

# Interstellar Medium (ISM)

Week 9

2025 May 09 (Friday), 3PM

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# Density Determination

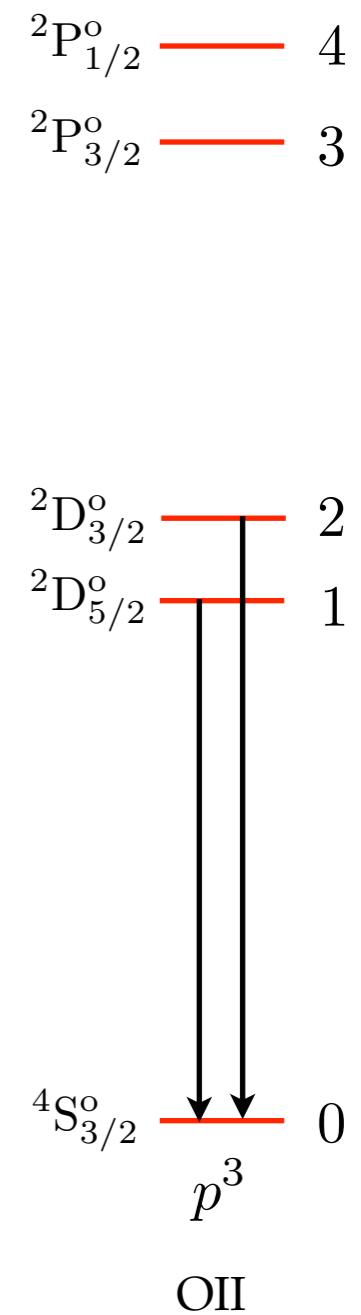
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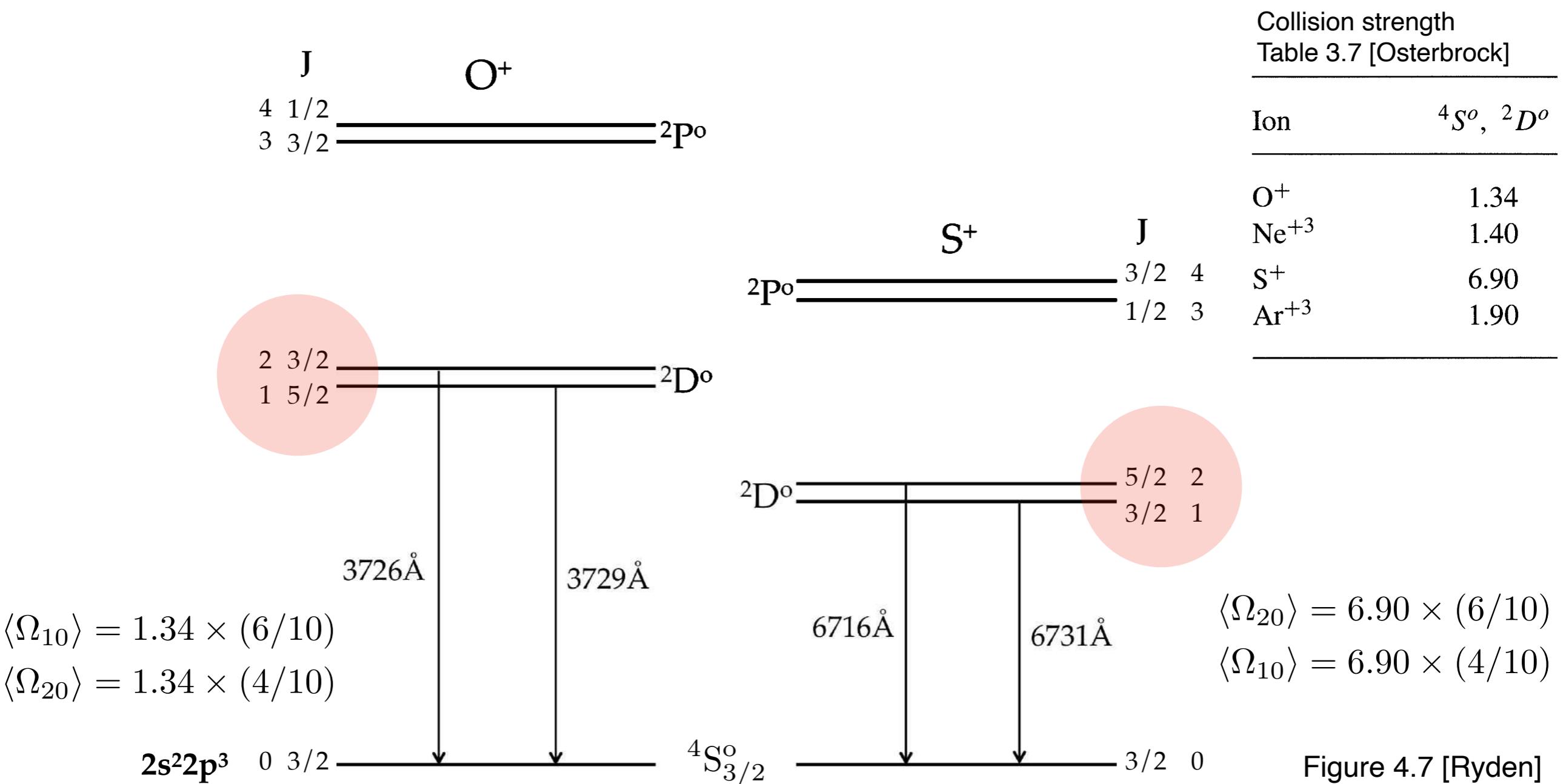
- Emission lines can also be used to estimate the free electron density of an ionized nebula. For this purpose, we need an ion which has ***two excited levels that are similar in energy, but which have different critical densities***. One of such systems is singly-ionized oxygen.

- Ions with 7 or 15 electrons have  $2s^22p^3$  and  $3s^23p^3$  configurations, with energy level structures that make them suitable for use as density diagnostics.

Density-sensitive nebular lines (Å).

$p^3$ Ions	[O II]	[S II]	[Ne IV]	[Ar IV]
$^2D_{3/2} \rightarrow ^4S_{3/2}$	3726	6731	2423	4740
$^2D_{5/2} \rightarrow ^4S_{3/2}$	3729	6716	2426	4711
	$2s^22p^2$	$3s^23p^2$	$2s^22p^2$	$3s^23p^2$
	Z = 8	Z = 16	Z = 10	Z = 18





Notice that energy ordering of the fine-structure levels are different between  $O^+$  and  $S^+$ . The  $p^3$  configuration for the two ions are half-filled, and thus **Hund's rule for the energy ordering is not applicable**.

[There are three typos in J values for  $S^+$  and in the notation for the lowest level in Figure 4.7 of Ryden.]

- Here, we will ignore the transition between 2 and 1 because the transition is very slow.
- $1 \rightarrow 0$  transition:
  - The emissivity of the  $1 \rightarrow 0$  transition, integrated over the entire line width, is

$$4\pi j(1 \rightarrow 0) = n_1 A_{10} h\nu_{10}$$

- In statistical equilibrium, the rate of collisional excitation from the ground state will be balanced by radiative and collisional de-excitation:

$$n_e n_0 k_{01} = n_1 (A_{10} + n_e k_{10})$$

Then,

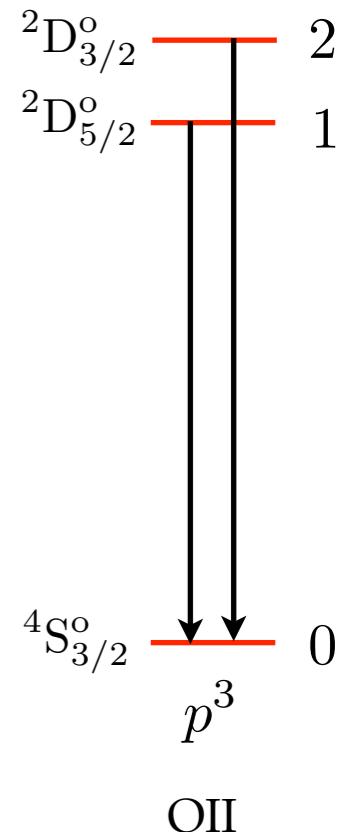
$$\begin{aligned} 4\pi j(1 \rightarrow 0) &= n_e n_0 \frac{k_{01}}{A_{10} + n_e k_{10}} A_{10} h\nu_{10} \\ &= n_e n_0 \frac{k_{01}}{1 + n_e / n_{\text{crit},1}} h\nu_{10} \end{aligned}$$

where  $n_{\text{crit},1} \equiv A_{10}/k_{10}$

- $2 \rightarrow 0$  transition:

- Similarly, we obtain

$$4\pi j(2 \rightarrow 0) = n_e n_0 \frac{k_{02}}{1 + n_e / n_{\text{crit},2}} h\nu_{20} \quad \text{where } n_{\text{crit},2} \equiv A_{20}/k_{20}$$



- The ratio of the strength of the two lines in the doublet is

$$\begin{aligned}\frac{j(2 \rightarrow 0)}{j(1 \rightarrow 0)} &= \frac{\nu_{20}}{\nu_{10}} \frac{k_{02}}{k_{01}} \frac{1 + n_e/n_{\text{crit},1}}{1 + n_e/n_{\text{crit},2}} \\ &= \frac{\nu_{20}}{\nu_{10}} \frac{\langle \Omega_{20} \rangle}{\langle \Omega_{10} \rangle} e^{-h\nu_{21}/kT} \frac{1 + n_e/n_{\text{crit},1}}{1 + n_e/n_{\text{crit},2}}\end{aligned}$$

$$\begin{aligned}k_{0u} &= \left( \frac{\beta}{T^{1/2}} \frac{1}{g_0} \right) \langle \Omega_{u0} \rangle e^{-h\nu_{u0}/kT} \\ \beta &= 8.62942 \times 10^{-6}\end{aligned}$$

- Thus, we can write the line ratio as

$$\frac{j(2 \rightarrow 0)}{j(1 \rightarrow 0)} \simeq \frac{\langle \Omega_{20} \rangle}{\langle \Omega_{10} \rangle} \frac{1 + n_e/n_{\text{crit},1}}{1 + n_e/n_{\text{crit},2}}$$

$h\nu_{20} \simeq h\nu_{10}$  Levels 1 and 2 are so close in energy.

$h\nu_{21} \equiv h\nu_{20} - h\nu_{10} \ll kT$

$h\nu_{21} \approx 2 \text{ meV}$  for O II ions

- In the low-density limit ( $n_e \ll n_{\text{crit},1}, n_{\text{crit},2}$ ),

$$\frac{j(2 \rightarrow 0)}{j(1 \rightarrow 0)} \simeq \frac{\langle \Omega_{20} \rangle}{\langle \Omega_{10} \rangle} = \frac{g_2}{g_1}$$

$$\Omega_{(\text{SLJ}, \text{ S'L'J'})} = \frac{(2J' + 1)}{(2S' + 1)(2L' + 1)} \Omega_{(\text{SL}, \text{ S'L'})}$$

Recall the sum rule for the collision strength for the fine-structure transitions.

[However, it is not clear that the sum rule is valid even beyond the LS-coupling scheme. Recent QM calculations show that the proportionality relation is only an approximation.]

- In high-density limit ( $n_e \gg n_{\text{crit},2}, n_{\text{crit},1}$ ),

$$\frac{j(2 \rightarrow 0)}{j(1 \rightarrow 0)} \simeq \frac{\langle \Omega_{20} \rangle}{\langle \Omega_{10} \rangle} \frac{n_{\text{crit},2}}{n_{\text{crit},1}} = \frac{\langle \Omega_{20} \rangle}{\langle \Omega_{10} \rangle} \frac{A_{20}/k_{20}}{A_{10}/k_{10}} = \frac{g_2}{g_1} \frac{A_{20}}{A_{10}}$$

$$k_{u0} = \frac{\beta}{T^{1/2}} \frac{\langle \Omega_{u0} \rangle}{g_u}$$

► For O II ion,

$$n_e \ll n_{\text{crit}} \rightarrow \frac{j(1 \rightarrow 0)}{j(2 \rightarrow 0)} \simeq \frac{g_1}{g_2}$$

$$n_e \gg n_{\text{crit}} \rightarrow \frac{j(1 \rightarrow 0)}{j(2 \rightarrow 0)} \simeq \frac{g_1}{g_2} \frac{A_{10}}{A_{20}}$$

$$\frac{j([\text{O II}] 3728.8)}{j([\text{O II}] 3726.1)} = 1.5$$

$$\frac{j([\text{O II}] 3728.8)}{j([\text{O II}] 3726.1)} = 0.3$$

$$g_1 = 6, \quad A_{10} = 3.59 \times 10^{-5} \text{ s}^{-1}$$

$$g_2 = 4, \quad A_{20} = 1.79 \times 10^{-4} \text{ s}^{-1}$$

$$\frac{j(3729)}{j(3726)} = 1.5 \frac{1 + (n_e / 1.55 \times 10^4 \text{ cm}^{-3}) T_4^{-1/2}}{1 + (n_e / 3.11 \times 10^3 \text{ cm}^{-3}) T_4^{-1/2}}$$

$$\langle \Omega_{10} \rangle = 1.34 \times (6/10)$$

$$\langle \Omega_{20} \rangle = 1.34 \times (4/10)$$

► For S II ion,

$$n_e \ll n_{\text{crit}} \rightarrow \frac{j(2 \rightarrow 0)}{j(1 \rightarrow 0)} \simeq \frac{g_2}{g_1}$$

$$n_e \gg n_{\text{crit}} \rightarrow \frac{j(2 \rightarrow 0)}{j(1 \rightarrow 0)} \simeq \frac{g_2}{g_1} \frac{A_{20}}{A_{10}}$$

$$\frac{j([\text{S II}] 6716)}{j([\text{S II}] 6731)} = 1.5$$

$$\frac{j([\text{S II}] 6716)}{j([\text{S II}] 6731)} = 0.44$$

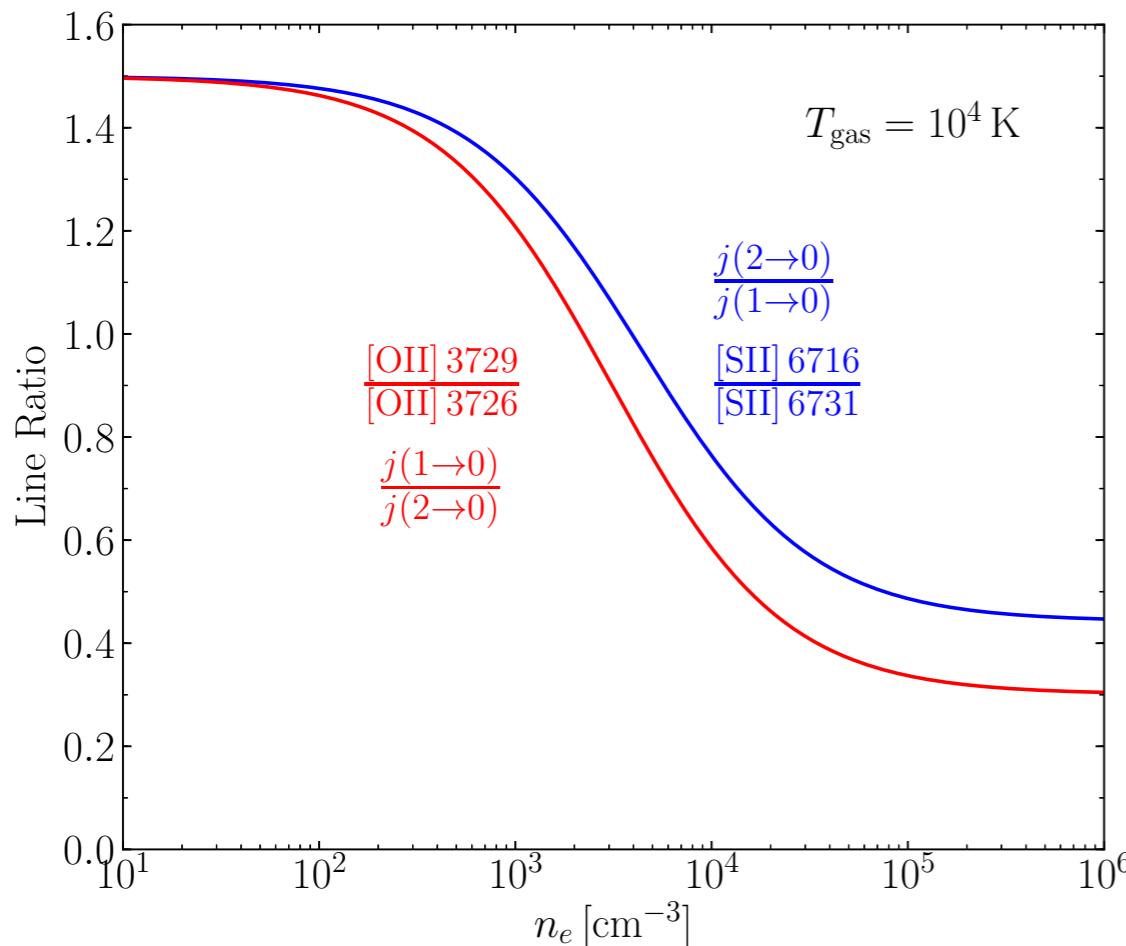
$$g_2 = 6, \quad A_{20} = 2.60 \times 10^{-4} \text{ s}^{-1}$$

$$g_1 = 4, \quad A_{10} = 8.82 \times 10^{-4} \text{ s}^{-1}$$

$$\frac{j(6716)}{j(6731)} = 1.5 \frac{1 + (n_e / 1.48 \times 10^4 \text{ cm}^{-3}) T_4^{-1/2}}{1 + (n_e / 4.37 \times 10^3 \text{ cm}^{-3}) T_4^{-1/2}}$$

