

# Modern Astronomy

## Part 1. Interstellar Medium (ISM)

Lecture 1

2025 March 5 (Wednesday), 9AM

updated on 03/05, 21:56

선광일 (Kwangil Seon)  
UST / KASI

# Professors & Classroom

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## **Professors:**

- 1- 4th weeks: Prof. Seon, Kwangil (선광일, [kiseon@kasi.re.kr](mailto:kiseon@kasi.re.kr))
- 5- 8th weeks: Prof. Kim, Sang Chul (김상철, [sckim@kasi.re.kr](mailto:sckim@kasi.re.kr))
- 9-12th weeks: Prof. Lee, Sang-Sung (이상성, [sslee@kasi.re.kr](mailto:sslee@kasi.re.kr))
- 13-16th weeks: Prof. Shin, Jihey (신지혜, [jhshin@kasi.re.kr](mailto:jhshin@kasi.re.kr))

## **Day & Time:**

Monday 2-5PM

## **Classroom:**

이원철홀(LWC) 220 (선광일 / 신지혜)

세종홀 (SJH) 1층 서회의실 (김상철 / 이상성)

# Syllabus

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Week	Date	
1	03-05	Introduction / Interstellar Medium (ISM)
2	03-10	Atomic Gas / Multiphase Medium
3	03-17	Molecular Clouds and Dust
4	03-24	Formation of Stars and Planets
5	03-31	Stars: The Hertzsprung-Russell Diagram
6	04-07	The Evolution of Stars
7	04-14	Star Deaths
8	04-21	The Milky Way Galaxy
9	04-28	Galaxies beyond the Milky Way
10	05-05	Hubble's Law and Distance Scale
11	05-12	Active Galaxies 1
12	05-19	Active Galaxies 2
13	05-26	General Relativity and Friedmann Equation
14	06-02	Evolution of Universe and Inflation Cosmology
15	06-09	Density Perturbations and Nonlinear Structure Formation
16	06-16	Survey and Computer Simulation

# Textbooks / Lecture Notes of Part 1

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- Main Textbooks
  - ▶ Introduction to the Interstellar Medium (Jonathan P. Williams)
  - ▶ Fundamentals of Astrophysics (Stan Owocki)
- Other References
  - ▶ Introductory
    - Introduction to the Interstellar Medium - Jonathan P. Williams
    - The Physics of the Interstellar Medium - J. E. Dyson & D.A. Williams
  - ▶ Intermediate
    - Interstellar and Intergalactic Medium - Barbara Ryden
- Lecture Notes
  - <https://seoncafe.github.io/Teaching.html>

# Unit of distance

- **AU (the Astronomical Unit)**: the average distance of the Earth from the Sun

$$\text{AU} = 1.50 \times 10^{11} \text{ m}$$

- ***parallax***: As the Earth goes around the Sun, the nearby stars change their positions very slightly with respect to the faraway stars. This phenomenon is known as parallax. The angle  $\theta$  is half of the angle by which this star appears to shift with the annual motion of the Earth and is defined to be the parallax.



- ***parsec (pc)***: the distance where the star has to be so that its parallax turns out to be  $1''$ .

As seen in January      As seen in July

$$\begin{aligned}
 \text{pc} &= 3.09 \times 10^{16} \text{ m} \\
 &= 3.26 \text{ light years} \\
 &= 206,265 \text{ AU}
 \end{aligned}$$

# Introduction to the Interstellar Medium

# What is the ISM?

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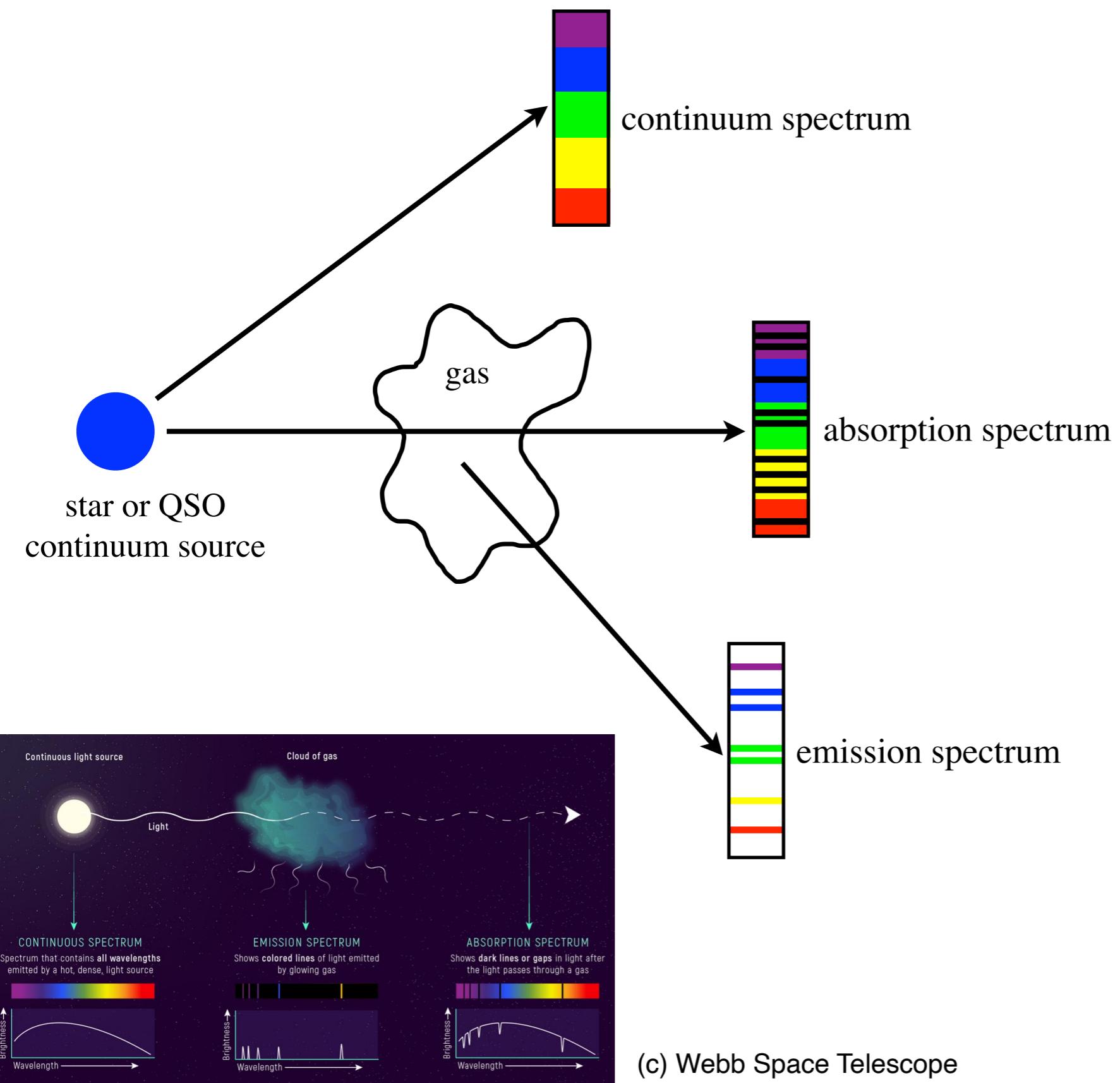
## What is the ISM?

- The ISM is anything not in stars. (D. E. Osterbrock)
- Just what it says: **The stuff between the stars** in and around galaxies, especially our own Milky Way.
- **Gas + Solid particles**
  - It is mad up almost entirely of gas with a tiny (solid) particles called dust grains.
  - In addition to these, the ISM includes radiation, cosmic rays, and magnetic fields.

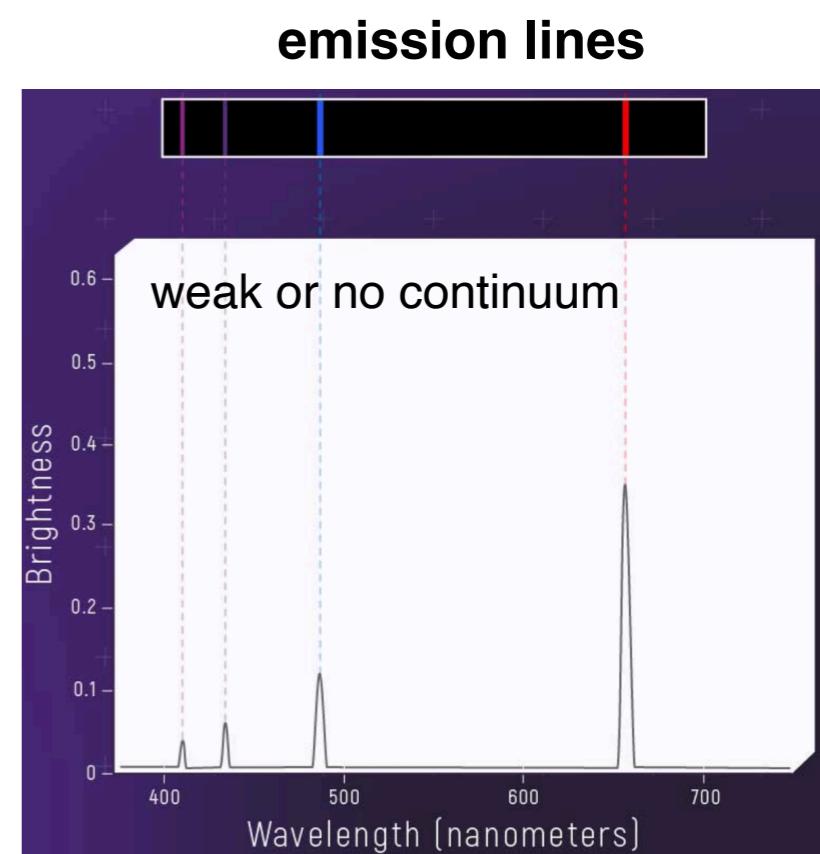
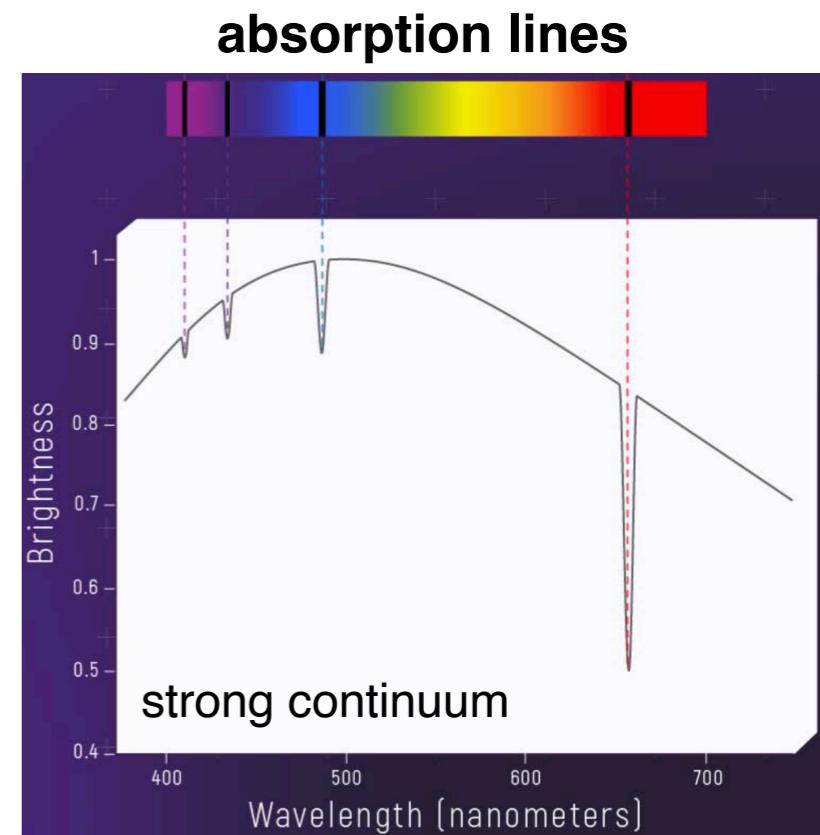
## Why do we study the ISM?

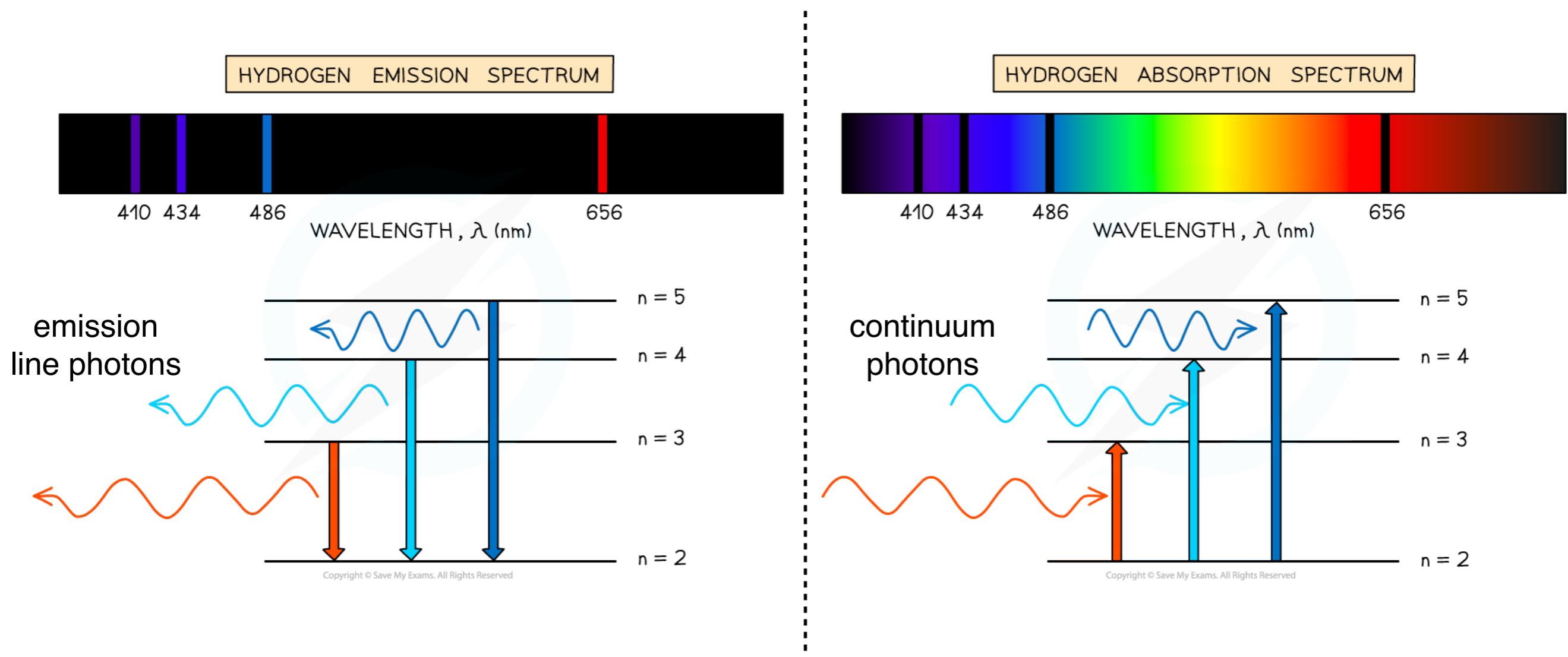
- The ISM is everywhere and it affects all sorts of observations, but more often as an essential complement for understanding the Galaxy.
- The ISM is the most important component of galaxies, for it is the ISM that is responsible for forming the stars that are the dominant sources of energy.
- *The ISM is the most beautiful component of galaxies. (B. T. Draine)*
- *The ISM is beautiful, both in the literal sense, as in images of colorful nebulae, and in the physics that helps us understand our origins and the way the Universe works. (J. P. Williams)*

