21-cm Line:
Wouthuysen-Field Effect

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21cm line in Astronomy

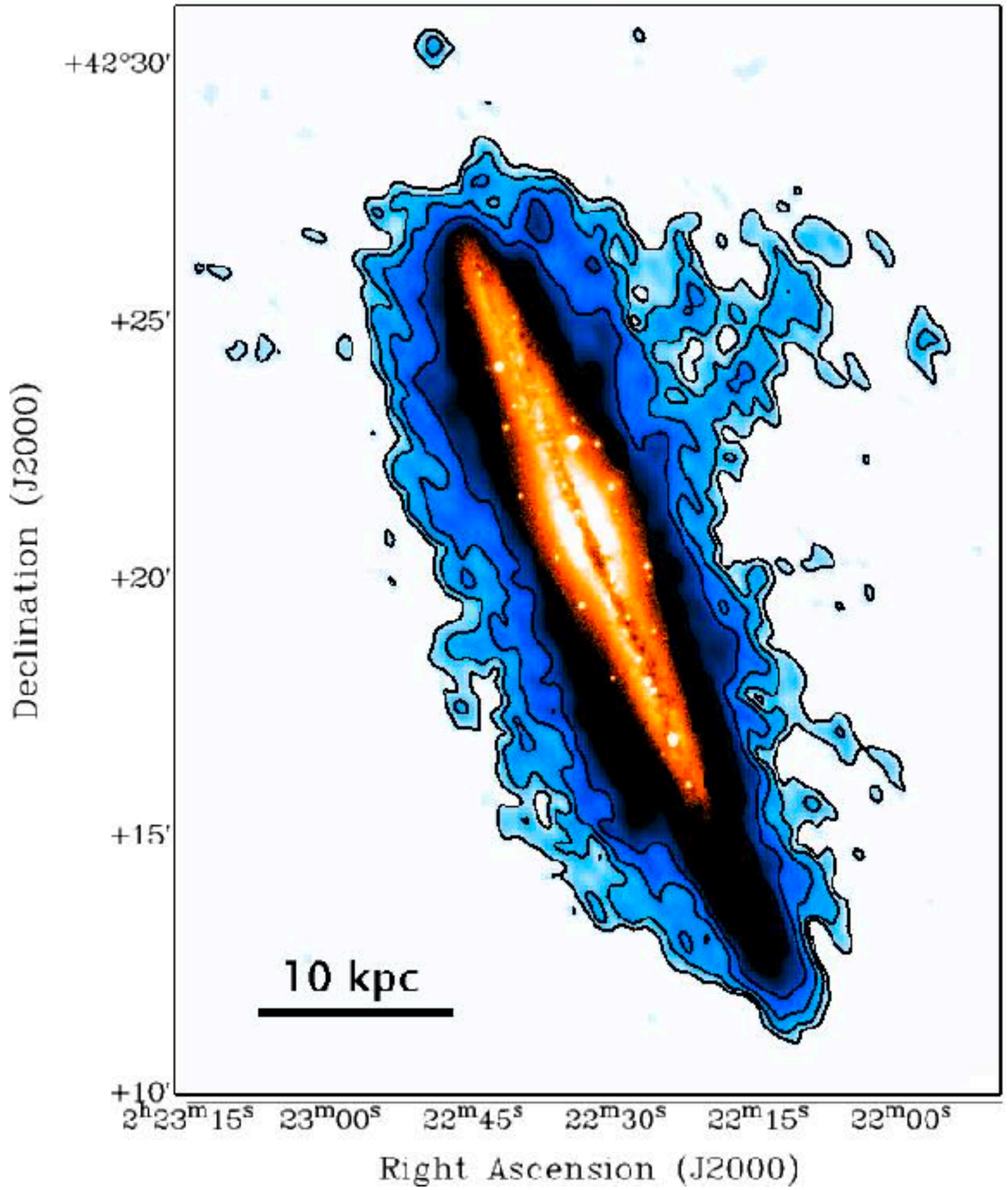
- **21 cm line is the most important tool in Radio Astronomy**
- 21cm line provides information about

    (1) Cold Neutral Medium & Warm Neutral Medium

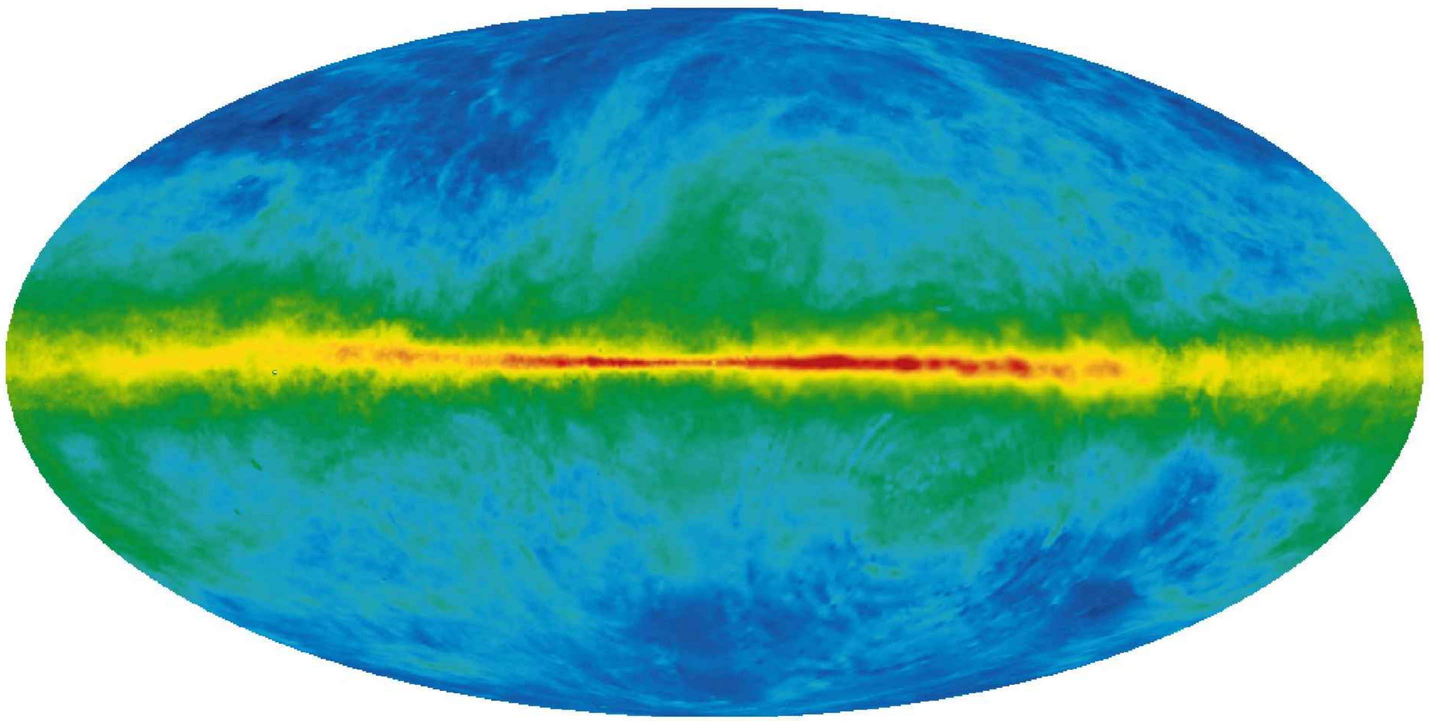
    (2) Circumgalactic & Intergalactic medium

