Interstellar Medium (ISM)

Week 8
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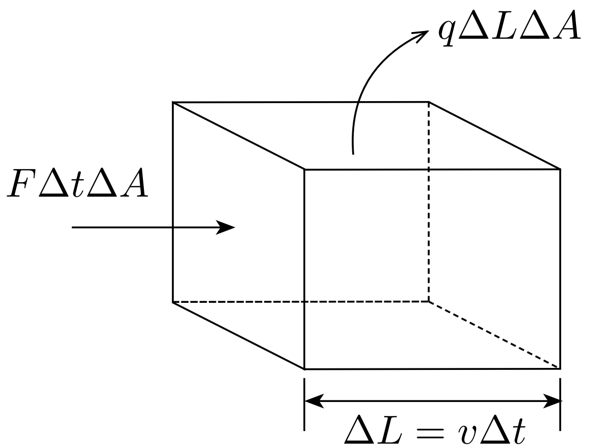
Hot Ionized Medium

- Gas Dynamics / Shock / Hot Gas Cooling
- Supernova Remnant
- Local Hot Bubble

Introduction to Gas Dynamics

- Assumption for hydrodynamics:
  - particle mean free path << size of the region
  - We will describe the equations for conservation of mass, momentum and energy, in 1D space.
- ***Definition***
  - Flux of a hydrodynamic quantity q :

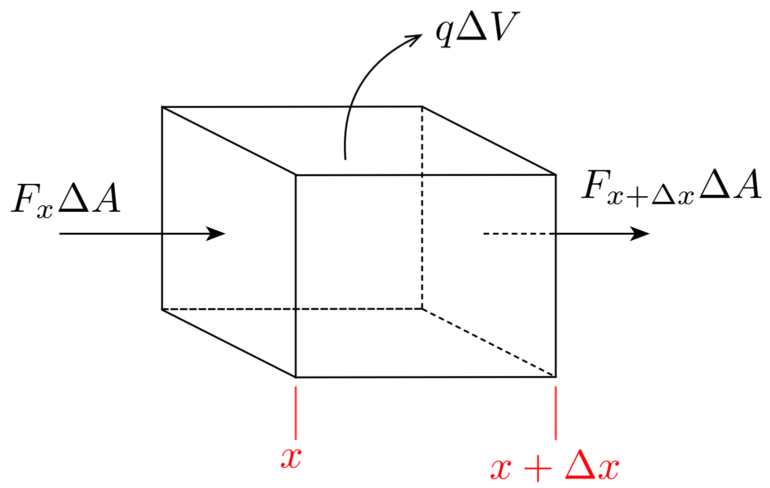
    Fluid moves a distance ΔL during a time interval Δt with a velocity v.



$$F\Delta t\Delta A = q\Delta L\Delta A \quad \rightarrow \quad \boxed{F = qv}$$

- ***Conservation equation for a quantity*** q
  - change of the quantity within a volume ΔV for a time interval Δt:

    Here, Δt and Δx are independent.



$$\frac{q\Delta V|_{t+\Delta t} - q\Delta V|_t}{\Delta t} = F\Delta A|_x - F\Delta A|_{x+\Delta x}$$

$$\frac{\partial q}{\partial t} = -\frac{\partial F}{\partial x} \longrightarrow \boxed{\frac{\partial q}{\partial t} = -\frac{\partial (qv)}{\partial x}}$$

  - Here, no sources or sinks of the quantity within ΔV were assumed. If any, the loss and gain terms could be added in the right-had side.

- ***Mass conservation (continuity equation)***

$$\frac{\partial \rho}{\partial t} = -\frac{\partial(\rho u)}{\partial x}$$

- ***Momentum conservation (Euler's equation)***
  - (temporal) change of the momentum within a volume

    = (spatial) change of momentum due to fluid flow + gas pressure acting on its surface

$$\frac{\partial}{\partial t}(\rho u) = -\frac{\partial}{\partial x}\left(\rho u^2 + P\right)$$

  - Further terms could be added in the right-hand side, accounting for forces due to gravity, magnetic fields, radiation field, and viscosity.
  - Viscous force is due to “internal friction” in the fluid (resistivity of the fluid to the flow), as two adjacent fluid parcels move relative to each other.)

$$\text{viscous force} \propto \frac{\partial^2 u}{\partial x^2}$$

The viscous force is usually much smaller than force due to gas pressure, but important in high-speed flows with large velocity gradients, as in accretion disks.

- *Energy conservation*

- The first law of thermodynamics states that

  heat added in a system = change in internal energy + work done on surroundings

$$dQ = dU + PdV$$

- Internal energy (per particle) for ideal gas is

$$U/N = \frac{3}{2}kT \quad \text{for monatomic gas (translation about 3 axes)}$$
$$U/N = \frac{5}{2}kT \quad \text{for diatomic gas} \quad \text{(+rotation about 2 axes)}$$
$$U/N = 3kT \quad \text{for polyatomic gas (+rotation about 3 axes)}$$

Here, N is the number of particles.

An ideal gas is a theoretical gas composed of many randomly moving point particles whose only interactions are perfectly elastic collisions (no viscosity or heat conduction).

- In general, the internal energy per particle is

$$U/N = \frac{f}{2}kT \quad (f = \text{degree of freedom})$$

At high temperature, molecules have access to an increasing number of vibrational degrees of freedom, as they start to bend and stretch.

- The ideal gas law (the equation of state) for a perfect Maxwellian distribution.

$$PV = NkT$$

- ***Specific heat capacity*** is the amount of *heat energy required to raise the temperature of a material per unit of mass*.
  - specific heat capacity at constant volume:

$$c_V \equiv \frac{1}{M}\left(\frac{\partial Q}{\partial T}\right)_V = \frac{1}{M}\left(\frac{\partial U}{\partial T}\right)_V$$

$$c_V = \frac{f}{2}\frac{k}{m}$$

M = total mass

$m = M/N$ = mass per particle

$m = \mu m_{\rm H}$

(μ = mean atomic weight per particle)

  - specific heat capacity at constant pressure:

$$c_P \equiv \frac{1}{M}\left(\frac{\partial Q}{\partial T}\right)_P = \frac{1}{M}\left(\frac{\partial U}{\partial T}\right)_P + \frac{P}{M}\left(\frac{\partial V}{\partial T}\right)_P = \frac{1}{M}\frac{f}{2}Nk + \frac{P}{M}\frac{Nk}{P}$$

$$\therefore \quad c_P = \frac{f+2}{2}\frac{k}{m} = c_V + \frac{k}{m}$$

  - Ratio of specific heat capacities:

$$\gamma \equiv \frac{c_P}{c_V} = \frac{f+2}{f} = \frac{5}{3} \quad \text{for monatomic gas}$$
$$= \frac{7}{5} \quad \text{for diatomic gas}$$
$$= \frac{4}{3} \quad \text{for polyatomic gas}$$

γ is called the adiabatic index.

$c_P > c_V$

This inequality implies that when pressure is held constant, some of the added heat goes into PdV work instead of into internal energy.

- Energy Conservation - limiting cases
  - ***Adiabatic flow*** - negligible heat transport (Internal energy is changed only by work).

$$dQ = dU + PdV = Mc_V dT + PdV$$
$$dQ = 0$$
$$\rightarrow PdV = -Mc_V dT$$

$$PV = NkT$$
$$\rightarrow VdP + PdV = NkdT$$

We combine two equation and eliminate dT term:

$$\begin{aligned} VdP + PdV &= -\frac{Nk}{Mc_V} PdV \\ &= -\frac{k}{m}\frac{1}{c_V} PdV \end{aligned} \longrightarrow \begin{aligned} VdP &= -\left(1 + \frac{k}{m}\frac{1}{c_V}\right) PdV \\ &= -\frac{1}{c_V}\left(c_V + \frac{k}{m}\right) PdV \\ &= -\gamma PdV \end{aligned}$$

$$\longrightarrow \frac{dP}{P} = -\gamma \frac{dV}{V}$$

We can rewrite this in terms of density:

$$\rho V = M$$
$$\rightarrow \rho dV + V d\rho = 0$$
$$\rightarrow \frac{d\rho}{\rho} = -\frac{dV}{V} \longrightarrow \frac{dP}{P} = \gamma \frac{d\rho}{\rho}$$

In summary,

$$\frac{dP}{P} = -\gamma \frac{dV}{V} \qquad P \propto \rho^{\gamma}$$
$$\frac{dP}{P} = \gamma \frac{d\rho}{\rho} \qquad P \propto V^{-\gamma}$$
$$\rightarrow T \propto V^{-(\gamma-1)}$$

adiabatic heating/cooling

- ***Isothermal flow*** - extremely efficient cooling (heat transport).

heat transport timescale << dynamic timescale

This implies the balance between heating and cooling, hence a constant temperature.

From the ideal gas law,

$$P = \frac{N}{V}kT = \rho\frac{kT}{m} \longrightarrow \begin{array}{l} P \propto \rho \\ P \propto V^{-1} \end{array}$$

- In general, we have

$$\begin{array}{l} P \propto \rho^\gamma \\ P \propto V^{-\gamma} \end{array} \quad (\gamma = 1 \text{ for isothermal gas})$$

A gas that has an equation of state with this power-law form is called a ***polytope***, from the Greek polytropos, meaning “turning many ways” or “versatile.”

(A polystrope should not be confused with a polytrope, which is the n-dimensional generalization of a 2D polygon and 3D polyhedron.)

- ***Specific internal energy*** of the gas (per unit mass):

$$\epsilon \equiv U/M$$
$$U/N = \frac{f}{2}kT$$

$$\longrightarrow \quad \epsilon = \frac{f}{2}\frac{kT}{m} \quad \text{or} \quad \epsilon = \frac{1}{\gamma - 1}\frac{kT}{m} = \frac{1}{\gamma - 1}\frac{P}{\rho}$$

- Total Energy (per unit volume):
  - ***Internal energy per unit volume***:
  - ***Kinetic energy due to bulk motion, per unit volume***:
  - ***Work on unit volume***:

$$\mathcal{E}_{\rm int} = \rho\epsilon = \frac{1}{\gamma - 1}P$$
$$\mathcal{E}_{\rm kin} = \rho\frac{u^2}{2}$$
$$\mathcal{E}_{\rm mech} = \frac{PdV}{dV} = P$$

$$\mathcal{E} = \mathcal{E}_{\rm int} + \mathcal{E}_{\rm kin} + \mathcal{E}_{\rm mech}$$
$$= \rho\left(\frac{u^2}{2} + \epsilon\right) + P$$

$$\longrightarrow \quad \mathcal{E} = \rho\frac{u^2}{2} + \frac{\gamma}{\gamma - 1}P$$

- ***Energy conservation***:

$$\frac{\partial \mathcal{E}}{\partial t} = -\frac{\partial (u\mathcal{E})}{\partial x}$$

$$\frac{\partial}{\partial t}\left(\rho\frac{u^2}{2} + \frac{\gamma}{\gamma - 1}P\right) = -\frac{\partial}{\partial x}\left[u\left(\rho\frac{u^2}{2} + \frac{\gamma}{\gamma - 1}P\right)\right]$$

Sound Wave

- Suppose that we are surrounded by an ideal gas with a plane parallel symmetry:
  - We consider a region where the gas has initially a uniform density, pressure, and no bulk velocity: $\rho_0, \; P_0, \; u_0 = 0$

    In the uniform gas, we introduce small perturbations of the form:

$$\begin{aligned} \rho(x,t) &= \rho_0 + \rho_1(x,t) \\ u(x,t) &= u_1(x,t) \\ P(x,t) &= P_0 + P_1(x,t) \end{aligned} \longrightarrow \begin{aligned} P_1 &= P - P_0 \\ &\propto (\rho_0 + \rho_1)^\gamma - \rho_0^\gamma \\ &\propto \gamma \rho_0^{\gamma-1} \rho_1 \end{aligned} \longrightarrow P_1 = \frac{\gamma P_0}{\rho_0} \rho_1$$

We obtain:

$$\begin{aligned} \frac{\partial \rho}{\partial t} &= -\frac{\partial(\rho u)}{\partial x} \\ \rho \frac{\partial u}{\partial t} &= -\rho u \frac{\partial u}{\partial x} - \frac{\partial P}{\partial x} \end{aligned} \longrightarrow \begin{aligned} \frac{\partial \rho_1}{\partial t} &= -\rho_0 \frac{\partial u_1}{\partial x} \\ \rho_0 \frac{\partial u_1}{\partial t} &= -\frac{\partial P_1}{\partial x} = -\frac{\gamma P_0}{\rho_0} \frac{\partial \rho_1}{\partial x} \end{aligned} \longrightarrow \boxed{\frac{\partial^2 \rho_1}{\partial t^2} = -\frac{\gamma P_0}{\rho_0} \frac{\partial^2 \rho_1}{\partial x^2}}$$

  - The resulting equation represents a sound wave (acoustic wave) with a constant sound speed:

$$\boxed{c_s = \left(\frac{\gamma P}{\rho}\right)^{1/2} = \left(\frac{\gamma k T}{m}\right)^{1/2} \qquad c_s \propto \rho^{(\gamma-1)/2}}$$

For $\gamma > 1$, sound travels more rapidly in a denser gas.

- The sound speed is of the same order as the mean thermal velocity:

$$c_s = 1.2\,\mathrm{km\,s^{-1}} \left(\frac{\gamma}{5/3}\right)^{1/2} \left(\frac{m}{m_p}\right)^{-1/2} \left(\frac{T}{100\,\mathrm{K}}\right)^{1/2}$$

$(m_p = \text{proton mass})$

- ***Sound crossing time***:
  - sound crossing time = time it takes for a signal to cross a region of size *L*:

$$t_{\mathrm{cross}} = L/c_s$$

  - A small pressure gradient tends to be smoothed out within the sound crossing time. Generally, when a stationary gas is disturbed, the resultant changes in velocity, density, pressure, and temperature are communicated downstream at the sound speed.

    Fast changes occurring on timescales $\ll t_{\mathrm{cross}}$ will survive.

    Slow changes occurring on timescales $\gg t_{\mathrm{cross}}$ will be damped.

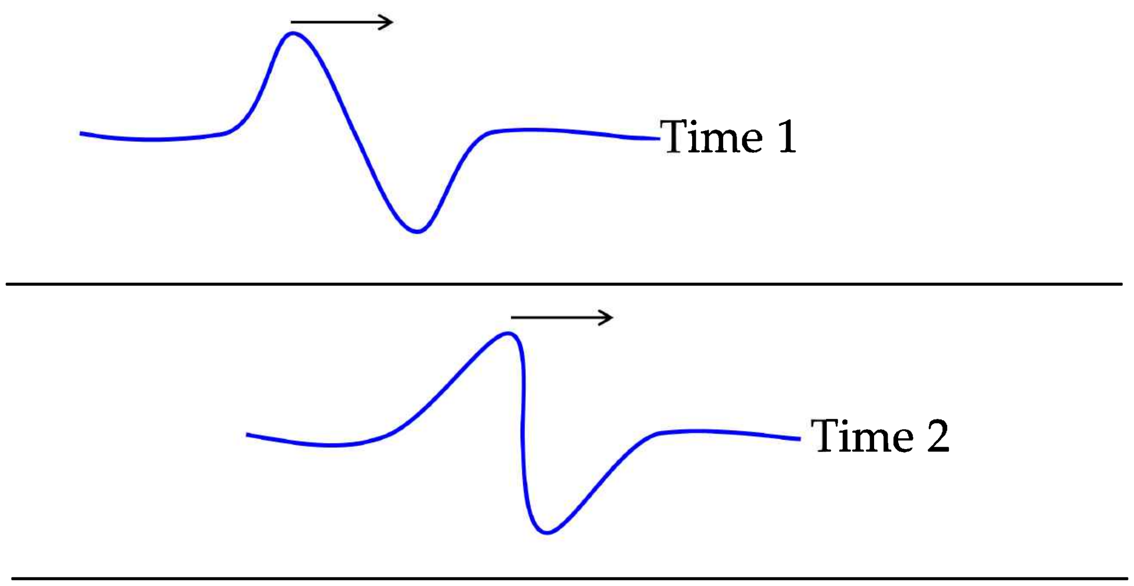
- ***Mach number*** = gas velocity / sound speed

$$\mathcal{M} \equiv u/c_s$$

Shock

- Shock
  - A low-amplitude sound wave traveling through a medium will be adiabatic; that is it will not increase the entropy of the gas through which it passes.
  - For an adiabatic process, the equation of state for the gas is

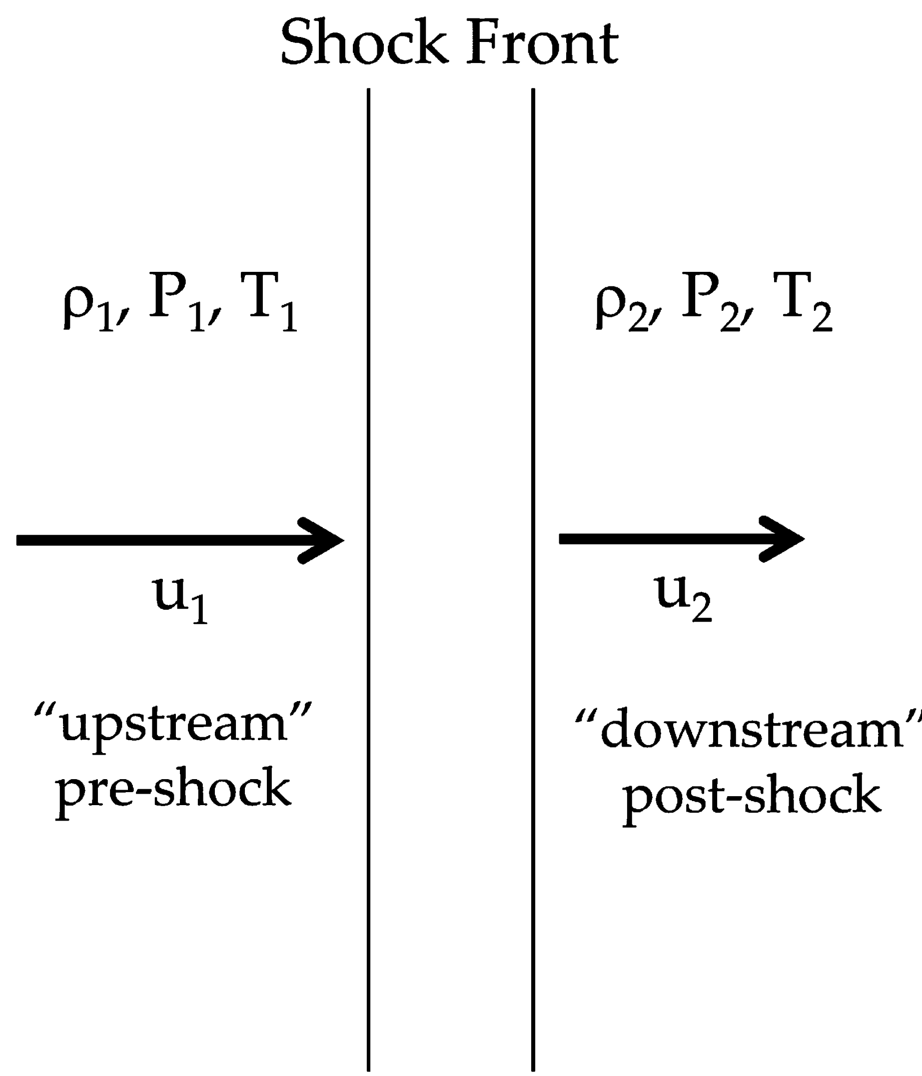
$$c_s \propto \rho^{(\gamma-1)/2}$$



  - Thus, for $\gamma > 1$, sound travels more rapidly in a denser gas.
  - ***For a supersonic gas, the motion itself is faster than the speed of communication, and instead of a smooth transition, the physical quantities (density, pressure, and temperature) undergo a sudden change in values over a small distance.*** This phenomenon is referred to as a shock.
  - We define the shock front as the region over which the velocity, density, and pressure of the gas undergo sudden changes. The shock front is a layer whose thickness is comparable to the mean free path between particle collisions.
  - The ordinary sound that we hear every day will not, in practice, steepen into shocks.
  - However, high amplitude pressure fluctuations will rapidly steepen into shocks.

