KAIST Astrophysics (PH481) - Part 1

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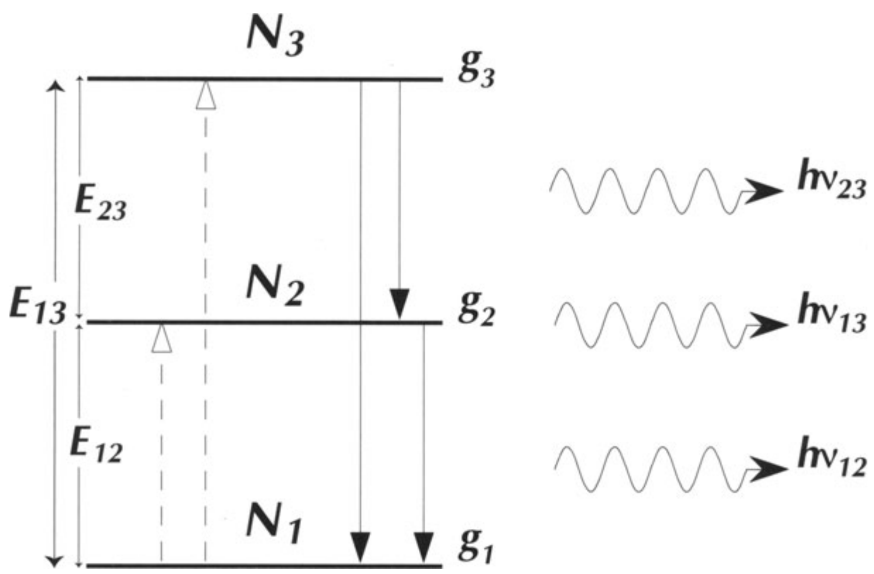
Nebular Diagnostics

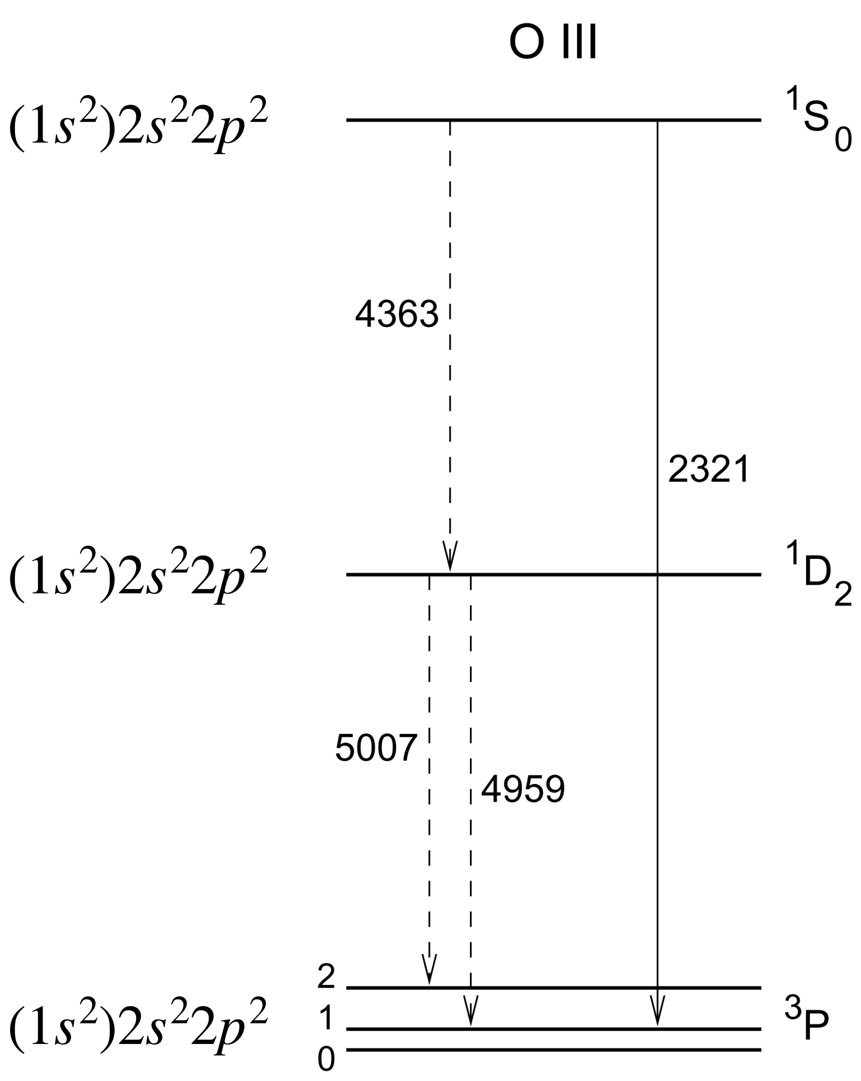
- ***The populations of excited states of atoms and ions depend on the local density and temperature.***
  - Therefore, if we can determine the level populations from observations, we can use atoms and ions as probes of the interstellar space.
  - To be a useful probe, an atom or ion must be sufficiently abundant to observe, must have energy levels that are at suitable energies, and must have radiative transitions that allow us to probe these levels, either through emission lines or absorption lines.
- There are two principal types of nebular diagnostics.
  - ***Temperature***: The first type uses ions with ***two excited levels*** that are both "energetically accessible" at the temperatures of interest, but ***with an energy difference between them that is comparable to*** kT***,*** so that the populations of these levels are sensitive to the gas temperature.
  - ***Density***: The second type uses ions with two or more "energetically accessible" ***energy levels that are at nearly the same energy***, so that the relative rates for populating these levels by collisions are nearly independent of temperature.
    - The ratio of the level populations will have one value in the low-density limit, where every collisional excitation is followed by spontaneous radiative decay, and another value in the high-density limit, where the levels are populated by the balance between the collisional excitation and deexcitation.
    - If the relative level populations in these two limits differ (in general, they do), then the relative level populations (determined by observed emission line ratios) can be used to determine the density in the emission region.

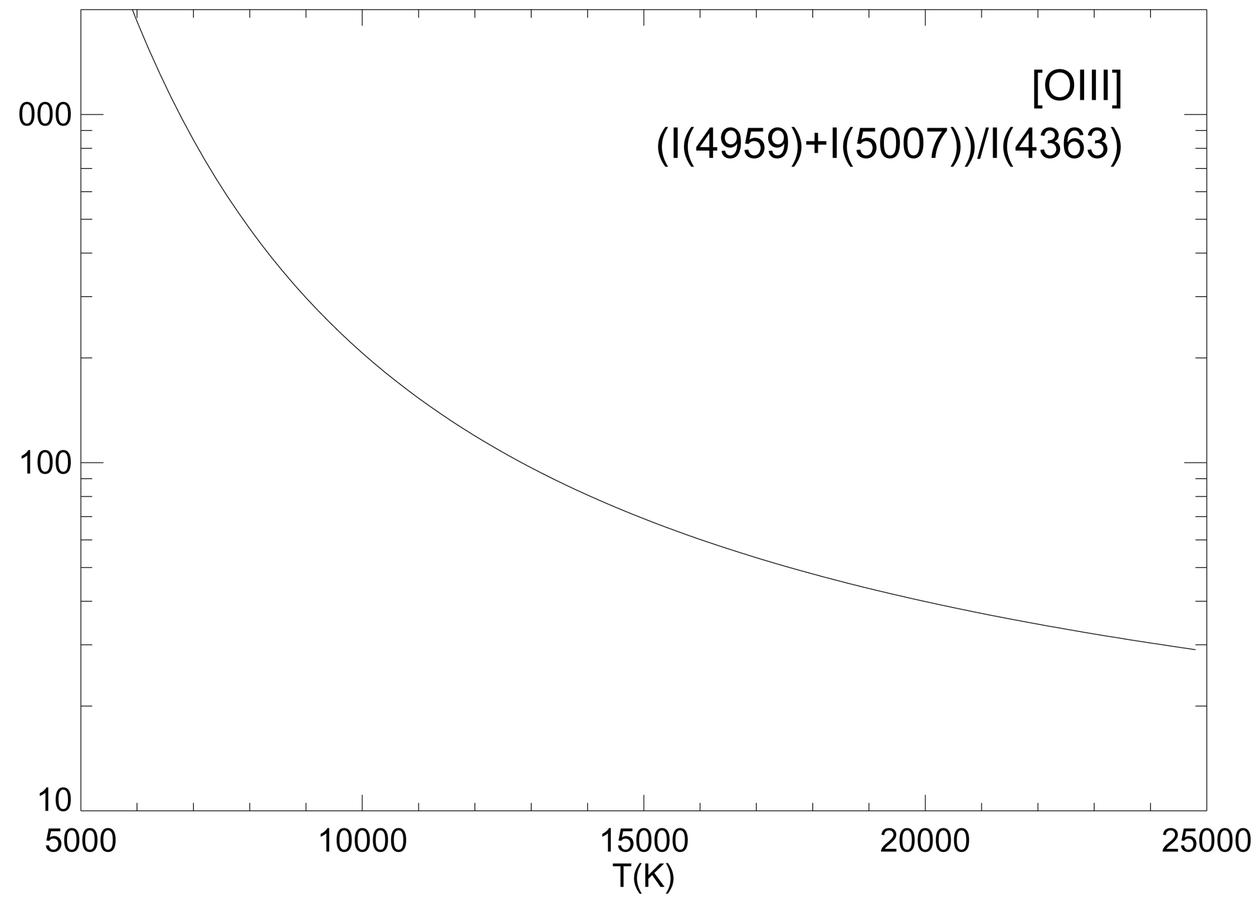
Electron Temperature



Choose an atom with two excited levels with different excitation energies, i.e., $E_{12} < E_{13}$, but $E_{23} \sim kT$.

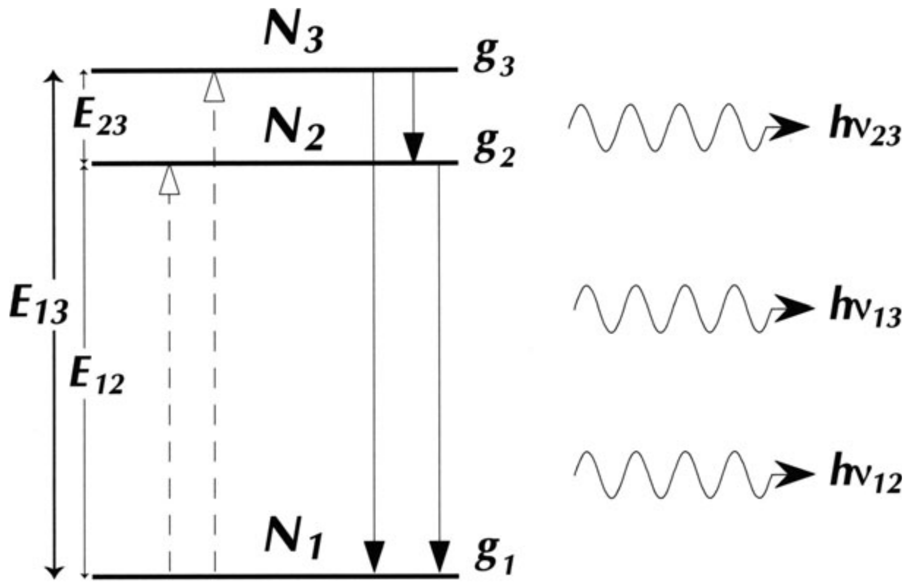
Note that the levels are at the same ground configuration. Thus, they are all forbidden lines.





