Interstellar Medium (ISM)

Lecture 13
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Circumgalactic & Intracluster Medium
Intergalactic Medium 1

- CGM (HVCs, starburst-driven galactic winds)
- ICM (dark matter, hot diffuse ICM)
- IGM (The Gunn-Peterson effect)
- Cosmic Recombination & Reionization

Introduction

- CGM and ICM
  - The ***circumgalactic medium*** is the diffuse gas (and some dust) that lies outside the main body of a galaxy's stellar distribution, but still lies inside the virial radius of the galaxy's dark halo.
  - The ***intracluster medium*** is the diffuse hot gas that lies inside the virial radius of a cluster of galaxies, but is not bound to any particular galaxy in the cluster.
- **Virial radius**
  - The ***virial*** radius of a galaxy or cluster is often defined as the radius inside which the virial theorem holds true, and where we can safely make a mass estimate by using:

$$M \sim \frac{\sigma_v^2 r}{G}$$

$$2\langle K \rangle + \langle U \rangle = 0$$

Recall the virial relation for a uniform, self-gravitating system: $$M = \frac{5\sigma_v^2 R}{G}$$

  - See the previous lecture note to see the proof of the viral theorem.
  - In a numerical simulation, finding the virial radius is fairly simple.
    - Just look inside a particular radius and see if the velocity dispersion, radius, and mass follow the virial relation.

- In reality, approximations must be made. The virial radius is usually approximated as ***the radius*** $r_{\rm vir}$ ***within the mean density*** $\rho(v_{\rm vir})$ ***is equal to 200 times the critical density*** $\rho_{\rm c}$ ***of the universe***:

$$\rho_{\rm c} \equiv \frac{3H^2}{8\pi G}$$

$$\rho(v_{\rm vir}) = 200\rho_{\rm c}$$

Here, H = the Hubble parameter
G = the Gravitational constant

- Critical density: the mean density of matter when the overall geometry of the universe if flat (Euclidean).
- From the 2015 Planck results, the current values of the Hubble parameter and the critical density are:

$$H_0 = 67.74 \pm 0.46 \ [\mathrm{kpc\,s^{-1}\,Mpc^{-1}}]$$
$$\rho_{\rm c,0} = (8.62 \pm 0.12) \times 10^{-30} \ [\mathrm{g\,cm^{-3}}]$$

Note that $\rho_{\rm c,0}$ corresponds to 5 hydrogen atoms per cubic meter.

- The virial radius of a galaxy or cluster is thus defined as the radius inside which:

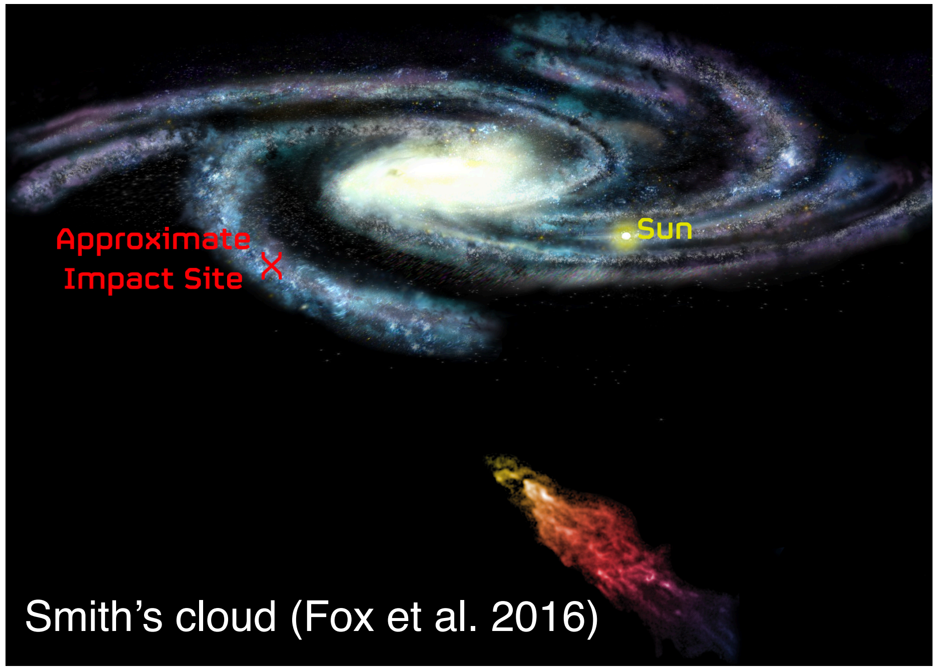
$$\rho(r_{\rm vir}) = 200\rho_{\rm c,0} \approx 1.7 \times 10^{-27} \ [\mathrm{g\,cm^{-3}}] \approx 2.5 \times 10^4 \, M_\odot \ [\mathrm{kpc^{-3}}]$$

- For example, $r_{\rm vir} \sim 250\,\mathrm{kpc}$ for the Milky Way. [Note: the scale length of the Galaxy's stellar disk ~ 3 kpc.]
  $\sim 3\,\mathrm{Mpc}$ for a cluster of galaxies (Coma Cluster).
- <u>The virial radius is also often referred to as r_{200}.</u>

CGM: High-Velocity Clouds

- ***H I HVCs***
  - History
    - The CGM around our galaxy was first detected in 21-cm emission (Muller et al. 1963).
    - The brightest clouds: HVC40-15+100 (Smith 1963; **Smith's cloud**), complexes A and C (Hulsbosch & Raimond 1966), "South Pole complex" (Dieter 1964), clouds M, AC I, AC II, AC III (Mathewson et al. 1966)
  - They appear to have a high radial velocity relative to the galactic disk, which is not consistent with the Galactic rotation. Hence, these fairly cool and dense gas clouds containing the neutral hydrogen are called ***high-velocity clouds (HVCs)***.
    - **Velocity**: Most of the HVCs are at galactocentric velocities between -250 km/s and +250km/s.
    - **Line broadening**: Their 21-cm emission has typical line broadening parameter $b \sim \sqrt{2}\sigma_v \sim 12\,\mathrm{km\,s^{-1}}$, indicating that the emission is from a warm neutral medium with a temperature of $T_{\rm gas} \sim 9000\,\mathrm{K}$.
    - **Column density**: A typical HVC column density is $N_{\rm HI} \sim 10^{19}\,\mathrm{cm^{-2}}$.
    - **Angular Size**: From enormous systems like Complex C (90°×20°) to subdegree-scale compact high-velocity clouds (CHVCs). Some of them are not resolved even with large single-dish radio telescopes.

- Distance
  - An upper limit to the distance of a HVC can be set by detecting absorption at the velocity of the HVC in the spectrum of a star with known distance.
  - A lower limit is set from stars not showing absorption by the HVC, with a detection limit below the expected line strength, found from an abundance determination based on an extra-galactic object.
  - Most of the HVCs are ***at distance d < 15 kpc from the Galactic center***, and are ***within ~ 30^o of the disk plane*** as seen from the Galactic center. They reside either in the extended Galactic halo or at extragalactic (Local Group) distances.
  - For example,
    - Complex A: 2.5 kpc < z < 6.5 kpc
    - Complex C: d ~ 10 kpc
    - The IV arch: z < 1.7 kpc
    - Complex H: z > 3-5 kpc
    - Smith's cloud: d ~ 12.4 kpc



Smith's cloud (Fox et al. 2016)

- Velocity
  - The sky coverage of high-velocity gas at negative velocities is greater than that at positive velocities. (Does this mean they are mostly falling into the Galaxy?)
  - See also next slide

Velocity (north-south, east-west asymmetry)

Wakker et al. (2003)

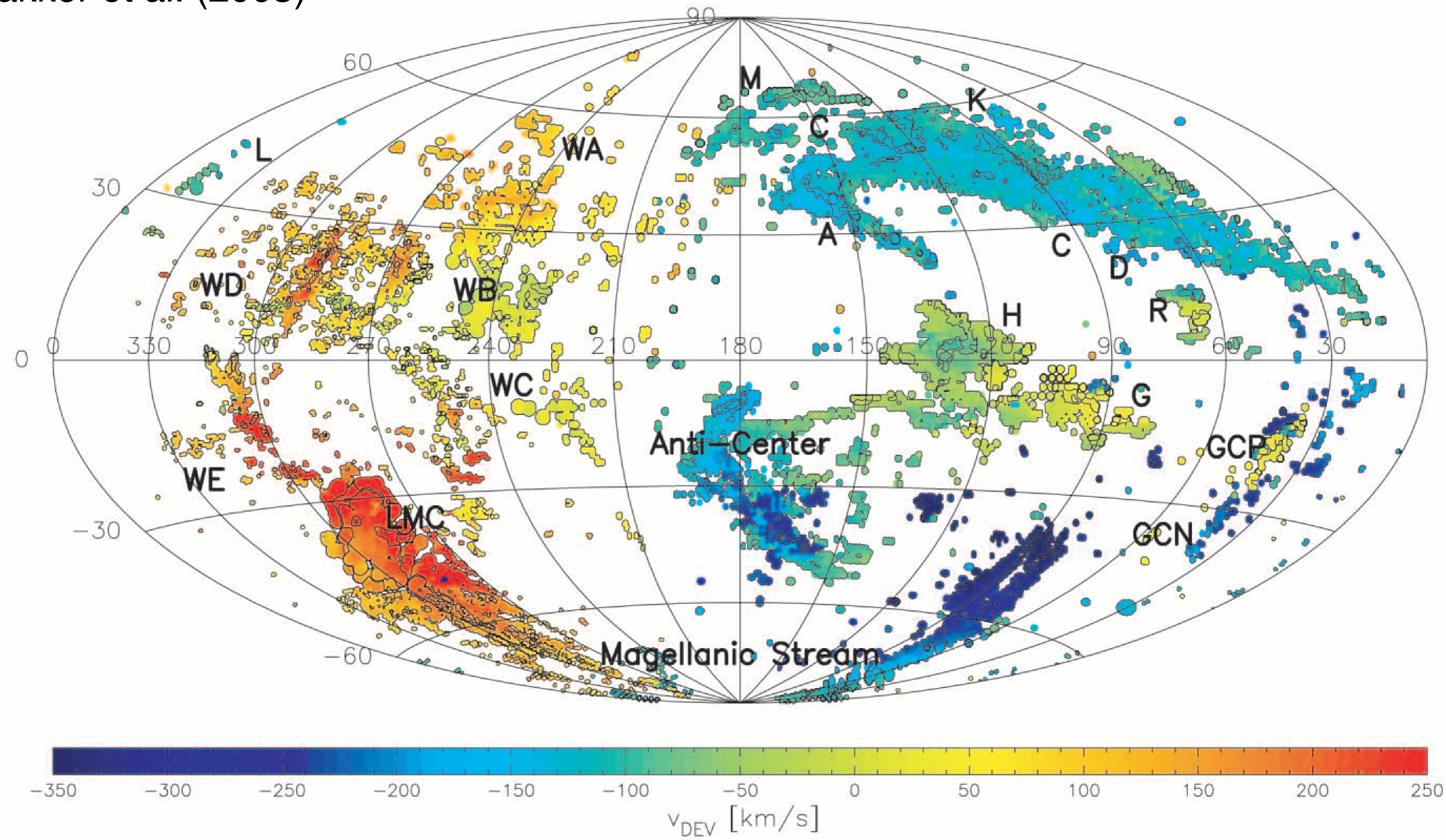


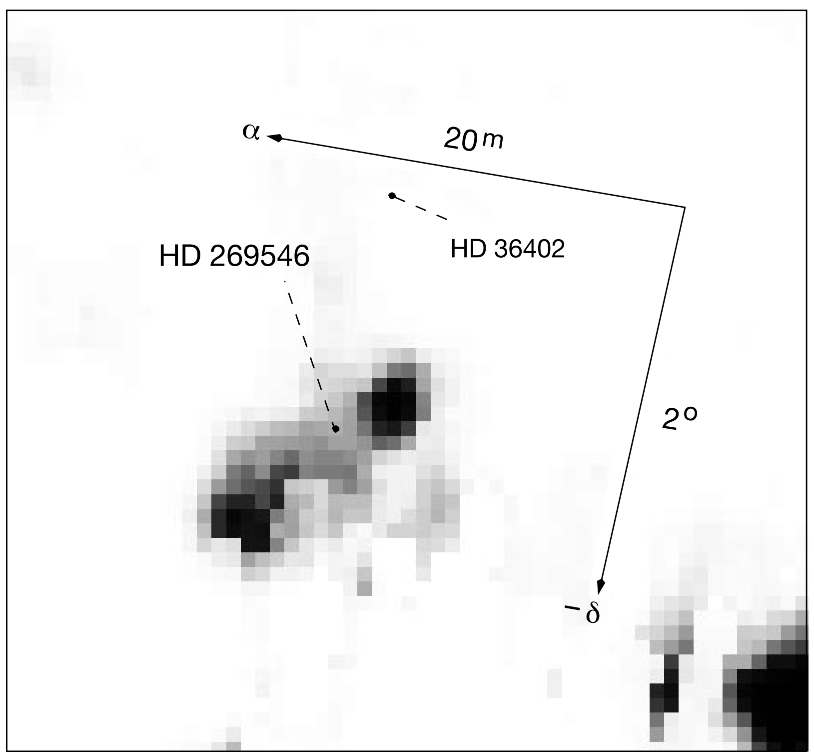
FIG. 16.—Map of the deviation velocities of the H I high-velocity gas. Data are from Hulsbosch & Wakker (1988) and Morras et al. (2000). The deviation velocity is the difference between the observed LSR velocity and the maximum velocity that can be easily understood in a simple model of Galactic differential rotation ($v_{\rm rot,\odot} = 220$ km s^{-1}, $R_\odot = 8.5$ kpc, $R_{\rm MW} = 26$ kpc, $z_{\rm ISM} = 2$ kpc at $R_\odot$, increasing to 6 kpc at $R_{\rm MW}$, see Wakker 1991). The names of the major complexes are indicated. Contour levels are at 0.05, 0.5, and 1 K brightness temperature, or ~2, 20, and 40 × 10^{18} cm^{-2}.

