

# Interstellar Medium (ISM)

Week 1

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# Syllabus

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## What is the ISM?

- The ISM is anything not in stars. (D. E. Osterbrock)
- Just what it says: The stuff between the stars in and around galaxies, especially our own Milky Way.
- **Gas, dust, radiation, cosmic rays, magnetic fields.**

## Why do we study the ISM?

- The ISM is the most beautiful component of galaxies. (B. T. Draine)
- The ISM is the most important component of galaxies, for it is the ISM that is responsible for forming the stars that are the dominant sources of energy.

## The objective of studying the ISM is to understand:

- how the ISM is organized and distribution in the Milky Way and other galaxies
- what are the conditions (temperature, density, ionization, etc) in different parts of it
- how it dynamically evolves.
- Eventually, we would like to understand star formation, the process responsible for the very existence of galaxies as luminous objects.

## Textbook

- Interstellar and Intergalactic Medium (Barbara Ryden & Richard Pogge)
- Physics of Interstellar and Intergalactic Medium (Bruce T. Draine)

# Syllabus

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Week	Date	
1	03-19	Introduction
2	03-26	Atomic Spectroscopy
3	04-02	Collisional Excitation/Ionization and Recombination
4	04-09	Cold Neutral Medium
5	04-16	Warm Neutral Medium
6	04-23	Warm Ionized Medium 1
7	04-30	Warm Ionized Medium 2
8	05-07	Midterm Exam
9	05-14	Hot Ionized Medium
10	05-21	Interstellar Dust 1
11	05-28	Interstellar Dust 2
12	06-04	Molecular Clouds 1 (Molecular Hydrogen)
13	06-11	Molecular Clouds 2
14	06-18	Circumgalactic & Intracluster Gas
15	06-25	Diffuse Intergalactic Medium / Warm-Hot Intergalactic Medium
16	07-02	Final Exam

# Introduction

# Baryonic Matter

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## Definitions:

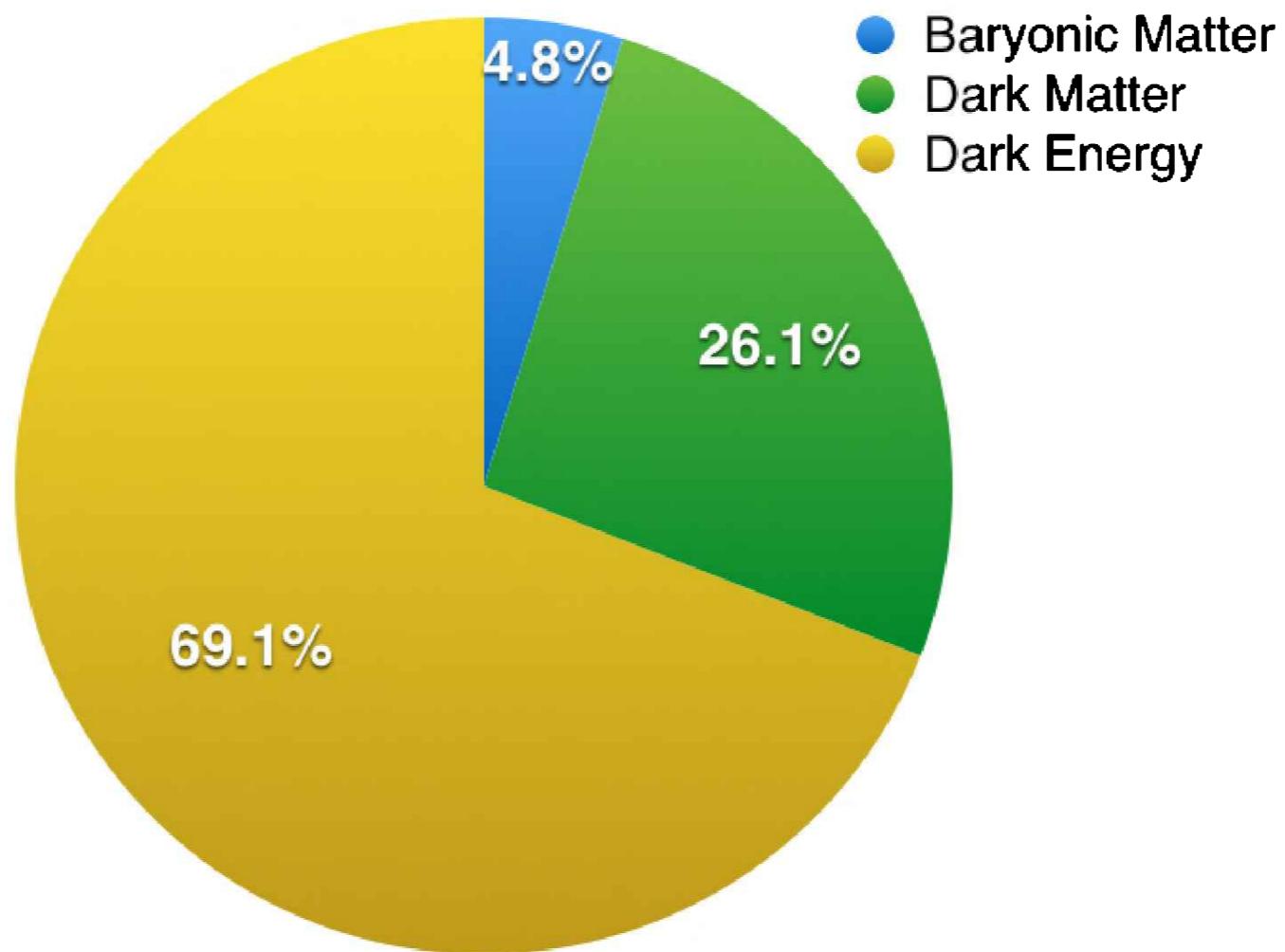
- Baryons = protons, neutrons and matter composed of them (i.e. atomic nuclei)
- Leptons = electrons, neutrinos
- In astronomy, however, the term '**baryonic matter**' is used more loosely to refer to **matter that is made of protons, neutrons, and electrons**, since protons and neutrons are always accompanied by electrons. Neutrinos, on the other hand, are considered non-baryonic by astronomers. (Note that black holes are also included as baryonic matter.)

# The mass-energy density

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Relative contribution of baryons, dark matter, and dark energy to the mass-energy density of the current universe (Planck 2015)

- The majority of the universe is made of dark energy and dark matter.
- Dark energy is ignored until we discuss cosmic evolution.
- Dark matter is important only because it provides potential wells for baryonic matter to be trapped in.

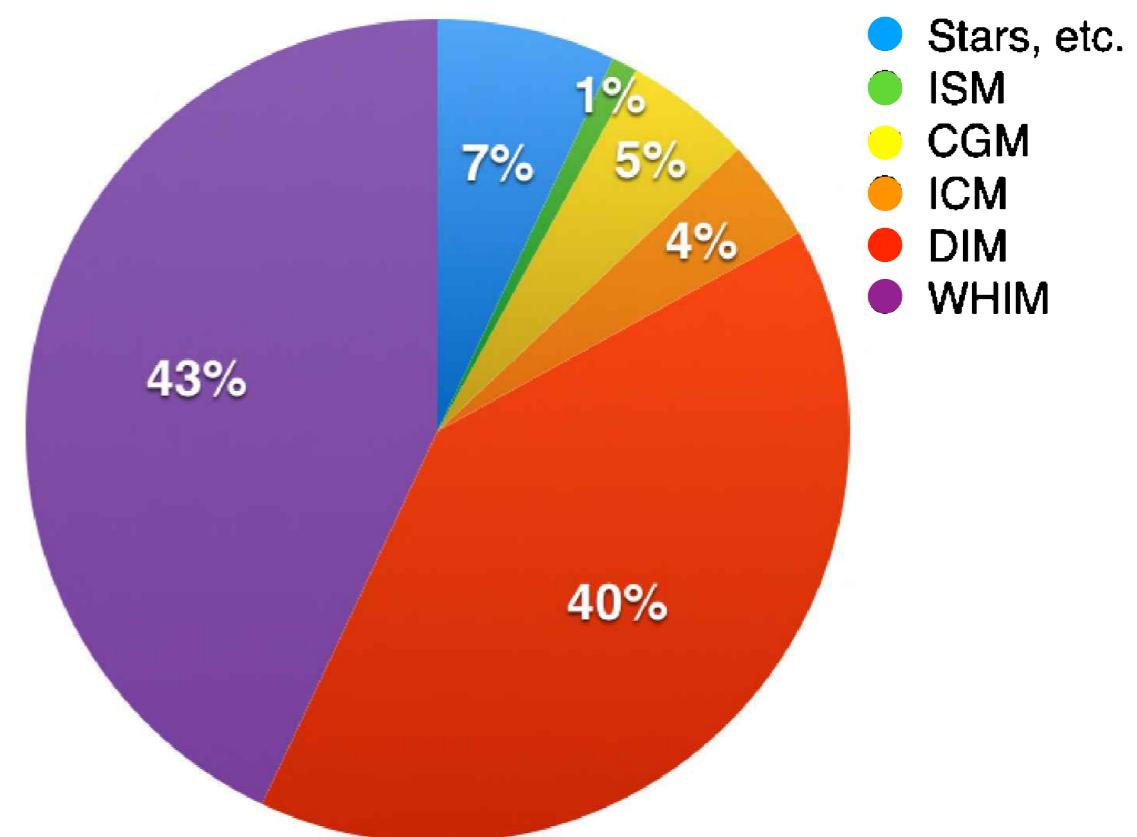


# The baryonic mass density

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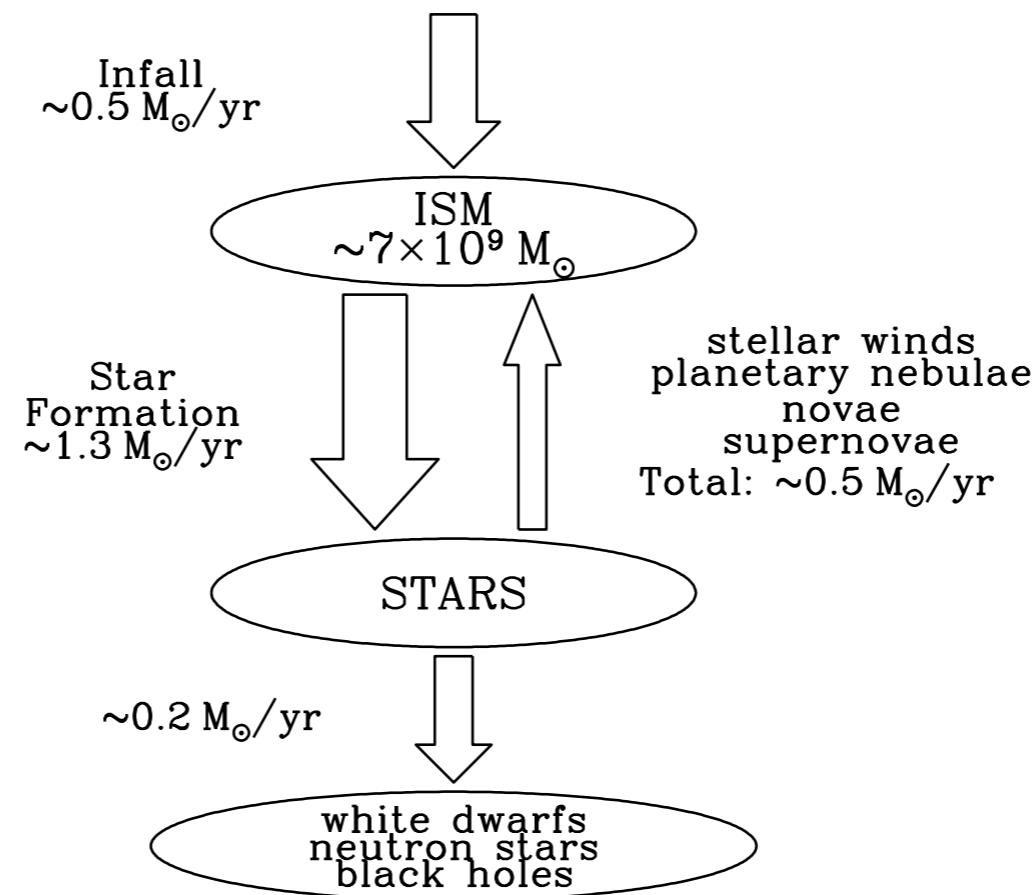
## The baryonic mass density in the current universe

- 7% : stars + compact objects (such as stellar remnants, brown dwarfs, and planets)
- 1% : interstellar medium (ISM), filling the volume between stars.
- 5%: circumgalactic medium (CGM), bound within the dark halo of a galaxy, but outside the main distribution of stars.
- 4% : intracluster medium (ICM) of clusters of galaxies, bound to the cluster as a whole, but not to any individual galaxy.
- 40%: diffuse intergalactic medium (DIM), made of low density, mostly photo-ionized gas ( $T < 10^5$  K).
- 43% : warm-hot intergalactic medium (WHIM), made of shock-heated gas ( $10^5 \text{ K} < T < 10^7$  K).