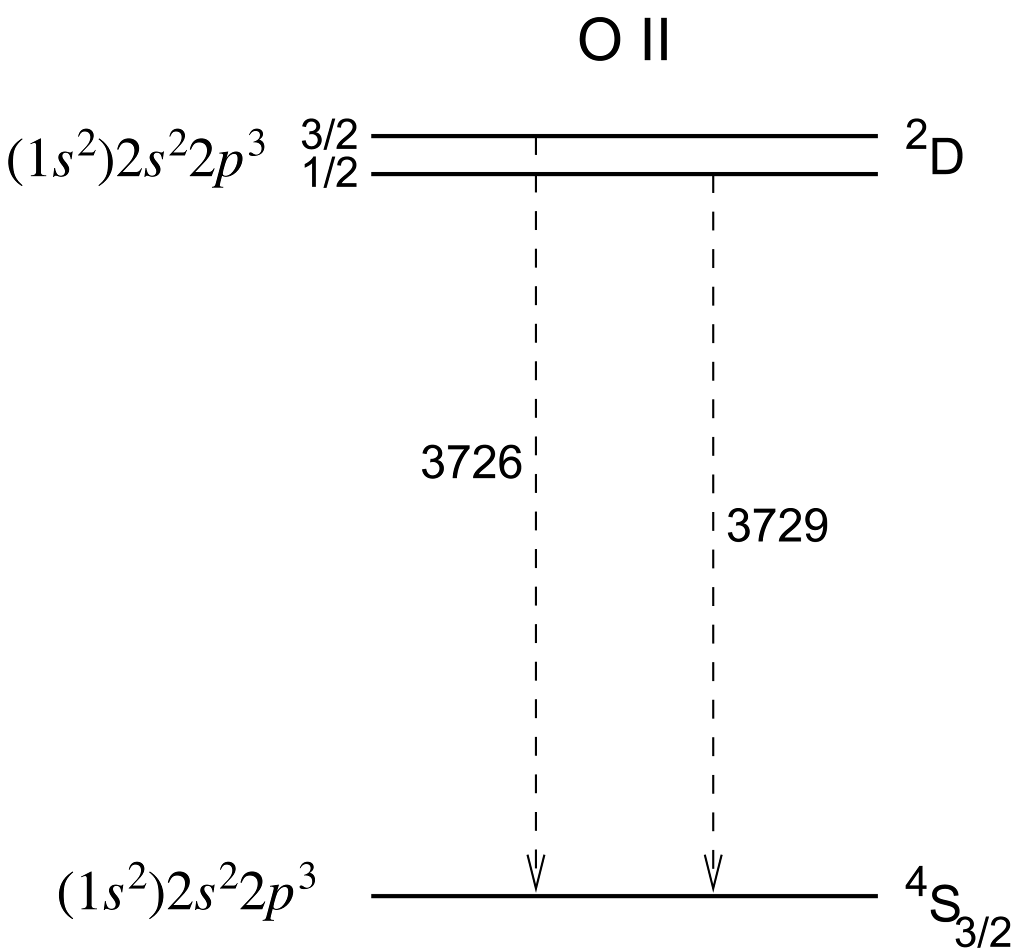
$$\frac{I(4959+5007)}{I(4363)} = \frac{7.7\exp(3.29\times 10^4/T)}{1+4.5\times 10^{-4} n_e T^{-1/2}}$$

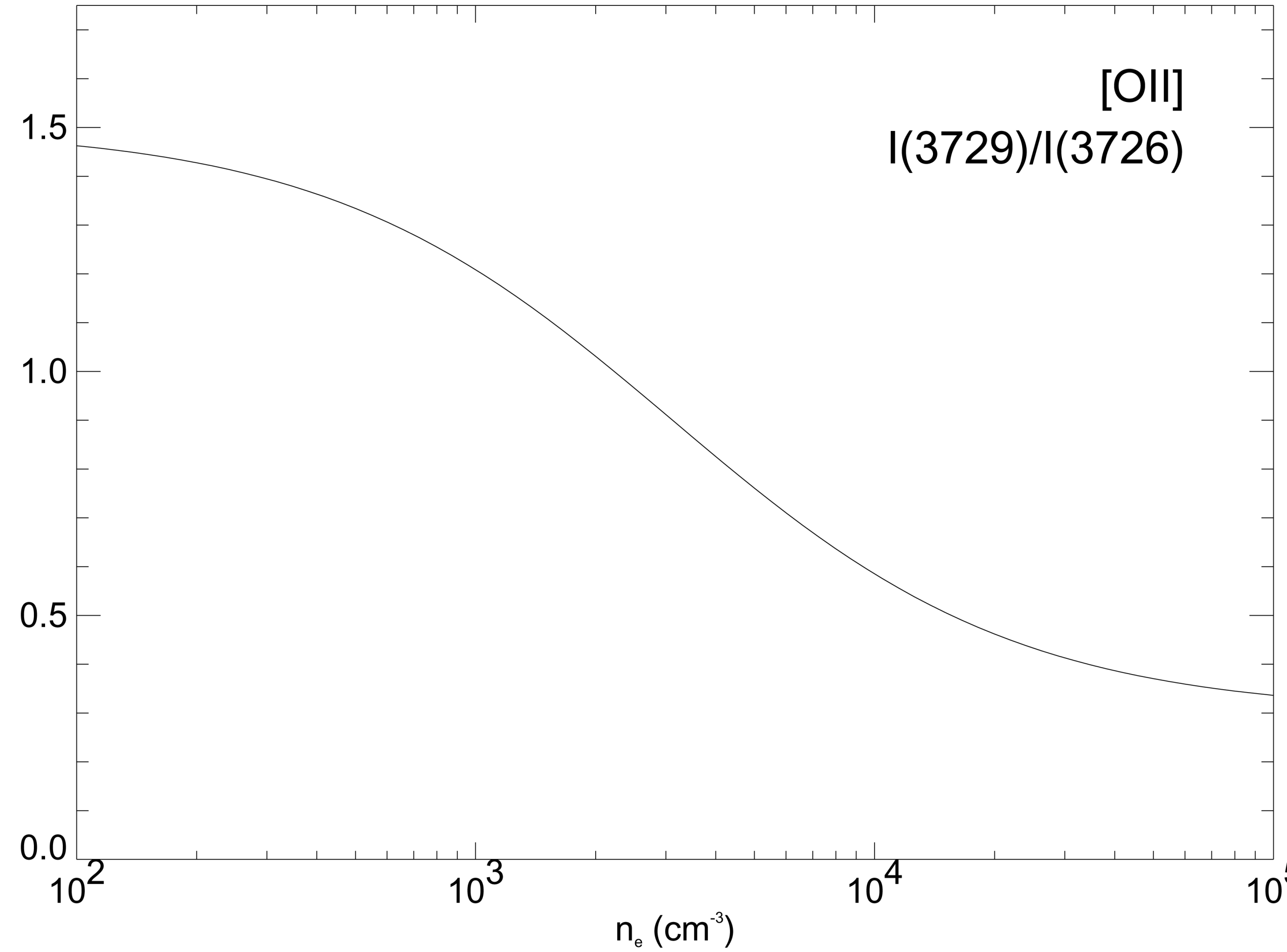
Electron Density



Choose an atom with two excited levels with almost same excitation energy i.e., $E_{12} \sim E_{13}$.

Note that the levels are at the same ground configuration. Thus, they are all forbidden lines.

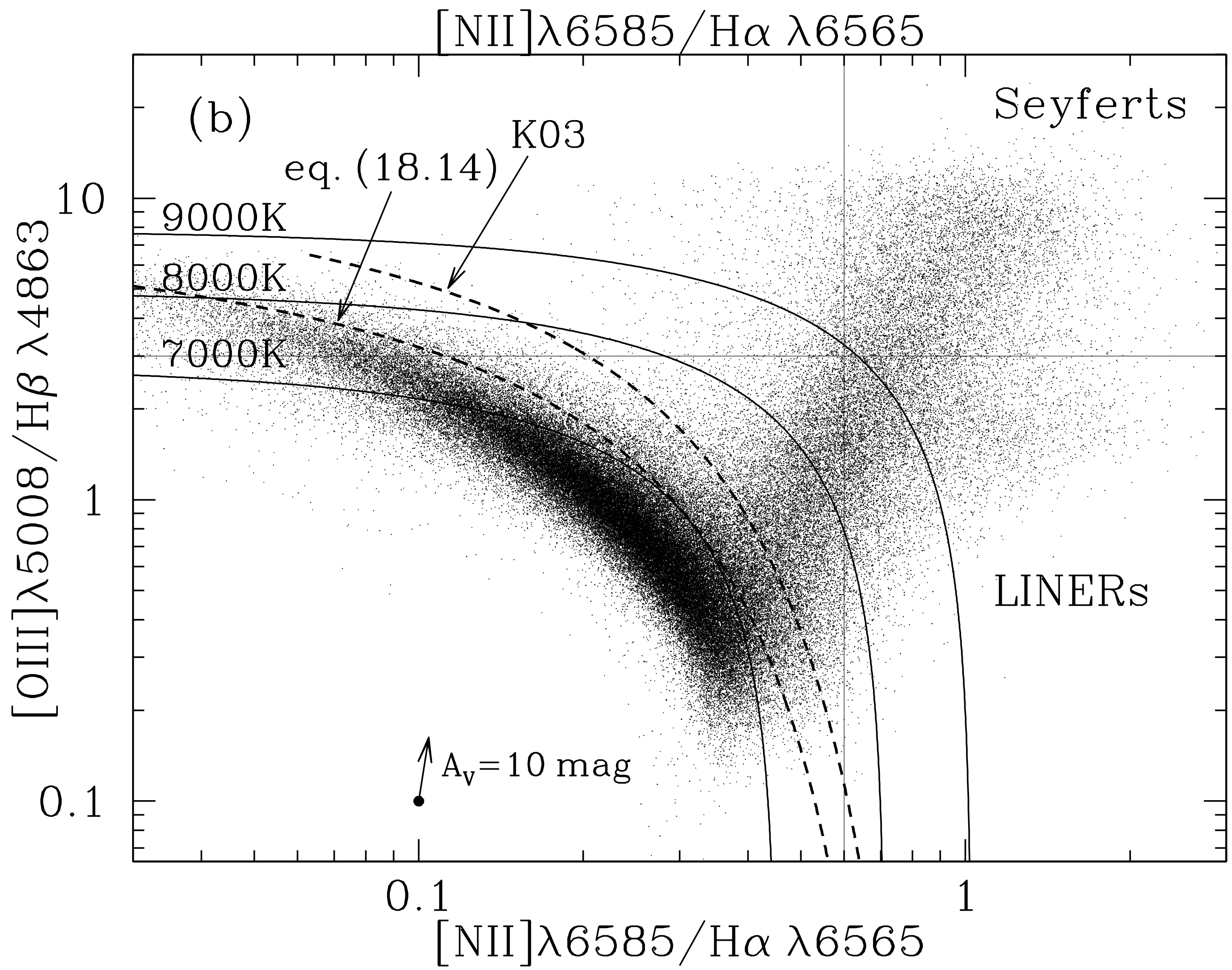




BPT diagram

- The optical line emission from star-forming galaxies is usually dominated by emission lines from H II regions. Some galaxies, however, have strong continuum and line emission from an active galactic nucleus (AGN). The line emission is thought come from gas that is heated and ionized by X-rays from the AGN.
- Baldwin, Phillips & Terlevich (1981) pointed out that one could distinguish star-forming galaxies from galaxies with spectra dominated by active galactic nuclei by plotting the ratio of [O III]/Hβ versus [N II]/Hα - this is now referred to as the BPT diagram.

- These lines have advantage of being among the strongest optical emission lines from H II regions.
- Furthermore, the line ratios employ pairs of lines with similar wavelengths so that the line ratios are nearly unaffected by whatever dust extinction may be present.
- Normal H II regions are photo-ionized by OB stars. On the other hand, In AGNs, the medium is photo-ionized by a hard radiation field, which can be produced by either power-law continuum from an AGN or from shock excitation.