Shock Front

- Jump condition (***Rankine-Hugoniot conditions***)
  - Let
    $$\rho = \text{mass density},\ T = \text{temperature},$$
    $$m = \text{mean molecular mass}$$
  - If a patch is small compared to the shock front's radius of curvature, then we can treat the shock front as if it has ***plane parallel*** symmetry.
  - ***It is convenient to use a frame of reference in which the shock front is stationary.***
  - Let us consider a shock propagating with velocity V_s into a gas that is previously at rest. In the frame of reference of the shock, the gas in the pre-shock region is approaching at a velocity of $-V_s$.
  - In this frame, the bulk velocity $u_1 = -V_s$ of the pre-shock (upstream) gas toward the shock front. The bulk velocity u_2 of the post-shock (downstream) gas points away from the shock front.



Plane parallel steady-state shock, in the reference frame of the shock front.

- Let's consider a steady state solution.
  - The gas properties immediately before being shocked ("1") and immediately after being shocked ("2") are obtained from the conservation laws:

$$\rho_1 u_1 = \rho_2 u_2$$
$$\rho_1 u_1^2 + P_1 = \rho_2 u_2^2 + P_2$$
$$u_1 \left(\rho_1 \frac{u_1^2}{2} + \frac{\gamma}{\gamma - 1} P_1 \right) = u_2 \left(\rho_2 \frac{u_2^2}{2} + \frac{\gamma}{\gamma - 1} P_2 \right)$$

Dividing the third equation with the first equation:

$$\frac{u_1^2}{2} + \frac{\gamma}{\gamma - 1} \frac{P_1}{\rho_1} = \frac{u_2^2}{2} + \frac{\gamma}{\gamma - 1} \frac{P_2}{\rho_2}$$

In summary,

$$\rho_1 u_1 = \rho_2 u_2$$
$$\rho_1 u_1^2 + P_1 = \rho_2 u_2^2 + P_2$$
$$\frac{u_1^2}{2} + \frac{\gamma}{\gamma - 1} \frac{P_1}{\rho_1} = \frac{u_2^2}{2} + \frac{\gamma}{\gamma - 1} \frac{P_2}{\rho_2}$$

Here, *we assume that an adiabatic index is the same on both sides of the shock front.*

- From the three equations, we should be able to derive the changes, ρ_2/ρ_1, u_2/u_1, and P_2/P_1 across the shock.

  It is convenient to use a dimensionless number, the Mach number of the upstream:

$$\mathcal{M}_1 = u_1/c_1,\ c_1^2 = \frac{\gamma P_1}{\rho_1} \quad \rightarrow \quad P_1 = \frac{u_1^2 \rho_1}{\gamma \mathcal{M}_1^2}$$

(1) To find the equation for densities:

$$\rho_1 u_1^2 + P_1 = \rho_2 u_2^2 + P_2$$

$$\rho_1 u_1 = \rho_2 u_2 \text{ and } P_1 = \frac{u_1^2 \rho_1}{\gamma \mathcal{M}_1^2} \quad \rightarrow \quad \rho_1 u_1^2 + u_1^2 \frac{\rho_1}{\gamma \mathcal{M}_1^2} = \frac{(\rho_1 u_1)^2}{\rho_2} + P_2$$

$$\rightarrow \quad P_2 = \rho_1 u_1^2 \left(1 + \frac{1}{\gamma \mathcal{M}_1^2} - \frac{\rho_1}{\rho_2}\right)$$

Inserting these relations into the energy conservation equation:

$$\frac{u_1^2}{2} + \frac{\gamma}{\gamma - 1} \frac{P_1}{\rho_1} = \frac{u_2^2}{2} + \frac{\gamma}{\gamma - 1} \frac{P_2}{\rho_2}$$

$$\rightarrow \quad \frac{u_1^2}{2} + \frac{1}{\gamma - 1} \frac{u_1^2}{\mathcal{M}_1^2} = \frac{1}{2}\left(\frac{\rho_1 u_1}{\rho_2}\right)^2 + \frac{\gamma}{\gamma - 1} \frac{\rho_1 u_1^2}{\rho_2} \left(1 + \frac{1}{\gamma \mathcal{M}_1^2} - \frac{\rho_1}{\rho_2}\right)$$

$$\rightarrow \quad \frac{1}{2} + \frac{1}{\gamma - 1} \frac{1}{\mathcal{M}_1^2} = \frac{1}{2}\left(\frac{\rho_1}{\rho_2}\right)^2 + \frac{\gamma}{\gamma - 1} \frac{\rho_1}{\rho_2} \left(1 + \frac{1}{\gamma \mathcal{M}_1^2} - \frac{\rho_1}{\rho_2}\right)$$

$$ax^2 + bx - c = 0$$

where $x = \dfrac{\rho_1}{\rho_2}$

$$a = \frac{1}{2} - \frac{\gamma}{\gamma - 1}$$

$$b = \frac{\gamma}{\gamma - 1} + \frac{1}{(\gamma - 1)\mathcal{M}_1^2}$$

$$c = \frac{1}{2} + \frac{1}{(\gamma - 1)\mathcal{M}_1^2}$$

$$x = \frac{b^2 \pm \sqrt{b^2 + 4ac}}{2a}$$

$$\frac{\rho_1}{\rho_2} = \frac{-\left[\frac{\gamma}{\gamma-1} + \frac{1}{(\gamma-1)\mathcal{M}_0^2}\right] \pm \frac{\mathcal{M}_1^2-1}{\mathcal{M}_1^2(\gamma-1)}}{1-\frac{2\gamma}{\gamma-1}}$$

$$= \frac{\gamma\mathcal{M}_1^2 + 1 \mp (\mathcal{M}_1^2 - 1)}{(\gamma+1)\mathcal{M}_1^2}$$

$\longrightarrow$

$$\frac{\rho_1}{\rho_2} = 1 \quad \text{or} \quad \frac{(\gamma-1)\mathcal{M}_1^2 + 2}{(\gamma+1)\mathcal{M}_1^2}$$

$$\therefore \ \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)\mathcal{M}_1^2}{(\gamma-1)\mathcal{M}_1^2 + 2}$$

(2) Now, we obtain the equation for pressures:

Divide the following equation

$$P_2 = \rho_1 u_1^2 \left(1 + \frac{1}{\gamma\mathcal{M}_1^2} - \frac{\rho_1}{\rho_2}\right)$$

with this

$$P_1 = \frac{u_1^2 \rho_1}{\gamma\mathcal{M}_1^2}$$

$\longrightarrow$

$$\frac{P_2}{P_1} = \gamma\mathcal{M}_1^2\left(1 + \frac{1}{\gamma\mathcal{M}_1^2} - \frac{\rho_1}{\rho_2}\right)$$

$$= \gamma\mathcal{M}_1^2\left(1 + \frac{1}{\gamma\mathcal{M}_1^2} - \frac{(\gamma-1)\mathcal{M}_1^2 + 2}{(\gamma+1)\mathcal{M}_1^2}\right)$$

$$\therefore \ \frac{P_2}{P_1} = \frac{2\gamma\mathcal{M}_1^2 - (\gamma-1)}{\gamma+1}$$

(3) Using the ideal gas law:

$$P = \frac{\rho k T}{m} \quad \longrightarrow \quad \frac{T_2}{T_1} = \frac{\rho_1}{\rho_2}\frac{P_2}{P_1}$$

Using the equations for densities and pressures:

$$\therefore \ \frac{T_2}{T_1} = \frac{\left[(\gamma-1)\mathcal{M}_1^2 + 2\right]\left[2\gamma\mathcal{M}_1^2 - (\gamma-1)\right]}{(\gamma+1)^2\mathcal{M}_1^2}$$

In summary, we obtain the jump conditions:

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)\mathcal{M}_1^2}{(\gamma-1)\mathcal{M}_1^2+2} = \frac{u_1}{u_2}$$

$$\frac{P_2}{P_1} = \frac{2\gamma\mathcal{M}_1^2-(\gamma-1)}{\gamma+1}$$

$$\frac{T_2}{T_1} = \frac{\left[(\gamma-1)\mathcal{M}_1^2+2\right]\left[2\gamma\mathcal{M}_1^2-(\gamma-1)\right]}{(\gamma+1)^2\mathcal{M}_1^2}$$

In the lab frame, let V_s = shock velocity, $v_1,\ v_2$ = gas velocities in upstream (pre-shock) and downstream (post-shock), respectively $(v_1 = 0)$.

Using $u_1 = -V_s$ and $u_2 = v_2 - V_s$, we have

$$\frac{-V_s}{v_2 - V_s} = \frac{(\gamma+1)\mathcal{M}_1^2}{(\gamma-1)\mathcal{M}_1^2+2}$$

$$v_2 = \frac{2(\mathcal{M}_1^2-1)}{(\gamma+1)\mathcal{M}_1^2} V_s$$

Note a typo in Equation (16.12) of Kwok's book.

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)\mathcal{M}_1^2}{(\gamma-1)\mathcal{M}_1^2+2} = \frac{u_1}{u_2}$$
$$\frac{P_2}{P_1} = \frac{2\gamma\mathcal{M}_1^2-(\gamma-1)}{\gamma+1}$$
$$\frac{T_2}{T_1} = \frac{\left[(\gamma-1)\mathcal{M}_1^2+2\right]\left[2\gamma\mathcal{M}_1^2-(\gamma-1)\right]}{(\gamma+1)^2\mathcal{M}_1^2}$$

monatomic gas: $\gamma = 5/3$

For a strong shock: $\mathcal{M}_1 \gg 1$

$$P_2 \approx \frac{2\gamma\mathcal{M}_1^2}{\gamma+1}P_1 \xrightarrow{P_1=c_1^2\frac{\rho_1}{\gamma}} \frac{2\gamma(u_1/c_1)^2}{\gamma+1} \quad c_1^2\frac{\rho_1}{\gamma}$$

$$T_2 \approx \frac{2(\gamma-1)\gamma}{(\gamma+1)^2}\mathcal{M}_1^2T_1 = \frac{2(\gamma-1)\gamma}{(\gamma+1)^2}\left(\frac{u_1}{c_1}\right)^2T_1$$

$$\frac{\rho_2}{\rho_1} \simeq \frac{\gamma+1}{\gamma-1} \qquad \frac{\rho_2}{\rho_1} \simeq 4$$
$$\frac{u_2}{u_1} \simeq \frac{\gamma-1}{\gamma+1} \qquad \frac{u_2}{u_1} \simeq \frac{1}{4}$$
$$P_2 \simeq \frac{2}{\gamma+1}\rho_1u_1^2 \qquad P_2 \simeq \frac{3}{4}\rho_1u_1^2$$
$$T_2 \simeq \frac{2(\gamma-1)}{(\gamma+1)^2}\frac{m}{k}u_1^2 \qquad T_2 \simeq \frac{3}{16}\frac{m}{k}u_1^2$$

speed of the downstream in the laboratory frame:

$$v_2 \simeq \frac{2}{(\gamma+1)}V_s \qquad v_2 \simeq \frac{3}{4}V_s$$

For an isothermal shock: $\gamma = 1$

$$\frac{\rho_2}{\rho_1} = \mathcal{M}_1^2 = \frac{u_1}{u_2}$$
$$P_2 = \mathcal{M}_1^2P_1 = \rho_1u_1^2 \longrightarrow u_1u_2 = c_1^2, \quad c_2 = c_1$$
$$T_2 = T_1$$

speed of the downstream in the laboratory frame:

$$v_2 = \left(1-\frac{1}{\mathcal{M}_1^2}\right)V_s$$

- Consider a strong shock
  - ***No matter how strong the shock is, the gas can only be compressed by a factor of at most 4***:

$$\frac{\rho_2}{\rho_1} \approx 4 \qquad \text{for } \gamma = 5/3$$

(monatomic gas)

$$P_2 \approx \frac{3}{4}\rho_1 u_1^2$$

$$T_2 \approx \frac{3}{16}\frac{m}{k}u_1^2$$

Note that

$$m = \frac{1.4 m_{\rm H}}{1.1} = 1.273 m_{\rm H} \quad \text{for neutral gas}$$

$$m = \frac{1.4 m_{\rm H}}{2.3} = 0.609 m_{\rm H} \quad \text{for ionized gas}$$

$$n \simeq 2.3 n_{\rm H}$$

for ionized gas,
one electron from an ionized hydrogen
two electrons from a doubly-ionized helium.

  - In the lab frame, $V_{\rm s}$ = shock velocity, $v_1, \; v_2$ = gas velocities in upstream and downstream, respectively.

$$u_1 = v_1 - V_s = -V_s \;\; (v_1 = 0)$$

$$u_2 = v_2 - V_s$$

  - Then, the post-shock velocity is

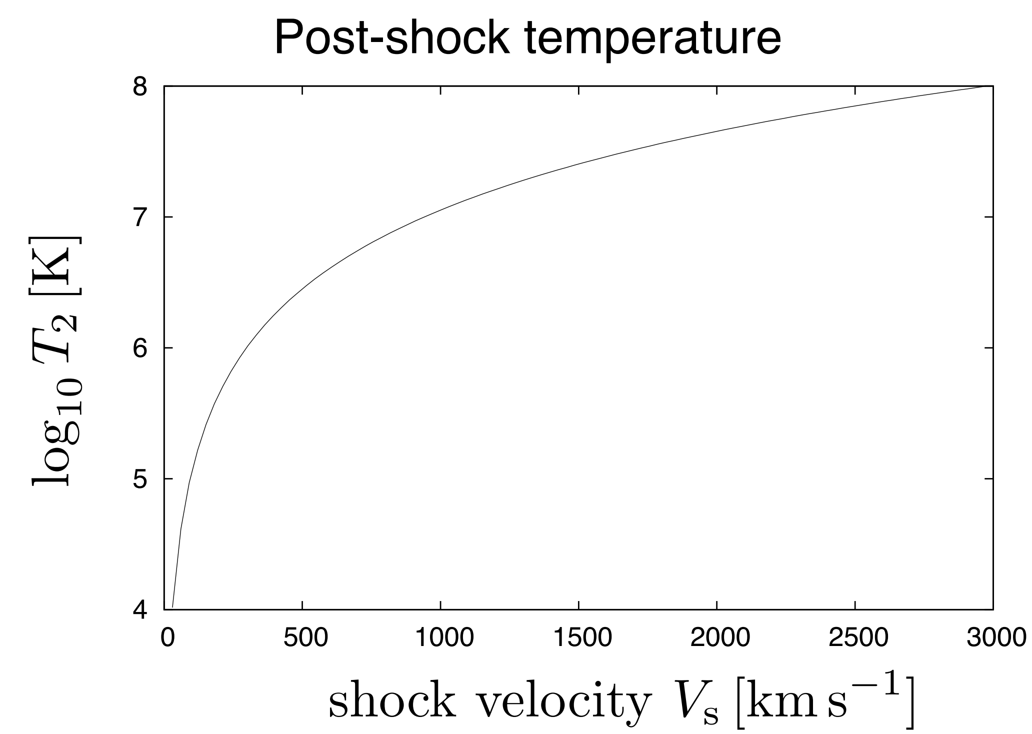
$$\frac{u_2}{u_1} = \frac{v_s - V_{\rm s}}{-V_{\rm s}} = \frac{1}{4} \quad \Rightarrow \quad \boxed{v_2 = \frac{3}{4}V_{\rm s}}$$

  - Hence, the post-shock moves in the same direction as the shock front with a velocity of 3/4 of the shock velocity.

- Then, the post-shock pressure, temperature, specific internal energy, and specific kinetic energy are, respectively,

$$P_2 = \frac{3}{4}\rho_1 V_\mathrm{s}^2$$
$$T_2 = \frac{3m}{16k}V_\mathrm{s}^2$$

for $\gamma = 5/3$



$$\frac{\mathcal{E}_{\mathrm{int},2}}{\rho_2} = \epsilon_2 = \frac{3}{2}\frac{P_2}{\rho_2} = \frac{3}{2}\frac{(3/4)\rho_1 V_\mathrm{s}^2}{4\rho_1}$$
$$\frac{\mathcal{E}_{\mathrm{kin},2}}{\rho_2} = \frac{1}{2}v_2^2$$

$$\frac{\mathcal{E}_{\mathrm{int},2}}{\rho_2} = \frac{9}{32}V_\mathrm{s}^2$$
$$\frac{\mathcal{E}_{\mathrm{kin},2}}{\rho_2} = \frac{9}{32}V_\mathrm{s}^2$$

- A strong shock can produce very high pressures and temperatures. An interstellar shock front with propagation speed $V_s \sim 1000$ km s^{-1} (typical for a supernova shock wave) produces shock heated gas with

$$T_2 \approx 1.38\times10^7\,\mathrm{K}\left(\frac{m}{0.609 m_\mathrm{H}}\right)\left(\frac{V_s}{1000\,\mathrm{km\,s^{-1}}}\right)^2$$ or $$T_2 \approx 1.38\times10^5\,\mathrm{K}\left(\frac{m}{0.609 m_\mathrm{H}}\right)\left(\frac{V_s}{100\,\mathrm{km\,s^{-1}}}\right)^2$$

assuming the shocks gas is fully ionized hydrogen.

- In general, shock fronts convert supersonic gas into subsonic gas in the shock's frame of reference. Shocks increase density, pressure, and temperature, and decrease bulk velocity relative to the shock front. Shocks act as entropy generators.

General Properties of the HIM

- Hot Ionized Medium, coronal gas
  - About half the volume of the ISM in our Galaxy is occupied by the HIM.
  - Temperature ~ 10^6 K.
  - Typical ion number density n ~ 0.004 cm^{-3}
  - It provides only ~ 0.2% of the mass of the ISM, despite being the largest contributor to its volume.

  - The HIM is hot because it has been heated by shock fronts that result from supernova explosions.

