Interstellar Medium (ISM)

Week 7
2025 April 14 (Monday), 2PM

updated 04/13, 09:22

선광일 (Kwangil Seon)
KASI / UST

[2] Spin Temperature Determination - Emission & Absorption

- To derive the spin temperature, **we need to combine the emission observation with an absorption observation,** which is so called ***"emission-absorption" method***.

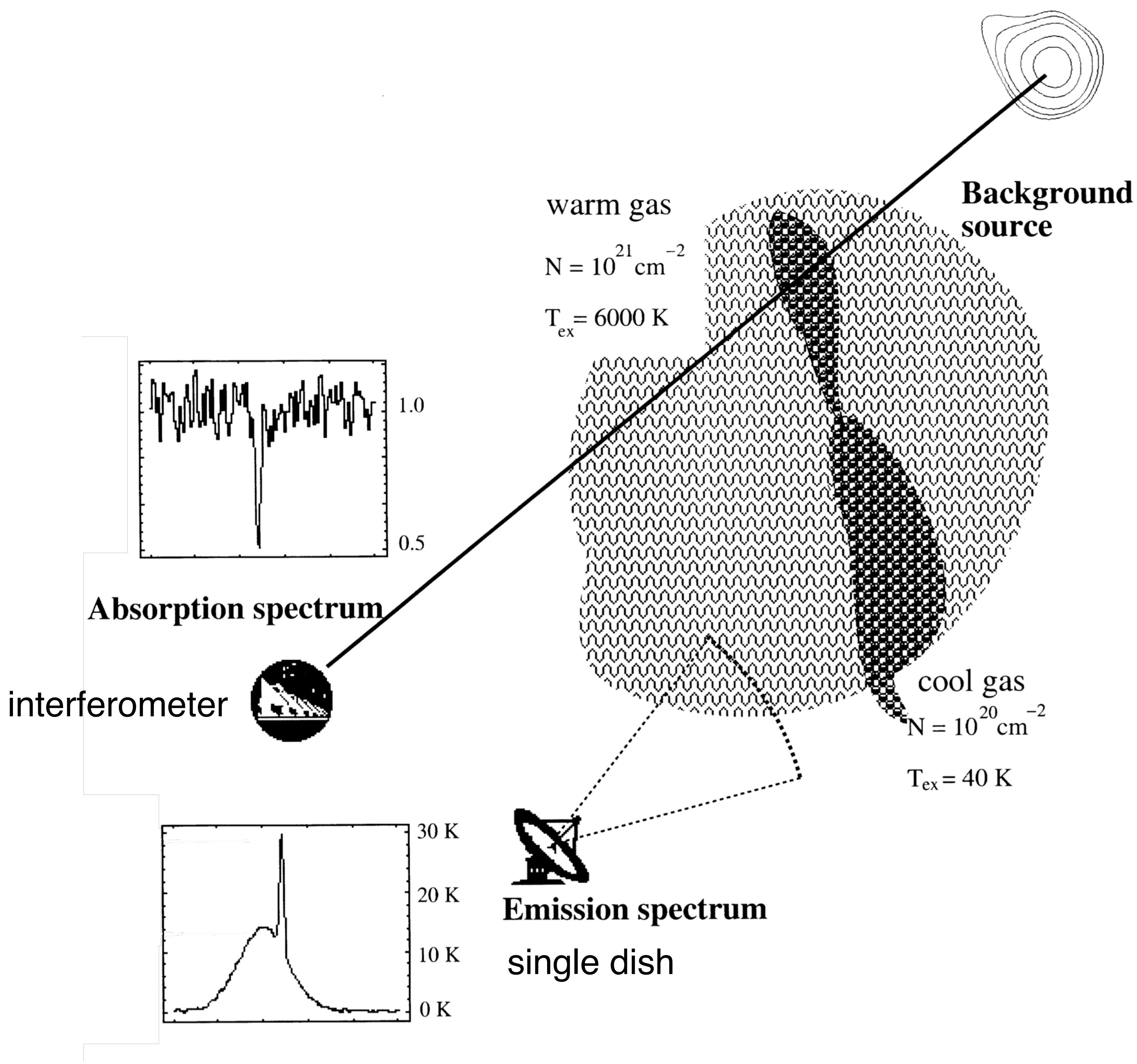
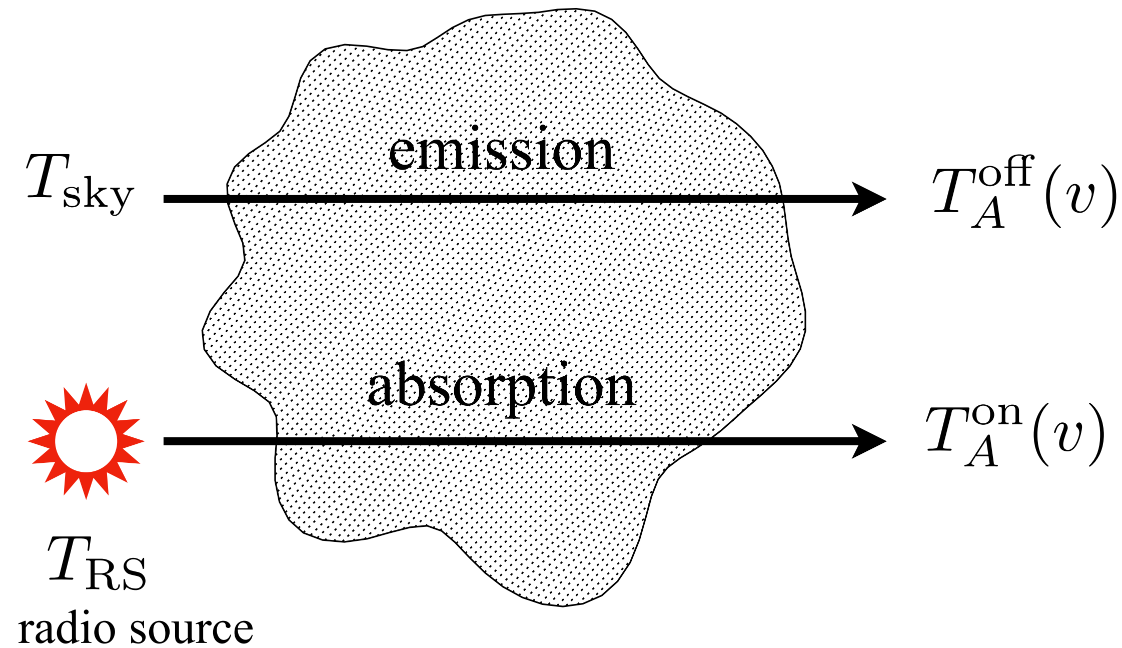


Figure 15 in Dickey et al. (2000, ApJ)

- In cases where we have a ***"bright" background radio source*** with a continuum spectrum (a typical radio-loud quasar or an active galactic nucleus, or a radio galaxy), we can study both emission and absorption by the foreground ISM in our galaxy by comparing ***"on-source" and "off-source" observations***.

- The spectra measured on the blank sky and on the radio source are, respectively,

$$T_A^{\rm on}(v) = T_{\rm RS} e^{-\tau_v} + T_{\rm spin}(v)\left(1 - e^{-\tau_v}\right)$$
$$T_A^{\rm off}(v) = T_{\rm sky} e^{-\tau_v} + T_{\rm spin}(v)\left(1 - e^{-\tau_v}\right)$$



- These two equation can be solved for the two unknowns, $\tau(v)$ and $T_{\rm spin}(v)$.

$$\tau(v) = \ln\left[\frac{T_{\rm RS} - T_{\rm sky}}{T_A^{\rm on}(v) - T_A^{\rm off}(v)}\right]$$

$$T_{\rm spin}(v) = \frac{T_A^{\rm off}(v) T_{\rm RS} - T_A^{\rm on}(v) T_{\rm sky}}{(T_{\rm RS} - T_{\rm sky}) - \left[T_A^{\rm on}(v) - T_A^{\rm off}(v)\right]}$$

The solutions give, in general, the spin temperature and the column density as functions of velocity.

We can derive the column density from these two quantities for an optically thick cloud.

- We usually consider a case where ***the radio source is "much" brighter than the spin temperature of the intervening hydrogen cloud***.
  - The RT equations for the "on-source" and "off-source" measurements can be written:

$$\text{assumptions}: T_{\rm RS} \gg T_{\rm spin} \gg T_{\rm sky}$$

$$(1) \quad T_A^{\rm on}(v) = T_{\rm RS} e^{-\tau_v} + T_{\rm spin}(v)\left(1 - e^{-\tau_v}\right)$$

$$(2) \quad T_A^{\rm off}(v) = T_{\rm sky} e^{-\tau_v} + T_{\rm spin}(v)\left(1 - e^{-\tau_v}\right)$$

$$\longrightarrow$$

$$\frac{T_{\rm RS} - T_A^{\rm on}(v)}{T_{\rm RS}} = 1 - e^{-\tau_v} - \frac{T_{\rm spin}(v)}{T_{\rm RS}}\left(1 - e^{-\tau_v}\right)$$

$$T_A^{\rm off}(v) = T_{\rm sky} + \left(T_{\rm spin} - T_{\rm sky}\right)\left(1 - e^{-\tau_v}\right)$$

$$(1) \quad \frac{T_{\rm RS} - T_A^{\rm on}(v)}{T_{\rm RS}} \approx 1 - e^{-\tau_v}$$

$$(2) \quad \Delta T_A^{\rm off}(v) = \Delta T_{\rm spin}(v)\left(1 - e^{-\tau_v}\right)$$

$$\text{or } T_A^{\rm off}(v) \approx T_{\rm spin}(v)\left(1 - e^{-\tau_v}\right)$$

Here, $\Delta T_A^{\rm off}(v) \equiv T_A^{\rm off}(v) - T_{\rm sky} \approx T_A^{\rm off}$

$\Delta T_{\rm spin}(v) \equiv T_{\rm spin}(v) - T_{\rm sky} \approx T_{\rm spin}$

$(T_{\rm sky} \approx 3\,{\rm K})$

Hereafter, $T_A^{\rm off}$ and $T_{\rm spin}$ denote $\Delta T_A^{\rm off}$ and $\Delta T_{\rm spin}$, respectively.

- ***Equivalent Width:***
  - Using the absorption spectrum from the "on-source" observation, we can "approximately" obtain the "**velocity equivalent width**."

$$W_v = \int dv \left(1 - e^{-\tau_v}\right) \approx \int dv \left[\frac{T_{\rm RS} - T_A^{\rm on}(v)}{T_{\rm RS}}\right]$$

$$\text{Note}: W_v = c \int \frac{d\nu}{\nu_{u\ell}} \left(1 - e^{-\tau_\nu}\right) = cW$$

For a weak absorption line, this approximation is **a lower limit.**

$$\frac{T_{\rm RS} - T_A^{\rm on}(v)}{T_{\rm RS}} = 1 - e^{-\tau_v} - \frac{T_{\rm spin}(v)}{T_{\rm RS}} \left(1 - e^{-\tau_v}\right) \le 1 - e^{-\tau_v}$$

In the optically thin case, we also note that $W_v \approx \int \tau_v dv$.

- ***Spin Temperature:***
  - Using the equation (2) in the previous page, we can obtain two spin temperatures. The first one is the line-of-sight average spin temperature, and the second the spin temperature in a velocity channel.

$$\langle T_{\rm spin} \rangle \approx \frac{\int T_A^{\rm off}(v) dv}{\int \left(1 - e^{-\tau_v}\right) dv} = \frac{\int T_A^{\rm off}(v) dv}{W_v} \longleftarrow$$

assuming $T_{\rm spin} =$ constant.

$$\int dv \left[T_A^{\rm off}(v)\right] = T_{\rm spin} \int dv \left(1 - e^{-\tau_v}\right)$$

$$T_{\rm spin}(v) = \frac{T_A^{\rm off}(v)}{\left(1 - e^{-\tau_v}\right)}$$

For a weak absorption line, this $T_{\rm spin}$ is a lower limit.

- ***In an optically thin limit,***
  - We know the relation between the antenna temperature and column density:

$$N_{\mathrm{HI}} = C_0 \int \left[T_A^{\mathrm{off}}(v) - T_{\mathrm{sky}}\right]$$
$$\approx C_0 \int T_A^{\mathrm{off}}(v) dv$$

$$\frac{dN_{\mathrm{HI}}}{dv} \approx C_0 T_A^{\mathrm{off}}(v)$$

  - Then, we can express the spin temperature in terms of column density and equivalent width (absorption profile):

$$\langle T_{\mathrm{spin}} \rangle = \frac{1}{W_v} \int T_A^{\mathrm{off}}(v) dv = \frac{C_0^{-1}}{W_v} N_{\mathrm{HI}}$$

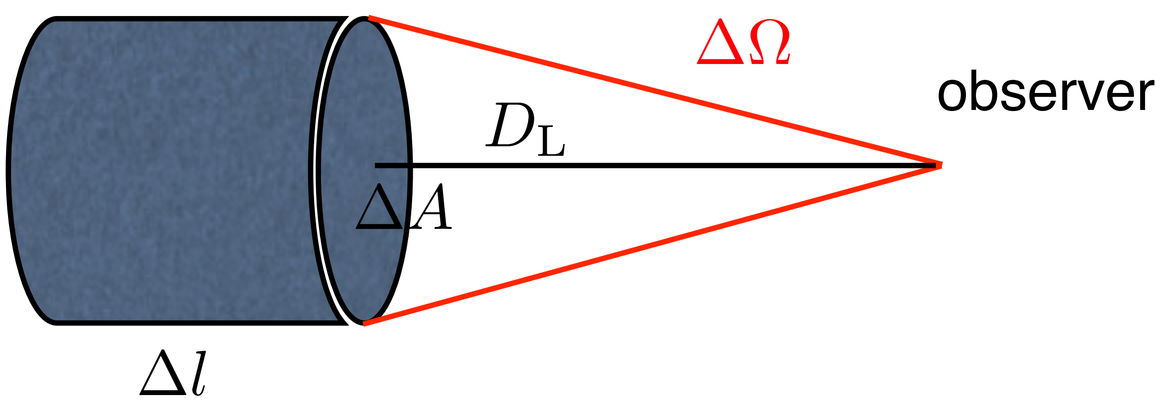
$$\langle T_{\mathrm{spin}} \rangle \approx 0.5516 \, \frac{N_{\mathrm{HI}}/10^{18}\,\mathrm{cm}^{-2}}{W_v/\mathrm{km\,s}^{-1}} \; [\mathrm{K}]$$

$$T_{\mathrm{spin}}(v) = \frac{C_0^{-1}}{(1 - e^{-\tau_v})} \frac{dN_{\mathrm{HI}}}{dv}$$

$$C_0^{-1} = 5.516 \times 10^{-19} \; \left[\frac{\mathrm{K\,km\,s}^{-1}}{\mathrm{cm}^{-2}}\right]$$

H I mass of an External Galaxy

- With the assumption that the emitting regions are optically thin, the total mass $M_{\rm HI}$ of H I in an external galaxy can be determined from the observed flux in the 21-cm line:



$F_\nu =$ observed flux density

$$F_{\rm obs} = \int F_\nu d\nu_{\rm obs} = I\Delta\Omega \qquad \leftarrow \cos\theta \approx 1$$

$$I = \int I_\nu d\nu_{\rm obs} = \frac{3}{16\pi} A_{u\ell} h\nu_{u\ell} N_{\rm HI}$$

$$I_\nu = \int j_\nu ds$$

- Here, $D_{\rm L}$ is the luminosity distance to the galaxy.

The luminosity distance is defined by the relationship between flux and luminosity:

$$F = \frac{L}{4\pi d_L^2}$$

$$n_{\rm H}\Delta l = N_{\rm HI}$$

$$\Delta A = D_{\rm L}^2 \Delta\Omega = D_{\rm L}^2 \frac{F_{\rm obs}}{I}$$

$$\begin{aligned} M_{\rm HI} &= m_{\rm H} n_{\rm H} \Delta V = m_{\rm H} n_{\rm H} \Delta l \Delta A \\ &= m_{\rm H} N_{\rm HI} D_{\rm L}^2 \frac{F_{\rm obs}}{I} \\ &= m_{\rm H} N_{\rm HI} D_{\rm L}^2 \frac{F_{\rm obs}}{(3/16\pi) A_{u\ell} h\nu_{u\ell} N_{\rm HI}} \end{aligned}$$

$$\begin{aligned} \therefore\ M_{\rm HI} &= \frac{16\pi}{3} \frac{m_{\rm H}}{A_{u\ell} h\nu_{u\ell}} D_{\rm L}^2 F_{\rm obs} \\ &= 4.945 \times 10^7 M_\odot \left(\frac{D_{\rm L}}{\rm Mpc}\right)^2 \left(\frac{F_{\rm obs}}{\rm Jy\ MHz}\right) \end{aligned}$$

- If the redshift of the galaxy is z :

$$\nu_{\rm obs} = \nu/(1+z) \longleftarrow \quad z = \frac{\nu_{\rm obs} - \nu_0}{\nu_0}$$

$$d\nu_{\rm obs} = \frac{\nu_{u\ell}}{(1+z)}\frac{dv}{c}$$

$$\begin{aligned} M_{\rm HI} &= \frac{16\pi}{3}\frac{m_{\rm H}}{A_{u\ell}h\nu_{u\ell}}D_{\rm L}^2\int F_\nu d\nu_{\rm obs} \\ &= \frac{16\pi}{3}\frac{m_{\rm H}}{A_{u\ell}h\nu_{u\ell}}D_{\rm L}^2\frac{\nu_{u\ell}}{c}\frac{1}{1+z}\int F_\nu dv \\ &= \frac{16\pi m_{\rm H}}{3A_{u\ell}hc}D_{\rm L}^2(1+z)^{-1}\int F_\nu dv \\ &= 2.343\times 10^5\, M_\odot (1+z)^{-1}\left(\frac{D_{\rm L}}{\rm Mpc}\right)^2\frac{\int F_\nu dv}{\rm Jy\,km\,s^{-1}} \end{aligned}$$

- Radio astronomers often report the integrated flux in “Jy km s^{-1}.”

Observations: Example 1

- All-sky map of H I 21-cm line intensity from the LAB survey (Kalberla et al. 2005), with an angular resolution ~ 0.6 deg.
    - Scale gives $\log_{10}$ N(HI) [cm^{-2}]. The LMC and SMC are visible, with a connecting H I "bridge".
    - The map was obtained by assuming the optically thin case.