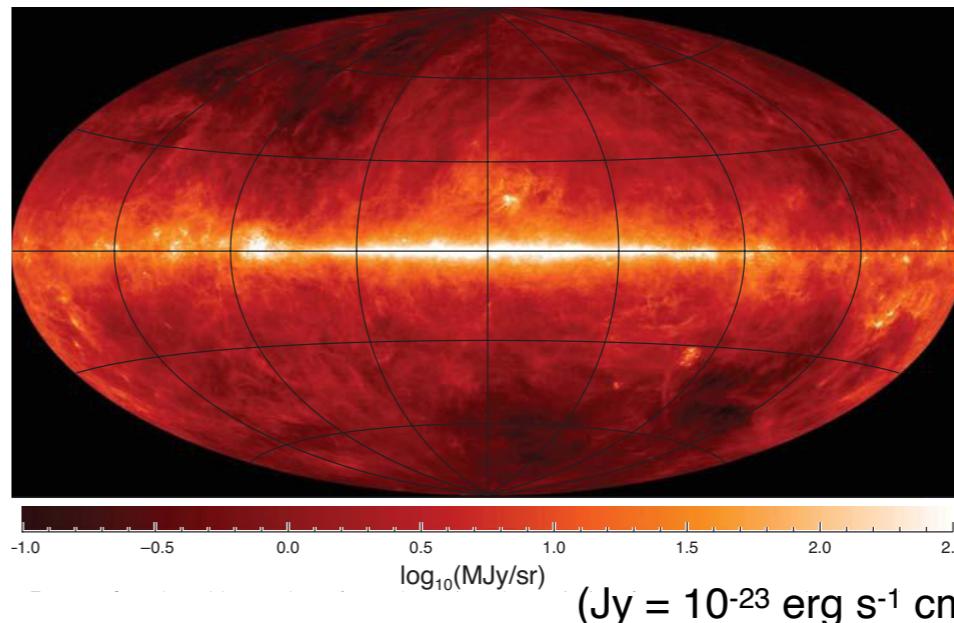
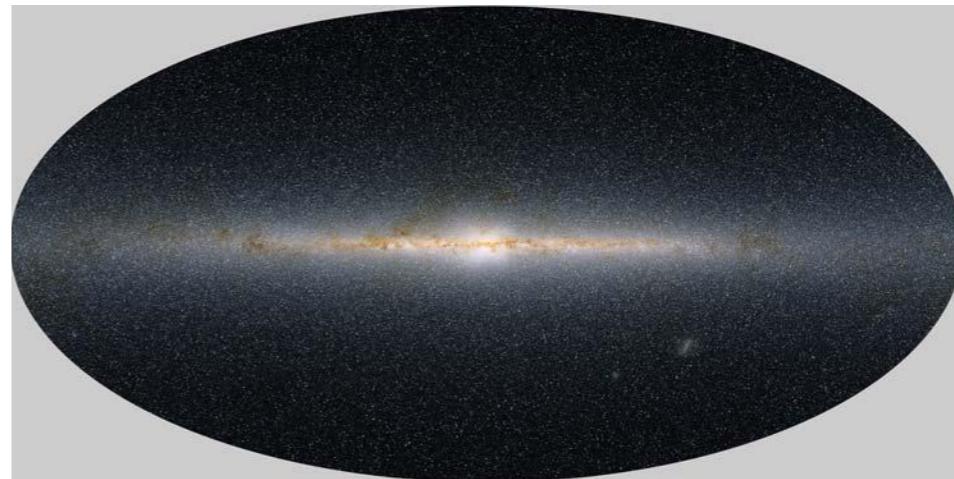
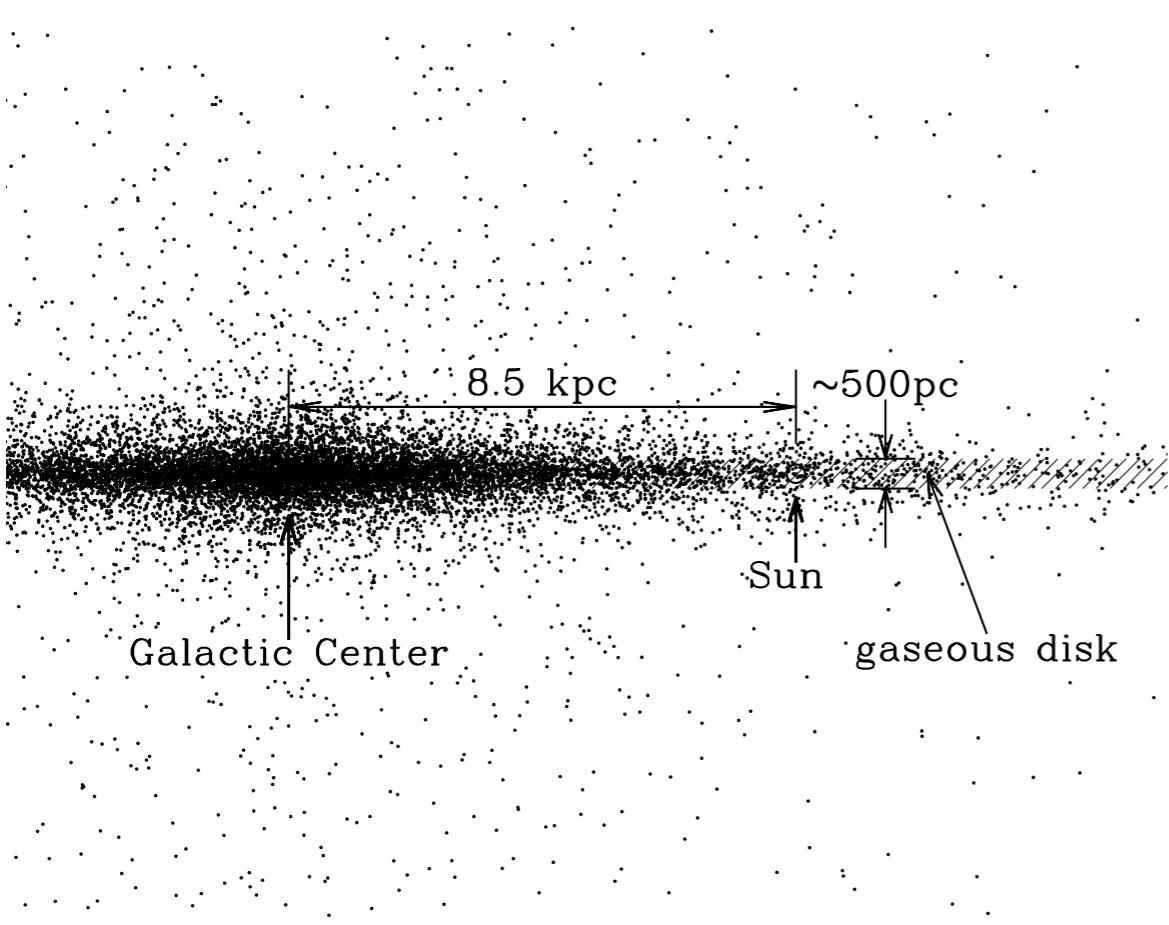
# Mass flow of the baryons in galaxies

- At early times, the baryonic mass in galaxies was primarily in the gas of the ISM. As galaxies evolve, the ISM is gradually converted to stars, and some part of the interstellar gas may be ejected from the galaxy in the form of galactic winds, or in some cases stripped from the galaxy by the IGM.
- About 10% of the baryons in the Milky Way are to be found in the ISM.



Flow of baryons in the Milky Way.

# Structure of the Milky Way



- Total mass of the Milky Way  $\sim 10^{11} M_\odot$  ( $M_\odot = 1.989 \times 10^{33} \text{ g}$ )

- stars  $\sim 5 \times 10^{10} M_\odot$
- dark matter  $\sim 5 \times 10^{10} M_\odot$
- interstellar gas  $\sim 7 \times 10^9 M_\odot$  (mostly H + He)

♦ Hydrogen mass: neutral H atoms ~ 60%, H<sub>2</sub> molecules ~ 20%, ionized H<sup>+</sup> atoms ~ 20%

Phase	$M(10^9 M_\odot)$	fraction
Total H II (not including He)	1.12	23%
Total H I (not including He)	2.9	60%
Total H <sub>2</sub> (not including He)	0.84	17%
<b>Total H II, H I and H<sub>2</sub> (not including He)</b>	<b>4.9</b>	
<b>Total gas (including He)</b>	<b>6.7</b>	

# ISM = dust + gas

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## Dust

- dust = tiny grains of solid material
  - Historically, courses on the ISM have dealt with “non-stellar stuffs.”
  - The dust and gas strongly influence each other.
    - ◆ Dust reprocesses sunlight, altering the radiation field passing through the gas.
    - ◆ Dust is made of refractory elements, so creating dust alters the chemical abundances of the surrounding gas.
    - ◆ Dust grains are a leading source of free electrons in the interstellar gas.
    - ◆ Gas molecules form on the surfaces of dust grains.

## Gas

- Interstellar gas occupies the same region as stars.
- Stars are made from interstellar gas, and emit stellar winds into the ISM over the course of their lives. When massive stars reach the end of their lifetimes, they inject enriched gas at high speeds into the surrounding interstellar gas.
- Stars emit photons that are capable of exciting the interstellar gas. The emission lines have strong diagnostic power, enabling us to determine densities, temperatures, and ionization states of interstellar gas.

# Abundance of elements in & Phases of the ISM

## Abundance of elements in the local ISM

Element	Abundance (ppm)	Atomic number	1 <sup>st</sup> ionization energy (eV)
hydrogen (H)	911,900	1	13.60
helium (He)	87,100	2	24.59
oxygen (O)	490	8	13.62
carbon (C)	270	6	11.26
neon (Ne)	85	10	21.56
nitrogen (N)	68	7	14.53
magnesium (Mg)	40	12	7.65
silicon (Si)	32	14	8.15
iron (Fe)	32	26	7.90
sulfur (S)	13	16	10.36

(ppm = parts per million)

H : 91.2% by number

He: 8.7%

others: 0.1%

The interstellar gas is primarily H and He resisting from the Big Bang.

A small amount of heavy elements was produced as the result of the return to the ISM of gas that has been processed in stars and stellar explosions.

$$M(Z > 2)/M_{\text{H}} = 0.0199; M(\text{total})/M_{\text{H}} = 1.402$$

## Five phases of the ISM

Name	T (K)	$n_{\text{H}}$ (cm <sup>-3</sup> )	Mass fraction	Volume fraction
Molecular Clouds	20	> 100	35%	0.1%
Cold Neutral Medium	100	30	35%	1%
Warm Neutral Medium	5000	0.6	25%	40%
Warm Ionized Medium	$10^4$	0.3	3%	10%
Hot Ionized Medium	$10^6$	0.004	0.2%	50%

# Phases of the ISM

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## Molecular clouds

- H<sub>2</sub> is the dominant form of molecules.
- **Number density  $\sim 10^6 \text{ cm}^{-3}$  in the molecular cloud cores**, which are self-gravitating and form stars. (Note that  $10^6 \text{ cm}^{-3}$  is comparable to the density in the most effective cryo-pumped vacuum chambers in laboratories.)
- How to observe: for instance, 2.6, 1.3 and 0.9 mm (115, 230 and 345 GHz) emission lines from CO.

## Cold neutral medium (CNM) ( $T \sim 10^2 \text{ K}$ )

- The dominant form of CNM is H I (atomic hydrogen).
- The CNM is distributed in sheets and filaments occupying  $\sim 1\%$  of the ISM volume.
- How to observe: UV and optical absorption lines in the spectra of background stars and quasars.

## Warm neutral medium (WNM) ( $T \sim 5 \times 10^3 \text{ K}$ )

- Its dominant form is H I (atomic hydrogen).
- A leading method of observing the WNM is using 21 cm emission.

## Warm ionized medium (WIM) or Diffuse ionized gas (DIG) ( $T \sim 10^4 \text{ K}$ )

- The dominant form is H II (ionized hydrogen or proton).
- The WIM is primarily photoionized by O and B stars.
- Observed using Balmer emission lines (H $\alpha$ ).

## Hot ionized medium (HIM) or coronal gas ( $T \gtrsim 10^{5.5} \text{ K}$ )

- The HIM is primarily heated by supernovae.
- HIM occupies  $\sim$  half of the ISM volume, but provides only 0.2% of the ISM mass.
- soft X-ray emission. O IV, N V, and C IV emission or absorption lines in the spectra of background stars.

