

KAIST Astrophysics

(PH481) - Part 1

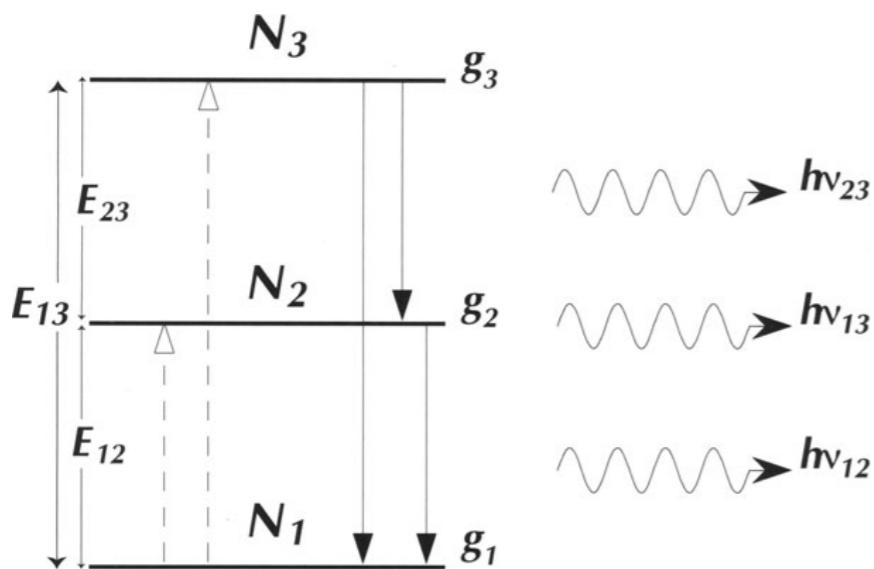
Week 4a
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Kwang-il Seon (선광일)
Korea Astronomy & Space Science Institute (KASI)
University of Science and Technology (UST)

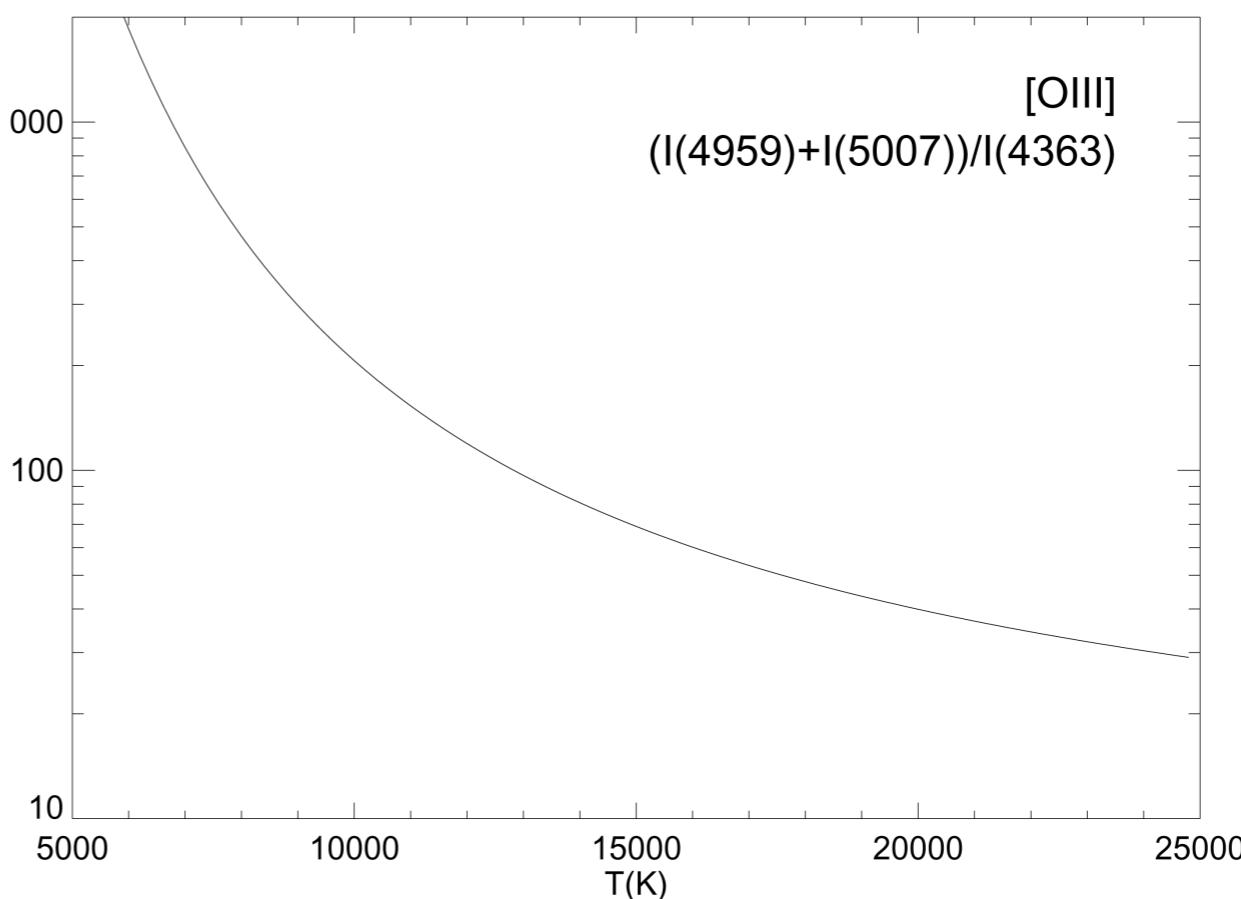
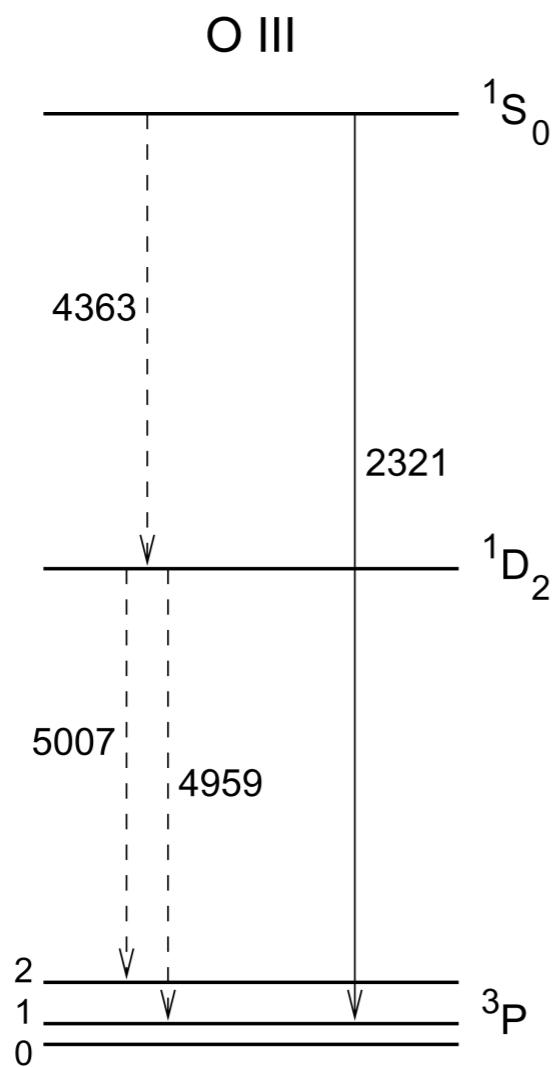
Nebular Diagnostics

- The populations of excited states of atoms and ions depend on the local density and temperature. Therefore, if we can determine the level populations from observations, we can use atoms and ions as probes of the interstellar space.
 - To be a useful probe, an atom or ion must be sufficiently abundant to observe, must have energy levels that are at suitable energies, and must have radiative transitions that allow us to probe these levels, either through emission lines or absorption lines.
- There are two principal types of nebular diagnostics.
 - **Temperature:** The first type uses ions with two excited levels that are both “energetically accessible” at the temperatures of interest, but ***with an energy difference between them that is comparable to kT*** , so that the populations of these levels are sensitive to the gas temperature.
 - **Density:** The second type uses ions with two or more “energetically accessible” ***energy levels that are at nearly the same energy***, so that the relative rates for populating these levels by collisions are nearly independent of temperature.
 - ▶ The ratio of the level populations will have one value in the low-density limit, where every collisional excitation is followed by spontaneous radiative decay, and another value in the high-density limit, where the levels are populated by the balance between the collisional excitation and deexcitation. If the relative level populations in these two limits differ (in general, they do), then the relative level populations (determined by observed emission line ratios) can be used to determine the density in the emission region.

Electron Temperature

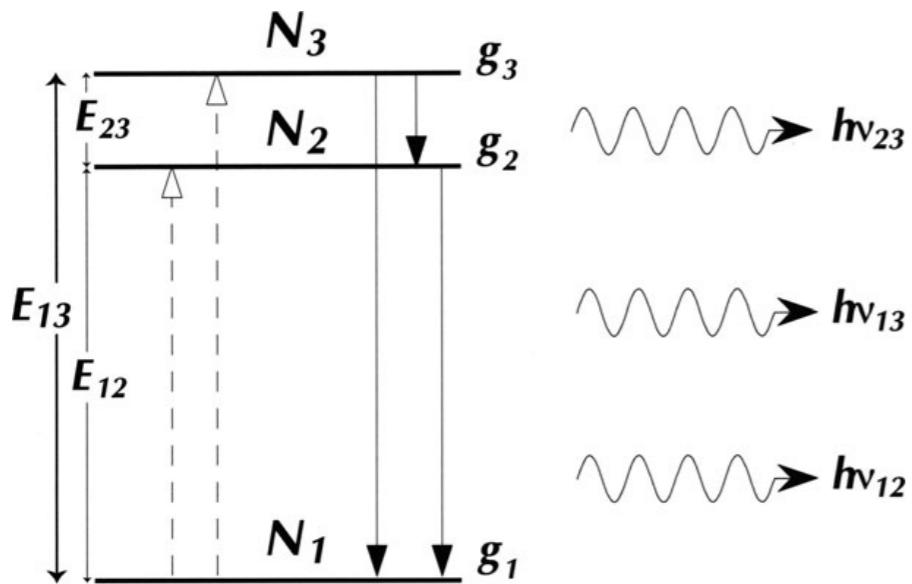


Choose an atom with two levels with different excitation energy, i.e., $E_{12} < E_{13}$, but $E_{23} \sim kT$.