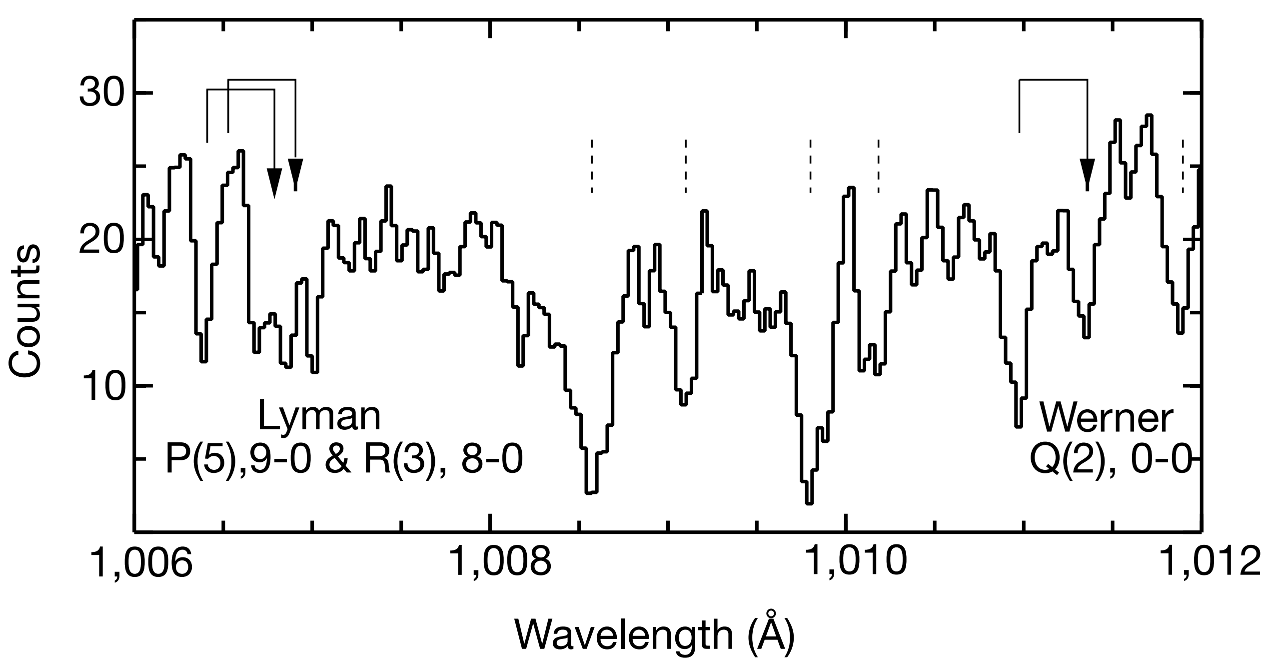
- Total Mass of HVCs
  - With distances known, we can determine the physical size and mass of individual HVCs.
  - The total H I mass of the HVCs around our Galaxy is $M_{\mathrm{HI}} \sim 3 \times 10^{7}\, M_{\odot}$.
  - Adding in helium, metals, and ionized hydrogen, its total mass is $M_{\mathrm{HVC}} \sim 7 \times 10^{7}\, M_{\odot}$.
- Ionized High-Velocity Clouds (IHVCs) invisible at 21 cm.
  - For instance, absorption lines of Si III, which traces $T_{\mathrm{gas}} \sim 2 \times 10^{4}\,\mathrm{K}$ gas, are seen over more than half the sight lines examined.
  - Absorption lines of O VI, tracing $T_{\mathrm{gas}} \sim 3 \times 10^{5}\,\mathrm{K}$ gas, are seen along more than half the sight lines.
  - The IHVCs are estimated to contain a total mass of $M_{\mathrm{IHVC}} \sim 10^{8}\, M_{\odot}$, comparable to the mass of the “traditional” HVCs seen in 21-cm emission.
- Hot Circumgalactic Medium
  - HVCs are surrounded by a hot ($T_{\mathrm{gas}} \sim 10^{6}\,\mathrm{K}$) circumgalactic medium, which is detected in X-rays and in O VII and O VIII absorption lines.
  - The density of the hot CGM is $n_{\mathrm{hot}} \sim 3 \times 10^{-5}\,\mathrm{cm}^{-3}$.
  - The hot CGM is thought to be extended ~ 100 kpc from the galactic center, with $M_{\mathrm{hot}} \sim 10^{10}\, M_{\odot}$

- Pressure Equilibrium between H I HVCs and the hot CGM
  - The hot CGM is of comparable temperature to the hot ISM, but its density is lower by two orders of magnitude.
  - Similarly, the temperature of H I HVCs is comparable to that of the warm ISM, but their density is lower by two orders of magnitude.
  - This suggests that HVCs are in approximate pressure equilibrium with the hot CGM.
- Total Mass of the CGM around our Galaxy is poorly known. Simulations indicates that the total mass of the CGM is $M_{\rm CGM} \sim 2 \times 10^{10}\, M_\odot \approx 5 M_{\rm ISM}$.
- Origin
  - Distance and angle from the Galactic disk indicates that ***most of the HVCs are definitely associated with the disk of our Galaxy***.
  - ***Some of the material in HVCs would be gas that has been stripped from satellite galaxies*** by ram pressure of the hot CGM (for instance, the Magellanic Stream).
    - The metallicity of the Magellanic Stream is ~10% of solar, consistent with the metallicity of the Small Magellanic Cloud. However, most of the HVCs can't be matched with any particular satellite galaxy.
  - ***Some of the CGM is pristine intergalactic gas falling inward*** through the virial radius.
    - Galaxy formation simulations show gas flowing inward along filaments to regions where galaxies are forming and evolving. The filamentary flow of gas tends to dilute the metallicity of circumgalactic gas.

HVCs: Molecular hydrogen

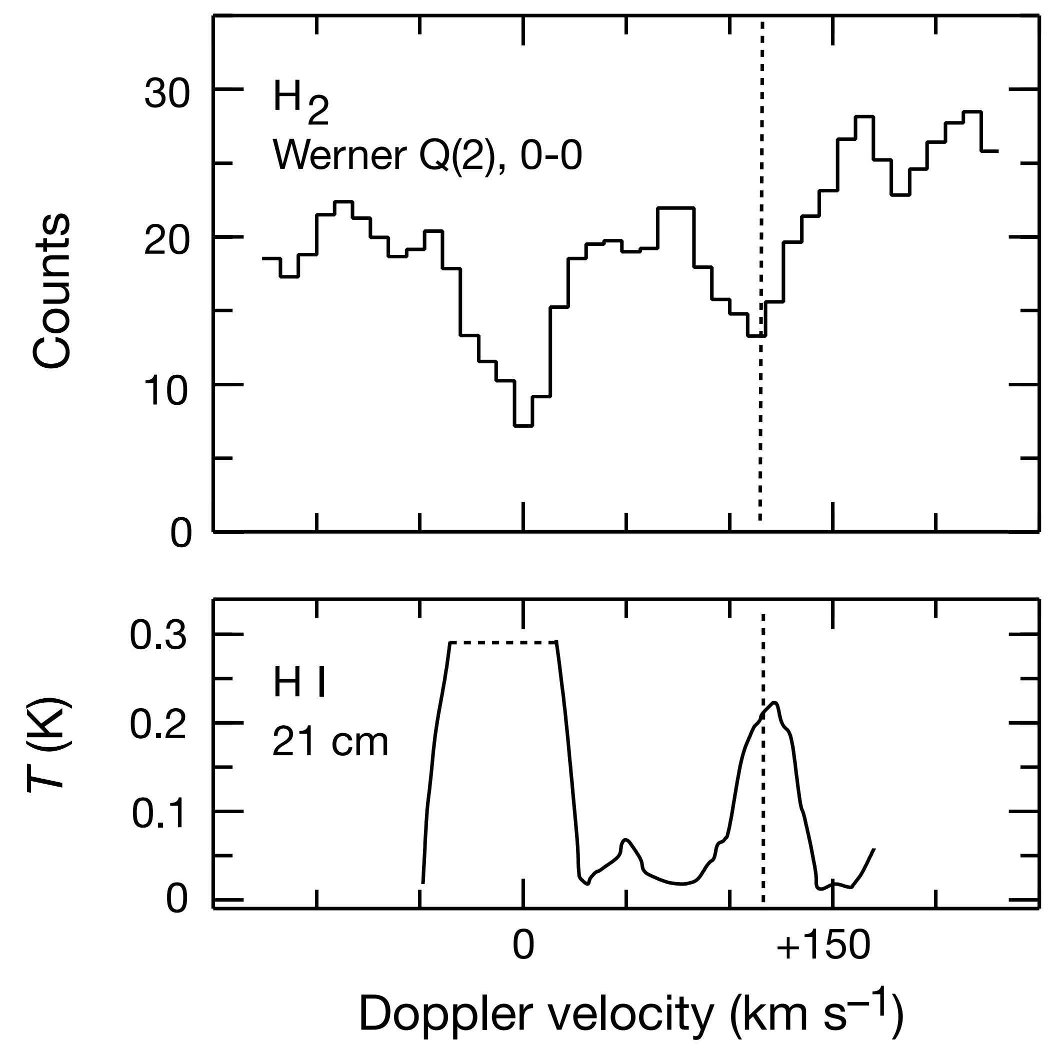
- Richter (1999, Nature) detected molecular hydrogen absorption in a HVC (with +120 km/s) along the sightline to LMC, using ORFEUS.





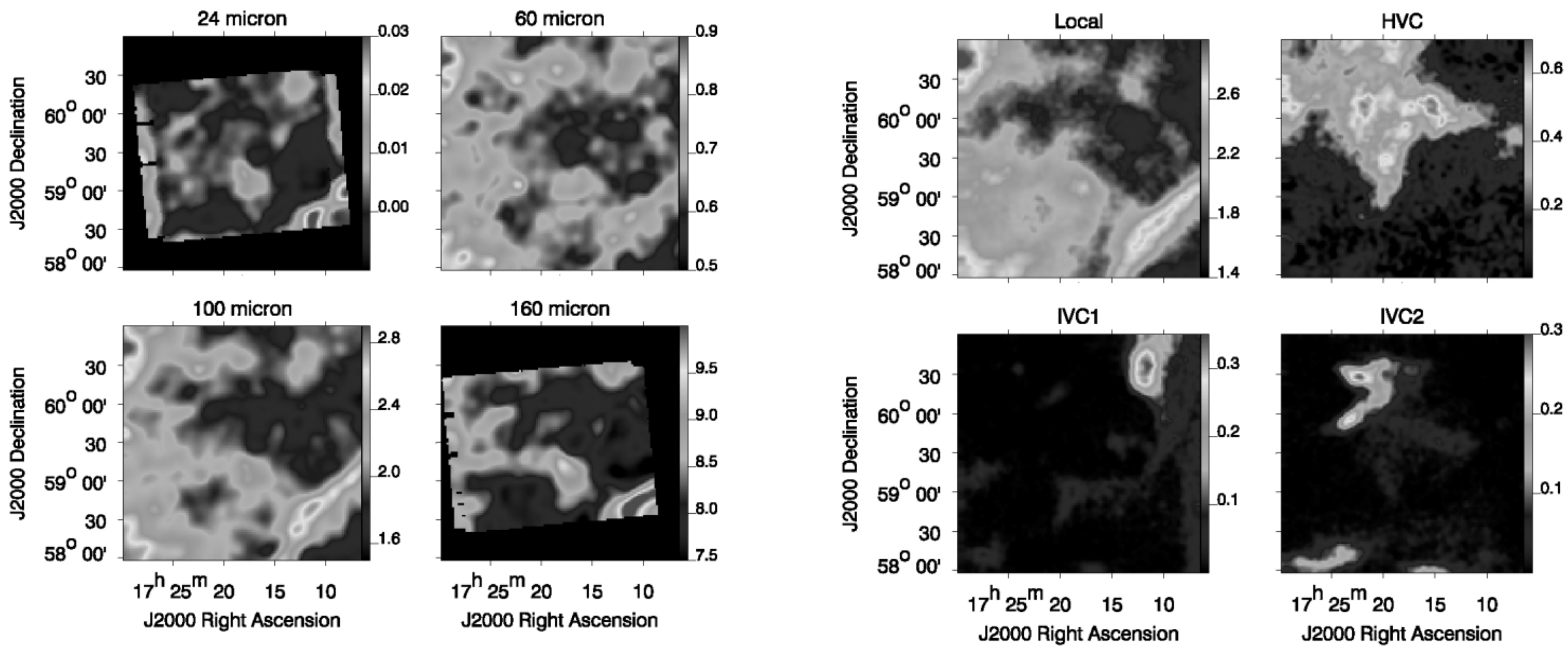
$$N(\mathrm{H_2}) = (2.2 - 3.6) \times 10^{15}\ \mathrm{cm}^{-2}$$

$$N(\mathrm{HI}) = 1.2 \times 10^{19}\ \mathrm{cm}^{-2}$$



HVCs: Dust

- Miville-Deschenes et al. (2005)
  - Complex C, T ~ 10.7 K (T ~ 17.5 K in the local ISM) is in accordance with its great distance from the Galactic distance.



- Peek et al. (2009)
  - couldn't detect dust in Complex C, not detected dust in Complex M.
  - There exist LVCs that have extremely low dust-to-gas ratios, consistent with being in the Galactic halo
- Plank Collaboration (2011)
  - found dust in Complex C at the 3 sigma level.

HVCs: Highly ionized gas - O VI absorption

Sembach et al. (2003)

small open circles - null detection

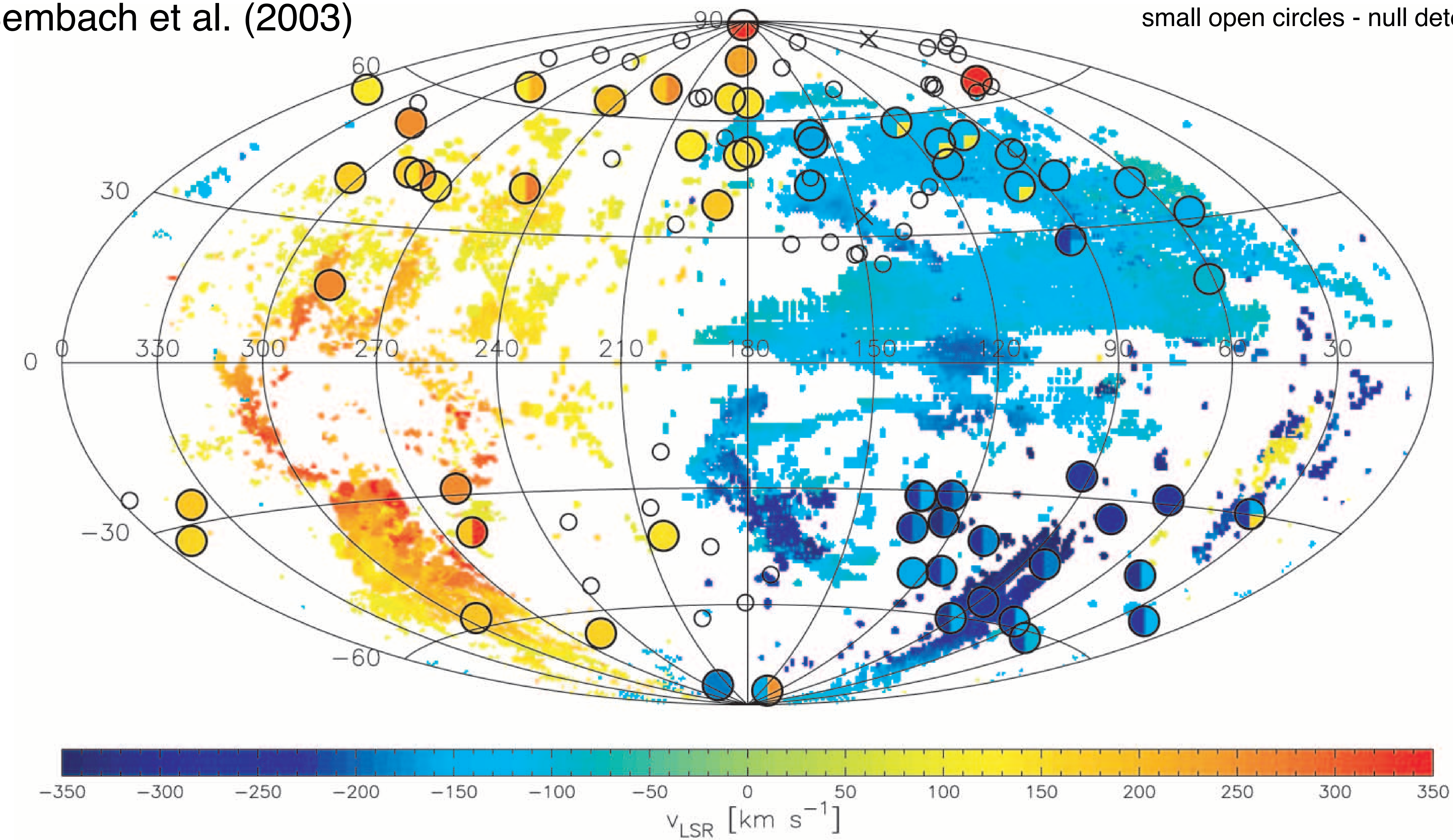


FIG. 11*a*

FIG. 11.—(*a*) Hammer-Aitoff projection of the high-velocity H I sky based on 21 cm emission measurements (adapted from Wakker et al. 2003). The H I data have a spatial resolution of approximately 36′ and are representative of gas with $N(\mathrm{H\,I}) > 2 \times 10^{18}$ cm^{-2}. Data for $|b| < 20°$ have been omitted for clarity. The positions of the high-velocity O VI features listed in Table 1 are denoted by the large circles, with the fill color indicating velocity on the same color scale used for the H I emission. If more than one high-velocity O VI feature is present, the circle is split and the velocities are color-coded in each section of the circle. Small open circles indicate null detections (Table 2). "X" marks indicate the locations of the two stellar sight lines in the sample. (*b*) Azimuthal equal-area projection of the data shown in (*a*), looking up toward the north Galactic pole. (*c*) Azimuthal equal-area projection of the data shown in (*a*), looking down toward the south Galactic pole.