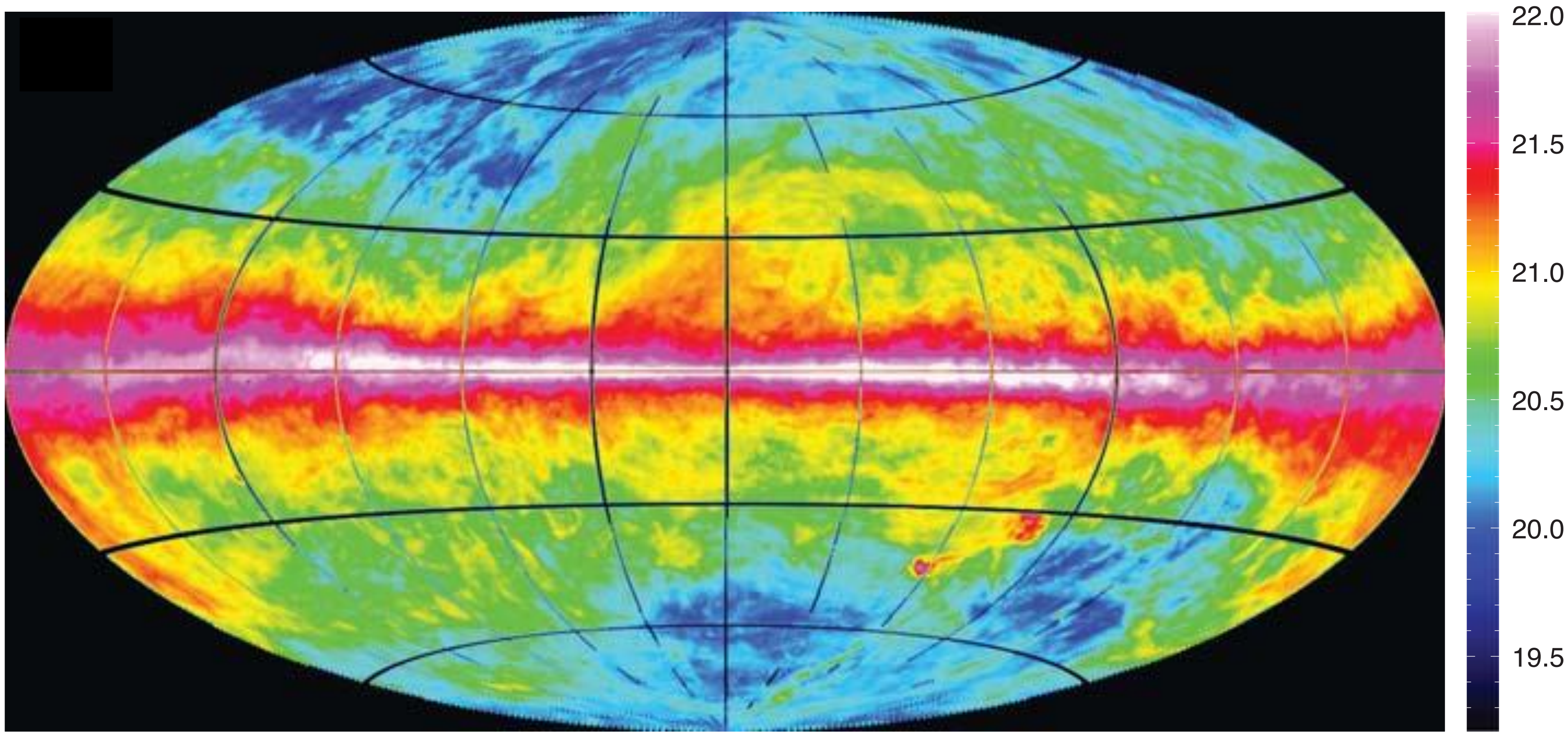
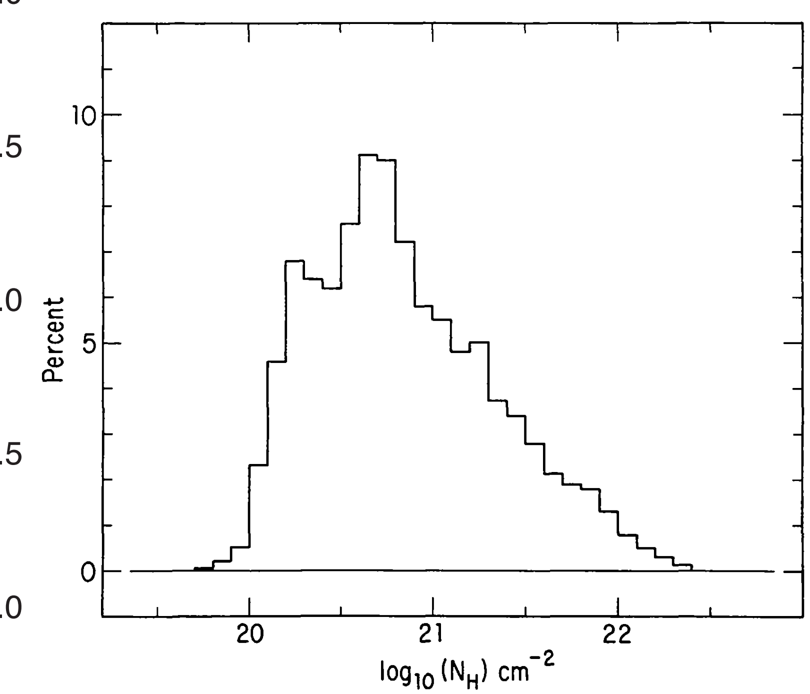


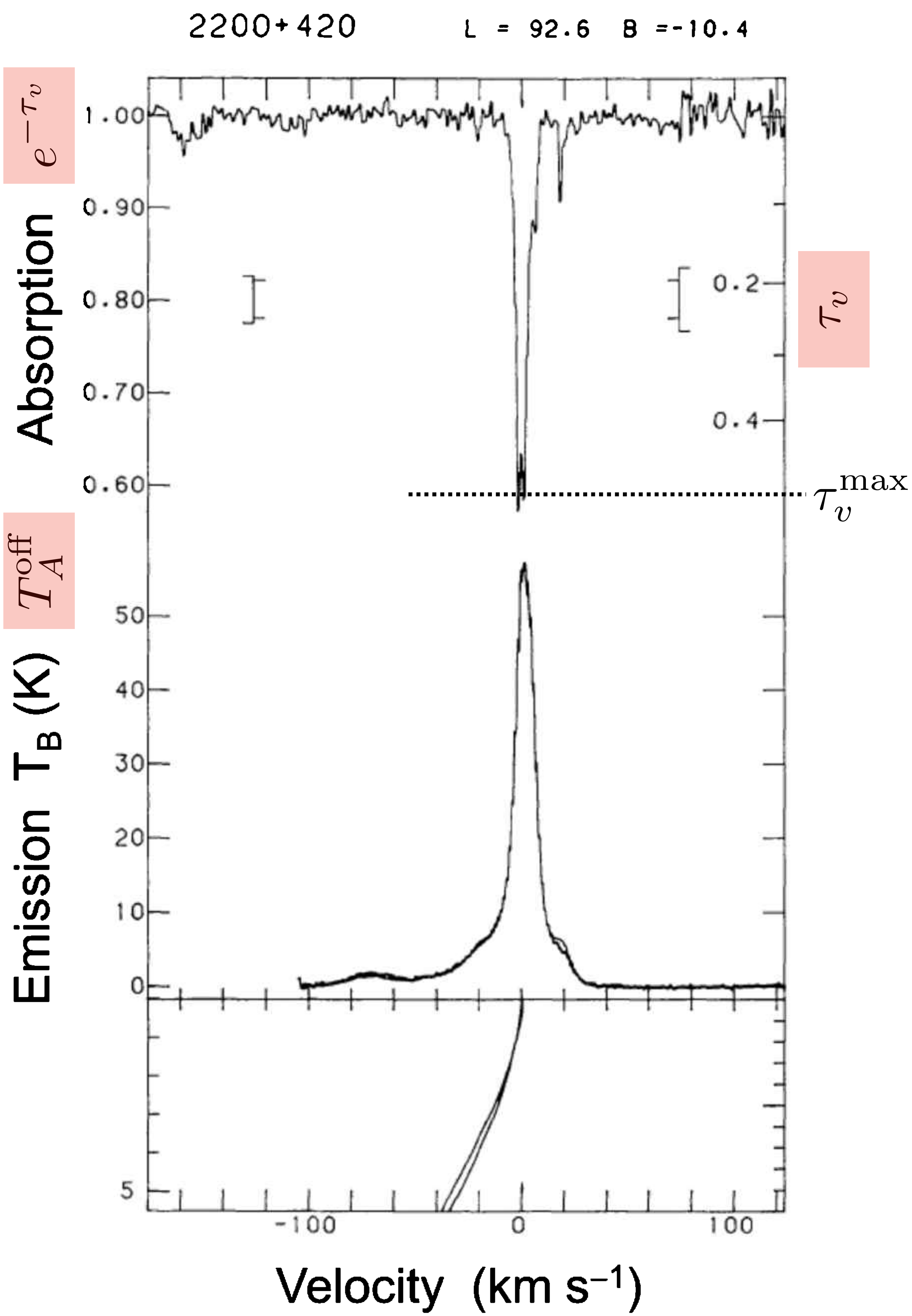
Plate 3 in [Draine]



The percentage of the sky covered by H I at a given N_H.

Figure 4 in Dickey & Lockman (1990, ARA&A)

Observations: Example 2



H I 21-cm absorption and emission along the line of sight towards BL Lacertae [Dickey et al. 1983; Figure 3.3 in Ryden]

[Absorption spectrum]

- Maximum optical depth :

$$\tau_v \sim 0.5 \qquad \text{optically thin}$$

- Equivalent width of the absorption line :

$$W_v = 7\,\mathrm{km\,s^{-1}}$$

[Emission spectrum]

- Integrated line intensity of the emission line :

$$\int T_A^{\rm off}(v)dv \approx 930\,\mathrm{K\,km\,s^{-1}}$$

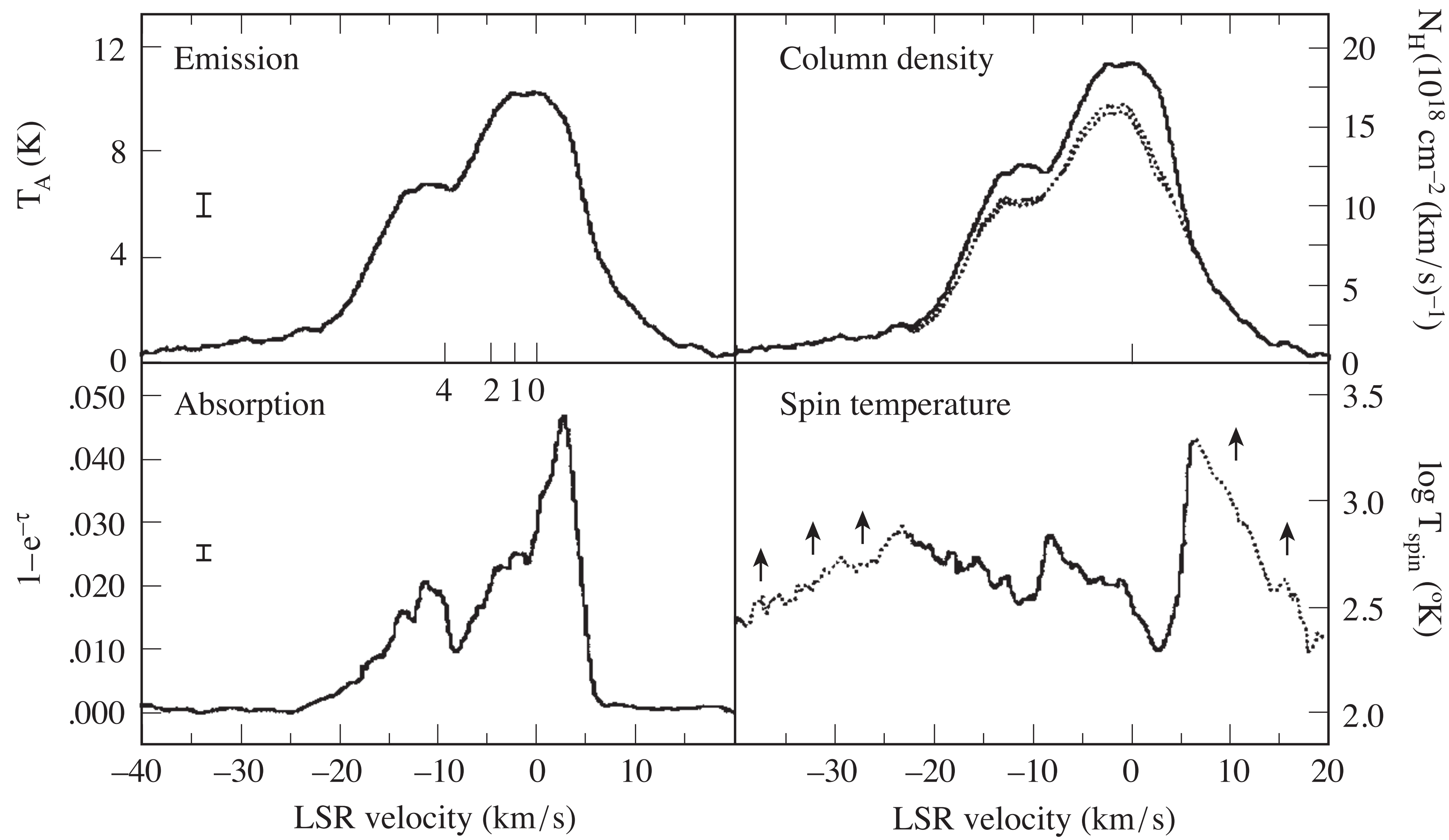
- Column density from the emission line :

$$N_{\rm HI} \approx 1.69 \times 10^{21}\,\mathrm{cm^{-2}}$$

- Now, the spin temperature is

$$\boxed{T_{\rm spin} = \frac{\int T_A^{\rm off}(v)dv}{W_v} = \frac{930\,\mathrm{K\,km\,s^{-1}}}{7\,\mathrm{km\,s^{-1}}} \approx 133\,\mathrm{K}.}$$

Observations: Example 3



Left panels: Observed H I emission (off the quasar 3C48) and absorption (toward 3C48, at $\ell = 134°$, $b = -28.7°$). Lower right: spin temperature $T_{\rm spin}(v)$ as a function of LSR velocity. Tick marks labeled 0, 1, 2, and 4 on abscissa of left panels show the LSR velocity expected for gas at a distance of 0,1,2,4 kpc (for an assumed Galactic rotation curve). Upper right: $dN(\mathrm{H\,I})/dv$ for different assumptions regarding the relative (foreground/background) locations of cold absorbing gas and warm gas seen only in emission. From Dickey et al. (1978).

[Figure 29.1 in Draine]

Define a point in space that is moving on a perfectly circular orbit around the center of the Galaxy at the Sun's galactocentric distance.

We measure all velocities of astronomical objects relative to this point, which is the **LSR (local standard of rest)**.

The Sun's orbital speed is 202-241 km/s (Majewski 2008, IAUS, 248).

Observations: CNM & WNM

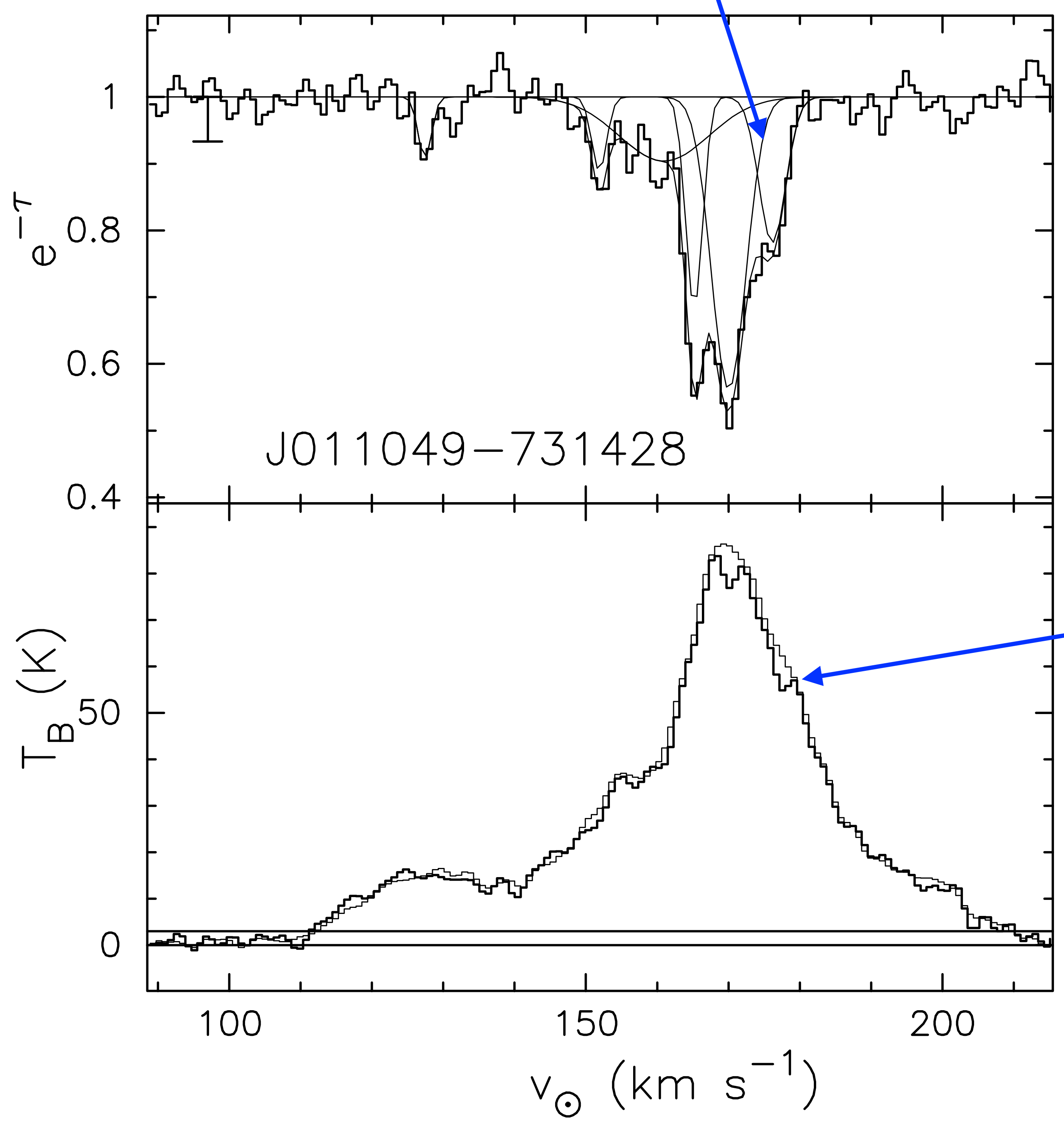
- The overall shape of the emission line profiles in our Galaxy is mainly determined by the large-scale distribution and kinematics of H I.

- Absorption lines are always narrower than emission lines.
  - Some velocities have H I emission but no detectable H I absorption.
  - Difference between emission and absorption spectrum:
    - Absorption spectra can usually be decomposed into Gaussian components.
    - However, emission spectra do not look like the superposition of a few Gaussians.
    - This is because the absorption lines arise only in regions of cool gas, which are more distinct along the line of sight, and thus have narrower intrinsic line widths than the gas that contributes to H I emission.

- The difference between emission and absorption results mainly from variation in the spin temperature of the H I along the line of sight.
  - Recall that the optical depth is inversely proportional to the spin temperature, indicating the difficulties in observing absorption spectra from the warm neutral medium, which has a temperature larger than 1000 K.

$$\boxed{\tau(v) = \frac{C_0^{-1}}{T_{\rm spin}(v)} \frac{dN_{\rm HI}}{dv}}$$

- Absorption and Emission spectra



Thin lines: sum of Gaussians

Absorption lines are mostly, if not all, caused by the CNM.

$$\tau(v) = \frac{C_0^{-1}}{T_{\rm spin}(v)} \frac{dN_{\rm HI}}{dv}$$

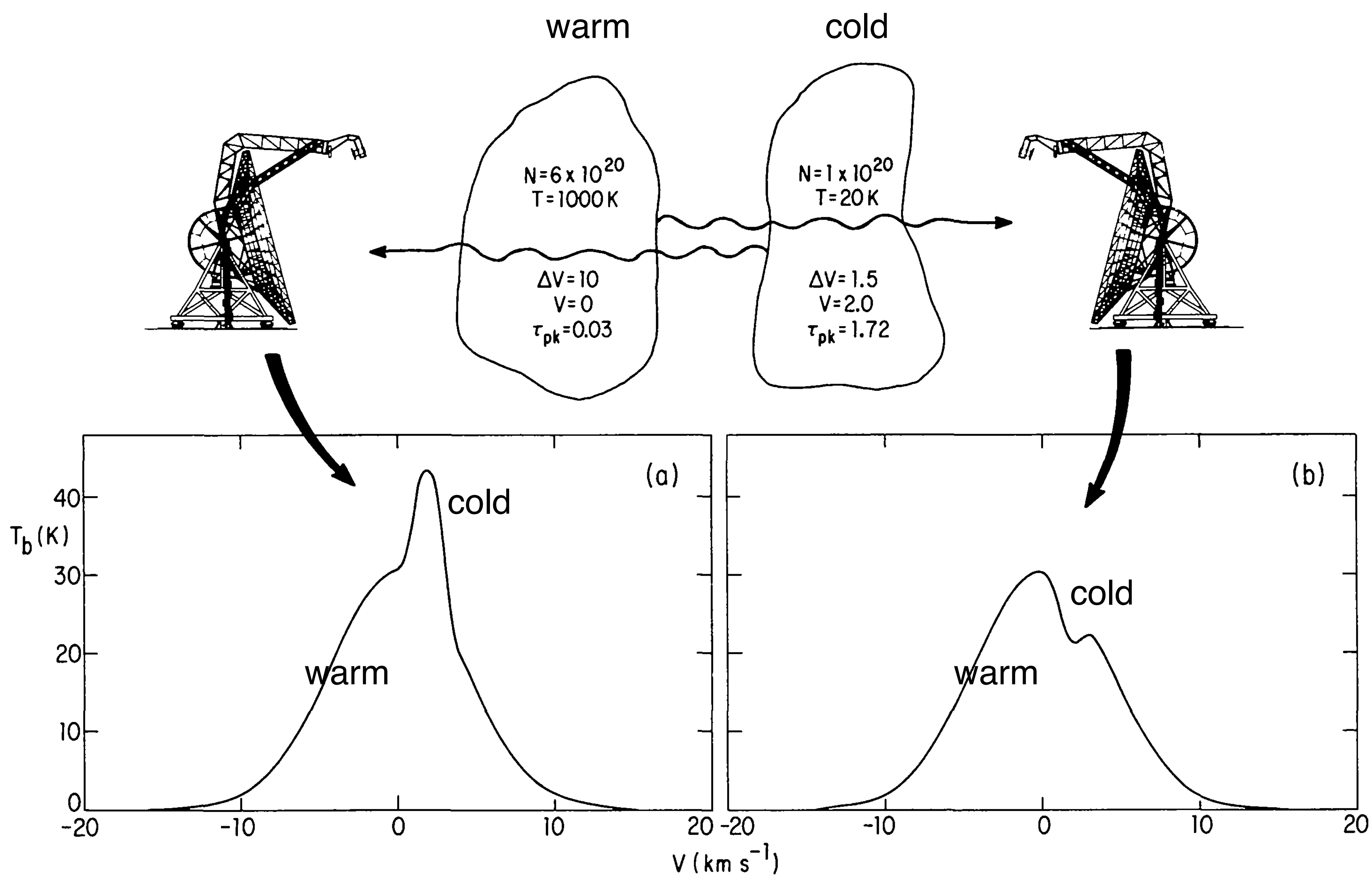
Emission lines are composed of emissions from the CNM and WNM.

Thin and Thick lines: two different methods to estimate the emission spectrum

(top) Absorption and (bottom) emission spectra in a direction of Small Magellanic Cloud.