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from B. T. Draine

Phase	$T$ (K)	$n_{\text{H}}$ (cm $^{-3}$ )	Comments
Coronal gas (HIM) $f_V \approx 0.5?$ $\langle n_{\text{H}} \rangle f_V \approx 0.002 \text{ cm}^{-3}$ ( $f_V \equiv$ volume filling factor)	$\gtrsim 10^{5.5}$	$\sim 0.004$	Shock-heated Collisionally ionized Either expanding or in pressure equilibrium Cooling by: ◊ Adiabatic expansion ◊ X ray emission Observed by: • UV and x ray emission • Radio synchrotron emission
H II gas $f_V \approx 0.1$ $\langle n_{\text{H}} \rangle f_V \approx 0.02 \text{ cm}^{-3}$	$10^4$	$0.3 - 10^4$	Heating by photoelectrons from H, He Photoionized Either expanding or in pressure equilibrium Cooling by: ◊ Optical line emission ◊ Free-free emission ◊ Fine-structure line emission Observed by: • Optical line emission • Thermal radio continuum
Warm HI (WNM) $f_V \approx 0.4$ $n_{\text{H}} f_V \approx 0.2 \text{ cm}^{-3}$	$\sim 5000$	0.6	Heating by photoelectrons from dust Ionization by starlight, cosmic rays Pressure equilibrium Cooling by: ◊ Optical line emission ◊ Fine structure line emission Observed by: • HI 21 cm emission, absorption • Optical, UV absorption lines

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from B. T. Draine

Phase	$T$ (K)	$n_{\text{H}}$ (cm $^{-3}$ )	Comments
Cool HI (CNM) $f_V \approx 0.01$ $n_{\text{H}} f_V \approx 0.3 \text{ cm}^{-3}$	$\sim 100$	30	Heating by photoelectrons from dust Ionization by starlight, cosmic rays Cooling by: ◊ Fine structure line emission Observed by: • HI 21-cm emission, absorption • Optical, UV absorption lines
Diffuse H <sub>2</sub> $f_V \approx 0.001$ $n_{\text{H}} f_V \approx 0.1 \text{ cm}^{-3}$	$\sim 50 \text{ K}$	$\sim 100$	Heating by photoelectrons from dust Ionization by starlight, cosmic rays Cooling by: ◊ Fine structure line emission Observed by: • HI 21-cm emission, absorption • CO 2.6-mm emission • optical, UV absorption lines
Dense H <sub>2</sub> $f_V \approx 10^{-4}$ $\langle n_{\text{H}} \rangle f_V \approx 0.2 \text{ cm}^{-3}$	$10 - 50$	$10^3 - 10^6$	Heating by photoelectrons from dust Ionization and heating by cosmic rays Self-gravitating: $p > p(\text{ambient ISM})$ Cooling by: ◊ CO line emission ◊ C I fine structure line emission Observed by: • CO 2.6-mm emission • dust FIR emission
Cool stellar outflows	$50 - 10^3$	$1 - 10^6$	Observed by: • Optical, UV absorption lines • Dust IR emission • HI, CO, OH radio emission

# Typical pressure & Energy densities

## Typical pressure of each phase

- $P = nk_B T \sim 4 \times 10^{-13} \text{ dyn cm}^{-2} \sim 4 \times 10^{-19} \text{ atm}$  (atmosphere)
- Here, Boltzmann constant,  $k_B = 1.38 \times 10^{-16} \text{ cm}^2 \text{ g s}^{-2} \text{ K}^{-1}$
- This is extremely low pressure compared to the atmospheric pressure around us.

## Energy density

$$\epsilon = \frac{3}{2} n k_B T$$
$$\sim 6 \times 10^{-13} \text{ erg cm}^{-3}$$
$$\sim 0.4 \text{ eV cm}^{-3}$$

Type	Energy density (eV cm <sup>-3</sup> )
Thermal energy	0.4
Turbulent kinetic energy	0.2
Cosmic microwave background	0.2606
Far-infrared from dust	0.3
Optical/near-IR from stars	0.6
Magnetic energy	0.9
Cosmic rays	1.4

- All of them are comparable in energy density.
- All energy densities in the ISM are roughly half an electron-volt per cubic centimeter.
- The near-equipartition is partly coincidental.
  - ◆ The fact that the energy density in the CMN is similar to the other energy densities is surely accidental.
  - ◆ But the other energy densities are in fact coupled, roughly regulated by feedback mechanisms between them.

# History of ISM Studies

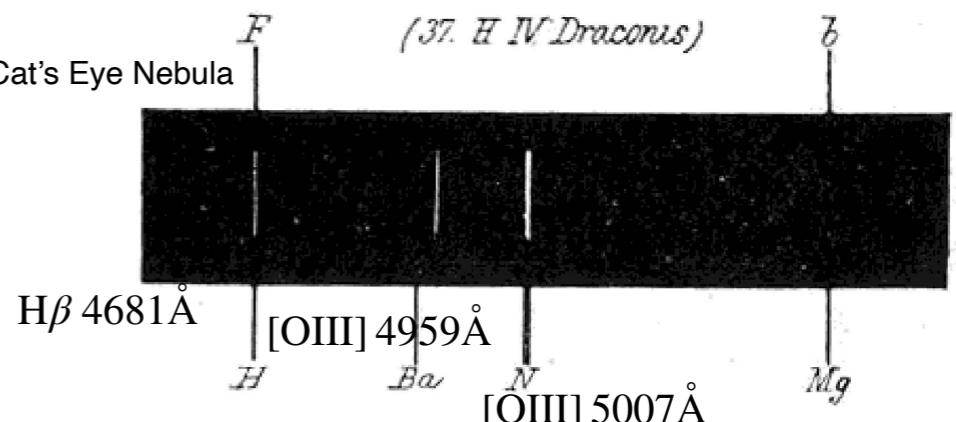
## Aether

- Early Greek astronomers believed that the volume inside the celestial sphere was filled with a diffuse aether, or quintessence.
- For centuries, the idea of a space-filling aether still lingered. Even Isaac Newton postulated “an aether medium,” which is so rare and subtle as to be undetectable, and strongly elastic.

the Cat's Eye Nebula (HST image)



spectrum of the Cat's Eye Nebula  
(W. Huggins)

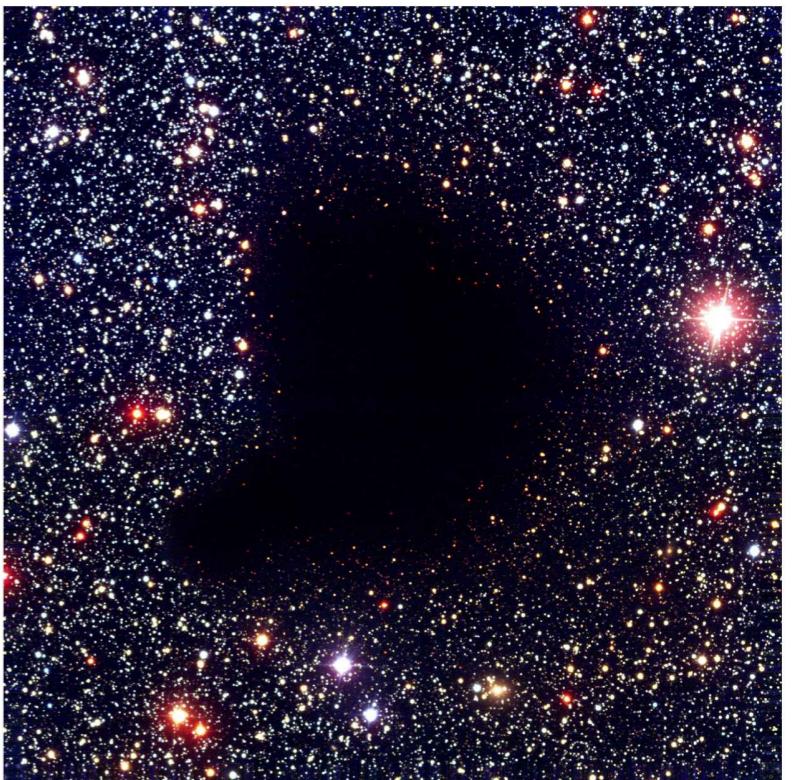


## Interstellar material

- The idea of visible, interstellar material arose in the 18th century, with the study of nebulae (Latin for “clouds”). “Nebula” was used to mean any extended luminous object.
- William Herschel resolved some nebulae into stars. In the 1980s, William Huggins demonstrated that some nebulae have emission line spectra, rather than the absorption line spectra.
- Hypothetical elements:
  - ◆ Huggins attributed 4959Å and 5007Å lines, observed in the Cat's Eye Nebula, to “nebulum” (or “nebulium”) => Ira Bowen discovered that these lines were actually forbidden [O III] lines.
  - ◆ aurorium : 5577Å in the spectrum of the aurora borealis => turned out to be [O I]
  - ◆ coronium: 5303Å in the spectrum of the Sun's corona => Fe XIV

# Interstellar Dust

- The existence of dust had been hinted at by the presence of dark nebulae (Barnard 68).
  - ◆ The dark nebulae were originally thought to be due to a lack of stars, but later recognized as being clouds of obscuring material.
- Vesto Slipher (1912) discovered that the spectrum of the nebula surrounding the Pleiades shows a continuum with absorption lines superposed.
  - ◆ He conjectured that this is light from stars, reflected from “fragmentary and disintegrated matter”, or dust.



Barnard 68 (at  $d \sim 150$  pc), in the constellation Ophiuchus.



The Pleiades cluster & surrounding reflection nebulae

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## Interstellar gas that is invisible to the eye

- Initially, bright nebulae were thought of as isolated clouds in (nearly) empty space.
- In 1901, Johannes Hartmann found:
  - ◆ the spectrum of binary Delta Orionis (a spectroscopy binary system) shows a narrow calcium absorption line (at  $\lambda 3924$ ) that is stationary in addition to the oscillating, broad absorption lines.
  - ◆ the Ca absorption line was caused by a gas cloud somewhere along the line of sight to Delta Orionis.
- Later, similar “stationary lines” were found along the sightlines to many other bright stars.
  - ◆ The lines were all narrow, and had strengths correlated with the distance to the background star.
  - ◆ Using higher resolution spectrographs, they had been revealed to have complex structures, consisting of many narrower lines with different radial velocities.
  - ◆ This indicates that the ISM has a complex structure, consisting neither of smooth uniform gas nor of isolated blobs drifting about in a vacuum.

# Ionized nebulae

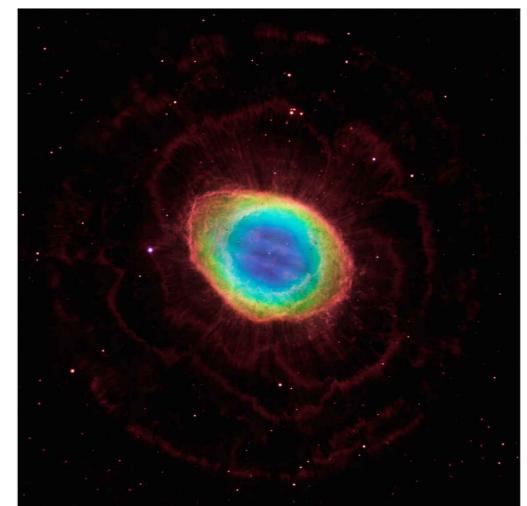
- H II regions
  - are regions of interstellar gas heated and photoionized by embedded O or B-type stars.
  - In 1939, Bengt Stromgren developed the idea that bright nebulae with strong emission lines are regions of photo ionized gas, surrounding hot star or other source of ionizing photons.
  - ex) Orion Nebula
- Planetary nebulae
  - are regions of ejected stellar gas heated and photoionized by the hot remnant stellar core, which is becoming a white dwarf.
  - ex) Ring Nebula, Cat's Eye Nebula
  - Ring Nebula:
    - ◆ central blue-green colors from [O III] 4595, 5007
    - ◆ outer reddish colors from Ha 6563, [N II] 6548, 6583



Orion Nebula ( $d \sim 410$  pc)  
HST image



Cat's Eye Nebula (HST image)



Ring Nebula (HST image)