    (3) Epoch of Reionization

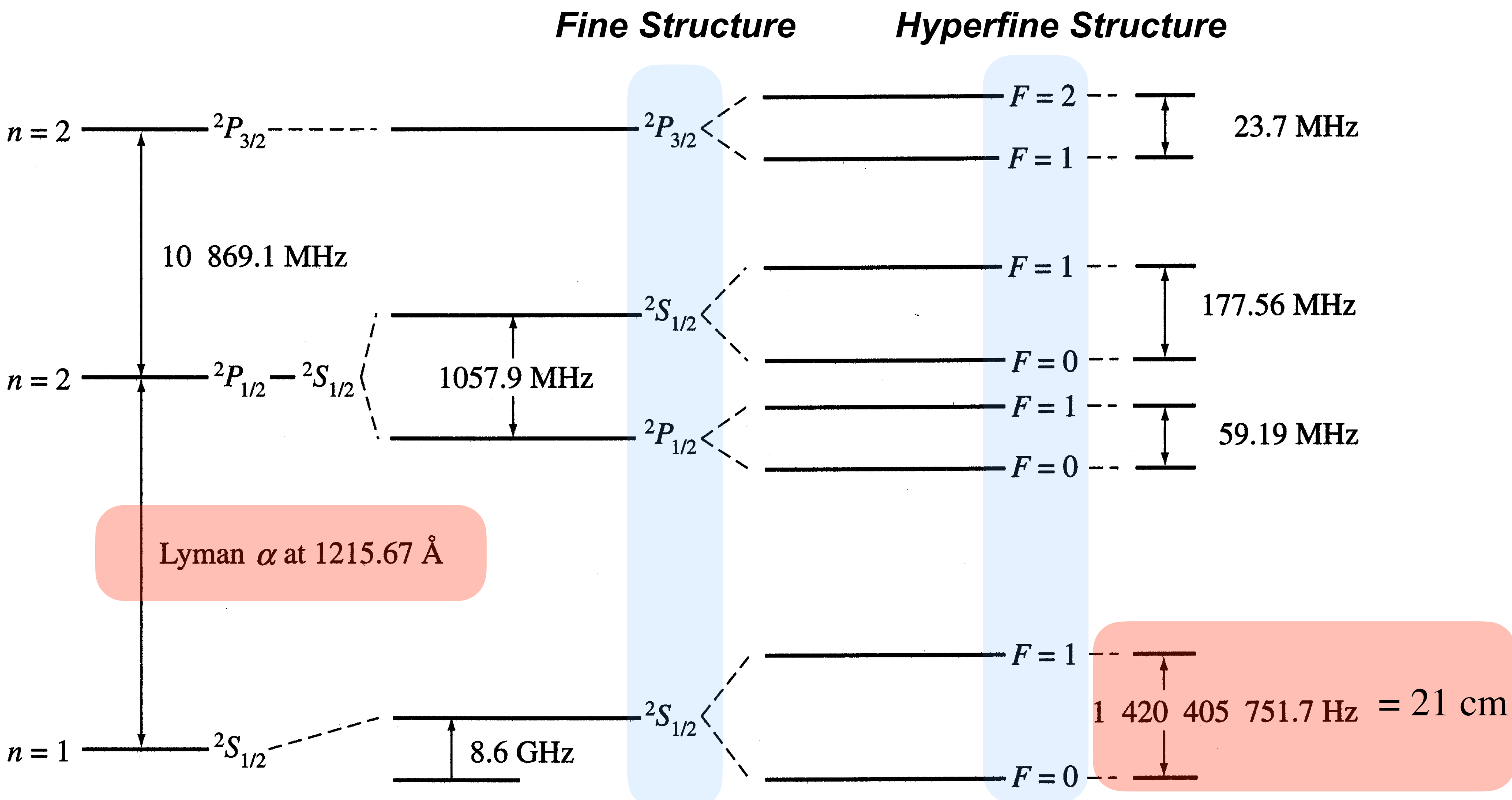
21 cm emission map of an edge-on galaxy NGC 891



All-sky image of emission at 21 cm



What is 21 cm Line?



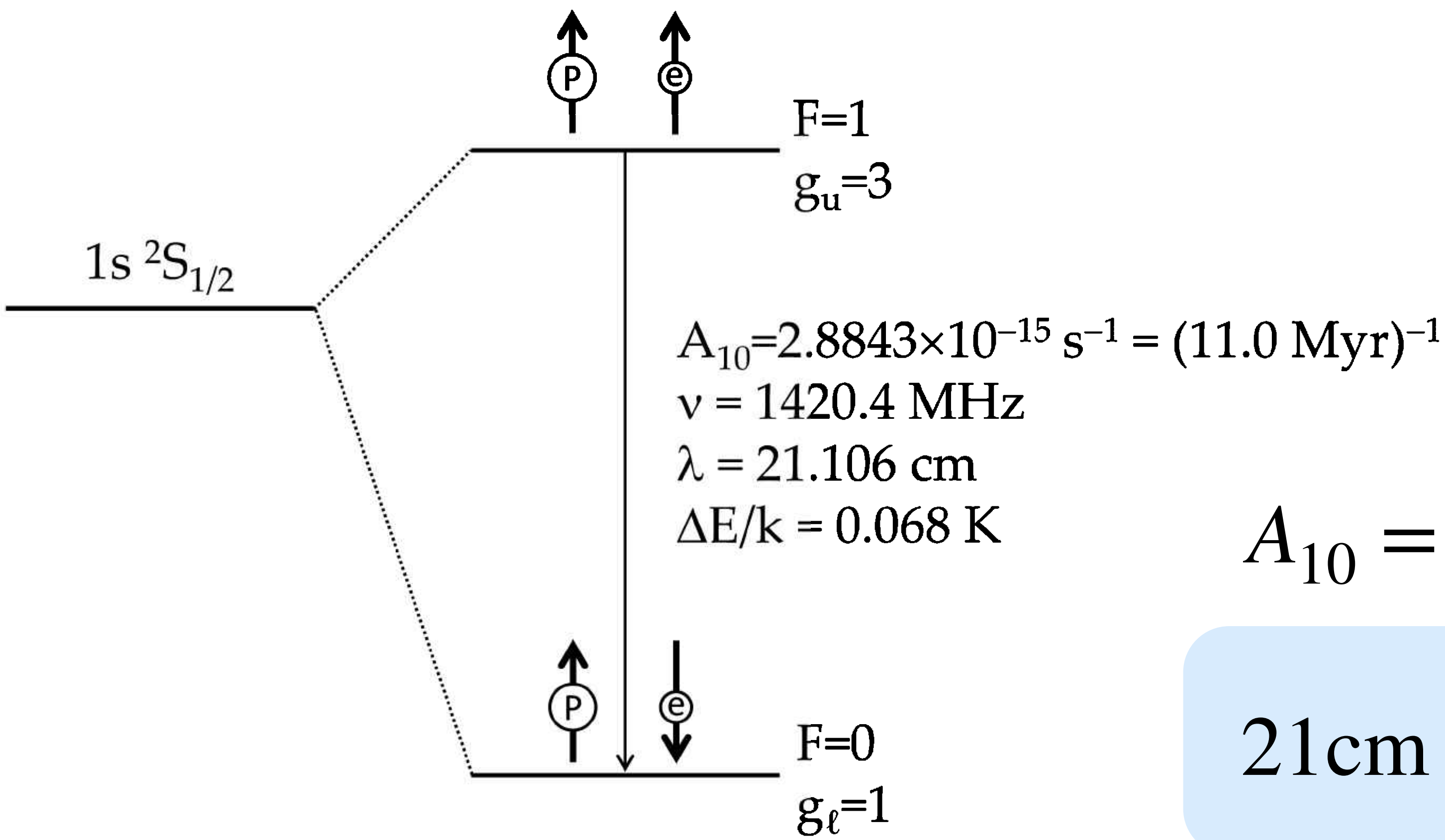
- ***Fine Structure***

  Coupling of the electron spin $S = 1/2$ with the electron angular momentum L gives the total angular momentum of electron: $J = L \pm 1/2$