# Observations of ISM: Absorption & Emission Lines



(c) Webb Space Telescope



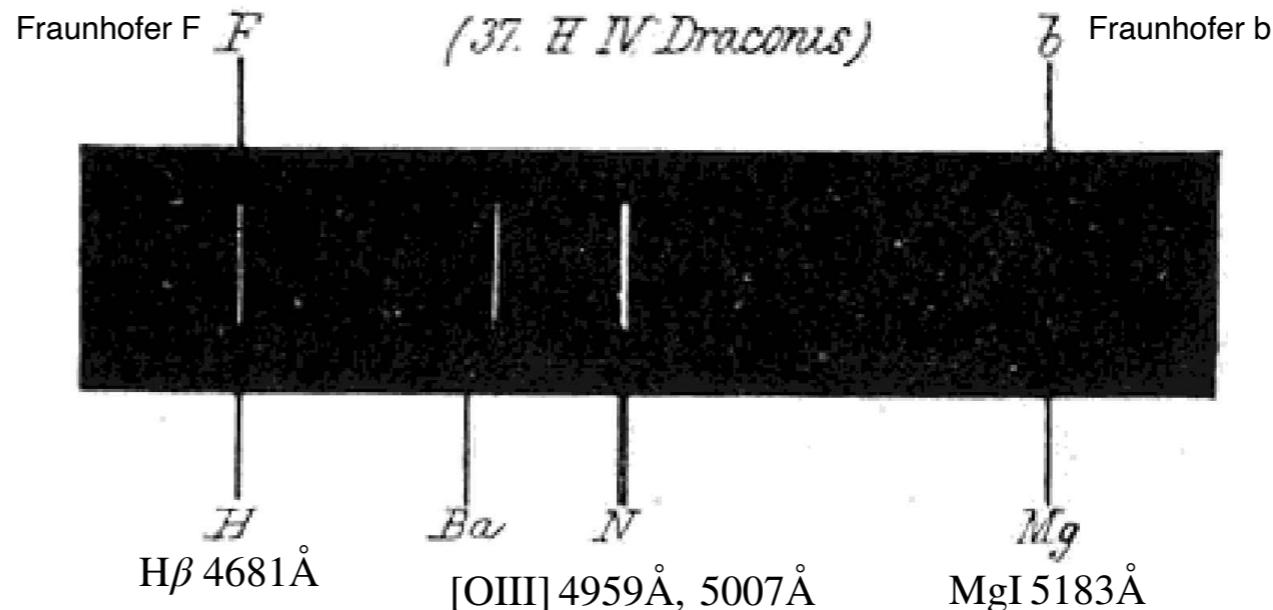


# History of ISM Studies

the Cat's Eye Nebula (planetary nebula/ HST image)



The first nebula spectrum: the Cat's Eye Nebula (NGC 6543; W. Huggins, 1864)



- William Herschel resolved some nebulae into stars. He thought that he had discovered star birth (it actually ejected by a dying star). In the 1860s, William Huggins demonstrated that some nebulae have **emission line spectra**, rather than the **absorption line spectra**.

- **Hypothetical elements:**

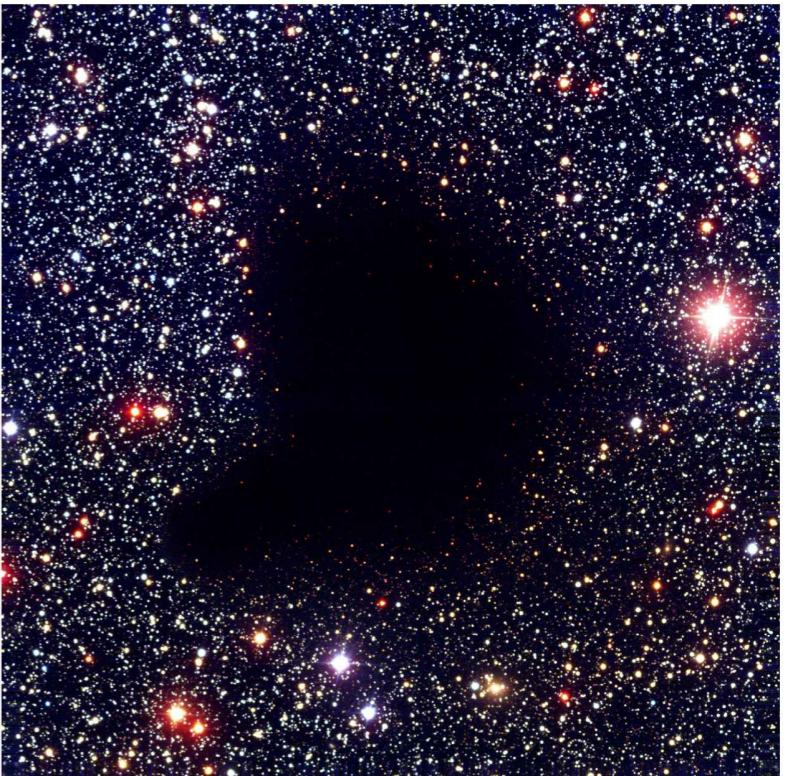
**Nebula means cloud in Latin.**

- ◆ Huggins attributed 4959Å, observed in the Cat's Eye Nebula, to "nebulum" (or "nebulium"), and 5007Å line to Nitrogen => Ira Bowen discovered that these lines were actually forbidden [O III] lines.
- ◆ aurorium : 5577Å in the spectrum of the aurora borealis => turned out to be [O I]
- ◆ coronium: 5303Å in the spectrum of the Sun's corona => Fe XIV

## Interstellar Dust

- The existence of dust had been hinted at by the presence of dark nebulae (Barnard 68).
  - ◆ The dark nebulae were originally thought to be due to a lack of stars, but later recognized as being clouds of obscuring material.
- Vesto Slipher (1912) discovered that the spectrum of the nebula surrounding the Pleiades shows a continuum with absorption lines superposed.
  - ◆ He conjectured that this is light from stars, reflected from “fragmentary and disintegrated matter”, or dust.

V. Slipher (/slaifer/ 1875-1969) is the first one who measured radial velocities for galaxies and discovered that distant galaxies are redshifted. He was also the first to relate these redshifts to velocity.



Barnard 68 (at  $d \sim 150$  pc), in the constellation Ophiuchus.



The Pleiades cluster & surrounding reflection nebulae

## Interstellar gas that is invisible to the eye

- Initially, bright nebulae were thought of as isolated clouds in (nearly) empty space.
- In 1901, Johannes Hartmann found:
  - ◆ the spectrum of binary Delta Orionis (a spectroscopy binary system) shows a narrow calcium absorption line (at  $\lambda 3934$ ) that is in **stationary** in addition to the **time-varying**, broad absorption lines due to the orbital motion of the stars
  - ◆ the stationary Ca absorption line was caused by a gas cloud somewhere along the line of sight to Delta Orionis.
- Later, similar “stationary lines” were found along the sightlines to many other bright stars.
  - ◆ The lines were all narrow, and had strengths correlated with the distance to the background star.
  - ◆ Using higher resolution spectrographs, they had been revealed to have complex structures, consisting of many narrower lines with different radial velocities.
  - ◆ This led to the realization that the ISM has a complex structure, consisting neither of smooth uniform gas nor of isolated blobs drifting about in a near-vacuum.

# Diversity of the ISM

## Ionized nebulae

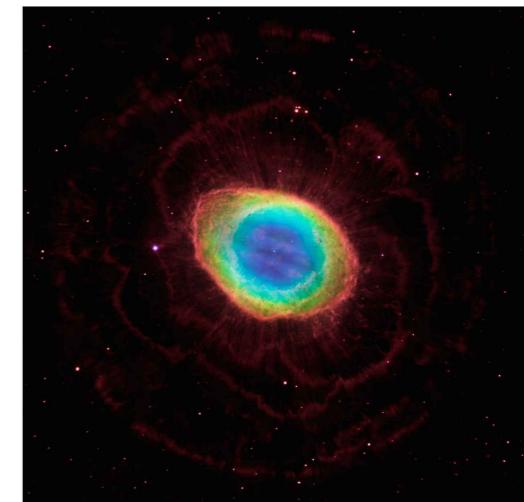
- H II regions
  - are regions of interstellar gas heated and photoionized by embedded O or B-type stars with  $T_{\text{eff}} > 25,000 \text{ K}$ .
  - In 1939, Bengt Stromgren developed the idea that bright nebulae with strong emission lines are regions of photoionized gas, surrounding hot star or other source of ionizing photons.
  - ex) Orion Nebula
- Planetary nebulae
  - are regions of ejected stellar gas heated and photoionized by the hot remnant stellar core, which is becoming a white dwarf.
  - ex) Ring Nebula, Cat's Eye Nebula
  - Ring Nebula:
    - ◆ central region: blue color, from He II 4686.
    - ◆ middle region: blue-green colors from [O III] 4959, 5007
    - ◆ outer reddish colors from H $\alpha$  6563, [N II] 6548, 6583



Orion Nebula ( $d \sim 410 \text{ pc}$ )  
HST image



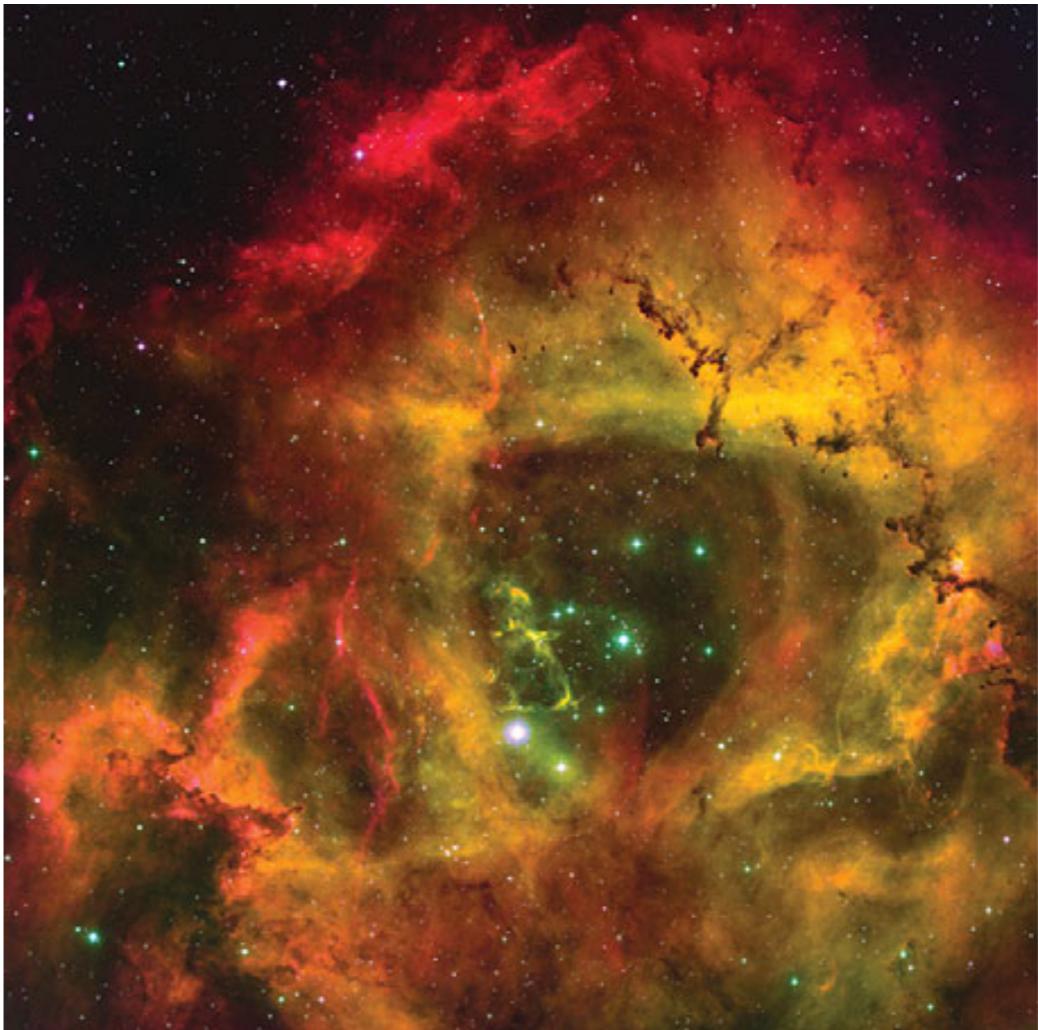
Cat's Eye Nebula (HST image)



Ring Nebula (HST image)

## H II regions

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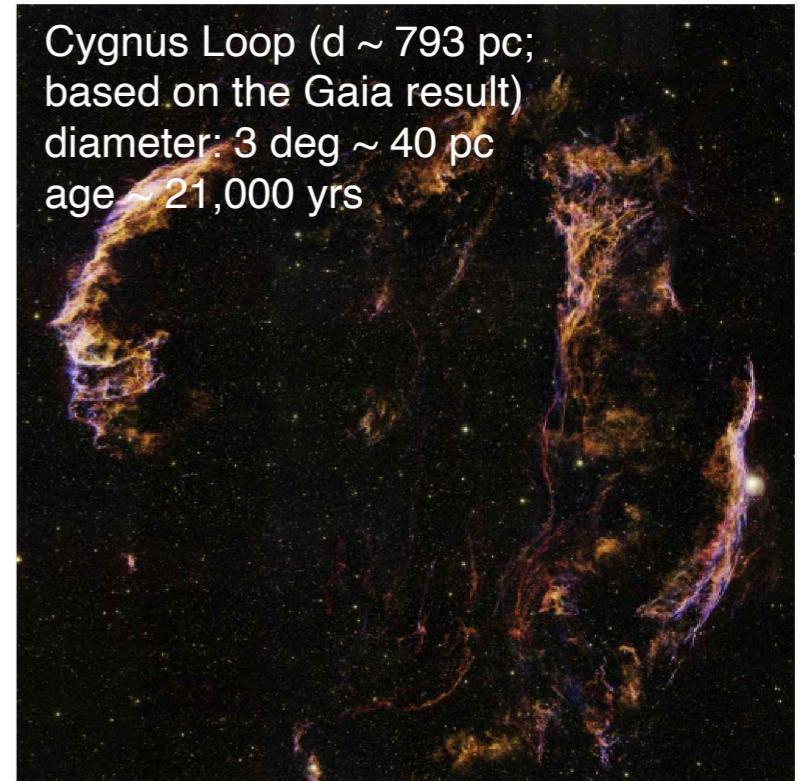
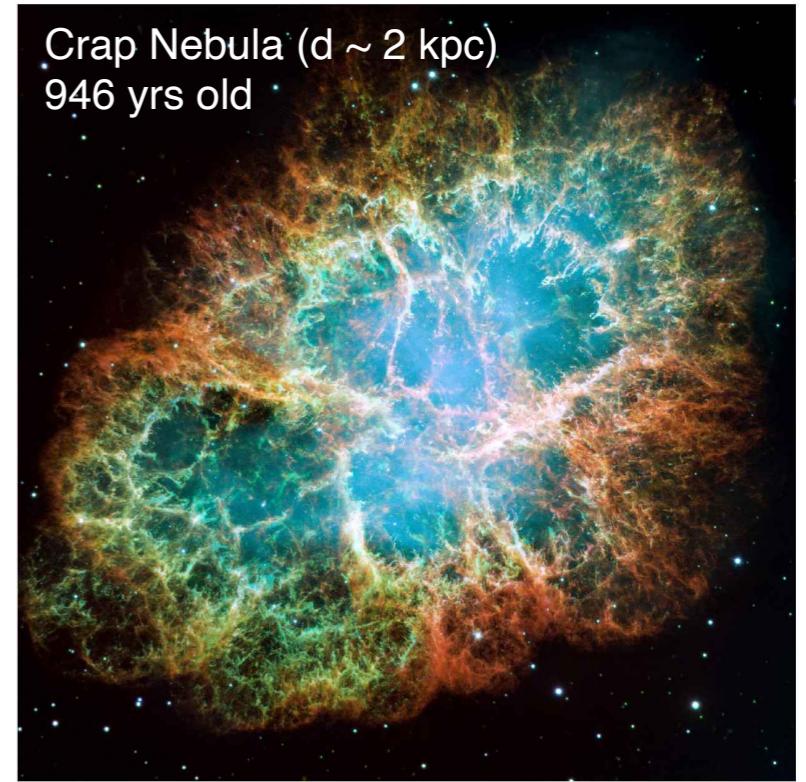
[Left] True-color optical image of the Rosetta Nebula. The reddish glow is from H $\alpha$  line emission from recombination of the ionized hydrogen. The central cavity has been evacuated by the strong, high-speed stellar winds from the central cluster of hot stars.

