Interstellar Medium (ISM)

Lecture 16
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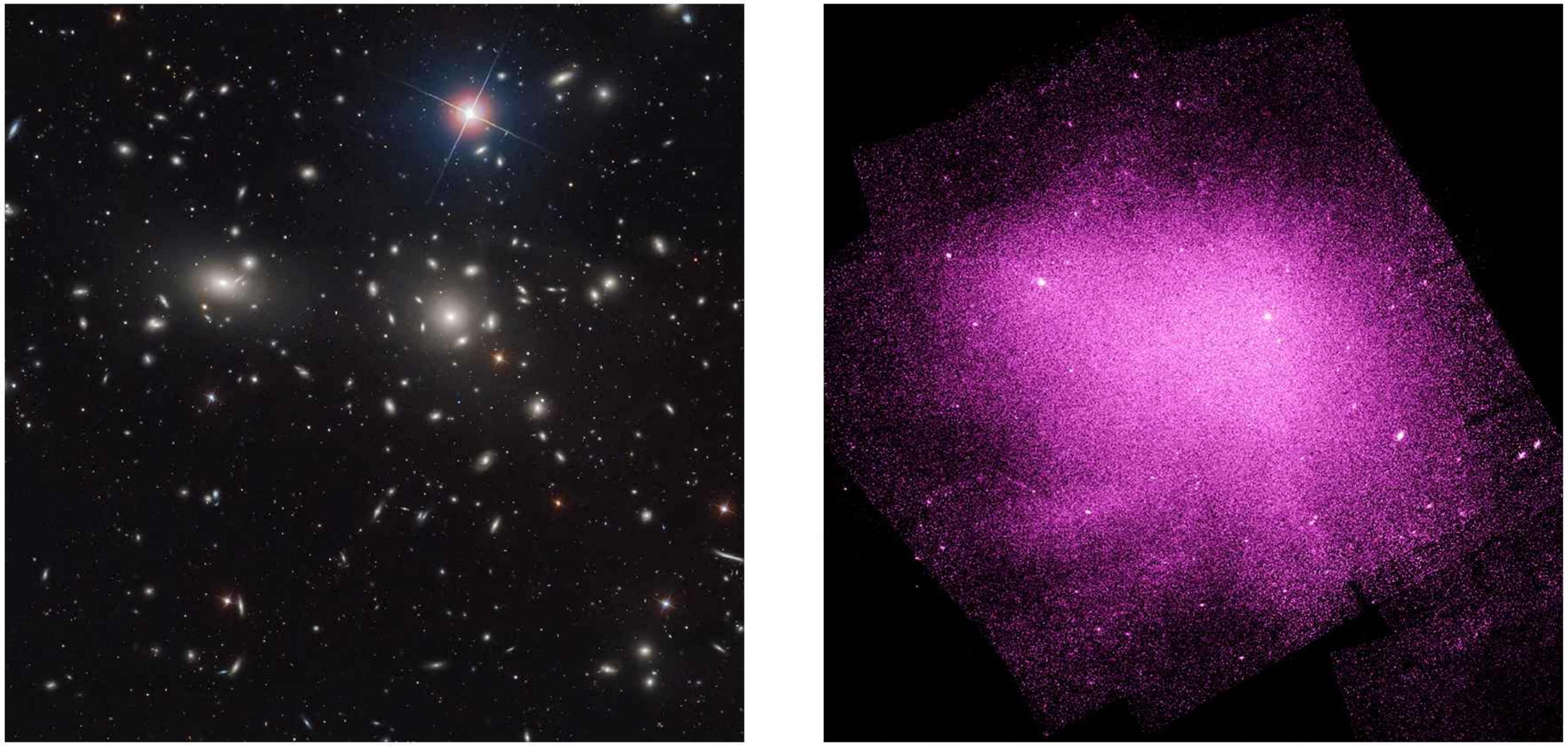
Intracluster Medium (ICM)

- ***Coma Cluster (Abell 1656)***
  - The Coma Cluster is a rich cluster located at a distance ~ 100 Mpc from our Galaxy (redshift z ~ 0.0231); at this distance, 1 arcsec ~ 0.48 kpc (10^8 AU).
  - About 1000 galaxies in the Coma Cluster have spectroscopic redshifts; taking into account of all the faint dwarf galaxies, there may be 10,000 or more galaxies in the Coma Cluster.



(a) Optical (b) X-ray

The Coma Cluster (a) at optical wavelengths [SDSS], and (b) at X-ray wavelengths [Chandra]. Two yellowish, brightest galaxies are NGC 4889 (left) and NGC 4874 (right), which are separated by 7 arcmin, corresponding to 200 kpc. [Fig 8.3, Ryden]

• *Dark Matter in the Coma Cluster*

- **Virial Mass**:

- In the central regions of the Coma Cluster, the line-of-sight velocity dispersion is found to be:

$$\sigma_v \approx 800\,\mathrm{km\,s^{-1}}$$

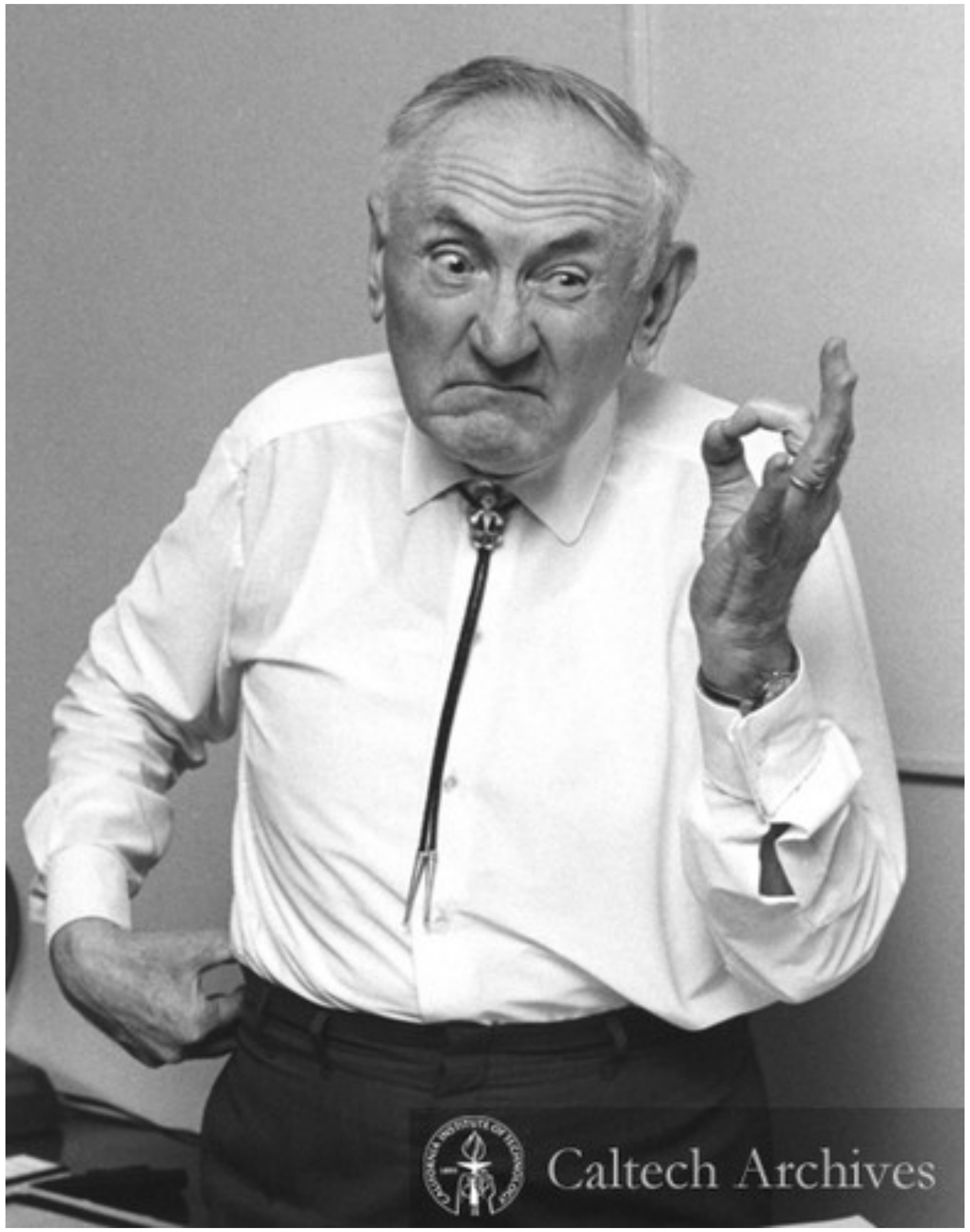
- The half-light radius, which is measured from the center of the two brightest cluster galaxies, that contains half the total flux of the galaxies in the cluster is:

$$r_\mathrm{h} \approx 52\,\mathrm{arcmin} \quad \xrightarrow{100\,\mathrm{Mpc} \times \tan(52'/60' \times \pi/180^\circ)} \quad r_\mathrm{h} \approx 1.5\,\mathrm{Mpc}$$

