Modern Astronomy

Part 1. Interstellar Medium (ISM)

Week 1
March 08 (Thuesday), 2022
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선광일 (Kwang-il Seon)
KASI / UST

Professors & Classroom

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Day & Time:

Tuesday 3-6PM

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LWC(이원철홀) 102 for weeks 1-8 & 13-16

JYS(장영실홀) 329 for weeks 9-12

Syllabus

Week	Date	
1	03-08	Introduction / Interstellar Medium (ISM)
2	03-15	Atomic Gas / Multiphase Medium
3	03-22	Molecular Clouds and Dust
4	03-29	Formation of Stars and Planets
5	04-05	Stars: The Hertzsprung-Russell Diagram
6	04-12	The Evolution of Stars
7	04-19	Star Deaths
8	04-26	The Milky Way Galaxy
9	05-03	Galaxies beyond the Milky Way
10	05-10	Hubble’s Law and Distance Scale
11	05-17	Active Galaxies
12	05-24	Active Galaxies
13	05-31	General Relativity and Friedmann Equation
14	06-07	Evolution of Universe and Inflation Cosmology
15	06-14	Density Pertubations and Nonlinear Structure Formation
16	06-21	Survey and Computer Simulation

Introduction to the Interstellar Medium

What is the ISM?

What is the ISM?

- The ISM is anything not in stars. (D. E. Osterbrock)
- Just what it says: The stuff between the stars in and around galaxies, especially our own Milky Way.
- It is made up almost entirely of gas with a tiny (solid) particles called dust grains.
  - In addition to these, the ISM includes radiation, cosmic rays, and magnetic fields.

Why do we study the ISM?

Why do we study the ISM?

- The ISM is the most beautiful component of galaxies. (B. T. Draine)

- The ISM is beautiful, both in the literal sense, as in images of colorful nebulae, and in the physics that helps us understand our origins and the way the Universe works. (J. P. Williams)

- The ISM is everywhere and it affects all sorts of observations, but more often as an essential complement for understanding the Galaxy.

- The ISM is the most important component of galaxies, for it is the ISM that is responsible for forming the stars that are the dominant sources of energy.

The objective of studying the ISM is to understand:

- how the ISM is organized and distributed in the Milky Way and other galaxies

- what are the conditions (temperature, density, ionization, etc) in different parts of it.

- how it dynamically evolves.

- And finally, we would like to understand star formation, the process responsible for the very existence of galaxies as luminous objects.

References

- Introductory
  - Introduction to the Interstellar Medium - Jonathan P. Williams
  - The Physics of the Interstellar Medium - J. E. Dyson & D.A. Williams
- Intermediate
  - Interstellar and Intergalactic Medium - Barbara Ryden
- Advanced
  - Radiative Processes in Astrophysics - George B. Rybicki & Alan P. Lightman
  - Physics and Chemistry of the Interstellar Medium - Sun Kwok
  - Astrophysics of the Diffuse Universe - Michael A. Dopita & Ralph S. Sutherland
  - Physics of the Interstellar and Intergalactic Medium - Bruce T. Draine
  - The Physics and Chemistry of the Interstellar Medium - A. G. G. M. Tielens

History of ISM Studies

Aether

- Early Greek astronomers believed that the volume inside the celestial sphere was filled with a diffuse aether, or quintessence.

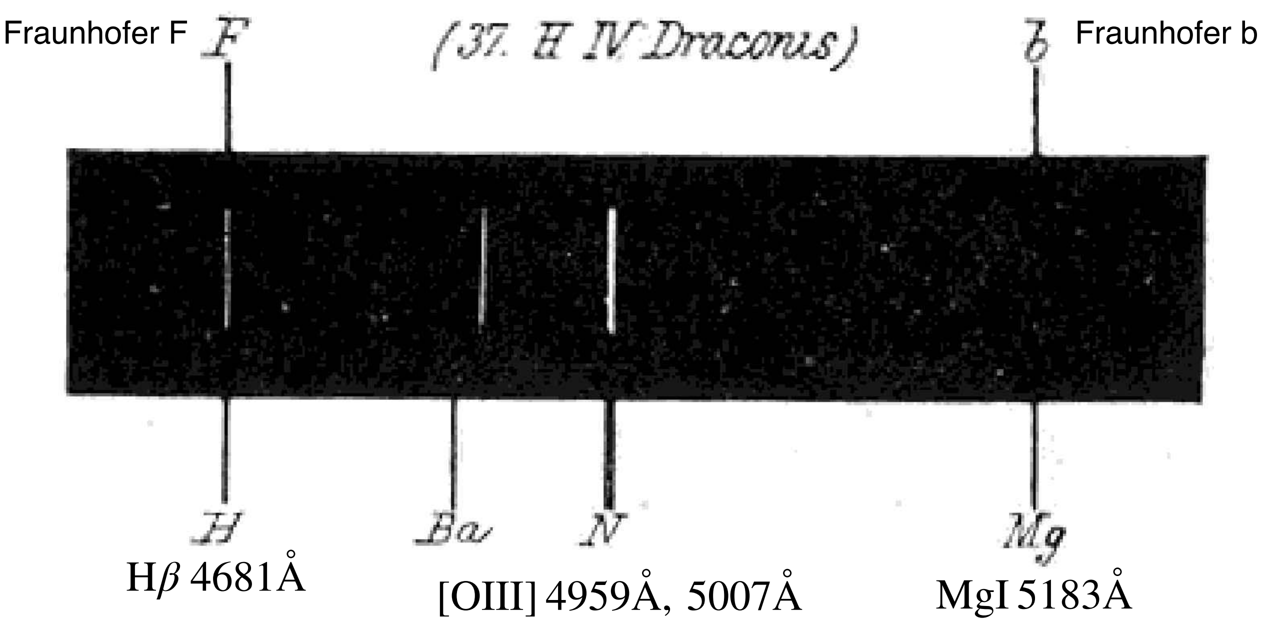
- For centuries, the idea of a space-filling aether still lingered. Even Issac Newton postulated **"an aether medium,"** which is so rare and subtle as to be undetectable, and strongly elastic.

Interstellar material

- The idea of visible, interstellar material arose in the 18th century, with the study of nebulae (Latin for "clouds"). "Nebula" was used to mean any extended luminous object.

the Cat's Eye Nebula (planetary neubla/ HST image)

The first nebula spectrum: the Cat's Eye Nebula (NGC 6543; W. Huggins, 1864)

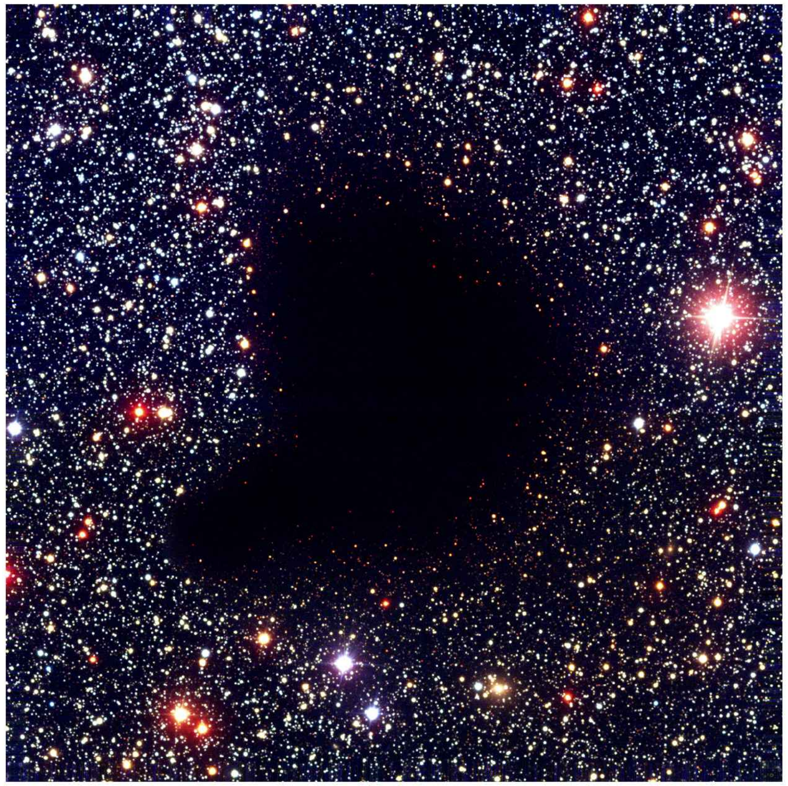


- William Herschel resolved some nebulae into stars. He though that he had discovered star birth (it actually ejected by a dying star). In the 1860s, William Huggins demonstrated that some nebulae have **emission line spectra**, rather than the **absorption line spectra**.
- ***Hypothetical elements***:
  - ✦ Huggins attributed 4959Å, observed in the Cat's Eye Nebula, to "nebulum" (or "nebulium"), and 5007Å line to Nitrogen => Ira Bowen discovered that these lines were actually forbidden [O III] lines.
  - ✦ aurorium : 5577Å in the spectrum of the aurora borealis => turned out to be [O I]
  - ✦ coronium: 5303Å in the spectrum of the Sun's corona => Fe XIV

Interstellar Dust

- The existence of dust had been hinted at by the presence of dark nebulae (Barnard 68).
  - ✦ The dark nebulae were originally thought to be due to a lack of stars, but later recognized as being clouds of obscuring material.
- Vesto Slipher (1912) discovered that the spectrum of the nebula surrounding the Pleiades shows a continuum with absorption lines superposed.
  - ✦ He conjectured that this is light from stars, reflected from “fragmentary and disintegrated matter”, or dust.

V. Slipher (/slaifer/ 1875-1969) is the first one who measured radial velocities for galaxies and discovered that distant galaxies are redshifted. He was also the first to relate these redshifts to velocity.



Barnard 68 (at d ~ 150 pc), in the constellation Ophiuchus.

The Pleiades cluster & surrounding reflection nebulae

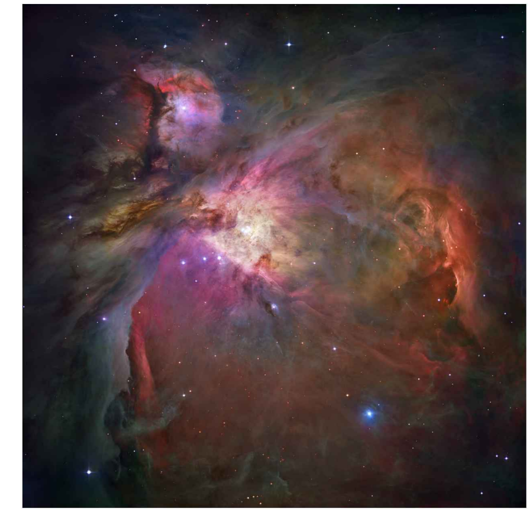
Interstellar gas that is invisible to the eye

- Initially, bright nebulae were thought of as isolated clouds in (nearly) empty space.
- In 1901, Johannes Hartmann found:
  - ✦ the spectrum of binary Delta Orionis (a spectroscopy binary system) shows a narrow calcium absorption line (at $\lambda 3924$) that is in **stationary** in addition to the **oscillating**, broad absorption lines.
  - ✦ the stationary Ca absorption line was caused by a gas cloud somewhere along the line of sight to Delta Orionis.
- Later, similar "stationary lines" were found along the sightlines to many other bright stars.
  - ✦ The lines were all narrow, and had strengths correlated with the distance to the background star.
  - ✦ Using higher resolution spectrographs, they had been revealed to have complex structures, consisting of many narrower lines with different radial velocities.
  - ✦ This indicates that the ISM has a complex structure, consisting neither of smooth uniform gas nor of isolated blobs drifting about in a vacuum.

Diversity of the ISM

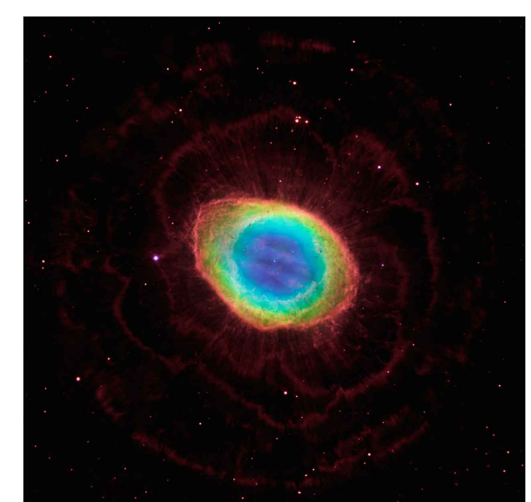
Ionized nebulae

- H II regions
  - are regions of interstellar gas heated and photoionized by embedded O or B-type stars.
  - In 1939, Bengt Stromgren developed the idea that bright nebulae with strong emission lines are regions of photoionized gas, surrounding hot star or other source of ionizing photons.
  - ex) Orion Nebula
- Planetary nebulae
  - are regions of ejected stellar gas heated and photoionized by the hot remnant stellar core, which is becoming a white dwarf.
  - ex) Ring Nebula, Cat's Eye Nebula
  - Ring Nebula:
    - ✦ central blue-green colors from [O III] 4959, 5007
    - ✦ outer reddish colors from Hα 6563, [N II] 6548, 6583



