semi-forbidden or intersystem) transitions*** or lines. They have **a typical lifetime of** $\sim 10^{-3}$ **s**. An intercombination transition is denoted with a single right bracket.

$$\text{C III] } 2s^2\ ^1S - 2s2p\ ^3P^o \text{ at } 1908.7\,\text{Å}. \quad (\Delta S = 1)$$

- If any one of the rules 1-4, 6-8 are violated, they are called ***forbidden transitions*** or lines. They have **a typical lifetime of** $\sim 1 - 10^3$ **s**. A forbidden transition is denoted with two square brackets.

$$1906.7\,\text{Å [C III] } 2s^2\ ^1S_0 - 2s2p\ ^3P^o_2, \quad (\Delta S = 1,\ \Delta J = 2)$$

- ***Resonance line*** denotes a dipole-allowed transition arising from the ground state of a particular atom or ion.

Forbidden Lines

- Forbidden lines are often difficult to study in the laboratory as collision-free conditions are needed to observe metastable states.
  - In this context, it must be remembered that laboratory ultrahigh vacuums are significantly denser than so-called dense interstellar molecular clouds.
  - ***Even in the best vacuum on Earth, frequent collisions knock the electrons out of these orbits (metastable states) before they have a chance to emit the forbidden lines.***
  - In astrophysics, low density environments are common. In these environments, the time between collisions is very long and an atom in an excited state has enough time to radiate even when it is metastable.
  - Forbidden lines of nitrogen ([N II] at 654.8 and 658.4 nm), sulfur ([S II] at 671.6 and 673.1 nm), and oxygen ([O II] at 372.7 nm, and [O III] at 495.9 and 500.7 nm) are commonly observed in astrophysical plasmas.
  - ***The forbidden 21-cm hydrogen line is particularly important for radio astronomy as it allows very cold neutral hydrogen gas to be seen.***
  - Since metastable states are rather common, forbidden transitions account for a significant percentage of the photons emitted by the ultra-low density gas in Universe.
  - ***Forbidden lines can account for up to 90% of the total visual brightness of objects such as emission nebulae.***

Notations

- Notations for Spectral Emission Lines and for Ions
  - There is a considerable confusion about the difference between these two ways of referring to a spectrum or ion, for example, C III or C^{+2}. These have very definite different physical meanings. However, in many cases, they are used interchangeably.
  - C^{+2} is a baryon and C III is a set of photons.
  - **C^{+2} refers to carbon with two electrons removed**, so that is doubly ionized, with a net charge of +2.
  - **C III is the spectrum produced by carbon with two electrons removed.** The C III spectrum will be produced by impact excitation of C^{+2} or by recombination of C^{+3}. So, depending on how the spectrum is formed. C III may be emitted by C^{+2} or C^{+3}.

collisional excitation: $C^{+2} + e^- \rightarrow C^{+2*} + e^- \rightarrow C^{+2} + e^- + h\nu$

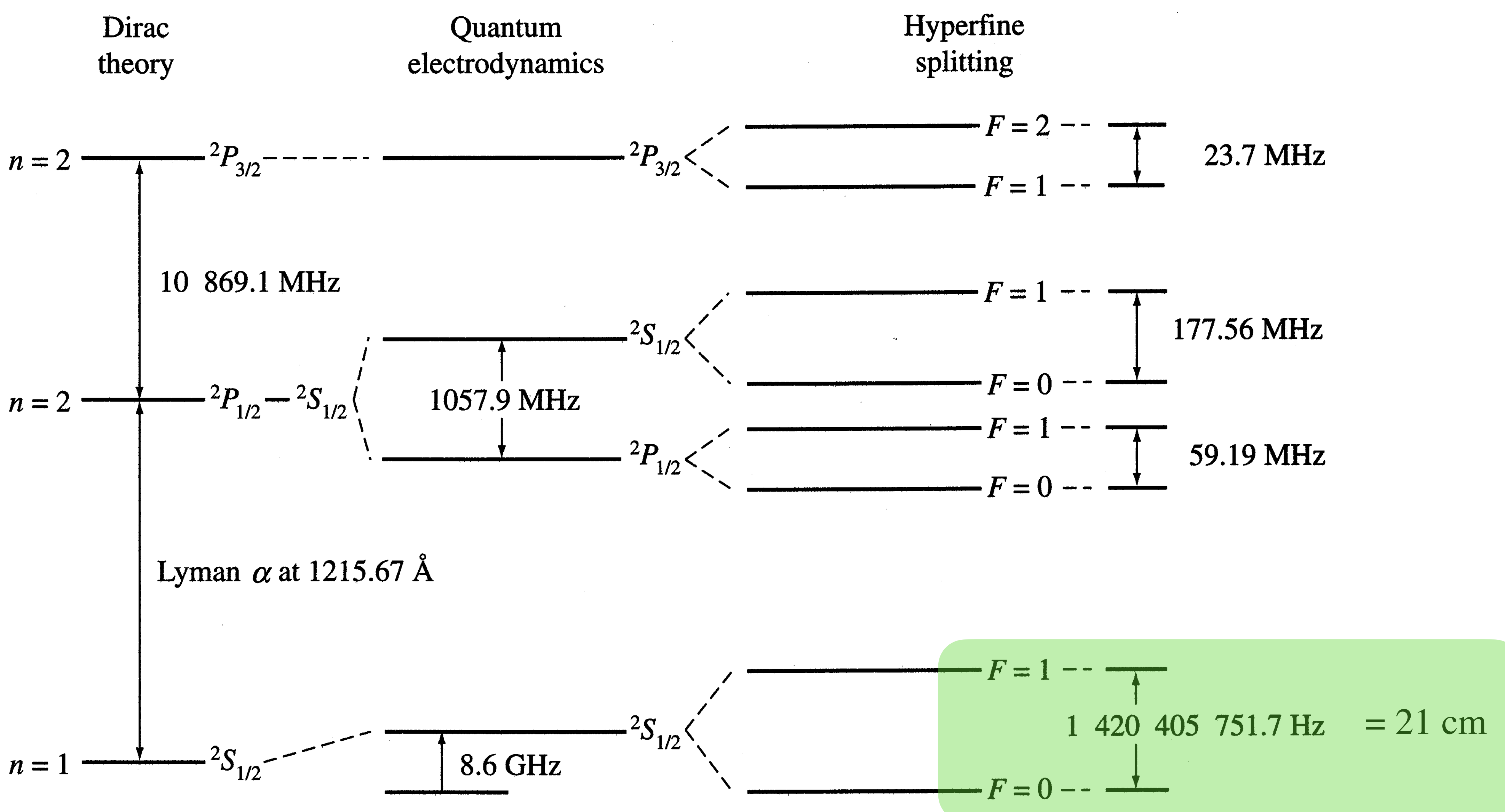
recombination: $C^{+3} + e^- \rightarrow C^{+2} + h\nu$

  - There is no ambiguity in absorption line studies - only C^{+2} can produce a C III absorption line. This had caused many people to think that C III refers to the matter rather than the spectrum.
  - But this notation is ambiguous in the case of emission lines.

