Interstellar Medium (ISM)

Lecture 11
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Hot Ionized Medium

- Hot Gas Cooling
- Supernova Remnant
- Local Hot Bubble

General Properties of the HIM

- Hot Ionized Medium, coronal gas
  - About half the volume of the ISM in our Galaxy is occupied by the HIM.
  - Temperature ~ 10^6 K.
  - Typical ion number density n ~ 0.004 cm^{-3}
  - It provides only ~ 0.2% of the mass of the ISM, despite being the largest contributor to its volume.

  - The HIM is hot because it has been heated by shock fronts that result from supernova explosions.

  - ***We live in the "Local Bubble", which is ~ 100 pc in size. The Local Bubble is thought to have been blown by a supernova that went off ~ 10 Myr ago.***

    See https://scivis.mpia.de/projects/dustribution/Eos_Cloud/interactive.html *and*

    https://scivis.mpia.de/projects/dustribution/Eos_Cloud/video.mp4

Collisional Ionization Equilibrium

- CIE
  - CIE assumes that the plasma is in a steady state, and that *collisional ionization, charge exchange, radiative recombination, and dielectronic recombination* are the only processes altering the ionization balance.
    - Note that the reverse process to collisional ionization is a three-body recombination, which is unlikely to occur.
  - **The ionization fractions for each element depend only on the gas temperature**, with no dependence on the gas density.
- Ionization fraction
  - For hydrogen, the balance equation is : ionization rate = recombination rate

$$n_e n(\mathrm{H}^0) k_{\mathrm{ci,H}} = n_e n(\mathrm{H}^+) \alpha_{\mathrm{A,H}} \qquad\qquad n(\mathrm{H}^0) + n(\mathrm{H}^+) = n(\mathrm{H})$$

  - The rate coefficients for collisional ionization and radiative recombination are:

$$k_{\mathrm{ci,H}} = 5.849 \times 10^{-9} \, T_4^{1/2} e^{-15.782/T_4} \, [\mathrm{cm}^3 \, \mathrm{s}^{-1}]$$

$$\alpha_{\mathrm{A,H}} = 4.13 \times 10^{-13} \, T_4^{-0.7131 - 0.0115 \ln T_4} \, [\mathrm{cm}^3 \, \mathrm{s}^{-1}] \quad \text{for} \quad 30\,\mathrm{K} < T < 3 \times 10^4\,\mathrm{K}$$

$$= 5 \times 10^{-16} T_7^{-1.5} \quad \text{for} \quad T > 10^6\,\mathrm{K}$$

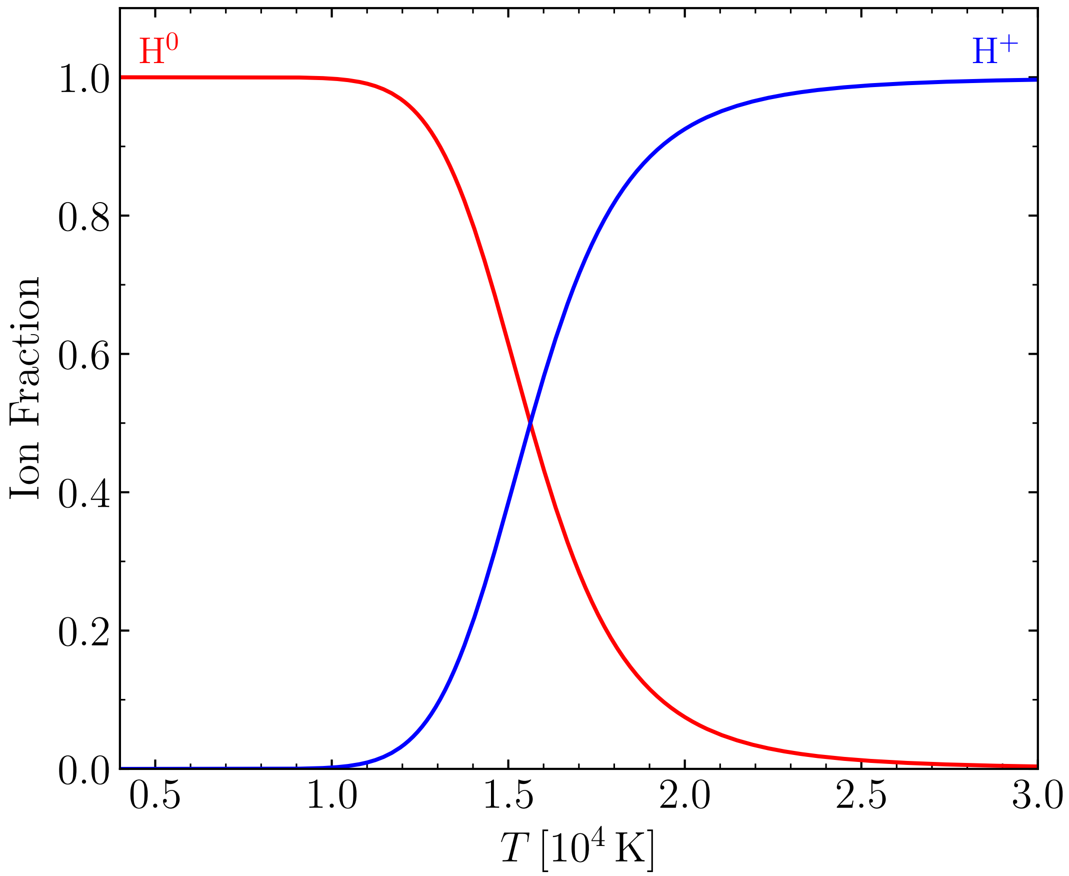
[from Draine]

$$\alpha_{\mathrm{A,H}} = 1.269 \times 10^{-13} \, [\mathrm{cm}^3 \, \mathrm{s}^{-1}] \, \frac{x^{1.503}}{(1 + (x/0.522)^{0.47})^{1.923}} \quad \text{where } x = 2 \times 157807\,\mathrm{K}/T$$

[Hui & Gendin 1997, MNRAS]

- The ionization fraction is

$$x \equiv \frac{n(\mathrm{H}^+)}{n(\mathrm{H}^0) + n(\mathrm{H}^+)}$$
$$= \frac{k_{\mathrm{ci,H}}}{k_{\mathrm{ci,H}} + \alpha_{\mathrm{A,H}}}$$



- The ion fractions are

$$x \approx 0.002 \quad \text{at} \quad T = 10^4\,\mathrm{K}$$
$$1 - x \approx 3 \times 10^{-7} \quad \text{at} \quad T = 10^6\,\mathrm{K}$$

H II regions with $T = 10^4$ K are photoionized by UV photons from hot stars.

Hydrogen gas with $T = 10^6$ K is almost entirely collisionally ionized.

- For Helium, the balance equations are:

$$n(\mathrm{He}^+)\alpha_{10} = n(\mathrm{He}^0)k_{01}$$
$$n(\mathrm{He}^+)k_{12} = n(\mathrm{He}^{2+})\alpha_{21}$$
$$n(\mathrm{He}) = n(\mathrm{He}^0) + n(\mathrm{He}^+) + n(\mathrm{He}^{2+})$$

Here, ij indicates $X^{i+} \rightarrow X^{j+}$.

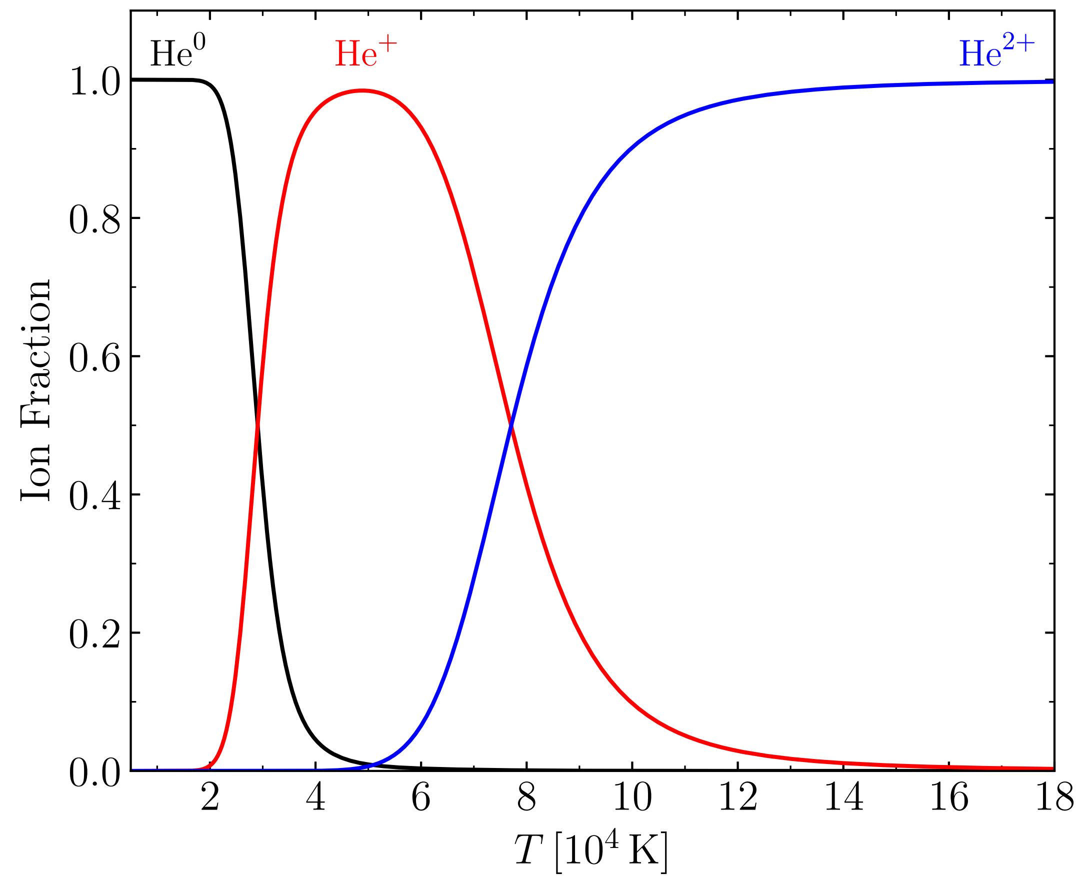
- The rate coefficients are

from R. Cen (1992, ApJS)

$$k_{01} = 2.39 \times 10^{-11}\, T^{1/2} e^{-285,335/T}$$
$$k_{12} = 5.68 \times 10^{-12}\, T^{1/2} e^{-631,515/T}$$
$$\alpha_{10} = 1.50 \times 10^{-10}\, T^{-0.6353}$$ radiative recombination
$$+\, 1.9 \times 10^{-3} T^{-1.5} e^{-470,000/T} (1 + 0.3 e^{-94,000/T})$$ dielectronic recombination (but not significant)
$$\alpha_{21} = 3.36 \times 10^{-10}\, T^{-1/2} T_3^{-0.2} / (1 + T_6^{0.7})$$