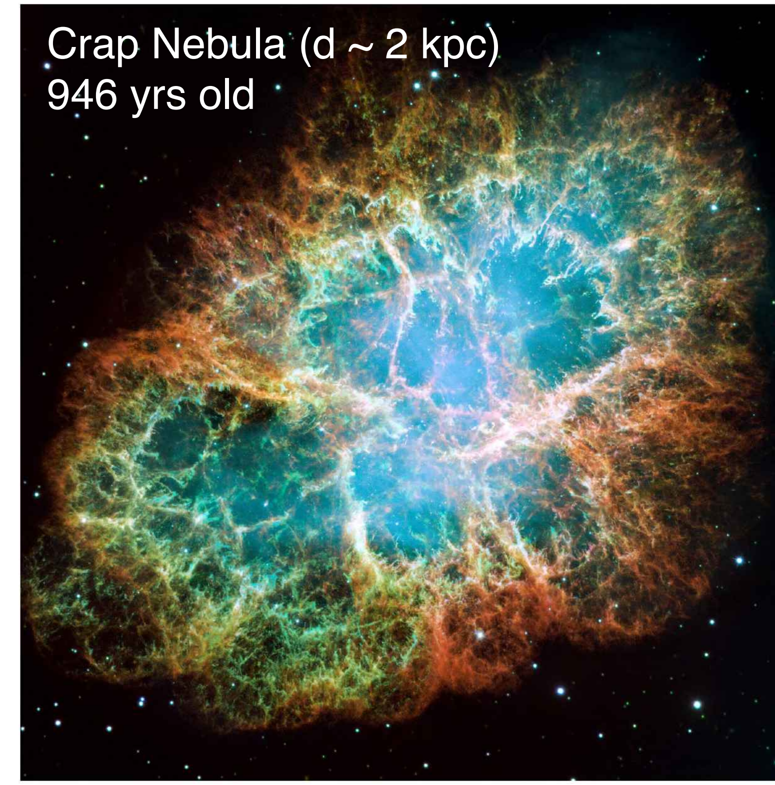
Orion Nebula (d ~ 410 pc)
HST image

Cat's Eye Nebula (HST image)

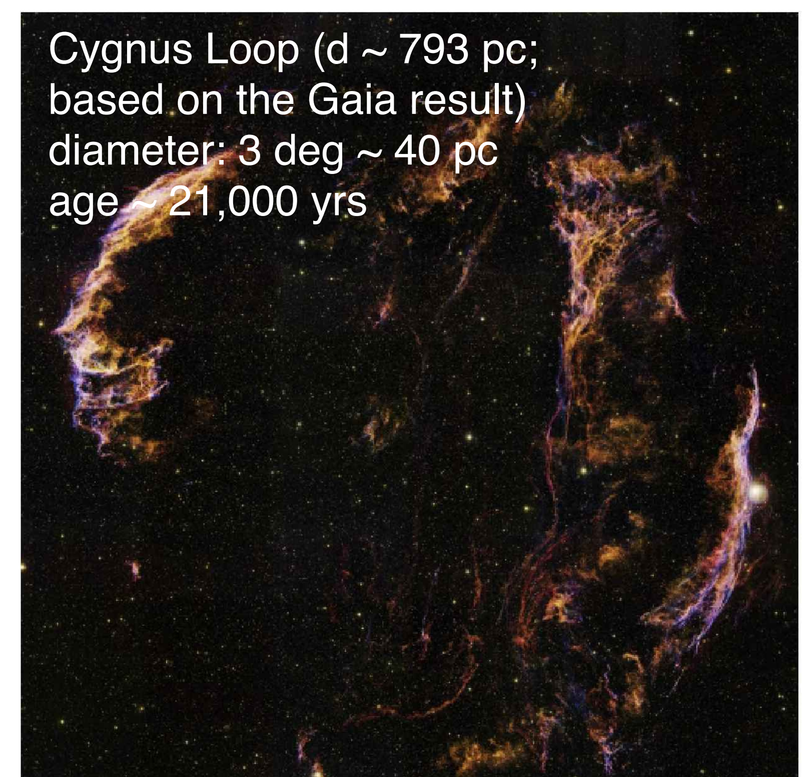


Ring Nebula (HST image)

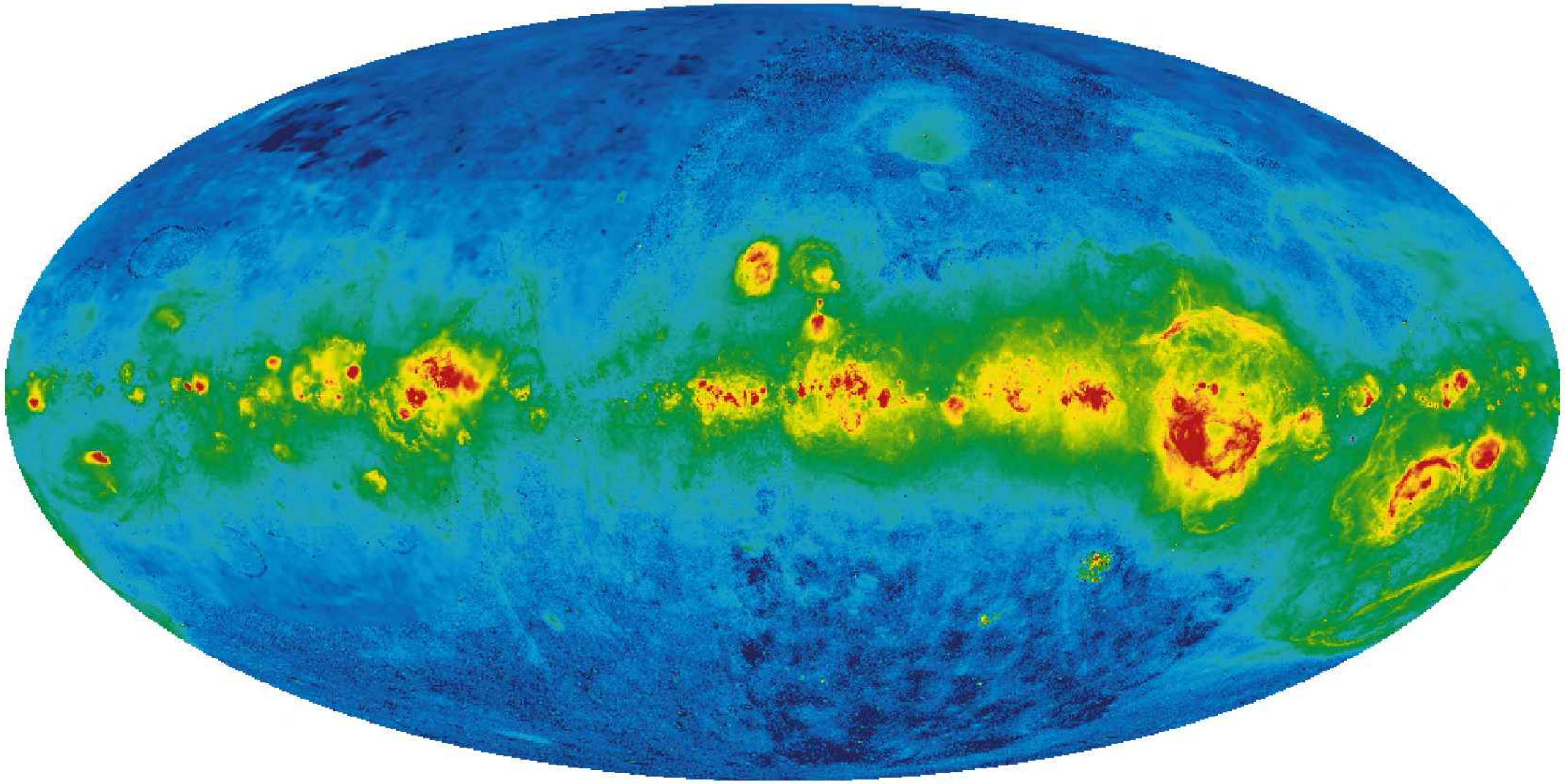
- Supernova remnants
  - are regions of gas heated by the blast wave from a supernova explosion.
  - Crab Nebula
    - ✦ a young pulsar-containing supernova remnant
    - ✦ are filled in with luminous gas.
    - ✦ are photoionized by its central pulsar.



  - Cygnus Loop (Veil Nebula)
    - ✦ most of the gas has been plowed up by the blast wave, leaving the center part empty.
    - ✦ The visible loop (or veil) is where the gas has cooled to T ~ 10,000 K.



- Warm Ionized Medium
  - About 20-80% of the ionized hydrogen in our galaxy lies in the relatively low density WIM.
  - Balmer line emission from recombining hydrogen fills the entire sky.
  - Although many ionized nebula (Orion, Crab, Cat's eye, etc) can be seen as the bright red blotches, they are not the dominant repository of recombining hydrogen in our galaxy.



All-sky map of Hα (6563Å) in a log scale from 0.03 Ry to 160 Ry.
Ry (rayleigh) = $10^6/4\pi$ photons cm^{-2} s^{-1} cm^{-2} Hz^{-1}

- Neutral Hydrogen Gas
  - All-sky map of H I 21-cm line intensity from the LAB survey (Kalberla et al. 2005), with an angular resolution ~ 0.6 deg.
  - Scale gives $\log_{10}$ N(HI) [cm^{-2}]. The LMC and SMC are visible, with a connecting H I “bridge”.

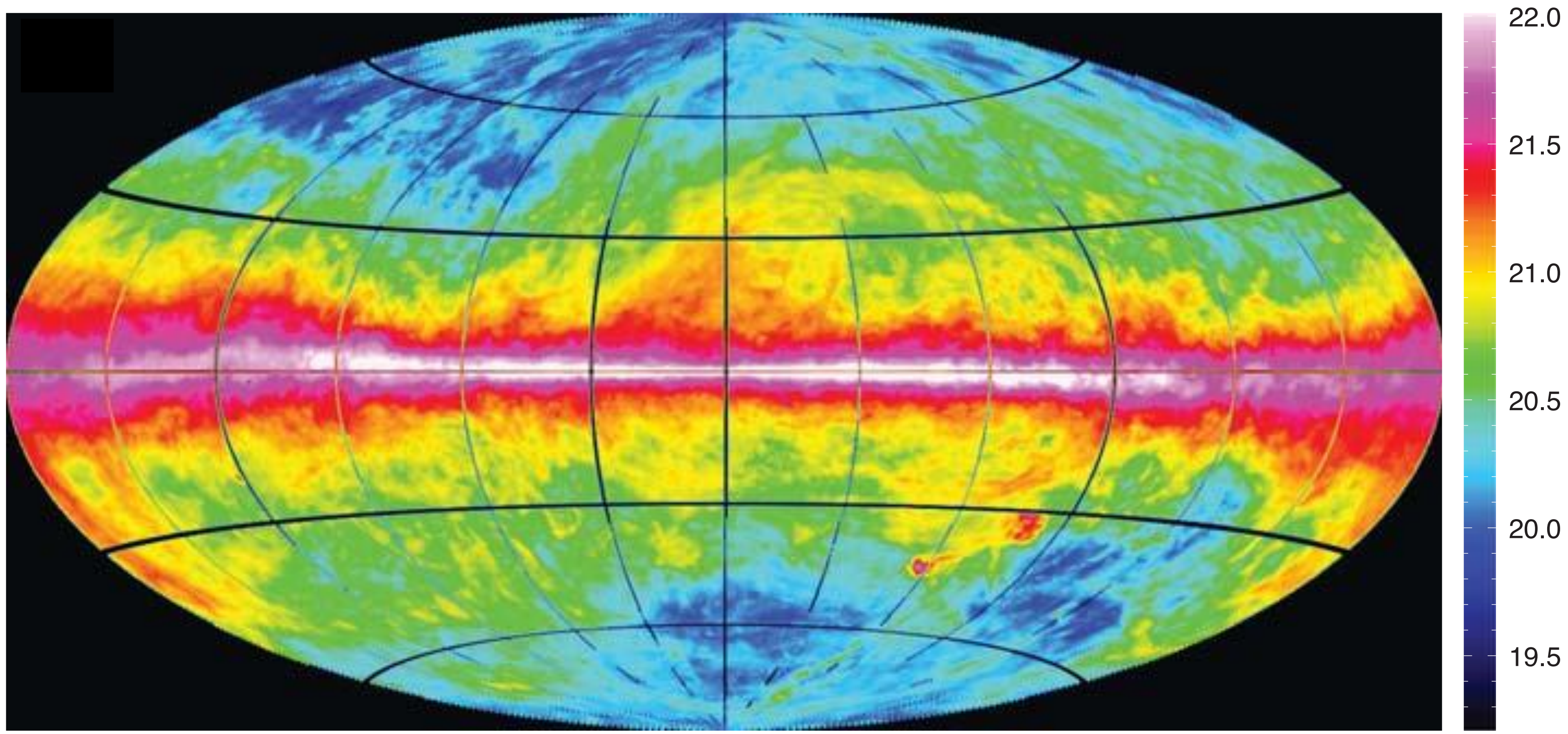


Plate 3 in [Draine]

Baryonic Matter

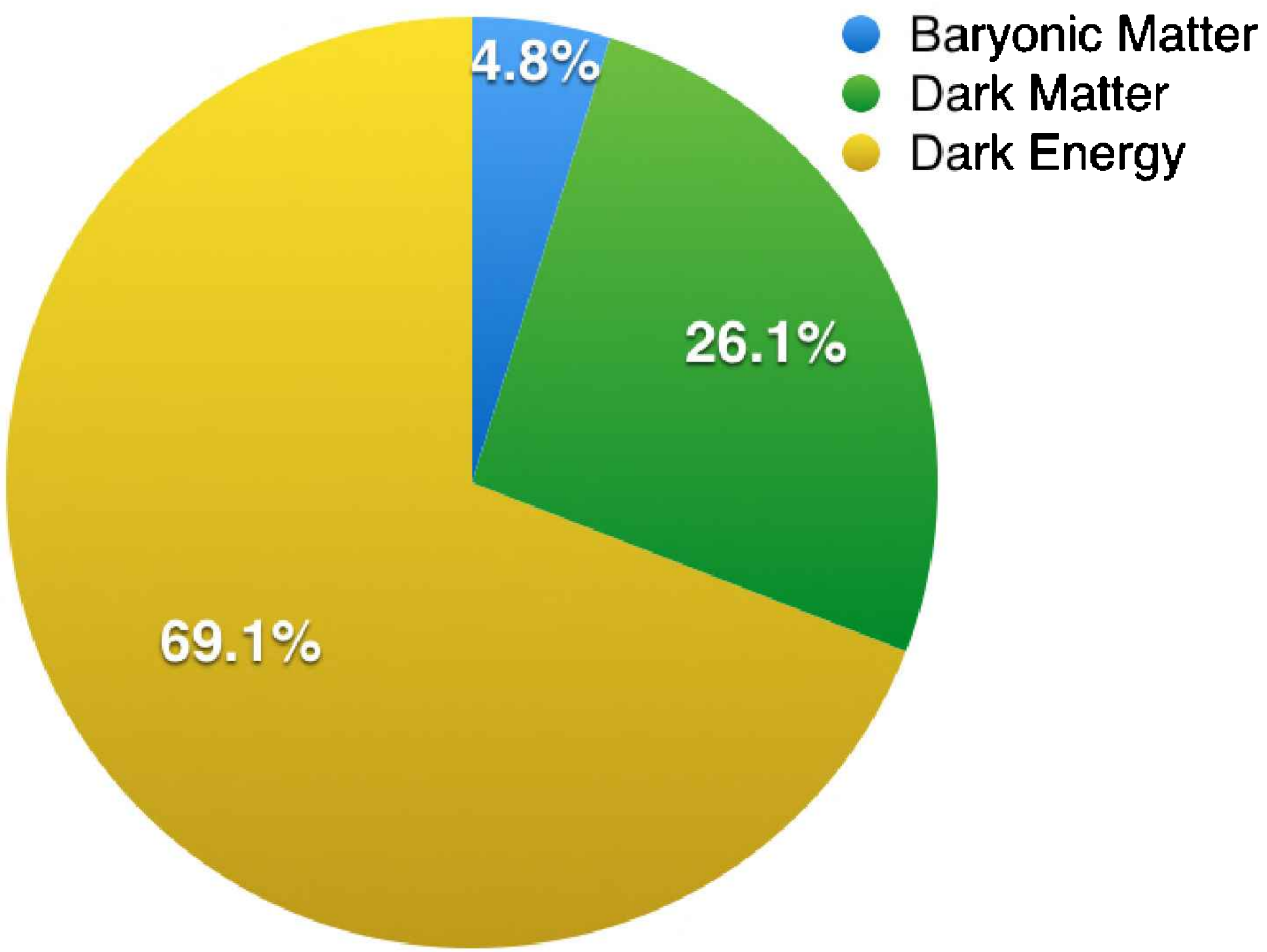
Definitions:

- Baryons = protons, neutrons and matter composed of them (i.e. atomic nuclei)
- Leptons = electrons, neutrinos
- In astronomy, however, the term ‘**baryonic matter**’ is used more loosely to refer to **matter that is made of protons, neutrons, and electrons**, since protons and neutrons are always accompanied by electrons. Neutrinos, on the other hand, are considered non-baryonic by astronomers. (Note that black holes are also included as baryonic matter.)