Hydrogen Atom : Fine & Hyperfine Structures

- ***Hyperfine Structure in the H atom***
  - Coupling the nuclear spin I to the total electron angular momentum J gives the final angular momentum F. For hydrogen this means

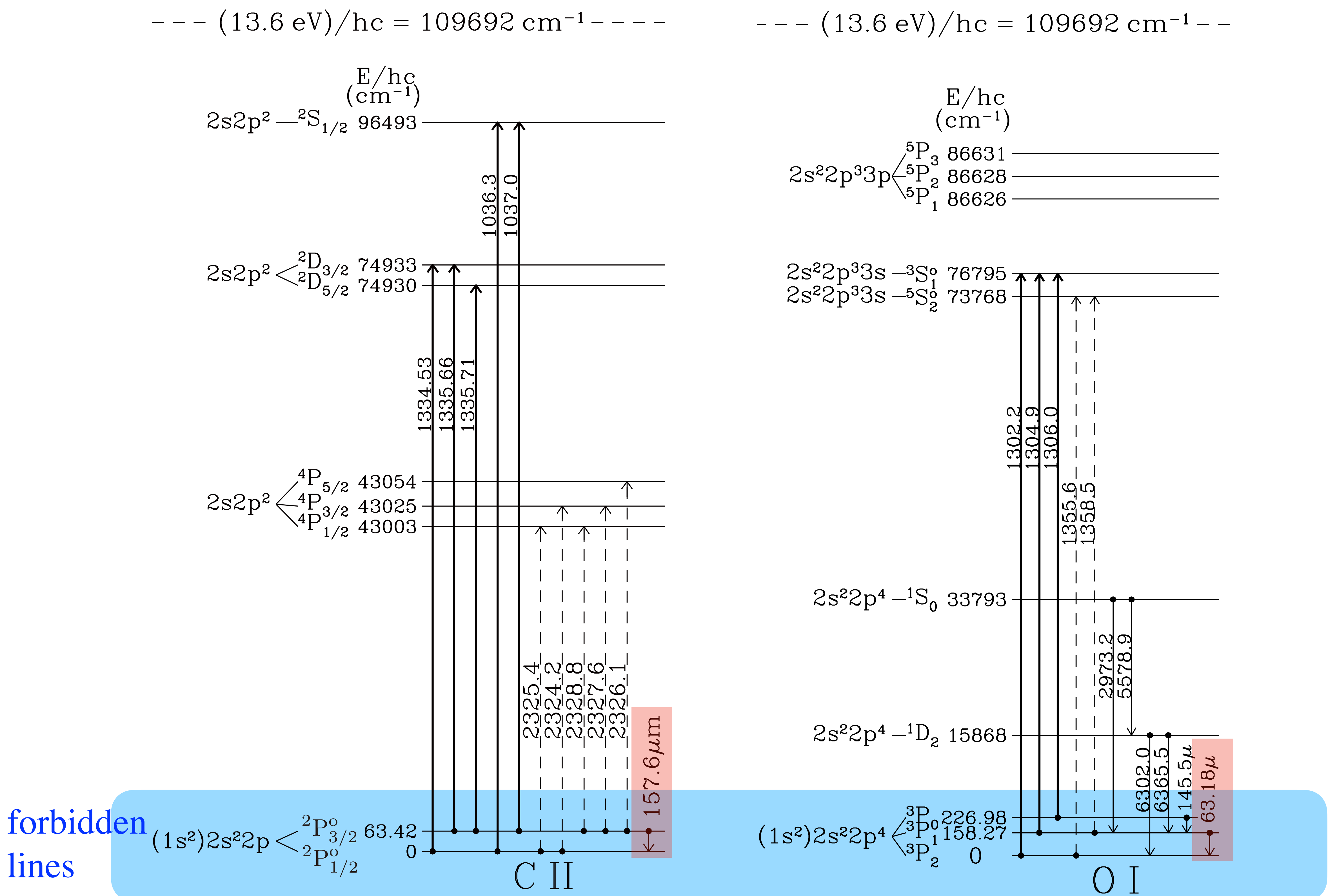
$$F = J + I = J \pm \frac{1}{2}$$



[Kwok] Physics and Chemistry of the ISM, [Bernath] Spectra of atoms and Molecules

- # 5 & 8 electrons

Upward heavy: allowed, Upward Dashed: intercombination, Downward solid: forbidden



[Draine] Physics of the Interstellar and Intergalactic Medium

Multiphase ISM

Five Phases of the ISM

Molecular clouds

- H_2 is the dominant form of molecules.
- **Number density ~ 10^6 cm^{-3} in the molecular cloud cores**, which are self-gravitating and form stars. (Note that 10^6 cm^{-3} is comparable to the density in the most effective cryo-pumped vacuum chambers in laboratories.)
- How to observe: for instance, 2.6, 1.3 and 0.9 mm (115, 230 and 345 GHz) emission lines from CO.

Cold neutral medium (CNM) ($T \sim 10^2$ K)

- The dominant form of CNM is H I (atomic hydrogen).
- The CNM is distributed in sheets and filaments occupying ~1% of the ISM volume.
- How to observe: UV and optical absorption lines in the spectra of background stars and quasars.

Warm neutral medium (WNM) ($T \sim 5 \times 10^3$ K)

- Its dominant form is H I (atomic hydrogen).
- A leading method of observing the WNM is using 21 cm emission.

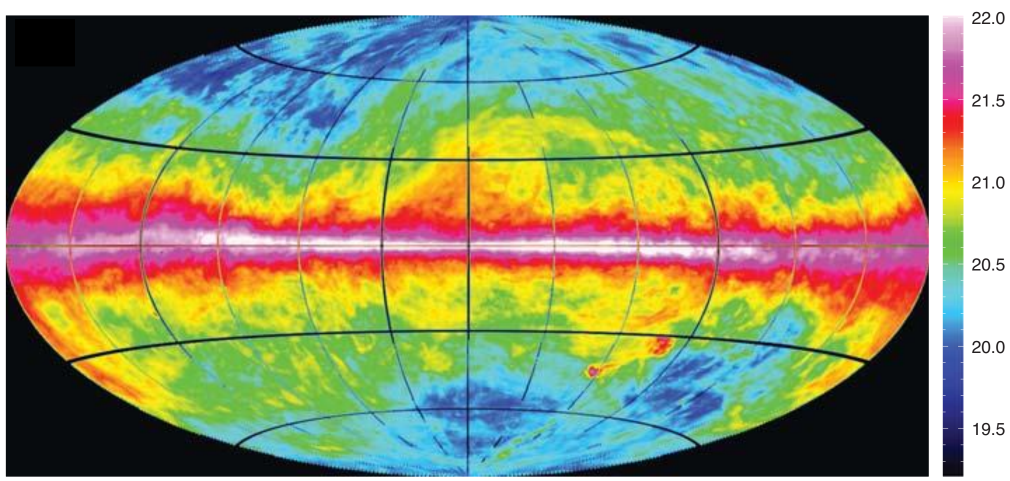
Warm ionized medium (WIM) or Diffuse ionized gas (DIG) ($T \sim 10^4$ K)

- The dominant form is H II (ionized hydrogen or proton).
- The WIM is primarily photoionized by O- and B- type stars.
- Observed using Balmer emission lines (Hα).