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- Supernova remnants

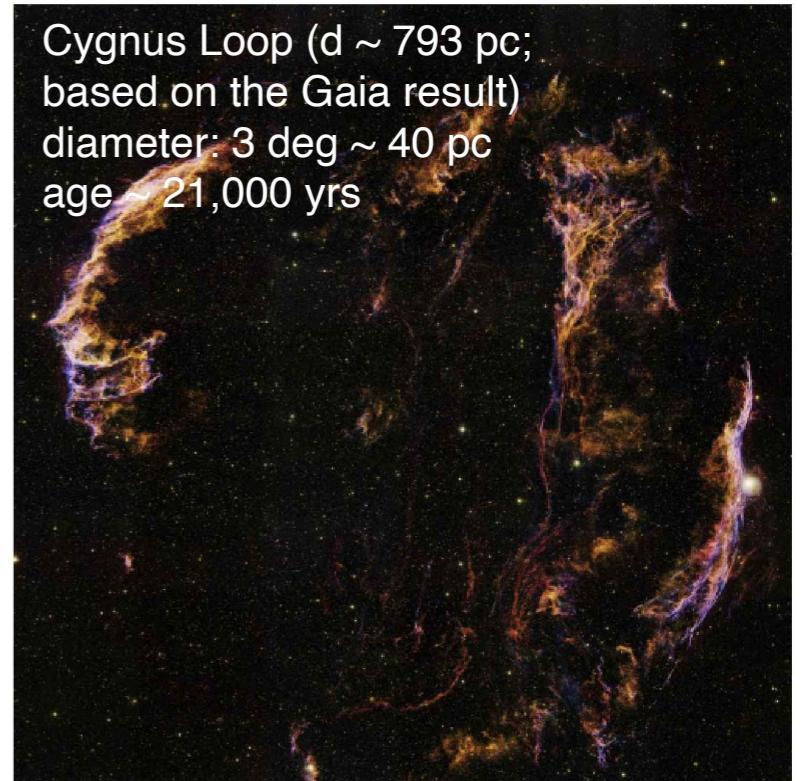
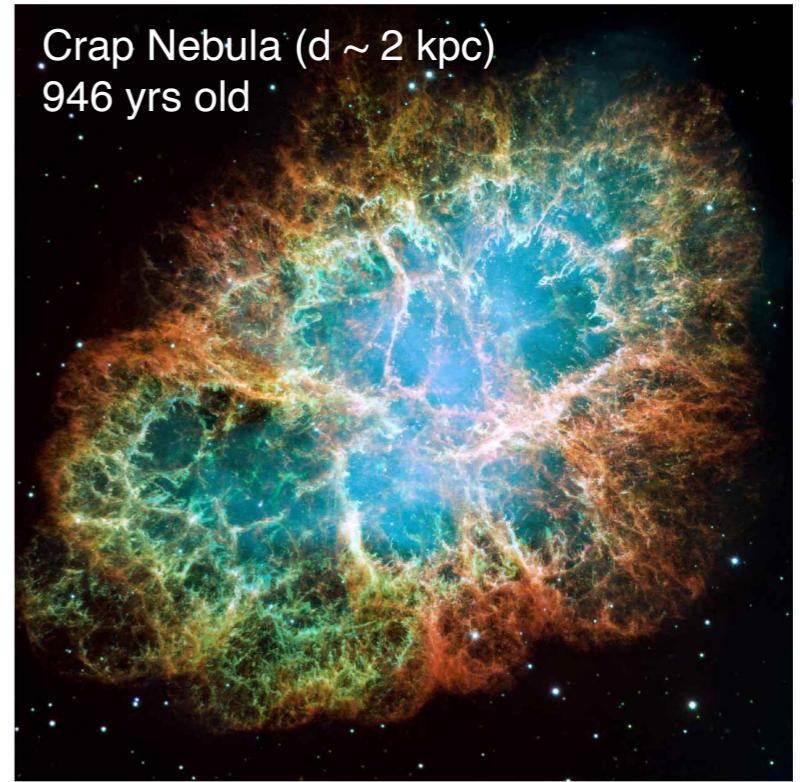
- are regions of gas heated by the blast wave from a supernova explosion.

- Crab Nebula

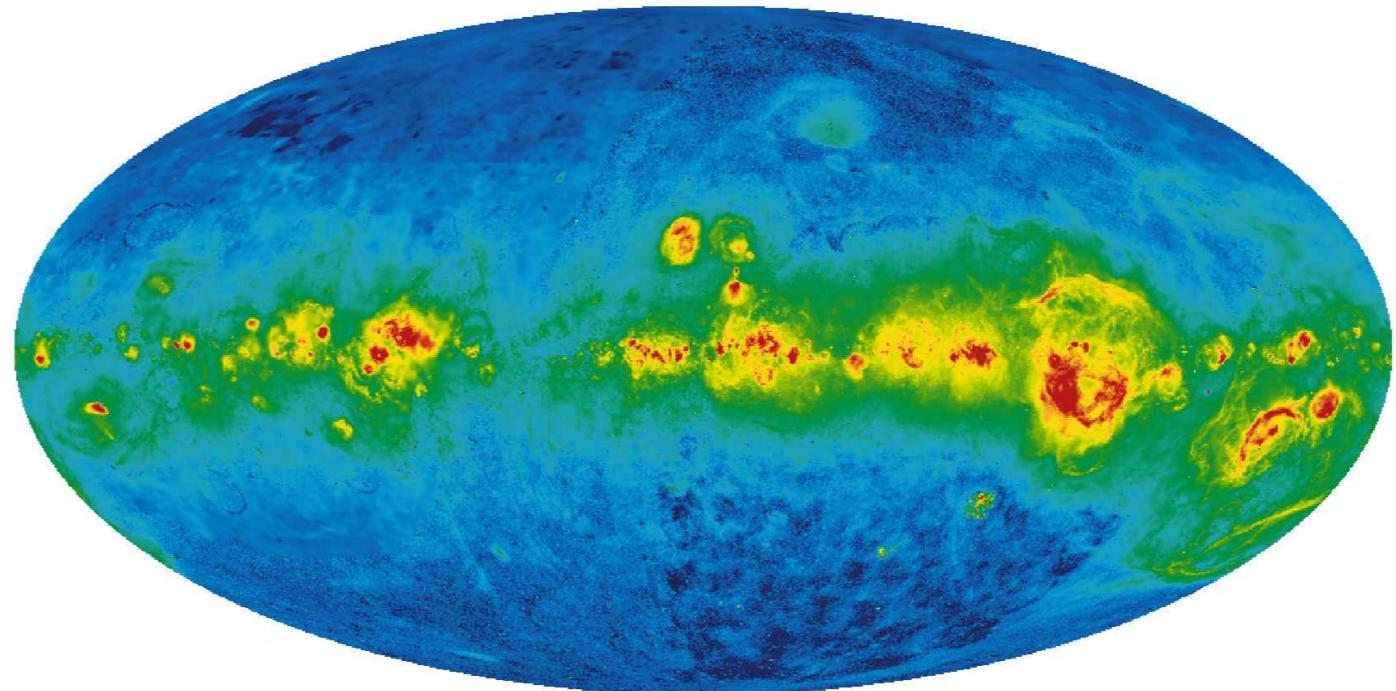
- ◆ a young pulsar-containing supernova remnant
    - ◆ are filled in with luminous gas.
    - ◆ are photoionized by its central pulsar.

- Cygnus Loop (Veil Nebula)

- ◆ most of the gas has been plowed up by the blast wave, leaving the center part empty.
    - ◆ The visible loop (or veil) is where the gas has cooled to  $T \sim 10,000$  K.



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- Warm Ionized Medium
    - About 90% of the ionized hydrogen in our galaxy lies in the relatively low density WIM.
    - Balmer line emission from recombining hydrogen fills the entire sky.
    - Although many ionized nebula (Orion, Crab, Cat's eye, etc) can be seen as the bright red blotches, they are not the dominant repository of recombining hydrogen in our galaxy.



All-sky map of H $\alpha$  (6563Å) in a log scale from 0.03 Ry to 160 Ry.  
Ry (rayleigh) =  $10^6/4\pi$  photons cm $^{-2}$  s $^{-1}$  cm $^{-2}$  Hz $^{-1}$

# Physical Description of the ISM

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- The ISM is described physically in terms of thermodynamic properties: density, temperature, pressure, etc.
- The gas of the ISM and IGM consists of individual atoms, molecules, ions, and electrons.
  - At low speeds, neutral atoms and molecules interact via elastic collisions.
  - At high speeds, atoms and molecules can undergo collisional excitation and ionization and thus the situation is complicated.
- Concept of equilibrium
  - “Equilibrium” means a state of balance.
  - The ISM and IGM are not in perfect equilibrium.
  - However, ***there are times when the assumption of some type of equilibrium is a useful approximation.***
- Types of equilibrium
  - kinetic equilibrium (thermal equilibrium)
  - excitation equilibrium
  - ionization equilibrium
  - pressure equilibrium

# Kinetic Equilibrium

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- A gas is in kinetic equilibrium (thermal equilibrium) when the individual particles have a Maxwellian distribution of velocities:

$$f(\mathbf{v})d^3\mathbf{v} = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) d^3\mathbf{v}$$

- The mean kinetic energy per gas particle will be, integrating over the distribution,  
 $\langle E \rangle = \frac{1}{2}m \langle v^2 \rangle = \frac{3}{2}kT$ , regardless of the particle mass.
- The mean particle energy in the ISM ranges over five orders of magnitude, from  $\sim 0.001$  eV in the coldest regions of molecular clouds to  $\sim 100$  eV in the HIM.
- The root mean square speed of each particle is  $v_{\text{rms}} = \langle v^2 \rangle^{1/2} = (3kT/m)^{1/2}$ .
- In the air around us:
  - The nitrogen and oxygen molecules are in kinetic equilibrium with each other at a temperature  $T \sim 300$  K and particle energy  $\langle E \rangle \sim 0.04$  eV.
  - This is much smaller than  $E = h\nu \sim 2$  eV of optical photons traversing the air and much larger than  $E \sim 0.001$  eV of the cosmic neutrinos traversing the universe.

- ***Requirement for the kinetic equilibrium***

- To come to kinetic equilibrium, particles need to collide with each other.
- The collisional timescales are sufficiently short that we can regard the ISM as being in kinetic equilibrium under most circumstances.

## - Order of magnitude estimation

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- Cross section for elastic collision
  - A small atom can be approximated as a “billiard ball” with a radius  $r \sim 3a_0$ , where  $a_0 = \hbar/(mke^2) \approx 5.29 \times 10^{-9} \text{ cm} = 0.529 \text{\AA}$  is the Bohr radius (the most probable distance between the nucleus and the electron in a hydrogen atom in its ground).
  - Two identical atoms will collide when they are separated by a distance  $d \leq 2r$ . The cross section for interactions is then  $\sigma \sim \pi(2r)^2 \sim 100a_0^2 \sim 3 \times 10^{-15} \text{ cm}^2$ .
- Mean free path
  - In air at sea level, the number density of molecules is  $n \sim 2.5 \times 10^{19} \text{ cm}^{-3}$  and the mean free path is  $\lambda_{\text{mfp}} = 1/(n\sigma) \sim 10^{-5} \text{ cm} \sim 0.1 \mu\text{m}$ .
  - In the CNM, with  $n \sim 30 \text{ cm}^{-3}$ ,  $\lambda_{\text{mfp}} \sim 10^{13} \text{ cm} \sim 1 \text{ AU}$ .
  - When we deal with a volume of gas that is larger than the mean free path, we can characterize that volume by its bulk properties, such as density, pressure, temperature.

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- (Elastic) Collisional time scale

- mean free path:  $\lambda_{\text{mfp}} \sim 1/(n\sigma)$ , typical velocity:  $v_{\text{rms}} \sim (3kT/m)^{1/2}$
  - collisional time scale (time required to come to kinetic equilibrium)

$$t_{\text{coll}} \sim \frac{\lambda_{\text{mf}}}{v_{\text{rms}}} \sim \frac{1}{n\sigma} \left( \frac{m}{2\langle E \rangle} \right)^{1/2}$$

- This indicates that a dense gas of energetic particles will come to kinetic equilibrium, thanks to the frequent collisions, more rapidly than a tenuous gas of slow-moving particles.

- Hydrogen-Hydrogen collision

- Assuming  $\sigma_{\text{HH}} \sim 3 \times 10^{-15} \text{ cm}^2$ ,  $t_{\text{coll}}(\text{HH}) \sim 2 \times 10^8 \text{ s} \left( \frac{n_{\text{H}}}{1 \text{ cm}^{-3}} \right)^{-1} \left( \frac{\langle E \rangle}{1 \text{ eV}} \right)^{-1/2}$
  - In a dense planetary (Earth) atmosphere,  $t_{\text{coll}}(\text{HH}) < 1 \text{ ns}$  ( $n_{\text{H}} > 10^{17} \text{ cm}^{-3}$ ).
  - In dense, molecular cores,  $t_{\text{coll}}(\text{HH}) > 1 \text{ min}$  ( $n_{\text{H}} \lesssim 10^6 \text{ cm}^{-3}$ ).
  - In the WNM,  $t_{\text{coll}}(\text{HH}) \sim 0.5 \text{ century}$  ( $n_{\text{H}} \sim 0.1 \text{ cm}^{-3}$ ).

