

Interstellar Medium (ISM)

Week 8

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선광일 (Kwangil Seon)
KASI / UST

Hot Ionized Medium

- Shock
- Supernova Remnant
- Local Hot Bubble

General Properties of the HIM

- Hot Ionized Medium, coronal gas
 - About half the volume of the ISM in our Galaxy is occupied by the HIM.
 - temperature $\sim 10^6$ K.
 - typical ion number density $n \sim 0.004 \text{ cm}^{-3}$
 - it provides only $\sim 0.2\%$ of the mass of the ISM, despite being the largest contributor to its volume.
 - The HIM is hot because it has been heated by shock fronts that result from supernova explosions.
 - We live in the “Local Bubble”, which is ~ 100 pc in size. The Local Bubble is thought to have been blown by a supernova that went off ~ 10 Myr ago.

Gas Dynamics

- Assumption for hydrodynamics:
 - particle mean free path << size of the region
 - We will describe the equations for conservation of mass, momentum and energy, in 1D space.
- Conservation equations
 - Mass conservation (continuity equation)
 - ▶ mass within a volume $dV = \rho dV/dA = \rho dx$ (per unit area)
 - ▶ no sources or sinks of material within dV

$$\begin{aligned}\frac{\partial}{\partial t}(\rho dx) &= \overbrace{\rho u}^{\text{incoming}} - \overbrace{(\rho + d\rho)(u + du)}^{\text{outgoing}} \\ &= -(\rho du + ud\rho + \cancel{d\rho du})\end{aligned}$$

$$\frac{\partial \rho}{\partial t} = -\frac{\partial(\rho u)}{\partial x}$$

- ▶ Mass loss and gain terms could be added in the right-hand side.

- Momentum conservation (Euler's equation)

- ▶ momentum within dV (per unit area) = $(\rho dV)u/dA = \rho dx u$
- ▶ = change of momentum due to fluid flow and gas pressure acting on the surface of dV

$$\begin{aligned}\frac{\partial}{\partial t}(\rho u dx) &= \overbrace{\rho u^2}^{\text{incoming}} - \overbrace{(\rho + d\rho)(u + du)^2}^{\text{outgoing}} + \overbrace{P}^{\text{incoming}} - \overbrace{P + dP}^{\text{outgoing}} \\ &= \rho u^2 - \left(\rho u^2 + 2\rho u du + \cancel{\rho du^2} + u^2 d\rho + \cancel{2ud\rho du} + \cancel{d\rho du^2} \right) - dP\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial t}(\rho u) &= -2\rho u \frac{\partial u}{\partial x} - u^2 \frac{\partial \rho}{\partial x} - \frac{\partial P}{\partial x} \\ \rho \frac{\partial u}{\partial t} + u \frac{\partial \rho}{\partial t} &= -\rho u \frac{\partial u}{\partial x} - u \left(\rho \frac{\partial u}{\partial x} + u \frac{\partial \rho}{\partial x} \right) - \frac{\partial P}{\partial x}\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial t}(\rho u) &= -2\rho u \frac{\partial u}{\partial x} - u^2 \frac{\partial \rho}{\partial x} - \frac{\partial P}{\partial x} \\ &= -\frac{\partial}{\partial x}(\rho u^2) - \frac{\partial P}{\partial x}\end{aligned}$$

or

Using mass conservation, $\frac{\partial u}{\partial t} = -\frac{\partial(\rho u)}{\partial x}$

$$\rho \frac{\partial u}{\partial t} = -\rho u \frac{\partial u}{\partial x} - \frac{\partial P}{\partial x}$$

$$\frac{\partial}{\partial t}(\rho u) = -\frac{\partial}{\partial x} (\rho u^2 + P)$$

- ▶ Further terms could be added in the right-hand side, accounting for forces due to gravity, magnetic fields, radiation field, and viscosity.

-
- ▶ Viscous force is due to “internal friction” in the fluid (resistivity of the fluid to the flow), as two adjacent fluid parcels move relative to each other.)

$$\text{viscous force} \propto \frac{\partial^2 u}{\partial x^2}$$

The viscous force is usually much smaller than force due to gas pressure, but important in high-speed flows with large velocity gradients, as in accretion disks.

- Energy conservation
 - ▶ The first law of thermodynamics states that
heat added in a system = change in internal energy + work done on surroundings

$$dQ = dU + PdV$$

An ideal gas is a theoretical gas composed of many randomly moving point particles whose only interactions are perfectly elastic collisions (no viscosity or heat conduction).

- ▶ Internal energy (per particle) for ideal gas is

$$U/N = \frac{3}{2}kT \text{ for monatomic gas (translation about 3 axes)}$$

$$U/N = \frac{5}{2}kT \text{ for diatomic gas (+rotation about 2 axes)}$$

$$U/N = 3kT \text{ for polyatomic gas (+rotation about 3 axes)}$$

N = number of particles

- ▶ In general, $U/N = \frac{f}{2}kT$ (f = degree of freedom)

At high temperature, molecules have access to an increasing number of vibrational degrees of freedom, as they start to bend and stretch.

- The ideal gas law (the equation of state) for a perfect Maxwellian distribution.

$$PV = NkT$$

- Specific heat capacity is the amount of heat energy required to raise the temperature of a material per unit of mass.

- ▶ specific heat capacity at constant volume:

$$c_V \equiv \frac{1}{M} \left(\frac{\partial Q}{\partial T} \right)_V = \frac{1}{M} \left(\frac{\partial U}{\partial T} \right)_V \quad c_V = \frac{f}{2} \frac{k}{m}$$

M = total mass

$m = M/N$ = mass per particle

- ▶ specific heat capacity at constant pressure:

$$c_P \equiv \frac{1}{M} \left(\frac{\partial Q}{\partial T} \right)_P = \frac{1}{M} \left(\frac{\partial U}{\partial T} \right)_P + \frac{P}{M} \left(\frac{\partial V}{\partial T} \right)_P$$

$$c_P = c_V + \frac{k}{m}$$

$$c_P = \frac{f+2}{2} \frac{k}{m}$$

- ▶ Ratio of specific heat capacities:

$$\gamma \equiv \frac{c_P}{c_V} = \frac{f+2}{f} = \frac{5}{3} \text{ for monatomic gas}$$

$$= \frac{7}{5} \text{ for diatomic gas}$$

$$= \frac{4}{3} \text{ for polyatomic gas}$$

γ is called the adiabatic index.

$$c_P > c_V$$

This inequality implies that when pressure is held constant, some of the added heat goes into PdV work instead of into internal energy.

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- Energy Conservation - limiting cases
 - Adiabatic flow - negligible heat transport. (Internal energy is changed only by work).

$$\begin{aligned} dQ &= 0 \\ \rightarrow PdV &= -Mc_VdT \end{aligned}$$

$$\begin{aligned} PV &= NkT \\ \rightarrow VdP + PdV &= NkdT \end{aligned}$$

We combine two equations and eliminate dT term:

$$\begin{aligned} VdP + PdV &= -\frac{Nk}{Mc_V} PdV \\ &= -\frac{k}{m c_V} PdV &\longrightarrow & VdP = -\left(1 + \frac{k}{m c_V}\right) PdV \\ &&& = -\frac{1}{c_V} \left(c_V + \frac{k}{m}\right) PdV \\ &&& = -\gamma PdV \\ &&\longrightarrow & \frac{dP}{P} = -\gamma \frac{dV}{V} \end{aligned}$$

We can rewrite this in terms of density:

$$\begin{aligned} \rho V &= M \\ \rightarrow \rho dV + Vd\rho &= 0 \\ \rightarrow \frac{d\rho}{\rho} &= -\frac{dV}{V} \end{aligned}$$

$$\longrightarrow \frac{dP}{P} = \gamma \frac{d\rho}{\rho}$$

$$\begin{aligned} P &\propto \rho^\gamma \\ P &\propto V^{-\gamma} \end{aligned}$$

-
- ▶ Isothermal flow - extremely efficient cooling (heat transport).

heat transport timescale << dynamic timescale

This implies the balance between heating and cooling, hence a constant temperature.

From the ideal gas law,

$$P = \frac{N}{V} = \rho \frac{kT}{m}$$



$$\begin{aligned} P &\propto \rho \\ P &\propto V^{-1} \end{aligned}$$

- ▶ In general, we have

$$\begin{aligned} P &\propto \rho^\gamma \\ P &\propto V^{-\gamma} \end{aligned}$$

$(\gamma = 1$ for isothermal gas)

A gas that has an equation of state with this power-law form is called a ***polytope***, from the Greek polytropos, meaning “turning many ways” or “versatile.”

(A polystrope should not be confused with a polytrope, which is the n-dimensional generalization of a 2D polygon and 3D polyhedron.)

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- Specific internal energy of the gas (per unit mass):

$$\begin{aligned} \epsilon &\equiv U/M \\ U/N &= \frac{f}{2}kT \end{aligned} \quad \longrightarrow \quad \epsilon = \frac{f}{2} \frac{kT}{m} \quad \text{or} \quad \epsilon = \frac{1}{\gamma - 1} \frac{kT}{m} = \frac{1}{\gamma - 1} \frac{P}{\rho}$$

- Total Energy (per unit volume):

- ▶ Internal energy per unit volume: $\rho\epsilon$
- ▶ Kinetic energy due to bulk motion, per unit volume: $\rho \frac{u^2}{2}$
- ▶ Work on unit volume: $\frac{PdV}{dV} = P$

$$\mathcal{E} = \rho \left(\frac{u^2}{2} + \epsilon \right) + P \quad \longrightarrow \quad \mathcal{E} = \rho \frac{u^2}{2} + \frac{\gamma}{\gamma - 1} P$$

- Energy conservation:

$$\frac{\partial \mathcal{E}}{\partial t} = - \frac{\partial(u\mathcal{E})}{\partial x}$$

Sound Wave

- Suppose that we are surrounded by an ideal gas with a plane parallel symmetry:
 - We consider a region where the gas has initially a uniform density, pressure, and no bulk velocity: $\rho_0, P_0, u_0 = 0$

In the uniform gas, we introduce small perturbations of the form:

$$\begin{array}{ll} \rho(x, t) = \rho_0 + \rho_1(x, t) & P_1 = P - P_0 \\ u(x, t) = u_1(x, t) & \propto (\rho_0 + \rho_1)^\gamma - \rho_0^\gamma \\ P(x, t) = P_0 + P_1(x, t) & \propto \gamma \rho_0^{\gamma-1} \rho_1 \end{array} \longrightarrow \quad \longrightarrow \quad P_1 = \frac{\gamma P_0}{\rho_0} \rho_1$$

We obtain The conservation equations give:

$$\begin{aligned} \frac{\partial \rho}{\partial t} &= -\frac{\partial(\rho u)}{\partial x} \\ \rho \frac{\partial u}{\partial t} &= -\rho u \frac{\partial u}{\partial x} - \frac{\partial P}{\partial x} \end{aligned} \quad \begin{aligned} \frac{\partial \rho_1}{\partial t} &= -\rho_0 \frac{\partial u_1}{\partial x} \\ \rho_0 \frac{\partial u_1}{\partial t} &= -\frac{\partial P_1}{\partial x} = -\frac{\gamma P_0}{\rho_0} \frac{\partial \rho_1}{\partial x} \end{aligned}$$

$$\frac{\partial^2 \rho_1}{\partial t^2} = -\frac{\gamma P_0}{\rho_0} \frac{\partial^2 \rho_1}{\partial x^2}$$

- The resulting equation represents a sound wave (acoustic wave) with a constant sound speed:

$$c_s = \left(\frac{\gamma P}{\rho} \right)^{1/2} = \left(\frac{\gamma k T}{m} \right)^{1/2} \quad c_s \propto \rho^{(\gamma-1)/2}$$

For $\gamma > 1$, sound travels more rapidly in a denser gas.

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- The sound speed is of the same order as the mean thermal velocity:

$$c_s = 1.2 \text{ km s}^{-1} \left(\frac{\gamma}{5/3} \right)^{1/2} \left(\frac{m}{m_p} \right)^{-1/2} \left(\frac{T}{100 \text{ K}} \right)^{1/2}$$

$(m_p = \text{proton mass})$

- Sound crossing time:

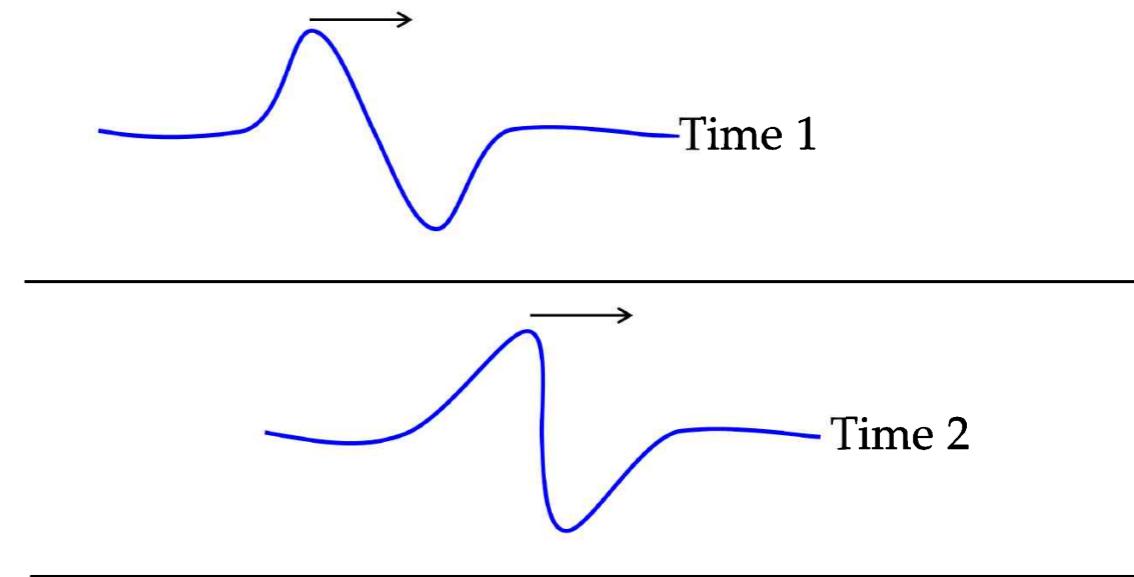
- ▶ sound crossing time = time it takes for a signal to cross a region of size L :

$$t_{\text{cross}} = L/c_s$$

- ▶ A small pressure gradient tends to be smoothed out within the sound crossing time.
 - ▶ Fast changes occurring on timescales $\ll t_{\text{cross}}$ will survive.
 - ▶ Slow changes occurring on timescales $\gg t_{\text{cross}}$ will be damped.
- Mach number = gas velocity / sound speed

$$\mathcal{M} \equiv u/c_s$$

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- Shock front
 - A low-amplitude sound wave traveling through a medium will be adiabatic; that is it will not increase the entropy of the gas through which it passes.
 - For an adiabatic process, the equation of state for the gas is
$$c_s \propto \rho^{(\gamma-1)/2}$$
 - Thus, for $\gamma > 1$, sound travels more rapidly in a denser gas. A sound wave, therefore, will steepen until a shock front forms.
 - The shock front is a layer whose thickness is comparable to the mean free path between particle collisions.
 - The ordinary sound that we hear every day will not, in practice, steepen into shocks.
 - However, high amplitude pressure fluctuations will rapidly steepen into shocks.
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Shock Front

- Jump condition (***Rankine-Hugoniot conditions***)