Baldwin, Phillips & Terlevich (1981, PASP, 93, 5)
Draine, Physics of the Interstellar and Intergalactic Medium

Lyman-Alpha, 21 cm Astrophysics

(1) Line Profile, Resonance Line (Mon)

(2) Lyα Forest (Mon)

(3) Cosmic Reionization Epoch, Gunn-Peterson Effect (Wed)

(4) Lyman Alpha Emitters (Wed)

(5) Lyα-21 cm coupling (Wouthuysen-Field Effect) (Wed)

Classical model to the motion of an electron in an atom

- ***Lorentz Oscillator Model*** **to describe the interaction between atoms and electric fields**
  - The electron (with a small mass) is bound to the nucleus of the atom (with a much larger mass) by a force that behaves according to Hooke's Law (a spring-like force).
  - An applied electric field would then interact with the charge of the electron, causing "stretching" or "compression" of the spring.
  - ***The electron's equation of motion:***

$$\boxed{m\ddot{\mathbf{x}} = -k\mathbf{x} + \mathbf{F}_{\text{ext}} + \mathbf{F}_{\text{rad}}}$$

$$k = m\omega_0^2 \;, \;\; \text{where } k = \text{spring constant}$$

$$\omega_0 = \text{natural (fundamental or resonant) freqency}$$

$\mathbf{F}_{\text{ext}} =$ external force, driving force, or external electric field

$\mathbf{F}_{\text{rad}} =$ radiation reaction force (radiation damping)
the damping of a charge's motion which arises because of the emission of radiation

[1] Spontaneous Emission : Damping, Free Oscillator

- **Undriven Harmonically Bound Particles** (free oscillator)
  - Since an oscillating electron represents a continuously accelerating charge, the electron will radiate energy.
  - The energy radiated away must come from the particle's own energy (energy conservation). In other words, **there must be a force acting on a particle by virtue of the radiation it produces. This is called the *radiation reaction force*.**
  - Let's derive the formula for the radiation reaction force from the fact that the energy radiated must be compensated for by the work done against the radiation reaction force.
  - On one hand, the radiative loss rate of energy, averaged over one cycle of the oscillating dipole, can be represented by the radiative reaction force:

$$\frac{dW}{dt} = \langle \mathbf{F}_{\mathrm{rad}} \cdot \dot{\mathbf{x}} \rangle$$

  - On the other hand, from the Larmor's formula for a dipole, the radiative loss will be:

$$\frac{dW}{dt} = -\frac{2e^2 \left\langle |\ddot{\mathbf{x}}|^2 \right\rangle}{3c^3}$$

[1] Spontaneous Emission : Abraham-Lorentz formula

$$\therefore \quad \langle \mathbf{F}_{\rm rad} \cdot \dot{\mathbf{x}} \rangle = -\frac{2e^2 \left\langle |\ddot{\mathbf{x}}|^2 \right\rangle}{3c^3}$$

Here, $\left\langle |\ddot{\mathbf{x}}|^2 \right\rangle \equiv \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} \ddot{\mathbf{x}} \cdot \ddot{\mathbf{x}} dt$ where τ is the oscillation period.

$$= \frac{1}{\tau} \ddot{\mathbf{x}} \cdot \dot{\mathbf{x}} \Big|_{-\tau/2}^{\tau/2} - \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} \dddot{\mathbf{x}} \cdot \dot{\mathbf{x}} dt$$

We assume that the initial and final states are the same: $\ddot{\mathbf{x}} \cdot \dot{\mathbf{x}}(-\tau/2) = \ddot{\mathbf{x}} \cdot \dot{\mathbf{x}}(\tau/2)$

Then,

$$\left\langle |\ddot{\mathbf{x}}|^2 \right\rangle = -\frac{1}{\tau} \int_{-\tau/2}^{\tau/2} \dddot{\mathbf{x}} \cdot \dot{\mathbf{x}} dt = -\left\langle \dddot{\mathbf{x}} \cdot \dot{\mathbf{x}} \right\rangle \;\rightarrow\; \langle \mathbf{F}_{\rm rad} \cdot \dot{\mathbf{x}} \rangle = \frac{2e^2 \left\langle \dddot{\mathbf{x}} \cdot \dot{\mathbf{x}} \right\rangle}{3c^3}$$

Therefore, we can obtain $\boxed{\mathbf{F}_{\rm rad} = \frac{2e^2 \dddot{\mathbf{x}}}{3c^3}}$: Abraham-Lorentz formula

- **Abraham-Lorentz formula**:

$$\mathbf{F}_{\rm rad} = \frac{2e^2 \dddot{\mathbf{x}}}{3c^3}$$

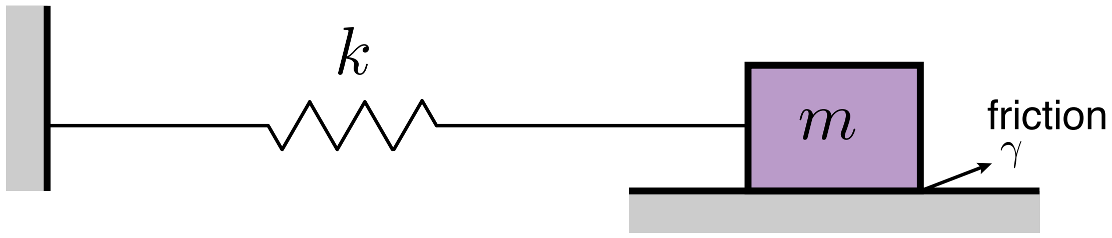
  - This formula depends on the derivative of acceleration. This increases the degree of the equation of motion of a particle and can lead to some nonphysical behavior if not used properly and consistently.
  - For a simple harmonic oscillator with a frequency ω_0, we can avoid the difficulty by using

$$\dddot{\mathbf{x}} = -\omega_0^2 \dot{\mathbf{x}}$$

  - ***This is a good assumption as long as the energy is to be radiated on a time scale that is long compared to the period of oscillation*** ($\gamma \ll \omega_0$)***.*** In this regime, ***radiation reaction may be considered as a perturbation on the particle's motion***. We then rewrite the radiation reaction force as

$$\mathbf{F}_{\rm rad} = -\frac{2e^2\omega_0^2}{3c^3}\dot{\mathbf{x}} = -m\gamma\dot{\mathbf{x}}, \qquad \gamma \equiv \frac{2e^2\omega_0^2}{3mc^3}$$

: damping constant



$$m\ddot{\mathbf{x}} + k\mathbf{x} + m\gamma\dot{\mathbf{x}} = 0$$

This is the equation for a string-mass system subject to friction damping.

- Therefore, the equation of motion of the electron in a Lorentz atom is

$$\ddot{\mathbf{x}} + \gamma\dot{\mathbf{x}} + \omega_0^2\mathbf{x} = 0$$

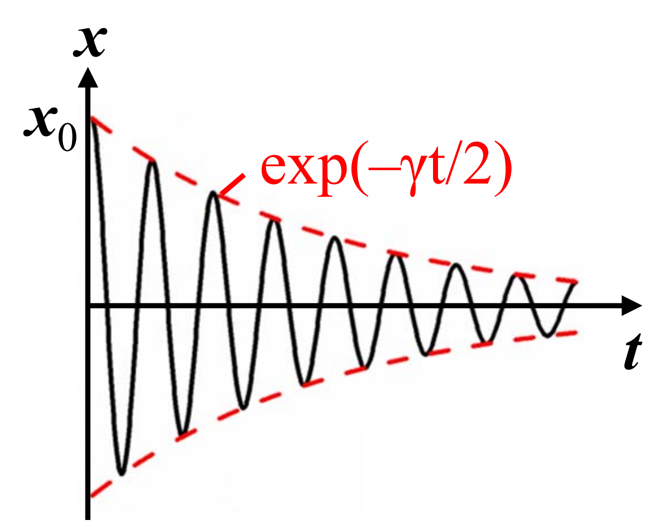
  - This equation may be solved by assuming that $x(t) \propto e^{\alpha t}$.

$$\alpha^2 + \gamma\alpha + \omega_0^2 = 0 \;\rightarrow\; \alpha = -(\gamma/2) \pm \sqrt{(\gamma/2)^2 - \omega_0^2}$$
$$= -\gamma/2 \pm i\omega_0 + \mathcal{O}(\gamma^2/\omega_0^2)$$

Here, we assumed $\gamma \ll \omega_0$.

  - Assuming initial conditions: $x(0) = x_0,\; \dot{x}(0) = 0 \text{ at } t = 0$
  - we have

$$x(t) = \frac{1}{2}x_0\left[e^{-(\gamma/2 - i\omega_0)t} + e^{-(\gamma/2 + i\omega_0)t}\right] = x_0 e^{-\gamma t/2}\cos\omega_0 t \;\longrightarrow$$



This is a damping oscillator.

- Power spectrum:

$$\bar{x}(\omega) = \frac{1}{2\pi}\int_0^\infty x(t)e^{i\omega t}dt = \frac{x_0}{4\pi}\left[\frac{1}{\gamma/2 - i(\omega+\omega_0)} + \frac{1}{\gamma/2 - i(\omega-\omega_0)}\right]$$

  - This becomes large in the vicinity of $\omega = \omega_0$ and $\omega = -\omega_0$.
  - We are ultimately interested only in positive frequencies, and only in regions in which the values become large. Therefore, we obtain

$$\bar{x}(\omega) \approx \frac{x_0}{4\pi}\frac{1}{\gamma/2 - i(\omega-\omega_0)}, \quad |\bar{x}(\omega)|^2 = \left(\frac{x_0}{4\pi}\right)^2 \frac{1}{(\omega-\omega_0)^2 + (\gamma/2)^2}$$

[1] Spontaneous Emission: Line profile

- Recall the Larmor's formula: $$\frac{dW}{d\omega} = \frac{8\pi\omega^4}{3c^3} e^2 |\bar{x}(\omega)|^2$$

- <u>Energy radiated per unit frequency</u>:

$$\frac{dW}{d\omega} = \frac{8\pi\omega^4}{3c^3}\frac{e^2 x_0^2}{(4\pi)^2}\frac{1}{(\omega-\omega_0)^2+(\gamma/2)^2} = \frac{1}{2} m \left(\frac{\omega^4}{\omega_0^2}\right) x_0^2 \frac{\gamma/2\pi}{(\omega-\omega_0)^2+(\gamma/2)^2}$$
$$\approx \frac{1}{2} m\omega_0^2 x_0^2 \frac{\gamma/2\pi}{(\omega-\omega_0)^2+(\gamma/2)^2}$$

- For a harmonic oscillator, note that the equation of motion is $\mathbf{F} = -k\mathbf{x} = -m\omega_0^2\mathbf{x}$, spring constant is $k = m\omega_0^2$, and the potential energy (energy stored in spring) is $(1/2)kx_0^2$.

- From $$\int_{-\infty}^{\infty} \frac{\gamma/2\pi}{(\omega-\omega_0)^2+(\gamma/2)^2} d\omega = \frac{1}{\pi}\tan^{-1}\{2(\omega-\omega_0)/\gamma\}|_{-\infty}^{\infty} = 1$$

- Note that the total emitted energy is equal to the initial potential energy of the oscillator:

$$W = \int_0^{\infty} \frac{dW}{d\omega} d\omega = \frac{1}{2} k\omega_0^2$$

- Profile of the emitted spectrum: $$\boxed{\phi(\omega) = \frac{\gamma/2\pi}{(\omega-\omega_0)^2+(\gamma/2)^2}}$$

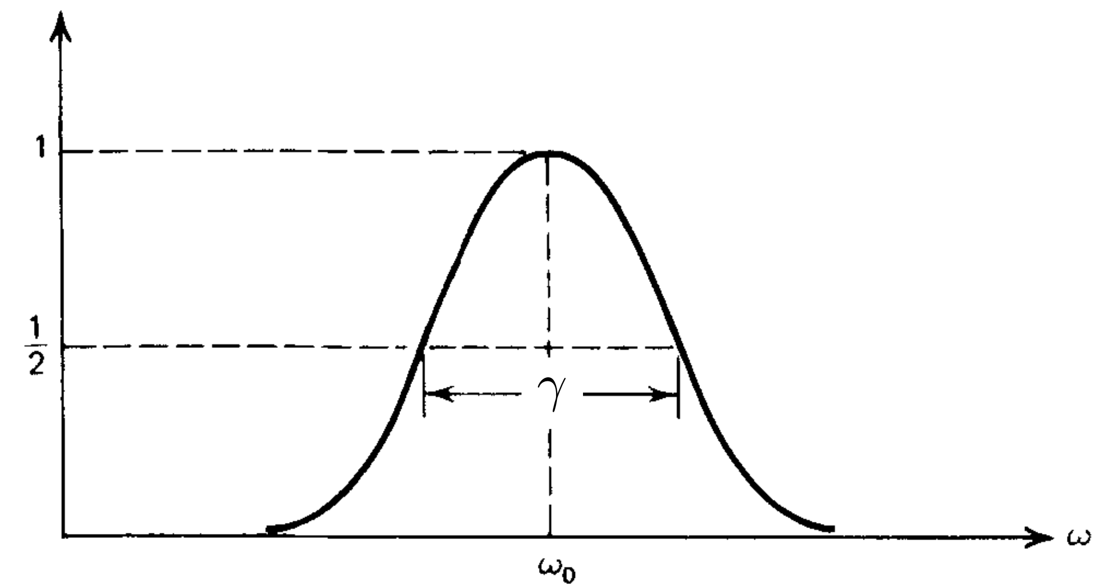
This is the Lorentz (natural) line profile.