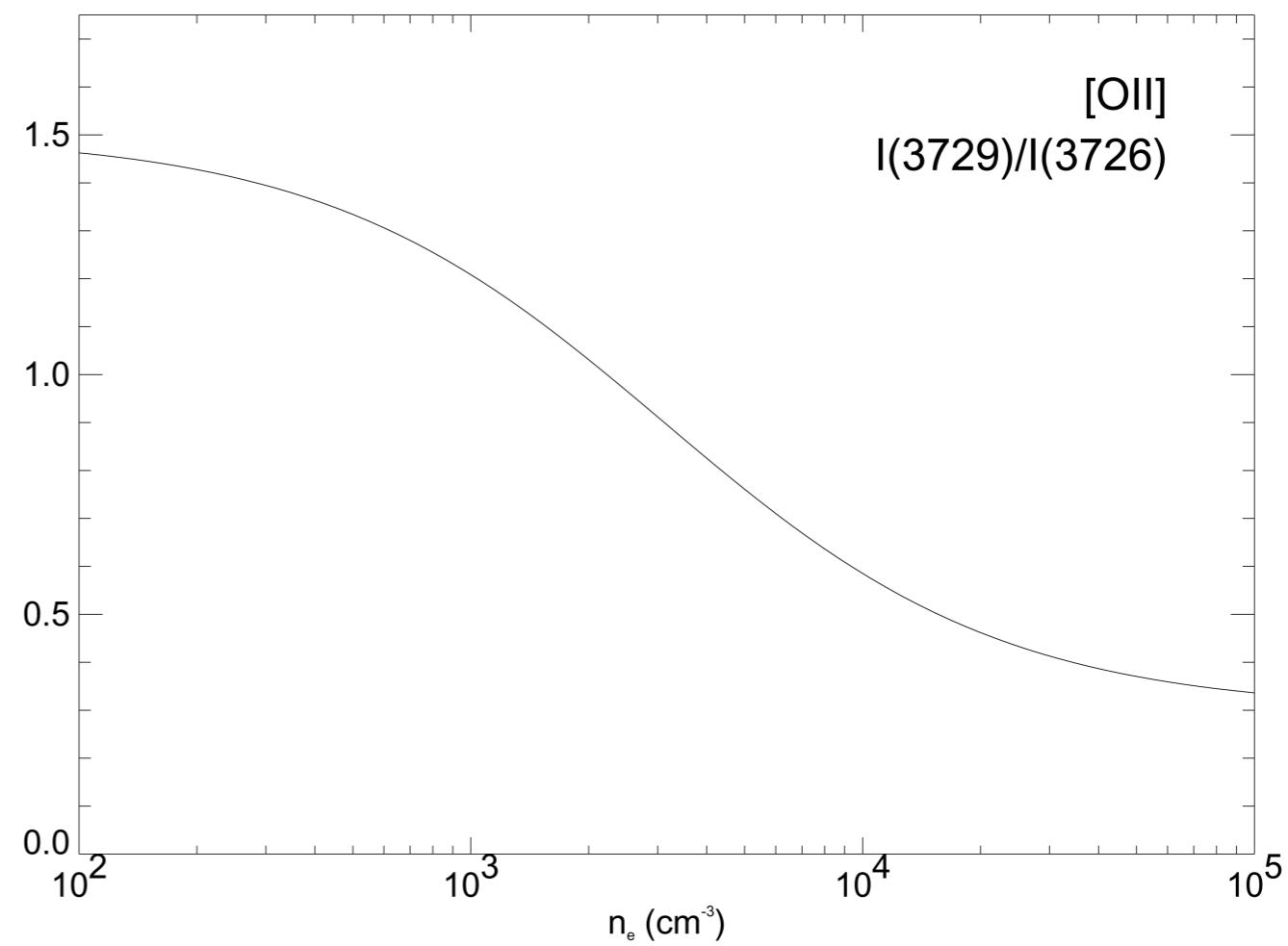
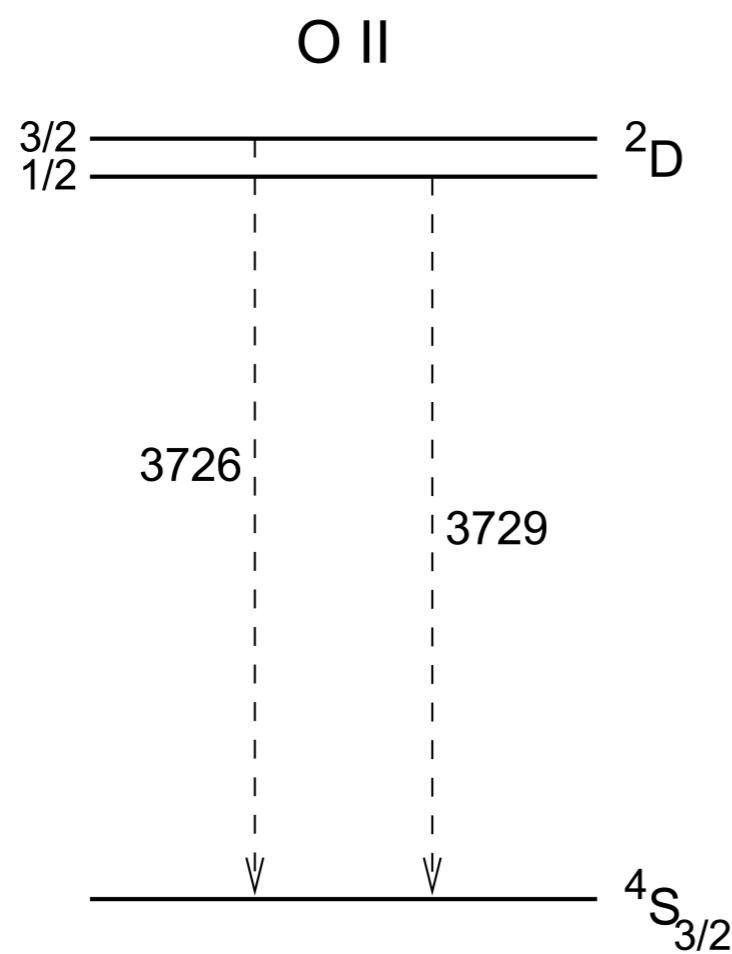


$$\frac{I(4959 + 5007)}{4363} = \frac{7.7 \exp(3.29 \times 10^4 / T)}{1 + 4.5 \times 10^{-4} n_e T^{-1/2}}$$

Electron Density

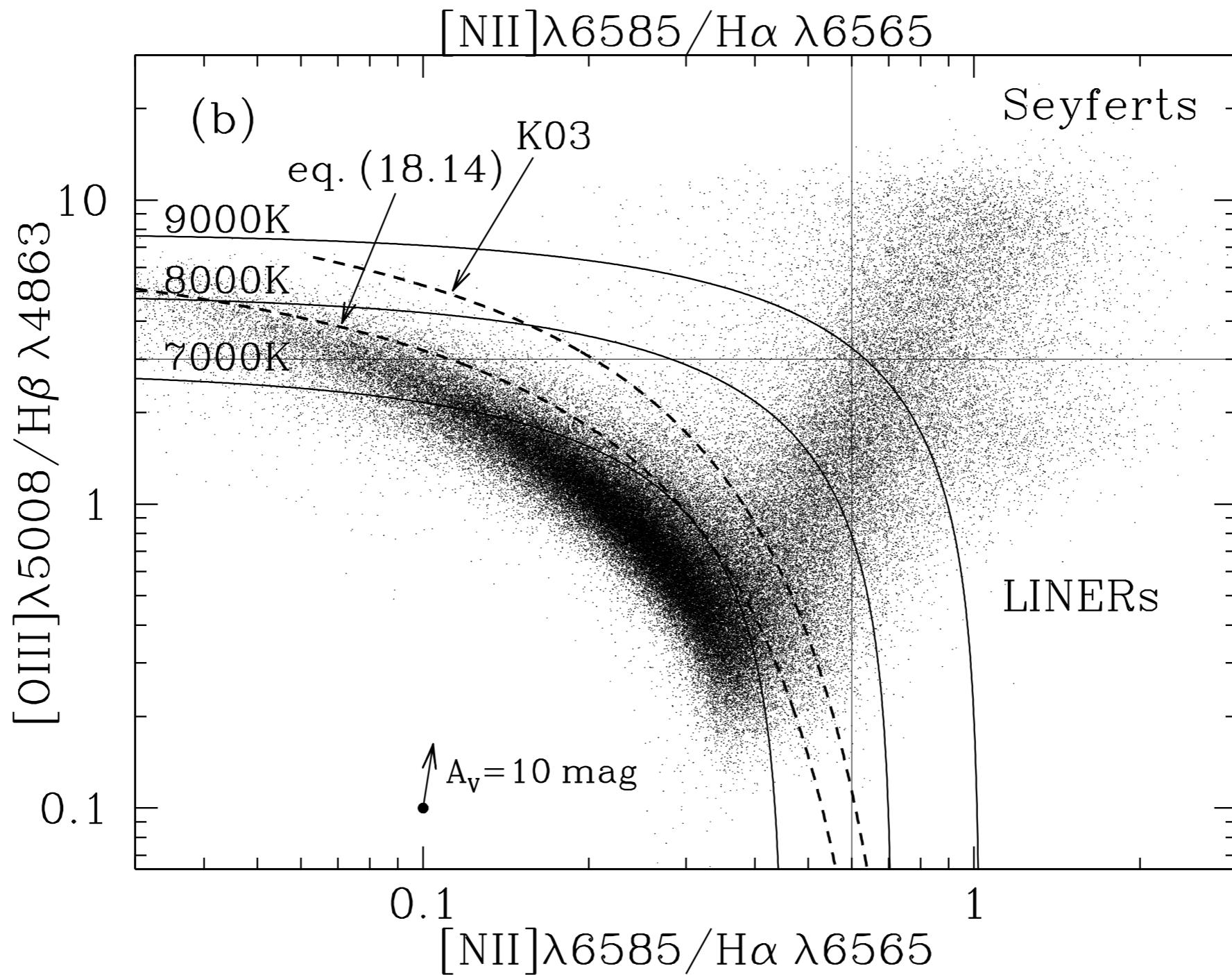


Choose an atom with two levels with almost same excitation energy i.e., $E_{12} \sim E_{13}$.



BPT diagram

Baldwin, Phillips & Terlevich (1981, PASP, 93, 5)



Lyman-Alpha, 21 cm Astrophysics

Harmonic Oscillator: classical model to the motion of an electron in an atom

- **Lorentz Oscillator Model** to describe the interaction between atoms and electric fields
 - The electron (with small mass) is bound to the nucleus of the atom (with a much larger mass) by a force that behaves according to Hooke's Law (a spring-like force).
 - An applied electric field would then interact with the charge of the electron, causing “stretching” or “compression” of the spring.
 - ***The electron's equation of motion:***

$$m\ddot{\mathbf{x}} = -k\mathbf{x} + \mathbf{F}_{\text{ext}} + \mathbf{F}_{\text{rad}}$$

$k = m\omega_0^2$, where k = spring constant

ω_0 = natural (fundamental or resonant) frequency

\mathbf{F}_{ext} = external force, driving force, or external electric field

\mathbf{F}_{rad} = radiation reaction force (radiation damping)
the damping of a charge's motion which arises because of
the emission of radiation

Free Oscillator - radiation damping; spontaneous emission

- **Undriven Harmonically Bound Particles** (free oscillator)
 - Since an oscillating electron represents a continuously accelerating charge, the electron will radiate energy.
 - The energy radiated away must come from the particle's own energy (energy conservation). In other words, **there must be a force acting on a particle by virtue of the radiation it produces. This is called the radiation reaction force.**
 - Let's derive the formula for the radiation reaction force from the fact that the energy radiated must be compensated for by the work done against the radiation reaction force.
 - On one hand, the radiative loss rate of energy, averaged over one cycle of the oscillating dipole, can be represented by the radiative reaction force:

$$\frac{dW}{dt} = \langle \mathbf{F}_{\text{rad}} \cdot \dot{\mathbf{x}} \rangle$$

- On the other hand, from the Larmor's formula for a dipole, the radiative loss will be:

$$\frac{dW}{dt} = -\frac{2e^2 \langle |\ddot{\mathbf{x}}|^2 \rangle}{3c^3}$$

Free Oscillator: Abraham-Lorentz formula

$$\therefore \langle \mathbf{F}_{\text{rad}} \cdot \dot{\mathbf{x}} \rangle = -\frac{2e^2 \langle |\ddot{\mathbf{x}}|^2 \rangle}{3c^3}$$

Here, $\langle |\ddot{\mathbf{x}}|^2 \rangle \equiv \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} \ddot{\mathbf{x}} \cdot \ddot{\mathbf{x}} dt$ $\tau = \text{oscillation period}$

$$= \frac{1}{\tau} \ddot{\mathbf{x}} \cdot \dot{\mathbf{x}} \Big|_{-\tau/2}^{\tau/2} - \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} \ddot{\mathbf{x}} \cdot \dot{\mathbf{x}} dt$$