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- Electron-Hydrogen collision
    - The mass of an electron is  $\sim 1/1836$  that of a proton. The typical electron speed will be greater than that of a hydrogen by a factor of  $(1836)^{1/2} \sim 43$ . Therefore, we can approximate the atom as standing still while an electron slams into it.
    - Assuming  $\sigma_{eH} \sim 10^{-15} \text{ cm}^2$ ,
  - Free electrons will do the thermalization everywhere but in the highest density ( $n_H > 0.5 \text{ cm}^{-3}$ ) portion of the ISM.
  - Electron-Electron collision
    - Two electrons collide when the electrostatic repulsion between them deflects their paths through a large angle (let's say 90 deg).
    - A large deflection requires that the electrostatic (Coulomb) force at closest approach is comparable to the initial kinetic energy.

$$\frac{e^2}{r_e} \approx E \quad (r_e \text{ is the separation at their closest approach.})$$

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- The effective cross section for electron-electron collisions is

$$\sigma_{ee} \approx \pi r_e^2 \sim \pi \frac{e^4}{\langle E \rangle^2}$$

- The typical collisional time scale is

$$t_{\text{coll}}(ee) \sim \frac{1}{n_e \sigma_{ee}} \left( \frac{m_e}{2 \langle E \rangle} \right)^{1/2} \sim \frac{\langle E \rangle^2}{\pi n_e e^4} \left( \frac{m_e}{2 \langle E \rangle} \right)^{1/2} \propto n_e^{-1} \langle E \rangle^{3/2}$$
$$\sim 4 \times 10^5 \text{ s} \left( \frac{n_e}{0.04 \text{ cm}^{-3}} \right)^{-1} \left( \frac{\langle E \rangle}{1 \text{ eV}} \right)^{3/2}$$

- Thus, electrons thermalize each other more rapidly than they thermalize the neutral atoms unless  $\langle E \rangle > 50 \text{ eV}$ , found in the HIM.
- The above collisional time scales are sufficiently short that we can regard the ISM as being in kinetic equilibrium under most circumstances.

# Excitation Equilibrium

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- Consider a system that has many energy states (for instance, electronic energy levels of an atom, or two rotational or vibrational states of a molecule, or the two hyper fine states of a hydrogen atom)
- A large population of such systems is said to be in **excitation equilibrium** if the relative level populations follows a Boltzmann distribution:  $\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} \exp\left(-\frac{E_{u\ell}}{kT}\right)$ , where  $g_i = 2J_i + 1$  is the statistical weight of the  $i^{\text{th}}$  energy level, and  $T$  is the kinetic temperature of the system.
  - In the limit  $kT \gg E_{u\ell}$ ,  $n_u/n_\ell = g_u/g_\ell$  (the two levels are populated according to their statistical weights.)
  - In the limit  $kT \ll E_{u\ell}$ ,  $n_u \approx 0$ .
  - The excitation equilibrium implies the thermal equilibrium.
  - However, not every system in kinetic equilibrium is in excitation equilibrium. (for instance, masers).
  - For any two energy states, we define an **excitation temperature** using the population ratio  $n_u/n_\ell$  such that  $\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} \exp\left(-\frac{E_{u\ell}}{kT_{\text{exc}}}\right)$ . In general,  $T_{\text{exc}} \neq T_{\text{kinetic}}$ .
  - The excitation temperature is nothing but a convenient way to parameterize the relative level populations.
  - For a system like a **maser** with inverted energy levels ( $n_u/n_\ell > g_u/g_\ell$ ), *the excitation temperature is negative*.

# Ionization Equilibrium

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- First ionization energy of the most abundant elements is  $I \sim 10$  eV.
  - hydrogen :  $I_{\text{H}} \sim 13.6$  eV, carbon:  $I_{\text{C}} \sim 11.26$  eV, magnesium:  $I_{\text{H}} \sim 7.65$  eV
  - The only regions where we expect collisional ionization of these neutral atoms are where  $T > 1.2 \times 10^5$  K and thus  $\langle E \rangle > 10$  eV. This temperature is attained only in the HIM.
- In practice, much of the ionization in the ISM is photoionization.



- For element  $X$  to be in the photoionization equilibrium, we require a balance between photoionization and radiative recombination:

$$n(X^r)n_\gamma\sigma_{\text{pho}}c = n(X^{r+1})n_e\sigma_{\text{rec}}v$$

$$\frac{n(X^{r+1})n_e}{n(X^r)} = \frac{n_\gamma \left\langle \sigma_{\text{pho}} \right\rangle_{h\nu} c}{\left\langle \sigma_{\text{rec}} v \right\rangle_v}$$

$\sigma_{\text{pho}}$  = cross-section for photoionization,  
depending on the photon energy  $h\nu$

$\sigma_{\text{rec}}$  = cross-section for recombination,  
depending on the electron velocity  $v$

averaged over the photon energy spectrum  
averaged over the Maxwellian velocity distribution

---

- Maxwellian distribution

$$\bar{f}(\mathbf{v})d^3\mathbf{v} = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{m\mathbf{v}^2}{2kT}\right) d^3\mathbf{v}$$

Here,  $E = \frac{1}{2}mv^2$  (the energy per particle)

$$f(E)dE = \frac{2}{\sqrt{\pi}} \left(\frac{E}{kT}\right)^{1/2} \exp\left(-\frac{E}{kT}\right) \frac{dE}{kT}$$

- Then, the radiative recombination rate,  $\alpha(T)$  is

$$\begin{aligned} \langle \sigma_{\text{rec}} v \rangle &= \left(\frac{2}{m_e}\right)^{1/2} \langle \sigma_{\text{rec}} E^{1/2} \rangle \\ &= \left(\frac{8kT}{\pi m_e}\right)^{1/2} \int_0^\infty \sigma_{\text{rec}} \frac{E}{kT} \exp\left(-\frac{E}{kT}\right) d(E/kT) \end{aligned}$$

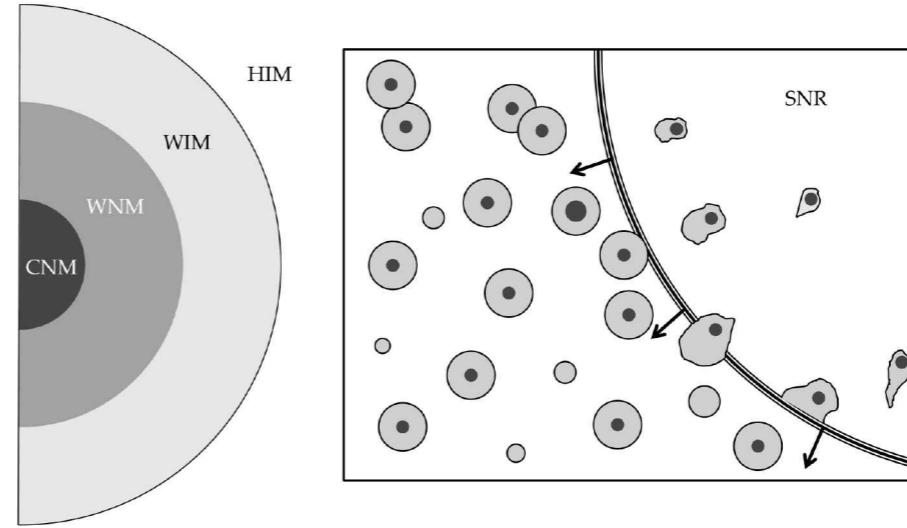
Note that  $\sigma_{\text{rec}} \approx \sigma_{\text{rec},0} \left(\frac{E}{I_{\text{H}}}\right)^{-1}$

$$\therefore \alpha(T) = \langle \sigma_{\text{rec}} v \rangle \approx \left(\frac{8}{\pi m_e kT}\right)^{1/2} \sigma_{\text{rec},0} I_{\text{H}} \propto T^{-1/2}$$

# Pressure Equilibrium

- We already pointed out that all five phases of the ISM have a pressure  $P \sim 4 \times 10^{-19}$  atm, equivalent to a thermal energy density  $(3/2)nkT \sim 0.4$  eV cm $^{-3}$ .
  - Thus, it is tempting to assume that the phases are in pressure equilibrium, with

$$n_1 k T_1 = n_2 k T_2 = 4 \times 10^{-19} \text{ atm}$$
$$n_1 T_1 = n_2 T_2 = 2,935 \text{ cm}^{-3} \text{ K}$$
$$(1 \text{ atm} = 1.013 \times 10^6 \text{ dyn cm}^{-2})$$



- Earlier views of the ISM did assume pressure equilibrium. Denser, cooler “clouds” in a tenuous, hotter “intercloud medium.”
- However, current studies of the ISM have rejected this simple picture. The ISM has indeed tendencies toward pressure equilibrium, but something always happens to throw things out of equilibrium.
  - ◆ The ubiquity of free electrons indicates that the ISM is coupled to the interstellar magnetic field. The turbulent energy density is not negligibly small. Thus, they have to be taken into account.
  - ◆ Supernova explosions are going off in the ISM, increasing the temperature  $T$ .
  - ◆ Hot young stars are pouring ionizing radiation into the ISM, splitting up atoms and increasing  $n$ .

# Heating and Cooling in the ISM

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- Balance between heating and cooling in our body.
  - Number density of molecules in your body is  $n \sim 3 \times 10^{22} \text{ cm}^{-3}$  and temperature is  $T \sim 310 \text{ K}$ .
  - If your temperature drops too low, your body increases the heating rate (by shivering) or decrease the cooling rate (by trying to fluff out fur).
  - If your temperature rises too high, your body increases the cooling rate (by sweating and thus increasing evaporative cooling) or decrease the heating rate (by stopping unnecessary activity).
- The temperature of the ISM is also determined by a balance between heating and cooling.
  - Each phase has a temperature where the balance is a stable one.
- Definitions
  - heating gain  $G$ , cooling loss  $L$  in units of  $\text{erg s}^{-1}$ .
  - volumetric heating rate  $g = nG$ , volumetric cooling rate  $\ell = nL$  in units of  $\text{erg cm}^{-3} \text{ s}^{-1}$ .
  - cooling function  $\Lambda$  in units of  $\text{erg cm}^3 \text{ s}^{-1}$ , which is useful for two-body interactions.
  - $\ell = nL = n^2\Lambda$ , where  $n$  is the total number density of gas particles.
  - Even when only one type of particle is losing energy, the energy loss is shared among all the gas particles due to the relatively short thermalization time scale in the ISM.

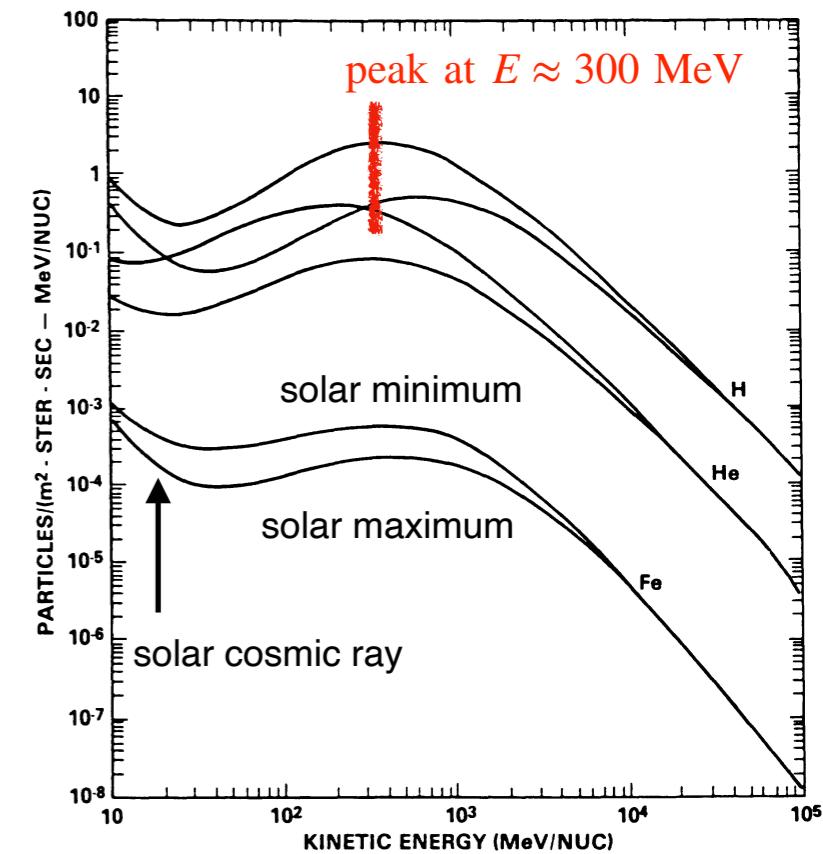
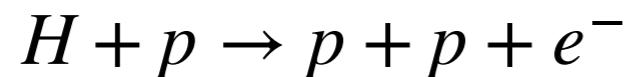
## - Heating -

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- Heating processes
  - The primary heating mechanisms of the ISM involve providing free electrons with high energies. Through collisions, the fast free electrons share their kinetic energy with other particles, and through further collisions, the distribution of velocities approaches a Maxwellian distribution.
  - **Ionization by cosmic rays**
  - **Photoionization of dust grains by starlight UV.**
  - **Photoionization of atoms (H, He, C, Mg, Si, Fe, etc) by X-rays or starlight UV.**
  - **Heating by shock waves and other MHD phenomena.**

# (1) Cosmic Ray Heating

- Cosmic Rays
  - Cosmic rays consist primarily of protons and helium nuclei, with heavier nuclei, electrons, positrons, and antiprotons making a small contribution (~1%). They have relativistic energies as high as  $10^{20}$  eV.
  - Cosmic rays are of galactic origin, and not universal except perhaps at very high energies.
  - The energy distribution of cosmic rays peaks at  $E \approx 300$  MeV.
  - The solar wind expels cosmic rays of low energy.
- The relatively low-energy cosmic rays ( $1 < E < 50$  MeV) can collisional ionize hydrogen atoms:



Differential energy spectra of cosmic-ray protons (H),  $\alpha$  particles (He), and iron, near solar minimum and maximum (Silberberg et al. 1987)

- The cosmic ray ionization produces electrons with a spectrum of energies. The ejected electron carries away a mean energy of  $\langle E \rangle \approx 35$  eV.
- Some of this kinetic energy will go into secondary ionization and excitation of H,  $H_2$ , and He that will then deexcite radiatively, but a fraction of the secondary electron energy will ultimately end up as thermal kinetic energy.