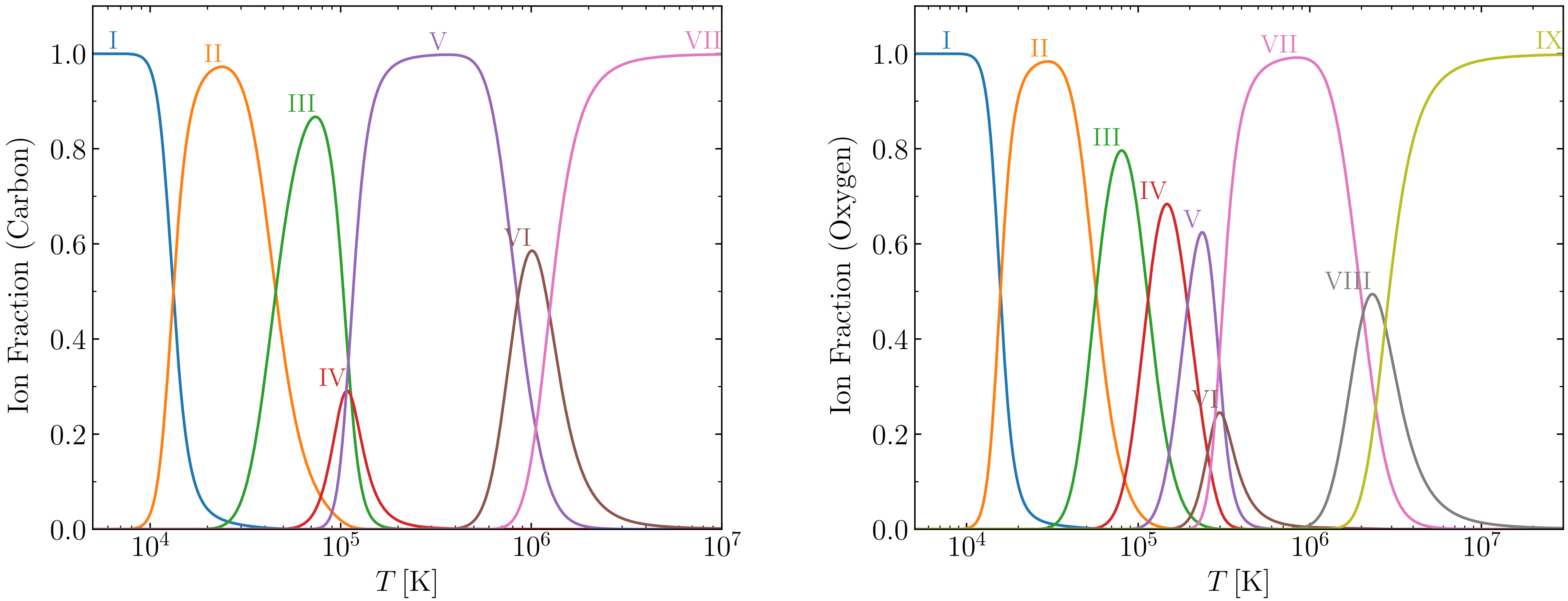
- Using the above rate coefficients, the ionization fractions can be estimated as follows:

$$x \equiv \frac{n(\mathrm{He}^+)}{n(\mathrm{He})} = \frac{1}{1 + \alpha_{10}/k_{01} + k_{12}/\alpha_{21}}$$
$$y \equiv \frac{n(\mathrm{He}^{2+})}{n(\mathrm{He})} = \frac{k_{12}}{\alpha_{21}} x$$



- Heavy Elements
  - The calculation is usually done numerically, for instance, using CHIANTI

    CHIANTI: https://www.chiantidatabase.org/
  - For instance, the ion fractions of carbon and oxygen as a function of temperature are:



- At $T \sim 10^6\,\mathrm{K}$, we expect a mix of C V, C VI, and C VII.
- At $T \sim 4 \times 10^6\,\mathrm{K}$ and higher, almost all the carbon will be in the form of fully ionized C VII.

The results were calculated using CHIANTI.

Cooling in CIE

- ***Cooling function***
  - At temperature $T > 10^4\,\mathrm{K}$, ionization of hydrogen provides enough free electrons so that collisional excitation of atoms or ions is dominated by electron collisions.
  - At low densities, every collisional excitation is followed by a radiative decay, and ***the rate of removal of thermal energy per unit volume*** can be written:

$$\boxed{\Lambda = n_e n_{\mathrm{H}} f_{\mathrm{cool}}(T)}$$

  The ***radiative cooling function*** $f_{\mathrm{cool}}(T) \equiv \Lambda / n_{\mathrm{H}} n_e$ is a function of ***temperature*** and of the ***elemental abundances*** relative to hydrogen.

  - *At high densities*, radiative cooling can be suppressed by collisional deexcitation, and the cooling function will then depend on density, in addition to T and elemental abundances.
  - *If ionizing radiation is present*, the ionization balance may depart from CIE, and the radiative cooling function will also depend on the spectrum and intensity of the ionizing radiation.

Fig 34.1
Draine

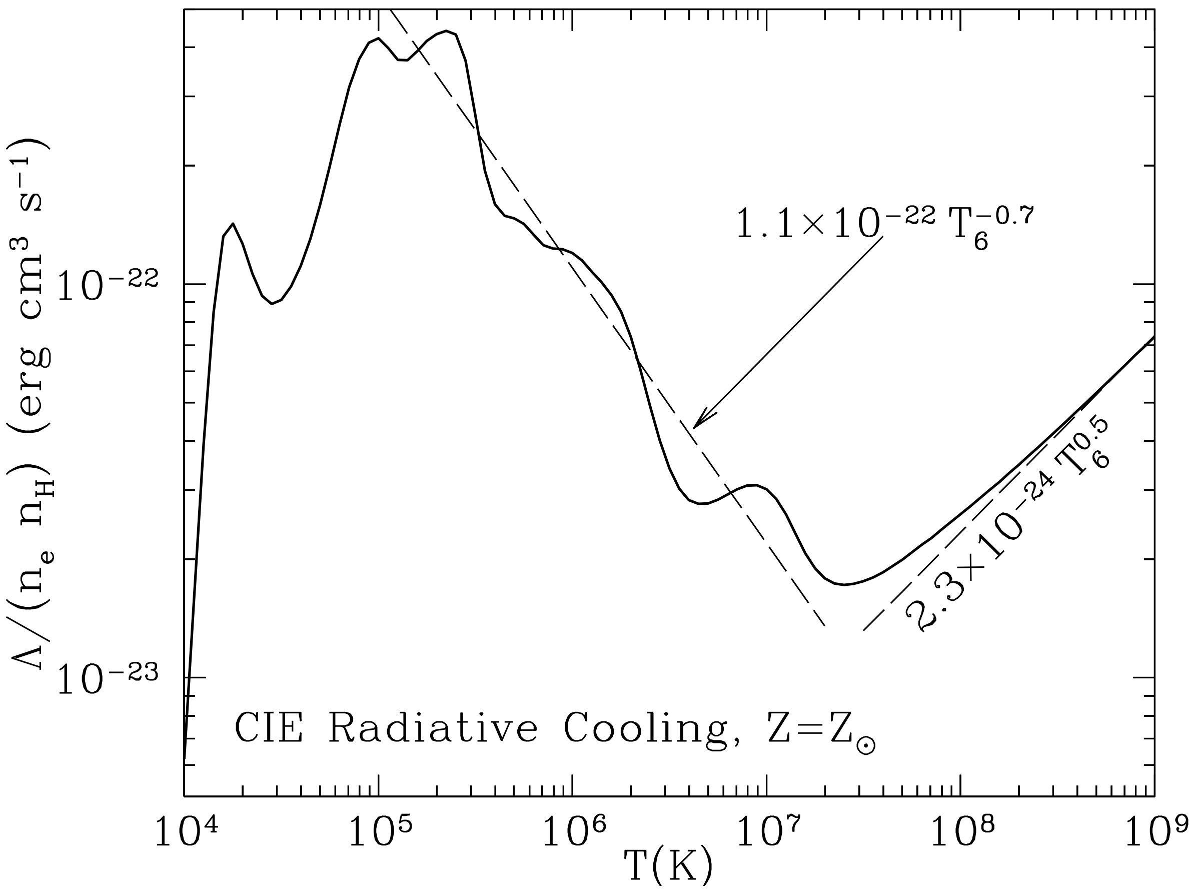
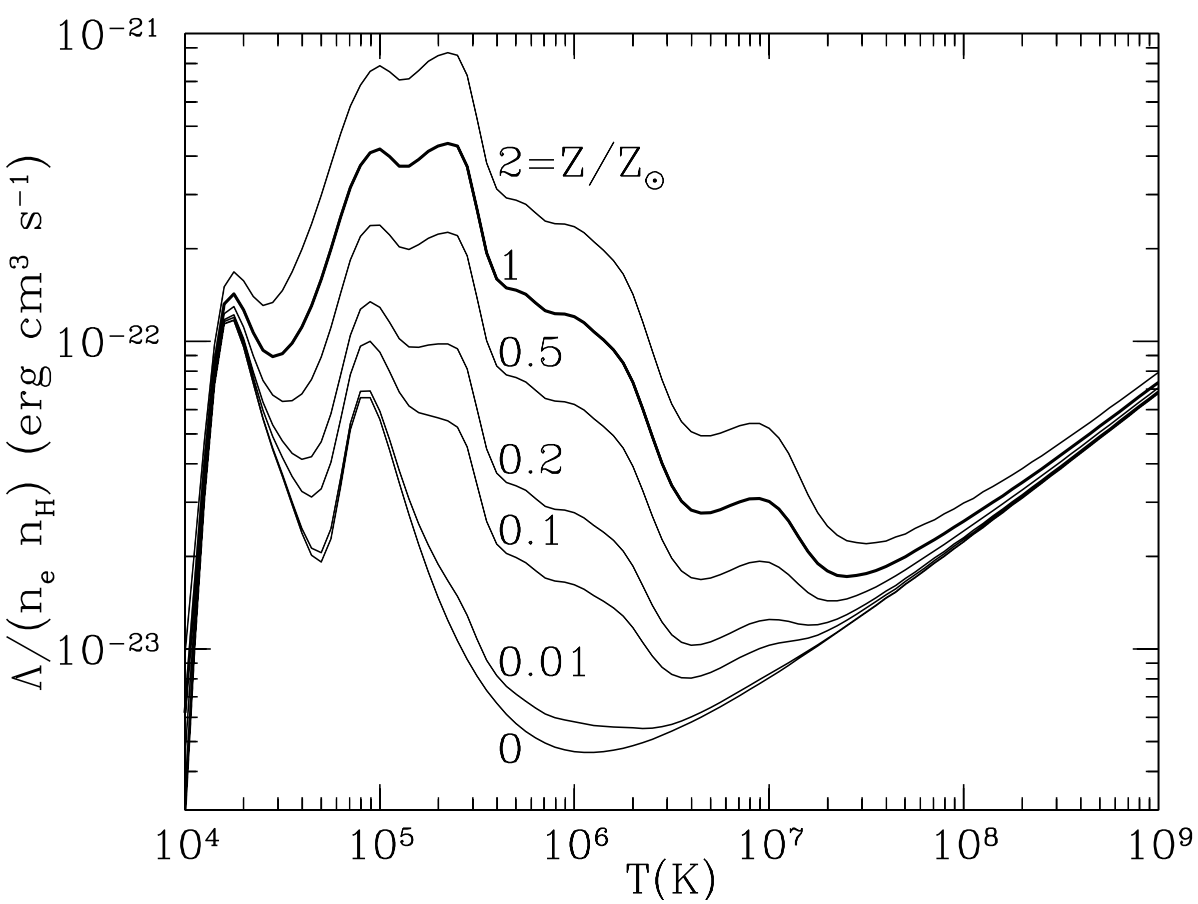