- Damping constant is the full width at half maximum (FWHM).

$$\phi(\omega) = \frac{\gamma/2\pi}{(\omega-\omega_0)^2 + (\gamma/2)^2}$$

$$\boxed{\phi(\nu) = \frac{\gamma/4\pi^2}{(\nu-\nu_0)^2 + (\gamma/4\pi)^2}}$$

Note $\phi(\omega)d\omega = \phi(\nu)d\nu$



- The line width $\Delta\omega = \gamma$ is a universal constant when expressed in terms of wavelength:

$$\lambda = \frac{2\pi c}{\omega}$$

$$\Delta\lambda = 2\pi c\frac{\Delta\omega}{\omega^2} = 2\pi c \frac{2}{3}\frac{r_e}{c} \longleftarrow \left(\Delta\omega = \gamma = \frac{2}{3} r_e \frac{\omega_0^2}{c}\right)$$

$$= \frac{4}{3}\pi r_e$$

$$= 1.2 \times 10^{-4}\text{Å}$$

However, in Quantum Mechanics, the line width is not a universal constant.

[2] Scattering : Driven Oscillator

- **Driven Harmonically Bound Particles** (forced oscillators)
  - Electron's equation of motion (electric charge = $-e$): $\mathbf{F}_{\rm ext} = -e\mathbf{E}_0 e^{i\omega t}$

$$\ddot{\mathbf{x}} + \gamma\dot{\mathbf{x}} + \omega_0^2\mathbf{x} = -\frac{e\mathbf{E}_0}{m}e^{i\omega t}$$

  - A particular solution for this inhomogeneous differential equation:

$$\mathbf{x} = \mathbf{x}_0 e^{i\omega t} \equiv |\mathbf{x}_0| e^{i(\omega t+\delta)} \quad \rightarrow \quad (-\omega^2 + i\omega\gamma + \omega_0^2)\mathbf{x}_0 e^{i\omega t} = -\frac{e\mathbf{E}_0}{m}e^{i\omega t}$$

$$\mathbf{x}_0 = \frac{(e/m)\mathbf{E}_0}{(\omega^2-\omega_0^2) - i\omega\gamma}$$

$$\mathbf{x}_0 = |\mathbf{x}_0|e^{i\delta} \propto (\omega^2-\omega_0^2) + i\omega\gamma \quad \rightarrow \quad \delta = \tan^{-1}\left(\frac{\omega\gamma}{\omega^2-\omega_0^2}\right)$$

The response is slightly out of phase with respect to the imposed field.

  - Time-averaged total power radiated is given by

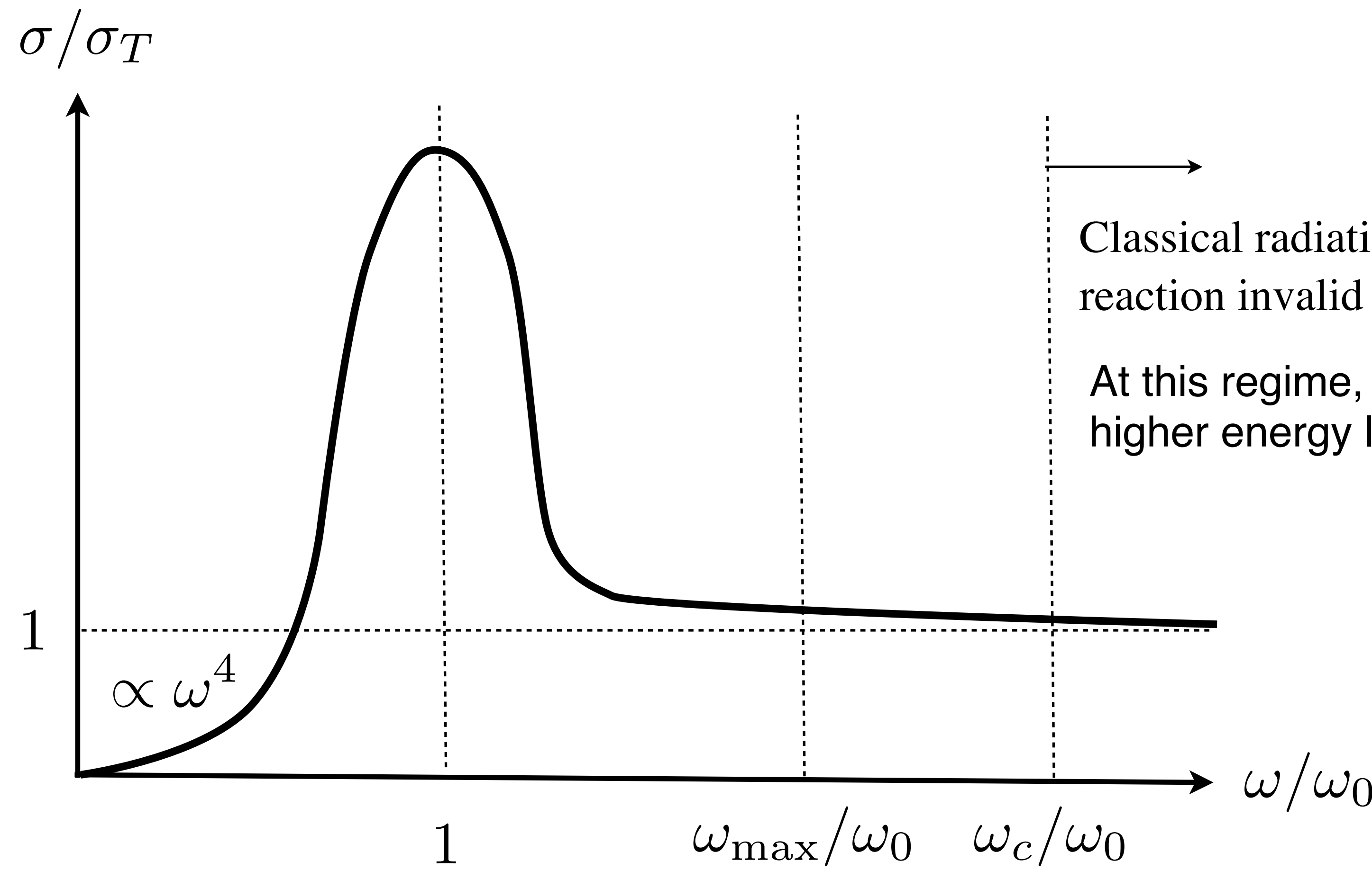
$$P = \left\langle \frac{dW}{dt} \right\rangle = \frac{2e^2\left\langle|\ddot{\mathbf{x}}|^2\right\rangle}{3c^3} = \frac{e^2\omega^4|\mathbf{x}_0|^2}{3c^3}$$

$$= \frac{e^4E_0^2}{3m^2c^3}\frac{\omega^4}{(\omega^2-\omega_0^2)^2 + (\omega\gamma)^2}$$

- Scattering cross section:

$$\sigma_{\rm sca} \equiv \frac{\langle P\rangle}{\langle S\rangle}, \quad \langle S\rangle = \frac{c}{8\pi}E_0^2 \longrightarrow \sigma_{\rm sca}(\omega) = \frac{8\pi e^4}{3m^2c^4}\frac{\omega^4}{(\omega^2-\omega_0^2)^2+(\omega\gamma)^2}$$

$$= \sigma_T \frac{\omega^4}{(\omega^2-\omega_0^2)^2+(\omega\gamma)^2}$$



At this regime, ionization and excitation to higher energy levels will occur.

- Limiting Cases of Interest

(a) $\omega \gg \omega_0$ (Thomson scattering by free electron)

$$\sigma_{\text{sca}} = \sigma_T = \frac{8\pi}{3} r_e^2$$

- At high incident energies, the binding becomes negligible. Therefore, this corresponds to the case of a free electron.

(b) $\omega \ll \omega_0$ (Rayleigh scattering by bound electron)

$$\sigma_{\text{sca}} = \sigma_T \left(\frac{\omega}{\omega_0} \right)^4 = \sigma_T \left(\frac{\lambda_0}{\lambda} \right)^4$$

- Rayleigh scattering refers to the ***scattering of light by particles smaller than the wavelength of the light.***
- The strong wavelength dependence of the scattering means that shorter (blue) wavelengths are scattered more strongly than longer wavelengths.
- (blue color of the sky) The dependence results in the indirect blue light coming from all regions of the sky.
- (red color of the sun at sunset) Conversely, glancing toward the Sun, the colors that were not scattered away - the longer wavelengths such as red and yellow light - are directly visible, giving the Sun itself a slightly yellowish color.
- However, viewed from space, the sky is black and the Sun is white.

(c) $\omega \approx \omega_0$ (resonance scattering of line radiation)

$$\sigma_{\rm sca}(\omega) \approx \sigma_T \frac{\omega_0^4}{(\omega-\omega_0)^2(2\omega_0)^2 + (\omega_0\gamma)^2}$$
$$= \sigma_T \frac{\omega_0^2/4}{(\omega-\omega_0)^2 + (\gamma/2)^2}$$

$$\sigma_T \frac{\omega_0^2}{4} = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 \times \frac{1}{4} \times \left(\gamma \frac{3}{2}\frac{mc^3}{e^2\omega_0^2}\right) = 2\pi^2 \frac{e^2}{mc}(\gamma/2\pi) \longrightarrow$$

Note that $\nu = 2\pi\omega$ and $\sigma_\nu = \sigma_\omega/2\pi$.

$$\sigma_\omega = \frac{2\pi^2 e^2}{mc} \frac{\gamma/2\pi}{(\omega-\omega_0)^2 + (\gamma/2)^2}$$

$$\boxed{\sigma_\nu = \frac{\pi e^2}{mc} \frac{\gamma/4\pi^2}{(\nu-\nu_0)^2 + (\gamma/4\pi)^2}}$$

- In the neighborhood of the resonance, ***the shape of the scattering cross-section is the same as the emission line profile from the free oscillator.***
- Total scattering cross section is
$$\int_0^\infty \sigma_\nu d\nu = \frac{\pi e^2}{mc}$$

- ***Resonance line***
  - A spectral line caused by an electron jumping between the ground state and the first energy level in an atom or ion. It is the longest wavelength line produced by a jump to or from the ground state.
  - Because the majority of electrons are in the ground state in many astrophysical environments, and because the energy required to reach the first level is the least needed for any transition, resonance lines are the strongest lines in the spectrum for any given atom or ion.

- ***In the quantum theory of spectral lines,***

  we obtain similar formulas, which are conveniently stated in terms of the classical results as

$$\sigma_\nu = f_{nn'} \frac{\pi e^2}{mc} \frac{\gamma/4\pi^2}{(\nu-\nu_0)^2 + (\gamma/4\pi)^2}$$

$$\int_0^\infty \sigma_\nu d\nu = f_{nn'} \frac{\pi e^2}{mc}$$

where $f_{nn'}$ is called the ***oscillator strength*** or ***f-value*** for the transition between states n and n'.