[Right] Composite false-color image showing the emission in H $\alpha$  (red), and lines of [O III] (green) and [S II] (blue). Credit: NASA/HST [Owocki]

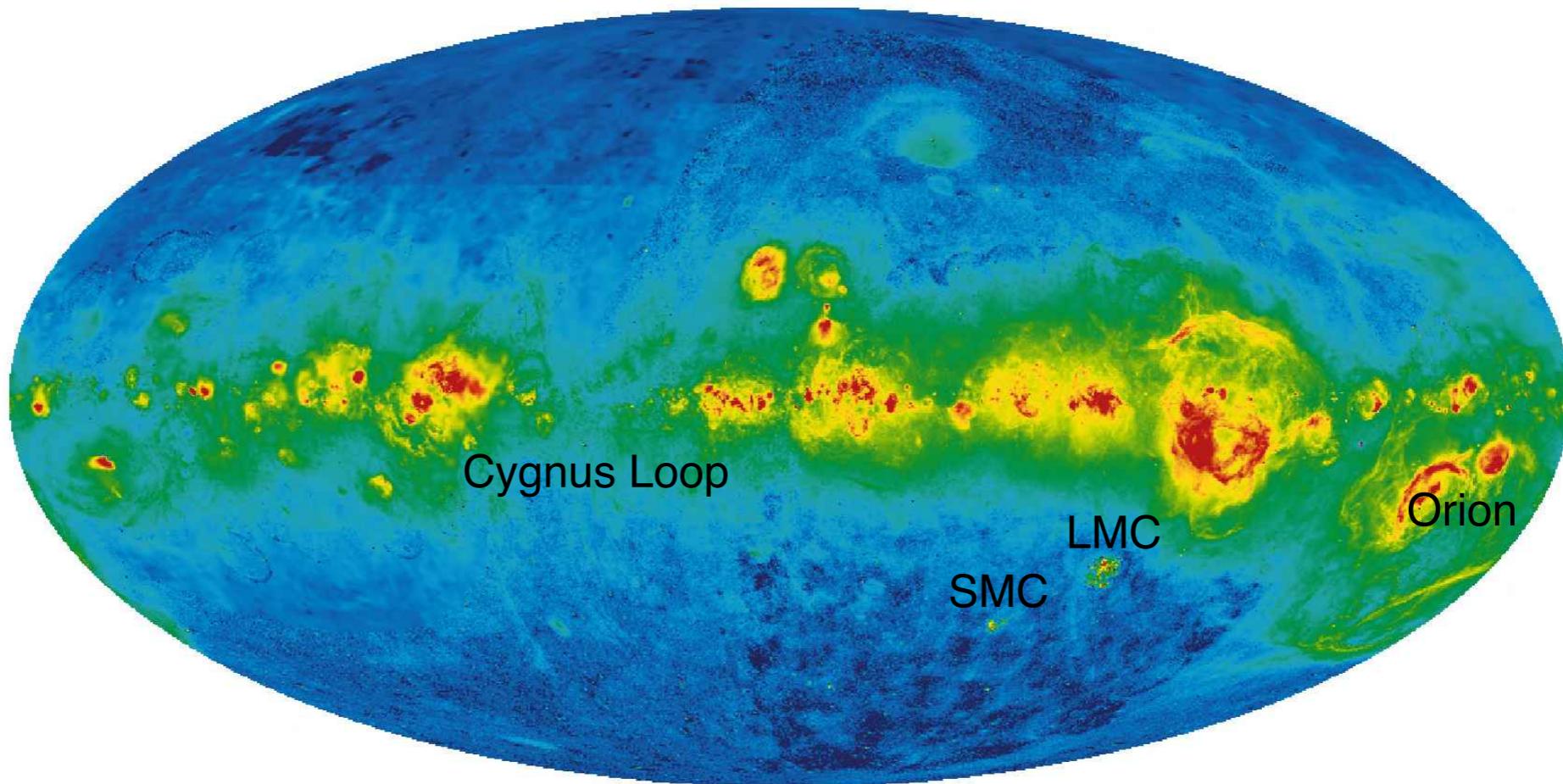
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- Supernova remnants

- are regions of gas heated by the blastwave from a supernova explosion.
- Crab Nebula
  - ◆ a young ( $t \sim 1000$  yr) pulsar-containing supernova remnant
  - ◆ are filled in with luminous gas.
  - ◆ are photoionized by its central pulsar.
  - ◆ are sometimes called ‘plerions’ meaning “full.”
- Cygnus Loop (Veil Nebula)
  - ◆ most of the gas has been plowed up by the blast wave, leaving the center part empty.
  - ◆ The visible loop (or veil) is where the gas has cooled to  $T \sim 10,000$  K.
  - ◆ is a middle-aged supernova remnant ( $t \sim 10^4$  yr).



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- Warm Ionized Medium (Diffuse Ionized Gas)
    - About 20-80% of the ionized hydrogen in our galaxy lies in the relatively low density WIM.
    - Balmer line emission from recombining hydrogen fills the entire sky.
    - Although many ionized nebula (Orion, Crab, Cat's eye, etc) can be seen as the bright red blotches, they are not the dominant repository of recombining hydrogen in our galaxy.



All-sky map of H $\alpha$  (6563Å) in a log scale from 0.03 Ry to 160 Ry.  
Ry (rayleigh) =  $10^6/4\pi$  photons cm $^{-2}$  s $^{-1}$  Hz $^{-1}$  sr $^{-1}$

- 
- Neutral Hydrogen Gas
    - All-sky map of H I 21-cm line intensity from the LAB survey (Kalberla et al. 2005), with an angular resolution  $\sim 0.6$  deg.
    - Scale gives  $\log_{10} N(\text{HI}) [\text{cm}^{-2}]$ . The LMC and SMC are visible, with a connecting H I “bridge”.

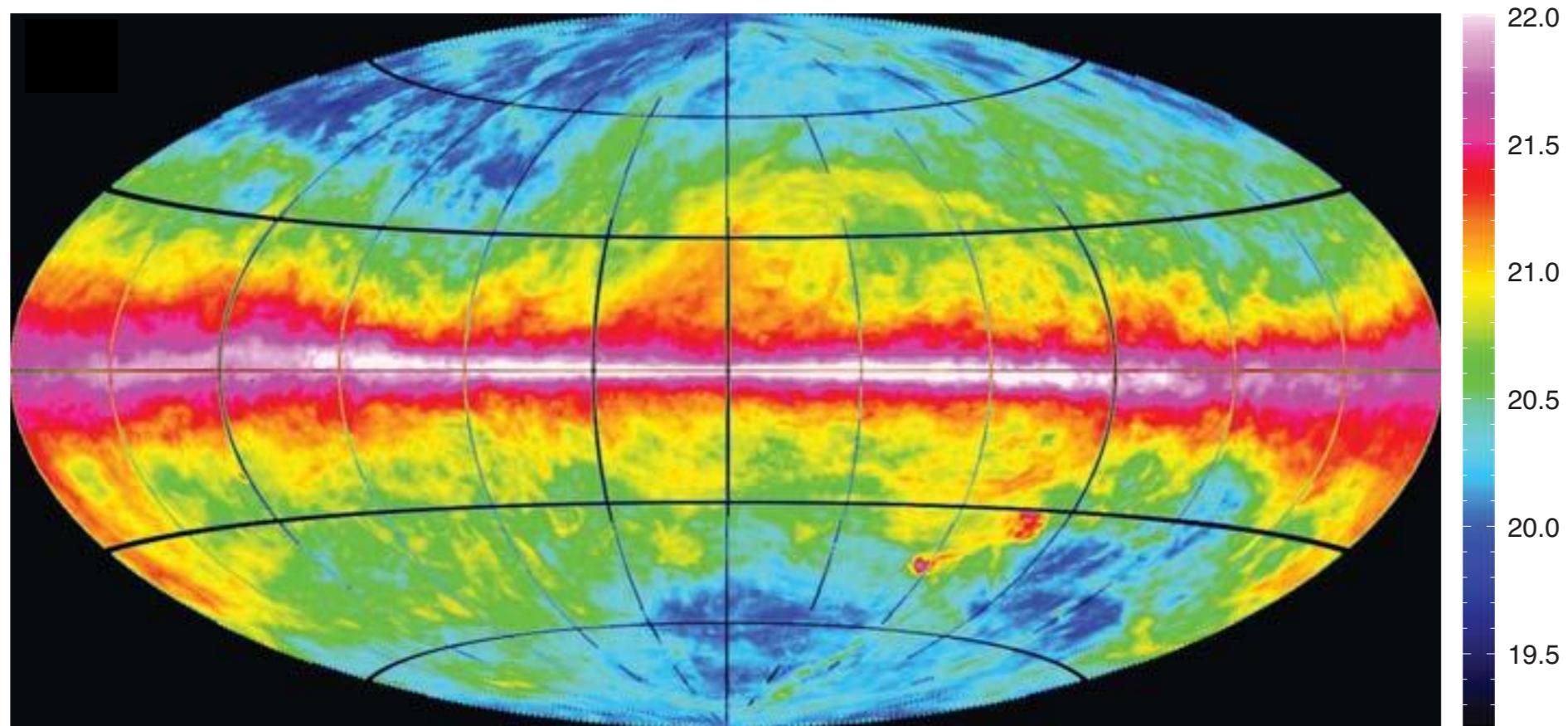


Plate 3 in [Draine]

# ISM in external galaxies



[Left] HST optical image of M51 (Whirlpool galaxy). The reddish blotches are from  $H\alpha$  line emission from giant H II regions, which arise when dense regions of hydrogen are photoionized by the UV radiation from numerous, recently formed, hot massive stars. Note their proximity to dark bands formed from absorption of background stellar light by cold interstellar dust, which outline the galactic spiral arms.

[Right] Composite image of M51 from 4 space missions. X-rays (purple) detected by the Chandra X-ray Observatory reveal point-like sources (black holes and neutron stars in binary systems) as well as diffuse gas. Optical data from HST (green) and infrared emission from the Spitzer Space Telescope (red) both highlight long lanes in the spiral arms that consist of stars and gas laced with dust. UV light (blue) from GALEX comes from hot, young stars, showing how well these track the H II giants and star-forming GMCs along the spiral arms.

Credit: NASA/HST/CXO/SST/GALEX [Owocki]

# Baryonic Matter

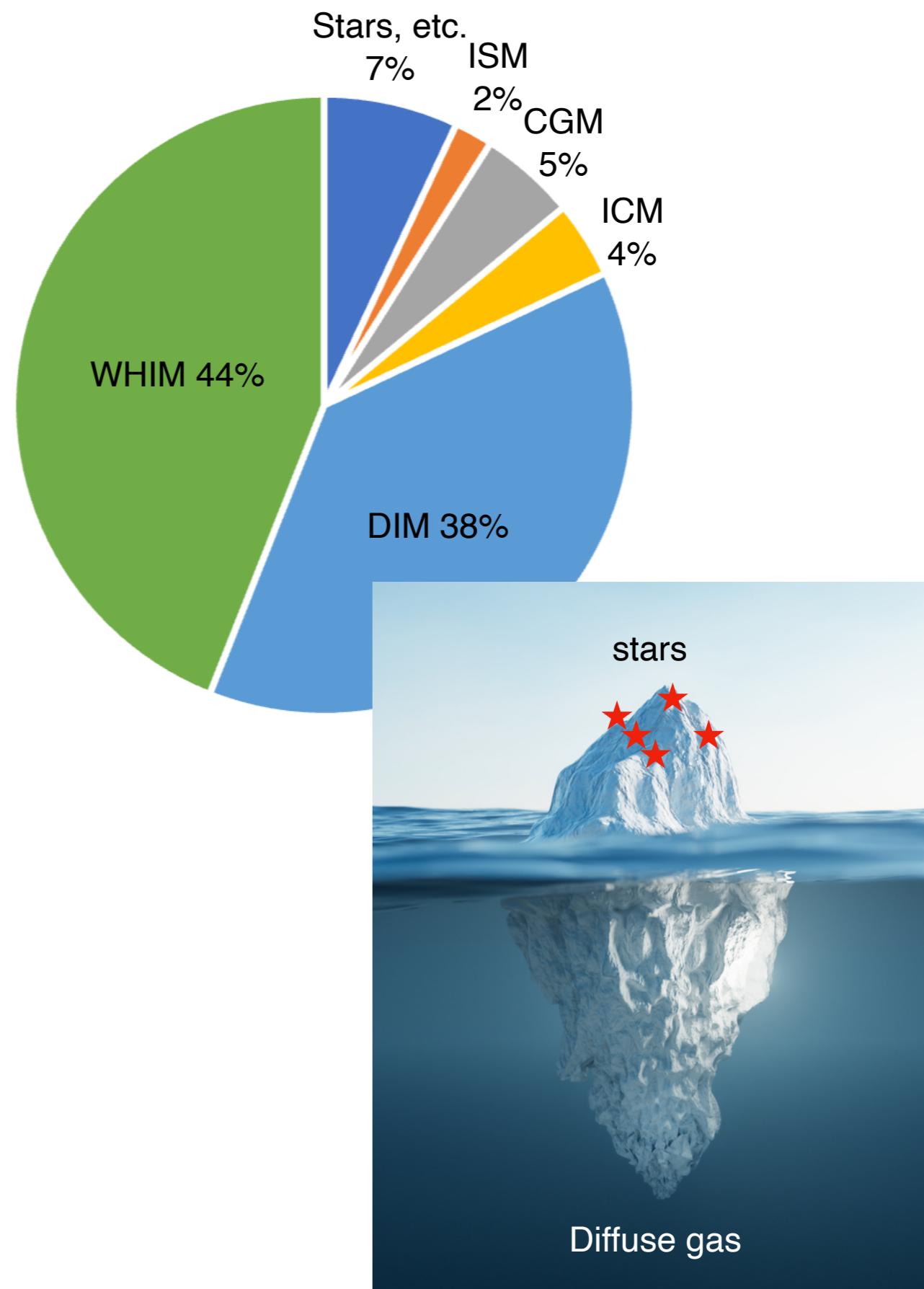
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## Definitions:

- Baryons = protons, neutrons and matter composed of them (i.e. atomic nuclei)
- Leptons = electrons, neutrinos
- In astronomy, however, the term '**baryonic matter**' is used more loosely to refer to **matter that is made of protons, neutrons, and electrons**, since protons and neutrons are always accompanied by electrons. Neutrinos, on the other hand, are considered non-baryonic by astronomers. (Note that black holes are also included as baryonic matter.)