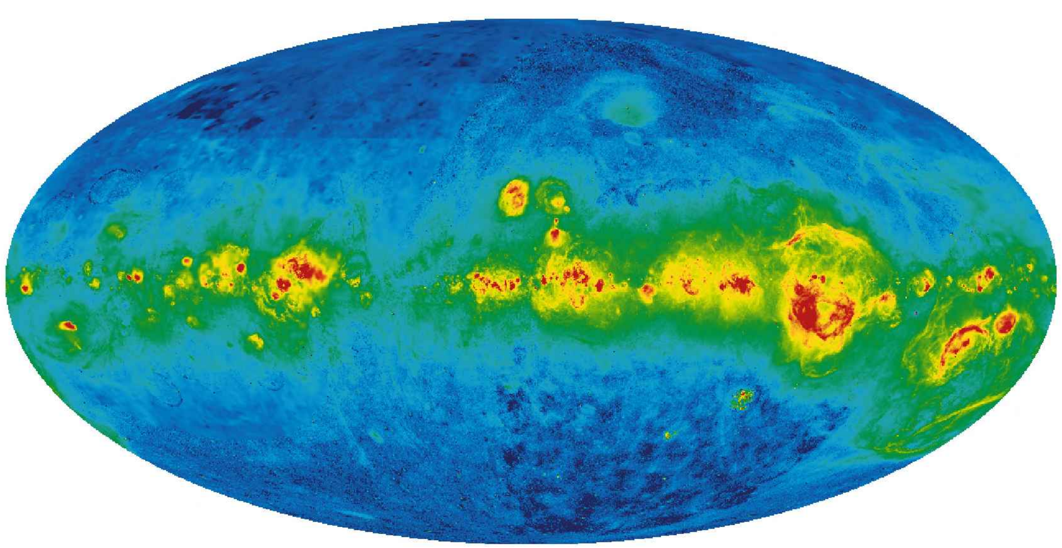
Hot ionized medium (HIM) or coronal gas ($T \gtrsim 10^{5.5}$ K)

- The HIM is primarily heated by supernovae.
- HIM occupies ~ half of the ISM volume, but provides only 0.2% of the ISM mass.
- soft X-ray emission. O VI, N V, and C IV emission or absorption lines in the spectra of background stars.

Name	T (K)	$n_{\mathrm{H}}(\mathrm{cm}^{-3})$	Mass fraction	Volume fraction
Molecular Clouds	20	> 100	35%	0.1%
Cold Neutral Medium	100	30	35%	1%
Warm Neutral Medium	5000	0.6	25%	40%
Warm Ionized Medium	10^4	0.3	3%	10%
Hot Ionized Medium	10^6	0.004	0.2%	50%

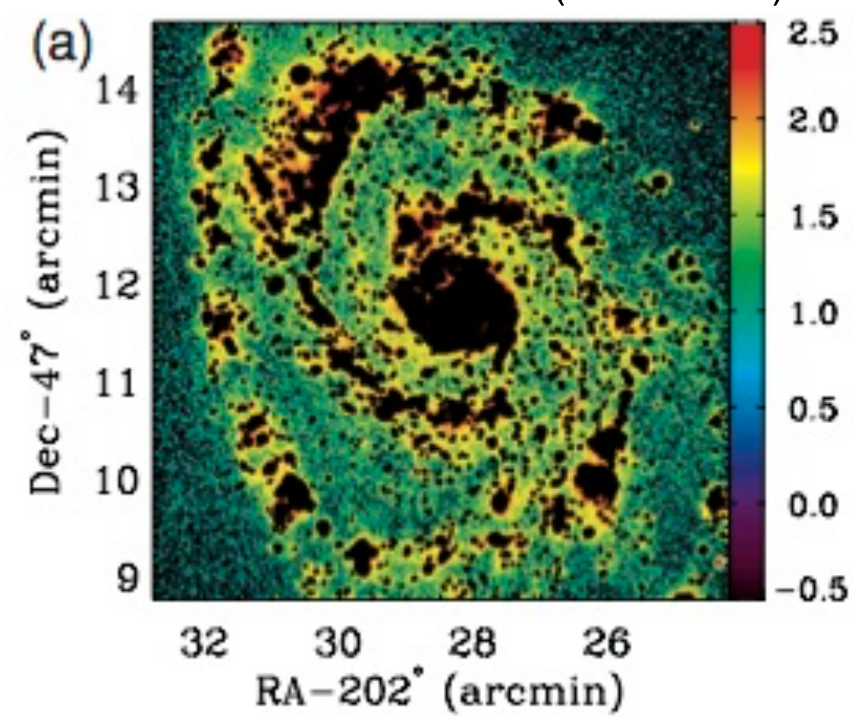


CNM + WNM: All-sky 21 cm map


WIM: All-sky map of Hα (6563Å)


M51 (NGC5195)
Plate 1 [Lequeux]

B band - blue
V band - green
Hα - red (DIG)

DIG: M 51 (Seon 2009)


< Temperature > - Heating and Cooling in the ISM

- ***The temperature of the ISM is determined by a balance between heating and cooling.***
  - Each phase has a temperature where the balance is a stable one.