Selected Resonance Lines[a] with $\lambda < 3000$ Å

	Configurations	ℓ	u	$E_\ell/hc(\mathrm{cm}^{-1})$	λ_{vac}(Å)	$f_{\ell u}$
C IV	$1s^22s - 1s^22p$	$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^o_{1/2}$	0	1550.772	0.0962
		$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^o_{3/2}$	0	1548.202	0.190
N V	$1s^22s - 1s^22p$	$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^o_{1/2}$	0	1242.804	0.0780
		$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^o_{3/2}$	0	1242.821	0.156
O VI	$1s^22s - 1s^22p$	$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^o_{1/2}$	0	1037.613	0.066
		$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^o_{3/2}$	0	1037.921	0.133
C III	$2s^2 - 2s2p$	$^1\mathrm{S}_0$	$^1\mathrm{P}^o_1$	0	977.02	0.7586
C II	$2s^22p - 2s2p^2$	$^2\mathrm{P}^o_{1/2}$	$^2\mathrm{D}^o_{3/2}$	0	1334.532	0.127
		$^2\mathrm{P}^o_{3/2}$	$^2\mathrm{D}^o_{5/2}$	63.42	1335.708	0.114
N III	$2s^22p - 2s2p^2$	$^2\mathrm{P}^o_{1/2}$	$^2\mathrm{D}^o_{3/2}$	0	989.790	0.123
		$^2\mathrm{P}^o_{3/2}$	$^2\mathrm{D}^o_{5/2}$	174.4	991.577	0.110
C I	$2s^22p^2 - 2s^22p3s$	$^3\mathrm{P}_0$	$^3\mathrm{P}^o_1$	0	1656.928	0.140
		$^3\mathrm{P}_1$	$^3\mathrm{P}^o_2$	16.40	1656.267	0.0588
		$^3\mathrm{P}_2$	$^3\mathrm{P}^o_2$	43.40	1657.008	0.104
N II	$2s^22p^2 - 2s2p^3$	$^3\mathrm{P}_0$	$^3\mathrm{D}^o_1$	0	1083.990	0.115
		$^3\mathrm{P}_1$	$^3\mathrm{D}^o_2$	48.7	1084.580	0.0861
		$^3\mathrm{P}_2$	$^3\mathrm{D}^o_3$	130.8	1085.701	0.0957
N I	$2s^22p^3 - 2s^22p^23s$	$^4\mathrm{S}^o_{3/2}$	$^4\mathrm{P}_{5/2}$	0	1199.550	0.130
		$^4\mathrm{S}^o_{3/2}$	$^4\mathrm{P}_{3/2}$	0	1200.223	0.0862
O I	$2s^22p^4 - 2s^22p^33s$	$^3\mathrm{P}_2$	$^3\mathrm{S}^o_1$	0	1302.168	0.0520
		$^3\mathrm{P}_1$	$^3\mathrm{S}^o_1$	158.265	1304.858	0.0518
		$^3\mathrm{P}_0$	$^3\mathrm{S}^o_1$	226.977	1306.029	0.0519
Mg II	$2p^63s - 2p^63p$	$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^o_{1/2}$	0	2803.531	0.303
		$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^o_{3/2}$	0	2796.352	0.608
Al III	$2p^63s - 2p^63p$	$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^o_{1/2}$	0	1862.790	0.277
		$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^o_{3/2}$	0	1854.716	0.557

Draine, Physics of the interstellar and intergalactic medium

[3] Line Broadening Mechanisms

- ***Atomic levels are not infinitely sharp***, nor are the lines connecting them.
  - (1) Doppler Broadening
  - (2) Natural Broadening
  - (3) Collisional Broadening
- **Doppler Broadening**
  - The simplest mechanism for line broadening in the Doppler effect. An atom is in thermal motion, so that the frequency of emission or absorption in its own frame corresponds to a different frequency for an observer.
  - Each atom has its own Doppler shift, so that the net effect is to spread the line out, but not to change its total strength.
  - The change in frequency associated with an atom with velocity component v_z along the line of sight (say, z axis) is, to lowest order in v_z/c, given by

$$\nu - \nu_0 = \nu_0 \frac{v_z}{c}$$

Recall Doppler shift: $\left[\frac{\nu}{\nu_0} = \frac{1}{\gamma\,(1-\beta\cos\theta)} \quad \rightarrow \quad \nu \approx \nu_0\,(1+\beta\cos\theta) \quad \rightarrow \quad \nu - \nu_0 = \frac{\nu_0 v_z}{c}\right]$

  - Here, ν_0 is the rest-frame frequency.

- We need to consider the velocity distribution of atoms. The number of atoms having velocities in the range $(v_z,\ v_z + dv_z)$ is proportional to

$$f(v_z)dz = \exp\left(-\frac{mv_z^2}{2kT}\right)dv_z$$

- From the Doppler shift formula, we have

$$v_z = \frac{c(\nu - \nu_0)}{\nu_0} \quad \rightarrow \quad dv_z = \frac{cd\nu}{\nu_0}$$

- Therefore, the strength of the emission is proportional to

$$\exp\left(-\frac{mv_z^2}{2kT}\right)dv_z \propto \exp\left[-\frac{mc^2(\nu-\nu_0)^2}{2\nu_0^2 kT}\right]d\nu$$

- Then, the normalized profile function is

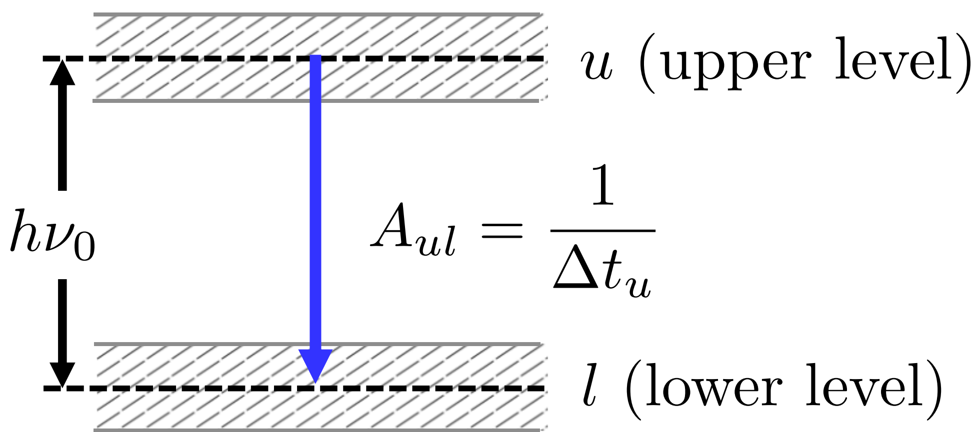
$$\phi(\nu) = \frac{1}{\Delta\nu_D\sqrt{\pi}}e^{-(\nu-\nu_0)^2/(\Delta\nu_D)^2} \qquad \text{where} \ \ \Delta\nu_D = \frac{\nu_0}{c}\sqrt{\frac{2kT}{m}}$$ is the Doppler width.

- In addition to thermal motions, there can be turbulent velocities associated with macroscopic velocity fields. The turbulent motions are accounted for by an effective Doppler width.

$$\Delta\nu_D = \frac{\nu_0}{c}\left(\frac{2kT}{m} + v_{\rm turb}^2\right)^{1/2}$$

where $v_{\rm turb}$ is a root-mean-square measure of the turbulent velocities. This assumes that the turbulent velocities also have a Gaussian distribution.

- **Natural Broadening**
  - A certain width to the atomic level is implied by ***the uncertainty principle***, namely, that the spread in energy ΔE and the duration Δt in the state must satisfy $\Delta E \Delta t \sim \hbar$ ($\hbar = h/2\pi$).



A_{ul} = decay rate
= decay probability per unit time, Einstein A coefficient.

$$\Delta E_u = \text{uncertainty in energy of } u$$

$$\Delta t_u = \text{the uncertainty in time of occupation of } u$$

$$\Delta \nu_u = \text{uncertainty in frequency}$$

$$= \Delta E_u / h = 1/(2\pi \Delta t_u) = A_{ul}/(2\pi)$$

$$\phi_\nu = \frac{\gamma/4\pi^2}{(\nu - \nu_0)^2 + (\gamma/4\pi)^2}$$

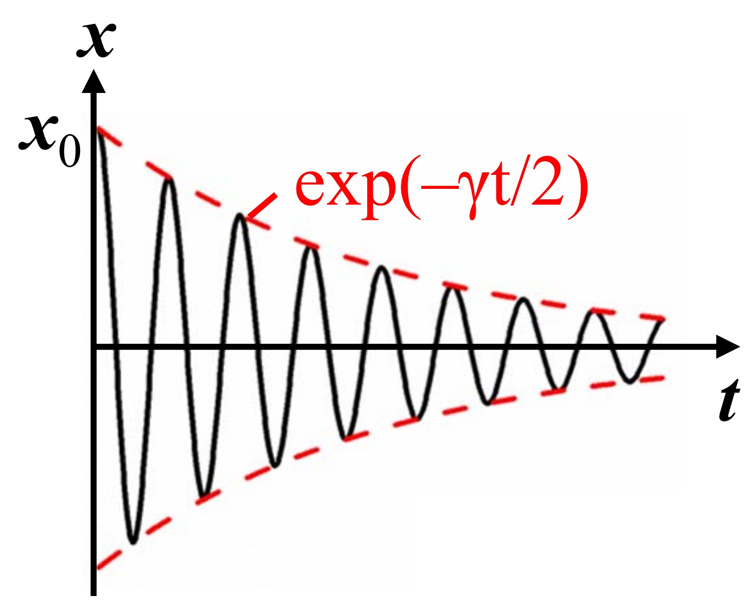
In terms of the line width $\Delta\nu_u$, the line profile can be rewritten as

$$\phi_\nu = \frac{1}{2\pi} \frac{\Delta\nu_u/2}{(\nu - \nu_0)^2 + (\Delta\nu_u/2)^2}$$

Therefore, γ is equivalent to the the Einstein A-coefficient., i.e., $\gamma = A_{ul}$.

The spontaneous decay of an atomic state n proceeds at a rate $\gamma = \sum_{n'} A_{nn'}$ where the sum is over all states n' of lower energy.

The electric field (the wave function of state n) is of the form $e^{-\gamma t/2}$ and then the energy decays proportional to $e^{-\gamma t}$. We then have an emitted spectrum determined by the decaying sinusoid type of electric field. The spectral profile is of the form, which is called a Lorentz (or natural, or Cauchy) profile,



$$\phi_\nu = \frac{\gamma/4\pi^2}{(\nu - \nu_0)^2 + (\gamma/4\pi)^2}$$

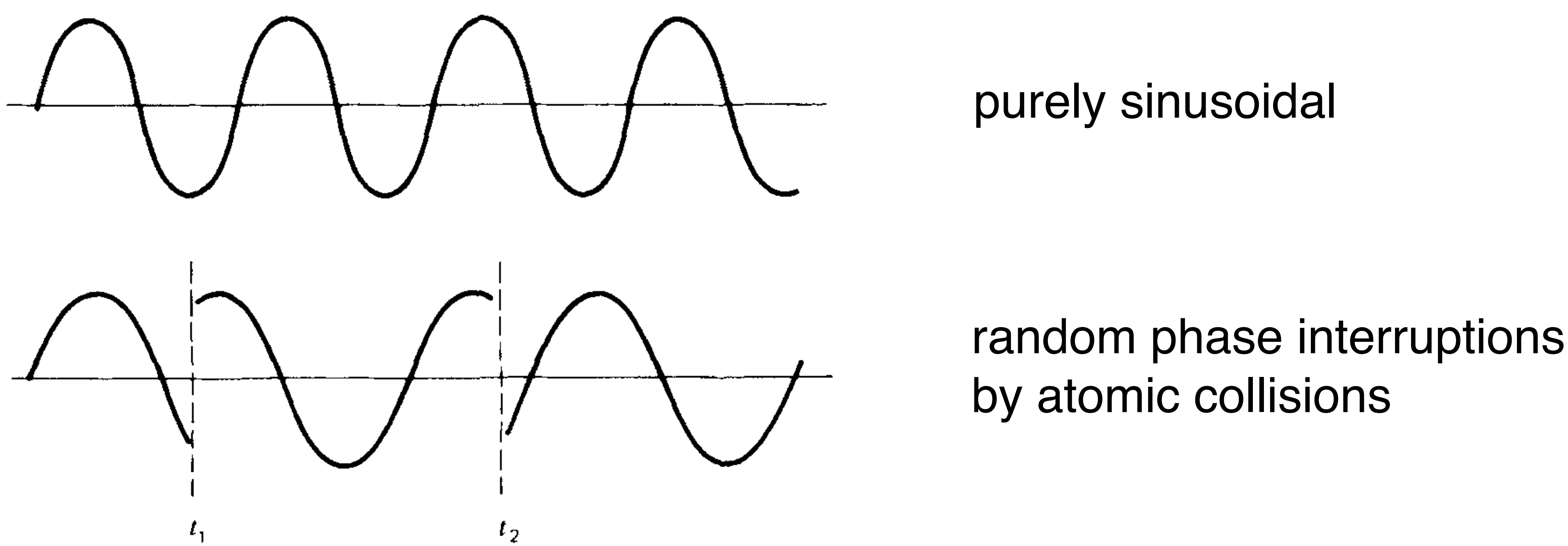
The above profile applies to cases in which only the upper state is broadened.

