Interstellar Medium (ISM)

Lecture 15
2025 June 09 (Monday), 2PM

updated 06/01, 22:37

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[Virial Theorem]

- For a collection of N point particles, the total moment of inertia is given by

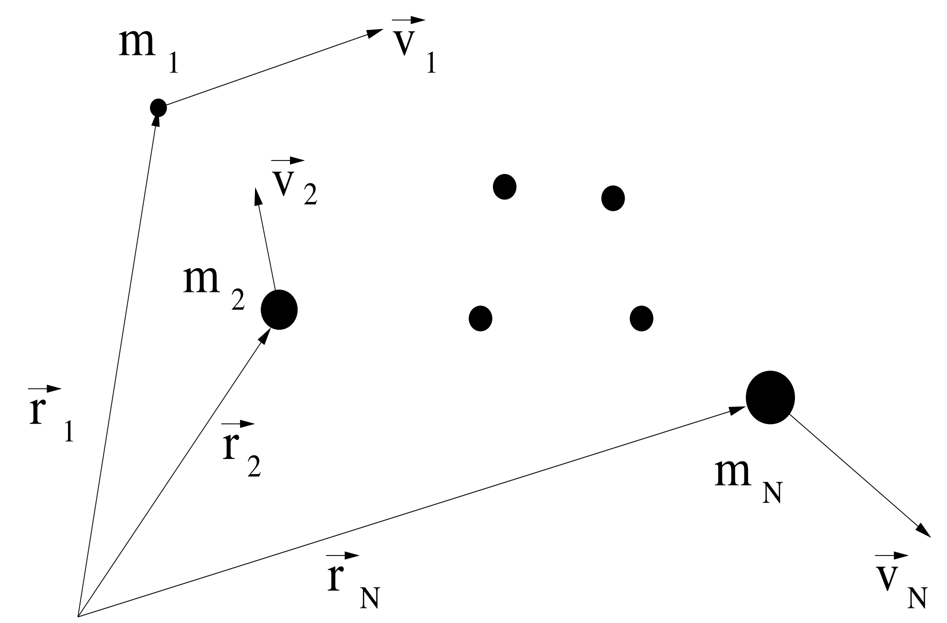
$$I = \sum_i m_i |\mathbf{r}_i|^2 = \sum_i m_i \mathbf{r}_i \cdot \mathbf{r}_i$$

- The time derivative of the moment of inertia is called the ***virial***:

$$Q = \frac{1}{2}\frac{dI}{dt} = \sum_i \mathbf{p}_i \cdot \mathbf{r}_i$$

Here, momentum vector:

$$\mathbf{p}_i = m_i \mathbf{v}_i \quad \left(= m_i \frac{d\mathbf{r}_i}{dt} \right)$$



Now take the time derivative of the viral

$$\frac{dQ}{dt} = \sum \mathbf{p}_i \cdot \mathbf{v}_i + \sum_i \frac{d\mathbf{p}_i}{dt} \cdot \mathbf{r}_i$$
$$= \sum_i m_i \mathbf{v}_i \cdot \mathbf{v}_i + \sum_i \mathbf{F}_i \cdot \mathbf{r}_i$$

Newton's second law:

$$\mathbf{F}_i = \frac{d\mathbf{p}_i}{dt} \quad \text{(the sum of all forces acting on particle } i\text{)}$$

The first term can be expressed in terms of the total kinetic energy of the system

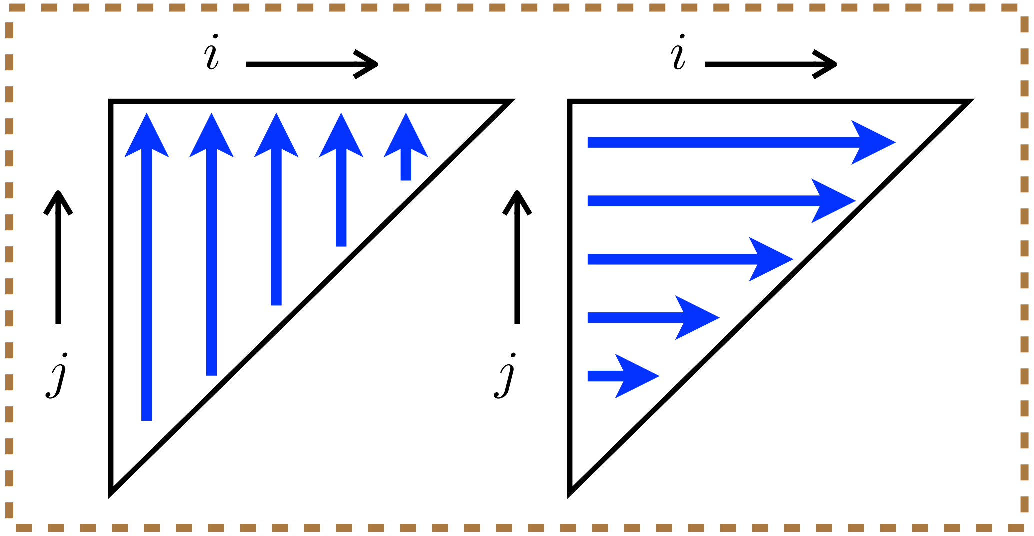
$$\frac{dQ}{dt} = 2K + \sum_i \mathbf{F}_i \cdot \mathbf{r}_i \quad \left(\text{where } K = \sum_i \frac{1}{2} m_i |\mathbf{v}_i|^2 \right)$$

- The total force on particle i is the sum of all the forces from the other particles j in the system:

$$\mathbf{F}_i = \sum_{j \neq i} \mathbf{f}_{j \to i}$$ Here, $\mathbf{f}_{j \to i}$ is the force on particle i from particle j

Then,

$$\sum_{i=1}^{N} \mathbf{F}_i \cdot \mathbf{r}_i = \sum_{i=1}^{N} \sum_{j \neq i} \mathbf{f}_{j \to i} \cdot \mathbf{r}_i$$
$$= \sum_{i=1}^{N} \sum_{j<i} \mathbf{f}_{j \to i} \cdot \mathbf{r}_i + \sum_{i=1}^{N} \sum_{j>i} \mathbf{f}_{j \to i} \cdot \mathbf{r}_i$$



The last term can be rewritten as follows:

$$\sum_{i=1}^{N} \sum_{j>i} \mathbf{f}_{j \to i} \cdot \mathbf{r}_i = \sum_{j=1}^{N} \sum_{i<j} \mathbf{f}_{j \to i} \cdot \mathbf{r}_i$$ interchange the ordering of the sums.

$$= \sum_{i=1}^{N} \sum_{j<i} \mathbf{f}_{i \to j} \cdot \mathbf{r}_j$$ interchange the name of the indices i and j.

$$= -\sum_{i=1}^{N} \sum_{j<i} \mathbf{f}_{j \to i} \cdot \mathbf{r}_j$$ Newton's third law: $\mathbf{f}_{i \to j} = -\mathbf{f}_{j \to i}$

Hence,

$$\sum_{i=1}^{N} \mathbf{F}_i \cdot \mathbf{r}_i = \sum_{i=1}^{N} \sum_{j<i} \mathbf{f}_{j \to i} \cdot (\mathbf{r}_i - \mathbf{r}_j)$$

- Now, consider the gravitational force:

$$\mathbf{f}_{j\to i} = \frac{Gm_i m_j}{r_{ij}^3}(\mathbf{r}_j - \mathbf{r}_i) \longrightarrow \sum_{i=1}^{N} \mathbf{F}_i \cdot \mathbf{r}_i = -\sum_{i=1}^{N}\sum_{j<i} \frac{Gm_i m_j}{r_{ij}}$$

$$\text{where } r_{ij} = |\mathbf{r}_j - \mathbf{r}_i|$$

Note that the gravitational potential energy between particle i and j is given by $U_{ij} = -\frac{Gm_i m_j}{r_{ij}}$.

$$\sum_{i=1}^{N} \mathbf{F}_i \cdot \mathbf{r}_i = \sum_{i=1}^{N}\sum_{j<i} U_{ij} = U$$

=> total potential energy of the system

The total potential energy of the system is the sum of the potential energy between all possible pairs of particles (note that one pair of particle should be counted only once, this is why there is a $j < i$ in the second sum).

- We finally obtain the following. Now we take the mean value of the derivative of the virial over a long period of time:

$$\frac{dQ}{dt} = 2K + U \longrightarrow \left\langle \frac{dQ}{dt} \right\rangle = 2\langle K \rangle + \langle U \rangle$$

- If ***the system is bounded and the particles have finite momentum***, the average value will go to zero and we obtain the virial theorem:

$$\left\langle \frac{dQ}{dt} \right\rangle = \lim_{T\to\infty} \frac{Q(T) - Q(0)}{T} = 0 \longrightarrow \boxed{2\langle K \rangle + \langle U \rangle = 0}$$

$\langle\rangle$ denotes both the ensemble average and time average.

Ergodic hypothesis: Averaging system variables over a long time period is equal to averaging them over the ensemble.

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T dt \to \lim_{N\to} \frac{1}{N}\sum_{i=1}^{N}$$

If a bound system has a huge number of particles, it is equivalent to seeing the system over a long period of time.

Virial Mass Estimate

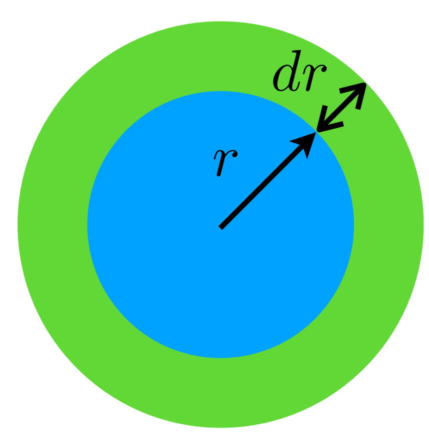
- A fundamental property is that GMCs are gravitationally bound and in viral equilibrium.
  - Their masses can then be estimated using line widths as a measure of the could velocities, following the arguments of Solomon et al.
  - The virial theorem provides a general equation that relates the average over time of the total kinetic energy of a stable, self-gravitating system of discrete particles, with the total potential energy of the system.

$$2\langle K\rangle + \langle U\rangle = 0$$

  - For a uniform density sphere with a mass M and radius R, the gravitational potential energy is

$$U = -\frac{3}{5}\frac{GM^2}{R}$$

$$\begin{aligned} U &= -\int_0^R \frac{GM_r dM_r}{r} \\ &= -\int_0^R \frac{G}{r}\left(\frac{4\pi}{3}r^3\rho\right) 4\pi r^2 \rho dr \\ &= -\frac{(4\pi)^2}{3\times 5}G\rho^2 R^5 = -\frac{3}{5}\frac{GM^2}{R} \end{aligned}$$



Density $\rho = \frac{M}{(4\pi/3)R^3}$

Mass within a radius r

$$M_r = \int_0^r \rho(4\pi r'^2 dr') = \frac{4\pi}{3}r^3\rho$$

Mass between a shell $(r, r+dr)$

$$dM_r = \rho(4\pi r^2 dr)$$

- If the cloud is in equilibrium, with self-gravity being balanced by turbulent pressure, the virial theorem states that the turbulent velocity, in one dimension, to be

$$\langle K \rangle = \sum_i \frac{3}{2} m_i \langle v_i^2 \rangle = \frac{3}{2} \sum_i m_i \sigma_v^2 = \frac{3}{2} M \sigma_v^2$$

Here, σ_v is the rms velocity dispersion in 1D.

$$2\langle K \rangle + \langle U \rangle = 0 \quad \rightarrow \quad \boxed{\sigma_v^2 = \frac{1}{5} \frac{GM}{R}}$$

velocity dispersion in 3D: $(\sigma_v^*)^2 = 3\sigma_v^2$

- Therefore, we obtain the total mass of the self-gravitation cloud in terms of the line width (broadening parameter):

virial mass: $$M = \frac{5b^2 R}{2G} \approx 600\, M_\odot \left(\frac{b}{1\,\mathrm{km\,s^{-1}}} \right)^2 \left(\frac{R}{1\,\mathrm{pc}} \right)$$

Here, $b = \sqrt{2}\sigma_v$

- Note that the virial theorem for a uniform density clouds indicates that the following size-linewidth correlation if the density is constant. But, it appears that the density is not a constant, but controlled by turbulence.

$$b^2 = \frac{2}{5} \frac{GM}{R} = \frac{8\pi}{15} G\rho R^2 \quad \rightarrow \quad \boxed{b = \left(\frac{8\pi}{15} G\rho \right)^{1/2} R}$$

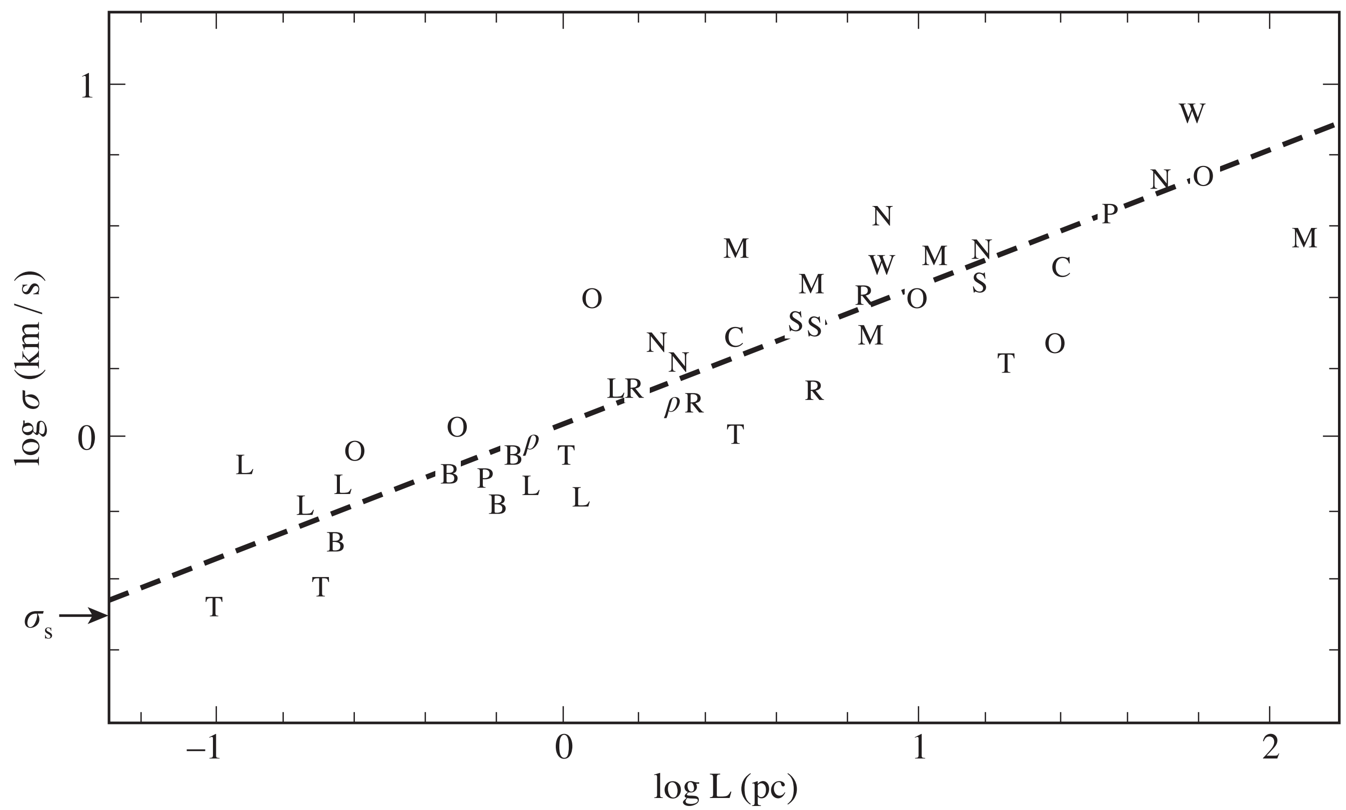
Size-Linewidth Relation in Molecular Clouds

- Larson (1981) noted that observations of molecular clouds in spectral lines of CO, H_2CO, NH_3, OH, and other species, were broadly consistent with a ***size-linewidth relation***, where a density peak of characteristic size *L* tends to have a 3D velocity dispersion given by

$$\sigma_v^* \approx 1.10\, L_{\rm pc}^{\gamma}\, {\rm km\, s^{-1}}, \quad \gamma \approx 0.38 \quad \text{for } 0.01 \lesssim L_{\rm pc} \lesssim 10^2 \quad (L_{\rm pc} = L/{\rm pc})$$

where *L* is the maximum projected dimension of the density peak.