- ***Hyperfine Structure***

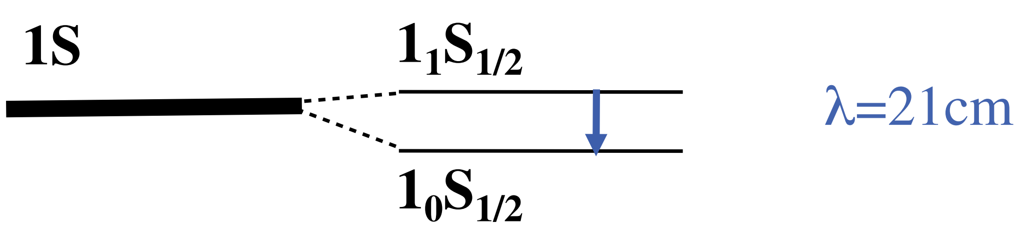
  Coupling of the proton spin $I = 1/2$ with the total electron angular momentum J gives the final angular momentum of hydrogen: $F = J \pm 1/2$.

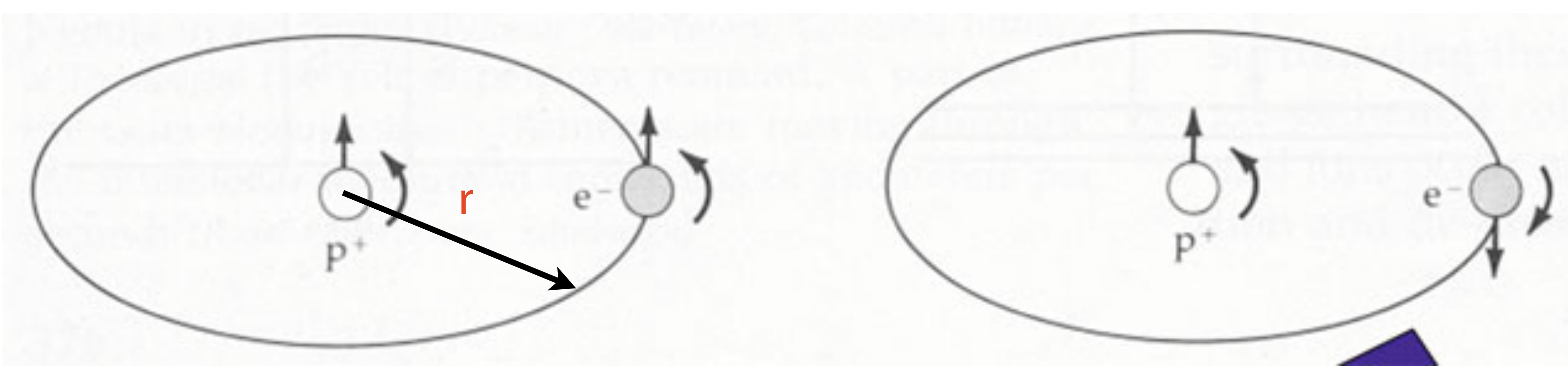
Population of the Hyperfine Levels



$$A_{10} = 2.88 \times 10^{-15}\,\mathrm{s}^{-1} = 1/(11\,\mathrm{Myr})$$

$$21\mathrm{cm} = 5.87\mu\mathrm{eV} = 0.068\,\mathrm{K}$$





- The transition time scale is very long.
- The energy difference is much lower than the kinetic temperature of the neutral ISM and temperature of the CMB.
- ***The relative population between the two levels in the ground state will be readily controlled by (1) the particle collisions and (2) the CMB.***
- The **21 cm spin temperature** (T_s) defines the relative population between the hyperfine levels in the ground state.

$$\frac{n_1}{n_0} = \frac{g_1}{g_0} \exp\left(-\frac{h\nu_{10}}{k_B T_s}\right)$$

$$= 3 \exp\left(-\frac{T_*}{T_s}\right)$$

$$T_* \equiv \frac{h\nu_{10}}{k_B} = 0.068\,\mathrm{K}$$

In a High-Density Medium

- ***Collisional Coupling***

Collisions between different particles can induce spin-flips in a hydrogen atom.

Collisions dominate the coupling in the early Universe (during the cosmic Dark Ages) where the gas density is high.

Three main channels are available.

- collisions between two hydrogen atoms
- collisions between a hydrogen atom and an electron
- collisions between a hydrogen atom and a proton

In low density medium: RT equation

- RT equation

$$I(\tau) = I(0)e^{-\tau} + S\left(1 - e^{-\tau}\right)$$

- excitation temperature (spin temperature for 21 cm)

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} \exp\left(-\frac{h\nu}{k_{\rm B} T_{\rm exc}}\right)$$

- Generalized Kirchhoff's law: The source function is given by

$$S = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(h\nu / k_{\rm B} T_{\rm exc}\right) - 1}$$

- In the Rayleigh-Jeans regime (at very low frequencies),

$$T_{\rm A} = T_{\rm A}(0)e^{-\tau} + T_{\rm exc}\left(1 - e^{-\tau}\right)$$

Here, the antenna temperature is defined as

$$I_\nu = \frac{2\nu^2}{c^2} k_{\rm B} T_b \quad \rightarrow \quad T_A \equiv \frac{c^2}{2k_{\rm B}\nu^2} I_\nu$$

In a Low Density Medium

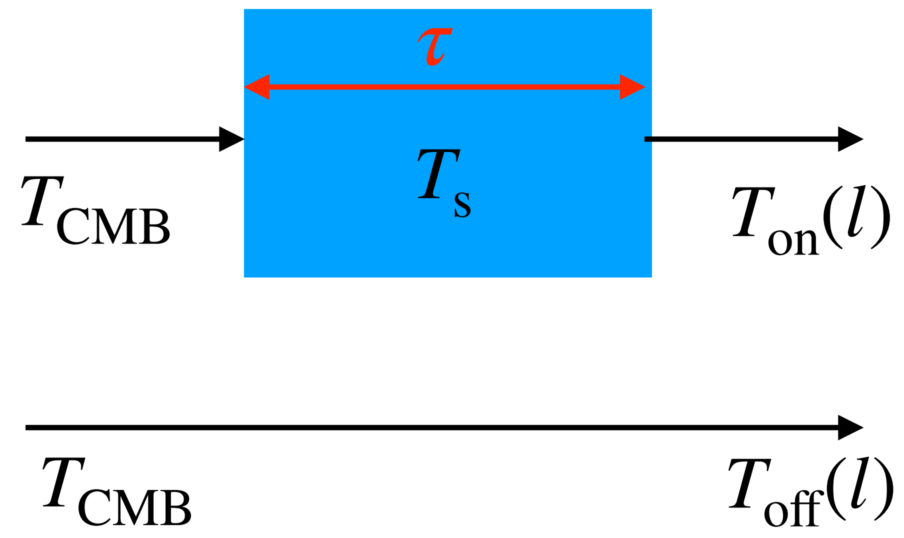
- In a very low density medium (diffuse ISM, CGM, IGM), the particle collisions are very rare.

- ***Direct Radiative Pumping:*** The radiative transition due to the CMB photons will control the relative population between the hyperfine structures.

This indicates $T_s = T_{\rm CMB}$.

RT equation in the Rayleigh-Jeans regime:

$$T_{\rm on}(l) = T_{\rm CMB} e^{-\tau} + T_s (1 - e^{-\tau})$$
$$T_{\rm off}(l) = T_{\rm CMB}$$
$$T_{\rm on}(l) - T_{\rm off}(l) = \left(T_s - T_{\rm CMB}\right)(1 - e^{-\tau})$$



Then, we have $T_{\rm on}(l) - T_{\rm off}(l) = 0$

Neither emission nor absorption feature from the hydrogen gas is detectable.