We assume that the initial and final states are the same: $\ddot{\mathbf{x}} \cdot \dot{\mathbf{x}}(-\tau/2) = \ddot{\mathbf{x}} \cdot \dot{\mathbf{x}}(\tau/2)$

Then,

$$\langle |\ddot{\mathbf{x}}|^2 \rangle = -\frac{1}{\tau} \int_{-\tau/2}^{\tau/2} \ddot{\mathbf{x}} \cdot \ddot{\mathbf{x}} dt = -\langle \ddot{\mathbf{x}} \cdot \dot{\mathbf{x}} \rangle \rightarrow \langle \mathbf{F}_{\text{rad}} \cdot \dot{\mathbf{x}} \rangle = \frac{2e^2 \langle \ddot{\mathbf{x}} \cdot \dot{\mathbf{x}} \rangle}{3c^3}$$

Therefore, we can obtain

$$\mathbf{F}_{\text{rad}} = \frac{2e^2 \ddot{\mathbf{x}}}{3c^3} : \text{Abraham-Lorentz formula}$$

- **Abraham-Lorentz formula:**

$$\mathbf{F}_{\text{rad}} = \frac{2e^2 \ddot{\mathbf{x}}}{3c^3}$$

- This formula depends on the derivative of acceleration. This increases the degree of the equation of motion of a particle and can lead to some nonphysical behavior if not used properly and consistently.
- For a simple harmonic oscillator with a frequency ω_0 , we can avoid the difficulty by using

$$\ddot{\mathbf{x}} = -\omega_0^2 \dot{\mathbf{x}}$$

- ***This is a good assumption as long as the energy is to be radiated on a time scale that is long compared to the period of oscillation.*** In this regime, radiation reaction may be considered as a perturbation on the particle's motion. We then rewrite the radiation reaction force as

$$\mathbf{F}_{\text{rad}} = -\frac{2e^2 \omega_0^2}{3c^3} \dot{\mathbf{x}} = -m\gamma \dot{\mathbf{x}}, \quad \gamma \equiv \frac{2e^2 \omega_0^2}{3mc^3} \quad : \text{ damping constant}$$

Note $\gamma = A_{21}$ in Quantum Mechanics

- Therefore, the equation of motion of the electron in a Lorentz atom is

$$\ddot{\mathbf{x}} + \gamma \dot{\mathbf{x}} + \omega_0^2 \mathbf{x} = 0$$

- This equation may be solved by assuming that $x(t) \propto e^{\alpha t}$

$$\begin{aligned}\alpha^2 + \gamma\alpha + \omega_0^2 &= 0 \rightarrow \alpha = -(\gamma/2) \pm \sqrt{(\gamma/2)^2 - \omega_0^2} \\ &= -\gamma/2 \pm i\omega_0 + \mathcal{O}(\gamma^2/\omega_0^2)\end{aligned}$$

- Assuming initial conditions

$$x(0) = x_0, \dot{x}(0) = 0 \text{ at } t = 0$$

- we have

$$x(t) = \frac{1}{2} x_0 \left[e^{-(\gamma/2 - i\omega_0)t} + e^{-(\gamma/2 + i\omega_0)t} \right] = x_0 e^{-\gamma t/2} \cos \omega_0 t$$

—————> This is a damping oscillator.

- Power spectrum:

$$\bar{x}(\omega) = \frac{1}{2\pi} \int_0^\infty x(t) e^{i\omega t} dt = \frac{x_0}{4\pi} \left[\frac{1}{\gamma/2 - i(\omega + \omega_0)} + \frac{1}{\gamma/2 - i(\omega - \omega_0)} \right]$$

- This becomes large in the vicinity of $\omega = \omega_0$ and $\omega = -\omega_0$.
- We are ultimately interested only in positive frequencies, and only in regions in which the values become large. Therefore, we obtain

$$\bar{x}(\omega) \approx \frac{x_0}{4\pi} \frac{1}{\gamma/2 - i(\omega - \omega_0)}, \quad |\bar{x}(\omega)|^2 = \left(\frac{x_0}{4\pi} \right)^2 \frac{1}{(\omega - \omega_0)^2 + (\gamma/2)^2}$$

Emission Line profile

- Recall the Larmor's formula:

$$\frac{dW}{d\omega} = \frac{8\pi\omega^4}{3c^3} e^2 |\bar{x}(\omega)|^2$$

- Energy radiated per unit frequency:

$$\begin{aligned}\frac{dW}{d\omega} &= \frac{8\pi\omega^4}{3c^3} \frac{e^2 x_0^2}{(4\pi)^2} \frac{1}{(\omega - \omega_0)^2 + (\gamma/2)^2} = \frac{1}{2} m \left(\frac{\omega^4}{\omega_0^2} \right) x_0^2 \frac{\gamma/2\pi}{(\omega - \omega_0)^2 + (\gamma/2)^2} \\ &\approx \frac{1}{2} m \omega_0^2 x_0^2 \frac{\gamma/2\pi}{(\omega - \omega_0)^2 + (\gamma/2)^2}\end{aligned}$$

- For a harmonic oscillator, note that the equation of motion is $\mathbf{F} = -k\mathbf{x} = -m\omega_0^2\mathbf{x}$, spring constant is $k = m\omega_0^2$, and the potential energy (energy stored in spring) is $(1/2)kx_0^2$.

- From

$$\int_{-\infty}^{\infty} \frac{\gamma/2\pi}{(\omega - \omega_0)^2 + (\gamma/2)^2} d\omega = \frac{1}{\pi} \tan^{-1} \{2(\omega - \omega_0)/\gamma\}|_{-\infty}^{\infty} = 1$$

- Note that the total emitted energy is equal to the initial potential energy of the oscillator:

$$W = \int_0^{\infty} \frac{dW}{d\omega} d\omega = \frac{1}{2} k \omega_0^2$$

- Profile of the emitted spectrum:

$$\phi(\omega) = \frac{\gamma/2\pi}{(\omega - \omega_0)^2 + (\gamma/2)^2}$$

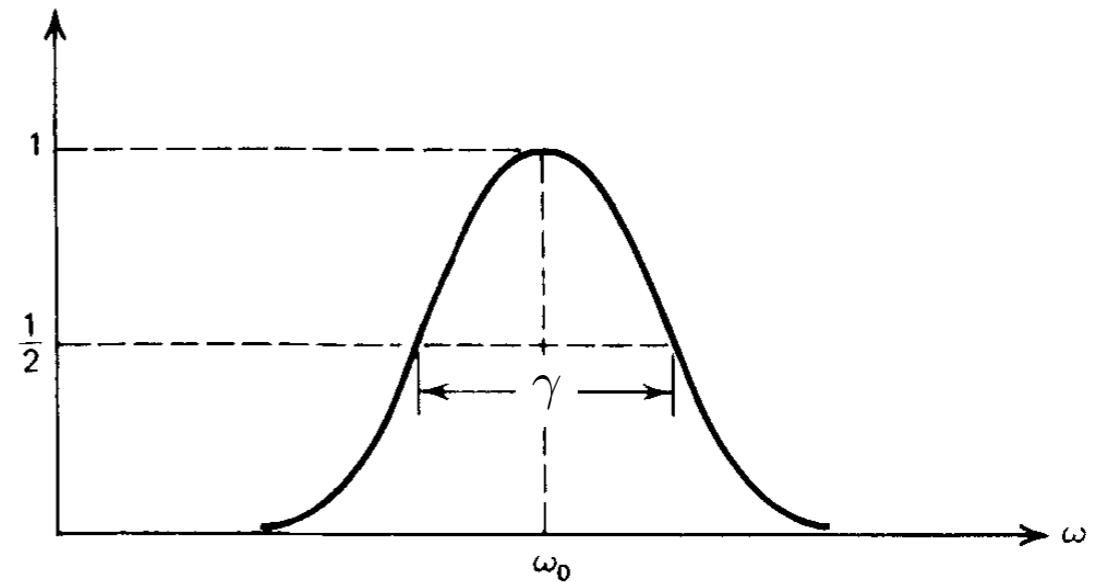
This is the Lorentz (natural) line profile.

- Damping constant is the full width at half maximum (FWHM).

$$\phi(\omega) = \frac{\gamma/2\pi}{(\omega - \omega_0)^2 + (\gamma/2)^2}$$

$$\phi(\nu) = \frac{\gamma/4\pi^2}{(\nu - \nu_0)^2 + (\gamma/4\pi)^2}$$

Note $\phi(\omega)d\omega = \phi(\nu)d\nu$



- The line width $\Delta\omega = \gamma$ is a universal constant when expressed in terms of wavelength:

$$\lambda = \frac{2\pi c}{\omega}$$

$$\Delta\lambda = 2\pi c \frac{\Delta\omega}{\omega^2} = 2\pi c \frac{2}{3} \frac{r_e}{c} \quad \leftarrow \quad \left(\Delta\omega = \gamma = \frac{2}{3} r_e \frac{\omega_0^2}{c} \right)$$

$$= \frac{4}{3} \pi r_e$$

$$= 1.2 \times 10^{-4} \text{ Å}$$

However, in Quantum Mechanics, the line width is not a universal constant.

Driven Oscillator (scattering)

- **Driven Harmonically Bound Particles** (forced oscillators)

- Electron's equation of motion (electric charge = $-e$): $\mathbf{F}_{\text{ext}} = -e\mathbf{E}_0 e^{i\omega t}$

$$\ddot{\mathbf{x}} + \gamma \dot{\mathbf{x}} + \omega_0^2 \mathbf{x} = -\frac{e\mathbf{E}_0}{m} e^{i\omega t}$$

- Steady-state solution of this equation:

$$\mathbf{x} = \mathbf{x}_0 e^{i\omega t} \equiv |\mathbf{x}_0| e^{i(\omega t + \delta)} \rightarrow (-\omega^2 + i\omega\gamma + \omega_0^2) \mathbf{x}_0 e^{i\omega t} = -\frac{e\mathbf{E}_0}{m} e^{i\omega t}$$

$$\mathbf{x}_0 = \frac{(e/m)\mathbf{E}_0}{(\omega^2 - \omega_0^2) - i\omega\gamma}$$

$$\mathbf{x}_0 = |\mathbf{x}_0| e^{i\delta} \propto (\omega^2 - \omega_0^2) + i\omega\gamma \rightarrow \delta = \tan^{-1} \left(\frac{\omega\gamma}{\omega^2 - \omega_0^2} \right)$$

The response is slightly out of phase with respect to the imposed field.