Figure 11 of Dickey et al. (2000)

Emission spectrum depending on the relative location of WNM and CNM



[Schematic of the geometry of 21-cm self-absorption]

The structure of an emission profile depends on the relative location of warm and cold clouds as viewed by the observer.

Figure 1 of Dickey & Lockman (1990)

A rough estimation of the fraction of gas in the cold phase

- The interstellar atomic hydrogen is in at least two thermal phases.
  - We assume that the warm gas temperature is large enough that no absorption is seen from the warm phase.
  - We further assume a value for the cold-phase temperature, for instance, $T_c \approx 55\,\mathrm{K}$.
  - Then, the fraction of gas in the cold phase is

The warm gas and cool gas are mixd so that $T_{\rm spin}$ is a weighted averaged, whether computed for individual velocity channels or line-of-sight integrals.

$$\boxed{f_c \equiv \frac{N_c}{N_w + N_c} \approx \frac{T_c}{\langle T_{\rm spin}\rangle}}$$

$$T_c = \frac{C_0^{-1}}{W_v^c} N_c$$

$$\langle T_{\rm spin}\rangle = \frac{C_0^{-1}}{W_v} N_{\rm HI}, \text{ where } W_v = W_v^c + W_c^w \approx W_v^c \text{ and } N_{\rm HI} = N_w + N_c$$

  - This gives us a rough estimation of the cold-phase H I fraction for galaxies.

Cool-Phase H I Fractions for Galaxies

Galaxy	Sample Size	$\langle T_s \rangle$ (K)	f_c ($T_c = 55$ K)
SMC	28	440	0.13
LMC	49	170	0.33
M31	16	150	0.37
M33	7	370	0.15
Milky Way	19	250	0.22

Table 3 of Dickey et al. (2000, ApJ, 536, 756)

Observations: CNM + WNM in our Galaxy

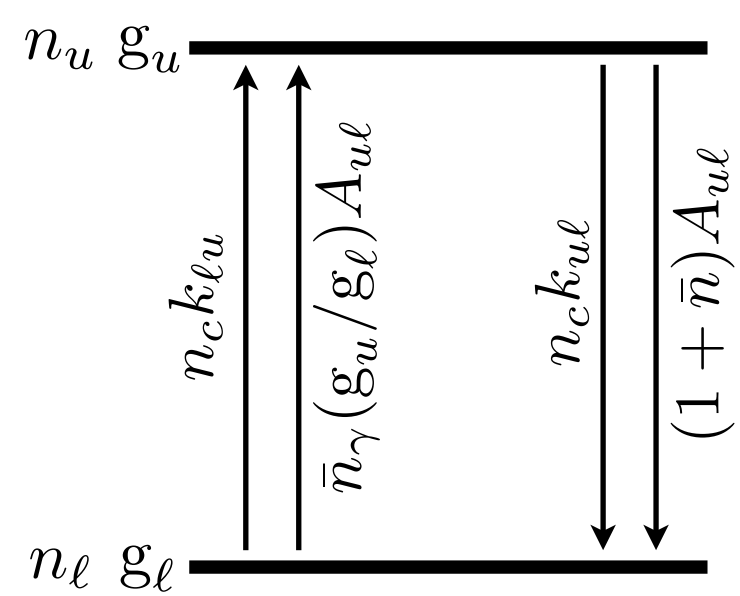
- The emission-absorption surveys (Heiles & Troland 2003) support the idea that, in the solar neighborhood (i.e., within ~ 500 pc of the Sun), interstellar H I is found primarily in two distinct phases: the CNM and the WNM.
  - About 40% of the H I (by mass) is in the CNM, with a median spin temperature $T_{\rm spin} \sim 70$ K. The remaining 60% of the H I is in the WNM phase, which appears to have a volume filling factor ~ 50% near the disk midplane.
  - Because warm H I absorbs very weakly, for some of the WNM material, it is only possible to determine a lower bound on $T_{\rm spin}$. Heiles & Troland (2003) conclude that > 48% of the WNM has $500 < T_{\rm spin} < 5000\,{\rm K}$, at these temperatures the gas is expected to be thermally unstable.

- Murray et al. (2014) detected a widespread warm neutral medium component with excitation temperature $\langle T_{\rm spin} \rangle = 7000^{+1800}_{-1200}\,{\rm K}$.
  - This temperature lies above theoretical predictions based on collisional excitation alone, implying that Lyα scattering, the most probable additional source of excitation, is more important in the ISM than previously assumed.

- Murray et al. (2018) found that the WNM makes up 52% of the total H I mass.
  - Following spectral modeling, they detect a stacked residual absorption feature corresponding to WNM with $T_{\rm spin} \sim 10^4\,{\rm K}$.

Excitation temperature = Gas kinetic temperature ?

- In some cases, it is sufficient to consider only the ground state and the first excited state.
  - Consider collisional excitation and de-excitation by some species (e.g., electrons) with density n_c, and suppose that radiation with the energy density u_ν .
  - The population of the excited state must satisfy:



$$\frac{dn_u}{dt} = n_\ell \left[n_c k_{\ell u} + \bar{n}_\gamma \frac{g_u}{g_\ell} A_{u\ell} \right] - n_u \left[n_c k_{u\ell} + (1 + \bar{n}_\gamma) A_{u\ell} \right]$$

Here, photon occupation number

$$\left(\bar{n}_\gamma \equiv \frac{c^3}{8\pi h \nu^3} u_\nu \right)$$

  - The steady-state solution with radiation and collision present is

$$\frac{n_u}{n_\ell} = \frac{n_c k_{\ell u} + \bar{n}_\gamma (g_u/g_\ell) A_{u\ell}}{n_c k_{u\ell} + (1 + \bar{n}_\gamma) A_{u\ell}}$$

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} \exp\left(-E_{u\ell}/k_B T_{\rm exc}\right)$$

By combining these two equations, we can calculate the *excitation temperature* between the two levels.

Here, by the principle of detailed balance, the upward collisional rate coefficient is given in term of the downward rate coefficient by

$$k_{\ell u} = \frac{g_u}{g_\ell} k_{u\ell} e^{-E_{u\ell}/kT_{\rm gas}} \qquad (T_{\rm gas} = \text{gas kinetic energy})$$

Excitation Temperature - Limiting Cases

- It is instructive to examine the population equation in various limits:
  - In the limit of $n_c \to \infty$ and no or weak radiation field $\bar{n}_\gamma = 0$:

$$\frac{n_u}{n_\ell} = \frac{n_c k_{\ell u}}{n_c k_{u\ell} + A_{u\ell}} = \frac{k_{\ell u}}{k_{u\ell}} = \frac{g_u}{g_\ell} e^{-E_{u\ell}/kT_{\rm gas}} \qquad \therefore\ T_{\rm exc} = T_{\rm gas}$$

  - If $n_c = 0$ and the radiation field has a brightness temperature of $T_b = T_{\rm rad}$ at the frequency $\nu = E_{u\ell}/h$:

$$\frac{n_u}{n_\ell} = \frac{\bar{n}_\gamma (g_u/g_\ell)}{(1+\bar{n}_\gamma)} = \frac{g_u}{g_\ell} e^{-E_{u\ell}/kT_{\rm rad}} \qquad \therefore\ T_{\rm exc} = T_{\rm rad}$$

$$\bar{n}_\gamma = \left(e^{E_{u\ell}/kT_{\rm rad}} - 1\right)^{-1} \quad \Longrightarrow \quad 1 + \bar{n}_\gamma = \bar{n}_\gamma\, e^{E_{u\ell}/kT_{\rm rad}}$$

  - If the radiation and gas has the same temperature $T_{\rm rad} = T_{\rm gas}$, then we can show that

$$\frac{n_u}{n_\ell} = \frac{n_c k_{\ell u} + \bar{n}_\gamma (g_u/g_\ell) A_{u\ell}}{n_c k_{u\ell} + (1+\bar{n}_\gamma) A_{u\ell}} = \frac{g_u}{g_\ell} e^{-E_{u\ell}/kT_{\rm gas}} \qquad \therefore\ T_{\rm exc} = T_{\rm rad} = T_{\rm gas}$$

  - In general, $T_{\rm exc} \neq T_{\rm gas}$ and $T_{\rm exc} \neq T_{\rm rad}$. What is the excitation temperature in the ISM?

Critical Density

- Interplay between the radiation and collisions determines the excitation temperature. Which one will be more important?
- For a collision partner c, we define the critical density $n_{\text{crit},\,u}$ for an excited state u to be the density for which collisional de-excitation equals radiative de-excitation, including stimulated emission:

$$n_{\text{crit},u}(c) \equiv \frac{\sum_{\ell<u}[1+(n_\gamma)_{u\ell}]\,A_{u\ell}}{\sum_{\ell<u}k_{u\ell}(c)}$$

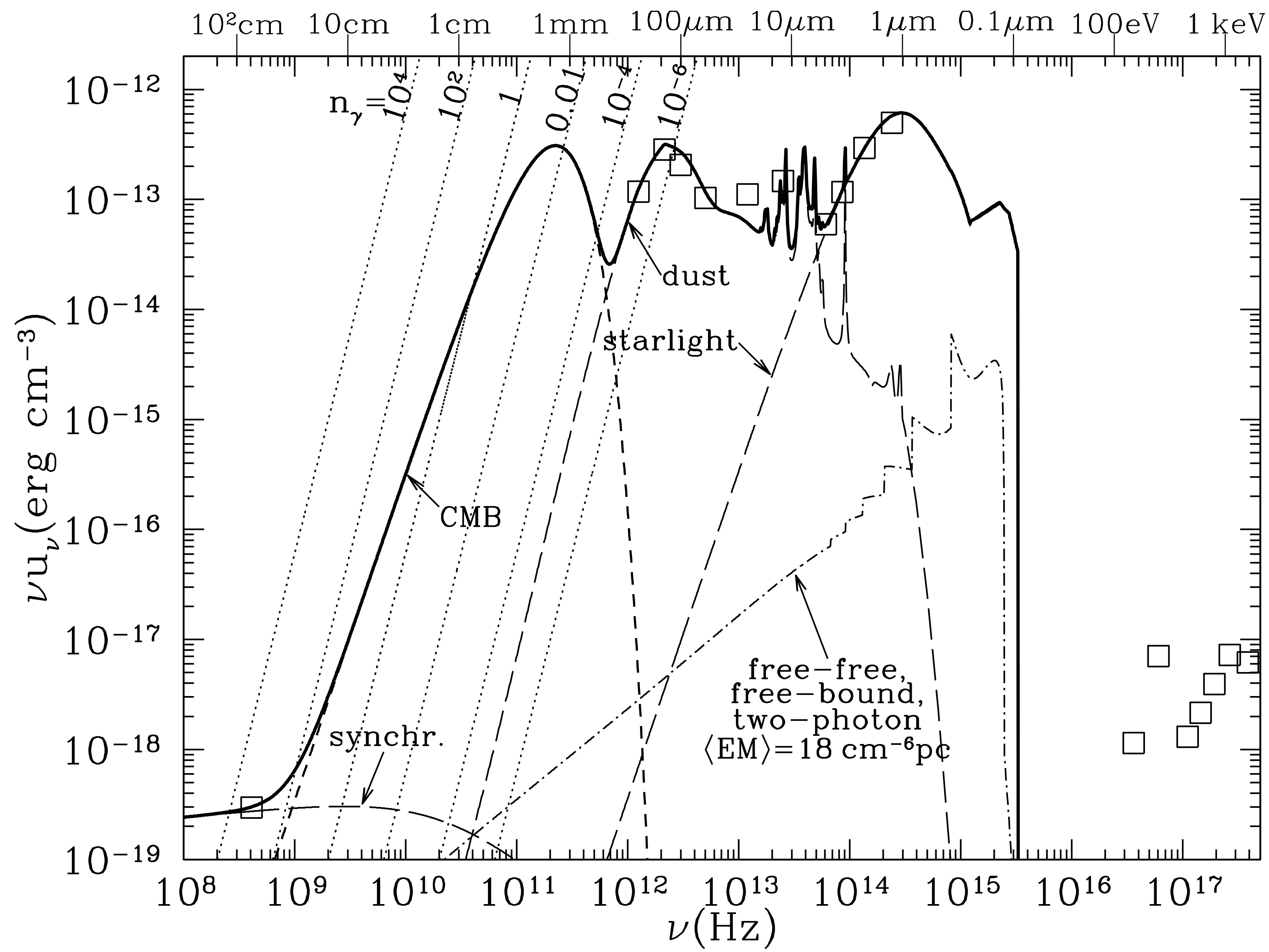
- Note that this definition applies to multilevel systems, but each excited level u may have a different critical density. The critical density depends on the intensity of ambient radiation. For many transitions, the correction by the stimulated emission is unimportant, but for 21-cm line, it is important.

Critical densities for fine-structure excitation [Table 17.1 in Draine, revised for both H and e$^-$, errata]

Ion	ℓ	u	E_ℓ/k (K)	E_u/k (K)	$\lambda_{u\ell}$ (μm)	$n_{\text{crit},u}(\text{H})$ $T=100$ K (cm^{-3})	$n_{\text{crit},u}(\text{H})$ $T=5000$ K (cm^{-3})	$n_{\text{crit},u}(e^-)$ $T=100$ K (cm^{-3})	$n_{\text{crit},u}(e^-)$ $T=5000$ K (cm^{-3})
C II	$^2\text{P}^\text{o}_{1/2}$	$^2\text{P}^\text{o}_{3/2}$	0	91.21	157.74	2.7×10^3	1.5×10^3	6.8	40.
C I	$^3\text{P}_0$	$^3\text{P}_1$	0	23.60	609.7	620	170	76.	6.4
	$^3\text{P}_1$	$^3\text{P}_2$	23.60	62.44	370.37	720	150	75.	6.3
O I	$^3\text{P}_2$	$^3\text{P}_1$	0	227.71	63.185	2.5×10^5	4.9×10^4	1.8×10^5	4.8×10^4
	$^3\text{P}_1$	$^3\text{P}_0$	227.71	326.57	145.53	2.4×10^4	8.6×10^3	2.3×10^4	5.8×10^3
Si II	$^2\text{P}^\text{o}_{1/2}$	$^2\text{P}^\text{o}_{3/2}$	0	413.28	34.814	2.5×10^5	1.2×10^5	140.	1.5×10^3
Si I	$^3\text{P}_0$	$^3\text{P}_1$	0	110.95	129.68	4.8×10^4	2.8×10^4	2.9×10^4	830.
	$^3\text{P}_1$	$^3\text{P}_2$	110.95	321.07	68.473	9.9×10^4	3.6×10^4	4.4×10^4	1.9×10^3

Interstellar Radiation Fields

To estimate the critical density, we need to know the radiation field strength.



Interstellar continuum radiation field in an H I cloud in the solar neighborhood (see text). Spectral lines are not included. Solid line is the sum of all components for $h\nu \leq 13.6\,\mathrm{eV}$. Squares show the measured sky brightness at 408 MHz (Haslam et al. 1982), the all-sky measurements by COBE-DIRBE in 10 broad bands from 240 μm to 1.25 μm (Arendt et al. 1998), and all-sky measurements by ROSAT between 150 eV and 2 keV (Snowden 2005, private communication). Dotted lines are contours of constant photon occupation number n_γ.

[Figure 12.1 in Draine]

Interstellar Radiation Field

- Starlight in an H I Region
  - There is very little radiation at energies of $13.6\,\mathrm{eV} \le E \lesssim 10^2\,\mathrm{eV}$, because they are very strongly absorbed by neutral H and He.
  - In the energy range 1 to 13.6 eV, most of the photons are starlight.
  - Mathis, Mezger, & Panagia (1983) have approximated the local starlight background as a sum of three dilute blackbodies and a piecewise power-law approximation:

$$\nu u_\nu = \sum_{j=1}^{3} \frac{8\pi h\nu^4}{c^3} \frac{W_j}{e^{h\nu/kT_j} - 1} \quad \text{for} \quad \lambda > 2450\ \text{Å}$$

$$\begin{aligned} T_1 &= 3000\,\mathrm{K}, \quad W_1 = 7 \times 10^{-13} \\ T_2 &= 4000\,\mathrm{K}, \quad W_2 = 1.65 \times 10^{-13} \\ T_3 &= 7500\,\mathrm{K}, \quad W_3 = 1 \times 10^{-14} \end{aligned}$$

$$\begin{aligned} \nu u_\nu &= 2.373 \times 10^{-14} (\lambda/1\,\mu\mathrm{m})^{-0.6678} \quad [\mathrm{erg\,cm^{-3}}] \qquad 1340 - 2450\text{Å} \\ &= 6.825 \times 10^{-13} (\lambda/1\,\mu\mathrm{m}) \quad [\mathrm{erg\,cm^{-3}}] \qquad 1100 - 1350\text{Å} \\ &= 1.287 \times 10^{-9} (\lambda/1\,\mu\mathrm{m})^{4.4172} \quad [\mathrm{erg\,cm^{-3}}] \qquad 912 - 1100\text{Å} \end{aligned}$$

- The earliest widely-cited estimate of the UV radiation field was made by Habing (1968).

$$\nu u_\nu \simeq 4 \times 10^{-14}\,\mathrm{erg\,cm^{-3}} \quad \text{at } \lambda = 1000\text{Å} \; (h\nu = 12.4\,\mathrm{eV})$$

  It is often convenient to reference other estimates to this value, so we define the dimensionless parameter:

$$\chi \equiv \frac{(\nu u_\nu)_{1000\text{Å}}}{4 \times 10^{-14}\,\mathrm{erg\,cm^{-3}}}$$

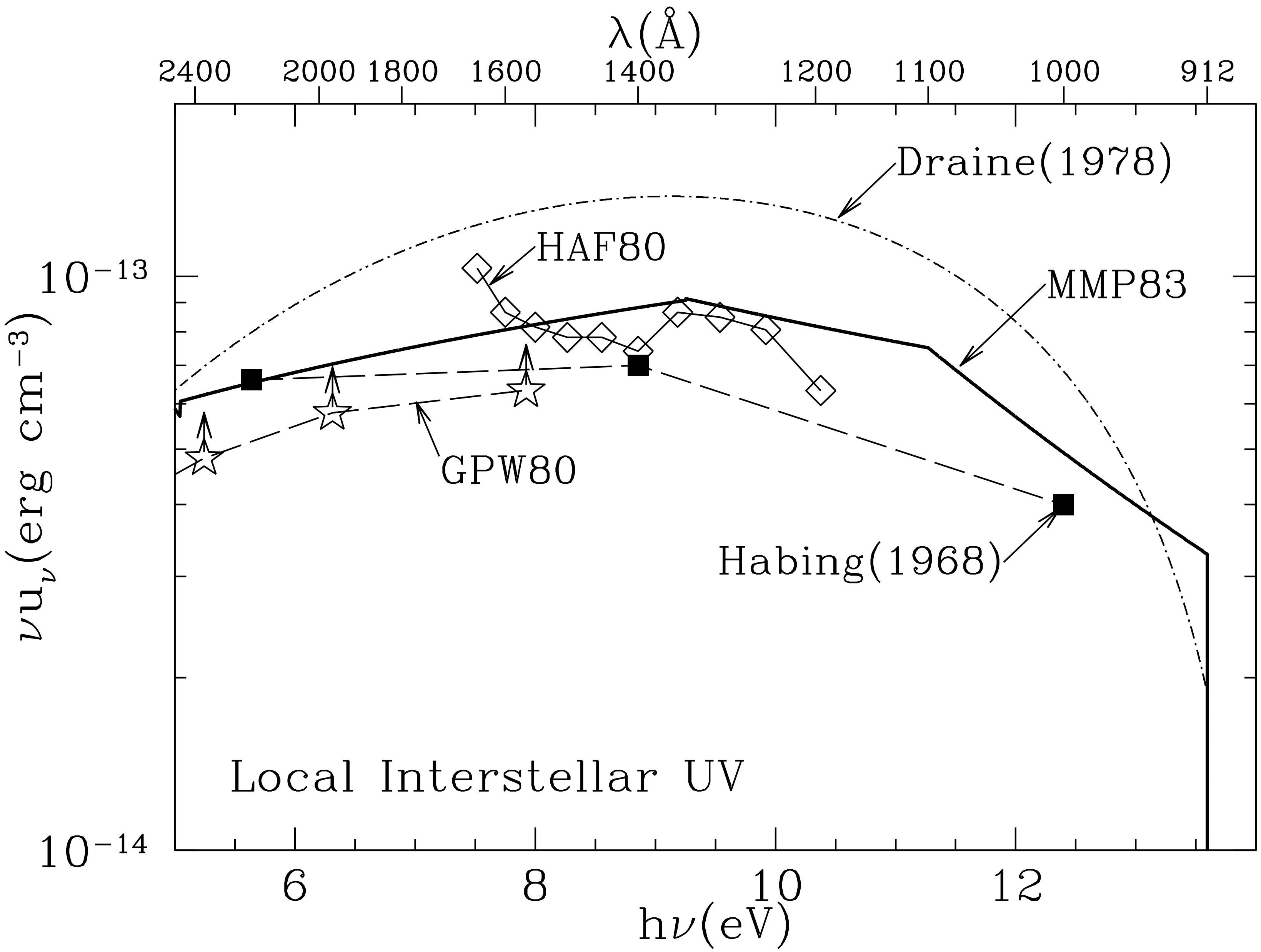
- Habing's UV spectrum, if integrated between 6.0 and 13.6 eV, gives an energy density:

$$u_{\mathrm{Hab}}(6 - 13.6\,\mathrm{eV}) = 5.29 \times 10^{-14}\,\mathrm{erg\,cm^{-3}}$$

  In this case, we define a different dimensionless parameter:

$$G_0 \equiv \frac{u(6 - 13.6\,\mathrm{eV})}{5.29 \times 10^{-14}\,\mathrm{erg\,cm^{-3}}}$$

- If we are interested in, e.g., the photodissociation of H_2 or CO, which requires photons with energies > 10 eV, then χ is the appropriate parameter to use.
- If we are interested in, e.g., the photoelectric heating rate, which is sensitive to a wider range of photon energies, then G_0 is more appropriate.