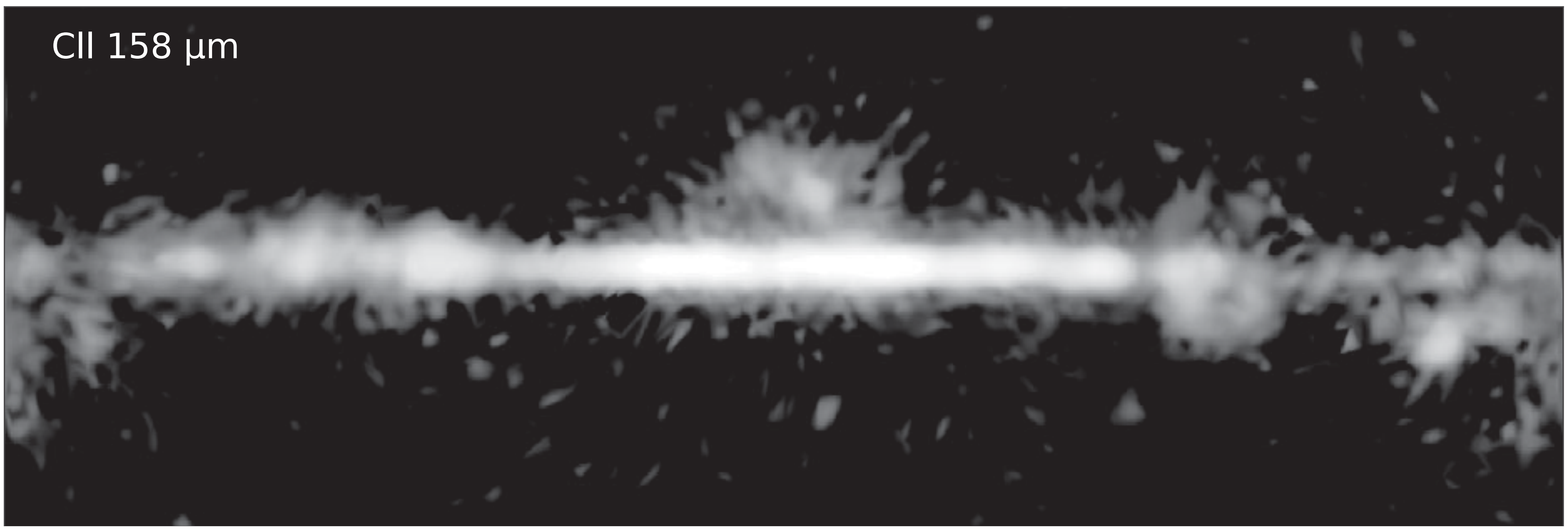
- Definitions
  - **Heating gain per atom** G, **Cooling loss per atom** L in units of erg s^{-1}.
  - **Volumetric heating rate** $g = nG$, **Volumetric cooling rate** $\ell = nL$ in units of erg cm^{-3} s^{-1}.
  - **Cooling function** Λ in units of erg cm^{3} s^{-1}, which is useful for ***two-body interactions***.
  - $\ell = nL = n^2\Lambda$, where n is the total number density of gas particles.
  - Even when only one type of particle is losing energy, the energy loss is shared among all the gas particles due to the relatively short thermalization time scale in the ISM.

Heating & Cooling in Neutral Medium

- Heating processes
  - The primary heating mechanisms of the ISM involve providing free electrons with high energies. Through collisions, the fast free electrons share their kinetic energy with other particles, and through further collisions, the distribution of velocities approaches a Maxwellian distribution.
  - **Source of free electrons**
    - ✦ Ionization by cosmic rays
    - ✦ **Photoionization of dust grains by starlight UV - the most important one.**
    - ✦ Photoionization of atoms (H, He, C, Mg, Si, Fe, etc) by X-rays or starlight UV.
  - **Other heating sources:**
    - ✦ **Heating by shock waves and other MHD phenomena.**
- Cooling processes
  - Collisional excited lines ([C II], [O I], Lyα, etc)
  - Thermal bremsstrahlung

- Heating & Cooling -

- Heating: Photoelectric heating by dust
  - UV and X-ray photons can knock electrons free from dust grains. The ejected electrons carry kinetic energy, which can be effective at heating the surrounding gas.
  - ***Photoelectrons emitted by dust grains dominate the heating of the diffuse neutral ISM (CNM and WNM) in the Milky Way.***
  - ***Photoelectric heating from dust may be an order of magnitude larger than the cosmic ray heating rate.***
- Cooling: Radiative cooling
  - Decreasing the average kinetic energy of particles in the ISM is usually done by ***radiative cooling***.
  - In the CNM, cooling is performed by infrared photons emitted by carbon and oxygen.
    - ✦ Oxygen is nearly all in the form of neutral O I. (the ionization energy = 13.26 eV)
    - ✦ Carbon will be nearly always in the form of singly ionized C II. (ionization energy = 11.26 eV) The background starlight in our galaxy has enough photons in the relevant energy range $11.26\,\mathrm{eV} < h\nu < 13.60\,\mathrm{eV}$ to keep the C atoms ionized.
  - [C II] 158µm (collisionally excited line emission)
  - [O I] 63.2µm (collisionally excited emission line)



C II 158 **µ**m line emission in the Galaxy. The map size is -180^{o} to 180^{o} in Galactic longitude and -60^{o} and 60^{o} in Galactic latitude. The data is from all-sky maps created by the Cosmic Microwave Background Explorer.

[Fig. 5.5. Introduction to the Interstellar Medium, J. P. Williams]

Two-Phase Model & Three-Phase Model

- As a result of stability analysis, Field, Goldsmith, and Habing (1969) created a two-phase model of the ISM, consisting of Cold Neutral Clouds, with n ~ 10 cm^{-3} and T ~ 100 K, embedded within a Warm Intercloud Medium, with n ~ 0.1 cm^{-3} and T ~ 10,000 K.

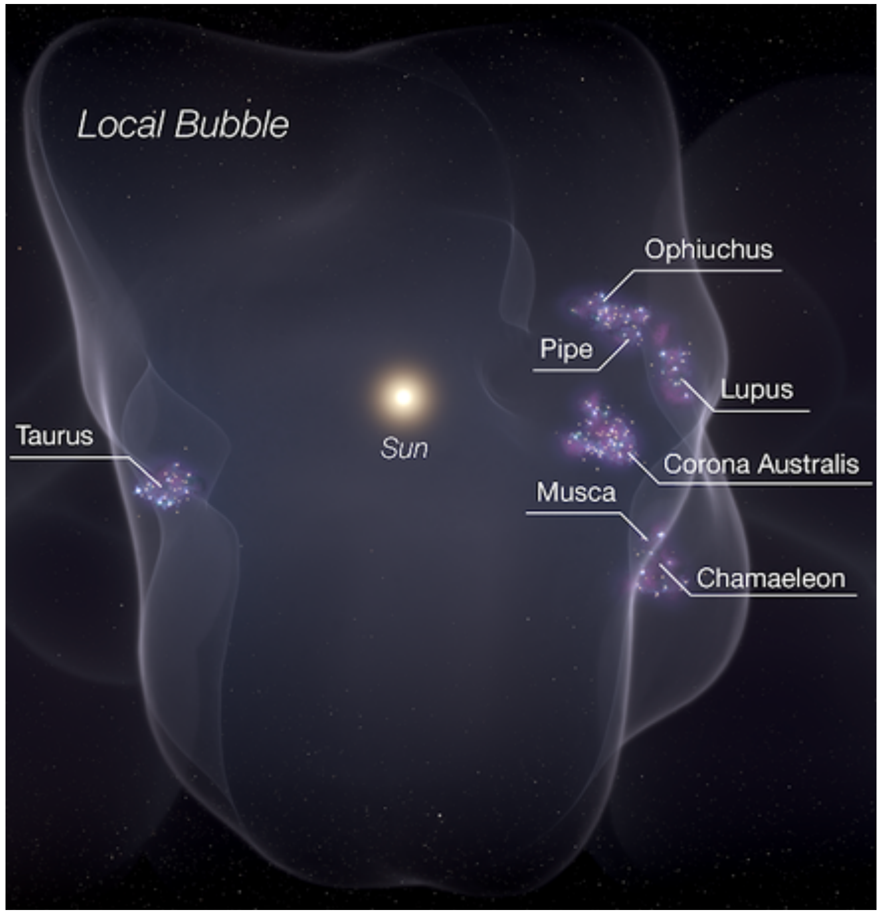
  ✦ They were unaware of the role played by dust in heating the ISM, assumed that ***collisional ionization by cosmic rays provided the bulk of the heating***.