- ***However, we observes galaxies with 21 cm. We need something that can make $T_s \neq T_{\rm CMB}$.***

Mechanisms that controls the spin temperature

- The spin temperature (T_s) is determined by three mechanisms.

(1) Direct Radiative Transitions by the background radiation field (Cosmic Microwave Background or Galactic Synchrotron)

$$I_\nu = \frac{2k_B T_R}{\lambda^2}$$

(Rayleigh-Jeans Law)

$$T_R = \text{brightness temperature} = 2.73\,\mathrm{K} \text{ or } 3.77\,\mathrm{K}$$

(2) Collisional Transitions (collision with other hydrogen and electron)

$$T_K = \text{gas kinetic temperature}$$

(3) Lyα pumping: Indirect Radiative Transitions involving intermediate levels caused by Lyα resonance scattering. The spin temperature becomes the same as the color temperature of the radiation field.

$$T_\alpha = \text{color temperature} \qquad J(\nu) \propto \exp\left(-\frac{h\nu}{k_B T_\alpha}\right)$$

The WF Effect

- Wouthuysen (1952, AJ, 57, 31)

"Wouthuysen" is pronounced as roughly "Vowt-how-sen." (바우타이슨)

Wouthuysen, S. A. On the excitation mechanism of the 21-cm (radio-frequency) interstellar hydrogen emission line.

The mechanism proposed here is a radiative one: as a consequence of absorption and re-emission of Lyman-α resonance radiation, a redistribution over the two hyperfine-structure components of the ground level will take place. Under the assumption—here certainly permitted—that induced emissions can be negelcted, it can easily be shown that the relative distribution of the two levels in question, under stationary conditions, will depend solely on the shape of the radiation spectrum in the Lα region, and not on the absolute intensity.

The shape of the spectrum of resonance radiation, quasi-imprisoned in a large gas cloud, could only be determined by a careful study of the "scattering" process (absorption and re-emission) in a cloud of definite shape and dimensions. The spectrum will turn out to depend upon the localization in the cloud.

Some features can be inferred from more general considerations. Take a gas in a large container, with perfectly reflecting walls. Let the gas be in equilibrium at temperature T, together with Planck radiation of that same temperature. The scattering processes will not affect the radiation spectrum. One can infer from this fact that the photons, after an infinite number of scattering processes on gas atoms with kinetic temperature T, will obtain a statistical distribution over the spectrum proportional to the Planck-radiation spectrum of temperature T. After a finite but large number of scattering processes the Planck shape will be produced in a region around the initial frequency.

Photons reaching a point far inside an interstellar gas cloud, with a frequency near the Lα resonance frequency, will have suffered on the average a tremendous number of collisions. Hence in that region, which is wider the larger the optical depth of the cloud is for the Lyman radiation, the Planck spectrum corresponding to the gas-kinetic temperature will be established as far as the shape is concerned. Because, however, the relative occupation of the two hyperfine-structure components of the ground state depends only upon the shape of the spectrum near the Lα frequency, this occupation will be the one corresponding to equilibrium at the gas temperature.

The conclusion is that the resonance radiation provides a long-range interaction between gas atoms, which forces the internal (spin-)degree of freedom into thermal equilibrium with the thermal motion of the atoms.

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From a thermodynamic argument, Wouthuysen speculated the followings:

A tremendous number of scattering will establish the Planck-like spectrum, at the Lyα line center, corresponding to the gas-kinetic temperature.

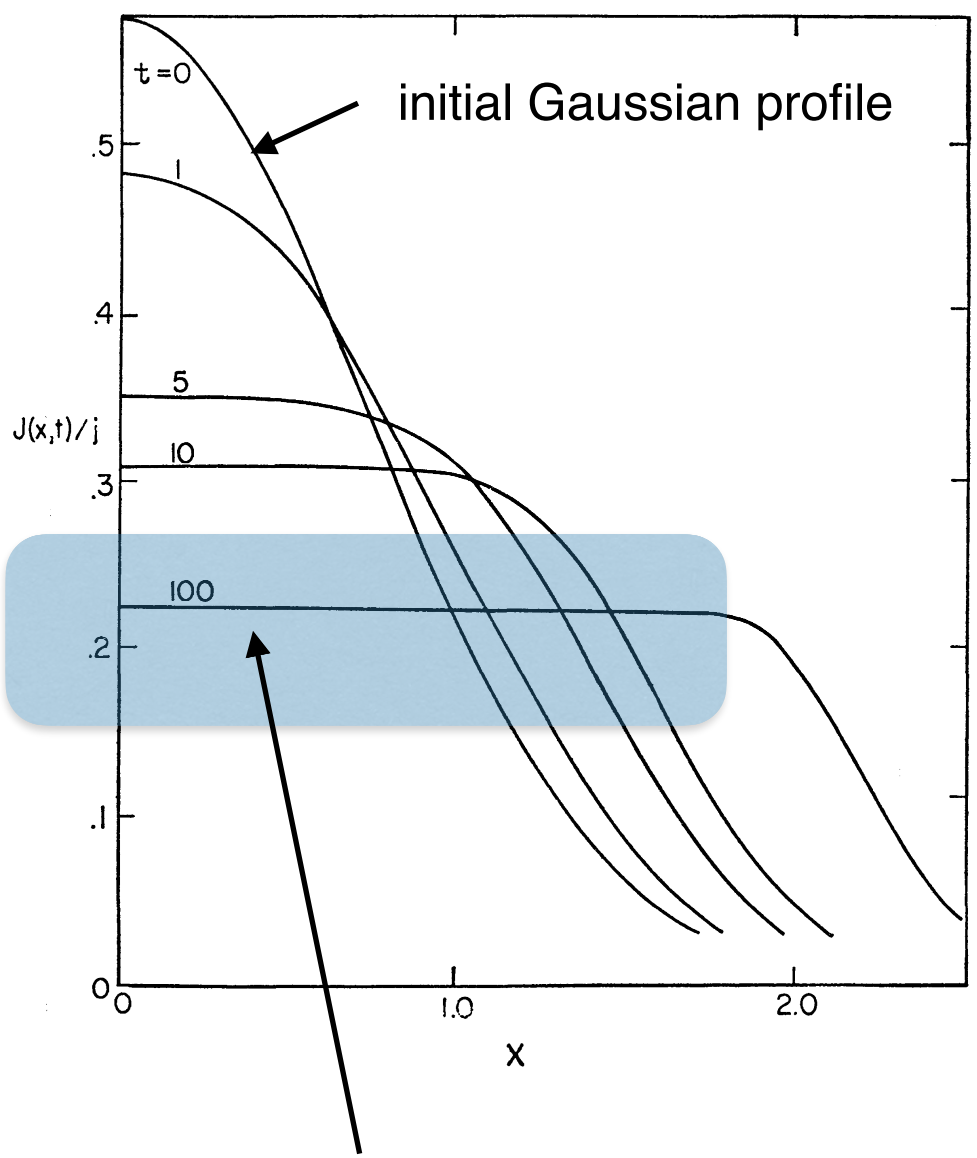
The Lyα radiation is coupled with the hyperfine state of the hydrogen atom.

In the end, ***the 21cm spin temperature will become equal to the kinetic temperature of the hydrogen gas.***

Relaxation of Lyα Profile

- Field (1958, PIRE, 46, 240; 1959, ApJ, 129, 551)

Recoil effect = momentum transfer between H atom and photon



spectral shape at the Lyα line center

Without recoil:
The spectral profile of Lyα, ***within the medium,*** becomes flat at the line center when the photons undergo a large number of resonance scatterings.

