Interstellar Medium (ISM)

Week 5
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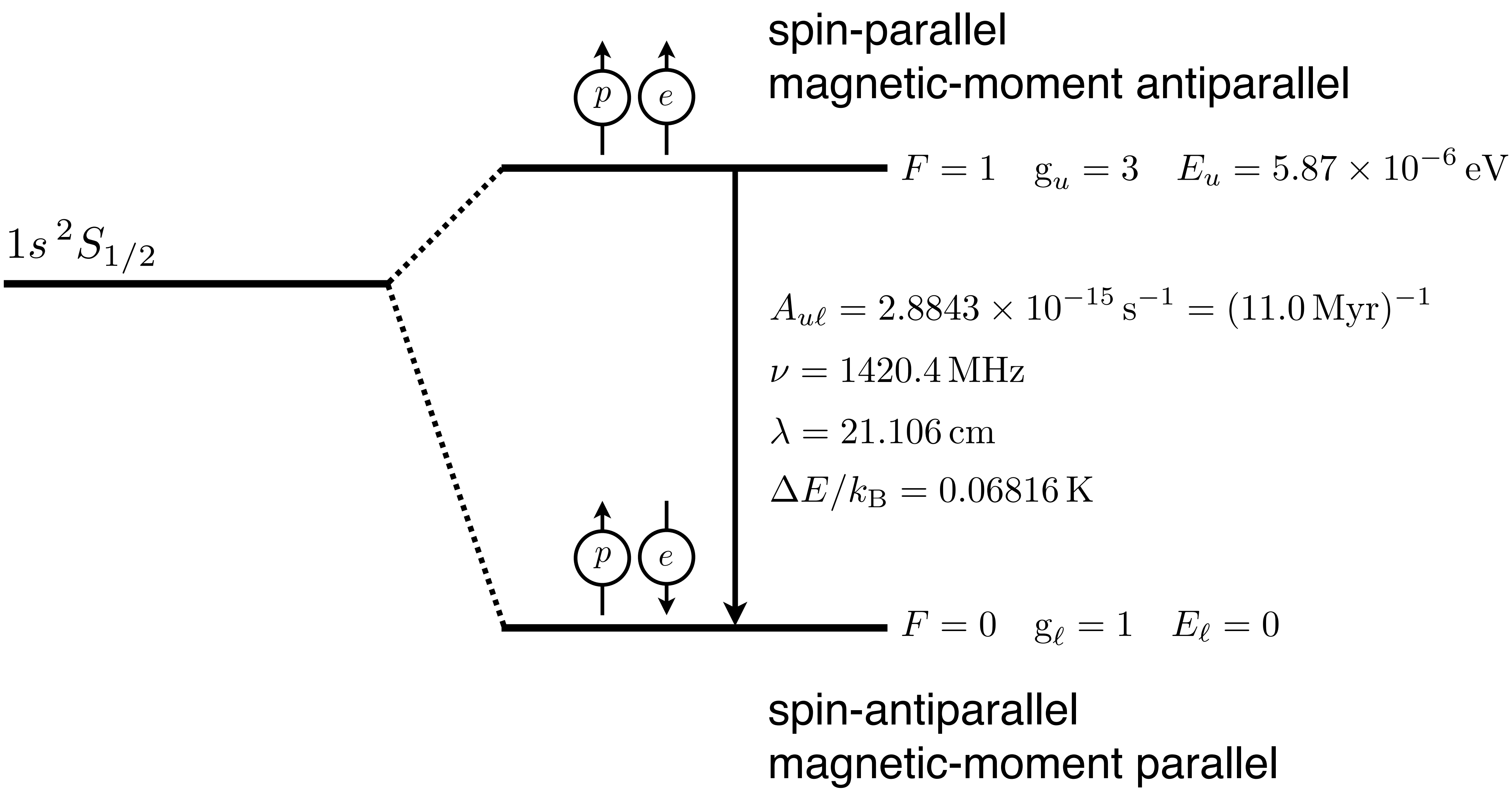
Neutral Medium 3

- H I 21 cm line
- Warm Neutral Medium

21 cm hyperfine line

- The CNM and WNM, taken together, provide over half the mass of the ISM.
  - H is the most abundant element in the universe. In the CNM and WNM, most of the hydrogen is in the form of neutral atoms.
  - The Lyα line of H provides a useful probe of the properties of the CNM and WNM. However, at its wavelength the Earth's atmosphere is highly opaque, and thus observing Lyα absorption requires orbiting UV satellites. In addition, Lyα can be seen in absorption only along those lines of sight toward sources with a high UV flux.
  - To do a global survey of atomic hydrogen in the galaxy, we need some way of easily detecting radiation from hydrogen, regardless of its kinetic temperature or number density.
  - Such a way was first found in 1944, by Henk van de Hulst. He attempted to find emission lines at the wavelengths ~ 1 cm to 20 m, at which the Earth's atmosphere is transparent. He then realized that the hyperfine structure line resulting from a flip of the electron spin within a hydrogen atom should have a wavelength of 21 cm. This was confirmed by Harold Ewen and Edward Purcell in 1951, when they first detected 21 cm emission from the Milky Way.

Hyperfine splitting of the 1s ground state of atomic H



Difference between Lyα and 21 cm transitions

- The excitation energy for Lyα (E = 10.2 eV, E/k = 118,000 K) is much higher than the kinetic temperature of the neutral ISM.

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} \exp\left(-\frac{118,000\,\mathrm{K}}{T}\right) \approx 1.7 \times 10^{-51} \text{ at } T = 1000\,\mathrm{K}$$

  - Collisional excitation is unimportant, and most hydrogen atoms are in the lower level of the Lyα transition.
  - The Lyα has a higher energy by a factor of $1.7\text{x}10^6$ than the 21 cm.

- The excitation energy for 21 cm is ~ 6μm, and its equivalent temperature E/k = 0.068 K is much lower than the temperature of the cosmic microwave background.
  - Even the CMB is able to populate the upper level.
  - Thus, there is ample opportunity to populate the upper energy level of the 21 cm hyperfine transition. In excitation equilibrium, the level populations for the 21 cm levels.

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} e^{-h\nu/kT_{\mathrm{exc}}} = 3\, e^{-0.068\,\mathrm{K}/T} \approx 3 \longrightarrow \boxed{n_u \simeq \frac{3}{4} n_{\mathrm{H}}, \; n_\ell \simeq \frac{1}{4} n_{\mathrm{H}}}$$

  - In fact, the hyperfine levels may not be in excitation equilibrium. Radio astronomers use the term ***spin temperature*** for 21 cm rather than the “excitation temperature.”

Emissivity and Optical Depth

- ***Emissivity***:
  - The upper level contains ~ 75% of the H I under all conditions of interest, and thus the 21-cm emissivity is effectively independent of the spin temperature.

$$j_\nu = n_u \frac{A_{u\ell}}{4\pi} h\nu_{u\ell}\phi_\nu \approx \frac{3}{16\pi} A_{u\ell} h\nu_{u\ell} n_{\rm H}\phi_\nu \qquad \left(n_u \approx \frac{3}{4} n_{\rm H}\right)$$

- ***Optical depth***

$$\kappa_\nu = n_\ell\sigma_{\ell u} - n_u\sigma_{u\ell} = n_\ell\sigma_{\ell u}\left(1 - e^{-h\nu_{u\ell}/kT_{\rm spin}}\right)$$

Because $h\nu_{u\ell}/kT_{\rm spin} \ll 1$ for all conditions of interest, the correction for stimulated emission is very important!

$$\kappa_\nu \approx n_\ell\sigma_{\ell u}\frac{h\nu_{u\ell}}{kT_{\rm spin}} \ll n_\ell\sigma_{\ell u} \quad \longleftarrow \quad e^{-h\nu_{u\ell}/kT_{\rm spin}} \approx 1 - k\nu_{u\ell}/kT_{\rm spin}$$

$$\begin{aligned}\kappa_\nu &\approx \left(\frac{1}{4}n_{\rm H}\right)\left(\frac{g_u}{g_\ell}\frac{c^2}{8\pi\nu_{u\ell}^2}A_\ell\phi_\nu\right)\frac{h\nu_{u\ell}}{kT_{\rm spin}} \quad \left(n_u \approx \frac{1}{4}n_{\rm H}\right)\\ &= \frac{3}{32\pi}A_{u\ell}\frac{hc\,\lambda_{u\ell}}{kT_{\rm spin}}n_{\rm H}\phi_\nu\end{aligned}$$

- The damping constant of the 21 cm line profile is extremely small, and thus we can assume that the line profile is a Gaussian.

$$a = \frac{\gamma_{u\ell}}{4\pi}\frac{\lambda_{u\ell}}{b} = 4.844\times10^{-20}\left(\frac{\gamma_{u\ell}}{2.8843\times10^{-15}\,\mathrm{s}^{-1}}\right)\left(\frac{\lambda_{u\ell}}{21.106\,\mathrm{cm}}\right)\left(\frac{1\,\mathrm{km\,s}^{-1}}{b}\right)$$

- Hence,

$$\phi_\nu = \frac{1}{\sqrt{\pi}\Delta\nu_D}H(u,a) \approx \frac{c}{\sqrt{\pi}\nu_{\ell u}b}e^{-u^2}$$

$$\left(u = v/b,\ b = \sqrt{2}v_{\rm rms} = \sqrt{2kT_{\rm gas}/m_{\rm H}}\right)$$

$$\tau_\nu = \kappa_\nu s = \frac{3}{32\pi}A_{u\ell}\frac{hc\,\lambda_{u\ell}}{kT_{\rm spin}}N_{\rm H\,I}\phi_\nu$$

$N_{\rm H\,I} \equiv \int n_{\rm H}ds$ is the column density of H I.

$\sim 10^{21}\,\mathrm{cm}^{-21}$ toward the Galactic disk.

$$= \frac{3}{32\pi}\frac{1}{\sqrt{\pi}}\frac{A_{u\ell}\lambda_{u\ell}^2}{b}\frac{hc}{kT_{\rm spin}}N_{\rm H\,I}e^{-u^2}$$

$$= 3.111\left(\frac{N_{\rm H\,I}}{10^{21}\,\mathrm{cm}^{-2}}\right)\left(\frac{100\,\mathrm{K}}{T_{\rm spin}}\right)\left(\frac{1\,\mathrm{km\,s}^{-1}}{b}\right)e^{-u^2}$$

or $$\tau_\nu = 2.201\left(\frac{N_{\rm H\,I}}{10^{21}\,\mathrm{cm}^{-2}}\right)\left(\frac{100\,\mathrm{K}}{T_{\rm spin}}\right)\left(\frac{1\,\mathrm{km\,s}^{-1}}{b/\sqrt{2}}\right)e^{-u^2}$$

Some lines of sight through our galaxy (at high galactic latitude) are optically thin and other lines of sight (at low galactic latitude) are optically thick at 21 cm.

- ***Self-absorption in the 21-cm line can be important*** in many sightlines in the ISM.
- ***The optical depth is inversely proportional to the spin temperature.***

- Typical optical depths of the 21-cm line:

$$\tau_0 = 0.311 \left(\frac{N_{\mathrm{H\,I}}}{10^{21}\,\mathrm{cm}^{-2}} \right) \left(\frac{100\,\mathrm{K}}{T_{\mathrm{spin}}} \right) \left(\frac{10\,\mathrm{km\,s}^{-1}}{b} \right)$$

- In the CNM, a typical spin temperature is $T_{\mathrm{spin}} \approx 50 - 100\,K$:

$$\tau_0^{\mathrm{CNM}} \approx 0.3 - 0.6$$
$$e^{-\tau_0} \approx 0.55 - 0.74$$

The CNM is in general optically thin, but show significant absorption.

- In the WNM, a typical spin temperature is $T_{\mathrm{spin}} \approx 5000 - 8000\,K$:

$$\tau_0^{\mathrm{WNM}} \approx 0.004 - 0.006$$
$$e^{-\tau_0} \approx 0.995$$

The 21-cm absorption is negligible in the WNM.

A typo in page 65 of Ryden's book:

cold

For thermal broadening b typical of the ~~warm~~ neutral medium, and excitation temperatures $T_{\mathrm{exc}} \sim 100\,\mathrm{K}$, lines of sight with $N_{\mathrm{HI}} > 10^{21}\,\mathrm{cm}^{-2}$ show significant absorption. (Remember that Lyman α becomes optically thick at a column

[1] Column Density Determination

- ***Radioastronomers express the line profile as a function of radial velocity rather than of frequency.*** This is logical because line broadening is only caused by the Doppler effect, its natural width being extremely narrow since the lifetime of the upper level is only limited by collisions which is rare in the diffuse medium.
- We first define the column density per velocity interval.

$$\frac{dN_{\rm HI}}{dv} = N_{\rm HI}\phi_v = N_{\rm HI}\frac{1}{\lambda_{u\ell}}\phi_\nu \qquad \phi_\nu = \phi_v\left|\frac{dv}{d\nu}\right| = \phi_v\frac{c}{\nu_{u\ell}} = \lambda_{u\ell}\phi_v$$

- The column density can be written:

$$\tau_\nu = \frac{3}{32\pi}A_{u\ell}\frac{hc\,\lambda_{u\ell}}{kT_{\rm spin}}N_{\rm H\,I}\phi_\nu \longrightarrow \tau(v) = \frac{3}{32\pi}A_{u\ell}\frac{hc\,\lambda_{u\ell}^2}{kT_{\rm spin}(v)}\frac{dN_{\rm HI}}{dv}$$

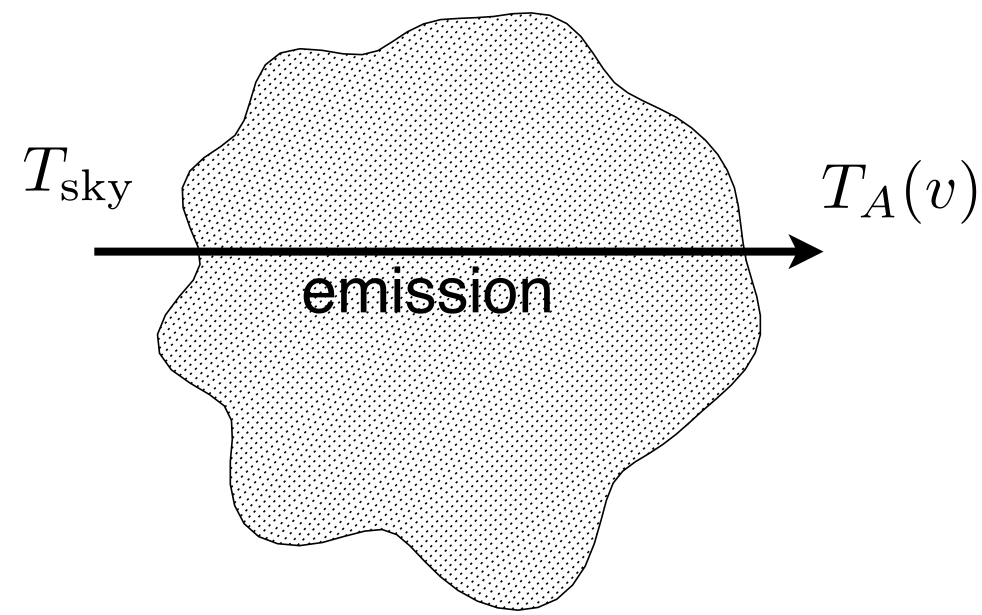
$$\begin{aligned}\frac{dN_{\rm HI}}{dv} &= \frac{32\pi}{3}\frac{k}{A_{u\ell}hc\lambda_{u\ell}^2}T_{\rm spin}(v)\tau(v) \\ &= 1.813\times10^{18}\,\frac{T_{\rm spin}(v)\tau(v)}{\rm K}\quad\left[\frac{\rm cm^{-2}}{\rm km\,s^{-1}}\right]\end{aligned} \qquad N_{\rm HI} = \int dv\,\frac{dN_{\rm HI}}{dv}$$

- This indicates that ***we need to know not only the optical depth but also the spin temperature to evaluate the column density***. However, in an optically thin limit, we will show that the dependency on the spin temperature is removed.

- ***Optically thin case:*** Suppose we are looking through an optically thin layer of neutral hydrogen toward a ***"dark sky"***, which is fainter than the hydrogen cloud, with an antenna temperature $T_{\rm sky}$.
  - In the optically thin limit, the RT equation becomes

$$\begin{aligned} T_A(v) &= T_{\rm sky} e^{-\tau_v} + T_{\rm spin}(v)\left(1 - e^{-\tau_v}\right) \\ &= T_{\rm sky} + \left(T_{\rm spin}(v) - T_{\rm sky}\right)\left(1 - e^{-\tau_v}\right) \\ &\approx T_{\rm sky} + T_{\rm spin}(v)\tau_v \quad \leftarrow \tau_\nu \ll 1, \;\; T_{\rm sky} \ll T_{\rm spin}(v) \end{aligned}$$

$$\tau(v) \approx \frac{T_A(v) - T_{\rm sky}}{T_{\rm spin}(v)}$$



  - The column density per unit velocity interval is

$$\begin{aligned} \frac{dN_{\rm HI}}{dv} &\approx \frac{32\pi}{3} \frac{k}{A_{u\ell} h c \lambda_{u\ell}^2} \left[T_A(v) - T_{\rm sky}\right] \\ &= 1.813 \times 10^{18} \frac{T_A(v) - T_{\rm sky}}{\rm K} \left[\frac{\rm cm^{-2}}{\rm km\, s^{-1}}\right] \end{aligned}$$

$$N_{\rm HI} = \int dv \frac{dN_{\rm HI}}{dv}$$

We measure the antenna temperature of the dark sky from the continuum at frequencies well above and below the 21-cm emission feature.

  - Therefore, the intensity integrated over the line profile gives us the total H I column density without need to know $T_{\rm spin}$, provided that self-absorption is not important.

- ***Alternative approach:***
  - If we now neglect absorption, then

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + j_\nu \approx j_\nu \longrightarrow I_\nu = I_\nu(0) + \int j_\nu ds = I_\nu(0) + \frac{3}{16\pi} A_{u\ell} h\nu_{u\ell} \phi_\nu N_{\rm HI}$$

  - Now suppose that $I_\nu(0)$ is known independently. We can then integrate the intensity over the line

$$\int [I_\nu - I_\nu(0)]\, d\nu = \frac{3}{16\pi} A_{u\ell} h\nu_{u\ell} N_{\rm HI}$$

  - This can be expressed in terms of antenna temperature T_A and relative velocity $v = [(\nu - \nu_{u\ell})/\nu_{u\ell}]\, c$

$$\begin{aligned} \int [T_A - T_A(0)]\, dv &= \int \frac{c^2}{2k\nu^2} [I_\nu - I_\nu(0)] \frac{c}{\nu_{u\ell}} d\nu \\ &\approx \frac{c^3}{2k\nu_{u\ell}^3} \frac{3}{16\pi} A_{u\ell} h\nu_{u\ell} N_{\rm HI} \\ &= C_0^{-1} N_{\rm HI} \end{aligned}$$

$$\begin{aligned} C_0 &\equiv \frac{32\pi}{3} \frac{k}{hc\lambda_{u\ell}^2 A_{u\ell}} \\ &= 1.813 \times 10^{18} \left[\frac{\rm cm^{-2}}{\rm K\, km\, s^{-1}}\right] \end{aligned}$$

$$C_0^{-1} = 5.516 \times 10^{-19} \left[\frac{\rm K\, km\, s^{-1}}{\rm cm^{-2}}\right]$$

- We, then, obtain the same equation as before:

$$
\begin{aligned}
N_{\mathrm{HI}} &\approx C_0 \int \left[T_A - T_A(0)\right] dv \\
&= 1.813 \times 10^{18} \int \frac{T_A - T_A(0)}{\mathrm{K\,km\,s^{-1}}} dv \quad \left[\mathrm{cm^{-2}}\right]
\end{aligned}
$$

- Here, we did not use the relation between the optical depth and column density.
  - In the first method, we assumed that $\tau_\nu \ll 1$ and $I_\nu(0) \ll S_\nu$:

$$
\begin{aligned}
I_\nu &= I_\nu(0) e^{-\tau_\nu} + S_\nu \left(1 - e^{-\tau_\nu}\right) \\
&= I_\nu(0) + \left[S_\nu - I_\nu(0)\right]\left(1 - e^{-\tau_\nu}\right) \\
&\approx I_\nu(0) + S_\nu \tau_\nu
\end{aligned}
$$

  - In the second method, we completely ignored the absorption.

$$
\begin{aligned}
\frac{dI_\nu}{ds} &= -\kappa_\nu I_\nu + j_\nu \\
&\approx j_\nu
\end{aligned}
$$

This may be a zeroth order approximation.