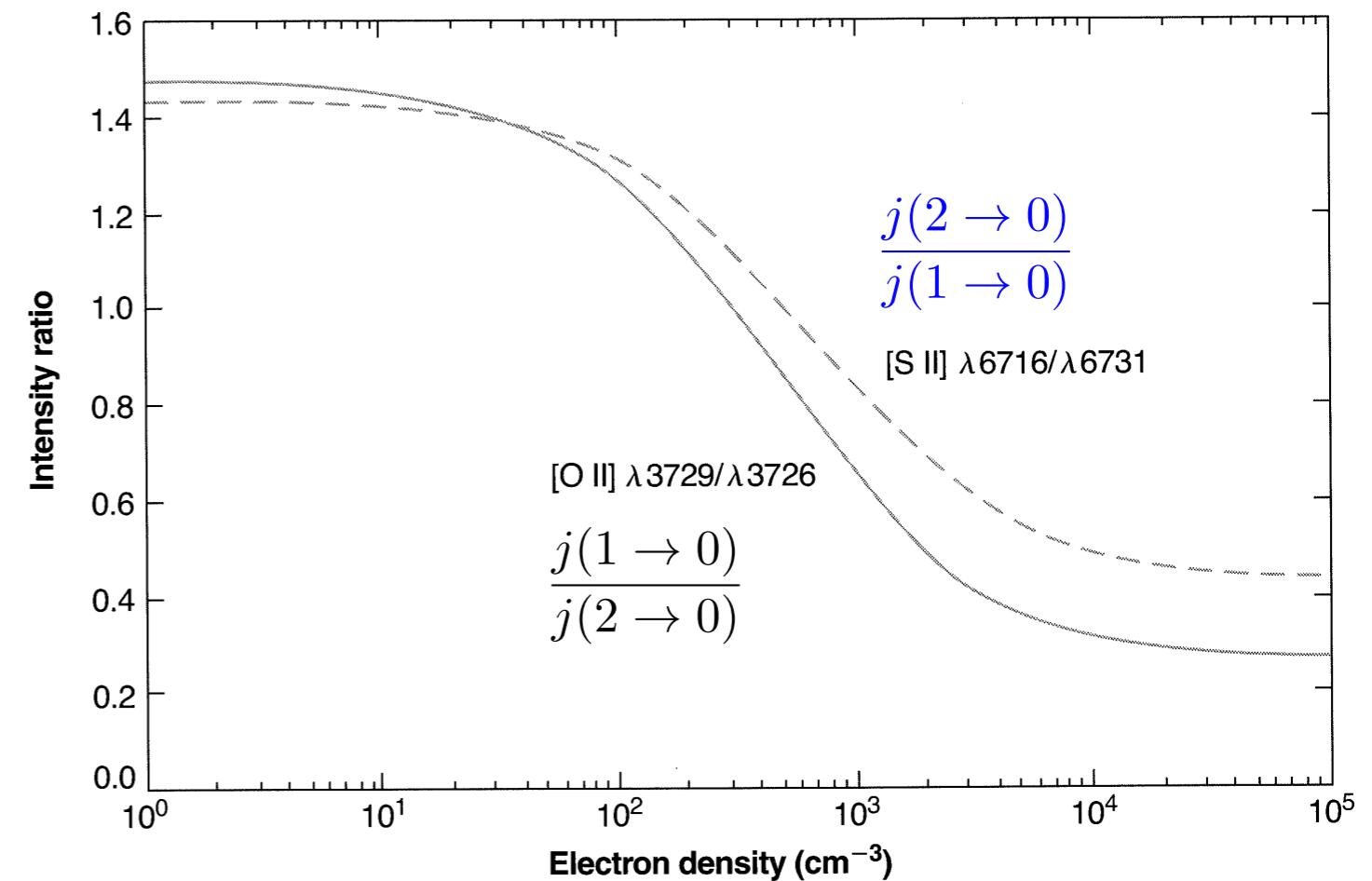
$$\langle \Omega_{20} \rangle = 6.90 \times (6/10)$$

$$\langle \Omega_{10} \rangle = 6.90 \times (4/10)$$



Obtained using the approximate equations in this lecture note.

**Notice differences between two figures.**



The full solution of the equilibrium equations, which also takes into account all transitions, including excitation to the  ${}^2\text{P}^o$  levels with subsequent cascading downward.

Figure 5.8 [Osterbrock]

# Abundance Determination

- Helium abundance
  - The abundance of He is determined from **comparison of the strengths of radiative recombination lines of H and He** in regions ionized by stars that are sufficiently hot ( $T_{\text{eff}} \gtrsim 3.9 \times 10^4 \text{ K}$ ) so that He is ionized throughout the H II regions.
- Heavy elements
  - The abundance of heavy elements can be inferred by **comparing the strengths of collisionally excited lines with recombination lines of H**.

**Oxygen:**  $(\lambda_{\text{H}\beta} = 4861.35 \text{ \AA})$

$$4\pi j([\text{OIII}] 5008) = n_e n(\text{O}^{+2}) k_{03} \frac{A_{32}}{A_{31} + A_{32}} E_{32}$$

$$4\pi j(\text{H}\beta) = n_e n(\text{H}^+) \alpha_{\text{eff}, \text{H}\beta} E_{\text{H}\beta}$$

where

$$\alpha_{\text{eff}, \text{H}\beta} \approx 3.03 \times 10^{-14} T_4^{-0.874 - 0.058 \ln T_4} \text{ cm}^3 \text{ s}^{-1}$$

$$k_{03} = 8.62942 \times 10^{-8} T_4^{-1/2} \frac{\Omega_{30}}{g_0} e^{-E_{30}/kT} \text{ cm}^3 \text{ s}^{-1} \quad (g_0 = 1)$$

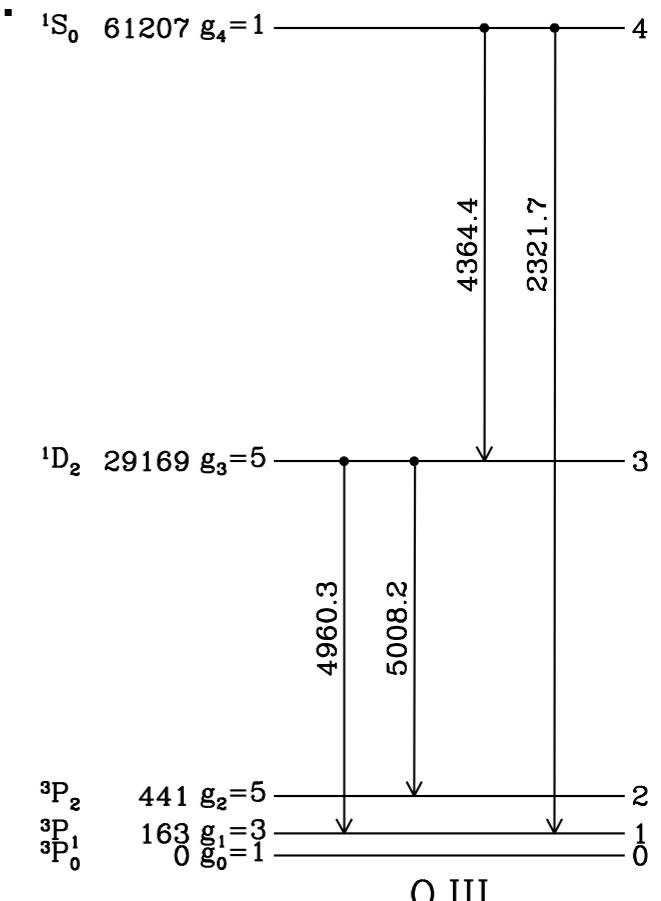
$$E_{32}/k = 29169 \text{ K}, \quad E_{\text{H}\beta}/k = 29588.5 \text{ K}$$

$$\Omega_{30} = 0.243 T_4^{0.120 + 0.031 \ln T_4}$$

$$A_{32} = 2.0 \times 10^{-2} \text{ [s}^{-1}]$$

$$A_{31} = 6.8 \times 10^{-3} \text{ [s}^{-1}]$$

$$\frac{[\text{OIII}] 5008}{\text{H}\beta} = 5.091 \times 10^5 T_4^{0.494 + 0.089 \ln T_4} e^{-2.917/T_4} \frac{n(\text{O}^{+2})}{n(\text{H}^+)}$$



- **Nitrogen:** ( $\lambda_{\text{H}\alpha} = 6562.79 \text{\AA}$ )

$$4\pi j(\text{[NII]} 6585) = n_e n(\text{N}^+) k_{03} \frac{A_{32}}{A_{31} + A_{32}} E_{32}$$

$$4\pi j(\text{H}\alpha) = n_e n(\text{H}^+) \alpha_{\text{eff}, \text{H}\alpha} E_{\text{H}\alpha}$$

where

$$\alpha_{\text{eff}, \text{H}\alpha} \approx 1.17 \times 10^{-13} T_4^{-0.942 - 0.031 \ln T_4} \text{ cm}^3 \text{ s}^{-1}$$

$$k_{03} = 8.62942 \times 10^{-8} T_4^{-1/2} \frac{\Omega_{30}}{g_0} e^{-E_{30}/kT} \text{ cm}^3 \text{ s}^{-1} \quad (g_0 = 1)$$

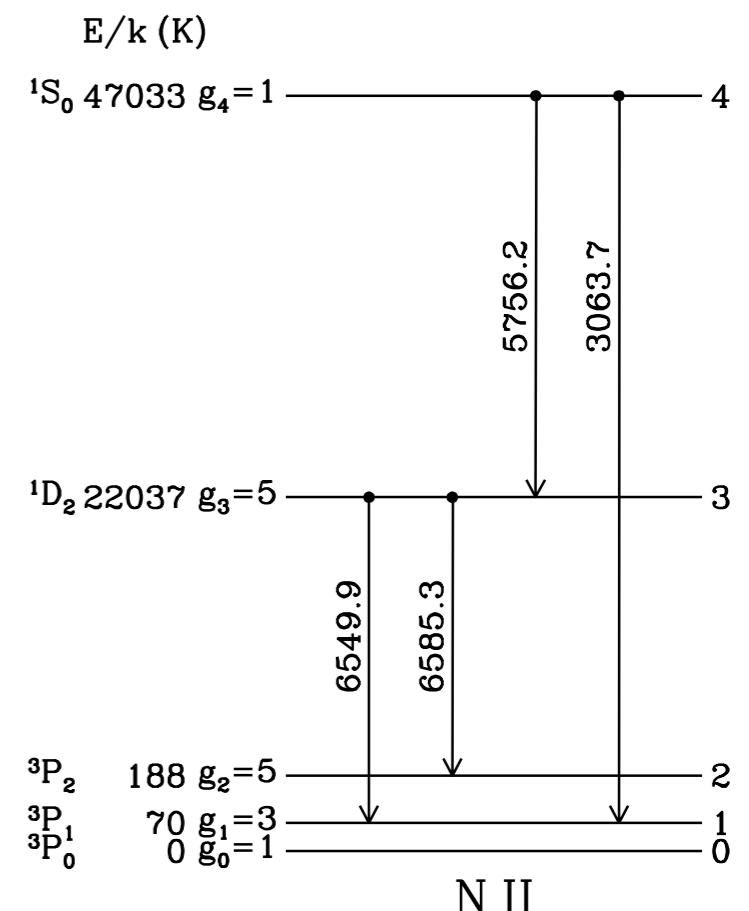
$$E_{32}/k = 21849 \text{ K}, \quad E_{\text{H}\alpha}/k = 21916.9 \text{ K}$$

$$\Omega_{30} = 0.303 T_4^{0.053 + 0.009 \ln T_4}$$

$$A_{32} = 3.0 \times 10^{-3} \text{ [s}^{-1}\text{]}$$

$$A_{31} = 9.8 \times 10^{-4} \text{ [s}^{-1}\text{]}$$

$$\frac{[\text{NII}]\ 6585}{\text{H}\alpha} = 1.679 \times 10^5 T_4^{0.495 + 0.040 \ln T_4} e^{-2.185/T_4} \frac{n(\text{N}^+)}{n(\text{H}^+)}$$



- Therefore, if the temperature  $T$  is known, the relative abundance of the ion can be obtained from the measured line ratio.
- The total elemental abundances are then obtained by applying **ionization correction factors (ICFs)**, which correct for abundances of unobserved ions.

# General Multilevel Atom

- It is easy to generalize the equations of statistical equilibrium up to an arbitrary number of levels.
  - In statistical equilibrium, the rate of collisional and radiative population of any level  $j$  is matched by the collisional and radiative depopulation rates of that same level.
  - When combined with the population normalization equation (the sum of the populations of all levels must add up to the total number of ions), we have a linear set of simultaneous equations which may be solved in the standard way.

$$\sum_{i \neq j}^M n_i n_e k_{ij} + \sum_{i=j+1}^M n_i A_{ij} - n_j \left( \sum_{i \neq j}^M n_e k_{ji} + \sum_{i=1}^{j-1} A_{ji} \right) = 0 \quad j = 1, 2, 3, \dots, M$$

$$\sum_{j=0}^M n_j = 1 \quad (\text{normalization}) \quad M + 1 \equiv \text{number of levels}$$

Useful softwares to calculate line ratios.

(1) PopRatio: <http://www.ignacioalex.com/popratio/>  
 Silva & Viegas (2001, Computer Physics Communications, 136, 319)

(2) PyNeb: <http://research.iac.es/proyecto/PyNeb/>; [https://github.com/Morisset/PyNeb\\_devel](https://github.com/Morisset/PyNeb_devel)  
 Luridiana, Morisset, & Shaw (2015, A&A, 573, A42)

# Ionization / Excitation Diagnostics: The BPT diagram

- Ionization / Excitation Mechanisms in galaxies
  - The optical line emission from star-forming galaxies is usually dominated by emission lines from H II regions.
  - Some galaxies have strong continuum and line emission from an active galactic nucleus (AGN). The line emission is thought to come from gas that is heated and ionized by X-rays from the AGN.
    - ▶ **Seyfert galaxies:** The AGN spectrum normally includes strong emission lines from high-ionization species like C IV and Ne V, which are presumed to be ionized by X-rays from the AGN. Seyfert (1943) discovered that some galaxies had extremely luminous, point-like nuclei, with emission line widths in some cases exceeding 4000 km/s.
    - ▶ **LINERs** (Low Ionization Nuclear Emission Region): In other cases, the nucleus has strong emission lines but primarily from low-ionization species. LINERs were first identified by Timothy Heckman (1980). [There are debates on the sources of ionization and line emission; AGN or star-forming regions, shock or photoionization]
- **BPT diagram:**
  - Baldwin, Phillips & Terlevich (1981) found that one could distinguish star-forming galaxies from galaxies with spectra dominated by AGNs by plotting the ratio of **[OIII]5008/H $\beta$  vs. [NII]6585/H $\alpha$ .**
  - These lines are among the strongest optical emission lines from H II regions.
  - The line ratios employ pairs of lines with similar wavelengths so that the line ratios are nearly unaffected by dust extinction.

- 
- Recall the structure of H and He ionization zone:
    - In H II regions where He is neutral (no photons with  $E > 24.6 \text{ eV}$ ), N and O will be essentially 100% singly ionized throughout the H ionization zone.
    - On the other hand, for O stars that are hot enough ( $24.6 < E < 54.6 \text{ eV}$ ) to have an appreciable zone of He ionization, the N and O in this zone can be doubly ionized.
    - Because **N and O have similar second ionization potentials** (29.6 and 35.1 eV for N and O, respectively), to a good approximation, HII regions will have:

$$\text{N}^+/\text{N} \approx \text{O}^+/\text{O} \quad \text{and} \quad \text{N}^{+2}/\text{N} \approx \text{O}^{2+}/\text{O}$$

- Essentially all of the gas-phase O and N in the H II region will be either singly or doubly ionized. Hydrogen will fully ionized in the H II region:

$$n(\text{N}) = n(\text{N}^+) + n(\text{N}^{+2}), \quad n(\text{O}) = n(\text{O}^+) + n(\text{O}^{+2}), \quad \text{and} \quad n(\text{H}) = n(\text{H}^+)$$

- Let's define the fraction of doubly-ionized ion.

$$\xi_{\text{N}} \equiv \frac{n(\text{N}^{+2})}{n(\text{N}^+) + n(\text{N}^{+2})} = \frac{n(\text{N}^{+2})}{n(\text{N})}$$

$$\xi_{\text{O}} \equiv \frac{n(\text{O}^{+2})}{n(\text{O}^+) + n(\text{O}^{+2})} = \frac{n(\text{O}^{+2})}{n(\text{O})}$$

Then, the fractions are approximately equal:

$$\xi_{\text{N}} \approx \xi_{\text{O}} \implies \xi$$

- We assume the N abundance to be solar, and the O abundance to be 80% solar (20% is presumed to be in silicate grains).
- In terms of the fraction,

$$\frac{n(\text{N}^+)}{n(\text{H}^+)} = (1 - \xi) \frac{n(\text{N})}{n(\text{H})} \quad \frac{n(\text{O}^{2+})}{n(\text{H}^+)} = \xi \frac{n(\text{O})}{n(\text{H})}$$

- Then, the line ratios can be written:

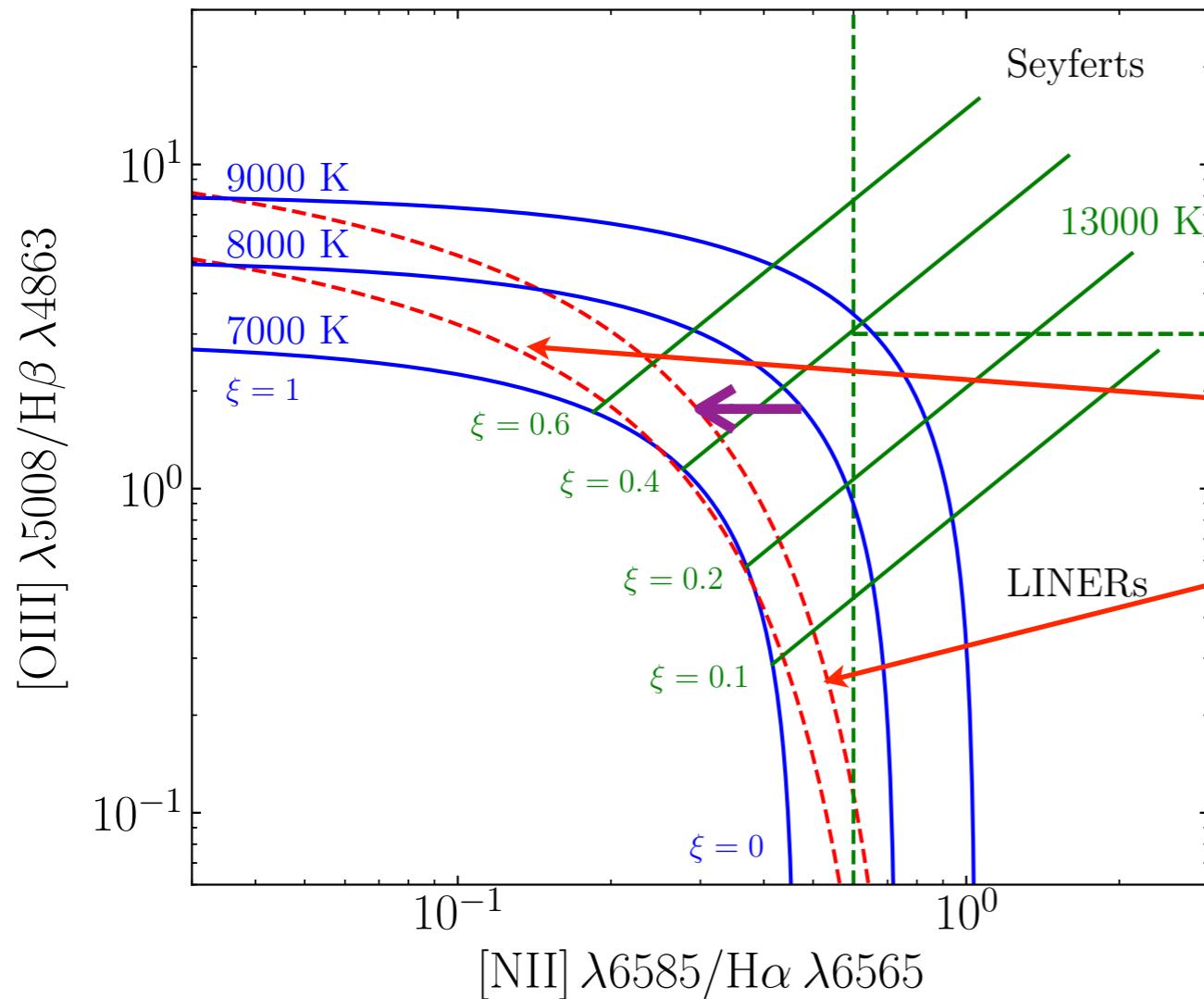
$$\frac{[\text{O III}] 5008}{\text{H}\beta} = 218.7 \xi T_4^{0.494 + 0.089 \ln T_4} e^{-2.917/T_4} \left( \frac{n_{\text{O}}/n_{\text{H}}}{0.8 \times 5.37 \times 10^{-4}} \right)$$

$$\frac{[\text{N II}] 6585}{\text{H}\alpha} = 12.44 (1 - \xi) T_4^{0.495 + 0.040 \ln T_4} e^{-2.185/T_4} \left( \frac{n_{\text{N}}/n_{\text{H}}}{7.41 \times 10^{-5}} \right)$$

- For an assumed temperature T, we can produce a theoretical curve of [OIII]5008/H $\beta$  versus [NII]6585/H $\alpha$  by varying the fraction  $\xi$  of the N and O that is doubly ionized.