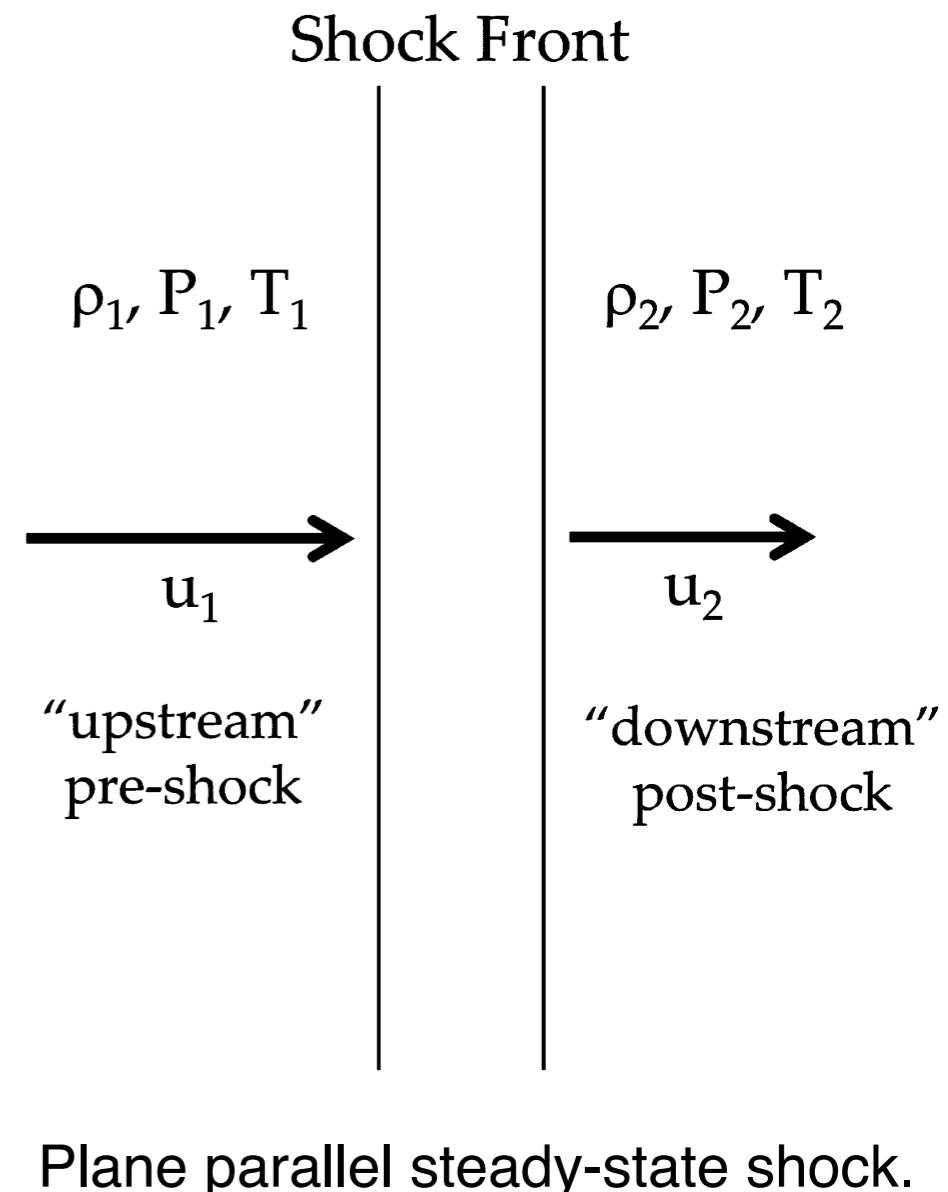
- Let

ρ = mass density, T = temperature,

m = mean molecular mass

- If a patch is small compared to the shock front's radius of curvature, then we can treat the shock front as if it has ***plane parallel*** symmetry.

- It is convenient to use ***a frame of reference in which the shock front is stationary***; in this frame, the bulk velocity u_1 of the pre-shock (upstream) gas toward the shock front. The bulk velocity u_2 of the post-shock (downstream) gas points away from the shock front.



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- Let's consider a steady state solution.
 - The gas properties immediately before being shocked (“1”) and immediately after being shocked (“2”) are obtained from the conservation laws:
 $\rho_1 u_1 = \rho_2 u_2$

$$\rho_1 u_1^2 + P_1 = \rho_2 u_2^2 + P_2$$

$$u_1 \left(\rho_1 \frac{u_1^2}{2} + \frac{\gamma}{\gamma - 1} P_1 \right) = u_2 \left(\rho_2 \frac{u_2^2}{2} + \frac{\gamma}{\gamma - 1} P_2 \right)$$

Dividing the third equation with the first equation:

$$\frac{u_1^2}{2} + \frac{\gamma}{\gamma - 1} \frac{P_1}{\rho_1} = \frac{u_2^2}{2} + \frac{\gamma}{\gamma - 1} \frac{P_2}{\rho_2}$$

Here, we assume that an adiabatic index is the same on both sides of the shock front.

- It is convenient to use a dimensionless number, the Mach number of the upstream:

$$\frac{P_2}{P_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma + 1}$$

$$\frac{T_2}{T_1} = \frac{[(\gamma - 1)M_1^2 + 2][2\gamma M_1^2 - (\gamma - 1)]}{(\gamma + 1)^2 M_1^2}$$

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- For a strong shock $\mathcal{M}_1 \gg 1$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)\mathcal{M}_1^2}{(\gamma - 1)\mathcal{M}_1^2 + 2} = \frac{u_1}{u_2}$$

$$\frac{P_2}{P_1} = \frac{2\gamma\mathcal{M}_1^2 - (\gamma - 1)}{\gamma + 1}$$

$$\frac{T_2}{T_1} = \frac{[(\gamma - 1)\mathcal{M}_1^2 + 2][2\gamma\mathcal{M}_1^2 - (\gamma - 1)]}{(\gamma + 1)^2\mathcal{M}_1^2}$$

$$\frac{\rho_2}{\rho_1} \approx \frac{\gamma + 1}{\gamma - 1}$$

$$P_2 \approx \frac{2\gamma\mathcal{M}_1^2}{\gamma + 1} P_1 \xrightarrow{P_1 = c_1^2 \frac{\rho_1}{\gamma}} \frac{2\gamma(u_1/c_1)^2}{\gamma + 1} c_1^2 \frac{\rho_1}{\gamma}$$

$$P_2 \approx \frac{2}{\gamma + 1} \rho_1 u_1^2$$

$$T_2 \approx \frac{2(\gamma - 1)\gamma}{(\gamma + 1)^2} \mathcal{M}_1^2 T_1 = \frac{2(\gamma - 1)\gamma}{(\gamma + 1)^2} \left(\frac{u_1}{c_1} \right)^2 T_1$$

$$T_2 \approx \frac{2(\gamma - 1)}{(\gamma + 1)^2} \frac{m}{k} u_1^2$$

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- Thus, no matter how strong the shock is, the density ratio has a finite value:

$$\frac{\rho_2}{\rho_1} \approx 4$$

$$\frac{\rho_2}{\rho_1} \approx 4 \text{ for } \gamma = 5/3 \text{ (monatomic gas)}$$

$$P_2 \approx \frac{3}{4} \rho_0 u_1^2$$

$$T_2 \approx \frac{3}{16} \frac{m}{k} u_1^2$$

- Consider a strong shock

- In the lab frame, V_s = shock velocity, v_1, v_2 = gas velocities in upstream and downstream, respectively.

$$u_1 = v_1 - V_s \approx -V_s \quad (V_s \gg v_1)$$

$$u_2 = v_2 - V_s$$

- Then, the post-shock velocity is

$$\frac{u_2}{u_1} = \frac{v_s - V_s}{-V_s} = \frac{1}{4} \quad \Rightarrow \quad v_2 = \frac{3}{4} V_s$$

- Hence, the post-shock moves in the same direction as the shock front with a velocity of 3/4 of the shock velocity.

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- Then, the post-shock pressure, temperature, specific internal energy, and specific kinetic energy are

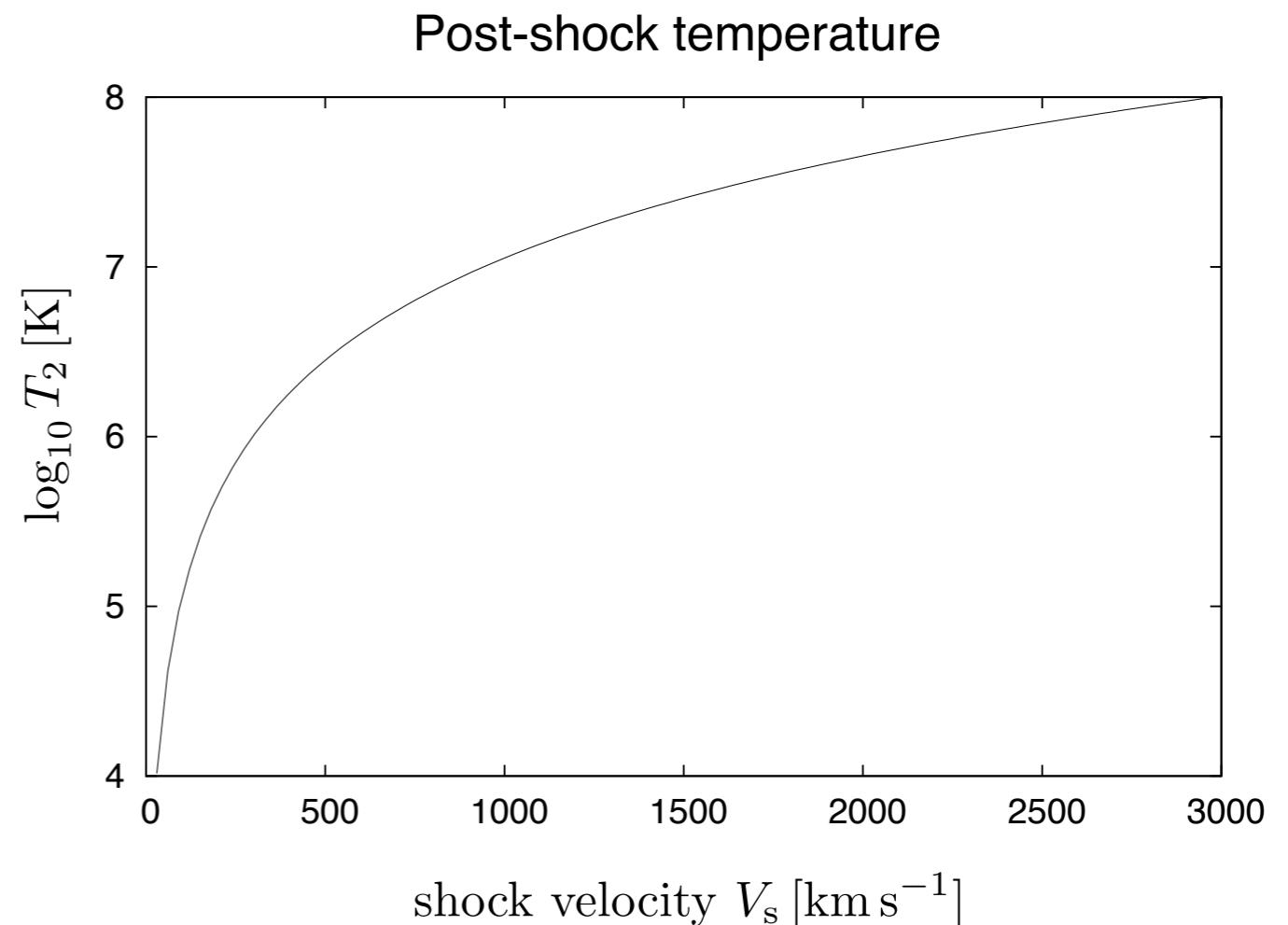
$$P_2 = \frac{3}{4} \rho_1 V_s^2$$

$$T_2 = \frac{3m}{16k} V_s^2$$

$$\epsilon_2 = \frac{3}{2} \frac{P_2}{\rho_2} = \frac{3}{2} \frac{(3/4)\rho_1 V_s^2}{4\rho_1} = \frac{9}{32} V_s^2$$

$$\mathcal{E}_{K,2} = \frac{1}{2} v_2^2 = \frac{9}{32} V_s^2$$

for $\gamma = 5/3$

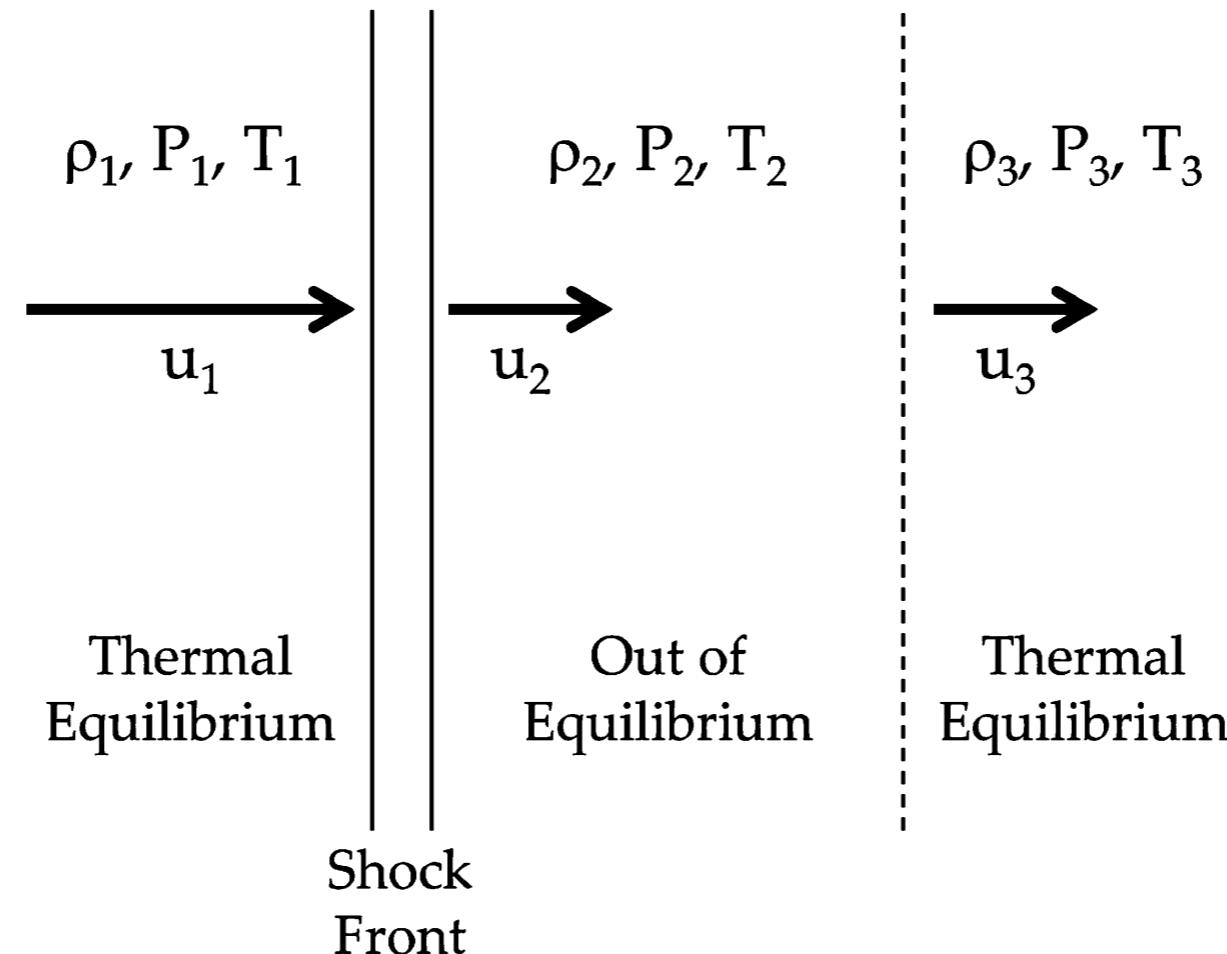


- A strong shock can produce very high pressures and temperatures. An interstellar shock front with propagation speed $u_1 \sim 1000 \text{ km s}^{-1}$ (typical for a supernova shock wave) produces shock heated gas with

$$T_2 \approx 1.1 \times 10^7 \text{ K} \left(\frac{u_2}{1000 \text{ km s}^{-1}} \right)^2$$

- assuming the shocks gas is fully ionized hydrogen.
- In general, shock fronts convert supersonic gas into subsonic gas in the shock's frame of reference. Shocks increase density, pressure, and temperature, and decrease bulk velocity relative to the shock front. Shocks act as entropy generators.

