KAIST Astrophysics (PH481) - Part 1

Week 1
Sep. 2 (Mon) & 4 (Wed), 2019

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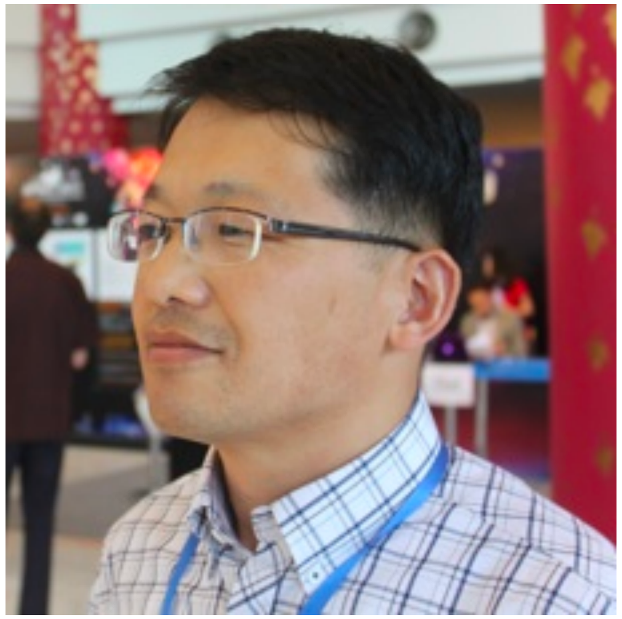
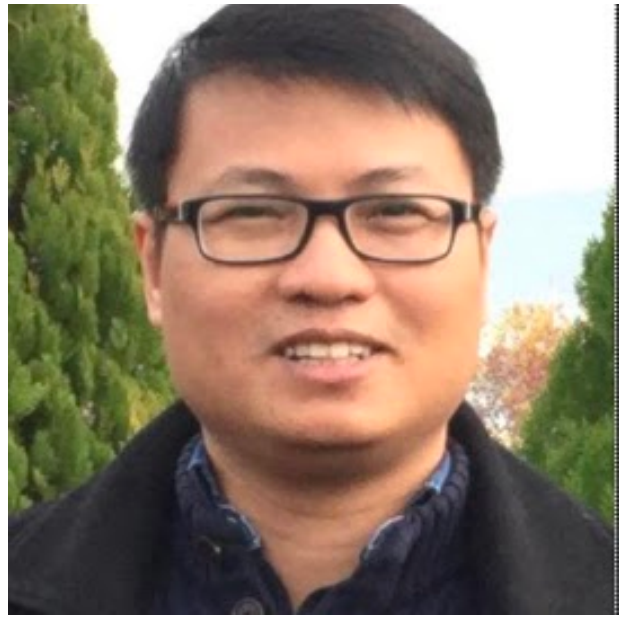
Instructors : UST faculties





Kwang-Il Seon
(선광일)

- Galaxies
- Interstellar Medium, Intergalactic Medium
- Ultraviolet, Optical, Far-Infrared astronomy
- Theory & Observation

Thiem Hoang

- Theoretical Astrophysics
- Dust Astrophysics
- CMB Foreground Polarization

Sang-Sung Lee
(이상성)

- Radio Astronomy
- Relativistic jets of Active Galactic Nuclei
- Observation

Junga Hwang
(황정아)

- Space Weather, Environments
- Earth Magnetosphere
- Earth Radiation Belt

- Astrophysics Website: https://sites.google.com/view/kaist-astroph.
- My lecture notes can be downloaded from https://seoncafe.github.io/Teaching.html.

Lecture Schedule

Week	Professor	Content
1	Kwang-il Seon	A brief introduction, Fundamentals of Radiative Transfer
2	Kwang-il Seon	Radiation Processes (Bremsstrahlung, Synchrotron, Compton Scattering)
3	Kwang-il Seon	Atomic Spectroscopy
4	Kwang-il Seon	Lyman-alpha, 21 cm Astrophysics
5	Thiem Hoang	Introduction to Dust Astrophysics
6	Thiem Hoang	Physics of Interstellar Dust
7	Thiem Hoang	Dust Evolution from Small Grains to Planets
8		Midterm examination

Week	Professor	Content
9	Thiem Hoang	Magnetic Fields in the Universe
10	Sang-Sung Lee	Galaxies beyond the Milky Way
11	Sang-Sung Lee	Hubble-Lemaitre Law and Distance Scale
12	Sang-Sung Lee	Active Galaxies and Radio Astronomy
13	Junga Hwang	Introduction to Near-Earth Space Environment
14	Junga Hwang	Particle Dynamics in Space Plasma Physics
15	Junga Hwang	Solarwind and Interplanetary Magnetic Field
16		Final examination

What is Astrophysics?

- The Universe is fascinating. Starting in ancient times, people have wondered about the nature of stars and how they might affect our lives.
- Our knowledge about them has progressed with time, often assisted by advances in fundamental science.
  - For example, in the nineteenth century, the source of solar energy was believed to be the gravitational potential energy and perhaps the chemical energy. These were the only possibilities known at that time. It was only later, after the discovery of nuclear fusion, that scientists realized that the Sun is power by nuclear energy.
- ***The branch of science that aims to understand the physics and chemistry of heavenly objects is called Astrophysics.***
- ***Astrophysics:*** application of the laws of physics to understand the behavior of astronomical objects, and to predict new phenomena that could be observed.
- This field has seen remarkable developments in the last century and has now reached ***the level of a precision science***.

Difference between astrophysics and other branches of physics

- Main difference between astrophysics and other branches of physics: **controlled experiments are (almost) never possible.**
- This means:
  - If many different physical effects are operating at the same time in a complex system, can't isolate them one by one.
  - Knowledge of rare events is limited - nearest examples will be distant. e.g., no supernova has exploded within the Milky Way since telescopes were invented.
  - Need to make best use of all the information available - many advances have come from opening up new regions of the electromagnetic spectrum.
  - Statistical arguments play a greater role than in many areas of lab physics.

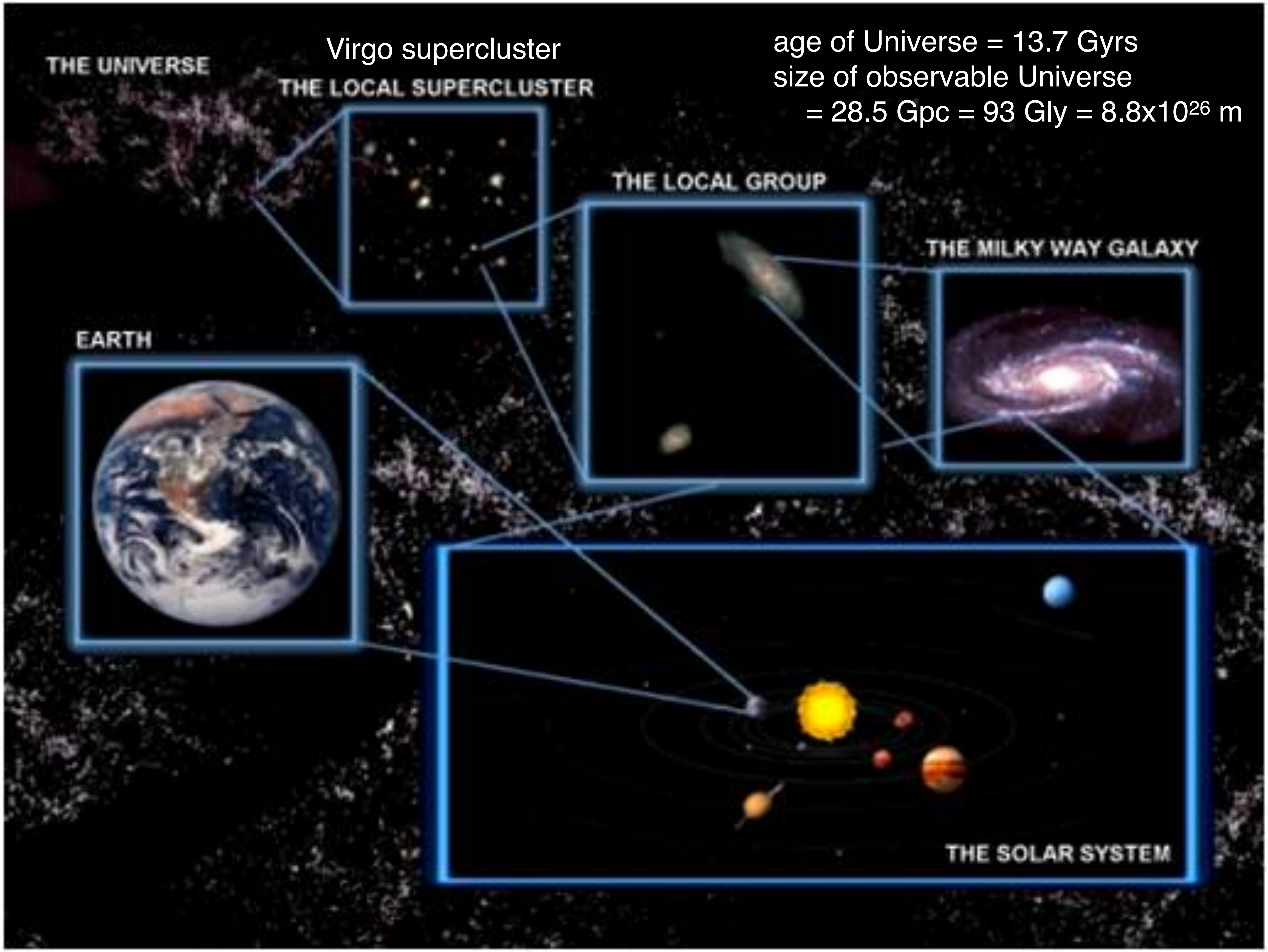
A brief introduction to Astronomy

Astronomical Objects

- The Universe contains structures on a wide range of scales.
  - These include the solar system and the planetary systems associated with other stars.
  - The stars themselves often form clusters that are part of bigger structures called galaxies.
  - Furthermore, the galaxies are also not found in isolation and form groups or clusters of galaxies that form larger clusters called superclusters.
  - The superclusters are the largest structures observed.
  - The size of the observable Universe is roughly 50 times larger than the size of the largest supercluster.

stars + planetary systems
    < star clusters
        < galaxies
            < groups or clusters of galaxies
                < superclusters

Tour of the Cosmos

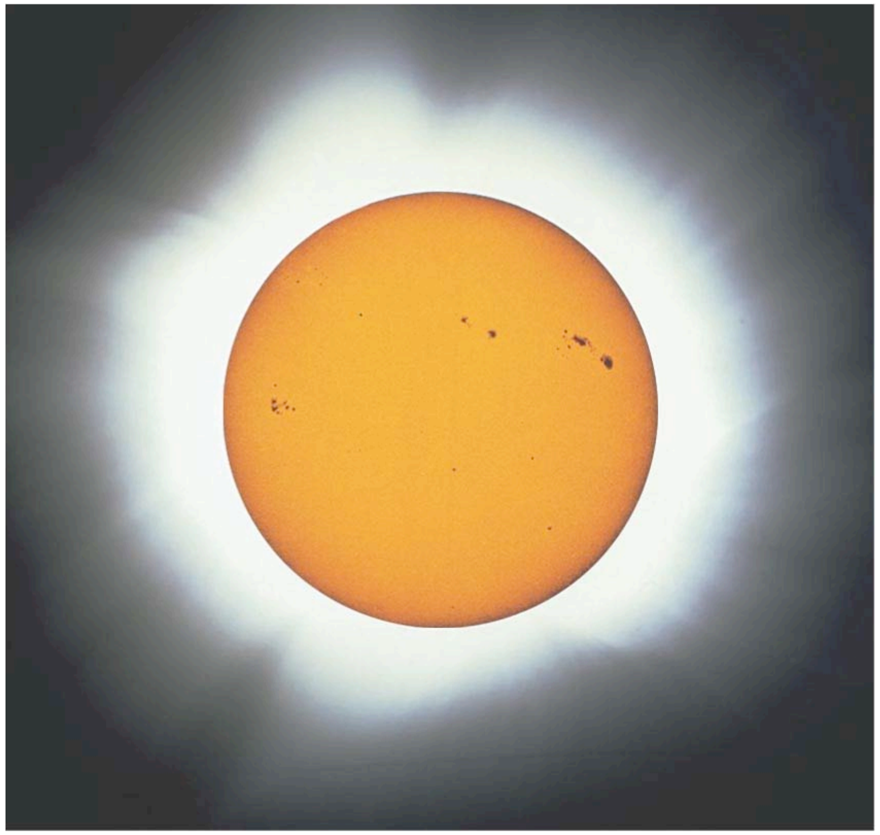


Examples: The Sun

- Radius & Mass: $R_\odot = 6.96 \times 10^8\,\mathrm{m} \sim 700{,}000\,\mathrm{km}$

$$M_\odot = 1.989 \times 10^{33}\,\mathrm{g} \sim 1000 \text{ Jupiter mass}$$

- Differential Rotation with a period about a month
- Temperature : 5800 K (a yellow star) at surface; $1.5\text{x}10^7$ K in the center.



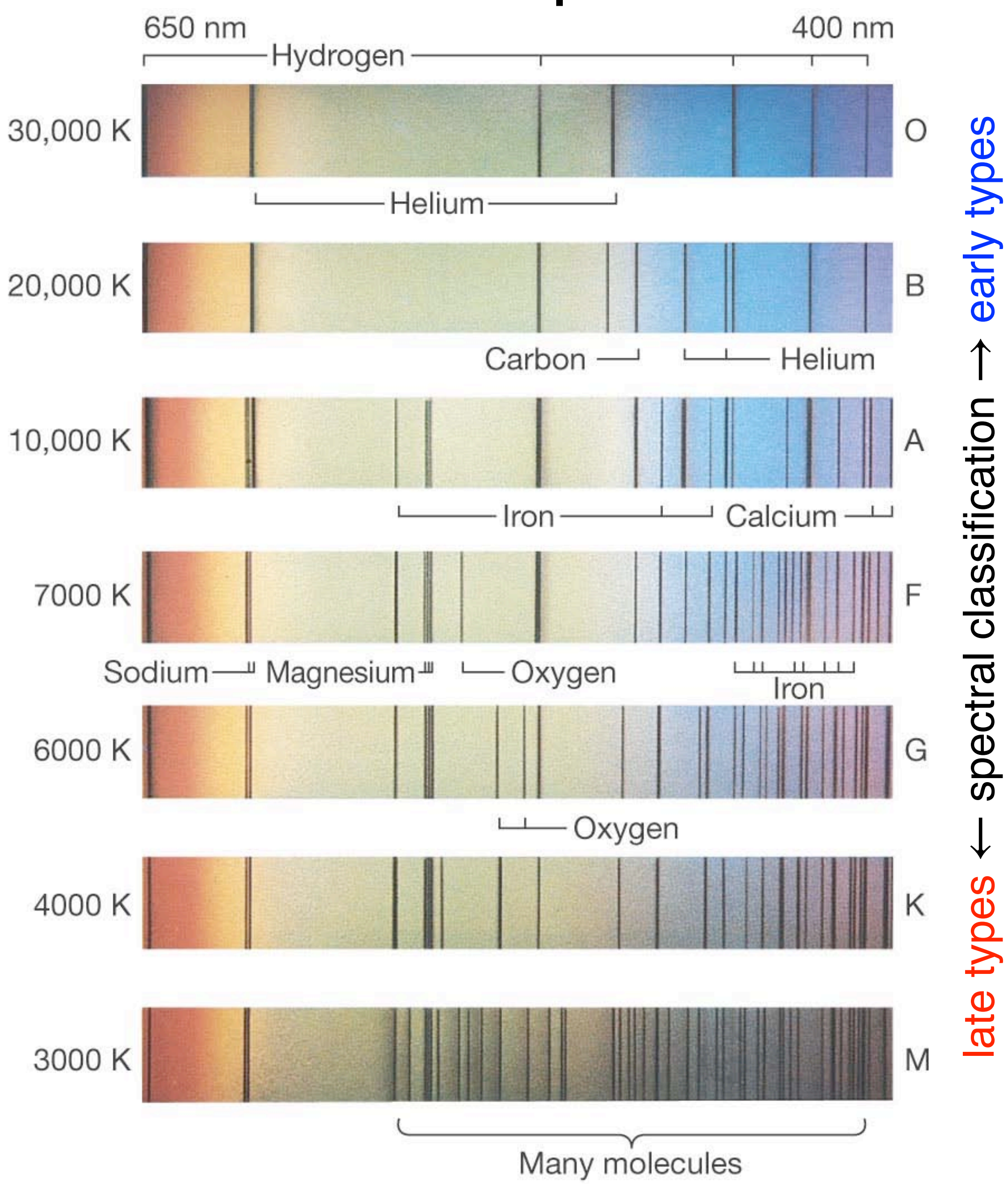
A V-band image of the Sun showing sunspots, the sharp edge of the Sun due to the thin photosphere, and the corona.

A UV image taken by SOHO (Solar and Heliospheric Observatory).

Examples: stars

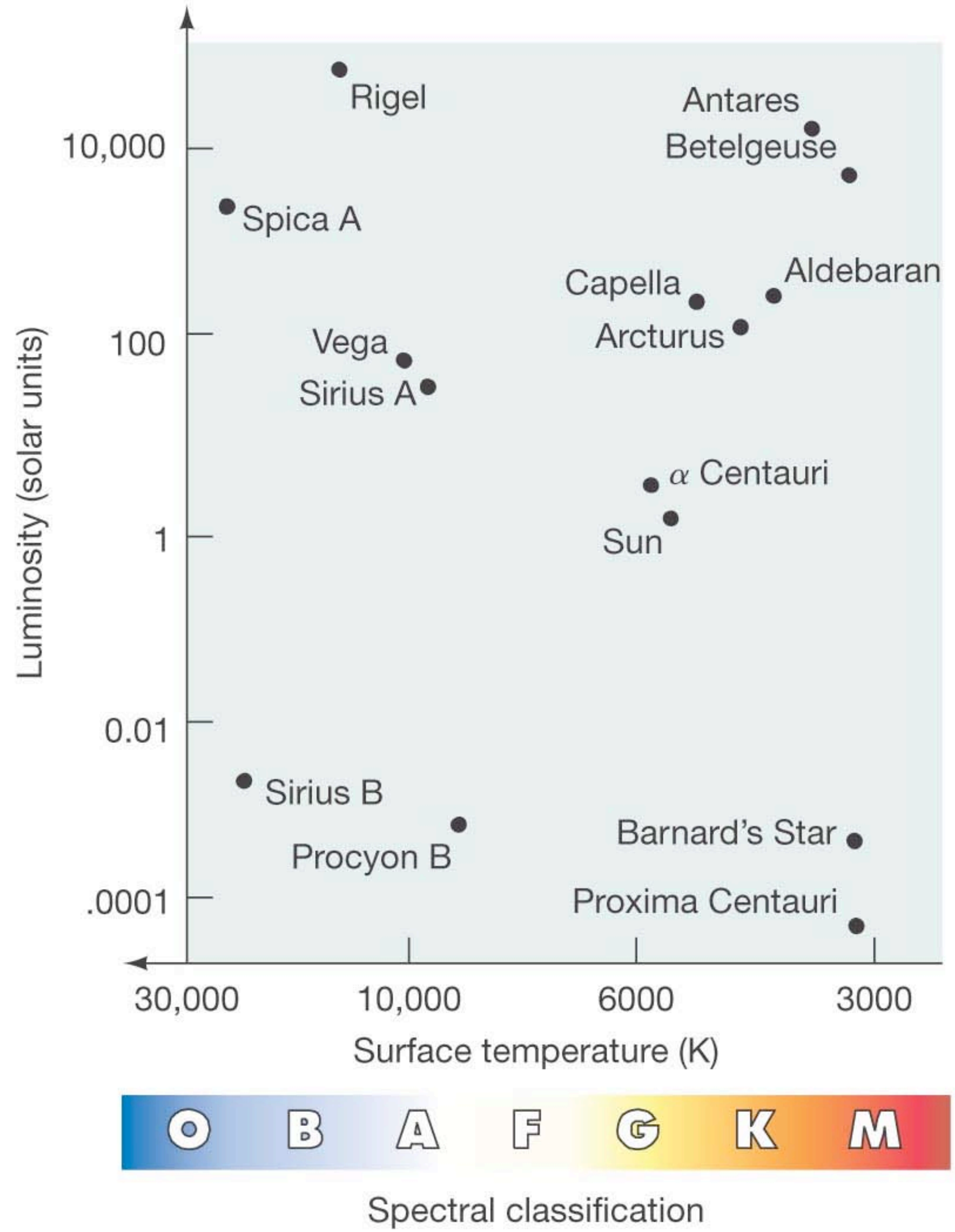
- Hertzsprung-Russell Diagram
  - The H-R diagram plots stellar luminosity against surface temperature.