The theoretical curve of [OIII]5008/H $\beta$  versus [NII]6585/H $\alpha$  that is obtained by varying the fraction is shown below:

Blue lines show the tracks of the equations calculated, by varying  $\xi$  from 0 to 1, for  $T = 7000, 8000$ , and  $9000$  K.



Empirical curves that discriminate the star-forming galaxies from AGNs.

$$\log_{10} ([\text{OIII}]/\text{H}\beta) = 1.10 - \frac{0.60}{0.01 - \log_{10} ([\text{NII}]/\text{H}\beta)}$$

$$\log_{10} ([\text{OIII}]/\text{H}\beta) = 1.3 - \frac{0.61}{0.05 - \log_{10} ([\text{NII}]/\text{H}\beta)}$$

Kauffmann et al. (2003, MNRAS, 346, 1055)

To derive the equation, we assumed that  $\xi_N = \xi_O = \xi$ . The discrepancy between the simple model with the observations would be due to the assumption. We need to note that  $E(\text{N}^+ \rightarrow \text{N}^{2+}) < E(\text{O}^+ \rightarrow \text{O}^{2+})$ , which implies that  $\xi_N \gtrsim \xi_O$ .

The arrow ← indicates the direction that is expected from  $\xi_N > \xi_O$ .

- Line ratios for 122,514 galaxies in SDSS DR7 with S/N > 5.

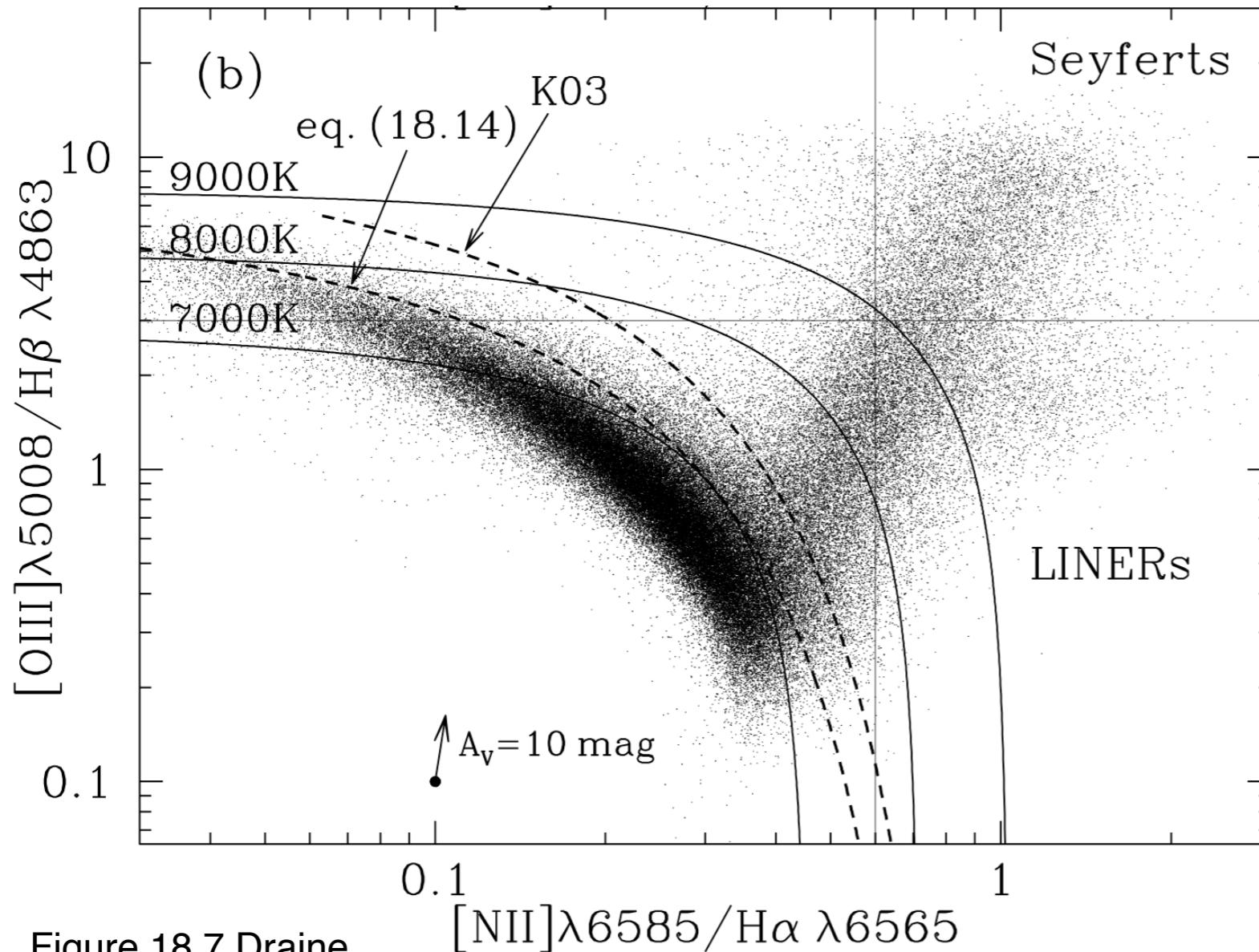


Figure 18.7 Draine



Flying seagull [Stasinska]

Photons from AGNs are harder than those from the massive stars that power H II regions.

***They induce more heating, implying that optical collisionally excited lines will be brighter with respect to recombination lines*** than in the case of ionization by massive stars.

The heating by an AGN boosts the [N II] line and creates a clear separation of the two wings.

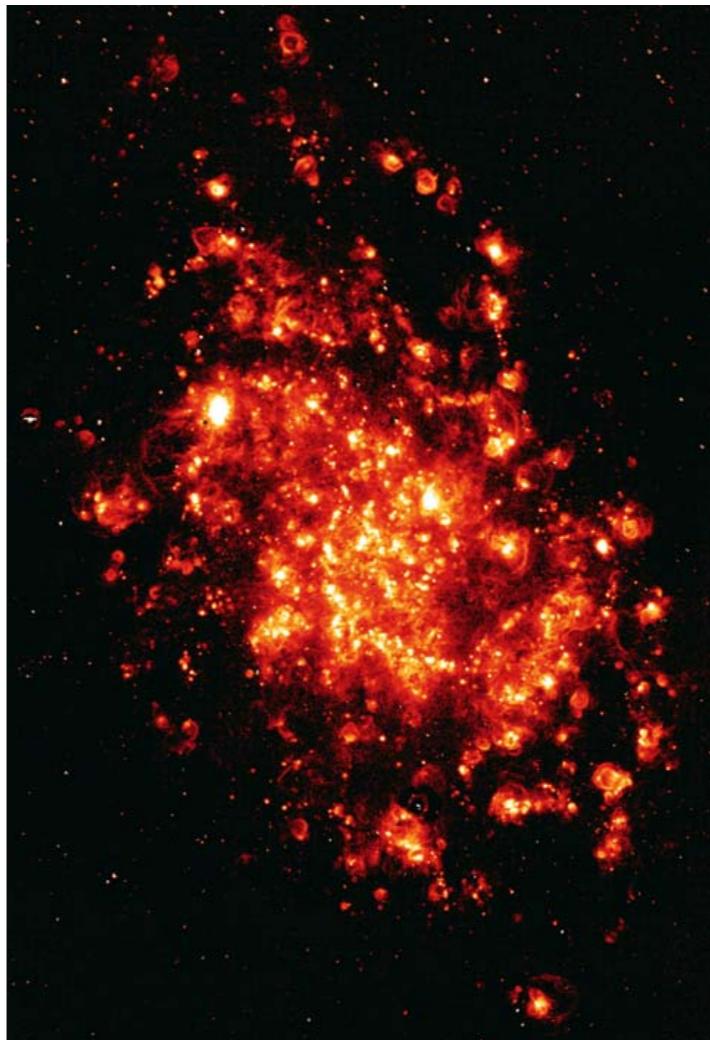
# Warm Ionized Medium / Diffuse Ionized Gas

- ***Outside well-defined H II regions, the ISM is known to contain diffuse ionized gas***
  - which can originate either from leaks of ionized gas out of H II regions due to the champagne effect, or from ionization by the UV radiation of isolated hot stars, and perhaps from other mechanisms.
  - In our Galaxy, the DIG is known to contain much more mass than the H II regions. Its total mass is of the order of 1/3 of that of H I.



M51 (NGC5195)  
Plate 1 [Lequeux]

B band - blue  
V band - green  
H $\alpha$  - red

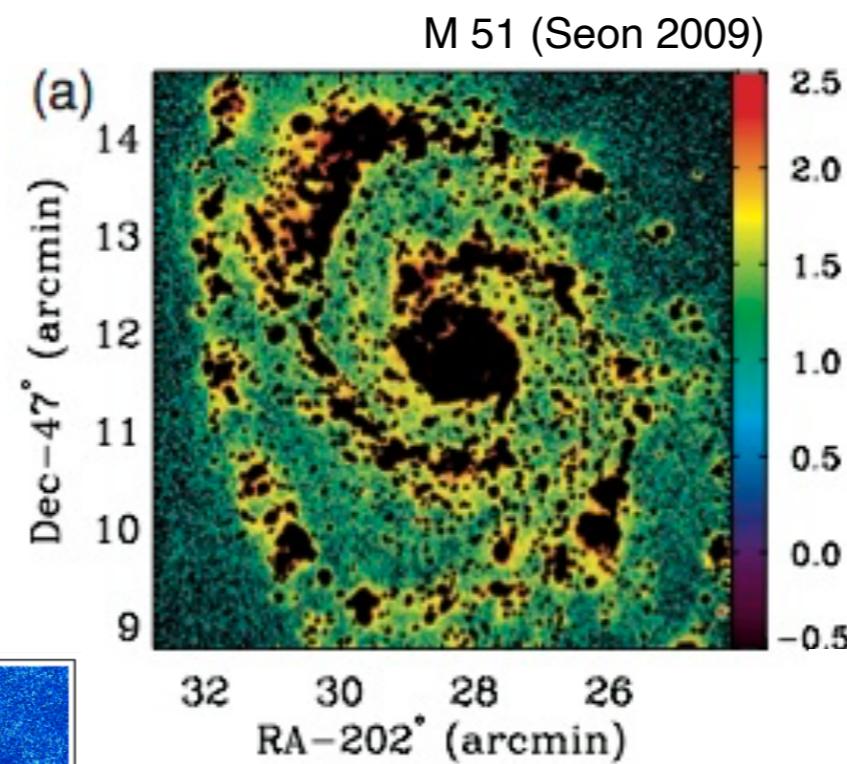
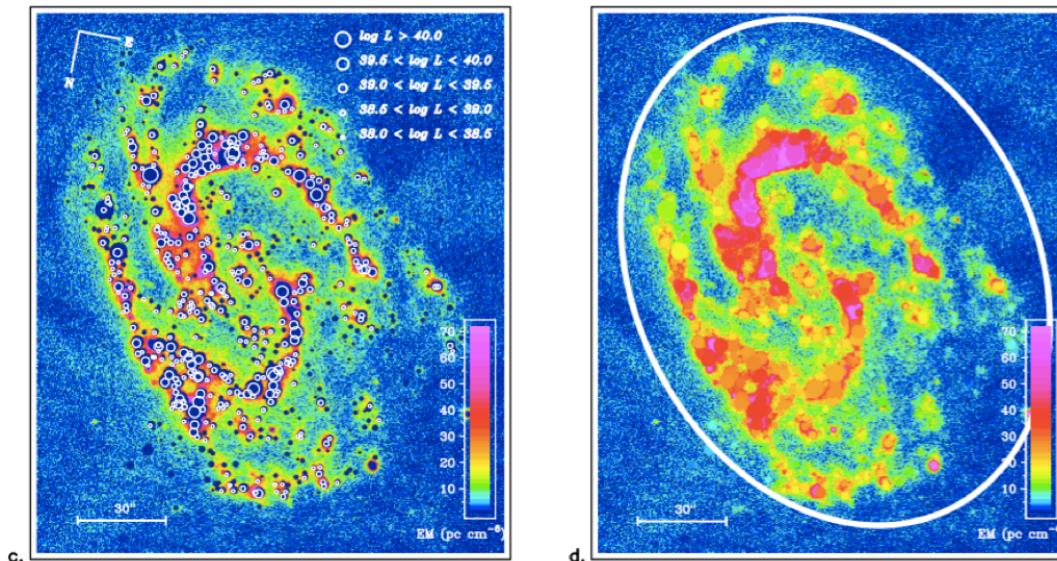


Ionized gas in M33  
Plate 10 [Lequeux]

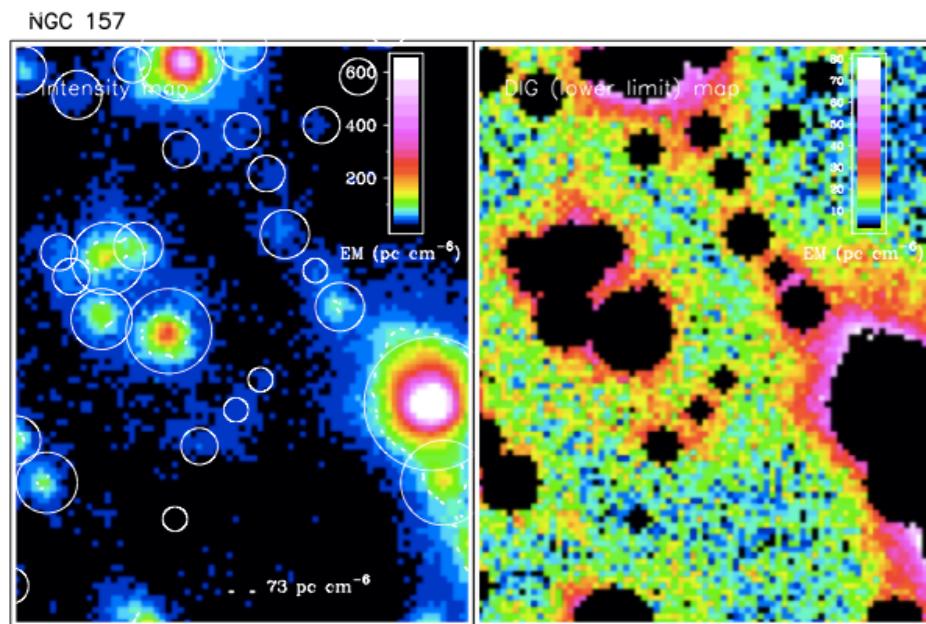
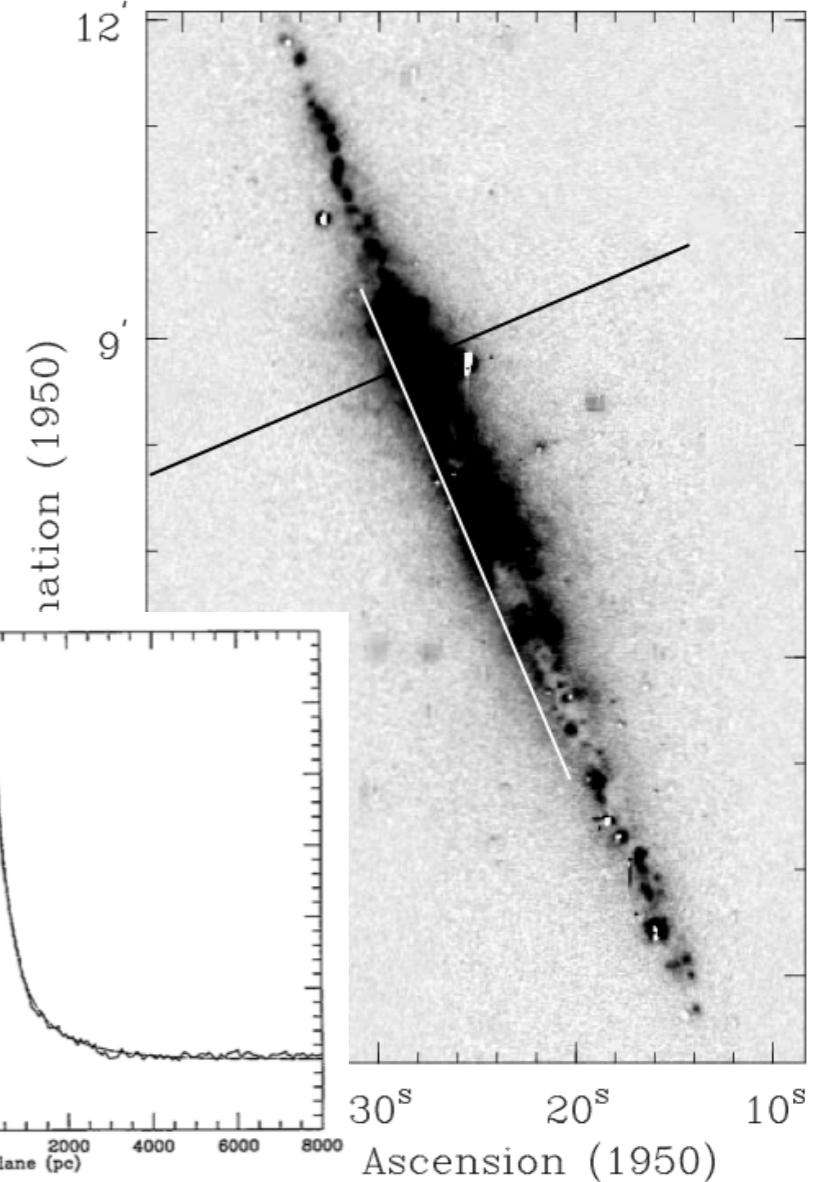
Many bubbles and  
the DIG present  
almost everywhere  
in the central  
region.