- 
- The heating efficiency depends on the fractional ionization.
    - ◆ If the ionization is high, then the primary electron has a high probability of losing its energy by long-range Coulomb scattering off free electrons, and  $\sim 100\%$  of the initial kinetic energy will be converted to heat.
    - ◆ When the gas is neutral, a fraction of the primary electron energy goes into secondary ionization or excitation of bound states.
    - ◆ Heat per primary ionization (Dalgarno & McCray 1972):

$$E_h \approx 6.5 \text{ eV} + 26.4 \text{ eV} \left( \frac{x_e}{x_e + 0.07} \right)^{1/2}, \quad x_e \equiv \frac{n_e}{n_{\text{H}}} \quad (\text{ionization fraction})$$

- ◆ Heating rate due to cosmic ray ionization:

$$g_{\text{CR}} \approx (n_{\text{H}} + n_{\text{He}}) \zeta_{\text{CR}} E_h$$

$$G_{\text{CR}} \approx 1.03 \times 10^{-27} \left( \frac{\zeta_{\text{CR}}}{10^{-16} \text{ s}^{-1}} \right) \left[ 1 + 4.06 \left( \frac{x_e}{x_e + 0.07} \right)^{1/2} \right] \text{ erg s}^{-1}$$

Here,  $\zeta_{\text{CR}}$  is the primary cosmic ray ionization rate (the average rate at which a hydrogen atom is ionized by cosmic rays).

- 
- Primary ionization rate by cosmic rays
    - From the observed spectrum of cosmic rays and the definition of the cosmic ray ionization rate:
$$\zeta_{\text{CR}} \gtrsim 7 \times 10^{-18} \text{ s}^{-1}$$
with a substantially larger rate being allowed by uncertainties
    - The observations of  $H_3^+$  appear to indicate a cosmic ray ionization rate, in diffuse molecular gas,
$$\zeta_{\text{CR}} \approx (0.5 - 3) \times 10^{-16} \text{ s}^{-1}$$
    - Note that  $\zeta_{\text{CR}} \approx 10^{-16} \text{ s}^{-1} \sim 3 \text{ Gyr}^{-1}$ .
  - ***Heating by cosmic rays is the dominant heating mechanism in molecular clouds***, where the dust opacity prevents high-energy photons from entering.

## (2) Photoelectric Heating by Dust

---

- UV and X-ray photons can knock electrons free from dust grains. The ejected electrons carry kinetic energy, which can be effective at heating the surrounding gas.
- ***Photoelectrons emitted by dust grains dominate the heating of the diffuse neutral ISM (CNM and WNM) in the Milky Way.***
- The work function, analogous to the ionization energy of an atom, for graphite is  $4.50 \pm 0.05$  eV. Therefore, UV photons with  $h\nu \gtrsim 5$  eV can kick out photoelectrons from dust grains. The photoelectric heating by dust is dominated by photons with  $h\nu \gtrsim 8$  eV.

$$G_{\text{pe}} \approx 1.4 \times 10^{-26} \frac{n_{\text{ph}}(8 - 13.6 \text{ eV})}{3 \times 10^{-3} \text{ cm}^{-3}} \frac{\langle \sigma_{\text{abs}} \rangle}{10^{-21} \text{ cm}^2} \frac{\langle Y \rangle}{0.1} \frac{\langle E_{\text{pe}} \rangle - \langle E_c \rangle}{1 \text{ eV}} \text{ erg s}^{-1}$$

Here,

$n_{\text{ph}}(8 - 13.6 \text{ eV})$  = number density of  $8 < h\nu < 13.6$  eV photons

$\langle \sigma_{\text{abs}} \rangle$  = total dust photo absorption cross section per H nucleon, averaged over the photon spectrum.

$\langle Y \rangle$  = photoelectric yield averaged over the spectrum of 8 to 13.6 eV photons absorbed by the interstellar grain mixture.

$\langle E_{\text{pe}} \rangle$  = mean kinetic energy of escaping photoelectrons.

$\langle E_c \rangle$  = mean kinetic energy of electrons captured from the plasma by grains.

- Photoelectric heating from dust may be an order of magnitude larger than the cosmic ray heating rate.

## (3) Photoionization Heating

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- Photons in the energy range  $11.26 \text{ eV} < h\nu < 13.60 \text{ eV}$  ( $911.6\text{\AA} < \lambda < 1101\text{\AA}$ ) are likely to end up ionizing a carbon atom. When carbon is photo ionized, a free electron is released.



- The released electron (photoelectron) carries away the energy between 0–2.34 eV.
- If there aren't many photons with  $h\nu > 13.6 \text{ eV}$ , hydrogen is predominantly in neutral form and most of positively charged ions are C II ( $C^+$ ).
- The heating gain from the photoionization of carbon is approximately:

$$G_{\text{CII}} = 2.2 \times 10^{-22} f(\text{CI}) \mathcal{A}_C \chi_0 \text{ erg s}^{-1}$$
$$\approx 10^{-29} \text{ erg s}^{-1}$$

Eq (3.8) of The Physics and Chemistry of the Interstellar Medium (A. G. G. M. Tielens)

Here,

$f(\text{CI})$  = neutral fraction of carbon ( $\sim 3.3 \times 10^{-4}$ )

$\mathcal{A}_C$  = atomic carbon abundance in the gas phase ( $\sim 2.70 \times 10^{-4} \times 0.5$ )

$\chi_0$  = intensity of the radiation field in units of the average interstellar radiation field.

- In the dusty ISM, the heating by carbon photoionization can't compete with the heating by electrons ejected from dust grains.
- H II regions and the diffuse IGM are the regions where photoionization becomes an important heating source.

## (4) Shock Heating

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- Shocks are propagating disturbances, characterized by abrupt, nearly discontinuous change in the temperature, pressure, and density.
  - ◆ In the ISM, shocks can be created by a supernova explosion or by the collision between molecular clouds.
  - ◆ On larger scales, shocks can be created by the collision of two galaxy clusters.
  - ◆ Shocks convert the kinetic energy of bulk flow into the thermal energy associated with random particle motion.
  - ◆ By a supernova shock, the temperature can rise to more than  $10^6$  K.
- Shock heating is the dominant heating mechanism in the HIM of the ISM and in the warm-hot intergalactic medium (WHIM).

## - Cooling -

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- Decreasing the average kinetic energy of particles in the ISM is usually done by ***radiative cooling***.
  - In the CNM, cooling is performed by infrared photons emitted by carbon and oxygen.
    - ◆ Oxygen is nearly all in the form of neutral O I. (the ionization energy = 13.26 eV)
    - ◆ Carbon will be nearly all in the form of singly ionized C II. (ionization energy = 11.26 eV) The background starlight in our galaxy has enough photons in the relevant energy range  $11.26 \text{ eV} < h\nu < 13.60 \text{ eV}$  to keep the C atoms ionized.
- [C II]  $158\mu\text{m}$  (collisionally excited line emission)
  - The electronic ground state of C II is split into two fine levels, separated by an energy  $E_{ul} = 7.86 \times 10^{-3} \text{ eV}$ , which corresponds to  $\lambda = 158 \mu\text{m}$  and  $T = E_{ul}/k = 91.2 \text{ K}$ .
  - The upper level is populated by collisions with hydrogen atoms and free electrons.
  - If C II is excited by collisions with free electrons, the cooling function is given by, for a C abundance  $n_{\text{C}}/n_{\text{H}} = 3 \times 10^{-4}$ ,

$$\frac{\Lambda_{[\text{CII}]}^e}{10^{-25} \text{ erg cm}^3 \text{ s}^{-1}} \approx 0.03 \left( \frac{x}{10^{-3}} \right) \left( \frac{T}{100 \text{ K}} \right)^{-1/2} \exp \left( -\frac{91.2 \text{ K}}{T} \right)$$

Here,  $x = n_e/n$  is the ionization fraction.

- 
- If the C II is excited by collisions with hydrogen atoms, the cooling function is

$$\frac{\Lambda_{\text{[CII]}}^{\text{H}}}{10^{-25} \text{ erg cm}^3 \text{ s}^{-1}} \approx 0.06 \left( \frac{T}{100 \text{ K}} \right)^{0.13} \exp \left( -\frac{91.2 \text{ K}}{T} \right)$$

- In the CNM, both contribute significantly to the excitation of C II.
- [O I] 63.2μm (collisionally excited emission line)
  - The electronic ground state of O I has a fine splitting of  $E_{ul}/k = 228 \text{ K}$ .
  - The upper level is populated primarily by collisions with hydrogen atoms.
  - The resulting cooling function due to the emission of 63.2μm is, for an abundance of  $n_{\text{O}}/n_{\text{H}} = 5.4 \times 10^{-4}$ ,

$$\frac{\Lambda_{\text{[OI]}}^{\text{H}}}{10^{-25} \text{ erg cm}^3 \text{ s}^{-1}} \approx 0.04 \left( \frac{T}{100 \text{ K}} \right)^{0.42} \exp \left( -\frac{228 \text{ K}}{T} \right)$$

- Note:
  - [C II] and [O I] are the dominant form of cooling in molecular clouds and the CNM.
  - Molecular clouds can also cool by emission from the vibrational and rotational transitions of molecules.
  - The critical densities for [C II] and [O I] are  $\sim 4 \times 10^3 \text{ cm}^{-3}$  and  $\sim 10^5 \text{ cm}^{-3}$ , respectively, implying that collisional deexcitation of these levels is unimportant in the diffuse ISM of the Milky Way.

- 
- Ly $\alpha$  1216Å
    - The first excited level of atomic hydrogen is  $E_{21} = 10.20 \text{ eV}$  above the ground state.
    - Although the first excited level will not be highly populated by collisions until the temperature reaches  $T \sim E_{21}/k = 118,000 \text{ K}$ , hydrogen is extremely abundant. Thus the cooling by Ly $\alpha$  can compete with cooling by IR fine-structure lines at temperature as low as  $T \sim 8000 \text{ K}$ .
    - The cooling function for H excited by collisions with free electrons is