HVCs: Ionized Gas

- Low ionized gas: Si II, Mg II

- Highly ionized gas:
  - Consideration of the possible sources of collisional ionization favors production of some of the O VI at the boundaries between cool/warm clouds of gas and a highly extended ($R > 70$ kpc), hot ($T > 10^6$ K), low- density ($n < 10^{-4}$ - 10^{-5} cm^{-3}) Galactic corona or Local Group medium.

- A bit low and inhomogeneous metallicity is usually found.
  - The most extended HVCs appear to be located relatively near the Galaxy, they may nonetheless be connected to extragalactic phenomena.
  - This is indicative of a combination of infalling primordial gas and outflowing enriched material.

HVCs in other galaxies

- HVCs appear to be a common feature around spiral galaxies (like M31 and M83).
- What is the origin of HVCs? There is no generally accepted theory regarding their origin.
  - cooling component of a supernova-driven “Galactic fountain”
  - inflows of neutral gas condensing from the local IGM
  - gaseous signatures of the “missing” dark matter subhalos around the Galaxy
- What is strange?
  - No stellar populations associated with HVCs have been found, unlike very low mass satellite galaxies which appear to harbor both stars and dark matter but little or no gas.
  - Aside from one case (Richter et al. 1999), HVCs appear to contain little or no molecular gas.

Galactic Wind

- The CGM is also fed by a galactic wind that blows interstellar gas outward, away from the disk.
  - M82 shows extremely strong galactic winds.
    - M82 has a star formation rate $R_{\rm SF} \sim 10\, M_\odot\, {\rm yr}^{-1}$. This high star formation rate provides a mechanism for driving the galactic wind.
    - When a large number of stars are formed within a short period of time, there will follow an epoch when multiple supernovae occur within a short period of time. The supernova remnants then merge to form a big hot bubble.
    - Active galactic nuclei can also inject large amounts of energy into the ISM, deriving a wind away from the galaxy.
    - The details of AGN physics are beyond the scope of this lecture, and we will focus on the case of the starburst-driven winds.

Optical, IR, and X-ray composite image of the starburst galaxy M81. The hot X-ray wind is in blue, the dusty outflow is red.

Fig 8.2, Ryden

Starburst-Driven Galactic Winds

- Galactic Winds
  - Starburst winds are driven by mechanical energy and momentum from stellar winds and supernovae (SNe).
  - For an instantaneous starburst, winds from OB stars dominate early on (< 3 Myr); next come Wolf-Rayet (WR) stars with mass-loss rates ~10 times higher (~3-6 Myr); and finally, core-collapse Type II SNe dominate until ~40 Myr when the least massive ($\sim 8 M_\odot$) ones explode.
  - In the real universe, star formation episodes last for more than 40 Myr.
  - Therefore, we may ***assume a constant star-formation rate*** for a time much longer than 40 Myr.
- A Simple Model:
  - We will assume that a star formation rate $\mathrm{SFR} = 1\, M_\odot\, \mathrm{yr}^{-1}$ leads to a SN rate of $R_{\mathrm{SN}} = 1/(50\, \mathrm{yr}) = 0.02\, \mathrm{yr}^{-1}$.
  - If each SN deposits an energy $E = 10^{51}$ erg into the ISM, this implies an ***energy injection rate***

$$
\begin{aligned}
\dot{E}_* &= 10^{51}\, \mathrm{erg} \times 0.02\, \mathrm{yr}^{-1} \left(\frac{\mathrm{SFR}}{1\, M_\odot\, \mathrm{yr}^{-1}} \right) \\
&= 6.3 \times 10^{41}\, \mathrm{erg\, s}^{-1} \left(\frac{\mathrm{SFR}}{1\, M_\odot\, \mathrm{yr}^{-1}} \right)
\end{aligned}
$$

- We now assume that the energy injection rate is constant with time. If the radiative losses of the overall system are negligible, the expanding bubble is energy-conserving (Sedov-Taylor solution).
    - In that case, the radius of the expanding shell of shocked ISM is given by

$$r_{\rm shell} = \xi \left(\frac{E_* t^2}{\rho}\right)^{1/5} \quad (\xi = 1.15167)$$

$$E_* = \dot{E}_* t$$

$$r_{\rm shell} = \xi \left(\frac{\dot{E}_* t^3}{\rho}\right)^{1/5}$$
$$\approx 520\,{\rm pc} \left(\frac{t}{1\,{\rm Myr}}\right)^{3/5} \left(\frac{\rm SFR}{1\,M_\odot\,{\rm yr}^{-1}}\right)^{1/5} \left(\frac{n_{\rm H}}{1\,{\rm cm}^{-3}}\right)^{-1/5}$$

    - The expansion rate of the shell is given by

$$v_{\rm shell} = \frac{dr_{\rm shell}}{dt} = \xi \frac{3}{5} \left(\frac{\dot{E}_*}{t^2 \rho}\right)^{1/5}$$
$$\approx 310\,{\rm km\,s}^{-1} \left(\frac{t}{1\,{\rm Myr}}\right)^{-2/5} \left(\frac{\rm SFR}{1\,M_\odot\,{\rm yr}^{-1}}\right)^{1/5} \left(\frac{n_{\rm H}}{1\,{\rm cm}^{-3}}\right)^{-1/5}$$

    - Energy losses from the hot gas inside the bubble are negligible until the temperature behind the expanding blastwave drops below ~ 2×10^7 K.

- When the temperature falls to this level, we can use the cooling time scale for collisionally excited line radiation: (see Lecture 8).

$$t_{\rm cool} \approx 0.19\,[{\rm Myr}]\left(\frac{T}{10^6\,{\rm K}}\right)^{1.7}\left(\frac{n_{\rm H}}{1\,{\rm cm}^{-3}}\right)^{-1}$$
$$\approx 6600\,[{\rm yr}]\left(\frac{v_{\rm shell}}{100\,{\rm km\,s^{-1}}}\right)^{3.4}\left(\frac{n_{\rm H}}{1\,{\rm cm}^{-3}}\right)^{-1}$$

$$T \simeq \frac{3}{16}\frac{m}{k}V_{\rm shock}^2$$
$$\approx 1.38\times10^5\,{\rm K}\left(\frac{m}{0.609m_{\rm H}}\right)\left(\frac{V_{\rm shock}}{100\,{\rm km\,s^{-1}}}\right)^2$$
$$m = \frac{1.4m_{\rm H}}{2.3} = 0.609m_{\rm H} \quad \text{for fully ionized gas}$$

- Putting in the relation for the expansion speed of the starburst-driven bubble, we find a cooling time:

$$t_{\rm cool} \approx 0.31\,{\rm Myr}\left(\frac{t}{1\,{\rm Myr}}\right)^{-1.36}\left(\frac{\rm SFR}{1\,M_\odot}\right)^{0.68}\left(\frac{n_{\rm H}}{1\,{\rm cm}^{-3}}\right)^{-1.68}$$

Solving the above equation for the instance when $t = t_{\rm cool}$, we find

$$t_{\rm cool} \approx 0.61\,{\rm Myr}\left(\frac{\rm SFR}{1\,M_\odot}\right)^{0.29}\left(\frac{n_{\rm H}}{1\,{\rm cm}^{-3}}\right)^{-0.71}$$

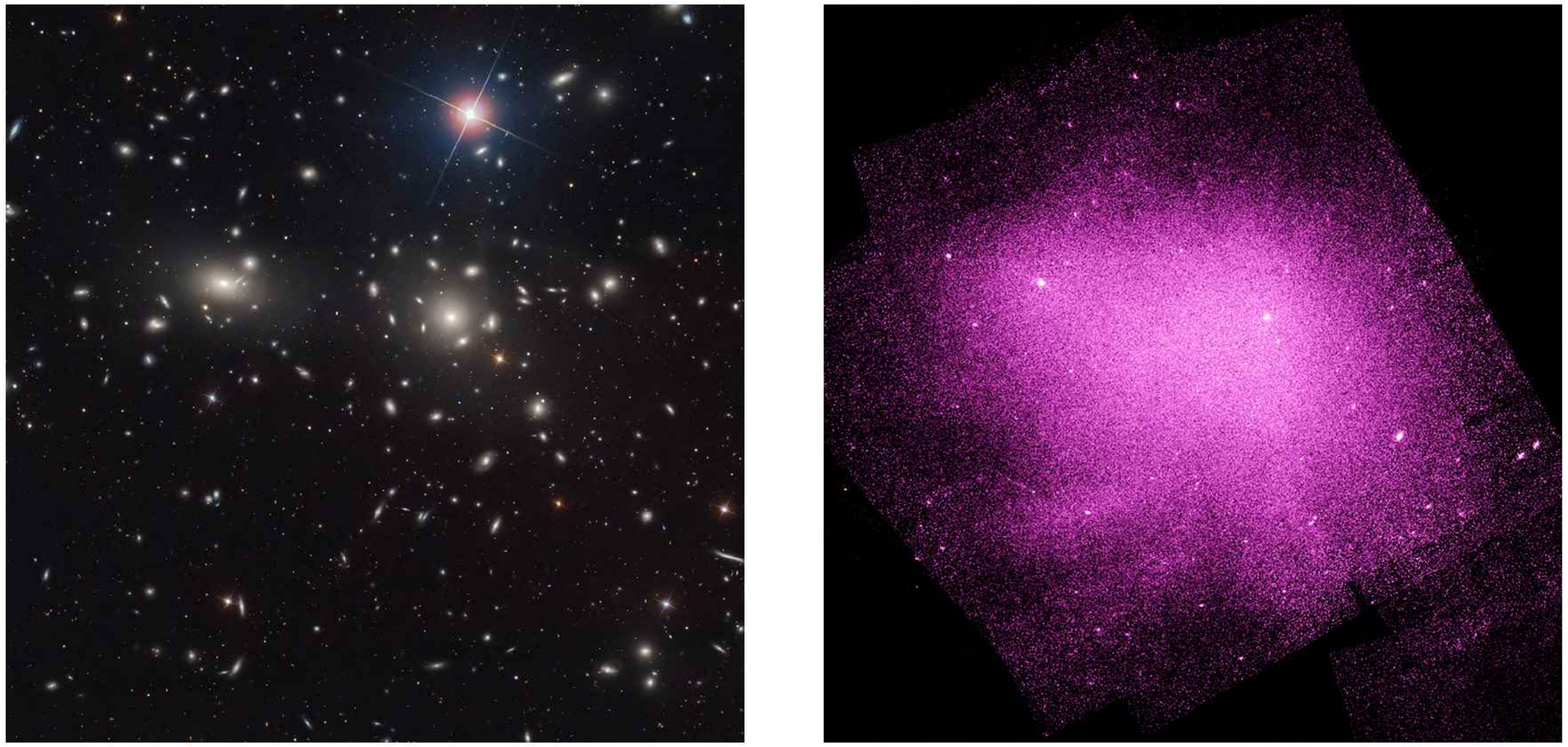
- By the time $t = t_{\rm cool}$, the bubble has expanded to a radius:

$$r_{\rm shell} \approx 390\,{\rm pc}\left(\frac{\rm SFR}{1\,M_\odot\,{\rm yr}^{-1}}\right)^{0.37}\left(\frac{n_{\rm H}}{1\,{\rm cm}^{-3}}\right)^{-0.63}$$

- The mean density of the WNM is ~ 0.6 cm^{-3}, and the scale height of the WNM, in the direction perpendicular to the Galactic disk, is ~ 350 pc. Thus, a superbubble is able to break free of the gaseous disk while still in its Sedov-Taylor phase.

Intracluster Medium (ICM)

- ***Coma Cluster***
  - The Coma Cluster is a rich cluster located at a distance ~ 100 Mpc from our Galaxy; at this distance, 1 arcsec ~ 0.48 kpc (10^8 AU).
  - About 1000 galaxies in the Coma Cluster have spectroscopic redshifts; taking into account of all the faint dwarf galaxies, there may be 10,000 or more galaxies in the Coma Cluster.



(a) Optical (b) X-ray

The Coma Cluster (a) at optical wavelengths [SDSS], and (b) at X-ray wavelengths [Chandra]. Two yellowish, brightest galaxies are NGC 4889 (left) and NGC 4874 (right), which are separated by 7 arcmin, corresponding to 200 kpc.