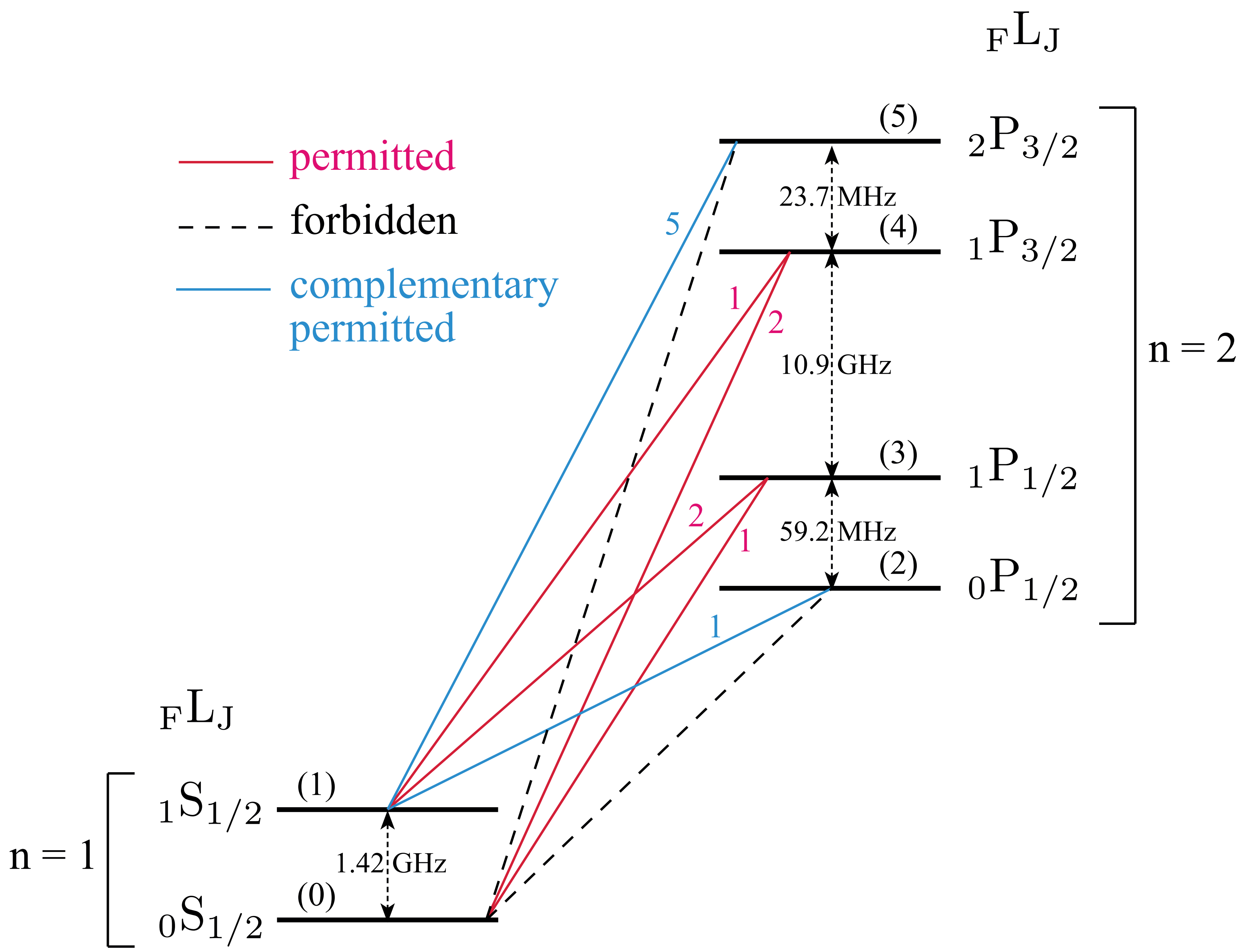
$$J(\nu,\, t \rightarrow \infty) = \text{constant}$$

With recoil:
Recoil of the scattering atom changes the slope of the Lyα central profile ***"at the limit of an infinite number of scattering"*** and gives a Boltzmann like (exponential) functional shape:

$$J(\nu,\, t \rightarrow \infty) \propto e^{-\frac{h(\nu - \nu_\alpha)}{kT_\mathrm{K}}}$$

The Wouthuysen-Field effect

The WF effect is a mechanism that the resonance scattering of Lyα photons indirectly control the relative populations between the hyperfine levels in the ground state (n = 1) via transitions involving the n = 2 state as an intermediate state.



Equation for spin temperature

- Balance equation for the population of the hyperfine states 0 and 1

$$n_0\left(P^R_{01}+P^c_{01}+P^\alpha_{01}\right)=n_1\left(P^R_{10}+P^c_{10}+P^\alpha_{10}\right)$$

P^R, P^c, and P^α are transition rates due to radio, collisions, and Lyα.

Two requirements for the WF effect:

(1) $T_\alpha = T_\mathrm{K}$
(2) The strength of Lyα radiation field must be strong.

Equation for spin temperature (a)

- In stationary state, rate equation for the population of the hyperfine states 0 and 1 can be written

$$n_0 \left(P_{01}^R + P_{01}^c + P_{01}^\alpha\right) = n_1 \left(P_{10}^R + P_{10}^c + P_{10}^\alpha\right) \qquad \text{Eq (1)}$$

where

$P^c,\ P^R,\ P^\alpha =$ transition rates (per sec) cause by collisions, radio, and Lyα

(1) Level Population in terms of spin temperature (T_S):

$$\frac{n_1}{n_0} = \frac{g_1}{g_0} \exp\left(-\frac{h\nu_{10}}{k_\mathrm{B} T_S}\right) \longleftarrow T_* \equiv \frac{h\nu_{10}}{k_\mathrm{B}} = 0.0681\,\mathrm{K}$$

$$\simeq 3\left(1 - \frac{T_*}{T_S}\right) \qquad \text{Eq (2)}$$

Equation for spin temperature (b)

(2) Ratio between the radiative transition rates in terms of the brightness temperature:

$$I_\nu = \frac{2\nu_{10}^2}{c^2} kT_R, \quad u_\nu = \frac{8\pi\nu_{10}^2}{c^3} kT_R$$

Using the definition of the brightness temperature, we obtain the radiative transition rates due to 21 cm radiation :

$$P_{01}^R = B_{01} u_\nu = \left(\frac{g_1}{g_0} \frac{c^3}{8\pi h \nu_{10}^3} A_{10} \right) \left(\frac{8\pi\nu_{10}^2}{c^3} kT_R \right) = 3 \frac{T_R}{T_*} A_{10}$$

$$P_{10}^R = A_{10} + B_{10} u_\nu = A_{10} + \left(\frac{c^3}{8\pi h \nu_{10}^3} A_{10} \right) \left(\frac{8\pi\nu_{10}^2}{c^3} kT_R \right) = \left(1 + \frac{T_R}{T_*} \right) A_{10}$$

$$\frac{P_{01}^R}{P_{10}^R} \simeq 3 \left(1 - \frac{T_*}{T_R} \right) \qquad \text{Eq (3)}$$

(3) Ratio between the collisional transition rates in terms of the gas kinetic temperature:

$$\frac{P_{01}^c}{P_{10}^c} = \frac{k_{01}}{k_{10}} = \frac{g_1}{g_0} \exp\left(-\frac{h\nu_{10}}{kT_K} \right) \simeq 3 \left(1 - \frac{T_*}{T_K} \right) \qquad \text{Eq (4)}$$

(T_K is the gas kinetic temperature.)

