$(\mathrm{AGN})^2$

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Introduction

- Temperature
    - The temperature in a static nebula is determined by the balance between **(1) heating by photoionization** and **(2) cooling by recombination and by collisional excitation emission** from the nebula.
- Energy gain and loss
    - **[Gain]** When a photon of energy $h\nu$ is absorbed, an electron (photoelectron) is created, having an energy $\frac{1}{2}mu^2 = h(\nu - \nu_0)$. The electrons produced are rapidly thermalized (see Chap. 2).
    - In ionization equilibrium, the photoionizations are balanced by an equal number of recombinations.
    - **[Loss]** In each recombination, a thermal electron with energy $\frac{1}{2}mu^2$ disappears. An average of this quantity over all recombinations represents the mean energy that "disappears" per recombination.
    - The difference between the mean energy of a newly created electron and the mean energy of a recombining electron represents the net gain in energy per ionization process.
    - **[Loss]** In equilibrium, this net energy gain is balanced by the energy lost by radiation, predominantly by electron collisional excitation of bound levels of abundant ions.
    - **[Loss]** Free-free emission (bremsstrahlung) is another, less important radiative energy-loss mechanism.

Energy Gain by Photoionizaiton

- In a pure H nebula, at any specific location in the nebula, the energy gain (per unit volume per unit time) is

$$G(\mathrm{H}) = n(\mathrm{H}^0)\int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} h(\nu-\nu_0)\sigma_\nu^{\mathrm{pi}}(\mathrm{H}^0)d\nu \quad [\mathrm{erg\ cm^{-3}\ s^{-1}}]$$

- In ionization equilibrium,

$$n(\mathrm{H}^0)\int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} \sigma_\nu^{\mathrm{pi}}(\mathrm{H}^0)d\nu = n_e n_p \alpha_{\mathrm{A}}(\mathrm{H}^0, T)$$

number of photoionization per unit volume per unit time = number of recombination per unit volume per unit time

- Then, the gain can be expressed as follows:

number of recombination

$$G(\mathrm{H}) = n_e n_p \alpha_{\mathrm{A}}(\mathrm{H}^0, T)\frac{\int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} h(\nu-\nu_0)\sigma_\nu^{\mathrm{pi}}(\mathrm{H}^0)d\nu}{\int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} \sigma_\nu^{\mathrm{pi}}(\mathrm{H}^0)d\nu} = n_e n_p \alpha_{\mathrm{A}}(\mathrm{H}^0, T)\frac{3}{2}kT_i$$

mean energy gain by electron per photoionization

- The mean energy of a newly created photoelectron depends on the form of the ionizing radiation field, but not on the absolute strength of the radiation.

- For a blackbody spectrum $J_\nu = B_\nu(T_*)$, the initial temperature $T_i \approx T_*$ when $kT_* < h\nu_0$.

Table 3.1

Mean input energy of photoelectrons

Model stellar atmosphere	T_i (K)			
T_* (K)	$\tau_0 = 0$	$\tau_0 = 1$	$\tau_0 = 5$	$\tau_0 = 10$
3.0×10^4	1.58×10^4	1.87×10^4	3.36×10^4	5.02×10^4
3.5×10^4	2.08×10^4	2.48×10^4	4.24×10^4	5.94×10^4
4.0×10^4	2.48×10^4	3.01×10^4	5.48×10^4	8.15×10^4
5.0×10^4	3.33×10^4	4.12×10^4	7.50×10^4	10.60×10^4

- At larger distances from the star, the spectrum of the ionizing radiation is attenuated by absorption in the nebula.
- The higher-energy photons penetrate further into the gas, and the mean energy of the photoelectrons produced at larger optical depths from the star is higher.

Energy Loss by Recombination

- The kinetic energy lost by the electron gas (per unit volume per unit time) in recombination is

$$L_R(\mathrm{H}) = n_e n_p \alpha_{\mathrm{A}}(\mathrm{H}^0, T)kT \quad [\mathrm{erg\ cm^{-3}\ s^{-1}}]$$

mean energy loss by a photon per recombination

$$\alpha_{\mathrm{A}}(\mathrm{H}^0, T) = \sum_{n=1}^{\infty} \alpha_n(\mathrm{H}^0, T) = \sum_{n=1}^{\infty}\sum_{L=0}^{n-1} \alpha_{nL}(\mathrm{H}^0, T) \quad [\mathrm{cm^3\ s^{-1}}]$$

$$\text{where} \quad \alpha_{nL}(\mathrm{H}^0, T) = \frac{1}{kT}\int_0^{\infty} u\sigma_{nL}(\mathrm{H}^0, T)\frac{1}{2}mu^2 f(u)du$$

  - α_{nL} is thus effectively a kinetic energy averaged recombination coefficient.