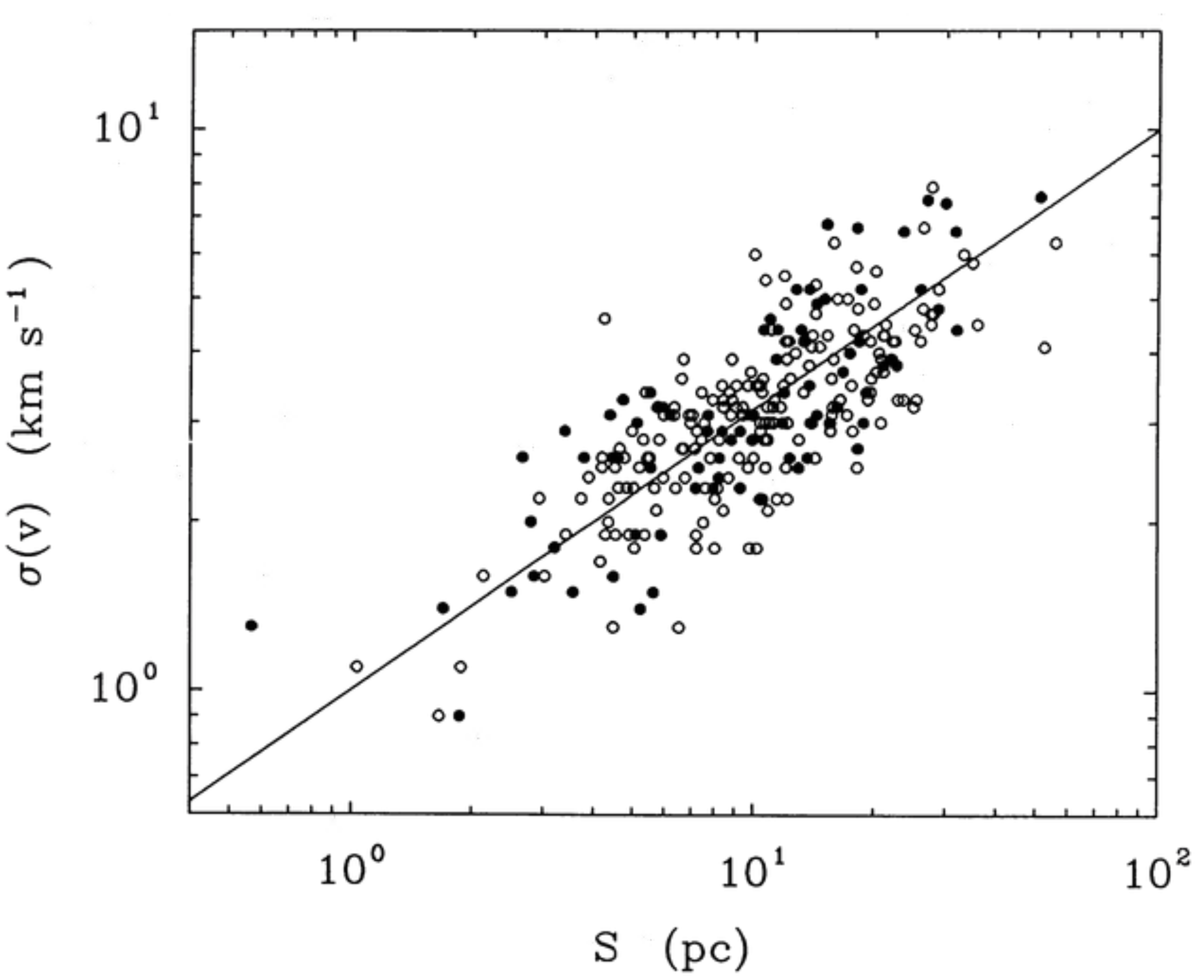
- Larson noted that the power-law index $\gamma \approx 0.38$ is curiously close to the index 1/3 found by **Kolmogorov** for a turbulent cascade in an incompressible fluid.
- It therefore is tempting to refer to the observed fluid motion as “turbulence,” although in realty *the motions are some combination of thermal motions, rotation, MHD waves, and turbulence.*



The 3D internal velocity dispersion versus maximum linear dimension *L* of the density peak.

Fig 32.5, Draine; Larson (1981)

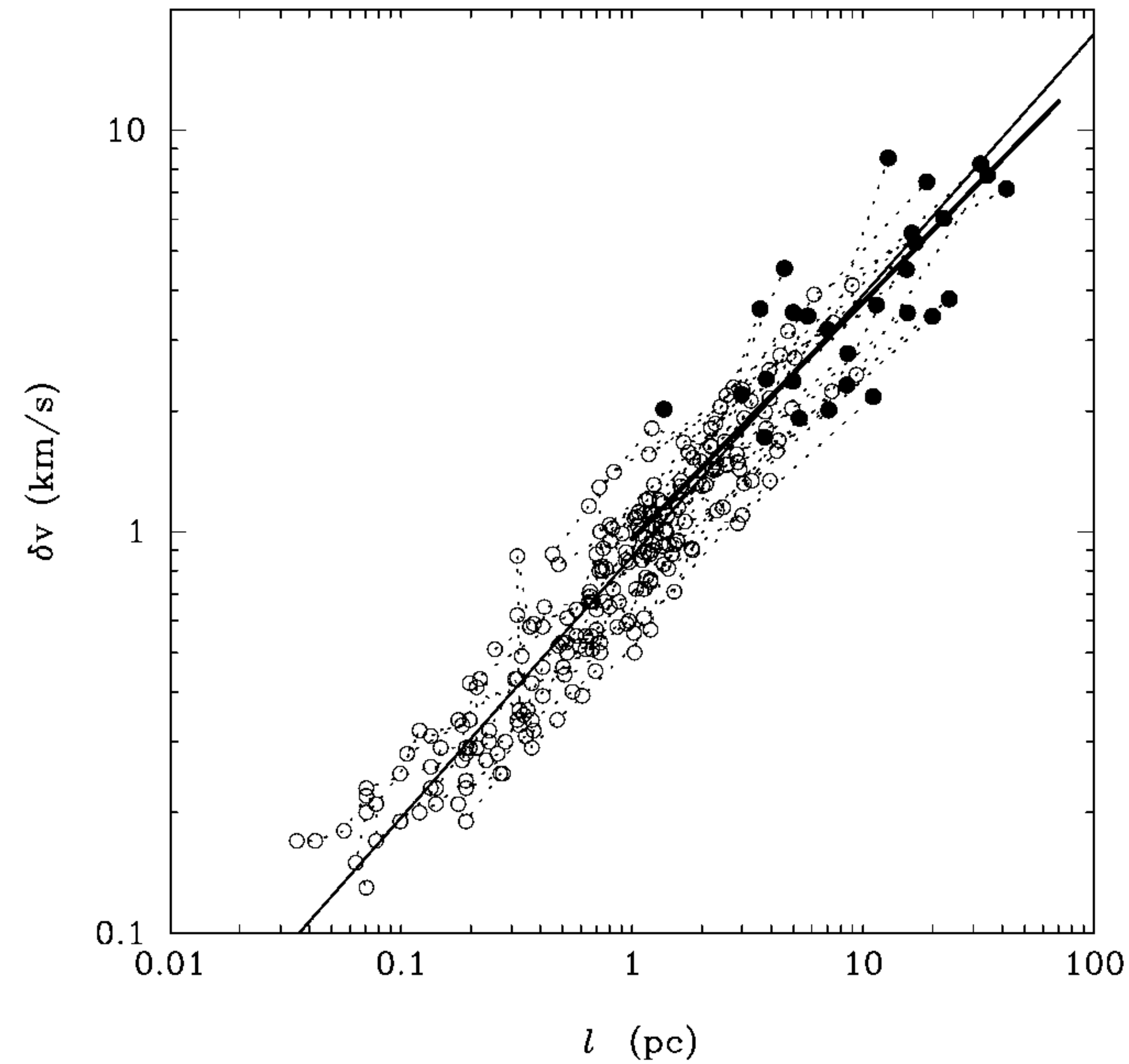
$$\sigma_v^* \approx 1.0\, L_{\rm pc}^{0.5}\, \rm km\, s^{-1}$$



Molecular cloud velocity dispersion as a function of size *S* for 273 clouds in the Galaxy

Fig 1, Solomon et al. (1987)

$$\sigma_v^* \approx 0.96\, L_{\rm pc}^{0.59}\, \rm km\, s^{-1}$$



Velocity dispersion versus size ℓ from PCA decomposition of ^{12}CO J = 1-0 imaging observations of 27 individual molecular clouds

Fig 1, Heyer & Brunt (2004)

- However, the power-law index $\gamma \approx 0.38$ has been questioned.
  - Solomon et al. (1987) found $\sigma_v^* \approx 1.0\, L_{\rm pc}^{0.5}\,{\rm km\,s^{-1}}$ from a study of 273 molecular clouds.
  - Heyer & Brunt (2004) found $\sigma_v^* \approx 0.96\, L_{\rm pc}^{0.59}\,{\rm km\,s^{-1}}$.
- Note that
  - For $L \gtrsim 0.02\,{\rm pc}$, $\sigma_v^* \gtrsim (kT/\mu)^{1/2} \approx 0.23(T/15\,{\rm K})^{1/2}\,{\rm km\,s^{-1}}$ $\Rightarrow$ The linewidth is supersonic.
  - For $L \lesssim 0.02\,{\rm pc}$, the line width is nearly thermal, with only a small contribution from rotation, waves or turbulence.
- The density peaks are generally self-gravitating. If we assume them to be in approximate virial equilibrium, and consider only the kinetic energy associated with fluid motions, we can estimate the clump mass using the virial theorem.
  - For a uniform density sphere with a diameter *L*, we can obtain the follows:

$$\sigma_v^{*2} = 3\sigma_v^2 = 6GM/5L \quad (L = 2R)$$

However, this gives $\sigma_v^* \propto L$ for a constant density, which is not consistent with the size-linewidth relation

$$\sigma_v^* \approx L_{\rm pc}^{\gamma}$$

			for $\gamma \approx 0.38$
clump mass:	$M \approx \dfrac{5\sigma_v^{*2}L}{6G} \approx 230\,L_{\rm pc}^{2\gamma+1}\,M_\odot$	$\rightarrow$	$230\,L_{\rm pc}^{1.76}\,M_\odot$
density:	$n_{\rm H} \approx M/[(4\pi/3)(L/2)^3 1.4 m_{\rm H}] \approx 1.3\times10^4\,L_{\rm pc}^{2\gamma-2}\,{\rm cm^{-3}}$	$\rightarrow$	$1.3\times10^4\,L_{\rm pc}^{-1.24}\,{\rm cm^{-3}}$
column density:	$N_{\rm H} = n_{\rm H}L \approx 4.0\times10^{22}\,L_{\rm pc}^{2\gamma-1}\,{\rm cm^{-2}}$	$\rightarrow$	$4.0\times10^{22}\,L_{\rm pc}^{-0.24}\,{\rm cm^{-2}}$

- We recall that for the dust in diffuse clouds, $A_V/N_{\mathrm{H}} = 1.87 \times 10^{21}\,\mathrm{cm}^2$, and we would have

$$A_V \approx 21\, L_{\mathrm{pc}}^{2\gamma-1}\,\mathrm{mag} \quad \rightarrow \quad 21\, L_{\mathrm{pc}}^{-0.24}\,\mathrm{mag}$$

The dust in dense clouds differs from that in the diffuse ISM, but this would give a reasonable estimate of the visual extinction through the cloud.

- If $\gamma \approx 0.38$, then **a GMC complex** with diameter $L \approx 50\,\mathrm{pc}$ would have

$$\boxed{\begin{aligned} M &\approx 2 \times 10^5\, M_\odot \\ n_{\mathrm{H}} &\approx 100\,\mathrm{cm}^{-3} \\ A_V &\approx 8\,\mathrm{mag} \end{aligned}}$$

whereas **a core** with diameter $L \approx 0.1\,\mathrm{pc}$ would have

$$\boxed{\begin{aligned} M &\approx 4\, M_\odot \\ n_{\mathrm{H}} &\approx 2 \times 10^5\,\mathrm{cm}^{-3} \\ A_V &\approx 40\,\mathrm{mag} \end{aligned}}$$

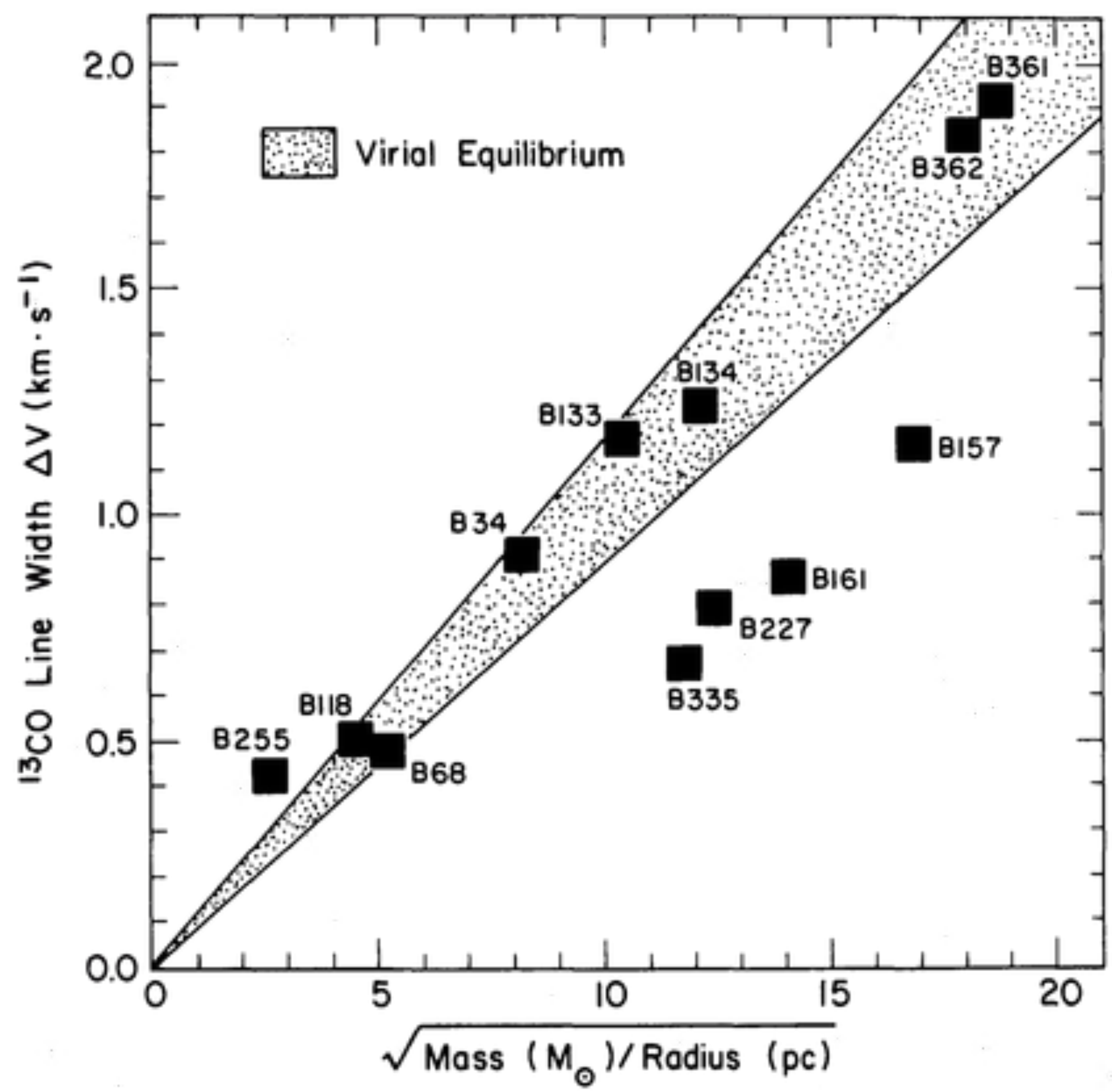
- Comments on the exponent: $N_{\rm H} \approx 4.0 \times 10^{22} L_{\rm pc}^{2\gamma-1}$
  - If $\gamma < 0.5$, smaller clouds tend to have a higher column density, meaning to be darker.
  - If $\gamma = 0.5$, small clouds and large clouds would all have the same column density and dust extinction (A_V).
  - However, **observationally, smaller structures tend to have higher A_V. This is consistent with exponent $\gamma < 0.5$.**
  - Therefore, ***the Larson's exponent would be better constrained with the observations***.

- Expressing the above relations with density as the independent variables, we obtain

$$
\begin{aligned}
L_{\rm pc} &= (n_3/13)^{1/(2\gamma-2)} && \rightarrow 7.94\, n_3^{-0.81} \quad , \\
\sigma_v &= 1.1(n_3/13)^{\gamma/(2\gamma-2)}\,{\rm km\,s^{-1}} && \rightarrow 2.43\, n_3^{-0.31}\,{\rm km\,s^{-1}} \quad , \\
M &= 230(n_3/13)^{(2\gamma+1)/(2\gamma-2)}\,M_\odot && \rightarrow 8940\, n_3^{-1.42}\,M_\odot \quad , \\
A_V &= 21(n_3/13)^{(2\gamma-1)/(2\gamma-2)}\,{\rm mag} && \rightarrow 13\, n_3^{0.19}\,{\rm mag} \quad ,
\end{aligned}
$$

- The ***line width - (mass/radius) correlation***:

$$b = \left(\frac{2G}{5}\right)^{1/2} \left(\frac{M}{R}\right)^{1/2}$$



The ^{13}CO line width versus (mass/radius)$^{1/2}$ for globules with $T < 10$K.

Fig 3, Leung et al. (1982, ApJ, 262, 583)

- The ***Mass - CO luminosity correlation***
  - Let $I_{\rm CO}$ the CO brightness temperature integrated over the line profile:

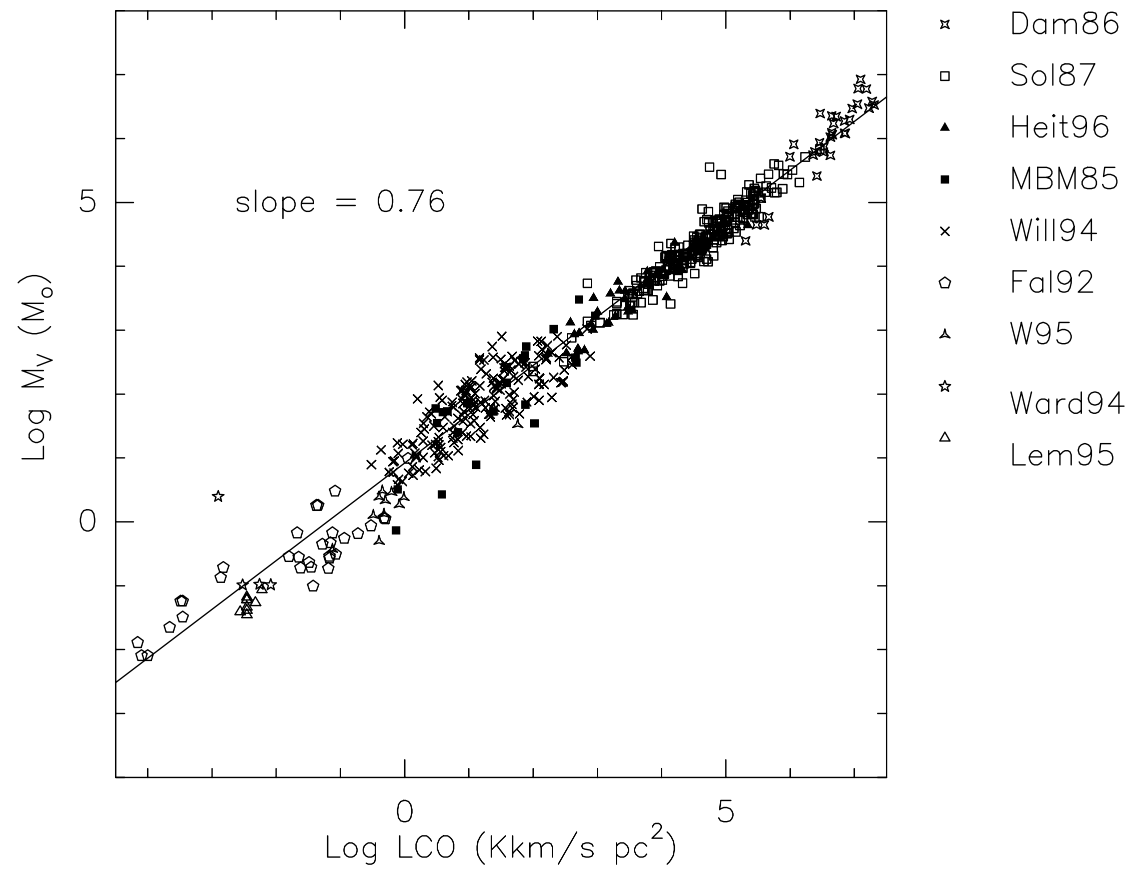
$$I_{\rm CO} \equiv \int T_{\rm CO} dv \approx T_{\rm CO} b \quad (b \approx \Delta v)$$

  - The CO luminosity of a cloud is:

$$\boxed{\begin{aligned} L_{\rm CO} &\equiv D^2 \int I_{\rm CO} d\Omega \approx \pi R^2 T_{\rm CO} b \\ &= T_{\rm CO}\sqrt{\frac{2\pi^2}{5} G M R^3} = \sqrt{\frac{6\pi}{20}G}\frac{T_{\rm CO}}{\sqrt{\rho}} M \end{aligned}}$$

Here, D = distance, and virial condition: $b \approx \sqrt{\frac{2}{5}\frac{GM}{R}}$

  - To the extent that the ratio $\sqrt{\rho}/T_{\rm CO}$ does not vary in the mean from cloud to cloud, this indicates that the total CO luminosity is directly proportional to the total mass of molecular clouds. However, in reality, the power-law slope is ~ 0.76.
  - ***This correlation supports the fundamental assumption that GMCs are in virial equilibrium.***



Virial mass versus CO luminosity relation taken from various sources.