- ▶ Face-on & edge-on galaxies
- ▶ H $\alpha$  flux of WIM  $\sim 20\%-60\%$  of the total H $\alpha$  flux.

NGC 157 (Zurita e al. 2000)

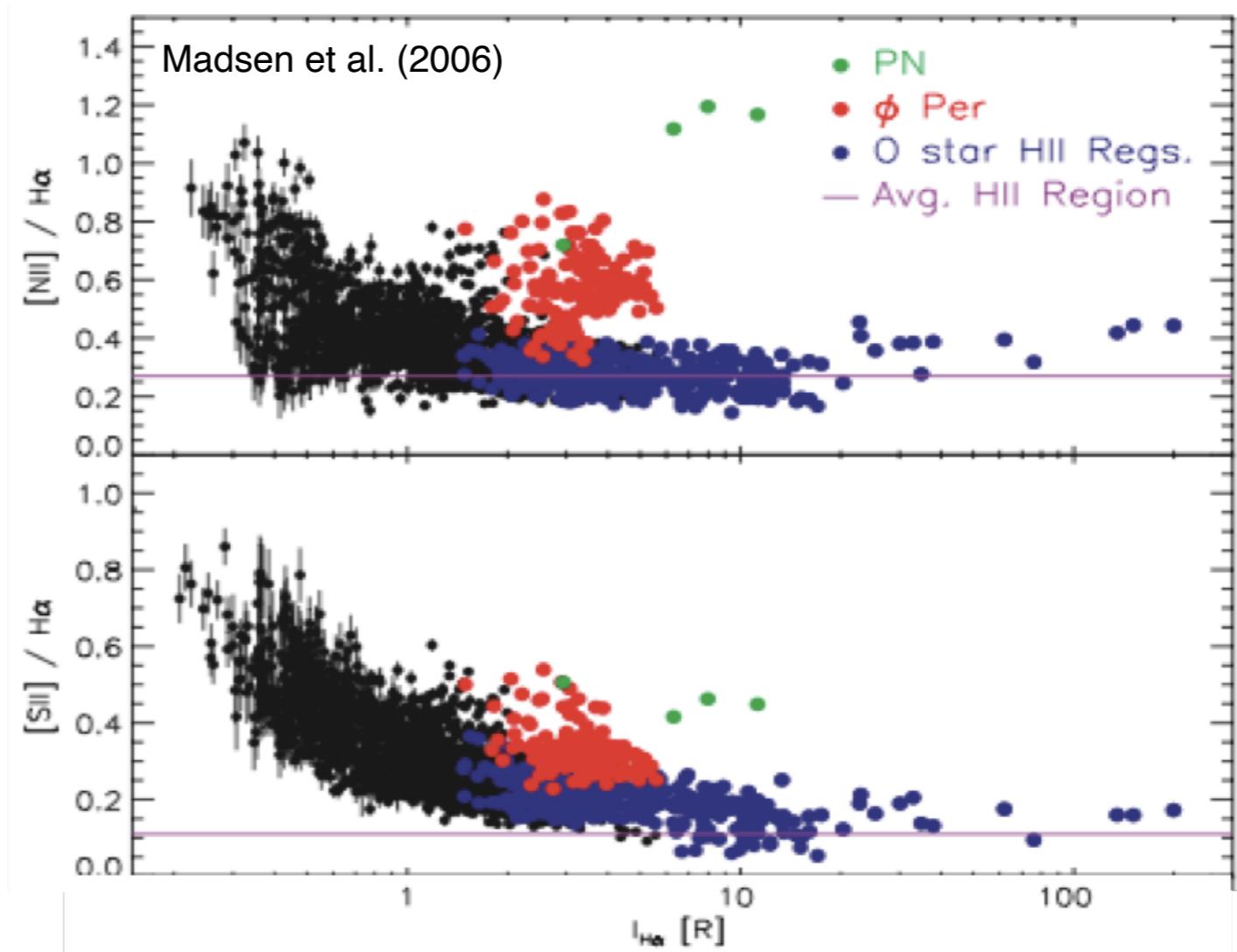
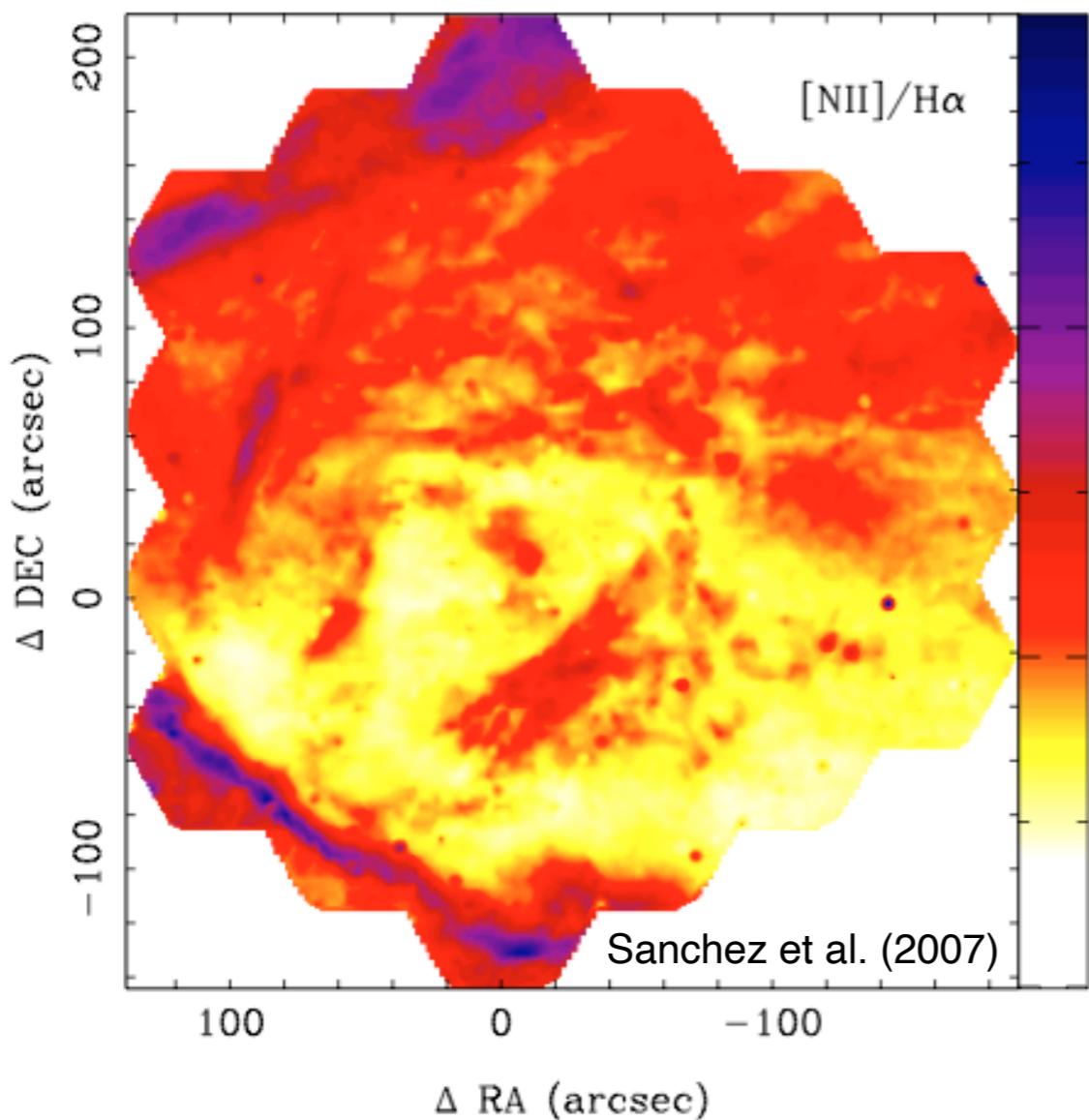


NGC 891 (Rand et al. 1998)

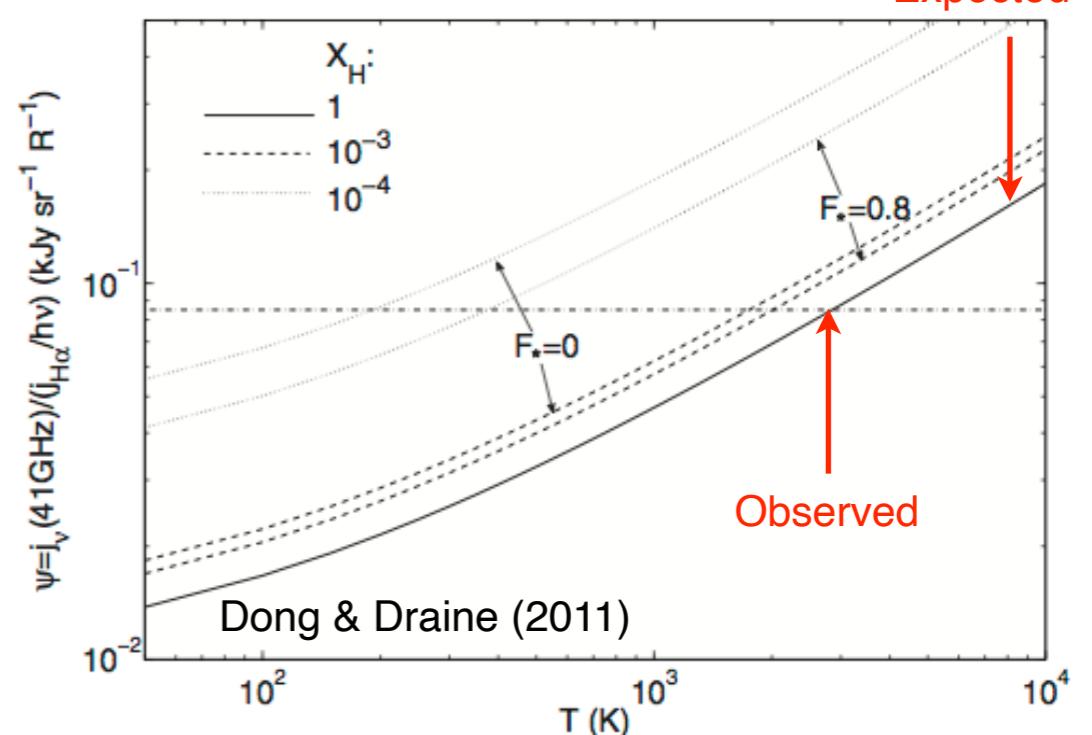
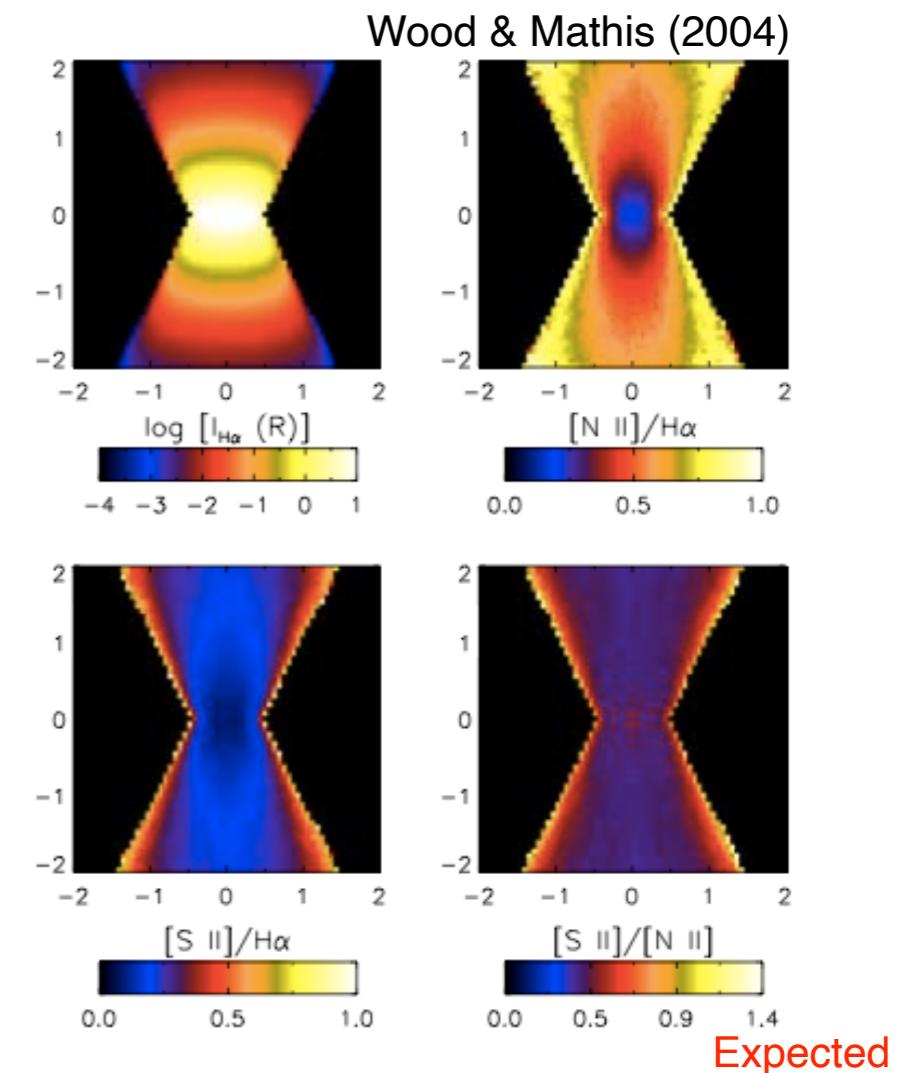


# Optical Line Ratios

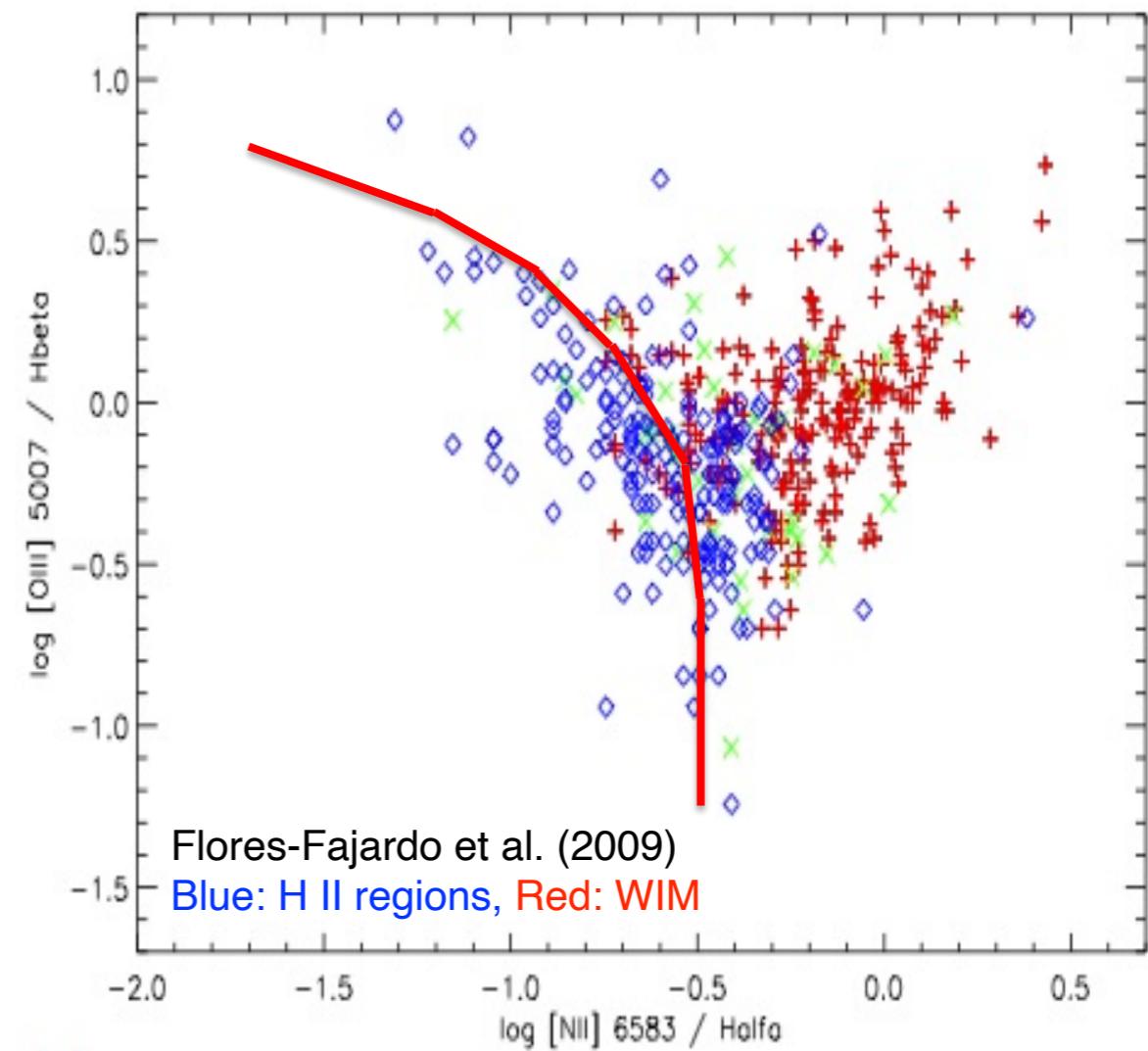
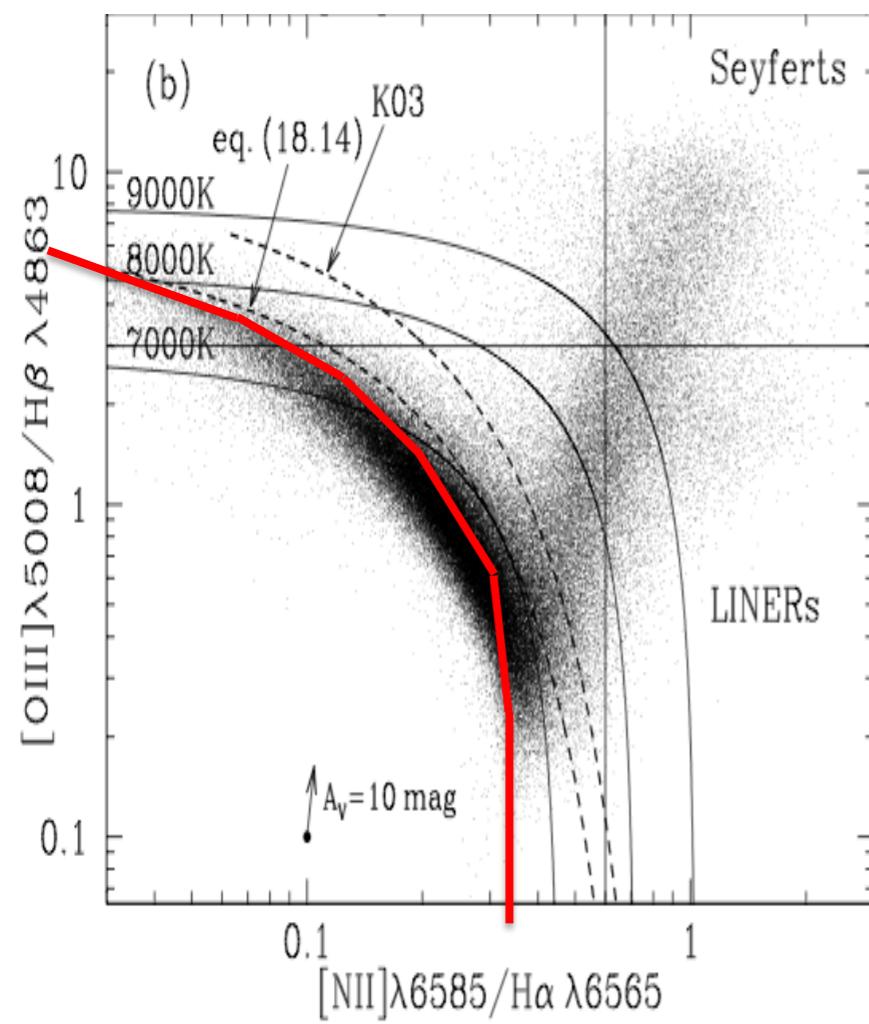
- $[\text{N II}] \lambda 6583/\text{H}\alpha$  and  $[\text{S II}] \lambda 6716/\text{H}\alpha$  in the diffuse regions are generally higher than the ratios in bright H II regions.
  - $[\text{N II}]/\text{H}\alpha \approx 0.25$  and  $[\text{S II}]/\text{H}\alpha \approx 0.1$  in bright H II regions
  - $[\text{N II}]/\text{H}\alpha \approx 0.3-0.6$  and  $[\text{S II}]/\text{H}\alpha \approx 0.2-0.4$  in the diffuse ISM regions.