The mass-energy density

Relative contribution of baryons, dark matter, and dark energy to the mass-energy density of the current universe (Planck 2015)

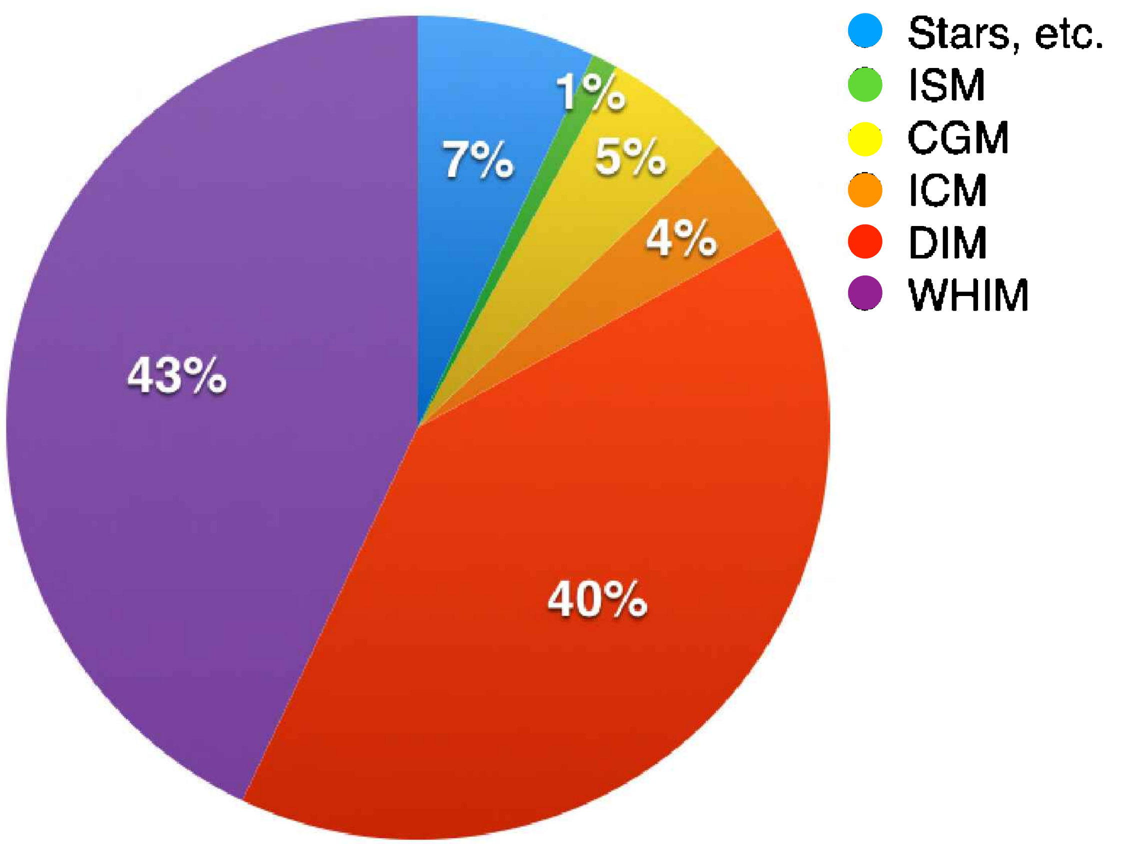
- The majority of the universe is made of dark energy and dark matter.
- Dark energy is ignored until we discuss cosmic evolution.
- Dark matter is important only because it provides potential wells for baryonic matter to be trapped in.



The baryonic mass density

The baryonic mass density in the current universe

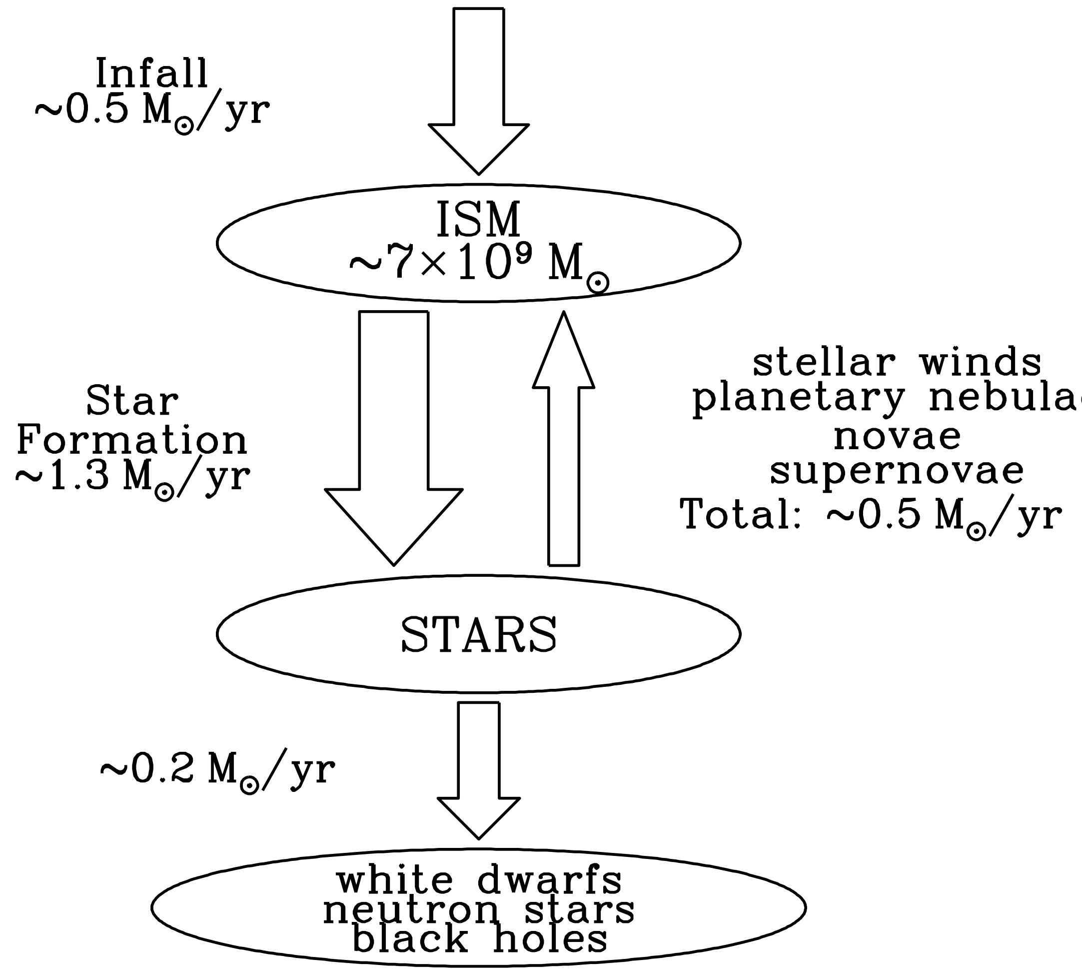
- 7% : stars + compact objects (such as stellar remnants, brown dwarfs, and planets)
- 1% : interstellar medium (ISM), filling the volume between stars.
- 5%: circumgalactic medium (CGM), bound within the dark halo of a galaxy, but outside the main distribution of stars.
- 4% : intracluster medium (ICM) of clusters of galaxies, bound to the cluster as a whole, but not to any individual galaxy.
- 40%: diffuse intergalactic medium (DIM), made of low density, mostly photo-ionized gas ($T < 10^5$ K).
- 43% : warm-hot intergalactic medium (WHIM), made of shock-heated gas (10^5 K $< T < 10^7$ K).



Mass flow of the baryons in galaxies

- At early times, the baryonic mass in galaxies was primarily in the gas of the ISM. As galaxies evolve, the ISM is gradually converted to stars, and some part of the interstellar gas may be ejected from the galaxy in the form of galactic winds, or in some cases stripped from the galaxy by the IGM.

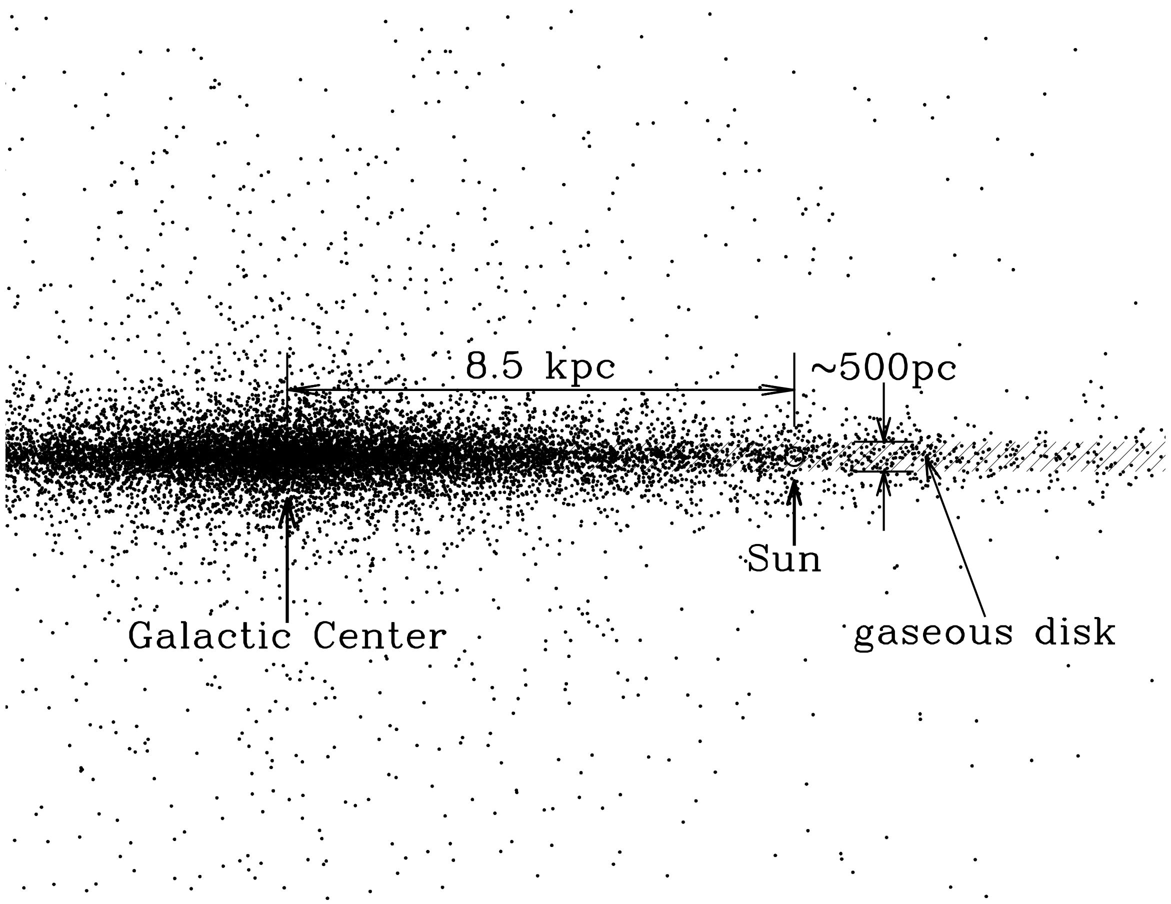
- About 10% of the baryons in the Milky Way are to be found in the ISM.

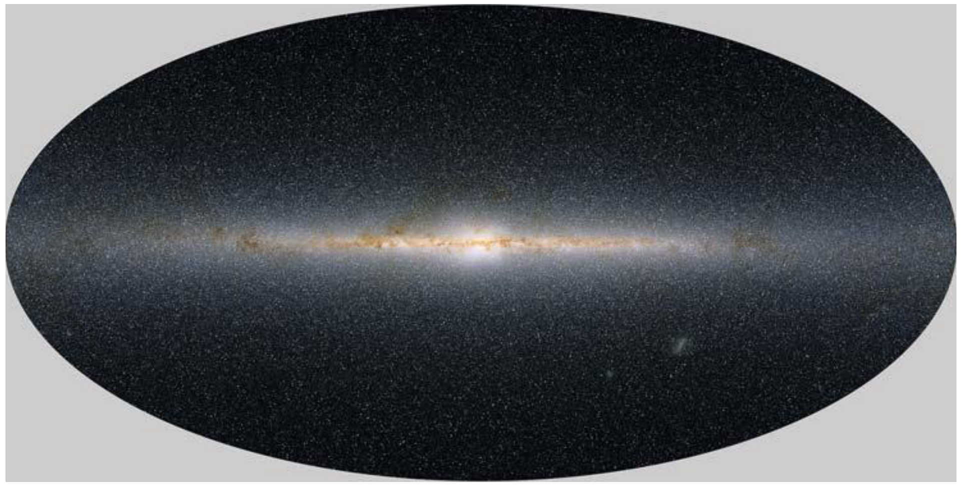


Flow of baryons in the Milky Way.