Fig 4, Chap 2, Blain, Combes, & Draine [The cold Universe]

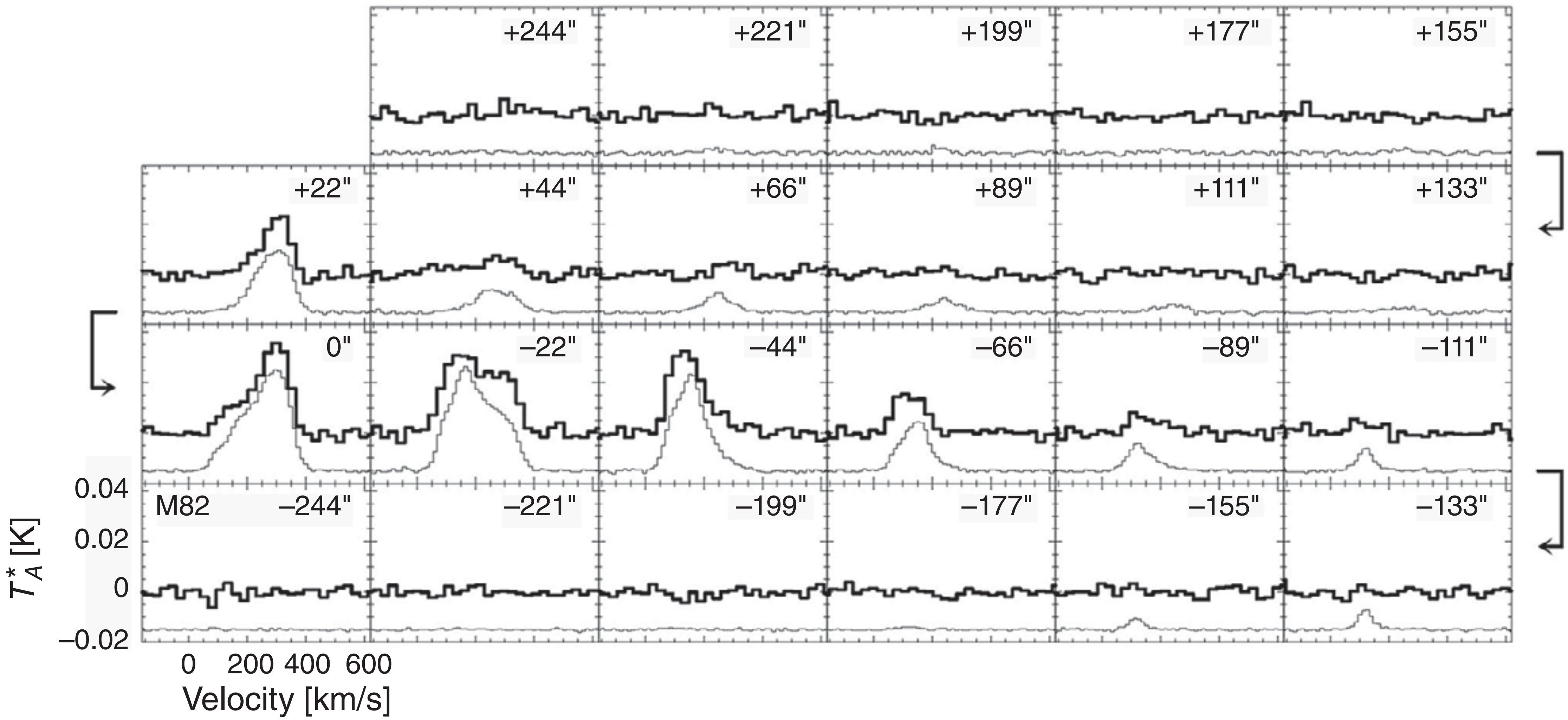
[Interstellar CO]

- Since the cold molecular hydrogen does not radiate (except the fluorescence UV line), the best tracer in galaxies is the most abundant molecule after H_2, i.e. the carbon monoxide CO. ***Much of what we know about molecular gas comes from observations of "tracer" molecules such as carbon monoxide (CO).***
- Fundamental frequencies
  - Vibrational frequency
    - The fundamental vibrational frequency corresponds to to a wavelength:

      $\lambda_0 = c/\nu_0 \approx 4.6\mu\mathrm{m}$ This energy is ~50% of the energy in the H_2 fundamental frequency.
  - Rotational frequency:
    - CO J=1-0, 2.60mm emission is the most important line used in studies of molecular gas.

      $$\lambda = 2.60\,\mathrm{mm} \rightarrow \nu = 115.3\,\mathrm{GHz} \rightarrow h\nu = 4.767 \times 10^{-4}\,\mathrm{eV} \rightarrow h\nu/k = 5.532\,\mathrm{K}$$
    - This energy level is therefore perfectly fitted to probe the cold molecular clouds.
    - The widely used CO lines (J = 1-0 at 2.6mm, J = 2-1 at 1.3 mm) are generally optically thick, for "standard" clouds of solar metallicity. This complicates the interpretation in terms of H_2 column densities and mass of the clouds.
- The line width b that we measure for molecular line emission is often much broader than the thermal value $b \sim 0.1(T_{\mathrm{gas}}/20\,\mathrm{K})^{1/2}\,\mathrm{km\,s^{-1}}$ that we expect. *The large broadening is similar for molecules of different molecular mass*, suggesting that ***the broadening is due to turbulent motions within the cloud.***

- ***Optical Depth & Velocity Structures***
  - Each individual molecular cloud is optically thick.
  - However, a giant molecular cloud is composed of many sub-structures, and there are many clouds along a line of sight. The "macroscopic" optical depth is in general not very large, due to velocity gradients: clouds do not hide each other at a given velocity, and therefore it is possible to "count" the clouds.
- ***Isotopic Species***
  - Since the CO lines are optically thick (on average $\tau_{\rm CO} \sim 10$), it might be interesting to observe the much less abundant isotope ^{13}CO.
  - At the solar neighborhood, the ^{12}CO/^{13}CO abundance ratio is ~90, and in individual molecular clouds, it is frequent that the ^{13}CO lines are optically thin.
    - However, when maps of the two isotopes are compared at millimeter wavelengths for well studied clouds, it appears that the ^{13}CO does not probe the entire extent of the clouds, but only the densest regions. This could be due to differences in excitation, self-shielding, or selective photo-dissociation.
  - The observed average ratio between integrated ^{12}CO and ^{13}CO intensities is of the order of 10.



^{13}CO emission lines (thin) and ^{12}CO emission lines divided by 10 (thick) for the $J = 1 \rightarrow 0$ transition, along the apparent major axis of the galaxy M82. At the $d \approx 3.5$ Mpc distance to M82, the 22 arcsec separation between the panels corresponds to ~370 pc.

[Paglione et al. 2001; Ryden]

Optical Thickness of CO (J = 1-0)

- The net absorption coefficient, taking into account both absorption and stimulated emission is:

$$\boxed{\kappa_\nu = n_0\sigma_{01}\left(1 - \frac{g_0}{g_1}\frac{n_1}{n_0}\right)}, \quad \sigma_{01} = \frac{g_1}{g_0}\frac{c^2}{8\pi\nu_{10}^2}A_{10}\phi_\nu, \quad \frac{n_1}{n_0} = \frac{g_1}{g_0}\exp\left(-\frac{h\nu_{10}}{kT_{\rm exc}}\right) \qquad g_J = 2J+1$$

- The optical depth of the thermally broadened CO line can be written as

$$\phi_\nu = \frac{1}{\sqrt{\pi}}\frac{\lambda}{b}e^{-\frac{v^2}{b^2}}$$

$$\tau_\nu = \int \kappa_\nu ds = \tau_0 e^{-v^2/b^2}$$

optical depth at line center =

$$\tau_0 = \frac{g_1}{g_0}\frac{c^2}{8\pi^{3/2}\nu_{10}^3}A_{10}\frac{c}{b}\left(1 - e^{-h\nu_{10}/kT_{\rm exc}}\right)N_0$$

$$A_{10} = 7.16\times 10^{-8}\,{\rm s}^{-1}$$

$$\tau_0 = 297\left(\frac{N_{\rm H}}{10^{21}\,{\rm cm}^{-2}}\right)\left[\frac{n({\rm CO})/n_{\rm H}}{7\times 10^{-5}}\right]\left[\frac{n(J=0)}{n({\rm CO})}\right]\left(\frac{2\,{\rm km\,s^{-1}}}{b}\right)\left(1 - e^{-5.532\,{\rm K}/{\rm T_{exc}}}\right)$$

Here, $N_{\rm H}$ is the column density of H nucleon.

Recall that *the cosmic abundance of carbon is* $n_{\rm C}/n_{\rm H} = 2.95\times 10^{-4}$ *and a fraction* $f_{\rm CO} \approx 0.25$ *of the carbon is in CO molecules.*

- The above equation seems to indicate that the CO J=1-0 line is optically thick. However, *we need to estimate the fraction of the CO that is in the J = 0 level.*

- The fraction of CO in a given rotational level J will be:

$$\frac{n(\mathrm{CO}, J)}{n(\mathrm{CO})} = \frac{(2J+1)e^{-B_0 J(J+1)/kT_{\mathrm{exc}}}}{\sum_J (2J+1)e^{-B_0 J(J+1)/kT_{\mathrm{exc}}}}$$

$$E_J = B_0 J(J+1)$$

Here, B_0 is the "rotation constant."

- We can approximate the partition function in the denominator by

$$\boxed{Z = \sum_{J=0}^{\infty} (2J+1)e^{-B_0 J(J+1)/kT_{\mathrm{exc}}} \approx \left[1 + (kT_{\mathrm{exc}}/B_0)^2\right]^{1/2}}$$

This approximation is exact in the limits $kT_{\mathrm{exc}}/B_0 \to 0$ and $kT_{\mathrm{exc}}/B_0 \gg 1$, and accurate to within 6% for all T_{exc}.

$$\tau_0 = 297 \left(\frac{N_{\mathrm{H}}}{10^{21}\,\mathrm{cm}^{-2}}\right) \left[\frac{n(\mathrm{CO})/n_{\mathrm{H}}}{7\times 10^{-5}}\right] \frac{\left(1 - e^{-5.532\,\mathrm{K}/\mathrm{T}_{\mathrm{exc}}}\right)}{\left[1 + (T_{\mathrm{exc}}/2.77\,\mathrm{K})^2\right]^{1/2}} \left(\frac{2\,\mathrm{km\,s}^{-1}}{b}\right)$$

Here, we used $B_0/k = 2.77\,\mathrm{K}$ for $^{12}\mathrm{C}^{16}\mathrm{O}$.

$$\tau_0 \approx 50 \left(\frac{N_{\mathrm{H}}}{10^{21}\,\mathrm{cm}^{-2}}\right) \left[\frac{n(\mathrm{CO})/n_{\mathrm{H}}}{7\times 10^{-5}}\right] \left(\frac{2\,\mathrm{km\,s}^{-1}}{b}\right)$$

for a typical CO rotation temperature $T_{\mathrm{exc}} \approx 8\,\mathrm{K}$.

- Thus, **the CO 1-0 transition is expected to be often quite optically thick.**

Radiative Trapping

- ***Radiative Trapping***:
  - In many situation of astrophysical interest, there is sufficient gas present so that, for some species X, a photon emitted in a transition $X_u \rightarrow X_\ell$ will have a high probability of being absorbed by another X_ℓ somewhere nearby, and, therefore, a low probability of escaping from the emitting region.
  - This phenomenon — referred to as radiative trapping — have two effects.

    (1) **it reduces the emission in the $X_u \rightarrow X_\ell$ photons emerging from the region**, and

    (2) **it acts to increase the level of excitation of species X** (relative to what it would be were the emitted photons to escape freely).
  - An exact treatment of the effects of radiative trapping is a complex problem of coupled radiative transfer and excitation — *it is nonlocal*, because photons emitted from one point in the cloud affect the level populations at other points.
  - The ***escape probability approximation*** is a simple way to take into account the effects of radiative trapping.

- ***Escape Probability Approximation***
  - Suppose that a some point $\mathbf{r}$ in the cloud, the optical depth $\tau_\nu(\hat{\mathbf{n}}, \mathbf{r})$ in direction $\hat{\mathbf{n}}$ and at frequency ν is known.
  - We define the direction-averaged "escape probability" for photons with frequency ν emitted from location $\mathbf{r}$:

$$\boxed{\bar{\beta}_\nu(\mathbf{r}) \equiv \int \frac{d\Omega}{4\pi} e^{-\tau_\nu(\hat{\mathbf{n}}, \mathbf{r})}}$$

Now define the direction-averaged and frequency-averaged escape probability:

$$\boxed{\langle \beta(\mathbf{r}) \rangle = \int \phi_\nu \bar{\beta}_\nu(\mathbf{r}) d\nu}$$

$$\phi_\nu = \text{normalized line profile}, \quad \int \phi_\nu d\nu = 1$$

  - Now we make **two approximations**:

(1) we will approximate **the excitation in the cloud as uniform**.

(2) we make the **"on-the-spot"** approximation: we assume that if a radiated photon is going to be absorbed, it will be absorbed so close to the point of emission that we can approximate it as being *absorbed (destroyed; not re-emitted)* at the point of emission.

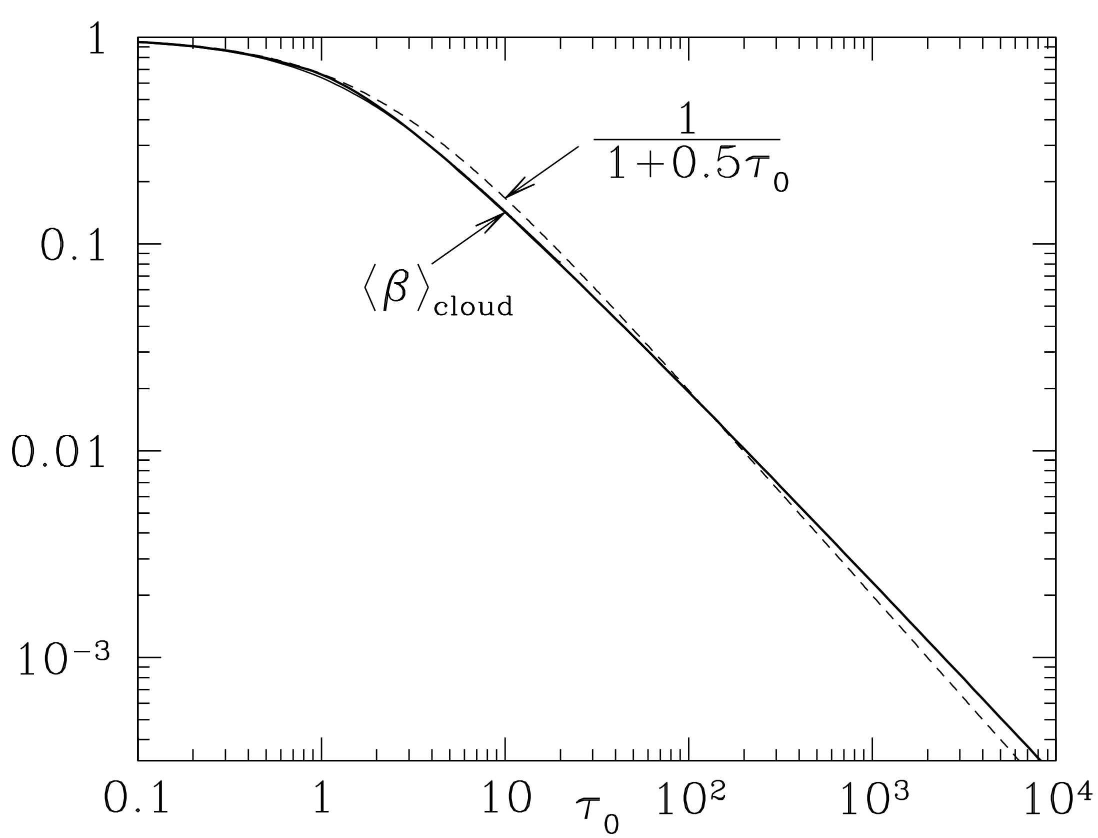
- These approximations replace a difficult nonlocal excitation problem with a much simpler local one! This is called the escape probability approximation.

Note that the "on-the-spot" approximation is not valid for resonance lines, such as Lyα.

- Homogeneous Static Spherical Cloud.
  - The angle-averaged escape probability $\bar{\beta}$ depends on the geometry and velocity structure of the region.