For further details about the interstellar radiation field, refer to Chapter 12 of Draine (2011).

Estimates for the UV background in the solar neighborhood, from Habing (1968), Draine (1978), and Mathis et al. (1983, MMP83).

The observational determination by Henry et al. (1980, HAF80), and the observational lower bound from Gondhalekar et al. (1980, GPW80) are also shown.

[Figure 12.2] Draine

H I Spin Temperature

- ***Collionsional rate coefficients***:
  - Collision with other H atoms

(Allison & Dalgarno 1969; Zygelman 2005)

$$k_{10}(\mathrm{H}) \approx \begin{cases} 1.19\times 10^{-10} T_2^{0.74-0.20\ln T_2}\,\mathrm{cm^3\,s^{-1}} & (20\,\mathrm{K} < T < 300\,\mathrm{K}) \\ 2.24\times 10^{-10} T_2^{0.207} \mathrm{e}^{-0.876/T_2}\,\mathrm{cm^3\,s^{-1}} & (300\,\mathrm{K} < T < 10^3\,\mathrm{K}) \end{cases}$$

$$k_{01}(\mathrm{H}) \approx 3k_{10}(\mathrm{H}) e^{-0.0682\,\mathrm{K}/T}$$

$T_2 \equiv T/100\,\mathrm{K}$

$$k_{\ell u} = \frac{g_u}{g_\ell} k_{u\ell} \exp\left(-\frac{E_{u\ell}}{k_B T}\right)$$

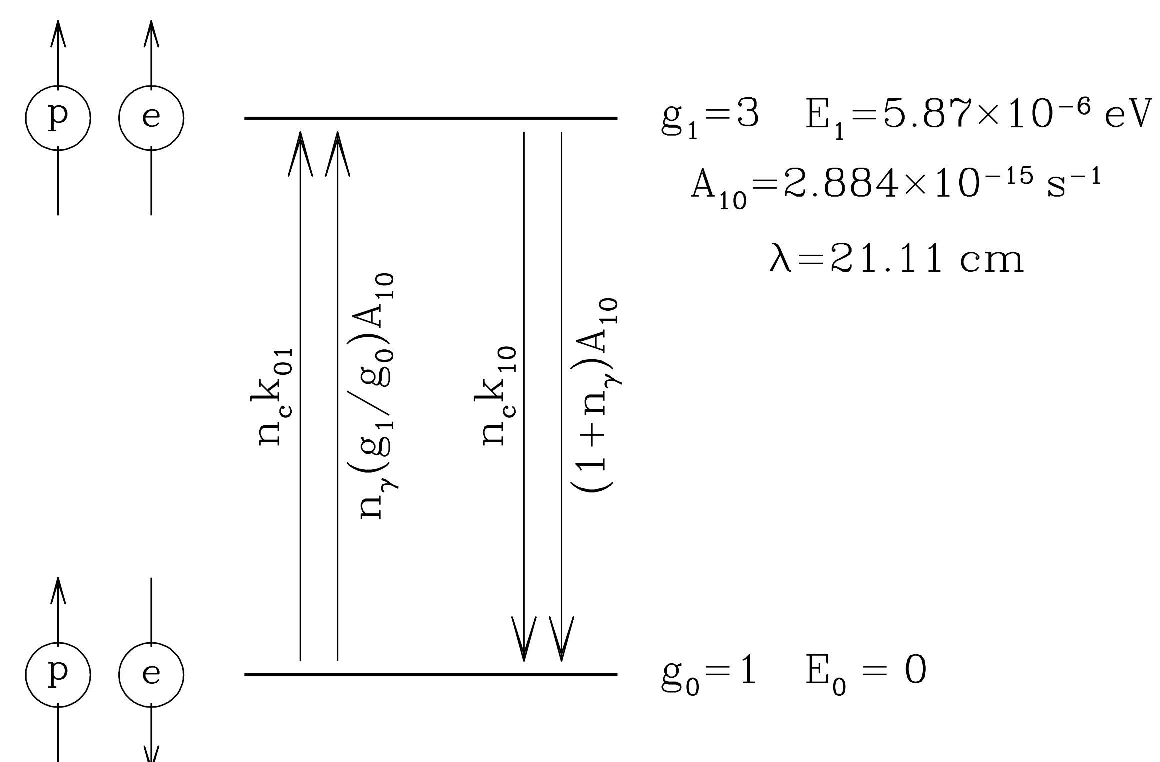
  - Collision with electrons

(Furlanetto & Furlanetto 2007)

$$k_{10}(e^-) \approx 2.26\times 10^{-9}\,(T/100\,\mathrm{K})^{0.5}\,\mathrm{cm^3\,s^{-1}} \quad (1 \lesssim T \lesssim 500\,\mathrm{K})$$

$$k_{01}(e^-) \approx 3k_{10}(e^-) e^{-0.0682\,\mathrm{K}/T}$$

  - This is a factor ~ 10 larger than that for H atoms. However, ***electrons will be minor importance in regions with a fractional ionization*** $x_e \lesssim 0.03$, such as the CNM and WNM.



[Figure 17.1 in Draine]

excitation (spin) temperature vs. kinetic temperature

$$\frac{n_1}{n_0} = \frac{n_c k_{01} + \bar{n}_\gamma (g_1/g_0) A_{10}}{n_c k_{10} + (1+\bar{n}_\gamma) A_{10}}$$

$$k_{10} = k_{10}(H) + k_{10}(e^-)$$

$$k_{01} = k_{01}(H) + k_{01}(e^-)$$

- Radiation Field strength
  - The radiation field near 21 cm is dominated by the cosmic microwave background plus Galactic synchrotron emission. The antenna temperature is

$$T_A \approx T_{\rm CMB} + T_{\rm syn} = 2.73\,{\rm K} + 1.04\,{\rm K} = 3.77\,{\rm K}$$

  - Photon occupation number:

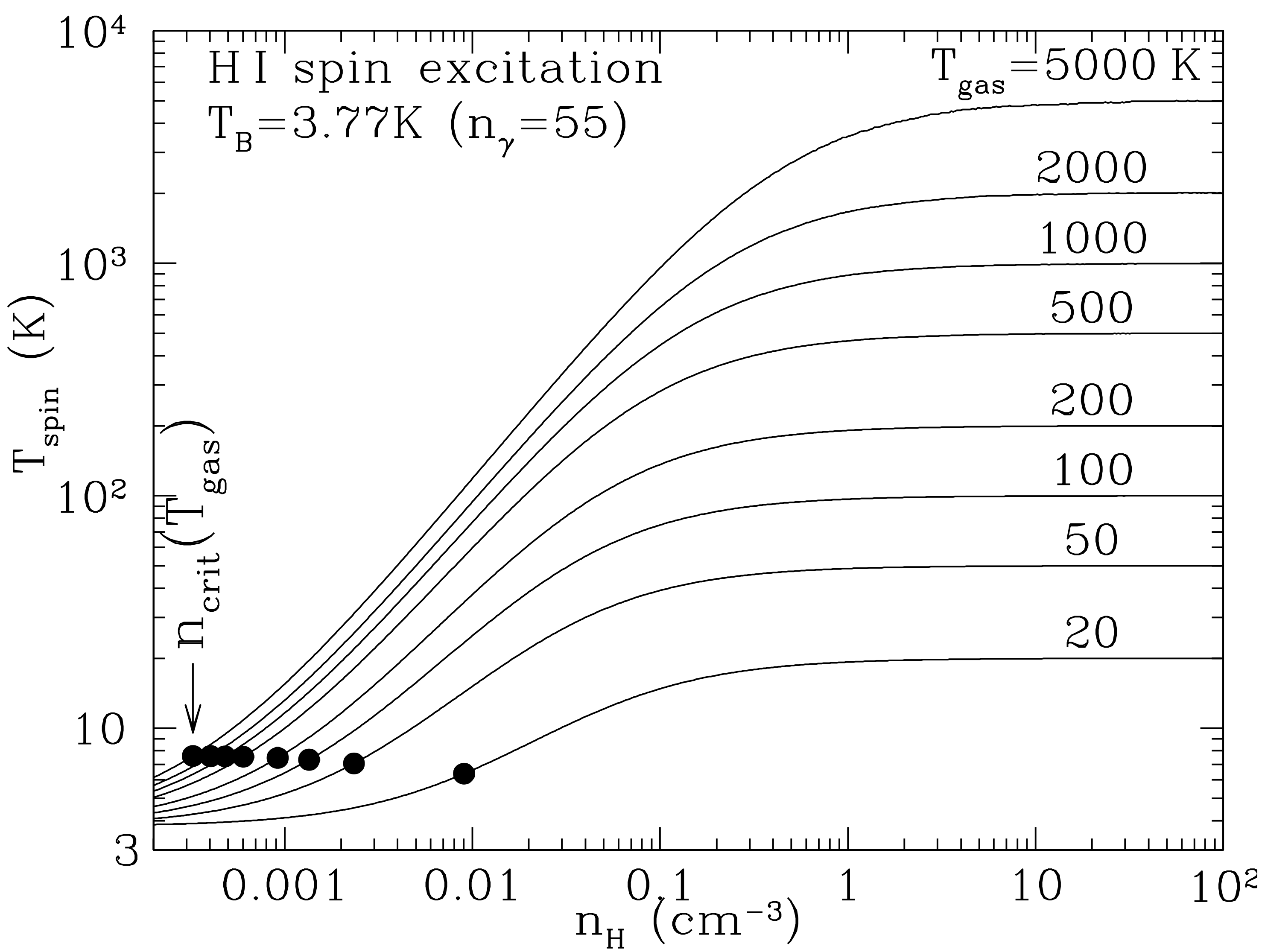
$$\bar{n}_\gamma = \left(e^{h\nu/kT_{\rm rad}} - 1\right)^{-1} \approx \frac{kT_A}{h\nu} \approx \frac{3.77\,{\rm K}}{0.0682\,{\rm K}} \approx 55$$

  - The critical density is then

$$\begin{aligned} n_{\rm crit}(H) &= \frac{(1+\bar{n}_\gamma)A_{10}}{k_{10}} \\ &\approx 0.02\,{\rm cm}^{-3} && \text{at } T \sim 10\,{\rm K} \\ &\approx 1.4\times 10^{-3}\,{\rm cm}^{-3} && \text{at } T \sim 100\,{\rm K} \\ &\approx 5\times 10^{-4}\,{\rm cm}^{-3} && \text{at } T \sim 1000\,{\rm K} \end{aligned}$$

- H I spin temperature as a function of density $n_{\rm H}$, including only 21 cm continuum radiation and collisions with H atoms. Lyα scattering is not included.
  - Filled circles show $n_{\rm crit}({\rm H})$ for each temperature.
  - It is important to note that one requires $n \gg n_{\rm crit}$ in order to have $T_{\rm spin}$ within, say, 10% of $T_{\rm gas}$, particularly at high temperatures.

$$\frac{n_u}{n_\ell} = \frac{n_c k_{\ell u} + \bar{n}_\gamma (g_u/g_\ell) A_{u\ell}}{n_c k_{u\ell} + (1+\bar{n}_\gamma) A_{u\ell}}$$
$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} \exp\left(-E_{u\ell}/k_{\rm B} T_{\rm exc}\right)$$



[Fig. 17.2 in Draine]

Note that Ryden states that "in the CNM and WNM, we expect the hyperfine levels of atomic hydrogen to be collisionally excited, and to have a spin temperature close to the gas temperature." based on that $n_{\rm crit} \sim 6\times10^{-4}$ cm^{-3} at T ~ 1000 K.

The collisional excitation is strong enough, only in the CNM, to bring the spin temperature close to the gas kinetic temperature.

However, this is not true in the WNM. In the WNM, the WF effect can thermalize the 21-cm spin temperature to the gas kinetic temperature.

In the CNM, the 21-cm spin temperature is a good tracer of the gas kinetic temperature. *However, this is not true for other levels in other atoms.*

C II Fine Structure Excitation

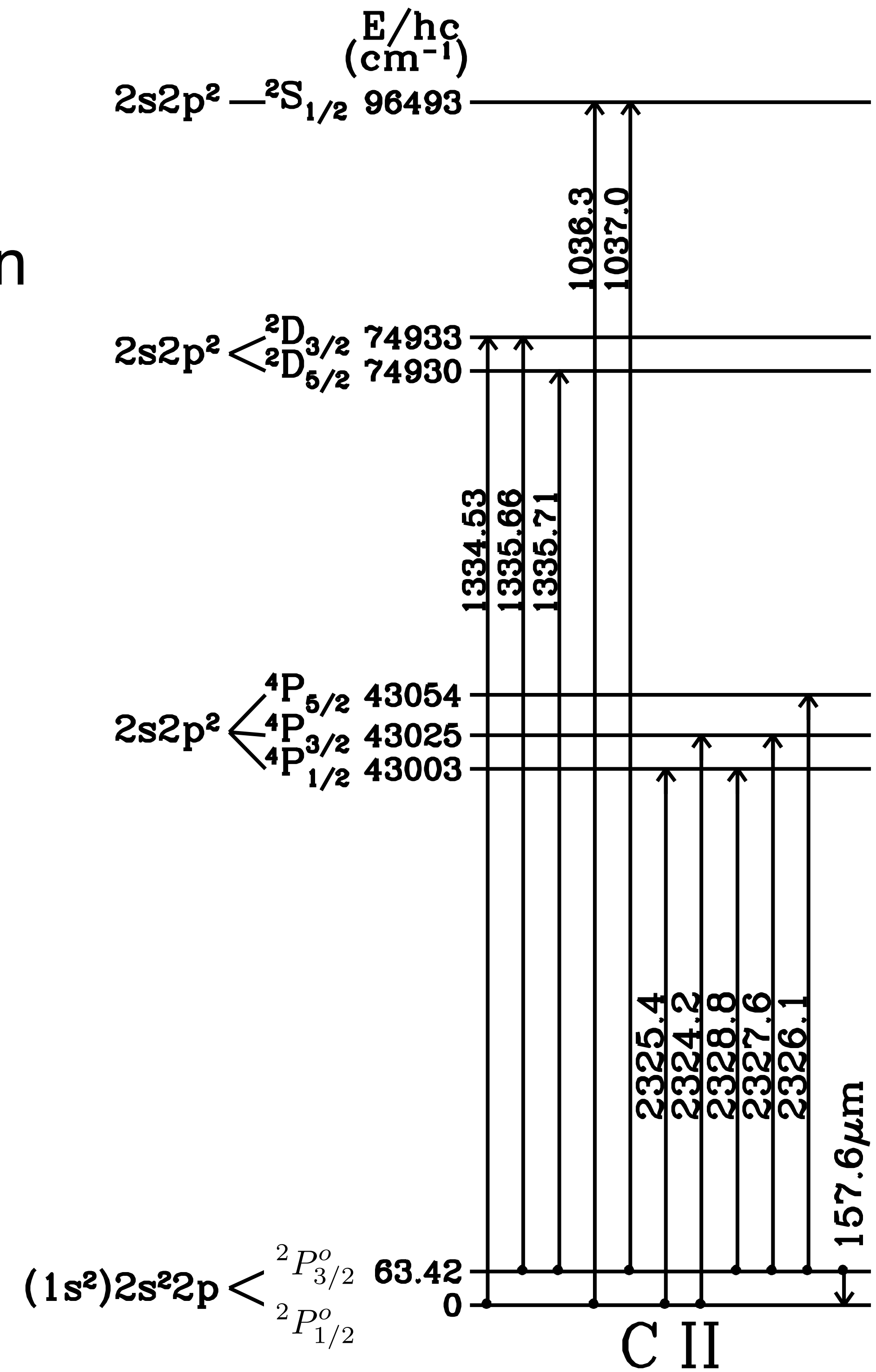
- The ground electronic state $1s^2 2s^2 2p\ {}^2P^o$ of C^+ contains two fine-structure levels.

- The electronically excited states have an excitation energy that is much higher than the kinetic temperature of the CNM.

$$2235\,\text{Å} \rightarrow E_{u\ell} = 0.56\,\text{eV} \rightarrow T = 6440\,\text{K}$$

- We may, therefore, consider the two fine-structure levels in the ground electronic state to be a two level atom.

- Will the populations of these two levels be thermalized in the ISM?



- Rate coefficients for collisional de-excitation:

$$\left\langle \Omega \left({}^2P^o_{1/2}, {}^2P^o_{3/2} \right) \right\rangle \approx 2.1 \qquad \left(T_4 = T/10^4\,\mathrm{K},\ T_2 = T/10^2\,\mathrm{K} \right)$$

$$k_{10}(e^-) \approx 4.53 \times 10^{-8}\, T_4^{-1/2}\, \mathrm{cm^3\,s^{-1}}$$

$$k_{10}(\mathrm{H}) \approx 7.58 \times 10^{-10}\, T_2^{0.1281+0.0087\ln T_2}\, \mathrm{cm^3\,s^{-1}}$$

(Barinovs et al. 2005)

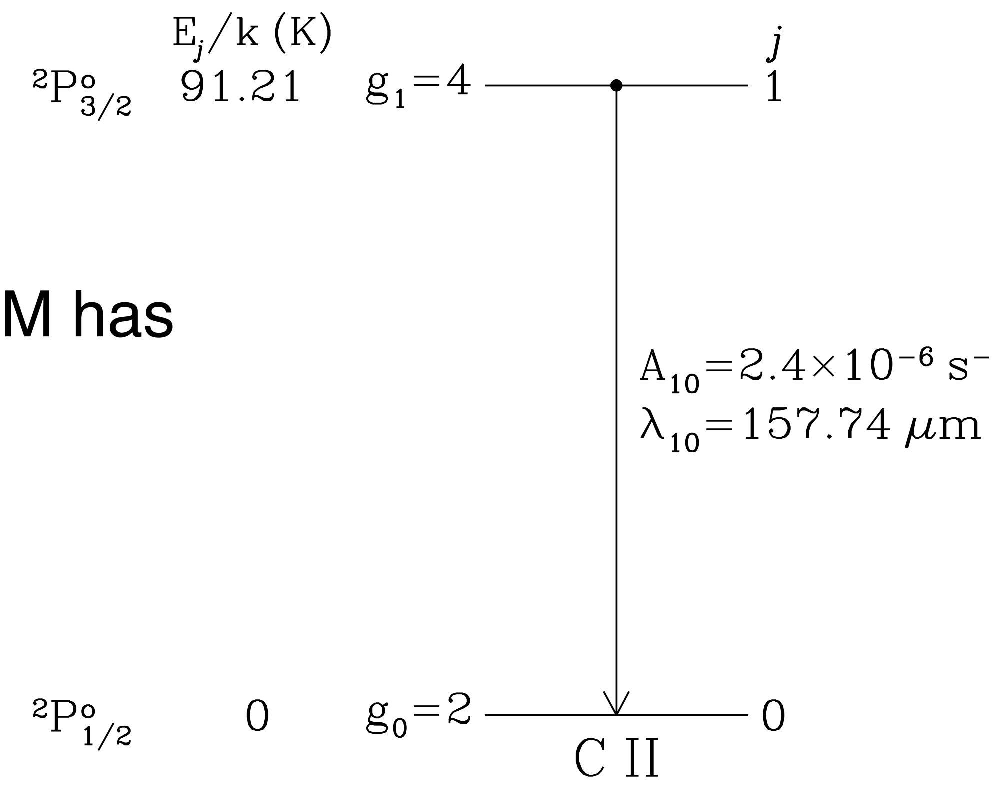
- At $\lambda = 158\,\mu\mathrm{m}$, the continuum background in the ISM has

$$\bar{n}_\gamma \approx 10^{-5} \ll 1 \longrightarrow n_\mathrm{crit} \simeq \frac{A_{10}}{k_{10}}$$

- Critical densities:

$$n_\mathrm{crit}(e^-) \approx 53\, T_4^{1/2}\, \mathrm{cm^{-3}}$$

$$n_\mathrm{crit}(\mathrm{H}) \approx 3.2 \times 10^3\, T_2^{-0.1281-0.0087\ln T_2}\, \mathrm{cm^{-3}}$$



[Figure 17.3 in Draine]

- The critical densities are much higher than the typical densities in both the CNM and WNM. Thus, **the C II fine-structure levels will be sub-thermally excited.**