Fig 34.2
Draine



Radiative Cooling Function for solar-abundance

- At $T < 10^7\,\mathrm{K}$, the cooling is dominated by collisional excitation of bound electrons.
- At high temperatures, the ions are fully stripped of electrons, and bremsstrahlung (free-free) cooling dominates.

$$\Lambda/n_e n_{\rm H} \approx 1.1 \times 10^{-22}\, T_6^{-0.7}\, [\mathrm{erg\,cm^3\,s^{-1}}]$$
$$(10^5 < T < 10^{7.3}\,\mathrm{K})$$

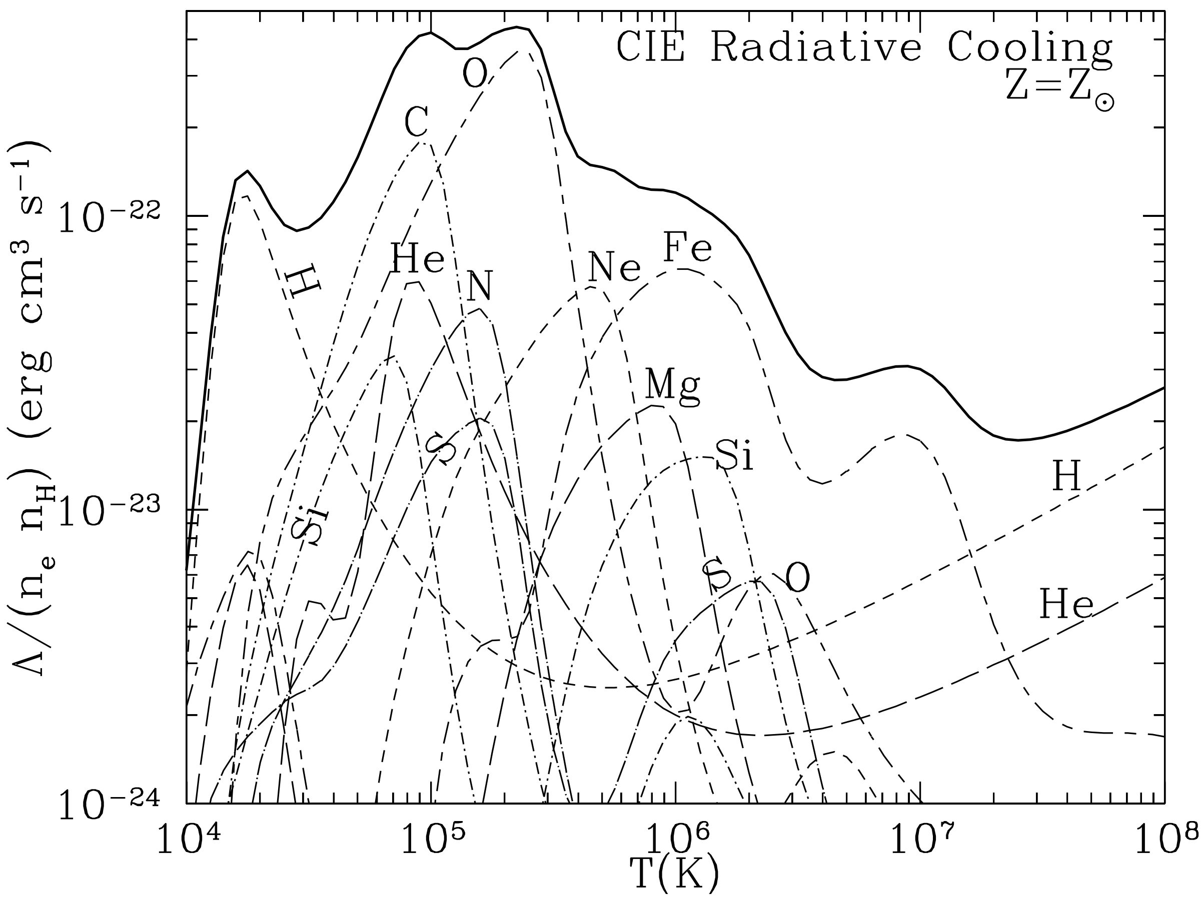
$$\Lambda/n_e n_{\rm H} \approx 2.3 \times 10^{-24}\, T_6^{0.5}\, [\mathrm{erg\,cm^3\,s^{-1}}]$$
$$(T > 10^{7.3}\,\mathrm{K})$$

Cooling Function for different abundances

- In most applications, the abundances of elements beyond He can be assumed to be scale up and down together.

The cooling functions were calculated using CHIANTI.

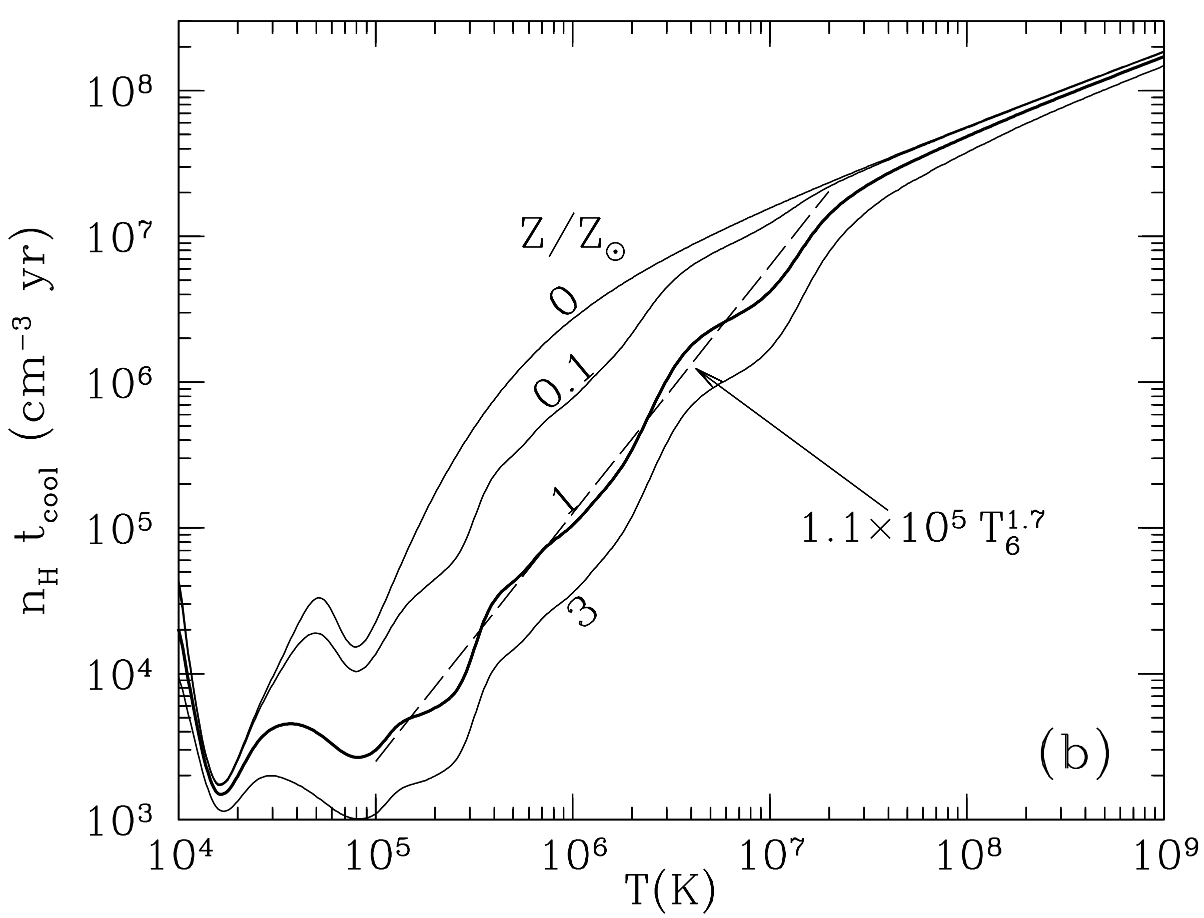
Fig 34.3
Draine



Radiative Cooling Function, with contributions from selected elements. In this calculation, the solar abundance is assumed.