  - ***We live in the "Local Bubble", which is ~ 100 pc in size. The Local Bubble is thought to have been blown by a supernova that went off ~ 10 Myr ago.***

Collisional Ionization Equilibrium

- CIE
  - CIE assumes that the plasma is in a steady state, and that collisional ionization, charge exchange, radiative recombination, and dielectronic recombination are the only processes altering the ionization balance.
    - Note that the reverse process to collisional ionization is a three-body recombination, which is unlikely to occur.
  - The ionization fractions for each element depend only on the gas temperature, with no dependence on the gas density.
- Ionization fraction
  - For hydrogen, the balance equation is : ionization rate = recombination rate

$$n_e n(\mathrm{H}^0) k_{\mathrm{ci,H}} = n_e n(\mathrm{H}^+) \alpha_{\mathrm{A,H}} \qquad n(\mathrm{H}^0) + n(\mathrm{H}^+) = n(\mathrm{H})$$

  - The rate coefficients for collisional ionization and radiative recombination are:

$$k_{\mathrm{ci,H}} = 5.849 \times 10^{-9} T_4^{1/2} e^{-15.782/T_4} \, [\mathrm{cm^3\,s^{-1}}]$$

$$\alpha_{\mathrm{A,H}} = 4.13 \times 10^{-13} T_4^{-0.7131 - 0.0115 \ln T_4} \, [\mathrm{cm^3\,s^{-1}}] \quad \text{for} \quad 30\,\mathrm{K} < T < 3 \times 10^4\,\mathrm{K}$$

$$= 5 \times 10^{-16} T_7^{-1.5} \quad \text{for} \quad T > 10^6\,\mathrm{K}$$

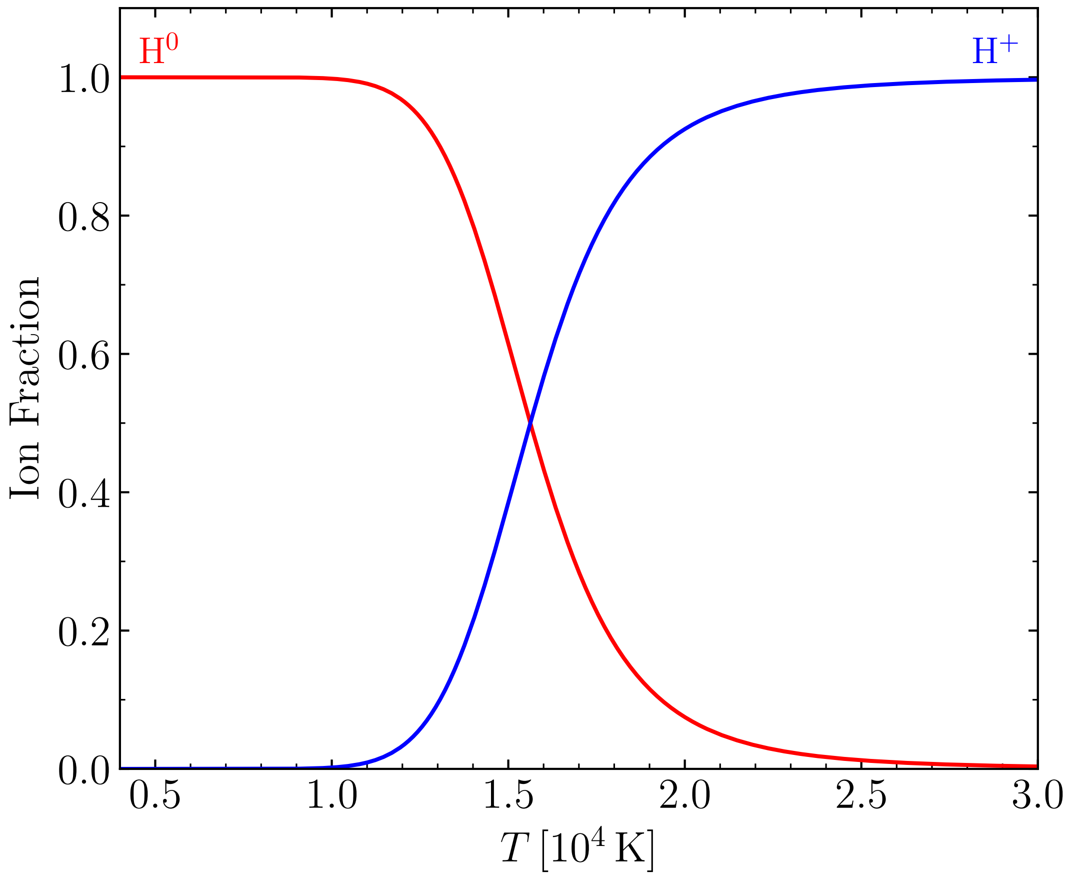
[from Draine]

$$\alpha_{\mathrm{A,H}} = 1.269 \times 10^{-13} \, [\mathrm{cm^3\,s^{-1}}] \frac{x^{1.503}}{(1 + (x/0.522)^{0.47})^{1.923}} \quad \text{where } x = 2 \times 157807\,\mathrm{K}/T$$

[Hui & Gendin 1997, MNRAS]

- The ionization fraction is

$$x \equiv \frac{n(\mathrm{H}^+)}{n(\mathrm{H}^0)+n(\mathrm{H}^+)} = \frac{k_{\mathrm{ci,H}}}{k_{\mathrm{ci,H}}+\alpha_{\mathrm{A,H}}}$$



- The ion fractions are

$$x \approx 0.002 \quad \text{at} \quad T = 10^4\,\mathrm{K}$$
$$1 - x \approx 3\times 10^{-7} \quad \text{at} \quad T = 10^6\,\mathrm{K}$$

H II regions with $T = 10^4$ K are photoionized by UV photons from hot stars.

Hydrogen gas with $T = 10^6$ K is almost entirely collisionally ionized.

- For Helium, the balance equations are:

$$n(\mathrm{He}^+)\alpha_{10} = n(\mathrm{He}^0)k_{01}$$
$$n(\mathrm{He}^+)k_{12} = n(\mathrm{He}^{2+})\alpha_{21}$$
$$n(\mathrm{He}) = n(\mathrm{He}^0) + n(\mathrm{He}^+) + n(\mathrm{He}^{2+})$$

Here, ij indicates $X^{i+} \rightarrow X^{j+}$.

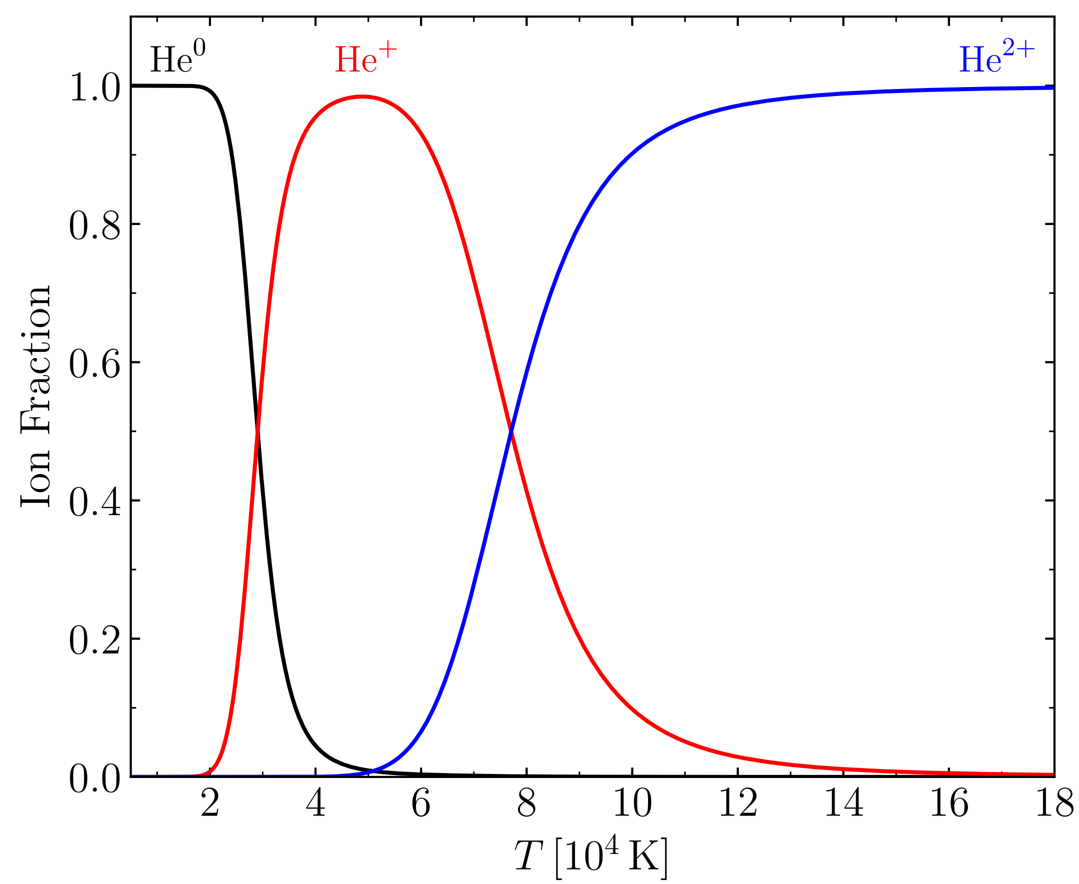
- The rate coefficients are

from Cen (1992, ApJS)

$$k_{01} = 2.39 \times 10^{-11}\, T^{1/2} e^{-285{,}335/T}$$
$$k_{12} = 5.68 \times 10^{-12}\, T^{1/2} e^{-631{,}515/T}$$
$$\alpha_{10} = 1.50 \times 10^{-10}\, T^{-0.6353}$$ radiative recombination
$$+\, 1.9 \times 10^{-3} T^{-1.5} e^{-470{,}000/T} (1 + 0.3 e^{-94{,}000/T})$$ dielectronic recombination (but not significant)
$$\alpha_{21} = 3.36 \times 10^{-10}\, T^{-1/2} T_3^{-0.2} / (1 + T_6^{0.7})$$

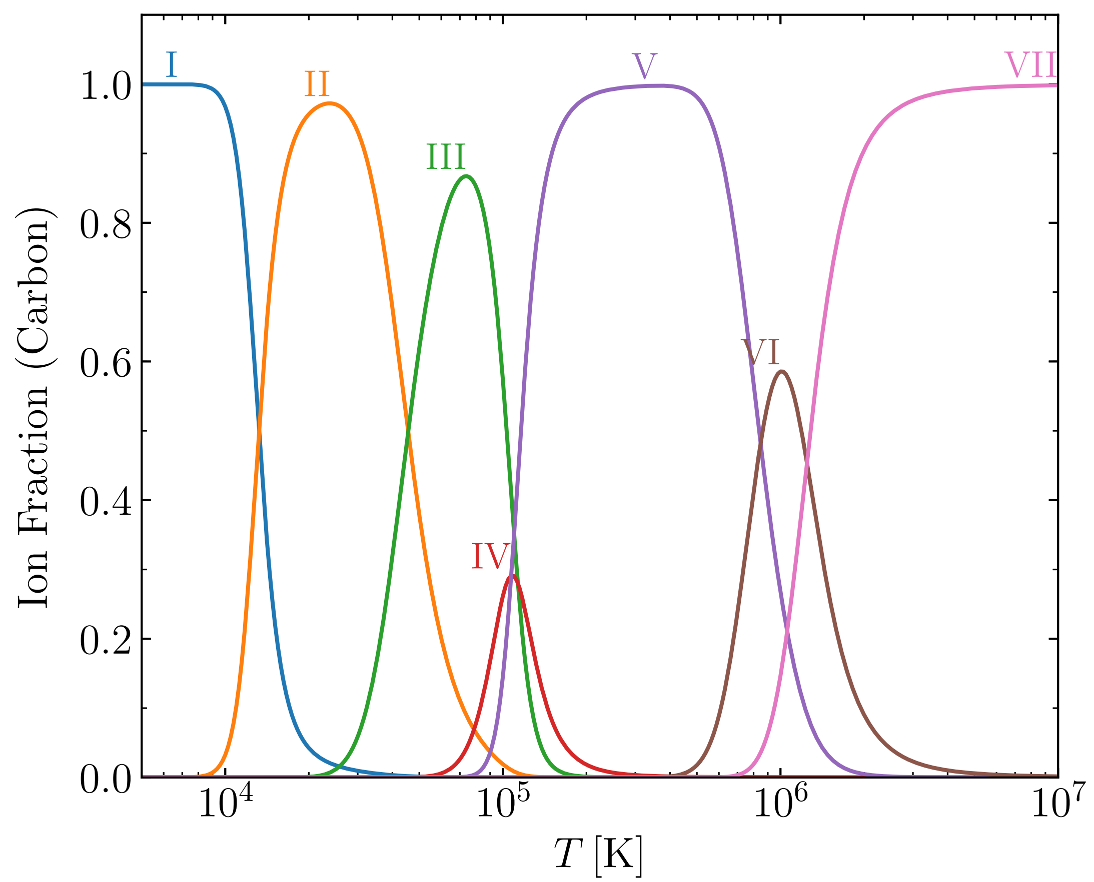
- Using the above rate coefficients, the ionization fractions can be estimated as follows:

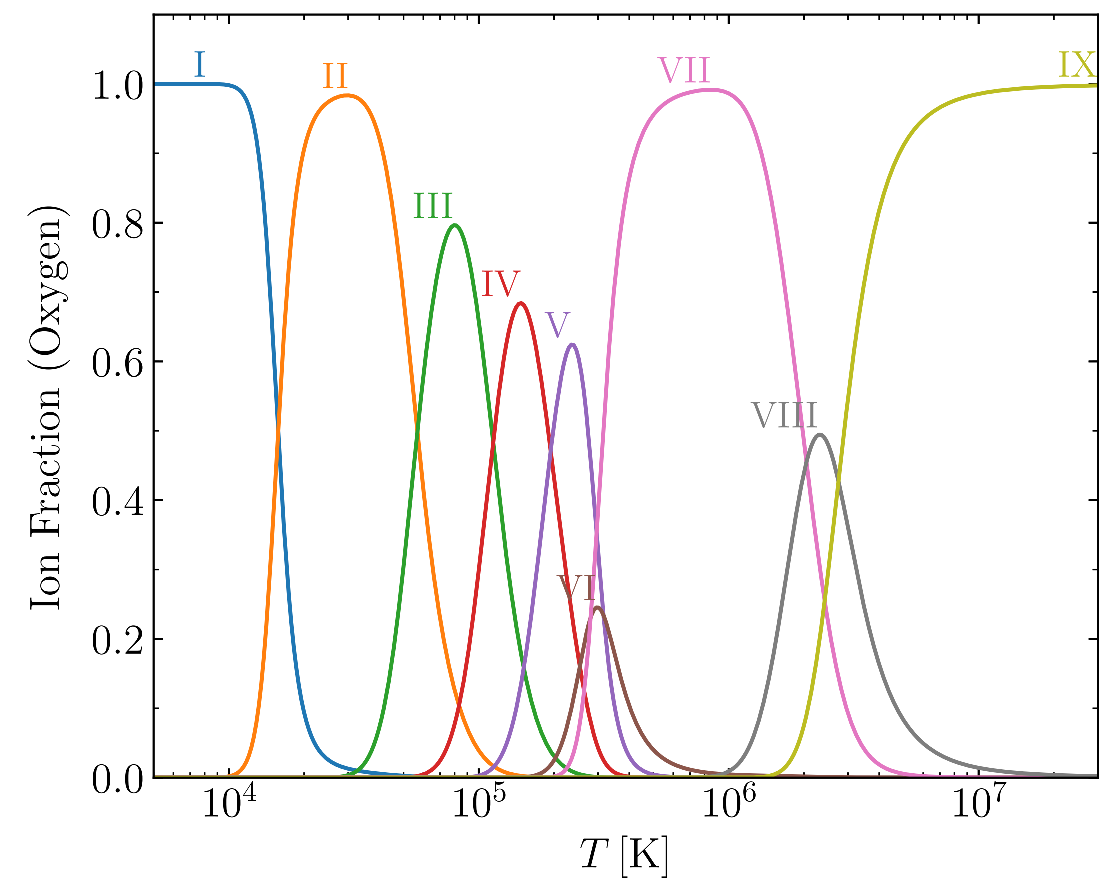
$$x \equiv \frac{n(\mathrm{He}^+)}{n(\mathrm{He})} = \frac{1}{1 + \alpha_{10}/k_{01} + k_{12}/\alpha_{21}}$$
$$y \equiv \frac{n(\mathrm{He}^{2+})}{n(\mathrm{He})} = \frac{k_{12}}{\alpha_{21}} x$$



- Heavy Elements
  - The calculation is usually done numerically, for instance, using CHIANTI

    CHIANTI: https://www.chiantidatabase.org/
  - For instance, the ion fractions of Carbon and Oxygen as a function of temperature are:





- At T ~ 10^6 K, we expect a mix of C V, C VI, and C VII.
- At T ~ 4×10^6 K and higher, almost all the carbon will be in the form of fully ionized C VII.

The figures were calculated using CHIANTI.

Cooling in CIE

- ***Cooling function***
  - At temperature T > 10^4 K, ionization of hydrogen provides enough free electrons so that collisional excitation of atoms or ions is dominated by electron collisions.
  - At low densities, every collisional excitation is followed by a radiative decay, and the rate of removal of thermal energy per unit volume can be written:

$$\Lambda = n_e n_{\rm H} f_{\rm cool}(T)$$

    The ***radiative cooling function*** $f_{\rm cool}(T) \equiv \Lambda / n_{\rm H} n_e$ is a function of ***temperature*** and of the ***elemental abundances*** relative to hydrogen.

  - At high densities, radiative cooling can be suppressed by collisional deexcitation, and the cooling function will then depend on density, in addition to T and elemental abundances.
  - If ionizing radiation is present, the ionization balance may depart from CIE, and the radiative cooling function will also depend on the spectrum and intensity of the ionizing radiation.