- Using the virial theorem, we can make a mass estimate for the Coma Cluster:

$$M_\mathrm{vir} = 5\frac{\sigma_v^2 r_\mathrm{h}}{G} \approx 2.7 \times 10^{48}\,\mathrm{g} \approx 1.4 \times 10^{15}\,M_\odot$$

In deriving the above virial equation, a constant density was assumed. But, the equation is correct within a factor of unity.



Fritz Zwicky (1898-1974)
Photo by Floyd Clark (1971)

1922 - Jacobus Kapteyn discovered evidence for galactic rotation and suggested the existence of dark matter using stellar velocities.
1933 - Fritz Zwicky applied the virial theorem to the Coma Cluster and obtained evidence of unseen mass.

- **Stellar Mass**:
  - Integrating over the luminosity function of the Coma galaxies, the total luminosity of all the stars in all the galaxies in the Coma Cluster is found to be:

$$L_{\mathrm{B}}^{\mathrm{Coma}} \approx 8 \times 10^{12}\, L_{\mathrm{B},\odot} \sim 400 L_{\mathrm{B}}^{\mathrm{Milky\ Way}}$$ in B-band

  - Most of the visible light from the Coma Cluster comes from elliptical galaxies, whose stellar populations have a ***mass-to-light ratio*** $\gamma_\star \approx 5 M_\odot / L_{\mathrm{B},\odot}$. This yields a total stellar mass for the Coma Cluster:

$$M_\star = \gamma_\star L_{\mathrm{B}} \approx (5 M_\odot / L_{\mathrm{B},\odot}) \left(8 \times 10^{12} L_{\mathrm{B},\odot}\right) \approx 4 \times 10^{13} M_\odot$$

- Thus, the total virial mass of the Coma Cluster is more than 30 times the mass of the stars that it contains:

$$\therefore M_{\mathrm{vir}} \approx 35 M_\star \ (\approx 1.4 \times 10^{15}\, M_\odot)$$

This discrepancy led Fritz Zwicky (1933) to deduce the existence of "dark matter."

  - Only eight redshifts of galaxies in the Coma Cluster were known at the time. However, it was enough for Zwicky to realize that the velocity dispersion was far too large for Coma to remain bound if stars were the only matter present.

Hot Diffuse Intracluster Medium

- Hot Diffuse ICM
  - Stars provide a minority of the baryons in a rich cluster like the Coma. As seen in X-ray, the Coma Cluster is revealed as containing a hot diffuse intracluster medium.
  - The X-ray spectrum has the characteristic shape of thermal bremsstrahlung, or free-free emission, with $kT \sim 10$ keV in the 2-20 keV range.
  - **Cooling Time Scale**:
    - For fully ionized hydrogen, the free-free emissivity is

$$j_{\nu,\mathrm{ff}} = C_{\mathrm{ff}} \left(\frac{m_e c^2}{kT}\right)^{1/2} n_e n_i e^{-h\nu/kT} g_{\nu,\mathrm{ff}}$$

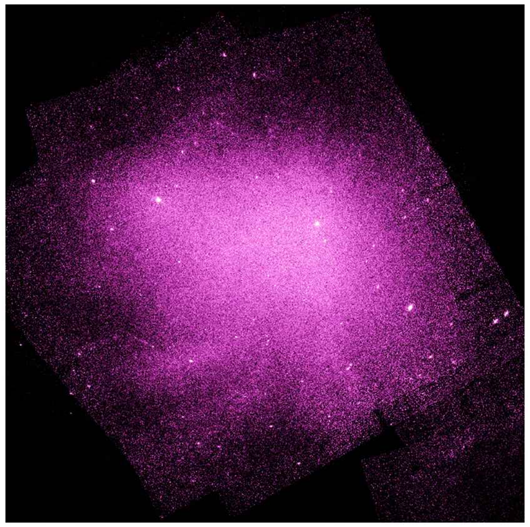
Here, $C_{\mathrm{ff}} \equiv \frac{8}{3}\left(\frac{2\pi}{3}\right)^{1/2} \frac{e^6}{m_e^2 c^4} = 7.070 \times 10^{-44}\,\mathrm{erg\,cm^3\,s^{-1}\,Hz^{-1}\,sr^{-1}}$

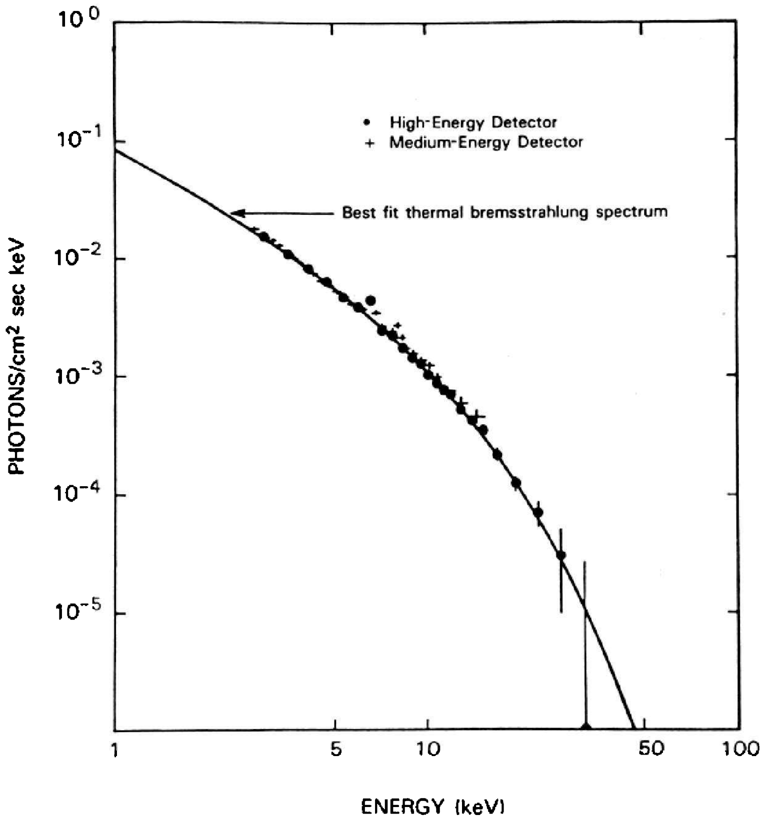
Gaunt factor: $g_{\nu,\mathrm{ff}} \approx \frac{\sqrt{3}}{\pi} \ln\left(\frac{4}{e^\gamma}\frac{kT}{h\nu}\right) \approx 0.551 \ln\left(2.25\frac{kT}{h\nu}\right)$

$$\text{for } kT \gg 13.6\,\mathrm{eV} \text{ and } h\nu < kT$$

(Note that the Gaunt factor has only a mild frequency dependence.)