  ✦ FGH (1969) advocated a two-phase model. However, they also speculated "an existence of a third stable phase at T > 10^6 K, with bremsstrahlung the chief cooling process."

- In the 1970s, detection of a diffuse soft X-ray background and of emission lines such as O VI 1032, 1038Å hinted at the existence of interstellar gas with T ~ 10^6 K. In fact, the Sun resides in a "***Local Bubble***" of hot gas, with T ~ 10^6 K and n ~ 0.004 cm^{-3}.

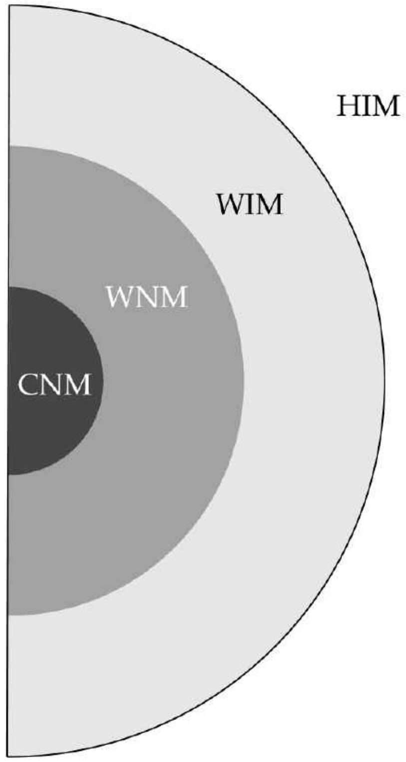
- Cox & Smith (1974) suggested that supernova remnants could produce a bubbly hot phase, and that the bubbles blown by supernovae would occupy a large volume fraction of the ISM.

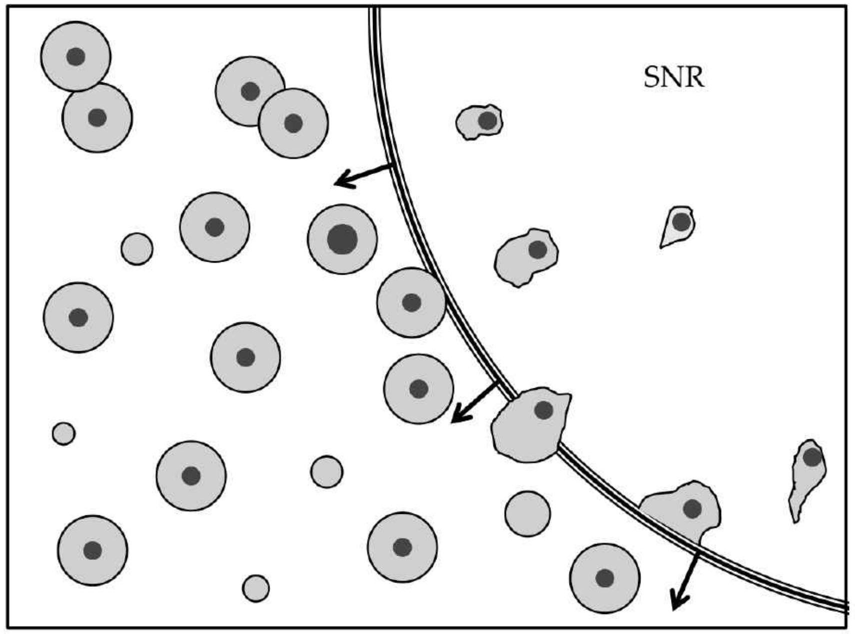
- A **superbubble or supershell** is a cavity which is ~100 pc across and is populated with hot (10^6 K) gas atoms, less dense than the surrounding ISM, blown against that medium and carved out by multiple supernovae and stellar winds.



McKee & Ostriker's Three-Phase Model

- McKee & Ostriker (1977)
  - They made a more elaborate argument for three phases within the ISM.
  - ***Cold Neutral Medium***, with $T \sim 80\,\mathrm{K}$, $n \sim 40\,\mathrm{cm}^{-3}$, and a low fractional ionization $x = n_e/n \lesssim 0.001$.
  - ***Warm Medium***, containing both ionized and neutral components, $T \sim 8000\,\mathrm{K}$ and $n \sim 0.3\,\mathrm{cm}^{-3}$, the ionization fraction ranging from $x \sim 0.02$ in the neutral component (WNM) to $x \sim 0.15$ in the ionized component (WIM).
  - ***Hot Ionized Medium***, consisting of the overlapping supernova bubbles, with $T \sim 10^6\,\mathrm{K}$ and $n \sim 0.002\,\mathrm{cm}^{-3}$, and $x \sim 1$ (nearly complete ionization).