Fig 34.1
Draine

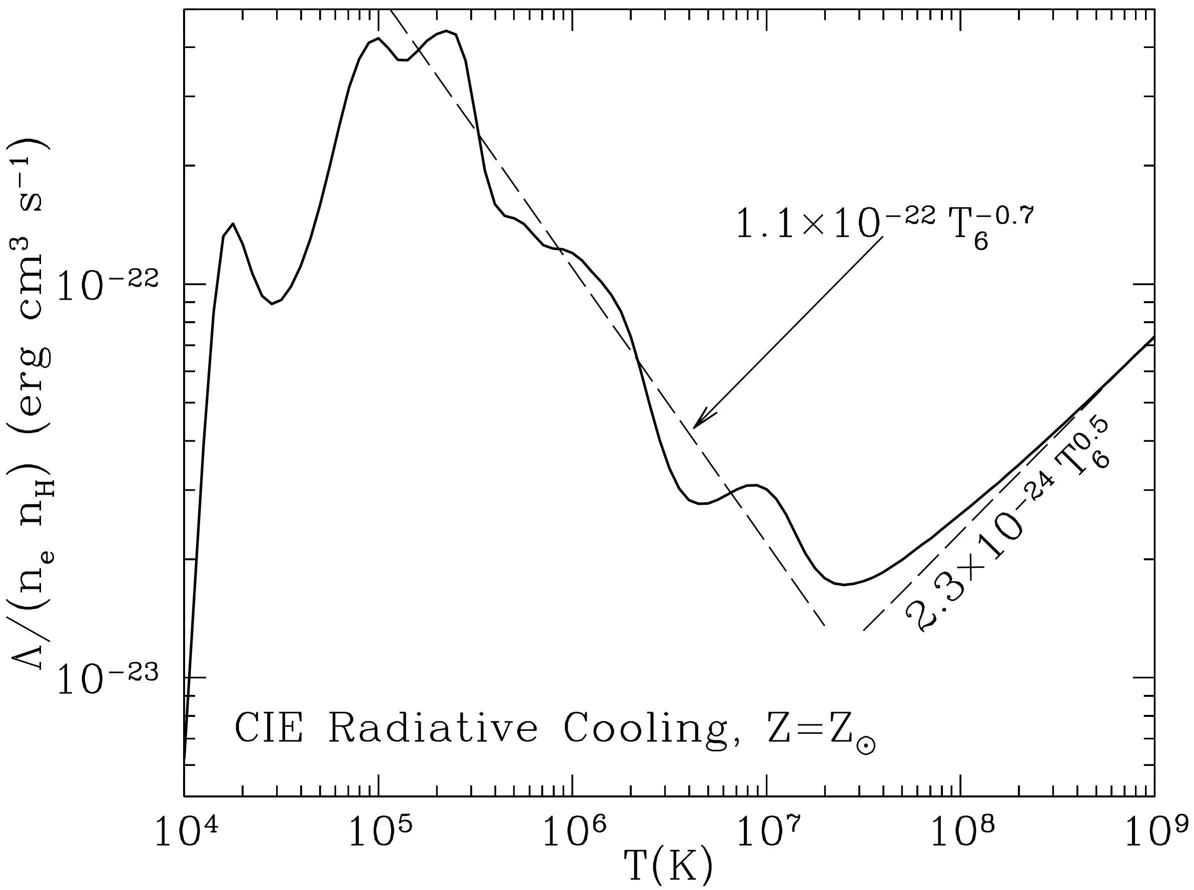
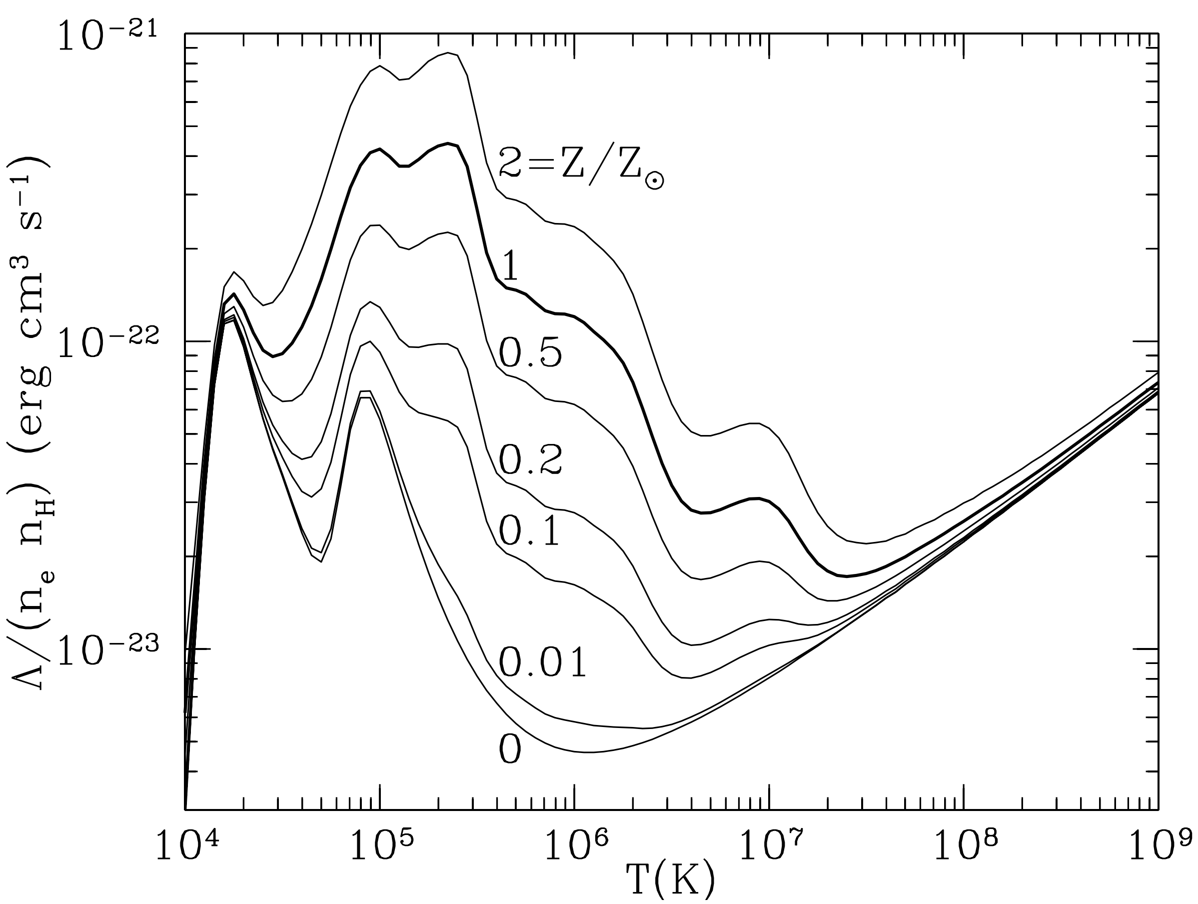


Fig 34.2
Draine



Radiative Cooling Function for solar-abundance

- At $T < 10^7$ K, the cooling is dominated by collisional excitation of bound electrons.
- At high temperatures, the ions are fully stripped of electrons, and bremsstrahlung (free-free) cooling dominates.

$$\Lambda/n_e n_{\rm H} \approx 1.1\times10^{-22}\,T_6^{-0.7}\,[\rm erg\,cm^3\,s^{-1}]$$
$$(10^5 < T < 10^{7.3}\,{\rm K})$$

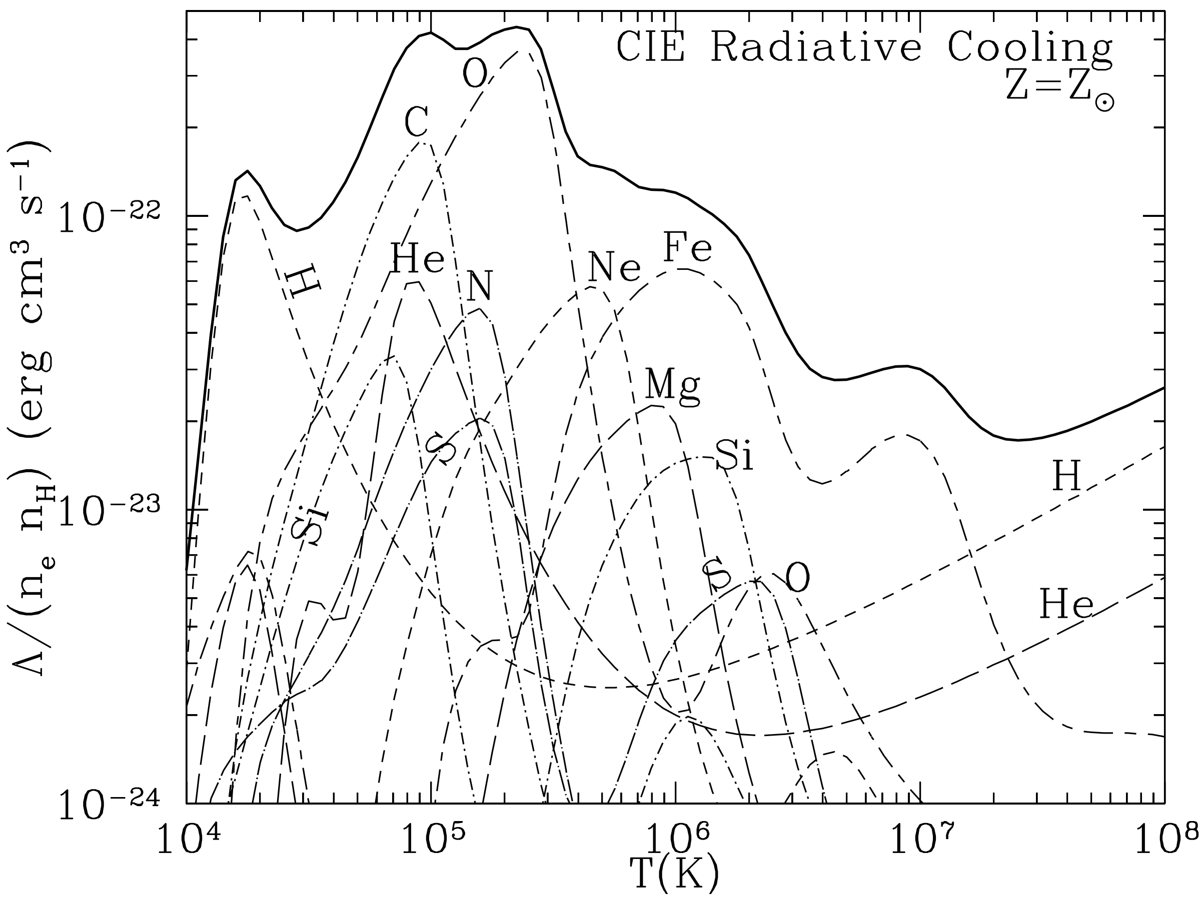
$$\Lambda/n_e n_{\rm H} \approx 2.3\times10^{-24}\,T_6^{0.5}\,[\rm erg\,cm^3\,s^{-1}]$$
$$(T > 10^{7.3}\,{\rm K})$$

Cooling Function for different abundances

- In most applications, the abundances of elements beyond He can be assumed to be scale up and down together.

The cooling functions were calculated using CHIANTI.

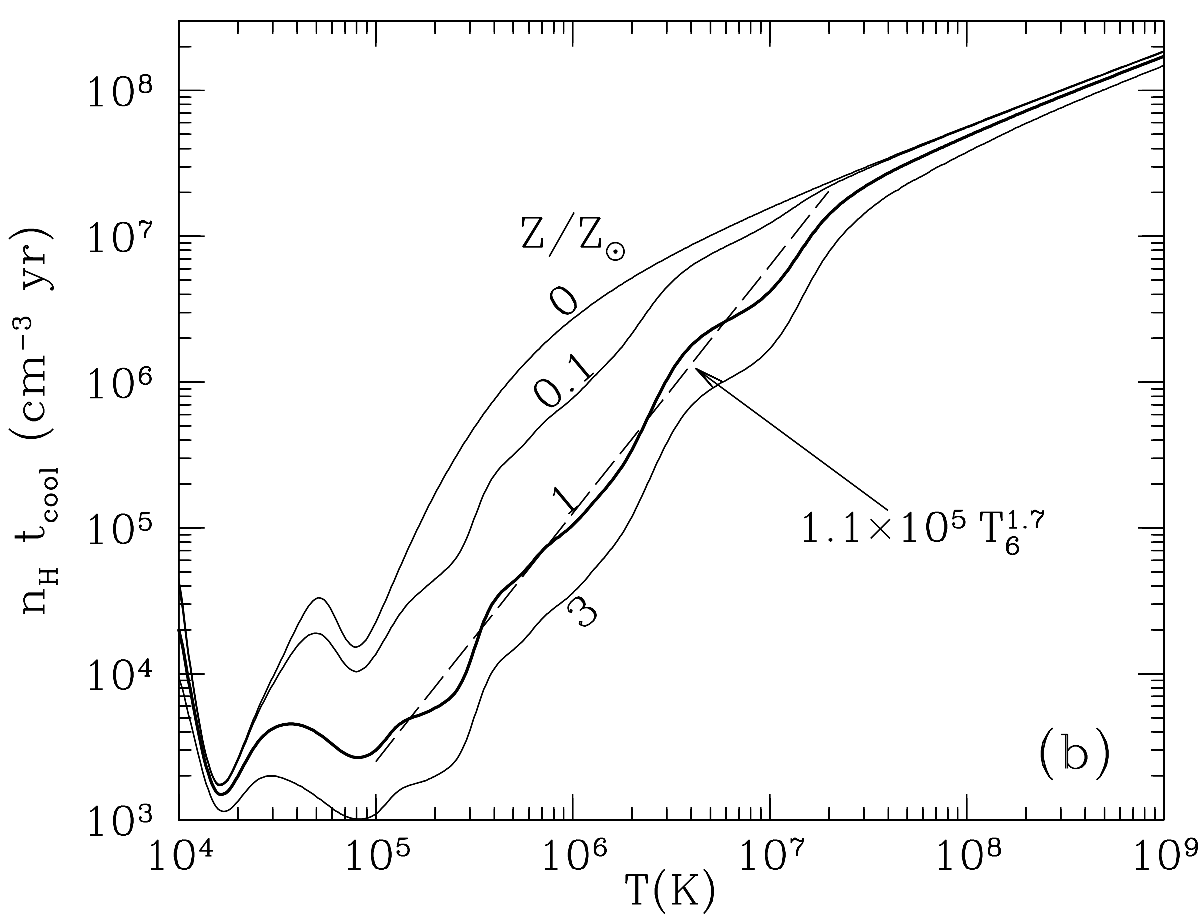
Fig 34.3
Draine



Radiative Cooling Function, with contributions from selected elements

- For $10^{5.8} < T < 10^{7.2}$ K, the cooling is dominated by Mg, Si, and Fe - elements that in cold gas are normally depleted by factors of 5 or more. In this calculation, the solar abundance is assumed.

Fig 34.4
Draine



Cooling time

$$t_{\rm cool} \approx 1.1 \times 10^5\, T_6^{1.7}\, (n_{\rm H}/{\rm cm}^{-3})^{-1}\, [{\rm yr}]$$
$$(10^5 \lesssim T \lesssim 10^{7.3}\, {\rm K})$$

for isochoric cooling (constant density)

Cooling Time Scale [isobaric / isochoric]

- Cooling Time scale for two important cases:
  - The first law of thermodynamics states that:

Heat added in a system: $dQ = dU + PdV$

  - Using a heating and cooling rate per volume Γ and Λ , the change in heat is

$$dQ = (\Gamma - \Lambda)V\,dt$$

  - The change in the internal energy is

$$dU = (\Gamma - \Lambda)V\,dt - PdV$$

  - When there is no external heating, the equation for an ideal gas with a degree of freedom f becomes:

$$\begin{aligned} U &= \frac{f}{2}NkT \\ PV &= NkT \end{aligned} \longrightarrow d\left(\frac{f}{2}NkT\right) = -\Lambda V\,dt - PdV$$

- Consider the case of constant pressure or constant volume:

$$PdV = d(PV) - VdP = d(NkT)$$ for ***isobaric cooling (constant pressure)***

$$PdV = 0$$ for ***isochoric cooling (constant density or volume)***

Therefore,

$$\frac{d}{dt}\left(\frac{f+2}{2}NkT\right) = -\Lambda V$$ for ***isobaric cooling (constant pressure)***

$$\frac{d}{dt}\left(\frac{f}{2}NkT\right) = -\Lambda V$$ for ***isochoric cooling (constant density or volume)***

The cooling time scale are then:

$$t_{\rm cool} \equiv \frac{T}{|dT/dt|} \quad \Rightarrow \quad t_{\rm cool} = \frac{f+2}{2}\frac{nkT}{\Lambda}$$ for isobaric cooling

$$n \equiv N/V \qquad\qquad = \frac{f}{2}\frac{nkT}{\Lambda}$$ for isochoric cooling

Here, the number density includes all particles (molecules, atoms, ions, electrons)

Time Scales in the HIM

- ***Cooling time scale***:
  - In the HIM with temperatures T ~ 10^6 - 10^7 K, the cooling time scale is:

$$t_{\rm cool} = \frac{5}{2}\frac{nkT}{\Lambda} \qquad n_e \approx 1.2 n_{\rm H} \qquad n \approx 2.3 n_{\rm H}$$

For fully ionized gas,
one electron from an ionized hydrogen
two electrons from a doubly-ionized helium.

  - The cooling time at T ~ 10^6 K is

$$t_{\rm cool} = \frac{5}{2}\frac{2.3}{1.2}\frac{kT}{\Lambda/(n_e n_{\rm H})}\frac{1}{n_{\rm H}}$$

$$t_{\rm cool} \approx 48\,[{\rm Myr}]\; T_6^{1.7}\left(\frac{n_{\rm H}}{0.004\,{\rm cm}^{-3}}\right)^{-1} \approx 0.19\,[{\rm Myr}]\; T_6^{1.7}\left(\frac{n_{\rm H}}{1\,{\rm cm}^{-3}}\right)^{-1} \longleftarrow \Lambda/n_e n_{\rm H} \approx 1.1\times 10^{-22}\, T_6^{-0.7}\,[{\rm erg\,cm^3\,s^{-1}}]$$

  - At T ~ 10^7 K, the cooling time is

$$t_{\rm cool} \approx 7.2\,[{\rm Gyr}]\; T_7^{1/2}\left(\frac{n_{\rm H}}{0.004\,{\rm cm}^{-3}}\right)^{-1} \approx 29\,[{\rm Myr}]\; T_7^{1/2}\left(\frac{n_{\rm H}}{1\,{\rm cm}^{-3}}\right)^{-1} \longleftarrow \Lambda/n_e n_{\rm H} \approx 2.3\times 10^{-24}\, T_6^{0.5}\,[{\rm erg\,cm^3\,s^{-1}}]$$

Given the low density of the HIM, the cooling time of gas is comparable to the age of our galaxy.

- ***Recombination and Ionization Time scale***:
  - If collisional ionization could somehow be turned off, the recombination time scale is

$$t_{\rm rec} = \frac{1}{n_{\rm e}\alpha_{\rm A,H}} \approx 0.6\,[{\rm Gyr}] \left(\frac{n_{\rm H}}{0.004\,{\rm cm}^{-3}}\right)^{-1} \qquad [T \approx 10^6\,{\rm K}] \qquad \alpha_{\rm A,H} \approx 1.5\times10^{-14}\,[{\rm cm}^3\,{\rm s}^{-1}]$$

  - If collisional ionization could suddenly switch on, the collisional ionization time scale is

$$t_{\rm ci} = \frac{1}{n_{\rm e}k_{\rm ci}} \approx 160\,[{\rm yr}] \left(\frac{n_{\rm H}}{0.004\,{\rm cm}^{-3}}\right)^{-1} \qquad [T \approx 10^6\,{\rm K}] \qquad k_{\rm ci,H} \approx 5.0\times10^{-8}\,[{\rm cm}^3\,{\rm s}^{-1}]$$

  - These times scales indicates that
    - If cold neutral hydrogen gas is shock-heated to ~ 10^6 K in a time $t_{\rm heat} \ll t_{\rm ci}$, it will take a time $t \sim t_{\rm ci}$ for hydrogen to become ionized. ***During this time interval, the hydrogen will be out of collisional ionization equilibrium (under-ionized than in CIE).***
    - If highly ionized gas at ~ 10^6 K is cooled on a timescale $t_{\rm cool} \ll t_{\rm rec}$, and the heating source is turned off, it will take a time $t \sim t_{\rm rec}$ for the hydrogen to recombine. ***During the intervening time, the gas will be out of CIE (over-ionized than in CIE).*** This is sometimes called “delayed recombination”.

- If a gradually cooling gas of the HIM to be remained in CIE, we require $t_{\rm rec} < t_{\rm cool}$.