Euler-Mascheroni constant: $\gamma = \lim_{n\to\infty}\left(-\ln n + \sum_{k=1}^{n}\frac{1}{k}\right) \approx 0.5772$





(top) X-ray image, (bottom) X-ray spectrum of the Coma Cluster [Henriksen & Mushotzky 1986]

$kT_{\mathrm{gas}} \sim 7.3\,\mathrm{keV}$ for Coma

- The ***power per unit volume*** produced by thermal bremsstrahlung is:

$$\mathcal{L}_{\rm ff} = 4\pi \int j_{\nu,\rm ff} d\nu = 4\pi C_{\rm ff} \frac{m_e c^2}{h} \left(\frac{kT}{m_e c^2}\right)^{1/2} n_e n_i \overline{g_{\rm ff}}$$

Here, $4\pi C_{\rm ff} \dfrac{m_e c^2}{h} = 1.10 \times 10^{-22}\,{\rm erg\,cm^3\,s^{-1}}$

Frequency-averaged Gaunt factor: $\overline{g_{\rm ff}} \approx 1.15$ $({\rm near}\ T \sim 10^8\,{\rm K})$

$$\longrightarrow \quad \boxed{\mathcal{L}_{\rm ff} \approx 1.77 \times 10^{-29}\,{\rm erg\,cm^{-3}} \left(\frac{kT}{10\,{\rm keV}}\right)^{1/2} \left(\frac{n_{\rm H}}{10^{-3}\,{\rm cm^{-3}}}\right)^2}$$

for a fully ionized hydrogen gas

- The ***mass density*** of the ICM is:

$$\rho_{\rm ICM} = m_{\rm H} n_{\rm H} \approx 1.67 \times 10^{-27}\,{\rm g\,cm^{-3}} \left(\frac{n_{\rm H}}{10^{-3}\,{\rm cm^{-3}}}\right)$$

- ***Mass-to-light ratio*** for the ICM (considering only the bremsstrahlung) is:

$$\gamma_{\rm ICM} = \frac{\rho_{\rm ICM}}{\mathcal{L}_{\rm ff}} \approx 180\,(M_\odot/L_\odot) \left(\frac{kT}{10\,{\rm keV}}\right)^{-1/2} \left(\frac{n_{\rm H}}{10^{-3}\,{\rm cm^{-3}}}\right)^{-1}$$

Note that $\gamma_{\rm ICM} \gg \gamma_\star$ $(\approx 5 M_\odot / L_{\rm B,\odot})$.
The bremsstrahlung is not an efficient cooling mechanism.

- The ***thermal energy density*** of the ICM is

$$\mathcal{E}_{\rm ff} = (2n_{\rm H})\frac{3}{2}kT \approx 4.81\times10^{-11}\,{\rm erg\,cm^{-3}}\left(\frac{n_{\rm H}}{10^{-3}\,{\rm cm^{-3}}}\right)\left(\frac{kT}{10\,{\rm keV}}\right)$$

- The ***cooling time*** for the ICM is then:

$$t_{\rm cool} = \mathcal{E}_{\rm ff}/\mathcal{L}_{\rm ff} \approx 86\,{\rm Gyr}\left(\frac{kT}{10\,{\rm keV}}\right)^{1/2}\left(\frac{n_{\rm H}}{10^{-3}\,{\rm cm^{-3}}}\right)^{-1}$$

The β-profile consists of a central core and a power-law outer part and is useful in the observational description of X-ray clusters.

- **Gas Mass**:
  - The ***gas density profile*** of clusters is often expressed with a β-profile:

$$\boxed{n_{\rm H} = n_0\left[1+\left(\frac{r}{r_c}\right)^2\right]^{-3\beta/2}}$$

$$\beta \approx 0.75$$
$$r_c \approx 0.3\,{\rm Mpc}$$
$$n_0 \approx 3\times10^{-3}\,{\rm cm^{-3}}$$

for the Coma Cluster

- The amount of gas within a radius r will then be: $n_{\rm H} \approx n_0\left(\frac{r}{r_c}\right)^{-3\beta}$

$$M_{\rm gas}(r) \approx \frac{4\pi}{3(1-\beta)}m_{\rm H}n_0r_c^3\left(\frac{r}{r_c}\right)^{3(1-\beta)} \quad \text{when } r \gg r_c \text{ and } \beta < 1$$

- We assume an abrupt cutoff at the virial radius ($r_{\rm vir} \approx 3\,{\rm Mpc}$). This gives a mass estimate:

$$M_{\rm gas}(r) \approx 2\times10^{14}M_\odot\left(\frac{r}{3\,{\rm Mpc}}\right)^{0.75} \longrightarrow M_{\rm gas} \approx 5M_\star \;\; (M_\star \approx 4\times10^{13}M_\odot)$$

- **Total Mass in the hydrostatic equilibrium**:
  - In X-rays, the central regions of the Coma Cluster look smooth. We thus expect that Coma, at least in its central regions, is ***a relaxed cluster, in hydrostatic equilibrium***.
  - Spherical objects in hydrostatic equilibrium obey the equation:

$$\boxed{\frac{dP}{dr} = -\frac{GM(r)\rho_{\rm gas}(r)}{r^2}}$$

$M(r)$ is the mass of everything inside a radius r, including gas, stars, and dark mass.
$\rho_{\rm gas}(r)$ is the gas density at a radius r.

  - The ideal gas law for an ionized hydrogen gas is:

$$\boxed{P = nkT = \frac{2\rho_{\rm gas}(r)}{m_{\rm H}} kT_{\rm gas}(r)}$$

$$n = n_e + n_p$$

$$\rho = n_e m_e + n_p n_{\rm H} \approx \frac{n_e + n_p}{2} m_{\rm H}$$

  - Combining the equation of hydrostatic equilibrium with the ideal gas law, we find the total mass $M(r)$ contained within a radius r :

$$\boxed{\begin{aligned} M(r) &= -\frac{r^2}{G\rho_{\rm gas}(r)} \frac{2k}{m_{\rm H}} \frac{d(\rho_{\rm gas} T_{\rm gas})}{dr} \\ &= \frac{2rkT_{\rm gas}}{Gm_{\rm H}} \left(-\frac{d\ln\rho_{\rm gas}}{d\ln r} - \frac{d\ln T_{\rm gas}}{d\ln r} \right) \end{aligned}}$$

- We assume that the intracluster gas is isothermal ($T_{\rm gas}$ = constant) as an first order approximation and use a beta profile for the gas density:

$$M(r) = \frac{2rkT_{\rm gas}}{Gm_{\rm H}}\left(-\frac{d\ln\rho_{\rm gas}}{d\ln r}\right)$$

$$n_{\rm H}(r) \approx n_0\,(r/r_c)^{-3\beta}$$

$$M(r) \approx \frac{6\beta kT_{\rm gas}}{Gm_{\rm H}}r \approx \frac{4.5kT_{\rm gas}}{Gm_{\rm H}}r \quad (\text{for r} \gg \text{r}_{\rm c} \text{ and } \beta \approx 0.75)$$

If a cluster is in hydrostatic equilibrium, and has a cutoff at the virial radius, then its total mass (including everything) is:

$$M_{\rm HE} \approx \frac{4.5kT_{\rm gas}}{Gm_{\rm H}}r_{\rm vir} \approx 3.0\times10^{15}\,M_\odot\left(\frac{kT_{\rm gas}}{10\,{\rm keV}}\right)\left(\frac{r_{\rm vir}}{3\,{\rm Mpc}}\right)$$

(Here, HE stands for the hydrostatic equilibrium.)

- For the Coma Cluster, with $kT_{\rm gas} = 7.3\,{\rm keV}$ and $r_{\rm vir} \approx 3\,{\rm Mpc}$, the total mass is:

$$M_{\rm HE} \approx 2.2\times10^{15}\,M_\odot$$

This is consistent with the virial estimate of $M_{\rm vir} \approx 1.4\times10^{15}\,M_\odot$.

In summary,

total mass	$M_{\rm tot} \approx 2\times10^{15}\,M_\odot$
stellar mass	$M_\star \approx 0.02M_{\rm tot}$
gas mass	$M_{\rm gas} \approx 0.1M_{\rm tot}$

Diffuse Intergalactic Medium - Introduction to Cosmology

- Cosmic baryon density:
  - The 2015 results from the Planck satellite tell us that the current baryonic mass density and number density of baryonic matter are, respectively:

$$\bar{\rho}_{\mathrm{bary},0} = (4.190 \pm 0.026) \times 10^{-31}\,\mathrm{g\,cm^{-3}} \approx 0.048\rho_{\mathrm{c},0}$$
$$\bar{n}_{\mathrm{bary},0} = \bar{\rho}_{\mathrm{bary},0}/m_{\mathrm{H}} = 2.5 \times 10^{-7}\,\mathrm{cm^{-3}}$$

ρ_c = critical density: the mean density of matter when the overall geometry of the universe is flat (Euclidean).