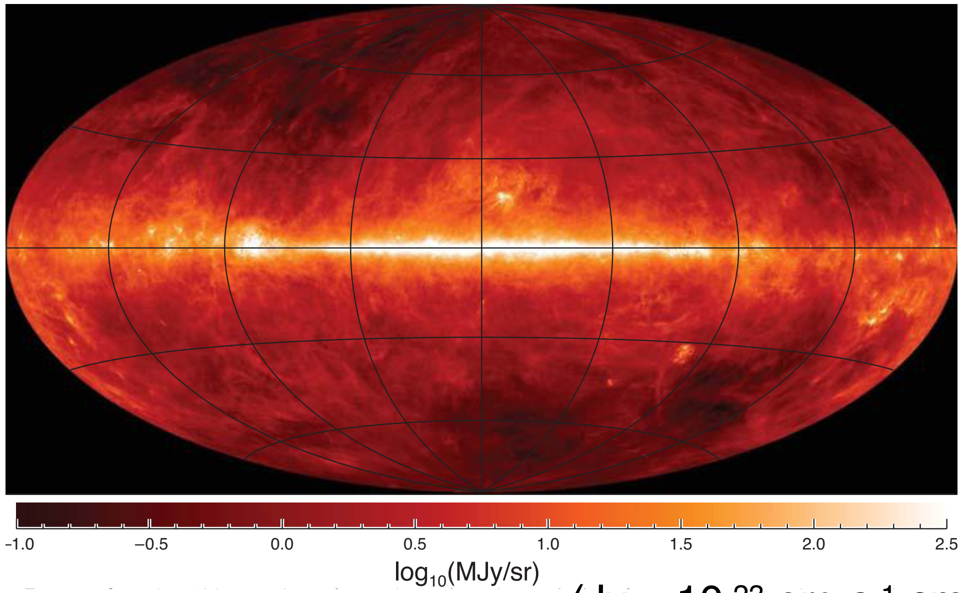
B. T. Draine

Structure of the Milky Way





2MASS survey
$\sim 5 \times 10^8$ stars
blue = 1.2 µm
green = 1.65 µm
red = 2.2 µm



IRAS+COBE
100 µm
dust emission

(Jy = 10^{-23} erg s^{-1} cm^{-2} Hz^{-1})

- Total mass of the Milky Way ~ $10^{11} M_\odot$ ($M_\odot = 1.989 \times 10^{33}$g)
  - stars $\sim 5 \times 10^{10} M_\odot$
  - dark matter $\sim 5 \times 10^{10} M_\odot$
  - interstellar gas $\sim 7 \times 10^{9} M_\odot$ (mostly H + He)
    - ✦ Hydrogen mass: neutral H atoms ~ 60%, H_2 molecules ~ 20%, ionized H^+ atoms ~ 20%

Phase	$M\,(10^9\ M_\odot)$	fraction
Total H II (not including He)	1.12	23%
Total H I (not including He)	2.9	60%
Total H_2 (not including He)	0.84	17%
Total H II, H I and H_2 (not including He)	4.9	
Total gas (including He)	6.7	

ISM = dust + gas

Dust

- dust = tiny grains of solid material
  - Historically, courses on the ISM have dealt with “non-stellar stuffs.”
  - The dust and gas strongly influence each other.
    - ✦ Dust reprocesses sunlight, altering the radiation field passing through the gas.
    - ✦ Dust is made of refractory elements, so creating dust alters the chemical abundances of the surrounding gas.
    - ✦ Dust grains are a leading source of free electrons in the interstellar gas (neutral gas).
    - ✦ Gas molecules form on the surfaces of dust grains.

Gas

- Interstellar gas occupies the same region as stars.
- Stars are made from interstellar gas, and emit stellar winds into the ISM over the course of their lives. When massive stars reach the end of their lifetimes, they inject enriched gas at high speeds into the surrounding interstellar gas.
- Stars emit photons that are capable of exciting the interstellar gas. The emission lines have strong diagnostic power, enabling us to determine densities, temperatures, and ionization states of interstellar gas.

Abundance of elements in the local ISM

Element	Abundance (ppm)	Atomic number	1st ionization energy (eV)
hydrogen (H)	911,900	1	13.60
helium (He)	87,100	2	24.59
oxygen (O)	490	8	13.62
carbon (C)	270	6	11.26
neon (Ne)	85	10	21.56
nitrogen (N)	68	7	14.53
magnesium (Mg)	40	12	7.65
silicon (Si)	32	14	8.15
iron (Fe)	32	26	7.90
sulfur (S)	13	16	10.36

(ppm = parts per million)

H : 91.2% by number
He: 8.7%
others: 0.1%

The interstellar gas is primarily H and He resisting from the Big Bang.

A small amount of heavy elements was produced as the result of the return to the ISM of gas that has been processed in stars and stellar explosions.

$M(Z > 2)/M_{\rm H} = 0.0199;\ M(\text{total})/M_{\rm H} = 1.402$

Density of the ISM

By terrestrial standards, the ISM is an almost perfect vacuum.

- The typical distance between stars is about 2 pc = 6x10^{16} m.
  - This is ~ 100 million times greater than the solar radius and 4000 times greater than the size of its heliosphere.

- ISM density
  - Total ISM mass is $\sim 7 \times 10^9 M_\odot$.
  - Approximating the Galaxy as a cylinder with a radius R ~ 10 kpc and a scale height H ~ 250 pc, this implies an average density

$$\rho = \frac{M}{\pi R^2 \times 2H} \approx 3 \times 10^{-21}\ \mathrm{kg\ m^{-3}}$$

$$\rho = n_\mathrm{H} m_\mathrm{H} + n_\mathrm{He} m_\mathrm{He} \approx \left(1 + \frac{1}{10} \times 4\right) n_\mathrm{H} m_\mathrm{H} \longleftarrow \begin{array}{l} n_\mathrm{He} \sim 0.1\ n_\mathrm{H} \\ m_\mathrm{He} \sim 4\ m_\mathrm{H} \end{array}$$

$$\boxed{n_\mathrm{H} \approx \frac{\rho}{1.4 m_\mathrm{H}} \simeq 1.3\ \mathrm{cm^{-3}}}$$

- Density of Air
  - From the ideal gas law using the pressure at sea level, $P = 10^5 \ \mathrm{N\ m^{-2}}\ (1\ \mathrm{bar})$, and temperature $T \approx 300\ \mathrm{K}$, we get

$$\boxed{n = P/k_{\mathrm{B}}T \approx 2.4 \times 10^{19}\ \mathrm{cm}^{-3}}$$

19 orders of magnitude higher than the average density in the ISM.

  - The extremely low density in the ISM mean that particle collisions are relatively rare, which allows us to observe some physical processes that we don't see on Earth (e.g., forbidden lines).

Typical pressure & Energy densities

Typical pressure of the ISM

- $P = nk_\mathrm{B}T \sim 4 \times 10^{-13}\,\mathrm{dyn\,cm^{-2}} \sim 4 \times 10^{-19}\,\mathrm{atm}$ (atmospheric pressure; 1 bar = 0.987 atm)
- Here, Boltzmann constant, $k_B = 1.38 \times 10^{-16}\ \mathrm{cm^2\ g\ s^{-2}\ K^{-1}}$
- This is extremely low pressure compared to the atmospheric pressure around us.

Energy density

$$\begin{aligned}\varepsilon &= \frac{3}{2} n k_\mathrm{B} T \\ &\sim 6 \times 10^{-13}\,\mathrm{erg\,cm^{-3}} \\ &\sim 0.4\,\mathrm{eV\,cm^{-3}}\end{aligned}$$

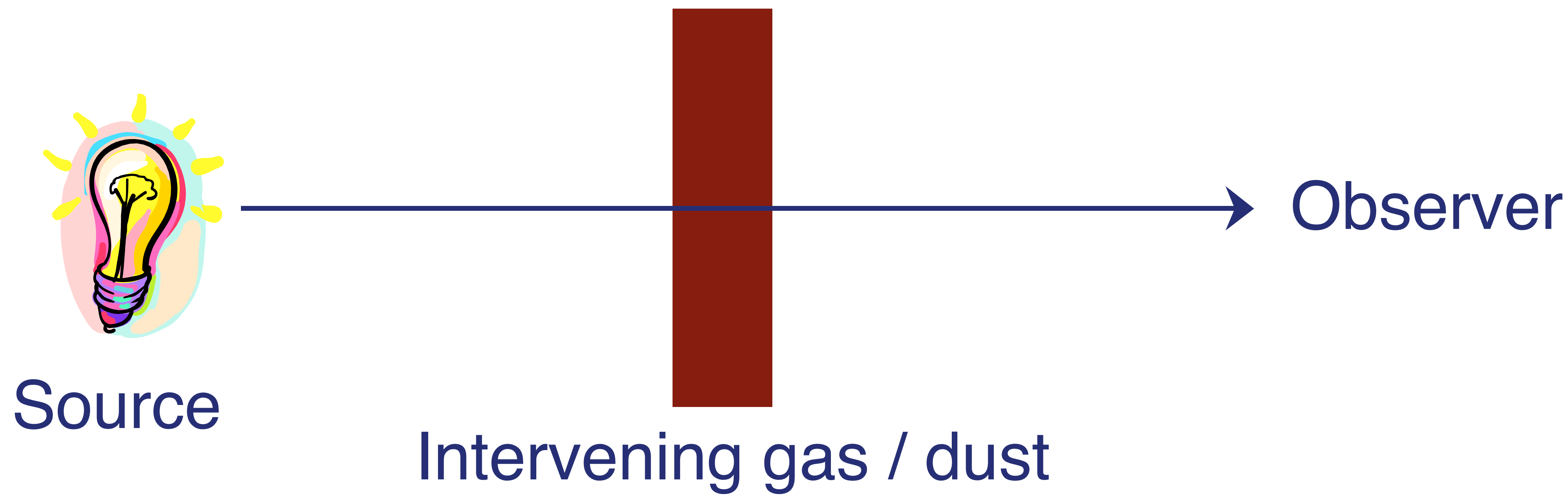
Type	Energy density ($\mathrm{eV\,cm^{-3}}$)
Thermal energy	0.4
Turbulent kinetic energy	0.2
Cosmic microwave background	0.2606
Far-infrared from dust	0.3
Optical/near-IR from stars	0.6
Magnetic energy	0.9
Cosmic rays	1.4

- All of them are comparable in energy density.
- All energy densities in the ISM are roughly half an electron-volt per cubic centimeter.
- The near-equipartition is partly coincidental.
  - ✦ The fact that the energy density in the CMB is similar to the other energy densities is surely accidental.
  - ✦ But the other energy densities are in fact coupled, roughly regulated by feedback mechanisms between them.

Radiative Transfer

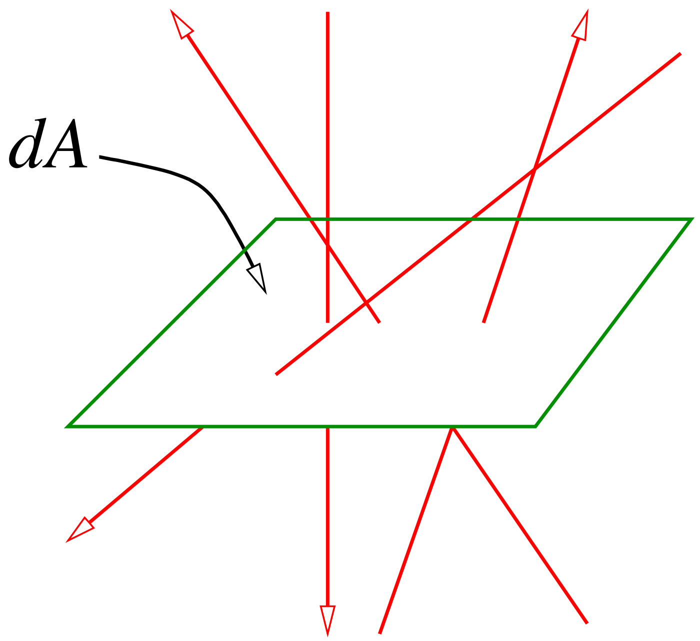
Radiative Transfer

- How is radiation affected as it propagates through intervening gas and dust media to the observer?



Simplication & Complexity

- Simplification:
  - Astronomical objects are normally much larger than the wavelength of radiation they emit.
  - Diffraction can be neglected.
  - Light rays travel to us along straight lines.