[Fig 8.3, Ryden]

- ***Dark Matter in the Coma Cluster***
  - **Virial Mass**:
    - In the central regions of the Coma Cluster, the line-of-sight velocity dispersion is found to be:

$$\sigma_v \approx 800\,\mathrm{km\,s^{-1}}$$

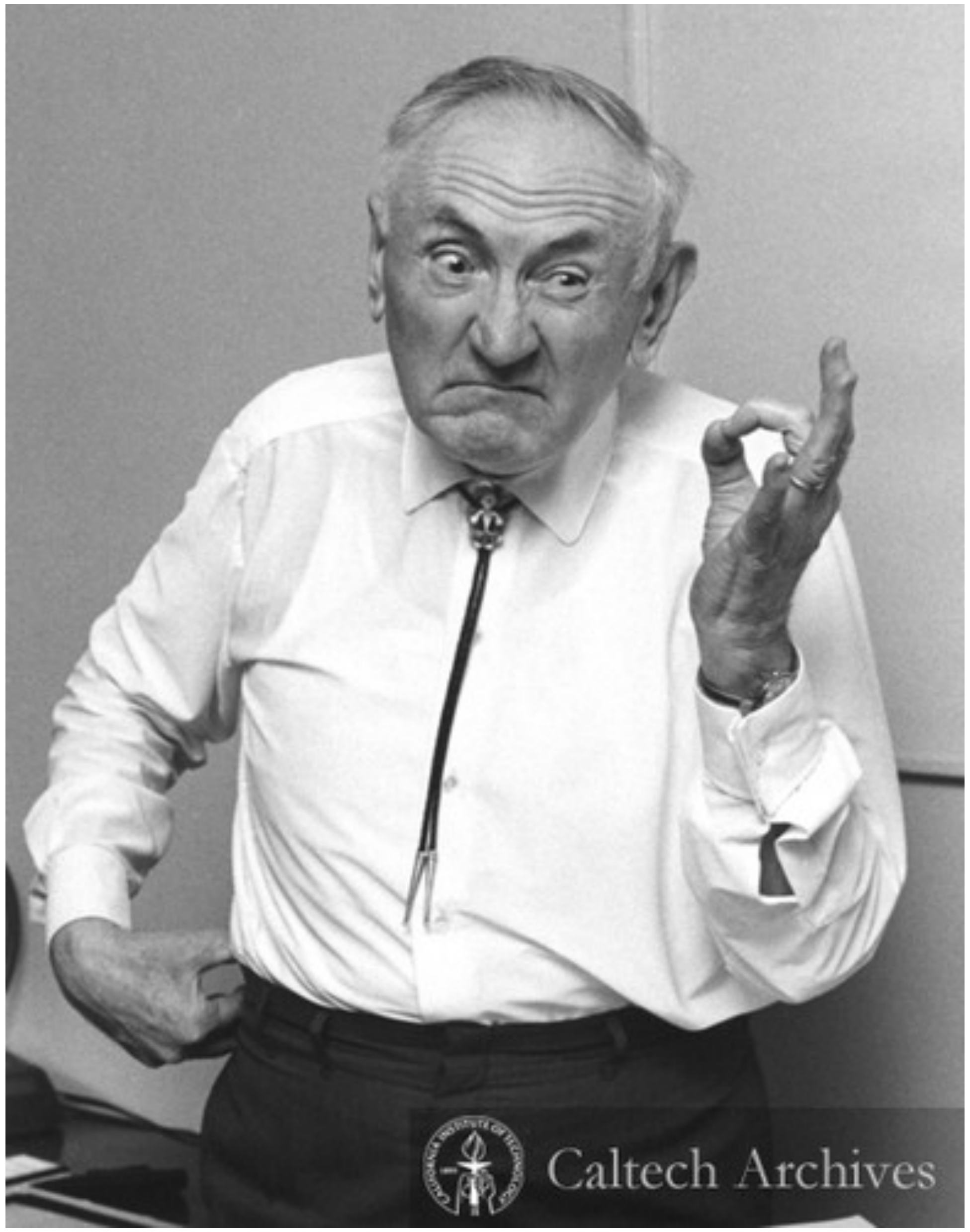
    - The half-light radius, which is measured from the center of the two brightest cluster galaxies, that contains half the total flux of the galaxies in the cluster is:

$$r_\mathrm{h} \approx 52\,\mathrm{arcmin} \quad \rightarrow \quad r_\mathrm{h} \approx 1.5\,\mathrm{Mpc}$$

    - Using the virial theorem, we can make a mass estimate for the Coma Cluster:

$$M_\mathrm{vir} = 5\frac{\sigma_v^2 r_\mathrm{h}}{G} \approx 2.7 \times 10^{48}\,\mathrm{g} \approx 1.4 \times 10^{15}\,M_\odot$$

In deriving the virial equation, a constant density was assumed. But, the equation is correct within a factor of unity.



Fritz Zwicky (1989-1974)
Photo by Floyd Clark (1971)

- **Stellar Mass**:
  - Integrating over the luminosity function of the Coma galaxies, the total luminosity of all the stars in all the galaxies in the Coma Cluster is found to be:

$$L_{\mathrm{B}}^{\mathrm{Coma}} \approx 8 \times 10^{12}\, L_{\mathrm{B},\odot} \sim 400 L_{\mathrm{B}}^{\mathrm{Milky\ Way}}$$ in B-band

  - Most of the visible light from the Coma Cluster comes from elliptical galaxies, whose stellar populations have a ***mass-to-light ratio*** $\gamma_\star \approx 5 M_\odot / L_{\mathrm{B},\odot}$. This yields a total stellar mass for the Coma Cluster:

$$M_\star = \gamma_\star L_{\mathrm{B}} \approx (5 M_\odot / L_{\mathrm{B},\odot}) \left(8 \times 10^{12} L_{\mathrm{B},\odot}\right) \approx 4 \times 10^{13} M_\odot$$

- Thus, the total virial mass of the Coma Cluster is more than 30 times the mass of the stars that it contains:

$$\therefore M_{\mathrm{vir}} \approx 35 M_\star$$

This discrepancy led Fritz Zwicky (1933) to deduce the existence of “dark matter.”

  - Only eight redshifts of galaxies in the Coma Cluster were known at the time. However, it was enough for Zwicky to realize that the velocity dispersion was far too large for Coma to remain bound if stars were the only matter present.

Hot Diffuse Intracluster Medium

- Hot Diffuse ICM
  - Stars provide a minority of the baryons in a rich cluster like the Coma. As seen in X-ray, the Coma Cluster is revealed as containing a hot diffuse intracluster medium.
  - The X-ray spectrum has the characteristic shape of thermal bremsstrahlung, or free-free emission, with kT ~ 10 keV in the 2-20 keV range.
  - **Cooling Time Scale**:
    - For fully ionized hydrogen, the free-free emissivity is

$$j_{\nu,\mathrm{ff}} = C_{\mathrm{ff}} \left(\frac{m_e c^2}{kT} \right)^{1/2} n_e n_i e^{-h\nu/kT} g_{\nu,\mathrm{ff}}$$

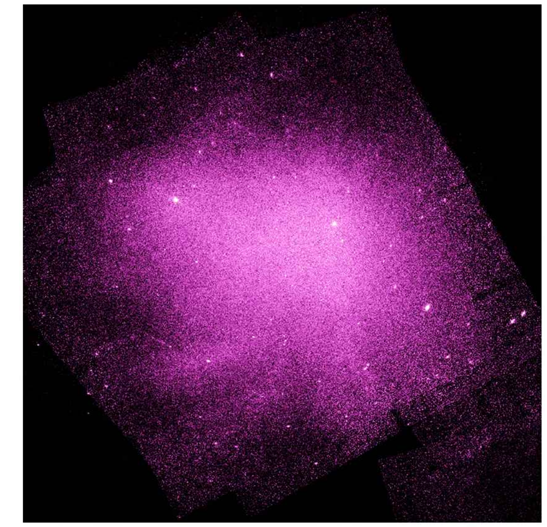
Here, $C_{\mathrm{ff}} \equiv \frac{8}{3} \left(\frac{2\pi}{3} \right)^{1/2} \frac{e^6}{m_e^2 c^4} = 7.070 \times 10^{-44}\,\mathrm{erg\,cm^3\,s^{-1}\,Hz^{-1}\,sr^{-1}}$

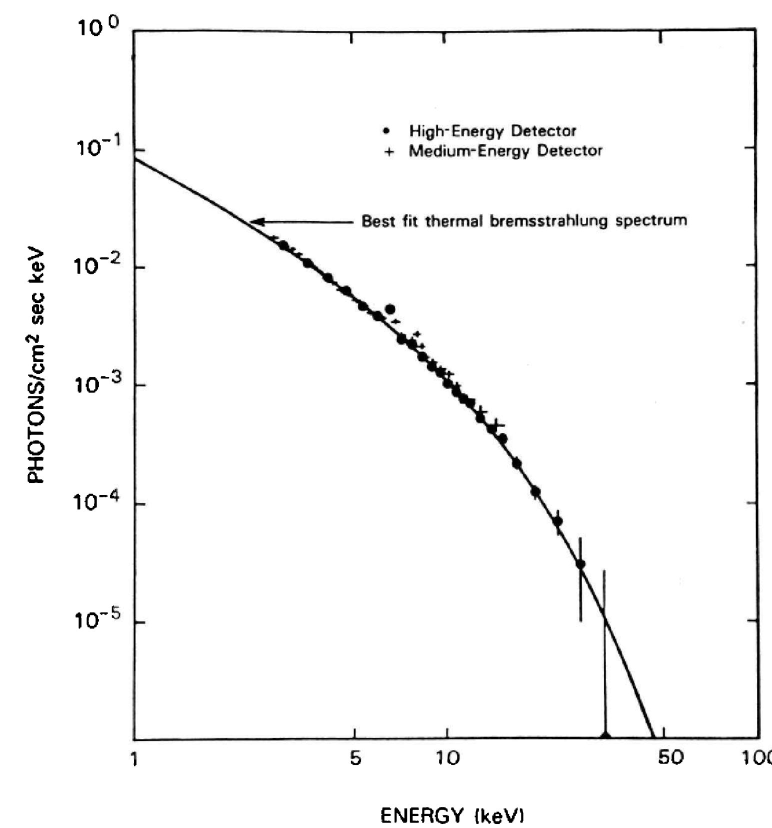
Gaunt factor: $g_{\nu,\mathrm{ff}} \approx \frac{\sqrt{3}}{\pi} \ln \left(\frac{4}{e^\gamma} \frac{kT}{h\nu} \right) \approx 0.551 \ln \left(2.25 \frac{kT}{h\nu} \right)$

$$\text{for } kT \gg 13.6\,\mathrm{eV} \text{ and } h\nu < kT$$

(Note that the Gaunt factor has only a mild frequency dependence.)

Euler-Mascheroni constant: $\gamma = \lim_{n\to\infty} \left(-\ln n + \sum_{k=1}^{n} \frac{1}{k} \right) \approx 0.5772$





(top) X-ray image, (bottom) X-ray spectrum of the Coma Cluster [Henriksen & Mushotzky 1986]

$kT_{\mathrm{gas}} \sim 7.3\,\mathrm{keV}$ for Coma

- The ***power per unit volume*** produced by thermal bremsstrahlung is:

$$\mathcal{L}_{\rm ff} = 4\pi \int j_{\nu,\rm ff} d\nu = 4\pi C_{\rm ff} \frac{m_e c^2}{h} \left(\frac{kT}{m_e c^2}\right)^{1/2} n_e n_i \overline{g_{\rm ff}}$$

Here, $4\pi C_{\rm ff} \frac{m_e c^2}{h} = 1.10 \times 10^{-22}\,{\rm erg\,cm^3\,s^{-1}}$

Frequency-averaged Gaunt factor: $\overline{g_{\rm ff}} \approx 1.15\ ({\rm near}\ T \sim 10^8\,{\rm K})$

$$\longrightarrow \quad \boxed{\mathcal{L}_{\rm ff} \approx 1.77 \times 10^{-29}\,{\rm erg\,cm^{-3}} \left(\frac{kT}{10\,{\rm keV}}\right)^{1/2} \left(\frac{n_{\rm H}}{10^{-3}\,{\rm cm^{-3}}}\right)^2}$$ for a fully ionized hydrogen gas

- The ***mass density*** of the ICM is:

$$\rho_{\rm ICM} = m_{\rm H} n_{\rm H} \approx 1.67 \times 10^{-27}\,{\rm g\,cm^{-3}} \left(\frac{n_{\rm H}}{10^{-3}\,{\rm cm^{-3}}}\right)$$

- ***Mass-to-light ratio*** for the ICM (considering only the bremsstrahlung) is:

$$\gamma_{\rm ICM} = \frac{\rho_{\rm ICM}}{\mathcal{L}_{\rm ff}} \approx 180\,(M_\odot/L_\odot) \left(\frac{kT}{10\,{\rm keV}}\right)^{-1/2} \left(\frac{n_{\rm H}}{10^{-3}\,{\rm cm^{-3}}}\right)^{-1}$$

Note that $\gamma_{\rm ICM} \gg \gamma_\star\ \ (\approx 5 M_\odot / L_{\rm B,\odot})$.
The bremsstrahlung is not an efficient cooling mechanism.

- The ***thermal energy density*** of the ICM is

$$\mathcal{E}_{\rm ff} = (2n_{\rm H})\frac{3}{2}kT \approx 4.81\times10^{-11}\,{\rm erg\,cm^{-3}}\left(\frac{n_{\rm H}}{10^{-3}\,{\rm cm^{-3}}}\right)\left(\frac{kT}{10\,{\rm keV}}\right)$$

- The ***cooling time*** for the ICM is then:

$$t_{\rm cool} = \mathcal{E}_{\rm ff}/\mathcal{L}_{\rm ff} \approx 86\,{\rm Gyr}\left(\frac{kT}{10\,{\rm keV}}\right)^{1/2}\left(\frac{n_{\rm H}}{10^{-3}\,{\rm cm^{-3}}}\right)^{-1}$$

- **Gas Mass**:

- The ***gas density profile*** of clusters is often expressed with a beta profile:

$$n_{\rm H} = \left[\frac{n_0}{1+(r/r_c)^2}\right]^{-3\beta/2} \qquad \begin{aligned}\beta &\approx 0.75\\ r_c &\approx 0.3\,{\rm Mpc}\\ n_0 &\approx 3\times10^{-3}\,{\rm cm^{-3}}\end{aligned}$$

for the Coma Cluster

- The amount of gas within a radius *r* will then be:

$$M_{\rm gas}(r) \approx \frac{4\pi}{3(1-\beta)}m_{\rm H}n_0r_c^3\left(\frac{r}{r_c}\right)^{3(1-\beta)} \quad \text{when } r\gg r_c \text{ and } \beta<1$$

- We assume an abrupt cutoff at the virial radius ($r_{\rm vir}\approx 3\,{\rm Mpc}$). This gives a mass estimate:

$$M_{\rm gas}(r) \approx 2\times10^{14}M_\odot\left(\frac{r}{3\,{\rm Mpc}}\right)^{0.75} \longrightarrow M_{\rm gas}\approx 5M_\star\ (M_\star\approx 4\times10^{13}M_\odot)$$

- **Total Mass in the hydrostatic equilibrium**:
  - In X-rays, the central regions of the Coma Cluster look smooth. We thus expect that Coma, at least in its central regions, is ***a relaxed cluster, in hydrostatic equilibrium***.
  - Spherical objects in hydrostatic equilibrium obey the equation:

$$\frac{dP}{dr} = -\frac{GM(r)\rho_{\rm gas}(r)}{r^2}$$

$M(r)$ is the mass of everything inside a radius r, including gas, stars, and dark mass.
$\rho_{\rm gas}(r)$ is the gas density.

  - The ideal gas law for an ionized hydrogen gas is:

$$P = nkT = \frac{2\rho_{\rm gas}(r)}{m_{\rm H}} kT_{\rm gas}(r)$$

  - Combining the equation of hydrostatic equilibrium with the ideal gas law, we find the total mass M contained within a radius r :

$$\begin{aligned} M(r) &= -\frac{r^2}{G\rho_{\rm gas}(r)} \frac{2k}{m_{\rm H}} \frac{d(\rho_{\rm gas} T_{\rm gas})}{dr} \\ &= \frac{2rkT_{\rm gas}}{Gm_{\rm H}} \left(-\frac{d\ln\rho_{\rm gas}}{d\ln r} - \frac{d\ln T_{\rm gas}}{d\ln r} \right) \end{aligned}$$

- We assume that the intracluster gas is isothermal ($T_{\rm gas}$ = constant) as an first order approximation and a beta profile for the gas density:

$$M(r) \approx \frac{6\beta kT_{\rm gas}}{Gm_{\rm H}} r \approx \frac{4.5kT}{Gm_{\rm H}} r \quad (\text{for r} \gg \text{r}_{\rm c} \text{ and } \beta \approx 0.75)$$

If a cluster is in hydrostatic equilibrium, and has a cutoff at the virial radius, then its total mass (including everything) is:

$$M_{\rm HE} \approx \frac{4.5kT_{\rm gas}}{Gm_{\rm H}} r_{\rm vir} \approx 3.0 \times 10^{15} M_{\odot} \left(\frac{kT_{\rm gas}}{10\,\text{keV}}\right) \left(\frac{r_{\rm vir}}{3\,\text{Mpc}}\right)$$

- For the Coma Cluster, with $kT_{\rm gas} = 7.3\,\text{keV}$ and $r_{\rm vir} \approx 3\,\text{Mpc}$, the total mass is:

$$M_{\rm HE} \approx 2.2 \times 10^{15} M_{\odot}$$

This is consistent with the virial estimate of $M_{\rm vir} \approx 1.4 \times 10^{15} M_{\odot}$.