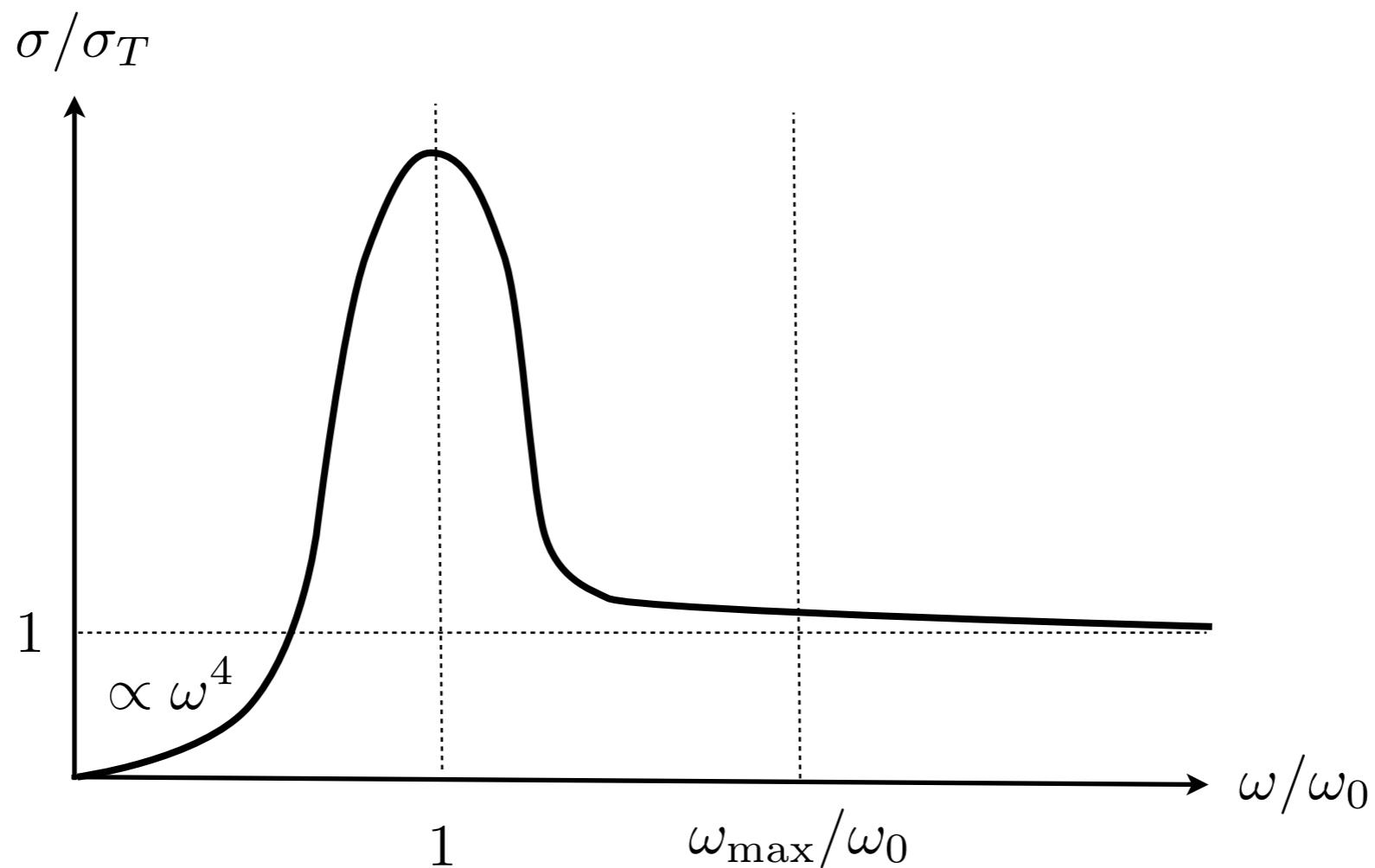
- Time-averaged total power radiated is given by

$$\begin{aligned} P &= \left\langle \frac{dW}{dt} \right\rangle = \frac{2e^2 \langle |\ddot{\mathbf{x}}|^2 \rangle}{3c^3} = \frac{e^2 \omega^4 |\mathbf{x}_0|^2}{3c^3} \\ &= \frac{e^4 E_0^2}{3m^2 c^3} \frac{\omega^4}{(\omega^2 - \omega_0^2)^2 + (\omega\gamma)^2} \end{aligned}$$

- Scattering cross section:

$$\sigma_{\text{sca}} \equiv \frac{\langle P \rangle}{\langle S \rangle}, \quad \langle S \rangle = \frac{c}{8\pi} E_0^2 \quad \longrightarrow \quad \sigma_{\text{sca}}(\omega) = \frac{8\pi e^4}{3m^2 c^4} \frac{\omega^4}{(\omega^2 - \omega_0^2)^2 + (\omega\gamma)^2}$$

$$= \sigma_T \frac{\omega^4}{(\omega^2 - \omega_0^2)^2 + (\omega\gamma)^2}$$



- Limiting Cases of Interest

- (a) $\omega \gg \omega_0$ (Thomson scattering by free electron)

$$\sigma_{\text{sca}} = \sigma_T = \frac{8\pi}{3} r_e^2$$

- ▶ At high incident energies, the binding becomes negligible. Therefore, this corresponds to the case of a free electron.
- (b) $\omega \ll \omega_0$ (Rayleigh scattering by bound electron)

$$\sigma_{\text{sca}} = \sigma_T \left(\frac{\omega}{\omega_0} \right)^4 = \sigma_T \left(\frac{\lambda_0}{\lambda} \right)^4$$

- ▶ Rayleigh scattering refers to the scattering of light by particles smaller than the wavelength of the light.
- ▶ The strong wavelength dependence of the scattering means that shorter (blue) wavelengths are scattered more strongly than longer wavelengths.
- ▶ (blue color of the sky) The dependence results in the indirect blue light coming from all regions of the sky.
- ▶ (red color of the sun at sunset) Conversely, glancing toward the Sun, the colors that were not scattered away - the longer wavelengths such as red and yellow light - are directly visible, giving the Sun itself a slightly yellowish color.
- ▶ However, viewed from space, the sky is black and the Sun is white.

- (c) $\omega \approx \omega_0$ (resonance scattering of line radiation)

$$\sigma_{\text{sca}}(\omega) \approx \sigma_T \frac{\omega_0^4}{(\omega - \omega_0)^2(2\omega_0)^2 + (\omega_0\gamma)^2}$$

$$= \sigma_T \frac{\omega_0^2/4}{(\omega - \omega_0)^2 + (\gamma/2)^2}$$

$$\sigma_T \frac{\omega_0^2}{4} = \frac{8\pi}{3} \left(\frac{e^2}{mc^2} \right)^2 \times \frac{1}{4} \times \left(\gamma \frac{3}{2} \frac{mc^3}{e^2 \omega_0^2} \right) = 2\pi^2 \frac{e^2}{mc} (\gamma/2\pi) \longrightarrow$$

Note that $\nu = 2\pi\omega$ and $\sigma_\nu = \sigma_\omega/2\pi$.

$$\sigma_\omega = \frac{2\pi^2 e^2}{mc} \frac{\gamma/2\pi}{(\omega - \omega_0)^2 + (\gamma/2)^2}$$

$$\sigma_\nu = \frac{\pi e^2}{mc} \frac{\gamma/4\pi^2}{(\nu - \nu_0)^2 + (\gamma/4\pi)^2}$$

- ▶ In the neighborhood of the resonance, ***the shape of the scattering cross section is the same as the emission line profile from the free oscillator.***
- ▶ Total scattering cross section is

$$\int_0^\infty \sigma_\nu d\nu = \frac{\pi e^2}{mc}$$

- ***In the quantum theory of spectral lines,***

we obtain similar formulas, which are conveniently stated in terms of the classical results as

$$\sigma_\nu = f_{nn'} \frac{\pi e^2}{mc} \frac{\gamma/4\pi^2}{(\nu - \nu_0)^2 + (\gamma/4\pi)^2}$$

$$\int_0^\infty \sigma_\nu d\nu = f_{nn'} \frac{\pi e^2}{mc}$$

where $f_{nn'}$ is called the ***oscillator strength*** or ***f-value*** for the transition between states n and n' .

Draine, Physics of the interstellar and intergalactic medium

Selected Resonance Lines^a with $\lambda < 3000 \text{ \AA}$

	Configurations	ℓ	u	$E_\ell/hc(\text{ cm}^{-1})$	$\lambda_{\text{vac}}(\text{\AA})$	$f_{\ell u}$
C IV	$1s^2 2s - 1s^2 2p$	$^2S_{1/2}$	$^2P_{1/2}^o$	0	1550.772	0.0962
		$^2S_{1/2}$	$^2P_{3/2}^o$	0	1548.202	0.190
N V	$1s^2 2s - 1s^2 2p$	$^2S_{1/2}$	$^2P_{1/2}^o$	0	1242.804	0.0780
		$^2S_{1/2}$	$^2P_{3/2}^o$	0	1242.821	0.156
O VI	$1s^2 2s - 1s^2 2p$	$^2S_{1/2}$	$^2P_{1/2}^o$	0	1037.613	0.066
		$^2S_{1/2}$	$^2P_{3/2}^o$	0	1037.921	0.133
C III	$2s^2 - 2s 2p$	1S_0	$^1P_1^o$	0	977.02	0.7586
C II	$2s^2 2p - 2s 2p^2$	$^2P_{1/2}^o$	$^2D_{3/2}^o$	0	1334.532	0.127
		$^2P_{3/2}^o$	$^2D_{5/2}^o$	63.42	1335.708	0.114
N III	$2s^2 2p - 2s 2p^2$	$^2P_{1/2}^o$	$^2D_{3/2}^o$	0	989.790	0.123
		$^2P_{3/2}^o$	$^2D_{5/2}^o$	174.4	991.577	0.110
CI	$2s^2 2p^2 - 2s^2 2p 3s$	3P_0	$^3P_1^o$	0	1656.928	0.140
		3P_1	$^3P_2^o$	16.40	1656.267	0.0588
		3P_2	$^3P_2^o$	43.40	1657.008	0.104
N II	$2s^2 2p^2 - 2s 2p^3$	3P_0	$^3D_1^o$	0	1083.990	0.115
		3P_1	$^3D_2^o$	48.7	1084.580	0.0861
		3P_2	$^3D_3^o$	130.8	1085.701	0.0957
NI	$2s^2 2p^3 - 2s^2 2p^2 3s$	$^4S_{3/2}^o$	$^4P_{5/2}$	0	1199.550	0.130
		$^4S_{3/2}^o$	$^4P_{3/2}$	0	1200.223	0.0862
OI	$2s^2 2p^4 - 2s^2 2p^3 3s$	3P_2	$^3S_1^o$	0	1302.168	0.0520
		3P_1	$^3S_1^o$	158.265	1304.858	0.0518
		3P_0	$^3S_1^o$	226.977	1306.029	0.0519
Mg II	$2p^6 3s - 2p^6 3p$	$^2S_{1/2}$	$^2P_{1/2}^o$	0	2803.531	0.303
		$^2S_{1/2}$	$^2P_{3/2}^o$	0	2796.352	0.608
Al III	$2p^6 3s - 2p^6 3p$	$^2S_{1/2}$	$^2P_{1/2}^o$	0	1862.790	0.277
		$^2S_{1/2}$	$^2P_{3/2}^o$	0	1854.716	0.557

Line Broadening Mechanisms

- ***Atomic levels are not infinitely sharp***, nor are the lines connecting them.
 - (1) Doppler Broadening
 - (2) Natural Broadening
 - (3) Collisional Broadening
- **Doppler Broadening**
 - The simplest mechanism for line broadening in the Doppler effect. An atom is in thermal motion, so that the frequency of emission or absorption in its own frame corresponds to a different frequency for an observer.
 - Each atom has its own Doppler shift, so that the net effect is to spread the line out, but not to change its total strength.
 - The change in frequency associated with an atom with velocity component v_z along the line of sight (say, z axis) is, to lowest order in v_z/c , given by

$$\nu - \nu_0 = \nu_0 \frac{v_z}{c}$$

Doppler shift: $\left[\frac{\nu}{\nu_0} = \frac{1}{\gamma(1 - \beta \cos \theta)} \rightarrow \nu \approx \nu_0 (1 + \beta \cos \theta) \rightarrow \nu - \nu_0 = \frac{\nu_0 v_z}{c} \right]$

- Here, ν_0 is the rest-frame frequency.

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- The number of atoms having velocities in the range $(v_z, v_z + dv_z)$ is proportional to

$$f(v_z)dv_z = \exp\left(-\frac{mv_z^2}{2kT}\right) dv_z$$

- From the Doppler shift formula, we have

$$v_z = \frac{c(\nu - \nu_0)}{\nu_0} \rightarrow dv_z = \frac{cd\nu}{\nu_0}$$

- Therefore, the strength of the emission is proportional to

$$\exp\left(-\frac{mv_z^2}{2kT}\right) dv_z \propto \exp\left[-\frac{mc^2(\nu - \nu_0)^2}{2\nu_0^2 kT}\right] d\nu$$

- Then, the normalized profile function is

$$\phi(\nu) = \frac{1}{\Delta\nu_D \sqrt{\pi}} e^{-(\nu - \nu_0)^2 / (\Delta\nu_D)^2} \quad \text{where } \Delta\nu_D = \frac{\nu_0}{c} \sqrt{\frac{2kT}{m}}$$

is the Doppler width.