[2] Spin Temperature Determination

- To derived the spin temperature, we need to combine the emission observation with an absorption observation, which is so called “emission-absorption” method.

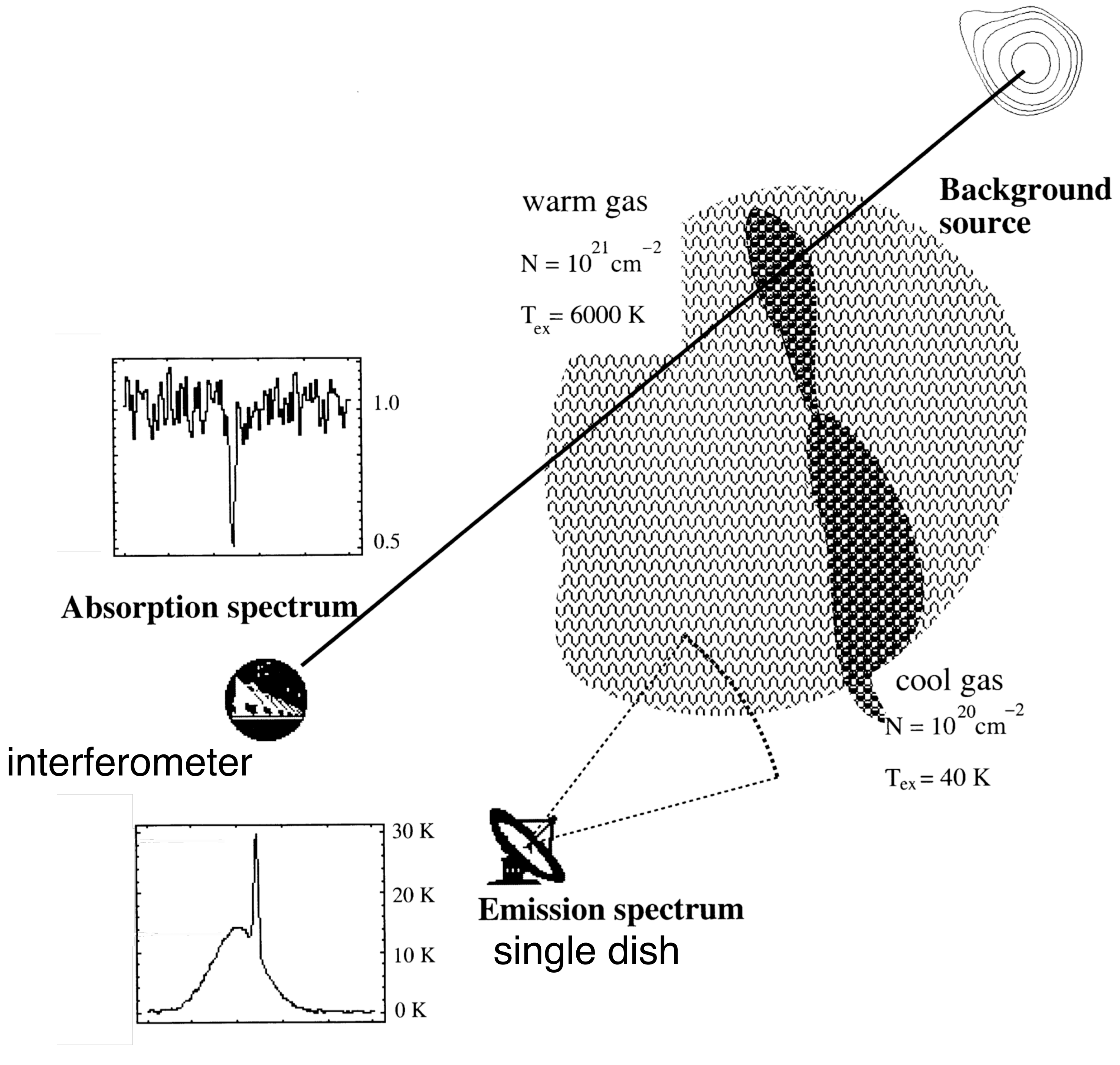
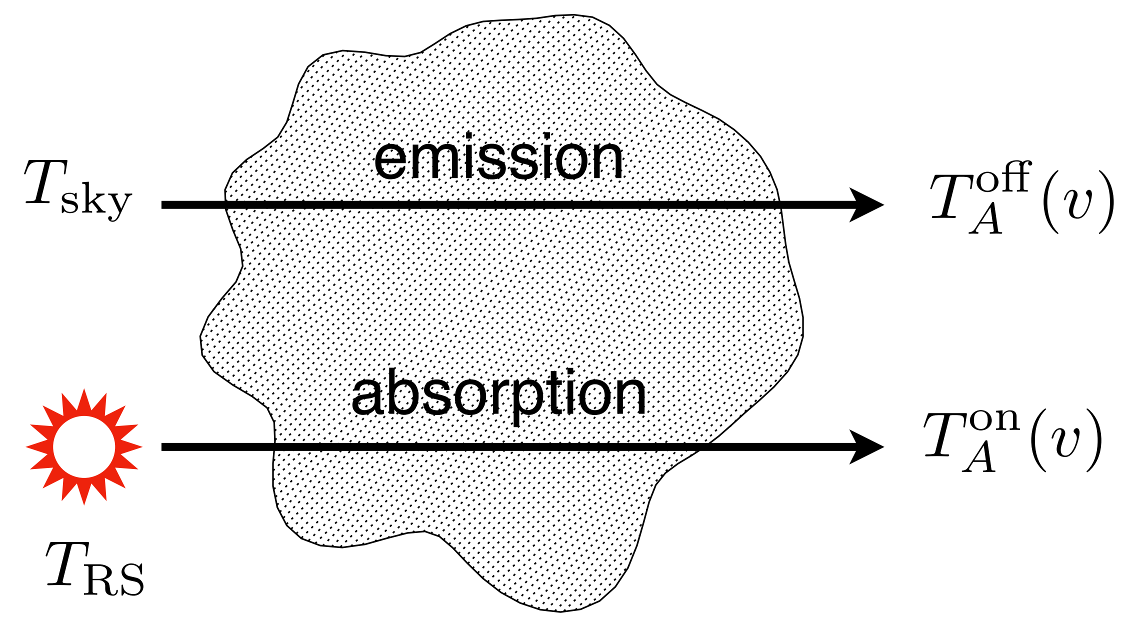


Figure 15 in Dickey et al. (2000, ApJ)

- In cases where we have a ***"bright" background radio source*** with a continuum spectrum (a typical radio-loud quasar or an active galactic nucleus, or a radio galaxy), we can study both emission and absorption by the foreground ISM in our galaxy by comparing ***"on-source" and "off-source" observations***.

- The spectra measured on the blank sky and on the radio source are, respectively,

$$T_A^{\rm on}(v) = T_{\rm RS}e^{-\tau_v} + T_{\rm spin}(v)\left(1 - e^{-\tau_v}\right)$$
$$T_A^{\rm off}(v) = T_{\rm sky}e^{-\tau_v} + T_{\rm spin}(v)\left(1 - e^{-\tau_v}\right)$$



- These two equation can be solved for the two unknowns, $\tau(v)$ and $T_{\rm spin}(v)$.

$$\tau(v) = \ln\left[\frac{T_{\rm RS} - T_{\rm sky}}{T_A^{\rm on}(v) - T_A^{\rm off}(v)}\right]$$

$$T_{\rm spin}(v) = \frac{T_A^{\rm off}(v)T_{\rm RS} - T_A^{\rm on}(v)T_{\rm sky}}{(T_{\rm RS} - T_{\rm sky}) - \left(T_A^{\rm on}(v) - T_A^{\rm off}(v)\right)}$$

The solution gives, in general, the spin temperature as a function of velocity.

- We can also derive the column density from these two quantities for an optically thick cloud.

- We usually consider a case where the radio source is "much" brighter than the spin temperature of the intervening hydrogen cloud.
  - The RT equations for the "on-source" and "off-source" measurements can be written:

$$\text{assumptions}: T_{\text{RS}} \gg T_{\text{spin}}$$

$$\begin{aligned}
(1)\quad & T_A^{\text{on}}(v) = T_{\text{RS}} e^{-\tau_v} + T_{\text{spin}}(v)\left(1 - e^{-\tau_v}\right) \\
(2)\quad & T_A^{\text{off}}(v) = T_{\text{sky}} e^{-\tau_v} + T_{\text{spin}}(v)\left(1 - e^{-\tau_v}\right)
\end{aligned}
\longrightarrow
\begin{aligned}
& T_A^{\text{on}}(v) \approx T_{\text{RS}}\, e^{-\tau_v} \\
& T_A^{\text{off}}(v) = T_{\text{sky}} + (T_{\text{spin}} - T_{\text{sky}})\left(1 - e^{-\tau_v}\right)
\end{aligned}$$

$$(1)\quad \frac{T_{\text{RS}} - T_A^{\text{on}}(v)}{T_{\text{RS}}} \approx 1 - e^{-\tau_v}$$

$$(2)\quad \Delta T_A^{\text{off}}(v) = \Delta T_{\text{spin}}(v)\left(1 - e^{-\tau_v}\right)$$

Here, $\Delta T_A^{\text{off}}(v) \equiv T_A^{\text{off}}(v) - T_{\text{sky}} \approx T_A^{\text{off}}$

$\Delta T_{\text{spin}}(v) \equiv T_{\text{spin}}(v) - T_{\text{sky}} \approx T_{\text{spin}}$

$(T_{\text{sky}} \approx 3\,\text{K})$

- ***Equivalent Width:***
  - Using the absorption spectrum from the "on-source" observation, we can "approximately" obtain the "velocity equivalent width."

$$W_v = \int dv \left(1 - e^{-\tau_v}\right)$$
$$\approx \int dv \left[\frac{T_{\rm RS} - T_A^{\rm on}(v)}{T_{\rm RS}}\right]$$

$$\text{Note}: W_v = c\int \frac{d\nu}{\nu_{u\ell}}\left(1 - e^{-\tau_\nu}\right) = cW$$

- ***Spin Temperature:***
  - Combining the two equations (1) and (2), we can obtain two spin temperatures. The first one is the line-of-sight average spin temperature, and the second the spin temperature in a velocity channel.

$$\langle \Delta T_{\rm spin}\rangle \approx \frac{\int \Delta T_A^{\rm off}(v)dv}{\int \left(1 - e^{-\tau_v}\right)dv}$$

$$\Delta T_{\rm spin}(v) = \frac{\Delta T_A^{\rm off}(v)}{\left(1 - e^{-\tau_v}\right)}$$

$\longleftarrow$ assuming $\Delta T_{\rm spin} = \ \text{constant.}$

$$\int dv \left[\Delta T_A^{\rm off}(v)\right] = \Delta T_{\rm spin}\int dv \left(1 - e^{-\tau_v}\right)$$

- ***In an optically thin limit,***
  - We know the relation between the antenna temperature and column density:

$$N_{\mathrm{HI}} \approx C_0 \int \Delta T_A^{\mathrm{off}}(v) dv \qquad \frac{dN_{\mathrm{HI}}}{dv} \approx C_0 \Delta T_A^{\mathrm{off}}(v)$$

  - Then, we can express the spin temperature in terms of column density and equivalent width (absorption profile):

$$\langle \Delta T_{\mathrm{spin}} \rangle = \frac{1}{W_v} \int \Delta T_A^{\mathrm{off}}(v) dv = \frac{C_0^{-1}}{W_v} N_{\mathrm{HI}}$$

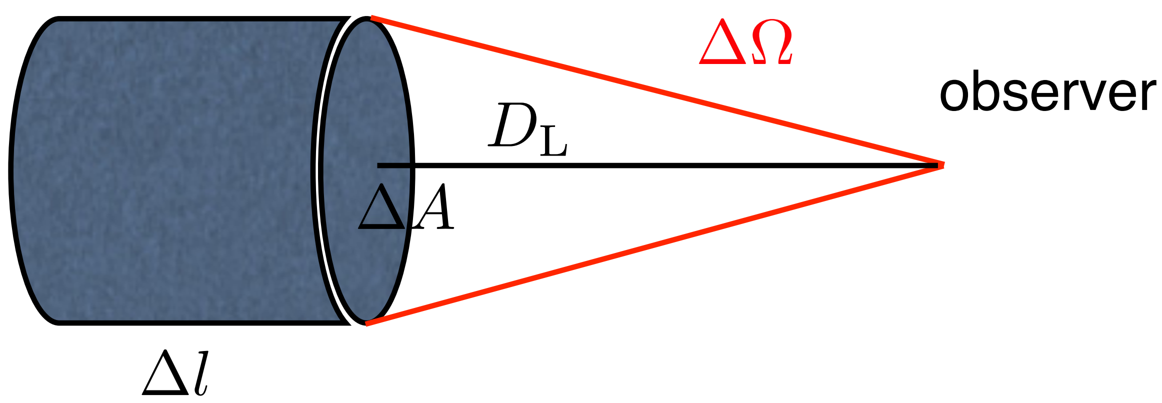
$$\langle \Delta T_{\mathrm{spin}} \rangle \approx 0.5516 \, \frac{N_{\mathrm{HI}}/10^{18}\,\mathrm{cm}^{-2}}{W_v/\mathrm{km\,s}^{-1}} \; [\mathrm{K}]$$

$$\Delta T_{\mathrm{spin}}(v) = \frac{C_0^{-1}}{(1 - e^{-\tau_v})} \frac{dN_{\mathrm{HI}}}{dv}$$

$$C_0^{-1} = 5.516 \times 10^{-19} \; \left[\frac{\mathrm{K\,km\,s}^{-1}}{\mathrm{cm}^{-2}} \right]$$

H I mass of an External Galaxy

- With the assumption that the emitting regions are optically thin, the total mass $M_{\rm HI}$ of H I in an external galaxy can be determined from the observed flux in the 21-cm line:



$F_\nu =$ observed flux density

$$F_{\rm obs} = \int F_\nu d\nu_{\rm obs} = I\Delta\Omega$$

$$I = \int I_\nu d\nu_{\rm obs} = \frac{3}{16\pi} A_{u\ell} h\nu_{u\ell} N_{\rm HI}$$

- Here, $D_{\rm L}$ is the luminosity distance to the galaxy.

$$n_{\rm H}\Delta l = N_{\rm HI}$$

$$\Delta A = D_{\rm L}^2 \Delta\Omega = D_{\rm L}^2 \frac{F_{\rm obs}}{I}$$

$$M_{\rm HI} = m_{\rm H} n_{\rm H} \Delta V = m_{\rm H} n_{\rm H} \Delta l \Delta A$$

$$= m_{\rm H} N_{\rm HI} D_{\rm L}^2 \frac{F_{\rm obs}}{I}$$

$$\therefore\ M_{\rm HI} = \frac{16\pi}{3} \frac{m_{\rm H}}{A_{u\ell} h\nu_{u\ell}} D_{\rm L}^2 F_{\rm obs}$$

$$= 4.945 \times 10^7 M_\odot \left(\frac{D_{\rm L}}{\rm Mpc}\right)^2 \left(\frac{F_{\rm obs}}{\rm Jy\,MHz}\right)$$

- If the redshift of the galaxy is z :

$$\nu_{\rm obs} = \nu/(1+z)$$
$$d\nu_{\rm obs} = \frac{\nu_{u\ell}}{(1+z)}\frac{dv}{c}$$

$$\begin{aligned}
M_{\rm HI} &= \frac{16\pi}{3}\frac{m_{\rm H}}{A_{u\ell}h\nu_{u\ell}}D_{\rm L}^2\int F_\nu d\nu_{\rm obs} \\
&= \frac{16\pi}{3}\frac{m_{\rm H}}{A_{u\ell}h\nu_{u\ell}}D_{\rm L}^2\frac{\nu_{u\ell}}{c}\frac{1}{1+z}\int F_\nu dv \\
&= \frac{16\pi m_{\rm H}}{3A_{u\ell}hc}D_{\rm L}^2(1+z)^{-1}\int F_\nu dv \\
&= 2.343\times 10^5\, M_\odot (1+z)^{-1}\left(\frac{D_{\rm L}}{\rm Mpc}\right)^2\frac{\int F_\nu dv}{\rm Jy\,km\,s^{-1}}
\end{aligned}$$

- Radio astronomers often report the integrated flux in “Jy km s^{-1}.”

Observations: Example 1

- All-sky map of H I 21-cm line intensity from the LAB survey (Kalberla et al. 2005), with angular resolution ~ 0.6 deg.
  - Scale gives $\log_{10}$ N(HI) [cm^{-2}]. The LMC and SMC are visible, with a connecting H I “bridge”.
  - The map was obtained by assuming the optically thin case.