- Coulomb focusing effect: the cross sections are approximately proportional to u^{-2}.
- Since the recombination cross sections are approximately proportional to u^{-2}, the electrons of lower kinetic energy are preferentially captured, and the mean energy of the captured electrons is somewhat less than $\frac{3}{2}kT_i$.
- In a pure H nebula that had no radiation losses, the thermal equilibrium equation would be $T_{\mathrm{eq}} > T_i$ because of the "heating" due to the preferential capture of the slower electrons.

$$G(\mathrm{H}) = L_R(\mathrm{H}) \quad \rightarrow \quad T_{\mathrm{eq}} > T_i$$

- **On-the-spot approximation:**
  - The radiation field J_ν should include the diffuse radiation as well as the stellar radiation modified by absorption ($e^{-\tau_\nu}$).
  - Every emission of an ionizing photon during a recombination to the level $n = 1$ is assumed to be balanced by absorption of the same photon at a nearby spot in the nebula.
  - Thus production of photons by the diffuse radiation field and recombination to the ground level is simply omitted from the gain and loss rates. Then, the equations are

$$G_{\mathrm{OTS}}(\mathrm{H}) = n(\mathrm{H}^0)\int_{\nu_0}^{\infty} \frac{4\pi J_{\nu s}}{h\nu} h(\nu-\nu_0)\sigma_\nu^{\mathrm{pi}}(\mathrm{H}^0)d\nu$$

$$= n_e n_p \alpha_{\mathrm{B}}(\mathrm{H}^0, T)\frac{\int_{\nu_0}^{\infty} \frac{4\pi J_{\nu s}}{h\nu} h(\nu-\nu_0)\sigma_\nu^{\mathrm{pi}}(\mathrm{H}^0)d\nu}{\int_{\nu_0}^{\infty} \frac{4\pi J_{\nu s}}{h\nu}\sigma_\nu^{\mathrm{pi}}(\mathrm{H}^0)d\nu}$$

$$L_{\mathrm{OTS}}(\mathrm{H}) = n_e n_p \alpha_{\mathrm{B}}(\mathrm{H}^0, T)kT \qquad \text{where} \quad \alpha_{\mathrm{B}}(\mathrm{H}^0, T) = \sum_{n=2}^{\infty} \alpha_n(\mathrm{H}^0, T)$$

  - The OTS approximation is not as accurate for the equilibrium as it is in the ionization equation, because of the fairly large difference in $h(\nu-\nu_0)$ between the ionizing photons in the stellar and diffuse radiation fields.

- Inclusion of He and heavy elements
  - Including He in the heating and recombination cooling rates is straightforward:

$$G = G(\mathrm{H}) + G(\mathrm{He})$$

$$G(\mathrm{He}) = n_e n(\mathrm{He}^+)\alpha_{\mathrm{A}}(\mathrm{He}^0, T)\frac{\int_{\nu_2}^{\infty}\frac{4\pi J_\nu}{h\nu}h(\nu-\nu_2)\sigma_\nu^{\mathrm{pi}}(\mathrm{He}^0)d\nu}{\int_{\nu_2}^{\infty}\frac{4\pi J_\nu}{h\nu}\sigma_\nu^{\mathrm{pi}}(\mathrm{He}^0)d\nu}$$

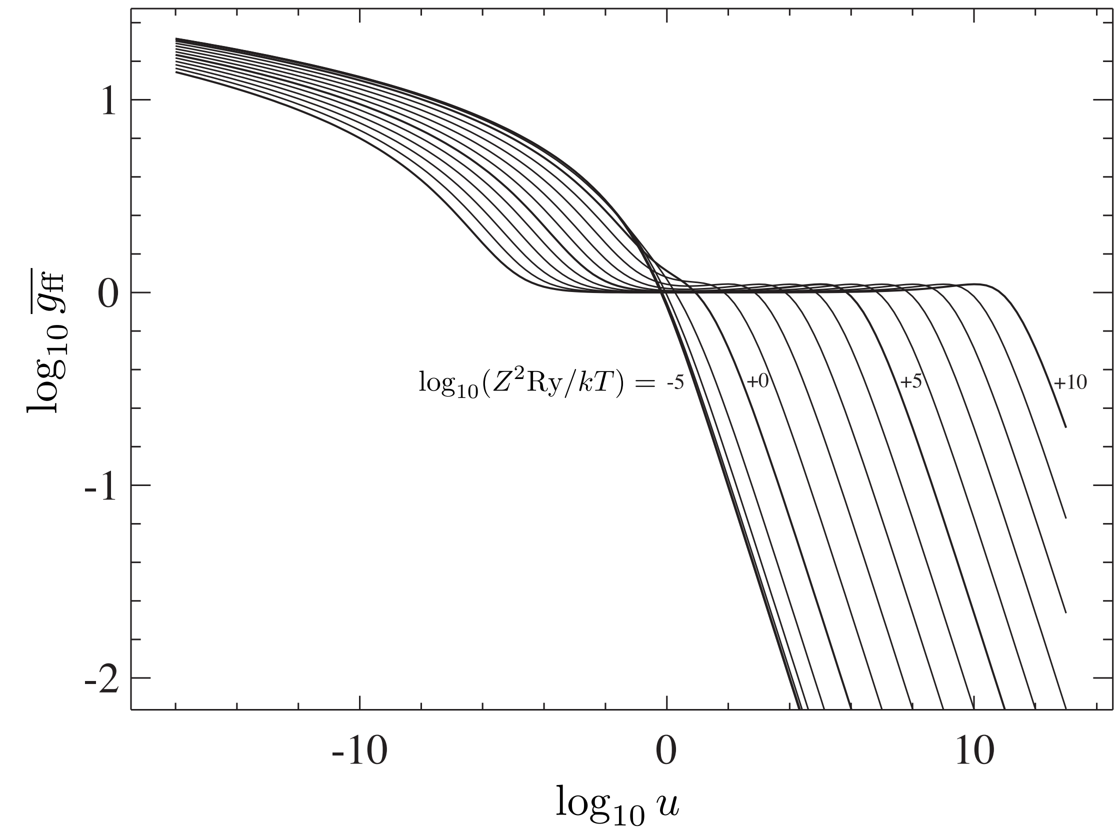
$$L = L_R(\mathrm{H}) + L_R(\mathrm{He})$$