Stellar Spectra



<How to memorize: Oh, Be A Fine Girl, Kiss Me>

H-R diagram of a few bright stars



Examples: globular clusters

- Globular clusters contain 10^5 - 10^6 old stars in a compact, often roughly spherical shape.

- But, they contain not much gas, dark matter.

47 Tucanae (NGC 104)

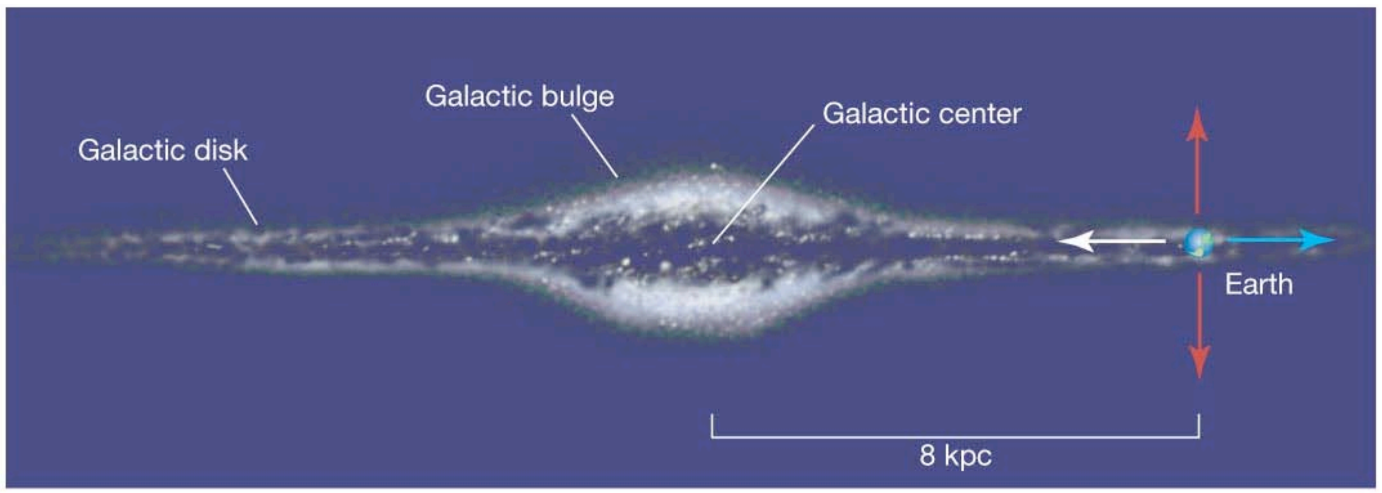
Credit: Michael Sidonio

Examples: Galaxies

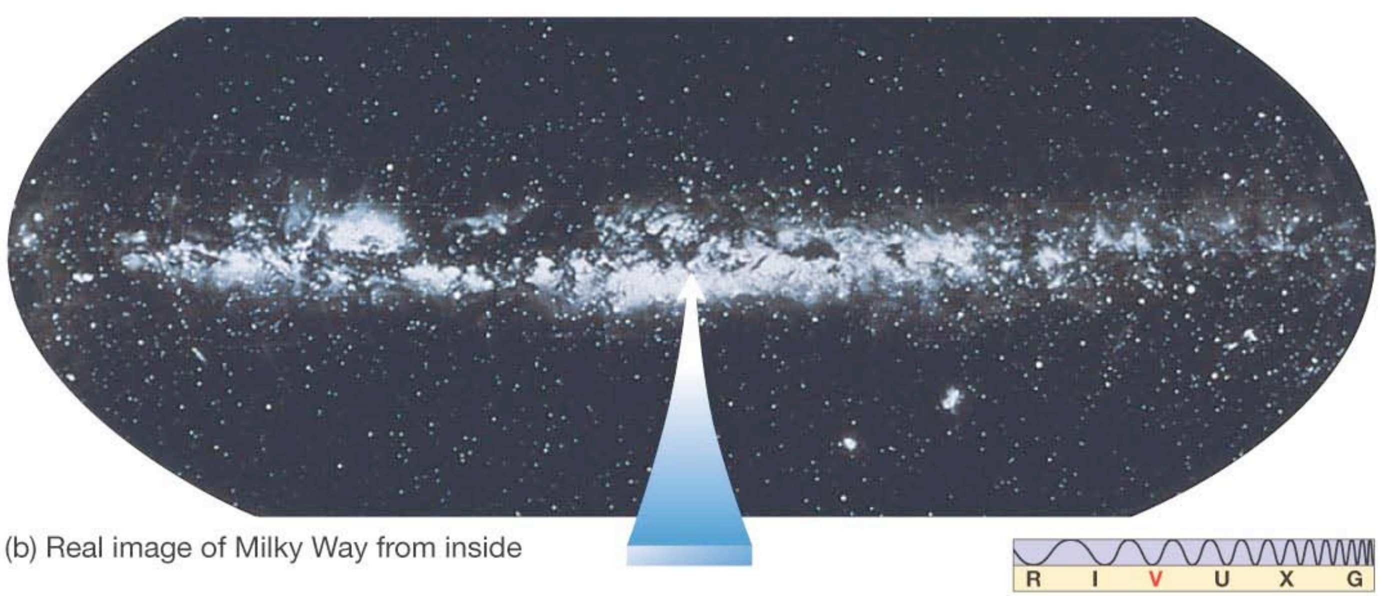
- Milky Way is what our galaxy appears as in the night sky.

Face-on view

Edge-on view



(a) Artist's view of Milky Way from afar



(b) Real image of Milky Way from inside

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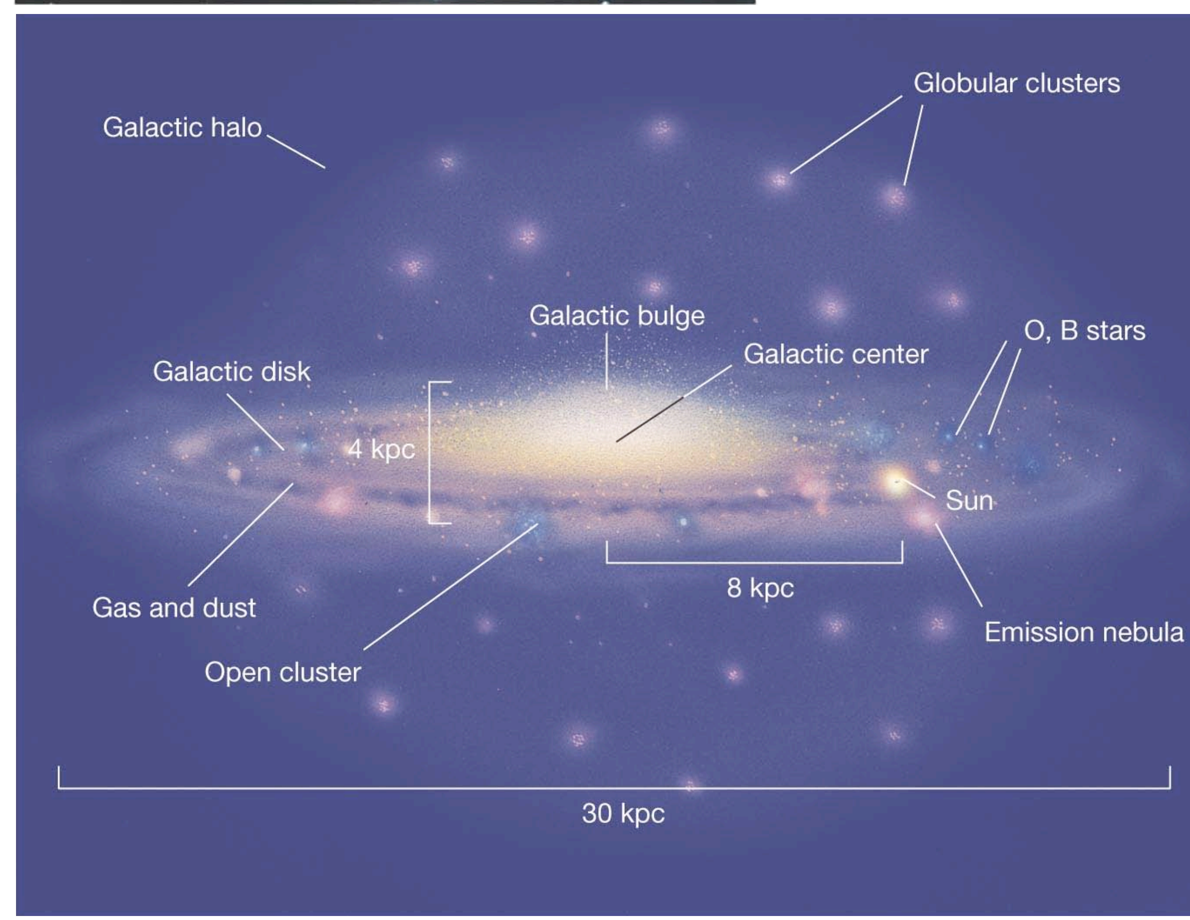
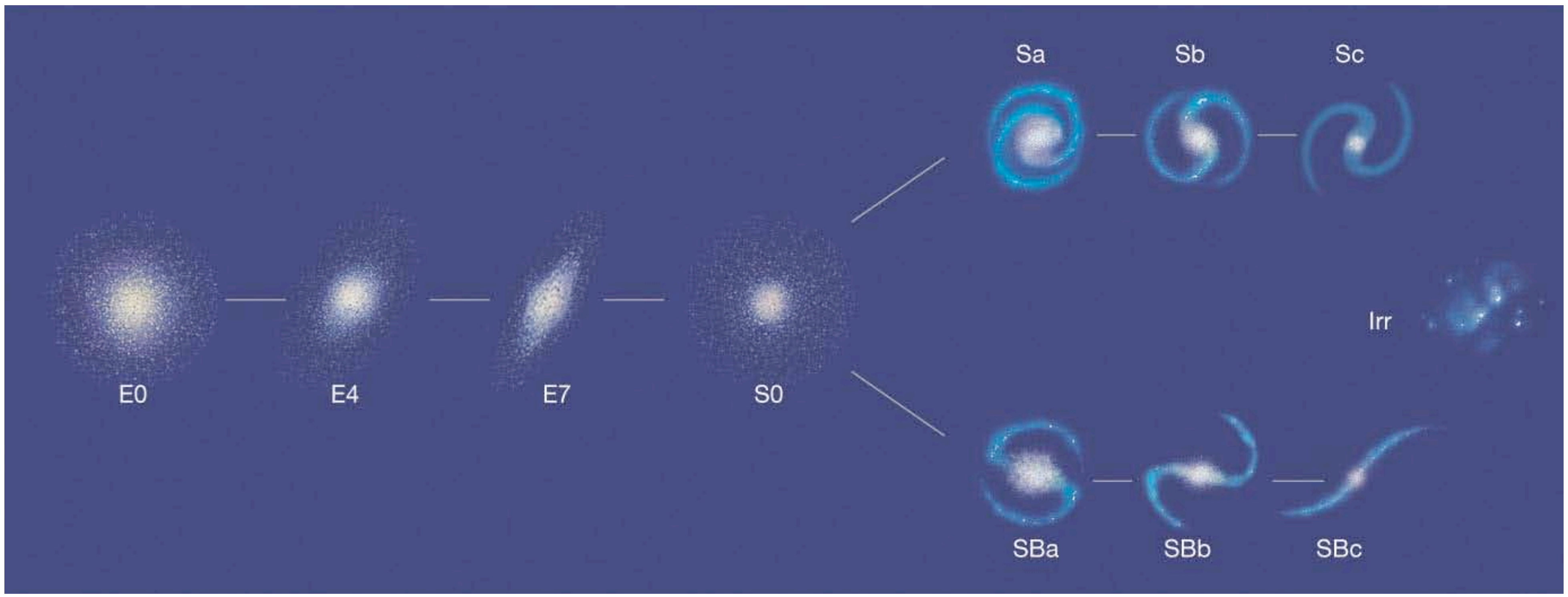
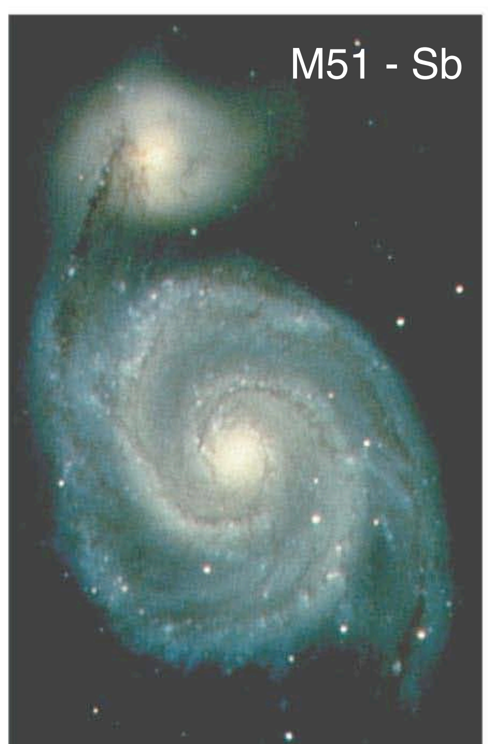


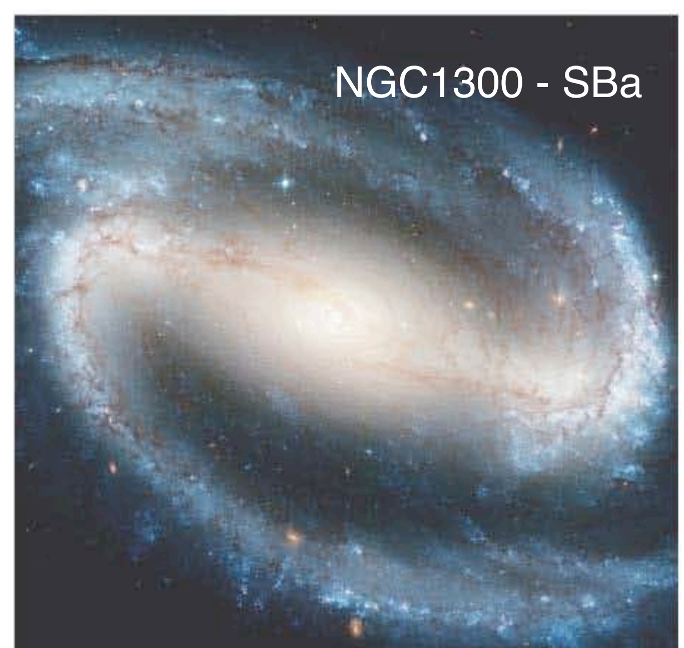
This artist's conception shows the various parts of our galaxy, and the position of our Sun.