Cooling in Shocked Gas



The structure of a plane parallel radiative shock [Figure 5.3 Ryden]

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- The hot shocked gas is out of equilibrium, and will start to cool. Thus, the shock will be followed by a radiative zone in which the shock heated gas cools down again by radiating away photons.
 - At high temperatures $T > 2 \times 10^7$ K
 - ▶ the cooling is dominated by bremsstrahlung (free-free radiation), for which the specific cooling rate is

$$\mathcal{L} = 2.7 \text{ [erg g}^{-1} \text{ s}^{-1}] \left(\frac{T}{10^7 \text{ K}} \right)^{1/2} \left(\frac{n_{\text{H}}}{1 \text{ cm}^{-3}} \right)$$

- ▶ assuming a gas of fully ionized hydrogen. The specific internal energy of ionized hydrogen is

$$\epsilon = \frac{3kT}{m_{\text{H}}} \approx 2.5 \times 10^{15} \text{ [erg g}^{-1}] \left(\frac{T}{10^7 \text{ K}} \right)$$

- ▶ Then, the bremsstrahlung cooling time is

$$\begin{aligned} t_{\text{cool}} &= \frac{\rho \epsilon}{\mathcal{L}} \\ &\approx 30 \text{ [Myr]} \left(\frac{T}{10^7 \text{ K}} \right)^{1/2} \left(\frac{n_{\text{H}}}{1 \text{ cm}^{-3}} \right)^{-1} \\ &\approx 30 \text{ [Myr]} \left(\frac{V_s}{1000 \text{ km s}^{-1}} \right) \left(\frac{n_{\text{H}}}{1 \text{ cm}^{-3}} \right)^{-1} \end{aligned}$$

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- ▶ During this time, the gas will move a distance, relative to the shock front:

$$R_{\text{cool}} \approx u_2 t_{\text{cool}} \approx \frac{u_1}{4} t_{\text{cool}}$$

$$\approx 8 \text{ [kpc]} \left(\frac{u_1}{1000 \text{ km s}^{-1}} \right)^2 \left(\frac{n_{\text{H}}}{1 \text{ cm}^{-3}} \right)^{-1}$$

- ▶ This tells us that the approximate thickness of the radiative zone for a strong shock is a long distance compared to the scale height of the ISM in our galaxy. Thus, the hot gas produced by high-speed shocks doesn't have time to cool before the shock runs out of gas to shock.
- At lower temperature ($10^5 \text{ K} < T < 2 \times 10^7 \text{ K}$) corresponding to slower shock speed ($80 \text{ km s}^{-1} < u_1 < 1200 \text{ km s}^{-1}$)
- ▶ collisionally excited line emission does most of the cooling, a useful approximation is

$$t_{\text{cool}} \approx 7000 \text{ [yr]} \left(\frac{u_1}{100 \text{ km s}^{-1}} \right)^{3.4} \left(\frac{n_{\text{H}}}{1 \text{ cm}^{-3}} \right)^{-1}$$

- ▶ This yields a thickness for the radiative zone.

$$R_{\text{cool}} \approx \frac{u_1}{4} t_{\text{cool}}$$

$$\approx 0.18 \text{ [pc]} \left(\frac{u_1}{100 \text{ km s}^{-1}} \right)^{4.4} \left(\frac{n_{\text{H}}}{1 \text{ cm}^{-3}} \right)^{-1}$$

- ▶ The shorter time scales and length scales mean that radiative cooling is more effective at changing the structure of slower shocks.

Collisional Ionization Equilibrium

- CIE
 - CIE assumes that the plasma is in a steady state, and that collisional ionization, charge exchange, radiative recombination, and dielectronic recombination are the only processes altering the ionization balance.
 - ▶ Note that the reverse process to collisional ionization is a three-body recombination, which is unlikely to occur.
 - The ionization fractions for each element depend only on the gas temperature, with no dependence on the gas density.
- Ionization fraction
 - For hydrogen, the balance equation is : ionization rate = recombination rate

$$n_e n(\text{H}^0) k_{\text{ci}, \text{H}} = n_e n(\text{H}^+) \alpha_{\text{A}, \text{H}} \quad n(\text{H}^0) + n(\text{H}^+) = n(\text{H})$$

- The rate coefficients for collisional ionization and radiative recombination are:

$$k_{\text{ci}, \text{H}} = 5.849 \times 10^{-9} T_4^{1/2} e^{-15.782/T_4} [\text{cm}^3 \text{s}^{-1}]$$

$$\begin{aligned} \alpha_{\text{A}, \text{H}} &= 4.13 \times 10^{-13} T_4^{-0.7131 - 0.0115 \ln T_4} [\text{cm}^3 \text{s}^{-1}] \quad \text{for } 30 \text{ K} < T < 3 \times 10^4 \text{ K} \\ &= 5 \times 10^{-16} T_7^{-1.5} \quad \text{for } T > 10^6 \text{ K} \end{aligned}$$

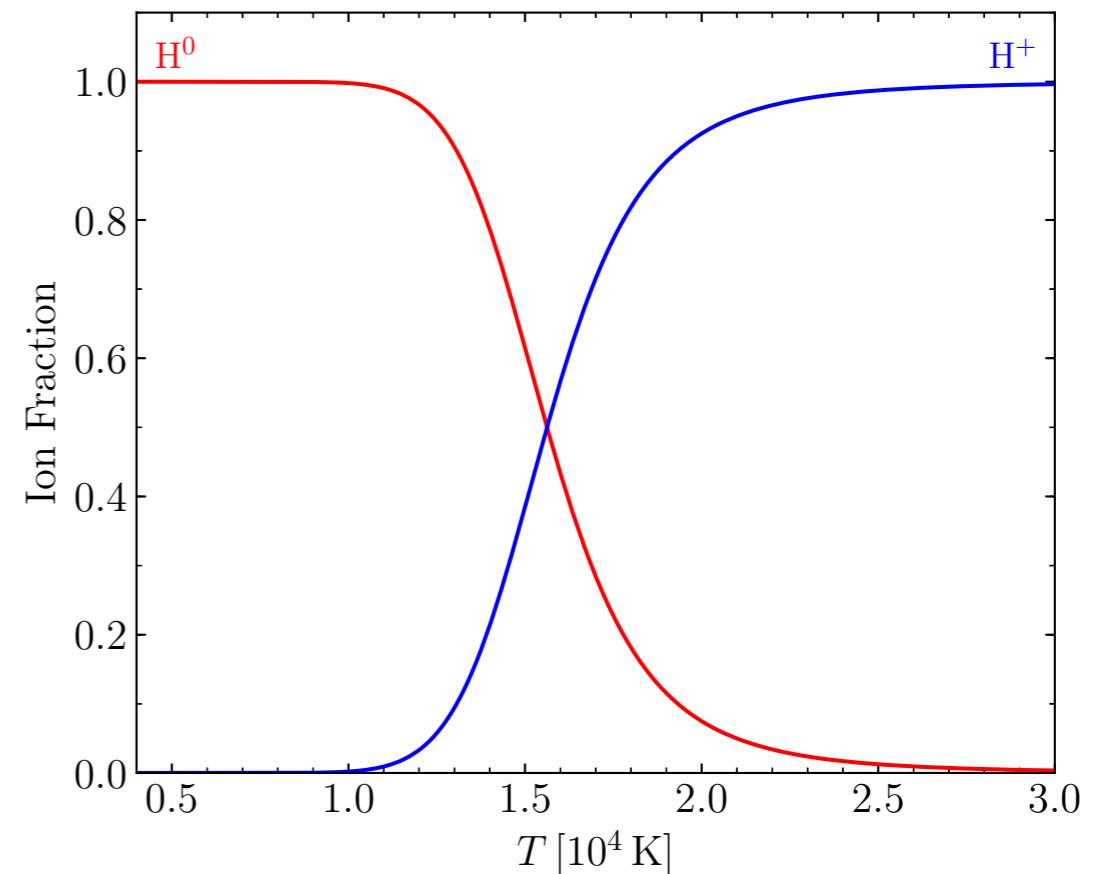
from Draine

- The ionization fraction is

$$\begin{aligned}x &\equiv \frac{n(\text{H}^+)}{n(\text{H}^0) + n(\text{H}^+)} \\&= \frac{k_{\text{ci},\text{H}}}{k_{\text{ci},\text{H}} + \alpha_{\text{A},\text{H}}}\end{aligned}$$

- The ion fractions are

$$\begin{aligned}x &\approx 0.002 \quad \text{at } T = 10^4 \text{ K} \\1 - x &\approx 3 \times 10^{-7} \quad \text{at } T = 10^6 \text{ K}\end{aligned}$$



H II regions with $T = 10^4$ K are photoionized by UV photons from hot stars.

Hydrogen gas with $T = 10^6$ K is almost entirely collisionally ionized.

- For Helium, the balance equations are:

$$n(\text{He}^+) \alpha_{10} = n(\text{He}^0) k_{01}$$

$$n(\text{He}^+) k_{12} = n(\text{He}^{2+}) \alpha_{21}$$

$$n(\text{He}) = n(\text{He}^0) + n(\text{He}^+) + n(\text{He}^{2+})$$

Here, ij indicates $X^{i+} \rightarrow X^{j+}$.

- The rate coefficients are

$$k_{01} = 2.39 \times 10^{-11} T^{1/2} e^{-285,335/T}$$

from Cen (1992, ApJS)

$$k_{12} = 5.68 \times 10^{-12} T^{1/2} e^{-631,515/T}$$

$$\alpha_{10} = 1.50 \times 10^{-10} T^{-0.6353} \text{ radiative recombination}$$

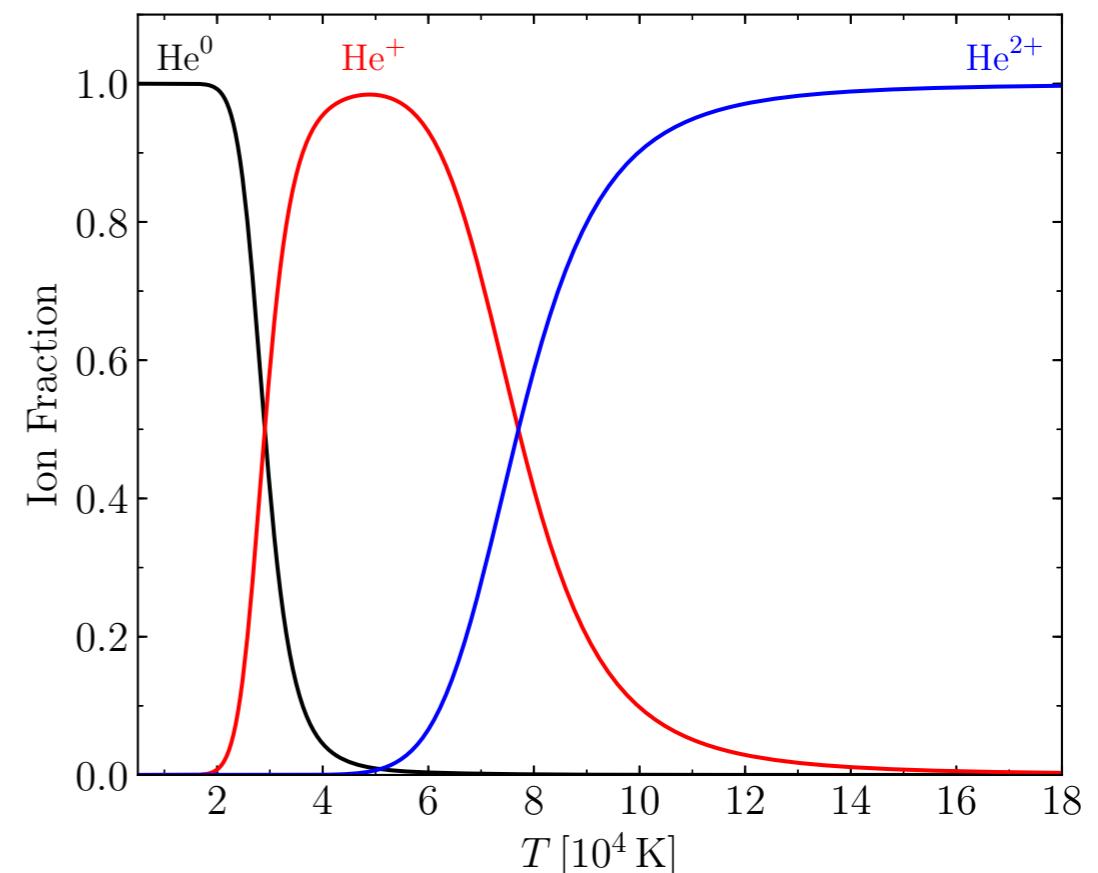
$$+ 1.9 \times 10^{-3} T^{-1.5} e^{-470,000/T} (1 + 0.3e^{-94,000/T}) \text{ dielectronic recombination (but not significant)}$$

$$\alpha_{21} = 3.36 \times 10^{-10} T^{-1/2} T_3^{-0.2} / (1 + T_6^{0.7})$$

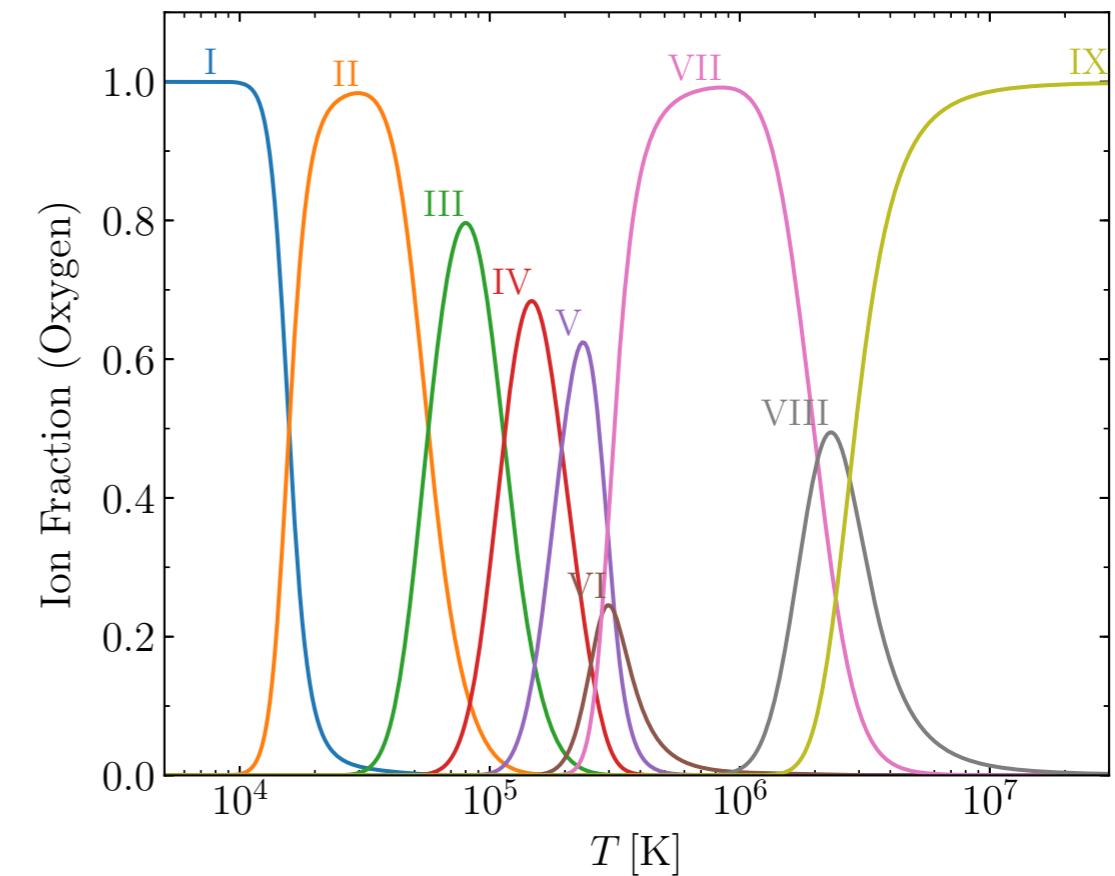
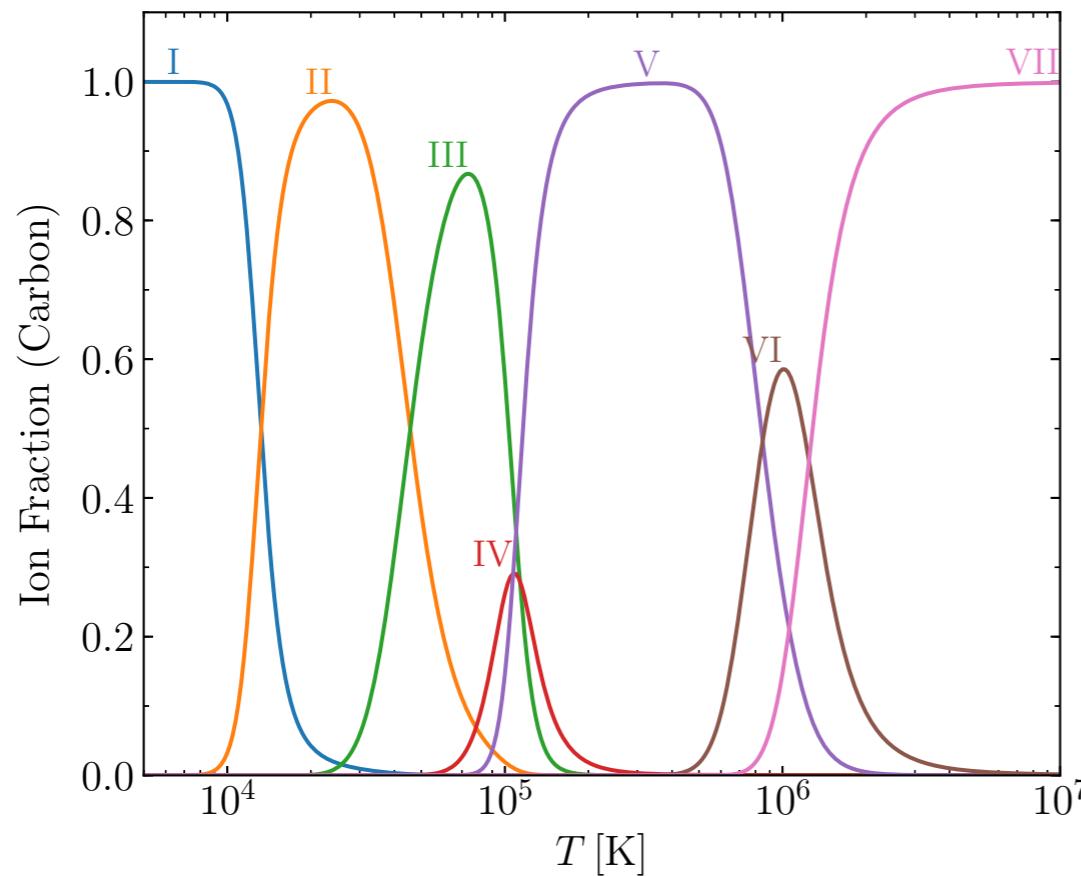
- Using the above rate coefficients, the ionization fractions can be estimated as follows:

$$x \equiv \frac{n(\text{He}^+)}{n(\text{He})} = \frac{1}{1 + \alpha_{10}/k_{01} + k_{12}/\alpha_{21}}$$

$$y \equiv \frac{n(\text{He}^{2+})}{n(\text{He})} = \frac{k_{12}}{\alpha_{21}} x$$