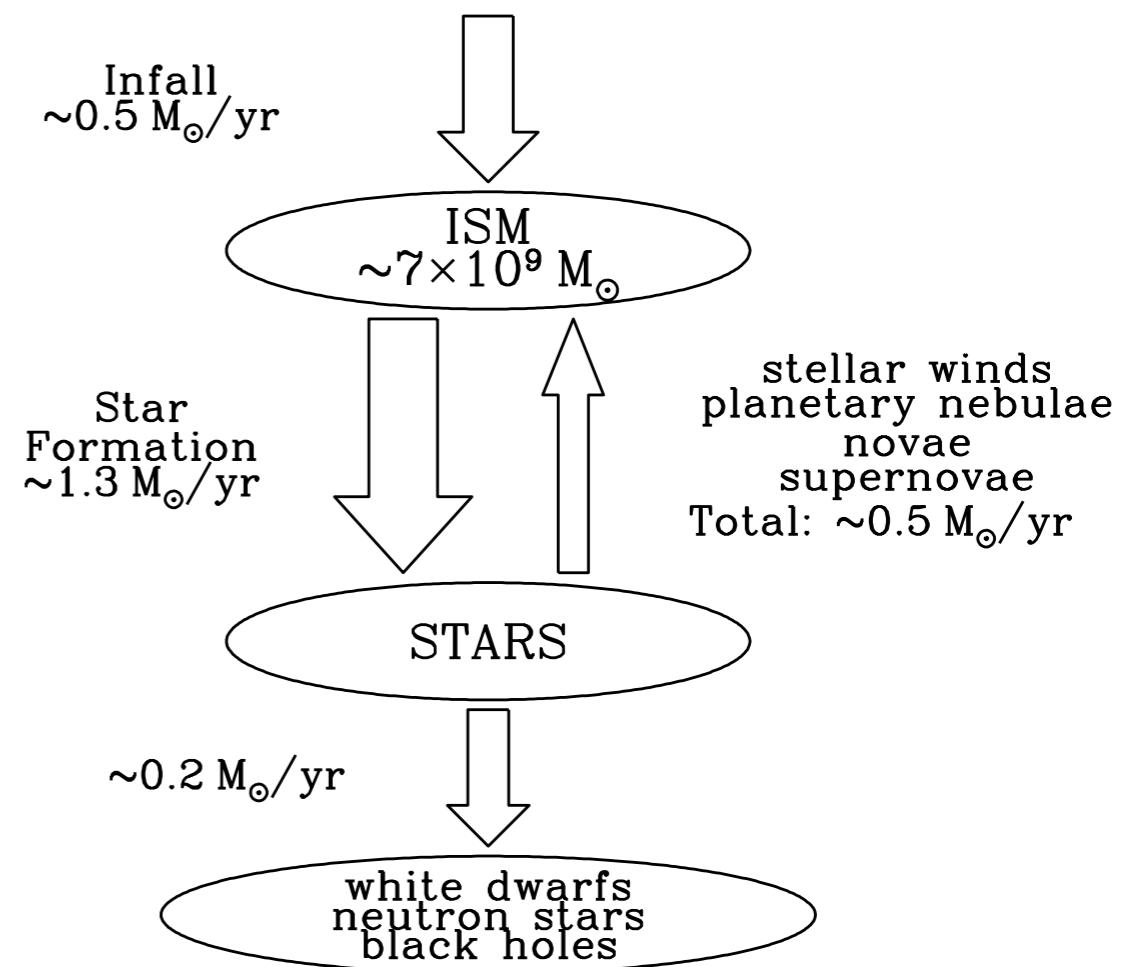
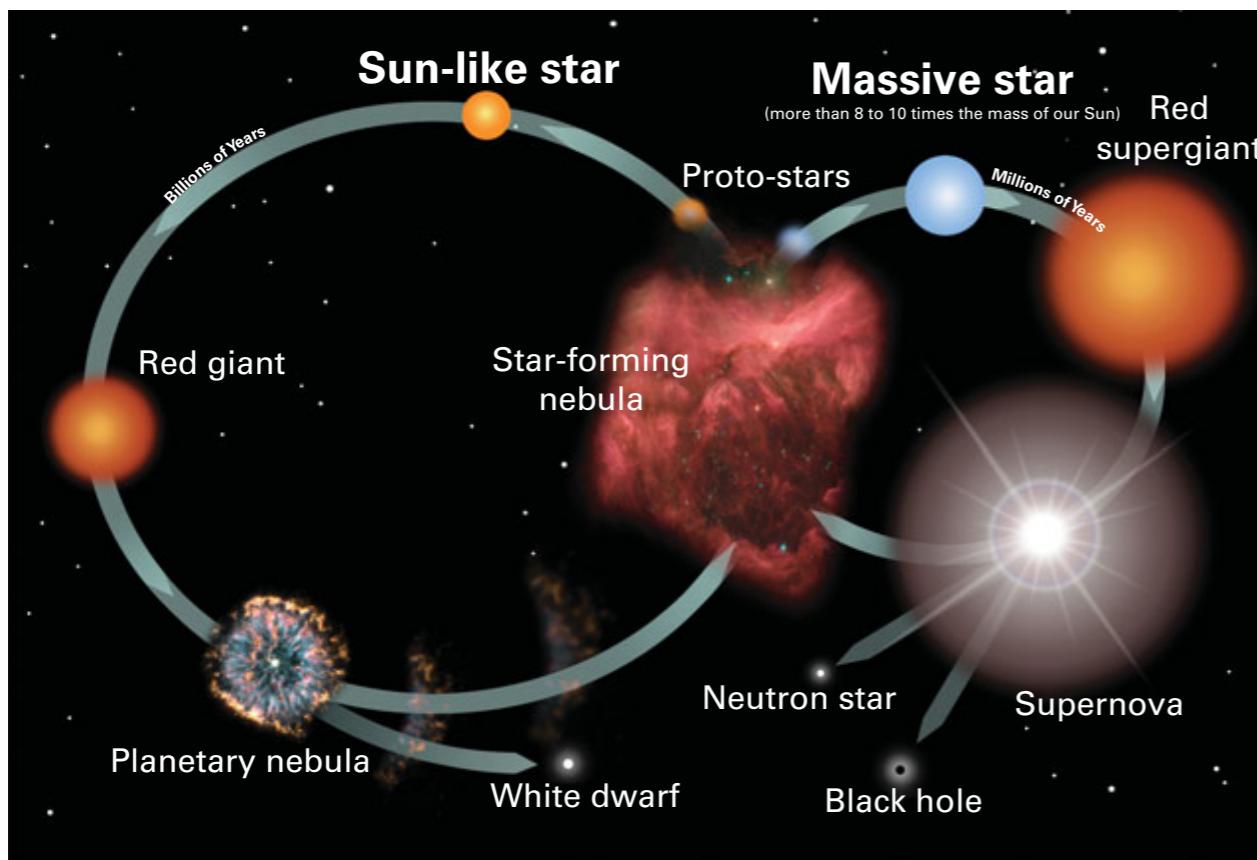
# The baryonic mass density

- 7% : stars + compact objects (such as stellar remnants, brown dwarfs, and planets)
- 2% : interstellar medium (ISM), filling the volume between stars.
- 5%: circumgalactic medium (CGM), bound within the dark halo of a galaxy, but outside the main distribution of stars.
- 4% : intracluster medium (ICM) of clusters of galaxies, bound to the cluster as a whole, but not to any individual galaxy.
- 38%: diffuse intergalactic medium (DIM), made of low density, mostly photo-ionized gas ( $T < 10^5$  K).
- 44% : warm-hot intergalactic medium (WHIM), made of shock-heated gas ( $10^5 \text{ K} < T < 10^7 \text{ K}$ ).



# Mass flow of the baryons in galaxies

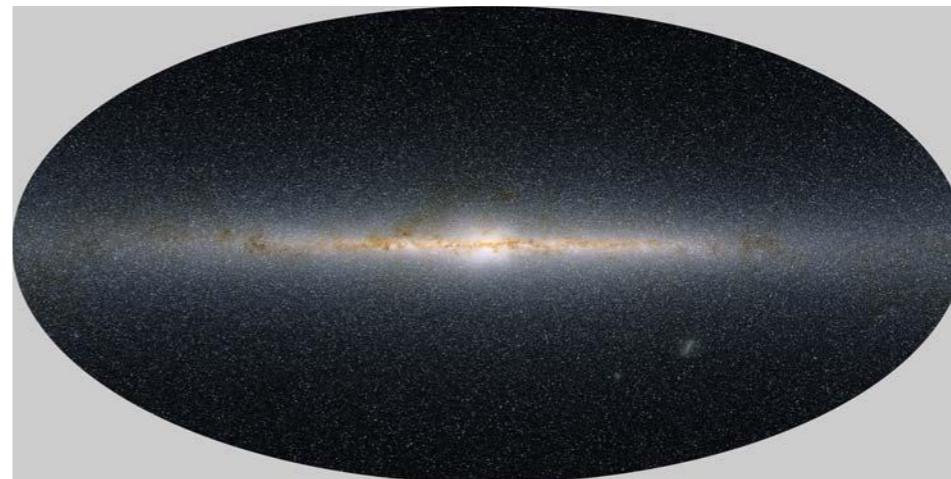
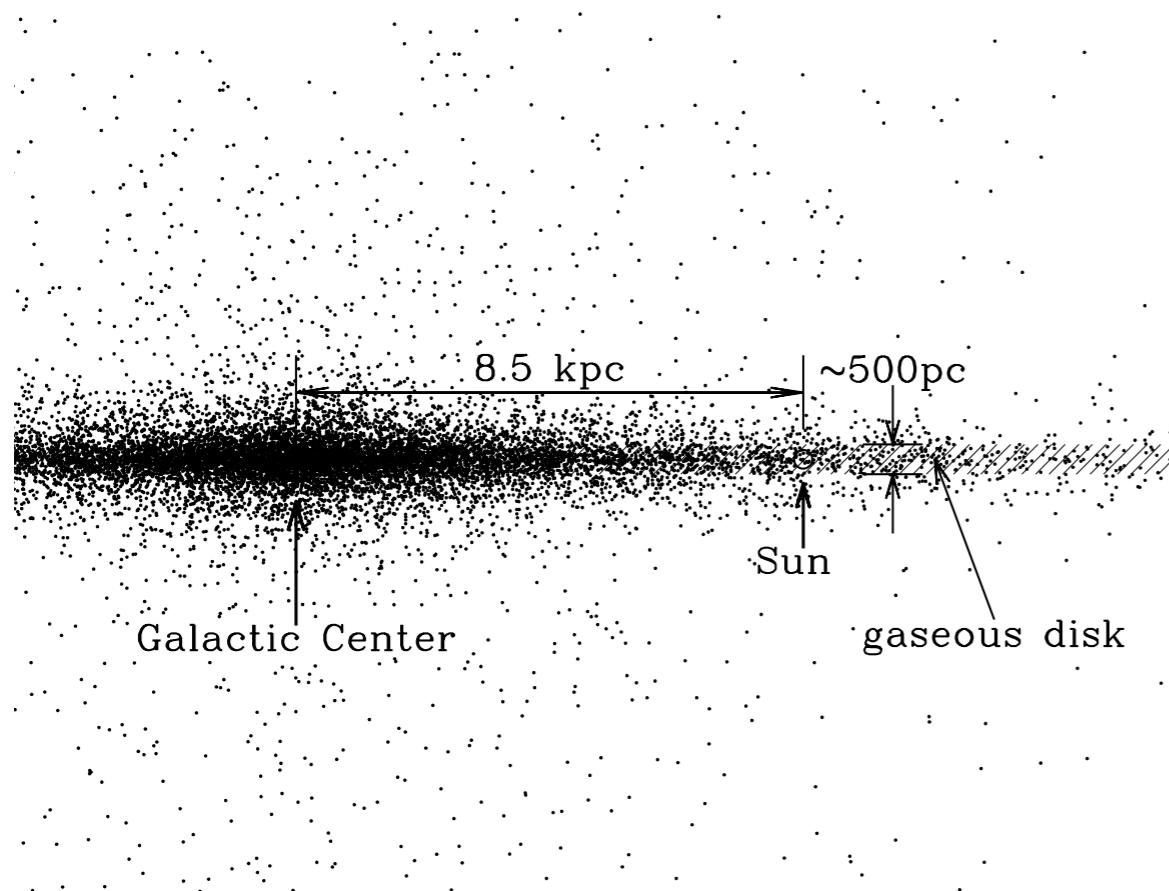
- At early times, the baryonic mass in galaxies was primarily in the gas of the ISM. As galaxies evolve, the ISM is gradually converted to stars, and some part of the interstellar gas may be ejected from the galaxy in the form of galactic winds, or in some cases stripped from the galaxy by the IGM.
- About 10% of the baryons in the Milky Way are to be found in the ISM.



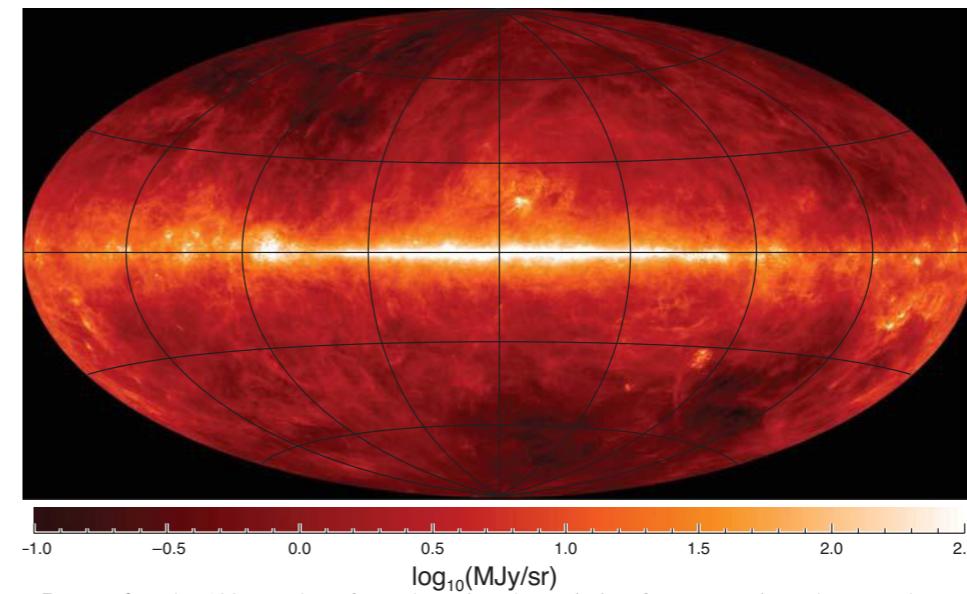
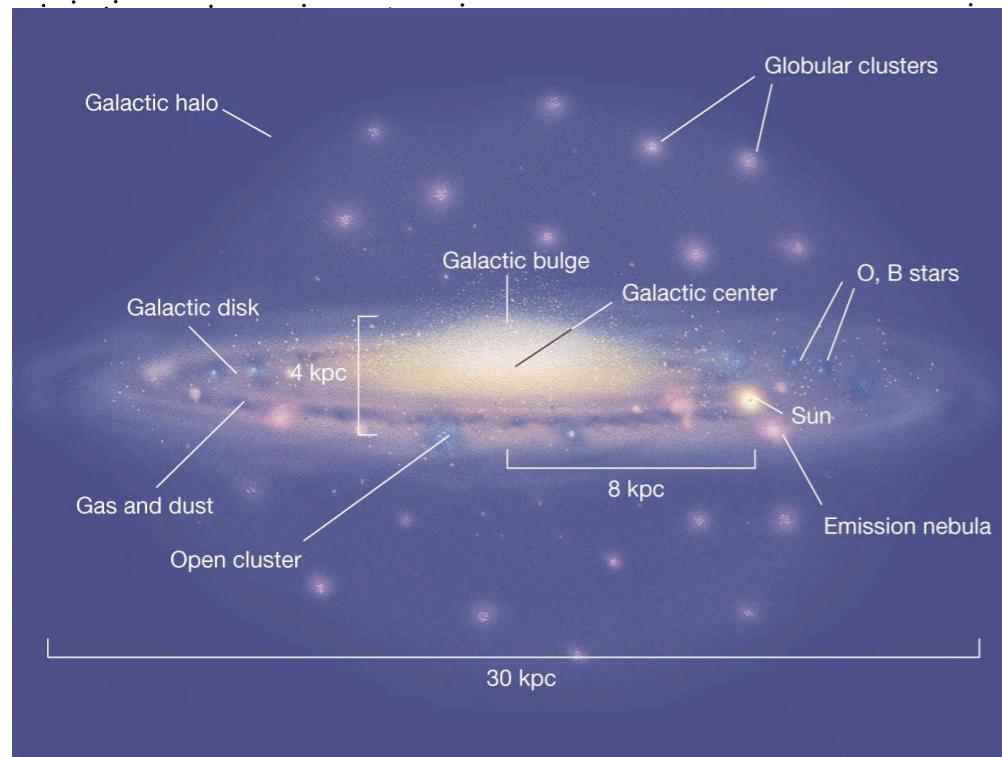
Credit: NASA, Night Sky Network

B. T. Draine

# Structure of the Milky Way



2MASS survey  
 $\sim 5 \times 10^8$  stars  
 blue = 1.2  $\mu\text{m}$   
 green = 1.65  $\mu\text{m}$   
 red = 2.2  $\mu\text{m}$



IRAS+COBE  
 100  $\mu\text{m}$   
 dust emission

(Jansky,  $\text{Jy} = 10^{-23} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}$ )

This artist's conception shows the various parts of our galaxy, and the position of our Sun.