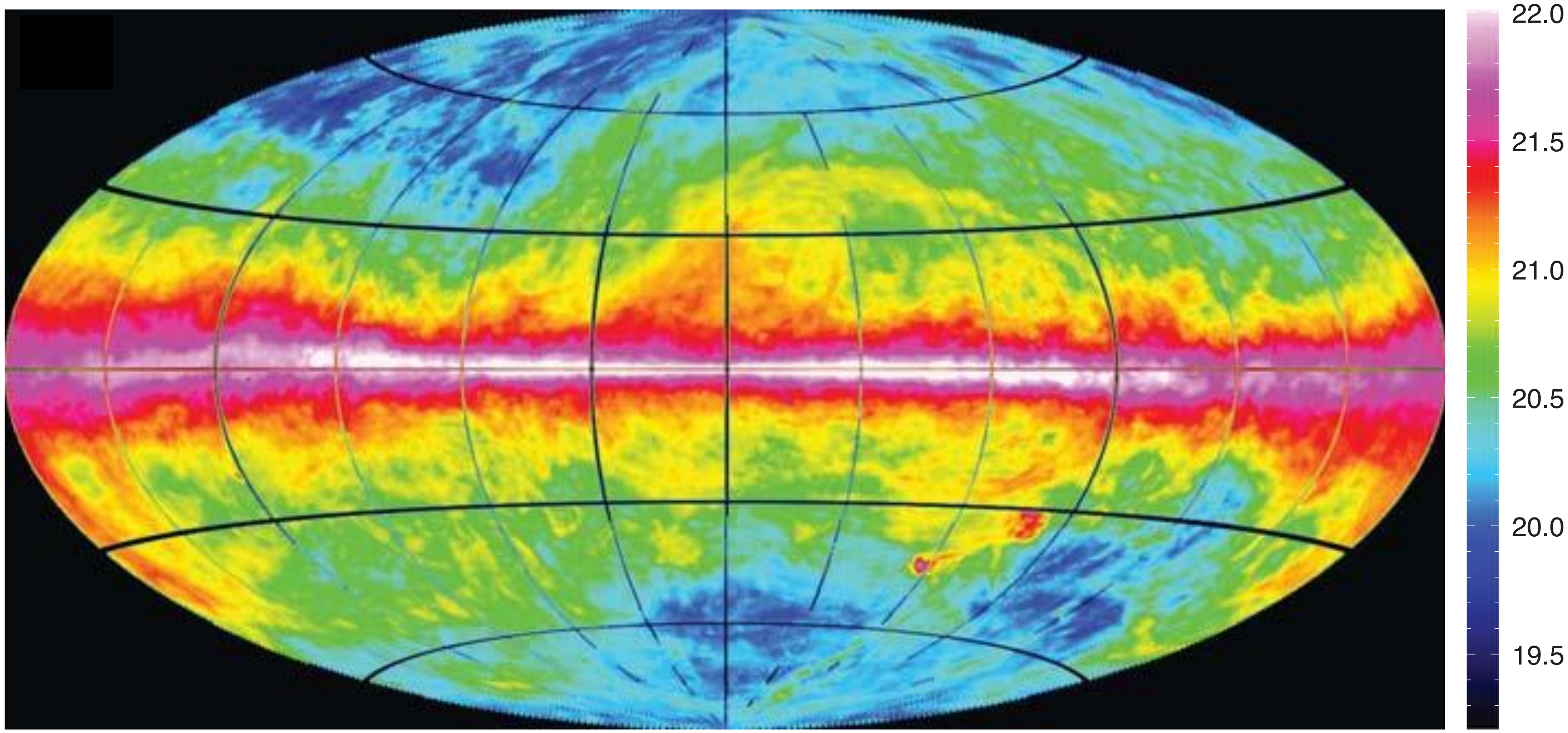
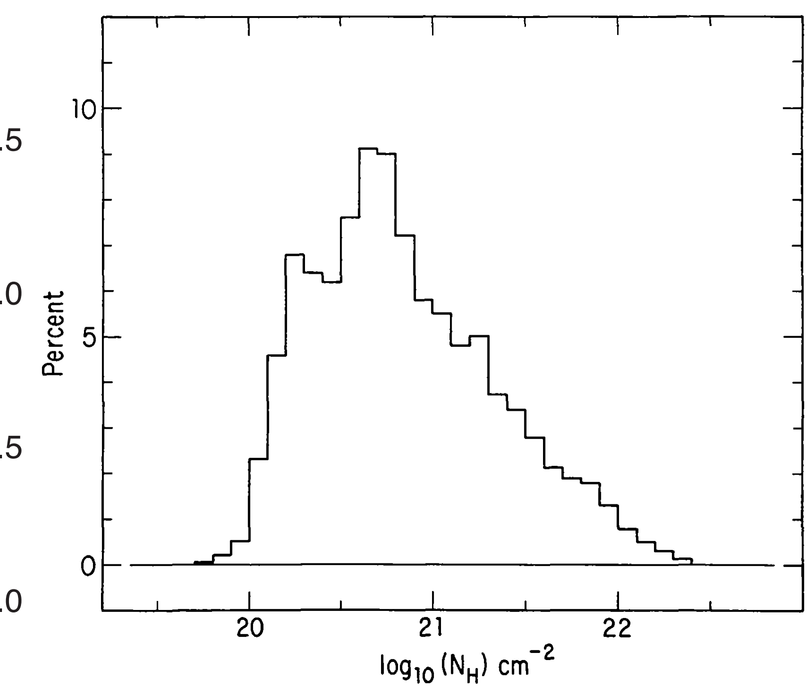


Plate 3 in [Draine]



The percentage of the sky covered by H I at a given N_H.

Figure 4 in Dickey & Lockman (1990, ARA&A)

Observations: Example 2

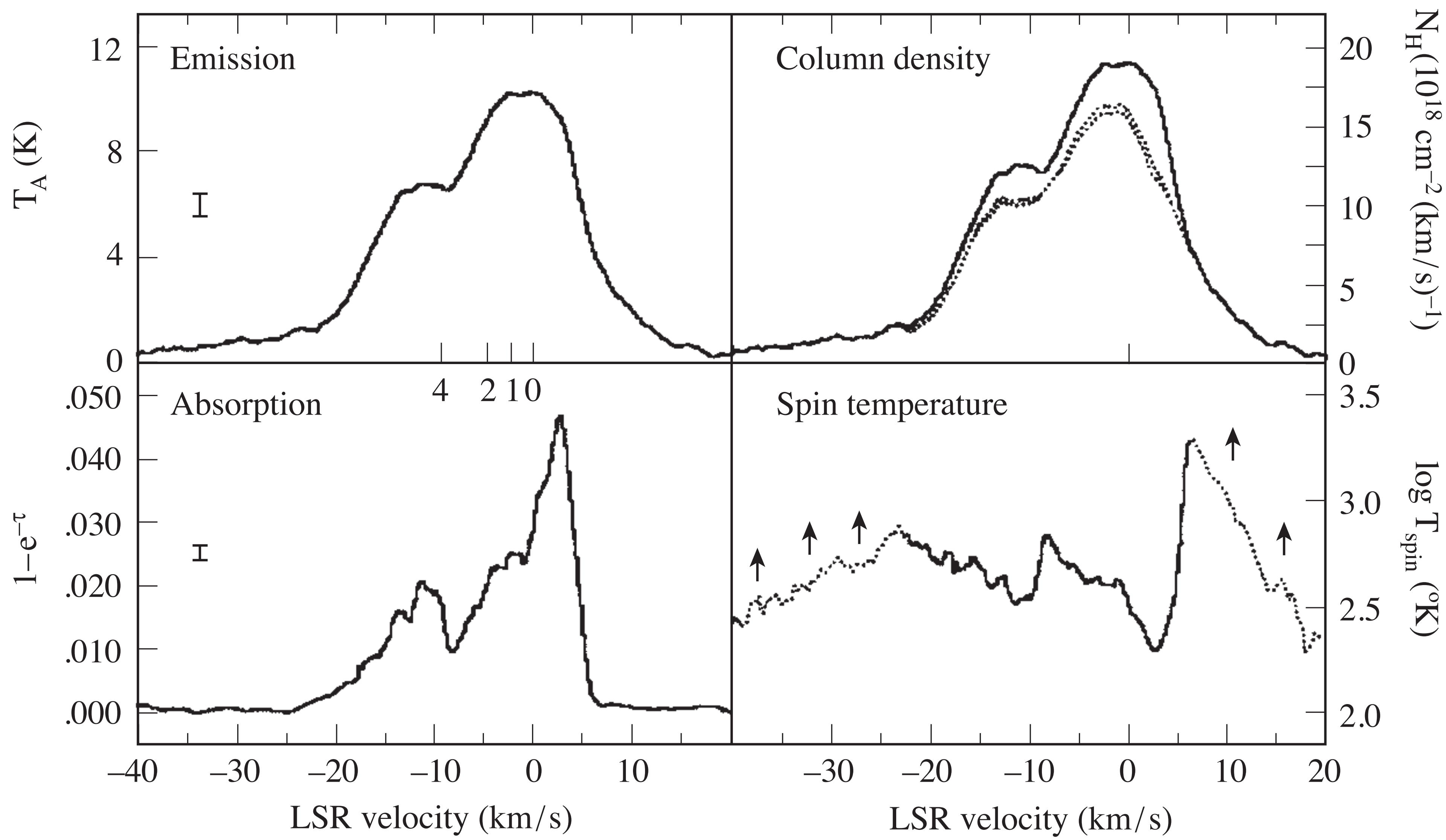
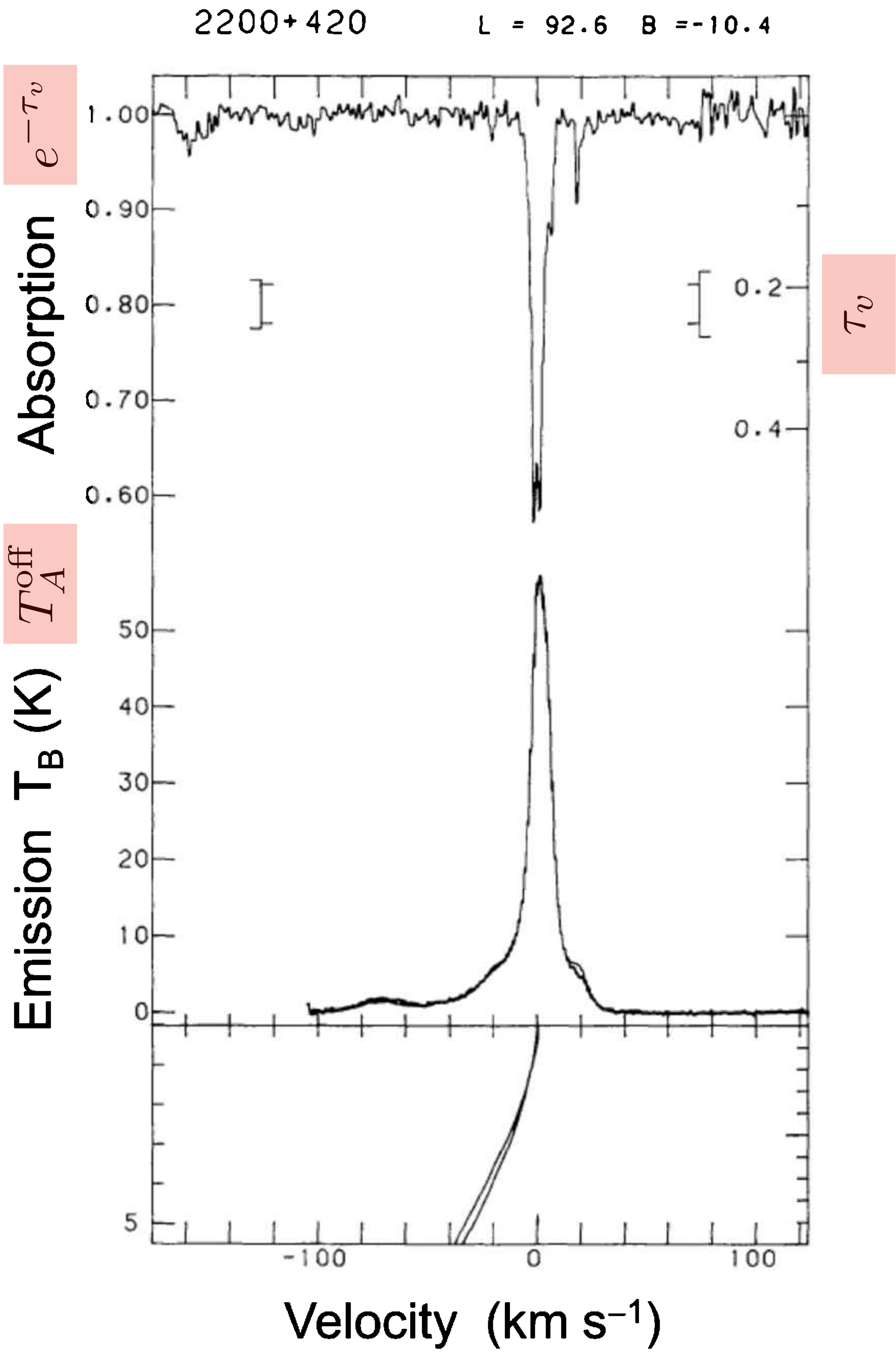


Figure 29.1 Left panels: Observed H I emission (off the quasar 3C48) and absorption (toward 3C48, at $\ell = 134°$, $b = -28.7°$). Lower right: spin temperature $T_{\rm spin}(v)$ as a function of LSR velocity. Tick marks labeled 0, 1, 2, and 4 on abscissa of left panels show the LSR velocity expected for gas at a distance of 0,1,2,4 kpc (for an assumed Galactic rotation curve). Upper right: $dN(\mathrm{H\,I})/dv$ for different assumptions regarding the relative (foreground/background) locations of cold absorbing gas and warm gas seen only in emission. From Dickey et al. (1978).

[Figure 29.1 in Draine]

Observations: Example 3



H I 21-cm absorption and emission along the line of sight towards BL Lacertae
[Dickey et al. 1983; Figure 3.3 in Ryden]

- maximum optical depth :

$$\tau_v \sim 0.5$$

- equivalent width of the absorption line :

$$W_v = 7\,\mathrm{km\,s^{-1}}$$

- integrated line intensity of the emission line :

$$\int \Delta T_A^{\rm off}(v)dv \approx 930\,\mathrm{K\,km\,s^{-1}}$$

- column density from the emission line :

$$N_{\rm HI} \approx 1.69 \times 10^{21}\,\mathrm{cm^{-2}}$$

- Now, the spin temperature is

$$T_{\rm spin} = \frac{\int \Delta T_A^{\rm off}(v)dv}{W_v} = \frac{930\,\mathrm{K,\ km\,s^{-1}}}{7\,\mathrm{km\,s^{-1}}}$$
$$\approx 133\,\mathrm{K}.$$

Observations: Cold Neutral Medium & Warm Neutral Medium

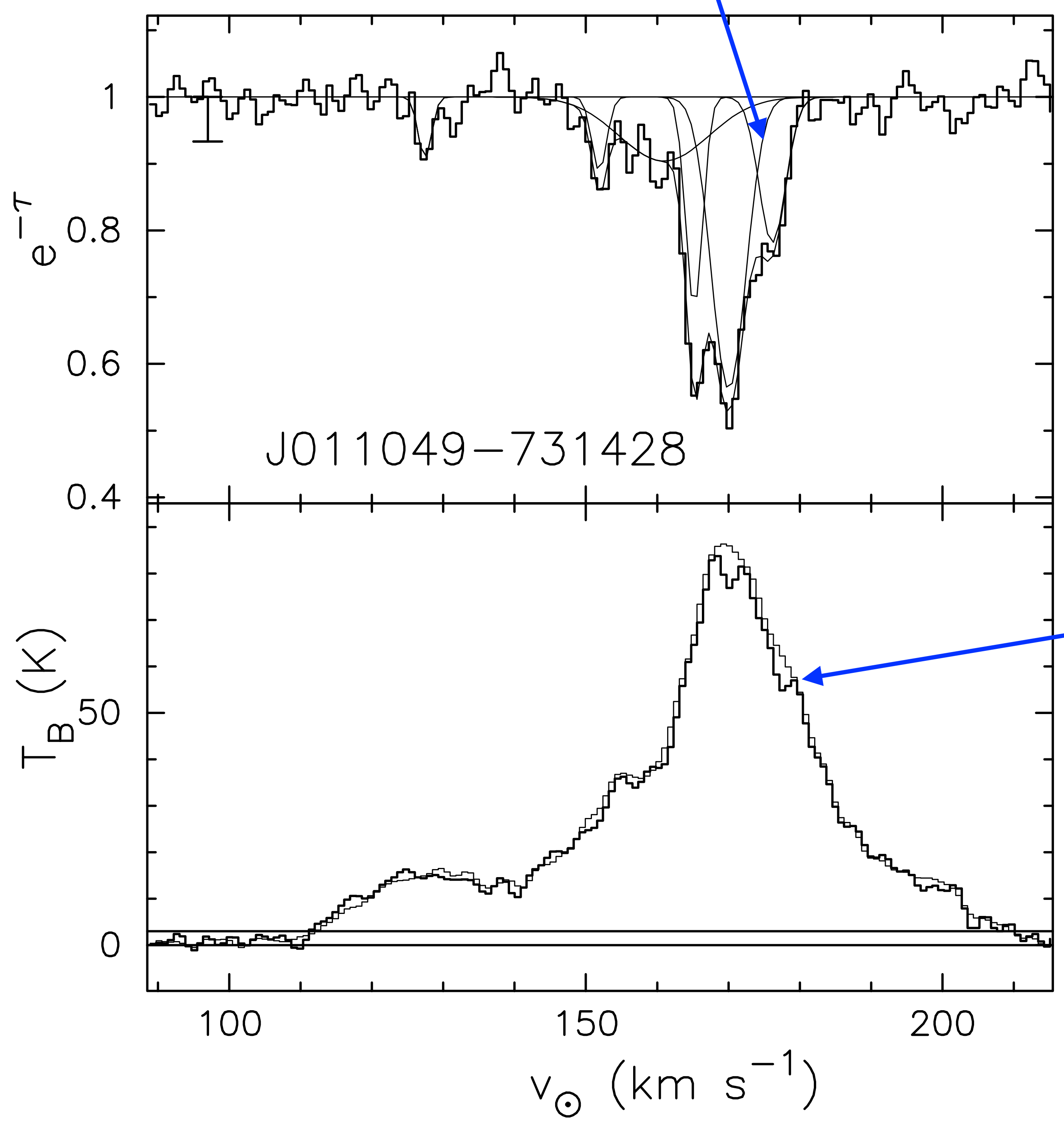
- The overall shape of the emission line profiles in our Galaxy is mainly determined by the large-scale distribution and kinematics of H I.

- Absorption lines are always narrower than emission lines.
  - Some velocities have H I emission but no detectable H I absorption.
  - Unlike emission spectra, which do not look like the superposition of a few Gaussians, absorption spectra can usually be decomposed into Gaussian components.
    - This is because the absorption lines arise only in regions of cool gas, which are more distinct along the line of sight, and which have narrower intrinsic line widths, than the gas that contributes to H I emission.
  - The difference between emission and absorption results mainly from variation in the spin temperature of the H I along the line of sight.
  - Recall that the optical depth is inversely proportional to the spin temperature, indicating the difficulties in observing absorption spectra from warm gas, which has a temperature larger than 1000 K.

$$\tau(v) = \frac{C_0^{-1}}{T_{\rm spin}(v)} \frac{dN_{\rm HI}}{dv}$$

- Absorption and Emission spectra

Thin lines: sum of Gaussians



Absorption lines are mostly, if not all, caused by the CNM.

$$\tau(v) = \frac{C_0^{-1}}{T_{\rm spin}(v)} \frac{dN_{\rm HI}}{dv}$$

Emission lines are composed of emissions from the CNM and WNM.

Thin and Thick lines: two different methods to estimate the emission spectrum

(top) Absorption and (bottom) emission spectra in a direction of Small Magellanic Cloud.