- Heavy Elements
 - ▶ The calculation is usually done numerically, for instance, using CHIANTI
[CHIANTI: https://www.chiantidatabase.org/](https://www.chiantidatabase.org/)
 - ▶ For instance, the ion fractions of Carbon and Oxygen as a function of temperature are:



- At $T \sim 10^6$ K, we expect a mix of C V, C VI, and C VII.
- At $T \sim 4 \times 10^6$ K and higher, almost all the carbon will be in the form of fully ionized C VII.

calculated using CHIANTI

Cooling in CIE

- ***Cooling function***

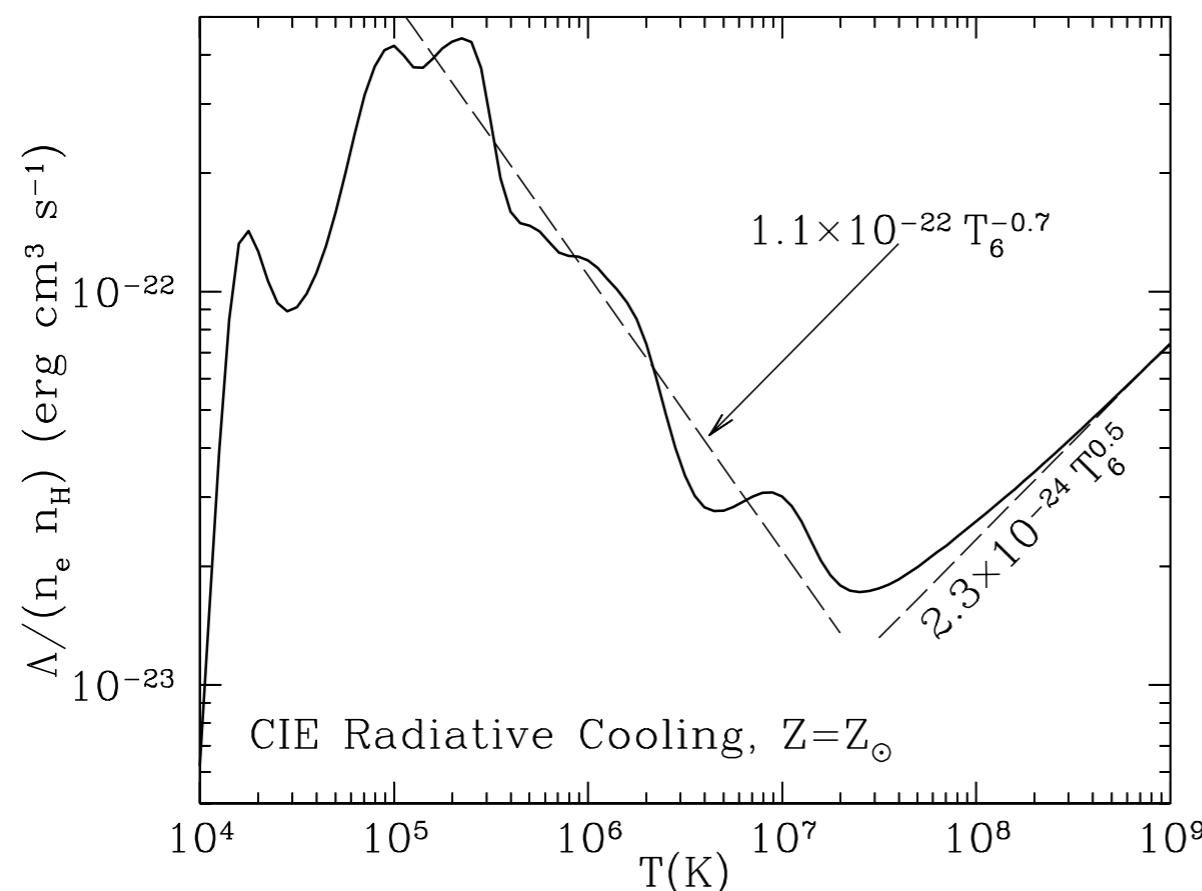
- At temperature $T > 10^4$ K, ionization of hydrogen provides enough free electrons so that collisional excitation of atoms or ions is dominated by electron collisions.
- At low densities, every collisional excitation is followed by a radiative decay, and the rate of removal of thermal energy per unit volume can be written:

$$\Lambda = n_e n_H f_{\text{cool}}(T)$$

The ***radiative cooling function*** $f_{\text{cool}}(T) \equiv \Lambda / (n_H n_e)$ is a function of ***temperature*** and of the ***elemental abundances*** relative to hydrogen.

- At high densities, radiative cooling can be suppressed by collisional deexcitation, and the cooling function will then depend on density, in addition to T and elemental abundances.
- If ionizing radiation is present, the ionization balance may depart from CIE, and the radiative cooling function will also depend on the spectrum and intensity of the ionizing radiation.

Fig 34.1
Draine



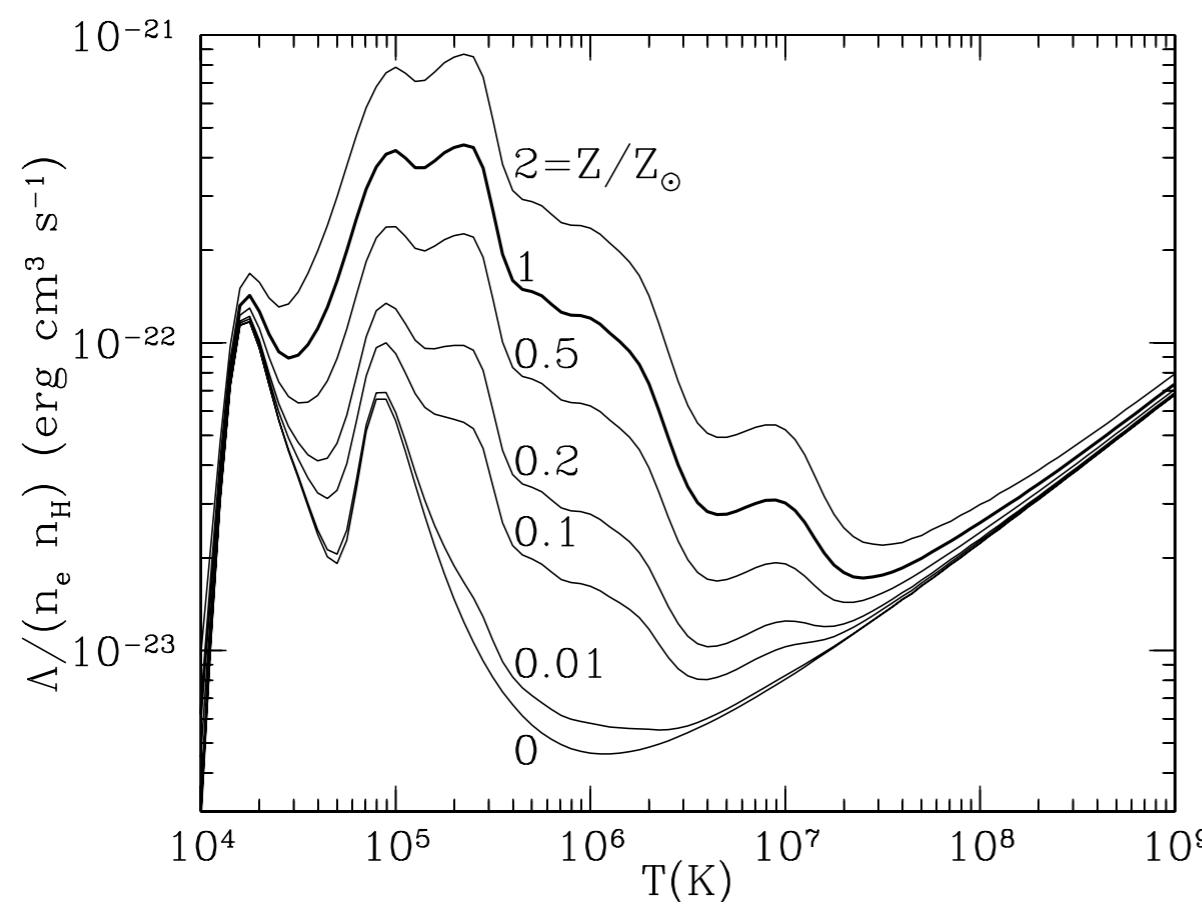
Radiative Cooling Function for solar-abundance

- At $T < 10^7$ K, the cooling is dominated by collisional excitation of bound electrons.
- At high temperatures, the ions are fully stripped of electrons, and bremsstrahlung (free-free) cooling dominates.

$$\Lambda/n_e n_H \approx 1.1 \times 10^{-22} T_6^{-0.7} \text{ [erg cm}^3 \text{ s}^{-1}\text{]}$$

$$(10^5 < T < 10^{7.3} \text{ K})$$

Fig 34.2
Draine

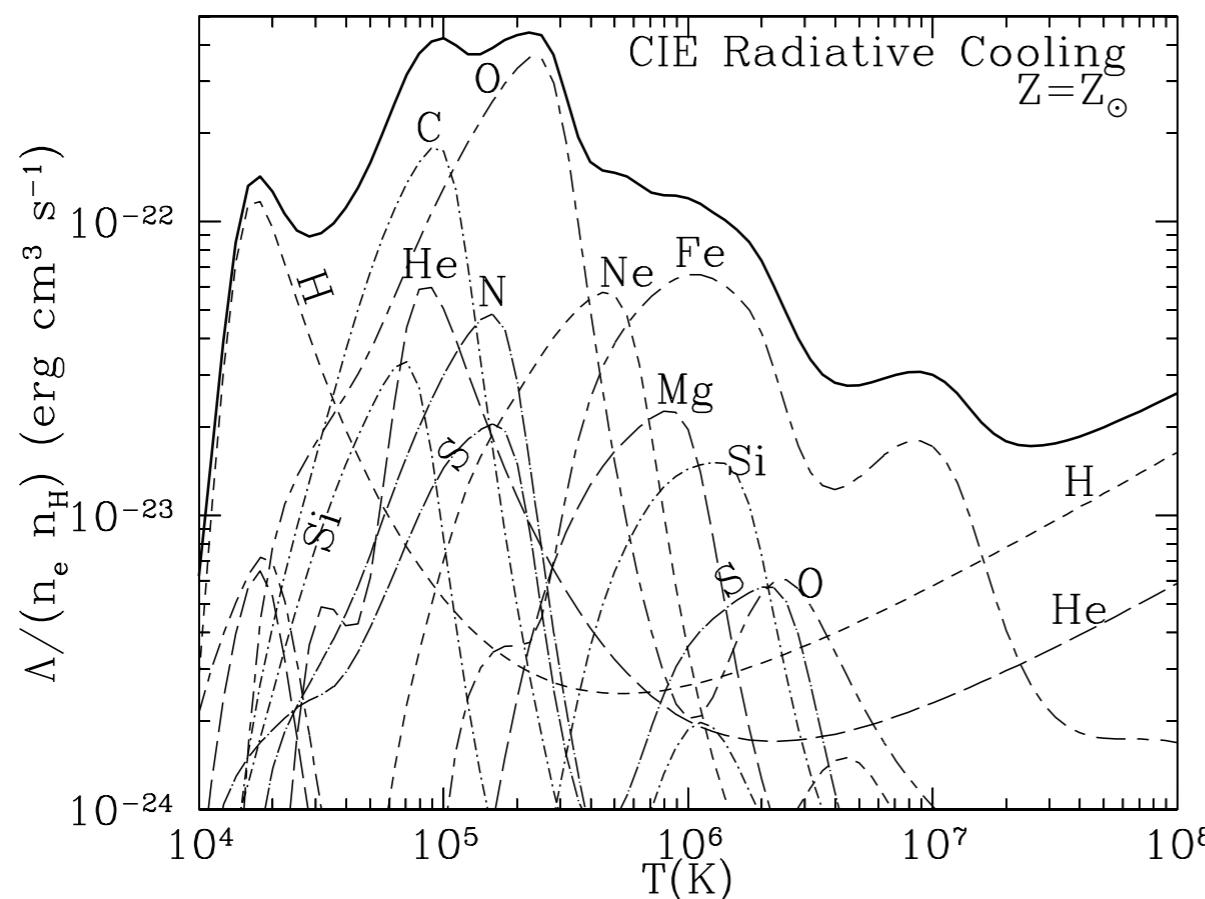


Cooling Function for different abundances

- In most applications, the abundances of elements beyond He can be assumed to be scaled up and down together.

The cooling functions were calculated using CHIANTI.

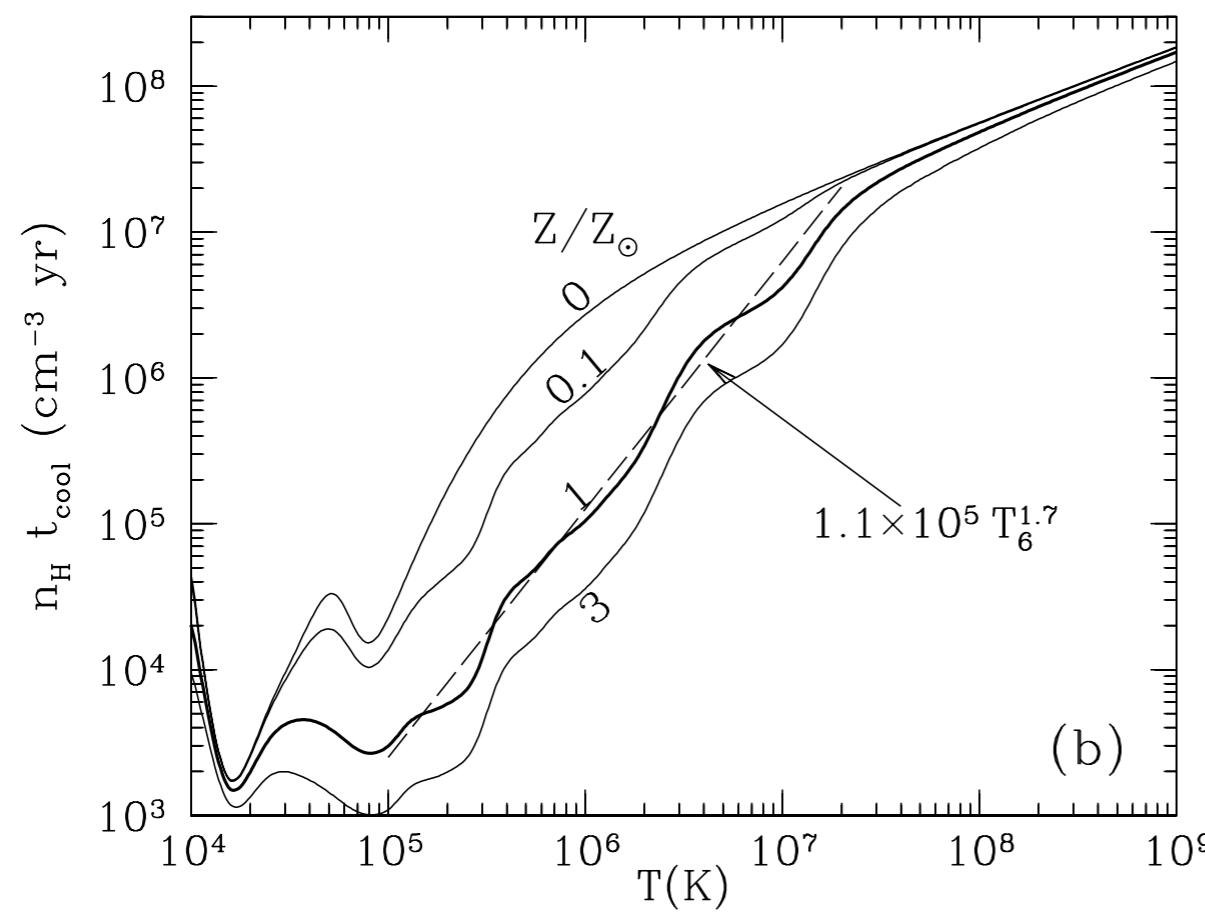
Fig 34.3
Draine



Radiative Cooling Function, with contributions from selected elements

- For $10^{5.8} < T < 10^{7.2}$ K, the cooling is dominated by Mg, Si, and Fe - elements that in cold gas are normally depleted by factors of 5 or more. In this calculation, the solar abundance is assumed.