- At $10^4\,\mathrm{K} < T < 3\times10^4\,\mathrm{K}$, cooling occurs mainly by Lyα emission from collisionally excited H atoms.
- At $3\times10^4\,\mathrm{K} < T < 2\times10^7\,\mathrm{K}$, cooling occurs mainly by permitted UV lines from collisionally excited heavy ions.
- For $10^{5.8}\,\mathrm{K} < T < 10^{7.2}\,\mathrm{K}$, the cooling is dominated by Mg, Si, and Fe - elements that in cold gas are normally depleted by factors of 5 or more.

Fig 34.4
Draine



Cooling time
(using the formula given in next slides)

$$t_{\rm cool} \approx 1.1\times10^5\,T_6^{1.7}\,(n_{\rm H}/{\rm cm}^{-3})^{-1}\,[{\rm yr}]$$
$$(10^5 \lesssim T \lesssim 10^{7.3}\,{\rm K})$$

for isochoric cooling (constant density)

Cooling Time Scale [isobaric / isochoric]

- Cooling Time scale for two important cases:
  - The **first law of thermodynamics** states that:

Heat added in a system: $$\boxed{dQ = dU + PdV}$$

  - Using a heating and cooling rate per volume Γ and Λ , the change in heat is

$$dQ = (\Gamma - \Lambda)V\,dt$$

  - The change in the internal energy is

$$dU = (\Gamma - \Lambda)V\,dt - PdV$$

  - When there is no external heating ($\Gamma = 0$), the equation for an ideal gas with a degree of freedom f becomes:

$$U = \frac{f}{2}NkT \qquad PV = NkT \longrightarrow \boxed{d\left(\frac{f}{2}NkT\right) = -\Lambda V\,dt - PdV}$$

- Consider the case of constant pressure or constant volume:

$$PdV = d(PV) - VdP = d(NkT)$$ for ***isobaric cooling (constant pressure)***

$$PdV = 0$$ for ***isochoric cooling (constant density or volume)***

Therefore,

$$\frac{d}{dt}\left(\frac{f+2}{2}NkT\right) = -\Lambda V$$ for ***isobaric cooling (constant pressure)***

$$\frac{d}{dt}\left(\frac{f}{2}NkT\right) = -\Lambda V$$ for ***isochoric cooling (constant density or volume)***

The cooling time scale are then:

$$t_{\rm cool} \equiv \frac{T}{|dT/dt|} \quad \Rightarrow \quad t_{\rm cool} = \frac{f+2}{2}\frac{nkT}{\Lambda}$$ for isobaric cooling

$$n \equiv N/V \qquad\qquad = \frac{f}{2}\frac{nkT}{\Lambda}$$ for isochoric cooling

Here, the number density includes all particles (molecules, atoms, ions, electrons)

Time Scales in the HIM

- ***Cooling time scale***:
  - In the HIM with temperatures $T \sim 10^6 - 10^7\,\mathrm{K}$, the cooling time scale is:

$$\boxed{t_{\rm cool} = \frac{5}{2}\frac{nkT}{\Lambda} \qquad n_e \approx 1.2 n_{\rm H} \quad n \approx 2.3 n_{\rm H}}$$

For fully ionized gas,
one electron from an ionized hydrogen
two electrons from a doubly-ionized helium.

  - The cooling time at $T \sim 10^6\,\mathrm{K}$ is

$$t_{\rm cool} = \frac{5}{2}\frac{2.3}{1.2}\frac{kT}{\Lambda/(n_e n_{\rm H})}\frac{1}{n_{\rm H}}$$

$$t_{\rm cool} \approx 48\,[\mathrm{Myr}]\; T_6^{1.7} \left(\frac{n_{\rm H}}{0.004\,\mathrm{cm^{-3}}}\right)^{-1}$$
$$\approx 0.19\,[\mathrm{Myr}]\; T_6^{1.7} \left(\frac{n_{\rm H}}{1\,\mathrm{cm^{-3}}}\right)^{-1}$$

$$\longleftarrow \quad \Lambda/n_e n_{\rm H} \approx 1.1 \times 10^{-22}\, T_6^{-0.7}\,[\mathrm{erg\,cm^3\,s^{-1}}]$$

The HIM frequently doesn't have time to cool thoroughly before another supernova shock wave comes through to heat it again.

  - At $T \sim 10^7\,\mathrm{K}$, the cooling time is

$$t_{\rm cool} \approx 7.2\,[\mathrm{Gyr}]\; T_7^{1/2} \left(\frac{n_{\rm H}}{0.004\,\mathrm{cm^{-3}}}\right)^{-1}$$
$$\approx 29\,[\mathrm{Myr}]\; T_7^{1/2} \left(\frac{n_{\rm H}}{1\,\mathrm{cm^{-3}}}\right)^{-1}$$

$$\longleftarrow \quad \Lambda/n_e n_{\rm H} \approx 2.3 \times 10^{-24}\, T_6^{0.5}\,[\mathrm{erg\,cm^3\,s^{-1}}]$$

Given the low density of the HIM, the cooling time of gas is comparable to the age of our galaxy (~13 Gyr, only after 0.8 Gyr after the Big Bang; Xiang & Rix, 2020, Nature, 603, 599).

- ***Recombination and Ionization Time scale***:
    - If collisional ionization could somehow be turned off, the recombination time scale is

$$t_{\rm rec} = \frac{1}{n_{\rm e}\alpha_{\rm A,H}} \approx 0.6\,[{\rm Gyr}]\left(\frac{n_{\rm H}}{0.004\,{\rm cm^{-3}}}\right)^{-1} \qquad [T \approx 10^6\,{\rm K}] \qquad \alpha_{\rm A,H} \approx 1.5\times10^{-14}\,[{\rm cm^3\,s^{-1}}]$$

    - If collisional ionization could suddenly switch on, the collisional ionization time scale is

$$t_{\rm ci} = \frac{1}{n_{\rm e}k_{\rm ci}} \approx 160\,[{\rm yr}]\left(\frac{n_{\rm H}}{0.004\,{\rm cm^{-3}}}\right)^{-1} \qquad [T \approx 10^6\,{\rm K}] \qquad k_{\rm ci,H} \approx 5.0\times10^{-8}\,[{\rm cm^3\,s^{-1}}]$$

    - These times scales indicates that
        - If cold neutral hydrogen gas is shock-heated to ~ 10^6 K in a time $t_{\rm heat} \ll t_{\rm ci}$, it will take a time $t \sim t_{\rm ci}$ for hydrogen to become ionized. ***During this time interval, the hydrogen will be out of collisional ionization equilibrium (under-ionized than in CIE).***
        - If highly ionized gas at ~ 10^6 K is cooled on a timescale $t_{\rm cool} \ll t_{\rm rec}$, and the heating source is turned off, it will take a time $t \sim t_{\rm rec}$ for the hydrogen to recombine. ***During the intervening time, the gas will be out of CIE (over-ionized than in CIE).*** This is sometimes called ***"delayed recombination"***.

- If a gradually cooling gas of the HIM to be remained in CIE, we require $t_{\rm rec} < t_{\rm cool}$.

  Assuming the recombination rate coefficient at high temperatures

$$\alpha_{\rm A,H} \approx 5 \times 10^{-16}\,{\rm cm^3\,s^{-1}}\,\left(T/10^7\,{\rm K}\right)^{-1.5}$$

- At T ~ 10^6 K,

$$t_{\rm rec} = \frac{1}{n_e \alpha_{\rm A,H}} \approx 0.5\,[{\rm Gyr}]\,\left(T/10^6\,{\rm K}\right)^{1.5}\left(n_{\rm H}/0.004\,{\rm cm^{-3}}\right)^{-1}$$

$$t_{\rm cool} \approx 48\,[{\rm Myr}]\,T_6^{1.7}\left(n_{\rm H}/0.004\,{\rm cm^{-3}}\right)^{-1}$$

$$\Rightarrow \quad \frac{t_{\rm rec}}{t_{\rm cool}} \approx 10\left(T/10^6\,{\rm K}\right)^{-0.2}$$

- At T ~ 10^7 K,

$$t_{\rm rec} = \frac{1}{n_e \alpha_{\rm A,H}} \approx 16\,[{\rm Gyr}]\,\left(T/10^7\,{\rm K}\right)^{1.5}\left(n_{\rm H}/0.004\,{\rm cm^{-3}}\right)^{-1}$$

$$t_{\rm cool} \approx 7.2\,[{\rm Gyr}]\,T_7^{0.5}\left(n_{\rm H}/0.004\,{\rm cm^{-3}}\right)^{-1}$$

$$\Rightarrow \quad \frac{t_{\rm rec}}{t_{\rm cool}} \approx 2.2\,(T/10^7\,{\rm K})$$

$$t_{\rm rec}(T \approx 10^6\,{\rm K}) \ll t_{\rm rec}(T \approx 10^7\,{\rm K}), \quad t_{\rm cool}(T \approx 10^6\,{\rm K}) \ll t_{\rm cool}(T \approx 10^7\,{\rm K})$$