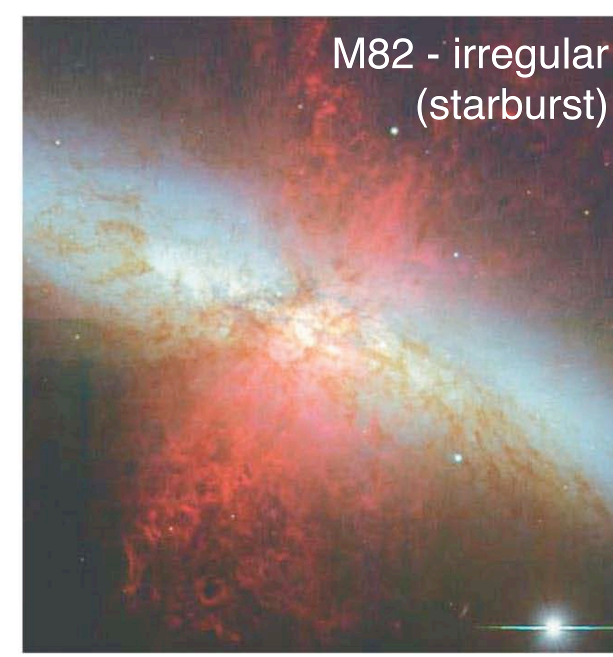
Examples: Hubble's Galaxy Classification

- Hubble's "tuning fork" is a convenient way to remember the galaxy classifications, although it has no deeper meaning:









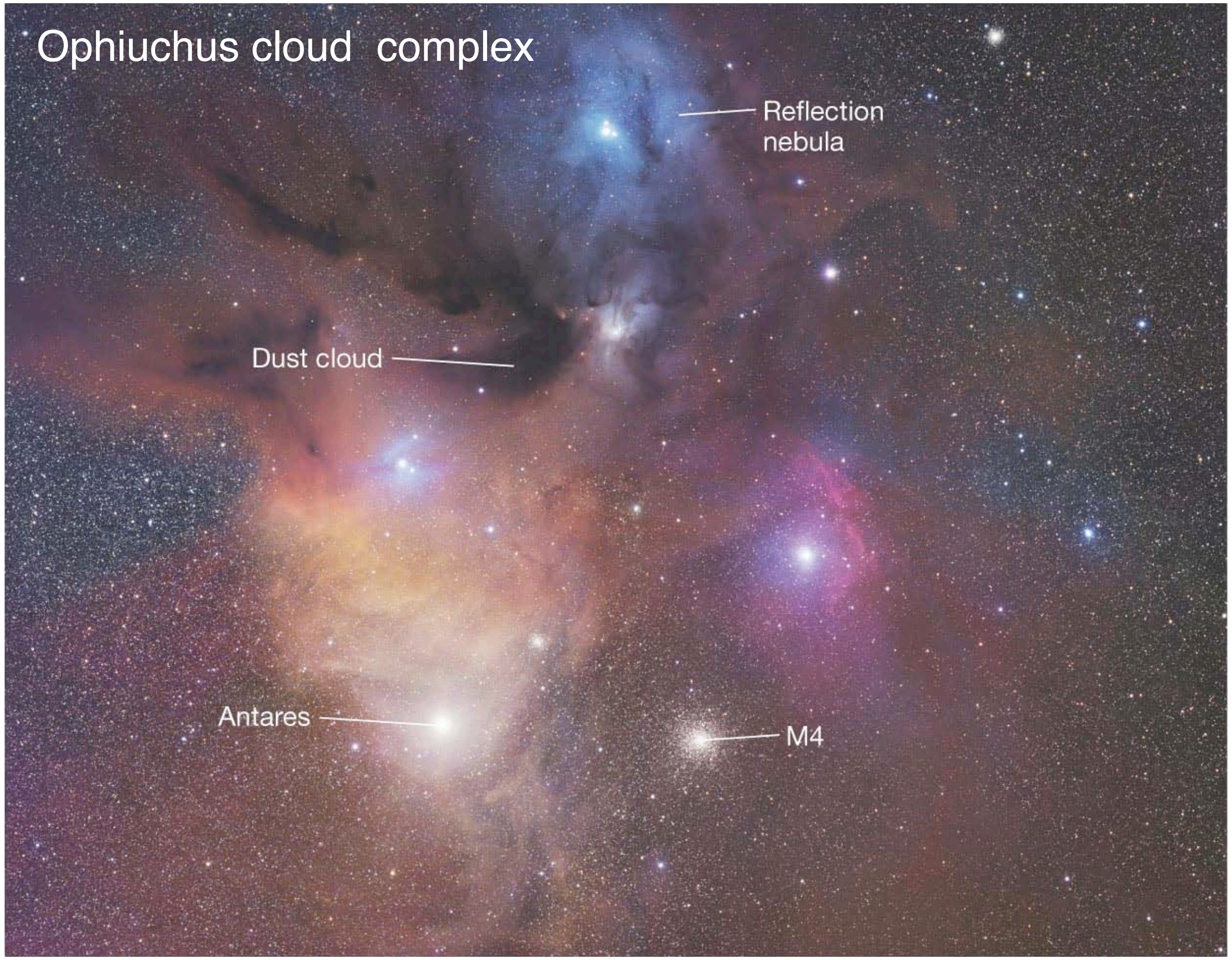
Examples: Interstellar Matter

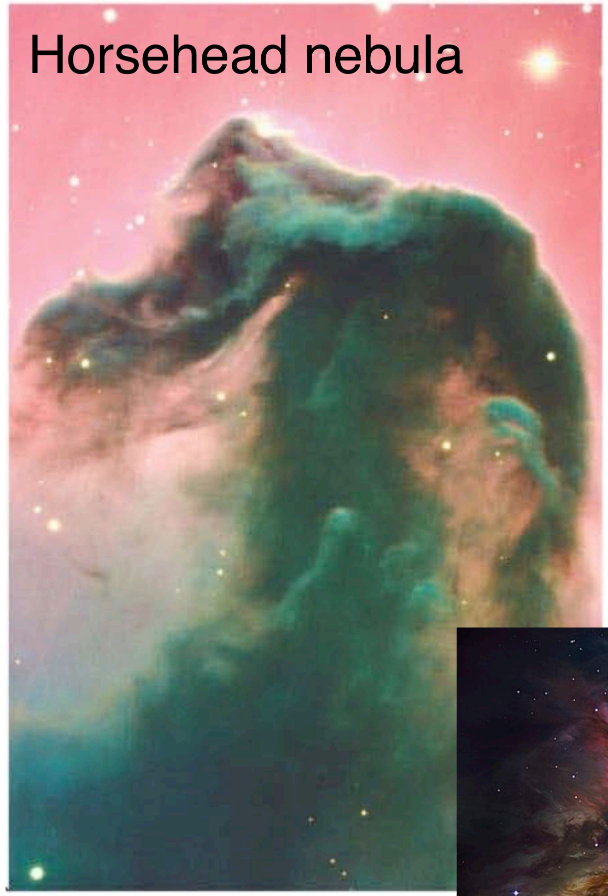
- The interstellar medium consists of gas and dust.

- ***Gas*** is atoms and small molecules, mostly hydrogen and helium, with less than a percent other elements.
  - The gas can be observed in three ways:
    - 21 cm line from neutral hydrogen atom (radio)
    - emission lines from atoms in cold, warm, and hot gas clouds (visible, UV)
    - spectral lines from molecules within cold dark clouds.
- ***Dust*** grains are solid-state particles and more like soot or smoke.
  - Dust absorbs light (“extinction”).
  - Dust preferentially absorbs shorter wavelengths, and thus reddens light that gets through (reddening = changes shape of spectrum, which is how color is defined).
  - Dust also emits its own light, a continuous spectrum, which always peaks somewhere in the infrared part of the spectrum, depending on the dust’s temperature.

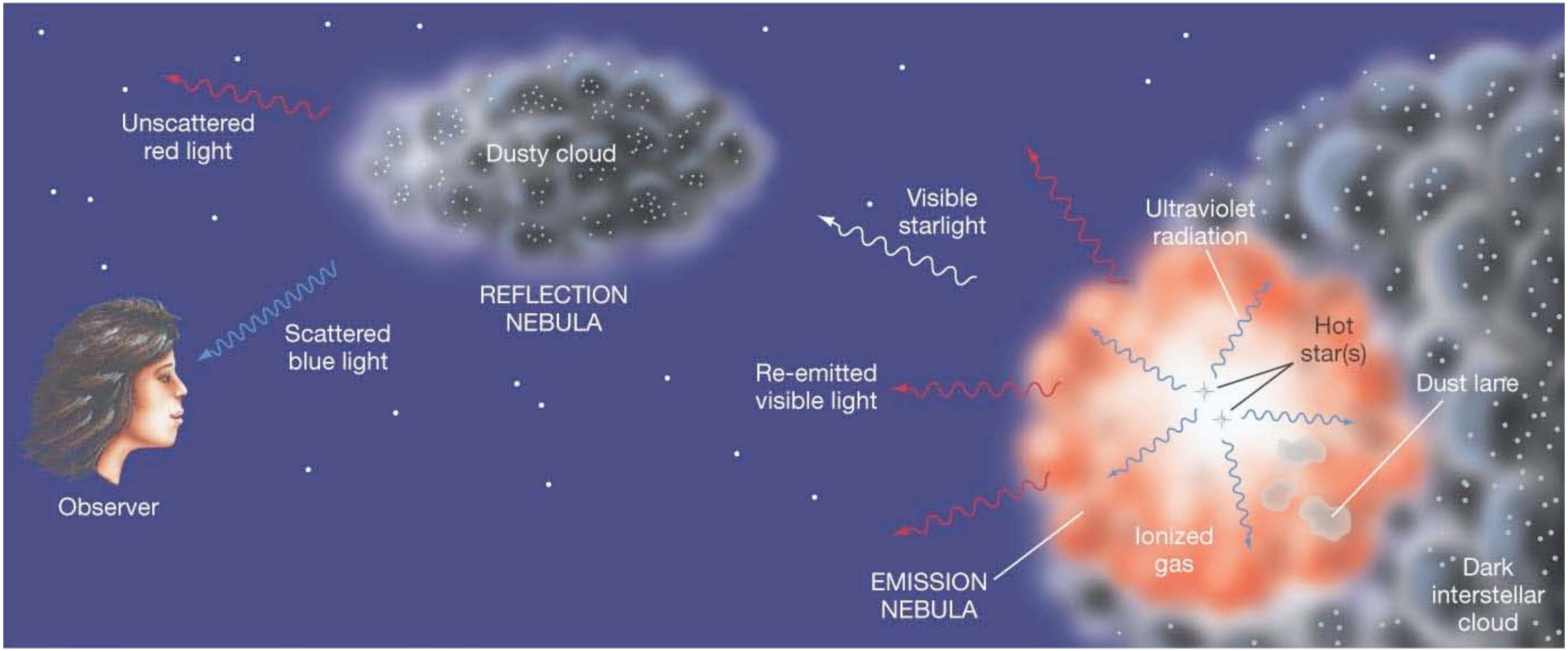
Examples: Nebulae

- “Nebula” is a general term used for fuzzy objects in the sky:
  - Emission nebula: Glows, due to emission lines from gas ionized and heated by hot young stars.
  - Dark & Reflection nebulae: dust cloud



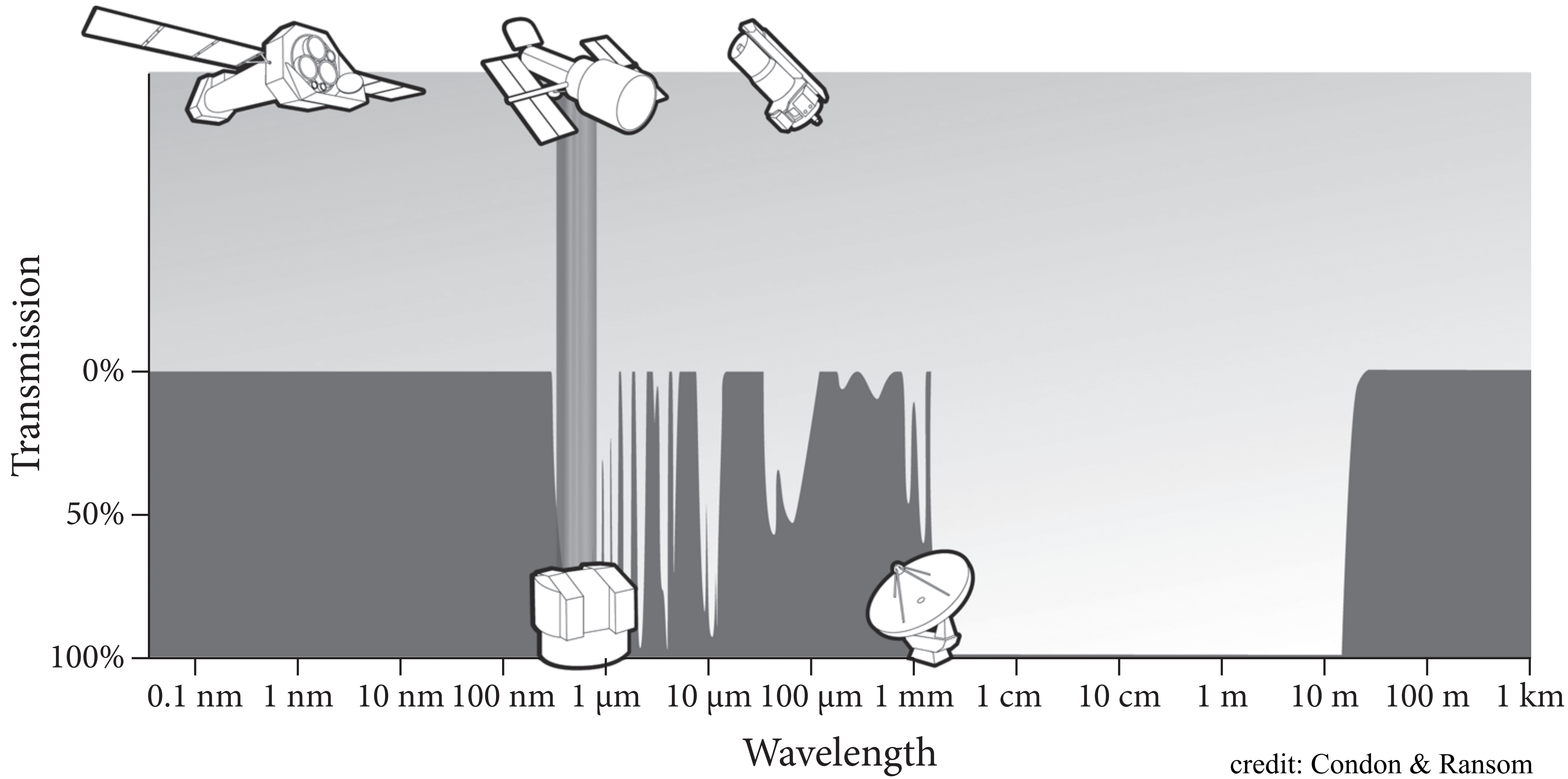


Examples: How nebulae work



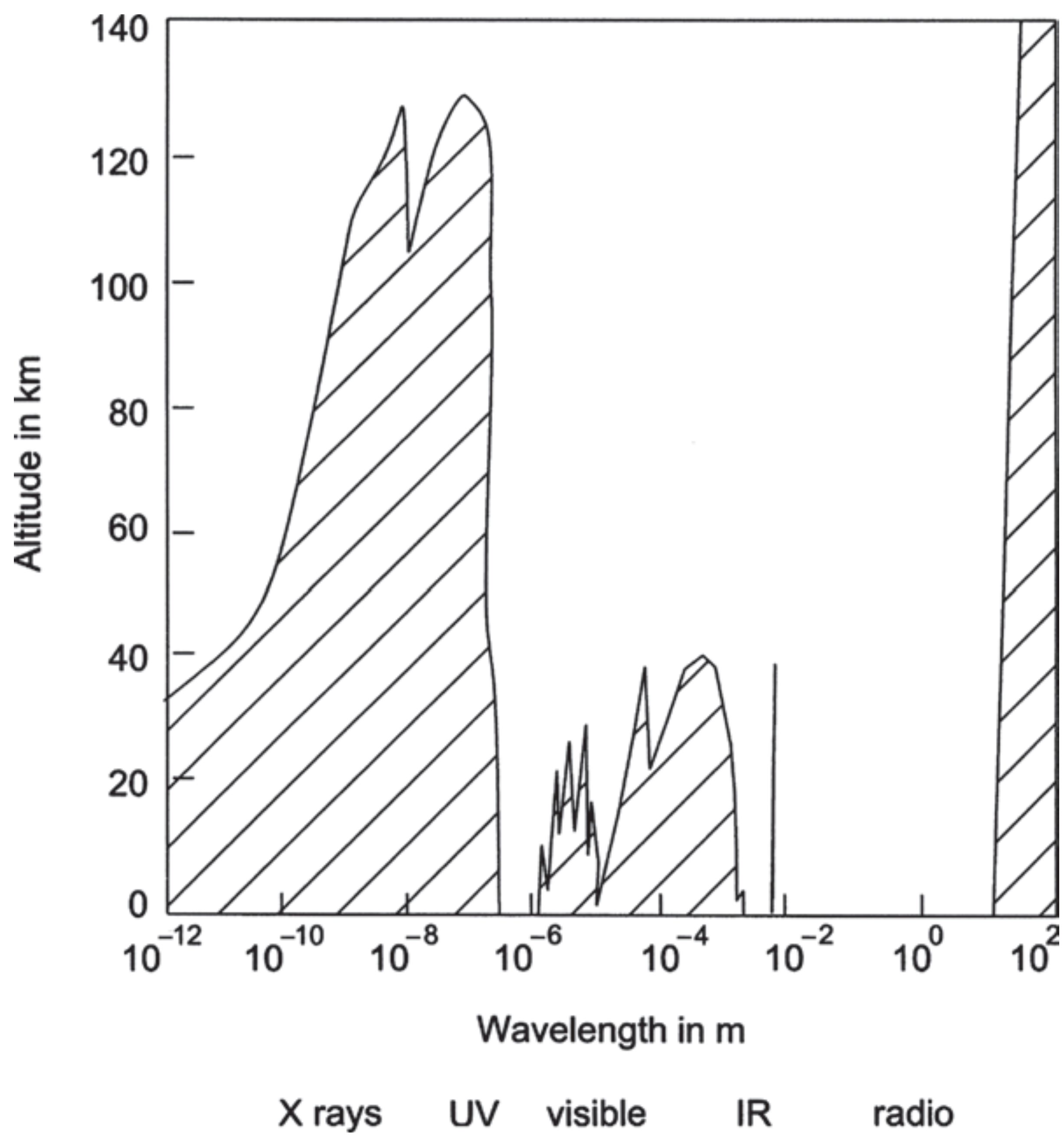
Atmospheric Windows

- The Earth's atmosphere absorbs electromagnetic radiation at most infrared (IR), ultraviolet (UV), X-ray, and gamma-ray wavelengths, so only optical/near-IR and radio observations can be made from the ground.
- The visible-light window is relatively narrow and spans the wavelengths of peak thermal emission from T ~ 3000 K to T ~ 10,000 K blackbodies.



credit: Condon & Ransom
Essential Radio Astronomy

- The penetrating ability of electromagnetic wave through the Earth's atmosphere.
  - The altitudes against different wavelengths indicate the heights above the sea level we have to climb to receive radiation of that wavelength.



credit: Shu (1982)

Astronomical Units: Unit of mass

- The radius and mass of the Earth are

$$R_{\mathrm{E}} = 6.378 \times 10^{6}\,\mathrm{m}$$
$$M_{\mathrm{E}} = 5.974 \times 10^{27}\,\mathrm{g}$$

- In comparison, the Sun is about a million times more massive with about 100 times larger radius:

$$R_{\odot} = 6.96 \times 10^{8}\,\mathrm{m}$$
$$M_{\odot} = 1.989 \times 10^{33}\,\mathrm{g}$$
$$L_{\odot} = 3.86 \times 10^{33}\,\mathrm{erg\,s^{-1}} = 3.86 \times 10^{26}\,\mathrm{W}$$

- The mass of the Sun is often used as the unit of mass in astrophysics.
  - Typical stellar mass = $0.1M_{\odot} - 20M_{\odot}$
  - Globular clusters (dense clusters of stars having nearly spherical shapes): $\sim 10^{5}M_{\odot}$
  - Mass of a typical galaxy: $\sim 10^{11}M_{\odot}$