Figure 11 of Dickey et al. (2000)

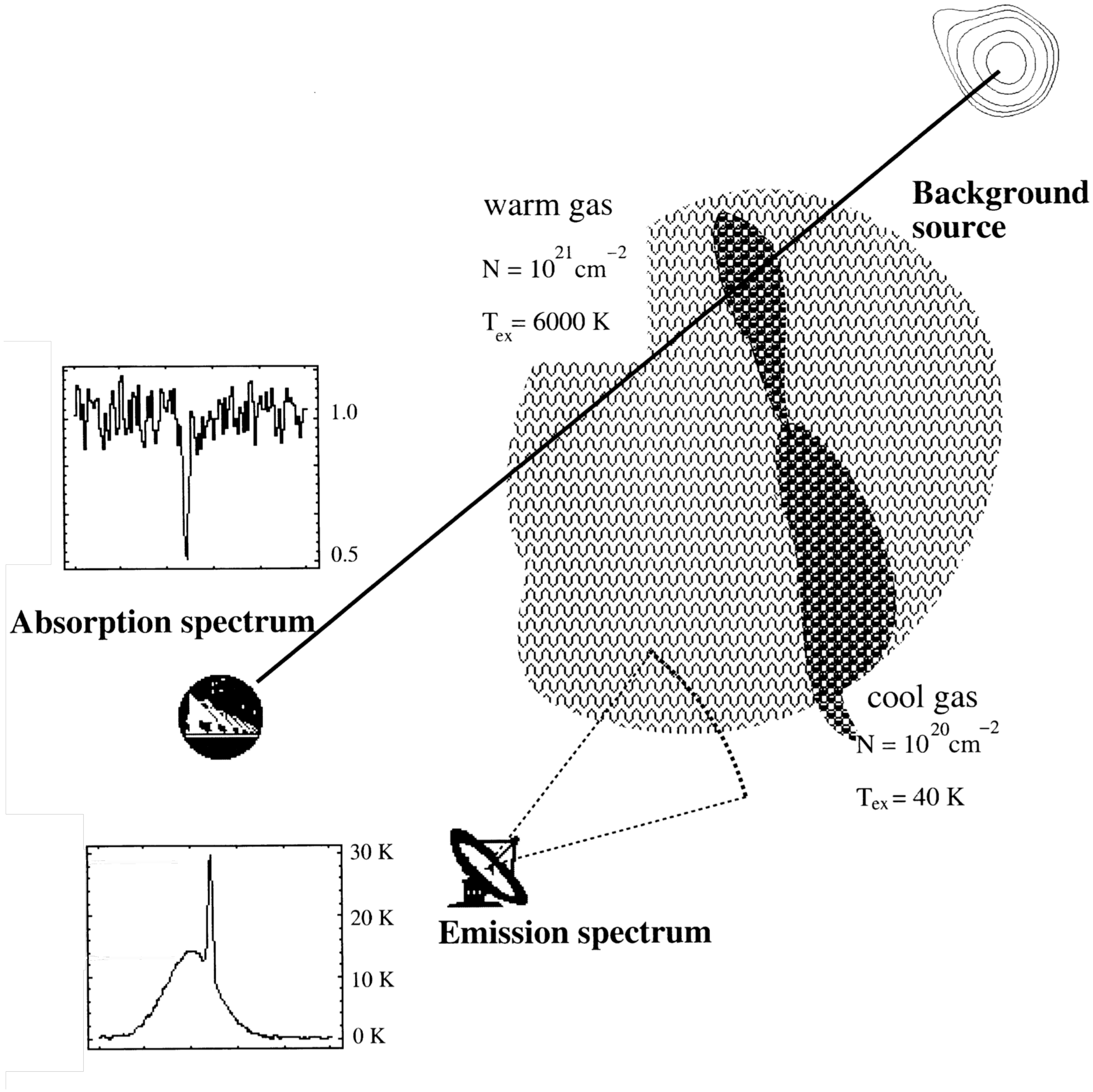
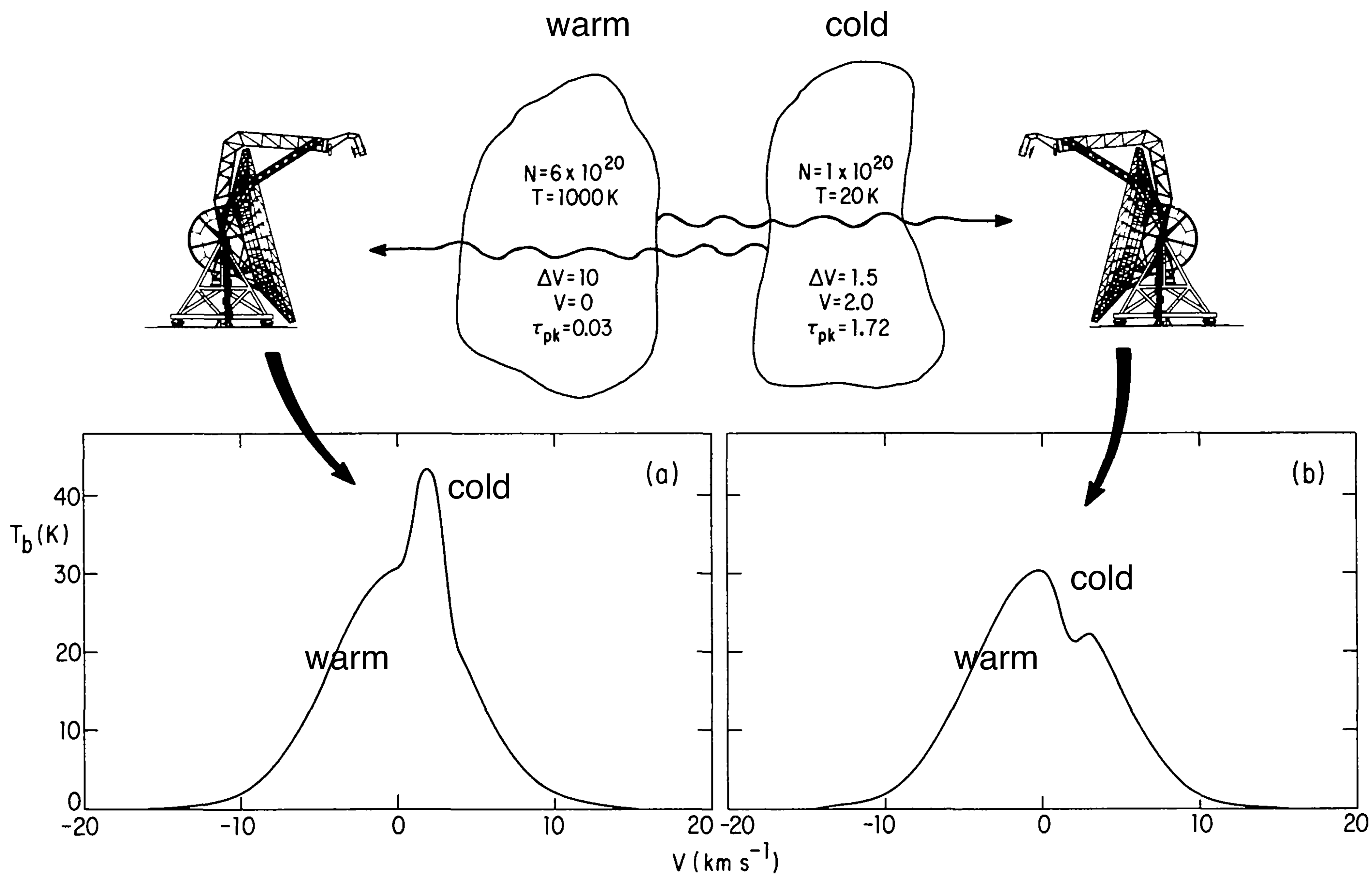


Figure 15 in Dickey et al. (2000, ApJ)



Schematic of the geometry of 21-cm self-absorption.
The structure of an emission profile depends on the relative location of warm and cold clouds as viewed by the observer.

Figure 1 of Dickey & Lockman (1990)

A rough estimation of the fraction of gas in the cool phase

- The interstellar atomic hydrogen is in at least two thermal phases.
  - We assume that the warm gas temperature is large enough that no absorption is seen from the warm phase.
  - We further assume a value for the cold-phase temperature, for instance, $T_c \approx 55\,\mathrm{K}$.
  - Then, the fraction of gas in the cold phase is

$$f_c \equiv \frac{N_c}{N_w + N_c} \approx \frac{T_c}{\langle T_{\rm spin} \rangle}$$

recall $\langle \Delta T_{\rm spin} \rangle = \frac{C_0^{-1}}{W_v} N_{\rm HI}$

  - This gives us a rough estimation of the cold-phase H I fraction for galaxies.

Cool-Phase H I Fractions for Galaxies

Galaxy	Sample Size	$\langle T_s \rangle$ (K)	f_c (T_c = 55 K)
SMC	28	440	0.13
LMC	49	170	0.33
M31	16	150	0.37
M33	7	370	0.15
Milky Way......	19	250	0.22

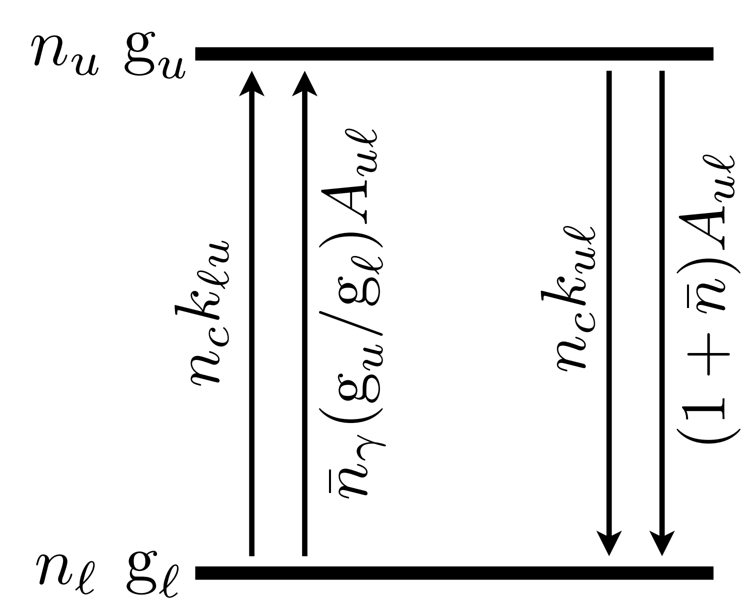
Table 3 of Dickey et al. (2000)

Observations: CNM + WNM in our Galaxy

- The most recent emission-absorption surveys (Heiles & Troland 2003) support the idea that, in the solar neighborhood (i.e., within ~ 500 pc of the Sun), interstellar H I is found primarily in two distinct phases: the CNM and the WNM.
  - About 40% of the H I (by mass) is in the CNM, with a median spin temperature T ~ 70 K. The remaining 60% of the H I is in the WNM phase, which appears to have a volume filling factor ~ 50% near the disk midplane.
  - Because warm H I absorbs very weakly, for some of the WNM material it is only possible to determine a lower bound on T_{spin}. Heiles & Troland (2003) conclude that > 48% of the WNM has 500 < T_{spin} < 5000 K, at these temperatures the gas is expected to be thermally unstable.
- Murray et al. (2014) detected a widespread warm neutral medium component with excitation temperature $\langle T_{\rm spin} \rangle = 7000^{+1800}_{1200}\,{\rm K}$.
  - This temperature lies above theoretical predictions based on collisional excitation alone, implying that Lyα scattering, the most probable additional source of excitation, is more important in the ISM than previously assumed.
- Murray et al. (2018) found that the WNM makes up 52% of the total H I mass.
  - Following spectral modeling, they detect a stacked residual absorption feature corresponding to WNM with $T_{\rm spin} \sim 10^4\,{\rm K}$.

What determines the 21-cm spin temperature? (Level Population)

- In some cases, it is sufficient to consider only the ground state and the first excited state.
  - Consider collisional excitation and de-excitation by some species (e.g., electrons) with density n_c, and suppose that radiation with the energy density u_ν .
  - The population of the excited state must satisfy:



$$\frac{dn_u}{dt} = n_\ell \left[n_c k_{\ell u} + \bar{n}_\gamma \frac{g_u}{g_\ell} A_{u\ell} \right] - n_u \left[n_c k_{u\ell} + (1 + \bar{n}_\gamma) A_{u\ell} \right]$$

photon occupation number

$$\left(\bar{n}_\gamma \equiv \frac{c^3}{8\pi h \nu^3} u_\nu \right)$$

  - The steady-state solution with radiation and collision present is

$$\frac{n_u}{n_\ell} = \frac{n_c k_{\ell u} + \bar{n}_\gamma (g_u/g_\ell) A_{u\ell}}{n_c k_{u\ell} + (1 + \bar{n}_\gamma) A_{u\ell}}$$

Using this equation, we can calculate the excitation temperature between the two levels.

Here, by the principle of detailed balance, the upward collisional rate coefficient is given in term of the downward rate coefficient by

$$k_{\ell u} = \frac{g_u}{g_\ell} k_{u\ell} e^{-E_{u\ell}/kT_{\rm gas}} \qquad (T_{\rm gas} = \text{gas kinetic energy})$$

- It is instructive to examine the population equation in various limits:
  - In the limit of $n_c \to \infty$ and no radiation field $\bar{n}_\gamma = 0$:

$$\frac{n_u}{n_\ell} = \frac{n_c k_{\ell u}}{n_c k_{u\ell} + A_{u\ell}} = \frac{k_{\ell u}}{k_{u\ell}} = \frac{g_u}{g_\ell} e^{-E_{u\ell}/kT_{\rm gas}} \qquad \therefore\ T_{\rm exc} = T_{\rm gas}$$

  - If $n_c = 0$ and the radiation field has a brightness temperature of $T_b = T_{\rm rad}$ at the frequency $\nu = E_{u\ell}/h$:

$$\frac{n_u}{n_\ell} = \frac{\bar{n}_\gamma (g_u/g_\ell)}{(1+\bar{n}_\gamma)} = \frac{g_u}{g_\ell} e^{-E_{u\ell}/kT_{\rm rad}} \qquad \therefore\ T_{\rm exc} = T_{\rm rad}$$

$$\bar{n}_\gamma = \left(e^{E_{u\ell}/kT_{\rm rad}} - 1\right)^{-1} \implies 1 + \bar{n}_\gamma = \bar{n}_\gamma\, e^{E_{u\ell}/kT_{\rm rad}}$$

  - If we have a radiation with the brightness temperature $T_b = T_{\rm rad} = T_{\rm gas}$, then we can show that

$$\frac{n_u}{n_\ell} = \frac{n_c k_{\ell u} + \bar{n}_\gamma (g_u/g_\ell) A_{u\ell}}{n_c k_{u\ell} + (1+\bar{n}_\gamma) A_{u\ell}} = \frac{g_u}{g_\ell} e^{-E_{u\ell}/kT_{\rm gas}} \qquad \therefore\ T_{\rm exc} = T_{\rm rad} = T_{\rm gas}$$

Critical Density

- For a collision partner c, we define the critical density $n_{\mathrm{crit},u}$ for an excited state *u* to be the density for which collisional de-excitation equals radiative de-excitation, including stimulated emission:

$$n_{\mathrm{crit},u}(c) \equiv \frac{\sum_{\ell<u} [1 + (n_\gamma)_{u\ell}] A_{u\ell}}{\sum_{\ell<u} k_{u\ell}(c)}$$

- Note that this definition applies to multilevel systems, but each excited level *u* may have a different critical density. The critical density depends on the intensity of ambient radiation. For many transitions, the correction is unimportant, but for 21-cm line, it is important.

Critical densities for fine-structure excitation [Table 17.1 in Draine, revised for both H and e$^-$, errata]

						$n_{\mathrm{crit},u}(\mathrm{H})$		$n_{\mathrm{crit},u}(e^-)$	
			E_ℓ/k	E_u/k	$\lambda_{u\ell}$	$T=100\,\mathrm{K}$	$T=5000\,\mathrm{K}$	$T=100\,\mathrm{K}$	$T=5000\,\mathrm{K}$
Ion	ℓ	u	(K)	(K)	(μm)	(cm^{-3})	(cm^{-3})	(cm^{-3})	(cm^{-3})
C II	$^2\mathrm{P}^\mathrm{o}_{1/2}$	$^2\mathrm{P}^\mathrm{o}_{3/2}$	0	91.21	157.74	2.7×10^3	1.5×10^3	6.8	40.
C I	$^3\mathrm{P}_0$	$^3\mathrm{P}_1$	0	23.60	609.7	620	170	76.	6.4
	$^3\mathrm{P}_1$	$^3\mathrm{P}_2$	23.60	62.44	370.37	720	150	75.	6.3
O I	$^3\mathrm{P}_2$	$^3\mathrm{P}_1$	0	227.71	63.185	2.5×10^5	4.9×10^4	1.8×10^5	4.8×10^4
	$^3\mathrm{P}_1$	$^3\mathrm{P}_0$	227.71	326.57	145.53	2.4×10^4	8.6×10^3	2.3×10^4	5.8×10^3
Si II	$^2\mathrm{P}^\mathrm{o}_{1/2}$	$^2\mathrm{P}^\mathrm{o}_{3/2}$	0	413.28	34.814	2.5×10^5	1.2×10^5	140.	1.5×10^3
Si I	$^3\mathrm{P}_0$	$^3\mathrm{P}_1$	0	110.95	129.68	4.8×10^4	2.8×10^4	2.9×10^4	830.
	$^3\mathrm{P}_1$	$^3\mathrm{P}_2$	110.95	321.07	68.473	9.9×10^4	3.6×10^4	4.4×10^4	1.9×10^3

Interstellar Radiation Fields

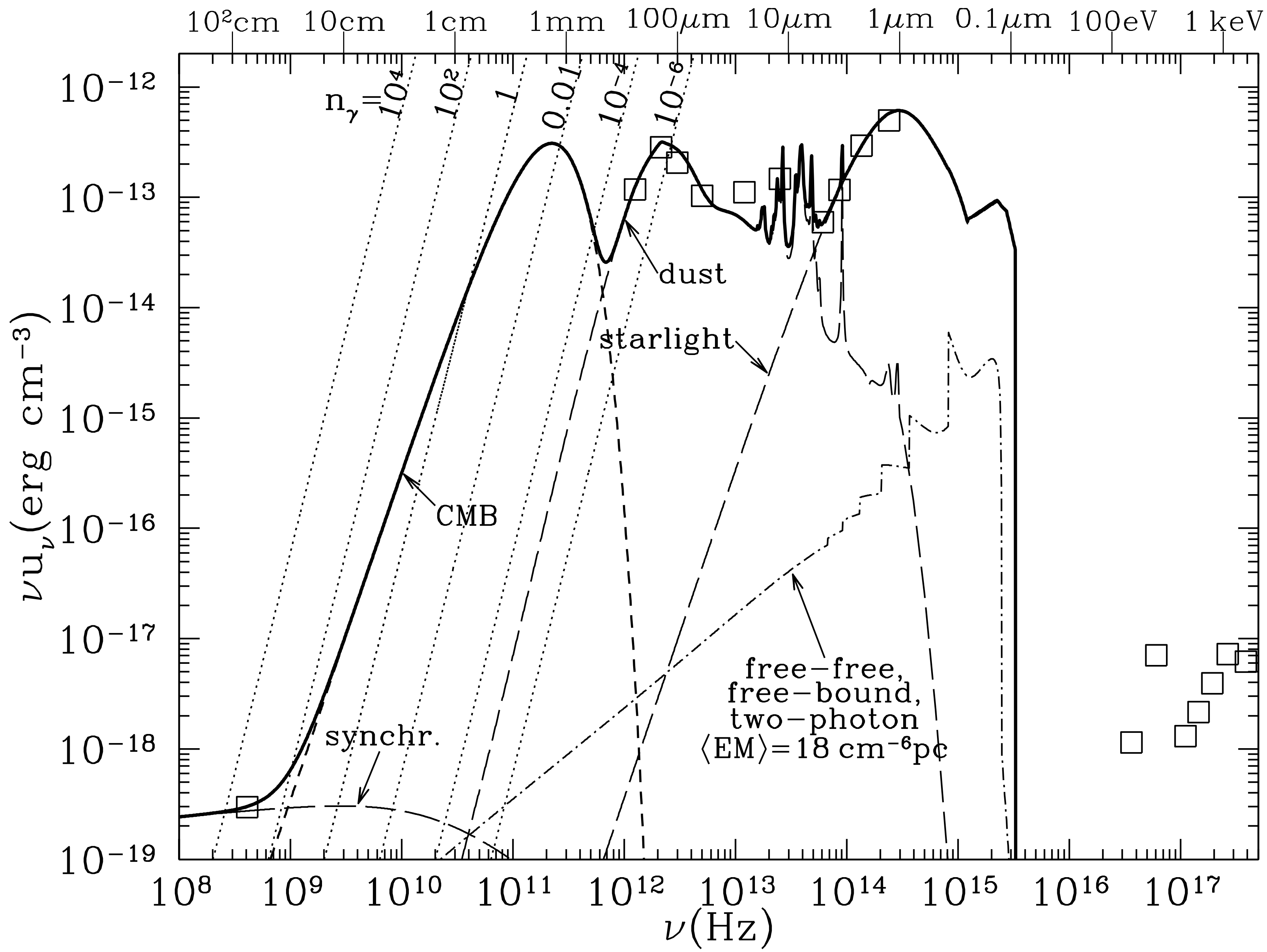


Figure 12.1 Interstellar continuum radiation field in an H I cloud in the solar neighborhood (see text). Spectral lines are not included. Solid line is the sum of all components for $h\nu \leq 13.6\,\mathrm{eV}$. Squares show the measured sky brightness at $408\,\mathrm{MHz}$ (Haslam et al. 1982), the all-sky measurements by COBE-DIRBE in 10 broad bands from 240 μm to 1.25 μm (Arendt et al. 1998), and all-sky measurements by ROSAT between 150 eV and 2 keV (Snowden 2005, private communication). Dotted lines are contours of constant photon occupation number n_γ.