Therefore, ***in the extremely hot regions, the hotter the gas is, the further away it is from CIE.***

Cooling in Shocked Gas

- ***The hot shocked gas is out of equilibrium***, and will start to cool. Thus, the shock will be followed by a radiative zone in which the shock heated gas cools down by radiating away photons.
  - At high temperatures $T > 2 \times 10^7\,\mathrm{K}$
    - The cooling is dominated by bremsstrahlung (free-free radiation), for which the *specific* cooling rate (per mass) is

$$\mathcal{L} = 2.7\,[\mathrm{erg\,g^{-1}\,s^{-1}}] \left(\frac{T}{10^7\,\mathrm{K}}\right)^{1/2} \left(\frac{n_\mathrm{H}}{1\,\mathrm{cm^{-3}}}\right)$$

assuming a gas of fully ionized hydrogen. The *specific* internal energy of ionized hydrogen (per mass) is

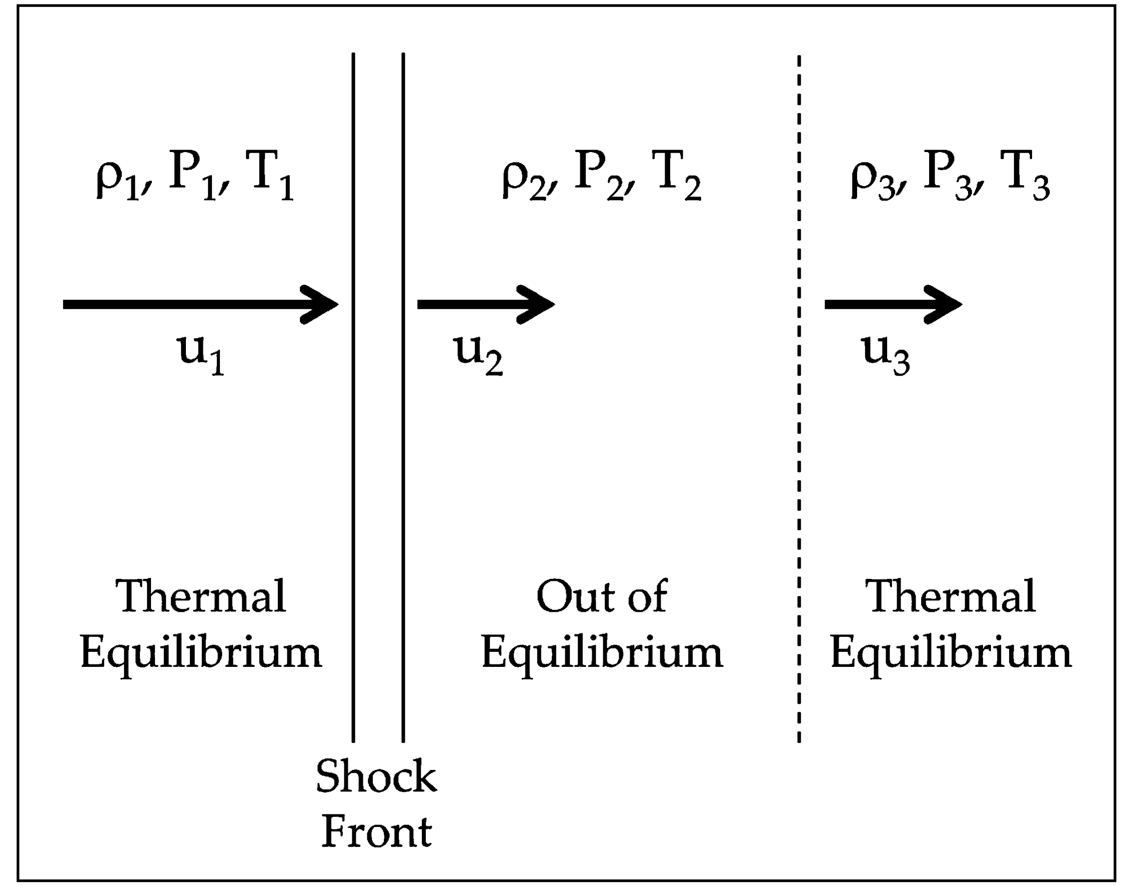
$$\epsilon = \frac{3}{2}\frac{(2n_\mathrm{H})kT}{n_\mathrm{H} m_\mathrm{H}} \approx 2.5 \times 10^{15}\,[\mathrm{erg\,g^{-1}}] \left(\frac{T}{10^7\,\mathrm{K}}\right)$$

    - Then, the bremsstrahlung cooling time is

$$t_\mathrm{cool} = \frac{\epsilon}{\mathcal{L}} \approx 29\,[\mathrm{Myr}] \left(\frac{T}{10^7\,\mathrm{K}}\right)^{1/2} \left(\frac{n_\mathrm{H}}{1\,\mathrm{cm^{-3}}}\right)^{-1}$$

$$\approx 34\,[\mathrm{Myr}] \left(\frac{V_\mathrm{s}}{1000\,\mathrm{km\,s^{-1}}}\right) \left(\frac{n_\mathrm{H}}{1\,\mathrm{cm^{-3}}}\right)^{-1}$$

$$T \simeq \frac{3}{16}\frac{m}{k}V_s^2$$



The structure of a plane parallel radiative shock [Figure 5.3 Ryden]

- During this time, the gas will move a distance, relative to the shock front:

$$R_{\rm cool} \approx u_2 t_{\rm cool} \approx \frac{u_1}{4} t_{\rm cool}$$
$$\approx 8.7\,[{\rm kpc}] \left(\frac{V_s}{1000\,{\rm km\,s^{-1}}}\right)^2 \left(\frac{n_{\rm H}}{1\,{\rm cm^{-3}}}\right)^{-1}$$

  This implies that ***the approximate thickness of the radiative zone for a strong shock is a long distance compared to the scale height of the ISM in our galaxy***. Thus, the hot gas produced by high-speed shocks doesn't have time to cool before the shock runs out of gas to shock.
  << No cooling of the shock gas in our galaxy >>

- At lower temperature $(10^5\,{\rm K} < T < 2\times 10^7\,{\rm K})$, corresponding to slower shock speeds $(80\,{\rm km\,s^{-1}} < u_1 = V_s < 1200\,{\rm km\,s^{-1}})$

  - The collisionally excited lines do most of the cooling. A useful approximation for the cooling rate gives

$$t_{\rm cool} \approx 6600\,[{\rm yr}] \left(\frac{V_s}{100\,{\rm km\,s^{-1}}}\right)^{3.4} \left(\frac{n_{\rm H}}{1\,{\rm cm^{-3}}}\right)^{-1}$$

  - This yields a thickness for the radiative zone.

$$R_{\rm cool} \approx \frac{V_s}{4} t_{\rm cool} = 0.17\,[{\rm pc}] \left(\frac{V_s}{100\,{\rm km\,s^{-1}}}\right)^{4.4} \left(\frac{n_{\rm H}}{1\,{\rm cm^{-3}}}\right)^{-1}$$

  - These shorter time scales and length scales mean that ***radiative cooling is more effective at changing the structure of slower shocks.***

Effects of Supernovae on the ISM

- Consider the simplest case of a spherically symmetric explosion of a star in a uniform density and temperature.
- ***Free-Expansion Phase***
  - Typical supernovae (SNe) explosion ejects a kinetic energy of

$$E_{51} \equiv E_0/(10^{51}\,\mathrm{erg}) \approx 1$$

$M_{\rm ej} \sim 10\,M_\odot$ for core collapse supernovae (Type II, Type Ib, and Type Ic), powered by the gravitational collapse of the dense core of an evolved massive star.
$M_{\rm ej} \sim 1\,M_\odot$ for the thermonuclear explosion of a white dwarf (Type Ia)

An explosion produces an energy $\sim 10^{53}\,\mathrm{erg}$ in neutrinos and $\sim 10^{49}$ erg in photons. These energies are not effectively deposited in the ISM.

  - The ejecta mass ranges from $M_{\rm ej} \sim 1.4\,M_\odot$ (Type Ia, white dwarf) to $M_{\rm ej} \sim 10 - 20\,M_\odot$ (Type II, core collapse of massive stars).
  - The ejecta will have a root-mean-square (rms) velocity of

$$v_{\rm ej} = \left(\frac{2E_0}{M_{\rm ej}}\right)^{1/2} = 1.00 \times 10^4\,\mathrm{km\,s^{-1}}\,E_{51}^{1/2}\left(\frac{M_{\rm ej}}{M_\odot}\right)^{-1/2}$$

$$\mathcal{M}_1 = \frac{v_{\rm ej}}{c_1} \approx \frac{1\times 10^4\,\mathrm{km/s}}{1\,\mathrm{km/s}} \gg 1$$

    - This velocity is far greater than the sound speed in the surrounding material, and the expanding ejecta will drive a fast shock into the circumstellar medium.
  - All of the matter interior to this shock surface is referred to as the ***supernova remnant*** (SNR).
  - **The density of the ejecta far exceeds the density of the circumstellar medium**, and the ejecta continue to **expand ballistically at nearly constant velocity**. — this is referred to as the “free expansion phase.” The ejecta are not slowed down significantly until they have swept up a mass of interstellar gas comparable to the mass of the ejecta.