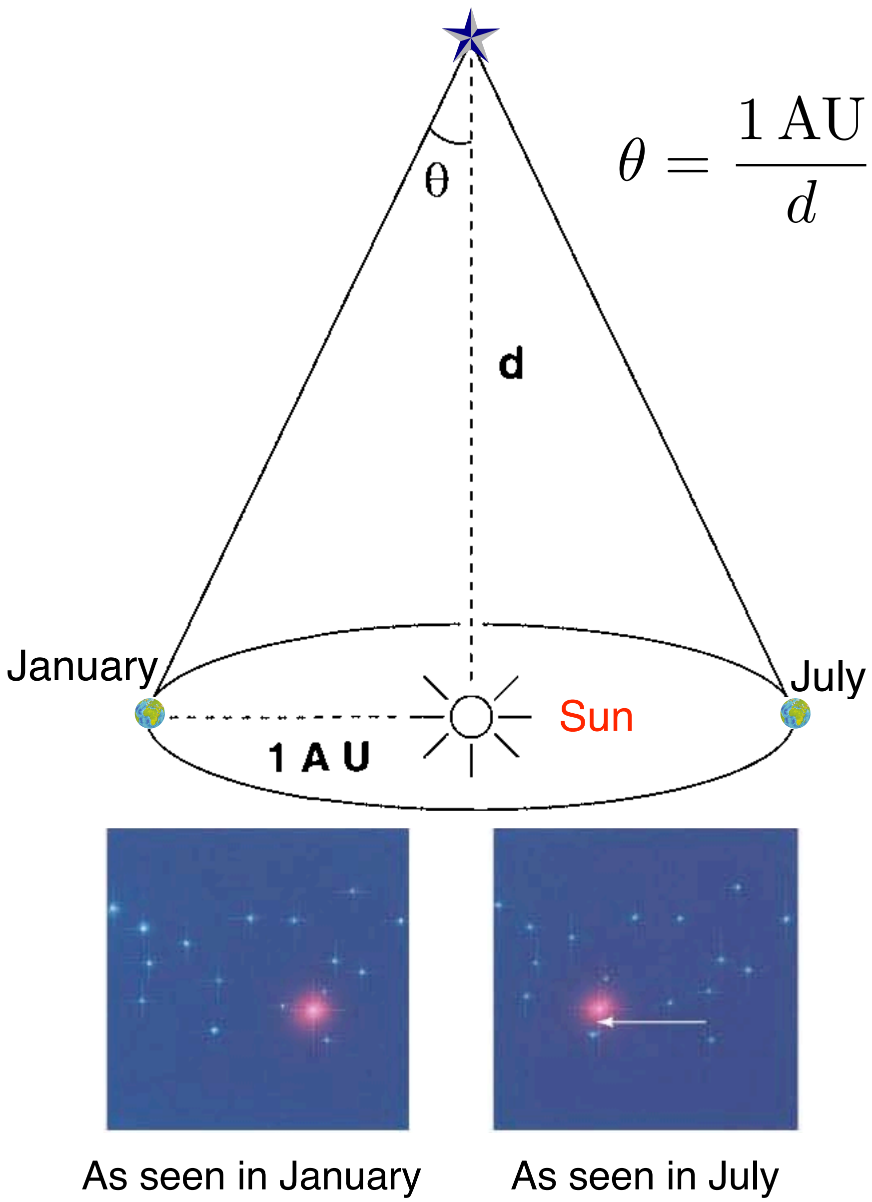
Unit of distance

- **AU (the Astronomical Unit)**: the average distance of the Earth from the Sun

$$\mathrm{AU} = 1.50 \times 10^{11}\,\mathrm{m}$$

- ***parallax***: As the Earth goes around the Sun, the nearby stars change their positions very slightly with respect to the faraway stars. This phenomenon is known as parallax. The angle θ is half of the angle by which this star appears to shift with the annual motion of the Earth and is defined to be the parallax.

$$\theta = \frac{1\,\mathrm{AU}}{d}$$



- ***parsec (pc)***: the distance where the star has to be so that its parallax turns out to be $1''$.

$$\begin{aligned}\mathrm{pc} &= 3.09 \times 10^{16}\,\mathrm{m} \\ &= 3.26\ \text{light years} \\ &= 206{,}265\ \mathrm{AU}\end{aligned}$$

Unit of time & Constellarations

- Time unit:
  - There is no special unit of time. Astrophysicists use years for large time scales and seconds for small time scales, the conversion factor being

$$\mathrm{yr} = 3.16 \times 10^7\,\mathrm{s}\,.$$

  - The age of the Sun in believed to be about 4.5 Gyr.
  - The age of the Universe is estimated to be ~ 13.78 Gyr (billion years).

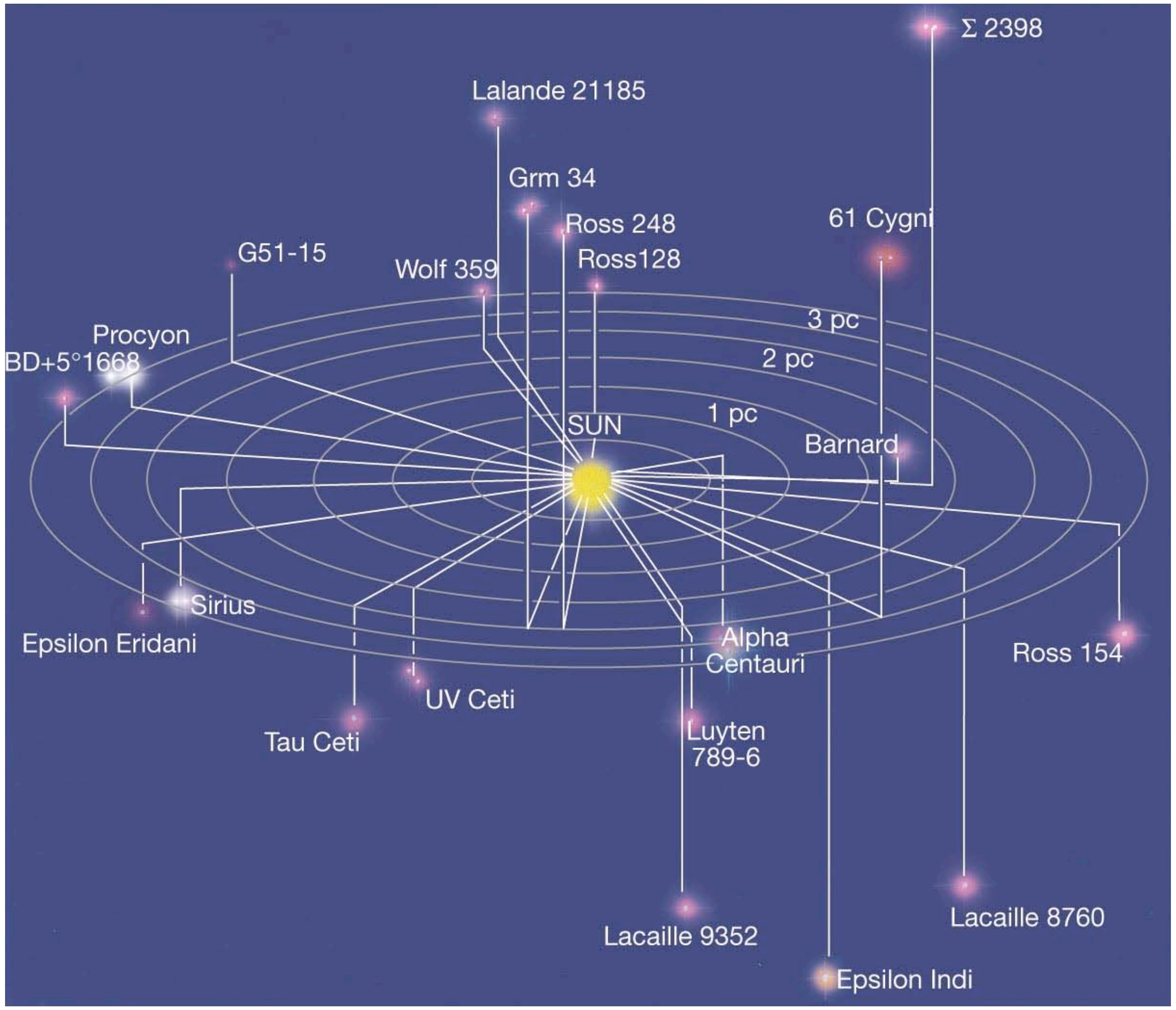
- Constellations:
  - These are just apparent groupings of stars in the sky; they are (usually) not physically associated, and could be at very different distances.

The solar neighborhood

- The star nearest to the Sun, Proxima Centauri, is at a distance of $\sim 1.31\,\mathrm{pc}$. Proxima Centauri is a member of the three-star system Alpha Centauri complex.

- Distance model:
  - Sun is a marble, Earth is a grain of sand orbiting 1 m away.
  - Nearest star is another marble 270 km away.
  - Solar system extends about 50 m from Sun.
  - Rest of distance to nearest star is basically empty.

The 30 closest stars to the Sun



Fundamentals of Radiative Transfer

Overview

- Why do we study radiation processes?
  - Radiative processes link astrophysical systems with astronomical observables.

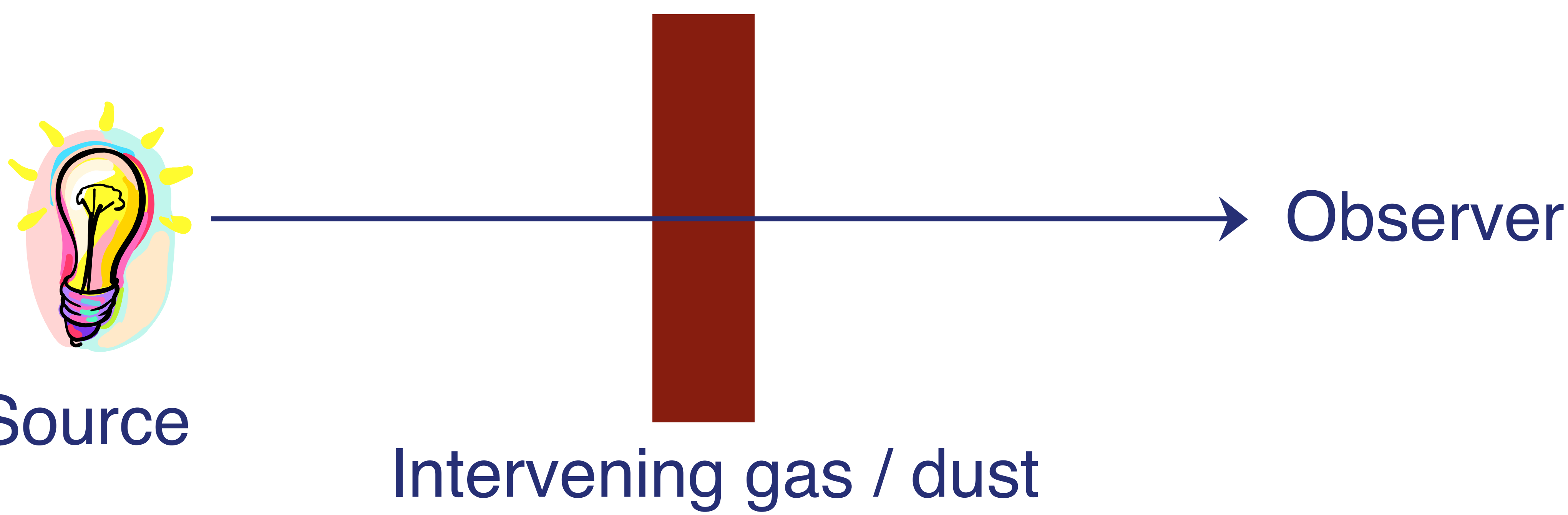
- Radiative processes
  - cover many areas of physics (electrodynamics, quantum mechanics, statistical mechanics, relativity...)

- References
  - Radiative Processes in Astrophysics (George Rybicki & Alan Lightman)
  - Astrophysics: Radiation and Gas Dynamics (천체물리학: 복사와 기체역학, 구본철, 김웅태, in Korean)
  - The Physics of Interstellar and Intergalactic Medium (Bruce T. Draine)
  - Astrophysics processes: The Physics of Astronomical Phenomena (Hale Bradt)

Topics to be covered

- Mechanisms that produce radiation: < next week >
  - Transitions within atoms (or molecules or dust)
    - Radiation from dust will be covered by Prof. Thiem Hoang.
  - Acceleration of electrons in a plasma by electric or magnetic fields.

- How is radiation affected as it propagates through intervening gas and dust media to the observer? < this week >



Basic properties of radiation

- Classically: electromagnetic waves
  - speed of light: $c = 3 \times 10^{10} \mathrm{cm\ s}^{-1}$
  - Electromagnetic radiation of frequency ν, wavelength λ in free space obeys:

$$\lambda = c/\nu$$

- Quantum mechanically: photons
  - quanta: massless, spin-1 particles (boson)
  - Individual photons have energy:

$$E = h\nu = hc/\lambda \ (h = 6.625 \times 10^{-27} \ \mathrm{ergs})$$

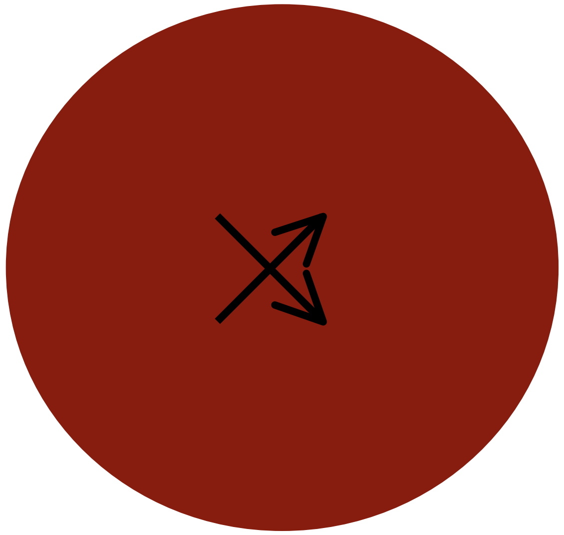
  - momentum:

$$E^2 = (m_\gamma c^2)^2 + (pc)^2$$

$$p = E/c$$

Simplication & Complexity

- Simplification:
  - Astronomical objects are normally much larger than the wavelength of radiation they emit.
  - Diffraction can be neglected.
  - Light rays travel to us along straight lines.



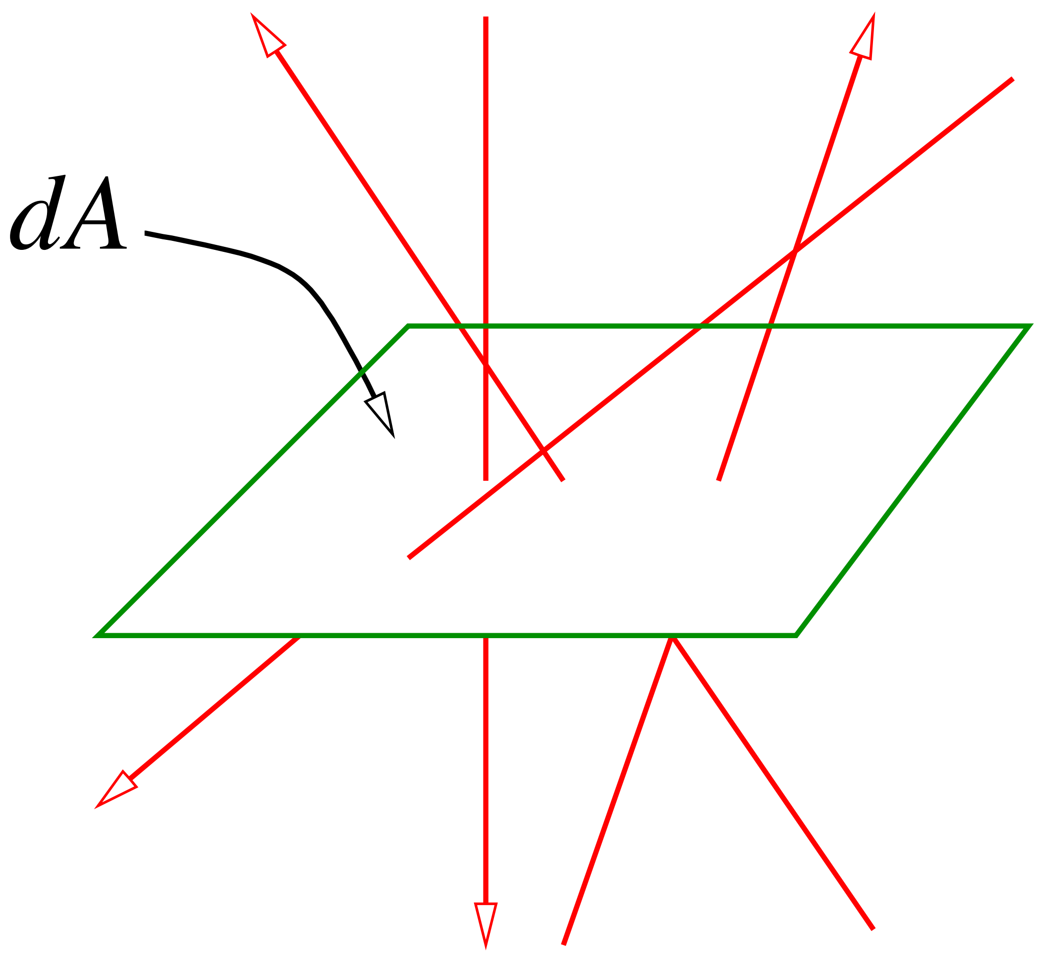
- Complexity:
  - At one point, photons can be traveling in several different directions.
  - For instance, at the center of a star, photons are moving equally in all directions. (However, radiation from a star seen by a distant observer is moving almost exactly radially.)
  - Full specification of radiation needs to say how much radiation is moving in each direction at every point. Therefore, we are dealing with the five- or six-dimensional problem. ($[x, y, z] + [\theta, \phi] + [t]$)

Energy Flux

- Definition
  - Consider a small area dA, exposed to radiation for a time dt.
  - Energy flux F is defined as ***the net energy*** dE ***passing through the element of area in all directions in the time interval*** so that

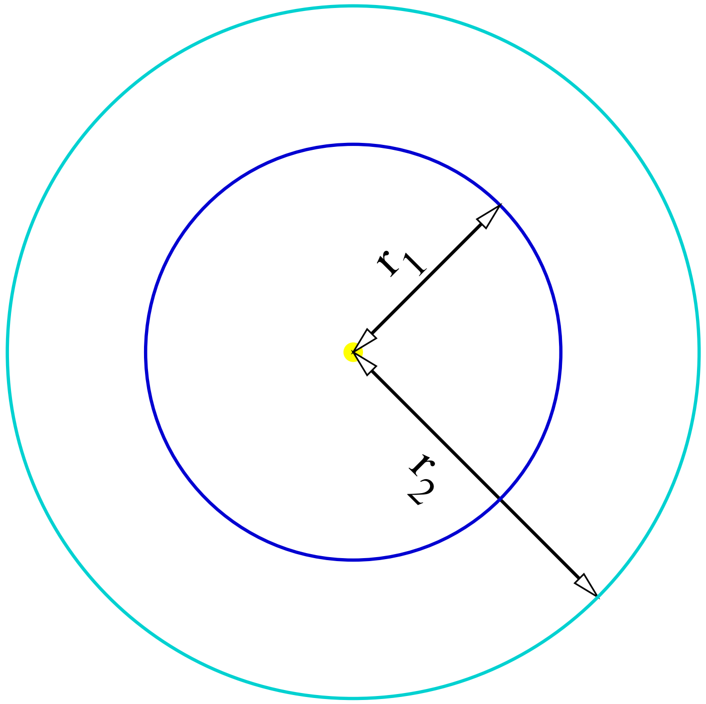
$$dE = F \times dA \times dt$$

  - Note that F ***depends on the orientation of the area element*** dA.
  - Unit: $\mathrm{erg\ cm^{-2}\ s^{-1}}$



Inverse Square Law

- Flux from an isotropic radiation source, i.e., a source emitting equal amounts of energy in all directions.



  - Because of energy conservation, flux through two shells around the source must be the same.

$$4\pi r_1^2 F(r_1) = 4\pi r_2^2 F(r_2)$$

  - Therefore, we obtain the inverse square law. $F = \dfrac{\text{const.}}{r^2}$

Energy Flux Density

- Real detectors are sensitive to a limited range of wavelengths. We need to consider how the incident radiation is distributed over frequency.