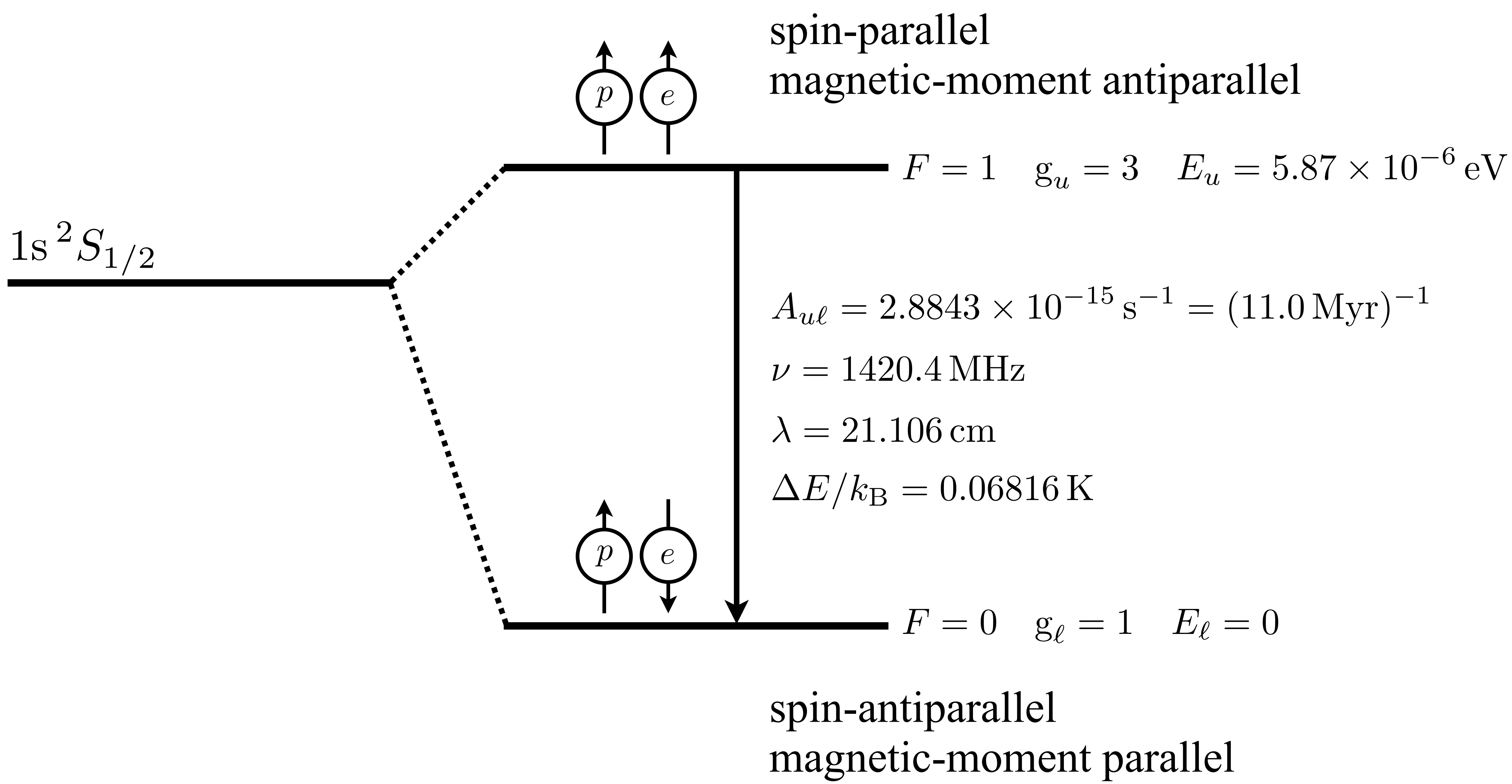
- However, in many ways, the ISM is a dynamic, turbulent, dusty, magnetized place.

Atomic Gas / Hydrogen Gas

Hydrogen Gas - 21 cm hyperfine line

- The CNM and WNM, taken together, provide over half the mass of the ISM.
  - H is the most abundant element in the universe. In the CNM and WNM, most of the hydrogen is in the form of neutral atoms.
  - **The Lyα line of H provides a useful probe of the properties of the CNM and WNM.** ***However, at its wavelength the Earth's atmosphere is highly opaque***, and thus observing Lyα absorption requires orbiting UV satellites. In addition, Lyα can be seen in absorption only along those lines of sight toward sources with a high UV flux.
  - Fortunately, H I can still be detected through the **hyperfine splitting** of the ground electronic state.
  - Such a way was first found in 1944, by Henk van de Hulst.
    - He attempted to find emission lines at the wavelengths ~ 1 cm to 20 m, at which the Earth's atmosphere is transparent. He then realized that the hyperfine structure line resulting from a flip of the electron spin within a hydrogen atom should have a wavelength of 21 cm.
    - This was confirmed by Harold Ewen and Edward Purcell in 1951, when they first detected 21 cm emission from the Milky Way.

Hyperfine splitting of the 1s ground state of atomic H



Note that the magnetic moment is proportional to the charge, so the electron and proton have opposite directions of the magnetic moments.

Difference between Lyα and 21 cm transitions

- The excitation energy for Lyα (E = 10.2 eV, E/k = 118,000 K) is much higher than the kinetic temperature of the neutral ISM.

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} \exp\left(-\frac{118,000\,\mathrm{K}}{T}\right) = 1.7 \times 10^{-51} \text{ at } T = 1000\,\mathrm{K} \qquad (g_u/g_\ell = 3)$$

  - Collisional excitation is unimportant, and most hydrogen atoms are in the lower level of the Lyα transition.
  - The Lyα has a higher energy by a factor of 1.7×10^6 than the 21 cm.

- The excitation energy for 21 cm is ~ 5.9 μeV, and its equivalent temperature E/k = 0.068 K is much lower than the temperature of the cosmic microwave background.
  - Even the CMB is able to populate the upper level.
  - If collisions are frequent, then **the spin temperature ($T_{\rm exc}$) will be solely determined by collisions**, and thus will be a good tracer of the gas kinetic temperature.
  - Thus, there is ample opportunity to populate the upper energy level of the 21 cm hyperfine transition. The level populations for the 21 cm levels, since $T_{\rm exc} \gg 0.068\,\mathrm{K}$ in all circumstances of the ISM.

$$\boxed{\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} e^{-h\nu/kT_{\rm exc}}} = 3\,e^{-0.068\,\mathrm{K}/T_{\rm exc}} \simeq 3 \longrightarrow n_u \simeq \frac{3}{4} n_{\rm H},\ n_\ell \simeq \frac{1}{4} n_{\rm H}$$

  - However, in many cases (in particular in WNM), the hyperfine levels may not be in excitation equilibrium. Radio astronomers use the term ***spin temperature*** for 21 cm rather than the “excitation temperature.”

Typical Optical Depths of the 21-cm line

- Typical optical depths of the 21-cm line:

$$\tau_\nu = \kappa_\nu \Delta s = \frac{3}{32\pi} A_{u\ell} \frac{hc\,\lambda_{u\ell}}{kT_{\rm spin}} (n_{\rm H}\Delta s)\,\phi_\nu$$

$$\tau_0 = 0.311 \left(\frac{N_{\rm H\,I}}{10^{21}\ {\rm cm^{-2}}}\right)\left(\frac{100\,{\rm K}}{T_{\rm spin}}\right)\left(\frac{10\,{\rm km\,s^{-1}}}{b}\right) \longleftarrow$$

column density:

$$N_{\rm HI} = n_{\rm HI}\Delta s \quad [{\rm cm^{-2}}]$$

- In the CNM, a typical spin temperature is $T_{\rm spin} \approx 50 - 100\,K$:

$$\tau_0^{\rm CNM} \approx 0.3 - 0.6$$
$$e^{-\tau_0} \approx 0.55 - 0.74$$

The CNM is in general optically thin, but show significant absorption.