    For the case of a finite cloud, $\bar{\beta}$ will depend on position — it will be highest at the cloud boundary, and smallest at the cloud center.
  - We now define the escape probability averaged over the line profile and over the cloud volume, for a uniform density spherical cloud.

$$\boxed{\langle\beta\rangle_{\mathrm{cloud}} = \frac{1}{4\pi R^3/3}\int_0^R \langle\beta(\mathbf{r})\rangle\, 4\pi r^2 dr}$$



The left figure shows the mass-averaged escape probability calculated numerically for the case of a homogeneous spherical cloud, as a function of the optical depth τ_0 at line-center from the center of the cloud to the surface. The gas is assumed to have a Gaussian velocity distribution. [Fig 19.1, Draine]

A satisfactory approximation is provided by the simple fitting function.

$$\langle\beta\rangle_{\mathrm{cloud}} = \frac{1}{1+0.5\tau_0}$$

Critical Density of CO (J = 1-0)

- Conditions that are favorable for formation of CO are usually favorable for the formation of H_2.
  - Therefore, the main colliders that excite and de-excite the rotational states of CO are hydrogen molecules rather than hydrogen atoms or free electrons.
  - The collisional de-excitation rate coefficient of CO (J = 1) for collisions with H_2 collisions is

$$\boxed{k_{10} \approx 6 \times 10^{-11}\,(T/100\,\mathrm{K})^{0.2}\,\mathrm{cm}^3\,\mathrm{s}^{-1}}$$

- The Einstein A coefficient for a rotational transition J → J-1 is given by

$$\boxed{A_{J\to J-1} = \frac{128\pi^3}{3\hbar}\left(\frac{B_0}{hc}\right)^3 \mu_0^2 \frac{J^4}{J+1/2} = 7.16 \times 10^{-8}\,\mathrm{s}^{-1}\ \text{ for } J = 1 \to 0}$$

- At the fundamental frequency, the dominant background radiation is the CMB.
  - The good news is that the 2.6 mm line of CO is radiatively excited by blackbody radiation, which is analytically tractable.
  - The bad news is that we can't use the Rayleigh-Jeans limit, as we did studying the 21 cm line.
  - The photon occupation number is: $\bar{n}_\gamma = \dfrac{1}{e^{5.532\,\mathrm{K}/2.725\,\mathrm{K}} - 1} = 0.151 \qquad (T_{\mathrm{rad}} = T_{\mathrm{CMB}} = 2.725\,\mathrm{K})$

- The critical density at which collisional de-excitation equals to radiative de-excitation is given by

$$\boxed{n_{\mathrm{crit,H_2}}(\mathrm{CO}, J=1) = \frac{(1+\bar{n}_\gamma)A_{10}}{k_{10}} \approx 1400(T/100\,\mathrm{K})^{-0.2}\,\mathrm{cm}^{-3}}$$

- Radiative trapping
  - The proceeding analysis has assumed that only photons from the external radiation field are used, ignoring contributions from emission line photons from the cloud material itself. This assumption is only strictly true in an optically thin cloud.
  - For lines in abundant molecular species, like the CO J = 1-0 line, the cloud will be optically thick, and we will have to include the effect of the radiative trapping.
  - Taking into account the radiative trapping, the actual Einstein A coefficient is replaced by an "effective" value $\langle \beta_{u\ell} \rangle_{\text{cloud}} A_{u\ell}$ (see Chap 19.1 of Draine).
  - Then, the critical density for the CO J = 1-0 line is:

$$\boxed{n_{\text{crit},\text{H}_2}(\text{CO}, J=1) = \frac{(1+\bar{n}_\gamma)\, \langle \beta_{10} \rangle_{\text{cloud}}\, A_{10}}{k_{10}} \approx 54 (T/100\,\text{K})^{-0.2}\,\text{cm}^{-3}}$$

$$\tau_0 \approx 50, \quad \langle \beta_{10} \rangle_{\text{cloud}} \approx 1/(1+0.5\tau_0) \approx 0.04$$

for $N_{\text{H}} = 10^{21}\,\text{cm}^{-2}, b = 2\,\text{km}\,\text{s}^{-1}, T_{\text{exc}} = 8\,\text{K}$

  - We, therefore, see that at least **the J = 1 level of CO is expected to be thermalized in the central dense cores of molecular clouds with** $n_{\text{H}} \gtrsim 10^2\,\text{cm}^{-3}$.

Excitation Temperature of CO (J = 1-0)

- No approximation is valid for CO.
  - We can't use the handy $kT_{\rm exc} \ll h\nu_{u\ell}$ approximation that was useful for Lyα line.
  - Neither can we use the $kT_{\rm exc} \gg h\nu_{u\ell}$ approximation that was used for 21-cm line.
- The radiative transfer equation is the usual

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

  - Emissivity and absorption coefficient are

$$j_\nu = n_1 \frac{A_{10}}{4\pi} h\nu_{10}\phi_\nu \qquad \kappa_\nu = n_0 \frac{g_1}{g_0} \frac{c^2}{8\pi\nu_{10}^2} A_{10}\phi_\nu \left(1 - \frac{g_0}{g_1}\frac{n_1}{n_0}\right), \quad \frac{n_1}{n_0} = \frac{g_1}{g_0} \exp\left(-\frac{h\nu_{10}}{kT_{\rm exc}}\right)$$

  - The source function can be written as

$$S_\nu \equiv \frac{j_\nu}{\kappa_\nu} = \frac{2h\nu_{10}^3}{c^2} \frac{1}{\exp\left(h\nu_{10}/kT_{\rm exc}\right) - 1} = B_\nu(T_{\rm exc})$$

  - If the excitation (rotation) temperature is constant over the entire region of emission, we have the solution:

$$I_\nu = I_\nu(0)e^{-\tau_\nu} + S_\nu\left(1 - e^{-\tau_\nu}\right)$$

  - The background radiation is the Cosmic Microwave Background: $I_\nu(0) = B_\nu(T_{\rm rad})$

- Now, we do perform the "on" observation toward a molecular cloud, and "off" observation over to a black sky.

cloud: $$I_\nu(\mathrm{on}) = \frac{2h\nu^3}{c^2}\left[\frac{e^{-\tau_\nu}}{\exp\left(h\nu/kT_{\mathrm{rad}}\right)-1} + \frac{1-e^{-\tau_\nu}}{\exp\left(h\nu/kT_{\mathrm{exc}}\right)-1}\right]$$

CMB: $$I_\nu(\mathrm{off}) = \frac{2h\nu^3}{c^2}\frac{1}{\exp\left(h\nu/kT_{\mathrm{rad}}\right)-1}$$

The difference between the two observations is

$$\begin{aligned}\Delta I_\nu &= I_\nu(\mathrm{on}) - I_\nu(\mathrm{off}) \\ &= \frac{2h\nu^3}{c^2}\left[\frac{1}{\exp\left(h\nu/kT_{\mathrm{exc}}\right)-1} - \frac{1}{\exp\left(h\nu/kT_{\mathrm{rad}}\right)-1}\right]\left(1-e^{-\tau_\nu}\right)\end{aligned}$$

- It is customary to express the intensity in terms of an antenna temperature:

$$T_{\mathrm{A}} \equiv \frac{c^2}{2k}\frac{\Delta I_\nu}{\nu^2} \approx \frac{h\nu}{k}\left[\frac{1}{\exp\left(h\nu/kT_{\mathrm{exc}}\right)-1} - \frac{1}{\exp\left(h\nu/kT_{\mathrm{rad}}\right)-1}\right]$$ for an optical thick cloud $\tau_\nu \gg 1$.

We can solve this for excitation temperature:

$$\frac{5.532\,\mathrm{K}}{T_{\mathrm{exc}}} = \ln\left(1 + \frac{1}{T_{\mathrm{A}}/5.532\,\mathrm{K} + 0.151}\right)$$

$$h\nu/k = 5.532\,\mathrm{K}$$
$$T_{\mathrm{rad}} = 2..725\,\mathrm{K}$$
$$\frac{1}{\exp\left(h\nu/kT_{\mathrm{rad}}\right)-1} = 0.151$$

- In our Galaxy, it is often found that $T_{\mathrm{A}} \sim 5\,\mathrm{K}$ at the line center, implying $T_{\mathrm{exc}} \sim 8\,\mathrm{K}$.

^{13}CO Column Density

- The ^{12}CO 2.6 mm line is optically thick. All the 2.6 mm emission that we see comes from a thin skin at the surface of the molecular cloud.
  - The column density of CO can be more easily derived from the optically thin ^{13}CO line. The rarer isotopes will refer to gas deeper in the molecular cloud.
  - The cosmic ratio of ^{13}CO to ^{12}CO is ~ 0.011. Hence, ^{13}CO is optically thin.
  - The frequency of ^{13}CO J = 1-0 transition is 110.20 GHz, 4.6% lower than that for ^{12}CO.

  ratio of the reduced masses:

$$\frac{\mu(^{13}\mathrm{C}^{16}\mathrm{O})}{\mu(^{12}\mathrm{C}^{16}\mathrm{O})} = \frac{13\times 16/(13+16)}{12\times 16/(12+16)} \approx 1.0460 \qquad \frac{B_0(^{13}\mathrm{C}^{16}\mathrm{O})}{B_0(^{12}\mathrm{C}^{16}\mathrm{O})} = \frac{\mu(^{12}\mathrm{C}^{16}\mathrm{O})}{\mu(^{13}\mathrm{C}^{16}\mathrm{O})} \approx 1/1.0460 = 0.9560$$

  - We assume that $T_{\rm exc}$ of ^{13}CO is the same as that of ^{12}CO. This is generally a safe assumption, since the two isotopes of carbon are intermingled in the ISM.

$$T_{\rm A}(^{13}\mathrm{CO}) \approx \frac{h\nu}{k}\left[\frac{1}{\exp\left(h\nu/kT_{\rm exc}\right)-1} - \frac{1}{\exp\left(h\nu/kT_{\rm rad}\right)-1}\right]\tau_\nu \qquad \leftarrow (1-e^{-\tau_\nu}) \approx \tau_\nu$$

$h\nu/k = 5.140\,\mathrm{K}$ for the ^{13}CO J = 1-0 transition.

  - The integrated line strength is then

$$\int T_{\rm A}(^{13}\mathrm{CO})dv \approx 5.140\,\mathrm{K}\left(\frac{1}{e^{5.140\,\mathrm{K}/T_{\rm exc}}-1} - 0.1787\right)\int \tau_\nu dv$$

- The optical depth, integrated over the thermally broadened line, is

$$\int \tau_\nu dv = \sqrt{\pi} b \tau_0 = \frac{1}{8\pi} \frac{g_1}{g_0} \frac{c^3}{\nu^3} A_{10} \left(1 - e^{-h\nu/kT_{\rm exc}}\right) N_0(^{13}{\rm CO})$$

$$\tau_0 = \frac{g_1}{g_0} \frac{c^2}{8\pi^{3/2}\nu_{10}^3} A_{10} \frac{c}{b} \left(1 - e^{-h\nu_{10}/kT_{\rm exc}}\right) N_0$$

Here, $N_0(^{13}{\rm CO})$ is the column density of ^{13}CO in its rotational ground state with J = 0.

Note: $$\begin{aligned} A_{10} &= 7.20 \times 10^{-8}\,{\rm s}^{-1} \quad \text{for } ^{12}{\rm CO} \\ &= 6.29 \times 10^{-8}\,{\rm s}^{-1} \quad \text{for } ^{13}{\rm CO} \end{aligned}$$

$$\int \tau_\nu dv \approx 1.52 \times 10^{-15} \left(1 - e^{-5.140\,{\rm K}/T_{\rm exc}}\right) N_0(^{13}{\rm CO}) \left[\frac{\rm km\,s^{-1}}{\rm cm^{-2}}\right]$$

- Therefore, we find:

$$\boxed{N_0(^{13}{\rm CO}) \approx 2.9 \times 10^{14} \frac{\int T_{\rm A}(^{13}{\rm CO}) dv}{1\,{\rm K\,km\,s^{-1}}} \left[{\rm cm^{-2}}\right] \quad \text{when} \quad T_{\rm exc} \approx 8\,{\rm K}}$$

X-factor (^{13}CO to H_2)

- X-factor (^{13}CO to H_2)
  - The fraction of ^{13}CO in its rotational ground state:

$$f_{J=0} = \frac{1}{\sum_J (2J+1) e^{-B_0 J(J+1)/kT_{\rm exc}}} \approx \left[\frac{1}{1 + \left(\frac{T_{\rm exc}}{2.766/1.0460\,{\rm K}} \right)^2} \right]^{1/2} \approx 0.314$$

  - Then, we obtain the CO to H_2 ratio:

$$N({\rm H_2}) = 1.8 \times 10^6 \times \frac{1}{2} \frac{n_{\rm H}/n_{\rm C}}{(1/3.0 \times 10^{-4})} \frac{n({\rm C})/n(^{12}{\rm CO})}{4} \frac{n(^{12}{\rm CO})/n(^{13}{\rm CO})}{90} \frac{N(^{13}{\rm CO})/N_0(^{13}{\rm CO})}{3} N_0(^{13}{\rm CO})$$

$$N({\rm H_2}) \approx 5 \times 10^{20} \left[\frac{\int T_{\rm A}(^{13}{\rm CO}) dv}{1\,{\rm K\,km\,s^{-1}}} \right] \quad [{\rm cm^{-2}}]$$

$$X_{^{13}{\rm CO}} \equiv \frac{N({\rm H_2})}{W_{^{13}{\rm CO}}} \approx 5 \times 10^{20} \left[\frac{\rm cm^{-2}}{\rm K\,km\,s^{-1}} \right]$$

$$W_{^{13}{\rm CO}} \equiv \int T_{\rm A}(^{13}{\rm CO}) dv$$

X-factor (^{12}CO to H_2) - Correct Approach

(Chapter 19 of Draine's book)

- X-factor
  - It would be useful if we had a reliable way to convert from something easy to measure (the antenna temperature integrated over the line of ^{12}CO) to something difficult to measure (the column density N(H_2) of molecular hydrogen).

$$W_{\rm CO} \equiv \int T_{\rm A}(^{12}{\rm CO})dv \qquad\qquad X_{\rm CO} \equiv \frac{N({\rm H}_2)}{W_{\rm CO}}$$

  - If the cloud is larger than our antenna beam, we can relate $W_{\rm CO}$ to $N({\rm H}_2)$.
    - Recall that the optical depth at line-center from center to edge of the spherical cloud:

$$\tau_0 = \frac{g_1}{g_0}\frac{A_{10}c^2}{8\pi^{3/2}\nu_{10}^3}\frac{c}{b}\left(1 - e^{-h\nu_{10}/kT_{\rm exc}}\right)n_0 R$$

Use the virial theorem:

$$b = \left(\frac{2G}{5}\frac{M}{R}\right)^{1/2}$$

$$\tau_0 = \frac{g_1}{g_0}\frac{A_{10}\lambda_{10}^3}{8\pi}\left(\frac{5}{2\pi G}\right)^{1/2}\frac{n_0 R^{3/2}}{M^{1/2}}\left(1 - e^{-h\nu_{10}/kT_{\rm exc}}\right)$$

    - The luminosity of the cloud in a spectral line (J = 1-0) is:

$$L_{10} = \int dr 4\pi r^2 n_1 A_{10} h\nu_{10} \langle\beta\rangle_{\rm cloud}$$
$$= \int dV 4\pi j_{10} \langle\beta\rangle_{\rm cloud}$$

Emissivity: $4\pi j_{10} = n_1 A_{10} h\nu_{10}$

Escape probability averaged over the line profile and over the cloud volume:

$$\langle\beta\rangle_{\rm cloud} = \frac{1}{4\pi R^3/3}\int_0^R \langle\beta(\mathbf{r})\rangle 4\pi r^2 dr$$

- Using the escape probability $\langle\beta\rangle_{\rm cloud} \approx \dfrac{1}{1+0.5\tau_0}$

$$L_{10} \approx \frac{4\pi}{3} R^3 n_1 A_{10} h\nu_{10} \frac{2}{\tau_0}$$

Here, we assume $\tau_0 \gg 1$, thus $\langle\beta\rangle_{\rm cloud} \approx 2/\tau_0$.

$$L_{10} \approx \frac{64\pi^2}{3} \left(\frac{2\pi G}{5}\right)^{1/2} \frac{hc}{\lambda_{10}^4} M^{1/2} R^{3/2} \frac{n_1}{n_0} \frac{g_0}{g_1} \frac{1}{1-e^{-h\nu_{10}/kT_{\rm exc}}}$$

$$\approx \frac{64\pi^2}{3} \left(\frac{2\pi G}{5}\right)^{1/2} \frac{hc}{\lambda_{10}^4} M^{1/2} R^{3/2} \frac{1}{e^{h\nu_{10}/kT_{\rm exc}}-1}$$

$$\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{h\nu_{10}/kT_{\rm exc}}$$

- The line luminosity per unit mass is:

$$\boxed{\frac{L_{10}}{M} \approx 32\pi^2 \left(\frac{2G}{15}\right)^{1/2} \frac{hc}{\lambda_{10}^4} \frac{1}{\rho^{1/2}} \frac{1}{e^{h\nu_{10}/kT_{\rm exc}}-1}} \longleftarrow \quad M = \frac{4\pi R^3}{3}\rho$$

Note that for a give spectral line, **the luminosity per unit mass depends on the cloud density and on the excitation temperature.**

- Now, we relate (1) the mass to the average column density of hydrogen nucleon and (2) the luminosity to the average antenna (brightness) temperature.