In summary,

total mass	$M_{\rm tot} \approx 2 \times 10^{15} M_{\odot}$
stellar mass	$M_{\star} \approx 0.02 M_{\rm tot}$
gas mass	$M_{\rm gas} \approx 0.1 M_{\rm tot}$

Diffuse Intergalactic Medium - Introduction to Cosmology

- Cosmic baryon density:
  - The 2015 results from the Planck satellite tell us that the current mass density and number density of baryonic matter are, respectively:

$$\bar{\rho}_{\mathrm{bary},0} = (4.190 \pm 0.026) \times 10^{-31}\,\mathrm{g\,cm^{-3}} \approx 0.048\rho_{\mathrm{c},0}$$
$$\bar{n}_{\mathrm{bary},0} = \bar{\rho}_{\mathrm{bary},0}/m_{\mathrm{H}} = 2.5 \times 10^{-7}\,\mathrm{cm^{-3}}$$

  - The amount of baryonic matter in gravitationally bound systems, including stars, the ISM, the CGM, and the ICM, provides only ~ 15% of this mean cosmic baryon density.
  - The remainder is provided by a very tenuous intergalactic medium (ICM).

- The Universe expands.
  - As a consequence, the mean baryon density decreases with time, since baryon number is conserved.
  - The expansion of the universe is homogeneous and isotropic on large scales, and thus can be described by a simple scale factor $a(t)$, which is customarily normalized so that $a(t_0) = 1$ at the present time $t_0 = 13.80\,\mathrm{Gyr}$ after the Big Bang.

$$a(t_0) = 1 \quad \text{at the present time} \quad t_0$$

  - The expansion of the universe is frequently described in terms of the Hubble parameter:

$$H(t) \equiv \frac{\dot{a}}{a}$$

Recall the Hubble's law: $V = H_0 R$

The Hubble parameter evaluated at the present time is the Hubble constant:

$$H_0 = H(t_0) = 67.74 \pm 0.46\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

The Hubble time and the Hubble distance are, respectively,

$$H_0^{-1} = 14.43 \pm 0.10\,\mathrm{Gyr}$$

$$cH_0^{-1} = 4425 \pm 30\,\mathrm{Mpc}$$

- Light emitted by a quasar at time t_e, with a redshift z_e is observed by us at $t_0\ (> t_e)$. The scale factor at the time the light was emitted is smaller than that at the present time:

$$a(t_e) < a(t_0) = 1$$

- The scale factor $a(t_e)$ can be expressed in terms of the redshift of the quasar:

$$a(t_e) = (1 + z_e)^{-1}$$

Recall that $z \equiv \frac{\nu - \nu_0}{\nu_0} = \frac{v}{c} \quad \Rightarrow \quad \frac{\nu}{\nu_0} = 1 + z \text{ and } \frac{\lambda}{\lambda_0} = \frac{1}{1+z}$

- Between the time of emission and the time of observation ($t_e < t < t_0$), the number density of baryons and the frequency of the emitted photons have been decreasing:

$$\bar{n}_{\rm bary}(t) = \bar{n}_{\rm bary,0}\, a(t)^{-3} = \bar{n}_{\rm bary,0}(1+z)^3 \qquad \nu(t) = \nu_0 a(t)^{-1} = \nu_0(1+z) \geq \nu_0$$

Absorption Lines as a Probe of the IGM

- The IGM is difficult to detect. A useful way of searching for intergalactic gas is to look for absorption lines from the IGM along the line of sight to a distant quasar.
  - Every parcel of gas along the line of sight to a distant quasar will selectively absorb certain wavelengths of continuum light of the quasar due to the presence of the various chemical elements in the gas.
  - Through the analysis of these quasar absorption lines we can study the spatial distributions, motions, chemical enrichment, and ionization histories of gaseous structures from redshift $z \sim 7$ until the present.
  - This structure includes the gas in galaxies of all morphological types as well as the diffuse gas in the IGM.

- The Lyα line (1216Å) would a particularly useful probe as long as it isn't too highly ionized.

- Consider the nearby quasar 3C273, at a redshift $z_e = 0.158$.
  - If 3C 273 produces a Lyα photon with an energy $h\nu_e = 10.20\,\mathrm{eV}$, by the time it reaches us, it will be redshifted to the lower energy:

$(1 + z_e = 1.158)$

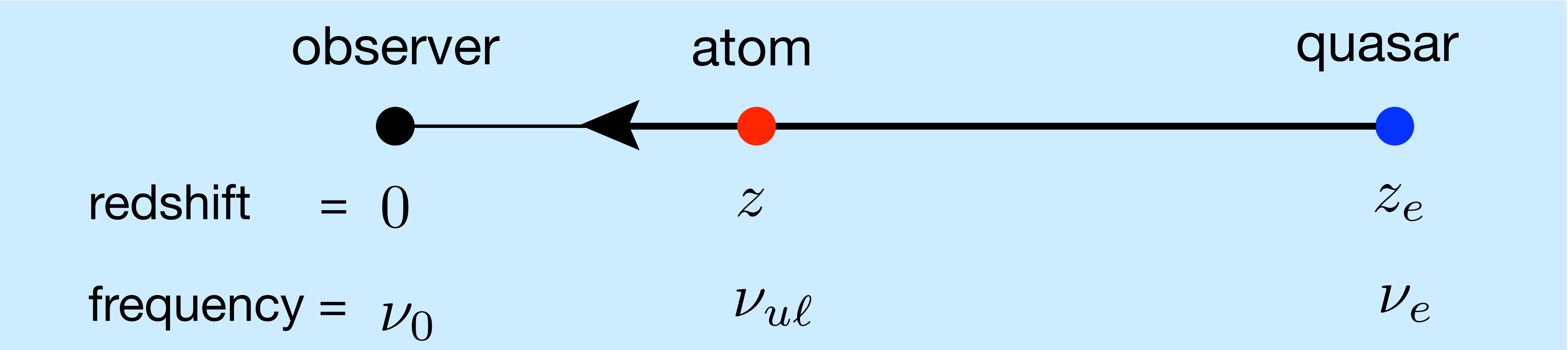
$$\boxed{h\nu_e = 10.20\,\mathrm{eV} \quad \rightarrow \quad h\nu_0 = h\nu_e a(t_e)/a(t_0) = 10.20\,\mathrm{eV}/1.158 = 8.81\,\mathrm{eV}}$$

  - If the continuum with an initial energy $h\nu_e = 10.20\,\mathrm{eV} \times 1.158$ at the redshift of 3C 272 will be redshifted to the lower energy:

$$\boxed{h\nu_e = 10.20\,\mathrm{eV} \times 1.158 = 11.81\,\mathrm{eV} \quad \rightarrow \quad h\nu_0 = h\nu_e/1.158 = 10.20\,\mathrm{eV}}$$

  This energy can be absorbed by neutral hydrogen in our own galaxy.

- In general, if a quasar is at a redshift z_e, photons with initial energy in the range $h\nu_{u\ell} < h\nu_e < (1 + z_e)h\nu_{u\ell}$ can be absorbed by a transition with energy $h\nu_{u\ell}$ somewhere along the line of sight from the quasar to us.

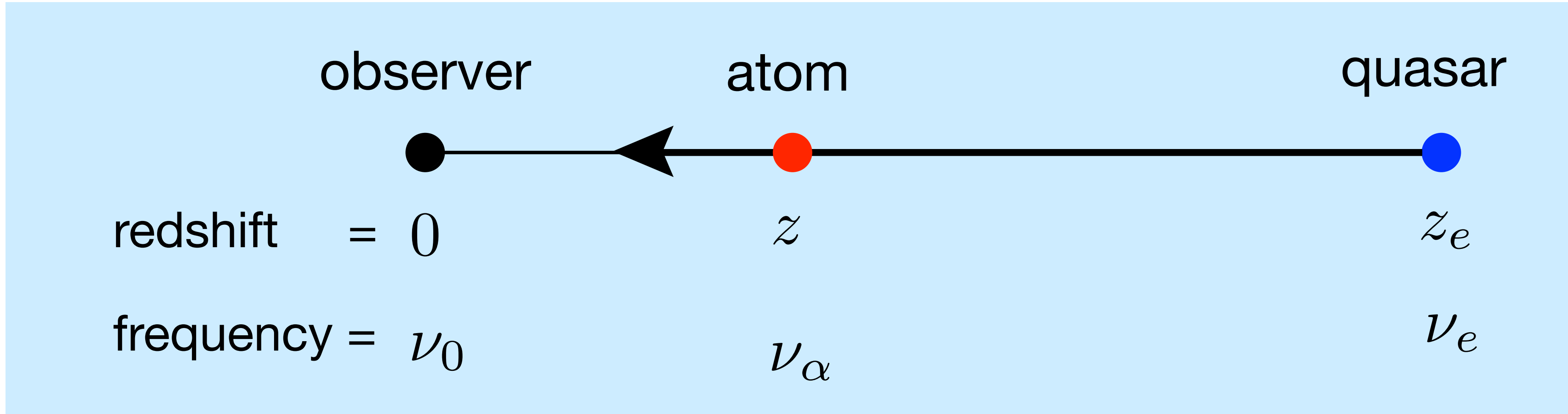


  - The redshift at which photons can be absorbed is:

$$\nu_e = \nu_0(1 + z_e)$$
$$\nu_{u\ell} = \nu_0(1 + z) \quad \longrightarrow \quad \boxed{z = \frac{\nu_{u\ell}}{\nu_0} - 1 \quad \text{or} \quad z = \frac{\nu_{u\ell}}{\nu_e}(1 + z_e) - 1}$$

Gunn-Peterson Effect

Jim Gunn & Bruce Peterson (1965)



- In order to understand the Gunn-Peterson effect, let's consider radiation observed at some frequency ν that was initially lying blueward of Lyα by a quasar at redshift z_e. The emitted photons pass through the local Lyα resonance as they propagates towards us through a smoothly distributed sea of neutral hydrogen atoms, and are scattered off the line-of-sight with a cross-section of

$$\sigma_\nu = \frac{\pi e^2}{m_e c} f_\alpha \phi_\nu = \chi_0 \phi_\nu \quad \left(\chi_0 \equiv \frac{\pi e^2}{m_e c} \right),$$

where ϕ_ν is the Voigt profile of the Lyα line, normalized so that $\int \phi_\nu d\nu = 1.$

- The total optical depth for resonant scattering at the observed frequency ν is given by the line integral of this cross-section times the neutral hydrogen density $n_{\rm HI}$ in the ground state,

$$\tau_\nu^{\rm GP} = \int_0^s \sigma_\nu n_{\rm HI} dl = \int_0^s \chi_0 \phi_\nu n_{\rm HI} dl.$$

- In an expanding Universe, the integral should be performed along the proper distance. We, therefore, want to use the redshift z instead of the proper length l travelled by light. Then, the optical depth becomes

$$\tau_\nu^{\mathrm{GP}} = \chi_0 \int_0^{z_e} \phi_\nu n_{\mathrm{HI}} \frac{dl}{dz} dz$$

- The expansion of the Universe is homogeneous and isotropic on large scale, and thus can be described by a simple scale factor $a(z)$.
- The scale factor today, $a(t_0) = 1$, is greater that the scale factor at the redshift z, $a(z) = 1/(1+z)$. We obtain the proper length element in terms of the redshift.

$$\frac{da}{a} = \frac{dz}{1+z}$$

Notice that the scale factor transforms like wavelength:

$$\lambda_0 = \lambda_z(1+z)$$
$$\lambda_z = \lambda_0/(1+z)$$

$$dl = c\,dt = c\frac{da}{\dot{a}} = c\frac{da}{Ha} = c\frac{dz}{H(1+z)} \quad \rightarrow \quad \frac{dl}{dz} = \frac{c}{H(1+z)}$$

- Then, the ***Gunn-Peterson optical depth*** is given by $$\tau_\nu^{\mathrm{GP}} = \chi_0 \int_0^{z_e} \phi_\nu n_{\mathrm{HI}} \frac{c}{H} \frac{dz}{1+z}.$$

- The thermal and natural broadening ($\Delta v_{\rm thermal} \sim 13~{\rm km~s^{-1}}$ for $T = 10^4 K$) is tiny compared to the "broadening" due to the Hubble expansion ($\Delta v = cz \sim 30{,}000~{\rm km~s^{-1}}$ for $z = 0.1$).

  Thus, we can treat the Voigt function as being very strongly peaked at the Lyα frequency ν_α in the local comoving frame. This resonance will occur at the redshift of z such that $\nu = \nu_\alpha/(1+z)$, i.e., at $z = \nu_\alpha/\nu - 1$.

- Then, ***in the local comoving frame at*** z, the frequency interval $d\nu$ can be expressed by $d\nu/\nu = dz/(1+z)$. Finally, we obtain

$$\tau_\nu^{\rm GP} = \chi_0 \int_0^{z_e} \phi_\nu n_{\rm HI} \frac{c}{H} \frac{d\nu}{\nu} \approx \frac{\chi_0 c}{\nu_\alpha} \frac{n_{\rm HI}(z)}{H(z)}.$$

Here, the density and Hubble parameter should be evaluated at the following z for the given ν.

$$\left(z = \frac{\nu_\alpha}{\nu} - 1 \right)$$

At low redshifts ($z \approx 0$), this gives an optical depth

$$\tau_\nu^{\rm GP} = 15{,}200 \frac{n_{\rm HI,0}}{\bar{n}_{\rm bary,0}},$$

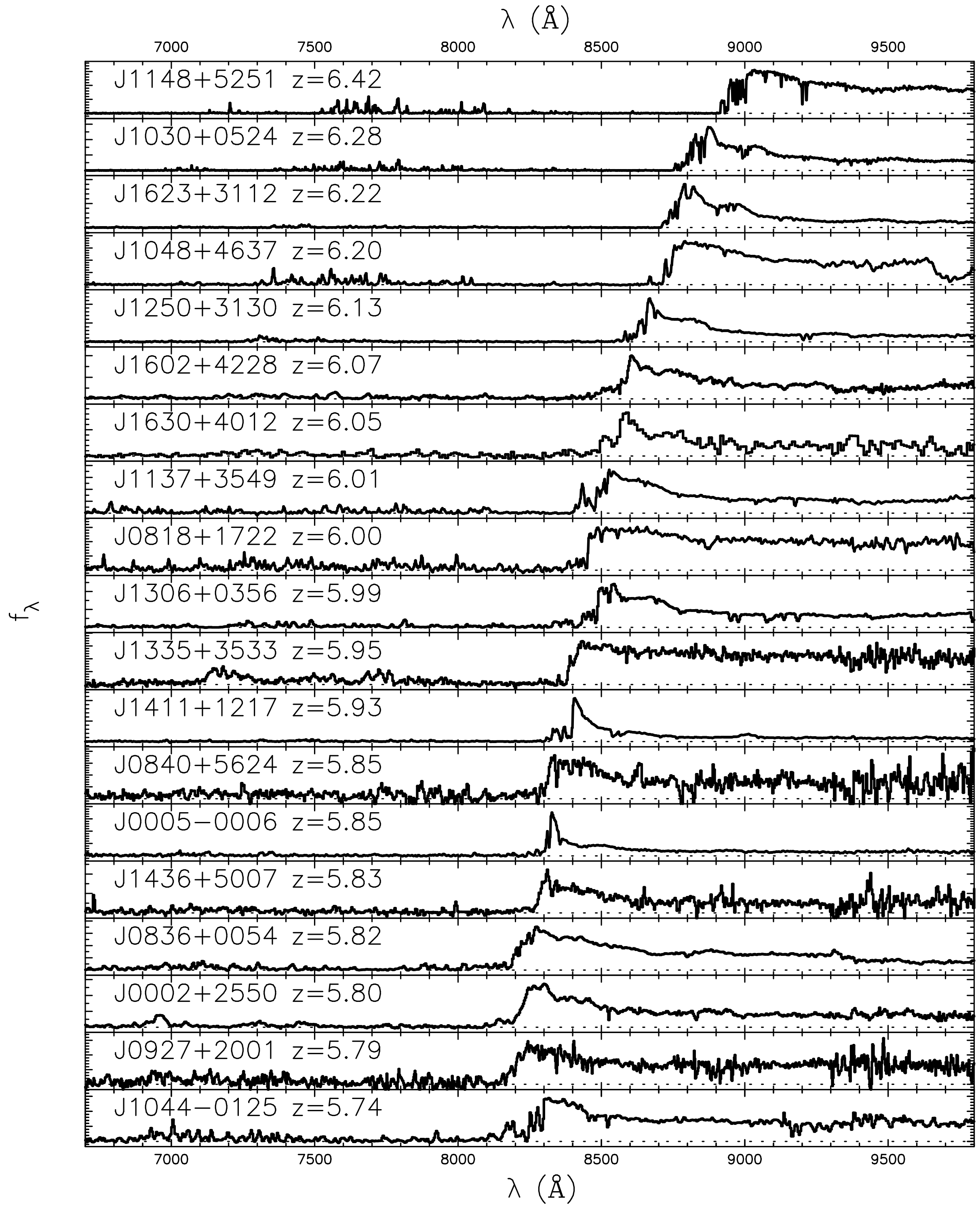
where the baryon number density at the present time is $\bar{n}_{\rm bary,0} = 2.5 \times 10^{-7}~{\rm cm^{-3}}$.