[Figure 12.1 in Draine]

H I Spin Temperature

- Collionsional rate coefficients: $$\frac{n_1}{n_0} = \frac{n_c k_{01} + \bar{n}_\gamma (g_1/g_0) A_{10}}{n_c k_{10} + (1+\bar{n}_\gamma) A_{10}}$$
  - Collision with other H atoms

(Allison & Dalgarno 1969; Zygelman 2005)

$$k_{10}(\mathrm{H}) \approx \begin{cases} 1.19\times 10^{-10} T_2^{0.74-0.20\ln T_2}\,\mathrm{cm^3\,s^{-1}} & (20\,\mathrm{K} < T < 300\,\mathrm{K}) \\ 2.24\times 10^{-10} T_2^{0.207} \mathrm{e}^{-0.876/T_2}\,\mathrm{cm^3\,s^{-1}} & (300\,\mathrm{K} < T < 10^3\,\mathrm{K}) \end{cases}$$

$$k_{01}(\mathrm{H}) \approx 3k_{10}(\mathrm{H}) e^{-0.0682\,\mathrm{K}/T}$$

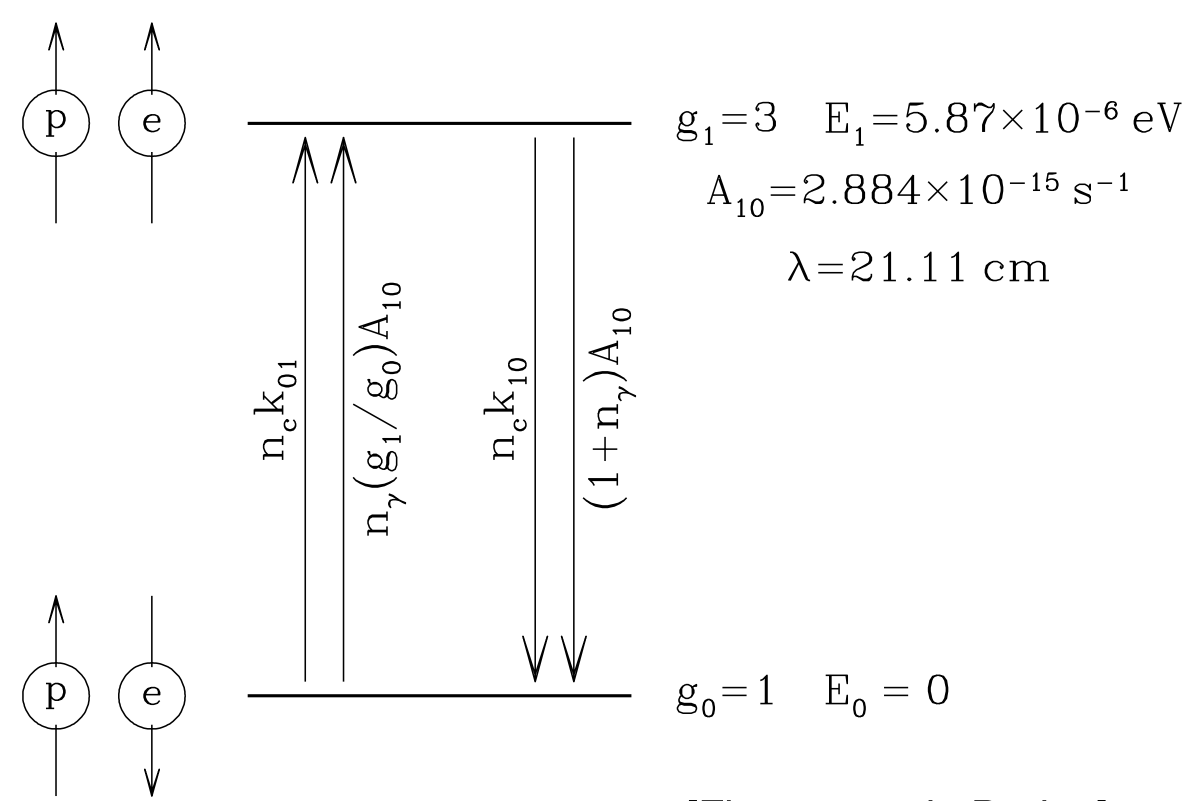
  - Collision with electrons

(Furlanetto & Furlanetto 2007)

$$k_{10}(e^-) \approx 2.26 \times 10^{-9}\,(T/100\,\mathrm{K})^{0.5}\,\mathrm{cm^3\,s^{-1}} \quad (1 \lesssim T \lesssim 500\,\mathrm{K})$$

$$k_{01}(e^-) \approx 3k_{10}(e^-) e^{-0.0682\,\mathrm{K}/T}$$

  - This is a factor ~ 10 larger than that for H atoms. However, ***electrons will be minor importance in regions with a fractional ionization*** $x_e \lesssim 0.03$, such as the CNM and WNM.



[Figure 17.1 in Draine]

- Radiation Field strength
  - The radiation field near 21 cm is dominated by the cosmic microwave background plus Galactic synchrotron emission. The antenna temperature is

$$T_A \approx T_{\rm CMB} + T_{\rm syn} = 2.73\,{\rm K} + 1.04\,{\rm K} = 3.77\,{\rm K}$$

  - Photon occupation number:

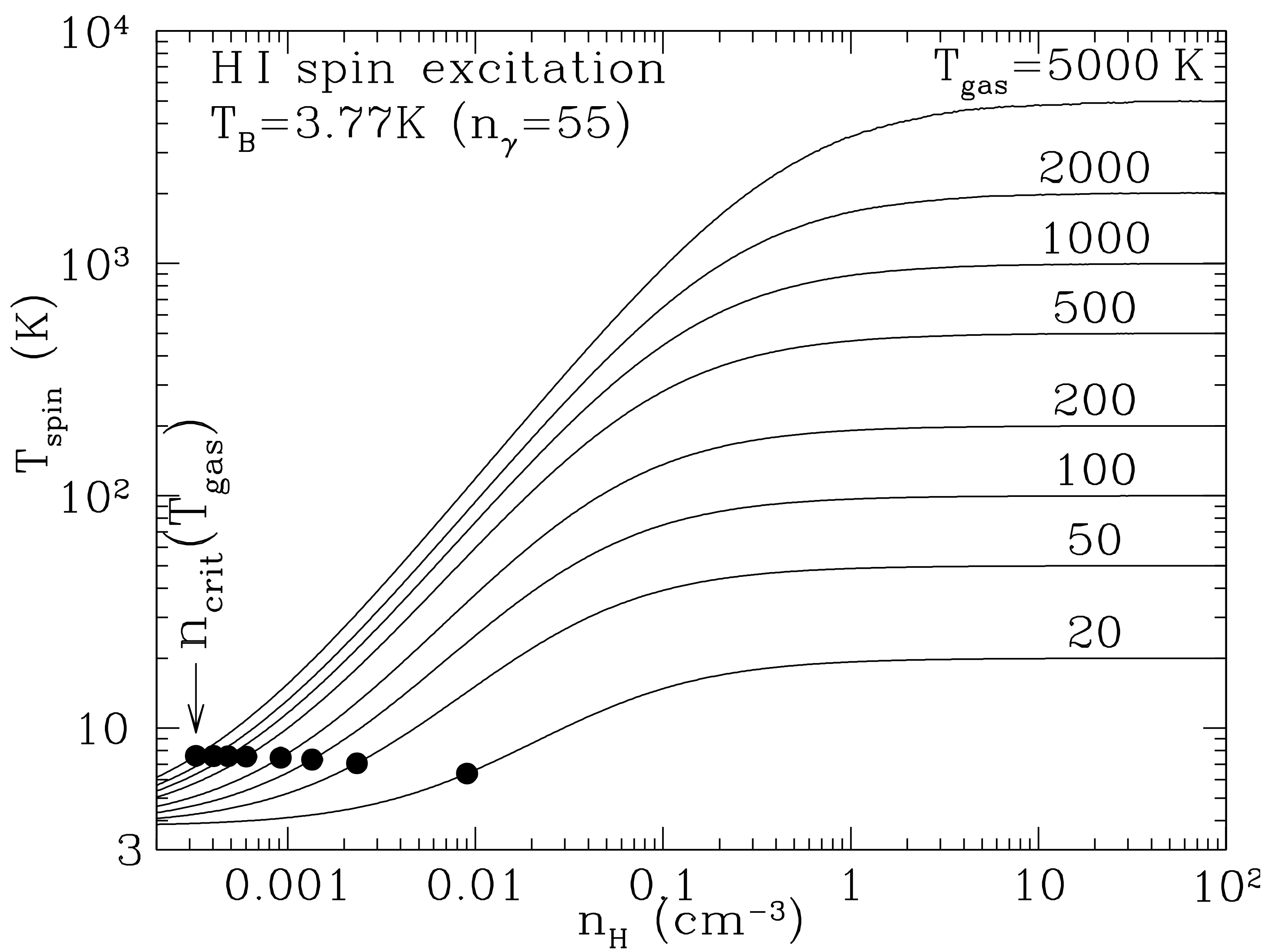
$$\bar{n}_\gamma = \left(e^{h\nu/kT_{\rm B}} - 1\right)^{-1} \approx \frac{kT_A}{h\nu} \approx \frac{3.77\,{\rm K}}{0.0682\,{\rm K}} \approx 55$$

  - The critical density is then

$$n_{\rm crit}(H) \approx 1.7 \times 10^{-3}\,(T/100\,{\rm K})^{-0.66}\,{\rm cm}^{-3}$$

$$\begin{aligned} n_{\rm crit} &\approx 0.07\,{\rm cm}^{-3} \text{ at } T \sim 10\,{\rm K} \\ &\approx 6 \times 10^{-4}\,{\rm cm}^{-3} \text{ at } T \sim 1000\,{\rm K} \end{aligned}$$

- H I spin temperature as a function of density n_H, including only 21 cm continuum radiation and collisions with H atoms. Lyα scattering is not included.
  - Filled circles show $n_{\rm crit}(\rm H)$ for each temperature.
  - It is important to note that one requires $n \gg n_{\rm crit}$ in order to have $T_{\rm spin}$ within, say, 10% of $T_{\rm gas}$, particularly at high temperatures.



[Fig. 17.2 in Draine]

Note that Ryden states that "in the CNM and WNM, we expect the hyperfine levels of atomic hydrogen to be collisionally excited, and to have a spin temperature close to the gas temperature." based on that $n_{\rm crit}$ ~ 6×10^{-4} cm^{-3} at T ~ 1000 K.

However, this is not true in the WNM.

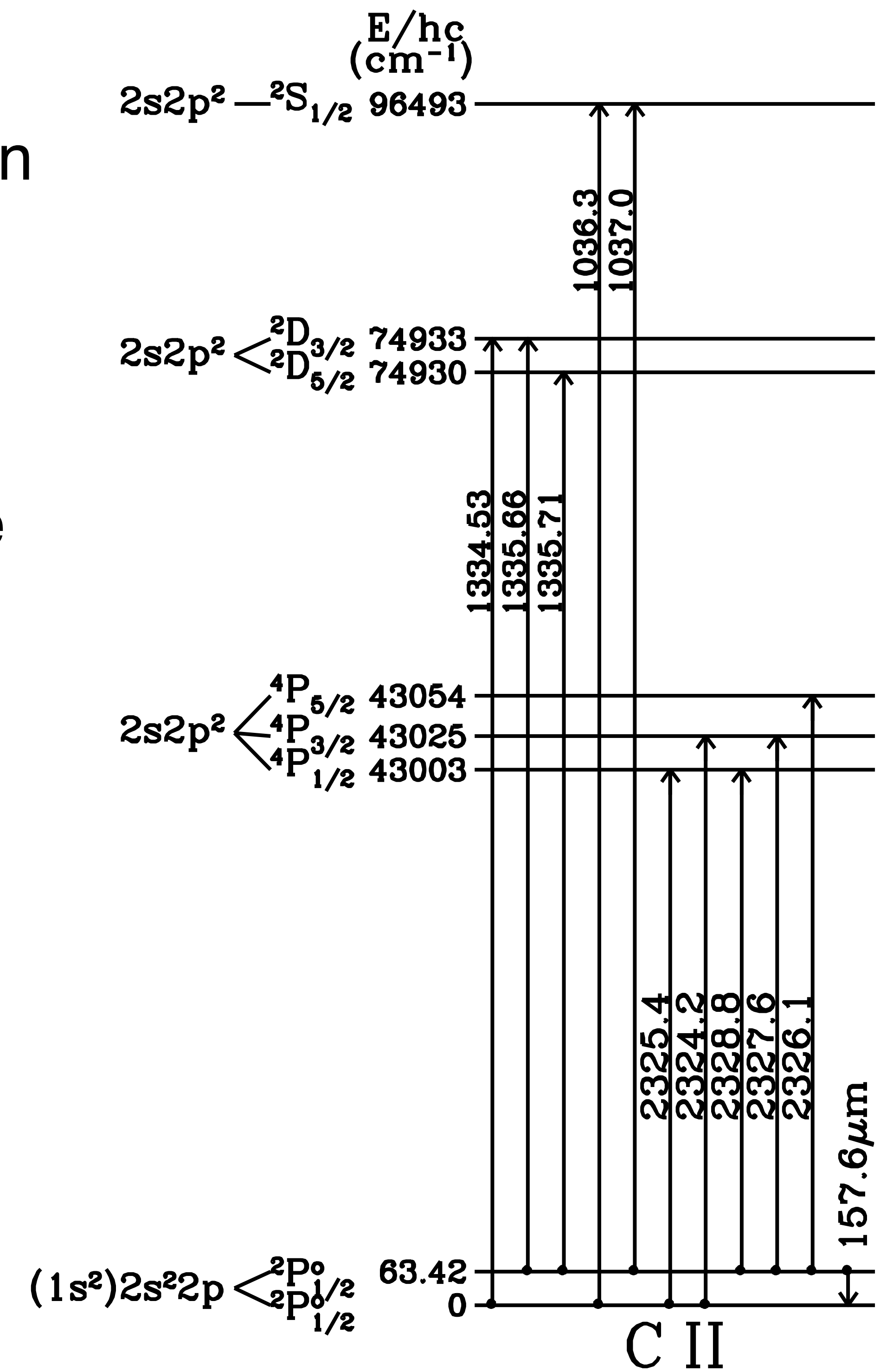
Only in the CNM, the collisional excitation is strong enough to bring the spin temperature close to the gas kinetic temperature.

C II Fine Structure Excitation

- The ground electronic state $1s^2 2s^2 2p\ ^2P^o$ of C^+ contains two fine-structure levels.
- The electronically excited states have an excitation energy that is much higher than the kinetic temperature of the CNM.

$$2235\,\text{Å} \rightarrow E_{u\ell} = 0.56\,\text{eV} \rightarrow T = 6440\,\text{K}$$

- We may, therefore, consider the two fine-structure levels in the ground electronic state to be a two level atom.
- Will the populations of these two levels be thermalized in the ISM?