- At the early times of the free-expanding phase, there is only one shock, which propagating outward into the ambient medium.
- *As the ejecta expands, its density drops and less hot, thanks to adiabatic cooling. Then, the pressure ($P_2 \approx \mathscr{M}_1^2 P_1$) of the shocked circumstellar/interstellar medium soon exceeds the thermal pressure in the ejecta.*

$$\rho_{\rm ej} = \frac{M_{\rm ej}}{(4\pi/3)R^3} = \frac{M_{\rm ej}}{(4\pi/3)(v_{\rm ej}t)^3} \propto t^{-3}$$

  - As the pressure in the ejecta drops, a reverse shock is driven into the ejecta. The SNR now contains two shocks: the ***original outward-propagating shock*** (the blastwave) and the ***reverse shock propagating inward***, slowing and shock-heating the ejecta (which had previously been cooled adiabatic expansion).
- ***End of the free-expanding phase***: ***The reverse shock becomes important when the expanding ejecta material has swept up a mass of circumstellar or interstellar matter comparable to the ejecta mass.***
  - The radius of the blastwave and the time when this occurs are:

$$R_1 = \left(\frac{M_{\rm ej}}{(4\pi/3)\rho_0}\right)^{1/3} = 5.88\times10^{18}\,{\rm cm}\left(\frac{M_{\rm ej}}{M_\odot}\right)^{1/3} n_0^{-1/3}$$

$$t_1 \approx \frac{R_1}{v_{\rm ej}} = 186\,{\rm yr}\left(\frac{M_{\rm ej}}{M_\odot}\right)^{5/6} E_{51}^{-1/2} n_0^{-1/3}$$

$$R_1 = 0.525\,{\rm pc}\,(M_{\rm ej}/M_\odot)^{1/3}\, n_0^{-1/3}$$

$\rho_0 \simeq 1.4 m_{\rm p} n_0 \;\rightarrow\;$ density of the ambient (neutral) medium

$n_0 \rightarrow$ number density of hydrogen in the ambient medium

  - The free-expansion phase applies only for $t \lesssim t_1$.

- ***Sedov-Taylor Phase (Energy-Conserving Phase, Adiabatic Phase)***
  - For $t \gtrsim t_1$, the reverse shock has reached the center of the remnant, all of the ejecta are now very hot, and the free-expansion phase is over.
    - The pressure in the SNR is far higher than the pressure in the surrounding medium.
    - The hot gas may have been emitting radiation, but if the density is low, the radiative losses at early times are negligible.
    - We can idealize the explosion as a large amount of energy released instantaneously at a point **(blast wave)**, in the form of kinetic energy plus radiation.
  - The mass of shocked gas is much greater than the ejecta mass. The pressure P_2 behind the shock is much greater than the pressure P_1 of the surrounding gas. Hence, the properties of the shock front no longer depend on the ejecta mass and the surrounding gas pressure. The shock expands as if it were traveling into a pressureless medium.
  - The SNR now enters a phase where
    - We can neglect (1) the mass of the ejecta ($M_{\rm ej} \ll (4\pi/3)R^3\rho_0$), (2) radiative losses, and (3) the pressure in the ambient medium.
    - ***The shock front properties depend only on the energy of explosion E and the density ρ_0 of the ambient medium.***
    - This phase can be approximated by idealizing the problem as ***<u>a "point explosion" injecting only energy E_0 (zero ejecta mass) into a uniform-density zero-temperature medium of density ρ_0, as in nuclear explosion.</u>***

- ***Self-similar Solution***: In this phase, only the scale of the pressure and the length scale evolve with time, but the shape of the pressure and density as a function of position remains unaltered. These motions are called self-similar.

- The evolution during this phase is determined only by ***the energy of explosion*** E_0, ***the density of interstellar gas*** ρ_0, ***and the elapsed time from the explosion t***.

  We do simple dimensional analysis to find out the form of the time evolution of the remnant. Let the explosion occur at $t = 0$ and the radius of the shock front be R_s. Suppose that

E = [mass]x[length]^2/[time]^2
ρ = [mass]/[length]^3

$$\boxed{R_s = AE^\alpha \rho_0^\beta t^\eta \qquad (A = \text{a dimensionless constant})}$$

- By equating the powers of mass, length, and time, we obtain

$$\begin{aligned} \text{mass} : 0 &= \alpha + \beta \\ \text{length} : 1 &= 2\alpha - 3\beta \\ \text{time} : 0 &= -2\alpha + \eta \end{aligned} \quad \longrightarrow \quad \alpha = 1/5,\ \beta = -1/5,\ \eta = 2/5$$

- Then, the solution for the shock-front radius is given by

$$\boxed{R_s = A\left(\frac{Et^2}{\rho_0}\right)^{1/5} \qquad A = 1.15167 \text{ for } \gamma = 5/3}$$

for a monatomic gas.
This value is obtained from the exact solution.

- The solution gives ***the radius and velocity of the shock front***, and ***temperature of the post-shock gas:***

$$R_s = 1.54 \times 10^{19}\,[\mathrm{cm}]\,E_{51}^{1/5} n_0^{-1/5} t_3^{2/5}$$
$$V_s = 1950\,[\mathrm{km\,s^{-1}}]\,E_{51}^{1/5} n_0^{-1/5} t_3^{-3/5}$$
$$T_s = 5.25 \times 10^{7}\,[\mathrm{K}]\,E_{51}^{2/5} n_0^{-2/5} t_3^{-6/5}$$

$$1.54 \times 10^{19}\,\mathrm{cm} = 5\,\mathrm{pc}$$
$$t_3 \equiv t/10^3\,\mathrm{yr}$$
$$V_s \equiv \frac{dR_s}{dt} = \frac{2}{5} A E^{1/5} \rho_0^{-1/5} t^{-3/5}$$
$$T_s = \frac{3}{16} m V_s^2 / k \quad \left(m \simeq \frac{1.4}{2.3} m_p \right) \quad \rho_0 = m n_0$$

m = mass per particle in the fully ionized, post-shock region.

- We have been assuming that the internal structure of the remnant is given by a similarity solution: by this we mean that the density, velocity, and pressure can be written

$$\rho(r) = \rho_0 f(x)$$
$$v(r) = \frac{R_s}{t} g(x)$$
$$P(r) = \frac{\rho_0 R_s^2}{t^2} h(x)$$

$$x \equiv \frac{r}{R_s}$$

$f(x), g(x), h(x)$ are dimensionless functions.

Inserting these into the fluid equations, with the Rankine-Hugonit relations for boundary conditions,Taylor (1950) and Sedov (1959) found the solution for the dimensionless functions [f(x), g(x), h(x), and A] independently, in connection with the development of nuclear weapons.

- ***Alternative approach***:
  - For the strong shock, the specific internal (thermal) and the kinetic energies just behind the strong shock are given by, respectively:

$$\epsilon_{\rm int} = \epsilon_{\rm kin} = \frac{9}{32} V_s^2$$

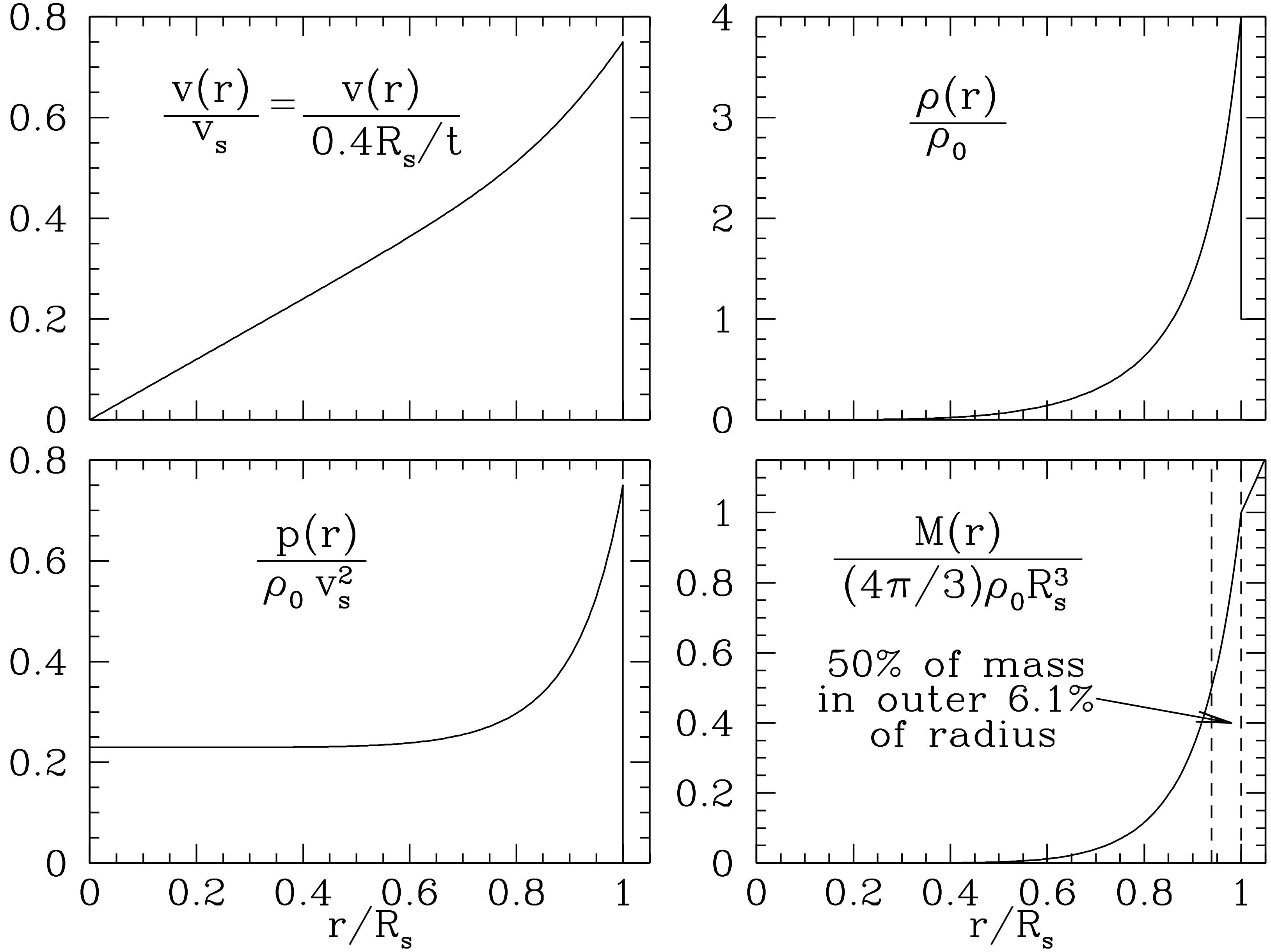
  - Since the blast wave is slowing down with time, the specific energy of the internal gas varies with radius within the bubble of hot gas.
  - Provided that most of the mass in the supernova remnant is made up of shocked interstellar gas and the mass of the stellar eject can be neglected, the total energy in the bubble of hot gas E_0, which is equal to the energy injected by the supernova explosion since the radiative losses are negligible, is therefore

$$E_0 = \phi \frac{4\pi}{3} R^3 \rho_0 \left(\epsilon_{\rm int} + \epsilon_{\rm kin}\right)$$
$$= \phi \frac{3\pi}{4} R^3 \rho_0 \left(\frac{dR}{dt}\right)^2$$

ρ_0 = density of ambient medium
R = the radius of the shock front at time t

where ϕ is a structure parameter, a numerical factor of order unity which accounts for the radial dependence of specific energy within the bubble. This equation represents the equation of motion of the shock front.

$$R = \left(\frac{25}{3\pi\phi}\right)^{1/5} \left(\frac{E_0 t^2}{\rho_0}\right)^{1/5} \qquad A \equiv \left(\frac{25}{3\pi\phi}\right)^{1/5} = 1.15167 \text{ for } \gamma = 5/3$$



as $r \to 0$

$\rho(r)/\rho_0 \to 0$

$T(r)/T_s \to \infty$

$P(r)/P_0 \to 0.306$

Sedov-Taylor solution for γ = 5/3.