- Thus, ***even when the number density of neutral hydrogen atoms is one part in 10,000 of the baryon density, the optical depth is larger than one.***
- ***This result implies the spectrum of a low redshift quasar, for instance 3C 273 at*** $z = 0.158$, ***should be black between the observed wavelengths*** $\lambda = 1216$Å ***and*** $1216 \times 1.158 = 1408$Å ***and emitted wavelengths*** $\lambda = 1050$Å ***and*** $1050 \times 1.158 = 1216$Å.

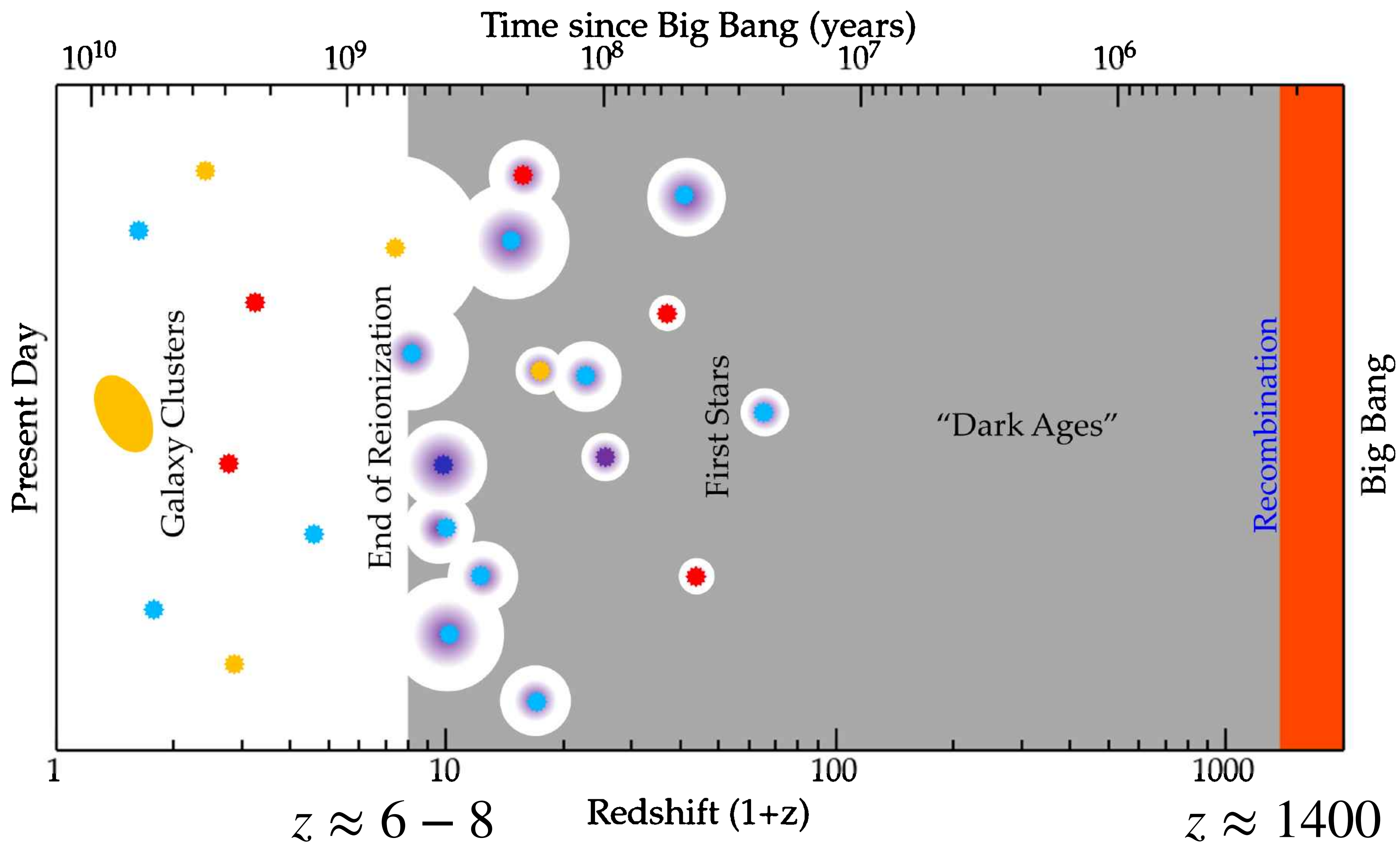
Absence of a Gunn-Peterson trough at z < 5

- However, it has been found that this is not the case. Therefore, ***either [1] intergalactic medium has a density very much lower than the mean baryon density of the Universe (gas is somehow segregated efficiently into galaxies) or [2] intergalactic gas is very highly ionized.***

- ***The absence of a <u>Gunn-Peterson trough</u> at redshifts*** $z < 5$ is now regarded as evidence that the IGM at low redshifts is highly ionized, not that it is absent.
- The figure shows spectra for high-redshift quasars. Notice that the Gunn-Peterson trough bluewards of the QSO Lyα emission is clearly apparent in the highest redshift ones.
- This indicates that the Universe has become somewhat more neutral at these redshifts. A similar behavior is also seen bluewards of the QSO Lyβ regions of the same spectra.
- These spectra show that ***the reionization of the IGM has ended at*** $z \approx 6$.



Dark Ages, Reionization Epoch



Schematic of the epoch of reionization [Barkana 2006; Fig 9.3 Ryden]

- In the early Universe, **some hundred million years after the Big Bang**, the temperature became low enough for electrons to combine with protons for the first time.
  - This is known as the epoch of recombination. This left the gas in the Universe in an overall neutral state.
- In today's Universe, however, nearly all of the gas between the galaxies is fully ionized.
  - ***There should have been a moment in the history of the Universe when it becomes ionized again.*** This period is known as the epoch of reionization.

- What caused the cosmic reionization?
  - How exactly this came to happens is still not fully understood, but one strong candidate for causing this to happen is the formation of ***the very first stars and galaxies***.
  - The other possible explanation would be strong high-energy radiation from quasars, and they do seem to have an effect, but the latest estimates indicate that they would contribute no more than ~ 10% to the total ionizing background radiation needed.
- When did the cosmic reionization happen?
  - One of the strongest pieces of evidence for the increasing fraction of neutral gas in the IGM is the Lyα forest and the so called ***Gunn-Peterson trough***.
  - The spectrum of a quasar, is intrinsically a bright continuum source with only a few very broad features. However, spectra of distant quasars show a large number of narrow absorption features, resembling the trunks of trees tightly packed together in a forest. This feature was therefore named the Lyα forest.
  - ***At higher redshifts, the absorption features appear closer together, until finally a completely absorbed trough is observed.*** This indicates that the universe was previously more filled with neutral gas, and ***at some point the IGM was completely neutral***. The term “Gunn-Peterson trough” is named after the study of Gunn and Peterson (1965).
  - The Gunn-Peterson trough is typically observed at $z \sim 6$, thus marks the end of the epoch of reionization.

More Details about the Gunn-Peterson Optical Depth

- A bit more details
  - Suppose that a fraction $f_{\rm H} \approx 0.9$ of all baryons are hydrogen nuclei. We can assume that $f_{\rm H}$ is constant after Big Bang Nucleosynthesis is complete because of inefficiency of stars at nucleosynthesis.
  - Let $f_{\rm n}(z)$ the fraction of neutral hydrogen in their ground state; this can be a function of redshift. Then, the density of neutral hydrogen in the ground state at a redshift *z*.

$$\overline{n}_{\rm bary} = \overline{n}_{\rm bary,0}\,(1+z)^3$$

$$\begin{aligned} n_{\rm HI}(z) &= \overline{n}_{\rm bary} f_{\rm H} f_{\rm n}(z) \\ &= \overline{n}_{\rm bary,0} f_{\rm H} f_{\rm n}(z)\,(1+z)^3 \approx 2.27 \times 10^{-7}\,[{\rm cm}^{-3}]\,\frac{f_{\rm H}}{0.9} f_{\rm n}(z)\,(1+z)^3 \end{aligned}$$

  - We can then write the optical depth at an arbitrary redshift:

$$\tau_\nu^{\rm GP} = \frac{\chi_0 c}{\nu_\alpha}\overline{n}_{\rm bary,0} f_{\rm H}\frac{f_{\rm n}\,(1+z)^3}{H(z)}$$

  - In a flat, LCDM universe, the Hubble parameter is given by (from the Friedmann equations)

$$\begin{aligned} H(z) &= H_0\left[\Omega_{\rm m,0}\,(1+z)^3 + \Omega_{\Lambda,0}\right]^{1/2} \qquad \text{Here, } \Omega_{\rm m,0} = 0.31 \text{ and } \Omega_{\Lambda,0} = 0.69 \\ &\approx H_0 \Omega_{\rm m,0}^{1/2}\,(1+z)^{3/2} \qquad \text{if } z \gg (\Omega_{\Lambda,0}/\Omega_{\rm m,0})^{1/3} - 1 \approx 0.31 \end{aligned}$$

Here, the density parameter is defined as: $\Omega \equiv \dfrac{\rho}{\rho_{\rm c}}$

- At redshifts $z \gg (\Omega_{\Lambda,0}/\Omega_{\mathrm{m},0})^{1/3} - 1 \approx 0.31$, we can use the approximation that the universe is matter dominated:

$$\begin{aligned}\tau_\nu^{\mathrm{GP}} &= \frac{\chi_0 c}{\nu_\alpha} \overline{n}_{\mathrm{bary},0} f_{\mathrm{H}} \frac{f_{\mathrm{n}} (1+z)^3}{H_0 \Omega_{\mathrm{m},0}^{1/2}} \\ &\approx 25,600 \, \frac{f_{\mathrm{H}}}{0.9} f_{\mathrm{n}}(z) \, (1+z)^{3/2}\end{aligned}$$

This indicates that, if we want a hope of seeing a Gunn-Peterson trough, we must go to high redshift. The appearance of the Gunn-Peterson effect at high redshifts have two reasons:

(1) the factor of $(1+z)^{3/2}$ is ~ 20 times bigger at z ~ 6 than at z << 1.

(2) we expect the neutral fraction $f_{\mathrm{n}}(z)$ for hydrogen to be larger at higher redshift.

Cosmology - Epoch of Recombination

- ***Beginning of the Big Bang Nucelosynthesis***:
  - The temperature of the Cosmic Background Radiation drops as the universe expands:

$$T = T_0/a = T_0(1+z), \quad \text{where } T_0 = 2.7255 \pm 0.0006\,\mathrm{K}$$ (temperature in the present time)

$$\left[kT_0 = (2.3486 \pm 0.0005) \times 10^{-4}\,\mathrm{eV}\right]$$

When $kT \sim 66\,\mathrm{keV}$ $(a \sim 3.6 \times 10^{-9},\ z \sim 2.8 \times 10^{8})$, deuterium nuclei could form without being photodossociated. This was the starting of Big Bang Nucleosynthesis, which dirted the hydrogen in the universe with significant amounts of helium and lithium.

In this **radiation dominated epoch**, the hydrogen (helium, lithium) was highly ionized.

- ***The Epoch of Recombination***:
  - Eventually, ***the temperature*** T ***reached a level low enough that the fractional ionization of hydrogen dropped below*** $x = 1/2$; the time when this happened is known as ***the epoch of recombination***.
  - Now we will find when the epoch of recombination began by using the *photoionization equilibrium condition for the pure hydrogen gas*.