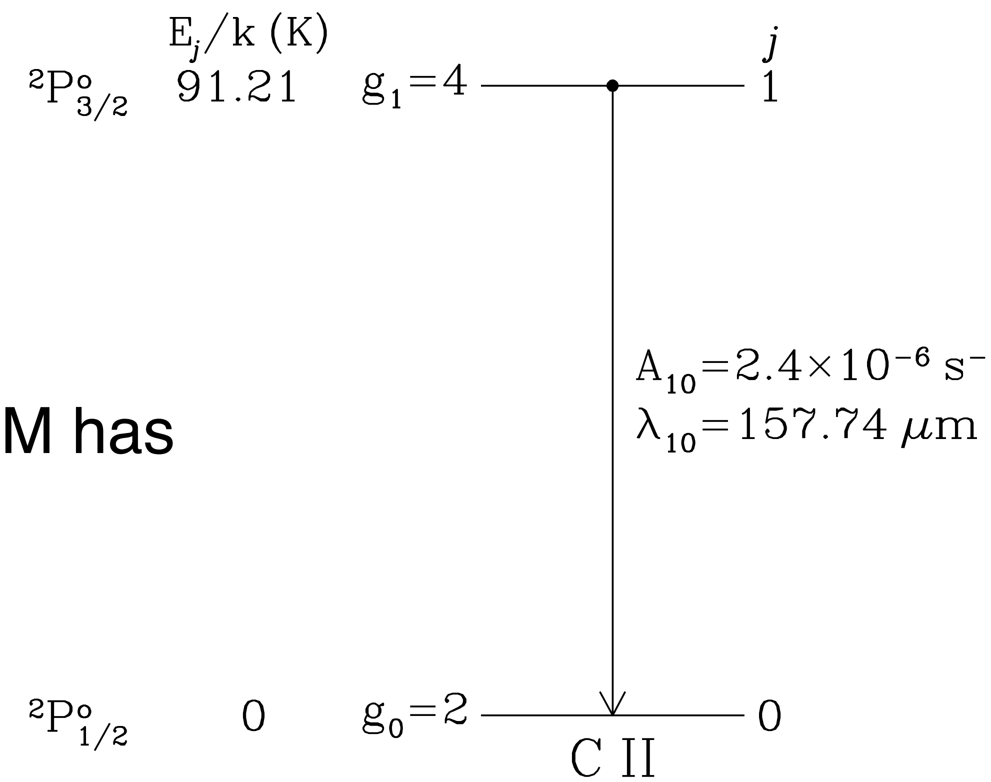


- Rate coefficients for collisional de-excitation:

$$\left\langle \Omega\left({}^2P^o_{1/2},\, {}^2P^o_{3/2}\right)\right\rangle \approx 2.1$$

$$k_{10}(e^-) \approx 4.53 \times 10^{-8}\, T_4^{-1/2}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$$

$$k_{10}(H) \approx 7.58 \times 10^{-10}\, T_2^{0.1281+0.0087\ln T_2}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$$



[Figure 17.3 in Draine]

- At $\lambda = 158\,\mu\mathrm{m}$, the continuum background in the ISM has

$$\bar{n}_\gamma \approx 10^{-5} \ll 1$$

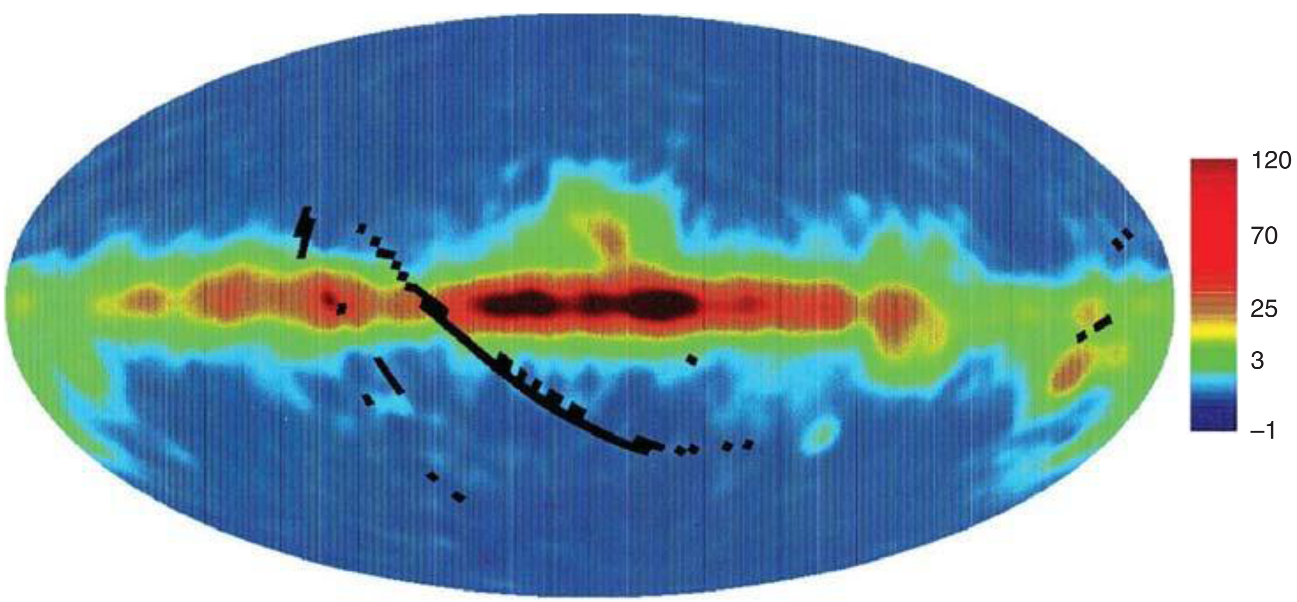
- Critical densities:

$$n_{\mathrm{crit}}(H) \approx 3.2 \times 10^3\, T_2^{-0.1281-0.0087\ln T_2}\,\mathrm{cm}^{-3}$$

$$n_{\mathrm{crit}}(e^-) \approx 53\, T_4^{1/2}\,\mathrm{cm}^{-3}$$ (Barinovs et al. 2005)

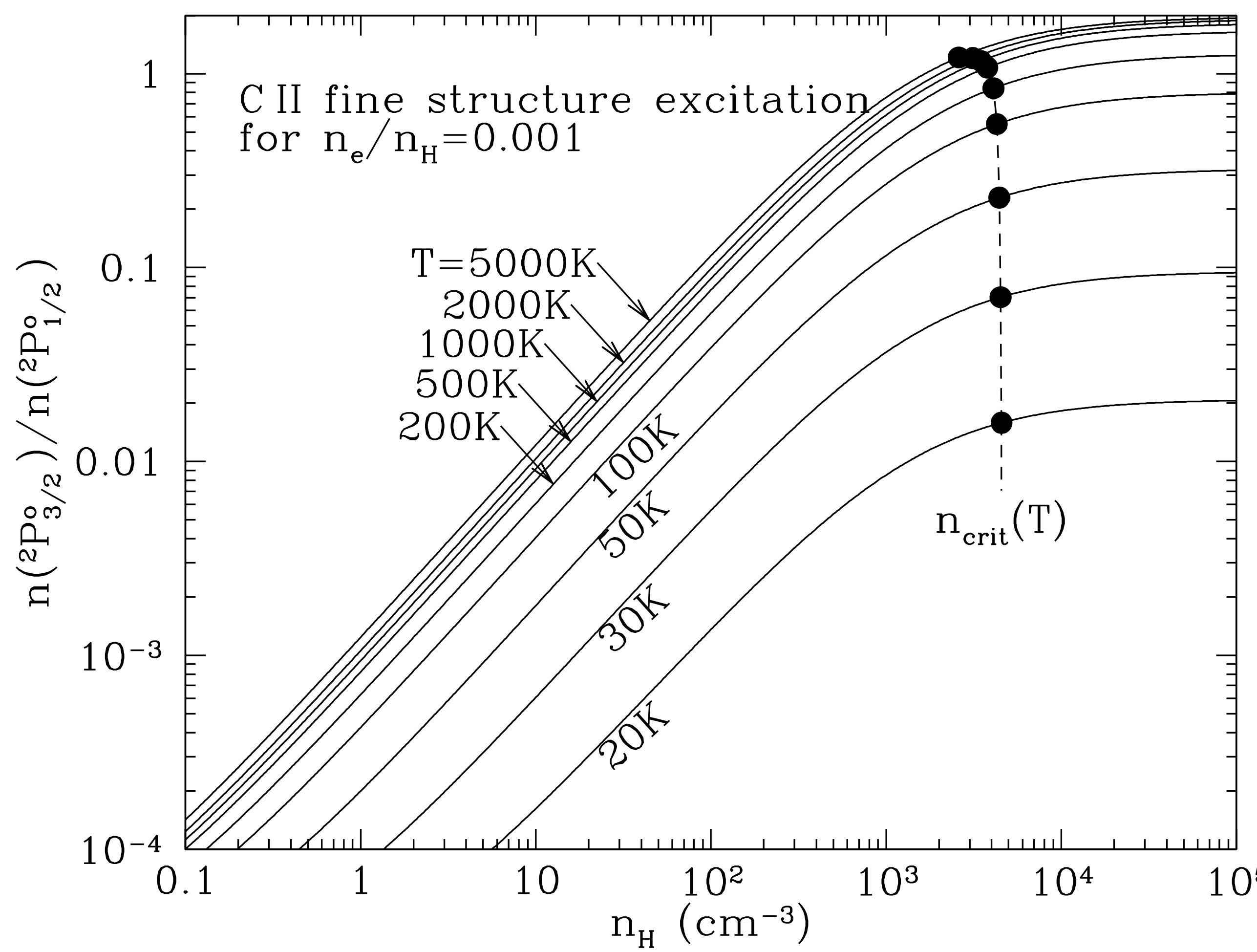
- The critical densities are much higher than the typical densities in the CNM and WNM. Thus, the C II fine-structure levels will be sub-thermally excited.

- The C II fine-structure levels will be sub-thermally excited. It follows that collisional excitations of the upper level will usually be followed by radiative decays, removing energy from the gas.
- The [C II] 158 µm transition is the principal cooling transition for the diffuse gas in star-forming galaxies.



All-sky map of [C II] 158 µm emission, made by Far InfraRed Absolute Spectrophotometer (FIRAS) on the COsmic Background Explorer (COBE) satellite (Fixsen et al. 1999).

[Plate 3 in Draine]



[Fig. 17.4 in Draine]

Equation for the 21-cm Spin Temperature

- We have derived the equation for the level populations in the presence of collision and radiation. Now, we will derive an intuitive equation for the spin temperature of the 21-cm line.

$$\frac{n_1}{n_0} = \frac{n_c k_{01} + \bar{n}_\gamma (g_1/g_0) A_{10}}{n_c k_{10} + (1+\bar{n}_\gamma) A_{10}}$$

- Let's define the temperature corresponding to the 21-cm transition.

$$T_* = E_{10}/k = 0.0682\,\mathrm{K}$$

- The temperatures of radiation and gas will be much higher than this:

$$T_{\rm gas} \approx 10 - 10^4\,\mathrm{K} \gg T_*, \quad T_{\rm rad} = 3.77\,\mathrm{K} \gg T_*, \quad T_{\rm spin} \gg T_*$$

- The population ratio can be written in terms of the excitation (spin) temperature:

$$\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-T_*/T_{\rm spin}} \simeq \frac{g_1}{g_0}\left(1 - \frac{T_*}{T_{\rm spin}}\right)$$

- Similarly,

$$k_{01} = \frac{g_1}{g_0} k_{10} e^{-T_*/T_{\rm gas}} \simeq \frac{g_1}{g_0} k_{10}\left(1 - \frac{T_*}{T_{\rm gas}}\right)$$

$$\bar{n}_\gamma = \frac{1}{e^{T_*/T_{\rm rad}} - 1} \simeq \frac{T_{\rm rad}}{T_*}$$

- Substituting these into the population equation, we obtain

$$1 - \frac{T_*}{T_{\rm spin}} = \frac{n_c k_{10}\left(1 - T_*/T_{\rm gas}\right) + \left(T_{\rm rad}/T_*\right) A_{10}}{n_c k_{10} + \left(1 + T_{\rm rad}/T_*\right) A_{10}}$$

- Finally, we obtain the following equation:

$$T_{\rm spin} = \frac{T_* + T_{\rm rad} + y_c T_{\rm gas}}{1 + y_c} \qquad \Longleftarrow \qquad y_c \equiv \frac{T_*}{T_{\rm gas}} \frac{n_c k_{10}}{A_{10}}$$

- Ignoring T_* term, we obtain an intuitive equation for the spin temperature.

$$T_{\rm spin} = \frac{T_{\rm rad} + y_c T_{\rm gas}}{1 + y_c} \qquad \Longleftarrow \qquad y_c \equiv \frac{T_*}{T_{\rm gas}} \frac{n_c k_{10}}{A_{10}}$$

This equation was first derived by G. Field (1958).

- This equation describes the spin temperature as ***a weighted mean of the radiation and gas temperatures with weights of 1 and y_c***.

- From the equation, we can show that

$$\begin{aligned} T_{\rm spin} &\simeq T_{\rm rad} \quad \text{if } y_c \ll 1 \\ T_{\rm spin} &\simeq T_{\rm gas} \quad \text{if } y_c \gg 1 \end{aligned}$$

- A new critical density of the colliding particle may be defined:

$$y_c = 1 \quad \Longrightarrow \quad n^*_{\rm crit} = \frac{T_{\rm gas}}{T_*}\frac{A_{10}}{k_{10}}$$

- Now, compare this density with the previous definition of the critical density.

$$\begin{aligned} n_{\rm crit} &\equiv \frac{[1+(n_\gamma)_{10}]\,A_{10}}{k_{10}} \\ &= \left[1+\frac{1}{e^{h\nu_{10}/kT_{\rm rad}}-1}\right]\frac{A_{10}}{k_{10}} \\ &\simeq \left(1+\frac{T_{\rm rad}}{T_*}\right)\frac{A_{10}}{k_{10}} \end{aligned}$$

$$\frac{n^*_{\rm crit}}{n_{\rm crit}} \approx \frac{T_{\rm gas}}{T_{\rm rad}}$$

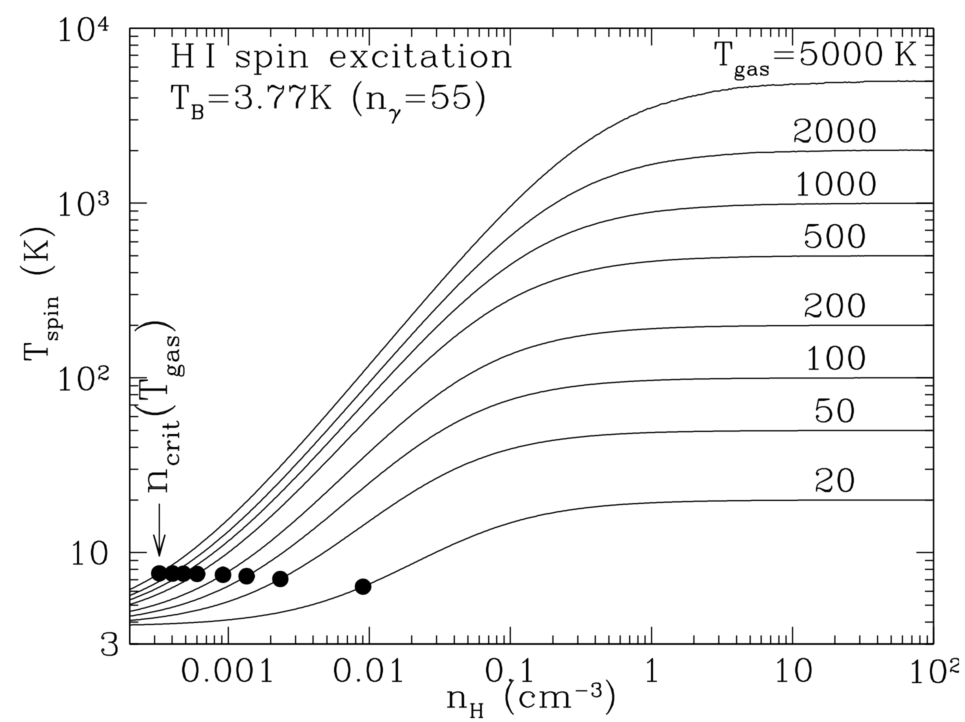
Detectability of Hydrogen in a Low Density Medium

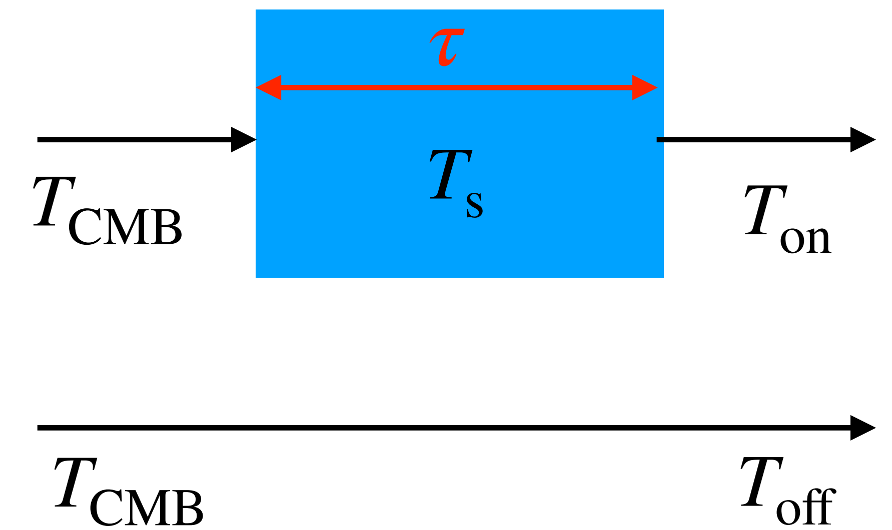
- In a very low density medium (WNM, CGM, IGM), the particle collisions are very rare ($n_{\rm HI} \ll n_{\rm crit}$).
- The radiative transition due to the CMB photons will control the relative population between the hyperfine structures.
  - This indicates $T_{\rm s} = T_{\rm CMB}$.
  - The RT equation in the Rayleigh-Jeans regime can be written in terms of temperature:

$$T_{\rm on} = T_{\rm CMB} e^{-\tau} + T_{\rm s} (1 - e^{-\tau}) = T_{\rm CMB}$$
$$T_{\rm off} = T_{\rm CMB}$$
$$T_{\rm on} - T_{\rm off} = 0$$

  - Then, we have $\boxed{T_{\rm on} = T_{\rm off} = T_{\rm CMB}}$.
  - ***Neither emission nor absorption feature from the hydrogen gas is detectable.***
- ***We need something that can make $T_{\rm s} \neq T_{\rm CMB}$.***

[Fig. 17.2 in Draine]





The Wouthuysen-Field effect: The Third Mechanism controlling the Spin Temperature

- Wouthuysen (1952, AJ, 57, 31)

“Wouthuysen” is pronounced as roughly “Vowt-how-sen.” (바우타이슨)

Wouthuysen, S. A. On the excitation mechanism of the 21-cm (radio-frequency) interstellar hydrogen emission line.

The mechanism proposed here is a radiative one: as a consequence of absorption and re-emission of Lyman-α resonance radiation, a redistribution over the two hyperfine-structure components of the ground level will take place. Under the assumption—here certainly permitted—that induced emissions can be negelcted, it can easily be shown that the relative distribution of the two levels in question, under stationary conditions, will depend solely on the shape of the radiation spectrum in the Lα region, and not on the absolute intensity.

The shape of the spectrum of resonance radiation, quasi-imprisoned in a large gas cloud, could only be determined by a careful study of the "scattering" process (absorption and re-emission) in a cloud of definite shape and dimensions. The spectrum will turn out to depend upon the localization in the cloud.

Some features can be inferred from more general considerations. Take a gas in a large container, with perfectly reflecting walls. Let the gas be in equilibrium at temperature T, together with Planck radiation of that same temperature. The scattering processes will not affect the radiation spectrum. One can infer from this fact that the photons, after an infinite number of scattering processes on gas atoms with kinetic temperature T, will obtain a statistical distribution over the spectrum proportional to the Planck-radiation spectrum of temperature T. After a finite but large number of scattering processes the Planck shape will be produced in a region around the initial frequency.

Photons reaching a point far inside an interstellar gas cloud, with a frequency near the Lα resonance frequency, will have suffered on the average a tremendous number of collisions. Hence in that region, which is wider the larger the optical depth of the cloud is for the Lyman radiation, the Planck spectrum corresponding to the gas-kinetic temperature will be established as far as the shape is concerned. Because, however, the relative occupation of the two hyperfine-structure components of the ground state depends only upon the shape of the spectrum near the Lα frequency, this occupation will be the one corresponding to equilibrium at the gas temperature.

The conclusion is that the resonance radiation provides a long-range interaction between gas atoms, which forces the internal (spin-)degree of freedom into thermal equilibrium with the thermal motion of the atoms.

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From a thermodynamic argument, Wouthuysen speculated the followings:

A tremendous number of scattering will establish the Planck-like spectrum, at the Lyα line center, corresponding to the gas-kinetic temperature.