Hydrogen column density: $N_{\rm H} = \dfrac{\Sigma}{1.4 m_{\rm H}} = \dfrac{\frac{M}{\pi R^2}}{1.4 m_{\rm H}}$

$\Sigma =$ surface density

Antenna temperature averaged over the projected surface area:

$$\int T_{\rm A} dv \equiv \frac{\int\int \frac{c^2}{2k\nu^2} I_\nu dv dA \left<\beta\right>_{\rm cloud}}{\int dA} \qquad \leftarrow I^{\rm obs} = I^{\rm intrinsic} \left<\beta\right>$$

Here,

$dA =$ projected area

$ds =$ pathlength along a sightline

From the radiative transfer equation: $I_\nu = \int j_\nu ds$

$$\boxed{\begin{aligned} \int T_{\rm A} dv &= \frac{1}{\pi R^2} \int \left[\int \frac{c^2}{2k\nu^2} j_\nu \left<\beta\right>_{\rm cloud} dv \right] dV \\ &= \frac{1}{\pi R^2} \frac{1}{4\pi} \frac{c^3}{2k\nu_{10}^3} L_{10} \end{aligned}}$$

$$\frac{v}{c} \equiv \frac{\nu - \nu_{10}}{\nu_{10}}$$

$$j_\nu dv = j_\nu \left| \frac{dv}{d\nu} \right| d\nu = j_\nu \frac{c}{\nu_{10}} d\nu$$

- Then, we can relate the antenna temperature to the total H column density (averaged over the beam solid angle):

$$\begin{aligned} \frac{N_{\rm H}}{\int T_{\rm A} dv} &= \frac{8\pi k}{1.4 m_{\rm H}} \frac{1}{\lambda_{10}^3} \frac{M}{L_{10}} \\ &= \frac{1}{4\pi} \frac{k\lambda_{10}}{hc} \left(\frac{15}{2.8 G m_{\rm H}} \right)^{1/2} (n_{\rm H})^{1/2} \left(e^{h\nu_{10}/kT_{\rm exc}} - 1 \right) \end{aligned}$$

Here, we used:
$\rho = 1.4 \times m_{\rm H} n_{\rm H}$

- If we assume $N(\mathrm{H_2}) = N_\mathrm{H}/2$, the above equation becomes:

$$X_\mathrm{CO} = \frac{N(\mathrm{H_2})}{\int T_\mathrm{A} dv} = \frac{1}{8\pi}\frac{k\lambda_{10}}{hc}\left(\frac{15}{2.8 G m_\mathrm{H}}\right)^{1/2} (n_\mathrm{H})^{1/2}\left(e^{h\nu_{10}/kT_\mathrm{exc}} - 1\right)$$
$$= 1.58 \times 10^{20} \left(\frac{n_\mathrm{H}}{10^3\,\mathrm{cm^{-3}}}\right)^{1/2}\left(e^{5.532\,\mathrm{K}/T_\mathrm{exc}} - 1\right)\left[\frac{\mathrm{cm^{-2}}}{\mathrm{K\,km\,s^{-1}}}\right]$$

- For $n_\mathrm{H} = 10^3\,\mathrm{cm^{-3}}$ and $T_\mathrm{exc} = 8\,\mathrm{K}$, this yields:

$$\boxed{X_\mathrm{CO} = \frac{N(\mathrm{H_2})}{\int T_\mathrm{A} dv} = 1.56 \times 10^{20} \left(\frac{n_\mathrm{H}}{10^3\,\mathrm{cm^{-3}}}\right)^{1/2}\left[\frac{\mathrm{cm^{-2}}}{\mathrm{K\,km\,s^{-1}}}\right]}$$

Dame et al. (2001) observationally find $X_\mathrm{CO} = (1.8 \pm 0.3) \times 10^{20}\,\mathrm{cm^{-2}/K\,km\,s^{-1}}$, where infrared emission from dust (Schlegel et al. 1998) was used as a mass tracer.

The most recent determination using gamma rays finds

$X_\mathrm{CO} = (1.76 \pm 0.04) \times 10^{20}\,\mathrm{cm^{-2}/K\,km\,s^{-1}}$ for the Orion A GMC (Okumura et al. 2009)

These results suggests that $n_\mathrm{H} = 10^3\,\mathrm{cm^{-3}}$ and $T_\mathrm{exc} = 8\,\mathrm{K}$ may be representative of self-gravitating molecular clouds in the local ISM.

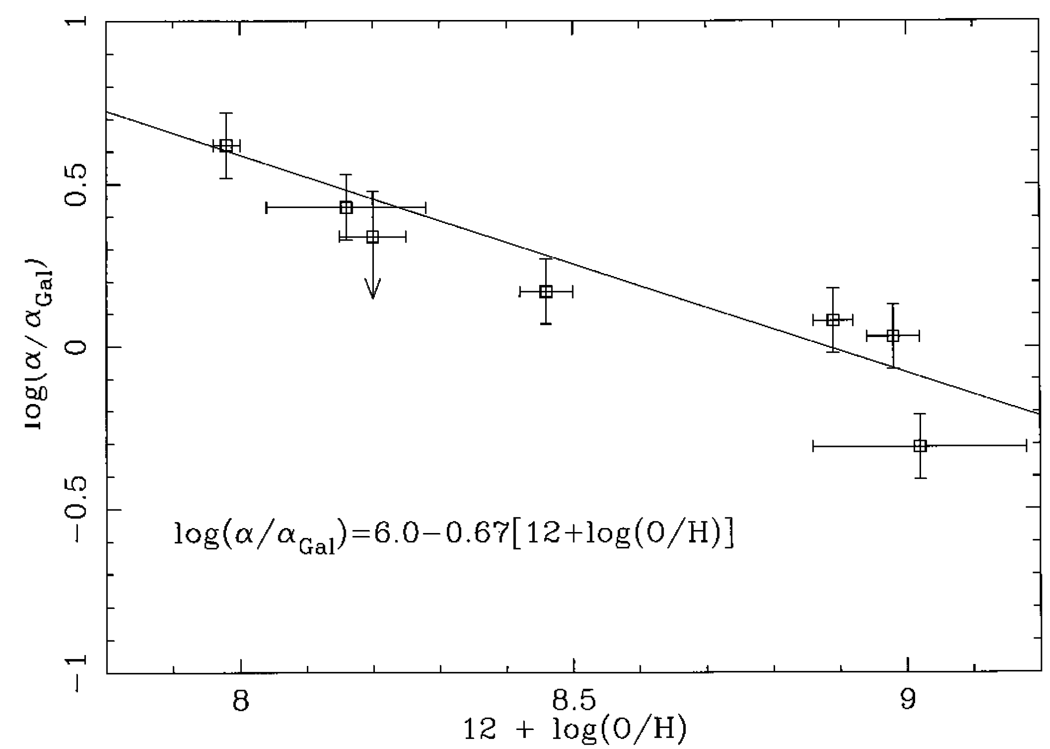
- Notes:
  - *Real molecular clouds have an outer layer where the hydrogen is molecular, but where the CO abundance is very low* because it is not sufficiently shielded from dissociating radiation.
  - *This outer layer does not show up in H I 21-cm or CO 1-0 surveys* — leading Wolfire et al. (2010) to refer to it as “the dark molecular gas” — but since it does contain dust, it does contribute to the Far-IR emission.
  - Wolfire et al. (2010) estimate that *~30% of the molecular mass in a typical Galactic molecular cloud may be in this envelope.*
  - At constant n_{H} and T_{exc}, *this low-CO layer will result in an increase in X_{CO}.* This is in agreement with recent observations.
  - **Other galaxies:** Recent observations indicates that X_{CO} in Local Group galaxies is of order

$$X_{\mathrm{CO}} \sim 4 \times 10^{20}\,\mathrm{cm}^{-2}\,/\mathrm{K\,km\,s}^{-1}$$ (Blitz et al. 2007)

with large variations from one galaxy to another.

An indirect estimate of the molecular gas mass based on modeling the IR emission from a sample of nearby galaxies also favored $X_{\mathrm{CO}} \approx 4 \times 10^{20}\,\mathrm{cm}^{-2}\,/\mathrm{K\,km\,s}^{-1}$ (Draine et al. 2007).

Metallicity dependence of the CO-to-H_2 conversion factor



The average CO-to-H_2 conversion factor in five Local Group galaxies (M31, M33, IC 10, NGC 6822, SMC) is plotted as a function of the oxygen abundance.

[Fig 2, Wilson, 1995, 448, L97]

X-factor (^{12}CO to H_2) - Approach of Ryden

- X-factor
  - Since a typical molecular cloud is optically thick at the J = 1-0 rotational transition of ^{12}CO, we use the radiative transfer solution for an opaque object:

$$I_\nu = \frac{2h\nu^3}{c^2}\left[\frac{e^{-\tau_\nu}}{\exp\left(h\nu/kT_{\rm rad}\right)-1} + \frac{1-e^{-\tau_\nu}}{\exp\left(h\nu/kT_{\rm exc}\right)-1}\right] \quad \rightarrow \quad \boxed{I_\nu \approx \frac{2h\nu^3}{c^2}\frac{1}{\exp\left(h\nu/kT_{\rm exc}\right)-1}}$$

  - Expressed in terms of the antenna temperature, this is

$$T_{\rm A} \equiv \frac{c^2}{2k\nu^2}I_\nu \approx \frac{h\nu}{k}\frac{1}{\exp\left(h\nu/kT_{\rm exc}\right)-1}$$

However, this is wrong by a geometrical factor because we are dealing with a sphere. Toward outer boundary of the sphere, the optical depth will be thin.

  - The observed emission lines is narrow, we can assume that $\nu = \nu_{10}$ over the entire line width.

$$\int T_{\rm A} dv \approx \sqrt{\pi}bT_{\rm A}(\nu_{10}) \approx \sqrt{\pi}\frac{h\nu_{10}}{k}\frac{b}{\exp\left(h\nu_{10}/kT_{\rm exc}\right)-1}$$

Here, we assume a Gaussian line profile

$$T_{\rm A} = T_{\rm A}(0)e^{-v^2/b^2}$$

  - Then, the X-factor is

$$X_{\rm CO} = \frac{N({\rm H_2})}{b}\frac{1}{\sqrt{\pi}}\frac{k}{h\nu_{10}}\left(e^{h\nu_{10}/kT_{\rm exc}}-1\right)$$

- Now let's use virial theorem for a uniform, spherical cloud:

$$\sigma_v^2 = \frac{GM_c}{5R_c},\ b = \sqrt{2}\sigma_v \quad \rightarrow \quad \boxed{M_c = \frac{5}{2}\frac{R_c b^2}{G}} \qquad M_c \equiv \frac{4\pi}{3}R_c^3\rho_c \quad \rightarrow \quad \boxed{\frac{b}{R_c} = \left(\frac{8\pi}{15}G\rho_c\right)^{1/2}}$$

- Surface density:

$$\Sigma = \frac{M_c}{\pi R_c^2} \xrightarrow{M_c = \frac{5}{2}\frac{R_c b^2}{G}} \Sigma = \frac{5}{2\pi G}\frac{b^2}{R_c}$$

- Column density of hydrogen nucleon in terms of velocity dispersion:

$$N(\mathrm{H}) \approx \frac{\Sigma}{1.4 \times m_{\mathrm{H}}} = \frac{5}{2.8\pi}\frac{1}{Gm_{\mathrm{H}}}\frac{b^2}{R_c} \qquad N(\mathrm{H}_2) = \frac{1}{2}N(\mathrm{H})$$

- The X-factor is:

$$\frac{N(\mathrm{H}_2)}{W_{\mathrm{CO}}} = \frac{5}{2 \times 2.8\pi\sqrt{\pi}}\frac{1}{Gm_{\mathrm{H}}}\frac{k}{h\nu_{10}}\frac{b}{R_c}\left(e^{h\nu_{10}/kT_{\mathrm{exc}}} - 1\right)$$

$$\frac{b}{R_c} = \left(\frac{8\pi}{15}G\rho_c\right)^{1/2} \approx \left(\frac{8\pi}{15}Gm_{\mathrm{H}}n_{\mathrm{H}} \times 1.4\right)^{1/2}$$

$$\frac{N(\mathrm{H}_2)}{W_{\mathrm{CO}}} = \frac{1}{2\pi}\frac{k}{h\nu_{10}}\left(\frac{20}{3}\frac{1}{2.8Gm_{\mathrm{H}}}\right)^{1/2} n_{\mathrm{H}}^{1/2}\left(e^{h\nu_{10}/kT_{\mathrm{exc}}} - 1\right)$$

This result gives a larger value than the correct one by a factor of 8/3 = 2.67.

Molecular Clouds: Chemistry and Ionization

- Molecular Hydrogen
  - In the Milky Way, **~ 22% of the interstellar gas is in molecular clouds**, where the bulk of the hydrogen is in H_2 molecules.
  - H_2 is formed primarily by due grain catalysis. Destruction of H_2 is primarily due to photodissociation, but self-shielding results in very low photodissociation rates in the central regions of molecular clouds.

- Other molecules
  - Once, the H_2 has been formed, other chemistry can follow.
  - Most of the gas will be neutral, but, *because of the presence of cosmic rays, there will always be some ions present in the gas.*
  - In the outer layers of molecular clouds, there may also be sufficient UV radiation to photoionize species with ionization potentials $E_I < 13.6\,\mathrm{eV}$.

- Five types of reactions
  - ***Photoionization***: $AB + h\nu \rightarrow AB^{+} + e^{-}$
    - Many molecules (but not H_2) can be photoionized by photons with $h\nu < 13.6\,\mathrm{eV}$ present in the ISRF in H I or diffuse H_2 clouds.
    - The ionization threshold for H_2 is 15.43 eV, hence H_2 will not be photoionized even in H I regions.
  - ***Photodissociation***: $AB + h\nu \rightarrow (AB)^{*} \rightarrow A + B$
    - For some molecules (including H_2 and CO), photoexciation leading to ***dissociation occurs via lines rather than continuum***.
    - This allows such species either to self-shield (H_2), or to be partially shielded if there is accidental overlap between important absorption lines with strong lines of H_2, which will be self-shielded. This is the case with CO.
  - ***Neutral-neutral exchange reactions***: $AB + C \rightarrow AC + B$
    - Most species in molecular clouds are neutral, and neutral-neutral collisions are, therefore, frequent.
    - However, even when a neutral-neutral reaction is exothermic, there will often (but not always) be an energy barrier that must be overcome for the reaction to proceed.

- For instance,

The following exothermic reaction has an energy barrier E/k_{B} = 1950 K, which causes the reaction to be negligibly slow at T < 100 K.

$$\mathrm{OH} + \mathrm{H} \rightarrow \mathrm{O} + \mathrm{H_2} \qquad k = 1.5 \times 10^{-12}\, T_2^{2.8} e^{-19.50/T_2}\, [\mathrm{cm^3\, s^{-1}}]\ (300\,\mathrm{K} < T < 2500\,\mathrm{K})$$

On the other hand, the reaction responsible for CO formation does not have any significant energy barrier, and can proceed at the low temperatures of molecular clouds.

$$\mathrm{C} + \mathrm{OH} \rightarrow \mathrm{CO} + \mathrm{H} \qquad k = 1 \times 10^{-10}\, [\mathrm{cm^3\, s^{-1}}]\ (10\,\mathrm{K} < T < 300\,\mathrm{K})$$

- ***Ion-neutral exchange reactions***: $AB^+ + C \rightarrow AC^+ + B$

- Ion-neutral reactions are very important in interstellar chemistry, for two reasons:

(1) exothermic reactions generally lack energy barriers, allowing the reaction to proceed rapidly even at low temperature, and

(2) the induced-dipole interaction results in ion-neutral rate coefficients that are relatively large, of order $\sim 2\times10^{-9}$ cm^3 s^{-1}.

- ***Radiative association reactions***: $$A + B \underset{k_d}{\overset{k_f}{\rightleftharpoons}} (AB)^* \xrightarrow{k_r} AB + h\nu$$

  - The reaction first creates an excited complex. The complex will fly apart in one vibrational period (~ 10^{-14} s), unless a photon is emitted first; it can be thought of as having a probability per unit time $k_d \approx 10^{14}\,\mathrm{s}^{-1}$ of spontaneously dissociating.
  - If AB has an appreciable electric dipole moment (i.e., is not homonuclear), the probability per unit time that (AB)* will emit a photon in an electronic transition is $k_r \approx 10^6\,\mathrm{s}^{-1}$. Thus, a fraction $k_r/(k_r + k_d)$ of the (AB)* complexes formed will result in formation of stable AB.
  - Then, the effective rate coefficient for radiative association will be:

$$k_{\rm ra} = k_f \frac{k_r}{k_r + k_d}$$

For ion-neutral reactions, the typical rate coefficient of "complex formation" (AB)* is $k_f \approx 10^{-9}[\mathrm{cm^3\,s^{-1}}]$, and hence the rate coefficient for radiative association will be $k_{\rm ra} \approx 10^{-17}\,[\mathrm{cm^3\,s^{-1}}]$.

  - For example, formation of CH^+ by radiative association:

$$\mathrm{C^+ + H \rightarrow CH^+} + h\nu \qquad k = 4.45 \times 10^{-17}\, T_2^{-1/2} e^{-0.229 T_2^{-1/2}}\, [\mathrm{cm^3\,s^{-1}}]$$

Formation of CO

- In diffuse molecular clouds, most of the gas-phase carbon is in the form of C^+, and most of the H is in the form of H_2 .
- Carbon chemistry is initiated by the radiative association reaction of C^+ with H_2 forming CH_2^+.
- CO is formed primarily by the sequence of the following reactions:

$$C^+ + H_2 \to CH_2^+ + h\nu \quad , \quad k_{33.6} = 5.0 \times 10^{-16} T_2^{-0.2}\,\mathrm{cm}^3\,\mathrm{s}^{-1} \ , \tag{33.6}$$

$$CH_2^+ + e^- \to \left\{ \begin{array}{ll} CH + H & (25\%) \\ C + H_2 & (12\%) \\ C + H + H & (63\%) \end{array} \right\} , k_{33.7} = 1.24 \times 10^{-6} T_2^{-0.60} \frac{\mathrm{cm}^3}{\mathrm{s}} , \tag{33.7}$$

$$CH + O \to CO + H \quad k_{33.8} = 6.6 \times 10^{-11}\,\mathrm{cm}^3\,\mathrm{s}^{-1} \ , \tag{33.8}$$

$$CO + h\nu \to C + O \quad k_{33.9} = 2.3 \times 10^{-10}\,\mathrm{s}^{-1} \times f_{\mathrm{shield}}(CO) \ , \tag{33.9}$$

$$C + h\nu \to C^+ + e^- \quad k_{33.10} = 2.6 \times 10^{-10}\,\mathrm{s}^{-1} \ . \tag{33.10}$$

From page 377 in [Draine]