Total energy flux: $F = \int F_\nu(\nu) d\nu$ Integral of F_ν over all frequencies

$\downarrow$

Units: $\mathrm{erg\ s^{-1}\ cm^{-2}\ Hz^{-1}}$

- F_ν is often called the “flux density.”

- Radio astronomers use a special unit to define the flux density: $1\ \mathrm{Jansky\ (Jy)} = 10^{-23}\ \mathrm{erg\ s^{-1}\ cm^{-2}\ Hz^{-1}}$

The magnitude scale

- For historical reasons, fluxes in the optical and infrared are measured in magnitudes.

- On the basis of naked eye observations, the Greek astronomer Hipparchus (190-120 BC) classified all the stars into six classes according to their apparent brightness.
  - The brighter ones belong to the first magnitude class. The faintest ones belong to the sixth magnitude class.

- Pogson (1856) noted that the faintest stars visible to the naked eye are about 100 times fainter compared to the brightest stars.
  - The brightest and faintest stars differ by five magnitude classes.
  - Therefore, stars in two successive classes should differ in apparent brightness by a factor $100^{1/5}$.

- Note that the human eye is more sensitive to a geometric progression $(I_0, 2I_0, 4I_0, 8I_0, \cdots)$ of intensity rather than an arithmetic progression $(I_0, 2I_0, 3I_0, 4I_0, \cdots)$. In other words, ***the apparent magnitude as perceived by the human eye scales roughly logarithmically with the radiation flux.***

- Suppose two stars have apparent brightnesses F_1 and F_2 and their magnitude classes are m_1 and m_2.

$$\frac{F_2}{F_1} = (100)^{\frac{1}{5}(m_1 - m_2)}.$$

- Then, on taking the logarithm of this, we find

$$m_1 - m_2 = 2.5 \log_{10}\left(\frac{F_2}{F_1}\right).$$

- This is the definition of ***apparent magnitude*** denoted by m, which is a measure of the apparent brightness of an object in the sky.
  - Note that the magnitude scale is defined in such a fashion that ***a fainter object has a higher value of magnitude.***

- We need a measure that quantifies the luminosity or intrinsic brightness of an object.
- The ***absolute magnitude*** of a celestial object is defined as the magnitude it would have if it were placed at a distance of 10 pc.
  - If the object is at a distance d pc, then $(10/d)^2$ is the ratio between its apparent brightness and the brightness it would have if it were at a distance of 10 pc

$$\frac{F(d)}{F(10)} = \left(\frac{10\,\mathrm{pc}}{d}\right)^2$$

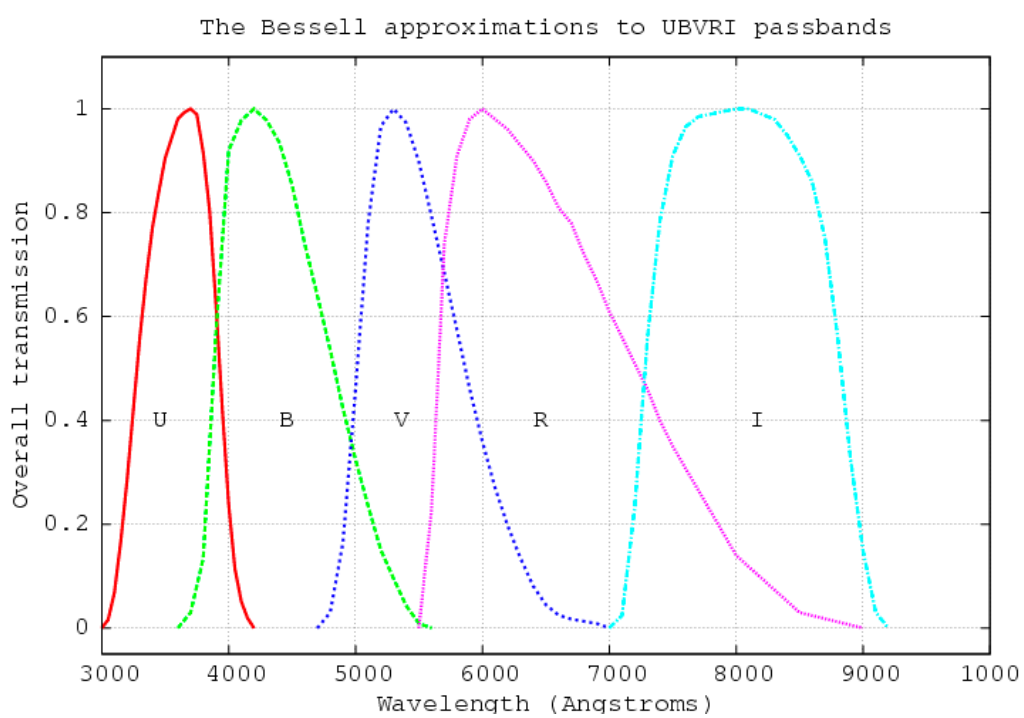
  - Then, the relation between apparent magnitude m and absolute magnitude M is

$$m - M = 2.5\log_{10}\left(\frac{d}{10\,\mathrm{pc}}\right)^2 = 5\log_{10}\left(\frac{d}{10\,\mathrm{pc}}\right)$$

  - The difference $m - M$ is called the ***distance modulus***.

Filters and Wavebands

- Common bandpasses



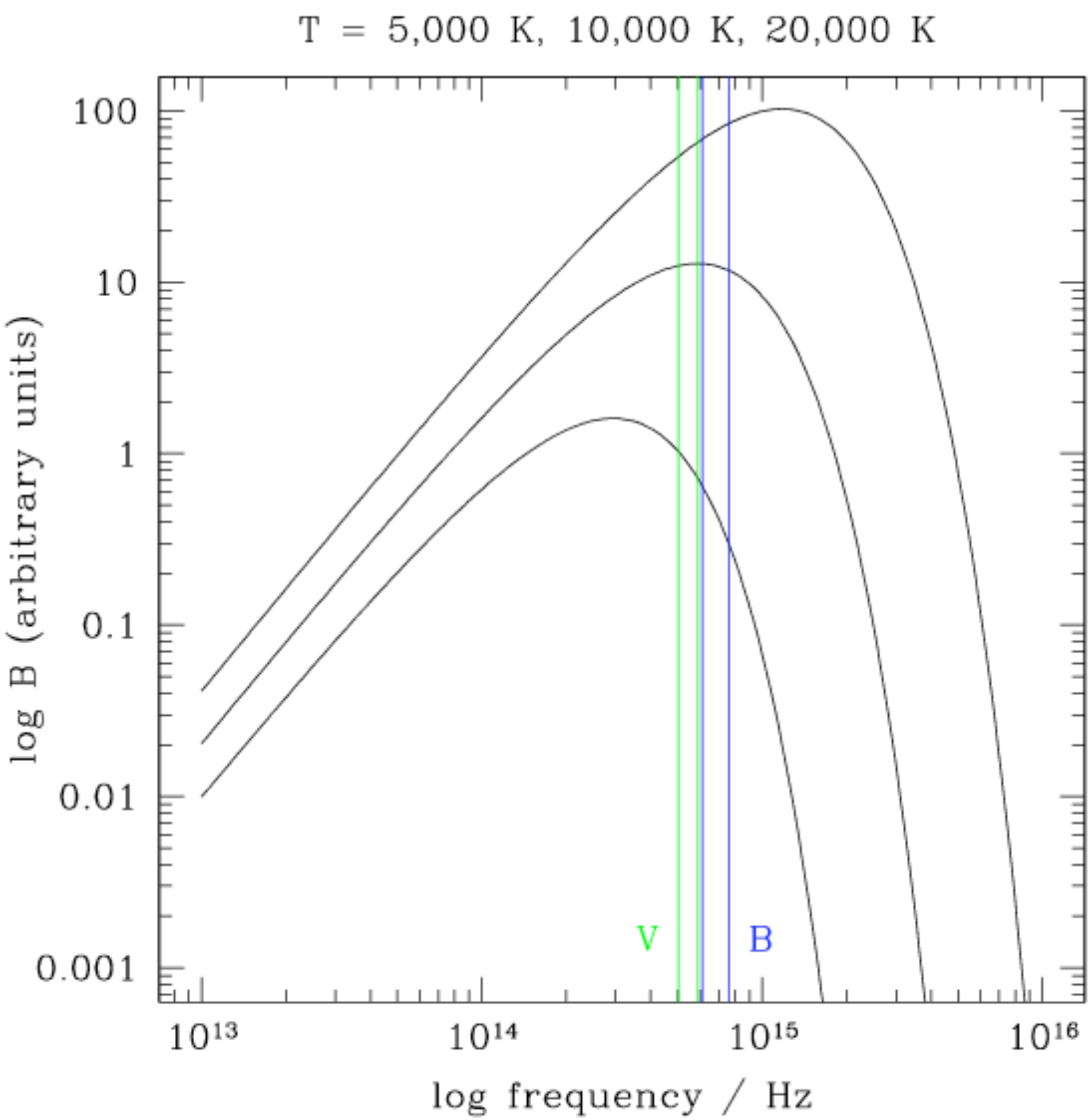
U (ultraviolet)	365 nm
B (blue)	440 nm
V (visible)	550 nm
R (red)	641 nm
I (near-infrared)	0.896 nm
J	0.900 nm
H	1.22 nm
K	2.19 nm

  - These are the central wavelengths of each band, which extend ~10% in wavelength to either side.
  - Magnitude at each bandpass is denoted by m_U, m_B, m_V, m_R, m_K, etc.

- Zero-points:
  - Note that the magnitude scale has been relatively defined.
  - ***The zero-points are defined such that the magnitude of a standard star (Vega) is zero in all wavebands.***

Colors

- The color of a star or other object is defined as the difference in the magnitude in each of two bandpasses:
  - e.g., the (B-V) color is : B-V = m_B-m_V

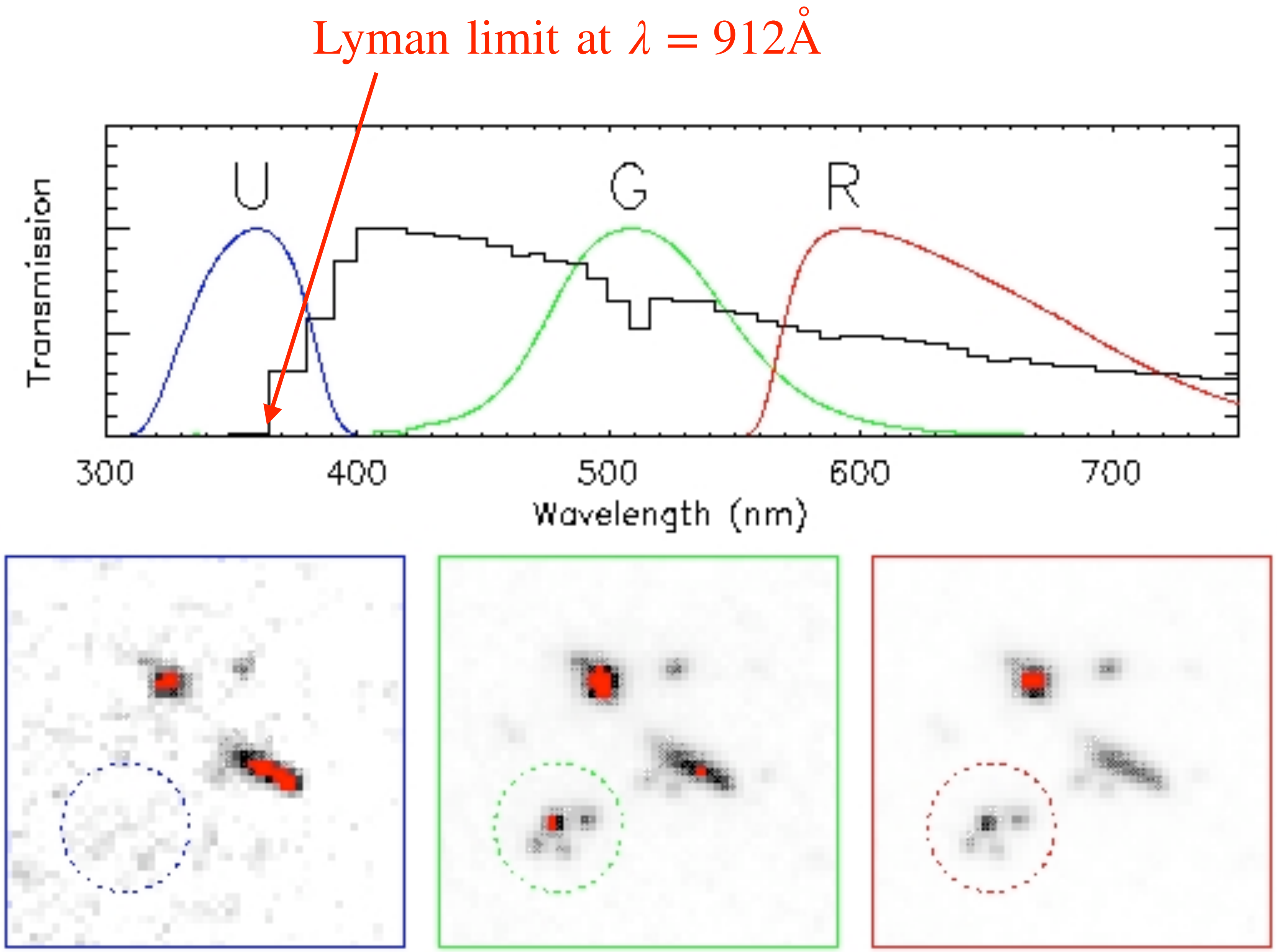


- Stars radiate roughly as blackbodies, so the color reflects surface temperature.

- The standard star “Vega” has T = 9500 K. By definition its color is zero.

The "Drop-out" Technique

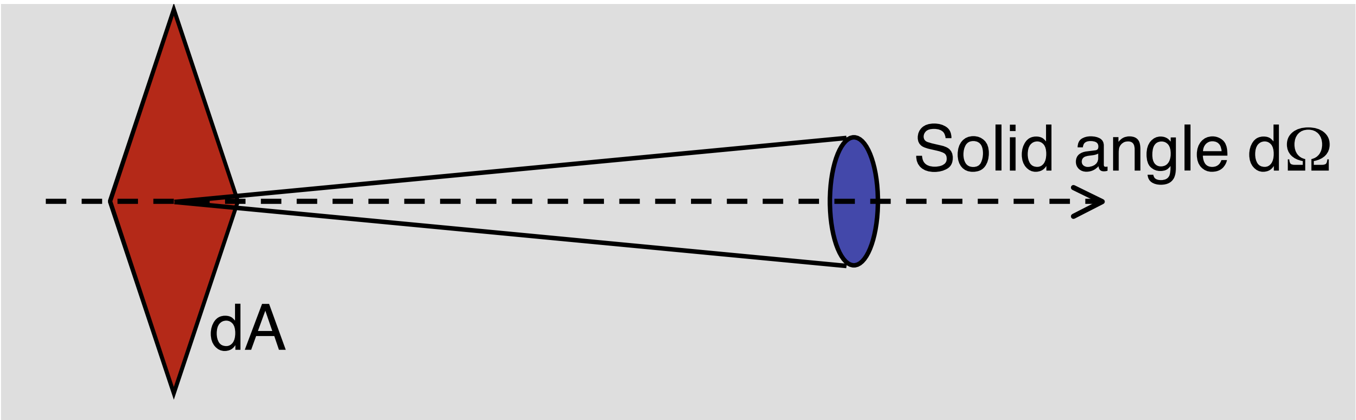
- The color can be useful - e.g., basis of most successful method for finding very distant (high redshift) galaxies:



- There is the large break in the continuum flux from an object that occurs at the 912Å Lyman limit from neutral hydrogen absorption.
- Observed galaxy spectrum shifts to the right for source at higher redshift.
- Because the spectrum has a sharp `break', the flux in U band drops off sharply.

(Specific) Intensity or (Surface) Brightness

- Recall that ***flux is a measure of the energy carried by all rays passing through a given area***
- ***Intensity is the energy carried along by individual rays.***



  - Let dE_ν be the amount of radiant energy which crosses the area dA in a direction within solid angle $d\Omega$ centered about $\mathbf{k}$ in a time interval dt with photon frequency between ν and $\nu + d\nu$.
  - The monochromatic specific intensity I_ν is then defined by the equation.

$$dE_\nu = I_\nu(\mathbf{k}, \mathbf{x}, t) dA_{\mathbf{k}} d\Omega d\nu dt$$

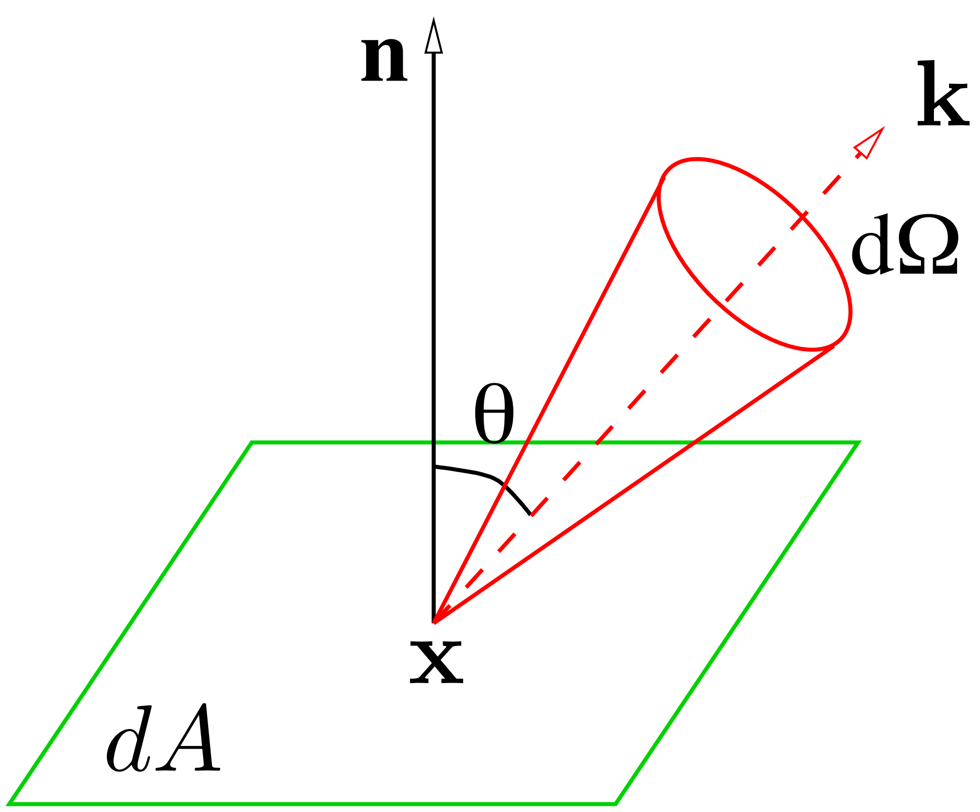
  - Unit: $\mathrm{erg\ s^{-1}\ cm^{-2}\ sr^{-1}\ Hz^{-1}}$
- Another (more intuitive) name for the specific intensity is ***brightness***.

Relation between the flux and the specific intensity

- Let's consider a small area dA, with light rays passing through it at all angles to the normal vector $\mathbf{n}$ of the surface.