Fig 34.4
Draine



Cooling time

$$t_{\text{cool}} \approx 1.1 \times 10^5 T_6^{1.7} (n_H/\text{cm}^{-3})^{-1} [\text{yr}]$$

$$(10^5 \lesssim 10^{7.3} \text{ K})$$

for isochoric cooling (constant density)

Cooling Time Scale [General Formulae]

- Cooling Time scale:

- The first law of thermodynamics is $dQ = dU + PdV$.
- Using a heating and cooling rate per volume Γ and Λ , the change in heat is

$$dQ = (\Gamma - \Lambda)Vdt$$

- The change in the internal energy is

$$dU = (\Gamma - \Lambda)Vdt - PdV$$

- For an ideal gas with a degree of freedom f , the equation becomes:

$$\begin{aligned} U &= \frac{f}{2}NkT \longrightarrow d\left(\frac{f}{2}NkT\right) = -\Lambda Vdt - PdV \\ PV &= NkT \end{aligned}$$

- Note that

$PdV = d(PV) - VdP = d(NkT)$ for isobaric cooling (constant pressure)

$PdV = 0$ for isochoric cooling (constant density or volume)

$$\therefore \frac{d}{dt} \left(\frac{f+2}{2}NkT \right) = -\Lambda V \rightarrow$$

$$\text{or } \frac{d}{dt} \left(\frac{f}{2}NkT \right) = -\Lambda V$$

$$t_{\text{cool}} \equiv \frac{T}{|dT/dt|} \Rightarrow t_{\text{cool}} = \frac{f+2}{2} \frac{nkT}{\Lambda} \text{ for isobaric cooling}$$

$$n \equiv N/V = \frac{f}{2} \frac{nkT}{\Lambda} \text{ for isochoric cooling}$$

Here, the number density includes all particles (molecules, atoms, ions, electrons)

Time Scales in the HIM

- In the HIM with temperatures $T \sim 10^6 - 10^7$ K, the cooling time scale is:

$$t_{\text{cool}} = \frac{5}{2} \frac{n k T}{\Lambda} \quad \begin{aligned} n_e &\approx 1.2 n_H \\ n &\approx 2.3 n_H \end{aligned} \quad \begin{aligned} &\text{one electron for an ionized hydrogen} \\ &\text{two electrons for a doubly-ionized helium.} \end{aligned}$$

- The cooling time at $T \sim 10^6$ K is

$$t_{\text{cool}} \approx 48 \text{ [Myr]} T_6^{1.7} \left(\frac{n_H}{0.004 \text{ cm}^{-3}} \right)^{-1} \quad \longleftarrow \quad \Lambda / n_e n_H \approx 1.1 \times 10^{-22} T_6^{-0.7} \text{ [erg cm}^3 \text{ s}^{-1}\text{]}$$

- At $T \sim 10^7$ K, the cooling time is

$$t_{\text{cool}} \approx 7.2 \text{ [Gyr]} T_7^{1/2} \left(\frac{n_H}{0.004 \text{ cm}^{-3}} \right)^{-1} \quad \longleftarrow \quad \Lambda / n_e n_H \approx 2.3 \times 10^{-24} T_6^{0.5} \text{ [erg cm}^3 \text{ s}^{-1}\text{]}$$

Given the low density of the HIM, the cooling time of gas is comparable to the age of our galaxy.

-
- Recombination and Ionization Time scale
 - If collisional ionization could somehow be turned off, the recombination time scale is

$$t_{\text{rec}} = \frac{1}{n_e \alpha_{A,\text{H}}} \approx 0.6 \text{ [Gyr]} \left(\frac{n_{\text{H}}}{0.004 \text{ cm}^{-3}} \right)^{-1} \quad [T \approx 10^6 \text{ K}] \quad \alpha_{A,\text{H}} \approx 1.5 \times 10^{-14} \text{ [cm}^3 \text{s}^{-1}\text{]}$$
 - If collisional ionization could suddenly switch on, the collisional ionization time scale is

$$t_{\text{ci}} = \frac{1}{n_e k_{\text{ci}}} \approx 160 \text{ [yr]} \left(\frac{n_{\text{H}}}{0.004 \text{ cm}^{-3}} \right)^{-1} \quad [T \approx 10^6 \text{ K}] \quad k_{\text{ci},\text{H}} \approx 5.0 \times 10^{-8} \text{ [cm}^3 \text{s}^{-1}\text{]}$$
 - These times scales indicates that
 - ▶ If cold neutral hydrogen gas is shock-heated to $\sim 10^6 \text{ K}$ in a time $t_{\text{heat}} \ll t_{\text{ci}}$, it will take a time $t \sim t_{\text{ci}}$ for hydrogen to become ionized. During this time interval, the hydrogen will be out of collisional ionization equilibrium (under-ionized than in CIE).
 - ▶ If highly ionized gas at $\sim 10^6 \text{ K}$ is cooled on a timescale $t_{\text{cool}} \ll t_{\text{ci}}$, and the heating source is turned off, it will take a time $t \sim t_{\text{rec}}$ for the hydrogen to recombine. During the intervening time, the gas will be out of CIE (over-ionized than in CIE). This is sometimes called “delayed recombination”.

-
- If a gradually cooling gas of the HIM to be remained in CIE, we require $t_{\text{rec}} < t_{\text{cool}}$.

Assuming the recombination rate coefficient at high temperatures

$$\alpha_{A,H} \approx 5 \times 10^{-16} \text{ cm}^3 \text{ s}^{-1} (T/10^7 \text{ K})^{-1.5}$$

- At $T \sim 10^6 \text{ K}$,

$$t_{\text{rec}} = \frac{1}{n_e \alpha_{A,H}} \approx 0.5 \text{ [Gyr]} (T/10^6 \text{ K})^{1.5} (n_{\text{H}}/0.004 \text{ cm}^{-3})^{-1}$$

$$t_{\text{cool}} \approx 48 \text{ [Myr]} T_6^{1.7} (n_{\text{H}}/0.004 \text{ cm}^{-3})^{-1}$$

$$\Rightarrow \frac{t_{\text{rec}}}{t_{\text{cool}}} \approx 10 (T/10^6 \text{ K})^{-0.2}$$

- At $T \sim 10^7 \text{ K}$,

$$t_{\text{rec}} = \frac{1}{n_e \alpha_{A,H}} \approx 16 \text{ [Gyr]} (T/10^7 \text{ K})^{1.5} (n_{\text{H}}/0.004 \text{ cm}^{-3})^{-1}$$

$$t_{\text{cool}} \approx 7.2 \text{ [Gyr]} T_7^{0.5} (n_{\text{H}}/0.004 \text{ cm}^{-3})^{-1}$$

$$\Rightarrow \frac{t_{\text{rec}}}{t_{\text{cool}}} \approx 2.2 (T/10^7 \text{ K})$$

Therefore, in the extremely hot regions, the hotter the gas is, the further away it is from CIE.

Effects of Supernovae on the ISM

- Consider the simplest case of a spherically symmetric explosion of a star in a uniform density and temperature.
- ***Free-Expansion Phase***
 - Typical supernovae (SNe) explosion ejects a kinetic energy of

$$E_{51} \equiv E_0 / (10^{51} \text{ erg}) \approx 1$$

- The ejecta mass ranges from $M_{\text{ej}} \sim 1.4 M_{\odot}$ (Type Ia, white dwarf) to $M_{\text{ej}} \sim 10 - 20 M_{\odot}$ (Type II, core collapse of massive stars).
- The ejecta will have a rms velocity of

$$v_{\text{ej}} = \left(\frac{2E_0}{M_{\text{ej}}} \right)^{1/2} = 1.00 \times 10^4 \text{ km s}^{-1} E_{51}^{1/2} \left(\frac{M_{\text{ej}}}{M_{\odot}} \right)^{-1/2}$$

- ▶ This velocity is far greater than the sound speed in the surrounding material, and the expanding ejecta will drive a fast shock into the circumstellar medium.
- All of the matter interior to this shock surface is referred to as the ***supernova remnant*** (SNR).
- The density of the ejecta far exceeds the density of the circumstellar medium, and the ejecta continue to **expand ballistically at nearly constant velocity**. — this is referred to as the “free expansion phase.”

- At the early times of the free-expanding phase, there is only one shock, which propagating outward into the ambient medium.
- As the ejecta expands, its density drops and the pressure of the shocked circumstellar medium soon exceeds the thermal pressure in the ejecta.

$$\rho_{\text{ej}} = \frac{M_{\text{ej}}}{(4\pi/3)R^3} = \frac{M_{\text{ej}}}{(4\pi/3)(v_{\text{ej}}t)^3} \propto t^{-3}$$

- ▶ As the pressure in the ejecta drops, a reverse shock is driven into the ejecta. The SNR now contains two shocks: the ***original outward-propagating shock*** (the blastwave) and the ***reverse shock propagating inward***, slowing and shock-heating the ejecta (which had previously been cooled adiabatic expansion).
- ***End of the free-expanding phase:*** The reverse shock becomes important when the expanding ejecta material has swept up a mass of circumstellar or interstellar matter comparable to the ejecta mass.
 - ▶ The radius of the blastwave and the time when this occurs are:

$$R_1 = \left(\frac{M_{\text{ej}}}{(4\pi/3)\rho_0} \right)^{1/3} = 5.88 \times 10^{18} \text{ cm} \left(\frac{M_{\text{ej}}}{M_{\odot}} \right)^{1/3} n_0^{-1/3}$$

$$t_1 \approx \frac{R_1}{v_{\text{ej}}} = 186 \text{ yr} \left(\frac{M_{\text{ej}}}{M_{\odot}} \right)^{5/6} E_{51}^{-1/2} n_0^{-1/3}$$

- ▶ The free-expansion phase applies only for $t_1 \lesssim t_1$.

$$R_1 = 0.525 \text{ pc} (M_{\text{ej}}/M_{\odot})^{1/3} n_0^{-1/3}$$

$\rho_0 \simeq 1.4m_p n_0$ → density of the ambient (neutral) medium
 n_0 → number density of hydrogen in the ambient medium

-
- **Sedov-Taylor Phase (Energy-Conserving Phase, Adiabatic Phase)**
 - For $t \gtrsim t_1$, the reverse shock has reached the center of the remnant, all of the ejecta are now very hot, and the free-expansion phase is over.
 - ▶ The pressure in the SNR is far higher than the pressure in the surrounding medium.
 - ▶ The hot gas may have been emitting radiation, but if the density is low, the radiative losses at early times are negligible.
 - The SNR now enters a phase where
 - ▶ we can neglect (1) the mass of the ejecta ($M_{\text{ej}} \ll (4\pi/3)R^3\rho_0 \rightarrow M_{\text{ej}} \approx 0$), (2) radiative losses, and (3) the pressure in the ambient medium.
 - ▶ This phase can be approximated by idealizing the problem as a “point explosion” injecting only energy E_0 (zero ejecta mass) into a uniform-density zero-temperature medium of density ρ_0 , as in nuclear explosion.
 - We do simple dimensional analysis to find out the form of the time evolution of the remnant. Let the explosion occur at $t = 0$ and the radius of the shock front be R_s . Suppose that

$$R_s = AE^\alpha \rho^\beta t^\eta \quad (A = \text{dimensionless constant})$$

- ▶ By equating the powers of mass, length, and time, we obtain

$$\text{mass : } 0 = \alpha + \beta$$

$$\text{length : } 1 = 2\alpha - 3\beta \quad \longrightarrow \quad \alpha = 1/5, \beta = -1/5, \eta = 2/5$$

$$\text{time : } 0 = -2\alpha + \eta$$

- ▶ The solution for the shock-front radius is

$$R_s = A \left(\frac{Et^2}{\rho_0} \right)^{1/5}$$

$$A = 1.15167 \text{ for } \gamma = 5/3$$

for a monatomic gas.

This value is obtained from the exact solution.

This gives ***the radius and velocity of the shock front, and temperature of the post-shock gas:***

$$R_s = 1.54 \times 10^{19} [\text{cm}] E_{51}^{1/5} n_0^{-1/5} t_3^{2/5}$$

$$V_s = 1950 [\text{km s}^{-1}] E_{51}^{1/5} n_0^{-1/5} t_3^{-3/5}$$

$$T_s = 5.25 \times 10^7 [\text{K}] E_{51}^{2/5} n_0^{-2/5} t_3^{-6/5}$$

$$1.54 \times 10^{19} \text{ cm} = 5 \text{ pc}$$

$$t_3 \equiv t/10^3 \text{ yr}$$

$$V_s \equiv \frac{dR_s}{dt} = \frac{2}{5} AE^{1/5} \rho_0^{-1/5} t^{-3/5}$$

$$T_s = \frac{3}{16} \mu V_s^2 / k \quad \left(\mu \simeq \frac{1.4}{2.3} m_p \right)$$

μ = mass per particle in the fully ionized, post-shock region.

- ▶ We have been assuming that the internal structure of the remnant is given by a similarity solution: by this we mean that the density, velocity, and pressure can be written

$$\rho(r) = \rho_0 f(x)$$

$$v(r) = \frac{R_s}{t} g(x)$$

$$P(r) = \frac{\rho_0 R_s^2}{t^2} h(x)$$

$$x \equiv \frac{r}{R_s}$$

$f(x), g(x), h(x)$ are dimensionless functions.

Inserting these into the fluid equations, with the Rankine-Hugonit relations for boundary conditions, Taylor (1950) and Sedov (1959) found the solution for the dimensionless functions [f(x), g(x), h(x), and A] independently, in connection with the development of nuclear weapons.