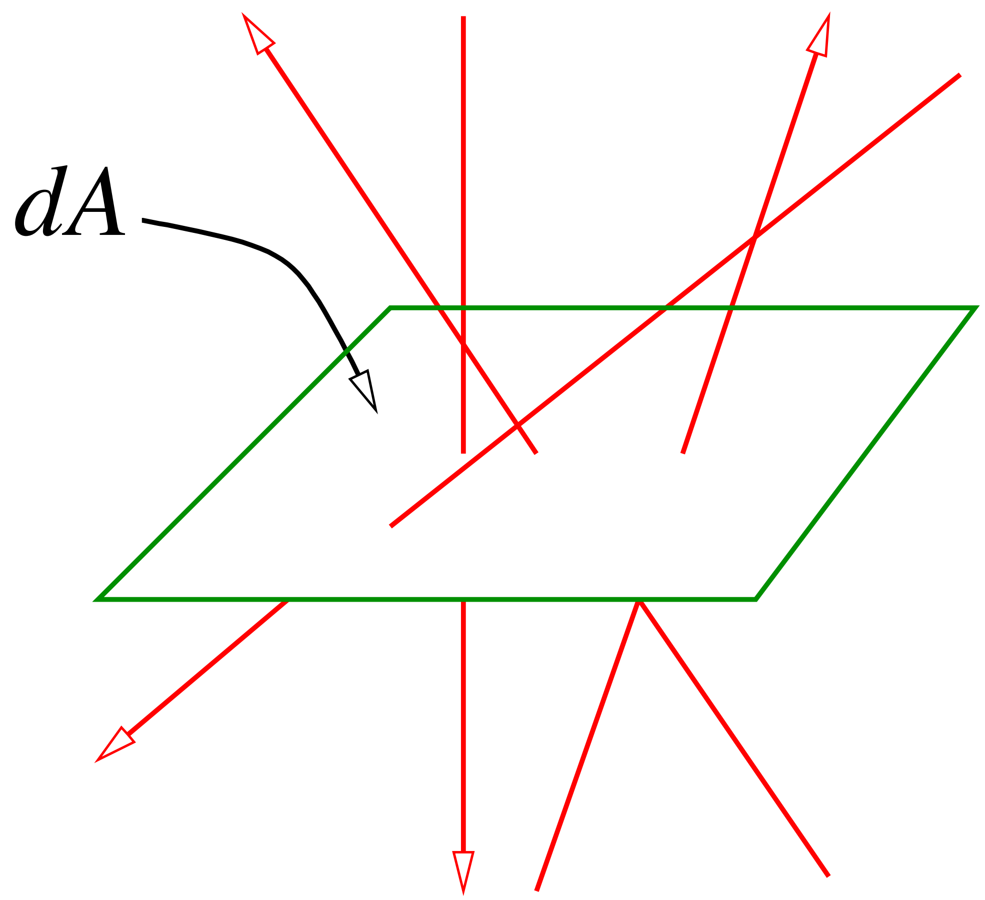


- Complexity:
  - At one point, photons can be traveling in several different directions.
  - For instance, at the center of a star, photons are moving equally in all directions. (However, radiation from a star seen by a distant observer is moving almost exactly radially.)
  - Full specification of radiation needs to say how much radiation is moving in each direction at every point. Therefore, we are dealing with the five- or six-dimensional problem. ($[x, y, z] + [\theta, \phi] + [t]$)

Energy Flux

- Definition
  - Consider a small area dA, exposed to radiation for a time dt.
  - Energy flux F is defined as ***the net energy dE passing through the element of area in all directions in the time interval*** so that

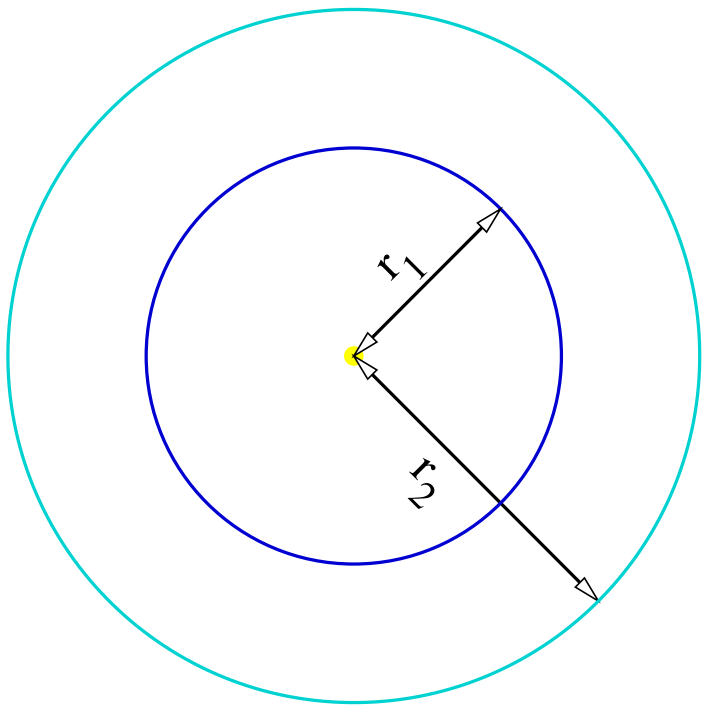
$$dE = F \times dA \times dt$$



  - Note that F ***depends on the orientation of the area element*** dA.
  - Unit: $\mathrm{erg\ cm^{-2}\ s^{-1}}$

Inverse Square Law

- Flux from an isotropic radiation source, i.e., a source emitting equal amounts of energy in all directions.



  - Because of energy conservation, flux through two shells around the source must be the same.

$$4\pi r_1^2 F(r_1) = 4\pi r_2^2 F(r_2)$$

  - Therefore, we obtain the inverse square law.

$$F = \frac{\text{const.}}{r^2}$$

Energy Flux Density

- Real detectors are sensitive to a limited range of wavelengths. We need to consider how the incident radiation is distributed over frequency.

Total energy flux: $F = \int F_\nu d\nu$ Integral of F_ν over all frequencies

$\downarrow$

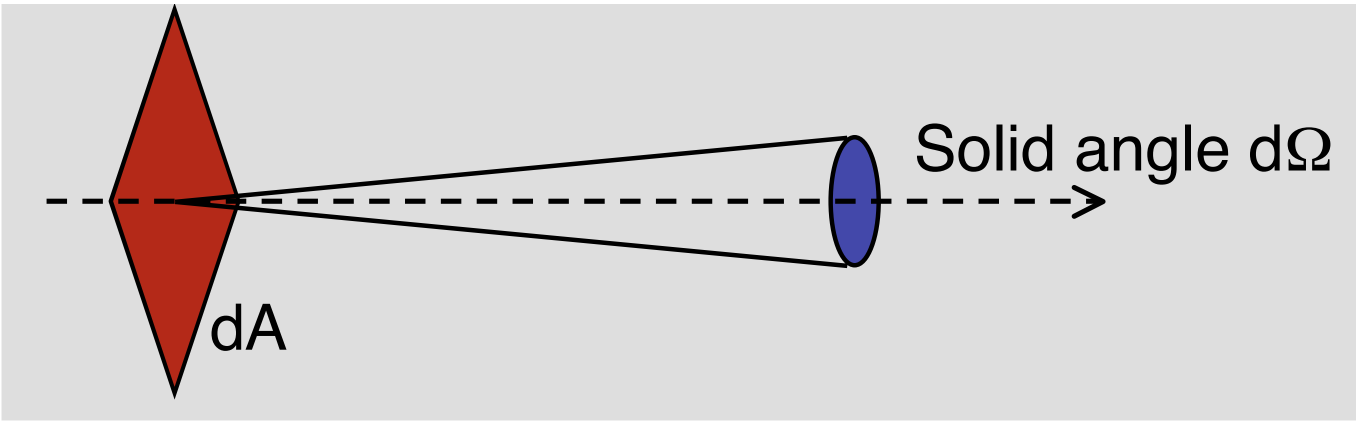
Units: $\mathrm{erg\ s^{-1}\ cm^{-2}\ Hz^{-1}}$

- F_ν is often called the "flux density."

- Radio astronomers use a special unit to define the flux density:
$1\ \mathrm{Jansky\ (Jy)} = 10^{-23}\ \mathrm{erg\ s^{-1}\ cm^{-2}\ Hz^{-1}}$

Specific Intensity or Surface Brightness

- Recall that ***flux is a measure of the energy carried by all rays passing through a given area***
- ***Intensity is the energy carried along by individual rays.***



$\mathbf{k}$ = direction of propagation

- Let dE_ν be the amount of radiant energy which crosses the area dA in a direction $\mathbf{k}$ within solid angle $d\Omega$ about in a time interval dt with photon frequency between ν and $\nu + d\nu$.
- The monochromatic specific intensity I_ν is then defined by the equation.

$$dE_\nu = I_\nu(\mathbf{k}, \mathbf{x}, t) dA_\mathbf{k} d\Omega d\nu dt$$

- Unit: $\mathrm{erg\ s^{-1}\ cm^{-2}\ sr^{-1}\ Hz^{-1}}$

- From the view point of an observer, the specific intensity is called ***surface brightness***.

Relation between the flux and the specific intensity

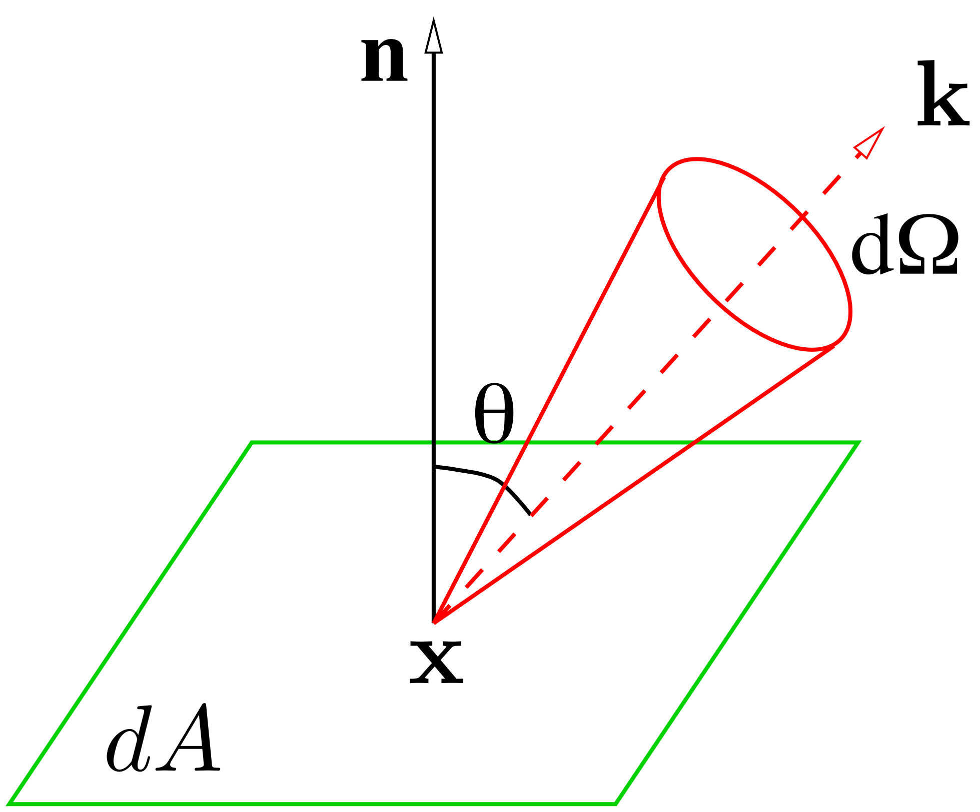
- Let's consider a small area dA, with light rays passing through it at all angles to the normal vector $\mathbf{n}$ of the surface.

- For a ray centered about $\mathbf{k}$, the area normal to $\mathbf{k}$ is

$$dA_{\mathbf{k}} = dA\cos\theta$$

  - By the definition,

$$F_\nu dA d\nu dt = \int I_\nu(\mathbf{k}, \mathbf{x}, t) dA_{\mathbf{k}} d\Omega d\nu dt$$



- Hence, net flux in the direction of $\mathbf{n}$ is given by integrating over all solid angles:

$$F_\nu = \int I_\nu \cos\theta d\Omega = \int_0^{2\pi}\int_0^{\pi} I_\nu \cos\theta \sin\theta d\theta d\phi$$

[Note] **flux = "sum of all ray vectors" which is then projected onto a normal vector**
intensity = magnitude of a single ray vector

Note

- Intensity can be defined as per wavelength interval.

$$I_\nu |d\nu| = I_\lambda |d\lambda| \qquad \leftarrow \left|\frac{d\nu}{d\lambda}\right| = \frac{c}{\lambda^2} = \frac{\nu}{\lambda}$$
$$\nu I_\nu = \lambda I_\lambda$$

- Integrated intensity is defined as the intensity over all frequencies.

$$I = \int_0^\infty I_\nu d\nu = \int_0^\infty I_\lambda d\lambda$$

- In astrophysics, we plot the ***spectral energy distribution (SED)*** as νI_ν versus ν or λI_λ versus λ.

Radiative Transfer Equation in free space

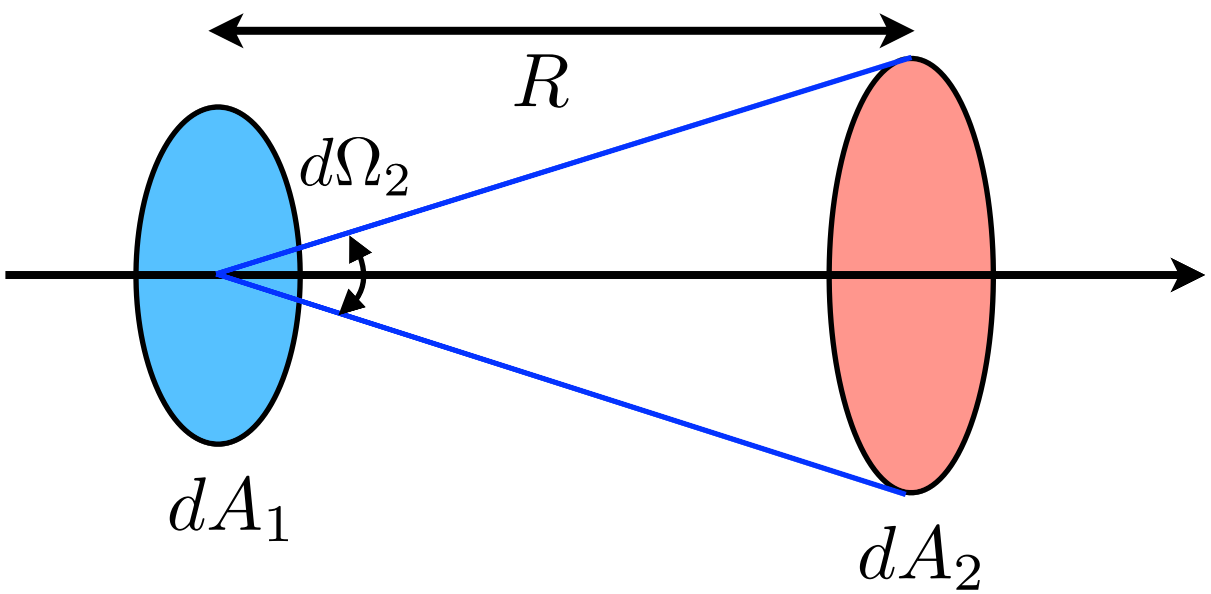
- How does specific intensity changes along a ray in free space
  - Suppose a bundle of rays and any two points along the rays and construct areas dA_1 and dA_2 normal to the rays at these points.
  - What are the energies carried by the rays passing through both areas?