- The number density of ionizing photons in the Cosmic Background Radiation:

$$n_\nu(T) = \frac{8\pi}{c^3}\frac{\nu^2}{\exp(h\nu/kT)-1} \quad \rightarrow \quad n_\nu(T) \approx \frac{8\pi}{c^3}\nu^2\exp\left(-\frac{h\nu}{kT}\right) \quad \text{for } h\nu \geq I_\mathrm{H} \gg kT$$

$$(I_\mathrm{H} = 13.6\,\mathrm{eV})$$

- The ***photoionization rate of hydrogen*** is

$$\zeta_\mathrm{pi} = \int_{\nu_0}^{\infty} n_\nu(T) c\sigma_\mathrm{pi}(\nu) d\nu \quad (\nu_0 \equiv I_\mathrm{H}/h)$$

- We will use the following approximations for the photoionization cross-section and the number density of photons:

$$\sigma_\mathrm{pi}(\nu) \approx \sigma_0 \left(\nu/\nu_0\right)^{-3}$$

$$\sigma_0 = 6.304 \times 10^{-18}\,\mathrm{cm}^{-2}$$

$$n_\nu(T) \approx \frac{8\pi}{c^3}\nu^2 e^{-\frac{h\nu}{kT}} \quad \text{for } h\nu \geq I_\mathrm{H} \gg kT$$

$$kT_0(1+z) \ll I_\mathrm{H}$$

- When $kT \ll I_\mathrm{H}$, corresponding to redshifts $z \ll (I_\mathrm{H}/kT_0) - 1 \approx 58,000$, the photoionization rate is

$$\begin{aligned}
\zeta_\mathrm{pi} &\approx \frac{8\pi}{c^2}\sigma_0\nu_0^3 \int_{\nu_0}^{\infty} e^{-h\nu/kT}\frac{d\nu}{\nu} \\
&= \frac{8\pi}{c^2}\sigma_0\nu_0^3 e^{-I_\mathrm{H}/kT} \int_0^{\infty} e^{-x}\frac{dx}{x + I_\mathrm{H}/kT} \quad \leftarrow \quad x = h(\nu-\nu_0)/kT \\
&\approx \frac{8\pi}{c^2}\sigma_0\nu_0^3 \left(\frac{kT}{I_\mathrm{H}}\right) e^{-I_\mathrm{H}/kT}
\end{aligned}$$

$$\begin{aligned}
\int_0^{\infty} e^{-x}\frac{dx}{x+a} &\approx \int_0^{\infty} e^{-x}\left(1+x/a\right)^{-1}\frac{dx}{a} \\
&\approx \frac{1}{a} - \frac{1}{a^2} + \frac{2}{a^3}\cdots (\text{if } a \gg 1)
\end{aligned}$$

- Numerically, this is

$$\zeta_{\rm pi} \approx 4.61 \times 10^8\,{\rm s}^{-1} \left(\frac{kT}{1\,{\rm eV}}\right) e^{-13.6\,{\rm eV}/kT}$$

- The total number density of blackbody photons is given by

$$n_\gamma = \int_0^\infty n_\nu(T) d\nu = \frac{8\pi}{c^3}\int_0^\infty \frac{\nu^2}{\exp(h\nu/kT)-1} d\nu$$
$$= 8\pi \left(\frac{kT}{hc}\right)^3 \int_0^\infty \frac{x^2 dx}{e^x - 1}$$

$$\int_0^\infty \frac{x^2 dx}{e^x-1} = \int_0^\infty dx x^2 e^{-x}\left(1-e^{-x}\right)^{-1} = \int_0^\infty dx x^2 \sum_{n=1}^\infty e^{-nx}$$
$$= \sum_{n=1}^\infty \frac{1}{n^3}\int_0^\infty dy y^2 e^{-y} \quad \leftarrow \quad y = nx$$
$$= \zeta(3)\Gamma(3)$$

$\zeta(3) \approx 1.202$ and $\Gamma(3) = 2$

$$n_\gamma = 16\pi\zeta(3)\left(\frac{kT}{hc}\right)^3 \approx 3.17 \times 10^{13} \left(\frac{kT}{1\,{\rm eV}}\right)^3 \ [{\rm cm}^{-3}]$$

- Using this, we can write the photoionization rate in the form:

$$\zeta_{\rm pi} \approx \frac{\sigma_0 c n_\gamma}{2\zeta(3)}\left(\frac{I_{\rm H}}{kT}\right)^2 e^{-I_{\rm H}/kT}$$

- ***Photoionization equilibrium for pure hydrogen gas***:

$$\zeta_{\rm pi} n_{\rm H^0} = n_e n_p \alpha_{\rm B,H}$$
$$\zeta_{\rm pi}(1-x) n_{\rm H} = x^2 n_{\rm H}^2 \alpha_{\rm B,H} \quad \longleftarrow$$

fractional ionization
$x \equiv n_e/n_{\rm H},\ n_e = n_p$

$$1 - x = x^2 \frac{n_{\rm H}\alpha_{\rm B,H}}{\zeta_{\rm pi}}$$

- The condition for the epoch of recombination is then given by:

$$1 - x = x^2 \frac{n_{\rm H}\alpha_{\rm B,H}}{\zeta_{\rm pi}} \quad \text{and} \quad x = 2 \quad \rightarrow \quad \frac{n_{\rm H}\alpha_{\rm B,H}}{\zeta_{\rm pi}} = 2$$

- From the above equation for the photoionization rate, we can rewrite the condition as follows:

$$\left(\frac{kT}{I_{\rm H}}\right)^2 e^{I_{\rm H}/kT} = \frac{\sigma_0 c}{\zeta(3)\alpha_{\rm B,H}} \frac{n_\gamma}{n_{\rm H}}$$

- ***The baryon-to-photon ratio*** in the hydrogen-only universe:

$$n_\gamma \propto T^3$$
$$T = T_0(1+z)$$
$$n_{\rm H} = \overline{n}_{\rm bary} = \overline{n}_{\rm bary,0}(1+z)^3$$

$$\longrightarrow \quad \frac{n_{\rm bary}}{n_\gamma} = \frac{n_{\rm bary,0}}{n_{\gamma,0}} \approx 6.1\times 10^{-10}$$

$$n_{\gamma,0} \approx 411\,[{\rm cm}^{-3}]$$
$$n_{\rm bary,0} \approx 2.50\times 10^{-7}\,[{\rm cm}^{-3}]$$

The ratio remains constant with time, unless stars were born and messed things up by generating non-CMB photons into the universe.

- Recall that ***the recombination rate coefficient*** is given by:

$$\alpha_{\rm B,H}(T) \approx 2.59\times 10^{-13}\, T_4^{-0.833-0.034\ln T_4} \quad [{\rm cm^3\,s^{-1}}]$$

- The condition for ***the epoch of recombination*** can be written as

$$\left(\frac{kT}{I_{\rm H}}\right)^{1.17} e^{I_{\rm H}/kT} = 9.8 \times 10^{15}$$

The solution to this equation is $kT \approx 0.024 I_{\rm H} \approx 0.33\,{\rm eV}$. This corresponds to the following temperature, redshift and mean baryon density:

the epoch of recombination: $T \approx 3800\,{\rm K},\ z \approx 1400,\ \overline{n}_{\rm bary} \approx 690\,{\rm cm}^{-3}$

- ***Dark Ages***
  - The recombination of hydrogen brings in ***the "Dark Ages", which is the period between recombination at z ~ 1400 and the formation of the first stars at z ~ 30***.
  - The Cosmic Background Radiation will have a temperature rating from $T \sim 3800$ K at $z \sim 1400$ to $T \sim 80$ K at $z \sim 30$.
  - At the beginning of the Dark Ages, the temperature is about that of an M star. By the end of the Dark Ages, the photons of the Cosmic Background Radiation would be too low in energy to photo-ionized hydrogen atoms.
  - ***The reionization of intergalactic hydrogen must be accomplished by UV photons that comes from massive stars and from active galactic nuclei (AGN).***

Epoch of Reionization

- ***The Epoch of Reionization***
  - The reionization is a patchy process. The individual H II regions around early (first) stars and those around AGNs gradually merge to form a single expanse of ionized gas.
  - Thus, we cannot speak of an instant of deionization, but rather an epoch of reionization as the patches of ionized gas take over more and more of the universe.

- We have some observational evidence on the ionization state of the baryonic gas as a function of time during the epoch of reionization, as seen in the quasar spectra at z > 6.

Fan et al. (2006) found that Lyα optical depth as a function of redshift, from a sample of 19 quasars.

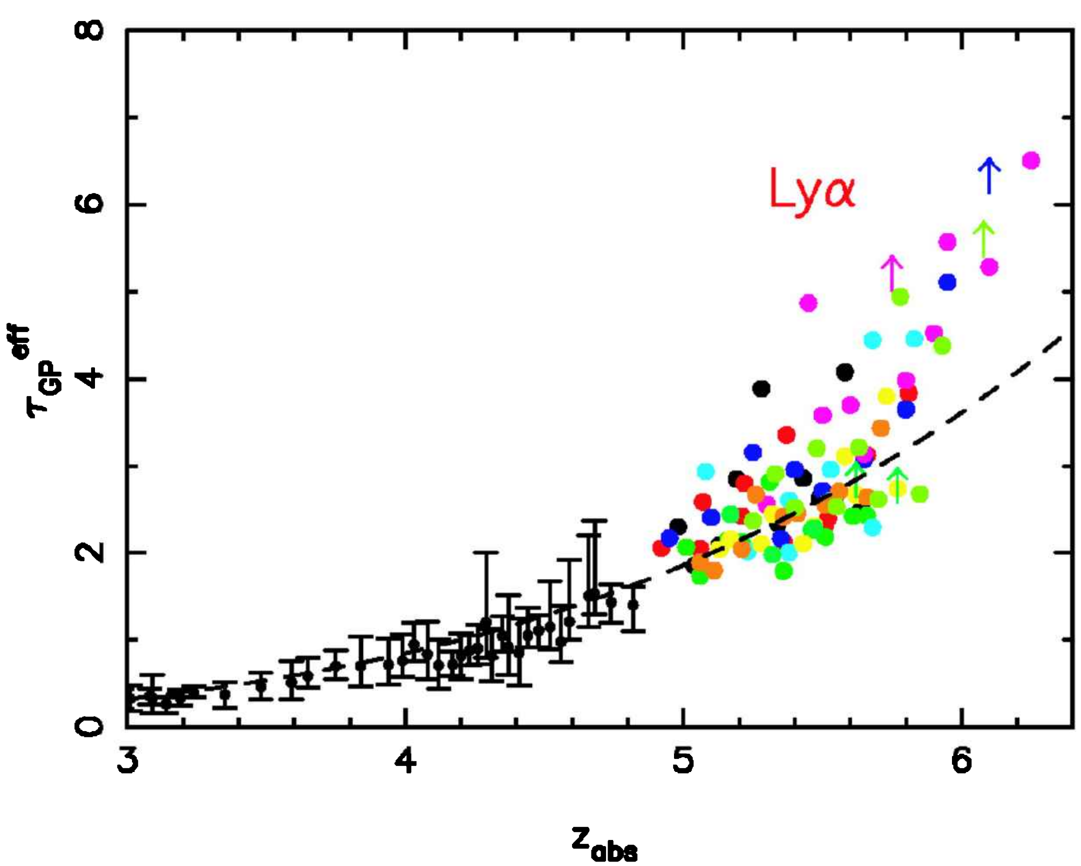
$$\tau_\nu \approx 2.6 \left(\frac{1+z}{6.5}\right)^{4.3} \quad \text{for } z < 5.5$$

We have previously obtained the following equation:

$$\tau_\nu^{\rm GP} \approx 25,600\, f_{\rm n}(z)\, (1+z)^{3/2}$$

Combining these two equations, we find ***the neutral fraction is small for z < 5.5***.

$$f_{\rm n}(z) \approx 6.2 \times 10^{-6} \left(\frac{1+z}{6.5}\right)^{2.8}$$



Lyα optical depth as a function of redshift.
[Fig 5, Fan et al. 2006, AJ, 132, 117]
[See also Fan et al. 2006, AR&AA]

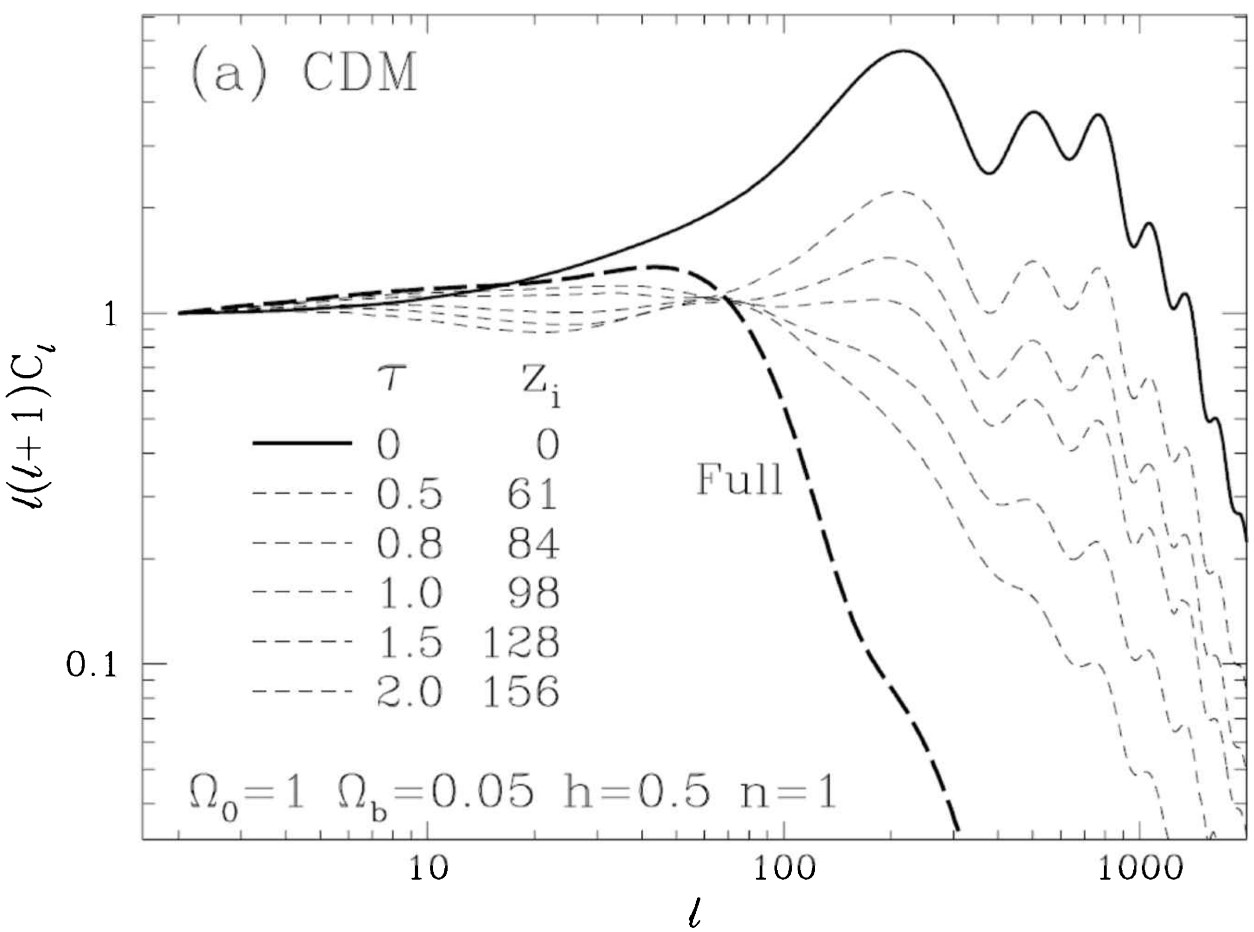
- Limitation of using Gunn-Peterson effect for determining the neutral fraction:
    - Quasars take us back only to z ~ 7. Thus, the Gunn-Peterson effect can tell us solely about the very late stages of reionization.
    - The optical depth higher than $\tau \sim 5$ corresponding to a transmitted fraction $e^{-\tau} \sim 0.007$, which is hard to distinguish from zero in a noisy spectrum.
    - To observe the earlier process of reionization, when the neutral fraction was still close to unity, we have to take a different approach, i.e., the CMB.

- ***The Epoch of Reionization, as probed by the CMB***.
    - The ***Cosmic Microwave Background*** contains information about the epoch of reionization.
    - The reionized gas of the IGM at low redshift provides free electrons between us and the CMB.
        - These free electrons scatter the photons of the CMB via Thomson scattering with cross-section:

$$\sigma_e = 6.652 \times 10^{-25}\,\mathrm{cm}^2$$

        - If the optical depth from Thomson scattering were $\tau_e \gg 1$, then the temperature fluctuations of the CMB would be thoroughly smeared out.

- ***The actual CMB spectrum shows only a modest suppression of the power spectrum of temperature fluctuations on small angular scales*** due to scattering from free electrons in the recognized IGM. Therefore, we expect that the free electrons in the recognized gas provide $\tau_e \ll 1$. ***The Planck results give*** $\tau_e = 0.066 \pm 0.016$.