  Assuming the recombination rate coefficient at high temperatures

$$\alpha_{\rm A,H} \approx 5 \times 10^{-16}\,{\rm cm^3\,s^{-1}}\left(T/10^7\,{\rm K}\right)^{-1.5}$$

  - At T ~ 10^6 K,

$$t_{\rm rec} = \frac{1}{n_e \alpha_{\rm A,H}} \approx 0.5\,[{\rm Gyr}]\ \left(T/10^6\,{\rm K}\right)^{1.5}\left(n_{\rm H}/0.004\,{\rm cm^{-3}}\right)^{-1}$$

$$t_{\rm cool} \approx 48\,[{\rm Myr}]\ T_6^{1.7}\left(n_{\rm H}/0.004\,{\rm cm^{-3}}\right)^{-1}$$

$$\Rightarrow \quad \frac{t_{\rm rec}}{t_{\rm cool}} \approx 10\left(T/10^6\,{\rm K}\right)^{-0.2}$$

  - At T ~ 10^7 K,

$$t_{\rm rec} = \frac{1}{n_e \alpha_{\rm A,H}} \approx 16\,[{\rm Gyr}]\ \left(T/10^7\,{\rm K}\right)^{1.5}\left(n_{\rm H}/0.004\,{\rm cm^{-3}}\right)^{-1}$$

$$t_{\rm cool} \approx 7.2\,[{\rm Gyr}]\ T_7^{0.5}\left(n_{\rm H}/0.004\,{\rm cm^{-3}}\right)^{-1}$$

$$\Rightarrow \quad \frac{t_{\rm rec}}{t_{\rm cool}} \approx 2.2\,(T/10^7\,{\rm K})$$

Therefore, ***in the extremely hot regions, the hotter the gas is, the further away it is from CIE.***

Cooling in Shocked Gas

- The hot shocked gas is out of equilibrium, and will start to cool. Thus, the shock will be followed by a radiative zone in which the shock heated gas cools down by radiating away photons.
  - At high temperatures $T > 2 \times 10^7\,\mathrm{K}$
    - The cooling is dominated by bremsstrahlung (free-free radiation), for which the specific cooling rate is

$$\mathcal{L} = 2.7\,[\mathrm{erg\,g^{-1}\,s^{-1}}] \left(\frac{T}{10^7\,\mathrm{K}}\right)^{1/2} \left(\frac{n_\mathrm{H}}{1\,\mathrm{cm^{-3}}}\right)$$

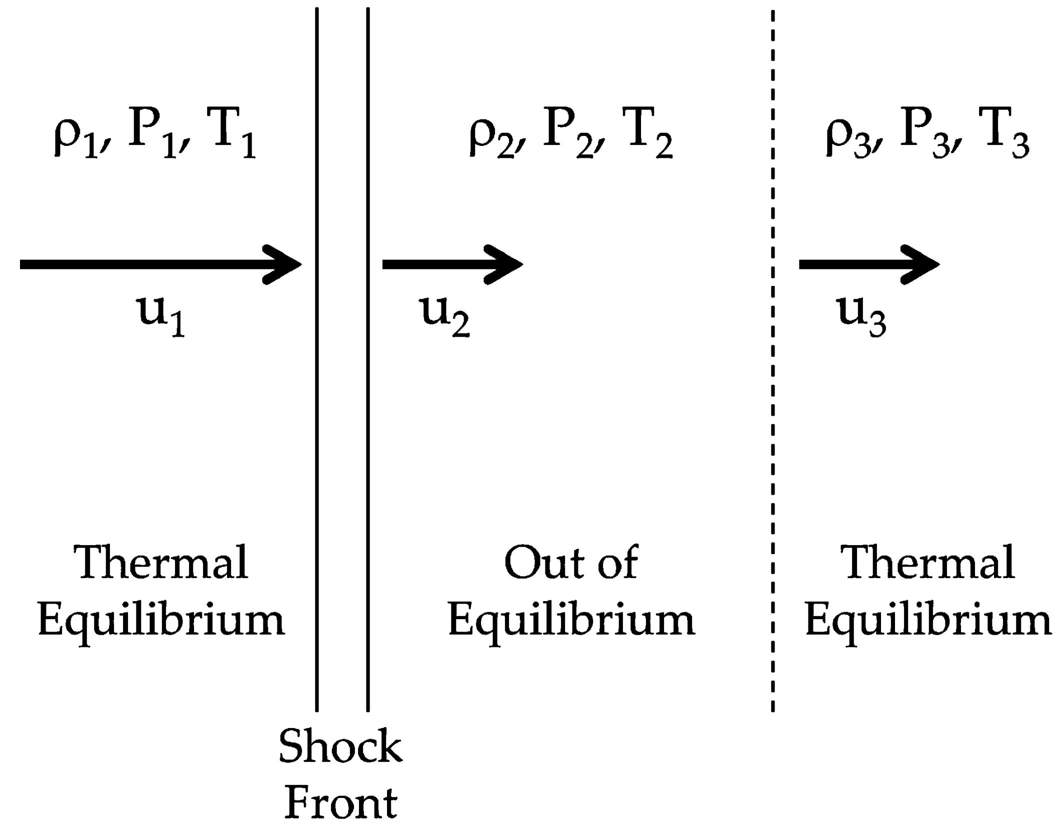
assuming a gas of fully ionized hydrogen. The specific internal energy of ionized hydrogen is

$$\epsilon = \frac{3kT}{m_\mathrm{H}} \approx 2.5 \times 10^{15}\,[\mathrm{erg\,g^{-1}}] \left(\frac{T}{10^7\,\mathrm{K}}\right)$$

    - Then, the bremsstrahlung cooling time is

$$t_\mathrm{cool} = \frac{\rho\epsilon}{\mathcal{L}} \approx 29\,[\mathrm{Myr}] \left(\frac{T}{10^7\,\mathrm{K}}\right)^{1/2} \left(\frac{n_\mathrm{H}}{1\,\mathrm{cm^{-3}}}\right)^{-1}$$
$$\approx 34\,[\mathrm{Myr}] \left(\frac{V_\mathrm{s}}{1000\,\mathrm{km\,s^{-1}}}\right) \left(\frac{n_\mathrm{H}}{1\,\mathrm{cm^{-3}}}\right)^{-1}$$

$$T \simeq \frac{3}{16}\frac{m}{k}V_s^2$$



The structure of a plane parallel radiative shock [Figure 5.3 Ryden]

- During this time, the gas will move a distance, relative to the shock front:

$$R_{\rm cool} \approx u_2 t_{\rm cool} \approx \frac{u_1}{4} t_{\rm cool}$$
$$\approx 8.7\,[\rm kpc] \left(\frac{V_s}{1000\,{\rm km\,s^{-1}}}\right)^2 \left(\frac{n_{\rm H}}{1\,{\rm cm^{-3}}}\right)^{-1}$$

This tells us that ***the approximate thickness of the radiative zone for a strong shock is a long distance compared to the scale height of the ISM in our galaxy***. Thus, the hot gas produced by high-speed shocks doesn't have time to cool before the shock runs out of gas to shock.

- At lower temperature $(10^5\,{\rm K} < T < 2\times 10^7\,{\rm K})$, corresponding to slower shock speed $(80\,{\rm km\,s^{-1}} < u_1 = V_s < 1200\,{\rm km\,s^{-1}})$

- The collisionally excited lines do most of the cooling, a useful approximation gives

$$t_{\rm cool} \approx 6600\,[\rm yr] \left(\frac{V_s}{100\,{\rm km\,s^{-1}}}\right)^{3.4} \left(\frac{n_{\rm H}}{1\,{\rm cm^{-3}}}\right)^{-1}$$

- This yields a thickness for the radiative zone.

$$R_{\rm cool} \approx \frac{V_s}{4} t_{\rm cool} = 0.17\,[\rm pc] \left(\frac{V_s}{100\,{\rm km\,s^{-1}}}\right)^{4.4} \left(\frac{n_{\rm H}}{1\,{\rm cm^{-3}}}\right)^{-1}$$

- The shorter time scales and length scales mean that radiative cooling is more effective at changing the structure of slower shocks.

Effects of Supernovae on the ISM

- Consider the simplest case of a spherically symmetric explosion of a star in a uniform density and temperature.
- ***Free-Expansion Phase***
  - Typical supernovae (SNe) explosion ejects a kinetic energy of

$$E_{51} \equiv E_0/(10^{51}\,\mathrm{erg}) \approx 1$$

  - The ejecta mass ranges from $M_{\rm ej} \sim 1.4\,M_\odot$ (Type Ia, white dwarf) to $M_{\rm ej} \sim 10-20\,M_\odot$ (Type II, core collapse of massive stars).
  - The ejecta will have a rms velocity of

$$v_{\rm ej} = \left(\frac{2E_0}{M_{\rm ej}}\right)^{1/2} = 1.00 \times 10^4\,\mathrm{km\,s^{-1}}\,E_{51}^{1/2}\left(\frac{M_{\rm ej}}{M_\odot}\right)^{-1/2}$$

    - This velocity is far greater than the sound speed in the surrounding material, and the expanding ejecta will drive a fast shock into the circumstellar medium.
  - All of the matter interior to this shock surface is referred to as the ***supernova remnant*** (SNR).
  - The density of the ejecta far exceeds the density of the circumstellar medium, and the ejecta continue to **expand ballistically at nearly constant velocity**. — this is referred to as the “free expansion phase.”

- At the early times of the free-expanding phase, there is only one shock, which propagating outward into the ambient medium.
- As the ejecta expands, its density drops and the pressure of the shocked circumstellar medium soon exceeds the thermal pressure in the ejecta.

$$\rho_{\rm ej} = \frac{M_{\rm ej}}{(4\pi/3)R^3} = \frac{M_{\rm ej}}{(4\pi/3)(v_{\rm ej}t)^3} \propto t^{-3}$$

  - As the pressure in the ejecta drops, a reverse shock is driven into the ejecta. The SNR now contains two shocks: the ***original outward-propagating shock*** (the blastwave) and the ***reverse shock propagating inward***, slowing and shock-heating the ejecta (which had previously been cooled adiabatic expansion).
- ***End of the free-expanding phase***: The reverse shock becomes important when the expanding ejecta material has swept up a mass of circumstellar or interstellar matter comparable to the ejecta mass.
  - The radius of the blastwave and the time when this occurs are:

$$R_1 = \left(\frac{M_{\rm ej}}{(4\pi/3)\rho_0}\right)^{1/3} = 5.88\times10^{18}\,{\rm cm}\left(\frac{M_{\rm ej}}{M_\odot}\right)^{1/3} n_0^{-1/3}$$

$$t_1 \approx \frac{R_1}{v_{\rm ej}} = 186\,{\rm yr}\left(\frac{M_{\rm ej}}{M_\odot}\right)^{5/6} E_{51}^{-1/2} n_0^{-1/3}$$

$$R_1 = 0.525\,{\rm pc}\,(M_{\rm ej}/M_\odot)^{1/3}\, n_0^{-1/3}$$

$\rho_0 \simeq 1.4 m_{\rm p} n_0 \;\rightarrow\;$ density of the ambient (neutral) medium

$n_0 \;\rightarrow\;$ number density of hydrogen in the ambient medium

  - The free-expansion phase applies only for $t \lesssim t_1$.

- ***Sedov-Taylor Phase (Energy-Conserving Phase, Adiabatic Phase)***
  - For $t \gtrsim t_1$, the reverse shock has reached the center of the remnant, all of the ejecta are now very hot, and the free-expansion phase is over.
    - The pressure in the SNR is far higher than the pressure in the surrounding medium.
    - The hot gas may have been emitting radiation, but if the density is low, the radiative losses at early times are negligible.
    - We can idealize the explosion as a large amount of energy released instantaneously at a point, in the form of kinetic energy plus radiation.
  - The SNR now enters a phase where
    - We can neglect (1) the mass of the ejecta ($M_{\rm ej} \ll (4\pi/3)R^3\rho_0$), (2) radiative losses, and (3) the pressure in the ambient medium.
    - This phase can be approximated by idealizing the problem as ***a "point explosion" injecting only energy E_0 (zero ejecta mass) into a uniform-density zero-temperature medium of density ρ_0, as in nuclear explosion.***

- ***Self-similar Solution***: In this phase, only the scale of the pressure and the length scale evolve with time, but the shape of the pressure and density as a function of position remains unaltered. These motions are called self-similar.
- The evolution during this phase is determined only by ***the energy of explosion E_0, the density of interstellar gas, and the elapsed time from the explosion t***.

  We do simple dimensional analysis to find out the form of the time evolution of the remnant. Let the explosion occur at *t* = 0 and the radius of the shock front be R_s. Suppose that

$$R_s = AE^\alpha \rho_0^\beta t^\eta \qquad \text{(}A\text{ = a dimensionless constant)}$$

- By equating the powers of mass, length, and time, we obtain

$$\begin{aligned} \text{mass} &: 0 = \alpha + \beta \\ \text{length} &: 1 = 2\alpha - 3\beta \\ \text{time} &: 0 = -2\alpha + \eta \end{aligned} \quad \longrightarrow \quad \alpha = 1/5,\ \beta = -1/5,\ \eta = 2/5$$

- Then, the solution for the shock-front radius is given by

$$R_s = A\left(\frac{Et^2}{\rho_0}\right)^{1/5} \qquad A = 1.15167 \text{ for } \gamma = 5/3$$

for a monatomic gas.
This value is obtained from the exact solution.

- The solution gives ***the radius and velocity of the shock front***, and ***temperature of the post-shock gas:***

$$R_s = 1.54 \times 10^{19}\,[\mathrm{cm}]\, E_{51}^{1/5} n_0^{-1/5} t_3^{2/5}$$
$$V_s = 1950\,[\mathrm{km\,s^{-1}}]\, E_{51}^{1/5} n_0^{-1/5} t_3^{-3/5}$$
$$T_s = 5.25 \times 10^{7}\,[\mathrm{K}]\, E_{51}^{2/5} n_0^{-2/5} t_3^{-6/5}$$

$$1.54 \times 10^{19}\,\mathrm{cm} = 5\,\mathrm{pc}$$
$$t_3 \equiv t/10^3\,\mathrm{yr}$$
$$V_s \equiv \frac{dR_s}{dt} = \frac{2}{5} A E^{1/5} \rho_0^{-1/5} t^{-3/5}$$
$$T_s = \frac{3}{16} m V_s^2 / k \quad \left(m \simeq \frac{1.4}{2.3} m_p \right) \quad \rho_0 = m n_0$$

m = mass per particle in the fully ionized, post-shock region.

- We have been assuming that the internal structure of the remnant is given by a similarity solution: by this we mean that the density, velocity, and pressure can be written

$$\rho(r) = \rho_0 f(x)$$
$$v(r) = \frac{R_s}{t} g(x)$$
$$P(r) = \frac{\rho_0 R_s^2}{t^2} h(x)$$

$$x \equiv \frac{r}{R_s}$$

$f(x), g(x), h(x)$ are dimensionless functions.

Inserting these into the fluid equations, with the Rankine-Hugonit relations for boundary conditions,Taylor (1950) and Sedov (1959) found the solution for the dimensionless functions [f(x), g(x), h(x), and A] independently, in connection with the development of nuclear weapons.

- ***Alternative approach***:
  - For the strong shock, the specific internal (thermal) and the kinetic energies just behind the strong shock are given by, respectively:

$$\epsilon_{\rm int} = \epsilon_{\rm kin} = \frac{9}{32} V_s^2$$

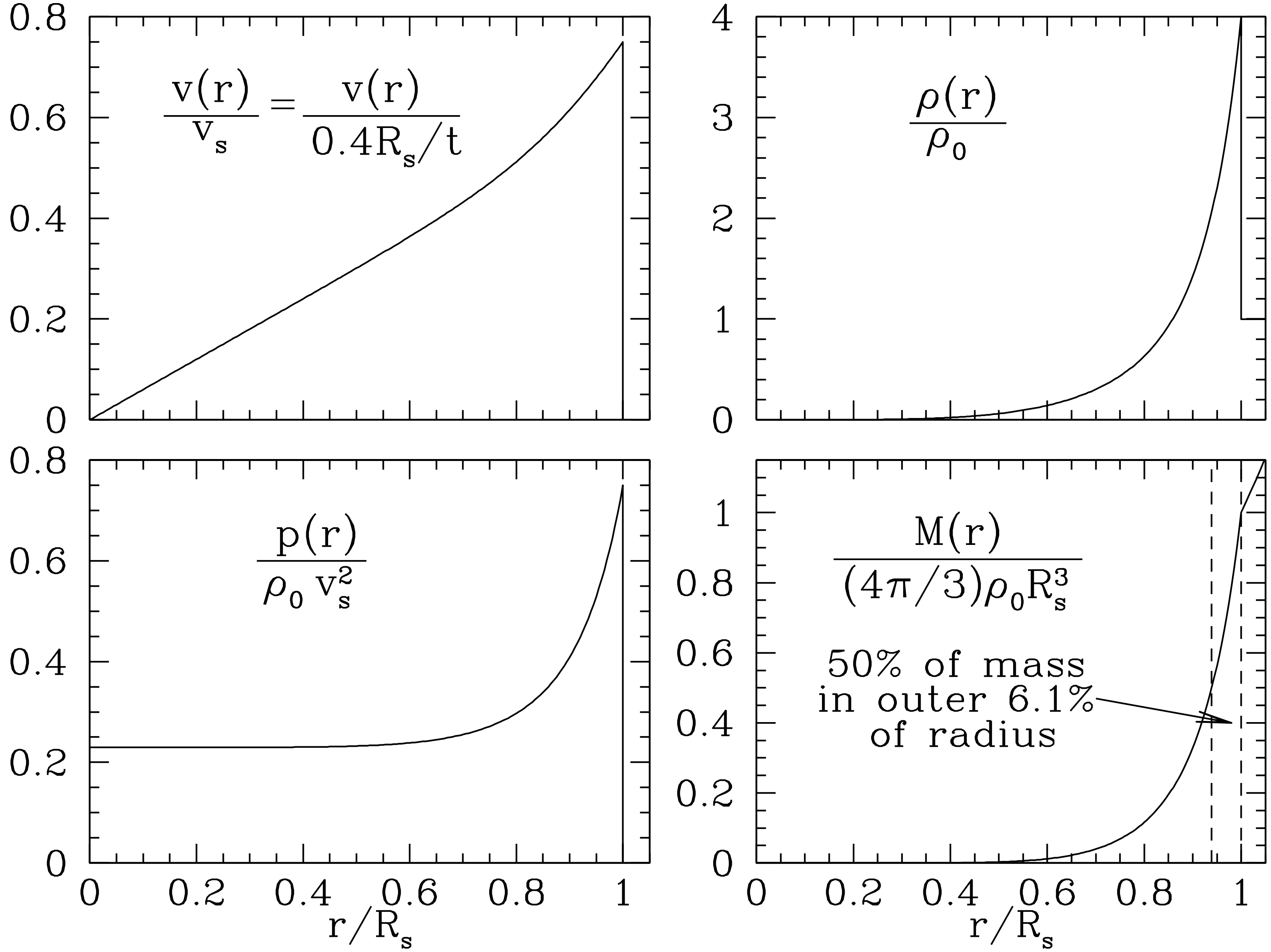
  - Since the blast wave is slowing down with time, the specific energy of the internal gas varies with radius within the bubble of hot gas.
  - Provided that most of the mass in the supernova remnant is made up of shocked interstellar gas and the mass of the stellar eject can be neglected, the total energy in the bubble of hot gas E_0, which is equal to the energy injected by the supernova explosion since the radiative losses are negligible, is therefore

$$\begin{aligned} E_0 &= \phi \frac{4\pi}{3} R^3 \rho_0 \left(\epsilon_{\rm int} + \epsilon_{\rm kin}\right) \\ &= \phi \frac{3\pi}{4} R^3 \rho_0 \left(\frac{dR}{dt}\right)^2 \end{aligned}$$

ρ_0 = density of ambient medium
R = the radius of the shock front at time t

where ϕ is a structure parameter, a numerical factor of order unity which accounts for the radial dependence of specific energy within the bubble. This equation represents the equation of motion of the shock front.

$$R = \left(\frac{25}{3\pi\phi}\right)^{1/5} \left(\frac{E_0 t^2}{\rho_0}\right)^{1/5} \qquad A \equiv \left(\frac{25}{3\pi\phi}\right)^{1/5} = 1.15167 \text{ for } \gamma = 5/3$$



as $r \to 0$

$$\rho(r)/\rho_0 \to 0$$
$$T(r)/T_s \to \infty$$
$$P(r)/P_0 \to 0.306$$

Sedov-Taylor solution for ɣ = 5/3.

The temperature profile (not shown) can be obtained from the ratio of the pressure and density profiles. The density falls inward, and the temperature rises inward.

[Figure 39.1, Draine]

- *End of the Sedov-Taylor phase*

- The hot gas interior to the shock front is, of course, radiating energy. When the radiative losses become important the SNR will leave the Sedov-Taylor phase and enter a "radiative" phase. In the radiative phase, the gas in the shell just interior to the shock front is now able to cool to temperatures much lower than the temperature $T_s = (3/16) m V_s^2 / k$ at the shock front.
- In order to calculate the time scale, we need to estimate the cooling rate:

cooling function at a radius r:

$$\Lambda(r) \approx C \left[T(r)/10^6\,\mathrm{K}\right]^{-0.7} n_{\mathrm{H}}(r) n_e(r), \quad C = 1.1 \times 10^{-22}\,\mathrm{erg\,cm^3\,s^{-1}}, \quad n_e \simeq 1.2 n_{\mathrm{H}}$$

cooling rate at a time t, integrated over the volume of the SNR:

$$\frac{dE}{dt} = -\int_0^{R_s} \Lambda(r) 4\pi r^2 dr \qquad \longleftarrow \qquad n_{\mathrm{H}} n_e = 1.2 n_{\mathrm{H}}^2 = 1.2 n_0^2 \left[\rho(r)/\rho_0(r)\right]^2$$

$$= -1.2 C n_0^2 \left(T_s/10^6\,\mathrm{K}\right)^{-0.7} \frac{4\pi}{3} R_s^3 \left\langle (\rho/\rho_0)^2 (T_s/T)^{0.7} \right\rangle$$

Here, $n_0 \equiv n_{\mathrm{H}}(r = R_s)$ is the hydrogen density of the ambient medium at r = R_s.