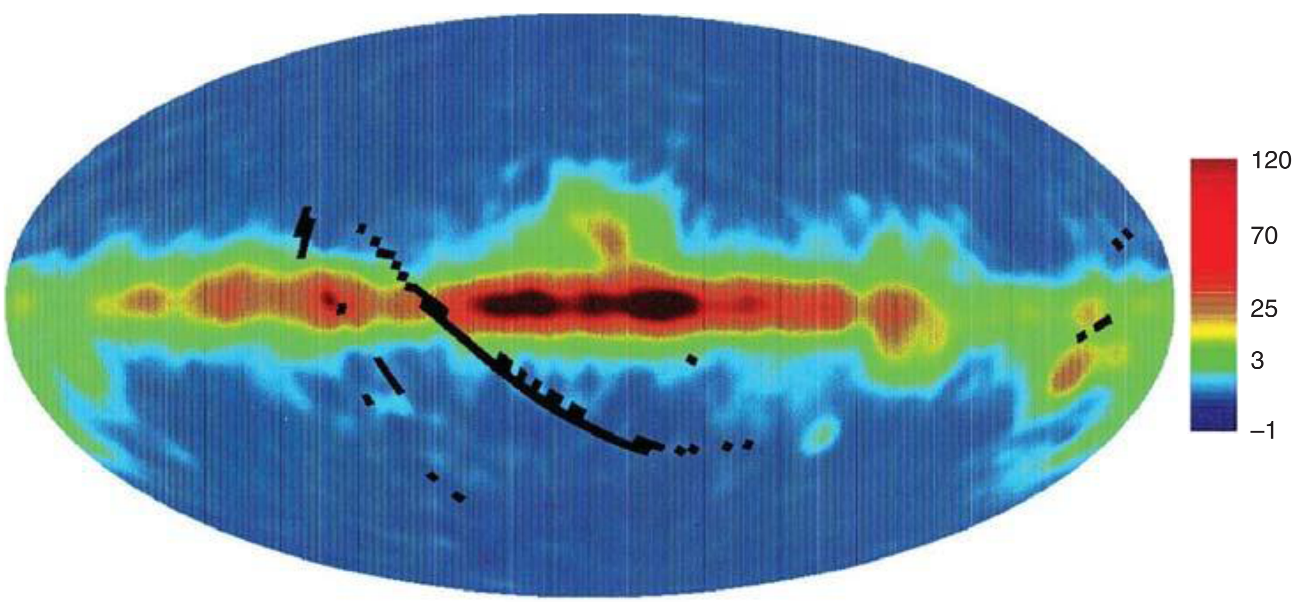
$$\frac{n_1}{n_0} = \frac{n_c k_{01}}{n_c k_{10} + A_{10}} \simeq n_c \frac{k_{01}}{A_{10}} = \frac{k_{01}}{k_{10}} \frac{n_c}{n_\mathrm{crit}} = 2e^{-91.21\,\mathrm{K}/T_\mathrm{gas}} \frac{n_\mathrm{H}}{n_\mathrm{crit}}$$

because $n_c \ll n_\mathrm{crit}$

$$\frac{n_1}{n_0} = 2e^{-91.21\,\mathrm{K}/T_\mathrm{exc}}$$

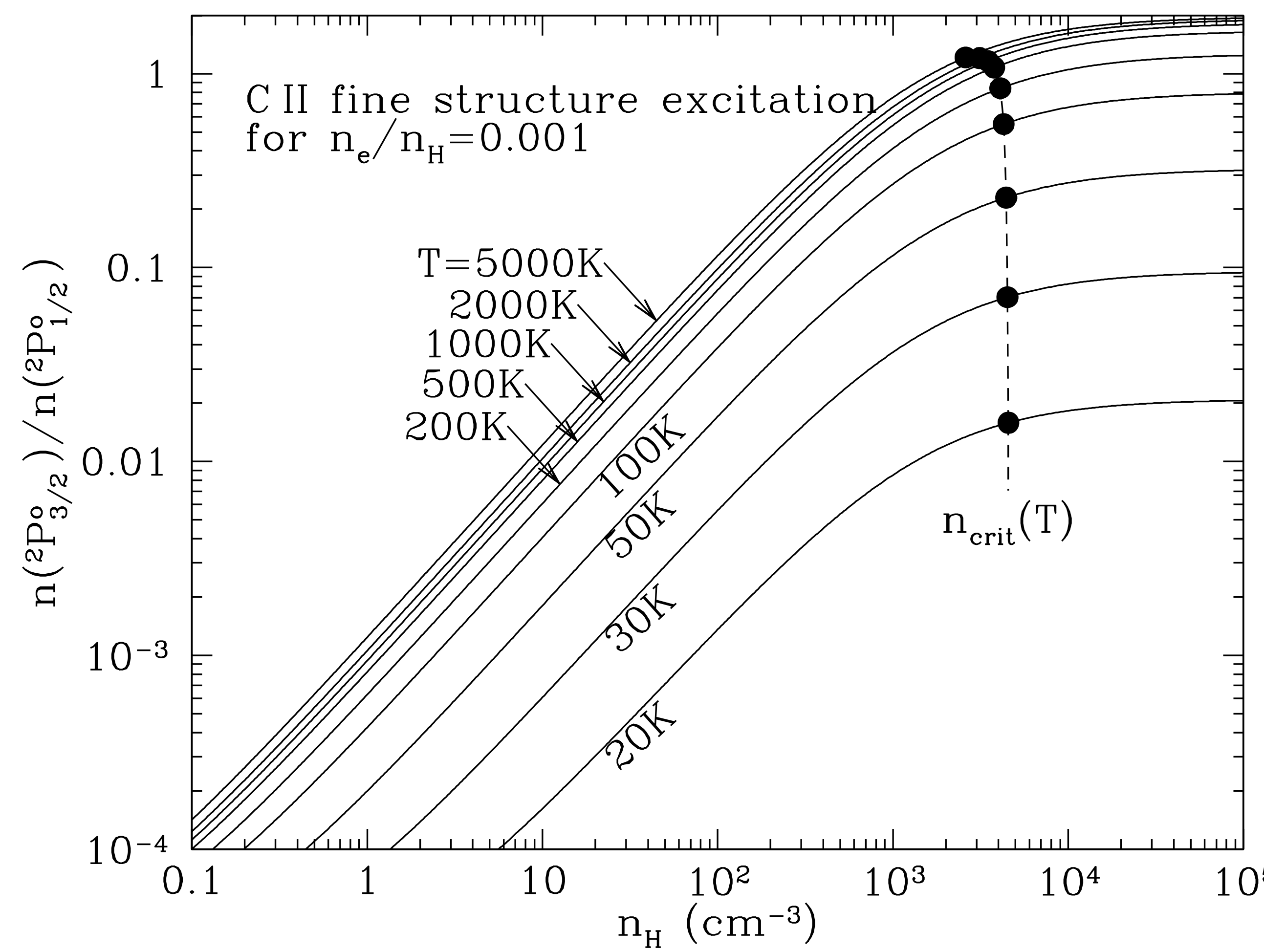
$$\rightarrow \frac{T_\mathrm{exc}}{T_\mathrm{gas}} \simeq \frac{1}{1 + (T_\mathrm{gas}/91.21\,\mathrm{K}) \ln(n_\mathrm{crit}/n_\mathrm{H})} < 1$$

- The C II fine-structure levels will be sub-thermally excited. Collisional excitations of the upper level ${}^2P^o_{3/2}$ will usually be followed by radiative decays, removing energy from the gas.
- The [C II] 158 μm transition is the principal cooling transition for the diffuse gas in star-forming galaxies.



All-sky map of [C II] 158 μm emission, made by Far InfraRed Absolute Spectrophotometer (FIRAS) on the COsmic Background Explorer (COBE) satellite (Fixsen et al. 1999).

[Plate 3 in Draine]



[Fig. 17.4 in Draine]

Equation for the 21-cm Spin Temperature

- We have derived the equation for the level populations in the presence of collision and radiation. Now, we will derive an intuitive equation for the spin temperature of the 21-cm line.

$$\frac{n_1}{n_0} = \frac{n_c k_{01} + \bar{n}_\gamma (g_1/g_0) A_{10}}{n_c k_{10} + (1 + \bar{n}_\gamma) A_{10}}$$

- Let's define the temperature corresponding to the 21-cm transition.

$$T_* = E_{10}/k = 0.0682\,\mathrm{K}$$

- The temperatures of radiation and gas will be much higher than this:

$$T_{\rm gas} \approx 10 - 10^4\,\mathrm{K} \gg T_*, \quad T_{\rm rad} = 3.77\,\mathrm{K} \gg T_*, \quad T_{\rm spin} \gg T_*$$

- The population ratio can be written in terms of the excitation (spin) temperature:

$$\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-T_*/T_{\rm spin}} \simeq \frac{g_1}{g_0} \left(1 - \frac{T_*}{T_{\rm spin}}\right)$$

- Similarly,

$$k_{01} = \frac{g_1}{g_0} k_{10} e^{-T_*/T_{\rm gas}} \simeq \frac{g_1}{g_0} k_{10} \left(1 - \frac{T_*}{T_{\rm gas}}\right)$$

$$\bar{n}_\gamma = \frac{1}{e^{T_*/T_{\rm rad}} - 1} \simeq \frac{T_{\rm rad}}{T_*}$$

- Substituting these into the population equation, we obtain

$$1 - \frac{T_*}{T_{\rm spin}} = \frac{n_c k_{10}\left(1 - T_*/T_{\rm gas}\right) + \left(T_{\rm rad}/T_*\right) A_{10}}{n_c k_{10} + \left(1 + T_{\rm rad}/T_*\right) A_{10}}$$

- Finally, we obtain the following equation:

$$T_{\rm spin} = \frac{T_* + T_{\rm rad} + y_c T_{\rm gas}}{1 + y_c} \quad \Longleftarrow \quad y_c \equiv \frac{T_*}{T_{\rm gas}} \frac{n_c k_{10}}{A_{10}}$$

- Ignoring T_* term, we obtain an intuitive equation for the spin temperature.

$$T_{\rm spin} = \frac{T_{\rm rad} + y_c T_{\rm gas}}{1 + y_c} \quad \Longleftarrow \quad y_c \equiv \frac{T_*}{T_{\rm gas}} \frac{n_c k_{10}}{A_{10}}$$

This equation was first derived by G. Field (1958).

- This equation describes the spin temperature as ***a weighted mean of the radiation and gas temperatures with weights of 1 and y_c***.
- From the equation, we can show that

$$\begin{aligned} T_{\rm spin} &\simeq T_{\rm rad} \quad \text{if } y_c \ll 1 \\ T_{\rm spin} &\simeq T_{\rm gas} \quad \text{if } y_c \gg 1 \end{aligned}$$

- A new critical density of the colliding particle may be defined:

$$\boxed{y_c = 1 \quad \Longrightarrow \quad n^*_{\rm crit} = \frac{T_{\rm gas}}{T_*}\frac{A_{10}}{k_{10}}}$$

- Now, compare this density with the previous definition of the critical density.

$$\begin{aligned} n_{\rm crit} &\equiv \frac{\left[1 + (n_\gamma)_{10}\right] A_{10}}{k_{10}} \\ &= \left[1 + \frac{1}{e^{h\nu_{10}/kT_{\rm rad}} - 1}\right] \frac{A_{10}}{k_{10}} \\ &\simeq \left(1 + \frac{T_{\rm rad}}{T_*}\right) \frac{A_{10}}{k_{10}} \end{aligned}$$

$$\frac{n^*_{\rm crit}}{n_{\rm crit}} \approx \frac{T_{\rm gas}}{T_{\rm rad}}$$

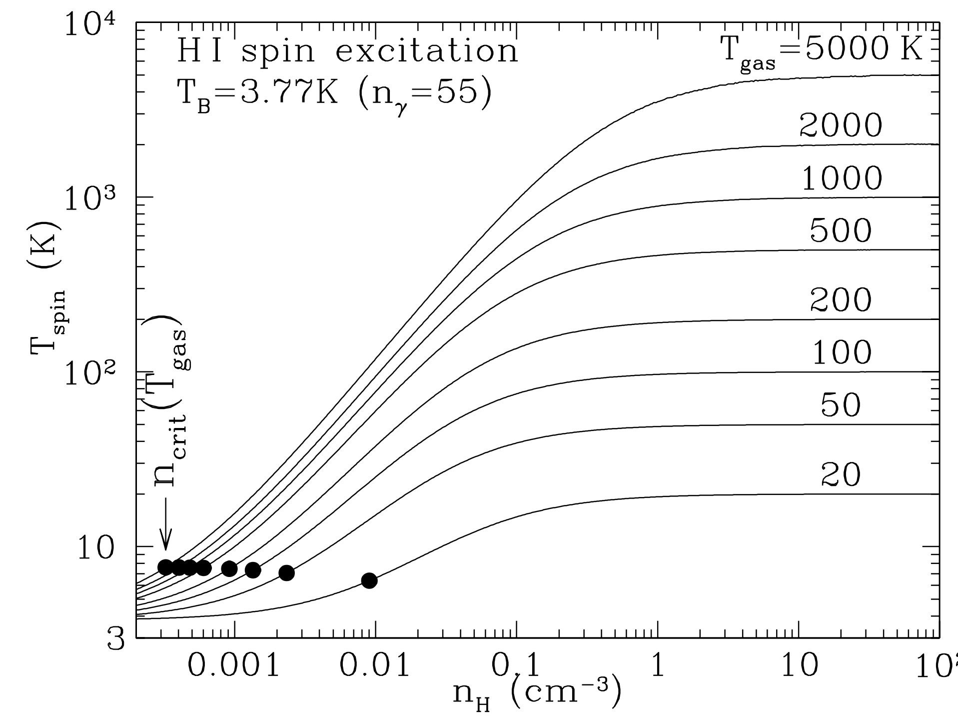
Detectability of Hydrogen in a Low Density Medium

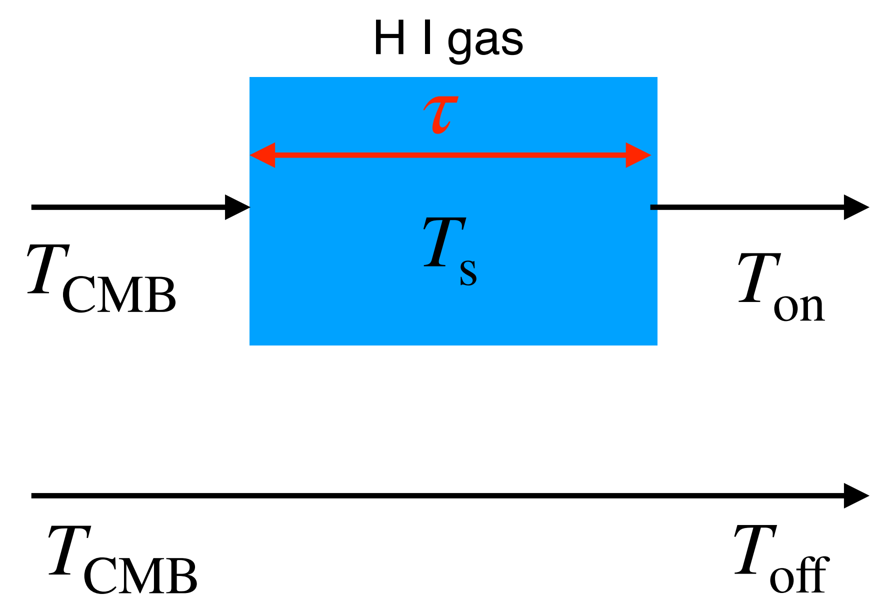
- In a very low density medium (WNM, CGM, IGM), the particle collisions are very rare ($n_{\rm HI} \ll n_{\rm crit}$).
- The radiative transition due to the CMB photons will control the relative population between the hyperfine structures.
  - This indicates $T_{\rm s} = T_{\rm CMB}$.
  - The RT equation in the Rayleigh-Jeans regime can be written in terms of temperature:

$$T_{\rm on} = T_{\rm CMB} e^{-\tau} + T_{\rm s} (1 - e^{-\tau}) = T_{\rm CMB}$$
$$T_{\rm off} = T_{\rm CMB}$$
$$T_{\rm on} - T_{\rm off} = 0$$

  - Then, we have $T_{\rm on} = T_{\rm off} = T_{\rm CMB}$.
  - ***Neither emission nor absorption feature from the hydrogen gas is detectable.***
- ***We need something that can make $T_{\rm s} \neq T_{\rm CMB}$.***

[Fig. 17.2 in Draine]





The Wouthuysen-Field effect: The Third Mechanism controlling the Spin Temperature

- Wouthuysen (1952, AJ, 57, 31)

Wouthuysen, S. A. On the excitation mechanism of the 21-cm (radio-frequency) interstellar hydrogen emission line.

The mechanism proposed here is a radiative one: as a consequence of absorption and re-emission of Lyman-α resonance radiation, a redistribution over the two hyperfine-structure components of the ground level will take place. Under the assumption—here certainly permitted—that induced emissions can be negelcted, it can easily be shown that the relative distribution of the two levels in question, under stationary conditions, will depend solely on the shape of the radiation spectrum in the Lα region, and not on the absolute intensity.

The shape of the spectrum of resonance radiation, quasi-imprisoned in a large gas cloud, could only be determined by a careful study of the "scattering" process (absorption and re-emission) in a cloud of definite shape and dimensions. The spectrum will turn out to depend upon the localization in the cloud.

Some features can be inferred from more general considerations. Take a gas in a large container, with perfectly reflecting walls. Let the gas be in equilibrium at temperature T, together with Planck radiation of that same temperature. The scattering processes will not affect the radiation spectrum. One can infer from this fact that the photons, after an infinite number of scattering processes on gas atoms with kinetic temperature T, will obtain a statistical distribution over the spectrum proportional to the Planck-radiation spectrum of temperature T. After a finite but large number of scattering processes the Planck shape will be produced in a region around the initial frequency.

Photons reaching a point far inside an interstellar gas cloud, with a frequency near the Lα resonance frequency, will have suffered on the average a tremendous number of collisions. Hence in that region, which is wider the larger the optical depth of the cloud is for the Lyman radiation, the Planck spectrum corresponding to the gas-kinetic temperature will be established as far as the shape is concerned. Because, however, the relative occupation of the two hyperfine-structure components of the ground state depends only upon the shape of the spectrum near the Lα frequency, this occupation will be the one corresponding to equilibrium at the gas temperature.

The conclusion is that the resonance radiation provides a long-range interaction between gas atoms, which forces the internal (spin-)degree of freedom into thermal equilibrium with the thermal motion of the atoms.

Institute for Theoretical Physics of the City University, Amsterdam.

"Wouthuysen" is pronounced as roughly "Vowt-how-sen." (바우타이슨)

From a thermodynamic argument, Wouthuysen speculated the followings:

A tremendous number of scattering will establish the Planck-like spectrum, at the Lyα line center, corresponding to the gas-kinetic temperature.

The Lyα radiation is coupled with the hyperfine state of the hydrogen atom.

In the end, ***the 21cm spin temperature will become equal to the kinetic temperature of the hydrogen gas.***

Mechanisms that controls the spin temperature

- The spin temperature (T_S) is determined by three mechanisms.

(1) Direct Radiative Transitions by the background radiation field (Cosmic Microwave Background or Galactic Synchrotron)

$$I_\nu = \frac{2k_B T_R}{\lambda^2}$$

(Rayleigh-Jeans Law)

$$T_R = \text{brightness temperature} = 2.73\,\text{K or } 3.77\,\text{K}$$

(2) Collisional Transitions (collision with other hydrogen and electron)

$$T_K = \text{gas kinetic temperature}$$

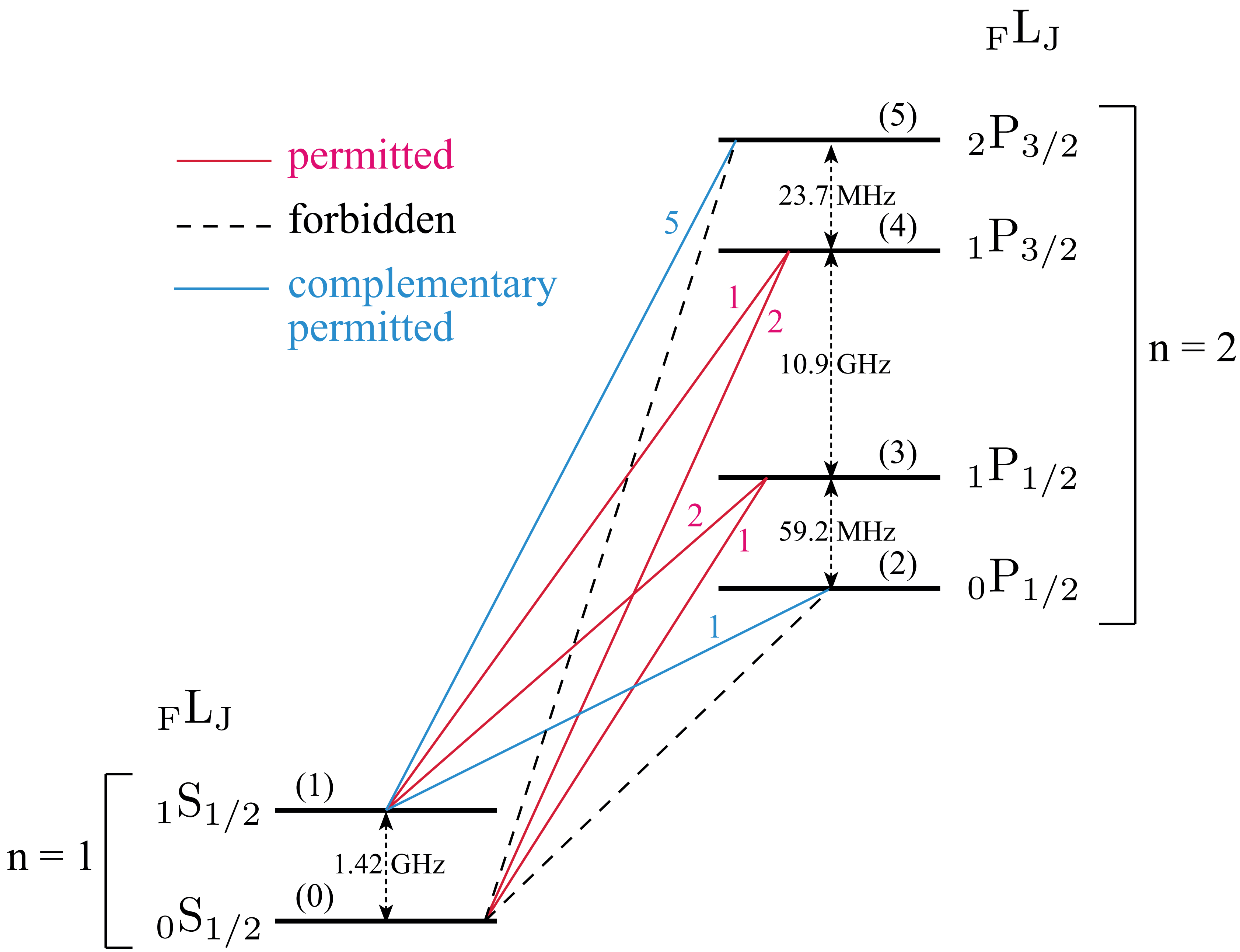
(3) Lyα pumping: Indirect Radiative Transitions involving intermediate levels caused by Lyα resonance scattering

$$T_\alpha = \text{color temperature}$$

$$J(\nu) \propto \exp\left(-\frac{h\nu}{k_B T_\alpha}\right)$$

Indirect Level Population by Lyα Scattering

The WF effect is a mechanism that the resonance scattering of Lyα photons indirectly control the relative populations between the hyperfine levels in the ground state (n = 1) via transitions involving the n = 2 state as an intermediate state.