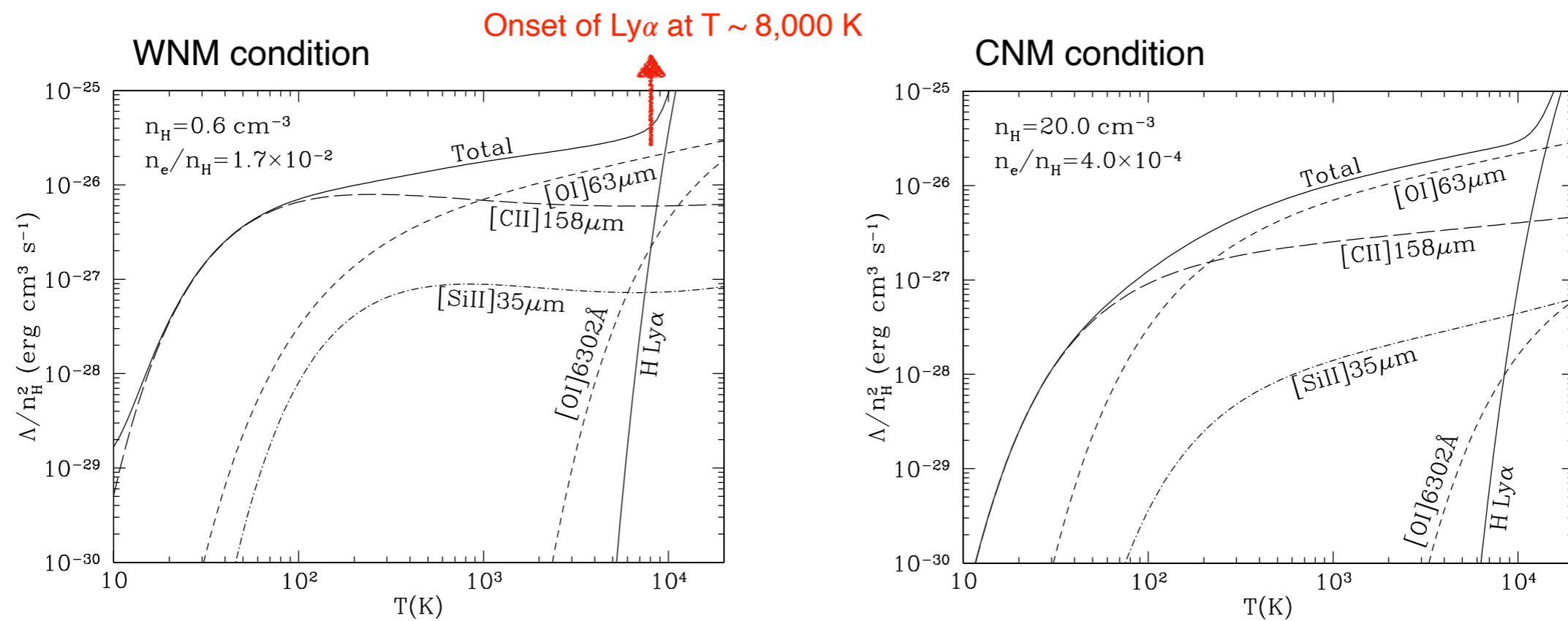
$$\frac{\Lambda_{[\text{Ly}\alpha]}^e}{10^{-25} \text{ erg cm}^3 \text{ s}^{-1}} \approx 7000 \left( \frac{x}{10^{-3}} \right) \left( \frac{T}{100 \text{ K}} \right)^{-0.5} \exp \left( -\frac{118,000 \text{ K}}{T} \right)$$

- When  $T > 15,000 \text{ K}$ ,
  - atomic hydrogen can be collisionally ionized, followed by radiative recombination to a high energy level, and followed by a cascade down to the ground state.
  - The recombination lines are an important cooling mechanism in the WNM and WIM.
  - These phases are also cooled by line emission from more highly ionized atoms such as O III, C IV, and O VI.

## • Thermal Bremsstrahlung

- In the HIM at  $T > 10^6$  K, the “braking radiation” emitted by electrons when they are accelerated by other charged particles can be a significant cooling mechanism.
- The cooling function is  $\Lambda \propto T^{1/2}$ .

## • Cooling Function

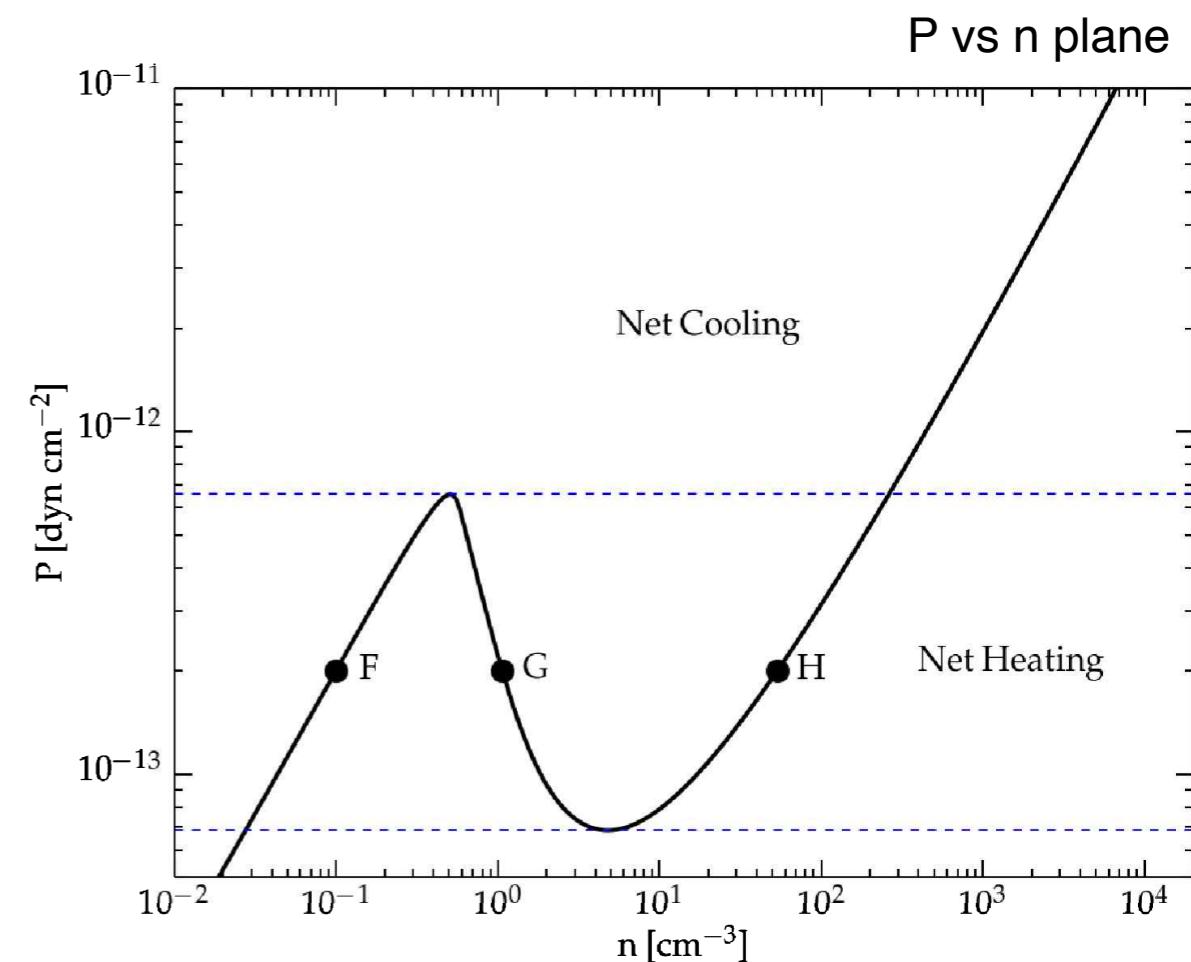
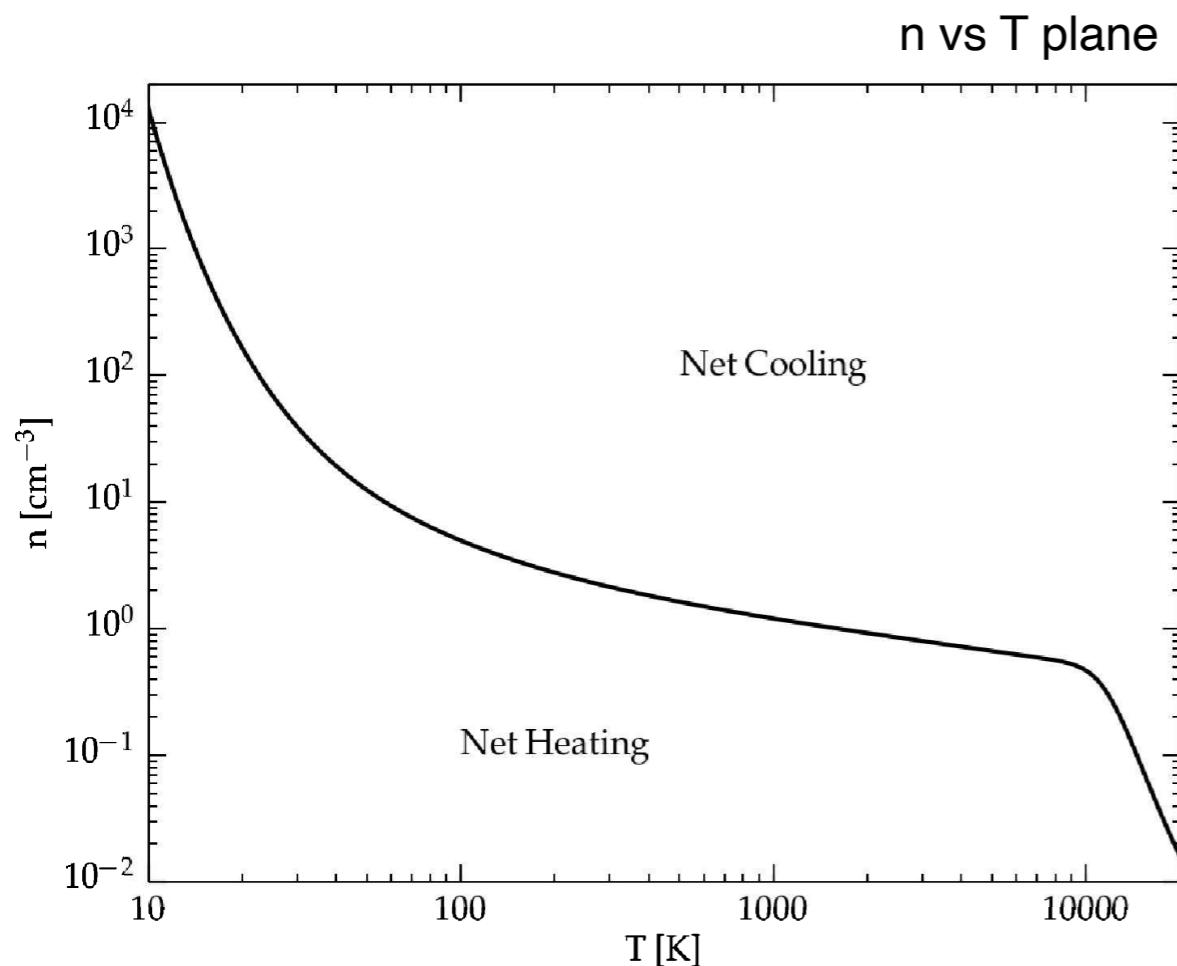


- For  $10 < T < 10^4$  K, [C II] 158 μm line is a major coolant. The [O I] 63 μm line is important for  $T > 100$  K. Ly $\alpha$  cooling dominates only at  $T > 10^4$  K.

# Stable & Unstable Equilibrium

- A thermal equilibrium must have heating and cooling balanced:  $g = \ell$ .
  - We assume **photoelectric heating by dust** and **cooling by [C II], [O I], and Ly $\alpha$** . Then, the equilibrium density is obtained by

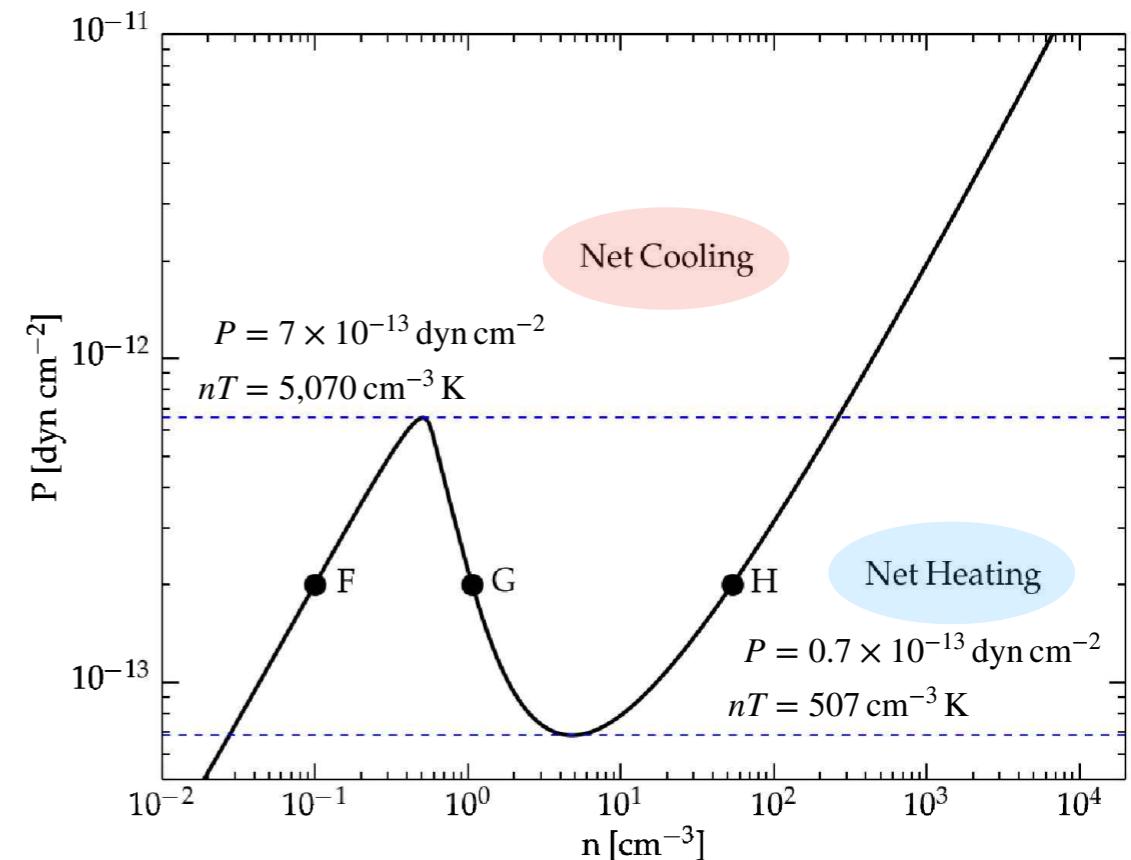
$$n_{\text{eq}} G = n_{\text{eq}}^2 \Lambda \quad \rightarrow \quad n_{\text{eq}}(T) = \frac{G}{\Lambda(T)} \quad \text{Note that } G \text{ is a (nearly) constant.}$$



- If every point along the above equilibrium line represented a stable equilibrium, then there could be a continuous distribution of temperatures, and thus of number densities.
- However, it's not the case. Not every equilibrium point is a stable equilibrium.