- 
- Total mass of the Milky Way  $\sim 10^{11} M_{\odot}$  ( $M_{\odot} = 1.989 \times 10^{33}$ g)
    - stars  $\sim 5 \times 10^{10} M_{\odot}$
    - dark matter  $\sim 5 \times 10^{10} M_{\odot}$
    - interstellar gas  $\sim 7 \times 10^9 M_{\odot}$  (mostly H + He)
      - ◆ Hydrogen mass: neutral H atoms  $\sim 60\%$ , H<sub>2</sub> molecules  $\sim 20\%$ , ionized H<sup>+</sup> atoms  $\sim 20\%$

Phase	$M(10^9 M_{\odot})$	fraction
Total H II (not including He)	1.12	23%
Total H I (not including He)	2.9	60%
Total H <sub>2</sub> (not including He)	0.84	17%
<b>Total H II, H I and H<sub>2</sub> (not including He)</b>	<b>4.9</b>	
<b>Total gas (including He)</b>	<b>6.7</b>	

# ISM = dust + gas

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## Dust

- dust = tiny grains of solid material
  - Historically, courses on the ISM have dealt with “non-stellar stuffs.”
  - The dust and gas strongly influence each other.
    - ◆ Dust reprocesses starlight, altering the radiation field passing through the gas.
    - ◆ Dust is made of **refractory** elements, so creating dust alters the chemical abundances of the surrounding gas.
    - ◆ Dust grains are a **leading source of free electrons** in the neutral interstellar gas.
    - ◆ **Gas molecules form on the surfaces of dust grains.**

## Gas

- Interstellar gas occupies the same region as stars.
- Stars are made from interstellar gas, and emit stellar winds into the ISM over the course of their lives. When massive stars reach the end of their lifetimes, they inject enriched gas at high speeds into the surrounding interstellar gas.
- Stars emit photons that are capable of exciting the interstellar gas. The emission lines have strong diagnostic power, enabling us to determine densities, temperatures, and ionization states of interstellar gas.

# Abundance of elements in the local ISM

Asplund (2009)

Element	Abundance (ppm)	Atomic number	1 <sup>st</sup> ionization energy (eV)
hydrogen (H)	911,900	1	13.60
helium (He)	87,100	2	24.59
oxygen (O)	490	8	13.62
carbon (C)	270	6	11.26
neon (Ne)	85	10	21.56
nitrogen (N)	68	7	14.53
magnesium (Mg)	40	12	7.65
silicon (Si)	32	14	8.15
iron (Fe)	32	26	7.90
sulfur (S)	13	16	10.36

(ppm = parts per million)

H : 91.2% by number

He: 8.7%

others: 0.1%

The interstellar gas is primarily H and He resisting from the Big Bang.

A small amount of heavy elements was produced as the result of the return to the ISM of gas that has been processed in stars and stellar explosions.

solar metallicity:

$$Z_{\odot} = M(Z > 2)/M_{\text{tot}} \approx 0.013 - 0.02$$

$$M(Z > 2)/M_{\text{H}} \approx 0.02$$

$$M(\text{total})/M_{\text{H}} \approx 1.40$$

X, Y, Z are often used to denote the mass fractions of hydrogen, helium, and metals, respectively.

Anders & Grevesse (1989)

$$X_{\odot} \approx 0.70$$

$$Y_{\odot} \approx 0.28$$

$$Z_{\odot} \approx 0.02$$

Asplund (2009)

$$X_{\odot} \approx 0.7380$$

$$Y_{\odot} \approx 0.2485$$

$$Z_{\odot} \approx 0.0134$$

Metallicity = the abundance of the elements heavier than hydrogen and helium

# Density of the ISM

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By terrestrial standards, the ISM is an almost perfect vacuum.

- The typical distance between stars is about  $2 \text{ pc} = 6 \times 10^{16} \text{ m}$ .
  - This is  $\sim 100$  million times greater than the solar radius and 4000 times greater than the size of its heliosphere.
  
  
  
  
  
  
- ISM density
  - Total ISM mass is  $\sim 7 \times 10^9 M_\odot$ .
  - Approximating the Galaxy as a cylinder with a radius  $R \sim 10 \text{ kpc}$  and a scale height  $H \sim 250 \text{ pc}$ , this implies an average density

$$\rho = \frac{M}{\pi R^2 \times 2H} \approx 3 \times 10^{-21} \text{ kg m}^{-3}$$

$$\rho = n_H m_H + n_{\text{He}} m_{\text{He}} \approx \left(1 + \frac{1}{10} \times 4\right) n_H m_H \quad \longleftarrow$$

$$\begin{aligned} n_{\text{He}} &\sim 0.1 n_H \\ m_{\text{He}} &\sim 4 m_H \end{aligned}$$

$$n_H \approx \frac{\rho}{1.4m_H} \simeq 1.3 \text{ cm}^{-3}$$

- Density of Air
  - From the ideal gas law using the pressure at sea level,  $P = 10^5 \text{ N m}^{-2}$  (1 bar), and temperature  $T \approx 300 \text{ K}$ , we get

$$n = P/k_B T \approx 2.4 \times 10^{19} \text{ cm}^{-3}$$

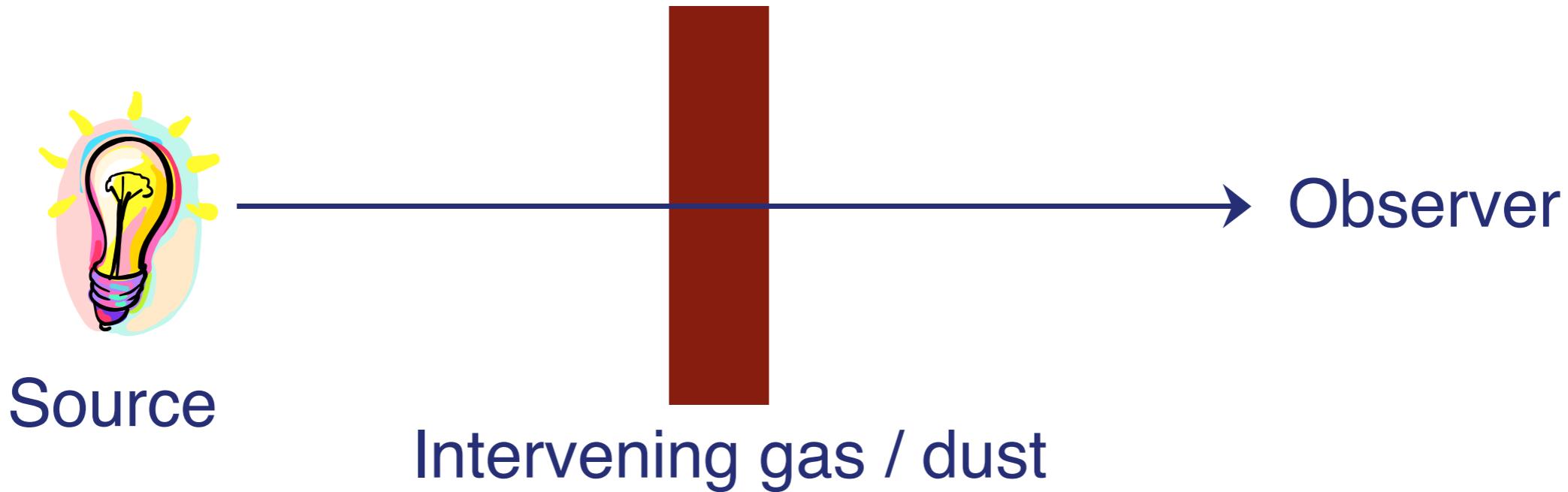
This density is **19 orders of magnitude higher than the average density in the ISM.**

- The extremely low density in the ISM mean that particle collisions are relatively rare, which allows us to observe some physical processes that we don't see on Earth (e.g., forbidden lines).

# Essential Background Physics: Radiative Transfer

# Radiative Transfer (복사전달)

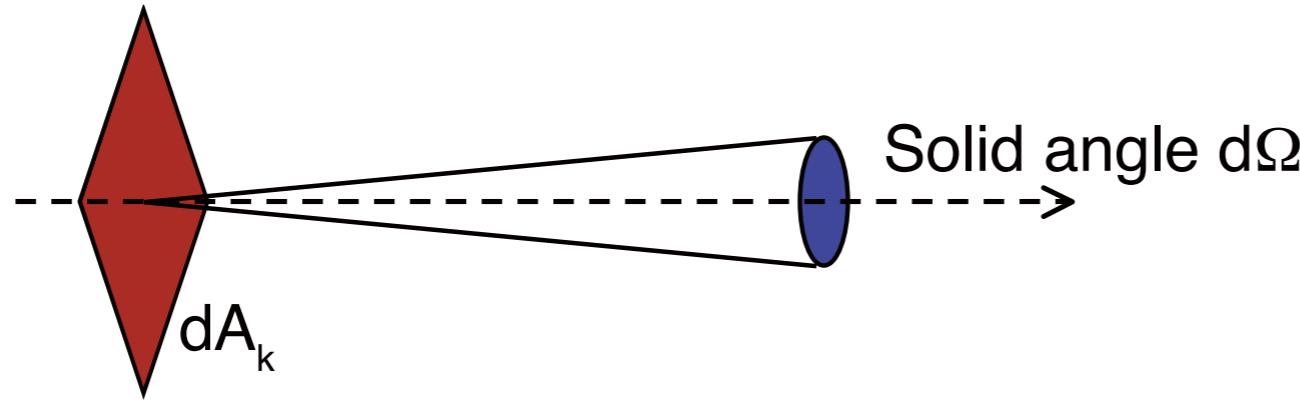
- How is radiation affected as it propagates through intervening gas and dust media to the observer?



- What we can learn?
  - gas/dust temperature, degree of ionization
  - gas kinematics (inflow, outflow)
  - elemental abundances
  - excitation sources

# ‘Specific’ Intensity (Surface Brightness)

- Intensity is the energy carried along by individual rays.



$\mathbf{k}$  = direction of propagation

- Let  $dE_\nu$  be the amount of radiant energy which crosses the area  $dA_k$  perpendicular to a direction  $\mathbf{k}$  within solid angle  $d\Omega$  about in a time interval  $dt$  with photon frequency between  $\nu$  and  $\nu + d\nu$ .
- The monochromatic specific intensity  $I_\nu$  is then defined by the equation.

$$I_\nu(\mathbf{k}, \mathbf{x}, t) = \frac{dE_\nu}{dA_k d\Omega d\nu dt}$$

- Unit:  $\text{erg s}^{-1} \text{cm}^{-2} \text{sr}^{-1} \text{Hz}^{-1}$
- From the view point of an observer, the specific intensity is called **surface brightness**.

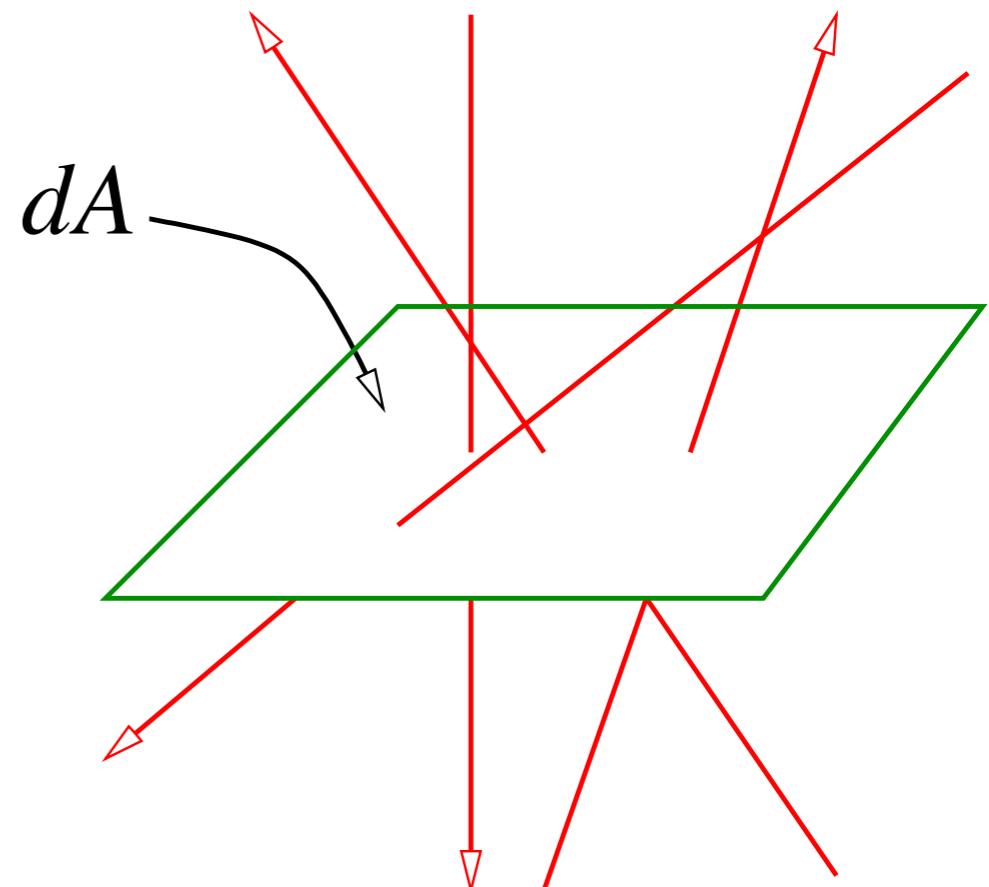
# Flux ‘Density’