$$L(\mathrm{He}) = n_e n(\mathrm{He}^+)\alpha_{\mathrm{A}}(\mathrm{He}^0, T)kT$$

  - The heating and recombination cooling rates are proportional to the densities of the ions involved, so the contributions of the heavy elements, which are much less abundant than H and He, can be negligible compared to those of H and He.

Energy Loss by Free-Free Radiation

van Hoof et al. (2014, MNRAS, 444, 420)



$u = h\nu/kT$

- Free-free radiation, in which a continuous spectrum is emitted, is a minor contributor to the cooling rate.
  - The cooling rate by ions of charge Z, integrated over all frequencies is approximately:

$$L_{FF}(Z) = 4\pi j_{ff}$$

$$= \frac{2^5 \pi e^6 Z^2}{3^{3/2} h m c^3} \left(\frac{2\pi k T}{m} \right) g_{ff} n_e n_+ \quad [\mathrm{erg\ cm^{-3}\ s^{-1}}]$$

$$= 1.42 \times 10^{-27} Z^2 T^{1/2} g_{ff} n_e n_+ \quad [\mathrm{erg\ cm^{-3}\ s^{-1}}]$$

$n_+ \approx n_p + n(\mathrm{He}^+)$ the number density of the ions

  - The numerical values of the mean Gaunt factor for free-free emission is a slowly varying function of n_e and T. For nebular conditions in the range $1.0 < g_{ff} < 1.5$.

$$\boxed{\overline{g_{\rm ff}} \sim \begin{cases} 1 & \text{for } u \sim 1 \\ 1-5 & \text{for } 10^{-4} < u < 1 \end{cases}}$$

Energy Loss by Collisionally Excited Line Radiation

- Collisional excitation of low-lying energy levels of common ions, such as O^+, O^{++}, and N^+, are the predominant source of radiative cooling.
  - These ions make a significant contribution in spite of their low abundance because they have energy levels with excitation potentials of the order of kT.
  - However, all the levels of H and He have much higher excitation potentials, and therefore are usually not important as collisionally excited coolants.

Collisional Excitation & De-excitation

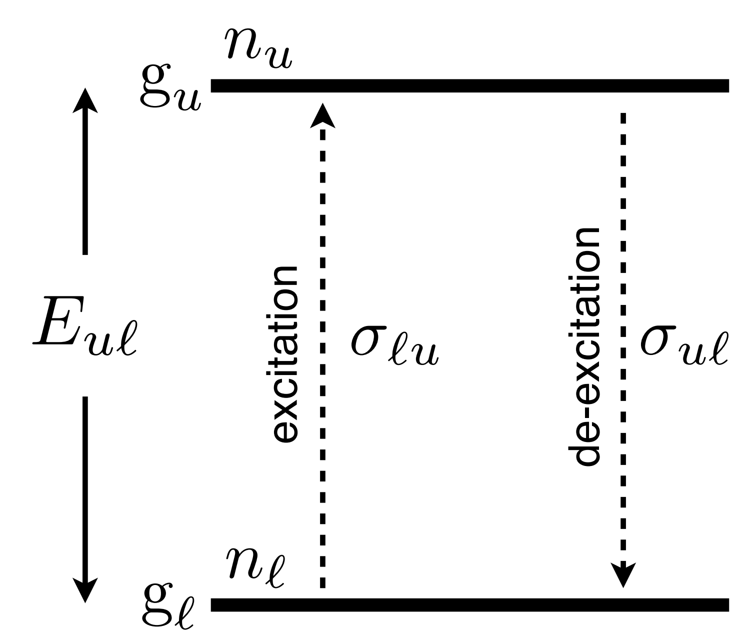
- **Collisional Rate (Two Level Atom)**
  - The cross section $\sigma_{\ell u}$ for collisional excitation from a lower level ℓ to an upper level u is, in general, inversely proportional to the impact energy (or v^2) above the energy threshold $E_{u\ell}$ and is zero below.
  - The collisional cross section can be expressed in the following form using a dimensionless quantity called the ***collision strength*** $\Omega_{\ell u}$:

$$\begin{aligned}\sigma_{\ell u}(v) &= (\pi a_0^2)\left(\frac{hR_{\rm H}}{\frac{1}{2}m_e v^2}\right)\frac{\Omega_{\ell u}}{{\rm g}_\ell} \quad {\rm cm}^2 \quad \text{for} \quad \frac{1}{2}m_e v^2 > E_{u\ell} \\ &= \frac{h^2}{4\pi m_e^2 v^2}\frac{\Omega_{\ell u}}{{\rm g}_\ell}\end{aligned}$$

$$\text{or} \quad \sigma_{\ell u}(E) = \frac{h^2}{8\pi m_e E}\frac{\Omega_{\ell u}}{{\rm g}_\ell} \qquad \left(E = \frac{1}{2}m_e v^2\right)$$

$$\text{where,} \quad a_0 = \frac{\hbar^2}{m_e e^2} = 5.12\times 10^{13} \quad \text{cm, Bohr radius}$$

$$R_{\rm H} = \frac{m_e e^4}{4\pi\hbar^3} = 109,737 \quad {\rm cm}^{-1}, \quad \text{Rydberg constant} \quad \left(\hbar = \frac{h}{2\pi}\right)$$



  - The collision strength $\Omega_{\ell u}$ is a function of electron velocity (or energy) but is often approximately constant near the threshold. Here, ${\rm g}_\ell$ and ${\rm g}_u$ are the statistical weights of the lower and upper levels, respectively.

- Advantage of using the collision strength is that (1) it removes the primary energy dependence for most atomic transitions and (2) they have the symmetry between the upper and the lower states.

The principle of detailed balance states that ***in thermodynamic equilibrium each microscopic process is balanced by its inverse.***

$$n_e n_\ell v_\ell \sigma_{\ell u}(v_\ell) f(v_\ell) dv_\ell = n_e n_u v_u \sigma_{u\ell}(v_u) f(v_u) dv_u$$

Here, v_ℓ and v_u are related by. $\frac{1}{2}m_e v_\ell^2 = \frac{1}{2}m_e v_u^2 + E_{u\ell}$, and $f(v)$ is a Maxwell velocity distribution of electrons. Using the Boltzmann equation of thermodynamic equilibrium,

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} \exp\left(-\frac{E_{u\ell}}{kT}\right)$$

we derive the following relation between the cross-sections for excitation and de-excitation are

$$\boxed{g_\ell v_\ell^2 \sigma_{\ell u}(v_\ell) = g_u v_u^2 \sigma_{u\ell}(v_u) \quad \text{Here,} \quad \frac{1}{2}m_e v_\ell^2 = \frac{1}{2}m_e v_u^2 + E_{u\ell}} \longrightarrow g_\ell \cdot (E + E_{u\ell}) \cdot \sigma_{\ell u}(E + E_{u\ell}) = g_u \cdot E \cdot \sigma_{u\ell}(E)$$

where $E = \frac{1}{2}m_e v_u^2$

and the symmetry of the collision strength between levels.

$$\boxed{\Omega_{\ell u} = \Omega_{u\ell}} \qquad \text{more precisely} \quad \boxed{\Omega_{\ell u}\left(E + E_{u\ell}\right) = \Omega_{u\ell}\left(E\right)}$$

These two relations were derived in the TE condition. However, ***the cross-sections are independent on the assumptions, and thus the above relations should be always satisfied.***

- Collisional excitation and de-excitation rates

The ***collisional de-excitation rate per unit volume per unit time, which is thermally averaged,*** is

$$\left(\frac{dn_\ell}{dt}\right)_{u\to\ell} = n_e n_u \int_0^\infty v\sigma_{u\ell}(v)f(v)dv = n_e n_u k_{u\ell} \quad [\mathrm{cm^{-3}\ s^{-1}}]$$

$$k_{u\ell} = \int_0^\infty v\sigma_{u\ell}(v)f(v)dv = \left(\frac{2\pi\hbar^4}{k_\mathrm{B} m_e^3}\right)^{1/2} T^{-1/2}\frac{\langle\Omega_{u\ell}\rangle}{g_u} = \frac{8.62942\times 10^{-6}}{T^{1/2}}\frac{\langle\Omega_{u\ell}\rangle}{g_u} \quad [\mathrm{cm^3\ s^{-1}}],$$

$$k_{u\ell} \equiv \langle\sigma v\rangle_{u\to\ell}$$

effective collision strength:

$$\langle\Omega_{u\ell}\rangle \equiv \int_0^\infty \Omega_{u\ell}(E)e^{-E/k_\mathrm{B}T}d(E/k_\mathrm{B}T)$$

and the ***collisional excitation rate per unit volume per unit time*** is

$$\left(\frac{dn_u}{dt}\right)_{\ell\to u} = n_e n_\ell k_{\ell u}$$

$$k_{\ell u} \equiv \langle\sigma v\rangle_{\ell\to u}$$

$$k_{\ell u} = \int_{v_\mathrm{min}}^\infty v\sigma_{\ell u}(v)f(v)dv \quad \text{Here, } \frac{1}{2}m_e v_\mathrm{min}^2 = E_{u\ell}$$

$$= \left(\frac{2\pi\hbar^4}{k_\mathrm{B} m_e^3}\right)^{1/2} T^{-1/2}\frac{\langle\Omega_{u\ell}\rangle}{g_\ell}\exp\left(-\frac{E_{u\ell}}{k_\mathrm{B}T}\right)$$