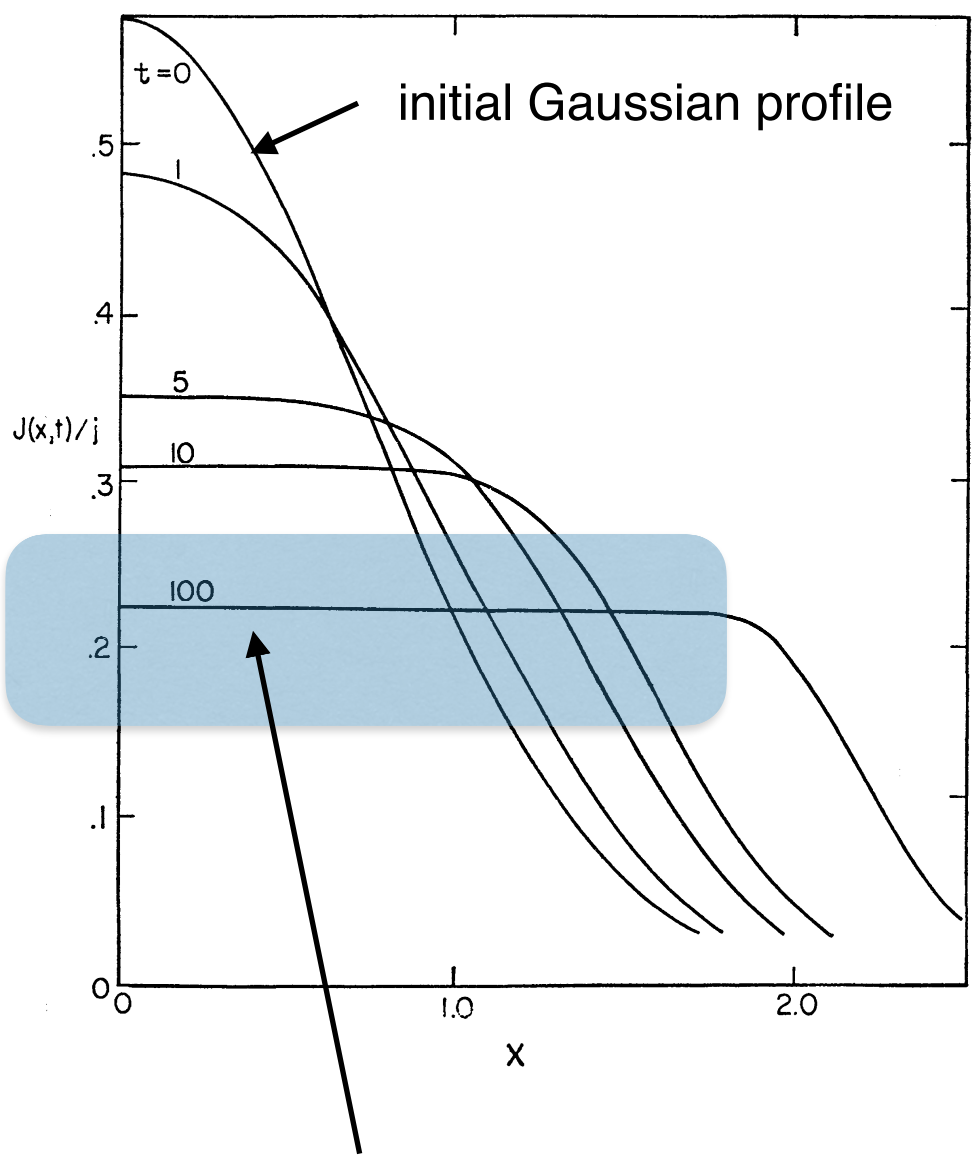
The Lyα radiation is coupled with the hyperfine state of the hydrogen atom.

In the end, ***the 21cm spin temperature will become equal to the kinetic temperature of the hydrogen gas.***

Relaxation of Lyα Profile

- Field (1958, PIRE, 46, 240; 1959, ApJ, 129, 551)

Recoil effect = momentum transfer between H atom and photon



spectral shape at the Lyα line center

Without recoil:
The spectral profile of Lyα, ***within the medium,*** becomes flat at the line center when the photons undergo a large number of resonance scatterings.

$$J(\nu,\, t \to \infty) = \text{constant}$$

With recoil:
Recoil of the scattering atom changes the slope of the Lyα central profile ***"at the limit of an infinite number of scattering"*** and gives a Boltzmann like (exponential) functional shape:

$$J(\nu,\, t \to \infty) \propto e^{-\frac{h(\nu-\nu_\alpha)}{kT_\mathrm{K}}}$$

Mechanisms that controls the spin temperature

- The spin temperature (T_S) is determined by three mechanisms.

(1) Direct Radiative Transitions by the background radiation field (Cosmic Microwave Background or Galactic Synchrotron)

$$I_\nu = \frac{2k_B T_R}{\lambda^2}$$

(Rayleigh-Jeans Law)

$$T_R = \text{brightness temperature} \\ = 2.73\,\text{K or } 3.77\,\text{K}$$

(2) Collisional Transitions (collision with other hydrogen and electron)

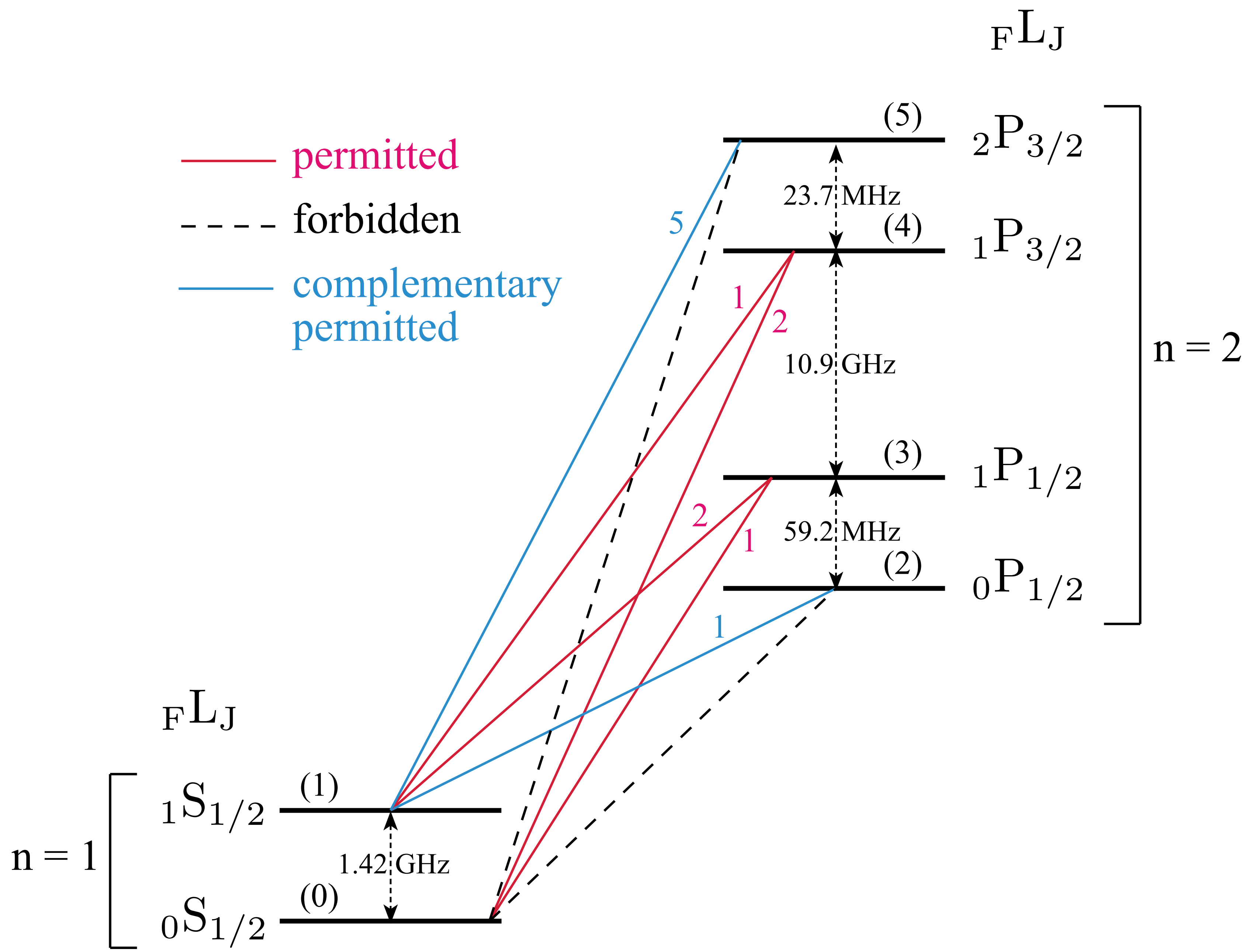
$$T_K = \text{gas kinetic temperature}$$

(3) Lyα pumping: Indirect Radiative Transitions involving intermediate levels caused by Lyα resonance scattering

$$T_\alpha = \text{color temperature} \qquad J(\nu) \propto \exp\left(-\frac{h\nu}{k_B T_\alpha}\right)$$

Indirect Level Population by Lyα Scattering

The WF effect is a mechanism that the resonance scattering of Lyα photons indirectly control the relative populations between the hyperfine levels in the ground state (n = 1) via transitions involving the n = 2 state as an intermediate state.



Equation for spin temperature (a)

- In stationary state, rate equation for the population of the hyperfine states 0 and 1 can be written

$$n_0\left(P_{01}^R + P_{01}^c + P_{01}^\alpha\right) = n_1\left(P_{10}^R + P_{10}^c + P_{10}^\alpha\right) \qquad \text{Eq (1)}$$

where

$P^c,\ P^R,\ P^\alpha =$ transition rates (per sec) cause by collisions, radio, and Lyα

(1) Level Population in terms of spin temperature (T_S):

$$\begin{aligned}\frac{n_1}{n_0} &= \frac{g_1}{g_0}\exp\left(-\frac{h\nu_{10}}{k_\mathrm{B}T_S}\right) \longleftarrow \quad T_* \equiv \frac{h\nu_{10}}{k_\mathrm{B}} = 0.0681\,\mathrm{K}\\ &\simeq 3\left(1-\frac{T_*}{T_S}\right) \qquad \text{Eq (2)}\end{aligned}$$

Equation for spin temperature (b)

(2) Ratio between the radiative transition rates in terms of the brightness temperature:

$$I_\nu = \frac{2\nu_{10}^2}{c^2} kT_R, \quad u_\nu = \frac{8\pi\nu_{10}^2}{c^3} kT_R$$

Using the definition of the brightness temperature, we obtain the radiative transition rates due to 21 cm radiation :

$$P_{01}^R = B_{01} u_\nu = \left(\frac{g_1}{g_0} \frac{c^3}{8\pi h \nu_{10}^3} A_{10} \right) \left(\frac{8\pi\nu_{10}^2}{c^3} kT_R \right) = 3 \frac{T_R}{T_*} A_{10}$$

$$P_{10}^R = A_{10} + B_{10} u_\nu = A_{10} + \left(\frac{c^3}{8\pi h \nu_{10}^3} A_{10} \right) \left(\frac{8\pi\nu_{10}^2}{c^3} kT_R \right) = \left(1 + \frac{T_R}{T_*} \right) A_{10}$$

$$\frac{P_{01}^R}{P_{10}^R} \simeq 3 \left(1 - \frac{T_*}{T_R} \right) \qquad \text{Eq (3)}$$

(3) Ratio between the collisional transition rates in terms of the gas kinetic temperature:

$$\frac{P_{01}^c}{P_{10}^c} = \frac{k_{01}}{k_{10}} = \frac{g_1}{g_0} \exp\left(-\frac{h\nu_{10}}{kT_K} \right) \simeq 3 \left(1 - \frac{T_*}{T_K} \right) \qquad \text{Eq (4)}$$

(T_K is the gas kinetic temperature.)

Equation for spin temperature (c)

(5) Ratio between the indirect transition rates due to Lyα pumping:

$$P_{01}^{\alpha} = \sum_{j=2}^{5} B_{0j} u(\nu_{0j}) \frac{A_{j1}}{\sum_{i=0}^{1} A_{ji}} = \sum_{j=2}^{5} \frac{g_j}{g_0} \frac{c^3}{8\pi h \nu_{0j}^3} A_{j0} u(\nu_{0j}) \frac{A_{j1}}{\sum_{i=0}^{1} A_{ji}}$$

photon occupation number

$$\simeq n_\gamma(\nu_{0\alpha}) \sum_{j=2}^{5} \frac{g_j}{g_0} A_{j0} \frac{A_{j1}}{\sum_{i=0}^{1} A_{ji}} \qquad \leftarrow \quad n_\gamma(\nu_{0j}) \simeq n_\gamma(\nu_{0\alpha}) \qquad \left(n_\gamma = \frac{c^3}{8\pi h \nu^3} u_\nu \right)$$

$$\simeq \exp\left(-\frac{h\nu_{0\alpha}}{kT_\alpha} \right) C_{01} \qquad \leftarrow \quad \text{Wiens Law}, \quad C_{01} \equiv \frac{1}{g_0} \sum_{j=2}^{5} g_j \frac{A_{j0} A_{j1}}{\sum_{i=0}^{1} A_{ji}}$$

assumption

$$P_{10}^{\alpha} = \sum_{j=2}^{5} B_{1j} u(\nu_{1j}) \frac{A_{j0}}{\sum_{i=0}^{1} A_{ji}} = \sum_{j=2}^{5} \frac{g_j}{g_1} \frac{c^3}{8\pi h \nu_{1j}^3} A_{j1} u(\nu_{1j}) \frac{A_{j0}}{\sum_{i=0}^{1} A_{ji}}$$

$$\simeq n_\gamma(\nu_{1\alpha}) \sum_{j=2}^{5} \frac{g_j}{g_1} A_{j1} \frac{A_{j0}}{\sum_{i=0}^{1} A_{ji}} \qquad \leftarrow \quad n_\gamma(\nu_{1j}) \simeq n_\gamma(\nu_{1\alpha})$$

$$\simeq \exp\left(-\frac{h\nu_{1\alpha}}{kT_\alpha} \right) C_{10} \qquad \leftarrow \quad \text{Wiens Law}, \quad C_{10} \equiv \frac{1}{g_1} \sum_{j=2}^{5} g_j \frac{A_{j1} A_{j0}}{\sum_{i=0}^{1} A_{ji}}$$

assumption

Notice that $\frac{C_{01}}{C_{10}} = \frac{g_1}{g_0} = 3$

$\nu_{0\alpha} - \nu_{1\alpha} = \nu_{10}$

Therefore, we have $$\frac{P_{01}^{\alpha}}{P_{10}^{\alpha}} \simeq 3 \exp\left(-\frac{h\nu_{10}}{kT_\alpha} \right) \simeq 3 \left(1 - \frac{T_*}{T_\alpha} \right)$$ Eq (5)

Equation for spin temperature (e)

Combining Eq (1) - (6), we obtain the following equation for the spin temperature in terms of the 21 cm brightness temperature, gas kinetic temperature, and Ly-alpha color temperature:

$$1 - \frac{T_*}{T_S} = \frac{\left(1 - \frac{T_*}{T_R}\right) P_{10}^R + \left(1 - \frac{T_*}{T_K}\right) P_{10}^c + \left(1 - \frac{T_*}{T_\alpha}\right) P_{10}^\alpha}{P_{10}^R + P_{10}^c + P_{10}^\alpha}$$

$$\frac{T_*}{T_S} = \frac{\frac{T_*}{T_R} P_{10}^R + \frac{T_*}{T_K} P_{10}^c + \frac{T_*}{T_\alpha} P_{10}^\alpha}{P_{10}^R + P_{10}^c + P_{10}^\alpha}$$

$$P_{10}^R = \left(1 + \frac{T_R}{T_*}\right) A_{10}$$

$$\frac{T_*}{T_S} = \frac{A_{10} + \frac{T_*}{T_K} P_{10}^c + \frac{T_*}{T_\alpha} P_{10}^\alpha}{\left(1 + \frac{T_R}{T_*}\right) A_{10} + P_{10}^c + P_{10}^\alpha}$$

$$\frac{T_S}{T_*} = \frac{\left(1 + \frac{T_R}{T_*}\right) A_{10} + P_{10}^c + P_{10}^\alpha}{A_{10} + \frac{T_*}{T_K} P_{10}^c + \frac{T_*}{T_\alpha} P_{10}^\alpha}$$

$$T_S = \frac{T_* + T_R + T_* \frac{P_{10}^c}{A_{10}} + T_* \frac{P_{10}^\alpha}{A_{10}}}{1 + \frac{T_*}{T_K} \frac{P_{10}^c}{A_{10}} + \frac{T_*}{T_\alpha} \frac{P_{10}^\alpha}{A_{10}}}$$

$$\boxed{T_S = \frac{T_* + T_R + y_c T_K + y_\alpha T_\alpha}{1 + y_c + y_\alpha}}$$

where

$$y_c \equiv \frac{T_*}{T_K} \frac{P_{10}^c}{A_{10}}$$

$$y_\alpha \equiv \frac{T_*}{T_\alpha} \frac{P_{10}^\alpha}{A_{10}}$$

$$T_* = \frac{h\nu_{10}}{k_B} = 0.0681\ {}^\circ\mathrm{K}$$ (This term is negligible in the above equation.)

Two requirements for the WF effect:

(1) $J_\nu \propto \exp\left(-\frac{h\nu}{kT_\alpha}\right)$ with $T_\alpha = T_{\mathrm{K}}$

(2) $y_\alpha \gg 1$ and $y_\alpha \gg y_c$

(Lyα radiation field should be strong.)