If both the upper and lower state are broadened, then the appropriate definition for γ is $\gamma = \gamma_u + \gamma_l$, where γ_u and γ_l are the widths of the upper and lower states involved in the transition. (Note that $\gamma_l = 0$ for the ground state.)

The longer the electron stays in the excited state, the narrower the line width so that metastable states can have very narrow lines (if the thermal Doppler broadening is not important).

- **Collisional Broadening**
  - The Lorentz profile applies even to certain types of collisional broadening mechanisms.
  - If the atom suffers collisions with other particles while it is emitting, the phase of the emitted radiation can be altered suddenly. If the phase changes completely randomly at the collision times, then information about the emitting frequencies is lost.
  - If the collisions occur with frequency $\nu_{\rm col}$, that is, each atom experiences $\nu_{\rm col}$ collisions per unit time on the average, then the profile is

$$\phi_\nu = \frac{\Gamma/4\pi^2}{(\nu - \nu_0)^2 + (\Gamma/4\pi)^2} \quad \text{where} \quad \Gamma = \gamma + 2\nu_{\rm col}$$



For derivation of the above formula, see Problem 10.7 of Rybiki & Lightman and Chapter 11 of Atomic Spectroscopy and Radiative Processes [Degl'Innocenti].

- **Voigt profile : Combined Doppler and Lorentz Profiles**
  - Atoms shows both a Lorentz profile plus the Doppler effect. In this case, we can write the profile as an average of the Lorentz profile over the various velocity states of the atom:

$$\phi(\nu) = \frac{\Gamma}{4\pi^2} \int_{-\infty}^{\infty} \frac{(m/2\pi kT)^{1/2} \exp\left(-mv_z^2/2kT\right)}{\left(\nu - \nu_0(1 + v_z/c)\right)^2 + (\Gamma/4\pi)^2} dv_z$$

  - The profile can be written using the Voigt function.

$$\phi(\nu) = \frac{1}{\Delta\nu_D \sqrt{\pi}} H(a,u)$$

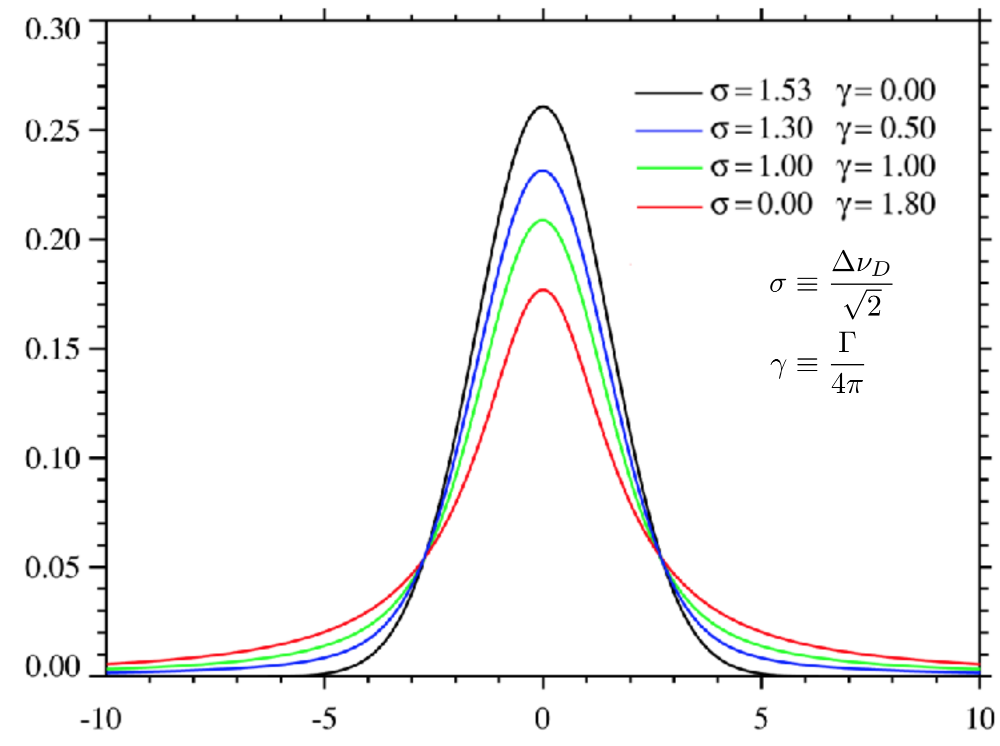
$$H(a,u) \equiv \int_{-\infty}^{\infty} \frac{e^{-y^2} dy}{a^2 + (u-y)^2}$$

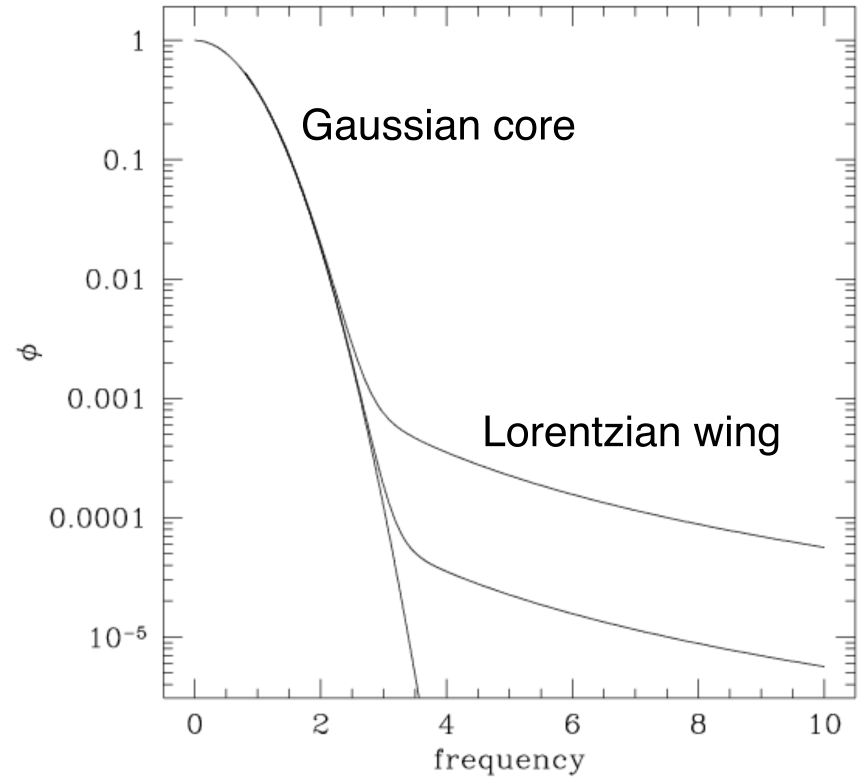
$$a \equiv \frac{\Gamma}{4\pi\Delta\nu_D}$$

$$u \equiv \frac{\nu - \nu_0}{\Delta\nu_D}$$

$$\Delta\nu_D = \frac{\nu_0}{c}\sqrt{\frac{2kT}{m}}$$

  - For small a, the "core" of the line is dominated by the Gaussian (Doppler) profile, whereas the "wings" are dominated by the Lorentz profile.





Diffuse Intergalactic Medium

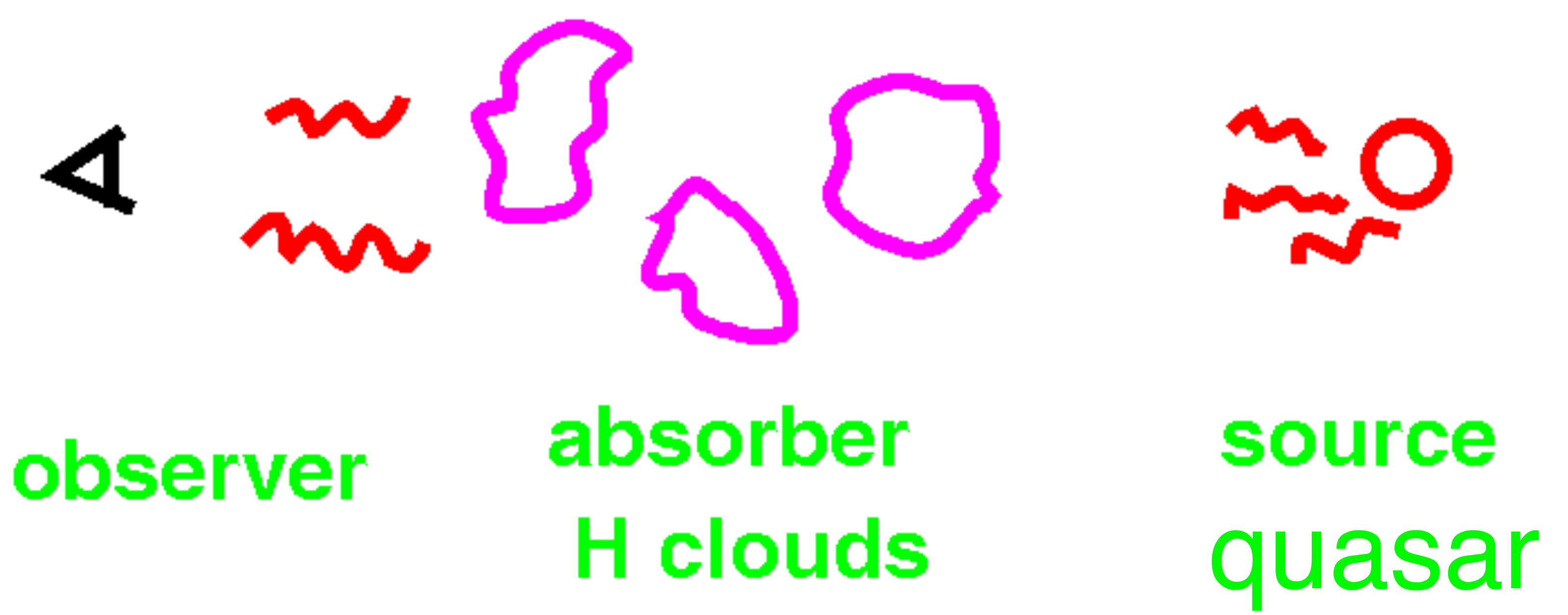
- The 2015 results from the Planck satellite tell us that the current mass density of baryonic matter ("dirty hydrogen") is

$$\langle \rho \rangle_{\mathrm{baryon}} = 4.2 \times 10^{-31} \mathrm{g\ cm}^{-3} \quad \rightarrow \quad \langle n \rangle_{\mathrm{baryon}} = 2.5 \times 10^{-7} \mathrm{cm}^{-3}$$

- The amount of baryonic matter in gravitationally bound systems, including stars, the Interstellar Medium (ISM), and the Circumgalactic Medium (CGM), provides only ~15% of this mean cosmic baryon density.
- The remainder is provided by a very tenuous intergalactic medium (IGM).

Absorption Lines as a Probe of the IGM

- Every parcel of gas along the line of sight to a distant quasar will selectively absorb certain wavelengths of continuum light of the quasar due to the presence of the various chemical elements in the gas.
- Through the analysis of these quasar absorption lines we can study the spatial distributions, motions, chemical enrichment, and ionization histories of gaseous structures from redshift $z \sim 6$ until the present.
- This structure includes the gas in galaxies of all morphological types as well as the diffuse gas in the IGM.