-
- For the strong shock, the specific internal (thermal) and the kinetic energies just behind the strong shock are given by, respectively:

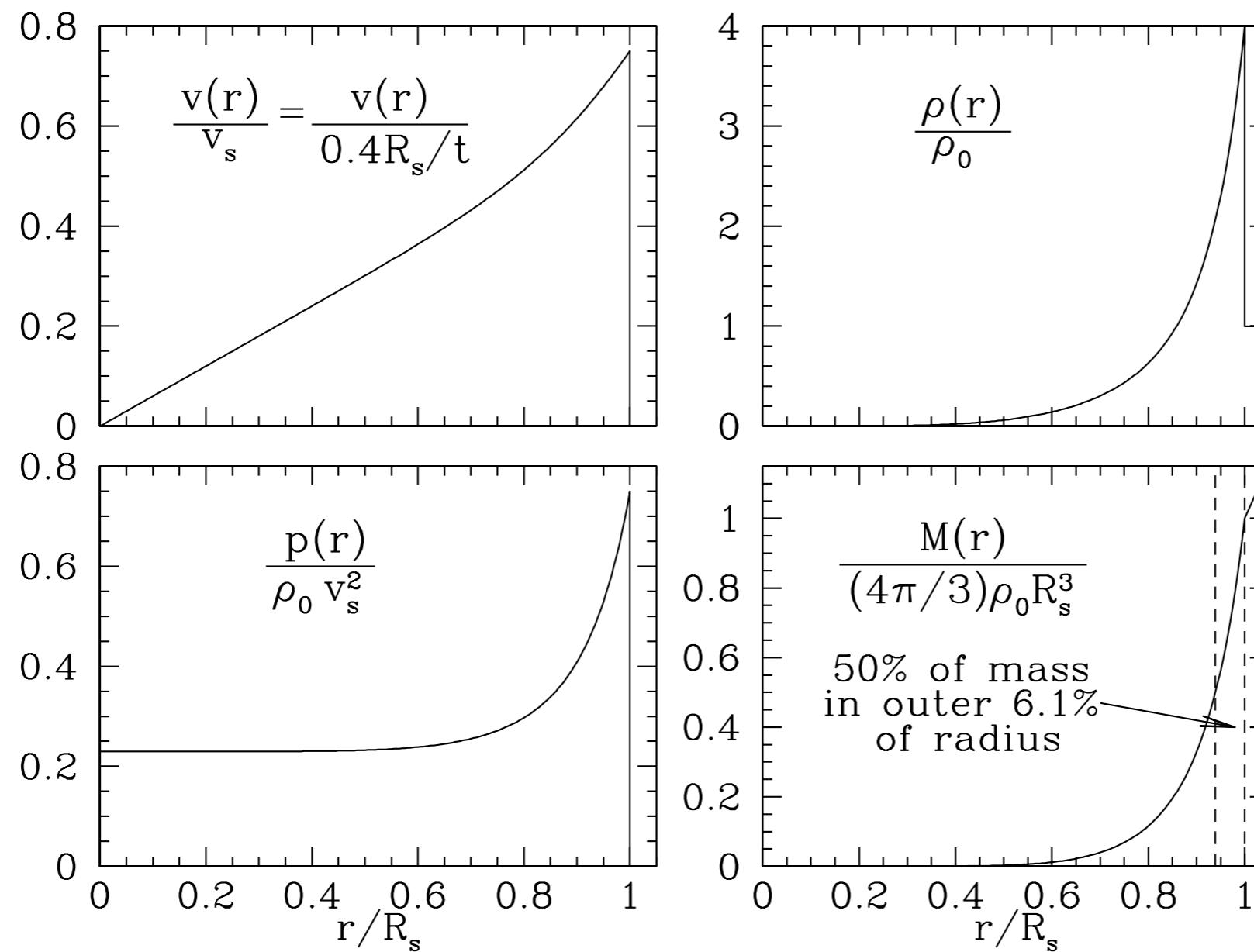
$$\mathcal{E}_{\text{int}} = \mathcal{E}_{\text{kin}} = \frac{9}{32} V_s^2$$

- Since the blast wave is slowing down with time, the specific energy of the internal gas varies with radius within the bubble of hot gas.
- The total energy in the bubble of hot gas E_0 , equal to the energy injected by the supernova explosion since the radiative losses are negligible, is therefore

$$\begin{aligned} E_0 &= \phi \frac{4\pi}{3} R^3 \rho_0 (\mathcal{E}_{\text{int}} + \mathcal{E}_{\text{kin}}) \\ &= \phi \frac{3\pi}{4} \rho_0 R^3 \left(\frac{dR}{dt} \right)^2 \end{aligned}$$

- where ϕ is a structure parameter, a numerical factor of order unity which accounts for the radial dependence of specific energy within the bubble. This equation represents the equation of motion of the bubble.

$$R = \left(\frac{25}{3\pi\phi} \right)^{1/5} \left(\frac{E_0 t^2}{\rho_0} \right)^{1/5} \quad A \equiv \left(\frac{25}{3\pi\phi} \right)^{1/5} = 1.15167 \text{ for } \gamma = 5/3$$



as $r \rightarrow 0$

$$\rho(r)/\rho_0 \rightarrow 0$$

$$T(r)/T_s \rightarrow \infty$$

$$P(r)/P_0 \rightarrow 0.306$$

Sedov-Taylor solution for $\gamma = 5/3$.

The temperature profile (not shown) can be obtained from the ratio of the pressure and density profiles. The density falls inward, and the temperature rises inward.

[Figure 39.1, Draine]

- ***End of the Sedov-Taylor phase***

- ▶ The hot gas interior to the shock front is, of course, radiating energy. When the radiative losses become important the SNR will leave the Sedov-Taylor phase and enter a “radiative” phase. In the radiative phase, the shock front is now able to cool to temperatures much lower than the temperature at the shock front.
- ▶ We need to estimate the cooling rate:

cooling function at a radius r :

$$\Lambda(r) \approx C [T(r)/10^6 \text{ K}]^{-0.7} n_{\text{H}}(r)n_e(r), \quad C = 1.1 \times 10^{-22} \text{ erg cm}^3 \text{ s}^{-1}, \quad n_e \simeq 1.2n_{\text{H}}$$

cooling rate at a time t , integrated over the volume of the SNR:

$$\begin{aligned} \frac{dE}{dt} &= - \int_0^{R_s} \Lambda(r) 4\pi r^2 dr \\ &= -1.2Cn_0^2 (T_s/10^6 \text{ K})^{-0.7} \frac{4\pi}{3} R_s^3 \langle (\rho/\rho_0)^2 (T_s/T)^{0.7} \rangle \end{aligned}$$

Here, $n_0 \equiv n_{\text{H}}(r = R_s)$ is the hydrogen density of the ambient medium at $r = R_s$.

$T_s \equiv T(r = R_s)$ is the temperature of the post-shock region at $r = R_s$.

$\langle \dots \rangle$ denotes a volume-weighted average over the SNR.

From the Sedov-Taylor solution, we obtain $\langle (\rho/\rho_0)^2 (T_s/T)^{0.7} \rangle = 1.817$.

-
- Now, integrate the energy loss rate over a time interval t :

$$\Delta E(t) = -1.2 \times (1.817) C \frac{4\pi}{3} n_0^2 \int_0^t dt R_s^3 (T_s/10^6 \text{ K})^{-0.7}$$

Using the previous solutions, we obtain the fractional energy loss by time t :

$$\begin{aligned} R_s &= 1.54 \times 10^{19} [\text{cm}] E_{51}^{1/5} n_0^{-1/5} t_3^{2/5} \\ T_s &= 5.25 \times 10^7 [\text{K}] E_{51}^{2/5} n_0^{-2/5} t_3^{-6/5} \end{aligned}$$

$$\frac{\Delta E(t)}{E_0} \approx -2.38 \times 10^{-6} n_0^{1.68} E_{51}^{-0.68} t_3^{3.04}$$

- If we suppose that the SNR enter the “radiative phase” when

$$\Delta E(t_{\text{rad}})/E_0 \approx -1/3$$

we can solve for the cooling time t_{rad} :

$$t_{\text{rad}} = 49.3 \times 10^3 [\text{yr}] E_{51}^{0.22} n_0^{-0.55}$$

- The radius and shock speed at the end of the Sedov-Taylor phase are:

$$7.32 \times 10^{19} \text{ cm} = 23.7 \text{ pc}$$

$$\begin{aligned} R_s(t_{\text{rad}}) &= 7.32 \times 10^{19} [\text{cm}] E_{51}^{0.29} n_0^{-0.42} \\ V_s(t_{\text{rad}}) &= 188 [\text{km s}^{-1}] (E_{51} n_0^2)^{0.07} \end{aligned}$$

Post-shock temperature is:

$$\begin{aligned} T_s(t_{\text{rad}}) &= 4.86 \times 10^5 [\text{K}] (E_{51} n_0^2)^{0.13} \\ kT_s(t_{\text{rad}}) &= 41 [\text{eV}] (E_{51} n_0^2)^{0.13} \end{aligned}$$

- ***Snowplow Phase (Momentum-Conserving Phase, Radiative Phase)***

- At $t \approx t_{\text{rad}}$,
 - ▶ Cooling cause the thermal pressure just behind the shock to drop suddenly, and the shock wave briefly stalls. However, the very hot gas in the interior of the SNR has not cooled.
 - ▶ The SNR now enters the snowplow phase, with a dense shell of cool gas enclosing a hot central volume where radiative cooling is unimportant.
 - ▶ This is called the snowplow phase because the mass of the dense shell increases as it “sweeps up” the ambient gas. Since the gas cools well the shell will be thin and so the gas in the shell has a radial velocity that is almost the same as the shock speed.
 - ▶ The gas in the hot center cools by adiabatic expansion:

$$P \propto V^{-\gamma} \propto R_s^{-3\gamma} = R_s^{-5}$$

- ▶ So that the pressure in the interior evolves as

$$P_i = P_0(t_{\text{rad}}) \left(\frac{R_{\text{rad}}}{R_s} \right)^5$$

- The pressure exerted by the hot center causes the “radial momentum” of the shell to increases:

M_s = mass of the shell

The mass density of ambient medium ρ_0 is constant.

$$\frac{d}{dt} (M_s V_s) \approx P_i 4\pi R_s^2 = 4\pi P_0(t_{\text{rad}}) R_{\text{rad}}^5 R_s^{-3}$$

$$\begin{aligned} M_s &= \left(\frac{4\pi}{3} R_s^3 \right) \rho_0 & \longrightarrow & \\ V_s &= \frac{dR_s}{dt} & & \frac{d^2 R_s^4}{dt^2} \propto R_s^{-3} \end{aligned}$$

Suppose that there is a power-law solution:

$$\begin{aligned} R_s &\propto t^\eta & \longrightarrow & \\ 4\eta - 2 = -3\eta &\Rightarrow \eta = 2/7 & & \end{aligned}$$

$$\begin{aligned} R_s &\approx R_s(t_{\text{rad}}) (t/t_{\text{rad}})^{2/7} \\ V_s &\approx \frac{2}{7} \frac{R_s}{t} = \frac{2}{7} \frac{R_s(t_{\text{rad}})}{t_{\text{rad}}} \left(\frac{t}{t_{\text{rad}}} \right)^{-5/7} \end{aligned}$$

- Because the effect of the internal pressure has been included, this solution is referred to as the pressure-modified (or pressure-driven) snowplow phase.
- Note that with this construction, $R_s(t)$ is continuous from the Sedov-Taylor phase to the pressure-modified snowplow phase, but $V_s(t)$ undergoes a discontinuous drop by $2/7 \sim 29\%$ at $t = t_{\text{rad}}$.

-
- ▶ In the late phase of evolution, the stored thermal energy in the central region has been entirely radiated away, and only the momentum of the dense shell keeps the remnant expanding into the ISM.

$$\frac{d}{dt} (M_s V_s) = 0$$

M_s = mass of the shell

$$M_s = \left(\frac{4\pi}{3} R_s^3 \right) \rho_0$$

$$V_s = \frac{dR_s}{dt}$$

The mass density of ambient medium ρ_0 is constant.

$$R_s^3 \frac{dR_s}{dt} = \text{constant}$$

$$R_s \approx R_s(t_{\text{rad}}) (t/t_{\text{rad}})^{1/4} \quad \text{for } t \gg t_{\text{rad}}$$

$$V_s \approx \frac{1}{4} \frac{R_s}{t} = \frac{1}{4} R_s(t_{\text{rad}}) (t/t_{\text{rad}})^{-3/4}$$

- ▶ Towards the end of this phase, the expansion velocity becomes sonic or subsonic with respect to the interstellar sound speed.
- In summary, the supernova expansion velocity is constantly slowed by its interaction with the surrounding medium.

$$R \propto t \rightarrow R \propto t^{2/5} \rightarrow R \propto t^{2/7} \rightarrow R \propto t^{1/4}$$

- ***Fadeaway (Merging with the ISM)***

- For typical ISM parameters, the shock speed at the beginning of the snowplow phase is

$$V_s(t_{\text{rad}}) = 188 \text{ [km s}^{-1}\text{]} (E_{51} n_0^2)^{0.07}$$

- This results in a very strong shock when propagating through interstellar gas with $T < 10^4$ K. However, the shock front gradually slows, and the shock compression declines. This proceeds until the shock speed approaches the effective sound speed in the gas and at this point the compression ratio approaches 1, and the shock wave turns into a sound wave.

$$V_s \approx \frac{2}{7} \frac{R_s(t_{\text{rad}})}{t_{\text{rad}}} \left(\frac{t}{t_{\text{rad}}} \right)^{-5/7}$$

- Setting $V_s \approx c_s$ gives the a “fadeaway time”

$$\begin{aligned} t_{\text{fade}} &\approx \left(\frac{(2/7)R_{\text{rad}}/t_{\text{rad}}}{c_s} \right)^{7/5} t_{\text{rad}} \\ &\approx 1.87 \times 10^6 \text{ [yr]} E_{51}^{0.32} n_0^{-32} \left(\frac{c_s}{10 \text{ km s}^{-1}} \right)^{-7/5} \end{aligned}$$

$$R_{\text{fade}} \equiv R_s(t_{\text{fade}}) \approx \frac{7}{2} V_s t_{\text{fade}} \approx \frac{7}{2} c_s t_{\text{fade}}$$

$$\approx 2.07 \times 10^{20} \text{ [cm]} E_{51}^{0.32} n_0^{-0.37} \left(\frac{c_s}{10 \text{ km s}^{-1}} \right)^{-2/5}$$

$$2.07 \times 10^{20} \text{ cm} = 67.1 \text{ pc}$$

Examples

- Free-expansion phase
 - The remnant associated with SN1987A is in the free expansion phase.
 - Cas A
 - In the case of Type II supernovae resulting from core collapse in massive stars, the supernova explosion is often preceded by a red supergiant phase, leaving a relatively dense circumstellar medium with a $\sim r^{-2}$ density profile.
 - The Cas A has been modeled with $M_{\text{ej}} \approx 4M_{\odot}$ and $E_{51} \approx 2$, expanding into a circumstellar medium with $n_{\text{H}} \approx 7(r/\text{pc})^{-2} \text{ cm}^{-3}$ left by a red supergiant phase (van Veelen et al. 2009). The reverse shock is now located at $\sim 60\%$ of the outer shock radius — much of Cas A ejecta is still in the free expansion phase.
 - The Cas A is at the end of its free expansion phase; it is ~ 300 years old. At visible wavelengths, many ‘knots’ of emission, moving radially outward with a velocity of $\sim 6000 \text{ km s}^{-1}$, can be seen. The knots are rich in oxygen, and are interpreted as being clumps of matter that have been ejected from the center of the star, where nucleosynthesis took place.

Examples

- Sedov-Taylor phase
 - The Tycho SNR and the Crab nebular are in the blast wave phase.
 - Light from the supernova that gave birth to the Crab Nebula was first seen on AD 1054 July 4. The SNR is at an age $t \approx 966 \text{ yr}$. The solution for the Sedov-Taylor phase implies

$$R_s = A \left(\frac{E_0 t^2}{\rho_0} \right)^{1/5}$$

$$V_s = \frac{dR_s}{dt} = \frac{2}{5} \frac{R_s}{t}$$

$$\frac{R_s}{V_s} = \frac{5}{2} t \approx 2400 \text{ yr}$$

- The azimuthally averaged radius of the Crab Nebula is $\theta \approx 150 \text{ arcsec}$ in angle. The proper motion of its expansion is $\mu \approx 0.16 \text{ arcsec yr}^{-1}$. This gives the ratio

$$\theta / \mu \approx 940 \text{ yr}$$
- which is smaller than required by the self-similar solution.
- This implies that V_s for the Crab Nebula is decreasing less rapidly than implied by the Sedov-Taylor solution.

$$V_s / R_s < \mu / \theta$$

- Snowplow phase
 - Cygnus Loop
 - The Cygnus Loop (aka the Veil Nebula) is in the snowplow phase. Its age is 40,000 yr, and its expansion velocity is $\sim 120 \text{ km s}^{-1}$. The dense shell is thermally unstable to the formation of filaments; such filaments can be seen in optically photographs of the Cygnus Loop.
 - Observationally, it is found that the internal pressure of SNRs in the snowplow phase is large enough to give a significant push to the shell.