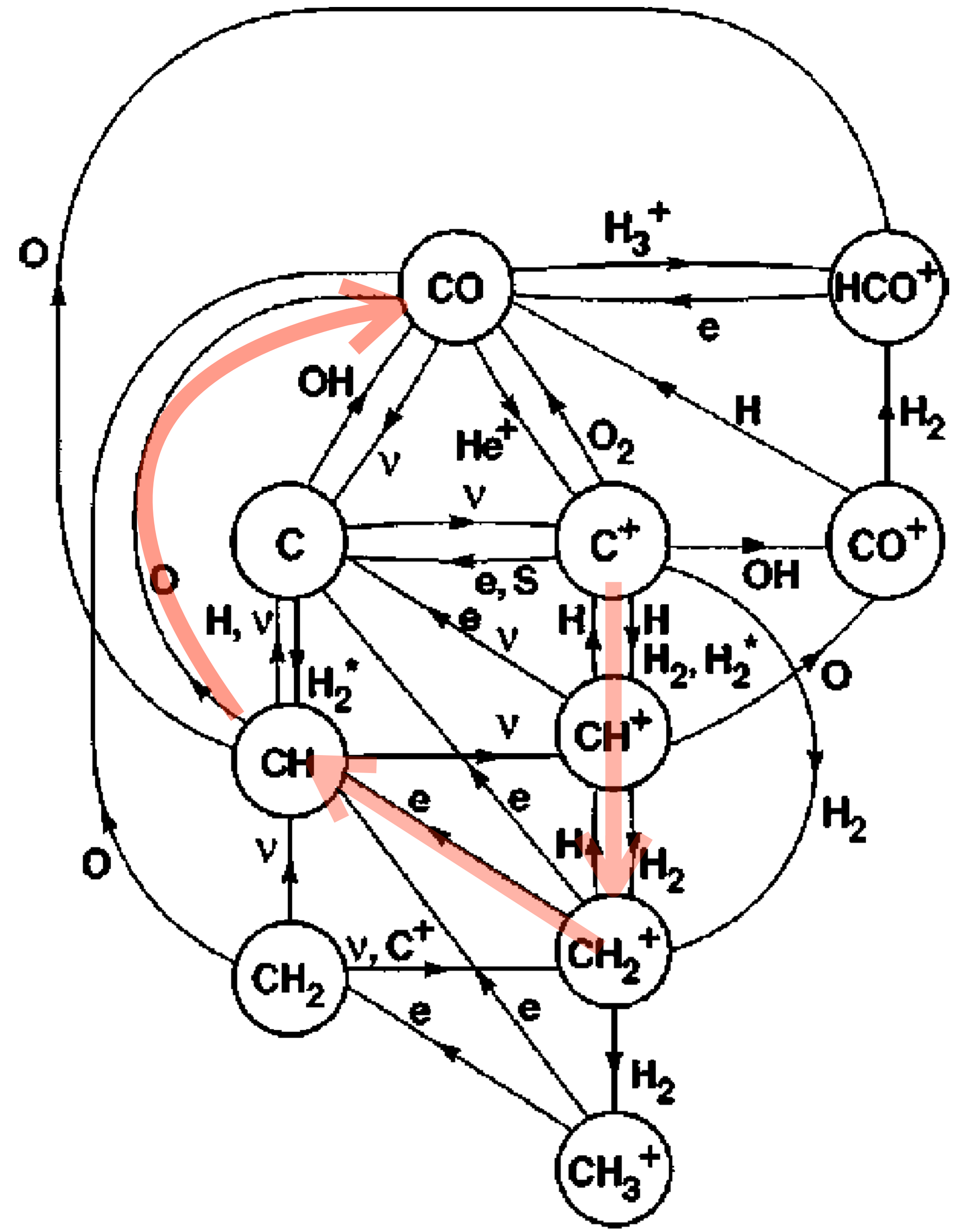


Fig 8.9, Tielens [The physics and chemistry of the interstellar medium]

Formation of OH and H_2O

- In diffuse H I clouds, cosmic-ray ionization produces H^+ — an important starting point of the O chemistry. Charge exchange of neutral O with H^+ yields O^+. Reaction of O^+ with H_2 then form OH^+.
- Deeper in the cloud, where H_2 is an important species, cosmic-ray ionization of H_2 leads to the formation of H_3^+, which readily reacts with O to form OH^+.
- This sequence of gas-phase reaction convert the gas-phase O to H_2O, yielding: $n(\mathrm{H_2O})/n_\mathrm{H} \approx 10^{-4}$
- However, observations indicate very low gas-phase abundance of H_2O:

$$n(\mathrm{H_2O})/n_\mathrm{H} \lesssim 3 \times 10^{-8}$$

far below the predicted abundance of H_2O.

- The gas-phase H_2O is removed by freeze-out on grains, producing H_2O ice mantles which can contain a substantial fraction of the total oxygen.

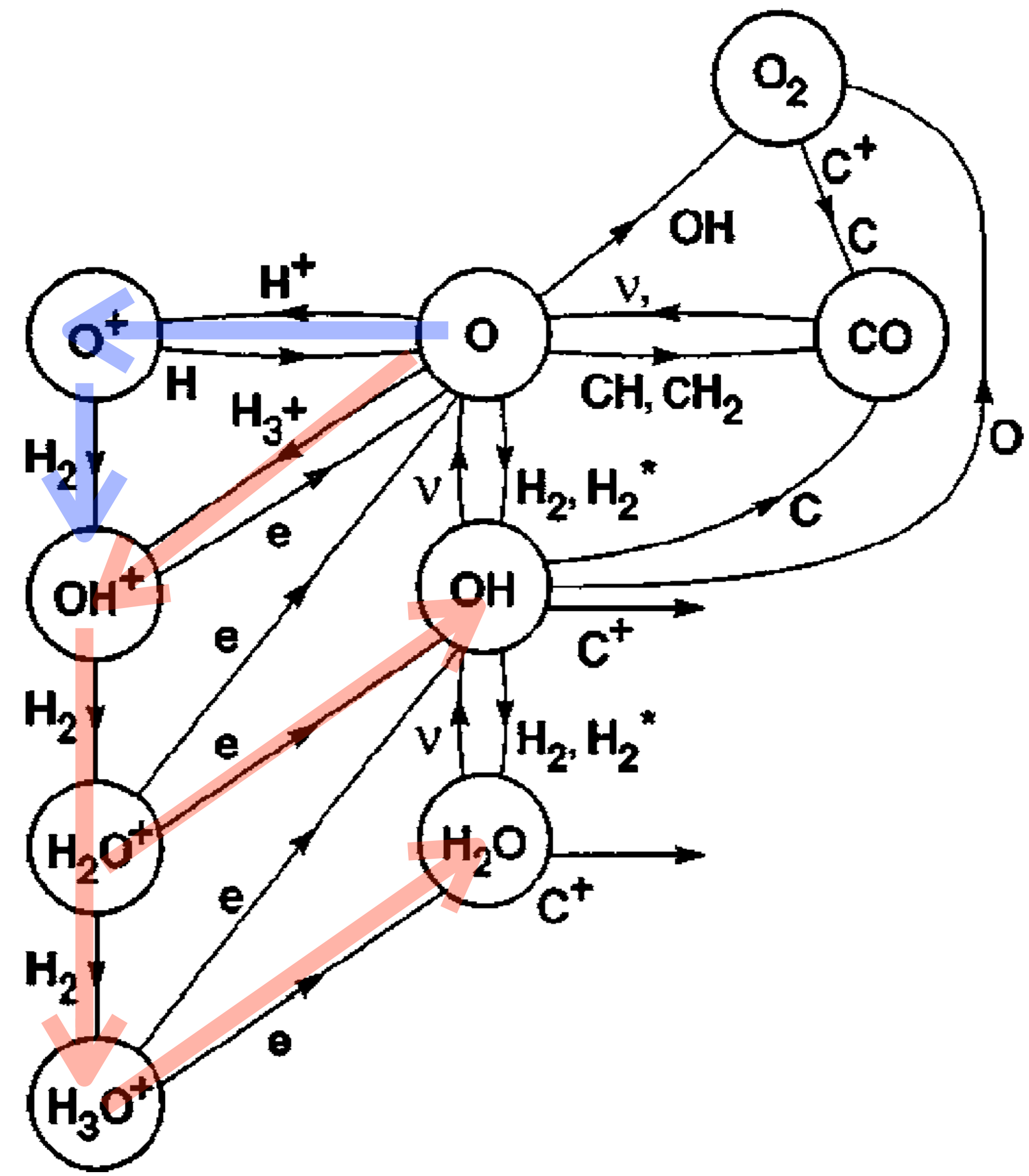


Fig 8.8, Tielens [The physics and chemistry of the interstellar medium]

Circumgalactic & Intracluster Medium

Introduction

- CGM and ICM
  - The ***circumgalactic medium*** is the diffuse gas (and some dust) that lies outside the main body of a galaxy's stellar distribution, but still lies inside the virial radius of the galaxy's dark halo.
  - The ***intracluster medium*** is the diffuse hot gas that lies inside the virial radius of a cluster of galaxies, but is not bound to any particular galaxy in the cluster.
- **Virial radius**
  - The ***virial*** radius of a galaxy or cluster is often defined as the radius inside which the virial theorem holds true, and where we can safely make a mass estimate by using:

$$M \sim \frac{\sigma_v^2 r}{G}$$

$$2\langle K \rangle + \langle U \rangle = 0$$

Recall the virial relation for a uniform, self-gravitating system: $M = \frac{5\sigma_v^2 R}{G}$

  - See the previous lecture note to see the proof of the viral theorem.
  - In a numerical simulation, finding the virial radius is fairly simple.
    - Just look inside a particular radius and see if the velocity dispersion, radius, and mass follow the virial relation.

- In reality, approximations must be made. The virial radius is usually approximated as ***the radius*** r_{vir} ***within the mean density*** $\rho(r_{\text{vir}})$ ***is equal to 200 times the critical density*** ρ_c ***of the universe***:

$$\rho_{\rm c} \equiv \frac{3H^2}{8\pi G}$$

$$\rho(r_{\rm vir}) = 200\rho_{\rm c}$$

Here, H = the Hubble parameter
G = the Gravitational constant

- Critical density: the mean density of matter when the overall geometry of the universe is flat (Euclidean).
- From the 2015 Planck results, the current values of the Hubble parameter and the critical density are:

$$H_0 = 67.74 \pm 0.46\ [{\rm kpc\,s^{-1}\,Mpc^{-1}}]$$
$$\rho_{\rm c,0} = (8.62 \pm 0.12) \times 10^{-30}\ [{\rm g\,cm^{-3}}]$$

Note that $\rho_{\rm c,0}$ corresponds to 5 hydrogen atoms per cubic meter.

- The virial radius of a galaxy or cluster is thus defined as the radius inside which:

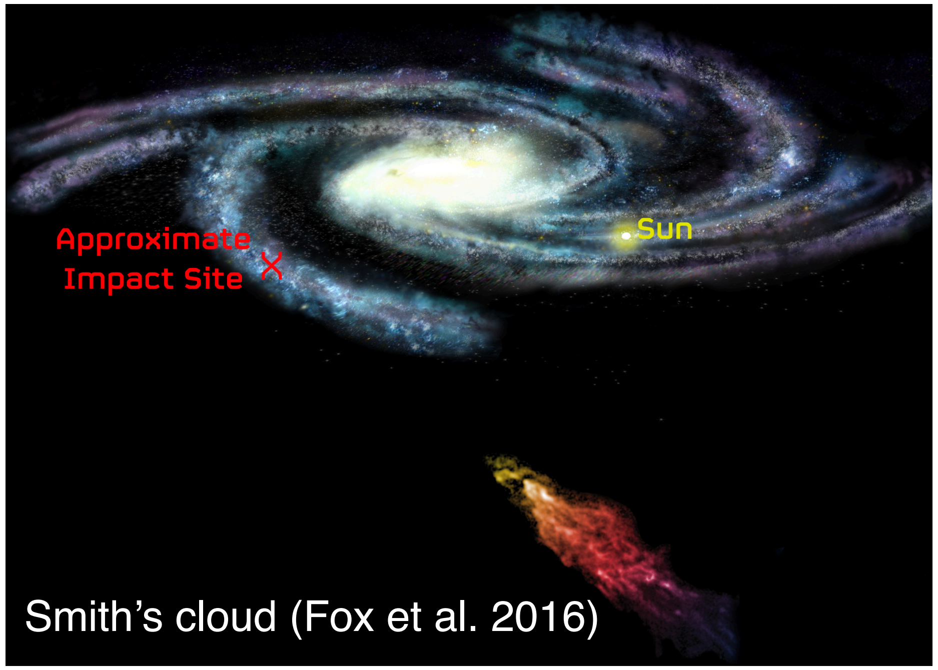
$$\rho(r_{\rm vir}) = 200\rho_{\rm c,0} \approx 1.7 \times 10^{-27}\ [{\rm g\,cm^{-3}}] \approx 2.5 \times 10^4\, M_\odot\ [{\rm kpc^{-3}}]$$

- For example, $r_{\rm vir} \sim 250\,{\rm kpc}$ for the Milky Way. [Note: the scale length of the Galaxy's stellar disk ~ 3 kpc.]
  $\sim 3\,{\rm Mpc}$ for a cluster of galaxies (Coma Cluster).
- **The virial radius is also often referred to as** r_{200}.

High-Velocity Clouds: CGM around our Galaxy

- ***H I HVCs***
  - History
    - The CGM around our galaxy was first detected in 21-cm emission (Muller et al. 1963).
    - The brightest clouds: HVC40-15+100 (Smith 1963; **Smith's cloud**), complexes A and C (Hulsbosch & Raimond 1966), "South Pole complex" (Dieter 1964), clouds M, AC I, AC II, AC III (Mathewson et al. 1966)
  - *They appear to have a high radial velocity relative to the galactic disk, which is not consistent with the Galactic rotation*. Hence, these fairly cool and dense gas clouds containing the neutral hydrogen are called ***high-velocity clouds (HVCs)***.
    - **Velocity**: Most of the HVCs are at galactocentric velocities between -250 km/s and +250km/s.
    - **Line broadening**: Their 21-cm emission has typical line broadening parameter $b \sim \sqrt{2}\sigma_v \sim 12\,\mathrm{km\,s^{-1}}$, indicating that the emission is from a warm neutral medium with a temperature of $T_{\mathrm{gas}} \sim 9000\,\mathrm{K}$.
    - **Column density**: A typical HVC column density is $N_{\mathrm{HI}} \sim 10^{19}\,\mathrm{cm^{-2}}$.
    - **Angular Size**: From enormous systems like Complex C (90°×20°) to subdegree-scale compact high-velocity clouds (CHVCs). Some of them are not resolved even with large single-dish radio telescopes.

- Distance
  - An upper limit to the distance of a HVC can be set by detecting absorption at the velocity of the HVC in the spectrum of a star with known distance.
  - A lower limit is set from stars not showing absorption by the HVC, with a detection limit below the expected line strength, found from an abundance determination based on an extra-galactic object.
  - Most of the HVCs are ***at distance d < 15 kpc from the Galactic center***, and are ***within ~ 30^o of the disk plane*** as seen from the Galactic center. They reside either in the extended Galactic halo or at extragalactic (Local Group) distances.
  - For example,
    - Complex A: 2.5 kpc < z < 6.5 kpc
    - Complex C: d ~ 10 kpc
    - The IV arch: z < 1.7 kpc
    - Complex H: z > 3-5 kpc
    - Smith's cloud: d ~ 12.4 kpc

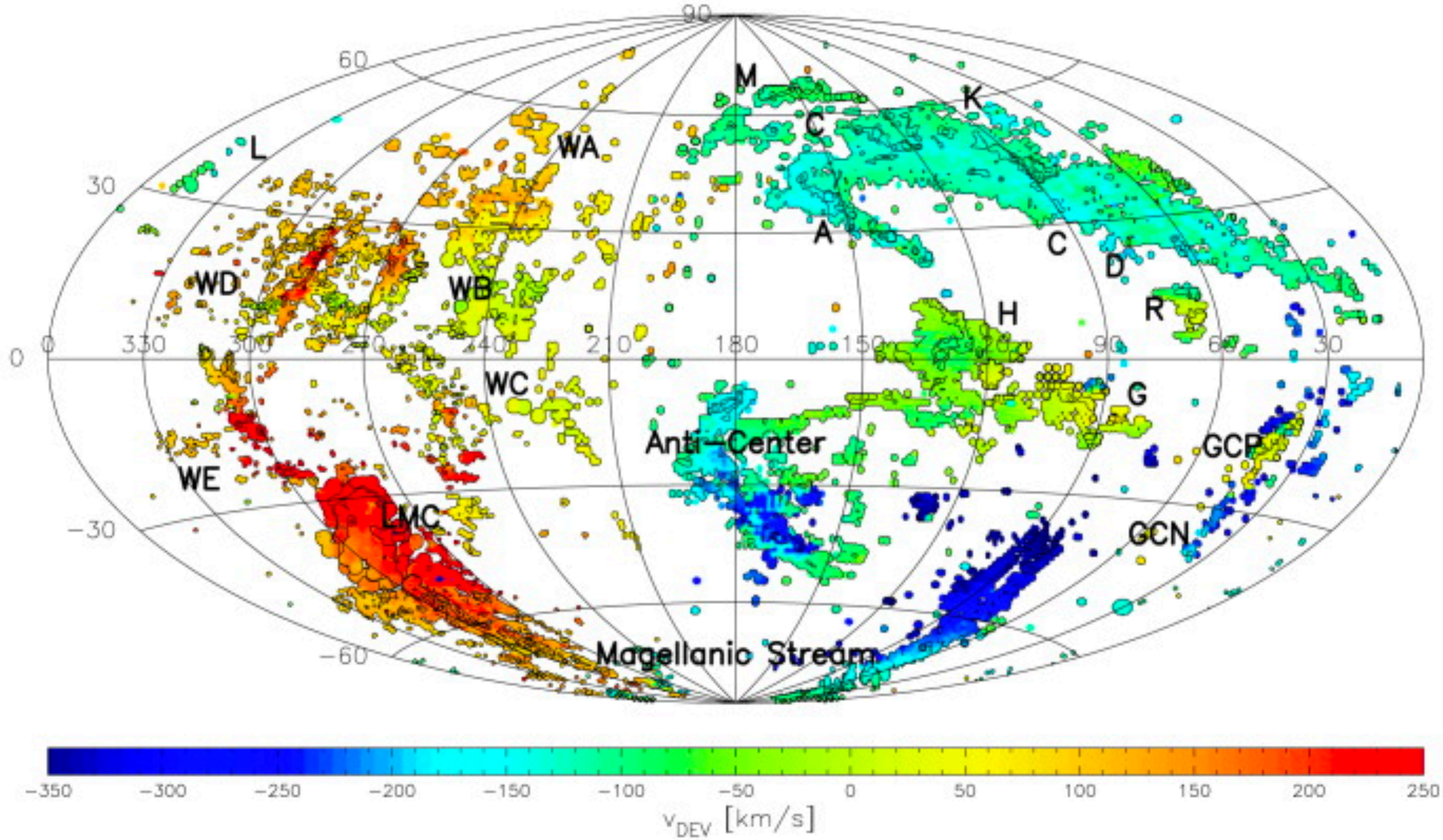


Smith's cloud (Fox et al. 2016)

- Velocity
  - *The sky coverage of high-velocity gas at negative velocities is greater than that at positive velocities.* (Does this mean they are mostly falling into the Galaxy?)
  - See also next slide

Velocity (north-south, east-west asymmetry)

Wakker et al. (2003, ApJS)



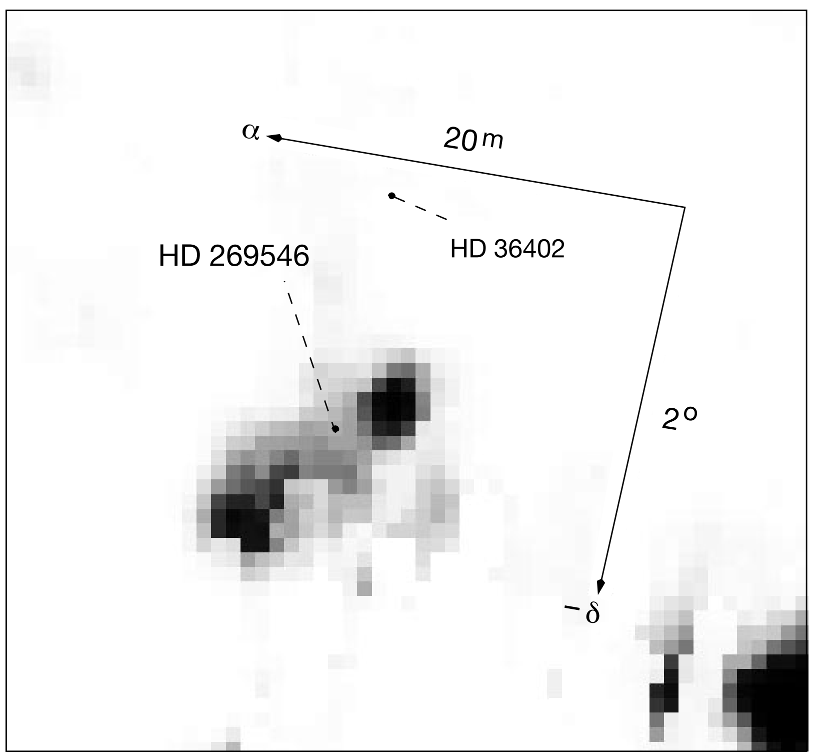
Map of the deviation velocities of the H I high-velocity gas. Data are from Hulsbosch & Wakker (1988) and Morras et al. (2000). The deviation velocity is the difference between the observed LSR (local standard of rest; mean motion of material in the Milky Way in the neighborhood of the sun) velocity and the maximum velocity that can be easily understood in a simple model of Galactic differential rotation ($v_{rot,}$ = 220 km s^{-1}, R = 8.5 kpc, R_{MW} = 26 kpc, z_{ISM} = 2 kpc at R, increasing to 6 kpc at R_{MW}, see Wakker 1991). The names of the major complexes are indicated. Contour levels are at 0.05, 0.5, and 1 K brightness temperature, or 2, 20, and 40 × 10^{18} cm^{-2}.