-
- **The line-center cross section for each atom**, neglecting stimulated emission, is therefore

$$\begin{aligned}\sigma_{\nu_0} &= B_{12} \frac{h\nu}{4\pi} \phi(\nu_0) = \frac{1}{\Delta\nu_D \sqrt{\pi}} \frac{h\nu_0}{4\pi} B_{12} \\ &= \frac{\pi e^2}{mc} f_{12} \frac{1}{\Delta\nu_D \sqrt{\pi}}\end{aligned}$$

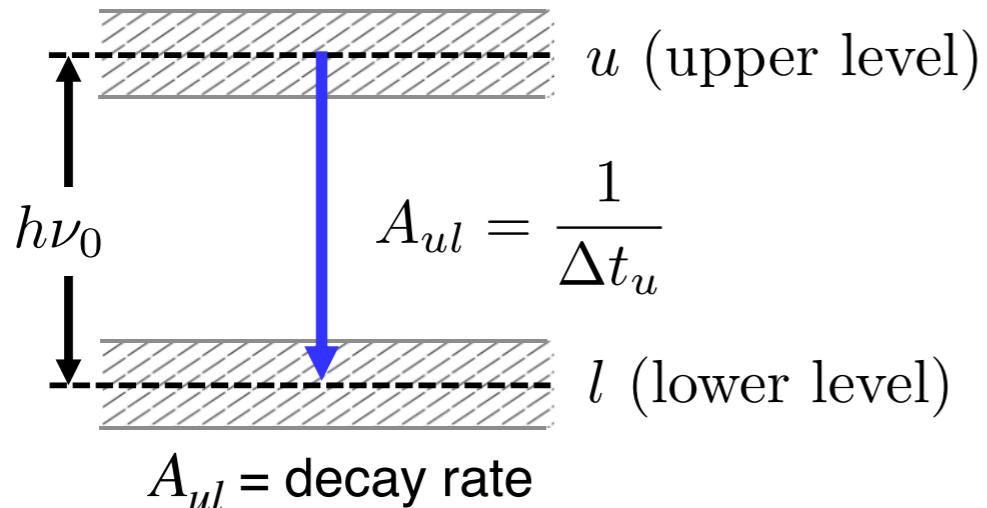
- In addition to thermal motions, there can be turbulent velocities associated with macroscopic velocity fields. **When the scale of the turbulence is small in comparison with a mean free path (microturbulence)**, the turbulent motions are accounted for by an effective Doppler width.

$$\Delta\nu_D = \frac{\nu_0}{c} \left(\frac{2kT}{m} + v_{\text{turb}}^2 \right)^{1/2}$$

- where v_{turb} is a root-mean-square measure of the turbulent velocities. This assumes that the turbulent velocities also have a Gaussian distribution.

• Natural Broadening

- A certain width to the atomic level is implied by the uncertainty principle, namely, that the spread in energy ΔE and the duration Δt in the state must satisfy $\Delta E \Delta t \sim \hbar = h/2\pi$.



ΔE_u = uncertainty in energy of u

Δt_u = the uncertainty in time of occupation of u

$\Delta\nu_u$ = uncertainty in frequency

$$= \Delta E_u/h = 1/(2\pi\Delta t_u) = A_{ul}/(2\pi)$$

$$\phi_\nu = \frac{\gamma/4\pi^2}{(\nu - \nu_0)^2 + (\gamma/4\pi)^2}$$

In terms of the line width $\Delta\nu_u$, the line profile can be represented by

$$\phi_\nu = \frac{1}{2\pi} \frac{\Delta\nu_u/2}{(\nu - \nu_0)^2 + (\Delta\nu_u/2)^2}$$

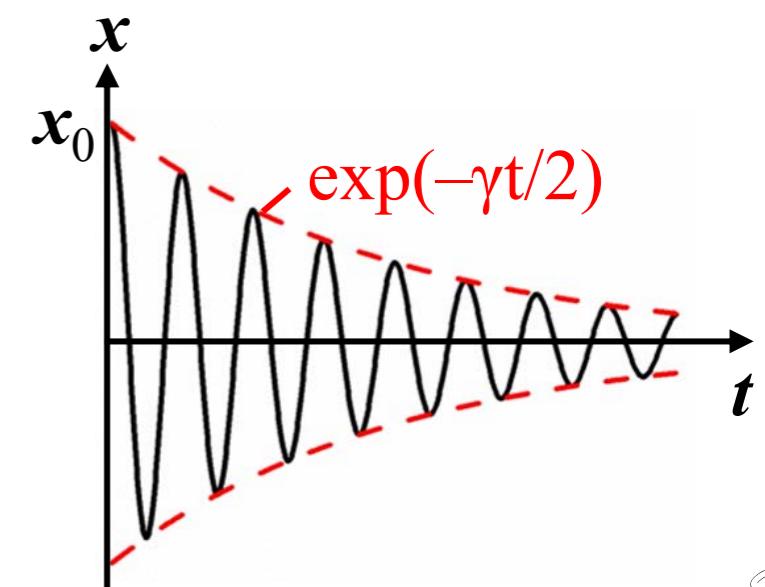
Therefore, γ is equivalent to the decay rate., i.e., $\gamma = A_{ul}$.

The spontaneous decay of an atomic state n proceeds at a rate where the sum is over all states n' of lower energy.

The electric field (the wave function of state n) is of the form $e^{-\gamma t/2}$ and then the energy decays proportional to $e^{-\gamma t}$. We then have an emitted spectrum determined by the decaying sinusoid type of electric field. The spectral profile is of the form, which is called a Lorentz (or natural, or Cauchy) profile,

$$\phi_\nu = \frac{\gamma/4\pi^2}{(\nu - \nu_0)^2 + (\gamma/4\pi)^2}$$

$$\gamma = \sum_{n'} A_{nn'}$$



The above profile applies to cases in which only the upper state is broadened.

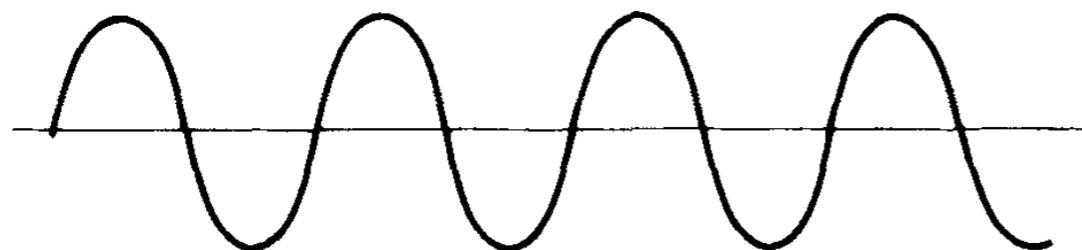
If both the upper and lower state are broadened, then the appropriate definition for γ is $\gamma = \gamma_u + \gamma_l$, where γ_u and γ_l are the widths of the upper and lower states involved in the transition. (Note that $\gamma_l = 0$ for the ground state.)

The longer an excited state exists, the narrower the line width so that metastable states can have very narrow lines (if the thermal Doppler broadening is not important).

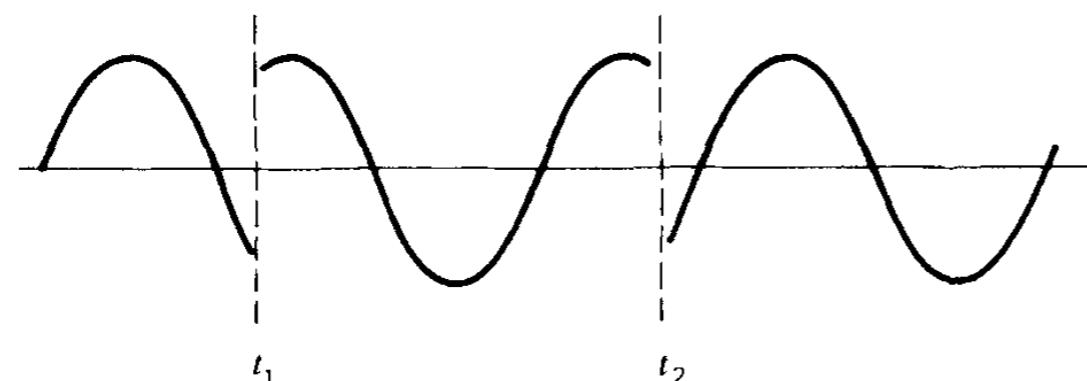
• Collisional Broadening

- The Lorentz profile applies even to certain types of collisional broadening mechanisms.
- If the atom suffers collisions with other particles while it is emitting, the phase of the emitted radiation can be altered suddenly. If the phase changes completely randomly at the collision times, then information about the emitting frequencies is lost.
- If the collisions occur with frequency ν_{col} , that is, each atom experiences ν_{col} collisions per unit time on the average, then the profile is

$$\phi_\nu = \frac{\Gamma/4\pi^2}{(\nu - \nu_0)^2 + (\Gamma/4\pi)^2} \quad \text{where } \Gamma = \gamma + 2\nu_{\text{col}}$$



purely sinusoidal



random phase interruptions
by atomic collisions

For derivation of the above formula, see Problem 10.7 of Rybiki & Lightman and Chapter 11 of Atomic Spectroscopy and Radiative Processes [Degl'Innocenti].

• Voigt profile : Combined Doppler and Lorentz Profiles

- Atoms shows both a Lorentz profile plus the Doppler effect. In this case, we can write the profile as an average of the Lorentz profile over the various velocity states of the atom:

$$\phi(\nu) = \frac{\Gamma}{4\pi^2} \int_{-\infty}^{\infty} \frac{(m/2\pi kT)^{1/2} \exp(-mv_z^2/2kT)}{(\nu - \nu_0(1 + v_z/c))^2 + (\Gamma/4\pi)^2} dv_z$$

- The profile can be written using the Voigt function.