Here, $k_{\ell u}$ and $k_{u\ell}$ are the collisional rate coefficient for excitation and de-excitation coefficients in units of $\mathrm{cm^3\ s^{-1}}$, respectively. We also note that ***the rate coefficients for collisional excitation and de-excitation are related by***

$$k_{\ell u} = \frac{g_u}{g_\ell}k_{u\ell}\exp\left(-\frac{E_{u\ell}}{k_\mathrm{B}T}\right) \qquad \langle\sigma v\rangle_{\ell\to u} = \frac{g_u}{g_\ell}\langle\sigma v\rangle_{u\to\ell}\exp\left(-\frac{E_{u\ell}}{k_\mathrm{B}T}\right)$$

Sum rule for collision strengths

- Quantum mechanical sum rule for collision strengths for the case where one term consists of a singlet ($S = 0$ or $L = 0$) and the second consists of a multiplet: the collision strength of each fine structure level J is related to the total collision strength of the multiplet by

$$\Omega_{(SLJ,\ S'L'J')} = \frac{(2J'+1)}{(2S'+1)(2L'+1)}\Omega_{(SL,\ S'L')}$$

  Here, $(2J'+1)$ is the statistical weight of an individual level in the multiplet, and $(2S'+1)(2L'+1)$ is the statistical weight of the multiplet term.

  We can regard the collision strength as "shared" amongst these levels in proportion to the statistical weights of the individual levels ($g_J = 2J+1$).

- ***The flux ratio between the lines in a multiplet is proportional to the ratio of their collision strengths, in a low density medium.*** Then, the flux ratio is determined by the ratio of their statistical weights.
  - C-like ions ($1s^2 2s^2 2p^2 \rightarrow 1s^2 2s^2 2p^2$) forbidden or inter combination transitions.

    ground states (triplet) - $^3P_0 : {}^3P_1 : {}^3P_2 = 1 : 3 : 5$

    excited states (singlets) - $^1D_2$, $^1S_1$
  - Li-like ions ($1s^2 2s^1 \rightarrow 1s^2 2p^1$) resonance transitions

    ground state (singlet) - $^2S_{1/2}$

    excited states (doublet) - $^2P_{3/2} : {}^2P_{1/2} = 2 : 1$

Collisionally-Excited Emission Line

- Emission line flux
  - **In the low density limit**, the collisional rate between atoms and electrons is much slower than the (spontaneous) radiative de-excitation rate of of the excited level. Thus, we can balance the collisional feeding into level u by the rate of radiative transition back down to level ℓ. The level population is determined by

$$n_e n_\ell k_{\ell u} = A_{u\ell} n_u \qquad \frac{n_u}{n_\ell} = \frac{n_e k_{\ell u}}{A_{u\ell}} = \frac{n_e}{A_{u\ell}} \beta \frac{\langle \Omega_{u\ell} \rangle}{g_\ell} T^{-1/2} \exp\left(-\frac{E_{u\ell}}{kT}\right)$$

where $A_{u\ell}$ is the Einstein coefficient for spontaneous emission. The line emissivity is given by

$$\begin{aligned} 4\pi j_{u\ell} &= E_{u\ell} A_{u\ell} n_u = E_{u\ell} n_e n_\ell k_{\ell u} \\ &= n_e n_\ell E_{u\ell} \frac{8.62942 \times 10^{-6}}{T^{1/2}} \frac{\langle \Omega_{u\ell} \rangle}{g_\ell} \exp\left(-\frac{E_{u\ell}}{kT}\right) \quad [\mathrm{erg\ cm^{-3}\ s^{-1}}] \\ &\simeq \beta \chi n_e^2 E_{\ell u} T^{-1/2} \frac{\langle \Omega_{u\ell} \rangle}{g_\ell} \exp\left(-\frac{E_{u\ell}}{kT}\right) \end{aligned}$$

Here, $\beta = \left(\frac{2\pi\hbar^4}{km_e^2}\right)^{1/2} = 8.62942 \times 10^{-6}$

$\chi = n_\ell / n_e$

For low temperature, the exponential term dominates because few electrons have energy above the threshold for collisional excitation, so that the line rapidly fades with decreasing temperature.

At high temperature, the $T^{-1/2}$ term controls the cooling rate, so the line fades slowly with increasing temperature.