Equation for spin temperature (c)

(5) Ratio between the indirect transition rates due to Lyα pumping:

$$P_{01}^{\alpha} = \sum_{j=2}^{5} B_{0j} u(\nu_{0j}) \frac{A_{j1}}{\sum_{i=0}^{1} A_{ji}} = \sum_{j=2}^{5} \frac{g_j}{g_0} \frac{c^3}{8\pi h \nu_{0j}^3} A_{j0} u(\nu_{0j}) \frac{A_{j1}}{\sum_{i=0}^{1} A_{ji}}$$

photon occupation number

$$\simeq n_\gamma(\nu_{0\alpha}) \sum_{j=2}^{5} \frac{g_j}{g_0} A_{j0} \frac{A_{j1}}{\sum_{i=0}^{1} A_{ji}} \qquad \leftarrow \quad n_\gamma(\nu_{0j}) \simeq n_\gamma(\nu_{0\alpha}) \qquad \left(n_\gamma = \frac{c^3}{8\pi h \nu^3} u_\nu \right)$$

$$\simeq \exp\left(-\frac{h\nu_{0\alpha}}{kT_\alpha}\right) C_{01} \qquad \leftarrow \quad \text{Wiens Law,} \quad C_{01} \equiv \frac{1}{g_0} \sum_{j=2}^{5} g_j \frac{A_{j0} A_{j1}}{\sum_{i=0}^{1} A_{ji}}$$

$$P_{10}^{\alpha} = \sum_{j=2}^{5} B_{1j} u(\nu_{1j}) \frac{A_{j0}}{\sum_{i=0}^{1} A_{ji}} = \sum_{j=2}^{5} \frac{g_j}{g_1} \frac{c^3}{8\pi h \nu_{1j}^3} A_{j1} u(\nu_{1j}) \frac{A_{j0}}{\sum_{i=0}^{1} A_{ji}}$$

$$\simeq n_\gamma(\nu_{1\alpha}) \sum_{j=2}^{5} \frac{g_j}{g_1} A_{j1} \frac{A_{j0}}{\sum_{i=0}^{1} A_{ji}} \qquad \leftarrow \quad n_\gamma(\nu_{1j}) \simeq n_\gamma(\nu_{1\alpha})$$

$$\simeq \exp\left(-\frac{h\nu_{1\alpha}}{kT_\alpha}\right) C_{10} \qquad \leftarrow \quad \text{Wiens Law,} \quad C_{10} \equiv \frac{1}{g_1} \sum_{j=2}^{5} g_j \frac{A_{j1} A_{j0}}{\sum_{i=0}^{1} A_{ji}}$$

Notice that $\frac{C_{01}}{C_{10}} = \frac{g_1}{g_0} = 3$

$\nu_{0\alpha} - \nu_{1\alpha} = \nu_{10}$

Therefore, we have $$\frac{P_{01}^{\alpha}}{P_{10}^{\alpha}} \simeq 3 \exp\left(-\frac{h\nu_{10}}{kT_\alpha}\right) \simeq 3\left(1 - \frac{T_*}{T_\alpha}\right)$$

Eq (5)

Equation for spin temperature (d)

Combining Eq (1) - (6), we obtain the following equation for the spin temperature in terms of the 21 cm brightness temperature, gas kinetic temperature, and Lyα color temperature:

$$1-\frac{T_*}{T_S}=\frac{\left(1-\frac{T_*}{T_R}\right)P_{10}^R+\left(1-\frac{T_*}{T_K}\right)P_{10}^c+\left(1-\frac{T_*}{T_\alpha}\right)P_{10}^\alpha}{P_{10}^R+P_{10}^c+P_{10}^\alpha}$$

$$\frac{T_*}{T_S}=\frac{\frac{T_*}{T_R}P_{10}^R+\frac{T_*}{T_K}P_{10}^c+\frac{T_*}{T_\alpha}P_{10}^\alpha}{P_{10}^R+P_{10}^c+P_{10}^\alpha}$$

$$P_{10}^R=\left(1+\frac{T_R}{T_*}\right)A_{10}$$

$$\frac{T_*}{T_S}=\frac{A_{10}+\frac{T_*}{T_K}P_{10}^c+\frac{T_*}{T_\alpha}P_{10}^\alpha}{\left(1+\frac{T_R}{T_*}\right)A_{10}+P_{10}^c+P_{10}^\alpha}$$

$$\frac{T_S}{T_*}=\frac{\left(1+\frac{T_R}{T_*}\right)A_{10}+P_{10}^c+P_{10}^\alpha}{A_{10}+\frac{T_*}{T_K}P_{10}^c+\frac{T_*}{T_\alpha}P_{10}^\alpha}$$

$$T_S=\frac{T_*+T_R+T_*\frac{P_{10}^c}{A_{10}}+T_*\frac{P_{10}^\alpha}{A_{10}}}{1+\frac{T_*}{T_K}\frac{P_{10}^c}{A_{10}}+\frac{T_*}{T_\alpha}\frac{P_{10}^\alpha}{A_{10}}}$$

$$T_S=\frac{T_*+T_R+y_cT_K+y_\alpha T_\alpha}{1+y_c+y_\alpha}$$

where $$y_c\equiv\frac{T_*}{T_K}\frac{P_{10}^c}{A_{10}}$$

$$y_\alpha\equiv\frac{T_*}{T_\alpha}\frac{P_{10}^\alpha}{A_{10}}$$

$$T_*=\frac{h\nu_{10}}{k_B}=0.0681\ ^\circ\mathrm{K}$$ (This term is negligible in the above equation.)

Two requirements for the WF effect:

(1) $J_\nu\propto\exp\left(-\frac{h\nu}{kT_\alpha}\right)$ with $T_\alpha=T_\mathrm{K}$

(2) $y_\alpha\gg1$ and $y_\alpha\gg y_c$
(Lyα radiation field should be strong.)