- For rays about $\mathbf{k}$ at angle θ, the area normal to $\mathbf{k}$ is

$$dA_{\mathbf{k}} = dA\cos\theta$$



  - If $\theta = 90^\circ$, then light rays in that direction contribute zero flux through area dA.

- Hence, net flux in the direction of $\mathbf{n}$ is given by integrating over all solid angles:

$$F_\nu dA d\nu dt = \int I_\nu(\mathbf{k}, \mathbf{x}, t) dA_{\mathbf{k}} d\Omega d\nu dt$$

$$\boxed{F_\nu = \int I_\nu \cos\theta d\Omega = \int_0^{2\pi}\int_0^{\pi} I_\nu \cos\theta \sin\theta d\theta d\phi}$$

Note

- Intensity can be defined as per wavelength interval.

$$I_\nu |d\nu| = I_\lambda |d\lambda| \qquad \leftarrow \left| \frac{d\nu}{d\lambda} \right| = \frac{c}{\lambda^2} = \frac{\nu}{\lambda}$$

$$\nu I_\nu = \lambda I_\lambda$$

- Integrated intensity is defined as the intensity over all frequencies.

$$I = \int_0^\infty I_\nu d\nu = \int_0^\infty I_\lambda d\lambda$$

How does specific intensity changes along a ray in free space

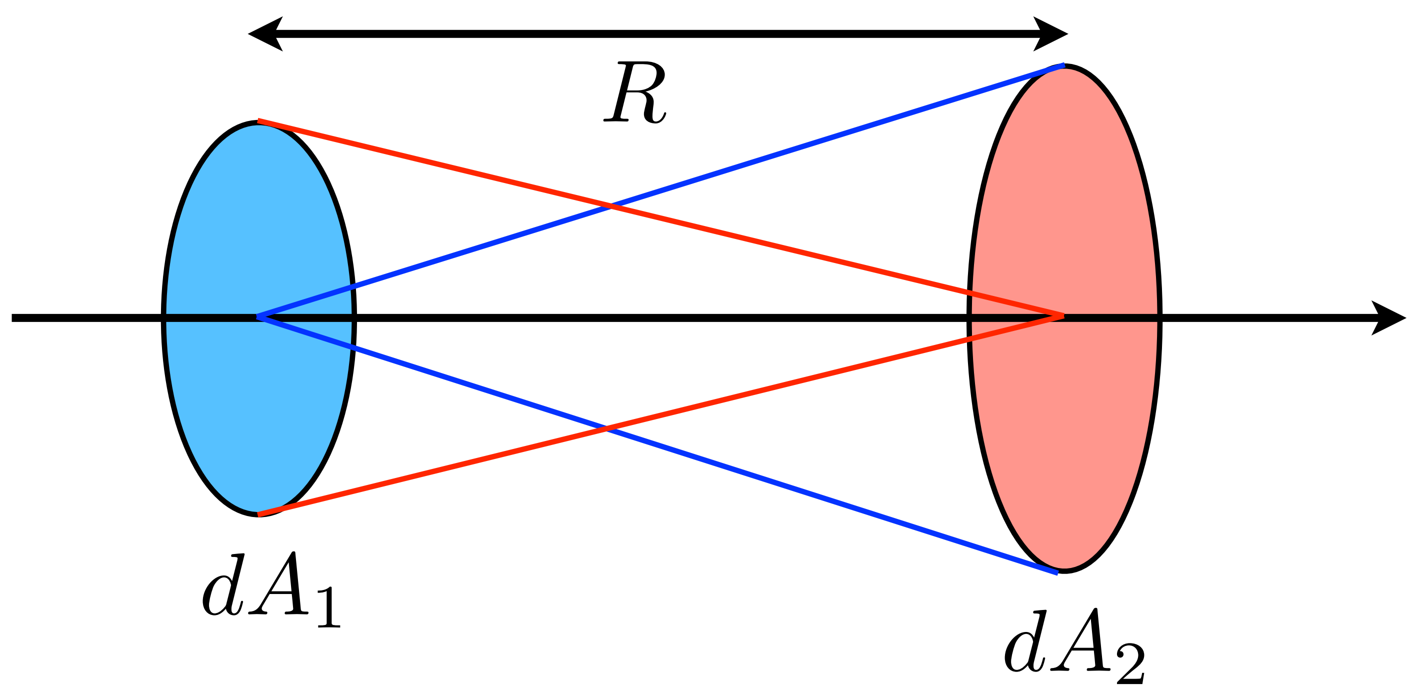
- Consider a bundle of rays and any two points along the rays. Construct areas dA_1 and dA_2 normal to the rays at these points.
- Consider the energy carried by the rays passing through both areas. Because energy is conserved,

$$dE_1 = I_1 dA_1 d\Omega_1 d\nu dt = dE_2 = I_2 dA_2 d\Omega_2 d\nu dt$$

- Here, $d\Omega_1$ is the solid angle subtended by dA_2 at the location 1 and so forth.

$$d\Omega_1 = \frac{dA_2}{R^2}$$
$$d\Omega_2 = \frac{dA_1}{R^2}$$

$\rightarrow$ $\boxed{I_1 = I_2}$



- Conclusion: the specific intensity remains the same as radiation propagates through free space.

$$I_1 = I_2$$

- If we measure the distance along a ray by variable s, we can express the result equivalently in differential form:

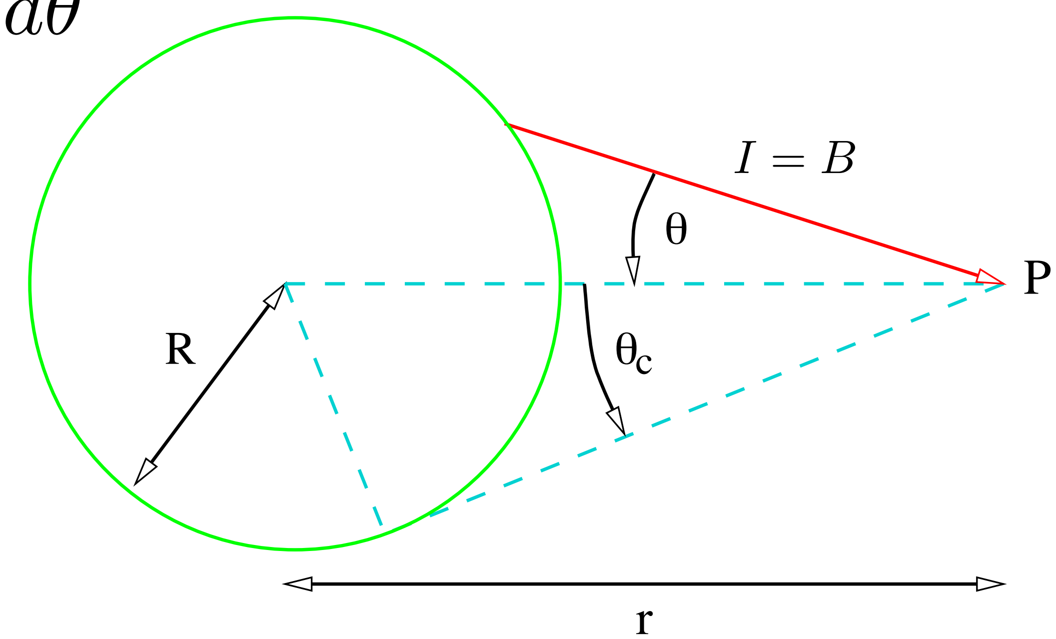
$$\boxed{\frac{dI}{ds} = 0}$$

Inverse Square Law for a Uniformly Bright Sphere

- Let's calculate the flux at P from a sphere of uniform brightness B.

$$F = \int I \cos\theta d\Omega = B \int_0^{\pi} d\phi \int_0^{\theta_c} \cos\theta \sin\theta d\theta$$
$$= \pi B(1 - \cos^2\theta_c) = \pi B \sin^2\theta_c$$

$$\sin\theta_c = \frac{R}{r} \rightarrow F = \pi B \left(\frac{R}{r}\right)^2$$



- Therefore, there is no conflict between the constancy of intensity and the inverse square law.
- Note
  - The flux at a surface of uniform brightness B is $F = \pi B$.
  - For stellar atmosphere, the astrophysical flux is defined by F/π .

(Specific) Energy Density

- Consider a bundle of rays passing through a volume element dV in a direction Ω .
- Then, the energy density per unit solid angle is defined by

$$dE = u_\nu(\Omega) dV d\Omega d\nu$$

- Since radiation travels at velocity c, the volume element is

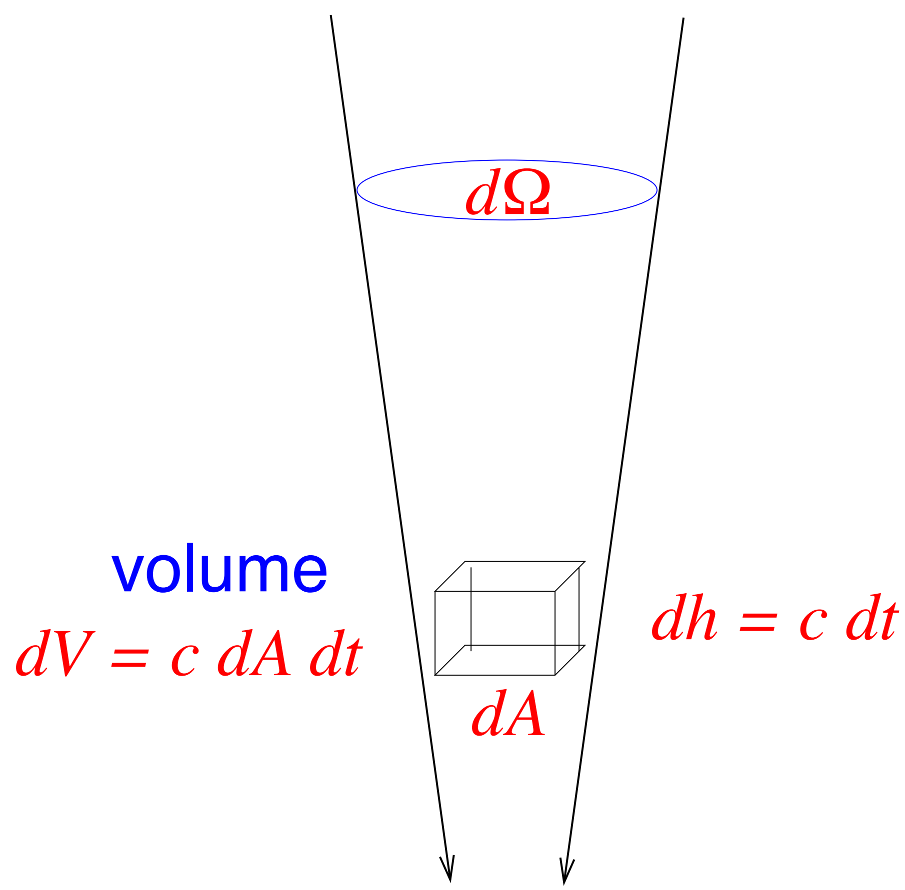
$$dV = dA(cdt)$$

- According to the definition of the intensity,

$$dE = I_\nu dA dt d\Omega d\nu$$

- Then, we have

$$u_\nu(\Omega) = I_\nu(\Omega)/c$$



Energy Density and Mean Intensity

- Integrating over all solid angle, we obtain

$$u_\nu = \int u_\nu(\Omega) d\Omega = \frac{1}{c} \int I_\nu d\Omega$$

- Mean intensity is defined by

$$J_\nu = \frac{1}{4\pi} \int I_\nu d\Omega$$

- Then, the energy density is

$$u_\nu = \frac{4\pi}{c} J_\nu$$

- Total energy density is obtained by integrating over all frequencies.

$$u = \int u_\nu d\nu = \frac{4\pi}{c} \int J_\nu d\nu$$

Radiative Transfer Equation

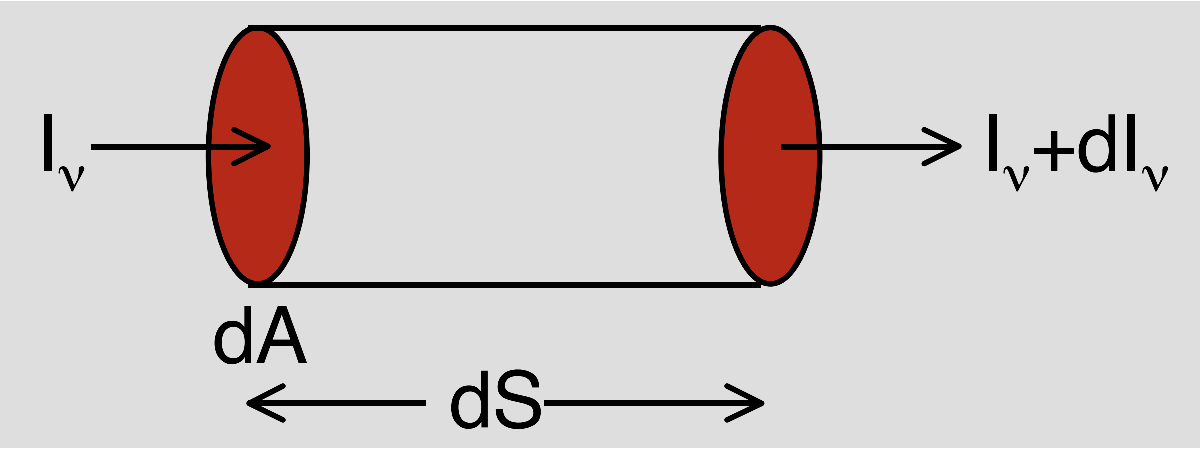
- As a ray passes through matter, energy may be added, subtracted, or scattered from it by emission, absorption, or scattering.

- The intensity will not in general remain constant.

- We need to derive the radiative transfer equation.

Emission

- If the radiation travels through a medium which is itself emitting radiation, that will add to the energy:



  - Spontaneous emission coefficient j_ν is the amount of energy emitted per unit time, per unit solid angle, per unit frequency, and per unit volume:

$$dE = j_\nu dV d\Omega dt d\nu \quad (j_\nu : \mathrm{erg\ cm^{-3}\ s^{-1}\ sr^{-1}\ Hz^{-1}})$$

  - In going a distance ds, a beam of cross section dA travels through a volume $dV = dAds$. Thus the intensity added to the beam is by ds spontaneous emission is:

$$dI_\nu = j_\nu ds$$

- Therefore, the equation of radiative transfer for pure emission becomes:

$$\frac{dI_\nu}{ds} = j_\nu$$

  - If we know what j_ν is, we can integrate this equation to find the change in specific intensity as radiation propagates through the medium:

$$I_\nu(s) = I_\nu(0) + \int_0^s j_\nu(s')ds'$$

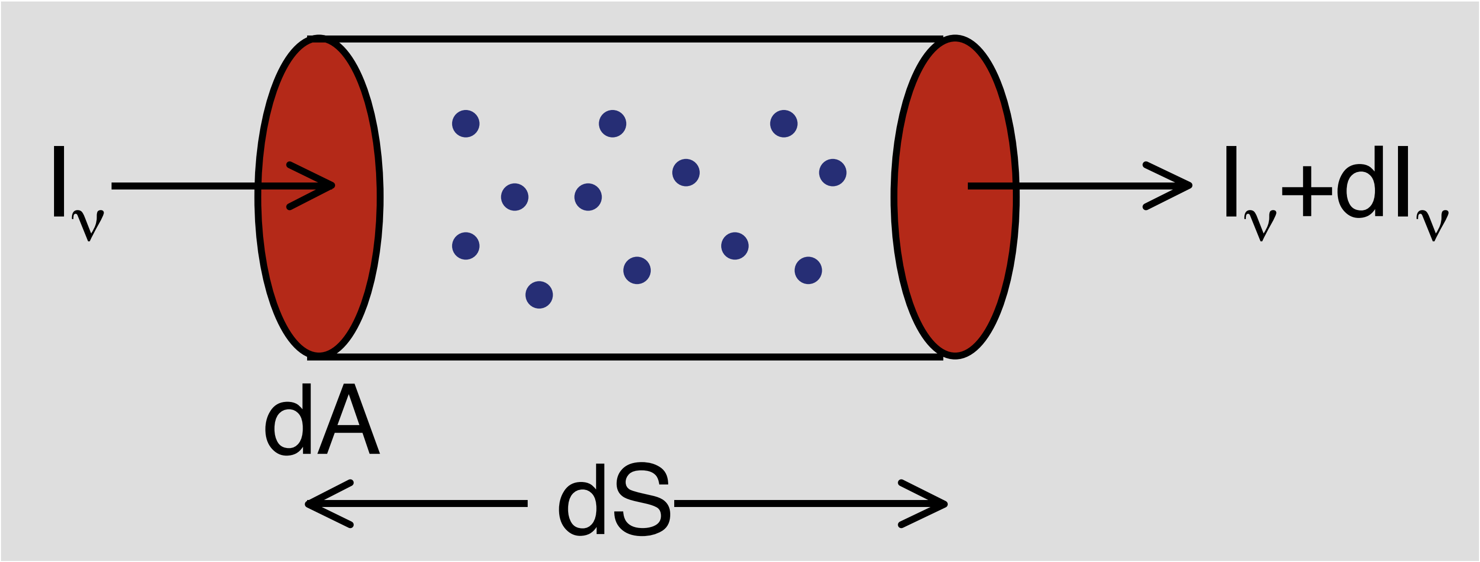
- We also define (angle-averaged, mass) emissivity ϵ_ν to be the energy emitted spontaneously per unit frequency, per unit time, and per unit mass. For isotropic emission,

$$dE = \epsilon_\nu \rho dV\, dt d\nu \frac{d\Omega}{4\pi}$$

$$j_\nu = \frac{\epsilon_\nu \rho}{4\pi} \quad \text{or} \quad \int j_\nu d\Omega = \epsilon_\nu \rho \qquad (\epsilon_\nu : \text{erg g}^{-1}\ \text{s}^{-1}\ \text{Hz}^{-1})$$

Absorption

- If the radiation travels through a medium which absorbs radiation, the energy in the beam will be reduced:



- Let n denote the number density of absorbers (particles per unit volume).
- Assume that each absorber has cross-sectional area = σ_ν (in units of cm^2).
- If a beam travels through ds, total area of absorbers is

$$\text{number of absorbers} \times \text{cross} - \text{section} = n \times dA \times ds \times \sigma_\nu$$

Fraction of radiation absorbed = fraction of area blocked:

$$\frac{dI_\nu}{I_\nu} = -\frac{ndAds\sigma_\nu}{dA} = -n\sigma_\nu ds$$

$$dI_\nu = -n\sigma_\nu I_\nu ds \equiv \alpha_\nu I_\nu ds$$

- ***Absorption coefficient*** is defined as $\alpha_\nu \equiv n\sigma_\nu$ (units: cm^{-1}).
- We can write the absorption coefficient in terms of mass:

$$\alpha_\nu \equiv \rho\kappa_\nu$$

- κ_ν is called the ***mass absorption coefficient*** or the ***opacity***.
- Opacity has units of $\mathrm{cm}^2\ \mathrm{g}^{-1}$ (i.e., the cross-section of a gram of gas).