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- Definition
  - ***Flux is a measure of the energy carried by all rays passing through a given area***
  - Consider a small area  $dA$ , exposed to radiation for a time  $dt$ .
  - Flux  $F_\nu$  is defined as ***the total (net) energy passing through a unit area in all directions within a unit time interval.***

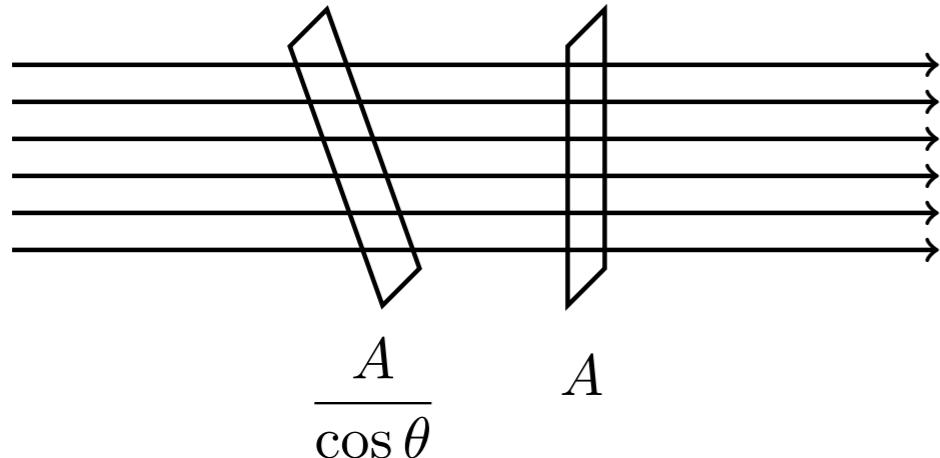
$$F_\nu = \frac{dE_\nu}{dAd\nu dt}$$

- Note that  $F_\nu$  ***depends on the orientation of the area element***  $dA$ .
- Unit: erg cm<sup>-2</sup> s<sup>-1</sup>
- $F_\nu$  is often called the “flux density.”
- Radio astronomers use a special unit to define the flux density: 1 Jansky (Jy) =  $10^{-23}$  erg s<sup>-1</sup> cm<sup>-2</sup> Hz<sup>-1</sup>



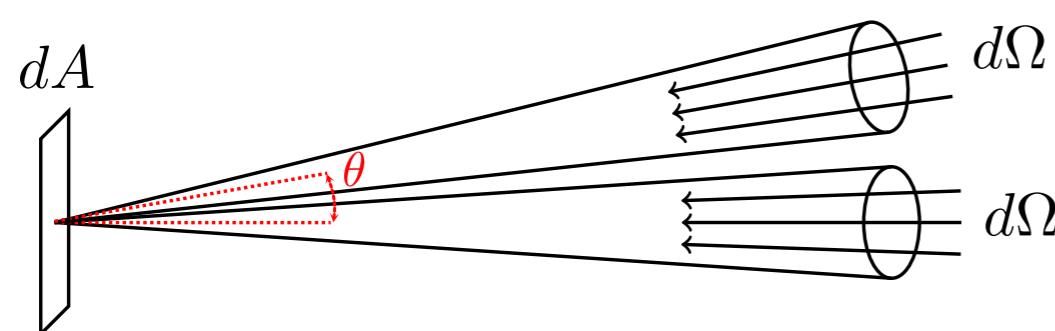
# Flux vs. Intensity

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The power delivered to the two surfaces are equal though their areas differ.

The flux is **the power per unit area** so the tilted surface gets less flux.

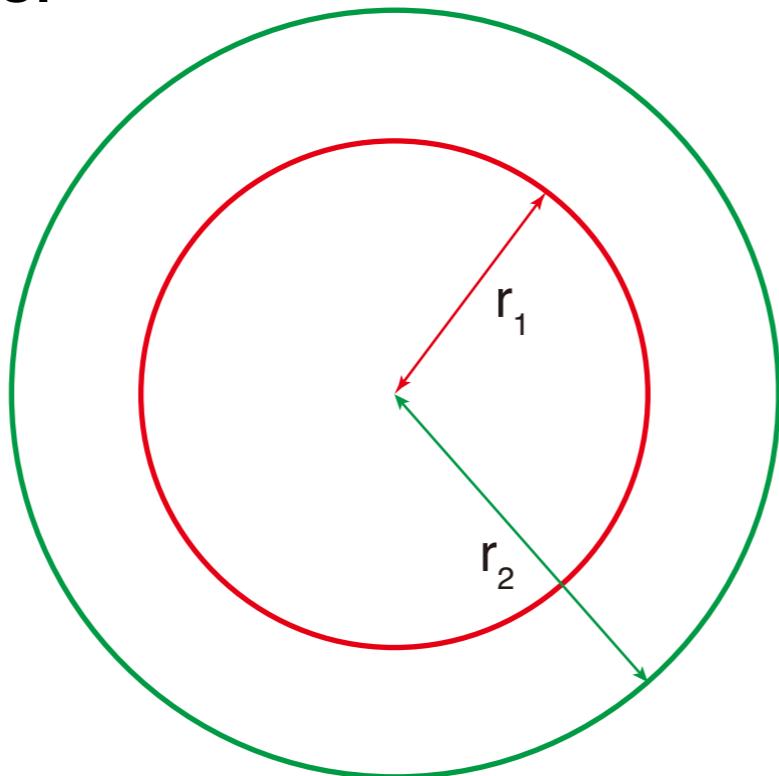


Two intensities are equal.  
The upper set of rays delivers less flux.

The rate that energy is delivered to a surface from light traveling around a direction  $\theta$  is  $I \cos \theta d\Omega$ .

## Inverse Square Law

- Flux from an isotropic radiation source, i.e., a source emitting equal amounts of energy in all directions.



- Because of energy conservation, flux through two shells around the source must be the same.

$$4\pi r_1^2 F(r_1) = 4\pi r_2^2 F(r_2)$$

- Therefore, we obtain the inverse square law.

$$F = \frac{\text{const.}}{r^2}$$

# Relation between the flux density and the specific intensity

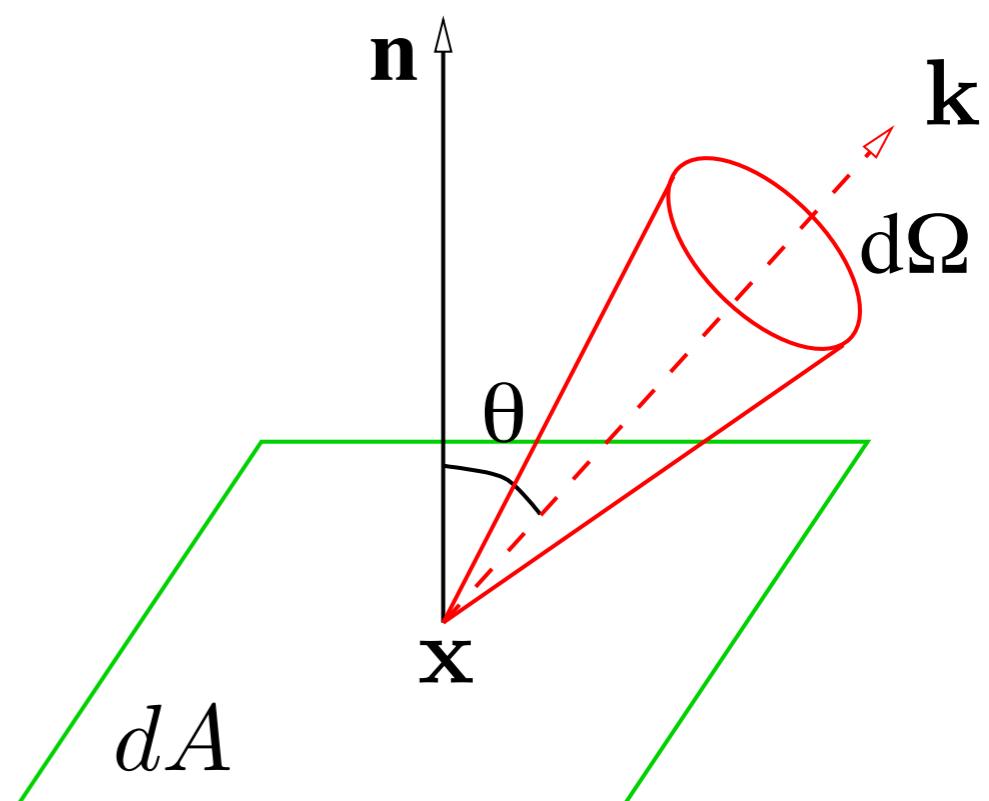
- Let's consider a small area  $dA$ , with light rays passing through it at all angles to the normal vector  $\mathbf{n}$  of the surface.
- For a ray centered about  $\mathbf{k}$ , the area normal to  $\mathbf{k}$  is

$$dA_{\mathbf{k}} = dA \cos \theta$$

- By the definition,

$$F_{\nu} dAd\nu dt = \int I_{\nu}(\mathbf{k}, \mathbf{x}, t) dA_{\mathbf{k}} d\Omega d\nu dt$$

- Hence, net flux in the direction of  $\mathbf{n}$  is given by integrating over all solid angles:

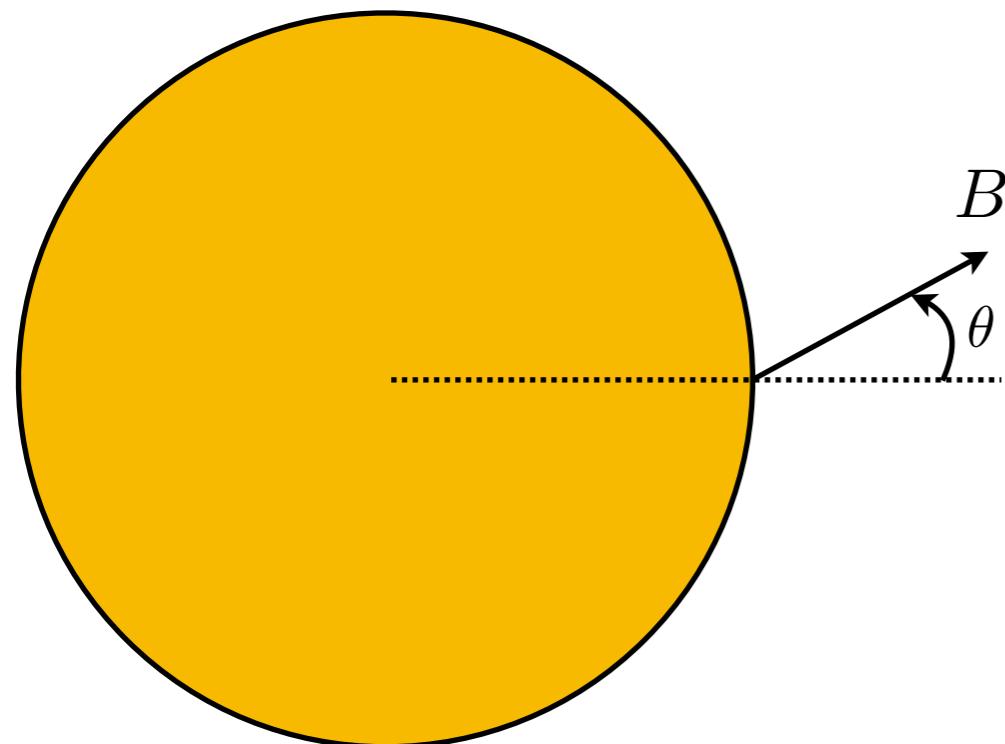


$$F_{\nu} = \int I_{\nu} \cos \theta d\Omega = \int_0^{2\pi} \int_0^{\pi} I_{\nu} \cos \theta \sin \theta d\theta d\phi$$

[Note] **flux** = sum of all ray vectors projected onto a normal vector  
**intensity** = absolute value of a single ray vector

## Flux from the surface of a uniformly bright sphere

- Let's calculate the flux at  $P$  on a sphere of uniform brightness  $B$



$$F = \int B \cos \theta d\Omega = \int_0^1 \int_0^{2\pi} B \mu d\mu d\phi$$

$$F = \pi B$$

The total luminosity from the sphere is then

$$L = (4\pi R^2)F = (4\pi R^2)\pi B$$

- In stellar atmosphere, the **astrophysical flux** is defined by  $F/\pi$ .

## Note — SED

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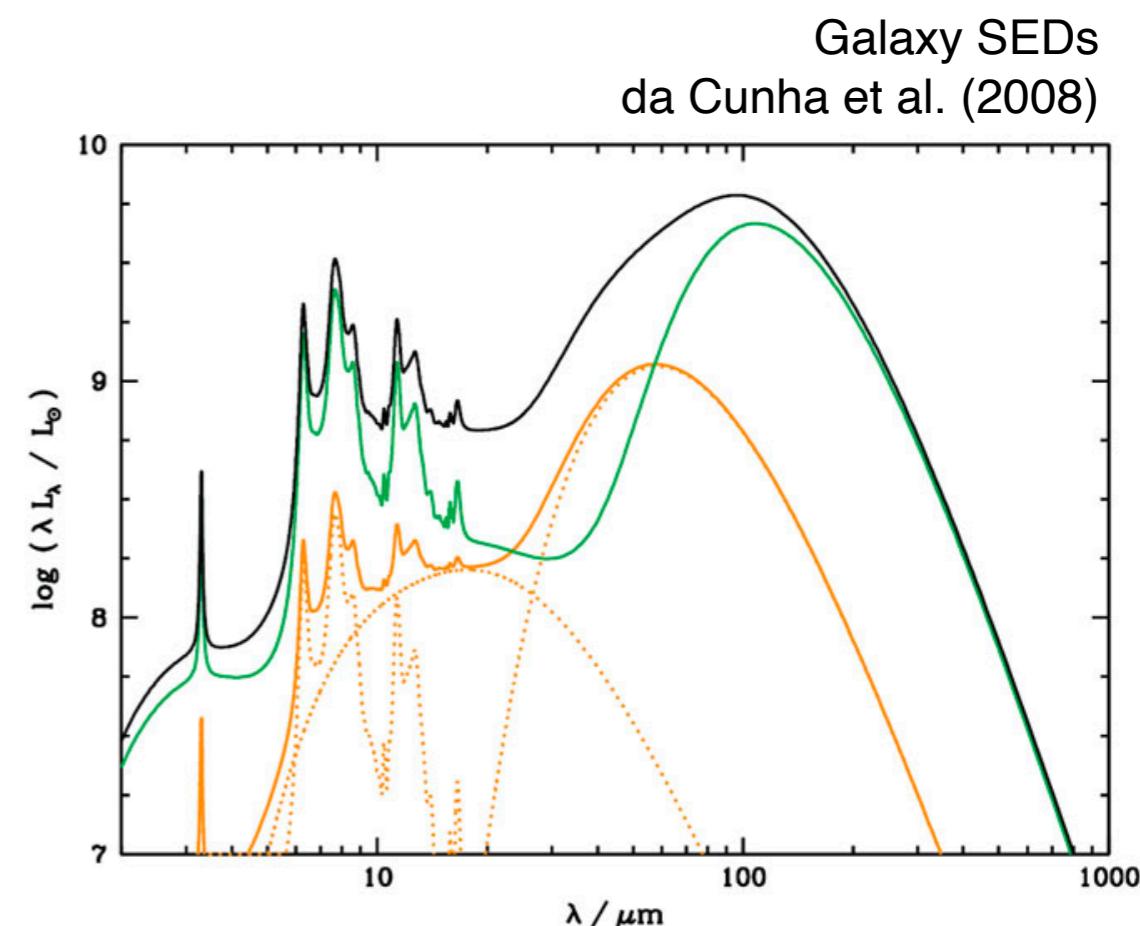
- Intensity can be defined as per wavelength interval.

$$\begin{aligned} I_\nu |d\nu| &= I_\lambda |d\lambda| & \leftarrow \left| \frac{d\nu}{d\lambda} \right| &= \frac{c}{\lambda^2} = \frac{\nu}{\lambda} & \leftarrow & \nu = \frac{c}{\lambda} \\ \nu I_\nu &= \lambda I_\lambda \end{aligned}$$

- Integrated intensity is defined as the intensity over all frequencies.

$$I = \int_0^\infty I_\nu d\nu = \int_0^\infty I_\lambda d\lambda$$

- In astrophysics, we plot the ***spectral energy distribution (SED)*** as  $\nu I_\nu$  versus  $\nu$  or  $\lambda I_\lambda$  versus  $\lambda$ .



