[2020. 3. 25 (수)]

* Barbara Ryden 의 책에서 hydrogen-hydrogen collision cross-section을 계산할 때 수소원자의 크기를 $r = 3a_0$ (a_0 = Bohr radius)로 가정함.

왜 $r = 3a_0$을 수소원자의 크기로 가정했을까?

수소원자의 radial wavefunction ($n=1, l=0$)은 아래와 같다.

$$R_{1,0} = \left(\frac{1}{a_0}\right)^{3/2} \cdot 2 \cdot e^{-(r/a_0)}$$

normalization은 $\int_0^\infty R_{n,l}^2 \, r^2 dr = 1$

전자가 $r \leq x$ 의 범위안에 존재할 확율은?

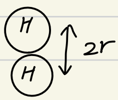
$$P(r \leq x) = \int_0^x R_{1,0}^2 \, r^2 dr = 1 - e^{-2(r/a_0)}\left(1 + 2(r/a_0) + 2(r/a_0)^2\right)$$

$$P(r \leq 3a_0) = 0.938$$

즉, $r \simeq 3a_0$는 전자가 94%의 확율로 존재하는 크기임.

이때, $\sigma_{HH} \approx \pi (2r)^2 = 36\pi a_0^2 = 113 a_0^2$ ($a_0 = 0.529 \mathring{A}$)

$$\boxed{\sigma_{HH} \approx 3.2 \times 10^{-15} \, cm^2}$$



* Ryden 의 책에서 hydrogen-electron collision 에 대한 cross-section은 $\sigma_{eH} \approx 10^{-15} cm^2$ 으로 가정함.

전자의 크기는 "0" 이므로 $\sigma_{eH} \approx \pi r^2 = 9\pi a_0^2$

$$\therefore \boxed{\begin{aligned} \sigma_{eH} &\approx 0.8 \times 10^{-15} \, cm^2 \\ &\approx 1 \times 10^{-15} \, cm^2 \end{aligned}}$$

* 그렇다면 electron-electron collision cross-section은?

$$\boxed{\frac{e^2}{r_e} \approx \langle E \rangle, \quad \sigma_{ee} = \pi r_e^2}$$

Coulomb potential energy = kinetic energy 조건이용

$$\sigma_{ee} \sim \pi \frac{e^4}{\langle E \rangle^2}$$

$1e = 4.80326 \times 10^{-10}\,\text{esu}$

$1\,\text{esu} = 1\,\text{statC} = \text{dyn}^{1/2}\,\text{cm} = \text{cm}^{3/2}\,\text{g}^{1/2}\,\text{s}^{-1}$

$1\,\text{eV} = 1.602 \times 10^{-12}\,\text{erg}$

$1\,\text{erg} = 1\,\text{dyn cm} = 1\,\text{g.cm}^2/\text{s}^2$

$e^2 = 2.3071307 \times 10^{-19} (\text{cm}^3\,\text{g}\,\text{s}^{-2}) \Rightarrow 1\,\text{erg cm}$

$\quad = 1.4401565 \times 10^{-7}\,\text{eV} \cdot \text{cm}$

$\sigma_{ee} \sim \pi \times 2.0740507 \times 10^{-14} (\langle E \rangle / \text{eV})^{-2}\,\text{cm}^2$

$\quad \sim 6.5158223 \times 10^{-14} (\langle E \rangle / \text{eV})^{-2}\,\text{cm}^2$

electron-electron collision cross-section은

$$\boxed{\sigma_{ee} \approx 6.52 \times 10^{-14}\,\text{cm}^2 (\langle E \rangle / \text{eV})^{-2}}$$

[2020.3.23(월)]

mean free path λ_{mfp}

H-H collision

$$\lambda_{mfp}^{(HH)} \sim 1/(n\sigma)$$

$$\sim \left(\frac{n_{atom}}{30\,cm^{-3}}\right)^{-1}\left(\frac{\sigma_{HH}}{3\times10^{-15}\,cm^2}\right)^{-1} \times (30\times3\times10^{-15})^{-1}\,cm$$

$$\sim 1.11\times10^{13}\,[cm]\,(n_{atom}/30\,cm^{-3})^{-1}(\sigma_{HH}/3\times10^{-15}\,cm^2)^{-1}$$

$$\sim 0.74\,AU\,(n_{atom}/30\,cm^{-3})^{-1}(\sigma_{HH}/3\times10^{-15}\,cm^2)^{-1}$$

e-H collision

$$\lambda_{mfp}^{(eH)} \sim \left(\frac{n_e}{0.04\,cm^{-3}}\right)^{-1}\left(\frac{\sigma_{eH}}{10^{-15}\,cm^2}\right)^{-1} \times (0.04\times10^{-15})^{-1}\,cm$$

$$\sim 2.5\times10^{16}\,[cm]\,(n_e/0.04\,cm^{-3})^{-1}(\sigma_{eH}/10^{-15}\,cm^2)^{-1}$$

$$\sim 1671\,AU\,(n_e/0.04\,cm^{-3})^{-1}(\sigma_{eH}/10^{-15}\,cm^2)^{-1}$$

e-e collision

$$\lambda_{mfp}^{(ee)} = \frac{1}{n\sigma_{ee}}$$

$$= \left(\frac{n_e}{0.04\,cm^{-3}}\right)^{-1}\left(\frac{\sigma_{ee}}{6.52\times10^{-14}\,cm^2}\right)^{-1}\left(\frac{\langle E\rangle}{1eV}\right)^{2}\,cm$$