- In the WNM, a typical spin temperature is $T_{\rm spin} \approx 5000 - 8000\,K$:

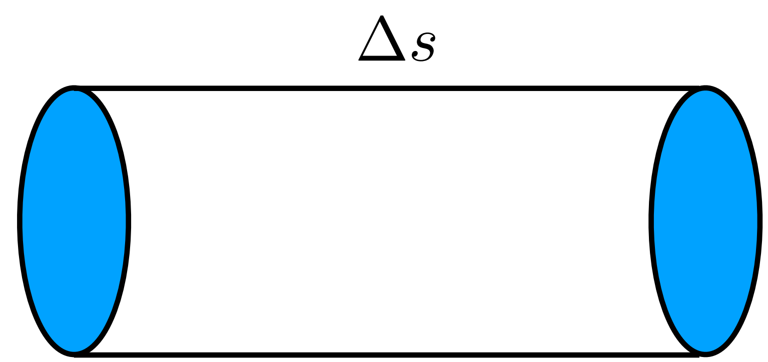
$$\tau_0^{\rm WNM} \approx 0.004 - 0.006$$
$$e^{-\tau_0} \approx 0.995$$

The 21-cm absorption is negligible in the WNM.

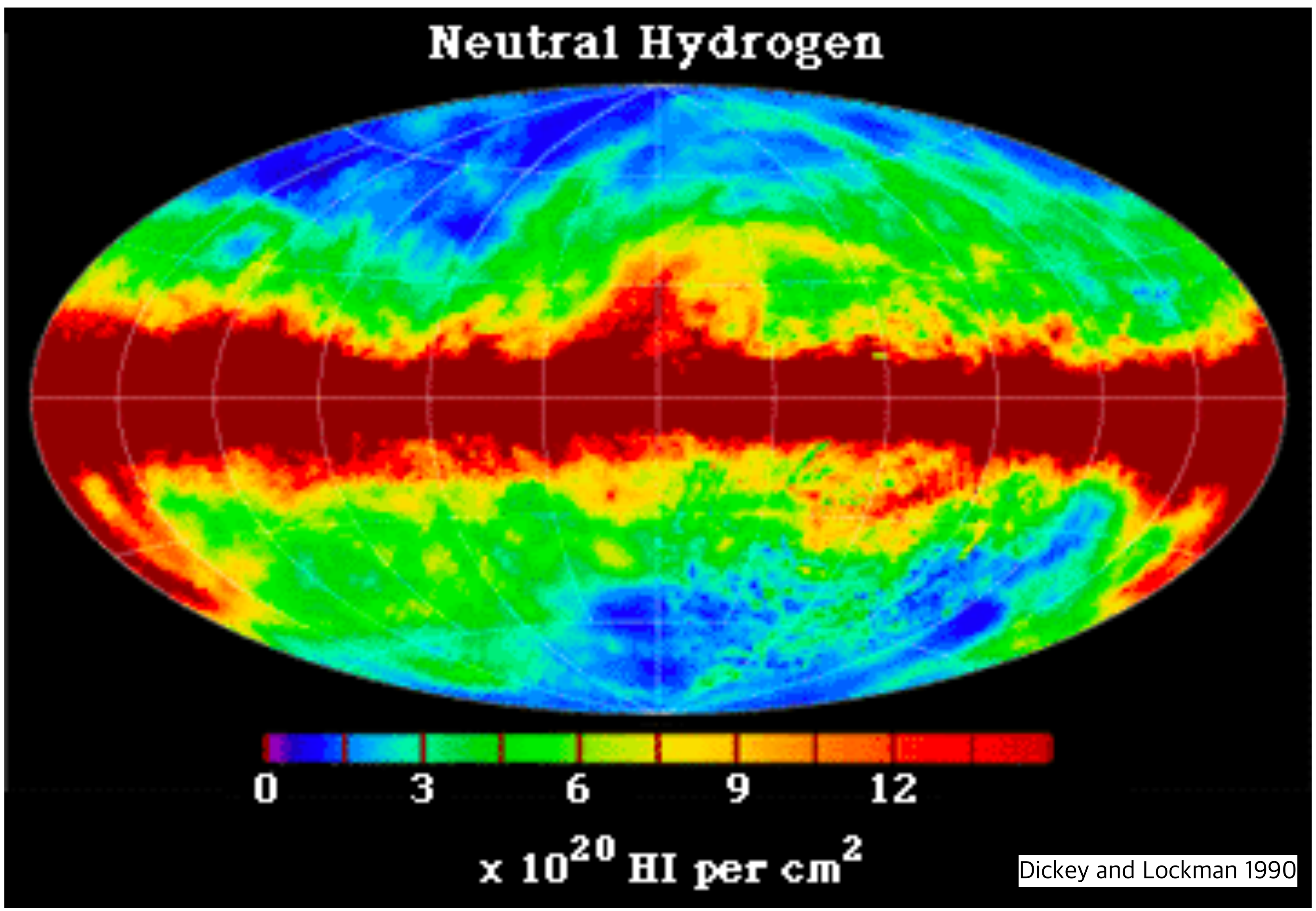
Typical H I column density in our Galaxy

- ***Column density:*** an amount of matter per unit of area along a line-of-sight

$$N_{\rm HI} = n_{\rm HI}\Delta s \quad [\rm cm^{-2}] \qquad \Leftarrow \qquad N_{\rm HI} = \frac{n_{\rm HI}V}{\Delta A}$$



$$\Delta A = 1\,\rm cm^2$$



Dickey and Lockman 1990

$$\boxed{N_{\rm HI} = 10^{20} - 10^{22}\,\rm cm^2}$$

in our Galaxy,
(except for the Lockman hole)