The temperature profile (not shown) can be obtained from the ratio of the pressure and density profiles. ***The density falls inward, and the temperature rises inward.***

[Figure 39.1, Draine]

- *End of the Sedov-Taylor phase*

- The hot gas interior to the shock front is, of course, radiating energy. When the radiative losses become important, a dense shell forms behind the shock and the SNR will leave the Sedov-Taylor phase and enter a "radiative" phase. In the radiative phase, the gas in the shell just interior to the shock front is now able to cool to temperatures much lower than the temperature $T_s = (3/16) m V_s^2 / k$ at the shock front.
- In order to calculate the time scale, we need to estimate the cooling rate:

cooling function at a radius r:

$$\Lambda(r) \approx C \left[T(r)/10^6\,\mathrm{K}\right]^{-0.7} n_\mathrm{H}(r) n_e(r), \quad C = 1.1 \times 10^{-22}\,\mathrm{erg\,cm^3\,s^{-1}}, \quad n_e \simeq 1.2 n_\mathrm{H}$$

cooling rate at a time t, integrated over the volume of the SNR:

$$\frac{dE}{dt} = -\int_0^{R_s} \Lambda(r) 4\pi r^2 dr \longleftarrow n_\mathrm{H} n_e = 1.2 n_\mathrm{H}^2 = 1.2 n_0^2 \left[\rho(r)/\rho_0(r)\right]^2$$

$$= -1.2 C n_0^2 \left(T_s/10^6\,\mathrm{K}\right)^{-0.7} \frac{4\pi}{3} R_s^3 \left\langle (\rho/\rho_0)^2 (T_s/T)^{0.7} \right\rangle$$

Here, $n_0 \equiv n_\mathrm{H}(r = R_s)$ is the hydrogen density of the ambient medium at $r = R_s$.

$T_s \equiv T(r = R_s)$ is the temperature of the post-shock region at $r = R_s$.

$\langle \cdots \rangle$ denotes a volume-weighted average over the SNR.

From the Sedov-Taylor solution, we obtain $\left\langle (\rho/\rho_0)^2 (T_s/T)^{0.7} \right\rangle = 1.817$.

- Now, integrate the energy loss rate over a time interval *t* :

$$\Delta E(t) = -1.2 \times (1.817) C \frac{4\pi}{3} n_0^2 \int_0^t dt R_s^3 (T_s/10^6\,\mathrm{K})^{-0.7}$$

Using the previous solutions, we obtain the fractional energy loss by time *t* :

$$R_s = 1.54 \times 10^{19}\,[\mathrm{cm}]\, E_{51}^{1/5} n_0^{-1/5} t_3^{2/5}$$
$$T_s = 5.25 \times 10^{7}\,[\mathrm{K}]\, E_{51}^{2/5} n_0^{-2/5} t_3^{-6/5}$$

$$\frac{\Delta E(t)}{E_0} \approx -2.38 \times 10^{-6}\, n_0^{1.68} E_{51}^{-0.68} t_3^{3.04}$$

- If we suppose that the SNR enter the "radiative phase" when

$$\Delta E(t_{\mathrm{rad}})/E_0 \approx -1/3$$

we can solve for the cooling time t_{rad} :

$$t_{\mathrm{rad}} = 49.3 \times 10^3\,[\mathrm{yr}]\, E_{51}^{0.22} n_0^{-0.55}$$

end of the Sedov-Taylor phase

- The radius and shock speed at the end of the Sedov-Taylor phase are:

$7.32 \times 10^{19}\,\mathrm{cm} = 23.7\,\mathrm{pc}$

$$R_s(t_{\mathrm{rad}}) = 7.32 \times 10^{19}\,[\mathrm{cm}]\, E_{51}^{0.29} n_0^{-0.42}$$
$$V_s(t_{\mathrm{rad}}) = 188\,[\mathrm{km\,s^{-1}}]\, \left(E_{51} n_0^2\right)^{0.07}$$

Post-shock temperature is:

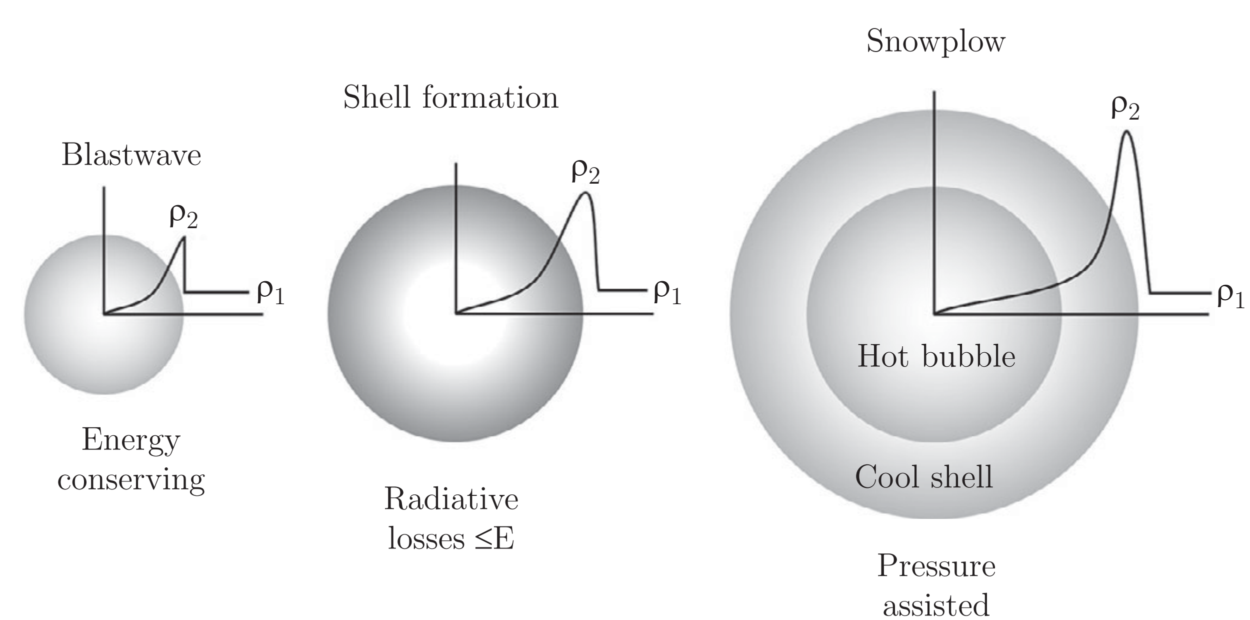
$$T_s(t_{\mathrm{rad}}) = 4.86 \times 10^5\,[\mathrm{K}]\, \left(E_{51} n_0^2\right)^{0.13}$$
$$kT_s(t_{\mathrm{rad}}) = 41\,[\mathrm{eV}]\, \left(E_{51} n_0^2\right)^{0.13}$$

- ***Snowplow Phase (Radiative Phase)***
  - At $t \approx t_{\rm rad}$,
    - Cooling cause the thermal pressure just behind the shock to drop suddenly, and the shock wave briefly stalls. However, ***the very hot gas in the interior of the SNR has not cooled***.
    - The SNR now enters the snowplow phase, with a dense shell of cool gas enclosing a hot central volume where radiative cooling is unimportant.
    - This is called the snowplow phase because the expanding dense shell scoops up additional mass as it expands (the mass of the dense shell increases as it "sweeps up" the ambient gas). Since the gas cools well the shell will be thin and so the gas in the shell has a radial velocity that is almost the same as the shock front speed.
    - Most of the mass of a supernova remnant in the snowplow phase is in the dense shell. The inner region contains gas at very high temperatures. Thus, despite its low density, ***the gas in the inner region has a significant pressure, which push the dense shell outward.***
    - The gas in the hot center cools by adiabatic expansion, and thus the pressure is given by

$$P \propto V^{-\gamma} \propto R_s^{-3\gamma} = R_s^{-5}$$

    - So that the pressure in the interior evolves as

$$P_i = P_0(t_{\rm rad}) \left(\frac{R_{\rm rad}}{R_s} \right)^5$$



Transition from the Sedov-Taylor phase to the snowplow phase [Figure 5.5, Ryden]

- In the initial phase, the pressure exerted by the hot center causes the “radial momentum” of the shell to increases:

M_s = mass of the shell = mass of the ambient medium swept by the SNR

$$\frac{d}{dt}(M_s V_s) \approx P_i 4\pi R_s^2 = 4\pi P_0(t_{\rm rad}) R_{\rm rad}^5 R_s^{-3}$$

$$M_s = \left(\frac{4\pi}{3} R_s^3\right)\rho_0 \qquad V_s = \frac{dR_s}{dt} \longrightarrow \frac{d}{dt}\left(R_s^3 \frac{dR_s}{dt}\right) \propto R_s^{-3}$$

Suppose that there is a power-law solution:

$$R_s \propto t^\eta$$

$$4\eta - 2 = -3\eta \ \Rightarrow \ \eta = 2/7 \longrightarrow$$

$$R_s \approx R_s(t_{\rm rad})\,(t/t_{\rm rad})^{2/7}$$

$$V_s \approx \frac{2}{7}\frac{R_s}{t} = \frac{2}{7}\frac{R_s(t_{\rm rad})}{t_{\rm rad}}\left(\frac{t}{t_{\rm rad}}\right)^{-5/7}$$

- Because the effect of the internal pressure has been included, this solution is referred to as the ***pressure-modified (or pressure-driven) snowplow phase***.
- Note that with this construction, $R_s(t)$ is continuous from the Sedov-Taylor phase to the pressure-modified snowplow phase, but $V_s(t)$ undergoes a discontinuous drop by 2/7 ~ 29% at $t = t_{\rm rad}$.