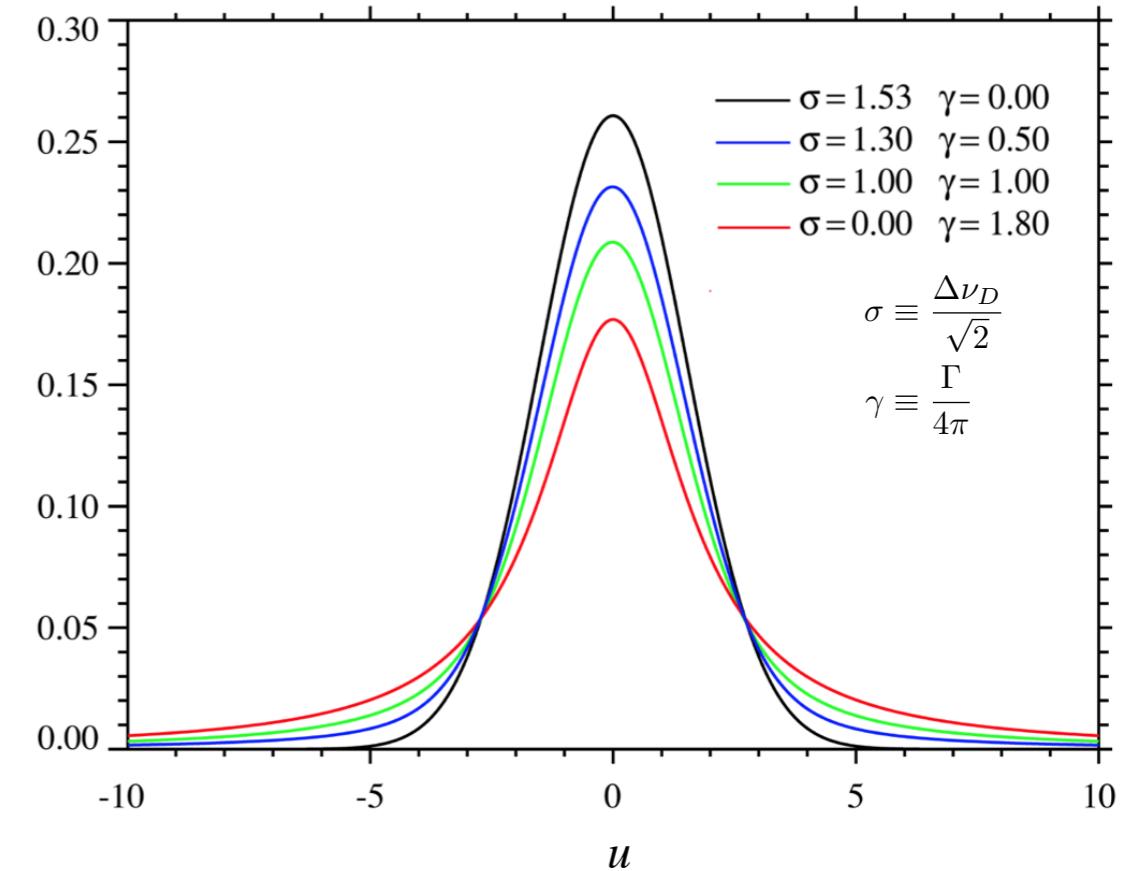
$$\phi(\nu) = \frac{1}{\Delta\nu_D \sqrt{\pi}} H(a, u)$$

$$H(a, u) \equiv \int_{-\infty}^{\infty} \frac{e^{-y^2} dy}{a^2 + (u - y)^2}$$

$$a \equiv \frac{\Gamma}{4\pi \Delta\nu_D}$$

$$u \equiv \frac{\nu - \nu_0}{\Delta\nu_D}$$

$$\nu_D = \frac{\nu_0}{c} \sqrt{\frac{2kT}{m}}$$



- For small a , the “core” of the line is dominated by the Gaussian (Doppler) profile, whereas the “wings” are dominated by the Lorentz profile.

Diffuse Intergalactic Medium

- The 2015 results from the Planck satellite tell us that the current mass density of baryonic matter (“dirty hydrogen”) is

$$\langle \rho \rangle_{\text{baryon}} = 4.2 \times 10^{-31} \text{ g cm}^{-3} \rightarrow \langle n \rangle_{\text{baryon}} = 2.5 \times 10^{-7} \text{ cm}^{-3}$$

- The amount of baryonic matter in gravitationally bound systems, including stars, the Interstellar Medium (ISM), the Circumgalactic Medium (CGM), and the Intracluster Medium (ICM), provides only $\sim 15\%$ of this mean cosmic baryon density.
- The remainder is provided by a very tenuous intergalactic medium (IGM).

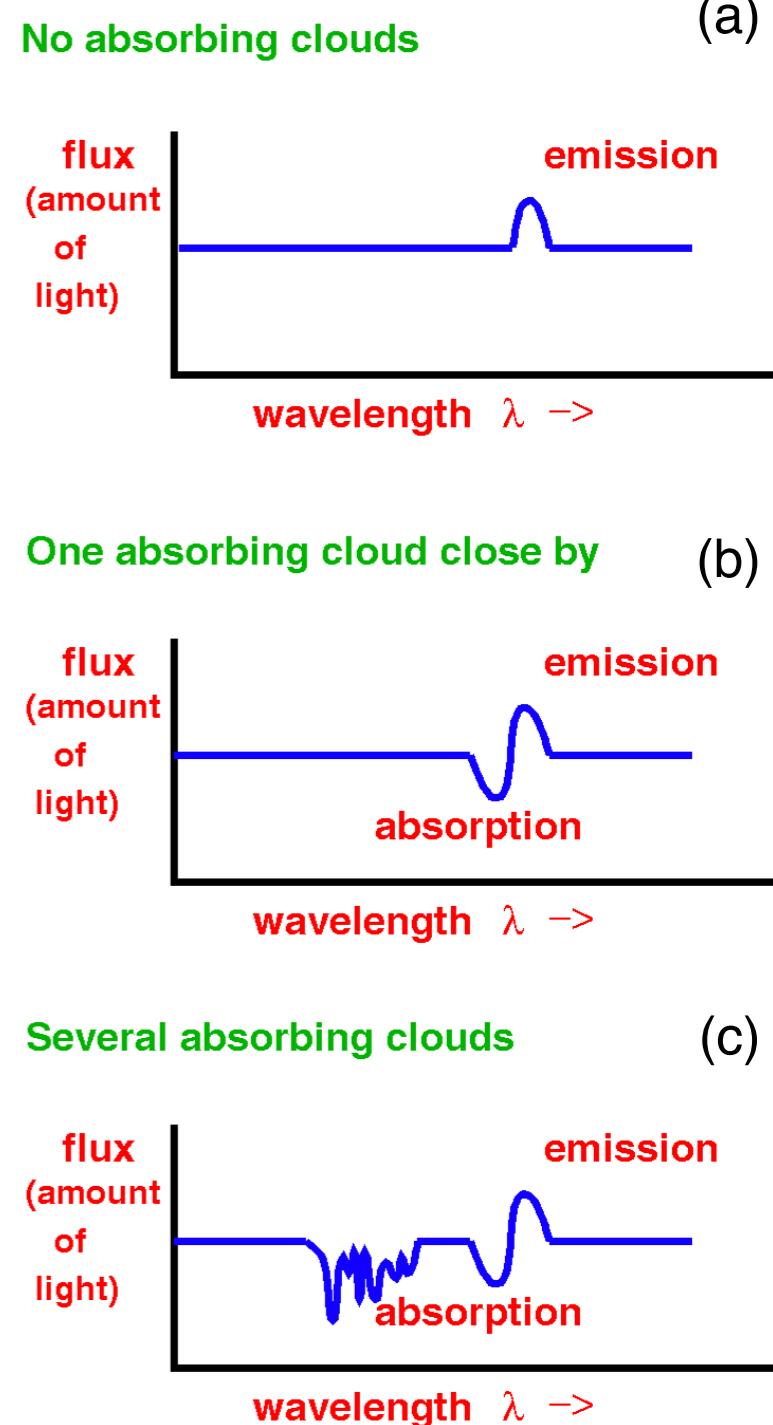
Absorption Lines as a Probe of the IGM

- Every parcel of gas along the line of sight to a distant quasar will selectively absorb certain wavelengths of continuum light of the quasar due to the presence of the various chemical elements in the gas.
- Through the analysis of these quasar absorption lines we can study the spatial distributions, motions, chemical enrichment, and ionization histories of gaseous structures from redshift $z \sim 6$ until the present.
- This structure includes the gas in galaxies of all morphological types as well as the diffuse gas in the IGM.



Ly α Forest

- Figure (a) shows a cartoon of how a quasar spectrum might look if there were no intervening neutral hydrogen between the quasar and us.
 - The quasar continuum is relatively flat. Broad emission features are produced by the quasar itself (near the black hole and its accretion disk).
- In some cases, gas near the quasar central engine also produces “intrinsic” absorption lines, most notably Ly α , and relatively high ionization metal transitions such as C IV, N V, and O VI.
- However, the vast majority of absorption lines in a typical quasar spectrum are “intervening”, produced by gas unrelated to the quasar that is located along the line of sight between the quasar and the Earth. Its wavelength is stretched by the expansion of the Universe from what it was initially at the quasar, and, if it had continued to travel to us, it would have been stretched some more from the 1216Å wavelength it had at the absorber.



- The cartoon below shows a quasar with its Ly α emission line redshifted from the UV into the red, and the Ly α absorption lines from four intervening clouds appearing as orange, yellow and green-blue.
- Each structure will produce an absorption line in the quasar spectrum at a wavelength of $\lambda_{\text{obs}} = \lambda_{\text{rest}}(1 + z_{\text{gas}})$, where z_{gas} is the redshift of the absorbing gas and $\lambda_{\text{rest}} = 1216\text{\AA}$ is the rest wavelength of the Ly α transition. Since $z_{\text{gas}} < z_{\text{quasar}}$, the redshift of the quasar, these Ly α absorption lines form a “forest” at wavelengths blueward of the Ly α emission of the quasar.
- The region reward of the Ly α emission will be populated only by absorption through other chemical transitions with longer $\lambda_{\text{Ly}\alpha}$.

Definition of redshift:

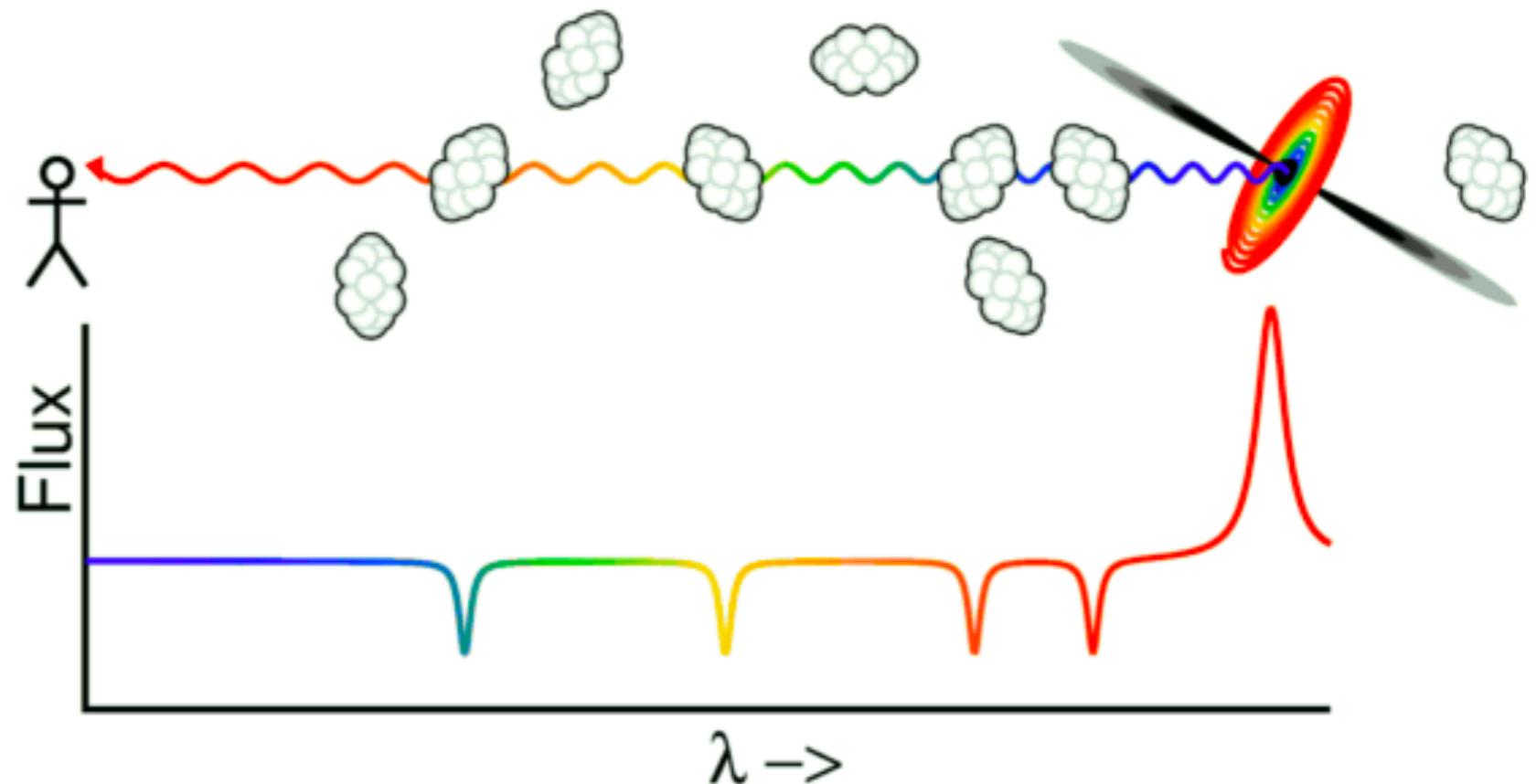
$$z \equiv \frac{\lambda_{\text{obs}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}}$$