$T_s \equiv T(r = R_s)$ is the temperature of the post-shock region at r = R_s.

$\langle \cdots \rangle$ denotes a volume-weighted average over the SNR.

From the Sedov-Taylor solution, we obtain $\left\langle (\rho/\rho_0)^2 (T_s/T)^{0.7} \right\rangle = 1.817$.

- Now, integrate the energy loss rate over a time interval *t* :

$$\Delta E(t) = -1.2 \times (1.817) C \frac{4\pi}{3} n_0^2 \int_0^t dt R_s^3 (T_s/10^6\,\mathrm{K})^{-0.7}$$

Using the previous solutions, we obtain the fractional energy loss by time *t* :

$$R_s = 1.54 \times 10^{19}\,[\mathrm{cm}]\, E_{51}^{1/5} n_0^{-1/5} t_3^{2/5}$$
$$T_s = 5.25 \times 10^7\,[\mathrm{K}]\, E_{51}^{2/5} n_0^{-2/5} t_3^{-6/5}$$

$$\frac{\Delta E(t)}{E_0} \approx -2.38 \times 10^{-6}\, n_0^{1.68} E_{51}^{-0.68} t_3^{3.04}$$

- If we suppose that the SNR enter the “radiative phase” when

$$\Delta E(t_{\mathrm{rad}})/E_0 \approx -1/3$$

we can solve for the cooling time t_{rad} :

$$t_{\mathrm{rad}} = 49.3 \times 10^3\,[\mathrm{yr}]\, E_{51}^{0.22} n_0^{-0.55}$$

end of the Sedov-Taylor phase

- The radius and shock speed at the end of the Sedov-Taylor phase are:

$$7.32 \times 10^{19}\,\mathrm{cm} = 23.7\,\mathrm{pc}$$

$$R_s(t_{\mathrm{rad}}) = 7.32 \times 10^{19}\,[\mathrm{cm}]\, E_{51}^{0.29} n_0^{-0.42}$$
$$V_s(t_{\mathrm{rad}}) = 188\,[\mathrm{km\,s^{-1}}]\, \left(E_{51} n_0^2\right)^{0.07}$$

Post-shock temperature is:

$$T_s(t_{\mathrm{rad}}) = 4.86 \times 10^5\,[\mathrm{K}]\, \left(E_{51} n_0^2\right)^{0.13}$$
$$kT_s(t_{\mathrm{rad}}) = 41\,[\mathrm{eV}]\, \left(E_{51} n_0^2\right)^{0.13}$$

- ***Snowplow Phase (Radiative Phase)***
  - At $t \approx t_{\rm rad}$,
    - Cooling cause the thermal pressure just behind the shock to drop suddenly, and the shock wave briefly stalls. However, ***the very hot gas in the interior of the SNR has not cooled***.
    - The SNR now enters the snowplow phase, with a dense shell of cool gas enclosing a hot central volume where radiative cooling is unimportant.
    - This is called the snowplow phase because the mass of the dense shell increases as it “sweeps up” the ambient gas. Since the gas cools well the shell will be thin and so the gas in the shell has a radial velocity that is almost the same as the shock front speed.
    - The gas in the hot center cools by adiabatic expansion:

$$P \propto V^{-\gamma} \propto R_s^{-3\gamma} = R_s^{-5}$$

    - So that the pressure in the interior evolves as

$$P_i = P_0(t_{\rm rad}) \left(\frac{R_{\rm rad}}{R_s} \right)^5$$

- In the initial phase, the pressure exerted by the hot center causes the "radial momentum" of the shell to increases:

M_s = mass of the shell = mass of the ambient medium swept by the SNR

$$\frac{d}{dt}\left(M_s V_s\right) \approx P_i 4\pi R_s^2 = 4\pi P_0(t_{\rm rad}) R_{\rm rad}^5 R_s^{-3}$$

$$M_s = \left(\frac{4\pi}{3} R_s^3\right) \rho_0$$

$$V_s = \frac{dR_s}{dt}$$

$$\longrightarrow \quad \frac{d}{dt}\left(R_s^3 \frac{dR_s}{dt}\right) \propto R_s^{-3}$$

Suppose that there is a power-law solution:

$$R_s \propto t^\eta$$

$$4\eta - 2 = -3\eta \;\Rightarrow\; \eta = 2/7$$

$$\longrightarrow$$

$$R_s \approx R_s(t_{\rm rad})\,(t/t_{\rm rad})^{2/7}$$

$$V_s \approx \frac{2}{7}\frac{R_s}{t} = \frac{2}{7}\frac{R_s(t_{\rm rad})}{t_{\rm rad}}\left(\frac{t}{t_{\rm rad}}\right)^{-5/7}$$

- Because the effect of the internal pressure has been included, this solution is referred to as the ***pressure-modified (or pressure-driven) snowplow phase***.
- Note that with this construction, $R_s(t)$ is continuous from the Sedov-Taylor phase to the pressure-modified snowplow phase, but $V_s(t)$ undergoes a discontinuous drop by 2/7 ~ 29% at $t = t_{\rm rad}$.

- In the late phase of evolution, the stored thermal energy in the central region has been entirely radiated away, and only the momentum of the dense shell keeps the remnant expanding into the ISM.

$$\frac{d}{dt}(M_s V_s) = 0 \qquad\qquad M_s = \text{mass of the shell}$$

$$M_s = \left(\frac{4\pi}{3} R_s^3\right)\rho_0$$

$$V_s = \frac{dR_s}{dt} \longrightarrow R_s^3 \frac{dR_s}{dt} = \text{constant}$$

$$R_s \approx R_s(t_{\rm rad})\,(t/t_{\rm rad})^{1/4} \quad \text{for } t \gg t_{\rm rad}$$

$$V_s \approx \frac{1}{4}\frac{R_s}{t} = \frac{1}{4} R_s(t_{\rm rad})\,(t/t_{\rm rad})^{-3/4}$$

- This phase is called the ***momentum-conserving snowplow phase***. Towards the end of this phase, the expansion velocity becomes sonic or subsonic with respect to the interstellar sound speed.

- ***In summary, the supernova expansion velocity is continuously slowed by its interaction with the surrounding medium.***

$$R \propto t \quad \rightarrow \quad R \propto t^{2/5} \quad \rightarrow \quad R \propto t^{2/7} \quad \rightarrow \quad R \propto t^{1/4}$$

- ***Fadeaway (Merging with the ISM)***
  - For typical ISM parameters, the shock speed at the beginning of the snowplow phase is

$$V_s(t_{\rm rad}) = 188\,[{\rm km\,s^{-1}}]\,\left(E_{51}n_0^2\right)^{0.07}$$

  - This results in a very strong shock when propagating through interstellar gas with T < 10^4 K. However, the shock front gradually slows, and the shock compression declines. This proceeds until the shock speed approaches the effective sound speed in the gas and at this point the compression ratio approaches 1, and the shock wave turns into a sound wave.

$$V_s \approx \frac{2}{7}\frac{R_s(t_{\rm rad})}{t_{\rm rad}}\left(\frac{t}{t_{\rm rad}}\right)^{-5/7} \longrightarrow \qquad t_{\rm fade} \approx \left(\frac{(2/7)R_{\rm rad}/t_{\rm rad}}{c_s}\right)^{7/5} t_{\rm rad}$$

  - Setting $V_s \approx c_s$ gives the a “fadeaway time”

$$\boxed{t_{\rm fade} \approx 1.87\times10^6\,[{\rm yr}]\,E_{51}^{0.32}n_0^{-32}\left(\frac{c_s}{10\,{\rm km\,s^{-1}}}\right)^{-7/5}}$$ end of the snowplow phase

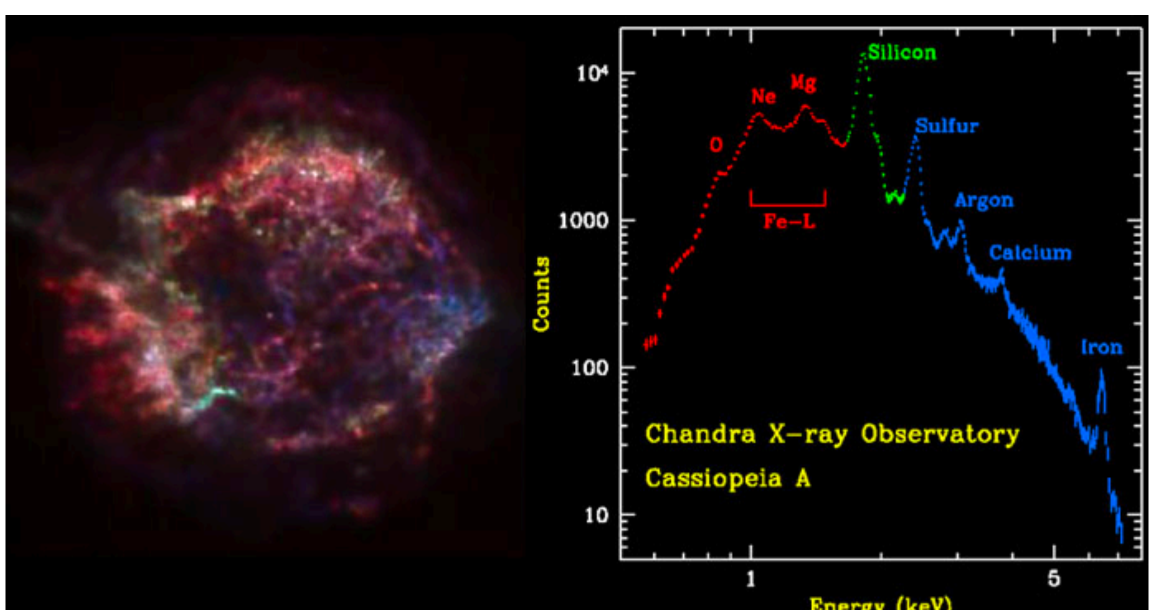
$$R_{\rm fade} \equiv R_s(t_{\rm fade}) \approx \frac{7}{2}V_s t_{\rm fade} \approx \frac{7}{2}c_s t_{\rm fade}$$

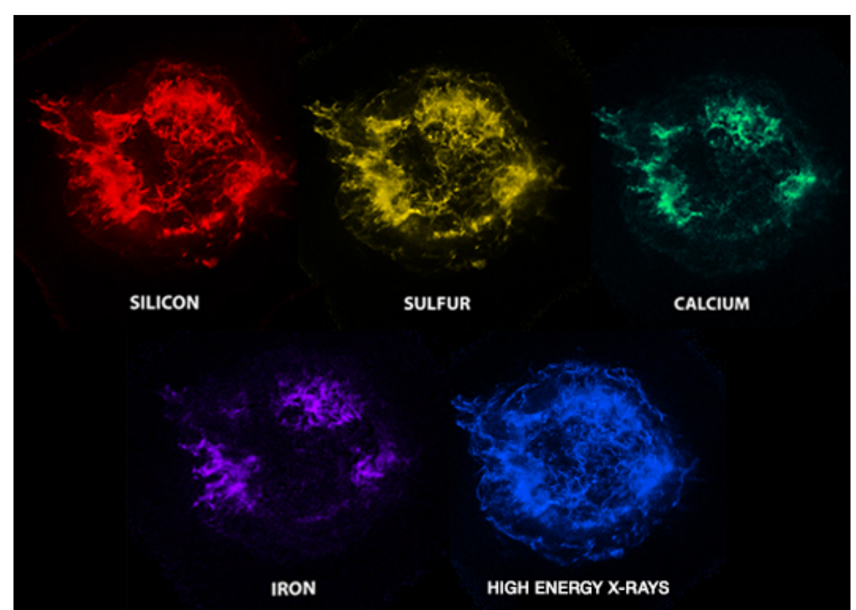
$$\approx 2.07\times10^{20}\,[{\rm cm}]\,E_{51}^{0.32}n_0^{-0.37}\left(\frac{c_s}{10\,{\rm km\,s^{-1}}}\right)^{-2/5}$$

$$2.07\times10^{20}\,{\rm cm} = 67.1\,{\rm pc}$$

SNR Examples

- Free-expansion phase
  - SN1987A, Cas A are in the free expansion phase.
  - Cas A
    - In the case of Type II supernovae resulting from core collapse in massive stars, the supernova explosion is often precede by a red supergiant phase, leaving a relatively dense circumstellar medium with a $\sim r^{-2}$ density profile.
    - The Cas A has been modeled with $M_{\rm ej} \approx 4 M_\odot$ and $E_{51} \approx 2$, expanding into a circumstellar medium with $n_{\rm H} \approx 7\,(r/{\rm pc})^{-2}\ {\rm cm}^{-3}$ left by a red supergiant phase (van Veelen et al. 2009). The reverse shock is now located at ~60% of the outer shock radius — much of Cas A ejecta is still in the free expansion phase.
    - The Cas A is at the end of its free expansion phase; it is ~ 300 years old (around 1680AD). At visible wavelengths, many ‘knots’ of emission , moving radially outward with a velocity of ~ 6000 km s^{-1}, can be seen. The knots are rich in oxygen, and are interpreted as being clumps of matter that have been ejected from the center of the star, where nucleosynthesis took place.





Cassiopeia A
Credit: NASA/CXC/SAO

Examples

- Sedov-Taylor phase
  - The Tycho SNR and the Crab nebular are in the blast wave phase.
    - Light from the supernova that gave birth to the Crab Nebula was first seen on AD 1054 July 4. The SNR is at an age $t \approx 966\,\mathrm{yr}$. The solution for the Sedov-Taylor phase implies

$$R_s = A\left(\frac{E_0 t^2}{\rho_0}\right)^{1/5}$$

$$V_s = \frac{dR_s}{dt} = \frac{2}{5}\frac{R_s}{t}$$

$$\frac{R_s}{V_s} = \frac{5}{2}t \approx 2400\,\mathrm{yr}$$

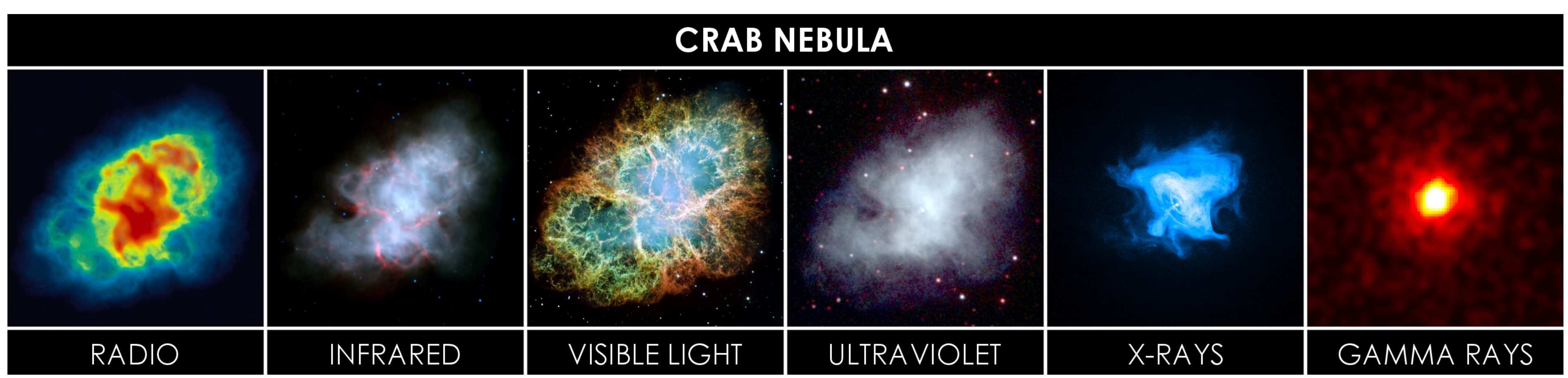
    - The azimuthally averaged radius of the Crab Nebula is $\theta \approx 150\,\mathrm{arcsec}$ in angle. The proper motion of its expansion is $\mu \approx 0.16\,\mathrm{arcsec\,yr^{-1}}$. This gives the ratio

$$\theta/\mu \approx 940\,\mathrm{yr} \qquad V_s/R_s < \mu/\theta$$

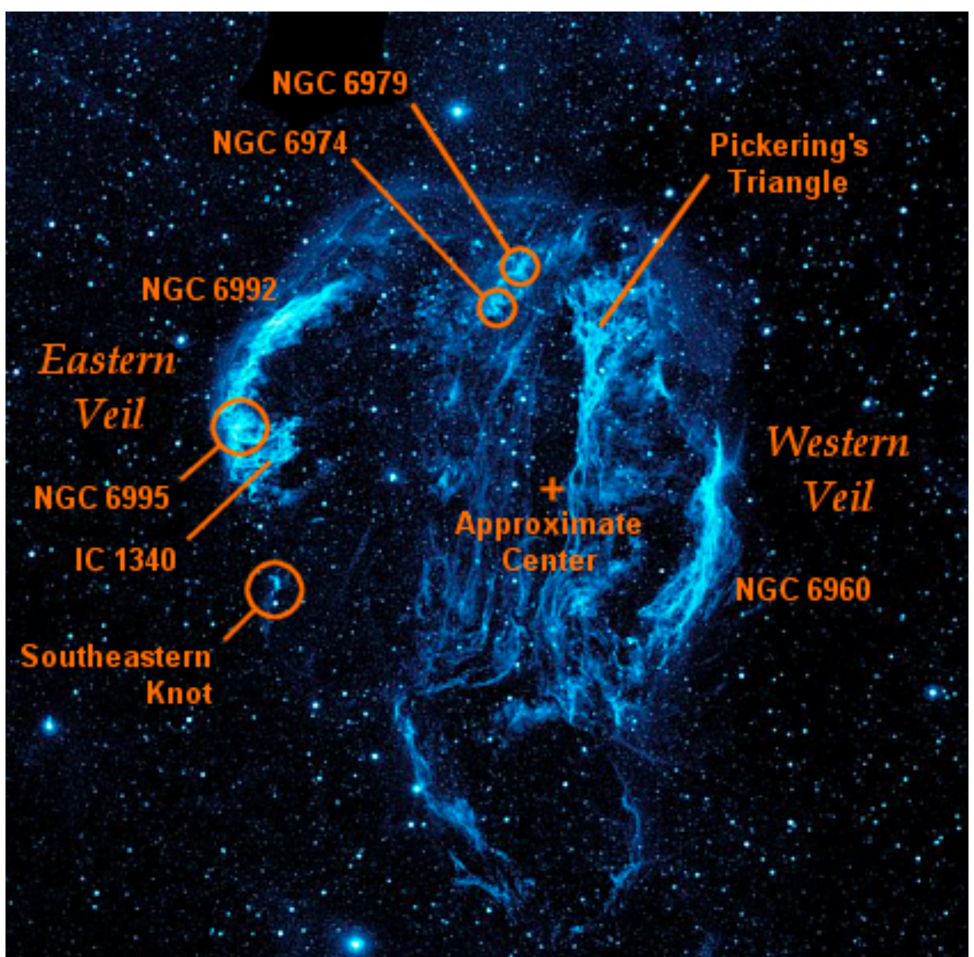
which is smaller than required by the self-similar solution.

    - This implies that V_s for the Crab Nebula is decreasing less rapidly than implied by the Sedov-Taylor solution.