- Ionizing Mechanism
  - Only the O stars meet and surpass the power requirements to ionize the diffuse ISM.
  - Density-bounded (leaky) H II regions
    - ▶ Turbulent or clumpy morphology of the ISM
    - ▶ Existence of enormous, H I-free bubbles/holes surrounding the O stars
- However, this mechanism can not explain the free-free radio emission
  - The WMAP data shows that the observed ratio of free-free radio continuum to H $\alpha$  is at least twice smaller than the expected value.
  - Davies et al. (2006, 2009), Dobler & Finkbeiner (2008a,b), Gold et al. (2011)
- See Seon & Witt (2012), and Dong & Draine (2011) for the possibility of alternative explanations.



# Heating source of the WIM



Detailed analyses of the line ratios  $[N II]/H\alpha$  and  $[S II]/H\alpha$  have been performed to obtain the temperature; it is found to be of the order of 8,000 K.

However, it is also known that WIM is being heated by additional sources. (see the above BPT diagram).

# Dynamics of H II regions

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- A new O star is presumably born within clouds of relatively dense cold gas. The appearance of a source of UV photons will have two effects.
  - First, the gas surrounding the new star will become ionized. Since the mean free path of an UV photon is very short in neutral hydrogen, the photons will be absorbed in a relatively thin surrounding shell of neutral hydrogen, producing new ionization. Thus ***the ionized and neutral gases are separated by an ionization front, which moves rapidly outward*** as more and more atoms become ionized by the stream of photons.
  - Second, the process increases the gas temperature from  $\sim 10^2$  K to  $\sim 10^4$  K, by a factor of about a hundred. the ionization process itself increases the number of gas particles, by a factor two. As a result, ***the pressure in the ionized gas is ~200 times greater than that in surrounding neutral material.*** This ionized gas cannot be confined and will expand. The ionized and neutral gas are set in motion.
  - Since the expansion velocity is likely to exceed the sound velocity in the surrounding H I region, ***a shock front may be expected to form***, moving out through the neutral gas. The dynamical analysis of H II regions must consider the interactions between the ionization front and the shock front, together with the equations of motion of the gas behind the two fronts.
- This process is not the only way in which ISM is set in motion by means of interaction with stars.
  - There are effects produced by the very high speed continuous mass loss - a ***stellar wind***.
  - Many massive stars terminate their existence in a violent explosive event - a ***supernova***.

- Basic Assumptions:
  - Any disturbances to the cloud structure produced by the formation of a star are neglected. *After a relatively short time ( $< 10^5$  yr), the star reaches a static configuration* in which it can remain for a much longer time ( $> 3 \times 10^6$  yr). The stellar radiant energy output rate and the spectral distribution of the radiation are more or less constant during this phase. The star then produces Lyman continuum photons at a constant rate. Since *the star formation time scale is so short, we may take the star to be ‘switched on’ instantaneously.*
  - The gas around the star will be assumed to be at rest (in the frame of reference of the star).
  - The gas has initially assumed to be uniform in density and temperature.
- Ionization front
  - The term “front” describes a more-or-less abrupt boundary between two regions of the ISM with very different properties.
  - An ionized nebula can be approximated as a region of highly ionized gas, separated from the surrounding neutral medium by a thin boundary region, of thickness  $\lambda_{\text{mfp}} \approx 0.002$  pc. Thus, an H II region is surrounded by an ionization front.

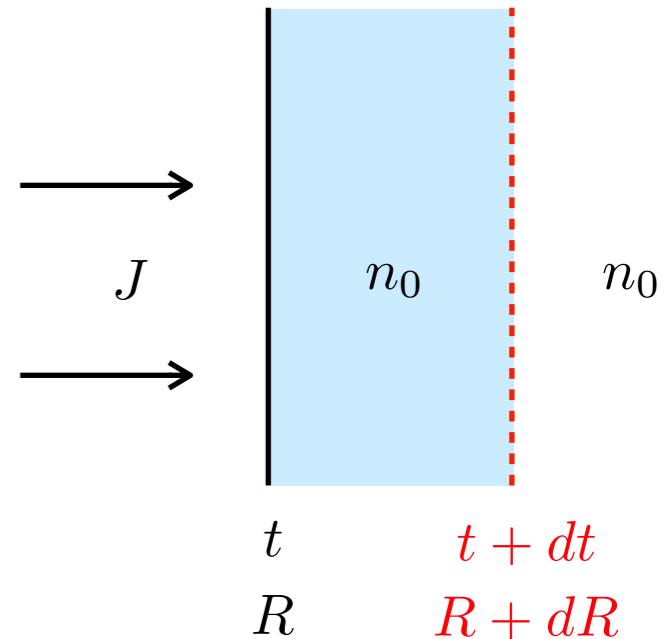
# The velocity of the ionization front

- Suppose that at time  $t$  the ionization front is located at a distance  $R$  from the star and at time  $t + dt$  it is at a distance  $R + dR$ .
  - Let  $n_0$  = number density of the undisturbed neutral hydrogen
  - $J$  = number of Lyman continuum photons incident normally on unit area of the ionization front per unit time.
- Ionization balance at the ionization front:** While the ionization front moves from  $R$  to  $R + dR$ , the photons will ionize all the neutral atoms lying between these two positions ( $R, R + dR$ ).
- We assume that only one photon is needed to ionize each atom as the front moves the distance  $dR$ . In other words, no recombination occurs within the distance interval  $dR$ . For unit area of the ionization front, the following relation must be satisfied:

$$J \Delta A dt = n_0 \Delta A dR$$

- Then, the velocity of the ionization front (in a fixed frame of reference) is:

$$\frac{dR}{dt} = \frac{J}{n_0}$$



# The initial stage of evolution of an ionized region

- Suppose that the UV source has been suddenly turned on.
- ***Ionization balance for the ionized region:*** We consider two factors:
  - *The radiation field at the ionization front is diluted because of the spherical geometry.*
  - Recombination takes place continuously inside the ionized region, and *some of the UV photons produced by the central source must go to reionize the atoms that have recombined.*
- Inside the ionized sphere, the fractional ionization is near unity. Thus,  $n_e = n_p = n_0$ . Using this condition, we obtain an equation for the expansion velocity of the ionization front.

$$\frac{J}{n_0} = \frac{dR}{dt} = \frac{Q_0}{4\pi R^2 n_0} - \frac{1}{3} R n_0 \alpha_B$$

- Let's define the following dimensionless quantities:

$$\rho \equiv R/R_s \quad \text{where } R_s \equiv \left( \frac{3}{4\pi} \frac{Q_0}{\alpha_B n_0^2} \right)^{1/3}$$

$$\tau \equiv t/t_{\text{rec}} \quad \text{where } t_{\text{rec}} \equiv \frac{1}{\alpha_B n_0}$$

Then, the equation in dimensionless form is

$$\frac{d\rho}{d\tau} = \frac{1}{3} \left( \frac{1}{\rho^2} - \rho \right)$$

- ▶ The equation can be written:

$$\frac{d\rho}{d\tau} = \frac{1}{3} \left( \frac{1}{\rho^2} - \rho \right) \rightarrow \frac{d\rho^3}{d\tau} = 1 - \rho^3$$

- ▶ It's solution is

$$\rho^3 = 1 - e^{-\tau}$$

$$R(t) = R_s \left( 1 - e^{-t/t_{\text{rec}}} \right)^{1/3}$$

initial condition:  $R(t = 0) = 0$

$$\begin{aligned} \frac{dx}{d\tau} + x &= 1 \\ e^\tau \frac{dx}{d\tau} + e^\tau x &= e^\tau \\ \frac{d(e^\tau x)}{d\tau} &= e^\tau \end{aligned} \quad \rightarrow \quad \begin{aligned} e^\tau x &= \int_0^\tau e^{\tau'} d\tau' = e^\tau - 1 \\ x &= 1 - e^{-\tau} \end{aligned}$$

- Scale Parameters:

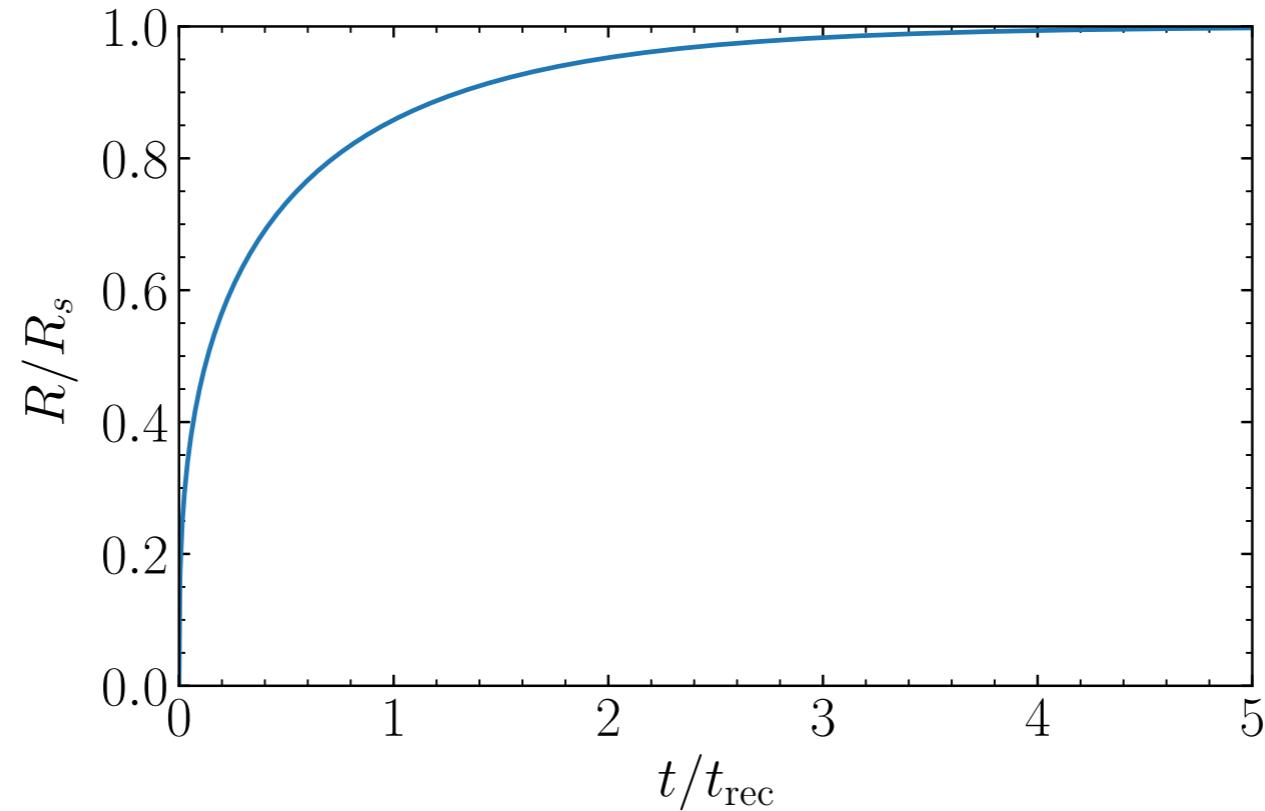
- The time scale introduced is the recombination time scale:

$$t_{\text{rec}} \equiv \frac{1}{\alpha_B n_0} \approx 4000 \text{ yr} \left( \frac{\alpha_B}{2.6 \times 10^{-3} \text{ cm}^3 \text{ s}^{-1}} \right)^{-1} \left( \frac{n_0}{30 \text{ cm}^{-3}} \right)^{-1}$$

the length scale introduced is the Strömgren radius:

$$R_s \equiv \left( \frac{3}{4\pi} \frac{Q_0}{\alpha_B n_0^2} \right)^{1/3} \approx 7 \text{ pc} \left( \frac{Q_0}{10^{49} \text{ s}^{-1}} \right)^{1/3} \left( \frac{\alpha_B}{2.6 \times 10^{-3} \text{ cm}^3 \text{ s}^{-1}} \right)^{-1/3} \left( \frac{n_0}{30 \text{ cm}^{-3}} \right)^{-2/3}$$

- 
- Hence, the time required to create a Strömgren sphere after turning on a hot star is an order of  $\sim 4000$  yr. This is also the time it takes the ionized Strömgren sphere to revert to neutral gas after the central UV source has been turned off.



- At times  $t \gg t_{\text{rec}} \sim 4000$  yr , the gas medium will be fully ionized with radius  $R \sim R_s \sim 7$  pc, surrounded by a partially ionized boundary of thickness  $\sim \lambda_{\text{mfp}} = (n_{\text{H}}\sigma_{\text{pi}})^{-1} \sim 0.002$  pc  $\ll R_s$ .

- We can compute the ***rate of expansion of the ionization front***:

$$\frac{dR}{dt} = \frac{R_s}{3t_{\text{rec}}} \frac{e^{-t/t_{\text{rec}}}}{(1 - e^{-t/t_{\text{rec}}})^{2/3}}$$

where the characteristic expansion velocity is

$$v_* \equiv \frac{R_s}{3t_{\text{rec}}} \simeq 560 \text{ km s}^{-1} \left( \frac{Q_0}{10^{49} \text{ s}^{-1}} \right)^{1/3} \left( \frac{n_0}{30 \text{ cm}^{-3}} \right)^{1/3}$$

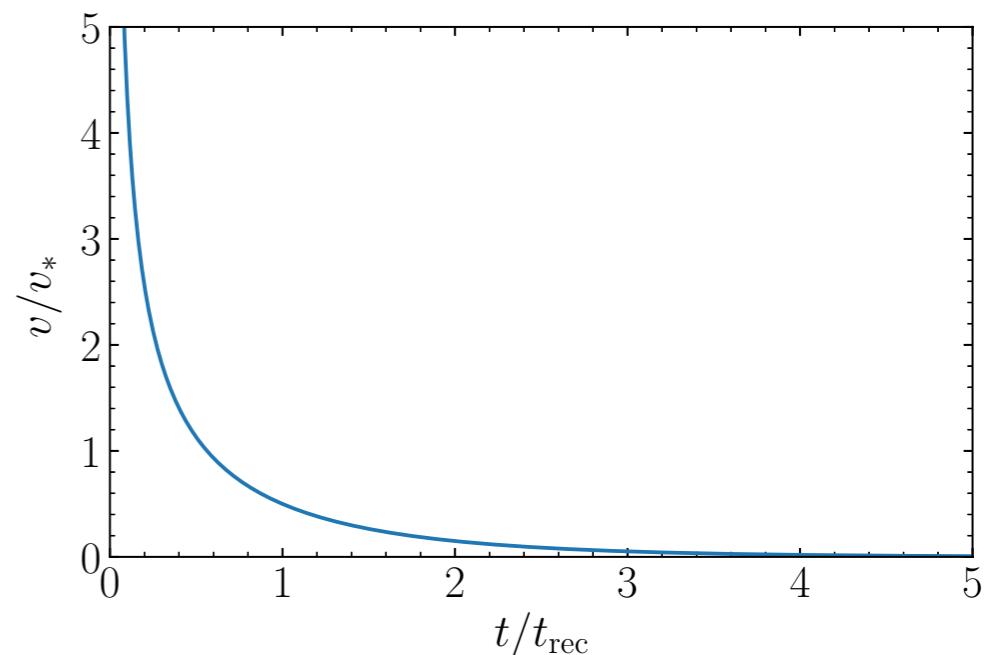
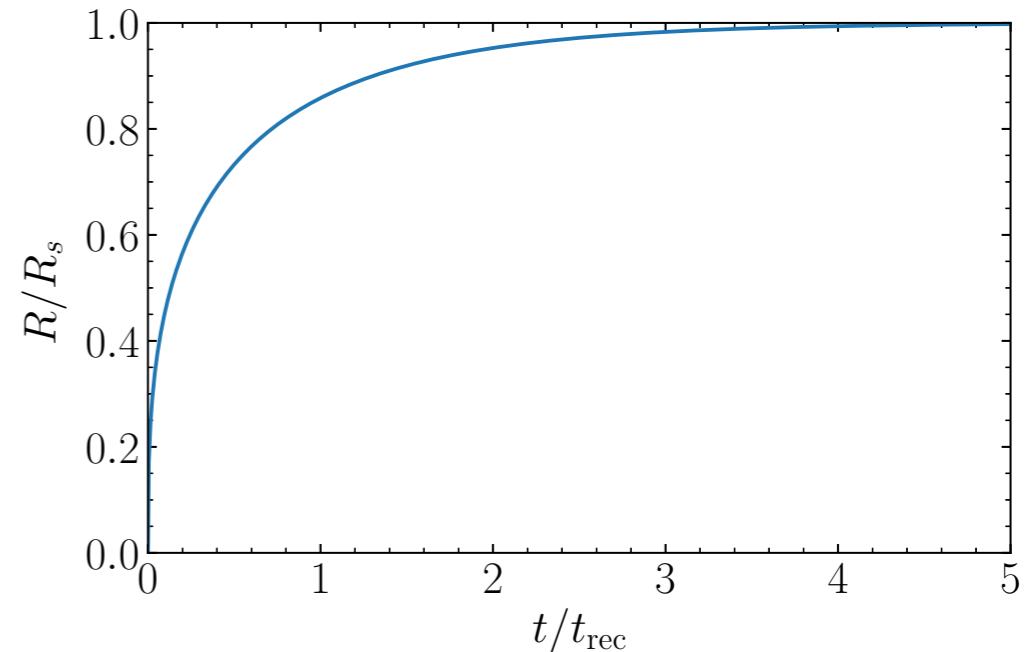
This is much larger than the sonic speed  $c_s \approx 1 \text{ km s}^{-1}$  in the neutral medium as well as  $c_s \approx 10 \text{ km s}^{-1}$  in the ionized medium.