Lyα Forest

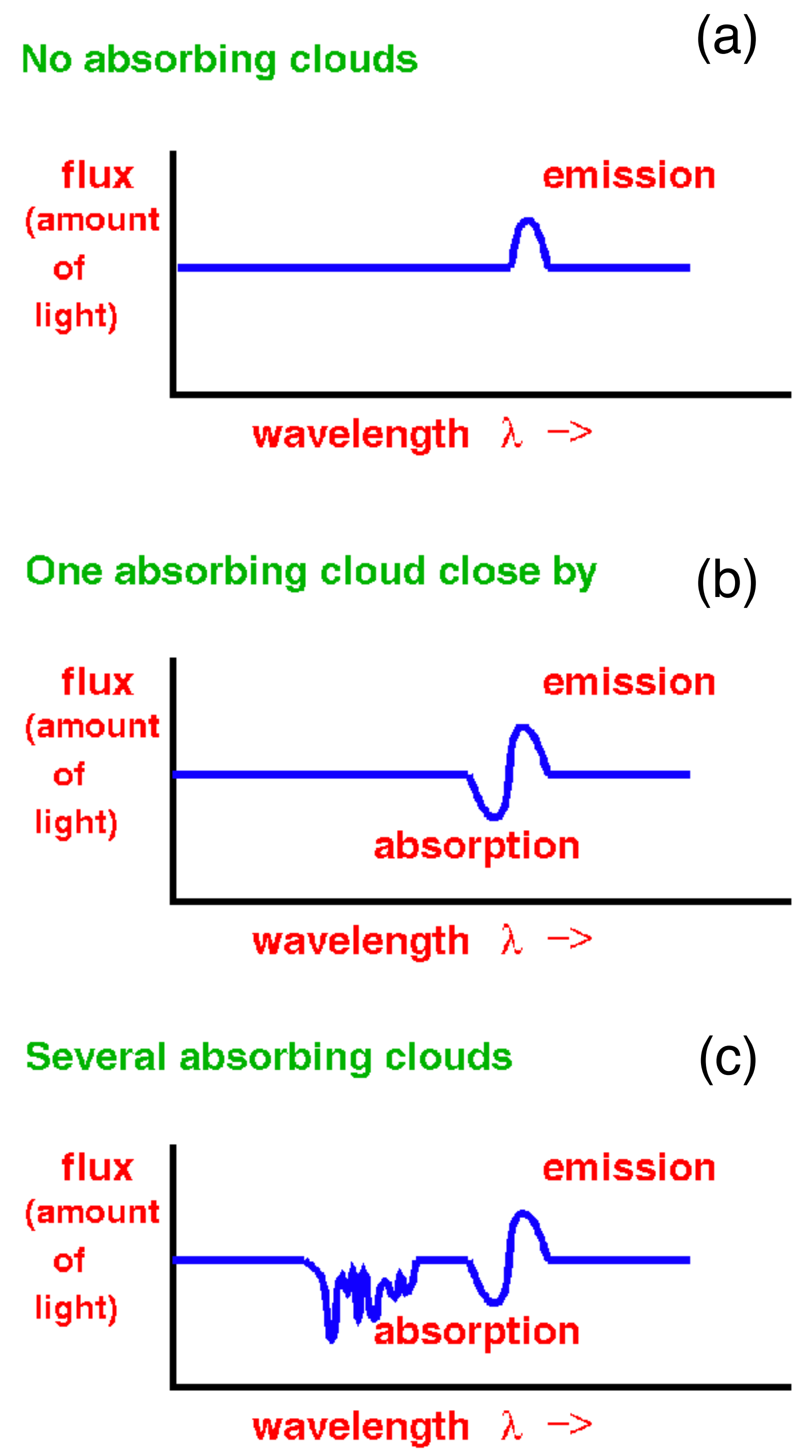
- Figure (a) shows a cartoon of how a quasar spectrum might look like if there were no intervening neutral hydrogen between the quasar and us.
  - The quasar continuum is relatively flat. Broad emission features are produced by the quasar itself (near the black hole and its accretion disk).

- In some cases, gas near the quasar central engine also produces “intrinsic” absorption lines, most notably Lyα, and relatively high ionization metal transitions such as C IV, N V, and O VI.

- However, the vast majority of absorption lines in a typical quasar spectrum are “intervening”, produced by gas unrelated to the quasar that is located along the line of sight between the quasar and the Earth.

- Its wavelength is stretched by the expansion of the Universe from what it was initially at the quasar, and, if it had continued to travel to us, it would have been stretched some more from the 1216Å wavelength it had at the absorber.

- The Lyα forest was first discovered by Roger Lynds in 1971.



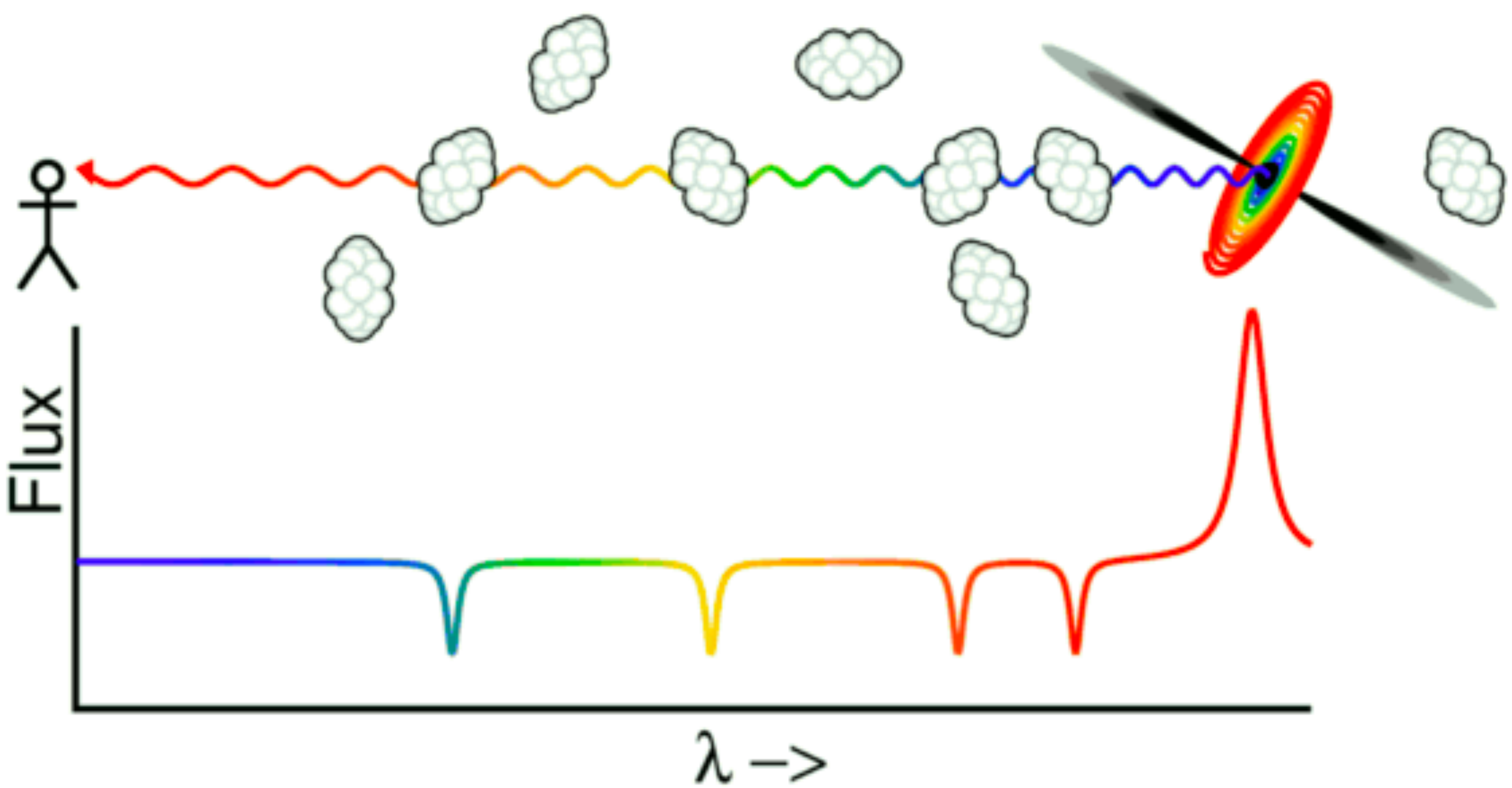
- The cartoon below shows a quasar with it Lyα emission line redshifted from the UV into the red, and the Lyα absorption lines from four intervening clouds appearing as orange, yellow and green-blue.
- Each structure will produce an absorption line in the quasar spectrum at a wavelength of $\lambda_{\rm obs} = \lambda_{\rm rest}(1 + z_{\rm gas})$, where $z_{\rm gas}$ is the redshift of the absorbing gas and $\lambda_{\rm rest} = 1216\mathring{A}$ is the rest wavelength of the Lyα transition. Since $z_{\rm gas} < z_{\rm quasar}$, the redshift of the quasar, these Lyα absorption lines form a "forest" at wavelengths blueward of the Lyα emission of the quasar.
- The region redward of the Lyα emission will be populated only by absorption through other chemical transitions with longer $\lambda_{Ly\alpha}$.

Definition of redshift:

$$z \equiv \frac{\lambda_{\rm obs} - \lambda_{\rm emit}}{\lambda_{\rm emit}}$$

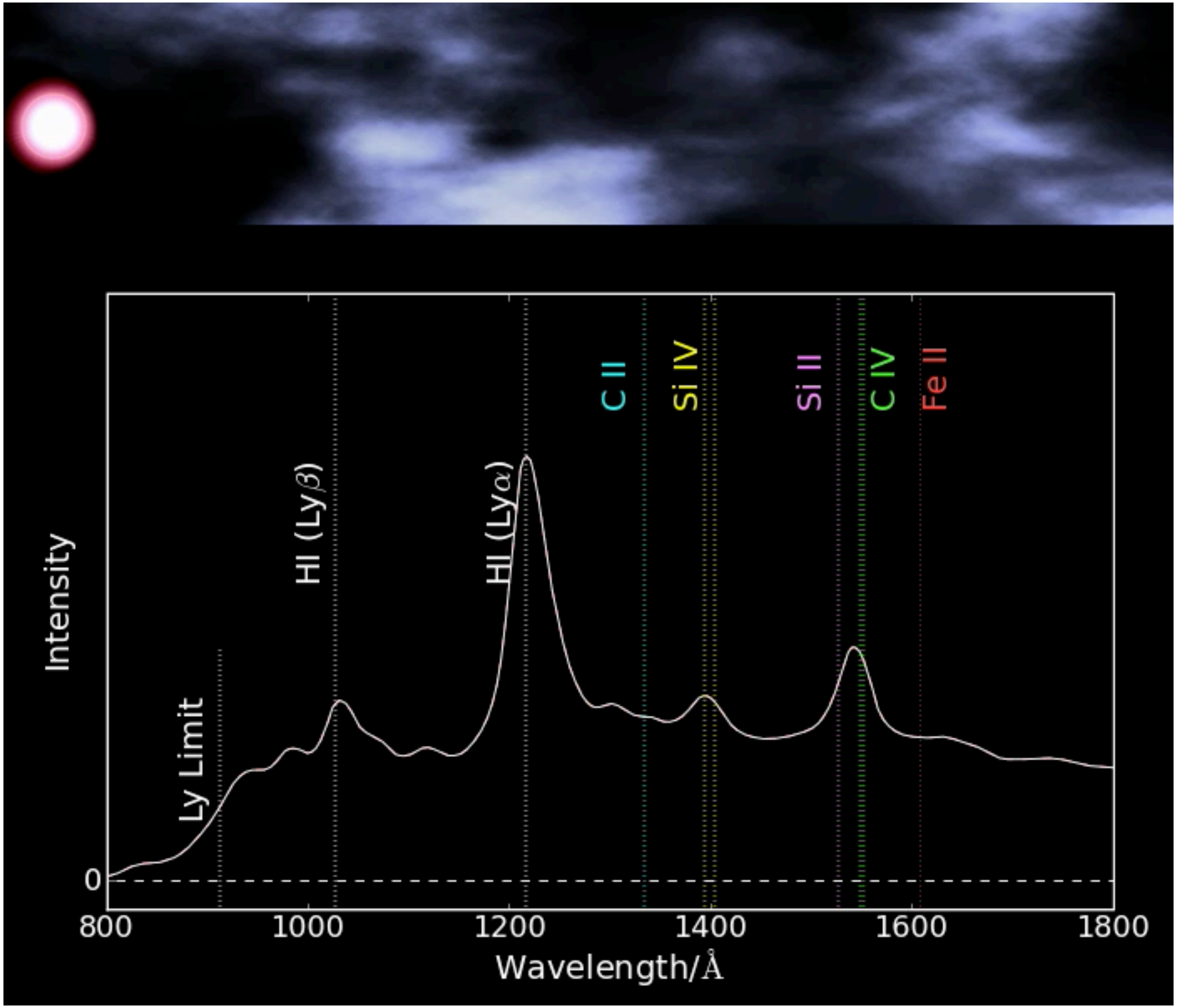
$$\lambda_{\rm obs} = \lambda_{\rm emit}(1 + z)$$