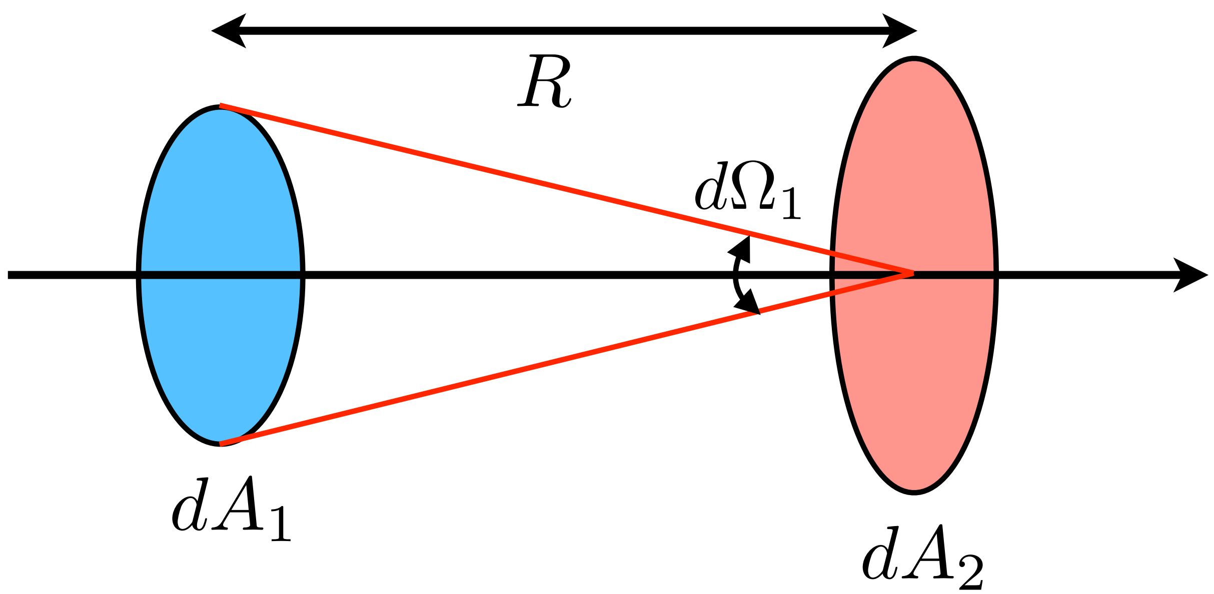
energy passing through 1

$$dE_1 = I_1 dA_1 d\Omega_2 d\nu dt$$

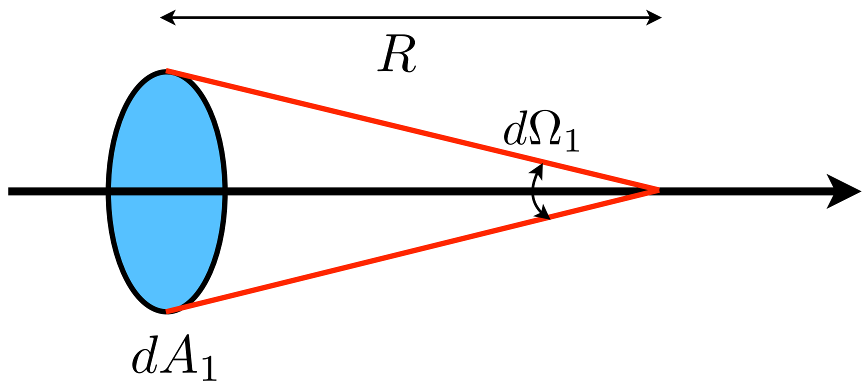
energy passing through 2

$$dE_2 = I_2 dA_2 d\Omega_1 d\nu dt$$





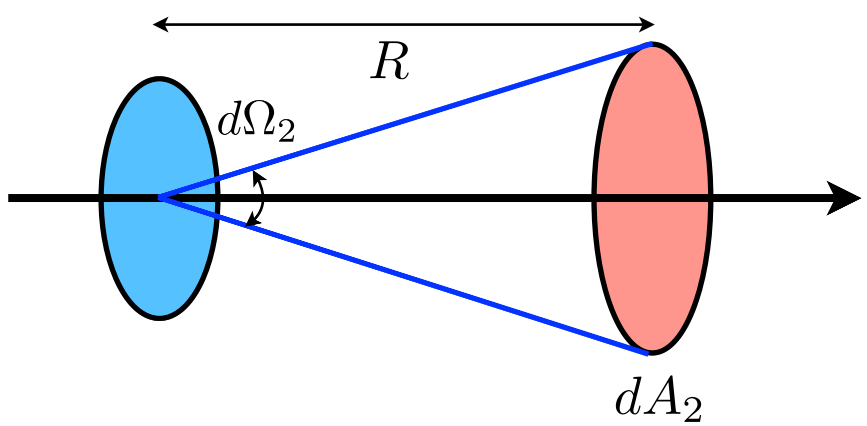
- Here, $d\Omega_1$ is the solid angle subtended by dA_2 at the location 1 and so forth.



$$d\Omega_1 = \frac{dA_1}{R^2}$$

conservation of energy:
Because energy is conserved,

$$\boxed{dE_1 = dE_2 \quad \rightarrow \quad I_1 = I_2}$$



$$d\Omega_2 = \frac{dA_2}{R^2}$$

- Conclusion (***the constancy of intensity***):
  - the specific intensity remains the same as radiation propagates through free space.

$$I_1 = I_2$$

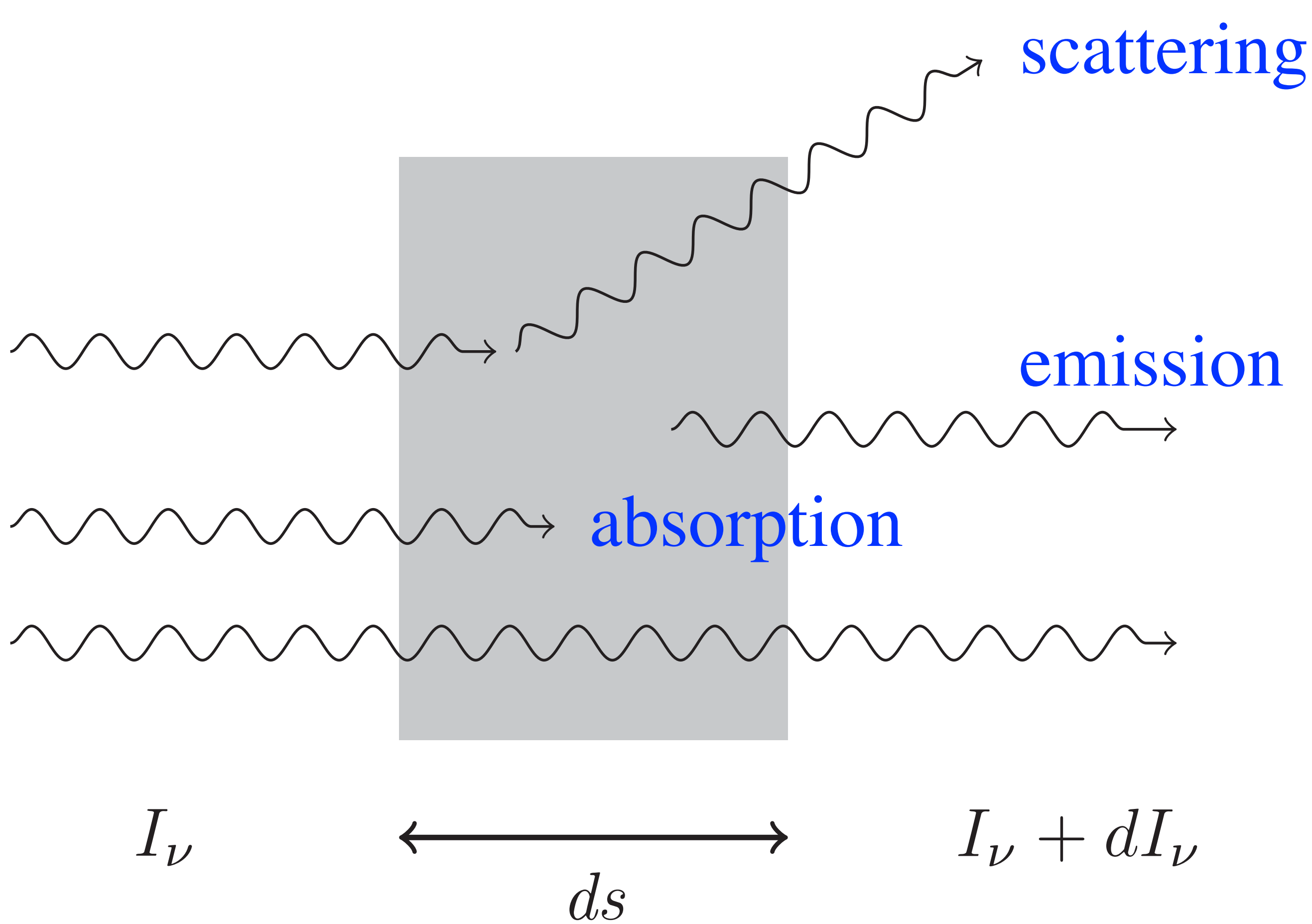
  - If we measure the distance along a ray by variable s, we can express the result equivalently in differential form:

$$\boxed{\frac{dI}{ds} = 0}$$

radiative transfer equation in free space

Radiative Transfer Equation in reality

- In reality, as a ray passes through matter, energy may be added, subtracted, or scattered from it by emission, absorption, or scattering.
  - The intensity will not in general remain constant.
  - These interactions are described by the ***radiative transfer equation***.

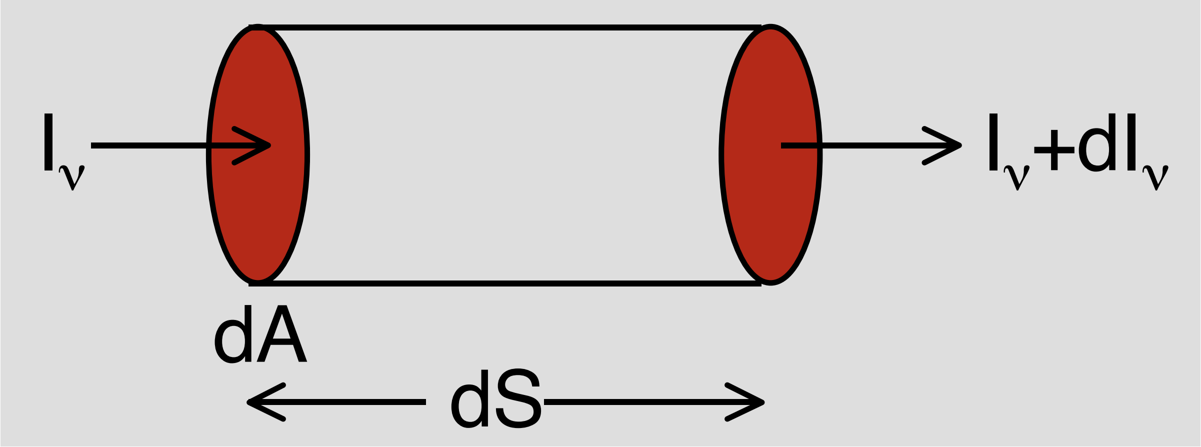


loss: scattering & absorption

gain: emission

Emission

- If the radiation travels through a medium which is itself emitting radiation, that will add to the energy:



- Spontaneous ***"emission coefficient" or "emissivity"*** j_ν is the amount of energy emitted per unit time, per unit solid angle, per unit frequency, and per unit volume:

$$dE = j_\nu dV d\Omega dt d\nu \quad (j_\nu : \mathrm{erg\ cm^{-3}\ s^{-1}\ sr^{-1}\ Hz^{-1}})$$

- In going a distance ds, a beam of cross section dA travels through a volume $dV = dAds$. Thus the intensity added to the beam is by ds is

$$dI_\nu = j_\nu ds \quad \longleftarrow \quad dE = (dI_\nu) dA d\Omega dt d\nu$$

- Therefore, the equation of radiative transfer for pure emission becomes:

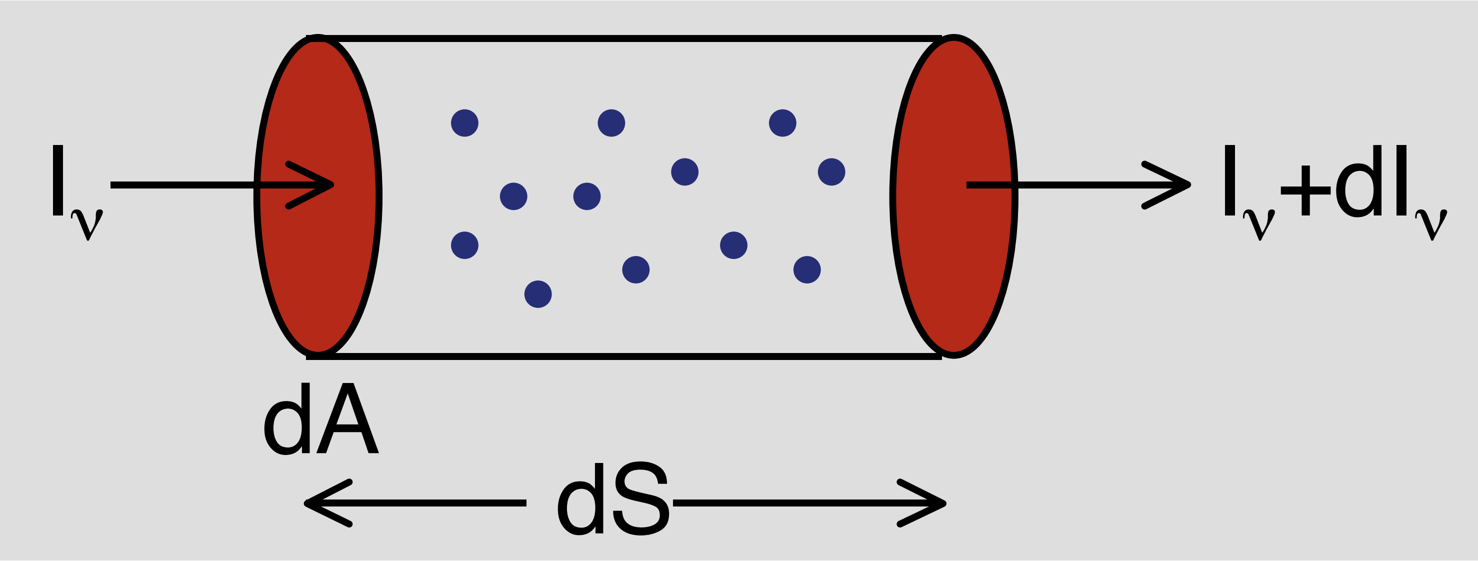
$$\frac{dI_\nu}{ds} = j_\nu$$

  - If we know what j_ν is, we can integrate this equation to find the change in specific intensity as radiation propagates through the medium:

$$I_\nu(s) = I_\nu(0) + \int_0^s j_\nu(s')ds'$$

Absorption

- If the radiation travels through a medium which absorbs radiation, the energy in the beam will be reduced:



- Let n denote the number density of absorbers (particles per unit volume).
- Assume that each absorber has a cross-sectional area of σ_ν (in units of cm^2).
- If a beam travels through ds, total area of absorbers is

$$\text{number of absorbers } \times \text{cross} - \text{section} = \left(n \times dA \times ds\right) \times \sigma_\nu$$

Fraction of radiation absorbed = fraction of area blocked:

$$\frac{dI_\nu}{I_\nu} = -\frac{n dA ds \sigma_\nu}{dA} = -n\sigma_\nu ds$$

$$dI_\nu = -n\sigma_\nu I_\nu ds \equiv -\kappa_\nu I_\nu ds$$

- ***Absorption coefficient*** is defined as $\kappa_\nu \equiv n\sigma_\nu$ (units: cm^{-1}), meaning the ***total cross-sectional area per unit volume***.

- If we include the effect of stimulated emission in the absorption coefficient, it may be referred to as the ***attenuation coefficient***. (as in Draine's book)

- Rearranging the previous equation, we obtain the equation of radiative transfer for pure absorption:

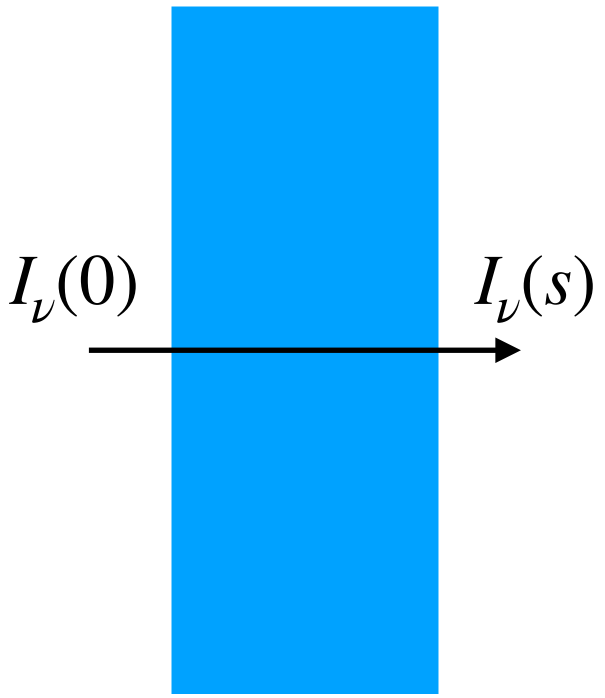
$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu$$

- The amount of reduced energy depends on how much radiation we already have.
- Integrate to find how radiation changes along path:

$$\int_0^s \frac{dI_\nu}{I_\nu} = -\int_0^s \kappa_\nu(s')ds'$$

$$\left[\ln I_\nu\right]_0^s = -\int_0^s \kappa_\nu(s')ds'$$

$$I_\nu(s) = I_\nu(0)\exp\left[-\int_0^s \kappa_\nu(s')ds'\right]$$



- If the absorption coefficient is a constant (example: a uniform density gas of ionized hydrogen), then we obtain

$$I_\nu(s) = I_\nu(0)e^{-\kappa_\nu s}$$

specific intensity after distance s

initial intensity at $s = 0$.

radiation exponentially absorbed with distance

- ***Optical depth***:
  - Imagine radiation traveling into a cloud of absorbing gas, exponential defines a scale over which radiation is attenuated.
  - When $\int_0^s \kappa_\nu(s')ds' = 1$, the intensity will be reduced to $1/e$ of its original value.

- We define the optical depth τ_ν as:

$$\tau_\nu(s) = \int_0^s \kappa_\nu(s')ds' \quad \text{or} \quad d\tau_\nu = \kappa_\nu ds$$

- A medium is said to be ***optically thick*** at a frequency ν if the optical depth for a typical path through the medium satisfies:

$$\tau_\nu(s) > 1$$

- The medium is ***optically thin*** if, instead:

$$\tau_\nu(s) < 1$$

- An optically thin medium is one which a typical photon of frequency ν can pass through without being (significantly) absorbed.

Radiative Transfer Equation

- ***Radiative transfer equation*** with both absorption and emission is

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + j_\nu$$

absorption emission

  - We can rewrite the radiative transfer equation using the optical depth as a measure of \`distance' rather than s:

$$\frac{dI_\nu}{\kappa_\nu ds} = -I_\nu + \frac{j_\nu}{\kappa_\nu}$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

  - where $S_\nu \equiv j_\nu/\kappa_\nu$ ***is called the source function***. This is an alternative and sometimes more convenient way to write the equation.

Mean Free Path

- From the exponential absorption law, the ***probability of a photon absorbed*** between optical depths τ_ν and $\tau_\nu + d\tau_\nu$ is

$$|dI_\nu| = \left|\frac{dI_\nu}{d\tau_\nu}\right| d\tau_\nu \quad \& \quad |dI_\nu| \propto P(\tau_\nu)d\tau_\nu \quad \rightarrow \quad \boxed{P(\tau_\nu) = e^{-\tau_\nu}}$$

= probability density function for the absorption at an optical depth τ_ν.

  - The mean optical depth traveled is thus equal to unity:

$$\langle\tau_\nu\rangle = \int_0^\infty \tau_\nu P(\tau_\nu)d\tau_\nu = \int_0^\infty \tau_\nu e^{-\tau_\nu} d\tau_\nu = 1$$

  - ***The mean free path is defined as the average distance a photon can travel through an absorbing material until it is absorbed.*** In a homogeneous medium, the mean free path is determined by

$$\langle\tau_\nu\rangle = \kappa_\nu \ell_\nu = 1 \quad \rightarrow \quad \boxed{\ell_\nu = \frac{1}{\kappa_\nu} = \frac{1}{n\sigma_\nu}}$$

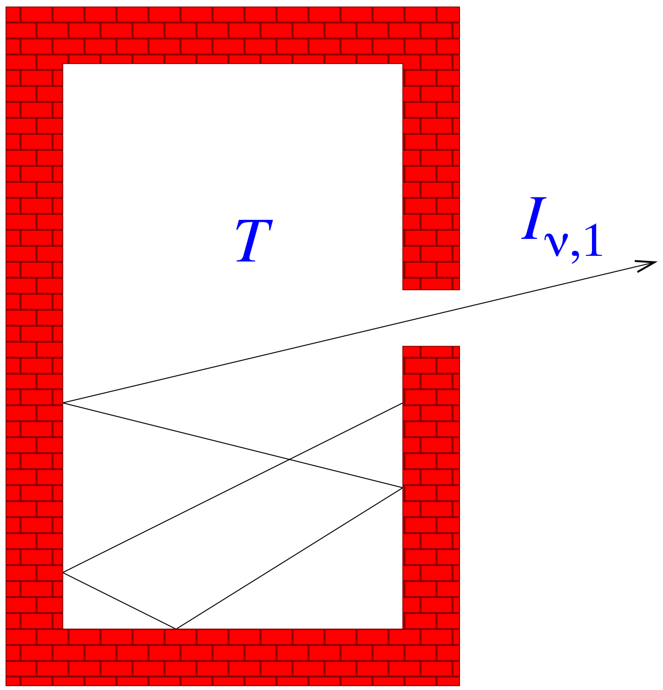
  - A local mean path at a point in an inhomogeneous material can be also defined.

Thermal equilibrium

- In general, equilibrium means a state of balance.
  - ***Thermal equilibrium* refers to steady states of temperature, which defines the average energy of material or photons.**
  - In a state of (complete) ***thermodynamic equilibrium (TE)***, no net flows of matter or of energy, no phase changes, and no unbalanced potentials (or driving forces), within the system. ***In TE, matter and radiation are in equilibrium at the same temperature T.***
  - If the material is (locally) in thermodynamic equilibrium at a well-defined temperature T, ***it is said to be in local thermodynamic equilibrium (LTE) even if it is not in equilibrium with the radiation field.***

Blackbody

- Imagine a container bounded by opaque walls with a very small hole.
  - ***Photons will be scattered and absorbed many times, (and eventually trapped and completely absorbed in the box).*** Under such conditions, the particles and photons continually share their kinetic energies. In perfect thermal equilibrium, ***the average particle kinetic energy will equal to the average photon energy, and a unique temperature T can be defined.***

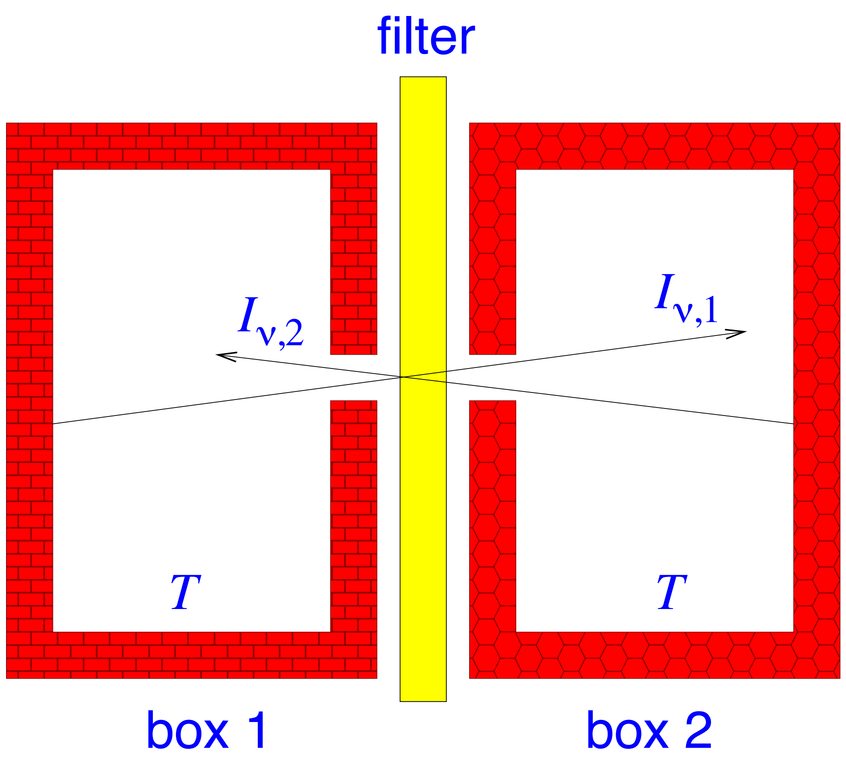


box 1

  - A blackbody is an idealized physical body that absorbs all incident radiation regardless of frequency or angle of incidence (i.e., perfect absorber). The above cavity can be regarded to be a blackbody.
  - Radiation from a blackbody in thermal equilibrium is called the blackbody radiation.

Blackbody radiation is the universal function.

- ***Now, consider another cavity (box 2), also at the same temperature***, but made of different material or shape and connect two cavities with a filter transparent only in the narrow frequency range ν and $\nu + d\nu$.
  - ***In equilibrium at T, radiation should transfer no net energy from one cavity to the other.*** Otherwise, one cavity will cool down and the other heats up; this violates the second law of thermodynamics.
  - Therefore, the intensity or spectrum that passes through the holes should be a universal function of T and should be isotropic.
  - The intensity and spectrum of the radiation emerging from the hole should be independent of the wall material (e.g., wood, copper, or concrete, etc) and any absorbing material that may be inside the cavity.



  - The universal function is called the Planck function $B_\nu(T)$.
  - This is the blackbody radiation.

Kirchhoff's Law in TE

- In (full) thermodynamic equilibrium at temperature T, by definition, we know that

$$\frac{dI_\nu}{ds} = 0 \quad \text{and} \quad I_\nu = B_\nu(T)$$

We also note that

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + j_\nu$$

- Then, we can obtain ***the Kirchhoff's law for a system in TE***:

$$\frac{j_\nu}{\kappa_\nu} = B_\nu(T), \qquad j_\nu = \kappa_\nu B_\nu(T)$$

Please please note that κ_ν is not the emissivity (nor emission coefficient), as often wrongly referred to in the literature of the external galaxies community.

$\kappa_\nu = \kappa_0 \nu^\beta$
$\kappa_\lambda = \kappa_0 \lambda^{-\beta}$ for dust absorption in Far-IR wavelengths

$j_\lambda = \kappa_0 \lambda^{-\beta} B_\lambda(T)$ is referred to as the modified blackbody.

- This is remarkable because it connects the properties $j_\nu(T)$ and $\kappa_\nu(T)$ of any kind of matter to the single universal spectrum $B_\nu(T)$.

Kirchhoff's Law in LTE

- Recall that Kirchhoff's law was derived for a system in thermodynamic equilibrium.