- The expansion speed of the ionization front at two limits:

$$\frac{dR}{dt} \approx \frac{R_s}{3t_{\text{rec}}} \left( \frac{t}{t_{\text{rec}}} \right)^{-2/3} \quad \text{for } t \ll t_{\text{rec}}$$

$$\frac{dR}{dt} \approx \frac{R_s}{3t_{\text{rec}}} e^{-t/t_{\text{rec}}} \quad \text{for } t \gg t_{\text{rec}}$$

Note that the expansion speed diverges at  $t = 0$ .



- 
- The ionization front will initially expand supersonically. When will the ionization front expand at subsonic speeds?

$$\frac{dR}{dt} = \frac{R_s}{3t_{\text{rec}}} e^{-t/t_{\text{rec}}} \lesssim c_i \quad c_i \approx 13 \text{ km s}^{-1} \quad \text{sound speed in the ionized medium}$$

$$t \lesssim t_{\text{sonic}} \equiv t_{\text{rec}} \ln \left( \frac{R_s}{3t_{\text{rec}}} \frac{1}{c_i} \right) \approx 3.8t_{\text{rec}} \simeq 15,000 \text{ yr}$$

- At this time, the ionization front will have a size of:

$$R(t = t_{\text{sonic}}) = R_s (1 - e^{-3.8})^{1/3} = 0.9925 R_s$$

- The ionization front will expand at a supersonic velocity until  $t \approx t_{\text{sonic}}$  ( $\sim 15,000$  yr). By that time, the ionized sphere has reached a radius  $R \sim 0.99R_s$  and then it starts to expand at subsonic speed.
- ***At  $t = R_s/c_s \sim 0.5$  Myr, the gas starts to flow outward as a result of the pressure gradient that has build up.***

# The final stage of evolution of an ionized region

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- Although the ionized sphere approaches ionization equilibrium at  $t \gtrsim t_{\text{rec}}$ , it would be still far from pressure equilibrium.
  - Outside the ionized zone, it will be embedded in the cold neutral medium with a temperature  $T \sim 100$  K.
  - Inside the sphere, the heating and cooling processes yield a temperature of  $T \sim 10,000$  K.
  - Also, the density of particles inside the ionized sphere will double when the hydrogen is ionized.
  - Thus, *the pressure inside the sphere will be  $\sim 200$  times higher than the pressure outside, meaning that the ionized gas will begin to expand.*
  - The ionized gas expands as long as it has a higher pressure than its surroundings. This expansion produces a shock and will cease when the hot ionized gas reaches pressure equilibrium with the surrounding cold neutral gas.
- ***The condition of final pressure equilibrium*** can be written in the form:

$$2n_f k T_i = n_0 k T_n$$

$n_f$  = number density of the ionized hydrogen.

$T_i$  and  $T_n$  = temperatures of the ionized and neutral gas, typically  $T_i = 10^4$  K,  $T_n = 10^2$  K .

- 
- The ionized gas sphere must still absorb all the stellar UV photons. Thus,

$$Q_0 = \frac{4}{3}\pi R_f^3 n_f^2 \alpha_B$$

Here,  $R_f$  is the final radius of the ionized gas sphere. From the pressure equilibrium condition, we obtain the final size:

$$n_f = (T_n/2T_i)n_0 \approx 0.005n_0 \quad \rightarrow \quad R_f = (2T_i/T_n)^{2/3} R_{s0} \approx 34R_{s0}$$

- The ratio of the mass of gas finally ionized to that contained within the initial Strömgren sphere is:

$$\frac{M_f}{M_s} = \frac{R_f^3 n_f}{R_{s0}^3 n_0} = \frac{2T_i}{T_n} \approx 200$$

- This indicates that *the initial Strömgren sphere contains only a very small fraction of the material which, in principle, a star could ultimately ionize.*

# The intermediate stage of evolution of an ionized region

- Before the pressure equilibrium is established, the gas density and temperature will be

$$n_i \approx 2n_0 > n_f \quad \text{and} \quad T_i = 10^4 \text{ K}$$

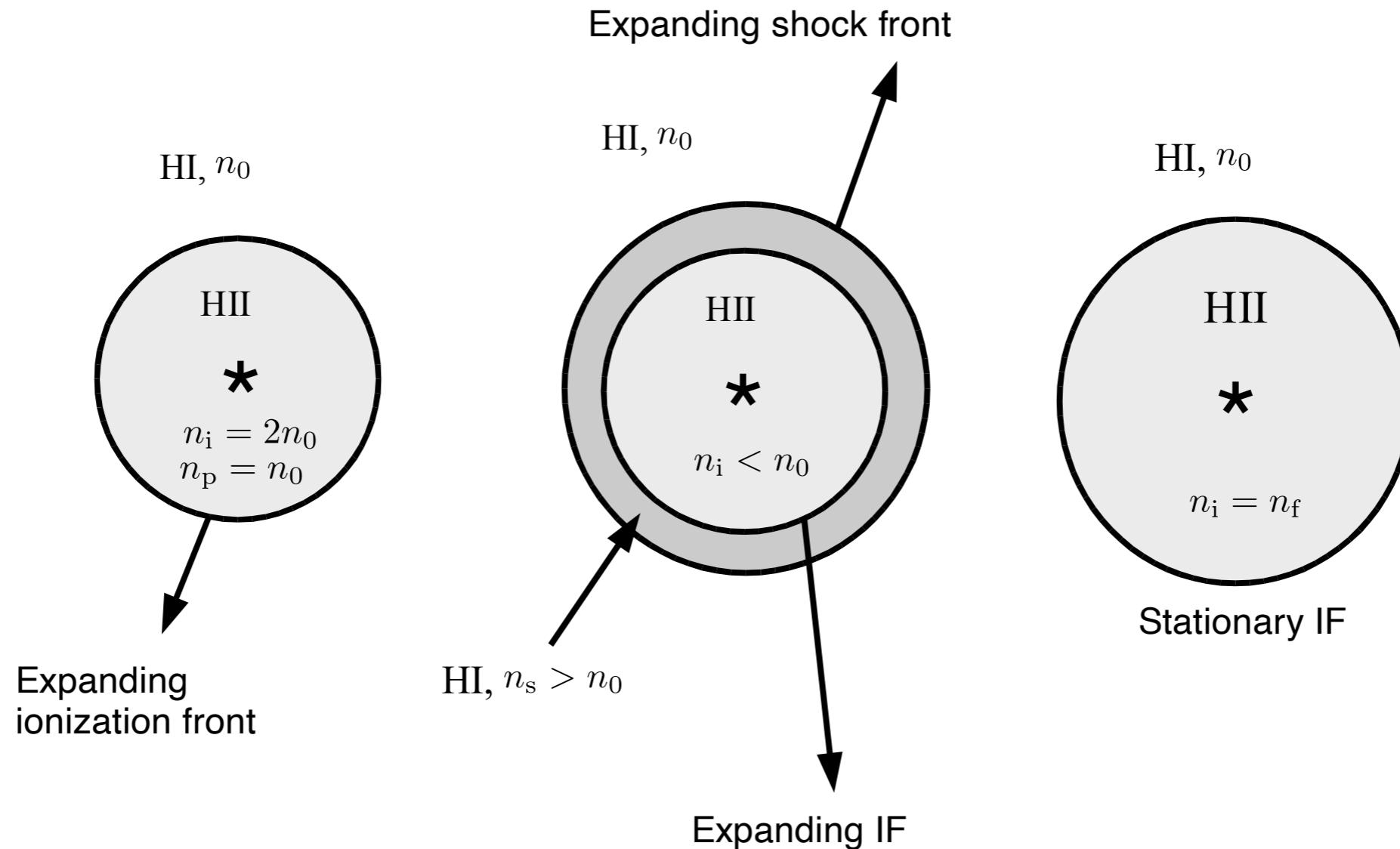
- Then the isothermal sound speeds of the ionized gas and neutral gas are, respectively:

$$c_i^2 = \frac{P_i}{\rho_i} \approx \frac{2n_0 k T_i}{n_0 m_H} \quad c_n^2 = \frac{P_n}{\rho_n} = \frac{n_0 k T_n}{n_0 m_H}$$

$$\frac{c_i}{c_n} = \left( \frac{n_i T_i}{n_0 T_n} \right)^{1/2} \approx \sqrt{200} = 14.14$$

- The sound speed of the ionized gas is much larger than that of the neutral gas.
- The ionized gas has a higher pressure and thus plays the role of a piston and pushes a shock wave into the neutral gas. *The expansion speed of the ionized gas is originally equal to about  $c_i$ , which is highly supersonic with respect to the sound speed in the neutral gas.*
- Note also that, at  $t \gtrsim t_{\text{sonic}} \approx 3.8t_{\text{rec}}$ , the expansion speed ( $c_i$ ) of ionized gas is larger than that of the ionization front.

$\frac{dR}{dt} > c_i$ at $t \lesssim t_{\text{sonic}}$	$\longrightarrow$	$\frac{dR}{dt} \approx c_i$ at $t \approx t_{\text{sonic}}$
initial stage		intermediate stage



Evolutionary scheme of an expanding H II region. (a) The initial stage, (b) expansion with a shock in the neutral gas, (c) the final equilibrium state.

[Figure 7.2 Dyson]

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- Sound crossing time
    - The ionized region will likely be overpressured relative to its surroundings, in which case it will expand on the sound crossing time.
    - The isothermal sound speed in fully ionized hydrogen is

$$c_s = (2kT/m_{\text{H}})^{1/2} = 13(T/10^4 \text{ K})^{1/2} \text{ km s}^{-1} \quad p = (n_{\text{HI}} + n_e)kT = 2n_{\text{H}}kT$$

- The time for a pressure wave to propagate a distance equal to Strömgren radius is

$$t_{\text{sound}} = \frac{R_s}{c_s} \approx 2.39 \times 10^5 \frac{Q_0/10^{49} \text{ s}^{-1}}{(n/10^2 \text{ cm}^{-3})^{2/3}} \text{ [yr]}$$

- This is about a hundred times longer than the recombination time (timescale of the expanding ionization front).

# Introduction to Gas Dynamics

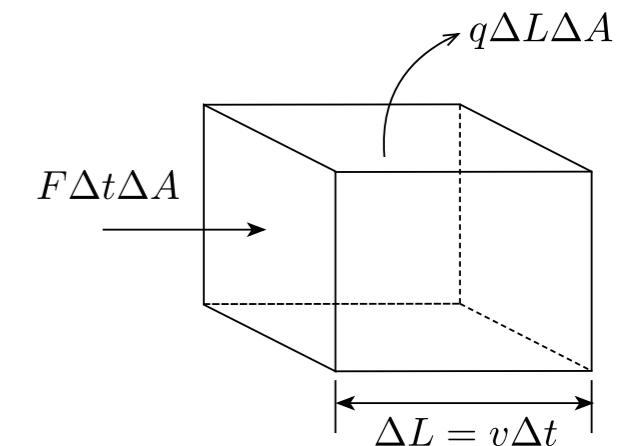
- Assumption for hydrodynamics:
  - particle mean free path << size of the region
  - We will describe the equations for conservation of mass, momentum and energy, in 1D space.

- ***Definition***

- Flux of a hydrodynamic quantity  $q$  (for instance, density):

Fluid moves a distance  $\Delta L$  during a time interval  $\Delta t$  with a velocity  $v$ .

$$F\Delta t\Delta A = q\Delta L\Delta A \rightarrow F = qv$$



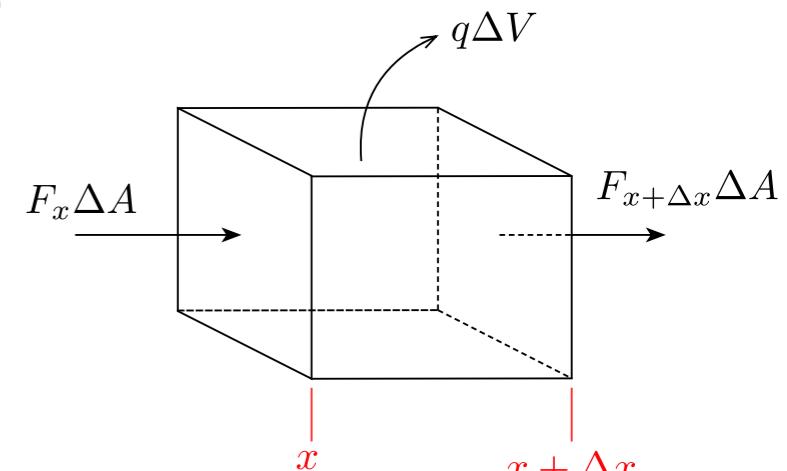
- ***Conservation equation for a quantity***  $q$

- change of the quantity within a volume  $\Delta V$  for a time interval  $\Delta t$ :

Here,  $\Delta t$  and  $\Delta x$  are independent.

$$\frac{q\Delta V|_{t+\Delta t} - q\Delta V|_t}{\Delta t} = F\Delta A|_x - F\Delta A|_{x+\Delta x}$$

$$\frac{\partial q}{\partial t} = -\frac{\partial F}{\partial x} \rightarrow \frac{\partial q}{\partial t} = -\frac{\partial(qv)}{\partial x}$$



- Here, no sources or sinks of the quantity within  $\Delta V$  were assumed. If any, the loss and gain terms should be added in the right-hand side.

# Mass Conservation

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- Conservation equations
  - ***Mass conservation (continuity equation)***
    - ▶ mass within a volume  $dV = \rho dV$
    - ▶ no sources or sinks of material within  $dV$
    - ▶ Consider the mass per unit area ( $dA$ ), contained in the volume

$$\rho dV/dA = \rho dx \quad \longrightarrow \quad \frac{\partial}{\partial t}(\rho dx) = \overbrace{\rho u}^{\text{incoming}} - \overbrace{(\rho + d\rho)(u + du)}^{\text{outgoing}}$$

$$= -(\rho du + ud\rho + d\rho du)$$

$$\frac{\partial \rho}{\partial t} = -\frac{\partial(\rho u)}{\partial x}$$

- ▶ Mass loss and gain terms should be added in the right-hand side, if necessary.

# Momentum Conservation

- **Momentum conservation (Euler's equation)**

- ▶ momentum within  $dV$  (per unit area) =  $(\rho dV)u/dA = \rho dx u$
- ▶ = change of momentum due to fluid flow and gas pressure acting on the surface of  $dV$

$$\begin{aligned}\frac{\partial}{\partial t}(\rho u dx) &= \overbrace{\rho u^2}^{\text{incoming}} - \overbrace{(\rho + d\rho)(u + du)^2}^{\text{outgoing}} + \overbrace{P}^{\text{incoming}} - \overbrace{P + dP}^{\text{outgoing}} \\ &= \rho u^2 - \left( \rho u^2 + 2\rho u du + \cancel{\rho du^2} + u^2 d\rho + \cancel{2ud\rho du} + \cancel{d\rho du^2} \right) - dP\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial t}(\rho u) &= -2\rho u \frac{\partial u}{\partial x} - u^2 \frac{\partial \rho}{\partial x} - \frac{\partial P}{\partial x} \\ \rho \frac{\partial u}{\partial t} + u \frac{\partial \rho}{\partial t} &= -\rho u \frac{\partial u}{\partial x} - u \left( \rho \frac{\partial u}{\partial x} + u \frac{\partial \rho}{\partial x} \right) - \frac{\partial P}{\partial x}\end{aligned}$$

Using mass conservation,  $\frac{\partial u}{\partial t} = -\frac{\partial(\rho u)}{\partial x}$

$$\rho \frac{\partial u}{\partial t} = -\rho u \frac{\partial u}{\partial x} - \frac{\partial P}{\partial x}$$

or

$$\begin{aligned}\frac{\partial}{\partial t}(\rho u) &= -2\rho u \frac{\partial u}{\partial x} - u^2 \frac{\partial \rho}{\partial x} - \frac{\partial P}{\partial x} \\ &= -\frac{\partial}{\partial x}(\rho u^2) - \frac{\partial P}{\partial x}\end{aligned}$$

$$\frac{\partial}{\partial t}(\rho u) = -\frac{\partial}{\partial x} (\rho u^2 + P)$$

- ▶ Further terms could be added in the right-hand side, accounting for forces due to gravity, magnetic fields, radiation field, and viscosity.

- ▶ The following quantity is sometimes known as **Bernoulli's constant**.

$$\rho u^2 + P$$

One may use it to understand why, for example, fast winds engulfing a house causes it to **explode**, rather than **implode**, because the pressure external to the house becomes lower than its value inside it.

- ▶ Viscous force is due to “internal friction” in the fluid (resistivity of the fluid to the flow), as two adjacent fluid parcels move relative to each other.)

$$\text{viscous force} \propto \frac{\partial^2 u}{\partial x^2}$$

The viscous force is usually much smaller than force due to gas pressure, but important in high-speed flows with large velocity gradients, as in accretion disks.