  - The amount of baryonic matter in gravitationally bound systems, including stars, the ISM, the CGM, and the ICM, provides only ~ 15% of this mean cosmic baryon density.

  - **The remainder is provided by a very tenuous intergalactic medium (IGM).**

- The Universe expands.
  - As a consequence, the mean baryon density decreases with time, since the baryon number is conserved.
  - The expansion of the universe is homogeneous and isotropic on large scales, and thus can be described by a simple scale factor $a(t)$, which is customarily normalized so that $a(t_0) = 1$ at the present time $t_0 = 13.8\,\mathrm{Gyr}$ after the Big Bang.

$$a(t_0) = 1 \quad \text{at the present time} \quad t_0$$

  - The expansion of the universe is frequently described in terms of the Hubble parameter:

$$H(t) \equiv \frac{\dot{a}}{a}$$

Recall the **Hubble-Lemaître law**: $V = H_0 R$

In 2018, IAU voted to recommend renaming the Hubble law as the Hubble-Lemaître law.

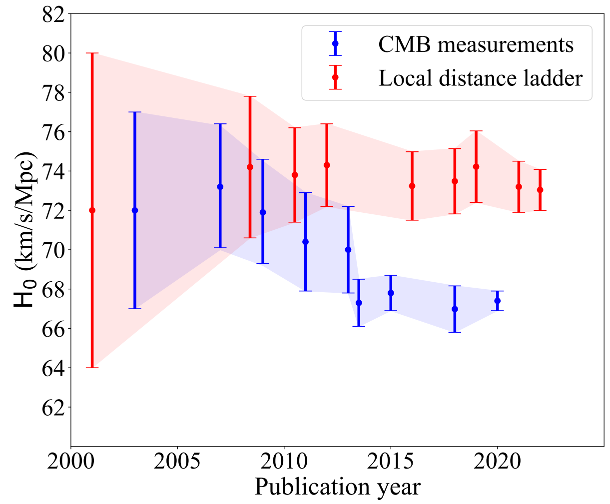
The ***Hubble parameter*** evaluated at the present time is the ***Hubble constant***:

$$H_0 = H(t_0) = 67.74 \pm 0.46\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$ from the CMB measurement

The Hubble time and the Hubble distance are, respectively,

$$H_0^{-1} = 14.43 \pm 0.10\,\mathrm{Gyr} \qquad cH_0^{-1} = 4425 \pm 30\,\mathrm{Mpc}$$

Hubble tension:



from Perivolaropoulos & Skara (2022)

- **Light emitted by a quasar at time t_e , with a redshift z_e is observed by us at $t_0\,(>t_e)$.** The scale factor at the time the light was emitted is smaller than that at the present time:

$$a(t_e) < a(t_0) = 1$$

- The scale factor $a(t_e)$ can be expressed in terms of the redshift of the quasar:

$$a(t_e) = (1 + z_e)^{-1}$$

$$\boxed{a(z) = \frac{1}{(1+z)}}$$

Recall that $z \equiv \dfrac{\nu - \nu_0}{\nu_0} = \dfrac{v}{c} \quad \Rightarrow \quad \dfrac{\nu}{\nu_0} = 1 + z$ and $\dfrac{\lambda}{\lambda_0} = \dfrac{1}{1+z}$

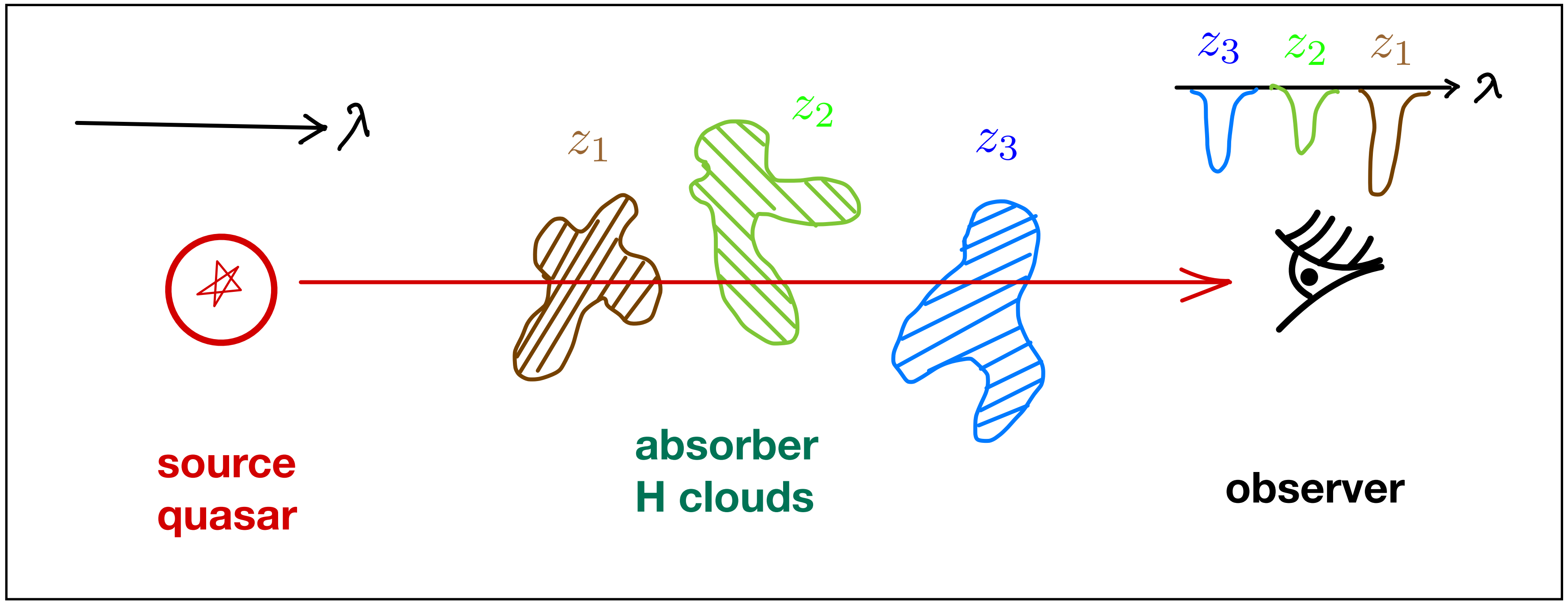
- Between the time of emission and the time of observation ($t_e < t < t_0$), ***the number density of baryons and the frequency of the emitted photons have been decreasing***:

$$\bar{n}_{\rm bary}(t) = \bar{n}_{\rm bary,0}\, a(t)^{-3} = \bar{n}_{\rm bary,0}(1+z)^3$$

$$\nu(t) = \nu_0 a(t)^{-1} = \nu_0(1+z) \geq \nu_0$$

Absorption Lines as a Probe of the IGM

- The IGM is difficult to detect. A useful way of searching for intergalactic gas is to look for **absorption lines from the IGM along the line of sight to a distant quasar**.
  - Every parcel of gas along the line of sight to a distant quasar will selectively absorb certain wavelengths of continuum light of the quasar due to the presence of the various chemical elements in the gas.
  - Through the analysis of these quasar absorption lines we can study the spatial distributions, motions, chemical enrichment, and ionization histories of gaseous structures from redshift $z \sim 7$ until the present.
  - This structure includes **the gas in galaxies** of all morphological types as well as **the diffuse gas** in the IGM.



- The Lyα line (1216Å) would be a particularly useful probe as long as the medium isn't too highly ionized.