- Total Mass of HVCs
  - With distances known, we can determine the physical size and mass of individual HVCs.
  - The total H I mass of the HVCs around our Galaxy is $M_{\rm HI} \sim 3 \times 10^7\,M_\odot$.
  - Adding in helium, metals, and ionized hydrogen, its total mass is $M_{\rm HVC} \sim 7 \times 10^7\,M_\odot$.
- Ionized High-Velocity Clouds (IHVCs) invisible at 21 cm.
  - For instance, absorption lines of Si III, which traces $T_{\rm gas} \sim 2 \times 10^4\,{\rm K}$ gas, are seen over more than half the sight lines examined.
  - Absorption lines of O VI, tracing $T_{\rm gas} \sim 3 \times 10^5\,{\rm K}$ gas, are seen along more than half the sight lines.
  - The IHVCs are estimated to contain a total mass of $M_{\rm IHVC} \sim 10^8\,M_\odot$, comparable to the mass of the "traditional" HVCs seen in 21-cm emission.
- Hot Circumgalactic Medium
  - HVCs are surrounded by a hot ($T_{\rm gas} \sim 10^6\,{\rm K}$) circumgalactic medium, which is detected in X-rays and in O VII and O VIII absorption lines.
  - The density of the hot CGM is $n_{\rm hot} \sim 3 \times 10^{-5}\,{\rm cm}^{-3}$.
  - The hot CGM is thought to be extended ~ 100 kpc from the galactic center, with $M_{\rm hot} \sim 10^{10}\,M_\odot$.

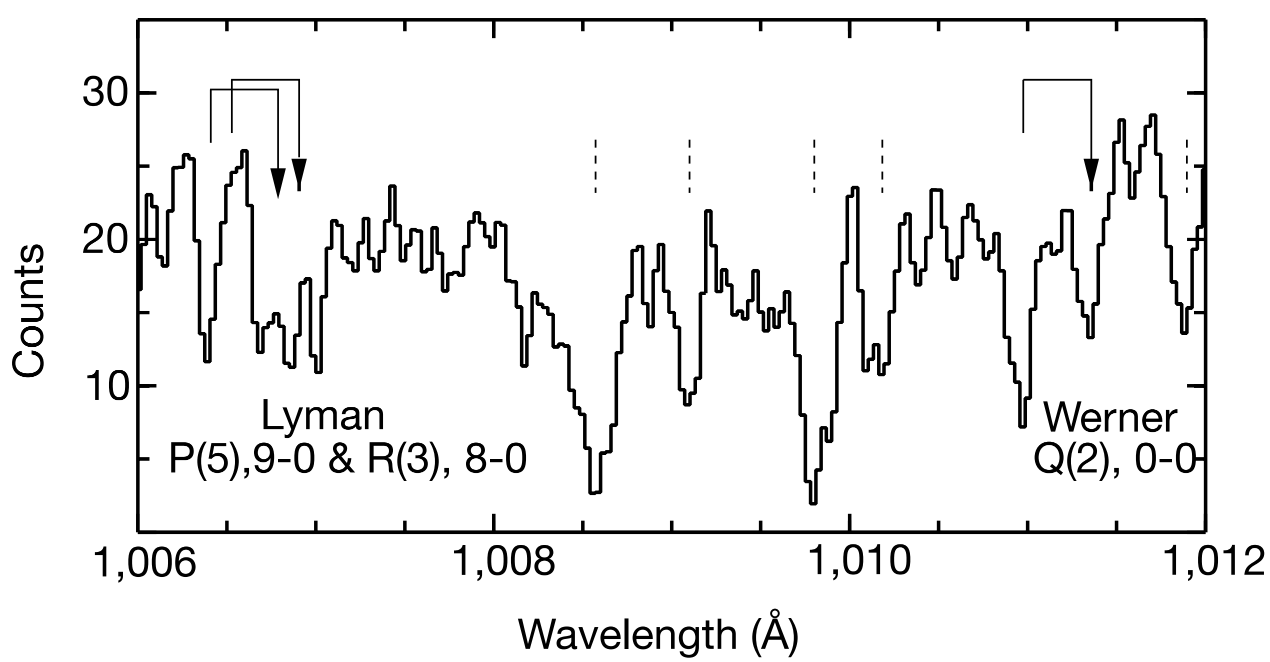
- Pressure Equilibrium between H I HVCs and the hot CGM
  - The hot CGM is of comparable temperature to the hot ISM, but its density is lower by two orders of magnitude.
  - Similarly, the temperature of H I HVCs is comparable to that of the warm ISM, but their density is lower by two orders of magnitude.
  - This suggests that *HVCs are in approximate pressure equilibrium with the hot CGM*.
- Total Mass of the CGM around our Galaxy is poorly known. Simulations indicates that the total mass of the CGM is $M_{\rm CGM} \sim 2 \times 10^{10}\, M_\odot \approx 5 M_{\rm ISM}$.
- Origin
  - Distance and angle from the Galactic disk indicates that ***most of the HVCs are definitely associated with the disk of our Galaxy***.
  - ***Some of the material in HVCs would be gas that has been stripped from satellite galaxies*** by ram pressure of the hot CGM (for instance, the Magellanic Stream).
    - The metallicity of the Magellanic Stream is ~10% of solar, consistent with the metallicity of the Small Magellanic Cloud. However, most of the HVCs can't be matched with any particular satellite galaxy.
  - ***Some of the CGM is pristine intergalactic gas falling inward*** through the virial radius.
    - Galaxy formation simulations show gas flowing inward along filaments to regions where galaxies are forming and evolving. The filamentary flow of gas tends to dilute the metallicity of circumgalactic gas.

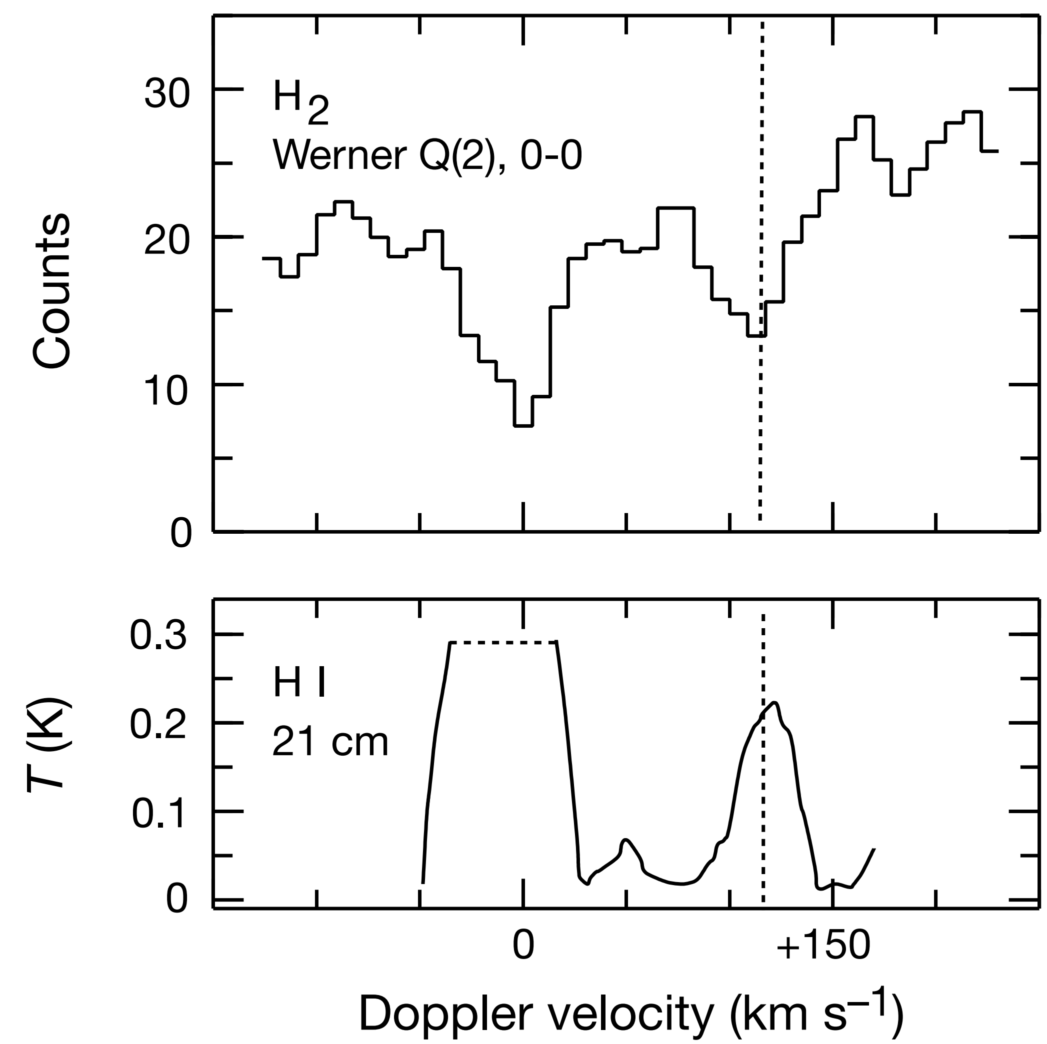
HVCs: Molecular hydrogen

- Richter (1999, Nature) detected molecular hydrogen absorption in a HVC (with +120 km/s) along the sightline to LMC, using ORFEUS.



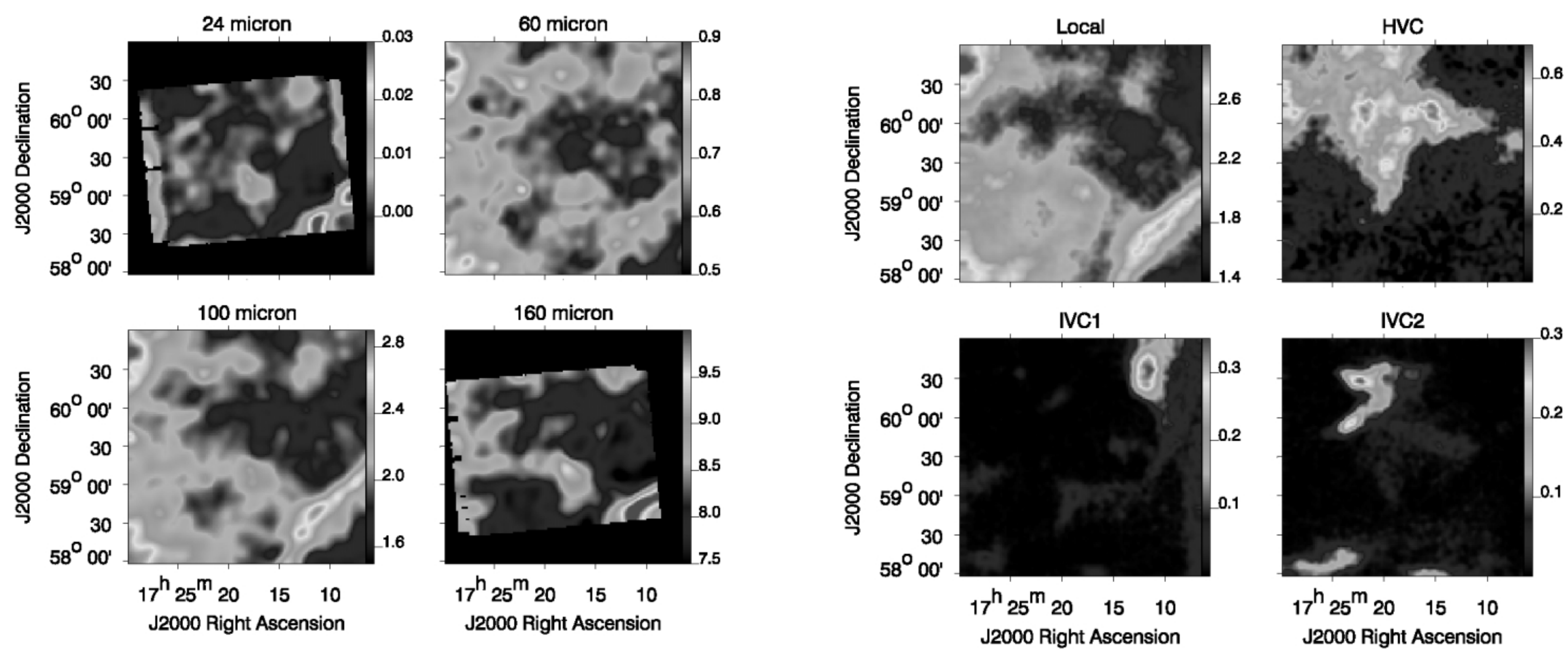
$$N(\mathrm{H}_2) = (2.2 - 3.6) \times 10^{15}\ \mathrm{cm}^{-2}$$
$$N(\mathrm{HI}) = 1.2 \times 10^{19}\ \mathrm{cm}^{-2}$$





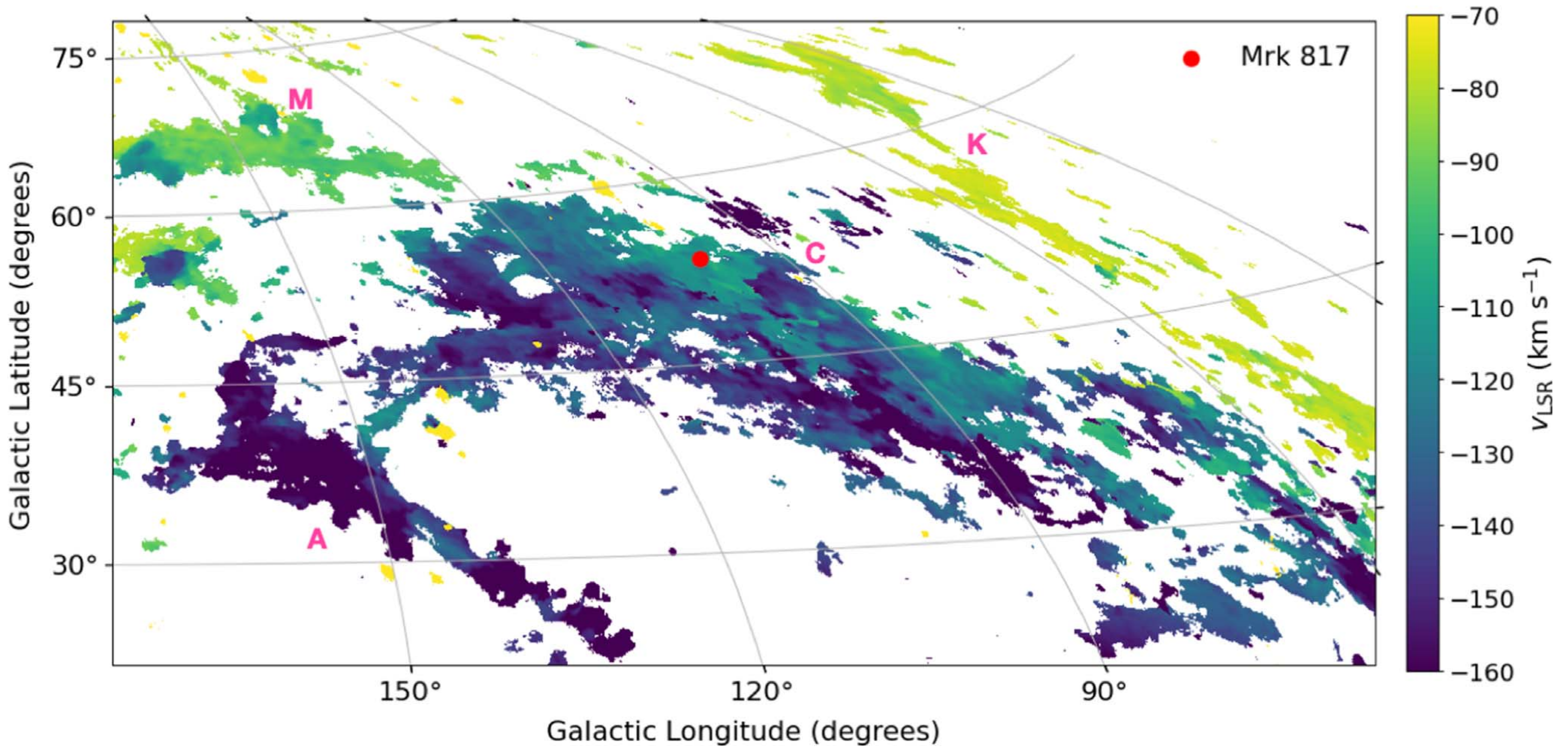
HVCs: Dust

- Miville-Deschenes et al. (2005)
  - Complex C, T ~ 10.7 K (T ~ 17.5 K in the local ISM) is in accordance with its great distance from the Galactic distance.