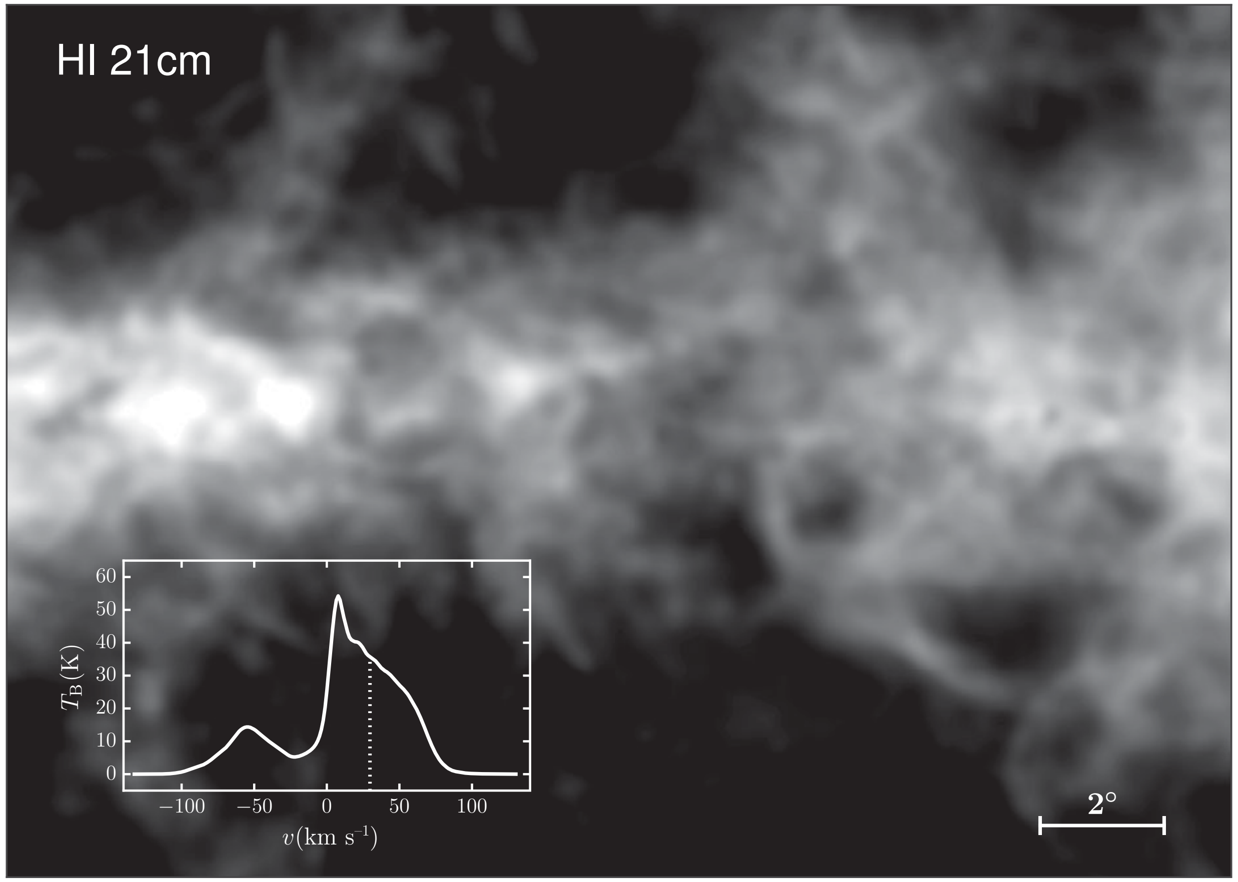
The Lockman hole is an area of the sky in which minimal amounts of neutral hydrogen gas are observed.

Column density in Lockman hole

$$N_{\rm HI} \approx 5 \times 10^{19}\ \rm cm^2$$

Coordinates:
(*l*, *b*) = (149.77, 52.03)
(RA, dec) = (10h45m, +58deg)
Size: ~ 15 square degrees

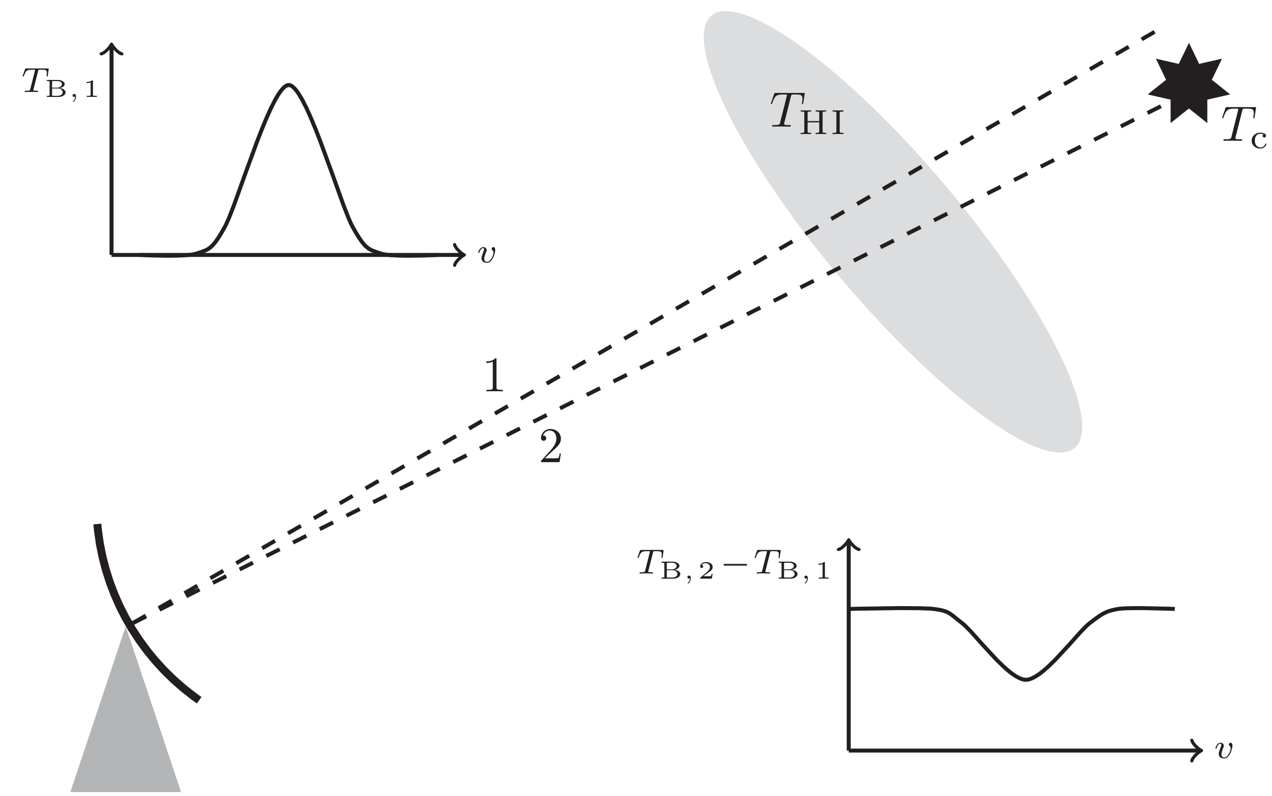
21 cm emission & absorption lines



Galactic 21 cm H I line emission from the HI4PI survey.

The inset shows the mean spectrum of this region in units of brightness temperature versus velocity.

The vertical dotted line shows the velocity slice shown in the image.



Schematic of an absorption experiment to measure the temperature of an atomic cloud.

The first line of sight measures the emission from the cloud only.
The second line of sight contains a background source with continuum (brightness) temperature T_c.

The difference between the two lines of sight reveals a dip in the continuum due to absorption by the cloud.

[Introduction to Interstellar Medium, J.P. Williams]

Homework (due date: 09/15)

[Q2] Consider an (isotropically emitting) star of uniform intensity $I_\nu = B$ at the surface. Show that the flux at the surface is

$$F_\nu = \int I_\nu \cos\theta d\Omega = \pi B$$

[reference] Radiative Processes in Astrophysics (Rybicki & Lightman)

[Q3] (a) The specific intensity at the surface of a star is, to first order, a blackbody. For a given effective temperature, $T_{\rm eff}$, and stellar radius, R, what is its bolometric luminosity?

(b) Look up values for these parameters and calculate this formula for the Sun.