- Consider the nearby quasar 3C273, at a redshift $z_e = 0.158$.
  - If 3C 273 produces a Lyα photon with an energy $h\nu_e = 10.20\,\mathrm{eV}$, by the time it reaches us, it will be redshifted to the lower energy:

$(1 + z_e = 1.158)$

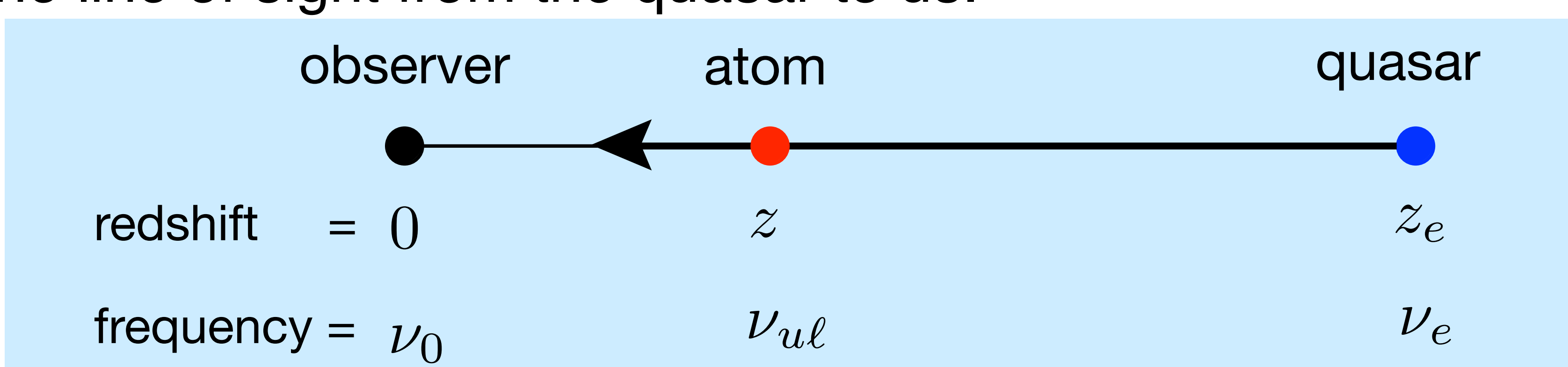
$$\boxed{h\nu_e = 10.20\,\mathrm{eV} \quad \rightarrow \quad h\nu_0 = h\nu_e a(t_e)/a(t_0) = 10.20\,\mathrm{eV}/1.158 = 8.81\,\mathrm{eV}}$$

  - If the continuum with an initial energy $h\nu_e = 10.20\,\mathrm{eV} \times 1.158$ at the redshift of 3C 272 will be redshifted to the lower energy:

$$\boxed{h\nu_e = 10.20\,\mathrm{eV} \times 1.158 = 11.81\,\mathrm{eV} \quad \rightarrow \quad h\nu_0 = h\nu_e/1.158 = 10.20\,\mathrm{eV}}$$

  This energy can be absorbed by neutral hydrogen in our own galaxy.

- In general, if a quasar is at a redshift z_e, photons with an initial energy in the range $h\nu_{u\ell} < h\nu_e < h\nu_{u\ell}(1 + z_e)$ can be absorbed by a transition with energy $h\nu_{u\ell}$ somewhere along the line of sight from the quasar to us.

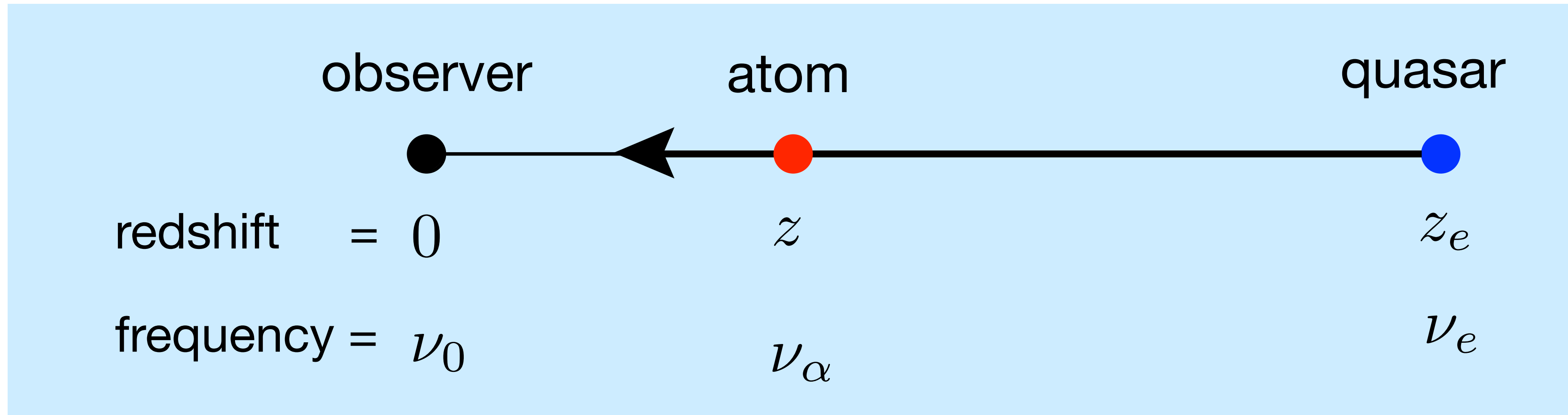


  - The redshift at which photons can be absorbed is:

$$\begin{aligned} \nu_e &= \nu_0(1 + z_e) \\ \nu_{u\ell} &= \nu_0(1 + z) \end{aligned} \quad \longrightarrow \quad \boxed{z = \frac{\nu_{u\ell}}{\nu_0} - 1 \quad \text{or} \quad z = \frac{\nu_{u\ell}}{\nu_e}(1 + z_e) - 1}$$

Gunn-Peterson Effect

Jim Gunn & Bruce Peterson (1965)



- In order to understand the Gunn-Peterson effect, let's consider radiation observed at some frequency ν that was initially lying blueward of Lyα by a quasar at redshift z_e. The emitted photons pass through the local Lyα resonance as they propagates towards us through a smoothly distributed sea of neutral hydrogen atoms, and are scattered off the line-of-sight with a cross-section of

$$\sigma_\nu = \frac{\pi e^2}{m_e c} f_\alpha \phi_\nu = \chi_0 \phi_\nu \quad \left(\chi_0 \equiv \frac{\pi e^2}{m_e c} f_\alpha \right),$$

where ϕ_ν is the Voigt profile of the Lyα line, normalized so that $\int \phi_\nu d\nu = 1.$

- The total optical depth for resonant scattering at the observed frequency ν is given by the line integral of this cross-section times the neutral hydrogen density $n_{\rm HI}$ in the ground state,

$$\boxed{\tau_\nu^{\rm GP} = \int_0^s \sigma_{\nu'} n_{\rm HI} dl = \int_0^s \chi_0 \phi_{\nu'} n_{\rm HI} dl.}$$

Note $\nu' = \nu(1+z)$ in the integral

- In an expanding Universe, ***the integral should be performed along the proper distance.*** We, therefore, want to use the redshift z instead of the proper length l travelled by light. Then, the optical depth becomes

$$\boxed{\tau_\nu^{\rm GP} = \chi_0 \int_0^{z_e} \phi_{\nu'}\, n_{\rm HI} \frac{dl}{dz} dz}$$

Here, $\boxed{\nu' = \nu(1+z)}$ in the integral.

proper distance = the distance between two points of space at a constant cosmological time.

comoving distance = the distance expressed in **comoving coordinates**. The comoving distance between two comoving points of space remains fixed at all times.

- The expansion of the Universe is homogeneous and isotropic on large scale, and thus can be described by a simple scale factor $a(z)$.
- The scale factor today, $a(t_0) = 1$, is greater that the scale factor at the redshift z, $a(z) = 1/(1+z)$. We obtain the proper length element in terms of the redshift.

$$\dot{a} = \frac{da}{dt} \qquad \frac{da}{a} = \frac{dz}{1+z}$$

$$dl = c\,dt = c\frac{da}{\dot{a}} = c\frac{da}{Ha} = c\frac{dz}{H(1+z)} \rightarrow \boxed{\frac{dl}{dz} = \frac{c}{H(1+z)}}$$

Notice that the scale factor transforms like wavelength:

$$\lambda_0 = \lambda_z(1+z)$$
$$\lambda_z = \lambda_0/(1+z)$$

- Then, the ***Gunn-Peterson optical depth*** is given by $$\boxed{\tau_\nu^{\rm GP} = \chi_0 \int_0^{z_e} \phi_{\nu'}\, n_{\rm HI} \frac{c}{H} \frac{dz}{1+z}.}$$

- The thermal and natural broadening ($\Delta v_{\rm thermal} \sim 13~{\rm km~s^{-1}}$ for $T = 10^4 K$) is tiny compared to the "broadening" due to the Hubble expansion ($\Delta v = cz \sim 30{,}000~{\rm km~s^{-1}}$ for $z = 0.1$).