# Ionization Front: Jump Condition

- Low density gas, like that of the ISM, can be treated as an ideal gas, with no viscosity with a pressure given by the ideal gas law:

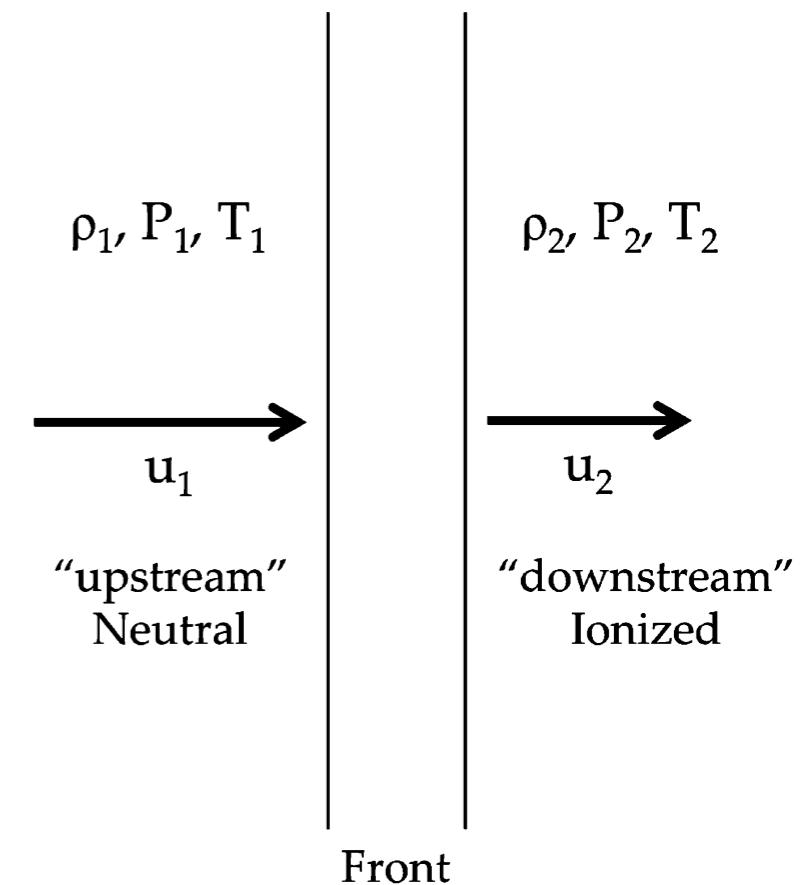
$$P = \frac{\rho k T}{m}$$

$\rho$  = mass density,  $T$  = temperature,  
 $m$  = mean molecular mass

$\rho, P, T, u$  = density, pressure, temperature, and bulk velocity

- Let's consider a small patch of the ionization front between the interior of an H II region and its exterior.

- If the patch is small compared to the ionization front's radius of curvature, then we can treat the ionization front as if it has **plane parallel** symmetry.
- It is convenient to use **a frame of reference in which the ionization front is stationary**; in this frame, the bulk velocity  $u_1$  of the neutral gas points toward the ionization front. The bulk velocity  $u_2$  of the ionized gas points away from the ionization front.



- 
- Let's consider a steady state solution.
    - We have seen that the speed of the ionization front surrounding a Strömgren sphere changes with time. However, the steady state solution gives us some intuition about the behavior of ionization fronts in general.
    - Then, the mass conservation and momentum conservation equation becomes:

$$\frac{d}{dx} (\rho u) = 0 \quad \frac{d}{dx} (\rho u^2 + P) = 0$$

- Let subscript **1** denote fluid variables in the neutral gas ahead of the I-front, and subscript **2** denotes fluid variables in the ionizing gas behind the I-front. Integrating these equations across the ionization front, we obtain:

$$\rho_1 u_1 = \rho_2 u_2$$

$$\rho_1 u_1^2 + P_1 = \rho_2 u_2^2 + P_2$$

- *The number of H atoms flowing through the ionization front per unit area per second must equal to J, the corresponding number of ionizing photons reaching the front.*  
Hence, the equation becomes

$$\rho_1 u_1 = \rho_2 u_2 = m_i J \quad \text{Here, } u_1 = \frac{dR}{dt} = \frac{J}{n_0}, \quad \rho_1 = m_i n_0$$

where  $m_i$  is the mean mass of the gas per newly created positive ion ( $m_i = m_H$  in a pure hydrogen gas). We may also write the equation of momentum conservation using the isothermal sound speeds:

$$\rho_1 (u_1^2 + c_1^2) = \rho_2 (u_2^2 + c_2^2)$$

$$c_s^2 = \frac{P}{\rho} \text{ for isothermal gas}$$

$$P = nkT$$

- We will consider a hydrogen gas.

$$c_1 = \left( \frac{kT_1}{m_H} \right)^{1/2} = 0.91 \text{ km s}^{-1} \left( \frac{T_1}{100 \text{ K}} \right)^{1/2} \quad \text{neutral hydrogen gas}$$

$$c_2 = \left( \frac{2kT_2}{m_H} \right)^{1/2} = 12.9 \text{ km s}^{-1} \left( \frac{T_2}{10^4 \text{ K}} \right)^{1/2} \quad \text{fully ionized gas}$$

Here, the number density of particles is  $2n_H$  in a fully-ionized hydrogen gas (downstream) and thus the factor 2 in  $c_2$ .

- In summary, the equations are

$$\begin{aligned}\rho_1 u_1 &= \rho_2 u_2 = m_i J \\ \rho_1 (u_1^2 + c_1^2) &= \rho_2 (u_2^2 + c_2^2)\end{aligned}$$

- We assume that  $\rho_1$  and  $u_1$  are known, and we seek to solve for the unknown  $\rho_2$  and  $u_2$ . We obtain a simple quadratic equation for  $x \equiv \rho_1/\rho_2 = u_2/u_1$ .

$$\begin{aligned}\frac{\rho_1}{\rho_2} (u_1^2 + c_1^2) &= \left(\frac{\rho_1}{\rho_2}\right)^2 u_1^2 + c_2^2 \\ u_1^2 x^2 - (u_1^2 + c_1^2) x + c_2^2 &= 0 \quad \longrightarrow \quad x = \frac{1}{2u_1^2} \left[ (u_1^2 + c_1^2) \pm \sqrt{(u_1^2 + c_1^2)^2 - 4u_1^2 c_2^2} \right]\end{aligned}$$

Then, the ratios between densities and velocities are:

$$\frac{u_2}{u_1} = \frac{\rho_1}{\rho_2} = \frac{1}{2u_1^2} \left[ (u_1^2 + c_1^2) \pm \sqrt{(u_1^2 + c_1^2)^2 - 4u_1^2 c_2^2} \right]$$

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{1}{2c_2^2} \left[ (u_1^2 + c_1^2) \mp \sqrt{(u_1^2 + c_1^2)^2 - 4u_1^2 c_2^2} \right]$$

- 
- The roots are real if and only if

$$\begin{aligned} f(u_1) &\equiv (u_1^2 + c_1^2)^2 - 4u_1^2 c_2^2 \\ &= (u_1^2 + c_1^2 + 2u_1 c_2)(u_1^2 + c_1^2 - 2u_1 c_2) \geq 0 \end{aligned}$$

This requires:

$$\begin{aligned} u_1^2 + c_1^2 - 2u_1 c_2 &\geq 0 \\ \left[ u_1 - \left( c_2 + \sqrt{c_2^2 - c_1^2} \right) \right] \left[ u_1 - \left( c_2 - \sqrt{c_2^2 - c_1^2} \right) \right] &\geq 0 \end{aligned}$$

Therefore,

$$u_1 \geq u_R \equiv c_2 + \sqrt{c_2^2 - c_1^2} \quad \text{or} \quad u_1 \leq u_D \equiv c_2 - \sqrt{c_2^2 - c_1^2}$$

We also note that

$$\begin{aligned} u_1^2 + c_1^2 + 2u_1 c_2 &= \left[ u_1 + \left( c_2 + \sqrt{c_2^2 - c_1^2} \right) \right] \left[ u_1 + \left( c_2 - \sqrt{c_2^2 - c_1^2} \right) \right] \\ \rightarrow \quad f(u_1) &= (u_1^2 - u_R^2)(u_1^2 - u_D^2) \end{aligned}$$

$$\begin{aligned} \frac{u_2}{u_1} &= \frac{\rho_1}{\rho_2} = \frac{1}{2u_1^2} \left[ (u_1^2 + c_1^2) \pm \sqrt{(u_1^2 - u_R^2)(u_1^2 - u_D^2)} \right] \\ \frac{\rho_2}{\rho_1} &= \frac{u_1}{u_2} = \frac{1}{2c_2^2} \left[ (u_1^2 + c_1^2) \mp \sqrt{(u_1^2 - u_R^2)(u_1^2 - u_D^2)} \right] \end{aligned}$$

- The rapidly propagating ionization fronts, with  $u_1 \geq u_R$  are called ***R-type fronts (R stands for “rarefied” or rapid)***. The dilatory ionization fronts are called ***D-type fronts (D stands for “dense” or dilatory)***.
  - ▶ An R-type front has  $u_1 \geq u_R > c_2 > c_1$ , and is supersonic with respect to the neutral medium.
  - ▶ A D-type front has  $u_1 \leq u_D < c_1 < c_2$ , and is subsonic with respect to the neutral medium.
- For a given front propagation speed  $u_1$ , there are two possible values of the density ratio  $\rho_2/\rho_1$  across the ionization front as a function of the propagation speed  $u_1$ .
  - ▶ The front that has the ***larger density contrast*** is called a ***strong*** front.
  - ▶ The front that has the ***smaller density contrast*** is called a ***weak*** front.
  - ▶ Thus, there are four types of ionization front: weak R, strong R, weak D, strong D.

$$\frac{\rho_2}{\rho_1} = \frac{1}{2c_2^2} \left[ (u_1^2 + c_1^2) \pm \sqrt{(u_1^2 - u_R^2)(u_1^2 - u_D^2)} \right]$$

R-front:  $u_1 \geq u_R$  weak –; strong +  
 D-front:  $u_1 \leq u_D$  weak +; strong –

- ▶ The solutions for  $u_1 = u_R$  and  $u_1 = u_D$  are called “R-critical” and “D-critical”, respectively.

- Since  $c_2$  exceeds  $c_1$  by about one or two order of magnitude in an interstellar ionization front ( $c_2 \gg c_1$ ),

$$u_R = c_2 + \sqrt{c_2^2 - c_1^2} \approx c_2 + c_2 \left( 1 - \frac{1}{2} \frac{c_1^2}{c_2^2} - \frac{1}{8} \frac{c_1^4}{c_2^4} \right)$$

$$u_D = c_2 - \sqrt{c_2^2 - c_1^2} \approx c_2 - c_2 \left( 1 - \frac{1}{2} \frac{c_1^2}{c_2^2} - \frac{1}{8} \frac{c_1^4}{c_2^4} \right)$$

$$u_R \approx 2c_2 \left( 1 - \frac{1}{4} \frac{c_1^2}{c_2^2} \right) > c_2 > c_1 > u_D$$

$$u_D \approx \frac{1}{2} \frac{c_1^2}{c_2} \left( 1 + \frac{1}{4} \frac{c_1^2}{c_2^2} \right) < c_1 < c_2 < u_R$$

- Approximate solutions:

**R-critical**       $\frac{\rho_2}{\rho_1} \approx 2 \left( 1 - \frac{1}{4} \frac{c_1^2}{c_2^2} \right)$       for  $u_1 = u_R$

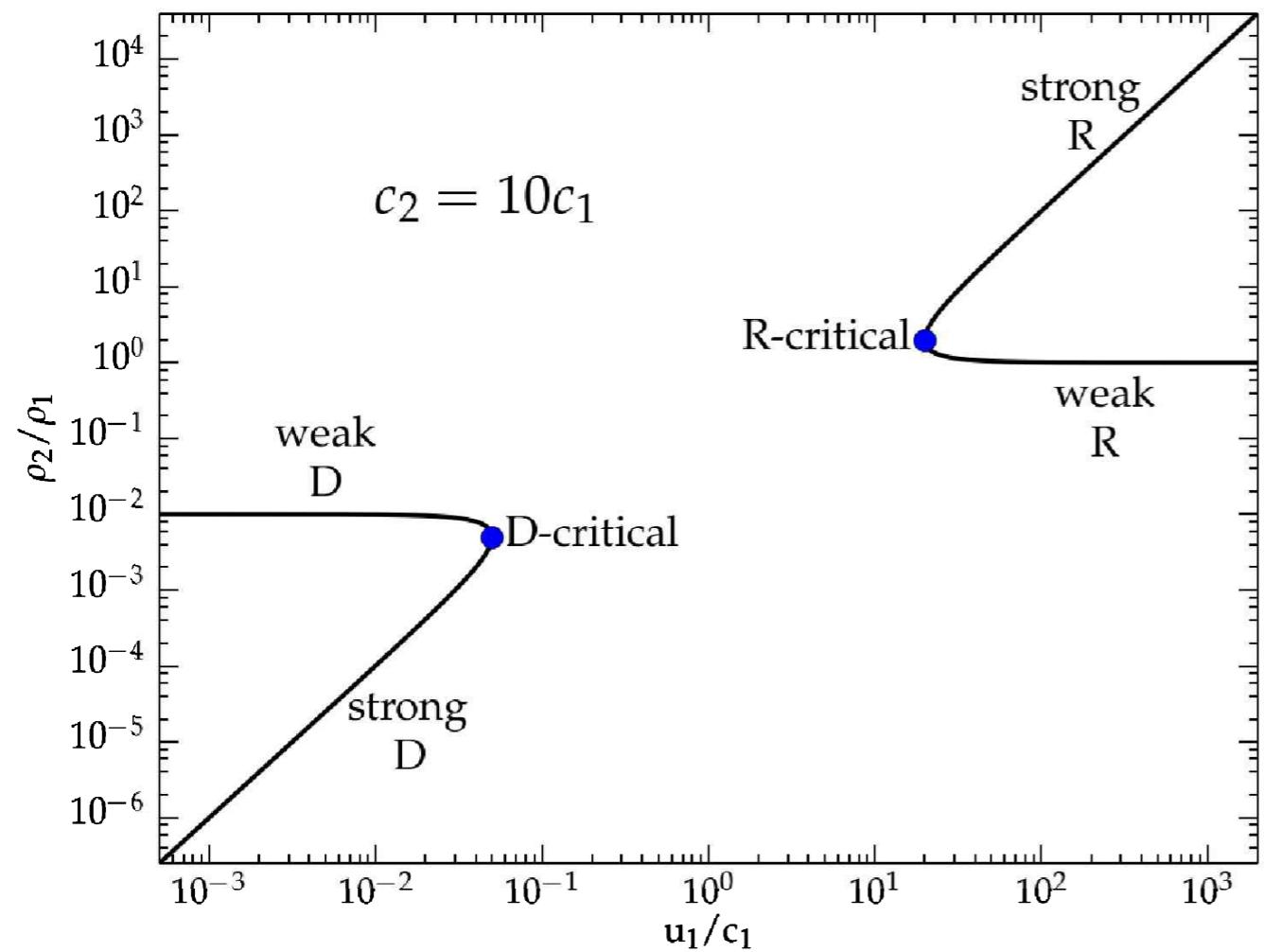
**D-critical**       $\frac{\rho_2}{\rho_1} \approx \frac{1}{2} \frac{c_1^2}{c_2^2} \left( 1 + \frac{1}{4} \frac{c_1^2}{c_2^2} \right)$       for  $u_1 = u_D$

**weak R-front**       $\frac{\rho_2}{\rho_1} \approx 1 + \frac{c_2^2}{u_1^2}$       for  $u_1 \gg u_R$

**strong R-front**       $\frac{\rho_2}{\rho_1} \approx \frac{u_1^2}{c_2^2} - 1$

**weak D-front**       $\frac{\rho_2}{\rho_1} \approx \frac{c_1^2}{c_2^2} - \frac{u_1^2}{c_1^2}$       for  $u_1 \ll u_D$

**strong D-front**       $\frac{\rho_2}{\rho_1} \approx \frac{u_1^2}{c_1^2} \left( 1 + \frac{c_2^2}{c_1^4} u_1^2 \right)$



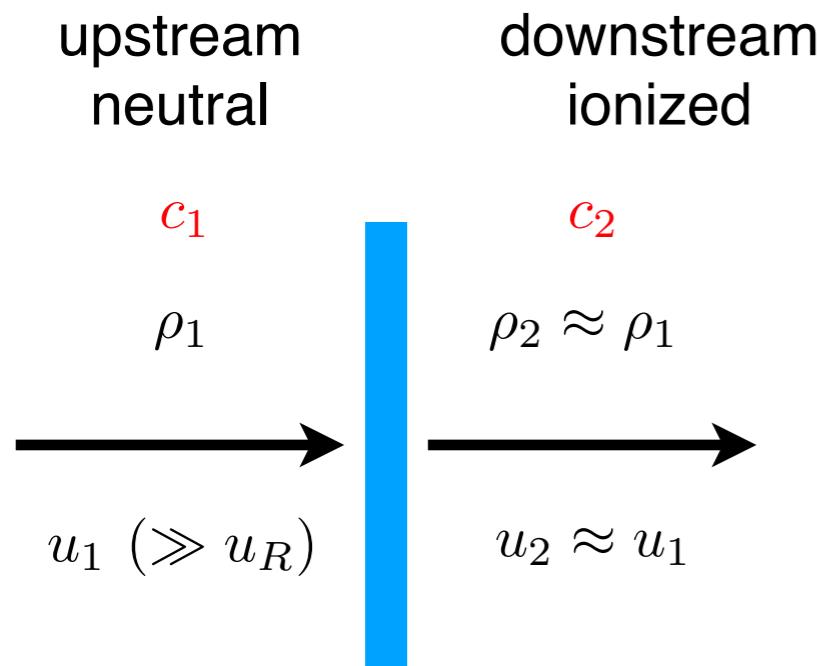
	$u_R \approx 2c_2, \quad u_D \approx \frac{1}{2} \frac{c_1^2}{c_2}$
R-critical	$\frac{\rho_2}{\rho_1} \approx 2$ for $u_1 = u_R$
D-critical	$\frac{\rho_2}{\rho_1} \approx \frac{1}{2} \frac{c_1^2}{c_2^2}$ for $u_1 = u_D$
strong R-front	$\frac{\rho_2}{\rho_1} \approx \frac{u_1^2}{c_2^2}$ for $u_1 \gg u_R$
weak R-front	$\frac{\rho_2}{\rho_1} \approx 1$
weak D-front	$\frac{\rho_2}{\rho_1} \approx \frac{c_1^2}{c_2^2} \Rightarrow \rho_1 c_1^2 \approx \rho_2 c_2^2$ for $u_1 \ll u_D$
strong D-front	$\frac{\rho_2}{\rho_1} \approx \frac{u_1^2}{c_1^2}$