Question related to the WF effect

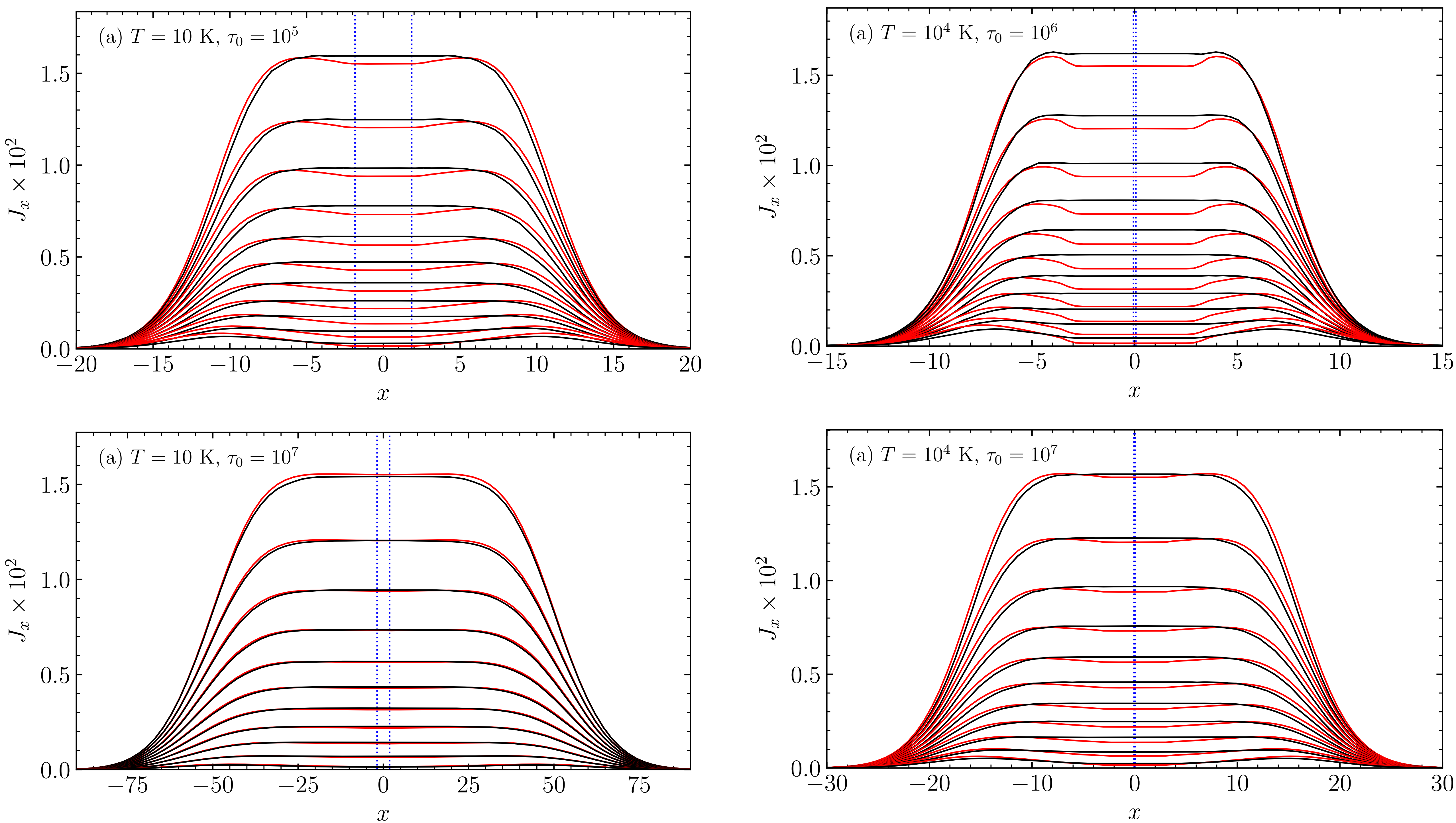
- Lyα line profile inside the medium: How many scatterings are required to make $T_\alpha = T_{\mathrm{K}}$?

  - Deguchi & Watson (1985) showed that an optical depth of 10^5 - 10^6 is required.

  - Shaw et al. (2017) claimed that the Lyα color temperature rarely traces the gas kinetic temperature.

LaRT Test: Line profile inside the medium

Without recoil effect



red lines: analytic solution
black lines: simulation results

Recoil Effect on Spectra inside the medium

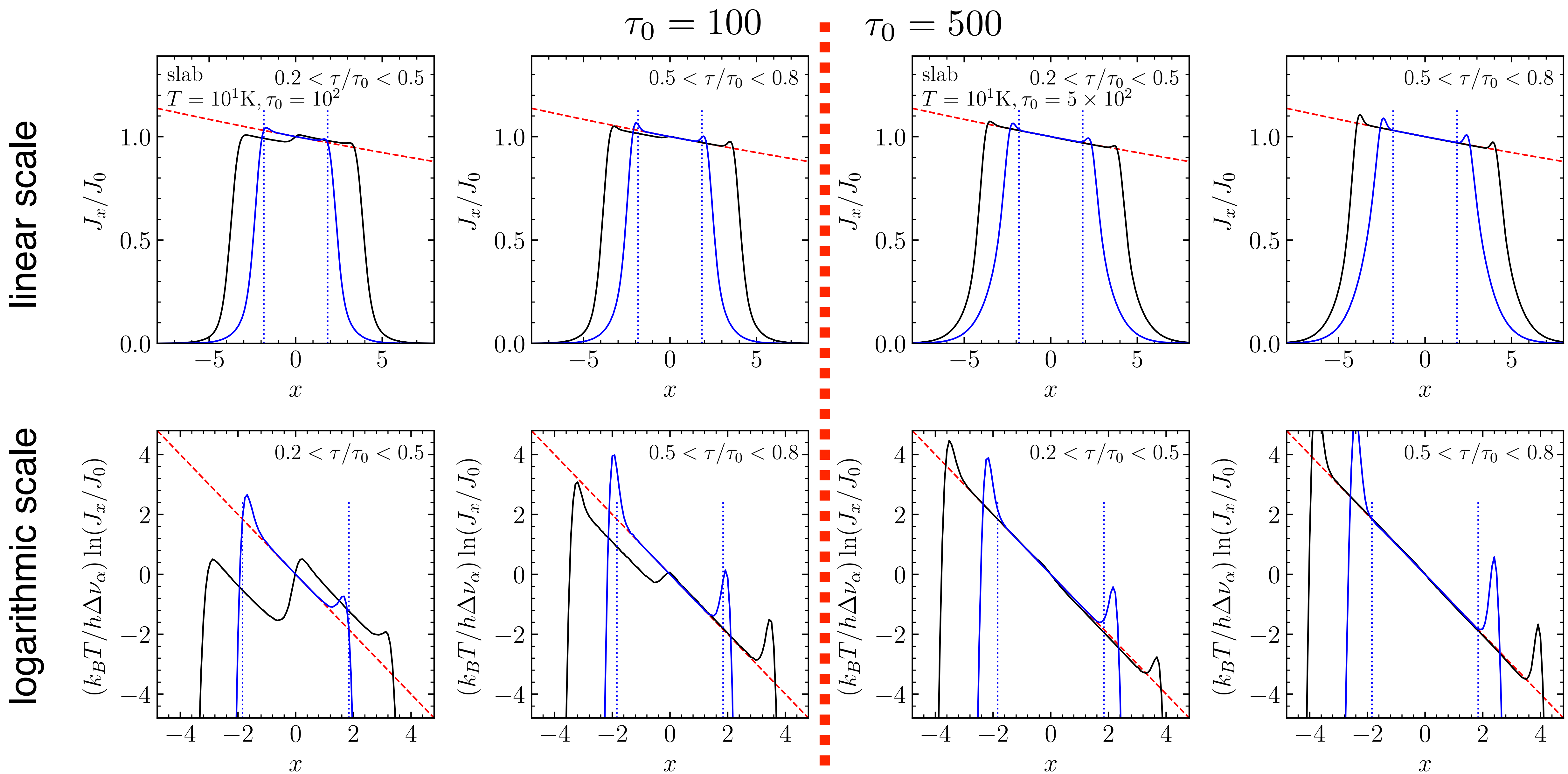
With the recoil effect

T = 10 K

red dashed lines: $J_x \propto \exp\left(-\frac{h(\nu-\nu_\alpha)}{k_B T_K}\right)$

black lines: included the fine structure splitting

blue lines: ignored the fine structure



The resonance-line profile at the line center approaches to the exponential function with the gas kinetic temperature, even in a system with an optical depth as low as 100-500.