Overlapping of SNRs

- By the time t_{fade} , a SNR has expanded to fill a volume $(4\pi/3)R_{\text{fade}}^3$
 - What is the probability that another SN will occur within this volume and affect the evolution of the original SNR before it has faded away?
 - Let a rate of occurrence of a SN per volume $S \equiv 10^{-13} S_{-13} \text{ pc}^{-3} \text{ yr}^{-1}$
 - The SN rate in the Milky Way has been estimated from records of historical SNe and from observations of similar galaxies. The SN frequency in the Galaxy is estimated to be ***one event every 30-50 yr*** (Tammann et al. 1994).
 - Consider the “distk” to have a radius of 15 kpc and a thickness 200 pc, and suppose that the SN rate within this volume is 1/60 yr. Then, this gives a SN rate per volume:

$$S = \frac{(60 \text{ yr})^{-1}}{\pi R^2 H} \quad R = 15 \text{ kpc}, \quad H = 200 \text{ pc}$$

$$\approx 1.2 \times 10^{-13} \text{ pc}^{-3} \text{ yr}^{-1}$$

$$S_{-13} \approx 1.2$$

- The expectation value for the number of additional SNe that will explode within this volume during the lifetime t_{fade} of the original SNR is

$$N_{\text{SN}} = S \frac{4\pi}{3} R_{\text{fade}}^3 t_{\text{fade}}$$

$$\approx 0.24 S_{-13} E_{51}^{1.26} n_0^{-1.47} \left(\frac{c_s}{10 \text{ km s}^{-1}} \right)^{-2.6}$$

- In the two-phase model of the ISM, the interstellar volume is filled by the WNM with density
- Then, $n_0 \approx 1 \text{ cm}^{-3}$ and $c \approx 6 \text{ km s}^{-1}$ ($T = 6000 \text{ K}$)

$$N_{\text{SN}} \approx 1.1 \quad t_{\text{fade}} \approx 2 \text{ Myr}$$
- Therefore, we conclude that if we were to start with the two-phase model, the initially near-uniform WNM will be destroyed by the effects of SNe. Massive stars tend to be born in stellar associations. They enter their supernova phase before they have a chance to drift apart. Thus, the SNRs of the neighboring supernovae will merge to form a single super bubble, which may be hundreds of parsec across. Such super bubbles are a primary source of the hot ionized component of the ISM.
- In other words, SNRs will overlap and occupy a major fraction of the disk volume. This implies that, at least for the Milky Way, SNRs will create a multiphase ISM, consisting of low-density regions in the interior of the SNRs, and dense regions containing most of the gas mass.

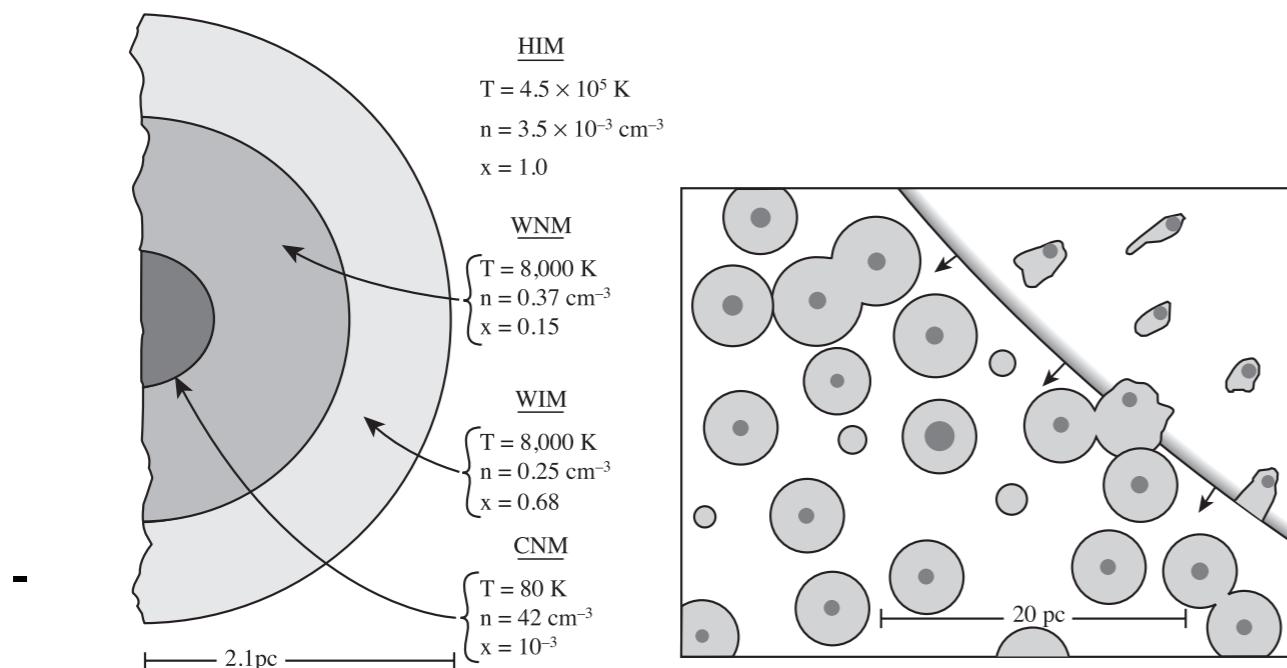


Figure 39.3 Draine

Left: Structure of a typical cold cloud in the three-phase model of McKee & Ostriker (1977)

Right: Close-up of a supernova blast wave.

$$N_{\text{SN}} \approx 0.24 S_{-13} E_{51}^{1.26} n_0^{-1.47} \left(\frac{c_s}{10 \text{ km s}^{-1}} \right)^{-2.6}$$

- We can eliminate the sound speed in favor of the pressure

$$P = 1.4 n_{\text{H}} m_{\text{H}} c_s^2$$

$$N_{\text{SN}} \approx 0.48 S_{-13} E_{51}^{1.26} n_0^{-1.47} \left(\frac{P/k}{10^4 \text{ cm}^{-3} \text{ K}} \right)^{-1.3}$$

- We can solve for the pressure

$$P/k = 5700 [\text{cm}^{-3} \text{ K}] S_{-13}^{0.77} E_{51}^{0.97} n_0^{-0.13} N_{\text{SN}}^{-0.77}$$

$$P/k \approx 6600 \text{ cm}^{-3} \text{ K} \text{ for } S_{-13} \approx 1.2$$

- This is comparable to the observed thermal pressure $P/k \approx 3800 \text{ cm}^{-3} \text{ K}$ in the ISM today.
- The principal shortcoming of the MO77 three-phase model is the failure to predict the substantial amount of the warm HI gas. The model predicts only 4.3% of the HI mass in the warm phase (WNM and WIM). The 21-cm line observations indicate that more than 60% of the HI within 500 pc of the Sun is actually in the warm phase (Heiles & Troland 2003).

Observing the HIM - Local Bubble

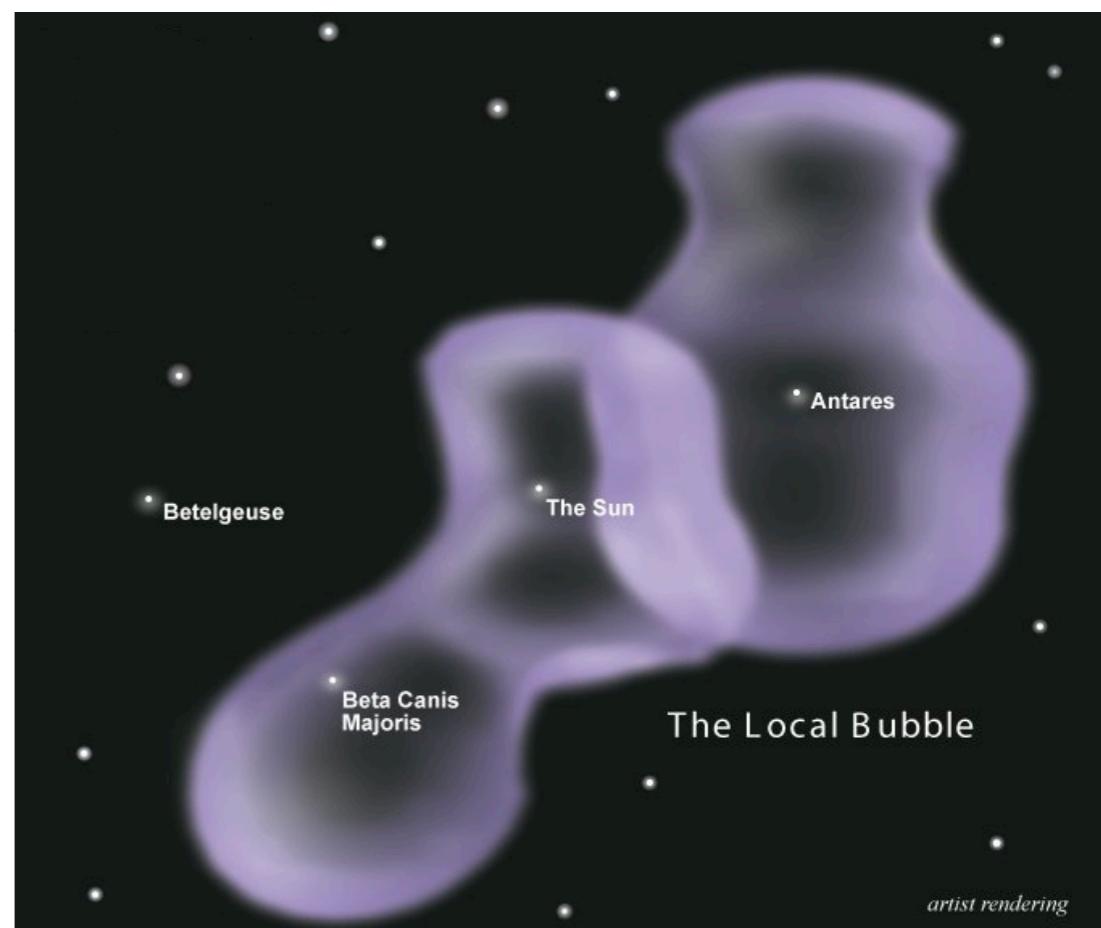
- Emission Line?
 - HII regions have readily observable emission lines that let us estimate their density and temperature.
 - However, finding the density and temperature of the HIM is more difficult.
- The temperature of the HIM, $T \sim 10^6$ K, corresponds to $h\nu \sim kT \sim 100$ eV. However, photons with an energy 100 eV ($\sim 120\text{\AA}$) are difficult to observe, even if we use space observatories.
 - ▶ This is primarily because of the large photoionization cross-section of hydrogen.

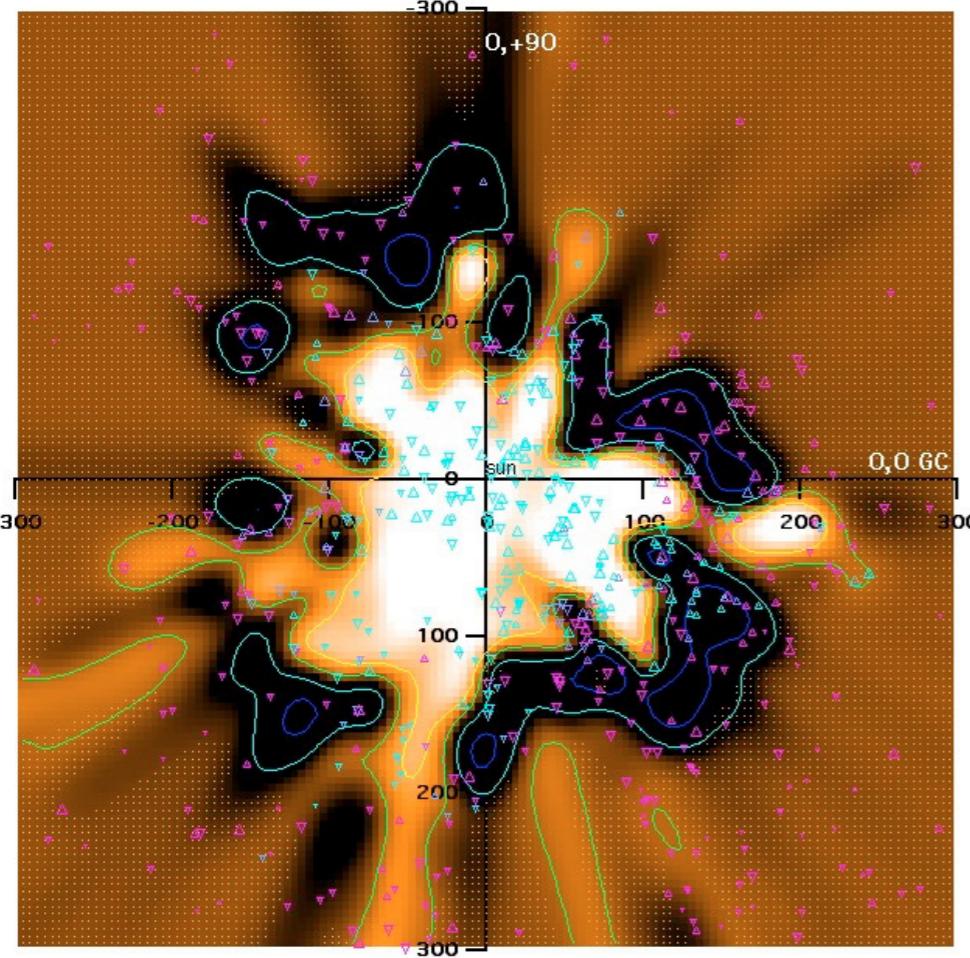
$$\sigma_{\text{pi}} \approx 6.3 \times 10^{-18} \text{ cm}^2 (h\nu/I_{\text{H}})^{-3}$$

- ▶ A column depth $N(\text{HI}) > 10^{20} \text{ cm}^{-2}$ (corresponding to ~ 20 pc through the WNM, ~ 1 pc through the CNM) will absorb nearly all 100 eV photon that is emitted by a bubble of hot gas.
- The only portion of the HIM from which we can observe 100 eV photon is the Local Bubble.

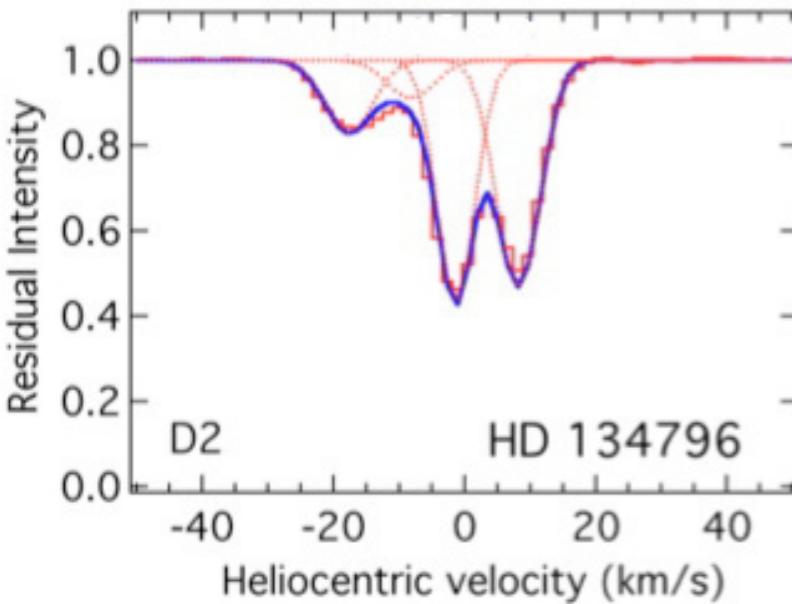
- Soft-Xray (1/4 keV) background, the Local Hot Bubble
 - The X-ray background is an unexpectedly intense diffuse flux of soft X-rays (Bowyer et al. 1968). X-ray at the low energies are easily absorbed by interstellar clouds, so it was concluded that they must have a local origin. Moreover, observations show that there is a suspicious lack of cool, absorbing gas with about ~ 100 pc of the Sun.
 - The existence of the local hot bubble has been proposed to explain the soft X-ray background. The local bubble is a cavity in the ISM filled with hot, X-ray-emitting gas.
 - A nearby supernova explosion could carve the bubble out millions of years ago, leaving a region filled with relatively little neutral hydrogen, but a lot of 10^6 K gas.
 - But this paradigm was challenged when X-rays emanating from a comet as it passed through the solar wind.
 - When the charged ions flowing out from the Sun collide with neutral hydrogen and helium atoms in interplanetary space, they can exchange electrons, producing soft X-ray photons in the process. If the X-ray emission caused by the solar wind could account for the entire X-ray background, then it would negate the need for a local hot bubble.
 - Galeazzi et al. (2014, Nature) combined observations from the “Diffuse X-rays from the Local galaxy” (DXL) rocket mission with the ROSAT data and found that the X-ray emission from solar wind could only account for about 40% of the diffuse X-ray background.