## - Pressure Equilibrium

- Let's assume that the interstellar gas is in pressure equilibrium.
  - For pressures in the range  $0.7 \times 10^{-13} \text{ dyn cm}^{-3} < P < 7 \times 10^{-13} \text{ dyn cm}^{-3}$ , bounded by the dashed lines, there are three possible values of  $n_{\text{eq}}$  at a fixed pressure.
  - Consider what happens at a point, for instance P, if you slightly change the temperature while keeping the pressure fixed.
    - If T increases, n must decrease, and you must move left from point F. This moves you into the net cooling portion, and T consequently decreases.
    - If T decrease, n must increase, and this moves you rightward into the net heating portion, and T consequently increases.
    - Thus, a negative feedback restores the original temperature.
  - A similar negative feedback maintains temperature stability at point H.
  - However, now consider what happens at G.
    - If T increases, n must decrease, and you must move left from point G. This moves you into the net heating portion, and T increases further, until you reach F.
    - If T decrease, n must increase, and this moves you rightward into the net cooling portion, and T decrease further, until you reach H.
    - Thus, a positive feedback makes the point unstable.
  - Consequently, we have two stable equilibrium points (F and H).**



## - History: Two-Phase Model & Three-Phase Model

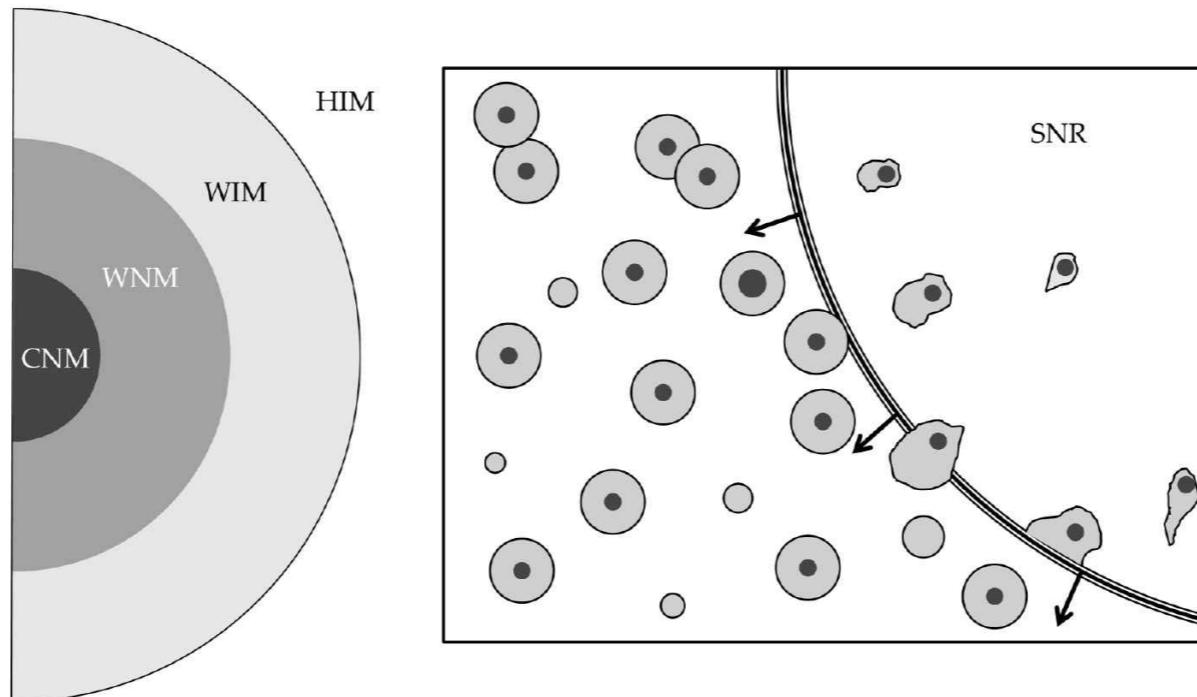
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- As a result of their analysis, Field, Goldsmith, and Hasinger created a two-phase model of the ISM, consisting of Cold Neutral Clouds, with  $n \sim 10 \text{ cm}^{-3}$  and  $T \sim 100 \text{ K}$ , embedded within a Warm Intercloud Medium, with  $n \sim 0.1 \text{ cm}^{-3}$  and  $T \sim 10,000 \text{ K}$ .
- FGH (1969) advocated a two-phase model. However, they also speculated “an existence of a third stable phase at  $T > 10^6 \text{ K}$ , with bremsstrahlung the chief cooling process.”
- In the 1970s, detection of a diffuse soft X-ray background and of emission lines such as O VI 1032, 1038Å hinted at the existence of interstellar gas with  $T \sim 10^6 \text{ K}$ . In fact, the Sun resides in a “Local Bubble” of hot gas, with  $T \sim 10^6 \text{ K}$  and  $n \sim 0.004 \text{ cm}^{-3}$ .
- Cox & Smith (1974) suggested that supernova remnants could produce a bubbly hot phase, and that the bubbles blown by supernovae would have a porosity factor (volume fraction of the ISM occupied by hot bubbles):

$$q > 0.1 \left( \frac{r_{\text{SN}}}{10^{-13} \text{ pc}^{-3} \text{ yr}^{-1}} \right)$$

## - History: McKee & Ostriker's Three-Phase Model

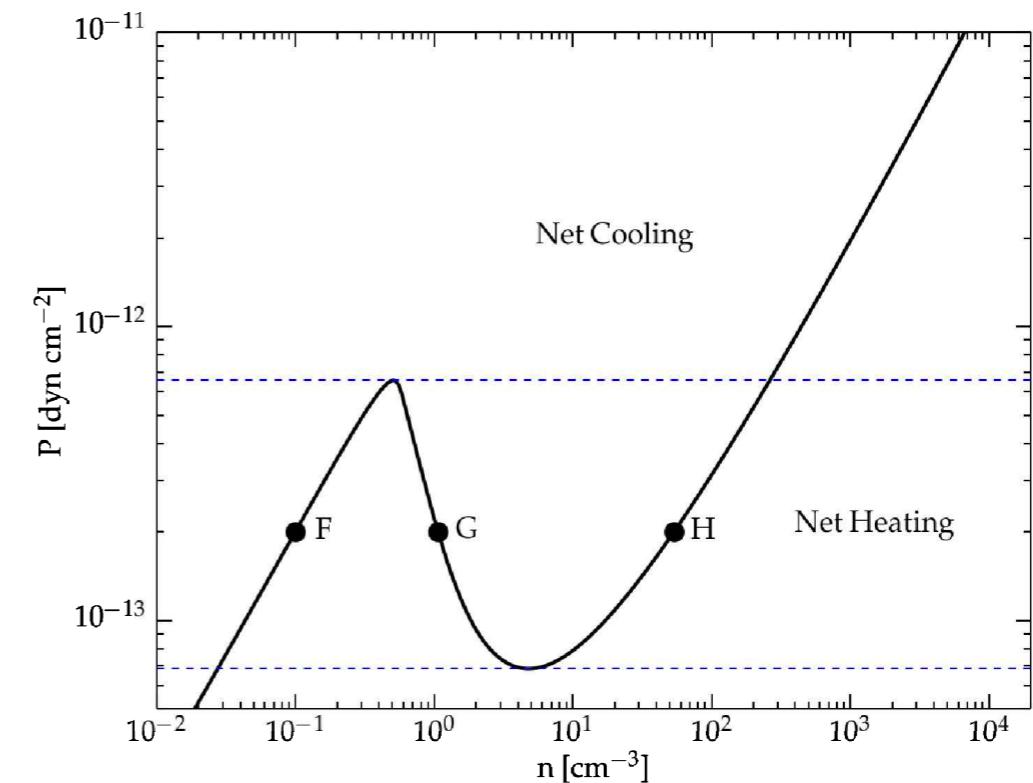
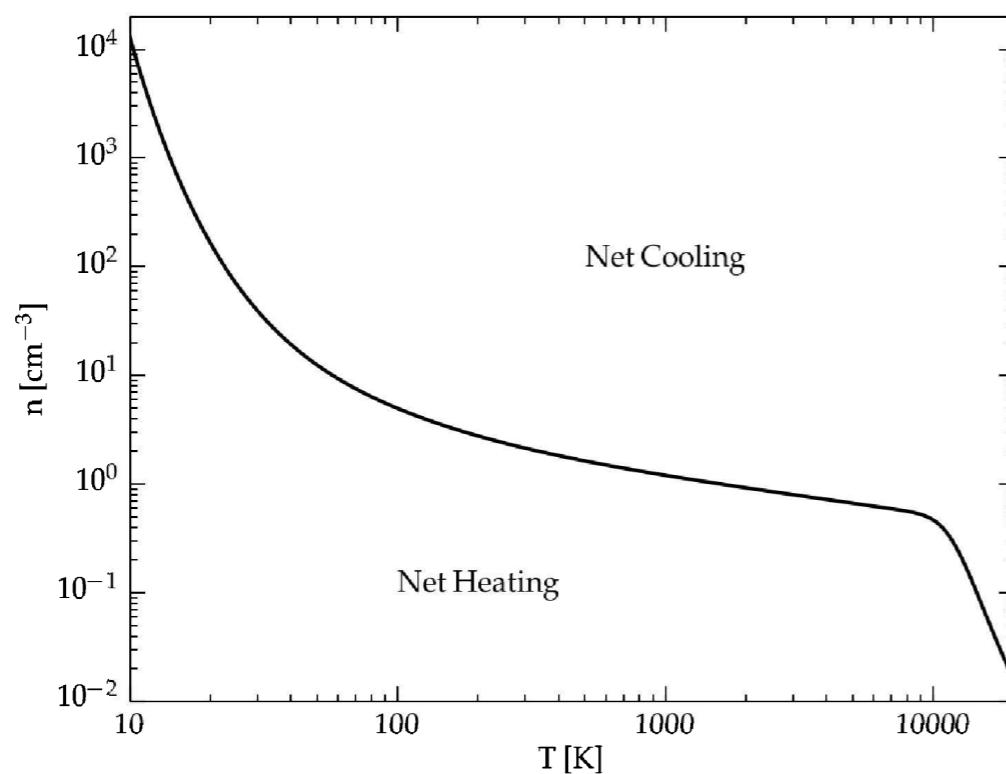
- McKee & Ostriker (1977)
  - They made a more elaborate argument for three phases within the ISM.
  - **Cold Neutral Medium**, with  $T \sim 80$  K,  $n \sim 40 \text{ cm}^{-3}$ , and a low fractional ionization  $x = n_e/n \sim 0.001$ .
  - **Warm Medium**, containing both ionized and neutral components,  $T \sim 8000$  K and  $n \sim 0.3 \text{ cm}^{-3}$ , the ionization fraction ranging from  $x \sim 0.15$  in the neutral component (WNM) to  $x \sim 0.15$  in the ionized component (WIM).
  - **Hot Ionized Medium**, consisting of the overlapping supernova bubbles, with  $T \sim 10^6$  K and  $n \sim 0.002 \text{ cm}^{-3}$ , and  $x \sim 1$  (nearly complete ionization).



- However, in many ways, the ISM is a dynamic, turbulent, dusty, magnetized place.

# Homework (assignment)

- Two stable phases
  - Use the formulae for the photoelectric heating rate by dust and the cooling rate by [CII] 158 $\mu$ m, [OI] 63.2 $\mu$ m, and Ly $\alpha$ , described in this lecture note (and textbook).
  - Use IDL, python, or whatever you can use.
    - ◆ Make the plots as shown below.



- ◆ Make a plot “P versus T,” in addition to the above plots.
- ◆ Compute the numerical values of the equilibrium **density** and **temperature** for a pressure  $P = 4 \times 10^{-13}$  dyn cm $^{-3}$ .