Crab Nebula in multi wavelength
https://en.wikipedia.org/wiki/Crab_Nebula#/media/File:Crab_Nebula_in_Multiple_Wavelengths.png

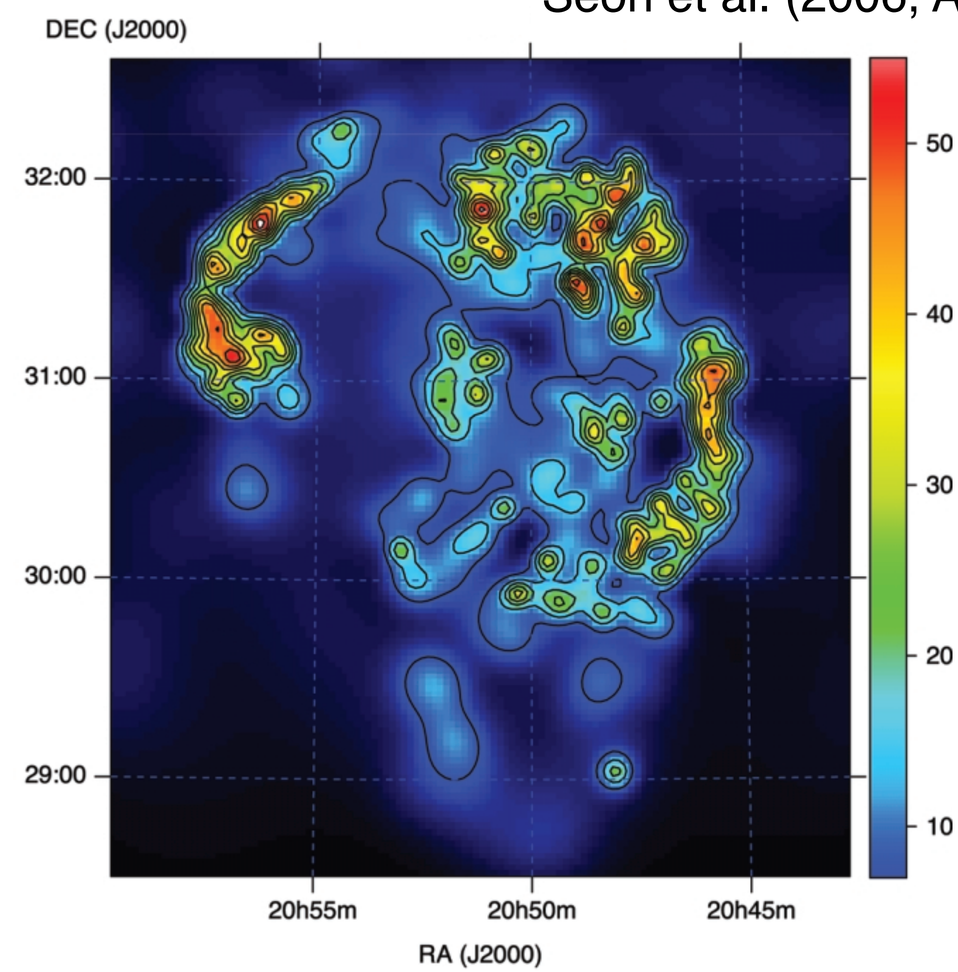


- Snowplow phase
  - Cygnus Loop
    - The Cygnus Loop (aka the Veil Nebula) is in the snowplow phase. Its age is 40,000 yr, and its expansion velocity is ~ 120 km s^{-1}. The dense shell is thermally unstable to the formation of filaments; such filaments can be seen in optically photographs of the Cygnus Loop.
    - Observationally, it is found that the internal pressure of SNRs in the snowplow phase is large enough to give a significant push to the shell.



NASA/GALEX

C IV (1550Å) image
FIMS/SPEAR, STSAT-1
Seon et al. (2006, ApJL)



Overlapping of SNRs

- By the fadeaway time $t_{\rm fade}$, a SNR has expanded to fill a volume $(4\pi/3)R_{\rm fade}^3$
  - What is the probability that another SN will occur within this volume and affect the evolution of the original SNR before it has faded away?
  - Let a rate of occurrence of a SN per volume $S \equiv 10^{-13} S_{-13}\,{\rm pc}^{-3}\,{\rm yr}^{-1}$
  - The SN rate in the Milky Way has been estimated from records of historical SNe and from observations of similar galaxies. The SN frequency in the Galaxy is estimated to be ***one event every 30-50 yr*** (Tammann et al. 1994).
  - Consider the Milky Way "disk" to have a radius of 15 kpc and a thickness 200 pc, and suppose that the SN rate within this volume is 1/60 yr. Then, this gives a SN rate per volume:

$$R = 15\,{\rm kpc} \qquad H = 200\,{\rm pc}$$

$$S = \frac{(60\,{\rm yr})^{-1}}{\pi R^2 H} \approx 1.2 \times 10^{-13}\,{\rm pc}^{-3}\,{\rm yr}^{-1} \longrightarrow S_{-13} \approx 1.2$$

  - The expectation value for the number of additional SNe that will explode within this volume during the lifetime $t_{\rm fade}$ of the original SNR is

$$\begin{aligned} N_{\rm SN} &= S \frac{4\pi}{3} R_{\rm fade}^3 t_{\rm fade} \\ &\approx 0.24 S_{-13} E_{51}^{1.26} n_0^{-1.47} \left(\frac{c_s}{10\,{\rm km\,s^{-1}}}\right)^{-2.6} \end{aligned}$$

In the two-phase model of the ISM, the interstellar volume is filled by the WNM with density

$$n_0 \approx 1\,\mathrm{cm}^{-3} \text{ and } c \approx 6\,\mathrm{km\,s}^{-1} \;(T = 6000\,\mathrm{K})$$

Then, the expectation value for the number of additional SNe that will explode within this volume during the lifetime $t_{\mathrm{fade}} \approx 2\,\mathrm{Myr}$ of the original SNR is

$$N_{\mathrm{SN}} \approx 1.1$$

- Therefore, we conclude that if we were to start with the two-phase model, the initially near-uniform WNM will be destroyed by the effects of SNe.
- Massive stars tend to be born in stellar associations. They enter their supernova phase before they have a chance to drift apart. Thus, the SNRs of the neighboring supernovae will merge to form a single super bubble, which may be hundreds of parsec across. Such super bubbles are a primary source of the hot ionized component of the ISM.
- In other words, SNRs will overlap and occupy a major fraction of the disk volume. This implies that, at least for the Milky Way, SNRs will create a multiphase ISM, consisting of low-density regions in the interior of the SNRs, and dense regions containing most of the gas mass.

Three-Phase Model of the ISM

- An initially uniform ISM consisting of warm HI would be transformed by SNRs into a medium consisting of low-density hot gas and dense shells of cold gas.
  - This transform would take place in just a few Myr. McKee & Ostriker (1977) developed a model of the ISM that took into account the effects of these SNRs.

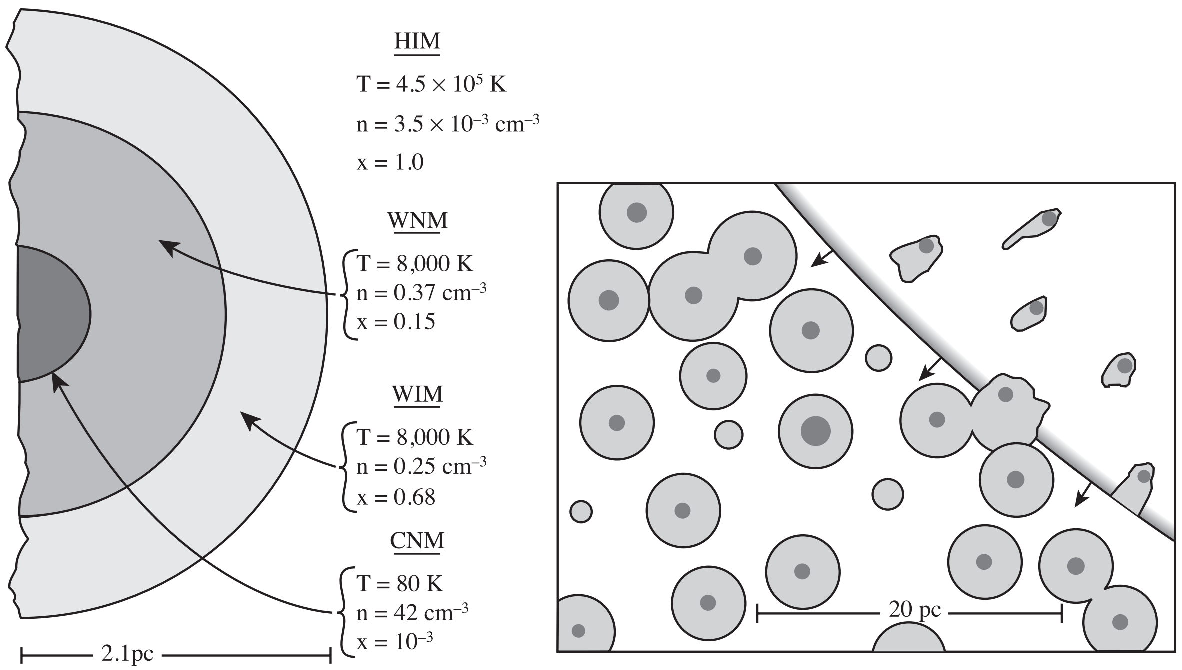


Figure 39.3 Draine

Left: Structure of a typical cold cloud in the three-phase model of McKee & Ostriker (1977)

Right: Close-up of a supernova blast wave.

  - MO1977 argued that the pressure in the ISM was maintained by SNe.
  - If initially the ISM had a low pressure, then SNRs would expand to large radii, with resulting overlap. The pressure in the ISM will rise until the SNRs tend to overlap just as they are fading, at which point the pressure in the ISM is the same as the pressure in the SNR.
  - According to this argument, the overlapping condition $N_{\rm SN} \approx 1$ can be used to predict the pressure of the ISM.

- We can express the expectation value for overlap in favor of the pressure

$$N_{\rm SN} \approx 0.24 S_{-13} E_{51}^{1.26} n_0^{-1.47} \left(\frac{c_s}{10\,{\rm km\,s^{-1}}}\right)^{-2.6}$$

We can eliminate the sound speed in favor of the pressure $P = 1.4\, n_{\rm H} m_{\rm H} c_s^2$

$$N_{\rm SN} \approx 0.48 S_{-13} E_{51}^{1.26} n_0^{-1.47} \left(\frac{P/k}{10^4\,{\rm cm^{-3}\,K}}\right)^{-1.3}$$

We can solve for the pressure

$$P/k = 5700\,[{\rm cm^{-3}\,K}]\, S_{-13}^{0.77} E_{51}^{0.97} n_0^{-0.13} N_{\rm SN}^{-0.77}$$

$$P/k \approx 6600\,{\rm cm^{-3}\,K}\ \text{for}\ S_{-13} \approx 1.2$$

This is comparable to the observed thermal pressure $P/k \approx 3800\,{\rm cm^{-3}\,K}$ in the ISM today.

- ***The principal shortcoming of the MO77 three-phase model is the failure to predict the substantial amount of the warm HI gas.*** The model predicts only 4.3% of the HI mass in the warm phase (WNM and WIM). The 21-cm line observations indicate that more than 60% of the HI within 500 pc of the Sun is actually in the warm phase (Heiles & Troland 2003).

Observing the HIM - Local Bubble

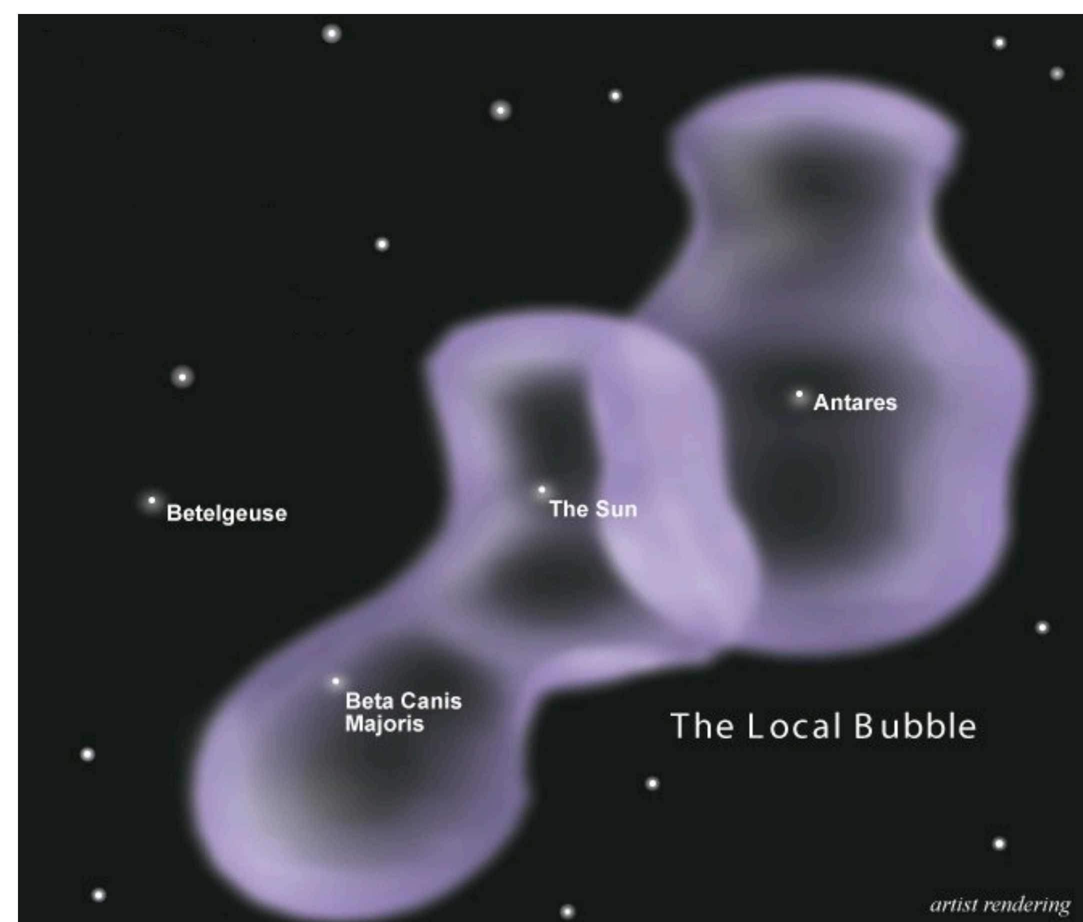
- How to observe the HIM - Emission Lines?
  - HII regions have readily observable emission lines that let us estimate their density and temperature.
  - However, finding the density and temperature of the HIM is more difficult.
  - The temperature of the HIM, T ~ 10^6 K, corresponds to $h\nu \sim kT \sim 100\,\mathrm{eV}$. However, photons with an energy 100 eV (~ 120Å) are difficult to observe, even if we use space observatories.
    - This is primarily because of the large photoionization cross-section of hydrogen.

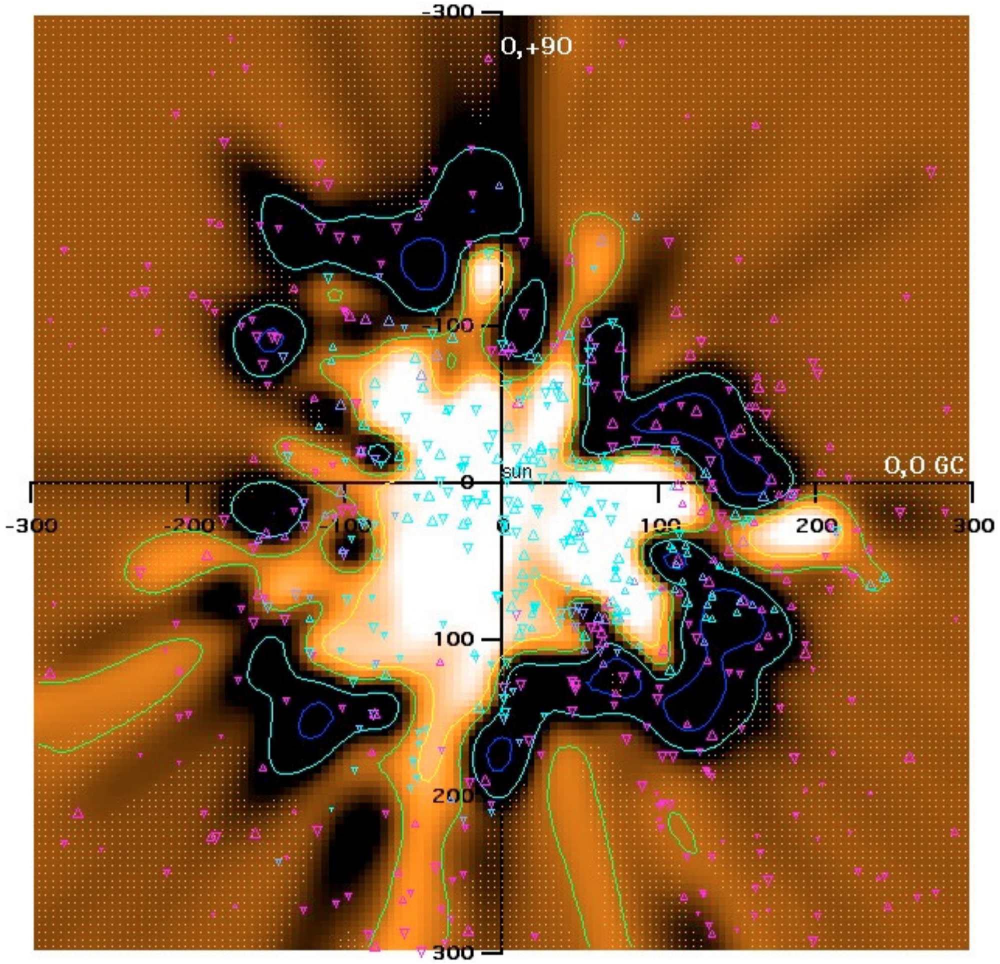
$$\sigma_{\mathrm{pi}} \approx 6.3 \times 10^{-18}\,\mathrm{cm}^2\ (h\nu/I_{\mathrm{H}})^{-3}$$

    - A column depth N(HI) > 10^{20} cm^{-2} (corresponding to ~ 20 pc through the WNM, ~ 1 pc through the CNM) will absorb nearly all 100 eV photon that is emitted by a bubble of hot gas.
  - The only portion of the HIM from which we can observe 100 eV photon is the Local Bubble.