# Luminosity

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- To determine the energy per unit time, we integrate flux over area.
  - Monochromatic luminosity:** Considering a sphere centered on a source with radius  $R$ , the monochromatic luminosity is

$$\begin{aligned} L_\nu &= R^2 \int d\Omega F_\nu \\ &= 4\pi R^2 F_\nu \quad \text{for an isotropic source} \end{aligned}$$

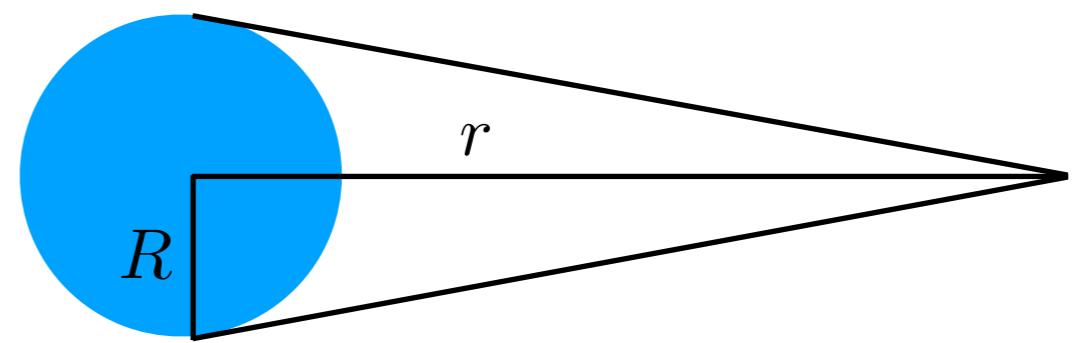
- The **bolometric luminosity** is

$$L_{\text{bol}} = \int L_\nu d\nu = \int L_\lambda d\lambda = 4\pi R^2 \int F_\nu d\nu$$

- Flux and Luminosity of an extended source

$$\begin{aligned} F &= \pi I \left( \frac{R}{r} \right)^2 = I \frac{A}{r^2} \\ &= I \Omega_{\text{source}} \end{aligned}$$

$$L = (4\pi r^2)F = (4\pi r^2)I \Omega_{\text{source}}$$



$$A = \pi R^2$$

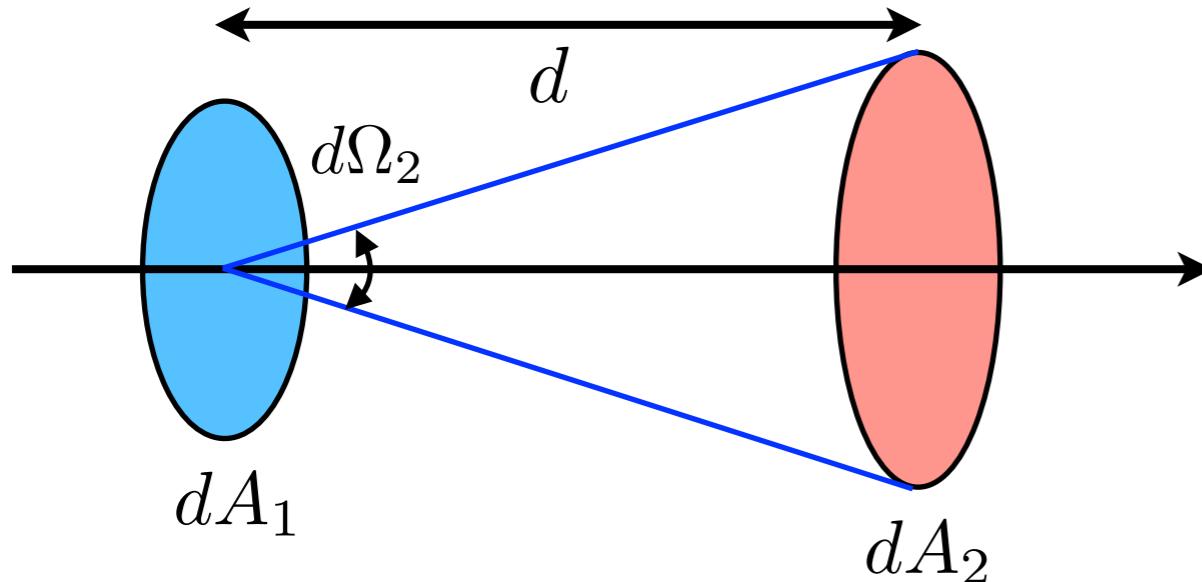
## < Radiative Transfer Equation > — in free space

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- How does intensity changes along a ray in free space
  - Suppose a bundle of rays and any two points along the rays and construct two “infinitesimal” areas  $dA_1$  and  $dA_2$  normal to the rays at these points.
  - What are the energies carried by the rays passing through both areas?

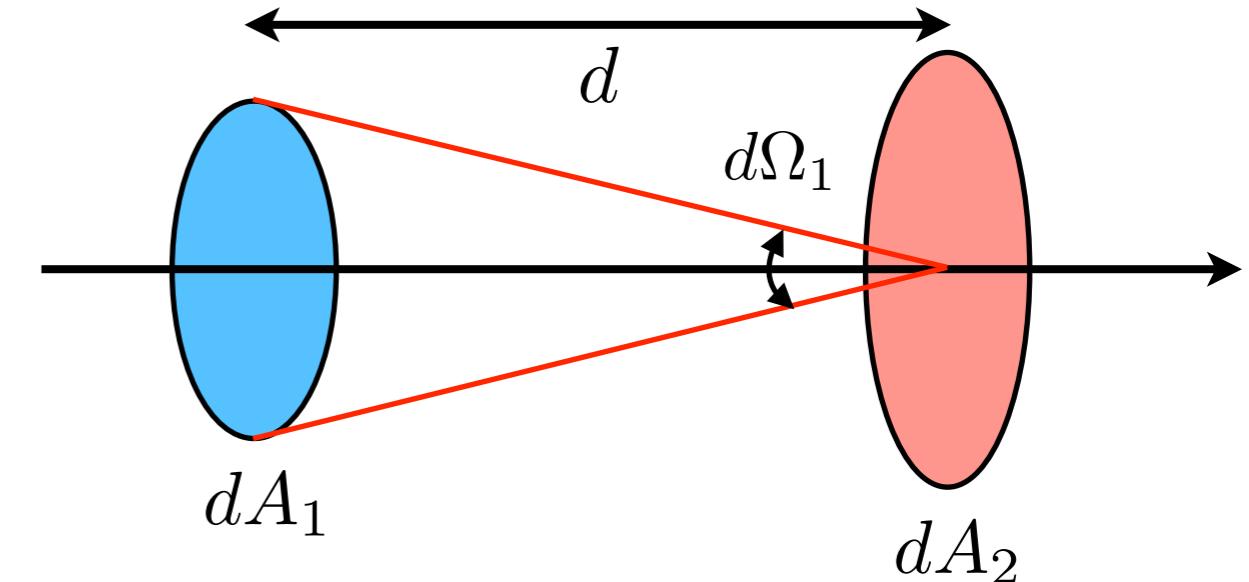
**energy passing through 1**

$$dE_1 = I_1 dA_1 d\Omega_2 d\nu dt$$

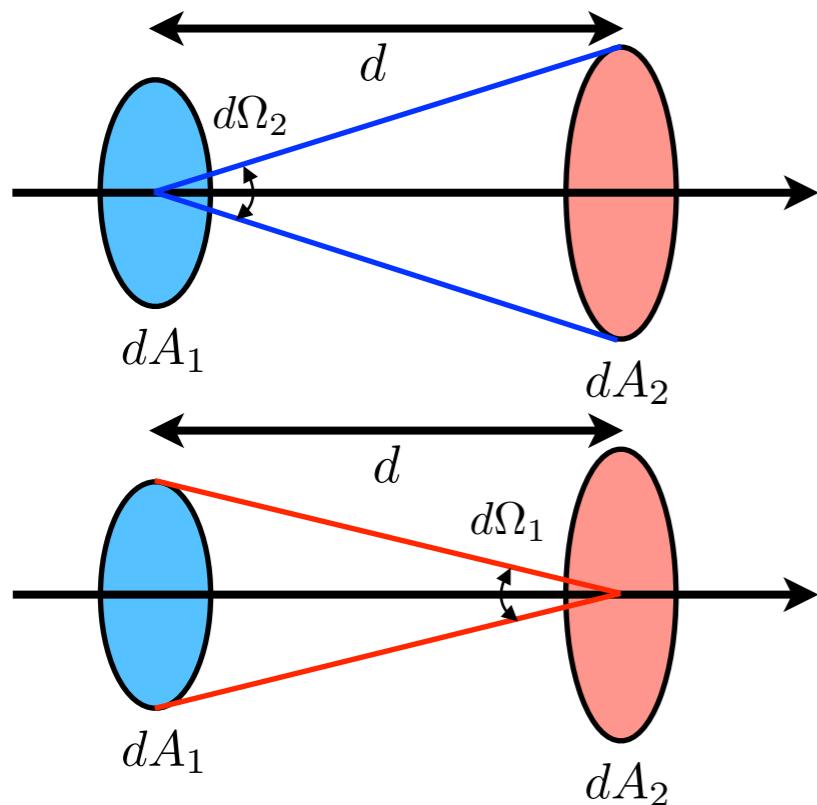


**energy passing through 2**

$$dE_2 = I_2 dA_2 d\Omega_1 d\nu dt$$



- Here,  $d\Omega_2$  is the solid angle subtended by  $dA_2$  at the location 1 and  $d\Omega_1$  is the solid angle subtended by  $dA_1$  at the location 2.



$$d\Omega_2 = \frac{dA_2}{d^2}$$

$$d\Omega_1 = \frac{dA_1}{d^2}$$

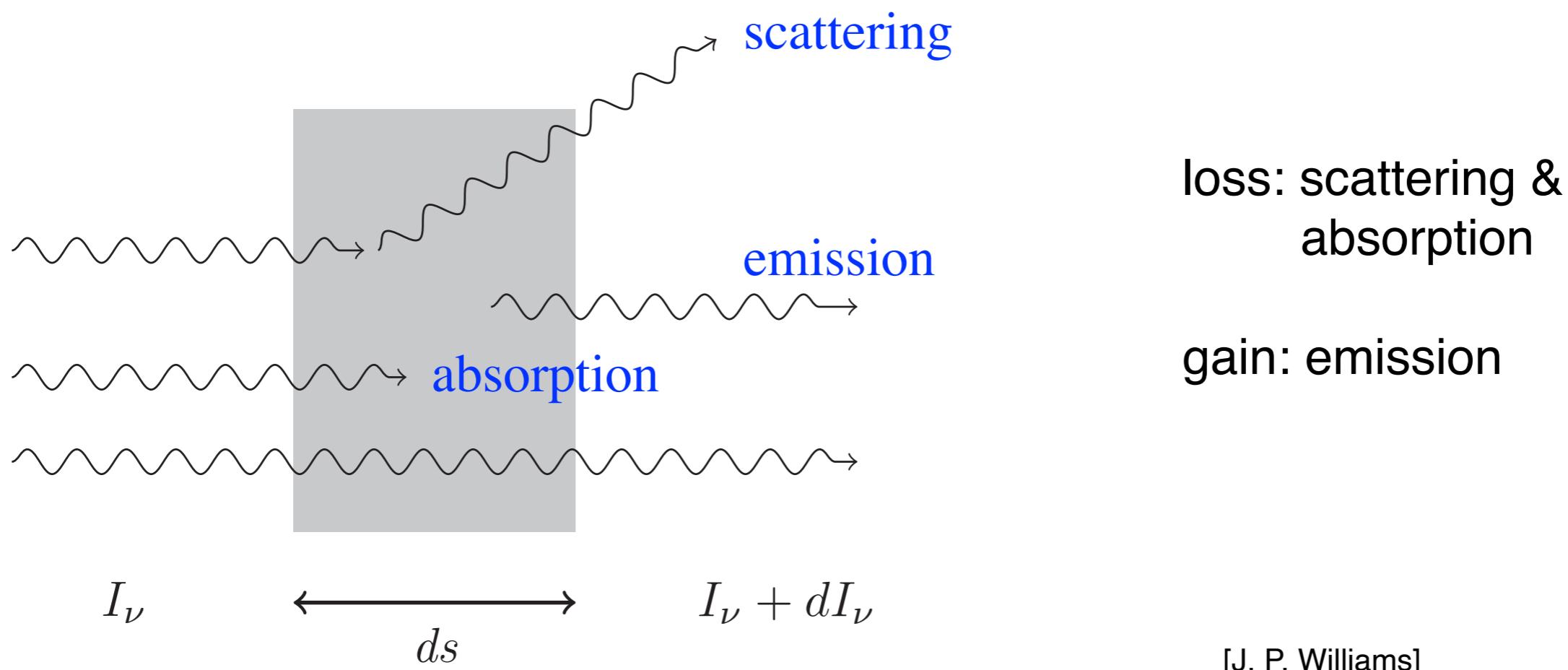
Conservation of energy:  
Because energy is conserved,

$$dE_1 = dE_2 \rightarrow I_1 = I_2$$

- Conclusion (***the constancy of intensity***):  $I_1 = I_2 \rightarrow \boxed{\frac{dI}{ds} = 0}$ 
  - the specific intensity remains the same as radiation propagates through free space.
- We receive the same specific intensity at the telescope as is emitted at the source.
  - Imagine looking at a uniformly lit wall and walking toward it. As you get closer, a field-of-view with fixed angular size will see a progressively smaller region of the wall, but this is exactly balanced by the inverse square law describing the spreading of the light rays from the wall.