Effect of reionization on the CMB power spectrum.
Solid line = CMB spectrum without Thomson scattering by reionized gas.
Dashed lines = CMB spectrum with different optical depths from Thomson scattering

[Hu 1995; Fig 9.5 Ryden]

- ***Thomson scattering***

  - The optical depth for Thomson scattering can be written as

$$\tau_e = c\int_{t_*}^{t_0} n_e(t)\sigma_e dt = c\sigma_e \int_{a(t_*)}^{1} n_e(a)\frac{da}{aH} = c\sigma_e \int_0^{z_*} n_e(z)\frac{dz}{H(z)(1+z)}$$

Here, t_* = time at which the reionization is completed.

- For a universe made of pure hydrogen, which undergoes complete reionization at the time t_*, the number density of free electrons can be expressed in terms of the Baryon density:

$$n_e = n_{\rm H} = \overline{n}_{\rm bary} = \overline{n}_{\rm bary,0}(1+z)^3$$

- Then after the complete reionization $(t > t_*)$, the optical depth for Thomson scattering is:

$$\boxed{\tau_e = c\sigma_e\overline{n}_{\rm bary,0}\int_0^{z_e}\frac{(1+z)^2 dz}{H(z)}} \approx 4.97\times 10^{-21}\,{\rm s}^{-1}\int_0^{z_*}\frac{(1+z)^2 dz}{H(z)}$$

$$H(z) = H_0\left[\Omega_{\rm m,0}(1+z)^3 + \Omega_{\Lambda,0}\right]^{1/2} \longrightarrow \tau_e = \frac{2}{3}\frac{c\sigma_e\overline{n}_{\rm bary,0}}{H_0\Omega_{\rm m,0}}\left\{\left[\Omega_{\rm m,0}(1+z_*)^3 + \Omega_{\Lambda,0}\right]^{1/2} - \left[\Omega_{\rm m,0}+\Omega_{\Lambda,0}\right]^{1/2}\right\}$$

$$\approx \frac{2}{3}\frac{c\sigma_e\overline{n}_{\rm bary,0}}{H_0\Omega_{\rm m,0}}\left\{\left[\Omega_{\rm m,0}(1+z_*)^3 + \Omega_{\Lambda,0}\right]^{1/2} - 1\right\}$$

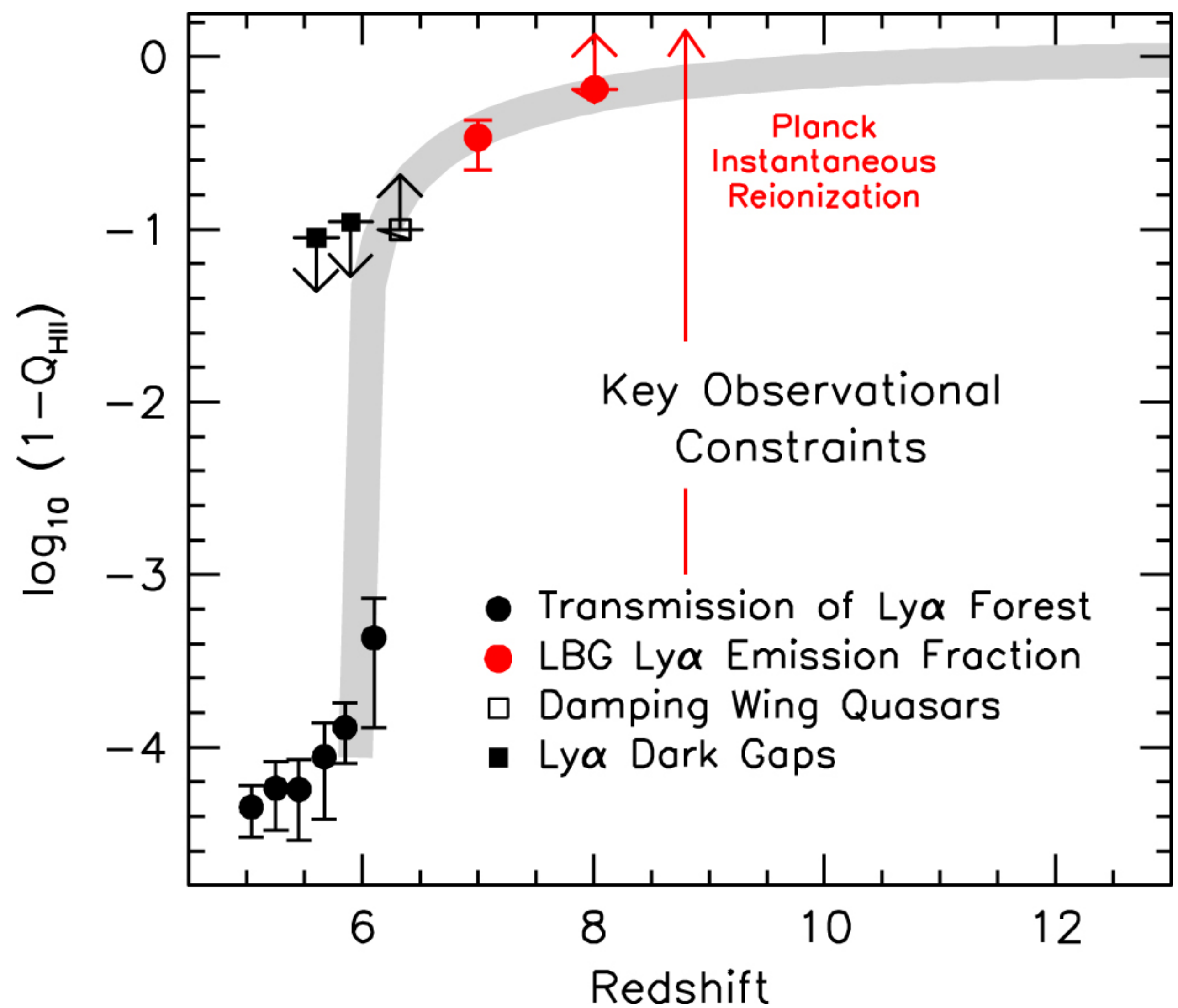
$$\boxed{\tau_e \approx 0.00486\left\{\left[0.31(1+z_*)^3 + 0.69\right]^{1/2} - 1\right\}}$$

- From the observed value of the optical depth, this gives a redshift of reionization:

$$\boxed{\tau_e \approx 0.066 \quad \rightarrow \quad z_* = 7.8}$$

Note that we assumed the pure-hydrogen universe and an instantaneous reionization, in deriving this result.

- In the figure, the filling factor of ionized hydrogen is denoted by Q_{HII}.
- The latest results (Planck Collaboration et al., 2015) places reionization at $z \sim 8.8$, assuming a model in which the universe is instantly reionized.
  - Studies of the cosmic microwave background (CMB) tell us of the column density of ionized material in front of the last scattering surface.
  - Thomson scattering of CMB photons upon free electrons causes the signal to become partially linearly polarized, allowing us to calculate a Thomson optical depth which in turn can be used to estimate when reionization took place.
  - The red arrow shows the instantaneous reionization redshift from Planck Collaboration et al. (2015).
- The gray shaded region schematically follows the evolution in the filling factor.



Summary of constraints on the redshift at which reionization took place. (Bouwens et al. 2015).

The points include Gunn-Peterson and Lyα dark gaps from Fan et al. (2006) and McGreer et al. (2015), quasar damping wings from Schroeder et al. (2013), and Lyα galaxies from Schenker et al. (2014).

Reionization Sources

- We need to find objects that were produce large quantities of ionizing photons with $h\nu > 13.6$ eV at a redshift z ~ 10.
  - Suppose that a single source of ionizing photons (either an AGN or a star-forming galaxy) is turned on. Then, a Strömgren sphere will form, with a radius:

$$R(t) = R_s \left(1 - e^{-t/t_{\rm rec}}\right)^{1/3}$$

    - The recombination time $t_{\rm rec}$ is given by

$$t_{\rm rec} \equiv \frac{1}{\alpha_{\rm B,H} n_{\rm H}} = \frac{1}{\alpha_{\rm H} \overline{n}_{\rm bary,0}} (1+z)^{-3} \approx 0.5\,{\rm Gyr} \left(\frac{1+z}{11}\right)^{-3}$$

$\alpha_{\rm B,H} \approx 2.59 \times 10^{-13}\,[{\rm cm^3\,s^{-1}}]$

    - The Strömgren radius for the AGN or a star-forming galaxy is

$$R_s \equiv \left(\frac{3Q_0}{4\pi \alpha_{\rm B,H} n_{\rm H}^2}\right)^{1/3} \approx 0.7\,{\rm Mpc} \left(\frac{Q_0}{10^{54}\,{\rm s^{-1}}}\right)^{1/3} \left(\frac{1+z}{11}\right)^{-2}$$

  - In order for the Strömgren spheres to overlap and fill the universe with ionized hydrogen, ***the number density of photo-ionizing sources*** must be

$$\frac{4\pi R_s^3}{3} n_{\rm source} > 1 \longrightarrow \boxed{n_{\rm source} > \frac{3}{4\pi R_s^3} \approx 0.6\,{\rm Mpc}^{-3} \left(\frac{Q_0}{10^{54}\,{\rm s^{-1}}}\right)^{-1} \left(\frac{1+z}{11}\right)^{6}}$$

- The above equation gives the required number density in physical units; the number density in coming length units, normalized to the present, is

$$n_{\rm source}\,(1+z)^{-3} = 5\times10^{-4}\,{\rm Mpc}^{-3}\left(\frac{Q_0}{10^{54}\,{\rm s}^{-1}}\right)^{-1}\left(\frac{1+z}{11}\right)^{3}$$

- Thus, the rate per comoving volume at which ionizing photons are produced must be greater than the following critical value:

$$\mathcal{F}_{\rm crit} \equiv Q_0 n_{\rm source}\,(1+z)^{-3} = 5\times10^{50}\,{\rm Mpc}^{-3}\,{\rm s}^{-1}\left(\frac{1+z}{11}\right)^{3}$$

  This is the equivalent of a dozen O3 main sequence stars per cubic Mpc, which is not a lot.

- Now examine the OB stars as a candidate for the cosmic reionization. We should notice that the lifetime of an O3 star is only ~ 1 Myr, which is much shorter than the recombination time $t_{\rm rec}\sim 500\,{\rm Myr}((1+z)/11)^{-3}$. Thus, we need continuous star-formation.
  - Using the starburst99 code (Leitherer et al. 1999; https://www.stsci.edu/science/starburst99/docs/default.htm), we obtain ***the production rate of ionizing photons, per a unit star-formation rate, for the continuous star-formation***:

$$Q_* = 10^{53.148}\,[{\rm s}^{-1}]\frac{\rm SFR}{1\,M_\odot\,{\rm yr}^{-1}}$$

adopting the Initial Mass Function of Kroupa (2001), for continuous star-formation

$\xi(m)\Delta m = m^{-\alpha}\Delta m$
$\alpha = 0.3$ for $m < 0.08M_\odot$
$\alpha = 1.3$ for $0.08M_\odot < m < 0.5M_\odot$
$\alpha = 2.3$ for $m > 0.5M_\odot$

  - Only a fraction $f_{\rm esc}$ of the photoionizing photons will escape the galaxy and enter the IGM.

- ***Comoving star-formation rate, required to keep the universe ionized*** is then:

$$Q_* f_{\rm esc} \geq \mathcal{F}_{\rm crit} \quad \longrightarrow \quad \boxed{\mathrm{SFR} \gtrsim 0.004\, M_\odot\, \mathrm{yr}^{-1}\, \mathrm{Mpc}^{-3} \left(\frac{1+z}{11}\right)^3 f_{\rm esc}^{-1}}$$

Here, $f_{\rm esc}$ is the escape fraction of ionizing photons (escape out of the host galaxy and enter the IGM).

The escape fraction is a critical parameter to identify the main source of the cosmic reionization. ***However, the escape fraction of ionizing photon is poorly known at all redshifts.***

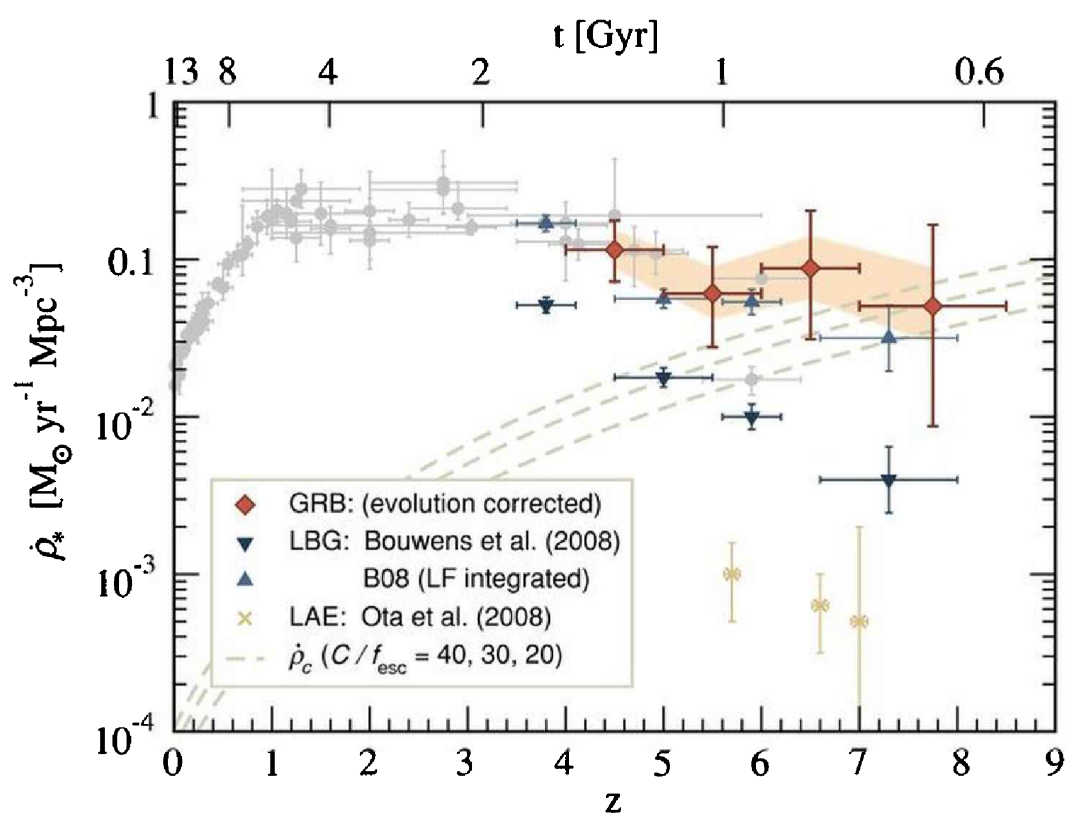
- **Observation of the cosmic star-formation rate**:

Galaxies

- The cosmic star-formation rate has a fairly broad peak in the redshift range $1 < z < 4$.
- The comoving SF rate at $z \sim 10$ was lower than that at $z \sim 2$, but how much lower is not clear.
- The earliest generation of stars (Pop III stars) would definitely play a significant role in ionizing the universe. But, they are still poorly understood.
- Galaxies are likely to be the dominant source of the cosmic reionization.

AGNs

- The observed comoving density of bright quasars has a narrower peak in the redshift range $2 < z < 3$.
- It is known that the number of AGNs wasn't enough to recognize the IGM at $z \sim 10$.
- However, the faintest AGNs may have been numerous enough to contribute significantly to the reionization. (This scenario is highly unlikely. But, it cannot be completely ruled out.)



The cosmic star-formation rate using different traces (GRB = Gamma-Ray Bursts; LBG = Luminous Blue Galaxies; LAE = Lyman-Alpha Emitters) [Kistler et al. 2009; Fig 9.6 Ryden]