Equation for spin temperature

We obtain the following equation for the spin temperature in terms of the 21 cm brightness temperature, gas kinetic temperature, and Ly-alpha color temperature:

$$T_{\rm S} = \frac{T_* + T_R + y_c T_K + y_\alpha T_\alpha}{1 + y_c + y_\alpha}$$

$$T_{\rm S} \simeq \frac{T_R + y_c T_K + y_\alpha T_\alpha}{1 + y_c + y_\alpha}$$

where $$y_c \equiv \frac{T_*}{T_K}\frac{P_{10}^c}{A_{10}}$$

$$y_\alpha \equiv \frac{T_*}{T_\alpha}\frac{P_{10}^\alpha}{A_{10}}$$

$$T_* = \frac{h\nu_{10}}{k_B} = 0.0681\ ^\circ{\rm K}$$ (This term is negligible in the above equation.)

Two requirements for the WF effect:

(1) $J_\nu \propto \exp\left(-\dfrac{h\nu}{kT_\alpha}\right)$ with $T_\alpha = T_{\rm K}$

(2) $y_\alpha \gg 1$ and $y_\alpha \gg y_c$
(Lyα radiation field should be strong.)

Read Seon & Kim (2020) for a details.
https://ui.adsabs.harvard.edu/abs/2020ApJS..250....9S/abstract

H II Regions

- Ionization and Recombination
- Strömgren Sphere
- Recombination Lines
- Heating & Cooling

Atomic Processes

- **Excitation and de-excitation (Transition)**
  - Radiative excitation (photoexcitation; photoabsorption)
  - Radiative de-excitation (spontaneous emission and stimulated emission)
  - Collisional excitation
  - Collisional de-excitation

- **Emission Line**
  - Collisionally-excited emission lines
  - Recombination lines (recombination following photoionization or collisional ionization)

- **Ionization**
  - Photoionization and Auger-ionization
  - Collisional Ionization (Direct ionization and Excitation-autoionization)
- **Recombination**
  - Radiative recombination $\Longleftrightarrow$ Photoionization
  - Dielectronic Recombination (not dielectric!)
  - Three-body recombination $\Longleftrightarrow$ Direct collisional ionization
- **Charge exchange**

Ionization - [Photoionization]

- Interstellar medium (ISM) is transparent to $h\nu < 13.6$ eV photons, but is very opaque to ionizing photons with $h\nu > 13.6$ eV. In fact, the ISM does not become transparent until $h\nu \sim 1$ keV.
  - Sources of ionizing photons include massive, hot young stars, hot white dwarfs, and supernova remnant shocks.
- ***From the Outer Shells***
  - Photoionization is the ionization of an atomic species by the absorption of a photon. Photoionization is the inverse process to radiative recombination.

$$X + h\nu \;\rightarrow\; X^+ + e^- + \Delta E$$

  - If the incoming photon has sufficient energy, it may leave the ionized species in an excited state.

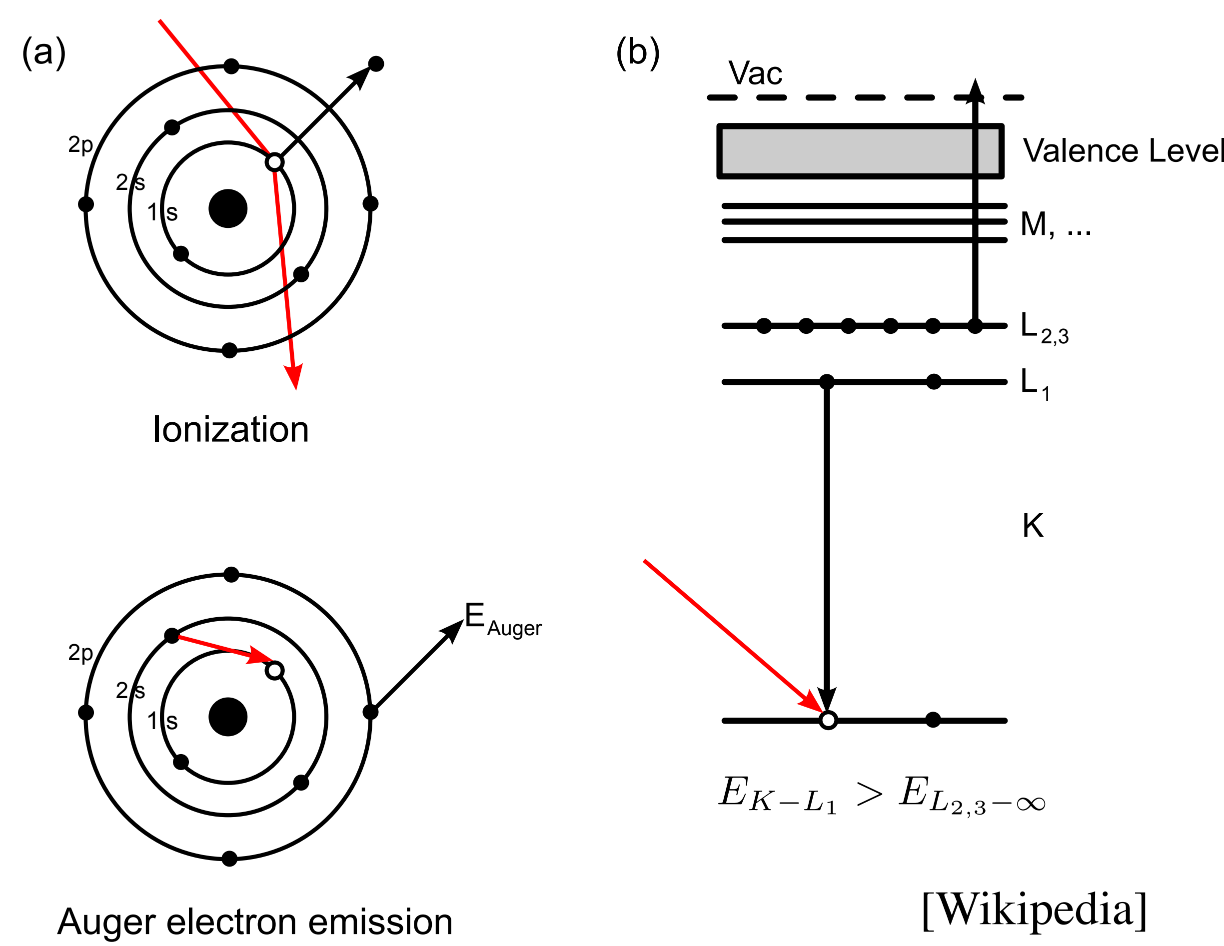
$$\begin{aligned} X + h\nu \;&\rightarrow\; X_*^+ + e^- + \Delta E \\ X_*^+ \;&\rightarrow\; X^+ + h\nu_1 + h\nu_2 + \cdots \end{aligned}$$

Here, *X* denotes an atom, molecule, or ion and subscript * indicates an excited states.

ΔE denotes energy carried by the electron.

- ***Inner Shell Photoionization***
    - If the energy of the incoming photon is still higher, it becomes possible to remove one of the inner shell electrons which also results in a change in the electron configuration in the excited species. This may be followed by a radiative readjustment back to the ground state.
    - However, in this case, **Auger ionization** becomes more probable. High energy photons may eject an inner shell electron from an ion or atom, and the resulting ion may then fill the gap in its inner shell with an outer electron, ejecting another outer electron to remove the energy in a ***radiationless*** transition called an Auger transition. Such a process will produce very energetic electrons which will lose their energy in heating up the gas.

$$
\begin{aligned}
X + h\nu &\rightarrow X_{**}^{+} + e^{-} + \Delta E_1 \\
X_{**}^{+} &\rightarrow X_{*}^{(m+1)+} + m e^{-} + \Delta E_2 \\
& \quad \text{(radiationless autoionization, } m \geq 1) \\
X_{*}^{(m+1)+} &\rightarrow X^{(m+1)+} + h\nu_1 + h\nu_2 + \cdots
\end{aligned}
$$



[Wikipedia]

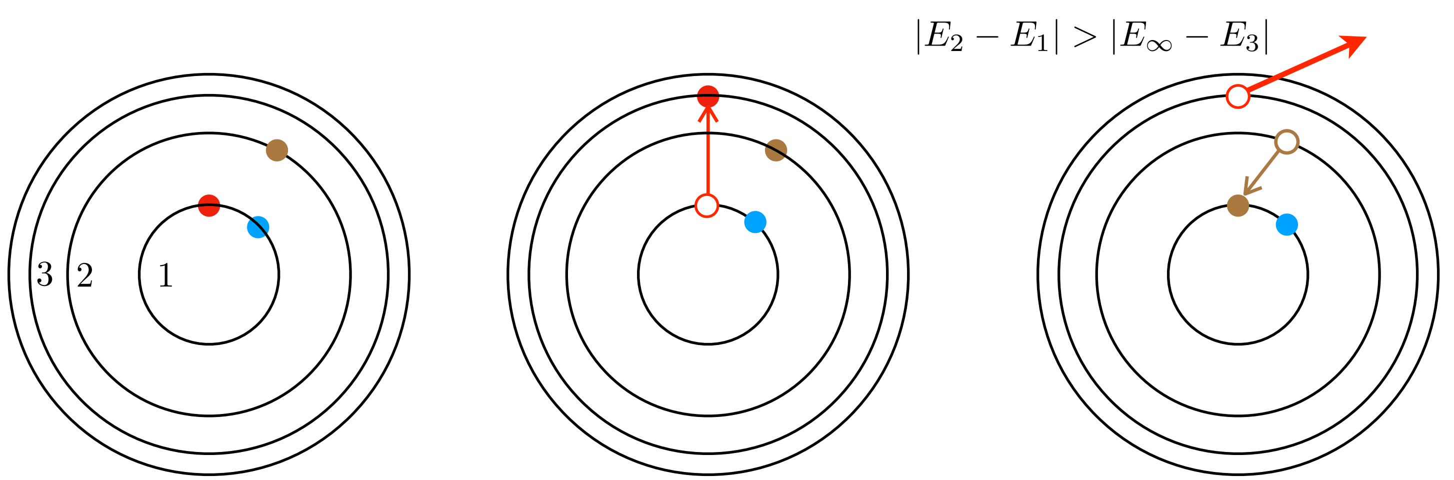
Ionization - [Collisional ionization]

- ***Direct collisional ionization***: The process whereby an electron strikes an atom or ion X, with sufficient energy to strip out a bound electron:

$$X + e^- \;\rightarrow\; X^+ + 2e^- - \Delta E$$

- ***Excitation-autoionization***: At sufficiently high electron impact energies, more than one electron of the target may be excited, leaving the atom in an unstable state, which is stabilized by the radiationless ejection of one of outer electrons, possibly followed by a radiative decay of the ionized atom back go its ground state. This process is favored in heavy elements which have a large number of inner shell electrons and only one or two electrons in the outer shell.

$$\begin{aligned} X + e^- &\rightarrow X_* + e^- - \Delta E_1 \\ X_* &\rightarrow X_*^+ + e^- + \Delta E_2 \\ X_*^+ &\rightarrow X^+ + h\nu \end{aligned}$$



- For example, in collisions of Li-like ions, excitation and autoionization can occur via excitation of the 1s-electron into states with principal quantum numbers $n \geq 2$. After the decay of a doubly excited state, one has an additional electron in the final channel.

$$\begin{aligned} X(1s^2 2s) + e^- &\rightarrow X_{**}(1s2snl) + e^- - \Delta E_1 \\ &\rightarrow X_*^+(1s^2) + (e^- + \Delta E_2) + (e^- - \Delta E_1) \end{aligned}$$

Recombination

- ***Radiative recombination***
  - Radiative recombination is the process of capture of an electron by an ion where the excess energy of the electron is radiated away in a photon.
  - The electron is captured into an excited state. The recombined but still excited ion radiates several photons in a radiative cascade, as it returns to the ground state:

$$X^+ + e^- \rightarrow X_* + h\nu \qquad \text{(recombination continuum)}$$
$$X_* \rightarrow X + h\nu_1 + h\nu_2 + h\nu_3 + \cdots \qquad \text{(recombination lines)}$$

  - The photon in the first line represents a **recombination continuum** ($h\nu$) photon. However, photons ($h\nu_1$, $h\nu_2$, $h\nu_3$) represent quantized transitions and are therefore termed **recombination lines**.
  - ***The total effective recombination rate is the sum of the recombination rate to each state.***

- ***Dielectronic recombination***
  - For an electron that is initially free to be captured to a bound state of an atom or ion, the electron must lose energy.
    - Radiative recombination is relatively slow because it is necessary to create a photon to remove this energy as part of the capture process. This can take place only during the brief time that the free electron is appreciably accelerated by the electric field of ion.
    - However, if an ion has at least one bound electron, then it is possible for the incoming electron to transfer energy to a bound electron, promoting the bound electron to an excited state, and removing enough energy from the first electron that it too can be captured in an excited state. Then, the ion now have two electrons in excited state.
    - Dielectronic recombination (DR) is a resonant two-step process.
      - The first step is a double-electron process, often called dielectronic capture, through which one free electron is captured and another core electron is simultaneously excited forming a doubly excited state. One of the electron is in an autoionizing state, n_1l_1, and the other is in an excited state, n_2l_2. In the second step, the ion in a doubly excited state emits a photon and decays into a stable state below the ionization limit.

$$
\begin{aligned}
X^+(1s,\ldots) + e^- &\rightarrow X_{**}(n_1l_1;\ n_2l_2) \\
X_{**}(n_1l_1;\ n_2l_2) &\rightarrow X_*(n_3l_3;\ n_2l_2) + h\nu \\
X_*(n_3l_3;\ n_2l_2) &\rightarrow X(n_3l_3;\ n_4l_4) + h\nu_1 + h\nu_2 + \cdots
\end{aligned}
$$

    - Dielectronic recombination is important in high-temperature plasmas, where it often exceeds the radiative recombination rate.

- Three-body Recombination
  - The combination process of an electron with a positive ion in a gas in such a way that the incoming free electron transfers energy and momentum to another free electron in the neighborhood of the ion.

$$X^+ + 2e^- \rightarrow X + e^- + \Delta E$$

  - Three-body recombination is the inverse process of collisional ionization.
  - In most interstellar medium, three-body recombination is unimportant.
  - In dense regions with electron densities above 10^4 cm^{-3}, three-body recombination into high levels of the hydrogen atom with principal quantum numbers (n > 100) can be important. (Compare Eq. (3.44) and (14.6) in Draine)

Charge Exchange

- During the collision of two ionic species, the charge clouds surrounding each interact, and it is possible that an electron is exchanged between them.

$$X^+ + Y \rightleftharpoons X + Y^+ + \Delta E$$

  - Since, in virtually all diffuse astrophysical plasmas, hydrogen and helium are overwhelmingly the most abundant species, the charge-exchange reactions which are significant to the ionization balance of the plasma are

$$X^+ + \mathrm{H}^0 \rightleftharpoons X + \mathrm{H}^+ + \Delta E$$
$$X^+ + \mathrm{He}^0 \rightleftharpoons X + \mathrm{He}^+ + \Delta E$$

  - The reactions are exothermic (발열) because of the lower ionization potential of the X^+ ion. Thus, the reverse reaction occurs only when $kT \gtrsim \Delta E$. In many cases we have to consider only the forward reaction.
  - Charge-exchange may also occur in collisions of molecules with atoms. i.e.,

$$\mathrm{CO}^+ + \mathrm{H}^0 \rightarrow \mathrm{CO} + \mathrm{H}^+$$

Ionization Equilibrium

- ***Photoionization Equilibrium***:
  - Balance between photo-ionization and the process of recombination.

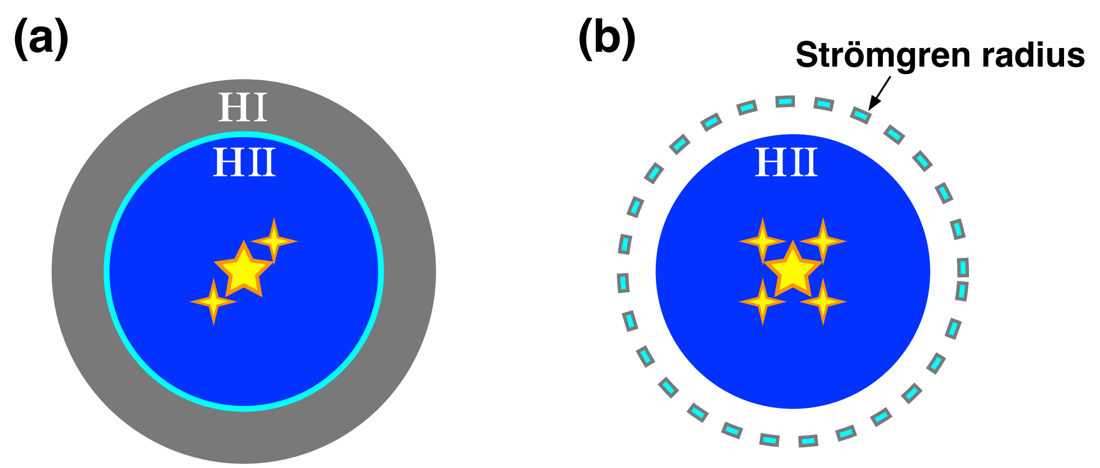
- ***Collisional Ionization Equilibrium (CIE)*** or coronal equilibrium
  - Balance at a given temperature between collisional ionization from the ground states of the various atoms and ions, and the process of recombination from the higher ionization stages.
  - In this equilibrium, effectively, all ions are in their ground state.

- Ionization balance under conditions of local thermodynamic equilibrium (LTE)
  - The ionization equilibrium in LTE is described by the ***Saha equation***.
  - In LTE, the excited states are all populated according to Boltzmann's law.

Ionization bounded vs. Density bounded

- Ionized atomic hydrogen regions, broadly termed "**H II regions**", are composed of gas ionized by photons with energies above the hydrogen ionization energy of 13.6 eV.
  - ***Ionization Bounded:*** These objects include "***classical H II regions***" ionized by hot O or B stars (or clusters of such stars) and associated with regions of recent massive-star formation, and "planetary nebulae", the ejected outer envelopes of AGB stars photoionized by the hot remnant stellar core.
  - ***Density Bounded (Matter Bounded): Warm Ionized Medium / Diffuse Ionized Gas***: Ionized Gas in the diffuse ISM, far away from OB associations.

(a) An ionization-bounded nebula whose radius is determined by the ionization equilibrium. The LyC is entirely consumed to ionized the surrounding H I gas.
(b) In a density-bounded nebula, the amount of the surrounding H I gas is not enough to consume all LyC photons. Some of the LyC escapes from the cloud, which is called the LyC leakage.



- The UV, visible and IR spectra of H II regions are very rich in emission lines, primarily collisional excited lines of metal ions and recombination lines of hydrogen and helium. H II regions are also observed at radio wavelengths, emitting radio free-free emission from thermalized electrons and radio recombination lines from highly excited states of H, He, and some metals (e.g., H109α and C lines).

• Three processes govern the physics of H II regions:
  - ***Photoionization Equilibrium***: the balance between photoionization and recombination. This determines the spatial distribution of ionic states of the elements in the ionized zone.
  - ***Thermal Balance*** between heating and cooling. Heating is dominated by photoelectrons ejected from hydrogen and helium with thermal energies of a few eV. Cooling is mostly dominated by electron-ion impact excitation of metal ion followed by emission of "forbidden" lines from low-lying fine structure levels. It is these cooling lines that give H II regions their characteristic spectra.
  - ***Hydrodynamics***, including shocks, ionization and photodissociation fronts, and outflows and winds from the embedded stars.