- ***Kirchhoff's law applies not only in TE but also in LTE***:
  - Recall that $B_\nu(T)$ is independent of the properties of the radiating /absorbing material.
  - In contrast, both $j_\nu(T)$ and $\kappa_\nu(T)$ depend only on the materials in the cavity and on the temperature of that material; they do not depend on the ambient radiation field or its spectrum.
  - Therefore, the Kirchhoff's law should be true even for the case of LTE.
  - ***In LTE, the ratio of the emission coefficient to the absorption coefficient is a function of temperature and frequency only.***
  - This generalized version of Kirchhoff's law is an exceptionally valuable tool for calculating the emission coefficient from the absorption coefficient or vice versa.

Implications of Kirchhoff's Law

- A good absorber is a good emitter, and a poor absorber is a poor emitter. (In other words, a good reflector must be a poor absorber, and thus a poor emitter.)

$$j_\nu = \kappa_\nu B_\nu(T) \quad \rightarrow \quad j_\nu \text{ increases as } \kappa_\nu \text{ increases}$$

  - It is not possible to thermally radiate more energy than a blackbody, at equilibrium.

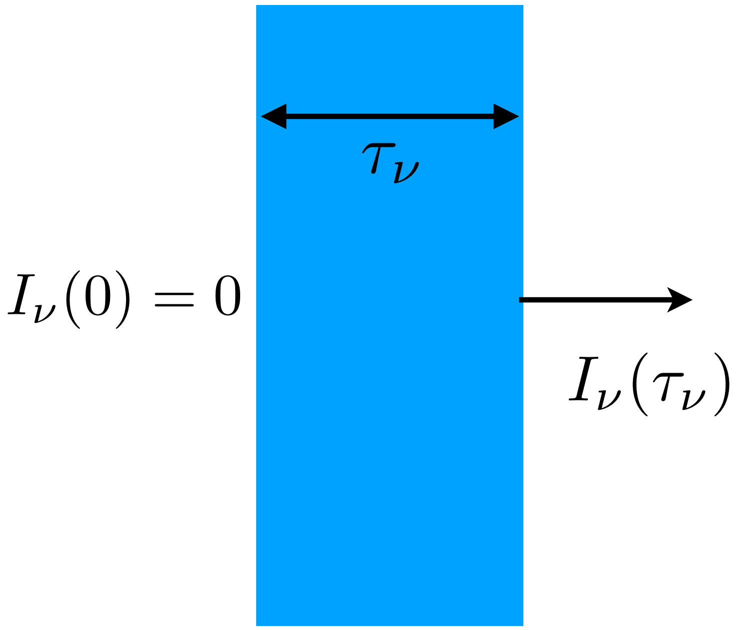
$$j_\nu ds \le B_\nu(T) \quad \text{because} \quad \kappa_\nu ds = \frac{\left| dI_\nu \right|_{\text{abs}}}{I_\nu} \le 1$$

  - The radiative transfer equation in LTE can be rewritten: $\dfrac{dI_\nu}{d\tau_\nu} = -I_\nu + B_\nu(T)$

- ***Remark***:
  - ***Blackbody radiation*** means $I_\nu = B_\nu(T)$. An object for which the intensity is the Planck function is emitting blackbody radiation.
  - ***Thermal radiation is defined to be radiation emitted by "matter" in LTE***. Thermal radiation means $S_\nu = B_\nu(T)$. An object for which the source function is the Planck function is emitting thermal radiation.
  - ***Thermal radiation becomes blackbody radiation only for optically thick media.***

- To see the difference between thermal and blackbody radiation,
  - consider a slab of material with optical depth τ_ν that is producing thermal radiation.
  - If no light is falling on the back side of the slab, the intensity that is measured on the front side of the slab is

$$\begin{aligned} I_\nu(\tau_\nu) &= I_\nu(0)e^{-\tau_\nu} + S_\nu\left(1 - e^{-\tau_\nu}\right) \\ &= B_\nu\left(1 - e^{-\tau_\nu}\right) \end{aligned}$$



  - If the slab is optical thick at frequency ν $(\tau_\nu \gg 1)$, then

$$I_\nu \approx B_\nu$$

  - If the slab is optically thin $(\tau_\nu \ll 1)$, then

$$I_\nu \approx \tau_\nu B_\nu \ll B_\nu$$

  This indicates that the radiation, although thermal, will not be blackbody.

  Thermal radiation becomes blackbody radiation only for optical thick media.

Spectrum of Blackbody Radiation

- There is no perfect blackbody.
  - However, the cosmic microwave background comes quite close; stars can sometimes be usefully approximated as blackbodies.
  - By the end of the 19th century, the blackbody spectrum was fairly well known empirically, from laboratory studies. In 1900, Max Planck, using his idea of quantized energies, derived the blackbody spectrum.

- The frequency dependence of blackbody radiation is given by the ***Planck function***:

$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp\left(h\nu/k_\mathrm{B}T\right) - 1} \quad \text{or} \quad B_\lambda(T) = \frac{2hc^2/\lambda^5}{\exp\left(hc/\lambda k_\mathrm{B}T\right) - 1}$$

$$h = 6.63 \times 10^{-27} \text{ erg s (Planck's constant)}$$

$$k_\mathrm{B} = 1.38 \times 10^{-16} \text{ erg K}^{-1} \text{ (Boltzmann's constant)}$$

Stefan-Boltzmann Law

- Emergent flux is proportional to T^4.

$$F = \pi \int B_\nu(T) d\nu = \pi B(T) \longleftarrow B(T) = \int B_\nu(T) d\nu = \frac{ac}{4\pi} T^4 = \frac{\sigma}{\pi} T^4$$

$$F = \sigma T^4$$

$$\text{Stephan} - \text{Boltzmann constant} : \sigma = \frac{2\pi^5 k_B^4}{15 c^2 h^3} = 5.67 \times 10^{-5} \text{ erg cm}^2 \text{ s}^{-1} \text{ K}^{-4} \text{ sr}^{-1}$$

- Total energy density ***(another form of the Stefan-Boltzmann law)***

$$u = \frac{4\pi}{c} \int B_\nu(T) d\nu = \frac{4\pi}{c} B(T) \qquad u(T) = \left(\frac{T}{3400 \text{ K}} \right)^4 \text{erg cm}^{-3}$$

$$u = aT^4$$

$$\text{radiation constant} : a \equiv \frac{4\sigma}{c} = 7.57 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4}$$

Rayleigh-Jeans Law & Wien Law

Rayleigh-Jeans Law (low-energy limit)

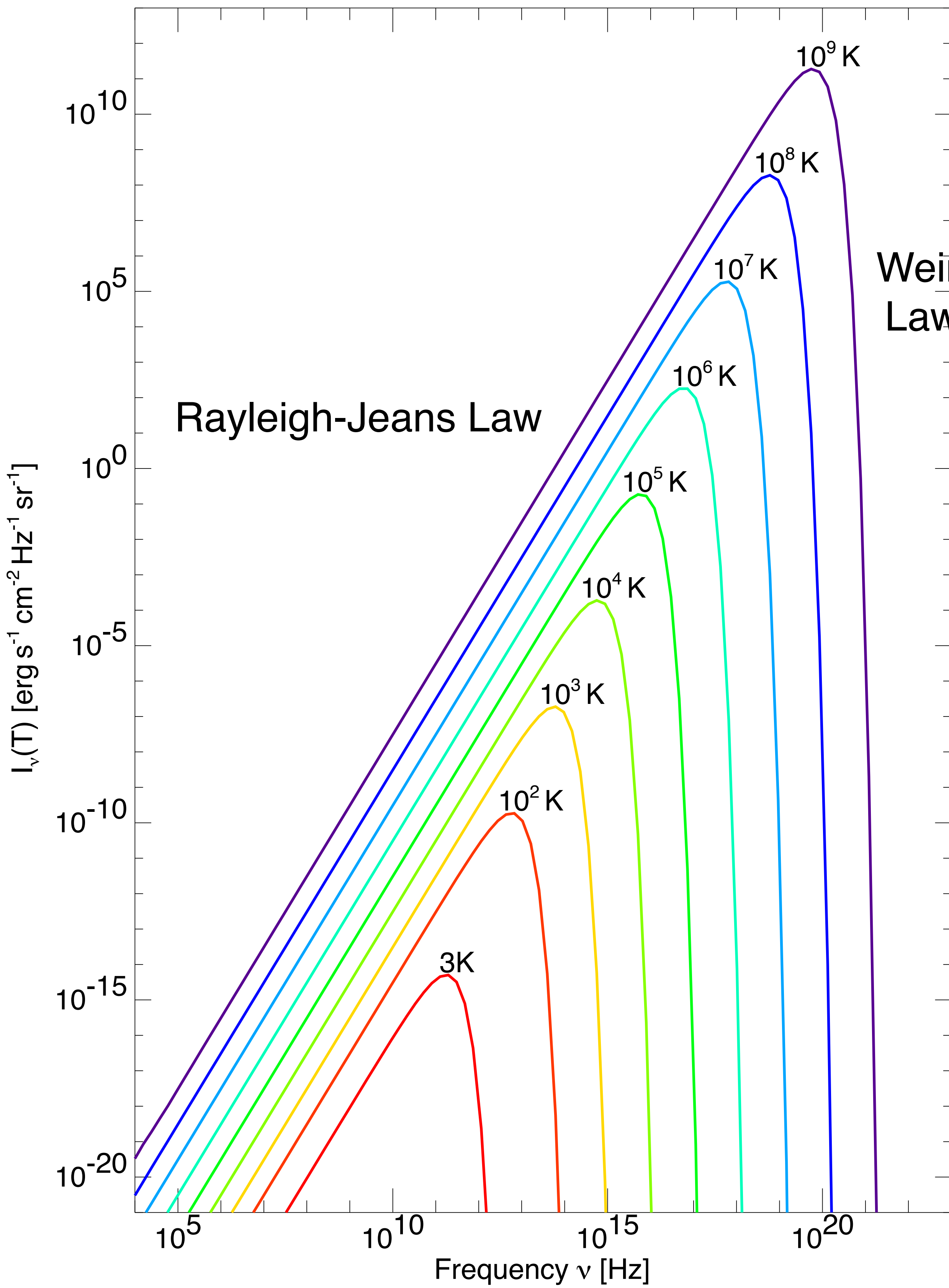
$$h\nu \ll k_{\rm B}T \quad (\nu \ll 2\times 10^{10}\,(T/1\,{\rm K})\,{\rm Hz})$$

$$\boxed{I_\nu^{RJ}(T) = \frac{2\nu^2}{c^2}k_{\rm B}T}$$

Wien Law (high-energy limit)

$$h\nu \gg k_{\rm B}T$$

$$\boxed{I_\nu^{W}(T) = \frac{2h\nu^3}{c^2}\exp\left(-\frac{h\nu}{k_{\rm B}T}\right)}$$



Characteristic Temperatures

- ***Brightness Temperature:***
  - The brightness temperature is defined to be the temperature such that a blackbody at that temperature would have specific intensity:

$$I_\nu = B_\nu(T_b) \quad \rightarrow \quad T_b(\nu) = \frac{h\nu/k_\mathrm{B}}{\ln\left[1 + 2h\nu^3/(c^2 I_\nu)\right]}$$

- ***Antenna Temperature:***
  - Radio astronomers are used to working at very low frequencies. They define the antenna temperature as being the brightness temperature in the Rayleigh-Jeans tail.

$$I_\nu = \frac{2\nu^2}{c^2} k_\mathrm{B} T_b \quad \rightarrow \quad T_A \equiv \frac{c^2}{2k_\mathrm{B}\nu^2} I_\nu$$

  - Radiative transfer equation in the RJ limit:
    - If the matter is in LTE and has its energy levels populated according to an excitation temperature $T_\mathrm{exc} \gg h\nu/k_\mathrm{B}$, then the source function is given by

$S_\nu(T_\mathrm{exc}) = (2\nu^2/c^2) k_\mathrm{B} T_\mathrm{exc}$

    - Then, RT equation becomes $\boxed{\dfrac{dT_A}{d\tau_\nu} = -T_A + T_\mathrm{exc} \quad \text{if } h\nu \ll k_\mathrm{B} T_\mathrm{exc}}$

$$T_A = T_A(0)e^{-\tau_\nu} + T_\mathrm{exc}(1 - e^{-\tau_\nu}) \quad \text{if } T_\mathrm{exc} \text{ is constant.}$$

- ***Color Temperature:***
  - By fitting the spectrum to a blackbody curve without regarding to vertical scale (absolute intensity scale), a color temperature T_c is obtained.
  - The color temperature correctly gives the temperature of a blackbody source of unknown absolute scale.
- ***Effective Temperature:***
  - The effective temperature of a source is obtained by equating the actual flux F to the flux of a blackbody at temperature T_{eff}.

$$F = \int \int I_\nu \cos\theta d\nu d\Omega = \sigma T_{\text{eff}}^4$$

- ***Excitation Temperature:***
  - The excitation temperature of level u relative to level ℓ is defined by

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} \exp\left(-\frac{E_{u\ell}}{k_B T_{\text{exc}}}\right) \quad \rightarrow \quad T_{\text{exc}} \equiv \frac{E_{u\ell}/k_B}{\ln\left(\frac{n_\ell/g_\ell}{n_u/g_u}\right)} \quad (E_{u\ell} \equiv E_u - E_\ell)$$

  - Radio astronomers studying the 21 cm line sometimes use the term "spin temperature" T_{spin} for excitation temperature.

Luminosity

- To determine the energy per unit time, we integrate flux over area.
  - Considering a sphere centered on a source with radius *R*, the monochromatic luminosity is

$$
\begin{aligned}
L_\nu &= R^2 \int d\Omega F_\nu \\
&= 4\pi R^2 F_\nu \qquad \text{for an isotropic source}
\end{aligned}
$$

  - The bolometric luminosity is

$$
\begin{aligned}
L_{\rm bol} &= \int L_\nu d\nu = \int L_\lambda d\lambda \\
&= 4\pi R^2 \int F_\nu d\nu
\end{aligned}
$$

Homework (due date: 03/15)

(1) Consider an (isotropically emitting) star of uniform intensity $I_\nu = B$ at the surface, show that the flux at the surface is

$$F_\nu = \int I_\nu \cos\theta d\Omega = \pi B$$

(2) The specific intensity of a star is, to first order, a blackbody. For a given effective temperature, $T_{\rm eff}$, and stellar radius, R, derive its bolometric luminosity.

(3) Look up values for these parameters and calculate this formula for the Sun.