$$\lambda_{\text{obs}} = \lambda_{\text{emit}}(1 + z)$$

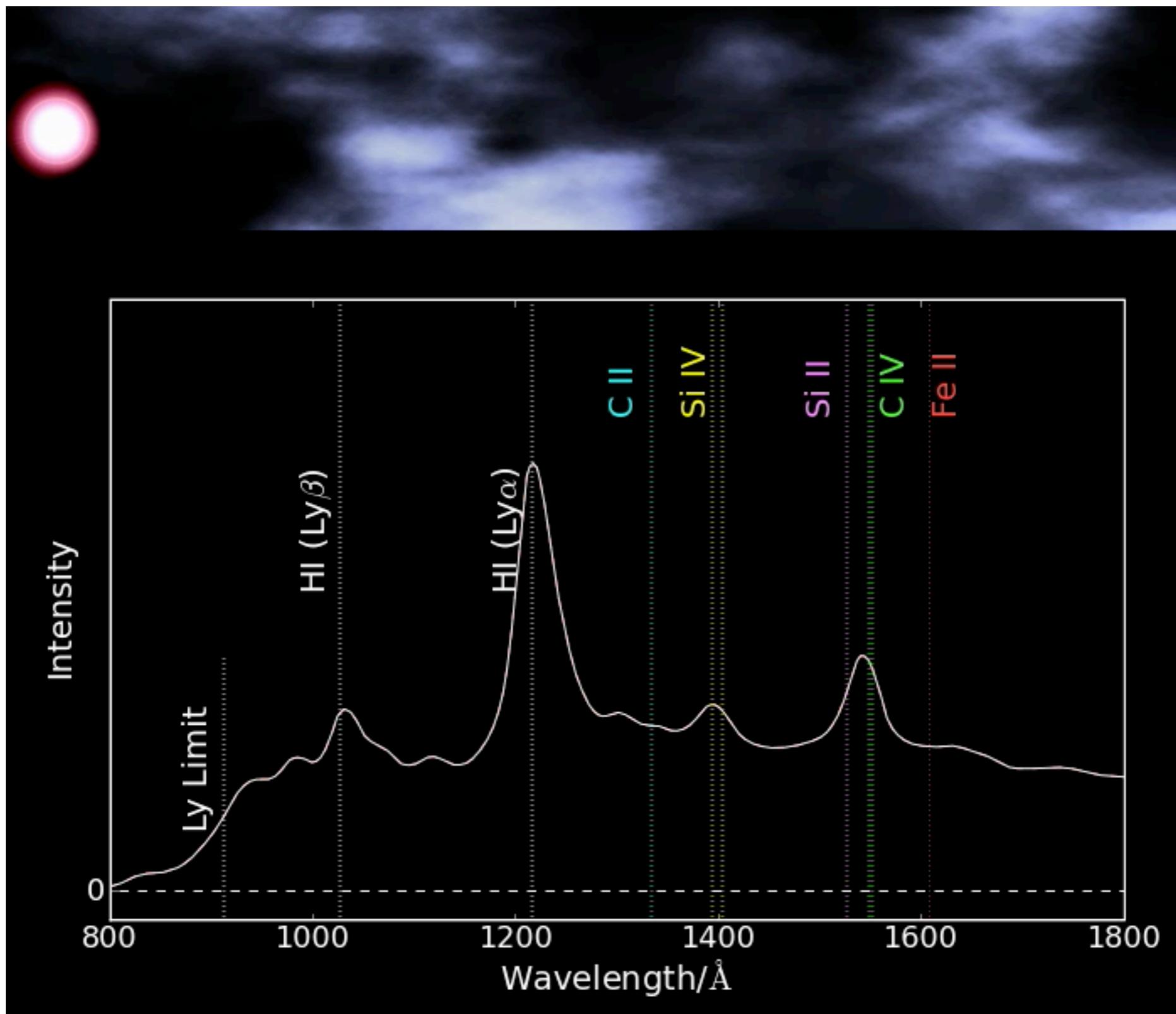
$$\lambda_{\text{obs}}^{\text{gas}} = \lambda_{\text{rest}}(1 + z_{\text{gas}})$$

$$\lambda_{\text{obs}}^{\text{quasar}} = \lambda_{\text{rest}}(1 + z_{\text{quasar}})$$

$$\therefore \lambda_{\text{obs}}^{\text{gas}} < \lambda_{\text{obs}}^{\text{quasar}}$$



A very nice visualization that shows how different systems absorb Lyman-alpha, made by Andrew Pontzen.
http://www.cosmocrunch.co.uk/media/dla_credited.mov

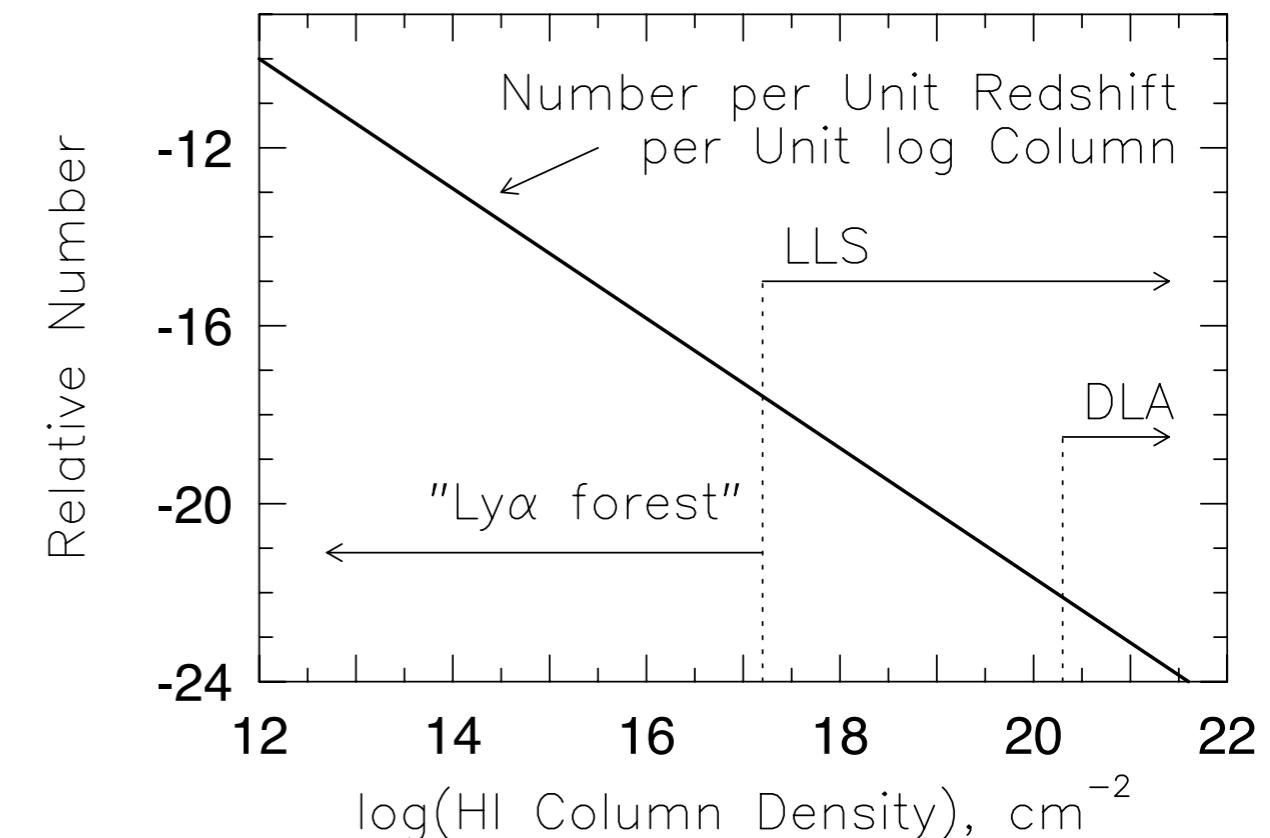


Ly α Absorption System

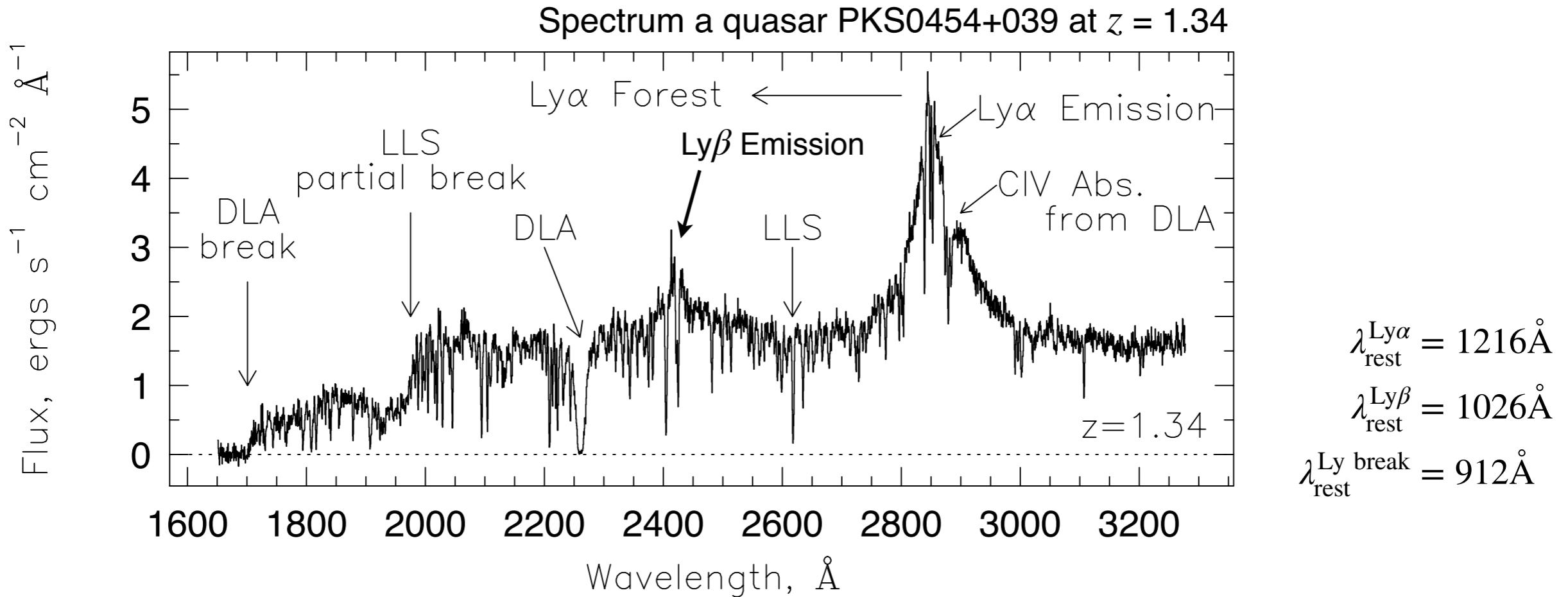
- A structure along the line of sight to the quasar can be described by its neutral Hydrogen column density $N(\text{H I})$, the product of the density of the material and the path length along the line of sight through the gas.
- Historical classification

$N(\text{H I}) < 10^{12} \text{ cm}^{-2}$	Currently not observable
$10^{12} < N(\text{H I}) < 10^{17.2} \text{ cm}^{-2}$	Ly α forest
$10^{17.2} < N(\text{H I}) < 10^{20.3} \text{ cm}^{-2}$	Lyman limit systems
$10^{20.3} < N(\text{H I})$	Damped Ly α systems

- Column density distribution
 - There are many more weak lines than strong lines.
 - The column density distribution roughly follows a power-law.