  Thus, we can treat **the Voigt function as being very strongly peaked at the Lyα frequency ν_α in the local comoving frame.** This resonance will occur at the redshift of z such that $\nu = \nu_\alpha/(1+z)$, i.e., at $z = \nu_\alpha/\nu - 1$.

- Then, ***in the local comoving frame at*** z, the frequency interval $d\nu'$ can be expressed by $d\nu'/\nu' = dz/(1+z)$. Finally, we obtain

$$\begin{aligned}
\tau_\nu^{\rm GP} &= \chi_0 \int_0^{z_e} \phi_{\nu'}\, n_{\rm HI} \frac{c}{H} \frac{d\nu'}{\nu'} \\
&\approx \chi_0 n_{\rm HI} \frac{c}{H} \int_0^{z_e} \phi_{\nu'} \frac{d\nu'}{\nu'} \\
&\approx \frac{\chi_0 c}{\nu_\alpha} \frac{n_{\rm HI}(z)}{H(z)}
\end{aligned}$$

Here, the density and Hubble parameter should be evaluated at the following z for the given ν.

$$z = \frac{\nu_\alpha}{\nu} - 1$$

At low redshifts ($z \approx 0$), this gives an optical depth $\tau_\nu^{\rm GP} = 15{,}200 \dfrac{n_{\rm HI,0}}{\overline{n}_{\rm bary,0}}$,

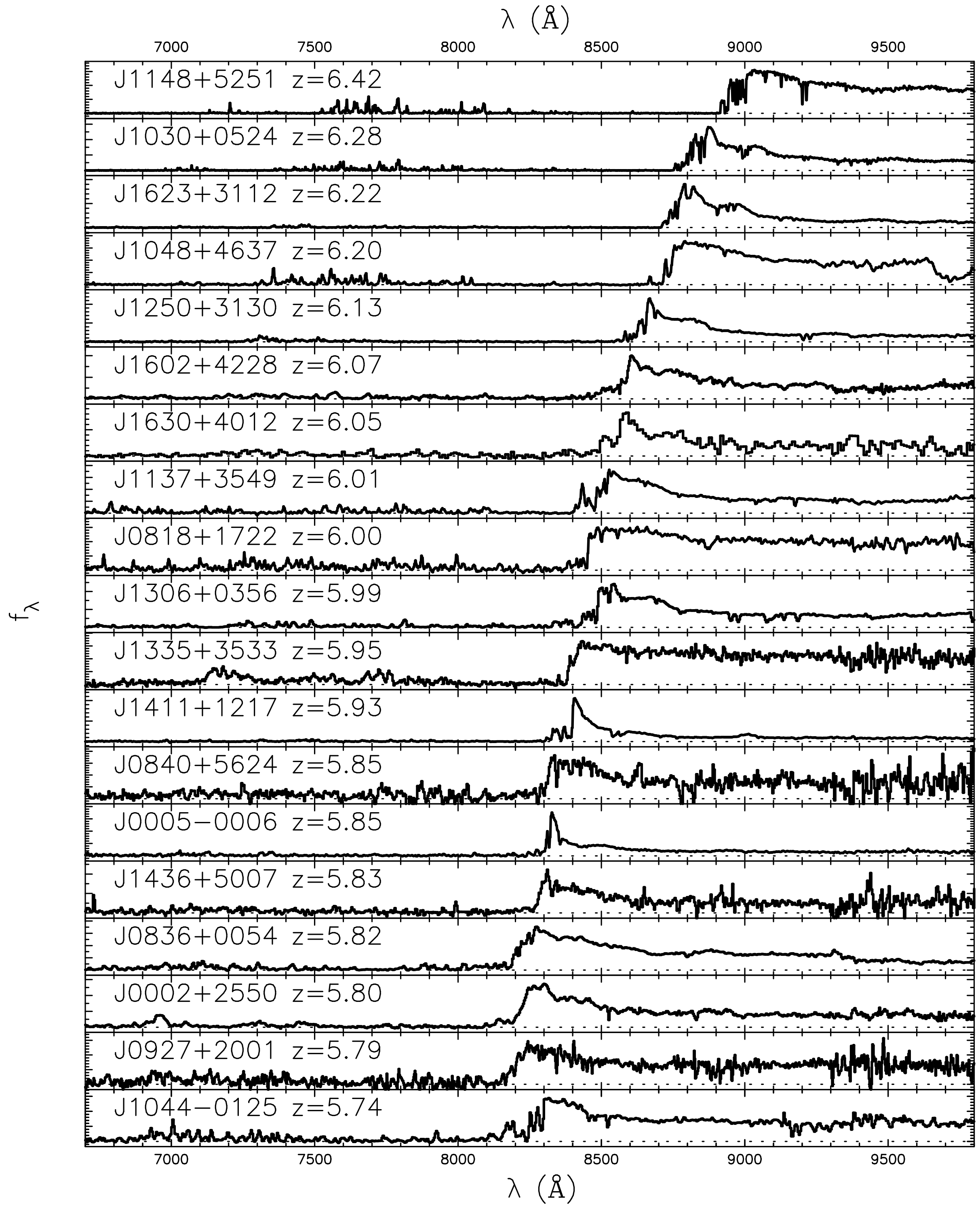
where the baryon number density at the present time is $\overline{n}_{\rm bary,0} = 2.5 \times 10^{-7}~{\rm cm^{-3}}$.

- Thus, ***even when the number density of neutral hydrogen atoms is one part in 10,000 of the baryon density, the optical depth is larger than one.***
- ***This result implies the spectrum of a low redshift quasar, for instance 3C 273 at $z = 0.158$, should be black between the observed wavelengths $\lambda = 1216$Å and $1216 \times 1.158 = 1408$Å and emitted wavelengths $\lambda = 1050$Å and $1050 \times 1.158 = 1216$Å.*** **This is called the Gunn-Peterson trough.**

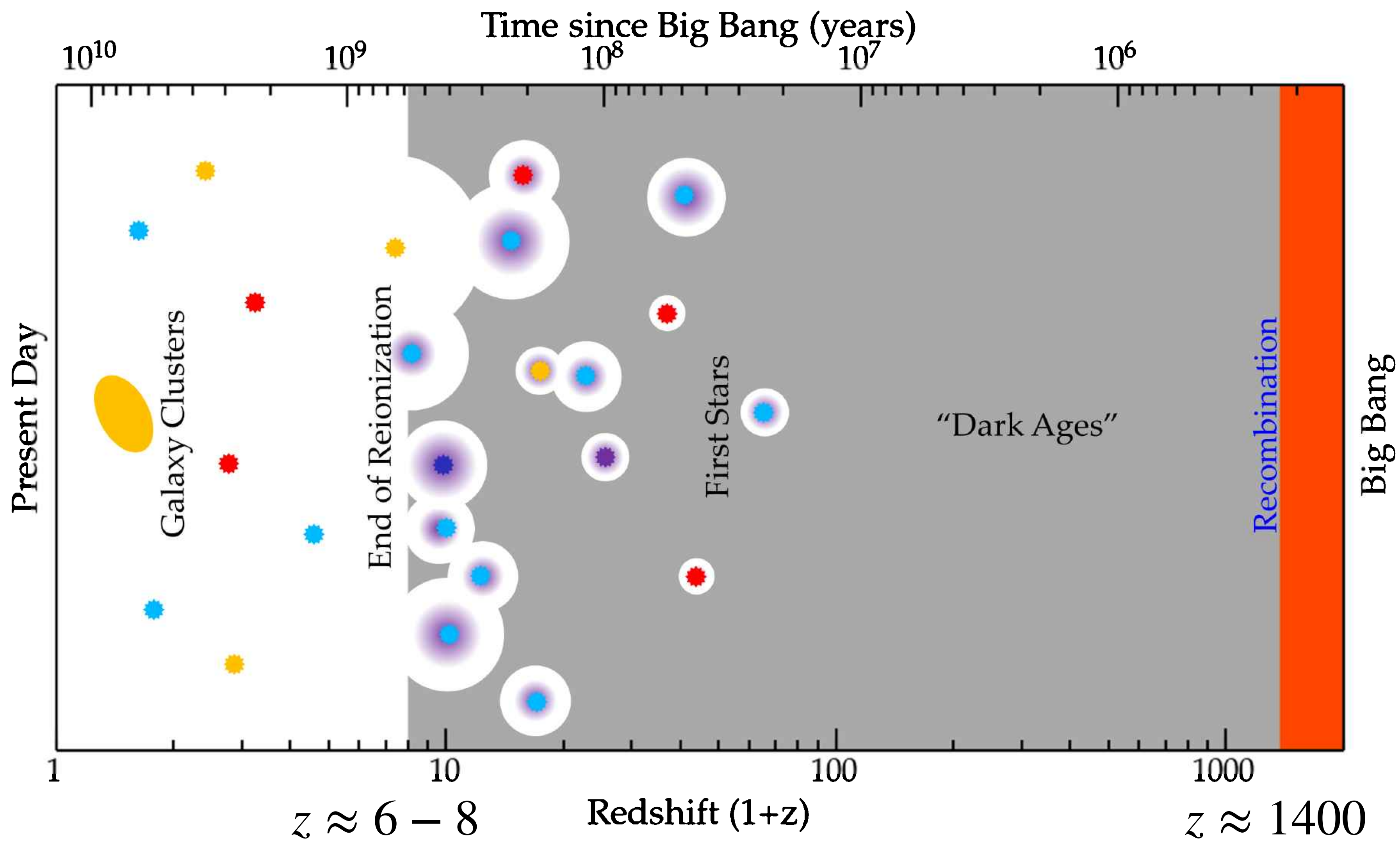
Absence of a Gunn-Peterson trough at $z < 5$