- Ionization fraction
  - In the CNM, the fractional ionization $x = n_e/n_{\rm H} \sim 0.001$. In the WNM, $x \sim 0.1$.
  - Although the number of free electrons are small in the neutral ISM, free electrons play a role in bringing the WNM to kinetic equilibrium. Free electrons photo-ejected from dust grains are the major heat source in the neutral medium.
  - In the WIM, $x \sim 0.7$. In the HIM, $x \sim 1.0$.

- H II regions, with $T \sim$ 10,000 K and $n \sim$ 0.3 cm^{-3}, contributes only a few percent of the mass of the ISM, and no more than ten percent of its volume.
  - Classical H II regions and planetary nebulae have a similar temperature, but planetary nebulae have a higher density.

Ionization Energy

- First ionization energy = energy required to remove the most loosely bound electron in a neutral atom in its ground state.
  - The first ionization energy of hydrogen is $I_{\rm H} = 13.59844\,{\rm eV}$.
  - The highest first ionization energy is that of He, with $I_{\rm He} = 24.6\,{\rm eV} = 1.91 I_{\rm H}$.
  - The second ionization energy of helium is $I_{\rm HeII} = 54.4\,{\rm eV} = 4 I_{\rm H}$.
  - The lowest first ionization energy of astrophysically interesting elements is that of potassium (K, Z = 19), with $I_{\rm K} = 4.3\,{\rm eV}$. (Rubidium, cesium, and francium have lower first ionization energies, but they are not much seen in the ISM.)
  - Thus, photoionization of neutral atoms will be done by UV photons in the wavelength range λ = 500 - 3000Å (corresponding to E = 4.1 - 24.8 eV).

- A hydrogenic (hydrogen-like) ion with atomic number Z has an ionization energy of $Z^2 I_{\rm H}$.
  - 54.4 eV for He II, 122.4 eV for Li III, and so forth.

Photoionization

- The (nonrelativistic) quantum mechanics of hydrogen-like ions (with only one electron) give an analytic expression for the ground-state photoionization (photoelectric) cross section.

$$\boxed{\sigma_{\rm pi}(\nu) = \sigma_0 \left(\frac{Z^2 I_{\rm H}}{h\nu}\right)^4 \frac{{\rm e}^{4-4\arctan(x)/x}}{1-{\rm e}^{-2\pi/x}}, \quad x \equiv \sqrt{\frac{h\nu}{Z^2 I_{\rm H}} - 1} \quad \text{for } h\nu > Z^2 I_{\rm H}}$$

- The cross section at threshold is

$$\sigma_0 \equiv \frac{2^9\pi}{3{\rm e}^4}\alpha\pi a_0^2 Z^{-2} = 6.304\times 10^{-18} Z^{-2}\,{\rm cm}^{-2}$$

fine-structure constant
$(\alpha \equiv e^2/\hbar c = 1/137.04,\ {\rm e} = 2.71828...)$

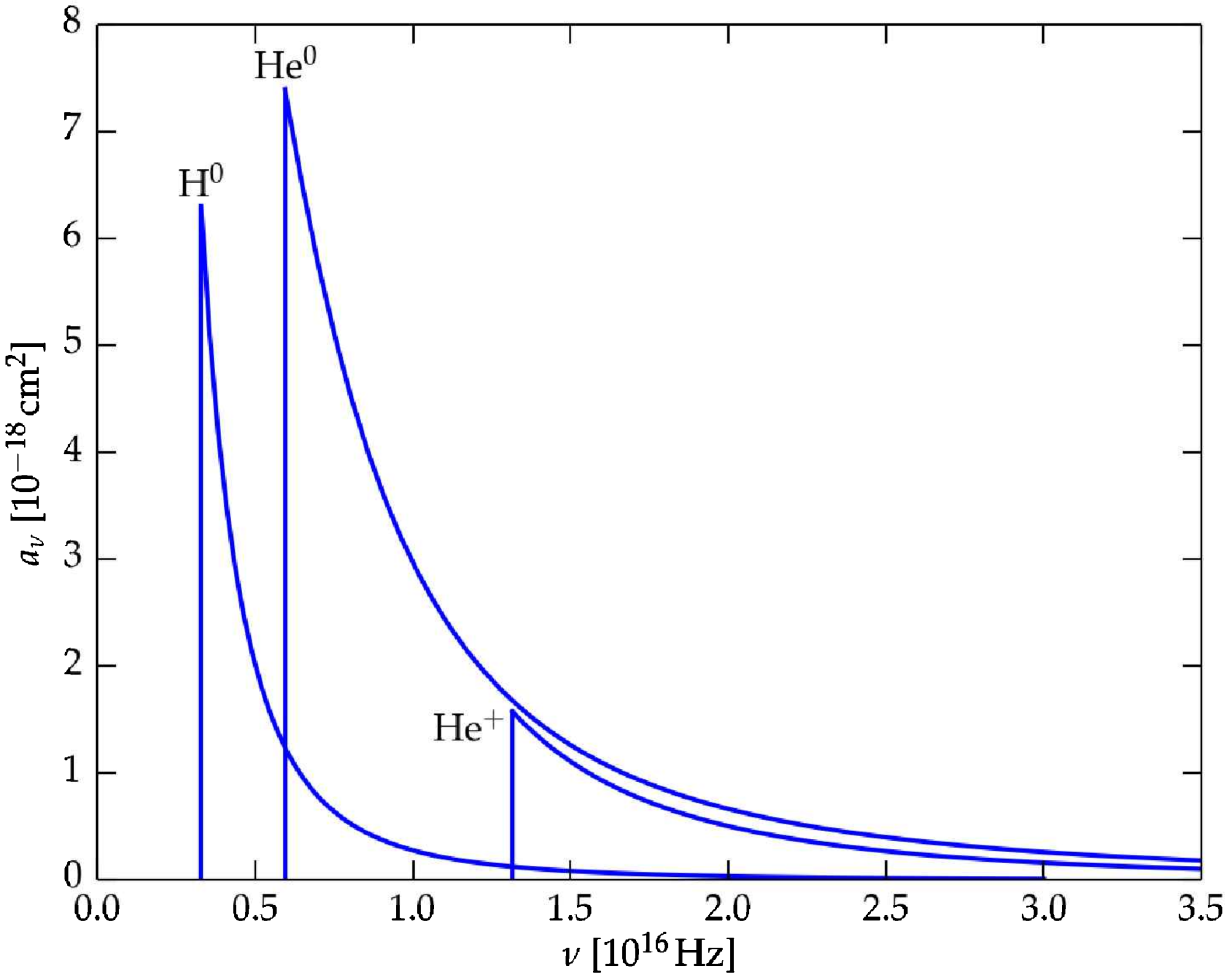
- The photoionization cross section is reasonably approximated by a power-law:

$$\boxed{\sigma_{\rm pi}(\nu) \approx \sigma_0 \left(\frac{h\nu}{Z^2 I_{\rm H}}\right)^{-3} \quad \text{for } Z^2 I_{\rm H} \lesssim h\nu \lesssim 100 Z^2 I_{\rm H}}$$

- At high energies, the asymptotic behavior is:

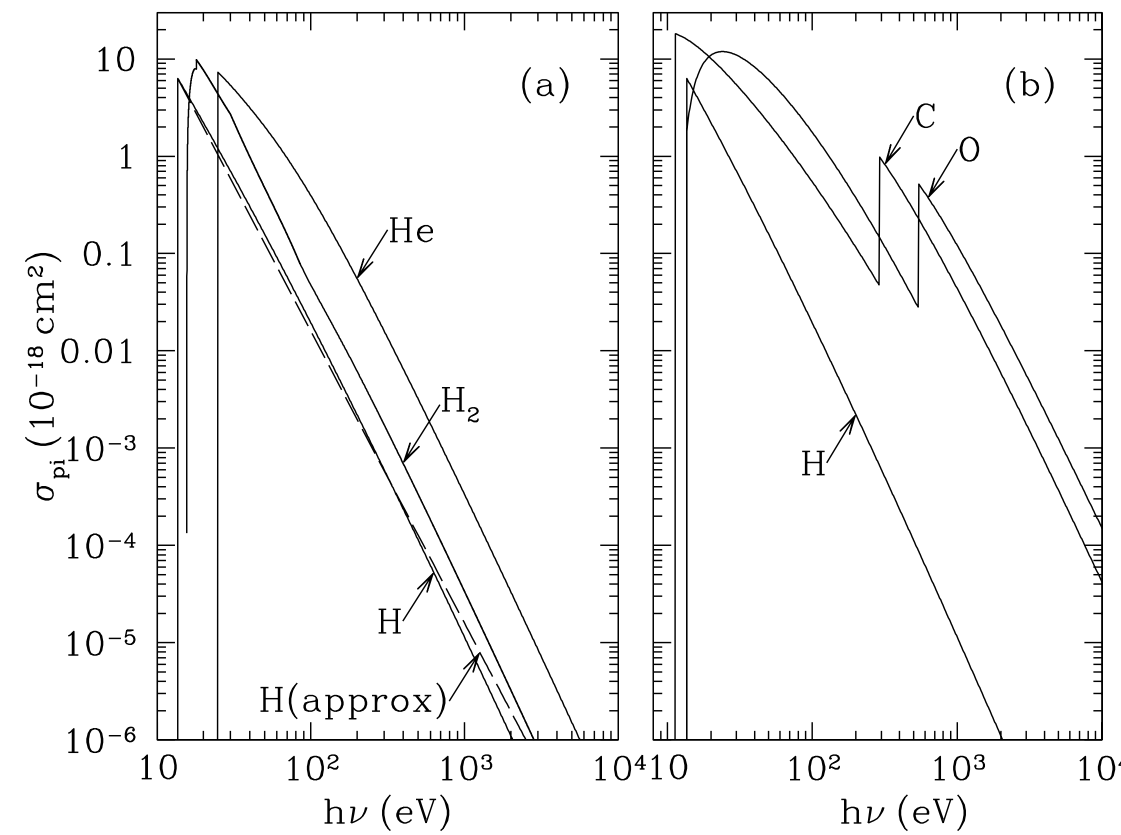
$$\sigma_{\rm pi}(\nu) \approx \frac{2^8}{3Z^2}\alpha(\pi a_0^2)\left(\frac{h\nu}{Z^2 I_{\rm H}}\right)^{-3.5} \quad \text{for} \quad h\nu \gg Z^2 I_{\rm H}$$

The hydrogen photoionization cross section becomes equal to the Thomson (Compton) Scattering cross section for $h\nu \approx 2.5\,{\rm keV}$; above this energy photoionization of H is dominated by Thomson scattering rather than photoelectric absorption.

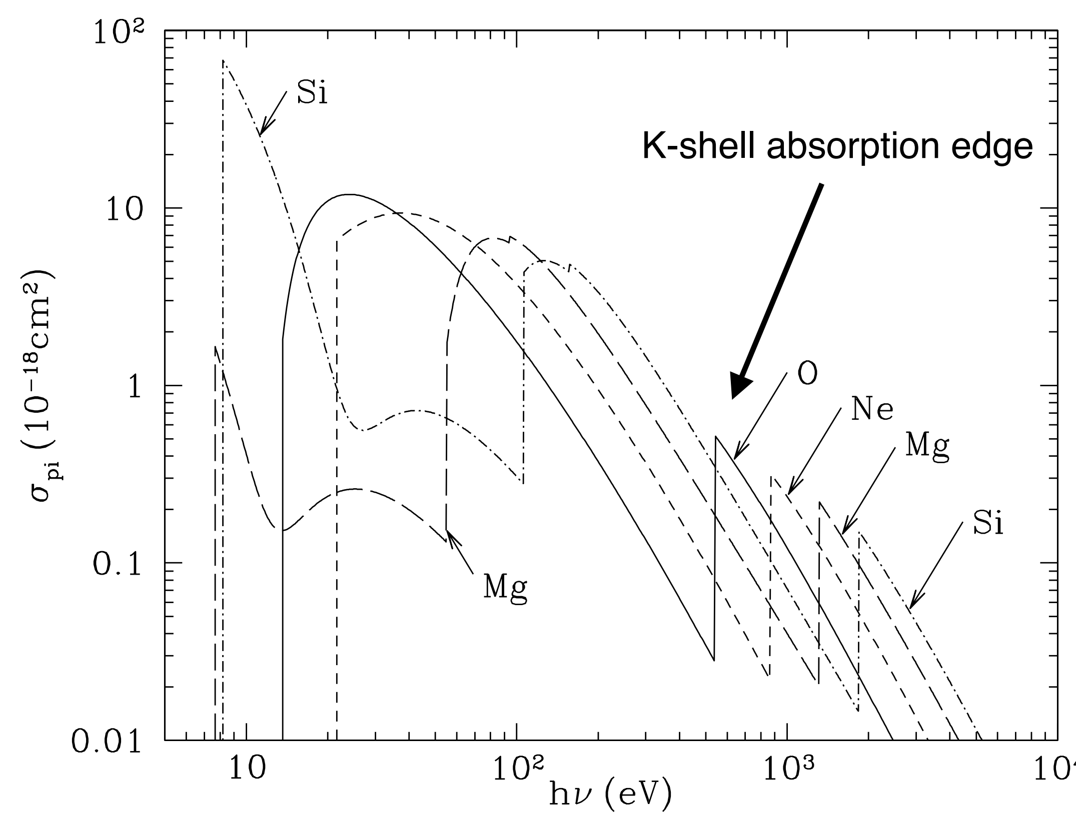


Photoionization cross section for hydrogen(H^0), hydrogenic helium (He$^+$), and neutral helium (He^0).
[Fig. 4.1 in Ryden]

- For atoms with three or more electrons, the energy dependence of the photoionization cross section is considerably more complicated because there is more than one available channel.
  - Convenient analytic fits to the contribution of individual shells to photoionization cross section are given by Verner & Yakovlev (1995) and Verner et al. (1996).



[Fig. 13.1 in Draine]



[Fig. 13.2 in Draine]

- Photoionization rate (the probability of photoionization per unit time, for a single atom that undergoes photoionization)

$$\zeta_{\rm pi} = \int_{\nu_1}^{\infty} \sigma_{\rm pi}(\nu) c \frac{u_\nu}{h\nu} d\nu \qquad \nu_1 = Z^2 I_{\rm H}/h = 3.29 \times 10^{15}\,{\rm Hz}\ (\text{for hydrogenic ions})$$

= (cross section) x (flux of ionizing photons by number)

$$\text{flux} \ = 4\pi \frac{J_\nu}{h\nu} = c \frac{u_\nu}{h\nu}$$

- Since the photoionization cross section decreases fairly steeply with increasing photon energy, **most photoionization occurs by photons with energies just above the ionization energy (13.6 eV for hydrogen)**, in a range of the spectrum where the background is produced mainly by hot stars.
- The volumetric photoionization rate, for instance, for hydrogen is

$$\frac{dn_p}{dt} = n_{\rm H^0} \zeta_{\rm pi} \quad [{\rm cm^{-3}\,s^{-1}}]$$

= (# of atoms/volume) x (ionization rate per atom)

where n_p and $n_{\rm H^0}$ are the number density of proton (ionized hydrogen) and the number of neutral hydrogen atom, respectively.

Radiative Recombination (RR)

- The cross section for the radiative recombination can be obtained using the photoionization cross section and the ***Milne relation***, which is derived from the principle of detailed balance.
- Consider an ion with its electron in some configuration that we will refer to as the "core". In a low-density plasma, free electrons can undergo transitions to bound states by emission of a photon. The electron is captured into some specific state $n\ell$ that will initially unoccupied.

$$X^+(\mathrm{core}) + e^- \rightarrow X(\mathrm{core} + n\ell) + h\nu$$

  - The RR rate coefficient for electron capture directly to level $n\ell$, with emission of a photon of energy $h\nu = I_{n\ell} + E$ (where $I_{n\ell}$ is the bounding energy required for ionization from level $n\ell$ and E is the captured electron energy), is

$$\alpha_{n\ell}(T) \equiv \langle \sigma_{\mathrm{rr,n\ell}} v \rangle = \left(\frac{8kT}{\pi m_e} \right)^{1/2} \int_0^\infty \sigma_{\mathrm{rr},n\ell}(E) \frac{E}{kT} e^{-E/kT} \frac{dE}{kT}$$

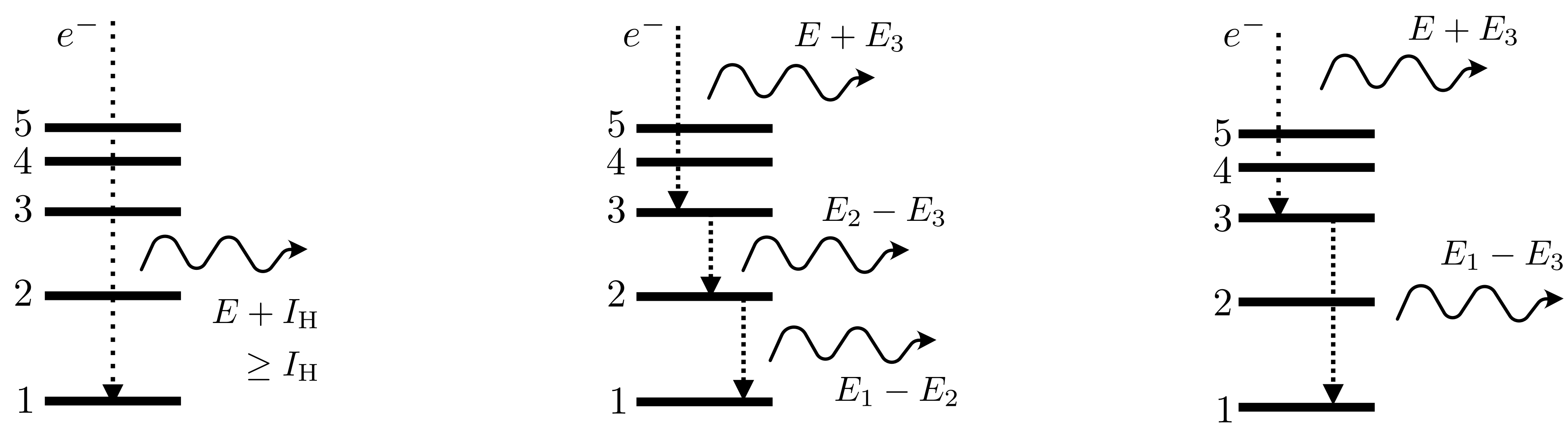
The integral indicates an average over the Maxwell distribution for electrons.

  - The volumetric rate of RR, for instance for hydrogen, can be written as

$$\frac{dn_p}{dt} = -n_e n_p \alpha$$

Notice that an electron of any energy can trigger a collisional de-excitation as well as RR.

- Properties of radiative recombination
  - In general, $\alpha_{n\ell}$ *is a decreasing function of* T, although it depends weakly on temperature. Therefore, ***it's easier to recombine with a slow electron than with a fast electron.***
  - In general, $\alpha_n = \sum_\ell \alpha_{n\ell}$ summed over all applicable values of ℓ, is a decreasing function of n, implying that ***it's easier to recombine to a low energy level than to a high energy level.***
- ***Recombination to the ground state:***
  - If the recombination is to the ground state of hydrogen ($n = 1$), the energy of the emitted photon is $E + I_\mathrm{H} \geq I_\mathrm{H}$. Thus, the emitted photon is guaranteed to have an energy of at least 13.6 eV, and will be capable of photoionizing any neutral hydrogen atom that it encounters. Thus, in regions that are optically thick to UV light at photon energies just above I_H, the emitted photon will be rapidly destroyed in photoionizing a nearby hydrogen atom.



$$n(X^r)n_\gamma\sigma_{\mathrm{pho}}c = n(X^{r+1})n_e\sigma_{\mathrm{rec}}\upsilon$$

$$\frac{n(X^{r+1})n_e}{n(X^r)} = \frac{n_\gamma \left\langle \sigma_{\mathrm{pho}} \right\rangle_{h\nu} c}{\left\langle \sigma_{\mathrm{rec}}\upsilon \right\rangle_{\upsilon}}$$

σ_{pho} = cross-section for photoionization, depending on the photon energy $h\nu$

σ_{rec} = cross-section for recombination, depending on the electron velocity υ

averaged over the photon energy spectrum

averaged over the Maxwellian velocity distribution

- Maxwellian distribution

$$\bar{f}(\mathbf{v})d^3\mathbf{v} = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{m\mathbf{v}^2}{2kT}\right) d^3\mathbf{v}$$

Here, $E = \frac{1}{2}mv^2$ (the energy per particle)

$$f(E)dE = \frac{2}{\sqrt{\pi}}\left(\frac{E}{kT}\right)^{1/2} \exp\left(-\frac{E}{kT}\right)\frac{dE}{kT}$$

- Then, the **radiative recombination rate coefficient**, $\alpha(T)$ is given by

$$\langle\sigma_{\rm rec}v\rangle = \left(\frac{2}{m_e}\right)^{1/2} \langle\sigma_{\rm rec}E^{1/2}\rangle$$

$$= \left(\frac{8kT}{\pi m_e}\right)^{1/2} \int_0^\infty \sigma_{\rm rec}\frac{E}{kT} \exp\left(-\frac{E}{kT}\right) d(E/kT)$$

Notice that Ryden's book call α the "recombination rate." But, the recombination rate is $n_e\langle\sigma_{\rm rec}v\rangle$.