- In the late phase of evolution, the stored thermal energy in the central region has been entirely radiated away, and only the momentum of the dense shell keeps the remnant expanding into the ISM.

$$\frac{d}{dt}(M_s V_s) = 0 \qquad M_s = \text{mass of the shell}$$

$$M_s = \left(\frac{4\pi}{3} R_s^3\right) \rho_0$$

$$V_s = \frac{dR_s}{dt} \longrightarrow R_s^3 \frac{dR_s}{dt} = \text{constant}$$

$$R_s \approx R_s(t_{\mathrm{rad}}) \, (t/t_{\mathrm{rad}})^{1/4} \quad \text{for } t \gg t_{\mathrm{rad}}$$

$$V_s \approx \frac{1}{4} \frac{R_s}{t} = \frac{1}{4} R_s(t_{\mathrm{rad}}) \, (t/t_{\mathrm{rad}})^{-3/4}$$

  - This phase is called the ***momentum-conserving snowplow phase***. Towards the end of this phase, the expansion velocity becomes sonic or subsonic with respect to the interstellar sound speed.

- ***In summary, the supernova expansion velocity is continuously slowed by its interaction with the surrounding medium.***

$$\boxed{R \propto t \quad \rightarrow \quad R \propto t^{2/5} \quad \rightarrow \quad R \propto t^{2/7} \quad \rightarrow \quad R \propto t^{1/4}}$$

- ***Fadeaway (Merging with the ISM)***
  - For typical ISM parameters, the shock speed at the beginning of the snowplow phase is

$$V_s(t_{\rm rad}) = 188\,[{\rm km\,s^{-1}}]\,\left(E_{51} n_0^2\right)^{0.07}$$

  - This results in a very strong shock when propagating through interstellar gas with $T <$ 10^4 K. However, the shock front gradually slows, and the shock compression declines. This proceeds until the shock speed approaches the effective sound speed in the gas and at this point the compression ratio approaches 1, and the shock wave turns into a sound wave.
  - Setting $V_s \approx c_s$ gives the a "fadeaway time"

$$V_s \approx \frac{2}{7}\frac{R_s(t_{\rm rad})}{t_{\rm rad}}\left(\frac{t}{t_{\rm rad}}\right)^{-5/7} \longrightarrow \qquad t_{\rm fade} \approx \left(\frac{(2/7)R_{\rm rad}/t_{\rm rad}}{c_s}\right)^{7/5} t_{\rm rad}$$

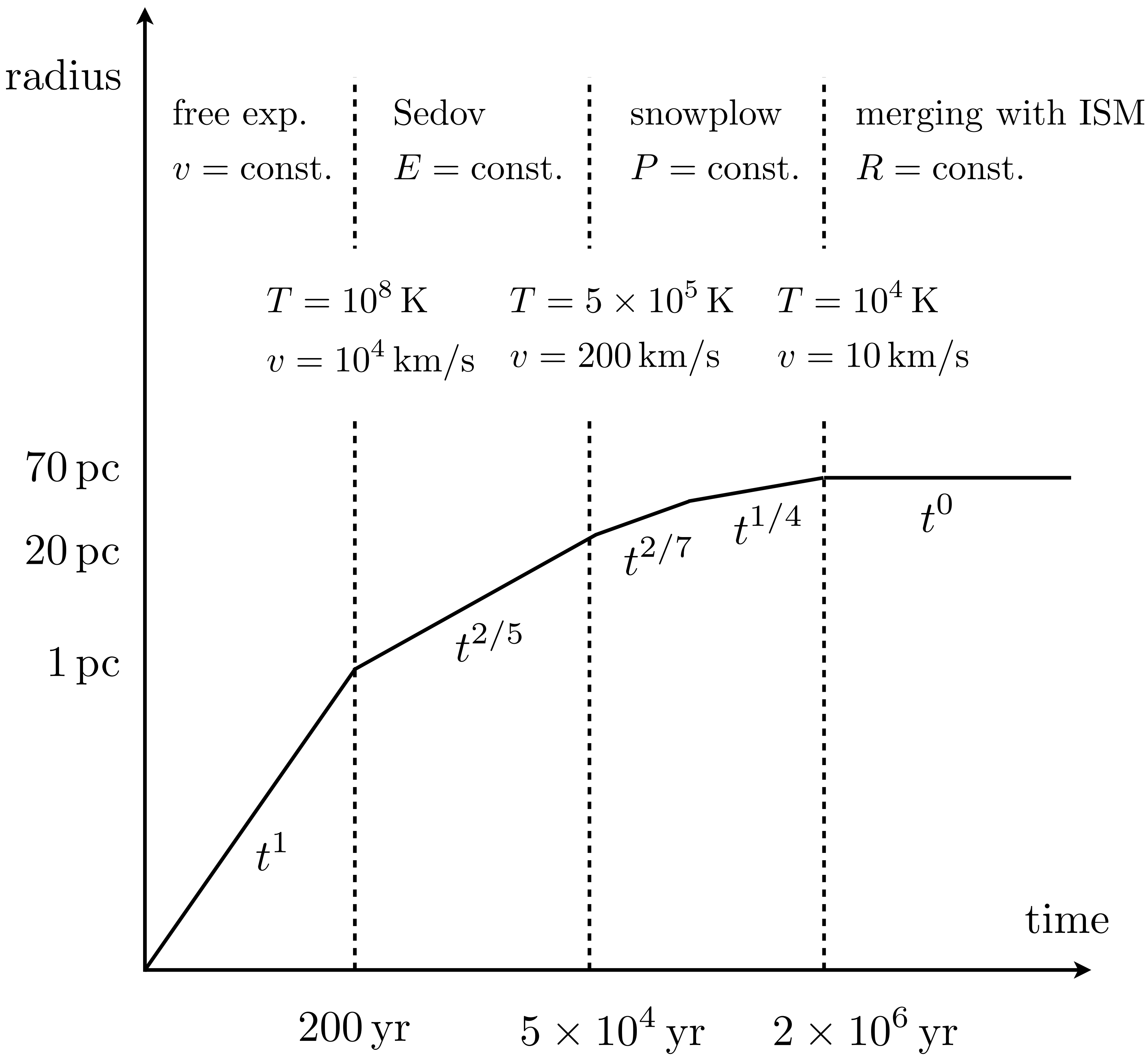
$$t_{\rm fade} \approx 1.87\times10^6\,[{\rm yr}]\,E_{51}^{0.32} n_0^{-32}\left(\frac{c_s}{10\,{\rm km\,s^{-1}}}\right)^{-7/5}$$ end of the snowplow phase

$$R_{\rm fade} \equiv R_s(t_{\rm fade}) \approx \frac{7}{2}V_s t_{\rm fade} \approx \frac{7}{2}c_s t_{\rm fade}$$

$$\approx 2.07\times10^{20}\,[{\rm cm}]\,E_{51}^{0.32} n_0^{-0.37}\left(\frac{c_s}{10\,{\rm km\,s^{-1}}}\right)^{-2/5}$$

$$2.07\times10^{20}\,{\rm cm} = 67.1\,{\rm pc}$$

Illustration of the phases in the dynamics of a SNR



Four Phases of the SNR

1. Free Expansion Phase
2. Sedov-Taylor Phase
3. Snowplow Phase
   (pressure-driven)
   (momentum-conserving)
4. Merging Phase

Rough estimates for the temperatures and velocities at the end of each phase are given.

Fig 4.9, Rosswog & Bruggen, Introduction to High-Energy Astrophysics (slightly modified)

Homework (due date: 06/02)

[Q15]

From Ryden's book

Technologically advanced (but ethically dubious) space aliens have found a method of converting mass M to energy E with 100% efficiency, using Einstein's relation $E = Mc^2$.

(a) The star Proxima Centauri ($M_{\rm pc} = 0.123\,{\rm M}_\odot$, $R_{\rm pc} = 0.143\,{\rm R}_\odot$) is at a distance $d = 1.301$ pc from the Earth. The space aliens convert 5% of Proxima Centauri's mass into energy, and use that energy to eject the remaining 95% spherically outward at an initial speed $u_{\rm ej}$. What is $u_{\rm ej}$ in kilometers per second?

(b) The gas between the solar system and Proxima Centauri is part of the Local Interstellar Cloud, with mean density $n_{\rm H} = 1\,{\rm cm}^{-3}$ and temperature $T = 8000$ K. Assuming uniform density and temperature, how long will it take the blastwave from the explosion in part (a) to reach the solar system? If the aliens beam us a taunting radio message at the same time they blow up Proxima Centauri, how long will we have to prepare for the arrival of the blastwave?

(c) What will be the speed $u_{\rm sh}$ of the blastwave as it passes through the solar system?

(d) What will be the pressure P_2 and the temperature T_2 right behind the blastwave as it passes through the solar system?

(e) What will be the total thermal energy of the portion of the blastwave that strikes the Earth? Is this greater than or less than the thermal energy of the Earth's atmosphere? [Hint: in addition to knowing the Earth's radius and average atmospheric temperature, you will need to know the mass of its atmosphere.]