- **In high-density limit**, the level population are set by the Boltzmann equilibrium, and the line emissivity is

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} \exp\left(-\frac{E_{u\ell}}{kT}\right)$$

$$\begin{aligned} 4\pi j_{u\ell} &= E_{\ell u} A_{u\ell} n_u \\ &= n_\ell E_{\ell u} A_{u\ell} \frac{g_u}{g_\ell} \exp\left(-\frac{E_{\ell u}}{kT}\right) \\ &\simeq \chi n_e E_{\ell u} A_{u\ell} \frac{g_u}{g_\ell} \exp\left(-\frac{E_{\ell u}}{kT}\right) \end{aligned}$$

Here, **the line flux scales as n_e rather than n_e^2, but the line flux tends to a constant value at high temperature.**

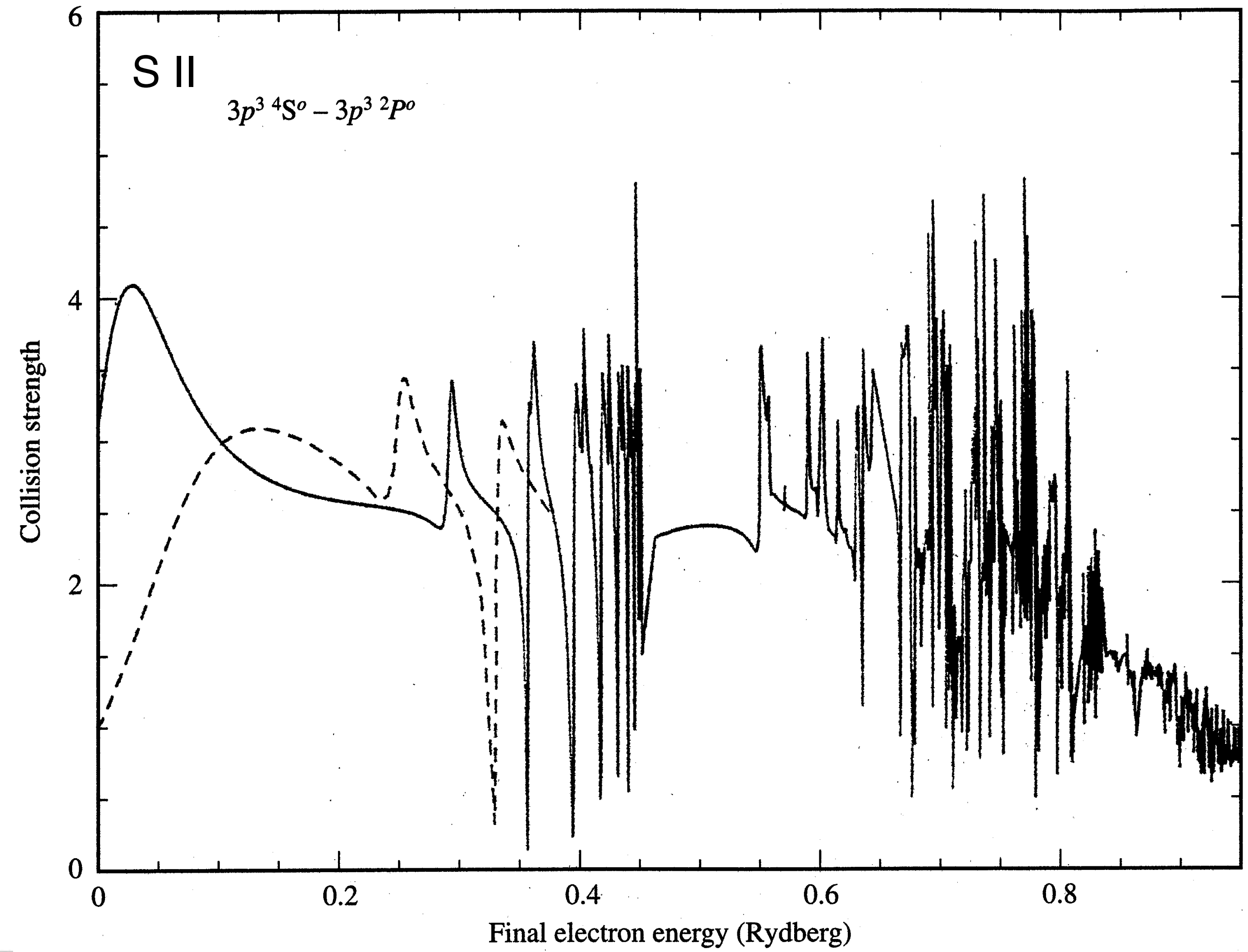
- **Critical density** is defined as **the density where the radiative depopulation rate matches the collisional de-excitation for the excited state.**

$$\begin{aligned} A_{u\ell} n_u &= n_e n_u k_{u\ell} \\ n_{\rm crit} &= \frac{A_{u\ell}}{k_{u\ell}} \end{aligned} \quad \rightarrow \quad \boxed{\begin{aligned} n_{\rm crit} &= A_{u\ell} \frac{g_u}{\beta \langle \Omega_{u\ell} \rangle} T^{1/2} \\ &= 1.2 \times 10^3 \frac{A_{u\ell}}{10^{-4}\,{\rm s}^{-1}} \frac{g_u}{\langle \Omega_{u\ell} \rangle} \left(\frac{T}{10^4\,{\rm K}}\right)^{1/2} \quad [{\rm cm}^{-3}] \end{aligned}}$$

- At densities higher than the critical density, collisional de-excitation becomes significant, and the forbidden lines will be weaker as the density increases.