$$\times (0.04\times6.52\times10^{-14})^{-1}$$

$$= 3.8368143\times10^{14}\,cm\,(n_e/0.04\,cm^{-3})^{-1}(\langle E\rangle/1eV)^2$$

$$= 25.6\,AU\,(n_e/0.04\,cm^{-3})^{-1}(\langle E\rangle/1eV)^2$$

$\lambda_{mfp}^{(ee)}$ in terms of T

$$\Rightarrow 3.84\times10^{10}\,\mathrm{cm}\,(n_e/0.04\,\mathrm{cm^{-3}})^{-1}(\langle E\rangle/0.01\,\mathrm{eV})^2$$
$$= 2.56\times10^{-3}\,\mathrm{AU}\,(n_e/0.04\,\mathrm{cm^{-3}})^{-1}(\langle E\rangle/0.01\,\mathrm{eV})^2$$

$\boxed{1\,\mathrm{eV} = 1.16\times10^4\,\mathrm{K}}$

$$\Rightarrow 2.85\times10^{10}\,\mathrm{cm}\,(n_e/0.04\,\mathrm{cm^{-3}})^{-1}(T/100\,\mathrm{K})^2$$
$$= 1.91\times10^{-3}\,\mathrm{AU}\,(n_e/0.04\,\mathrm{cm^{-3}})^{-1}(T/100\,\mathrm{K})^2$$

thermal velocity

$$v_{th} = (3kT/m)^{1/2} \qquad k_B = 1.38064852\times10^{-16}\,\mathrm{cm^2\,g\,s^{-2}\,K^{-1}}$$
$$m_H = 1.6735575\times10^{-24}\,\mathrm{g}$$

$$v_{th}^{(H)} = 1.57\,\mathrm{km\,s^{-1}}\,(T/100\,\mathrm{K})^{1/2}$$

$$v_{th}^{(e)} = (1836)^{1/2}\,v_{th}^{(H)} = 67.4\,\mathrm{km\,s^{-1}}\,(T/100\,\mathrm{K})^{1/2}$$

$\boxed{\text{Collision time scale}}$ for CNM

$\boxed{\text{H-H}}$

$$t_{coll}(HH) = \frac{\lambda_{mfp}^{(HH)}}{v_{th}^{(H)}}$$

$$= \frac{1.11\times10^{8}\ \mathrm{km}}{1.57\quad \mathrm{km\ s^{-1}}} (n_H/30\ \mathrm{cm^{-3}})^{-1} (\sigma_{HH}/3\times10^{-15}\mathrm{cm^2})^{-1} (T/100\mathrm{K})^{-1/2}$$

$(1\,\mathrm{yr} = 3.15\times10^{7}\,\mathrm{s})$

$$= 7.07\times10^{7}\,\mathrm{s}\ (n_H/30\ \mathrm{cm^{-3}})^{-1} (\sigma_{HH}/3\times10^{-15}\mathrm{cm^2})^{-1} (T/100\mathrm{K})^{-1/2}$$

$$= 2.24\ \mathrm{yr}$$

$\boxed{\text{e-H}}$

$$t_{coll}^{(eH)} = \frac{\lambda_{mfp}^{(eH)}}{v_{th}^{(e)}} = \frac{2.5\times10^{11}\ \mathrm{km}}{67.4\quad \mathrm{km s^{-1}}} \frac{(n_e/0.04\mathrm{cm^{-3}})^{-1} (\sigma_{eH}/10^{-15}\mathrm{cm^2})^{-1}}{(T/100\mathrm{K})^{1/2}}$$

$$= 3.709\times10^{9}\ \mathrm{s}\ (n_e/0.04\ \mathrm{cm^{-3}})^{-1} (\sigma_{eH}/10^{-15}\mathrm{cm^2})^{-1} (T/100\mathrm{K})^{-1/2}$$

$$= 118\ \mathrm{yr}\ (n_e/0.04\ \mathrm{cm^{-3}})^{-1} (\sigma_{eH}/10^{-15}\mathrm{cm^2})^{-1} (T/100\mathrm{K})^{-1/2}$$

$\boxed{\text{e-e}}$

$$t_{coll}^{(ee)} = \frac{\lambda_{mfp}^{(ee)}}{v_{th}^{(e)}} = \frac{3.84\times10^{5}\ \mathrm{km}}{67.4\quad \mathrm{km\ s^{-1}}} \frac{(n_e/0.04\ \mathrm{cm^{-3}})^{-1} (E/0.01\mathrm{eV})^{2}}{(T/100\mathrm{K})^{1/2}}$$

$$\approx 5.7\times10^{3}\,\mathrm{s} \times 0.743\ (n_e/0.04\ \mathrm{cm^{-3}})^{-1} (T/100\mathrm{K})^{3/2}$$

$$= 4.23\times10^{3}\,\mathrm{s}\ (n_e/0.04\ \mathrm{cm^{-3}})^{-1} (T/100\mathrm{K})^{3/2}$$

$$= 1.18\ \mathrm{hr}\ (n_e/0.04\ \mathrm{cm^{-3}})^{-1} (T/100\mathrm{K})^{3/2}$$