First Requirement: Spectral Shape of Lyα inside the medium

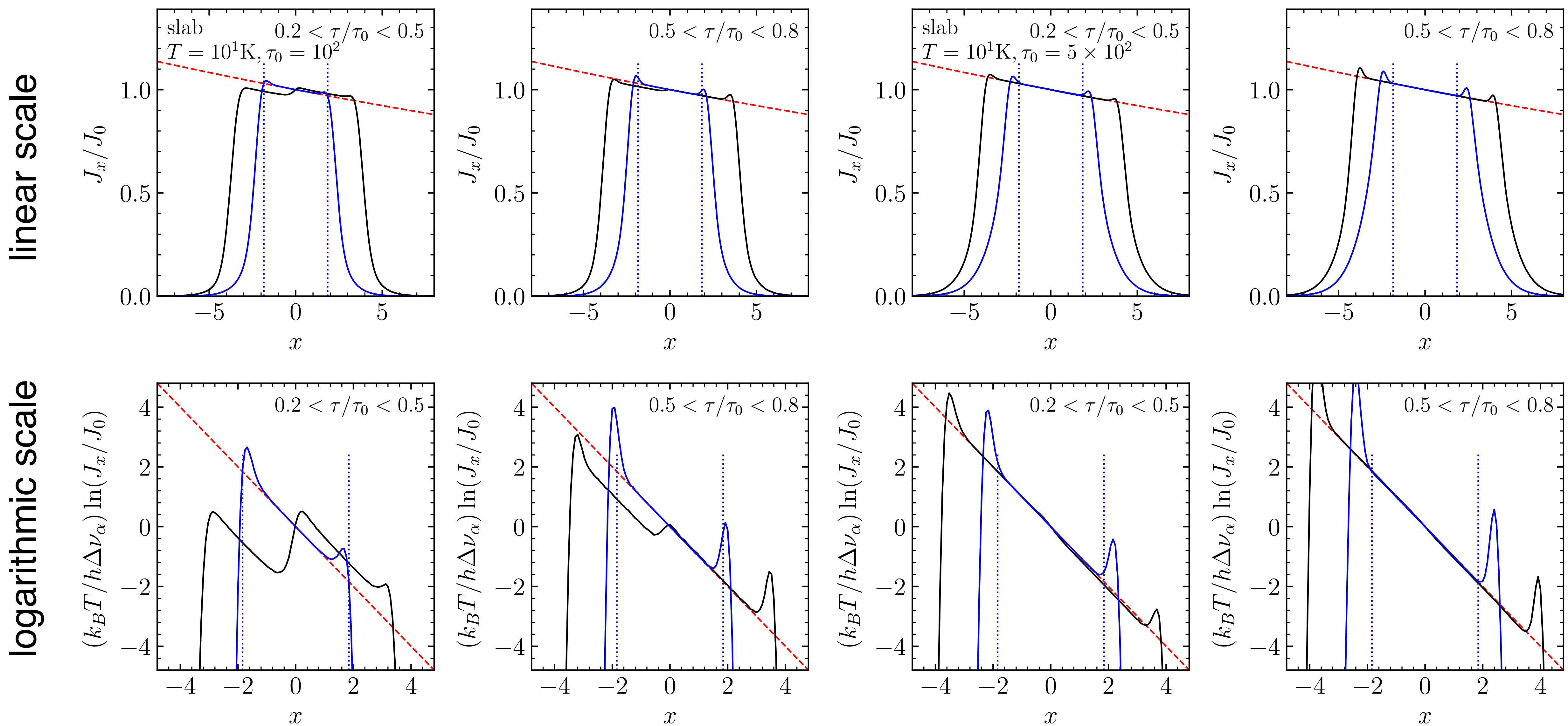
With the recoil effect

T = 10 K

red dashed lines: $J_x \propto \exp\left(-\frac{h(\nu-\nu_\alpha)}{k_B T_K}\right)$

black lines: included the fine structure splitting

blue lines: ignored the fine structure

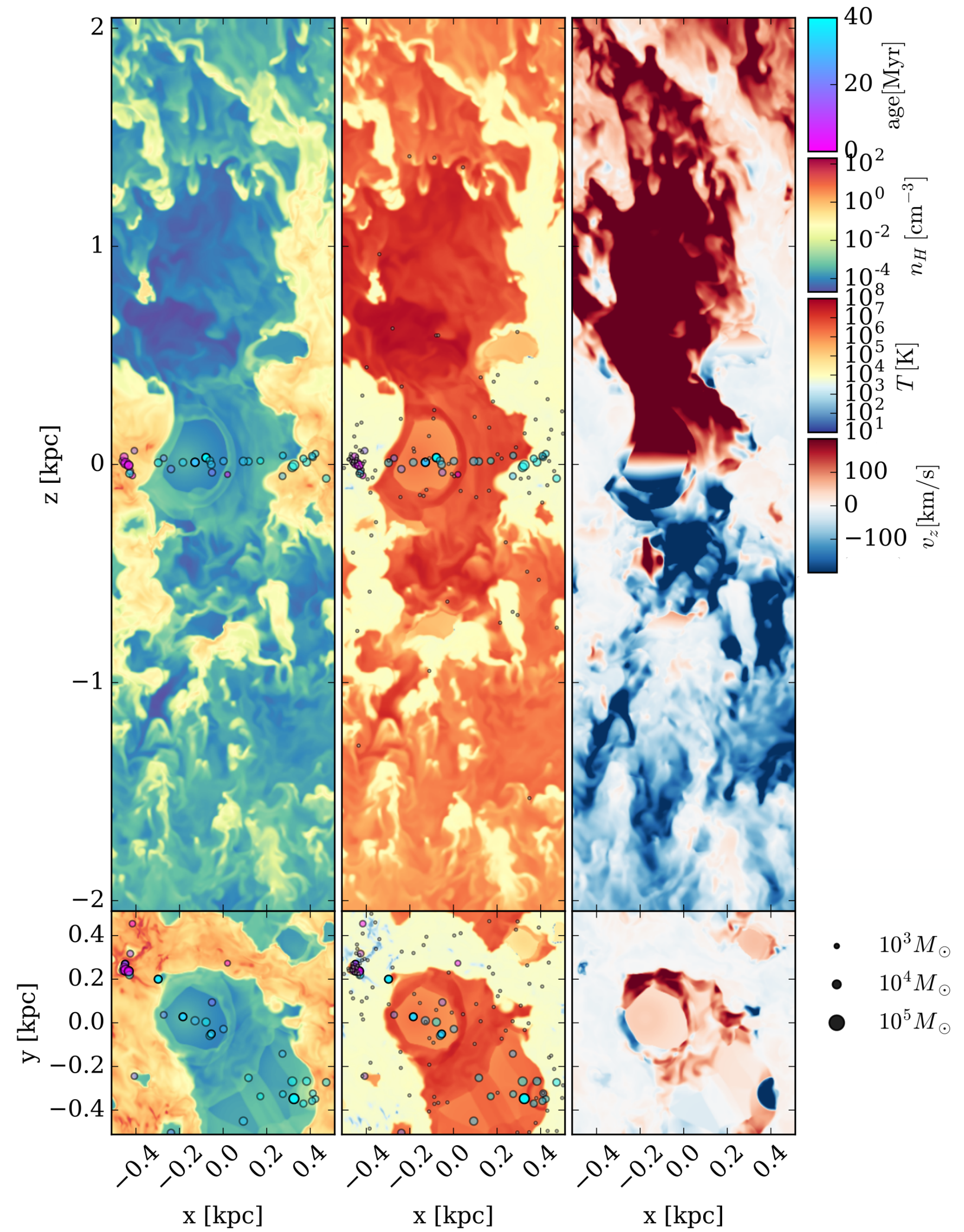


The resonance-line profile at the line center approaches to the exponential function with the gas kinetic temperature, even in a system with an optical depth as low as 100-500.

Second Requirement: Strength of Lyα Radiation

- ***Is Lyα radiation strong enough to make y_α large (in our Galaxy)?***
- We need to perform simulations in realistic ISM models to address this question.
- The TIGRESS frame work.
  - TIGRESS = Three-phase Interstellar Medium in Galaxies Resolving Evolution with Star Formation and Supernova Feedback
  - In the TIGRESS framework, the ideal MHD equations are solved in a local, shearing box, representing a small patch of a differential rotating galactic disk.
  - TIGRESS yields self-consistent 3D ISM models with self-regulated star formation.

TIGRESS
Kim & Ostriker (2017; 2018)



Lyα Sources

- ***Recombination line from H II regions (Galactic Disk)***
  - Vacca et al. (1996) gives the ionizing UV flux from 429 O- and early B-stars within 2.5 kpc of the Sun. From this, the Lyα production rate is calculated.

$$\psi_{\mathrm{Ly}\alpha} = 2.52 \times 10^7 \,\mathrm{photons\,cm^{-2}\,s^{-1}}$$

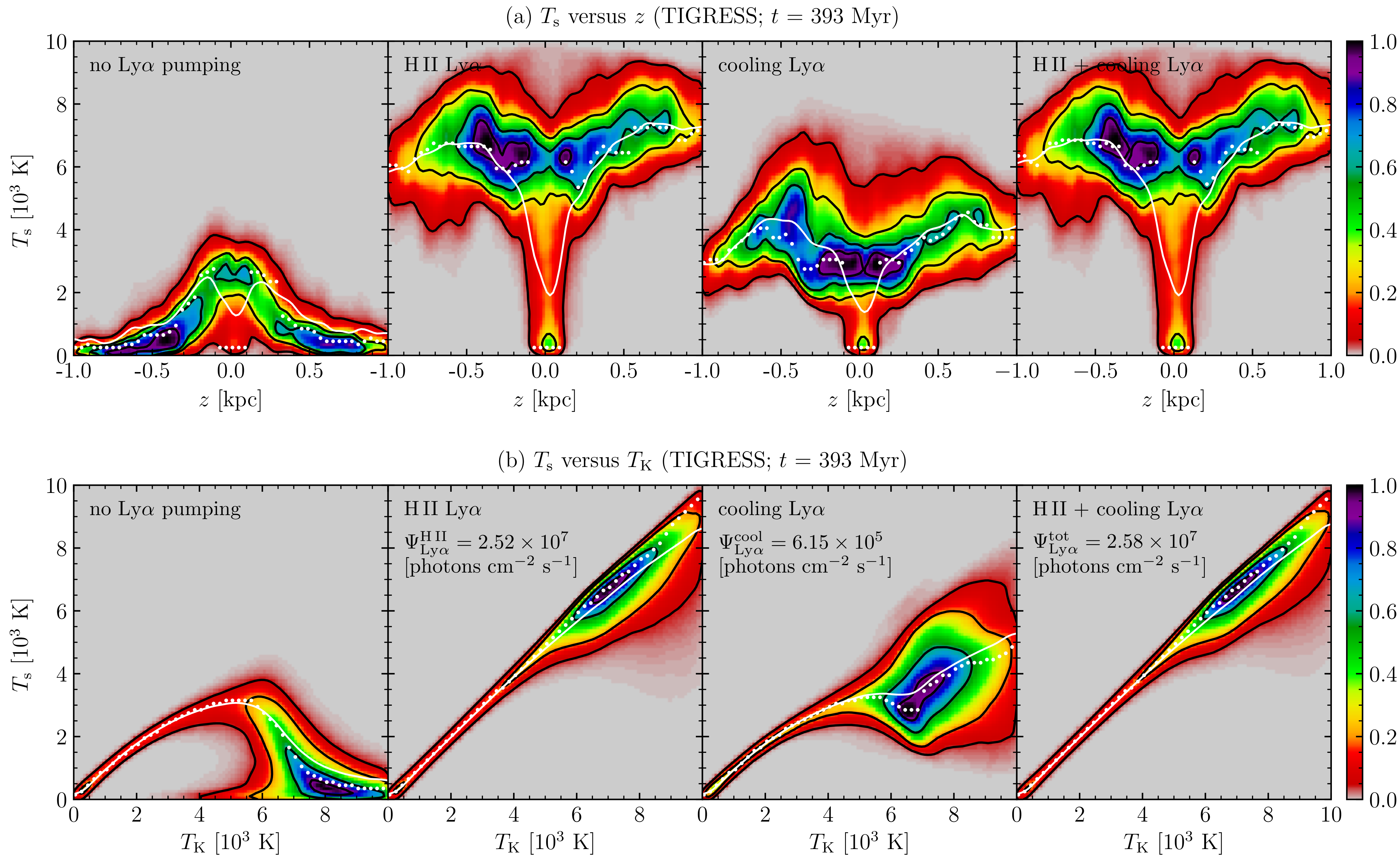
  - H II regions are assumed to be distributed as follows:

$$j_{\mathrm{Ly}\alpha}(z) \propto \exp\left(-\frac{z^2}{2\sigma^2}\right) \text{ where } \sigma = 81 \text{ pc}$$

- ***Collisional cooling Lyα lines from hot gas***
  - Assuming the collisional ionization equilibrium (CIE), the total Lyα production rate caused by the collisional excitation and ionization is calculated.

$$\frac{4\pi j_{\mathrm{Ly}\alpha}}{h\nu_\alpha} = n_e n_{\mathrm{H}} \frac{C_{\mathrm{Ly}\alpha}(T)}{h\nu_\alpha} + n_e n_p \alpha_{\mathrm{B}} P_{\mathrm{B}}(Ly\alpha)$$

Result: Spin vs. Kinetic Temperature



The Lyα radiation, at least in our Galaxy, is strong enough to thermalize the spin temperature to the gas kinetic temperature.

Homework

(1) If we consider only the background radiation field and collisions with hydrogen, the spin temperature of the 21-cm transition is given by

Eq(a): $$T_{\rm spin} = \frac{T_{\rm rad} + y_c T_{\rm gas}}{1 + y_c} \quad \text{where} \quad y_c \equiv \frac{T_*}{T_{\rm gas}} \frac{n_c k_{10}}{A_{10}}$$

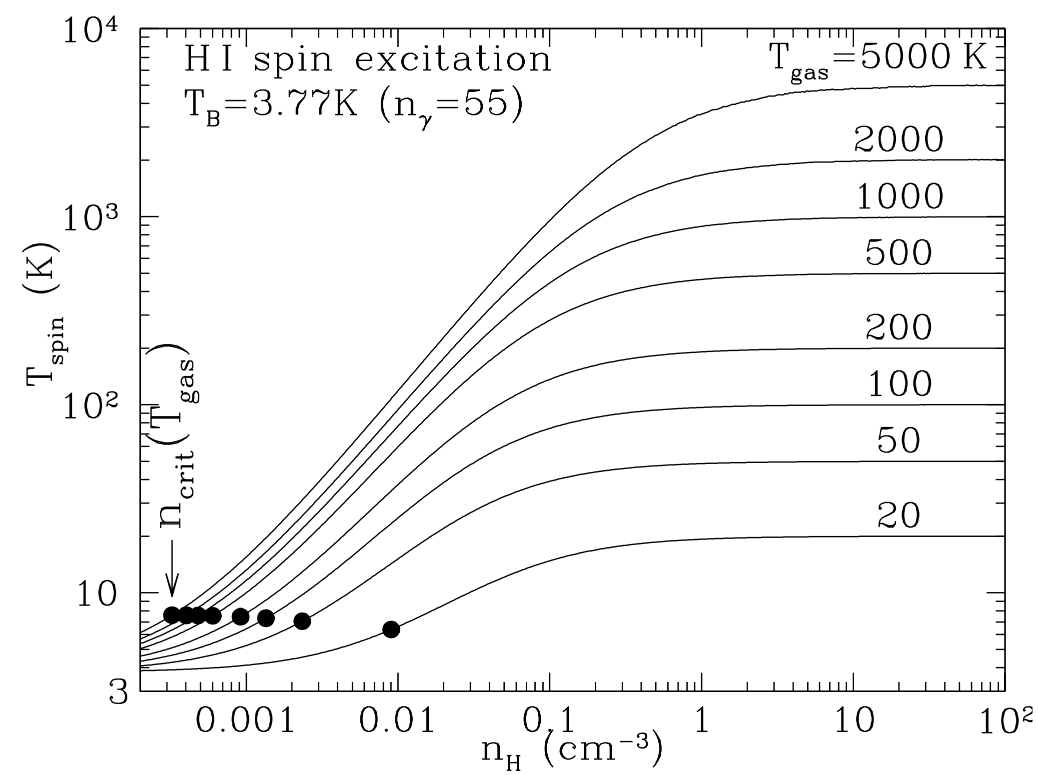
- Using the above equation, make a plot similar to the right side figure.

- Denote the two critical densities, for each gas temperature, defined by

Eq(b): $$n^*_{\rm crit} = \frac{T_{\rm gas}}{T_*} \frac{A_{10}}{k_{10}} \quad \text{and} \quad n_{\rm crit} = \frac{(1 + n_\gamma) A_{10}}{k_{10}}$$

(2) Discuss whether Eq(a) for the spin temperature for the 21-cm transition can be applied to the [C II] 158μm line or not.

Explain why the equation cannot be applied?



Supp: (1) Line strengths

- We know how to calculate $P_{10}^c = n_c k_{10}^c$ for a colliding partner with a density n_c. But, we need to find a way to calculate P_{10}^α , which is proportional to the radiation field strength.
  - The sum of all transitions from a given nJF to all the $n'J'$ levels (over all possible F') for a given $n'J'$ is proportional to $g_F = 2F + 1$.

$$\sum_{F'} S(nJF \to n'J'F') \propto 2F + 1, \quad \text{independent of } nJ$$

  - Applying the sum rule to four sets of downward transitions from $n = 2$ to $n' = 1$, the line strengths S_{ji} are

$$S_{51} : S_{41} + S_{40} : S_{31} + S_{30} : S_{21} = 5 : 3 : 3 : 1$$

  - Applying the sum rule to four sets of upward transitions from $n = 1$ to $n' = 2$,

$$S_{04} : S_{14} + S_{15} = 1 : 3$$
$$S_{03} : S_{12} + S_{13} = 1 : 3$$

  - Note $S_{ij} = S_{ji}$. Then, solving the above relations gives

$$S_{51} : S_{41} : S_{40} : S_{31} : S_{30} : S_{21} = 5 : 1 : 2 : 2 : 1 : 1$$

Ref: Condon & Shortley, The Theory of Atomic Spectra, Chap. IX, p236

Supp: (2) Einstein A-coefficients

- Note

$$S_{ji} \propto g_j A_{ji} \quad \text{or} \quad \frac{S_{ji}}{S_\alpha} = \frac{g_j}{g_{\rm tot}} \frac{A_{ji}}{A_\alpha}$$

$$S_\alpha = \sum_{j=2}^{5} \sum_{i=0}^{1} S_{ji} \rightarrow \text{total line strength}$$

$$g_{\rm tot} = \sum_{j=2}^{5} g_j = 12 \rightarrow \text{total statistical weight of n} = 2 \text{ level}$$

$$A_\alpha = \text{A coefficient for Ly}\alpha \text{ transition from n} = 2 \text{ to n} = 1$$

- From the above relations,

$$\frac{A_{20}}{A_\alpha} = \frac{A_{50}}{A_\alpha} = 0$$

$$\frac{A_{21}}{A_\alpha} = \frac{A_{51}}{A_\alpha} = 1$$

$$\frac{A_{30}}{A_\alpha} = \frac{A_{41}}{A_\alpha} = \frac{1}{3}$$

$$\frac{A_{31}}{A_\alpha} = \frac{A_{40}}{A_\alpha} = \frac{2}{3}$$

Supp: (3) Scattering Rate

It is convenient to introduce the “scattering rate,” which means the number of scatterings per an atom. By the definition of scattering cross section, the scattering rate is P_α.

$$P_\alpha = 4\pi \int \frac{J_\nu}{h\nu}\sigma_\nu d\nu \longleftarrow \left(\sigma_\nu = \frac{\mathrm{g}_u}{\mathrm{g}_\ell}\frac{c^2}{8\pi\nu_\alpha^2}A_\alpha\phi_\nu,\ n_\gamma = \frac{c^2}{2h\nu^3}J_\nu\right)$$

$$\begin{aligned} P_\alpha &= 4\pi \int \left(\frac{2\nu_\alpha^2}{c^2}n_\gamma\right)\left(\frac{\mathrm{g}_u}{\mathrm{g}_\ell}\frac{c^2}{8\pi\nu_\alpha^2}A_\alpha\phi_\nu\right)d\nu \\ &= \frac{\mathrm{g}_u}{\mathrm{g}_\ell}A_\alpha \int n_\gamma\phi_\nu d\nu \qquad \leftarrow \quad \left(\frac{g_u}{g_\ell} = \frac{g_{n=2}}{g_{n=1}} = \frac{5+3+3+1}{3+1} = 3\right) \\ &= 3A_\alpha n_\gamma(\nu_\alpha) \end{aligned}$$

$$\begin{aligned} P_{10}^\alpha &\simeq n_\gamma(\nu_\alpha)\sum_{j=2}^{5}\frac{\mathrm{g}_j}{\mathrm{g}_1}A_{j1}\frac{A_{j0}}{\sum_{i=0}^{1}A_{ji}} \\ &= n_\gamma(\nu_\alpha)\left(\frac{\mathrm{g}_3}{\mathrm{g}_1}A_{31}\frac{A_{30}}{A_{30}+A_{31}} + \frac{\mathrm{g}_4}{\mathrm{g}_1}A_{41}\frac{A_{40}}{A_{40}+A_{41}}\right) \\ &= n_\gamma(\nu_\alpha)\left(\frac{3}{3}\frac{2}{3}A_\alpha\frac{1}{3} + \frac{3}{3}\frac{1}{3}A_\alpha\frac{2}{3}\right) \qquad \leftarrow \quad (\mathrm{g}_j = 2F_j + 1) \\ &= \frac{4}{9}A_\alpha n_\gamma(\nu_\alpha) \end{aligned}$$

Therefore, we can express the downward transition rate coefficient in terms of the scattering rate:

$$P_{10}^\alpha = \frac{4}{27}P_\alpha$$

The scattering rate can be readily obtained in a Monte-Carlo simulation.