- Rearranging the previous equation, we obtain the equation of radiative transfer for pure absorption:

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu$$

  - The amount of reduced energy depends on how much radiation we already have.
  - Integrate to find how radiation changes along path:

$$\int_0^s \frac{dI_\nu}{I_\nu} = -\int_0^s \alpha_\nu(s')ds'$$

$$\left[\ln I_\nu\right]_0^s = -\int_0^s \alpha_\nu(s')ds'$$

$$I_\nu(s) = I_\nu(0)\exp\left[-\int_0^s \alpha_\nu(s')ds'\right]$$

- If the absorption coefficient is a constant (example: a uniform density gas of ionized hydrogen), then we obtain

$$I_\nu(s) = I_\nu(0)e^{-\alpha_\nu s}$$

specific intensity after distance s

initial intensity

radiation exponentially absorbed with distance

- ***Optical depth***:
  - Imagine radiation traveling into a cloud of absorbing gas, exponential defines a scale over which radiation is attenuated.
  - When $\int_0^s \alpha_\nu(s')ds' = 1$, the intensity will be reduced to 1/e of its original value.

- We define the optical depth τ_ν as:

$$\tau_\nu(s) = \int_0^s \alpha_\nu(s')ds' \quad \text{or} \quad d\tau_\nu = \alpha_\nu ds$$

- A medium is ***optically thick*** at a frequency ν if the optical depth for a typical path through the medium satisfies:

$$\tau_\nu(s) > 1$$

- The medium is said to be ***optically thin*** if, instead:

$$\tau_\nu(s) < 1$$

- An optically thin medium is one which a typical photon of frequency ν can pass through without being (significantly) absorbed.

Radiative Transfer Equation

- ***Radiative transfer equation*** with both absorption and emission is

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

absorption emission

- We can rewrite the radiative transfer equation using the optical depth as a measure of `distance' rather than s:

$$\frac{dI_\nu}{\alpha_\nu ds} = -I_\nu + \frac{j_\nu}{\alpha_\nu}$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

- where $S_\nu \equiv j_\nu / \alpha_\nu$ is the source function. This is an alternative and sometimes more convenient way to write the equation.

Example: Thomson Scattering

- A free electron has a cross-section to radiation given by

$$\sigma_\nu^{\mathrm{T}} = 6.7 \times 10^{-25}\ \mathrm{cm}^2$$

independent of frequency. The opacity is therefore

$$\kappa_\nu = \frac{n}{\rho}\sigma_\nu = \frac{N_A}{1g}\sigma_\nu = 0.4\ \mathrm{cm}^2\ \mathrm{g}^{-1}$$

$$(N_A = 6.022 \times 10^{23}\ \text{particles/mole} \quad : \quad \text{Avogardo constant})$$

if the gas is pure hydrogen (protons and electrons only).

Mean Free Path

- From the exponential absorption law, the probability of a photon absorbed between optical depths τ_ν and $\tau_\nu + d\tau_\nu$:

$$|dI_\nu| = \left|\frac{dI_\nu}{d\tau_\nu}\right| d\tau_\nu \quad \& \quad |dI_\nu| \propto P(\tau_\nu)d\tau_\nu \quad \rightarrow \quad P(\tau_\nu) = e^{-\tau_\nu}$$

  - The mean optical depth traveled is thus equal to unity:

$$\langle\tau_\nu\rangle = \int_0^\infty \tau_\nu P(\tau_\nu)d\tau_\nu = \int_0^\infty \tau_n u e^{-\tau_\nu} d\tau_\nu = 1$$

  - The mean free path is defined as the average distance a photon can travel through an absorbing material without being absorbed. In a homogeneous medium, the mean free path is determined by

$$\langle\tau_\nu\rangle = \alpha_\nu \ell_\nu = 1 \quad \rightarrow \quad \boxed{\ell_\nu = \frac{1}{\alpha_\nu} = \frac{1}{n\sigma_\nu}}$$

  - A local mean path at a point in an inhomogeneous material can be also defined.

Formal Solution of the RT equation

$$\frac{dI_\nu}{d\tau_\nu} = S_\nu - I_\nu$$

$$e^{\tau_\nu}\frac{dI_\nu}{d\tau_\nu} + e^{\tau_\nu} I_\nu = e^{\tau_\nu} S_\nu$$

$$\frac{d}{d\tau_\nu}\left(e^{\tau_\nu} I_\nu\right) = e^{\tau}_{\nu} S_\nu$$

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} S_\nu(\tau'_\nu) d\tau'_\nu$$

- The solution is easily interpreted as the sum of two terms:
  - the initial intensity diminished by absorption
  - the integrated source diminished by absorption.

- For a constant source function, the solution becomes

$$\begin{aligned} I_\nu(\tau_\nu) &= I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu}) \\ &= S_\nu + e^{-\tau_\nu}\left(I_\nu(0) - S_\nu\right) \end{aligned}$$

Relaxation

- "Relaxation"

$$I_\nu > S_\nu \quad \rightarrow \quad \frac{dI_\nu}{d\tau_\nu} < 0, \quad \text{then } I_\nu \text{ tends to decrease along the ray}$$

$$I_\nu < S_\nu \quad \rightarrow \quad \frac{dI_\nu}{d\tau_\nu} > 0, \quad \text{then } I_\nu \text{ tends to increase along the ray}$$

- The source function is the quantity that the specific intensity tries to approach, and does approach if given sufficient optical depth.

$$\text{As} \quad \tau_\nu \rightarrow \infty, \quad I_\nu \rightarrow S_\nu$$

Thermal equilibrium

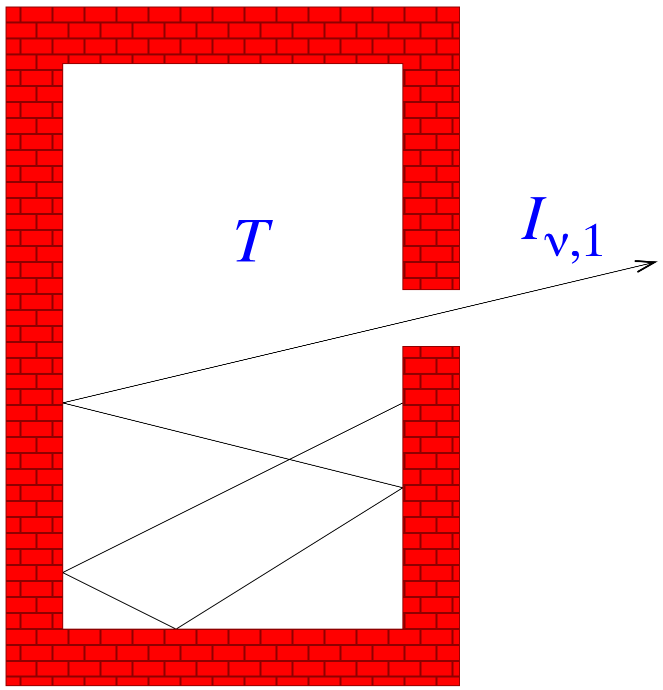
- In general, equilibrium means a state of balance.
- ***Thermal equilibrium refers to steady states of temperature, which defines the average energy of material or photons.***

- In a state of (complete) ***thermodynamic equilibrium (TE)***, no net flows of matter or of energy, no phase changes, and no unbalanced potentials (or driving forces), within the system. ***In TE, matter and radiation are in equilibrium at the same temperature T.***

- If the material is (locally) in thermodynamic equilibrium at a well-defined temperature T, ***it is said to be in local thermodynamic equilibrium (LTE) even if it is not in equilibrium with the radiation field.***

Blackbody

- Imagine a container bounded by opaque walls with a very small hole.
  - ***Photons will be scattered and absorbed many times, (and eventually trapped and completely absorbed in the box).*** Under such conditions, the particles and photons continually share their kinetic energies. In perfect thermal equilibrium, ***the average particle kinetic energy will equal to the average photon energy, and a unique temperature T can be defined.***
  - The intensity and spectrum of the radiation emerging from the hole would be independent of the wall material (e.g., wood, copper, or concrete, etc) and any absorbing material that may be inside the cavity.
  - A blackbody is an idealized physical body that absorbs all incident radiation regardless of frequency or angle of incidence (i.e., perfect absorber). The cavity can be regarded to be a blackbody.

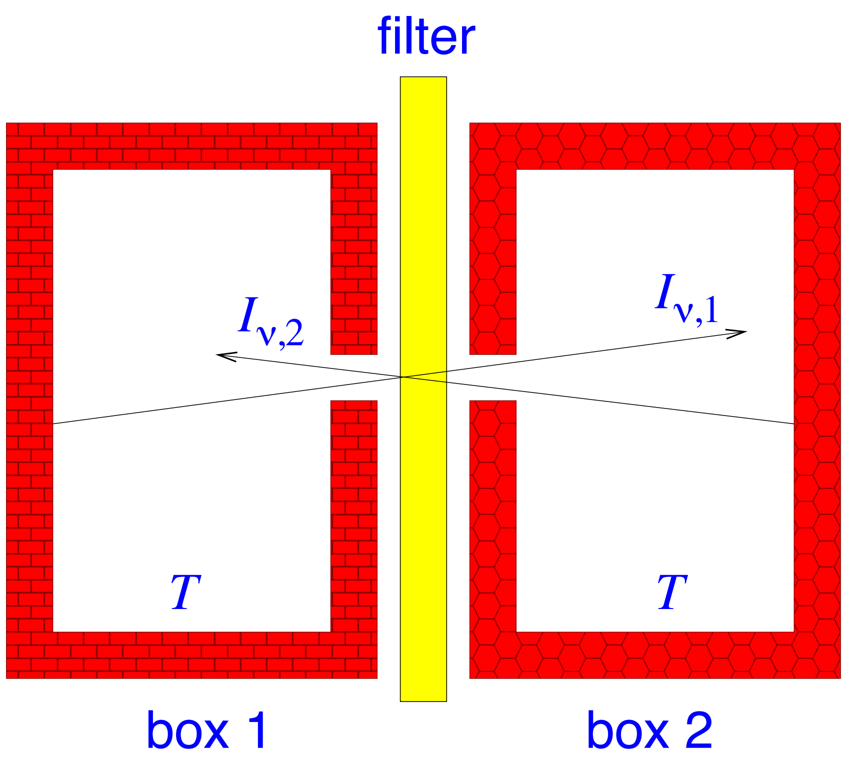


box 1

- Radiation from a blackbody in thermal equilibrium is called the blackbody radiation.

Blackbody radiation if the universal function.

- ***Now, consider another cavity (box 2), also at the same temperature***, but made of different material or shape and connect two cavities with a filter transparent only in the narrow frequency range ν and $\nu + d\nu$.
  - ***In equilibrium at T, radiation should transfer no net energy from one cavity to the other.*** Otherwise, one cavity will cool down and the other heats up; this violates the second law of thermodynamics.
  - Therefore, the intensity or spectrum that passes through the holes should be a universal function of *T* and should be isotropic.
  - The universal function is called the Planck function $B_\nu(T)$.
  - This is the blackbody radiation.



Kirchhoff's Law in TE

- In (full) thermodynamic equilibrium at temperature *T*, we know that

$$\frac{dI_\nu}{ds} = 0 \quad \text{and} \quad I_\nu = B_\nu(T)$$

  - We also note that $$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

  - Then, we can obtain ***the Kirchhoff's law for a system in TE***:

$$\frac{j_\nu(T)}{\alpha_\nu(T)} = B_\nu(T)$$

  - This is remarkable because it connects the properties $j_\nu(T)$ and $\alpha_\nu(T)$ of any kind of matter to the single universal spectrum $B_\nu(T)$.

Kirchhoff's Law in LTE

- Note that Kirchhoff's law was derived for a system in thermodynamic equilibrium.

- ***Kirchhoff's law applies in LTE as well as in TE***:
  - Recall that $B_\nu(T)$ is independent of the properties of the radiating/ absorbing material.
  - In contrast, both j_ν and $\alpha_\nu(T)$ depend only the materials in the cavity and on the temperature of that material; they do not depend on the ambient radiation field or its spectrum.

  - ***In LTE, the ratio of the emission coefficient to the absorption coefficient is a function of temperature and frequency only.***
  - This generalized version of Kirchhoff's law is an exceptionally valuable tool for calculating the emission coefficient from the absorption coefficient or vice versa.

Implications of Kirchhoff's Law

- A good absorber is a good emitter, and a poor absorber is a poor emitter. (In other words, a good reflector must be a poor absorber, and thus a poor emitter.)

$$j_\nu = \alpha_\nu B_\nu(T) \quad \rightarrow \quad j_\nu \text{ increases as } \alpha_\nu \text{ increases}$$

  - It is not possible to thermally radiate more energy than a blackbody, at equilibrium.

$$j_\nu < B_\nu(T) \quad \text{because} \quad \alpha_\nu < 1$$

  - The radiative transfer equation in LTE can be rewritten:

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + B_\nu(T)$$

  - Note:
    - Blackbody radiation means $I_\nu = B_\nu(T)$.
    - Thermal radiation is defined to be radiation emitted by "matter" in LTE. Thermal radiation means $S_\nu = B_\nu(T)$.
    - Thermal radiation becomes blackbody radiation only for optically thick media.

Spectrum of blackbody radiation

- The frequency dependence of blackbody radiation is given by the ***Planck function***:

$$B_\nu = \frac{2h\nu^3/c^2}{\exp(h\nu/k_\mathrm{B}T) - 1} \text{ or } B_\lambda = \frac{2hc^2/\lambda^5}{\exp(hc/\lambda k_\mathrm{B}T) - 1}$$

$$h = 6.63 \times 10^{-27} \text{ erg s (Planck's constant)}$$

$$k_\mathrm{B} = 1.38 \times 10^{-16} \text{ erg K}^{-1} \text{ (Boltzmann's constant)}$$

See “Fundamentals of Statistical and Thermal Physics” (Federick Reif) or “Astrophysical Concepts” (Harwit) for the derivation.

Stefan-Boltzmann Law

- Emergent flux is proportional to T^4.

$$F = \pi \int B_\nu(T) d\nu = \pi B(T) \longleftarrow B(T) = \int B_\nu(T) d\nu = \frac{ac}{4\pi} T^4 = \frac{\sigma}{\pi} T^4$$

$$F = \sigma T^4$$

$$\text{Stephan} - \text{Boltzmann constant} : \sigma = \frac{2\pi^5 k_B^4}{15 c^2 h^3} = 5.67 \times 10^{-5} \text{ erg cm}^2 \text{ s}^{-1} \text{ K}^{-4} \text{ sr}^{-1}$$

- Total energy density ***(another form of the Stefan-Boltzmann law)***

$$u = \frac{4\pi}{c} \int B_\nu(T) d\nu = \frac{4\pi}{c} B(T)$$

$$u = aT^4$$

$$u(T) = \left(\frac{T}{3400 \text{ K}}\right)^4 \text{ erg cm}^{-3}$$

$$\text{radiation constant} : a \equiv \frac{4\sigma}{c} = 7.57 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4}$$

Rayleigh-Jeans Law & Wien Law

- ***Rayleigh-Jeans Law (low-energy limit)***

$$h\nu \ll k_{\mathrm{B}}T \;\; (\nu \ll 2 \times 10^{10}\mathrm{Hz}(T/1\mathrm{K})) \quad \rightarrow \boxed{I_\nu^{RJ}(T) = \frac{2\nu^2}{c^2}k_{\mathrm{B}}T}$$

  - ultraviolet catastrophe: if the equation is applied to all frequencies, the total amount of energy would diverge. $\int \nu^2 d\nu \rightarrow \infty$

- ***Wien Law (high-energy limit)***

$$h\nu \gg k_{\mathrm{B}}T \quad \rightarrow \boxed{I_\nu^{W}(T) = \frac{2h\nu^3}{c^2}\exp\left(-\frac{h\nu}{k_{\mathrm{B}}T}\right)}$$

Wien Displacement Law

- Frequency at which the peak occurs:

$$\frac{\partial B_\nu}{\partial \nu}|_{\nu=\nu_{\rm max}} = 0 \quad \rightarrow \quad x = 3(1-e^{-x}), \text{ where } x = h\nu_{\rm max}/k_{\rm B}T$$

$$\boxed{h\nu_{\rm max} = 2.82 k_{\rm B}T \quad \text{or} \quad \frac{\nu_{\rm max}}{T} = 5.88 \times 10^{10} \text{ Hz deg}^{-1}}$$

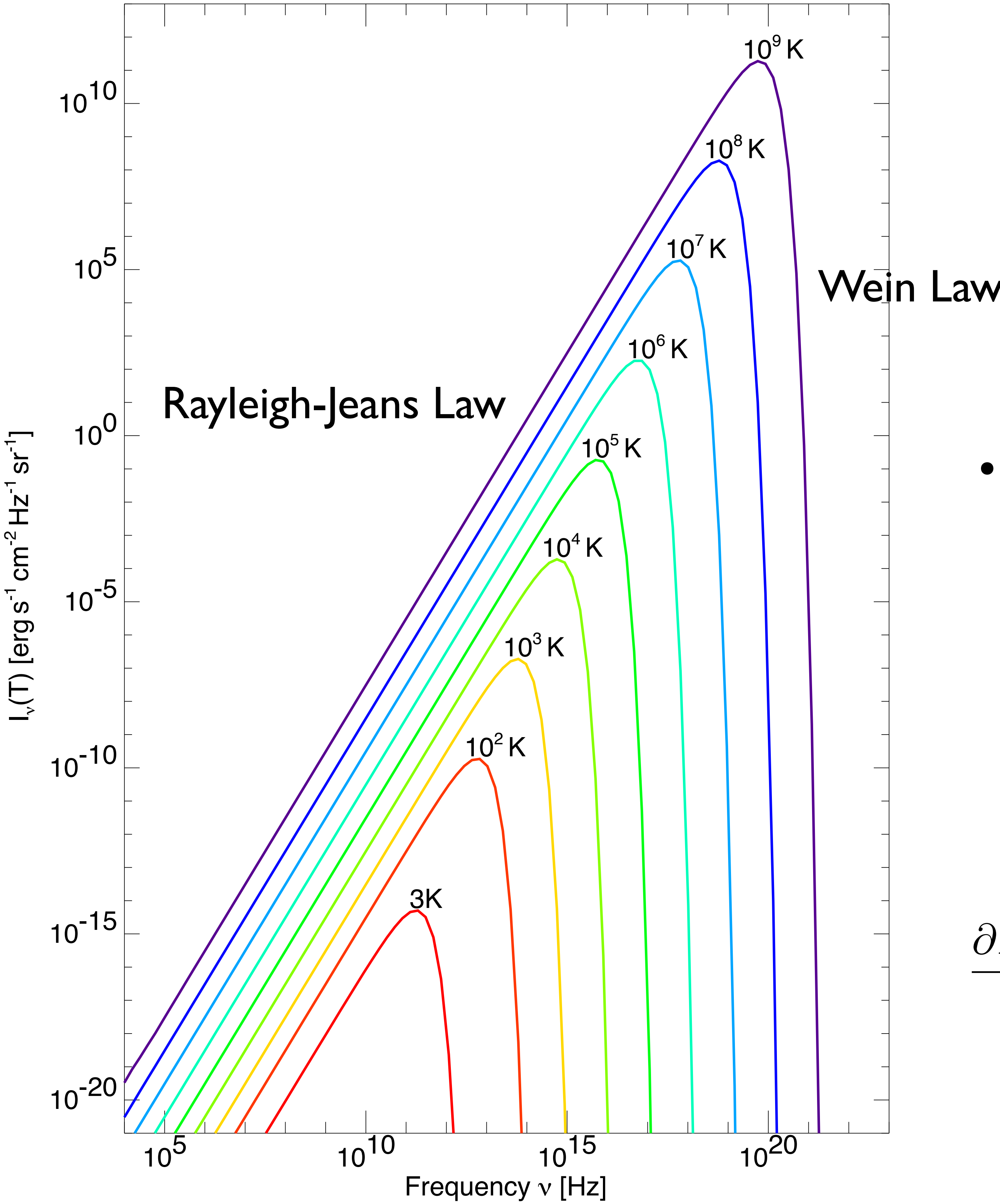
- Wavelength at which the peak occurs:

$$\frac{\partial B_\lambda}{\partial \lambda}|_{\lambda=\lambda_{\rm max}} = 0 \quad \rightarrow \quad y = 5(1-e^{-y}), \text{ where } y = hc/(\lambda_{\rm max} k_{\rm B}T)$$

$$y = 4.97 \quad \text{and} \quad \lambda_{\rm max}T = 0.290 \text{ cm deg}$$

- Note that $\nu_{\rm max} \neq c/\lambda_{\rm max}$

Monotonicity with Temperature



- Monotonicity:
  - Of two blackbody curves, the one with higher temperature lies entirely above the other.

$$\frac{\partial B_\nu(T)}{\partial T} = \frac{2h^2\nu^4}{c^2 k_\mathrm{B} T^2} \frac{\exp(h\nu/k_\mathrm{T})}{\left[\exp(h\nu/k_\mathrm{B}T) - 1\right]^2} > 0$$

Characteristic Temperatures

- ***Brightness Temperature:***

$$I_\nu = B_\nu(T_b)$$

  - The definition is used especially in radio astronomy, where the RJ law is usually applicable. In the RJ limit,

$$T_b = \frac{c^2}{2\nu^2 k_\mathrm{B}} I_\nu$$

  - Radiative transfer equation in the RJ limit:

$$\frac{dT_b}{d\tau_\nu} = -T_b + T \quad (T = \text{the temperature of the material})$$

$$T_b = T_b(0)e^{-\tau_\nu} + T(1 - e^{-\tau_\nu}) \quad \text{if } T \text{ is constant.}$$

  - In the Wien region, the concept is not so useful.