- However, it has been found that this is not the case. Therefore, ***either [1] intergalactic medium has a density very much lower than the mean baryon density of the Universe (gas is somehow segregated efficiently into galaxies) or [2] intergalactic gas is very highly ionized.***

- ***The absence of a Gunn-Peterson trough at redshifts*** $z < 5$ is now regarded as evidence that the IGM at low redshifts is highly ionized, not that it is absent.
- The figure shows spectra for high-redshift quasars. Notice that **the Gunn-Peterson trough bluewards of the QSO Lyα emission is clearly apparent in the highest redshift ones.**
- This indicates that **the Universe has become somewhat more neutral at these redshifts.** A similar behavior is also seen bluewards of the QSO Lyβ regions of the same spectra.
- These spectra show that ***the reionization of the IGM has ended at*** $z \approx 6$.



Dark Ages, Reionization Epoch



Schematic of the epoch of reionization [Barkana 2006; Fig 9.3 Ryden]

- In the early Universe, **some hundred million years after the Big Bang**, the temperature became low enough for electrons to combine with protons for the first time.
  - This is known as the epoch of recombination. This left the gas in the Universe in an overall neutral state.
- In today's Universe, however, nearly all of the gas between the galaxies is fully ionized.
  - ***There should have been a moment in the history of the Universe when it becomes ionized again.*** This period is known as the epoch of reionization.

- What caused the cosmic reionization?
  - How exactly this came to happens is still not fully understood, but one strong candidate for causing this to happen is the formation of ***the very first stars and galaxies***.
  - The other possible explanation would be strong high-energy radiation from quasars, and they do seem to have an effect, but the latest estimates indicate that they would contribute no more than ~ 10% to the total ionizing background radiation needed.
- When did the cosmic reionization happen?
  - One of the strongest pieces of evidence for the increasing fraction of neutral gas in the IGM is the Lyα forest and the so called ***Gunn-Peterson trough***.
  - The spectrum of a quasar, is intrinsically a bright continuum source with only a few very broad features. However, spectra of distant quasars show a large number of narrow absorption features, resembling the trunks of trees tightly packed together in a forest. This feature was therefore named ***the Lyα forest***.
  - ***At higher redshifts, the absorption features appear closer together, until finally a completely absorbed trough is observed.*** This indicates that the universe was previously more filled with neutral gas, and ***at some point the IGM was completely neutral***. The term "Gunn-Peterson trough" is named after the study of Gunn and Peterson (1965).
  - The Gunn-Peterson trough is typically observed at $z \sim 6$, thus marks the end of the epoch of reionization.

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- A bit more details
  - Suppose that a fraction $f_{\rm H} \approx 0.9$ of all baryons are hydrogen nuclei. We can assume that $f_{\rm H}$ is constant after Big Bang Nucleosynthesis is complete because of inefficiency of stars at nucleosynthesis.
  - Let $f_{\rm n}(z)$ the fraction of neutral hydrogen in their ground state; this can be a function of redshift. Then, the density of neutral hydrogen in the ground state at a redshift z.

$$\overline{n}_{\rm bary} = \overline{n}_{\rm bary,0}\,(1+z)^3$$

$$\begin{aligned} n_{\rm HI}(z) &= \overline{n}_{\rm bary} f_{\rm H} f_{\rm n}(z) \\ &= \overline{n}_{\rm bary,0} f_{\rm H} f_{\rm n}(z)\,(1+z)^3 \approx 2.27\times 10^{-7}\,[{\rm cm}^{-3}]\,\frac{f_{\rm H}}{0.9} f_{\rm n}(z)\,(1+z)^3 \end{aligned}$$

  - We can then write the optical depth at an arbitrary redshift:

$$\boxed{\tau_\nu^{\rm GP} = \frac{\chi_0 c}{\nu_\alpha}\overline{n}_{\rm bary,0} f_{\rm H} \frac{f_{\rm n}\,(1+z)^3}{H(z)}} \qquad \leftarrow \qquad \tau_\nu^{\rm GP} \simeq \frac{\chi_0 c}{\nu_\alpha}\frac{n_{\rm HI}(z)}{H(z)}$$

  - In a flat, ΛCDM universe, the Hubble parameter is given by (from the Friedmann equations)

$$\boxed{\begin{aligned} H(z) &= H_0\left[\Omega_{\rm m,0}\,(1+z)^3 + \Omega_{\Lambda,0}\right]^{1/2} \\ &\approx H_0 \Omega_{\rm m,0}^{1/2}\,(1+z)^{3/2} \end{aligned}}$$

ordinary + dark matter *dark energy*

$$\boxed{\text{Here, } \Omega_{\rm m,0} = 0.31 \text{ and } \Omega_{\Lambda,0} = 0.69}$$

$$\text{if } z \gg (\Omega_{\Lambda,0}/\Omega_{\rm m,0})^{1/3} - 1 \approx 0.31$$

Here, the ***density parameter*** is defined as: $\boxed{\Omega \equiv \frac{\rho}{\rho_{\rm c}}}$

standard model (Wilkinson Microwave Anisotropy Probe)

$$H(z) = H_0\left[\Omega_{\rm m,0}\,(1+z)^3 + \Omega_{\Lambda,0}\,(1+z)^{3(1+w)}\right]^{1/2}$$

$$w = -1 \;\; (w = p/\rho, \text{ the equation of state of the dark energy})$$

- At redshifts $z \gg (\Omega_{\Lambda,0}/\Omega_{\mathrm{m},0})^{1/3} - 1 \approx 0.31$, we can use the approximation that ***the universe is matter dominated***:

$$\boxed{\begin{aligned}\tau_\nu^{\mathrm{GP}} &= \frac{\chi_0 c}{\nu_\alpha} \bar{n}_{\mathrm{bary},0} f_{\mathrm{H}} \frac{f_{\mathrm{n}}\,(1+z)^3}{H_0 \Omega_{\mathrm{m},0}^{1/2}} \\ &\approx 25,600\, \frac{f_{\mathrm{H}}}{0.9} f_{\mathrm{n}}(z)\,(1+z)^{3/2}\end{aligned}}$$

This indicates that, if we want a hope of seeing a Gunn-Peterson trough, we must go to high redshift.

The appearance of the Gunn-Peterson effect at high redshifts have two reasons:

(1) the factor of $(1+z)^{3/2}$ is ~ 20 times bigger at $z \sim 6$ than that at $z \ll 1$.

(2) we expect the neutral fraction $f_{\mathrm{n}}(z)$ for hydrogen to be larger at higher redshift.