$$\lambda^{\rm gas}_{\rm obs} = \lambda_{\rm rest}(1 + z_{\rm gas})$$
$$\lambda^{\rm quasar}_{\rm obs} = \lambda_{\rm rest}(1 + z_{\rm quasar})$$
$$\therefore \lambda^{\rm gas}_{\rm obs} < \lambda^{\rm quasar}_{\rm obs}$$



© Edward Wright: http://www.astro.ucla.edu/~wright/Lyman-alpha-forest.html

A very nice visualization that shows how different systems absorb Lyman-alpha, made by Andrew Pontzen.
To see this movie, please download from http://www.cosmocrunch.co.uk/media/dla_credited.mov

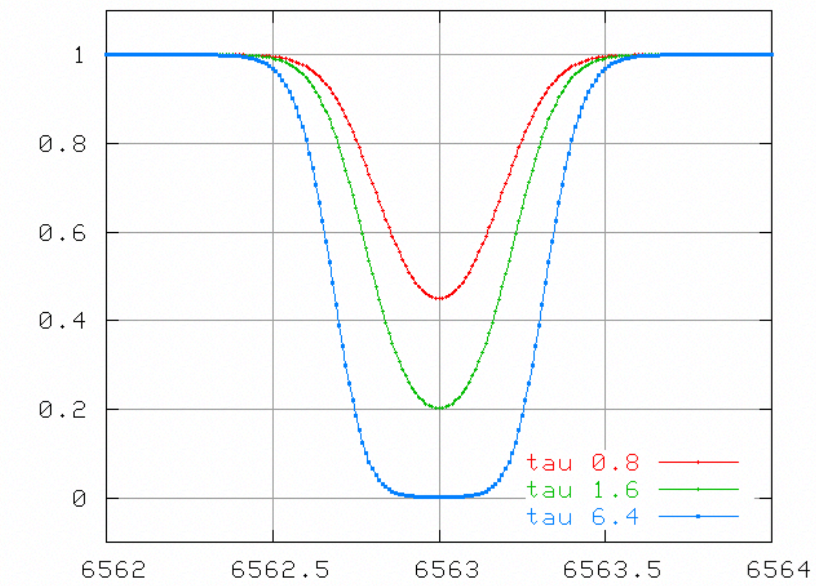


Lyα Absorption System

- A structure along the line of sight to the quasar can be described by its neutral Hydrogen column density N(H I), the produce of the density of the material and the path length along the line of sight through the gas.
- Historical classification

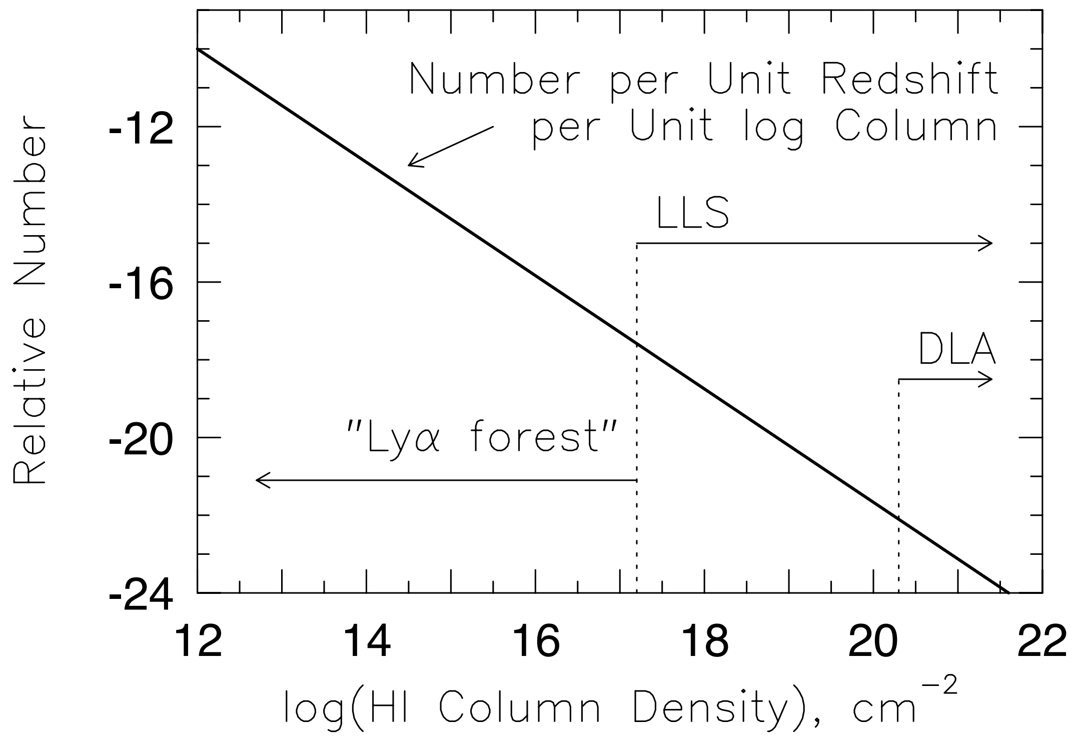
$$N(\mathrm{HI}) = n_H L$$

N (H I) < 10^{12} cm^{-2}	Currently not observable
10^{12} < N (H I) < $10^{17.2}$ cm^{-2}	Lyα forest
$10^{17.2}$ < N (H I) < $10^{20.3}$ cm^{-2}	Lyman limit systems
$10^{20.3}$ < N (H I)	Dampled Lyα systems

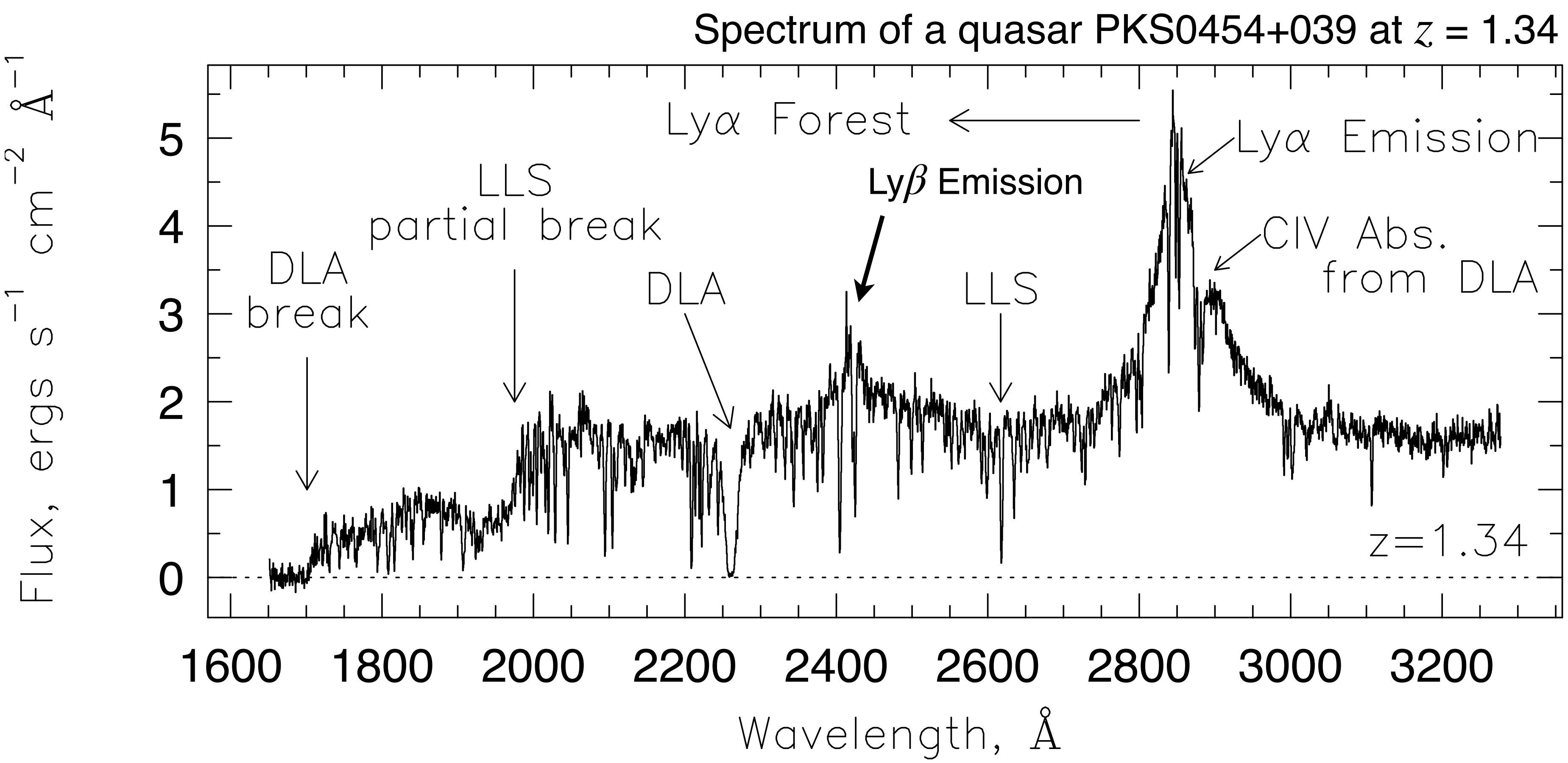


As N (HI) increases, the absorption line depth and width increase.

- Column density distribution
  - There are many more weak lines than strong lines.
  - The column density distribution roughly follows a power-law.



Typical spectrum of a quasar



$$\lambda_{\rm rest}^{\rm Ly\alpha} = 1216\text{Å}$$

$$\lambda_{\rm rest}^{\rm Ly\beta} = 1026\text{Å}$$

$$\lambda_{\rm rest}^{\rm Lyman\ break} = 912\text{Å}$$

- Typical spectrum of a quasar, showing the quasar continuum and emission lines, and the absorption lines produced by galaxies and IGM that lie between the quasar and the observer.
  - The Lyα forest, absorption produced by various intergalactic clouds, is apparent at wavelengths blueward of the Lyα emission line.
  - The two strongest absorbers, due to galaxies, are a damped Lyα absorber at z = 0.86 ($(1+0.86)\times 1216 = 2262\text{Å}$) and a Lyman limit system at z = 1.15 ($(1+1.15)\times 1216 = 2614\text{Å}$).
  - The damped Lyα absorber produces a Lyman limit break at ~ 1700Å ($(1+0.86)\times 912 = 1696\text{Å}$).
  - The Lyman limit system a partial Lyman limit break at ~ 1960Å ($(1+1.15)\times 912 = 1961\text{Å}$) since the neutral Hydrogen column density is not large enough for it to absorb all ionizing photons.