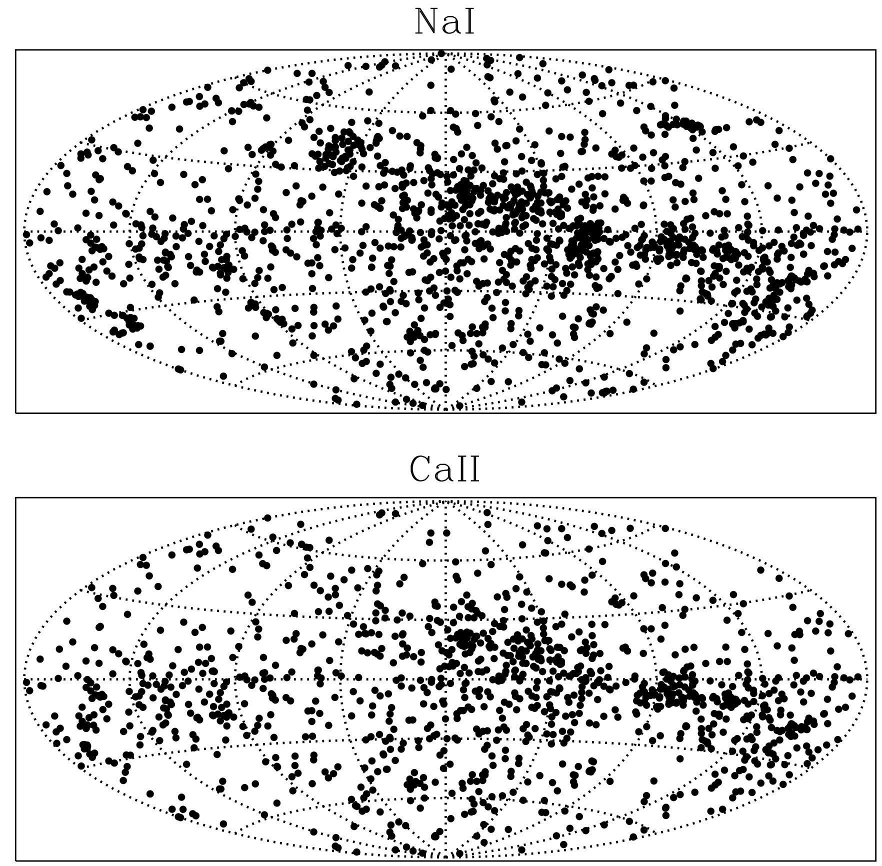
- Soft-Xray (1/4 keV) background, the Local Hot Bubble
  - The X-ray background is an unexpectedly intense diffuse flux of soft X-rays (Bowyer et al. 1968). X-ray at the low energies are easily absorbed by interstellar clouds, so it was concluded that they mush have a local origin. Moreover, observations show that there is a suspicious lack of cool, absorbing gas with about ~ 100 pc of the Sun.
  - The existence of the local hot bubble has been proposed to explain the soft X-ray background. The local bubble is a cavity in the ISM filled with hot, X-ray-emitting gas.
  - A nearby supernova explosion could carve the bubble out millions of years ago, leaving a region filled with relatively little neutral hydrogen, but a lot of 10^6 K gas.
  - But this paradigm was challenged when X-rays emanating from a comet as it passed through the solar wind.
  - When the charged ions flowing out from the Sun collide with neutral hydrogen and helium atoms in interplanetary space, they can exchange electrons, producing soft X-ray photons in the process. If the X-ray emission caused by the solar wind could account for the entire X-ray background, then it would negate the need for a local hot bubble.
  - Galeazzi et al. (2014, Nature) combined observations from the “Diffuse X-rays from the Local galaxy” (DXL) rocket mission with the ROSAT data and found that the X-ray emission from solar wind could only account for about 40% of the diffuse X-ray background.

The Local Bubble (NASA CHIPS)

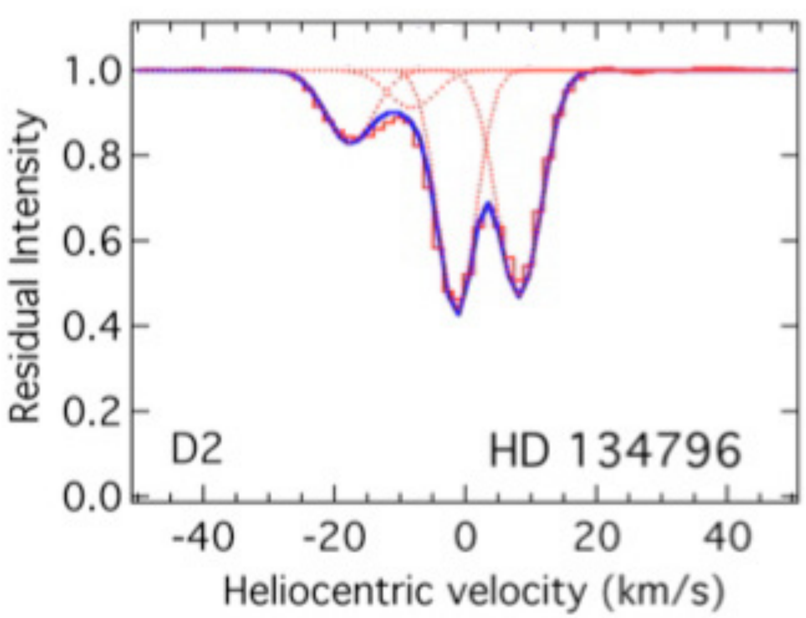




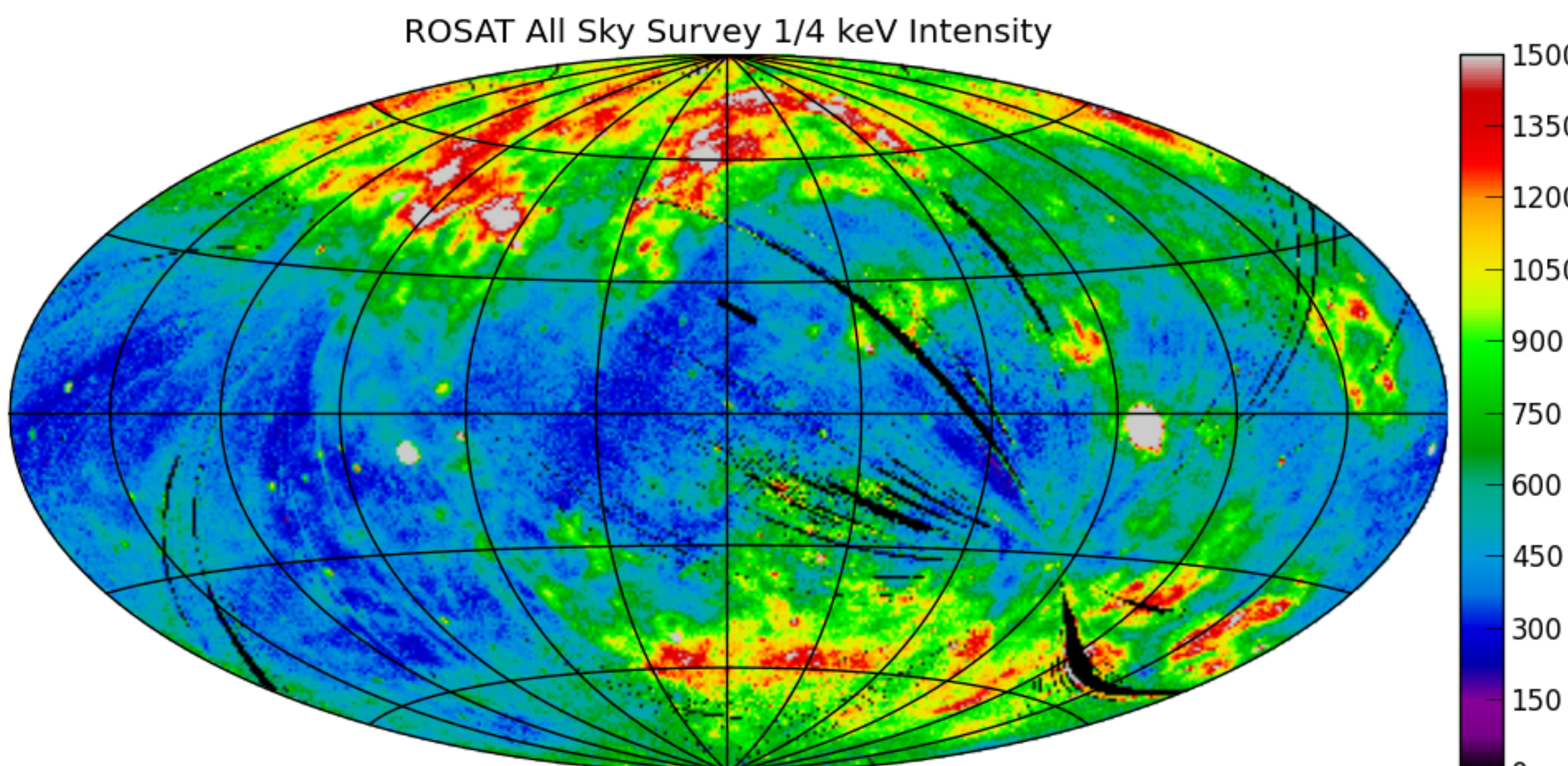
3D spatial distribution of Na I absorption, as viewed in the Galactic plane projection. (Welsh et al. 2010)



Sightlines for Na I and Ca II absorption line observations (Welsh et al. 2010)

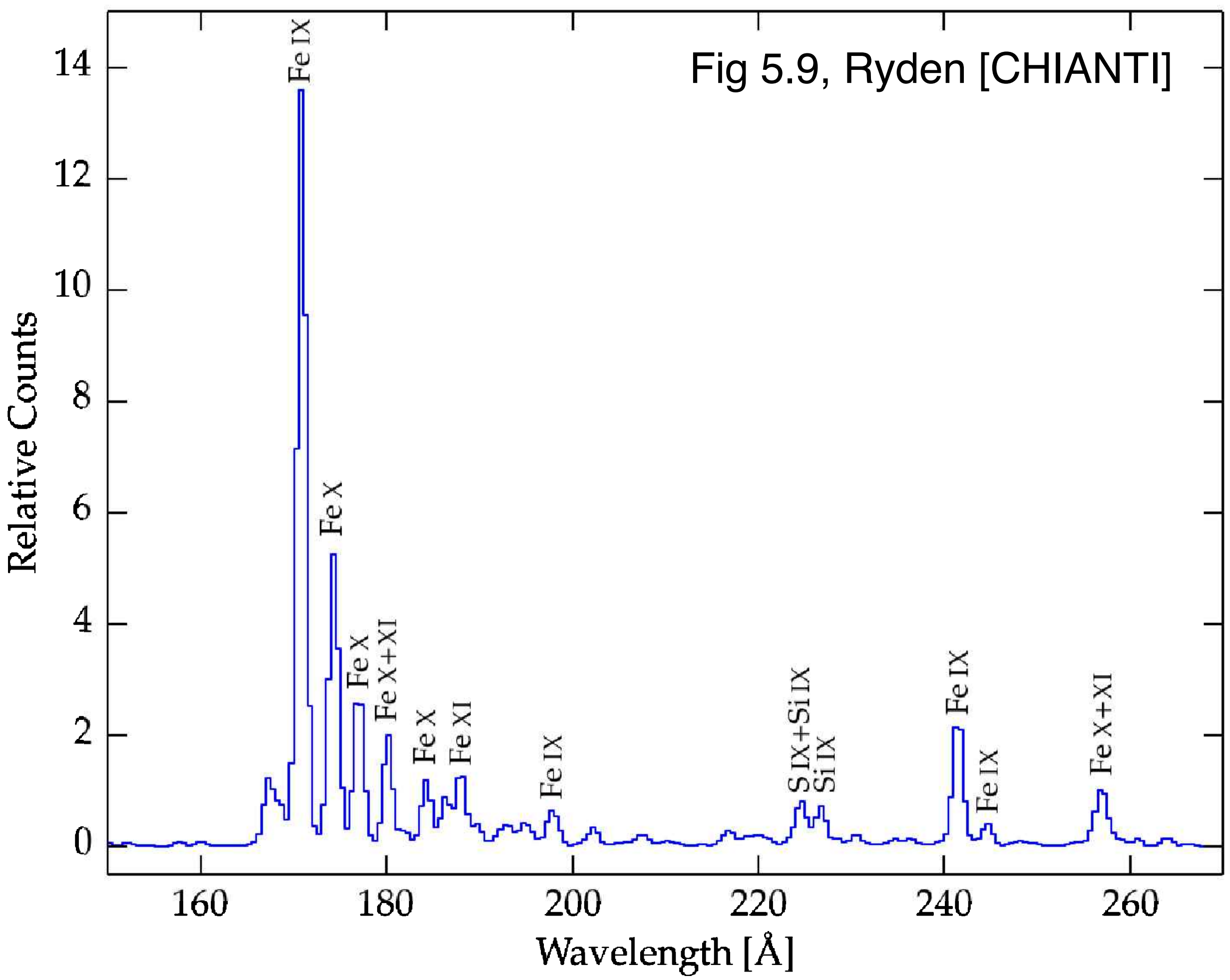


An example spectrum of Na I absorption line (Welsh et al. 2010)



intensity unit = 10^{-6} counts s^{-1} $arcmin^{-2}$ (Slavin, 2017)

- Emission Lines from the Local Bubble?
  - At T ~ 10^6 K, Hydrogen and helium is fully ionized. The most abundant heavy elements (oxygen, carbon, and neon) are primarily helium-like. The dominant forms of iron are Fe IX, X, and XI.
  - The iron ions from Fe IX to Fe XI produce a cluster of emission lines near (λ ~ 180Å). About half of the radiated power come out in the form of these iron emission lines.
  - Iron isn't fully ionized until T > 2×10^8 K. At T ~ 10^7 K, the dominant forms are Fe XX and XXI.



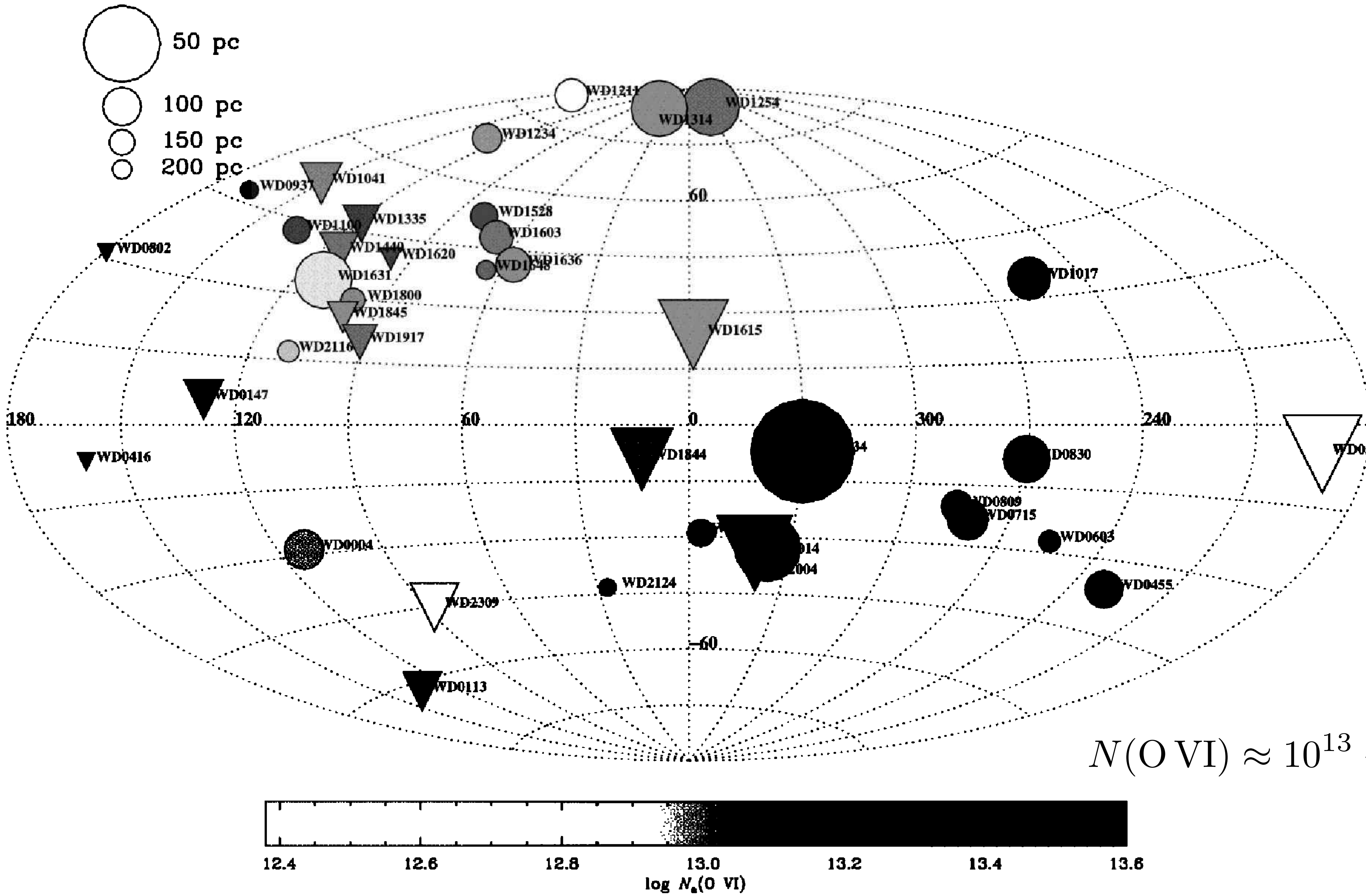
- However, observations at λ ~ 180Å by the Cosmic Hot Interstellar Plasma Spectrometer (CHIPS) showed that the Fe emission lines from the Local Bubble are much weaker than originally predicted.

- This might be attributable to (1) depletion of iron onto dust grains, (2) a somewhat lower temperature (7×10^5 K), and (3) delayed recombination.

- The Local Bubble is cooling, and Fe is more highly ionized than CIE tells us.

- Absorption Lines from the Local Bubble.
  - One important pair of UV lines is the Lithium-like OVI doublet 1032Å (12.01 eV) and 1038Å (11.95 eV). These lines are a major coolant for interstellar gas at T ~ 3×10^5 K.
  - At T ~ 3×10^5 K, only ~ 25% of the total oxygen abundance is O VI (~35% is OV and ~35% is OVII). However, the O VI doublet lines have a large cross section, and thus are readily observed even if the number density of OVI ions is small.
  - To detect the OVI lines in absorption, we need background sources with $T_{eff} > 1.4 \times 10^5$ K (12 eV). Main sequence stars are not this hot. So, we have to look at nearby, young, hot white dwarfs. In particular, white dwarfs of spectral type DA, which have only hydrogen lines in their spectra, are the best targets.
  - The distribution of OVI in the Local Bubble appears to be patchy, and the mean density of OVI lies in the range N(OVI)/d ~ $(0.7 - 13) \times 10^{-8}$ cm^{-3}. (See Savage & Lehner 2006).
  - The thermal broadening parameter was found to range from b = 15 km s^{-1} to 36 km s^{-1}. From this, we can find a temperature:

$$T = 2.5 \times 10^5\,\mathrm{K}\left(\frac{b}{16\,\mathrm{km\,s^{-1}}}\right)^2\left(\frac{m}{16\,m_{\mathrm{H}}}\right)$$



$$N(\mathrm{O\,VI}) \approx 10^{13} - 10^{13.6}\,\mathrm{cm}^{-2}$$

Location in galactic coordinates of 39 nearby hot DA white dwarfs (Savage & Lehner 2006).
Circles have OVI detection; triangles are non-detections at 2σ level.
Grayscale indicates the column density of OVI toward white dwarfs with detections.

Fig 5.11 [Ryden]

Homework

Q1.

The "cooling time" $\tau_{\rm cool} \equiv |d\ln T/dt|^{-1}$. Suppose the power radiated per unit volume Λ can be approximated by

$$\Lambda \approx A n_{\rm H} n_e \left[T_6^{-0.7} + 0.021 T_6^{1/2}\right]$$

for gas of cosmic abundances, where $A = 1.1\times10^{-22}\,{\rm erg\,cm^3\,s^{-1}}$, and $T_6 \equiv T/10^6\,{\rm K}$. Assume the gas to have $n_{\rm He} = 0.1 n_{\rm H}$, with both H and He fully ionized.

Compute the cooling time (at constant pressure) due to radiative cooling

(a) in a supernova remnant at $T = 10^7\,{\rm K}$, $n_{\rm H} = 10^{-2}\,{\rm cm^{-3}}$.

(b) for intergalactic gas within a dense galaxy cluster (the "intracluster medium") with $T = 10^8\,{\rm K}$, $n_{\rm H} = 10^{-3}\,{\rm cm^{-3}}$.

Q2.

Consider a strong shock wave propagating into a medium that was initially at rest. Assume the gas to be monatomic ($\gamma = 5/3$). Consider the material just behind the shock front. The gas has an energy density $u_{\rm thermal}$ from random thermal motions, and an energy density $u_{\rm flow}$ from the bulk motion of the shocked gas. If cooling is negligible, calculate the ratio $u_{\rm flow}/u_{\rm thermal}$ in the frame of reference where the shock front is stationary.