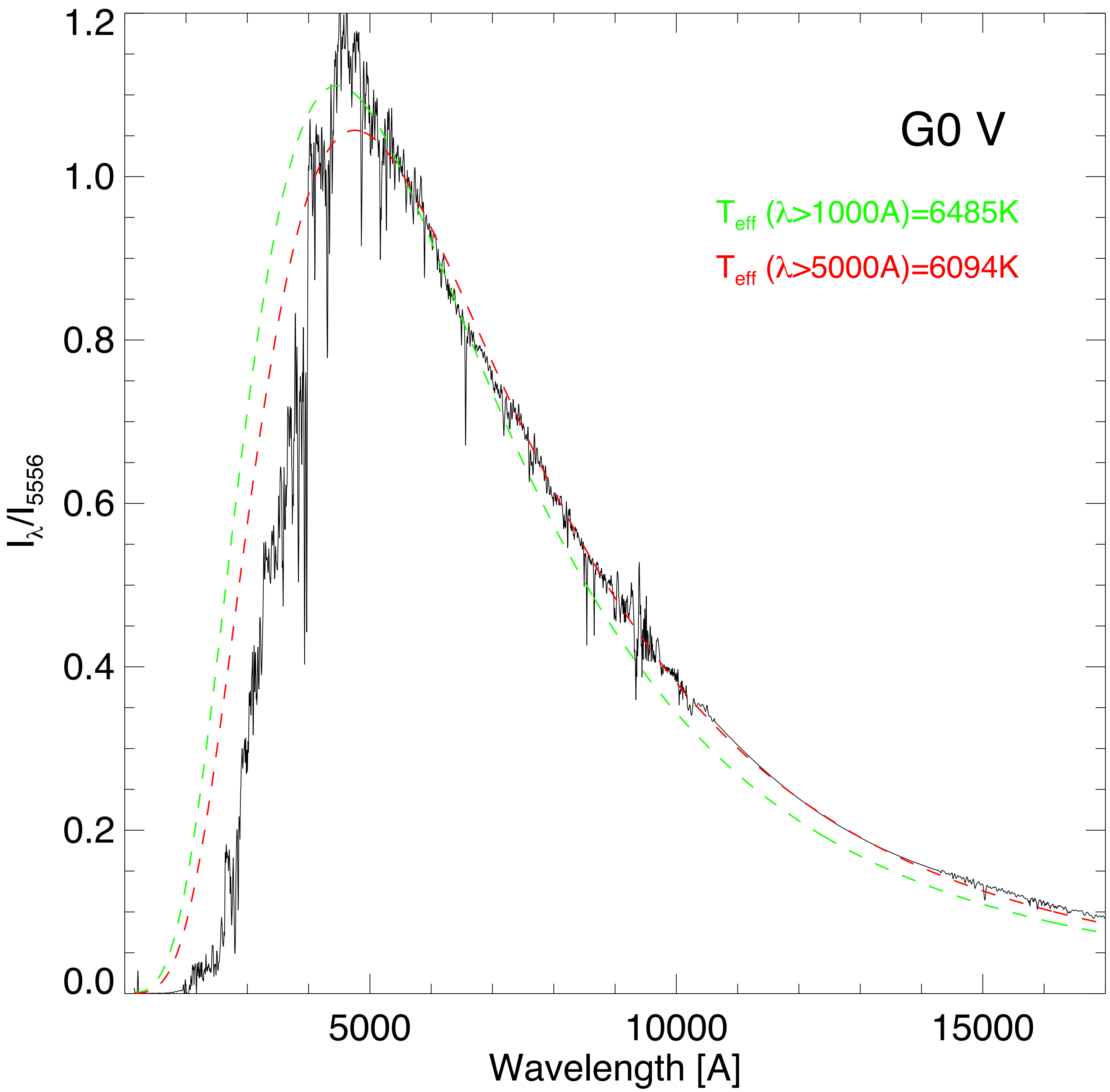
- ***Color Temperature:***
  - By fitting the spectrum to a blackbody curve without regarding to vertical scale (absolute intensity scale), a color temperature T_c is obtained.
  - The color temperature correctly gives the temperature of a blackbody source of unknown absolute scale.

- ***Effective Temperature:***
  - The effective temperature of a source is obtained by equating the actual flux F to the flux of a blackbody at temperature $T_{\rm eff}$.

$$F = \int \cos\theta I_\nu d\nu d\Omega = \sigma T_{\rm eff}^4$$

- ***Excitation Temperature:***

$$\frac{n_u}{n_l} = \frac{g_u}{g_l} \exp\left(-\frac{(E_u - E_l)}{k_{\rm B} T_{\rm ex}}\right)$$

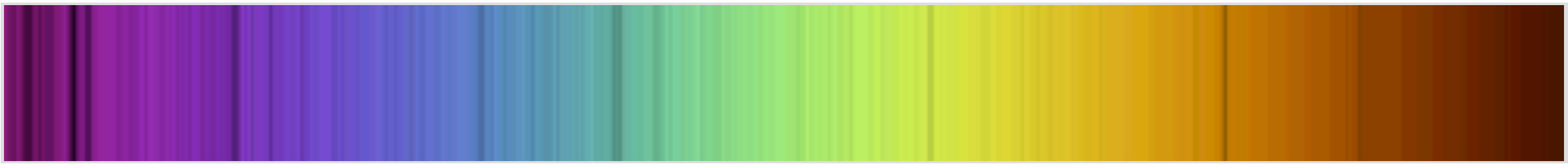


G0V spectrum (Pickles 1998, PASP, 110, 863)

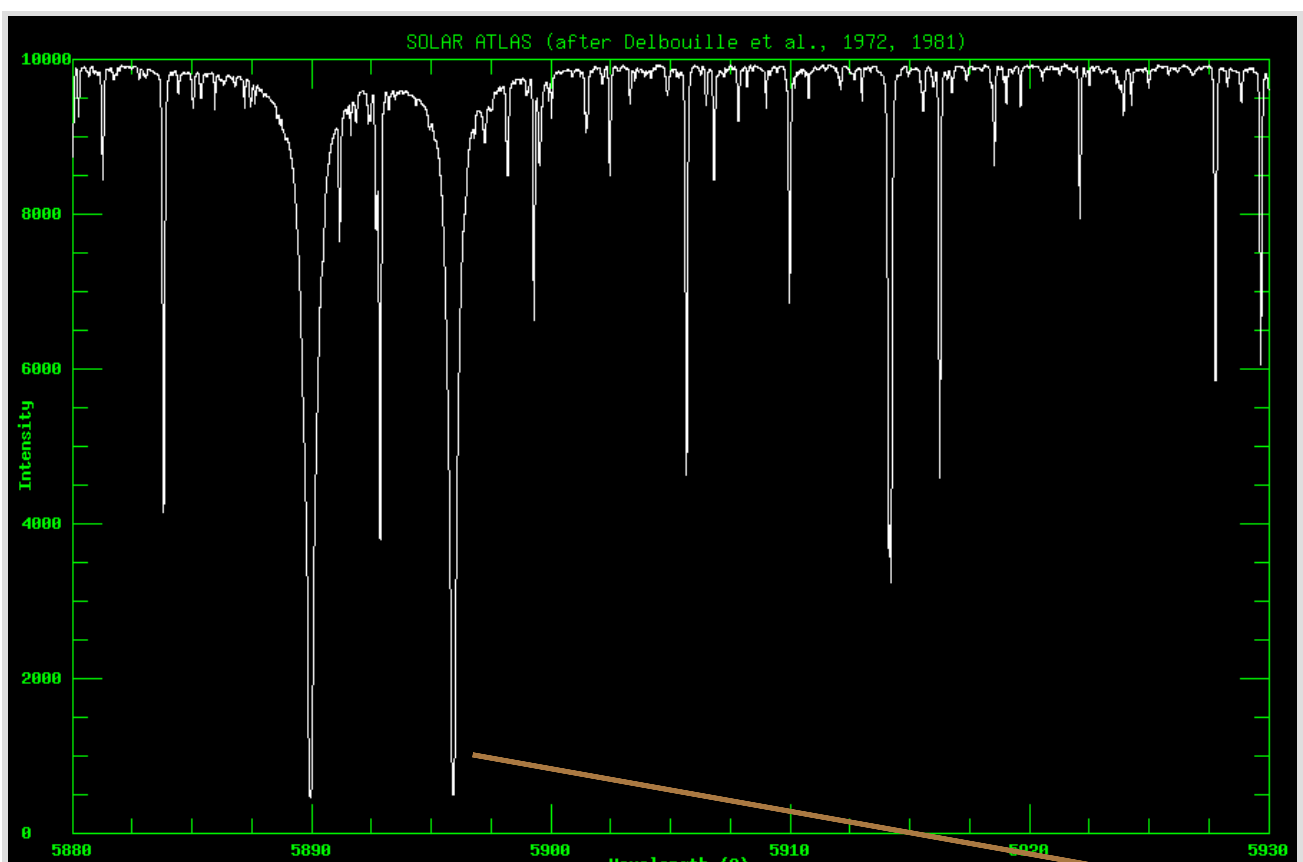
(Note that the solar spectral type is G2V.)

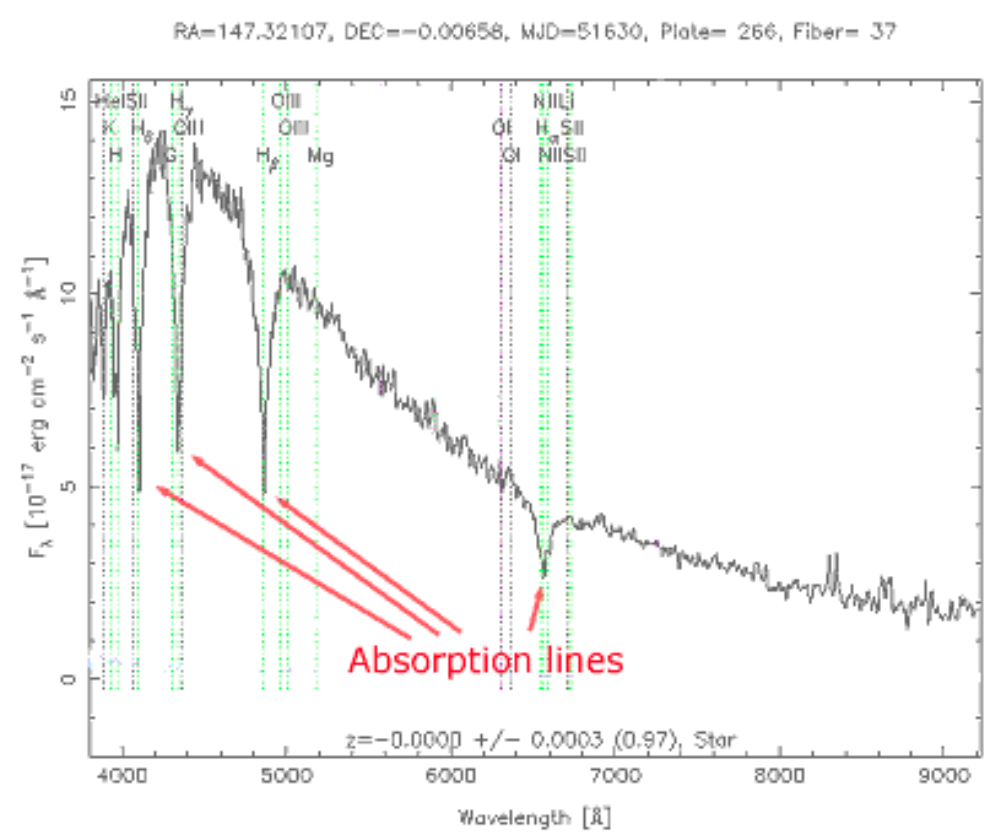
[Homework] Absorption line and emission line spectra

- Temperature of the Solar photosphere is ~ 6000 K. Lots of spectral lines of different elements are observed.



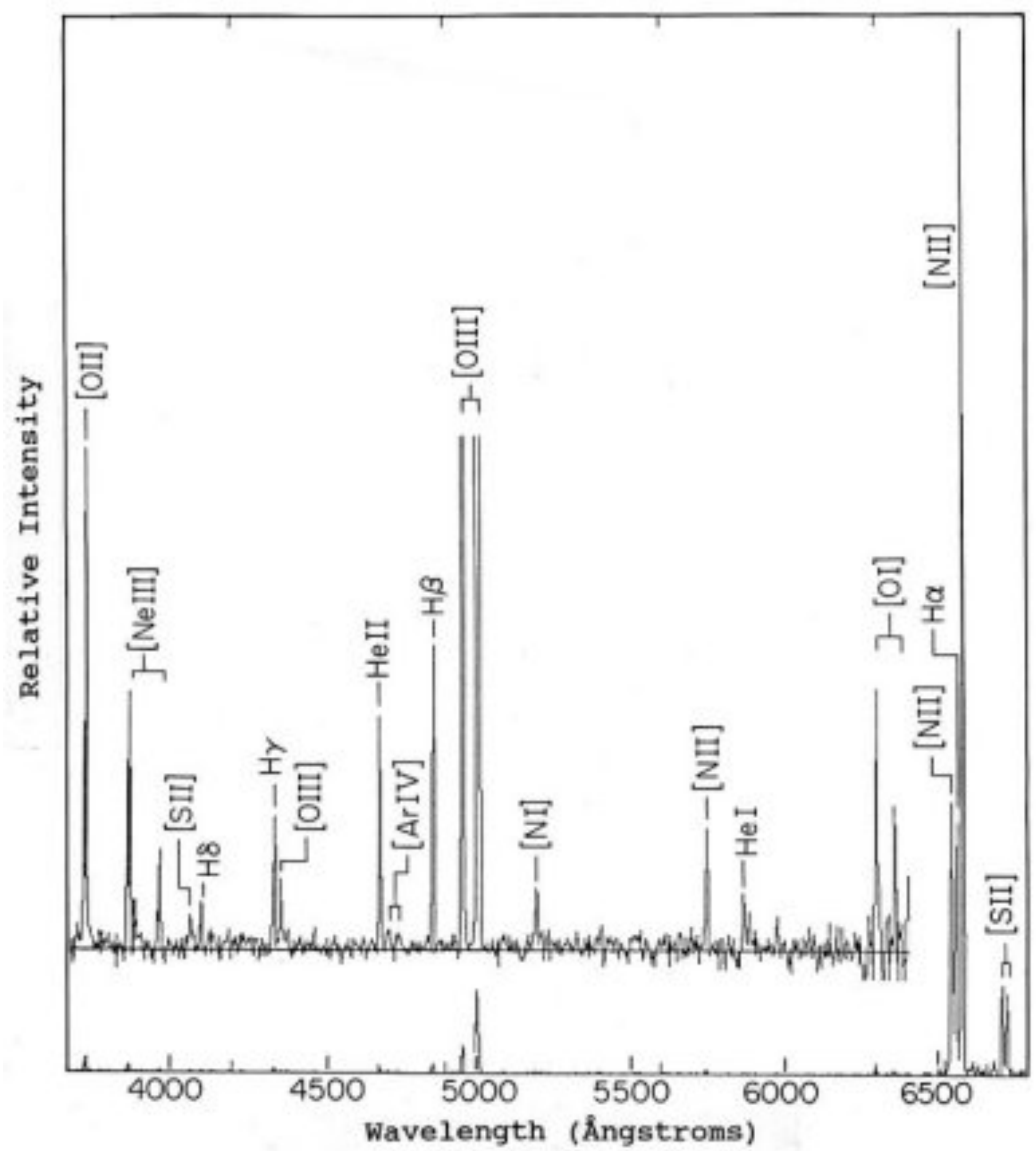
- ***Optical spectrum of stars is an absorption line spectrum*** - see dark absorption lines superimposed on a bright continuum.





Two strong absorption lines are Na I D lines due to sodium.

- However, ***emission nebulae typically show emission line spectra***: (Spectral lines are stronger than the continuum.)



Spectrum from an emission nebula

Homework:
- Explain why this difference happens?
- Deadline: Sept. 11 (Wed.)

Hint for homework

- Recall the solution for RT equation when the source function is constant.

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

- Assume optically thin regions, $\tau_\nu \ll 1$ and show that the above equation becomes

$$I_\nu(D) \approx I_\nu(0) + \tau_\nu(D)\big(S_\nu - I_\nu(0)\big) \quad \text{at } \tau_\nu = D$$

- See the following three figures, and explain why some objects show absorption line spectra, but some show emission line spectra.
- Note that $T(\tau_0 = 0) > T(\tau_0 = D)$ for the case of the stellar atmosphere.