Figure 4.11 [Ryden]

- We note that the four types are not all relevant to H II regions.
  - ▶ For instance, the strong R type means a lower density in the upstream (neutral gas). The strong R-type fronts are in fact unstable (Rayleigh-Taylor instability). In H II regions, the neutral gas has a higher density than the ionized gas. (or the same density at the initial stage).
  - ▶ The strong D type implies that the density in neutral gas increases forever when the ionization front slows down.
- **The fronts relevant to the H II regions are weak R-front and weak D-front.**

# Evolution of Ionization Front

## [1] Weak R front



We will assume that

$$c_1 = (kT_1/m_H)^{1/2}$$

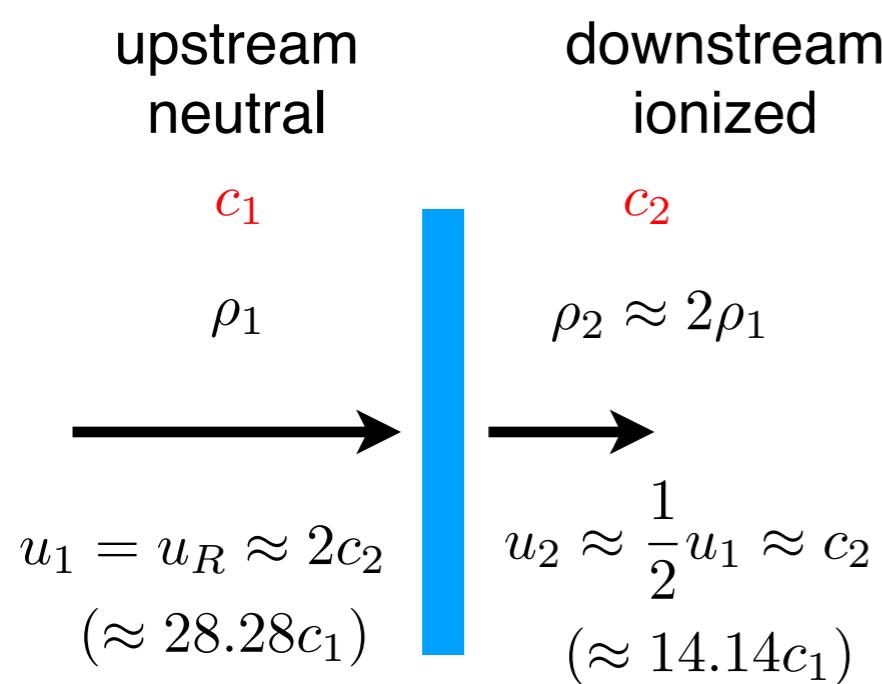
$$c_2 = (2kT_2/m_H)^{1/2} = (2T_2/T_1)^{1/2} c_1 = \sqrt{200} c_1$$

for  $T_1 = 10^2 \text{ K}$ ,  $T_2 = 10^4 \text{ K}$

### (1) Weak R front:

- Initially, the photon flux  $J$  is very large. Thus,  $u_1$  is very large, and the ionization front is initially a weak R-type front. The densities of neutral gas and ionized gas are nearly the same:  $\rho_2/\rho_1 \approx 1$ . (A weak R-type front compresses the gas only slightly.)
- As the ionization front expands, the flux of ionizing photons steadily decreases, and the propagation speed  $u_1$  of the front slows down.

## [2] R-critical front

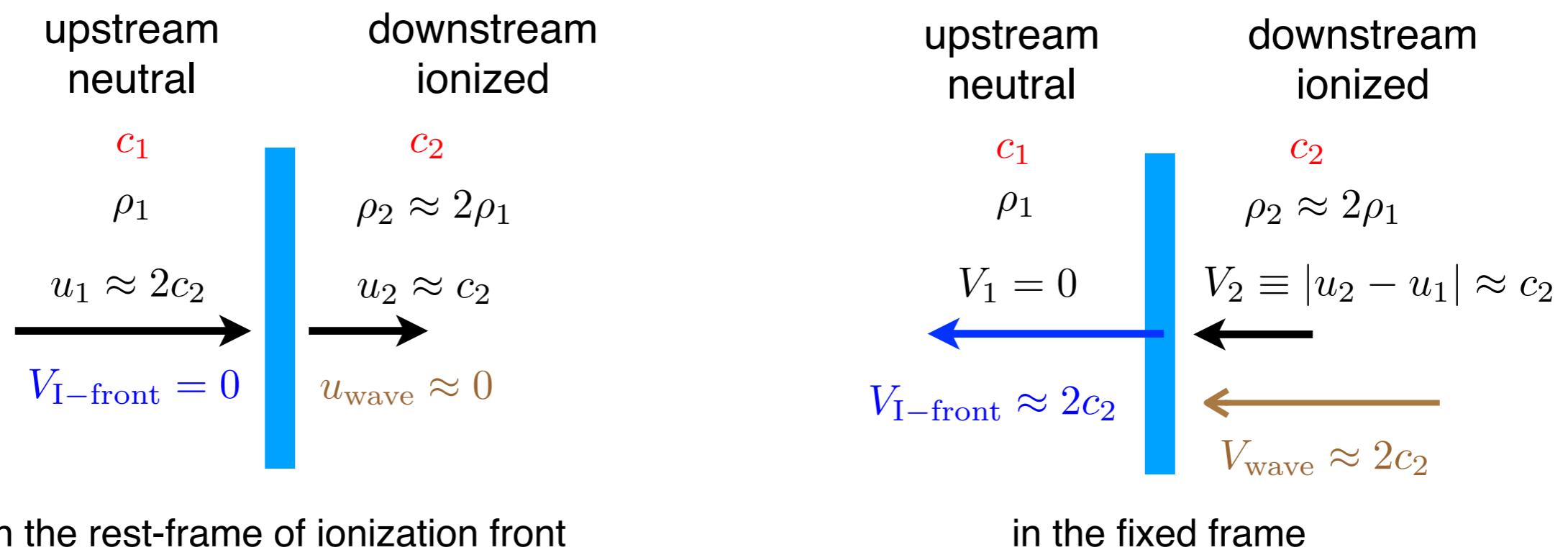


### (2) R-critical front:

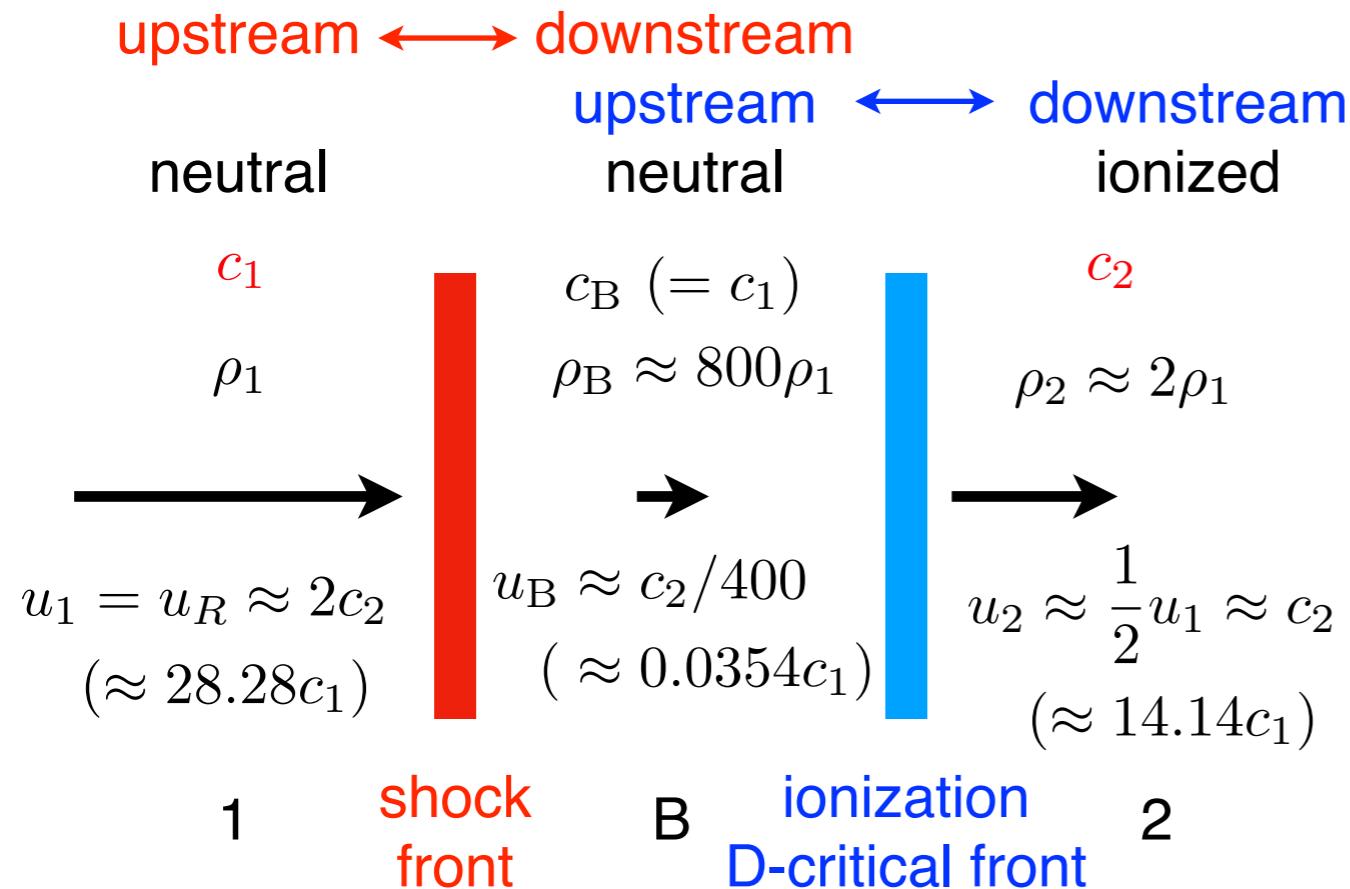
- Eventually, the speed drops to a value  $u_1 = u_R \approx 2c_2$ .
- At this point, the density ratio has risen to  $\rho_2/\rho_1 \approx 2$ .
- The speed of the ionized gas is  $u_2 \approx (1/2)u_R \approx c_2$  relative to the ionization front, or  $u_2 - u_1 \approx -c_2$  in a fixed frame of reference.
- As the ionization front slows down further, the R-type front can no longer exist.

● How does the evolution proceed once the ionization front becomes R-critical?

- When the R-critical condition is reached, the gas in the H II region just behind the front is moving at a speed equal to  $c_2 \gg c_1$ .
- This should derive a shock wave into the pre-ionization front gas. Before this point, the large pressure discrepancy between the H II region and the H I region ahead of it has no chance to act dynamically, because the ionization front races ahead with speed  $u_1$  so much faster than a pressure wave can catch it.
- When the ionization front slows down to a speed  $u_1 = u_R \approx 2c_2$ , however, the pressure wave (moving at a speed  $c_2$  on top of the speed  $u_2 \approx c_2$  that the H II fluid itself moves) can catch up with the ionization front and overtake it.
- In doing so, the pressure wave will steepen into a shock wave, thereby compressing the atomic gas behind it into a denser state that the lagging ionization front then has to eat into.



### [3] D-critical front



### (3) D-critical front:

- As the ionization front slows down further, the R-type front can no longer exit. What happens next is that ***the R-critical ionization front splits into a pair of fronts (shock front + ionization front)***.
- **A leading shock front is followed by a D-critical ionization front.** The shock front is the boundary between two regions of gas with different density, pressure, and temperature, but no necessarily different ionization states. The shock front propagates with a supersonic speed relative to the gas in the upstream of the shock front.

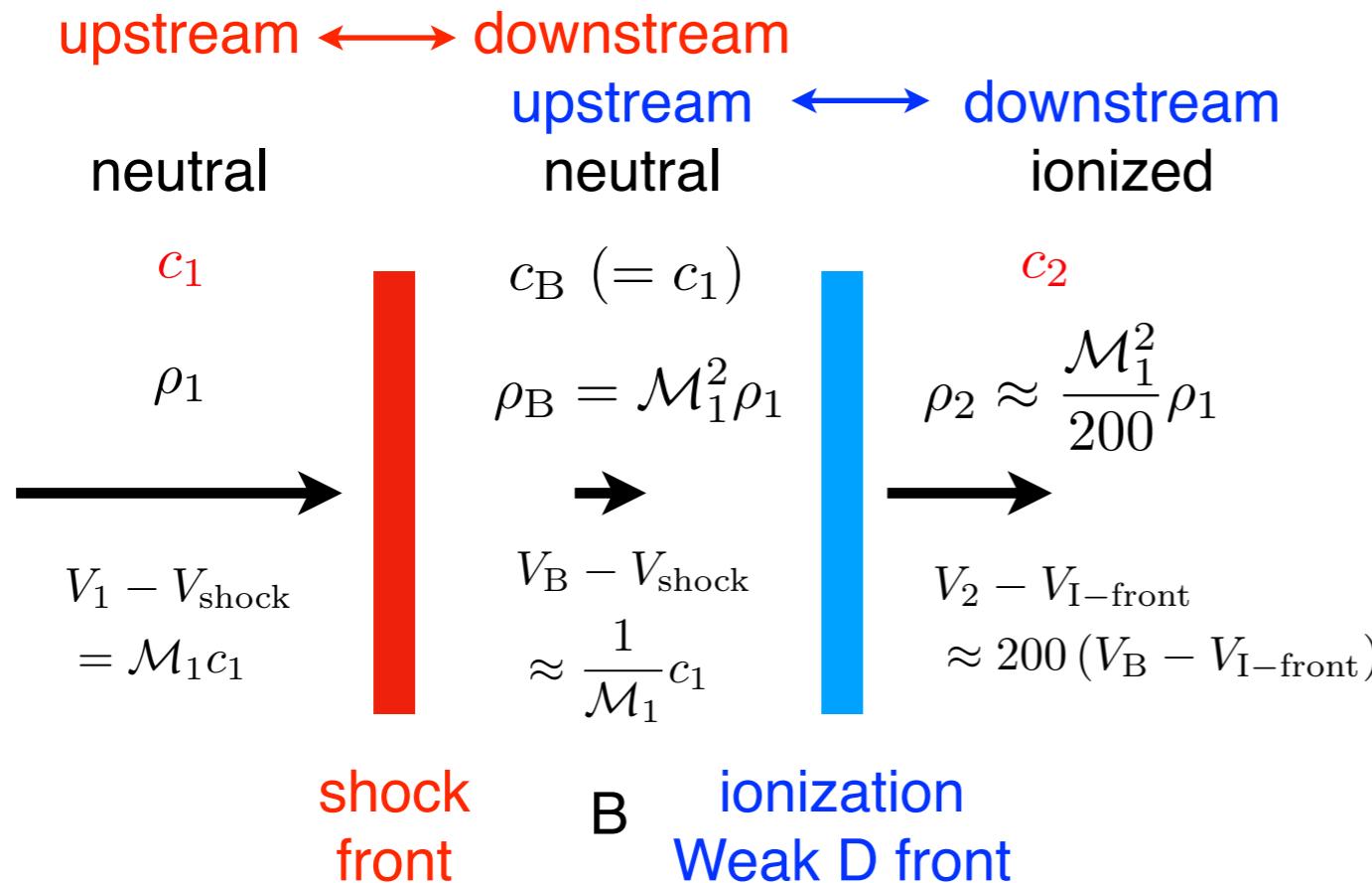
- We will assume ***an isothermal shock***. Then, the sound speed of the shocked region (B) must be  $c_s = c_1$  (from the Rankin-Hugoniot jump condition). Then, using the condition for the D-critical, we obtain the density and speed of the shocked region (B):

$$\begin{aligned}\frac{\rho_2}{\rho_s} &\approx \frac{1}{2} \frac{c_1^2}{c_2^2} = \frac{1}{400} \\ \rho_s &\approx 400\rho_2 \approx 800\rho_1\end{aligned}\quad \begin{aligned}\frac{u_s}{u_2} &= \frac{\rho_2}{\rho_s} \\ u_s &\approx \frac{1}{400}u_2 \approx \frac{1}{400}c_2 \approx \frac{1}{\sqrt{800}}c_1 = 0.0354c_1\end{aligned}$$

$$\longrightarrow \begin{aligned}\rho_s &\approx 800\rho_1 \\ u_s &\approx 0.0354c_1 \\ (\mathcal{M}_1 &= \sqrt{800})\end{aligned}$$

- The shocked region (B) has a very high density, and is almost stationary relative to the ionization front. The velocities  $u_1, u_s, u_2$  are measured in the rest-frame of I-front. The R-critical condition between 1 and 2 is still satisfied.

## [4] Weak D front



### (4) Weak D front:

- As the H II region expands still further, the leading shock front gradually weakens and the trailing D-critical front develops into a weak D-type front.
- Notice that the weakest of weak-D ionization fronts corresponds to the density discontinuity:

$$\frac{\rho_2}{\rho_B} = \frac{c_1^2}{c_2^2}$$

This is the condition for the static pressure equilibrium in isothermal gas,

$$\rho_2 c_2^2 = \rho_B c_1^2 \quad (P = \rho c_s^2)$$

the state that we expect for the final Strömgren sphere.

- The condition for the weak D-type front must be satisfied between the regions “B” and “2”.
- In addition to this condition, The shock jump condition should be satisfied between the regions “1” and “B”.
- However, notice that the velocities of the shock front and the ionization front can be different, in general.

$$V_{\text{shock}} \neq V_{\text{I-front}}$$

# Homework (due date: 05/15)

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[Q13]

- The observed spectrum of an HII region has

$$\frac{I([\text{O III}]4364.4 \text{\AA})}{I([\text{O III}]5008.2 \text{\AA})} = 0.003 ,$$

$$\frac{I([\text{O II}]3729.8 \text{\AA})}{I([\text{O II}]3727.1 \text{\AA})} = 1.2 .$$

- If interstellar reddening is assumed to be negligible, estimate the electron temperature  $T$  and the electron density  $n_e$ .
- Now suppose that it is learned that there is reddening due to intervening dust with

$$A(4364.4 \text{\AA}) - A(5008.2 \text{\AA}) = 0.31 \text{ mag}$$

Re-estimate  $T$  and  $n_e$ . You may find it convenient to use the following equations.

$$\left. \frac{F_{\lambda_2}}{F_{\lambda_1}} \right|_{\text{observed}} = \left. \frac{F_{\lambda_2}}{F_{\lambda_1}} \right|_{\text{intrinsic}} \exp [-(\tau_{\lambda_2} - \tau_{\lambda_1})] \quad \frac{A_{\lambda}}{\text{mag}} = 1.086 \tau_{\lambda}$$