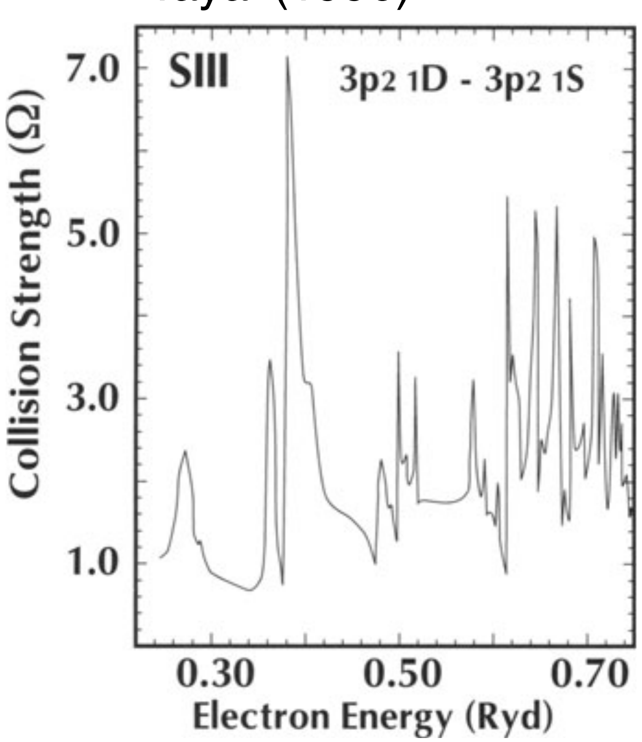
  At around the critical density, the "line emissivity vs density" plotted in log-log scale changes slope from +2 to +1.

- Collision Strength
  - Quantum mechanical calculations show that (1) the resonance structure in the collision strengths is important and (2) the collision strength increases with energy for neutral species.

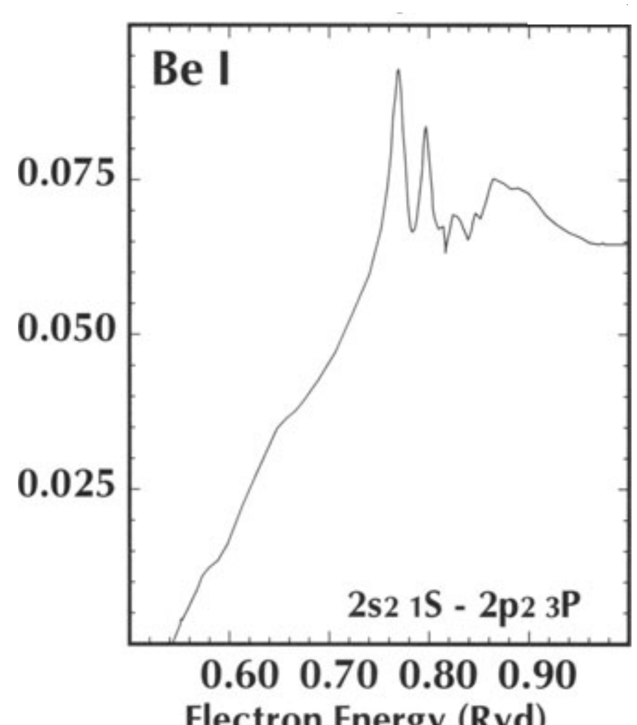


solid - Ramsbottom et al. (1996)
dashed - Cai & Pradhan (1993)

Tayal (1996)



Badnell (1999)



The ***effective collision strength,*** which is thermally averaged, has a value in a range of

$$\langle \Omega_{u\ell} \rangle = \int_0^\infty \Omega_{u\ell}(E) e^{-E/k_{\rm B}T} d(E/k_{\rm B}T)$$

$$10^{-2} < \langle \Omega_{u\ell} \rangle < 10$$

See Table F.1 to F.5 in [Draine]

- As can be seen in Tables and the formula, collisional de-excitation is negligible for resonance and most forbidden lines in the ISM.