Typical spectrum of a quasar



- Typical spectrum of a quasar, showing the quasar continuum and emission lines, and the absorption lines produced by galaxies and IGM that lie between the quasar and the observer.
 - The Ly α forest, absorption produced by various intergalactic clouds, is apparent at wavelengths bluebird of the Ly α emission line.
 - The two strongest absorbers, due to galaxies, are a damped Ly α absorber at $z = 0.86$ and a Lyman limit system at $z = 1.15$.
 - The former produces a Lyman limit break at $\sim 1700 \text{\AA}$ and the latter a partial Lyman limit break at $\sim 1950 \text{\AA}$ since the neutral Hydrogen column density is not large enough for it to absorb all ionizing photons. (Note that $\lambda_{\text{rest}}^{\text{Ly break}} = 912 \text{\AA}$).

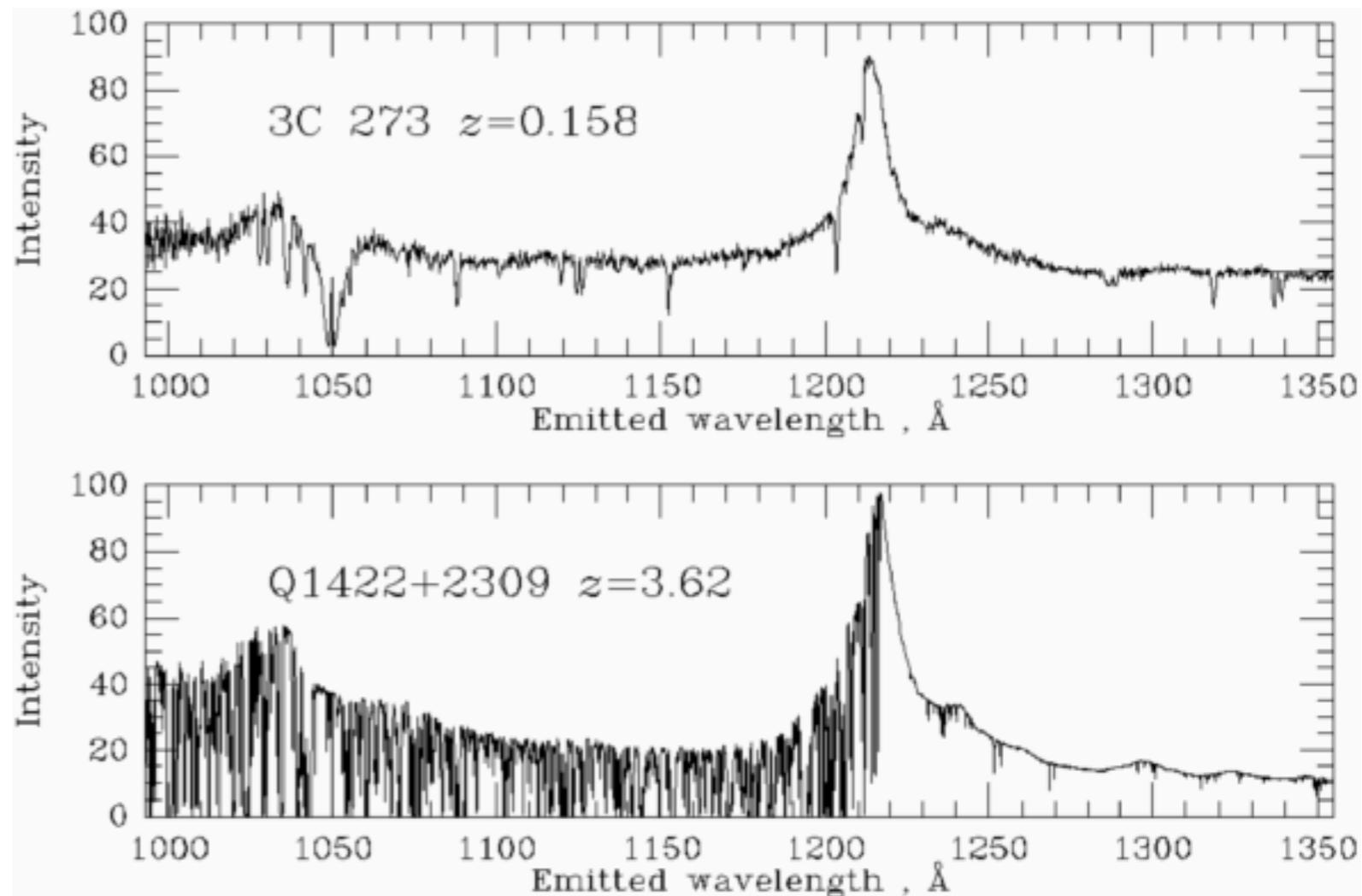
What are the Ly α absorption systems?

- They are probably hydrogen clouds and galaxies.
- At low redshift, many of the galaxies that are responsible for the DLA absorption can be directly identified. These galaxies are a heterogeneous population. They are not just the most luminous galaxies, but include dwarf and low surface brightness galaxies, and even cases where no galaxy has been identified to sensitivity limits.

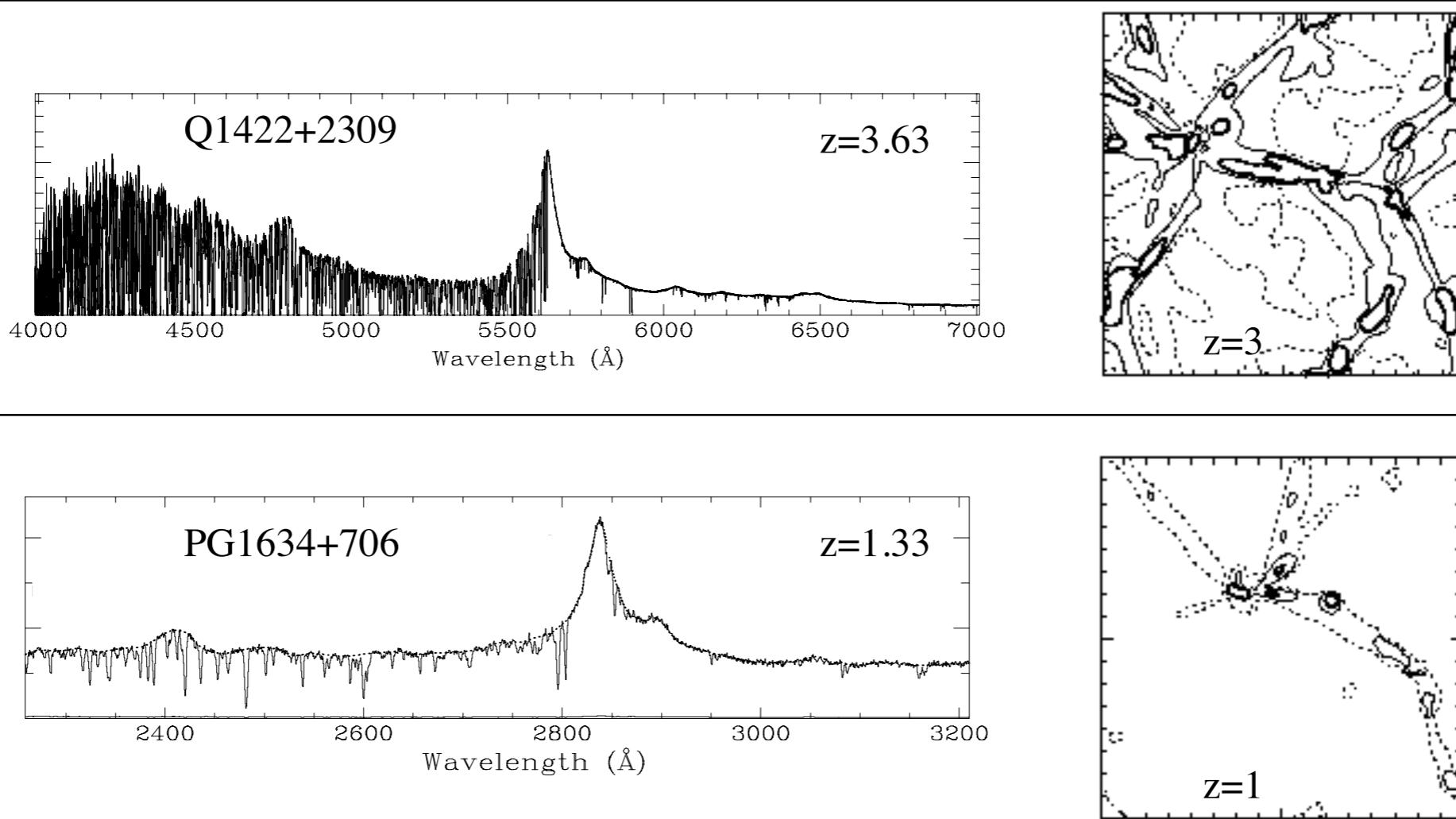
Evolution of Ly α Absorption Systems

- The Ly α absorption component evolves strongly with cosmic time.
 - We see dramatically more absorbers (be they clouds, filaments, or even crowding in velocity rather than in space) toward higher redshifts.
 - However, they have not completely disappeared at low redshifts. When the launch of HST provided the first capability of measuring Ly α at low redshifts to the required accuracy, it was found that a few of these absorbers remain in the local Universe.
 - They are generally but not specifically associated with galaxies - for instance, 3C 273 lies behind the Virgo cluster of galaxies, and has a couple of absorbers in the cluster's redshift range, but they cannot be cleanly identified in position and redshift with specific neighboring galaxies.
- The evolution of the Ly α forest may be intimately connected with the history of galaxy formation.
- This dramatic evolution in the number of forest clouds is mostly due to the expansion of the Universe, with a modest contribution from structure growth.

- This figure compares two quasars at very different redshifts, 3C 273 at $z = 0.158$ and 1422+2309 at $z= 3.62$, shifted to a common scale in emitted wavelength.



- At low redshift, 3C 273 shows only a handful Lyman alpha absorbers, including the strong and broad absorption from its light intercepting the disk of a foreground spiral galaxy (ours). Our galaxy also produces absorption in the C IV lines around 1550 Å, which appear at 1337 Å in the quasar's emitted frame.
- Hundreds of lines can be identified in the spectrum of 1422+2309, with the densest concentration near the quasar redshift - this is a very general feature showing how the density of Ly α absorbers decreases with cosmic time. The strong and broad emission peak is Ly α , which is almost chopped in half by the onset of the Ly α forest in the high-redshift quasar.



- Illustration of structure evolution of intergalactic gas from high to low redshift.
 - Higher redshift quasars show a much thicker forest of Ly α lines.
- The right-hand panels show slices through N-body/hydrodynamic simulation results at two epochs $z = 3$ and $z = 1$.
 - Three contour levels are shown : 10^{11} cm^{-2} (dotted lines), 10^{12} cm^{-2} (solid lines) and 10^{13} cm^{-2} (thick solid lines).
 - Evolution proceeds so that the voids become more empty and even lower column density material is found in filamentary structures at low redshifts.