The Local Bubble (NASA CHIPS)

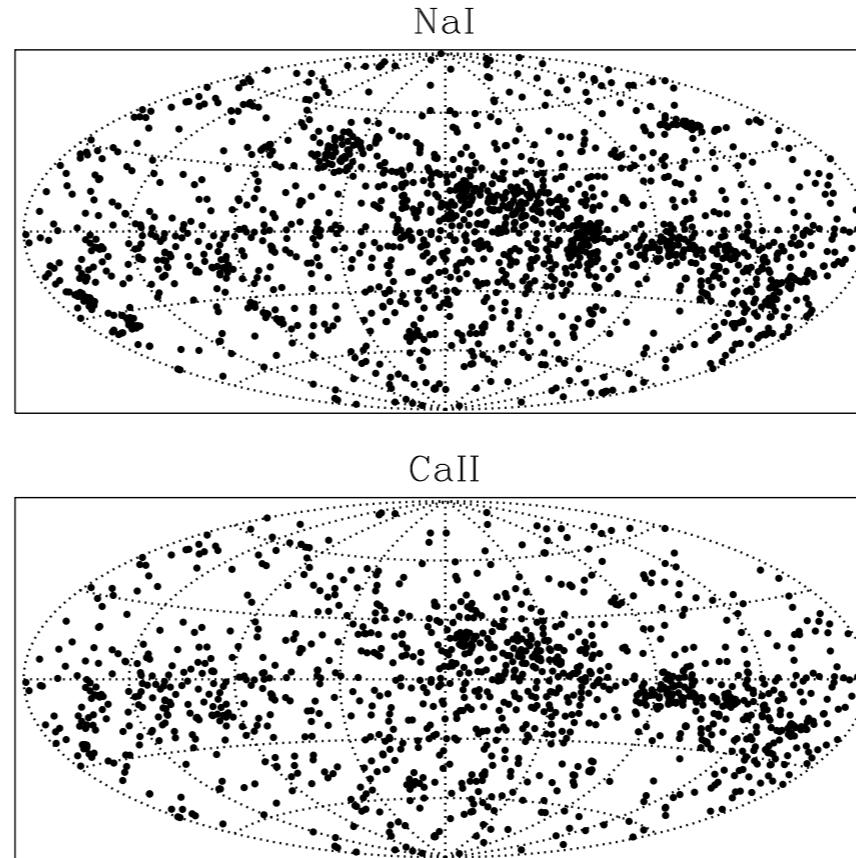




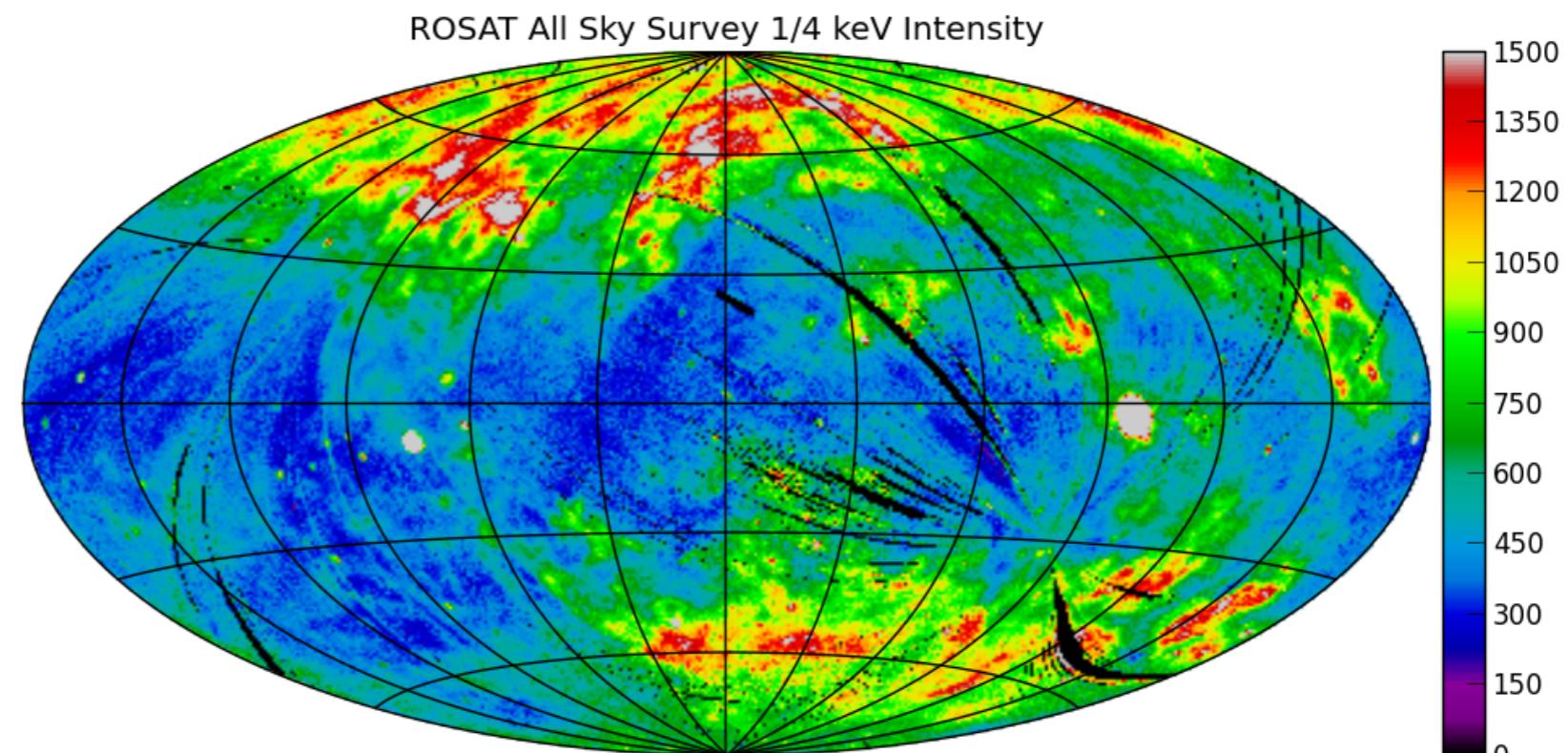
3D spatial distribution of Na I absorption,
as viewed in the Galactic plane
projection. (Welsh et al. 2010)



An example spectrum of Na I absorption
line (Welsh et al. 2010)

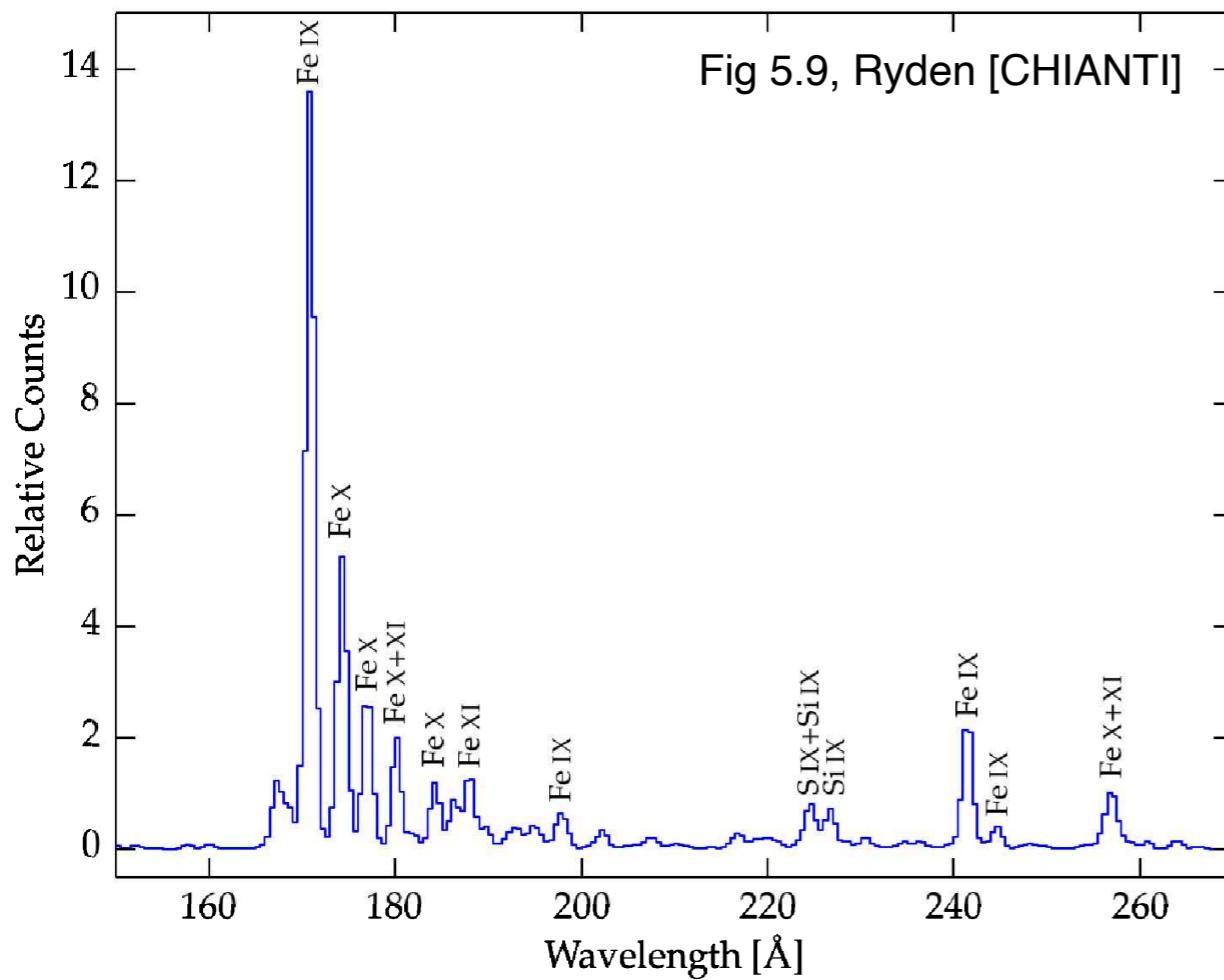


Sightlines for Na I and Ca II
absorption line observations
(Welsh et al. 2010)



intensity unit = 10^{-6} counts s⁻¹ arcmin⁻² (Slavin, 2017)

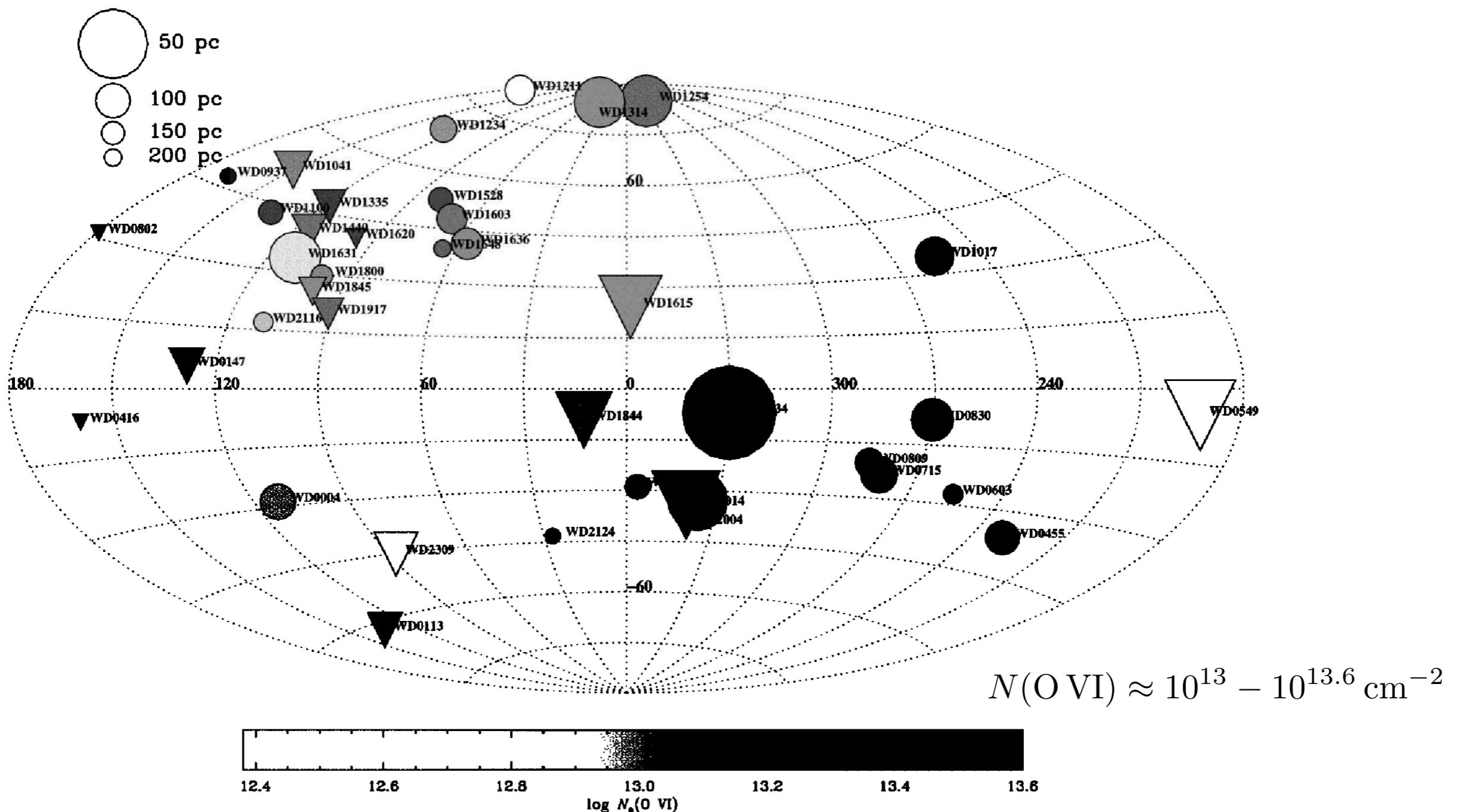
- Emission Lines from the Local Bubble?
 - At $T \sim 10^6$ K, Hydrogen and helium is fully ionized. The most abundant heavy elements (oxygen, carbon, and neon) are primarily helium-like. The dominant forms of iron are Fe IX, X, and XI.
 - The iron ions from Fe IX to Fe XI produce a cluster of emission lines near ($\lambda \sim 180\text{\AA}$). About half of the radiated power come out in the form of these iron emission lines.
 - Iron isn't fully ionized until $T > 2 \times 10^8$ K. At $T \sim 10^7$ K, the dominant forms are Fe XX and XXI.



- However, observations at $\lambda \sim 180\text{\AA}$ by the Cosmic Hot Interstellar Plasma Spectrometer (CHIPS) showed that the Fe emission lines from the Local Bubble are much weaker than originally predicted.
- This might be attributable to (1) depletion of iron onto dust grains, (2) a somewhat lower temperature (7×10^5 K), and (3) delayed recombination.
- The Local Bubble is cooling, and Fe is more highly ionized than CIE tells us.

-
- Absorption Lines from the Local Bubble.
 - One important pair of UV lines is the Lithium-like OVI doublet 1032Å (12.01 eV) and 1038Å (11.95 eV). These lines are a major coolant for interstellar gas at $T \sim 3 \times 10^5$ K.
 - At $T \sim 3 \times 10^5$ K, only $\sim 25\%$ of the total oxygen abundance is O VI ($\sim 35\%$ is OV and $\sim 35\%$ is OVII). However, the O VI doublet lines have a large cross section, and thus are readily observed even if the number density of OVI ions is small.
 - To detect the OVI lines in absorption, we need background sources with $T_{\text{eff}} > 1.4 \times 10^5$ K (12 eV). Main sequence stars are not this hot. So, we have to look at nearby, young, hot white dwarfs. In particular, white dwarfs of spectral type DA, which have only hydrogen lines in their spectra, are the best targets.
 - The distribution of OVI in the Local Bubble appears to be patchy, and the mean density of OVI lies in the range $N(\text{OVI})/d \sim (0.7 - 13) \times 10^{-8} \text{ cm}^{-3}$. (See Savage & Lehner 2006).
 - The thermal broadening parameter was found to range from $b = 15 \text{ km s}^{-1}$ to 36 km s^{-1} . From this, we can find a temperature:

$$T = 2.5 \times 10^5 \text{ K} \left(\frac{b}{16 \text{ km s}^{-1}} \right)^2 \left(\frac{m}{16 m_{\text{H}}} \right)$$



Location in galactic coordinates of 39 nearby hot DA white dwarfs (Savage & Lehner 2006). Circles have OVI detection; triangles are non-detections at 2σ level. Grayscale indicates the column density of OVI toward white dwarfs with detections.

Fig 5.11 [Ryden]