Note that $\sigma_{\rm rec} \approx \sigma_{\rm rec,0}\left(\frac{E}{I_{\rm H}}\right)^{-1}$ for hydrogen

$$\therefore\ \alpha(T) = \langle\sigma_{\rm rec}v\rangle \approx \left(\frac{8}{\pi m_e kT}\right)^{1/2} \sigma_{\rm rec,0}I_{\rm H} \propto T^{-1/2}$$

Case A and B (Radiative Recombination of Hydrogen)

- ***On-the-spot approximation:***
  - In optically thick regions, it is assumed that every photon produced by radiative recombination to the ground state of hydrogen is immediately, then and there, destroyed in photoionizing other hydrogen atom.
  - In the on-the-spot approximation, recombination to the ground state has no net effect on the ionization state of the hydrogen gas.
- Baker & Menzel (1938) proposed two limiting cases:
  - ***Case A:*** ***Optically thin*** to ionizing radiation, so that every ionizing photon emitted during the recombination process escapes. For this case, we sum the radiative capture rate coefficient $\alpha_{n\ell}$ over all levels $n\ell$.
  - ***Case B:*** ***Optically thick*** to radiation just above $I_{\rm H} = 13.60\,{\rm eV}$, so that ionizing photons emitted during recombination are immediately reabsorbed, creating another ion and free electron by photoionization. In this case, the recombinations directly to $n = 1$ do not reduce the ionization of the gas: ***only recombinations to*** $n \geq 2$ ***act to reduce the ionization.***
  - **Case B in photoionized gas**: Photoionized nebulae around OB stars (H II regions) usually have large enough densities of neutral H. For this situation, case B is an excellent approximation.
  - **Case A in collisionally ionized gas**: Regions where the hydrogen is collisional ionized are typically very hot ($T > 10^6$ K) and contain a very small density of neutral hydrogen. For these shock-heated regions, case A is an excellent approximation.

- ***Radiative recombination rate coefficients***:
  - In Case A, the relevant radiative recombination rate coefficient is found by summing over all energy levels of the hydrogen atom:

$$\alpha_{\mathrm{A,H}}(T) \equiv \sum_{n=1}^{\infty} \sum_{\ell=0}^{n-1} \alpha_{n\ell}(T)$$

$$\approx 4.18 \times 10^{-13}\, T_4^{-0.721-0.021\ln T_4}\ [\mathrm{cm}^3\,\mathrm{s}^{-1}]\quad \text{for } 0.3 \lesssim T_4 \lesssim 3 \quad \left(T_4 \equiv T/10^4\,\mathrm{K}\right)$$

  - In Case B, the relevant radiative recombination rate coefficient is found by summing over all energy levels other than the ground state:

$$\alpha_{\mathrm{B,H}}(T) \equiv \sum_{n=2}^{\infty} \sum_{\ell=0}^{n-1} \alpha_{n\ell}(T) = \alpha_{\mathrm{A,H}}(T) - \alpha_{1\mathrm{s}}(T)$$

$$\approx 2.59 \times 10^{-13}\, T_4^{-0.833-0.034\ln T_4}\ [\mathrm{cm}^3\,\mathrm{s}^{-1}]\quad \text{for } 0.3 \lesssim T_4 \lesssim 3$$

  - The percentage of radiative recombinations that go directly to the ground state is 30% at T = 3000 K but increases to 46% at T = 30,000 K. Thus, the distinction between Case A and Case B becomes increasingly important at higher temperatures.

$$\frac{\alpha_{1\mathrm{s,H}}}{\alpha_{\mathrm{A,H}}} = 1 - \frac{\alpha_{\mathrm{B,H}}}{\alpha_{\mathrm{A,H}}} = 1 - 0.0619\, T_4^{-0.112-0.013\ln T_4}$$

H II Regions and Strömgren Spheres

- ***Strömgren Sphere***:
  - Following Strömgren (1939), we consider the simple idealized problem of a fully ionized, spherical region of uniform medium plus a central source of ionizing photons.
  - The ionization is assumed to be maintained by absorption of the ionizing photons radiated by a central hot star. The central source produces ionizing photons, with energy $\nu > \nu_0 = I_{\rm H}/h$ at a constant rate Q_0 [photons s^{-1}].
  - At a distance r from the central star, the balance equation between ionization and recombination balance is

$$n_{\rm H^0}\int_{\nu_0}^{\infty}\frac{4\pi J_\nu}{h\nu}\sigma_\nu d\nu = n_p n_e \alpha_{\rm B,H}$$

L_ν = luminosity of the central star at frequency ν.

From the RT equation,

$$4\pi J_\nu = \frac{L_\nu}{4\pi r^2}e^{-\tau_\nu}, \quad \text{where } \tau_\nu = \int_0^r n_{\rm H^0}\sigma_\nu dr$$

geometrical attenuation + radiative absorption

Integrating the balance equation over the whole volume:

$$\int_0^\infty\int_{\nu_0}^\infty \frac{L_\nu/h\nu}{4\pi r^2}e^{-\tau_\nu}n_{\rm H^0}\sigma_\nu d\nu(4\pi r^2)dr = \int_0^\infty n_p n_e \alpha_{\rm B,H}(4\pi r^2)dr$$

$$\int_{\nu_0}^\infty L_\nu/h\nu\left[\int_0^\infty e^{-\tau_\nu}n_{\rm H^0}\sigma_\nu dr\right]d\nu = \int_0^\infty n_p n_e \alpha_{\rm B,H}(4\pi r^2)dr$$

The square bracket term in the left side is

$$\int_0^\infty e^{-\tau_\nu} n_{\rm H^0}\sigma_\nu dr = \int_0^\infty e^{-\tau_\nu} d\tau_\nu = 1$$

Then, we obtain

total number of ionizing photons

$$Q_0 = \int_0^\infty n_p n_e \alpha_{\rm B,H} dV, \quad \text{where } Q_0 \equiv \int_{\nu_0}^\infty \frac{L_\nu}{h\nu} d\nu \text{ and } dV = 4\pi r^2 dr$$

- Assuming that ***the ionization is nearly complete (*** $n_p = n_e = n_{\rm H}$ ***) within*** R_s, and nearly zero ($n_{\rm H^0} = n_{\rm H},\ n_e = 0$) outside R_s, we obtain the size of the ionized region.

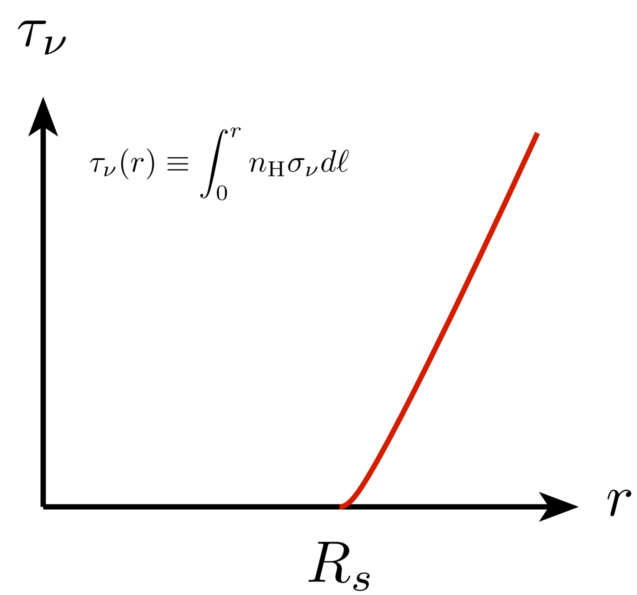
$$Q_0 = n_{\rm H}^2 \alpha_{\rm B,H} \frac{4\pi}{3} R_s^3$$

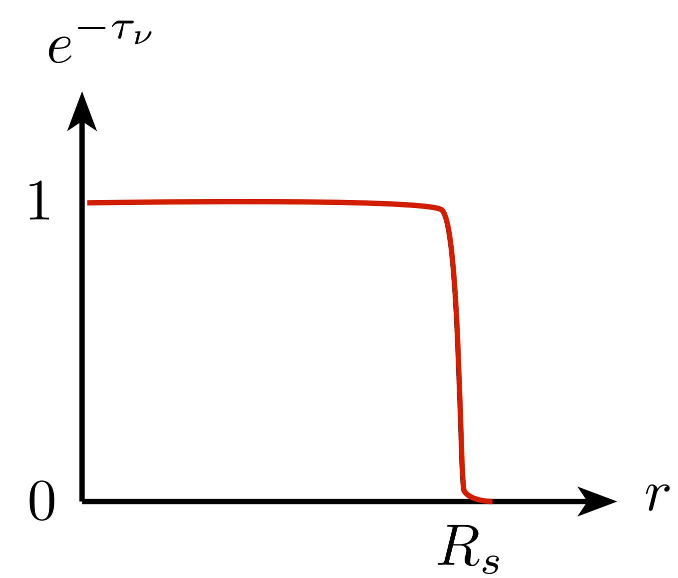
$$R_s = \left(\frac{3}{4\pi}\frac{Q_0}{\alpha_{\rm B,H} n_{\rm H}^2}\right)^{1/3}$$

$$= 3.17 \left(\frac{Q_0}{10^{49}\ \text{photons s}^{-1}}\right)^{1/3} \left(\frac{n_{\rm H}}{10^2\ \text{cm}^{-3}}\right)^{-2/3} \left(\frac{T}{10^4\ \text{K}}\right)^{0.28} \quad [\text{pc}]$$

The physical meaning of this is that ***the total number of ionizing photons emitted by the star balances the total number of recombinations within the ionized volume*** $(4\pi/3)R_s^3$, often called the Strömgren sphere. It's radius R_s is called the Strömgren radius.

- ***Opacity as a function of distance***
  - We note that the medium is fully ionized within the Strömgren sphere. Thus, within the Strömgren sphere, the opacity is nearly zero. The opacity suddenly increases at the boundary of the ionized region.





- ***Mean free path***
  - The mean free path of an ionizing photon is

$$\lambda_{\rm mfp} = \frac{1}{n_{\rm H}\sigma_{\rm pi}} \sim 5\times10^{-4}\,{\rm pc}\left(\frac{n_{\rm H}}{10^2\,{\rm cm}^{-2}}\right)^{-1}\left(\frac{\sigma_{\rm pi}}{6.304\times10^{-18}\,{\rm cm}^{-2}}\right)^{-1}$$

  This tells us that the transition from ionized gas to neutral gas at the boundary of the H II region will occur over a distance that is very small compared to the Strömgren radius.

- Time Scales:
  - ***Ionization time scale***: The Strömgren sphere analysis assumes a steady state solution. What is the time scale for approach to the steady state? Suppose that we start with a neutral region, and the ionizing source is suddenly turned on.

$$t_{\rm ioniz.} = \frac{\text{total number of ions to be created}}{\text{number of photons supplied per unit time}}$$
$$= \frac{(4\pi/3)R_s^3 n_{\rm H}}{Q_0} = \frac{1}{\alpha_{\rm B,H} n_{\rm H}} = 1.22 \times 10^3 \left(\frac{10^2\,{\rm cm}^{-3}}{n_{\rm H}}\right) \quad [{\rm yr}]$$

  - ***Recombination time scale***: Suppose that the ionizing source suddenly turns off. The ionized region will recombine on the recombination time scale:

$$t_{\rm rec} = \frac{1}{\alpha_{\rm B,H} n_{\rm H}} = 1.22 \times 10^3 \left(\frac{10^2\,{\rm cm}^{-3}}{n_{\rm H}}\right) \quad [{\rm yr}]$$

Note that the recombination time scale is identical to the ionization time scale!

The ionization/recombination time scale is shorter than the main-sequence lifetime > 5 My for a massive star.

Ionization of Helium

- Now, what about helium?
  - Out of every 1000 atoms, there are on average 912 hydrogen atoms, 87 helium atoms and one heavy atom.
    - Looking at the photoionization cross sections for H^0, He^0, He^{+1}, we see that above the 24.6 eV threshold for ionizing He^0, the photoionization cross section for helium is larger than that for hydrogen.

$$\begin{aligned}\sigma_{\rm pi,He^0} &\approx 6.5\,\sigma_{\rm pi,H^0} \quad \text{at } h\nu \sim 24.6\,\text{eV}\\ &\approx 14\,\sigma_{\rm pi,H^0} \quad \text{at } h\nu \sim 54.5\,\text{eV}\end{aligned}$$

    - Thus, the photoionization cross section for He is ~ 10 times that of H, while the number density of He is ~ 0.1 times that of H.
    - This implies that if we suddenly turn on a hot star, ***the initial photons in the range*** $24.6\,\text{eV} < h\nu < 54.4\,\text{eV}$ ***will be about as likely to photoionize a helium atom as a hydrogen atom.***
    - ***In the range of*** $13.6\,\text{eV} < h\nu < 24.6\,\text{eV}$***, on the other hand, nearly all the photons go to ionize H***; scarcer atoms (metals like O and C) account for only a tiny fraction of the ionizations.

- ***Radiative Recombination of Helium***

$$\mathrm{He}^{++} + e^- \rightarrow \mathrm{He}^+$$

$$\alpha_\mathrm{A}(T) \approx 4.13 \times 10^{-13}\, Z(T_4/Z^2)^{-0.7131-0.0115\ln(T_4/Z^2)} \quad [\mathrm{cm}^3\,\mathrm{s}^{-1}] \quad (30\,\mathrm{K} < T/Z^2 < 3\times 10^4\,\mathrm{K})$$

$$\alpha_\mathrm{B}(T) \approx 2.54 \times 10^{-13}\, Z(T_4/Z^2)^{-0.8163-0.0208\ln(T_4/Z^2)} \quad [\mathrm{cm}^3\,\mathrm{s}^{-1}]$$

$$\mathrm{He}^{+} + e^- \rightarrow \mathrm{He}^0$$

$$\alpha_{1\mathrm{s}^2,\mathrm{He}} = 1.54 \times 10^{-13}\, T_4^{-0.486} \quad [\mathrm{cm}^3\,\mathrm{s}^{-1}] \qquad (0.5 < T_4 < 2)$$

$$\alpha_{\mathrm{B},\mathrm{He}} = 2.72 \times 10^{-13}\, T_4^{-0.789} \quad [\mathrm{cm}^3\,\mathrm{s}^{-1}]$$

Here, $\alpha_{1\mathrm{s}^2,\mathrm{He}}$ is the recombination rate to the ground state $1\mathrm{s}^2\,{}^1S_0$.,

and $\alpha_{\mathrm{B},\mathrm{He}}$ is the recombination rate coefficient to all states except the ground state.

Note: $\alpha_{\mathrm{B},\mathrm{H}} \approx \alpha_{\mathrm{B},\mathrm{He}}$ and $\alpha_{\mathrm{A},\mathrm{H}} \approx \alpha_{\mathrm{A},\mathrm{He}}$.

Homework (due date: 05/08)

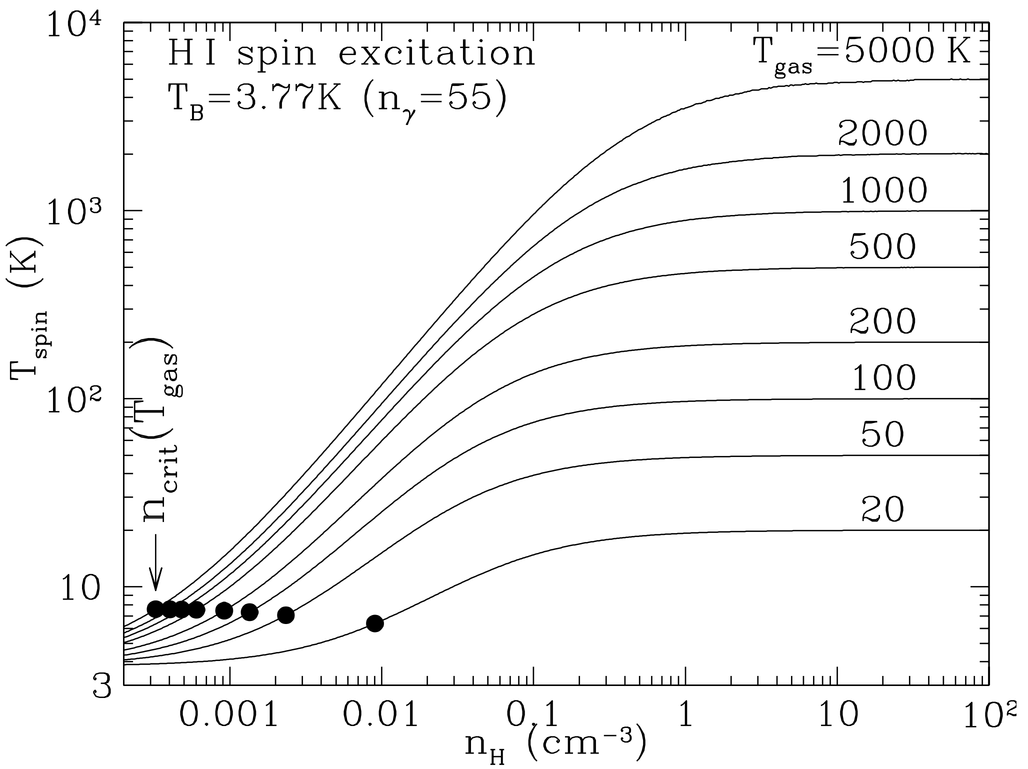
[Q9]

(1) If we consider only the background radiation field and collisions with hydrogen, the spin temperature of the 21-cm transition is given by

$$\text{Eq(a):} \quad T_{\rm spin} = \frac{T_{\rm rad} + y_c T_{\rm gas}}{1 + y_c} \quad \text{where} \quad y_c \equiv \frac{T_*}{T_{\rm gas}} \frac{n_c k_{10}}{A_{10}}$$

- Using the above equation, make a plot similar to the right side figure. (Extrapolate the approximate formula for k_{10} down below 20 K and up above 10^3 K.)
- Denote the two critical densities, for each gas temperature, defined by

$$\text{Eq(b):} \quad n^*_{\rm crit} = \frac{T_{\rm gas}}{T_*} \frac{A_{10}}{k_{10}} \quad \text{and} \quad n_{\rm crit} = \frac{(1+n_\gamma) A_{10}}{k_{10}}$$



(2) Discuss whether Eq(a) for the spin temperature for the 21-cm transition can be applied to the [C II] 158μm line or not.

Explain why the equation cannot be applied?

- [Q10]
  - Define E_x to be the energy at which the photoionization cross section for a hydrogenic ion is equal to the Thomson scattering cross section:

$$\sigma_T = (8\pi/3)(e^2/m_e c^2)^2 = (8\pi/3)(\alpha^2 a_0)^2$$

(a) Express E_x/I_{H} in terms of Z and the fine structure constant $\alpha \equiv e^2/\hbar c = 1/137.04$.

(b) For hydrogen, calculate E_x in eV.

Hint: use the cross section formula for high energies.

- [Q11]
  - The Einstein A coefficients for all the allowed transitions of hydrogen from levels $n \leq 3$ are given in the table below:

u	ℓ	$A_{u\ell}(\mathrm{s}^{-1})$	$\lambda_{u\ell}(\text{Å})$	
$3d$	$2p$	6.465×10^7	6564.6	Hα
$3p$	$2s$	2.245×10^7	6564.6	Hα
$3s$	$2p$	6.313×10^6	6564.6	Hα
$3p$	$1s$	1.672×10^8	1025.7	Lyβ
$2p$	$1s$	6.265×10^8	1215.7	Lyα

(a) Consider a hydrogen atom in the $3p$ state as the result of radiative recombination:

$$p + e^- \rightarrow H(3p)$$

What is the probability p_β that this atom will emit a Lyman β photon?

(b) In an H II region where hydrogen is the only important opacity source, what is the mean number of times a Lyman β photon, produced as the result of $p + e^- \rightarrow H(3p)$, is “scattered” (that is, absorbed and then re-emitted) before an Hα photon is emitted?

Hint: you may want to use the following formula:

$$\sum_{n=1}^{\infty} nq^n = q\sum_{n=1}^{\infty} nq^{n-1} = q\frac{d}{dq}\sum_{n=1}^{\infty} q^n = q\frac{d}{dq}\left[\frac{q}{1-q}\right] = \frac{q}{(1-q)^2}$$

- [Q12]
  - Absorption line observations of an interstellar cloud measure column densities:

$$N(\mathrm{CaI}) = 1.00 \times 10^{12}\,\mathrm{cm}^{-2}$$
$$N(\mathrm{CaII}) = 3.08 \times 10^{14}\,\mathrm{cm}^{-2}$$

The gas temperature is estimated to be $T = 50$ K. At this temperature the radiative recombination coefficient for $\mathrm{Ca\,II} + e^- \rightarrow \mathrm{Ca\,I} + h\nu$ is $\alpha = 1.3 \times 10^{-11}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$. The starlight within the cloud can photoionize $\mathrm{Ca\,I} + h\nu \rightarrow \mathrm{Ca\,II} + e^-$ with a photoionization rate $\zeta = 1.2 \times 10^{-10}\,\mathrm{s}^{-1}$. Estimate the electron density n_e in the cloud.