## < Radiative Transfer Equation > — Emission & Absorption

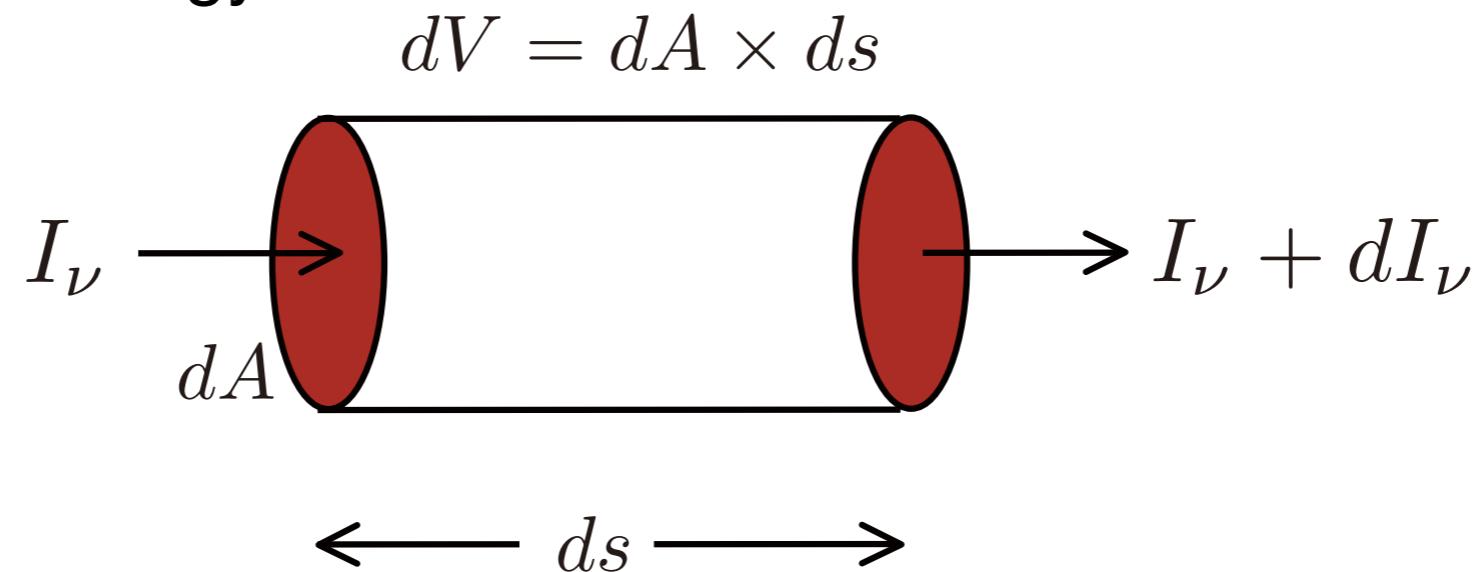
- In reality, as a ray passes through matter, energy may be added, subtracted, or scattered from it by emission, absorption, or scattering.
  - The intensity will not in general remain constant.
  - These interactions are described by the ***radiative transfer equation***.



# Emission

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- If the radiation travels through a medium which is itself emitting radiation, that will add to the energy:



- Spontaneous “**emission coefficient**” or “**emissivity**”  $j_\nu$  is the amount of energy emitted per unit time, per unit solid angle, per unit frequency, and per unit volume:

$$dE = j_\nu dV d\Omega dt d\nu \quad (j_\nu : \text{erg cm}^{-3} \text{ s}^{-1} \text{ sr}^{-1} \text{ Hz}^{-1})$$

- In going a distance  $ds$ , a beam of cross section  $dA$  travels through a volume  $dV = dA ds$ . Thus the intensity added to the beam is by  $ds$  is

$$dI_\nu = j_\nu ds \qquad \longleftrightarrow \qquad dE = (dI_\nu) dA d\Omega dt d\nu$$

- 
- Therefore, the equation of radiative transfer for pure emission becomes:

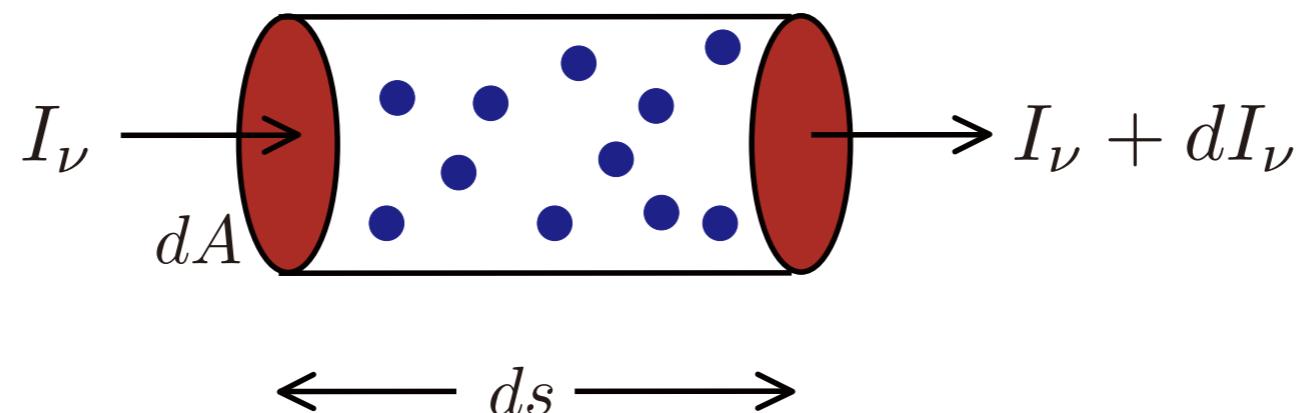
$$\frac{dI_\nu}{ds} = j_\nu$$

- If we know what  $j_\nu$  is, we can integrate this equation to find the change in specific intensity as radiation propagates through the medium:

$$I_\nu(s) = I_\nu(0) + \int_0^s j_\nu(s')ds'$$

# Absorption

- If the radiation travels through a medium which absorbs radiation, the energy in the beam will be reduced:



- Let  $n$  denote the number density of absorbers (particles per unit volume).
- Assume that each absorber has a cross-sectional area of  $\sigma_\nu$  (in units of  $\text{cm}^2$ ).

'geometric' cross section:  $\sigma = \pi r^2$  for a spherical particle with a radius  $r$

- If a beam travels through  $ds$ , total area of absorbers is

$$\text{number of absorbers} \times \text{cross section} = (n \times dA \times ds) \times \sigma_\nu$$

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Fraction of radiation absorbed = Fraction of area blocked

$$\frac{dI_\nu}{I_\nu} = - \frac{ndAds\sigma_\nu}{dA} = - n\sigma_\nu ds \quad \longrightarrow \quad \frac{dI_\nu}{ds} = - \alpha_\nu I_\nu$$

$$dI_\nu = - n\sigma_\nu I_\nu ds \equiv - \alpha_\nu I_\nu ds$$

- **Absorption coefficient** is defined as  $\alpha_\nu \equiv n\sigma_\nu$  (units:  $\text{cm}^{-1}$ ), meaning the **total cross-sectional area per unit volume**.

$$\alpha_\nu = n\sigma_\nu \quad [\text{cm}^{-1}]$$

$$= \rho\kappa_\nu$$

where  $\rho$  is the mass density and  $\kappa_\nu$  is called the **mass absorption coefficient** or the **opacity coefficient**.

# Emission + Absorption

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- **Radiative transfer equation** with both absorption and emission is

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

absorption      emission

- We can rewrite the radiative transfer equation using the optical depth as a measure of ‘distance’ rather than  $s$ :

$$\frac{dI_\nu}{\alpha_\nu ds} = -I_\nu + \frac{j_\nu}{\alpha_\nu}$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

- where  $S_\nu \equiv j_\nu/\alpha_\nu$  **is called the source function**. This is an alternative and sometimes more convenient way to write the equation.

## Solution: Emission Only

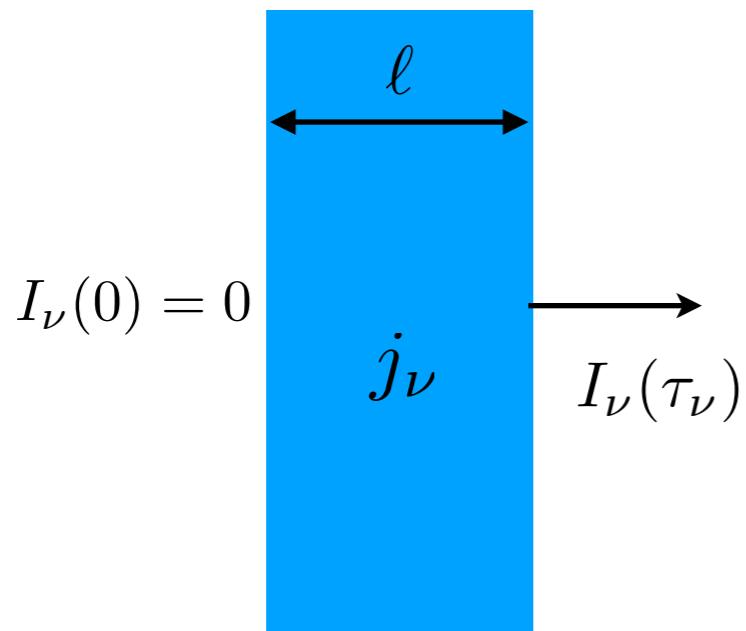
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- For pure emission,  $\alpha_\nu = 0$

$$\frac{dI_\nu}{ds} = j_\nu$$

$$I_\nu(s) = I_\nu(0) + \int_0^s j_\nu(s') ds'$$

- The brightness increase is equal to the emission coefficient integrated along the line of sight.



$$I_\nu = j_\nu \ell$$

if  $I_\nu(0) = 0$  and  $j_\nu = \text{constant}$

## Solution: Absorption Only

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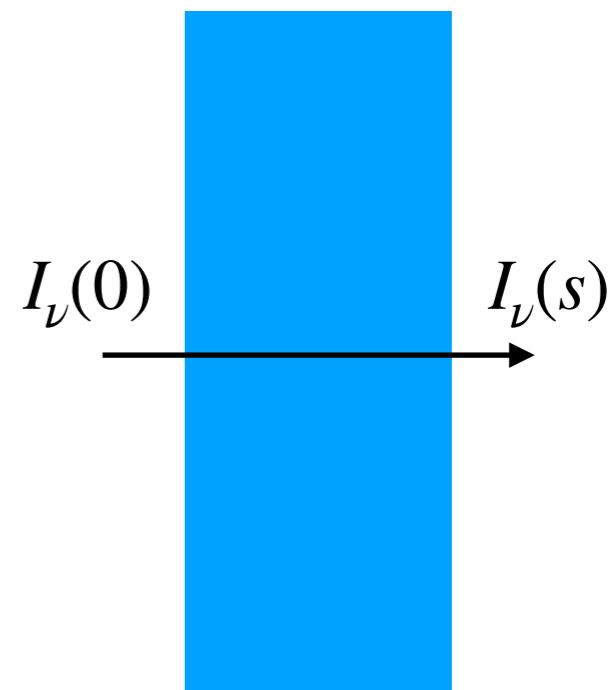
- Pure absorption:  $j_\nu = 0$

Rearranging the previous equation, we obtain the equation of radiative transfer for pure absorption:

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu$$

- The amount of reduced energy depends on how much radiation we already have.
- Integrate to find how radiation changes along path:

$$I_\nu(s) = I_\nu(0) \exp \left[ - \int_0^s \alpha_\nu(s') ds' \right]$$



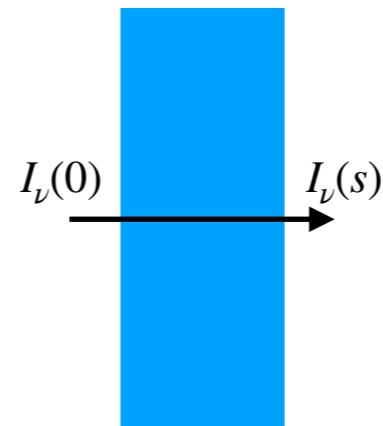
- The brightness decreases along the ray by the exponential of the absorption coefficient integrated along the line of sight.

- **Optical depth:**

Imagine radiation traveling into a cloud of absorbing gas, the exponential defines a scale over which radiation is attenuated.

We define the optical depth  $\tau_\nu$  as:

$$\tau_\nu(s) = \int_0^s \alpha_\nu(s')ds' \quad \text{or} \quad d\tau_\nu = \alpha_\nu ds$$



Transmitted Light

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu}$$

Absorbed Light

$$I_\nu^{\text{abs}}(\tau_\nu) = I_\nu(0)(1 - e^{-\tau_\nu})$$

- A medium is said to be **optically thick** at a frequency  $\nu$  if the optical depth for a typical path through the medium satisfies:

$$\tau_\nu(s) > 1$$

$$I_\nu(\tau_\nu) \rightarrow 0$$

$$I_\nu^{\text{abs}}(\tau_\nu) \rightarrow I_\nu(0)$$

- The medium is **optically thin** if, instead:

$$\tau_\nu(s) < 1$$

$$I_\nu(\tau_\nu) \rightarrow I_\nu(0)$$

$$I_\nu^{\text{abs}}(\tau_\nu) \rightarrow 0$$

An optically thin medium is one which a typical photon of frequency  $\nu$  can pass through without being (significantly) absorbed.

# Mean Free Path

- From the exponential absorption law, the **probability of a photon absorbed** between optical depths  $\tau_\nu$  and  $\tau_\nu + d\tau_\nu$  is

$$|dI_\nu| = \left| \frac{dI_\nu}{d\tau_\nu} \right| d\tau_\nu \Rightarrow |dI_\nu| = P(\tau_\nu) d\tau_\nu$$

$e^{-\tau_\nu}$

- The mean optical depth traveled is thus equal to unity:

$$\int_0^\infty P(\tau_\nu) d\tau_\nu = 1$$

$$P(\tau_\nu) = e^{-\tau_\nu}$$

= probability density function for the absorption at an optical depth  $\tau_\nu$ .

$$\langle \tau_\nu \rangle = \int_0^\infty \tau_\nu P(\tau_\nu) d\tau_\nu = \int_0^\infty \tau_\nu e^{-\tau_\nu} d\tau_\nu = 1$$

- The mean free path is defined as the average distance a photon can travel through an absorbing material until it is absorbed.** In a homogeneous medium, the mean free path is determined by

$$\langle \tau_\nu \rangle = \alpha_\nu \ell_{\text{mfp}} = 1 \rightarrow \ell_{\text{mfp}} = \frac{1}{\alpha_\nu} = \frac{1}{n\sigma_\nu}$$

- A local mean path at a point in an inhomogeneous material can be also defined.

# Mean Free Path

- From the exponential absorption law, the **probability of a photon absorbed** between optical depths  $\tau_\nu$  and  $\tau_\nu + d\tau_\nu$  is

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} \quad \xrightarrow{\hspace{1cm}}$$

**probability** = 
$$\frac{|dI_\nu|}{I_\nu(0)} = \left| \frac{dI_\nu}{\tau_\nu} \right| d\tau_\nu = e^{-\tau_\nu} d\tau_\nu \quad \rightarrow \quad P(\tau_\nu) = e^{-\tau_\nu}$$

- The mean optical depth traveled is thus equal to unity:

$$\langle \tau_\nu \rangle = \int_0^\infty \tau_\nu P(\tau_\nu) d\tau_\nu = \int_0^\infty \tau_\nu e^{-\tau_\nu} d\tau_\nu = 1$$

= **probability density function** for  
the absorption at an optical depth  $\tau_\nu$

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$$\langle \tau_\nu \rangle = \alpha_\nu \ell_{\text{mfp}} = 1 \quad \rightarrow \quad \ell_{\text{mfp}} = \frac{1}{\alpha_\nu} = \frac{1}{n\sigma_\nu}$$

- A local mean path at a point in an inhomogeneous material can be also defined.
- The **probability of a photon being absorbed within an optical depth  $\tau_\nu$**  is

$$\int_0^{\tau_\nu} P(\tau'_\nu) d\tau'_\nu = 1 - e^{-\tau_\nu}$$

# Homework (due date: 03/12)

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## [Q1] Optical Depth

- a. Suppose spherical dust grains have a radius  $r = 0.1 \text{ cm}$  and individual mass density  $\rho_{\text{grain}} = 1 \text{ g cm}^{-3}$ . What is their ‘geometric’ cross section  $\sigma$ , mass  $m$ , and associated opacity coefficient  $\kappa$ ?
- c. If the number density of these grains is  $n_{\text{dust}} = 1 \text{ cm}^{-3}$ , what is the mass density of dust  $\rho_{\text{dust}}$  and the mean free path  $\ell_{\text{mfp}}$  for light?
- e. What is the optical depth at a physical depth 1 m of such dust absorbers
- g. What fraction of impinging intensity  $I_0$  reaches this depth?