Question with regards to the WF effect

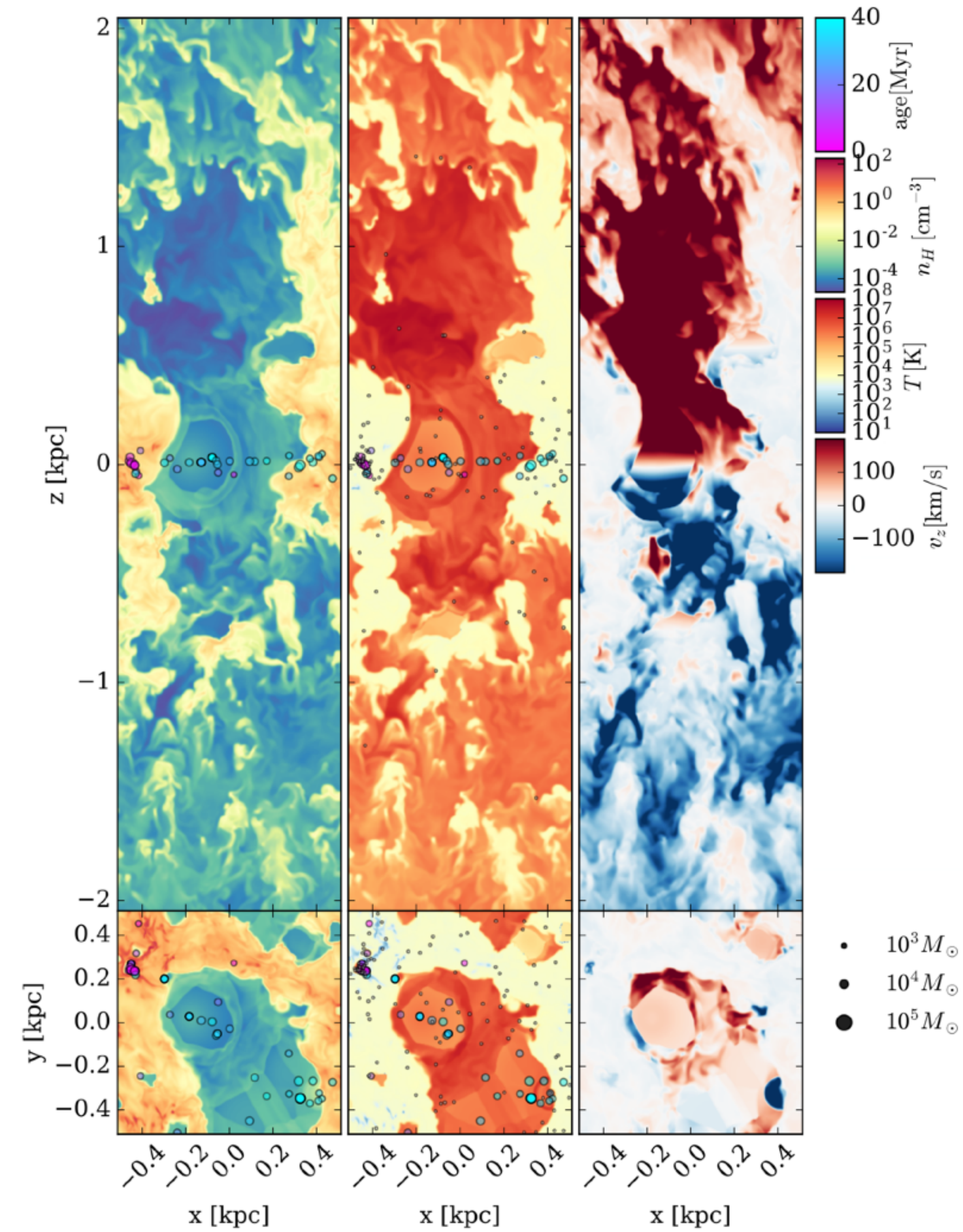
- ***Is Lyα radiation strong enough to make y_α large (in our Galaxy)?***
- We need to perform simulations in realistic ISM models to address this question.
- The TIGRESS frame work.

  TIGRESS = Three-phase Interstellar Medium in Galaxies Resolving Evolution with Star Formation and Supernova Feedback

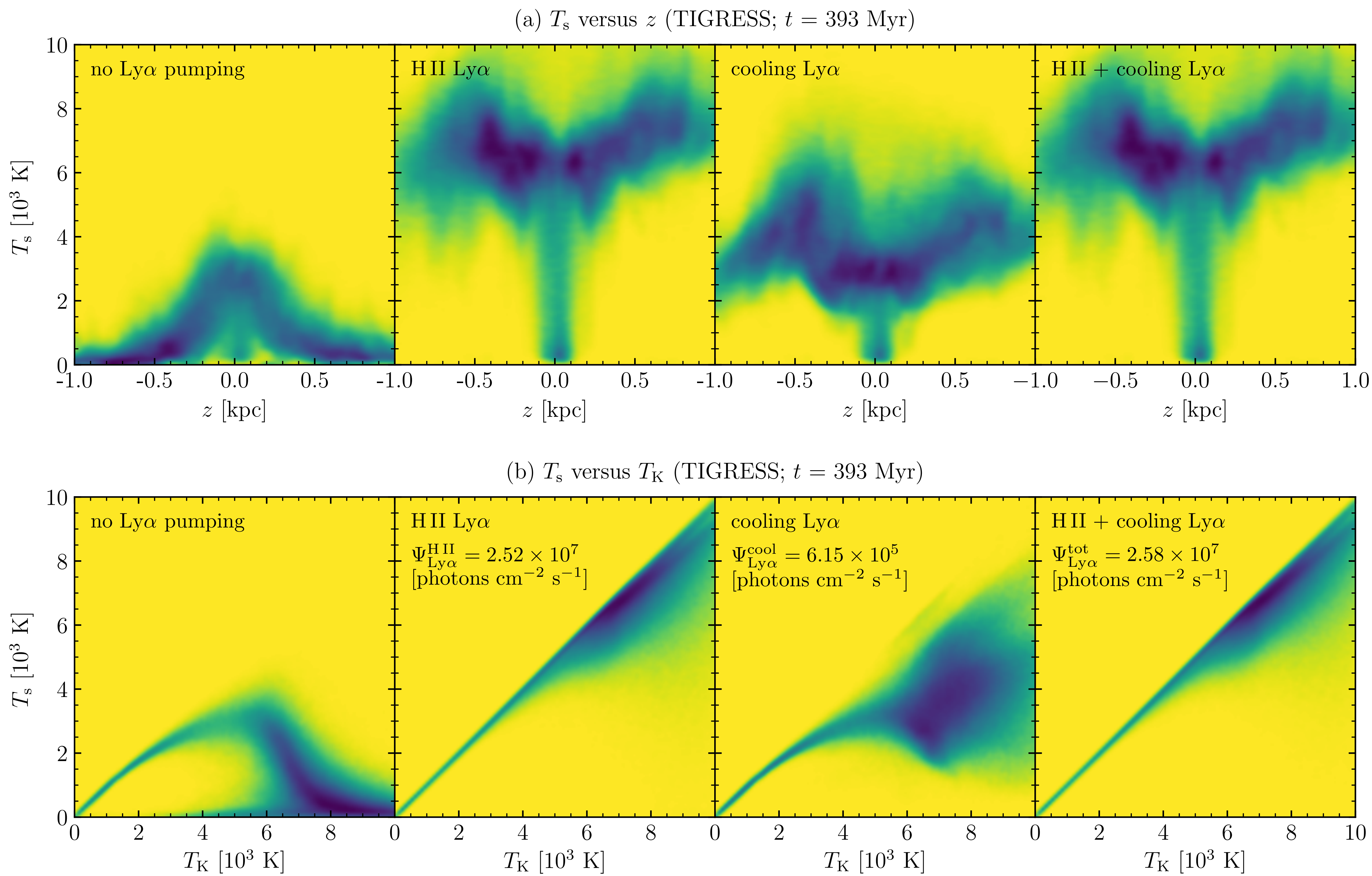
  In the TIGRESS framework, the ideal MHD equations are solved in a local, shearing box, representing a small patch of a differential rotating galactic disk.

  TIGRESS yields self-consistent 3D ISM models with self-regulated star formation.

TIGRESS
Kim & Ostriker (2017; 2018)



TIGRESS: Kinetic & Spin Temperatures



Summary

- The WF effect

(1) The color temperature approaches to the kinetic temperature of the gas, even in a system with an optical depth as low as $\tau_0 \approx 100-500$.

(2) The Lyα radiation field is, in general, likely to be strong enough to bring the 21cm spin temperature of the warm neutral medium close to the kinetic temperature.