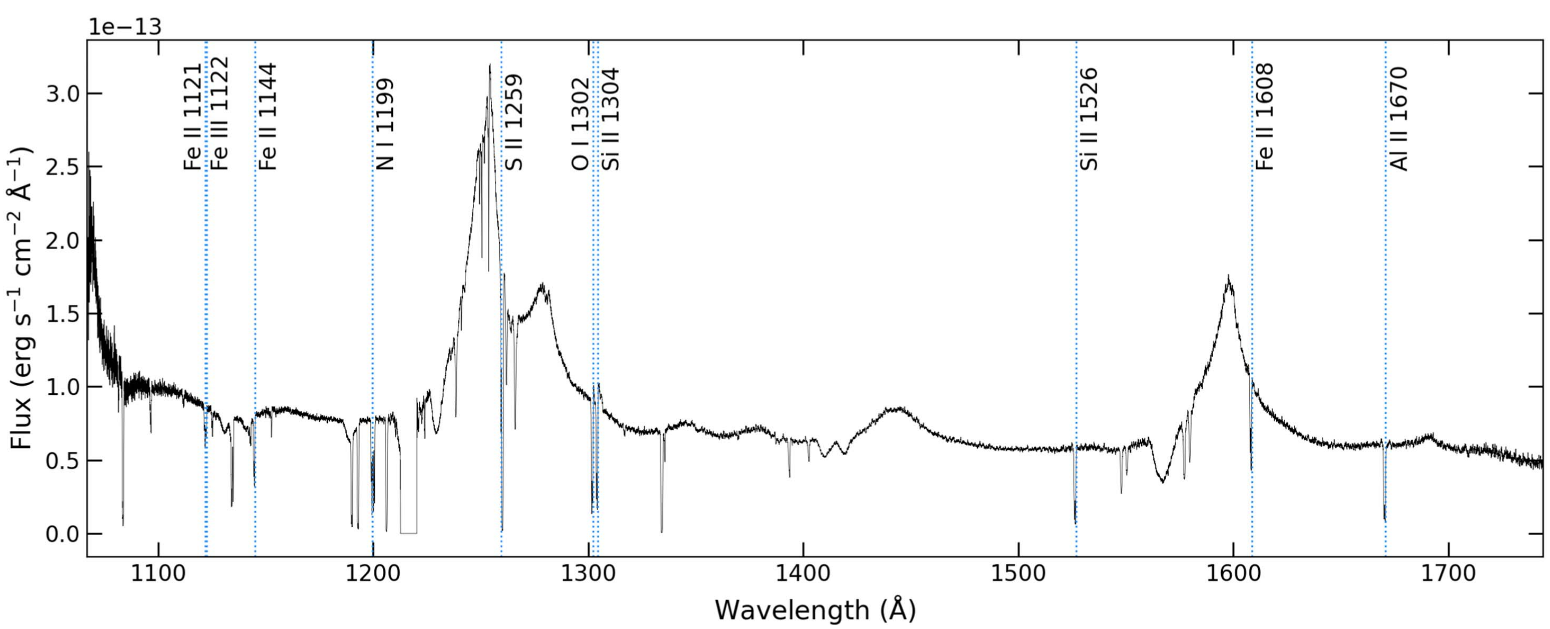


- Peek et al. (2009)
  - couldn't detect dust in Complex C, not detected dust in Complex M.
  - There exist LVCs that have extremely low dust-to-gas ratios, consistent with being in the Galactic halo
- Plank Collaboration (2011)
  - found dust in Complex C at the 3 σ level.

- Fox et al. (2023, ApJL, 946, L48)
    - They reports the detection of dust depletion in Complex C, a massive, infalling, low-metallicity HVC in the northern Galactic hemisphere.



[Top] H I 21 cm emission map of Complex C in Aitoff projection based on HI4PI data (Westmeier 2018). The map is integrated over the velocity range $-175 < v_{\rm LSR} < -50$ km s^{-1}. The location of Mrk 817 is marked with a red circle, and major HVC complexes are labeled in pink.



[Bottom] full coadded COS/FUV spectrum of Mrk 817 labeled with MW lines used in this analysis. The extremely high S/N of this spectrum enables high-precision measurements of Complex C absorption.

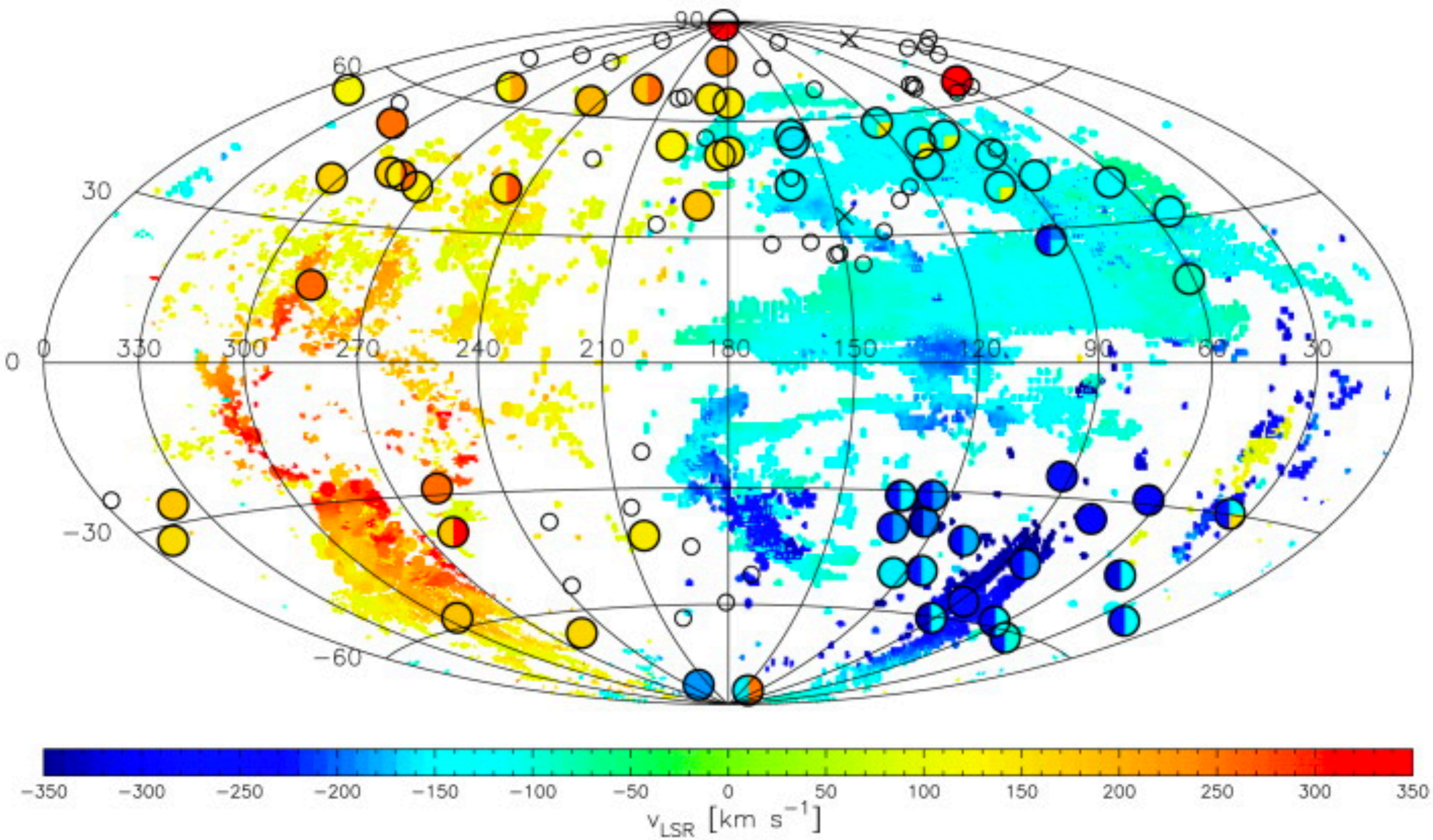
They found subsolar elemental abundance ratios of [Fe/ S] = –0.42, [Si/S] = –0.29, and [Al/S] = –0.53.

These ratios indicate the depletion of Fe, Si, and Al into dust grains, since S is mostly undepleted.

HVCs: Highly ionized gas - O VI absorption

Sembach et al. (2003)

small open circles - null detection



Hammer-Aitoff projection of the high-velocity H I sky based on 21 cm emission measurements (adapted from Wakker et al. 2003). The H I data have a spatial resolution of approximately 36′ and are representative of gas with N(H I) > 2 × 10^{18} cm^{-2}. Data for |b| < 20^{o} have been omitted for clarity. The positions of the high-velocity O VI features are denoted by the large circles, with the fill color indicating velocity on the same color scale used for the H I emission. If more than one high-velocity O VI feature is present, the circle is split and the velocities are color-coded in each section of the circle. Small open circles indicate null detections. "X" marks indicate the locations of the two stellar sight lines in the sample.

HVCs: Ionized Gas

- Low ionized gas: Si II, Mg II

- Highly ionized gas:
  - Consideration of the possible sources of collisional ionization favors production of some of the O VI at the boundaries between cool/warm clouds of gas and a highly extended ($R > 70\,\mathrm{kpc}$), hot ($T > 10^6\,\mathrm{K}$), low- density ($n < 10^{-4} - 10^{-5}\,\mathrm{cm}^{-3}$) Galactic corona or Local Group medium.

- A bit low and inhomogeneous metallicity is usually found.
  - The most extended HVCs appear to be located relatively near the Galaxy, they may nonetheless be connected to extragalactic phenomena.
  - This is indicative of a combination of infalling primordial gas and outflowing enriched material.

HVCs in other galaxies

- HVCs appear to be a common feature around spiral galaxies (like M31 and M83).

- What is the origin of HVCs? There is no generally accepted theory regarding their origin.
  - cooling component of a supernova-driven "Galactic fountain"
  - inflows of neutral gas condensing from the local IGM
  - gaseous signatures of the "missing" dark matter subhalos around the Galaxy

- What is strange?
  - *No stellar populations associated with HVCs have been found*, unlike very low mass satellite galaxies which appear to harbor both stars and dark matter but little or no gas.
  - Aside from one case (Richter et al. 1999), HVCs appear to contain little or no molecular gas.

Galactic Wind

- The CGM is also fed by a galactic wind that blows interstellar gas outward, away from the disk.

  - M82 shows extremely strong galactic winds.
    - M82 has a star formation rate $\mathrm{SFR} \sim 10\, M_{\odot}\, \mathrm{yr}^{-1}$. This high star formation rate provides a mechanism for driving the galactic wind.
    - When a large number of stars are formed within a short period of time, there will follow an epoch when multiple supernovae occur within a short period of time. The supernova remnants then merge to form a big hot bubble.
    - Active galactic nuclei can also inject large amounts of energy into the ISM, deriving a wind away from the galaxy.
    - The details of AGN physics are beyond the scope of this lecture, and we will focus on the case of the starburst-driven winds.

Optical, IR, and X-ray composite image of the starburst galaxy M81. The hot X-ray wind is in blue, the dusty outflow is red.

Fig 8.2, Ryden

Starburst-Driven Galactic Winds

- Galactic Winds
  - Starburst winds are driven by mechanical energy and momentum from stellar winds and supernovae (SNe).
  - For an instantaneous starburst, winds from OB stars dominate early on (< 3 Myr); next come Wolf-Rayet (WR) stars with mass-loss rates ~10 times higher (~3-6 Myr); and finally, core-collapse Type II SNe dominate until ~ 40 Myr when the least massive ($\sim 8 M_\odot$) ones explode.
  - In the real universe, star formation episodes last for more than 40 Myr.
  - Therefore, we may ***assume a constant star-formation rate*** for a time much longer than 40 Myr.
- A Simple Model:
  - We will assume that a star formation rate $\mathrm{SFR} \sim 1\, M_\odot\, \mathrm{yr}^{-1}$ leads to a SN rate of $R_{\mathrm{SN}} \sim 1/(50\,\mathrm{yr}) = 0.02\,\mathrm{yr}^{-1}$.
  - If each SN deposits an energy $E = 10^{51}\,\mathrm{erg}$ into the ISM, this implies an ***energy injection rate***

$$\begin{aligned}\dot{E}_* &= 10^{51}\,\mathrm{erg} \times 0.02\,\mathrm{yr}^{-1} \left(\frac{\mathrm{SFR}}{1\,M_\odot\,\mathrm{yr}^{-1}}\right) \\ &= 6.3 \times 10^{41}\,\mathrm{erg\,s}^{-1} \left(\frac{\mathrm{SFR}}{1\,M_\odot\,\mathrm{yr}^{-1}}\right)\end{aligned}$$

- We now assume that the energy injection rate is constant with time. If the radiative losses of the overall system are negligible, the expanding bubble is energy-conserving (Sedov-Taylor solution).
  - In that case, the radius of the expanding shell of shocked ISM is given by

$$r_{\rm shell} = \xi \left(\frac{E_* t^2}{\rho} \right)^{1/5} \quad (\xi = 1.15167)$$

$$E_* = \dot{E}_* t$$

$$r_{\rm shell} = \xi \left(\frac{\dot{E}_* t^3}{\rho} \right)^{1/5}$$
$$\approx 520\,{\rm pc} \left(\frac{t}{1\,{\rm Myr}} \right)^{3/5} \left(\frac{\rm SFR}{1\,M_\odot\,{\rm yr}^{-1}} \right)^{1/5} \left(\frac{n_{\rm H}}{1\,{\rm cm}^{-3}} \right)^{-1/5}$$

  - The expansion rate of the shell is given by

$$v_{\rm shell} = \frac{dr_{\rm shell}}{dt} = \xi \frac{3}{5} \left(\frac{\dot{E}_*}{t^2 \rho} \right)^{1/5}$$
$$\approx 310\,{\rm km\,s^{-1}} \left(\frac{t}{1\,{\rm Myr}} \right)^{-2/5} \left(\frac{\rm SFR}{1\,M_\odot\,{\rm yr}^{-1}} \right)^{1/5} \left(\frac{n_{\rm H}}{1\,{\rm cm}^{-3}} \right)^{-1/5}$$

  - Energy losses from the hot gas inside the bubble are negligible until the temperature behind the expanding blastwave drops below $\sim 2 \times 10^7\,{\rm K}$.

- When the temperature falls to this level, we can use the cooling time scale for collisionally excited line radiation: (see Lecture 10).

$$t_{\rm cool} \approx 0.19\,[\mathrm{Myr}]\left(\frac{T}{10^6\,\mathrm{K}}\right)^{1.7}\left(\frac{n_{\rm H}}{1\,\mathrm{cm}^{-3}}\right)^{-1}$$

$$\approx 6600\,[\mathrm{yr}]\left(\frac{v_{\rm shell}}{100\,\mathrm{km\,s}^{-1}}\right)^{3.4}\left(\frac{n_{\rm H}}{1\,\mathrm{cm}^{-3}}\right)^{-1}$$

$$T \simeq \frac{3}{16}\frac{m}{k}V_{\rm shock}^2$$

$$\approx 1.38\times 10^5\,\mathrm{K}\left(\frac{m}{0.609 m_{\rm H}}\right)\left(\frac{V_{\rm shock}}{100\,\mathrm{km\,s}^{-1}}\right)^2$$

$$m = \frac{1.4 m_{\rm H}}{2.3} = 0.609 m_{\rm H} \quad \text{for fully ionized gas}$$

- Putting in the relation for the expansion speed of the starburst-driven bubble, we find a cooling time:

$$t_{\rm cool} \approx 0.31\,\mathrm{Myr}\left(\frac{t}{1\,\mathrm{Myr}}\right)^{-1.36}\left(\frac{\mathrm{SFR}}{1\,M_\odot\,\mathrm{yr}^{-1}}\right)^{0.68}\left(\frac{n_{\rm H}}{1\,\mathrm{cm}^{-3}}\right)^{-1.68}$$

Solving the above equation for the instance when $t = t_{\rm cool}$, we find

$$t_{\rm cool} \approx 0.61\,\mathrm{Myr}\left(\frac{\mathrm{SFR}}{1\,M_\odot\,\mathrm{yr}^{-1}}\right)^{0.29}\left(\frac{n_{\rm H}}{1\,\mathrm{cm}^{-3}}\right)^{-0.71}$$

- By the time $t = t_{\rm cool}$, the bubble has expanded to a radius:

$$r_{\rm shell} \approx 390\,\mathrm{pc}\left(\frac{\mathrm{SFR}}{1\,M_\odot\,\mathrm{yr}^{-1}}\right)^{0.37}\left(\frac{n_{\rm H}}{1\,\mathrm{cm}^{-3}}\right)^{-0.63}$$

- The mean density of the WNM is ~ 0.6 cm^{-3}, and the scale height of the WNM, in the direction perpendicular to the Galactic disk, is ~ 350 pc. Thus, a superbubble is able to break free of the gaseous disk while still in its Sedov-Taylor phase.

Homework (due date: 06/18)

[Q16] (Lecture 14)

The mass distribution of GMCs in the Galaxy is given by [eq. (32.1) in the textbook]:

$$\frac{dN_{\rm GMC}}{d\ln M_{\rm GMC}} \approx N_u \left(\frac{M_{\rm GMC}}{M_u}\right)^{-\alpha} \quad 10^3\,M_\odot \lesssim M_{\rm GMC} < M_u$$

with $M_u \approx 6 \times 10^6\,M_\odot$, $N_u \approx 63$, and $\alpha \approx 0.6$ (Williams & McKee 1997, Astrophys. J. 476, 166).

(a) Calculate the total mass in GMCs in the Galaxy.

(b) Calculate the number of GMCs in the Galaxy with $M > 10^6\,M_\odot$.

[Q17] (Lecture 15 & Lecture 5)

An absorption line is observed in the spectrum of a quasar at an observed wavelength $\lambda = 5000.$ Å. The absorption is produced by an intergalactic cloud of gas somewhere between us and the quasar. The observer measures an equivalent width $W_\lambda = 1.0 \times 10^{-2}$ Å. The absorption line is resolved, with an observed $\mathrm{FWHM}_\lambda = 0.50$ Å. The line is assumed to be H I Lyman α, with rest wavelength $\lambda_0 = 1215.7$ Å and oscillator strength $f_{\ell u} = 0.4164$.

(a) What is the redshift z of the absorber?

(b) What is the column density of H I in the absorbing cloud?

(c) In the rest frame of the cloud, the H I has a one-dimensional velocity distribution $\propto e^{-(\Delta v/b)^2}$. What is the value of b for this cloud?