Ion	ℓ	u	E_ℓ/k (K)	E_u/k (K)	$\lambda_{u\ell}$ (μm)	$n_{\rm H,crit}(u)$ $T = 100\,$K (cm^{-3})	$T = 5000\,$K (cm^{-3})
C II	$^2P^o_{1/2}$	$^2P^o_{3/2}$	0	91.21	157.74	2.0×10^3	1.5×10^3
C I	$^3P_0$	$^3P_1$	0	23.60	609.7	620	160
	$^3P_1$	$^3P_2$	23.60	62.44	370.37	720	150
O I	$^3P_2$	$^3P_1$	0	227.71	63.185	2.5×10^5	4.9×10^4
	$^3P_1$	$^3P_0$	227.71	326.57	145.53	2.3×10^4	8.4×10^3
Si II	$^2P^o_{1/2}$	$^2P^o_{3/2}$	0	413.28	34.814	1.0×10^5	1.1×10^4
Si I	$^3P_0$	$^3P_1$	0	110.95	129.68	4.8×10^4	2.7×10^4
	$^3P_1$	$^3P_2$	110.95	321.07	68.473	9.9×10^4	3.5×10^4

Table 17.1 in [Draine]

- However, it is not true for the 21 cm hyperfine structure line of hydrogen.
  - The critical density for 21cm line is

$$n_{\rm crit} \sim 10^{-3}\,(T/100\,{\rm K})^{-1/2}\ \ [{\rm cm}^{-3}]$$

$$A_{u\ell} = 2.88\times10^{-15}\ [{\rm s}^{-1}]$$

  - The hyperfine levels are thus essentially in collisional equilibrium in the CNM.

The collisional strengths and other atomic data are available in the CHIANTI atomic database (https://www.chiantidatabase.org/).

Collision strengths at T = 10^4 K

Table 4.1 in The Interstellar Medium [Lequeux]

Ion	Transition l–u	λ µm	A_{ul} s^{-1}	Ω_{ul}	n_{crit} cm^{-3}
C I	$^3P_0-^3P_1$	609.1354	7.93×10^{-8}	–	(500)
	$^3P_1-^3P_2$	370.4151	2.65×10^{-7}	–	(3000)
C II	$^2P_{1/2}-^2P_{3/2}$	157.741	2.4×10^{-6}	1.80	47 (3000)
N II	$^3P_0-^3P_1$	205.3	2.07×10^{-6}	0.41	41
	$^3P_1-^3P_2$	121.889	7.46×10^{-6}	1.38	256
	$^3P_2-^1D_2$	0.65834	2.73×10^{-3}	2.99	7700
	$^3P_1-^1D_2$	0.65481	9.20×10^{-4}	2.99	7700
N III	$^2P_{1/2}-^2P_{3/2}$	57.317	4.8×10^{-5}	1.2	1880
O I	$^3P_2-^3P_1$	63.184	8.95×10^{-5}	–	2.3×10^4 (5×10^5)
	$^3P_1-^3P_0$	145.525	1.7×10^{-5}	–	3400 (1×10^5)
	$^3P_2-^1D_2$	0.63003	6.3×10^{-3}	–	1.8×10^6
O II	$^4S_{3/2}-^2D_{5/2}$	0.37288	3.6×10^{-5}	0.88	1160
	$^4S_{3/2}-^2D_{3/2}$	0.37260	1.8×10^{-4}	0.59	3890
O III	$^3P_0-^3P_1$	88.356	2.62×10^{-5}	0.39	461
	$^3P_1-^3P_2$	51.815	9.76×10^{-5}	0.95	3250
	$^3P_2-^1D_2$	0.50069	1.81×10^{-2}	2.50	6.4×10^5
	$^3P_1-^1D_2$	0.49589	6.21×10^{-3}	2.50	6.4×10^5
	$^1D_2-^1S_0$	0.43632	1.70	0.40	2.4×10^7
Ne II	$^2P_{1/2}-^2P_{3/2}$	12.8136	8.6×10^{-3}	0.37	5.9×10^5
Ne III	$^3P_2-^3P_1$	15.5551	3.1×10^{-2}	0.60	1.27×10^5
	$^3P_1-^3P_0$	36.0135	5.2×10^{-3}	0.21	1.82×10^4
Si II	$^2P_{1/2}-^2P_{3/2}$	34.8152	2.17×10^{-4}	7.7	(3.4×10^5)
S II	$^4S_{3/2}-^2D_{5/2}$	0.67164	2.60×10^{-4}	4.7	1240
	$^4S_{3/2}-^2D_{3/2}$	0.67308	8.82×10^{-4}	3.1	3270
S III	$^3P_0-^3P_1$	33.4810	4.72×10^{-4}	4.0	1780
	$^3P_1-^3P_2$	18.7130	2.07×10^{-3}	7.9	1.4×10^4
S IV	$^2P_{1/2}-^2P_{3/2}$	10.5105	7.1×10^{-3}	8.5	5.0×10^4
Ar II	$^2P_{1/2}-^2P_{3/2}$	6.9853	5.3×10^{-2}	2.9	1.72×10^6
Ar III	$^3P_2-^3P_1$	8.9914	3.08×10^{-2}	3.1	2.75×10^5
	$^3P_1-^3P_0$	21.8293	5.17×10^{-3}	1.3	3.0×10^4
Fe II	$^6D_{7/2}-^6D_{5/2}$	35.3491	1.57×10^{-3}	–	(3.3×10^6)
	$^6D_{9/2}-^6D_{7/2}$	25.9882	2.13×10^{-3}	–	(2.2×10^6)