Cosmology - Epoch of Recombination

- ***Beginning of the Big Bang Nucelosynthesis***:
  - The temperature of the Cosmic Background Radiation drops as the universe expands:

$$T = T_0/a = T_0(1+z), \quad \text{where } T_0 = 2.7255 \pm 0.0006\,\mathrm{K}$$ (temperature in the present time)

Temperature is equivalent to energy and thus to frequency.

$$\left[kT_0 = (2.3486 \pm 0.0005) \times 10^{-4}\,\mathrm{eV}\right]$$

When $kT \sim 66\,\mathrm{keV}$ ($a \sim 3.6 \times 10^{-9}, z \sim 2.8 \times 10^{8}$), deuterium nuclei could form without being photodissociated. This was the starting of Big Bang Nucleosynthesis, which dirtied the hydrogen in the universe with significant amounts of helium and lithium.

In this **radiation dominated epoch**, the hydrogen (helium, lithium) was highly ionized.

- ***The Epoch of Recombination***:
  - Eventually, ***the temperature*** T ***reached a level low enough that the fractional ionization of hydrogen dropped below*** $x = 1/2$; the time when this happened is known as ***the epoch of recombination***.
  - Now we will find when the epoch of recombination began by using the *photoionization equilibrium condition for the pure hydrogen gas*.

- The number density of ionizing photons in the Cosmic Background Radiation:

$$n_\nu(T) = \frac{8\pi}{c^3}\frac{\nu^2}{\exp(h\nu/kT)-1} \quad \rightarrow \quad n_\nu(T) \approx \frac{8\pi}{c^3}\nu^2 \exp\left(-\frac{h\nu}{kT}\right) \quad \text{for } h\nu \geq I_{\rm H} \gg kT$$

$$(I_{\rm H} = 13.6\,{\rm eV})$$

- The ***photoionization rate of hydrogen*** is

$$\zeta_{\rm pi} = \int_{\nu_0}^{\infty} n_\nu(T) c\sigma_{\rm pi}(\nu) d\nu \quad (\nu_0 \equiv I_{\rm H}/h)$$

- We will use the following approximations for the photoionization cross-section and the number density of photons:

$$\sigma_{\rm pi}(\nu) \approx \sigma_0 \left(\nu/\nu_0\right)^{-3} \qquad n_\nu(T) \approx \frac{8\pi}{c^3}\nu^2 e^{-\frac{h\nu}{kT}} \quad \text{for } h\nu \geq I_{\rm H} \gg kT$$

$$\sigma_0 = 6.304 \times 10^{-18}\,{\rm cm}^{-2}$$

$$kT = kT_0(1+z) \ll I_{\rm H}$$

- When $kT \ll I_{\rm H}$, corresponding to redshifts $z \ll (I_{\rm H}/kT_0) - 1 \approx 58,000$, the photoionization rate is

$$\begin{aligned}
\zeta_{\rm pi} &\approx \frac{8\pi}{c^2}\sigma_0\nu_0^3 \int_{\nu_0}^{\infty} e^{-h\nu/kT}\frac{d\nu}{\nu} \\
&= \frac{8\pi}{c^2}\sigma_0\nu_0^3 e^{-I_{\rm H}/kT}\int_0^{\infty} e^{-x}\frac{dx}{x + I_{\rm H}/kT} \quad \leftarrow \quad x = h(\nu-\nu_0)/kT \\
&\approx \frac{8\pi}{c^2}\sigma_0\nu_0^3 \left(\frac{kT}{I_{\rm H}}\right) e^{-I_{\rm H}/kT}
\end{aligned}$$

$$\int_0^{\infty} e^{-x}\frac{dx}{x+a} \approx \int_0^{\infty} e^{-x}\left(1+x/a\right)^{-1}\frac{dx}{a} \approx \frac{1}{a} - \frac{1}{a^2} + \frac{2}{a^3}\cdots (\text{if } a \gg 1)$$

- Numerically, this is

$$\zeta_{\rm pi} \approx 4.61 \times 10^8\,{\rm s}^{-1}\left(\frac{kT}{1\,{\rm eV}}\right)e^{-13.6\,{\rm eV}/kT}$$

- The total number density of blackbody photons is given by

$$n_\gamma = \int_0^\infty n_\nu(T)d\nu = \frac{8\pi}{c^3}\int_0^\infty \frac{\nu^2}{\exp\left(h\nu/kT\right)-1}d\nu$$
$$= 8\pi\left(\frac{kT}{hc}\right)^3\int_0^\infty \frac{x^2dx}{e^x-1}$$

$$\int_0^\infty \frac{x^2dx}{e^x-1} = \int_0^\infty dx x^2 e^{-x}\left(1-e^{-x}\right)^{-1} = \int_0^\infty dx x^2\sum_{n=1}^\infty e^{-nx}$$
$$= \sum_{n=1}^\infty \frac{1}{n^3}\int_0^\infty dy y^2 e^{-y} \qquad \leftarrow \quad y = nx$$
$$= \zeta(3)\Gamma(3)$$
$$\zeta(3) \approx 1.202 \quad \text{and} \quad \Gamma(3) = 2$$

$$n_\gamma = 16\pi\zeta(3)\left(\frac{kT}{hc}\right)^3 \approx 3.17\times 10^{13}\left(\frac{kT}{1\,{\rm eV}}\right)^3\ [{\rm cm}^{-3}]$$

- Using this, we can write the photoionization rate in the form:

$$\zeta_{\rm pi} \approx \frac{\sigma_0 c n_\gamma}{2\zeta(3)}\left(\frac{I_{\rm H}}{kT}\right)^2 e^{-I_{\rm H}/kT} \qquad \leftarrow \qquad \zeta_{\rm pi} \approx \frac{8\pi}{c^2}\sigma_0\nu_0^3\left(\frac{kT}{I_{\rm H}}\right)e^{-I_{\rm H}/kT}$$

- ***Photoionization equilibrium for pure hydrogen gas***:

$$\zeta_{\rm pi} n_{\rm H^0} = n_e n_p \alpha_{\rm B,H}$$
$$\zeta_{\rm pi}(1-x)n_{\rm H} = x^2 n_{\rm H}^2\alpha_{\rm B,H} \quad \longleftarrow$$

fractional ionization
$x \equiv n_e/n_{\rm H},\ n_e = n_p$

$$1 - x = x^2\frac{n_{\rm H}\alpha_{\rm B,H}}{\zeta_{\rm pi}}$$

- The condition for the epoch of recombination is then given by:

$$1 - x = x^2 \frac{n_\mathrm{H} \alpha_\mathrm{B,H}}{\zeta_\mathrm{pi}} \quad \text{and} \quad x = \frac{1}{2} \quad \rightarrow \quad \frac{n_\mathrm{H} \alpha_\mathrm{B,H}}{\zeta_\mathrm{pi}} = 2$$

- From the above equation for the photoionization rate, we can rewrite the condition as follows:

$$\left(\frac{kT}{I_\mathrm{H}}\right)^2 e^{I_\mathrm{H}/kT} = \frac{\sigma_0 c}{\zeta(3) \alpha_\mathrm{B,H}} \frac{n_\gamma}{n_\mathrm{H}}$$

- ***The baryon-to-photon ratio*** in the hydrogen-only universe:

$$n_\gamma \propto T^3$$
$$T = T_0(1+z)$$
$$n_\mathrm{H} = \overline{n}_\mathrm{bary} = \overline{n}_\mathrm{bary,0}(1+z)^3$$

$$\longrightarrow \quad \frac{n_\mathrm{bary}}{n_\gamma} = \frac{n_\mathrm{bary,0}}{n_{\gamma,0}} \approx 6.1 \times 10^{-10}$$

$$n_{\gamma,0} \approx 411\,[\mathrm{cm}^{-3}]$$
$$n_\mathrm{bary,0} \approx 2.50 \times 10^{-7}\,[\mathrm{cm}^{-3}]$$

The ratio remains constant with time, unless stars were born and messed things up by generating non-CMB photons into the universe.

- Recall that ***the recombination rate coefficient*** is given by:

$$\alpha_\mathrm{B,H}(T) \approx 2.59 \times 10^{-13}\, T_4^{-0.833-0.034\ln T_4}\ [\mathrm{cm}^3\,\mathrm{s}^{-1}]$$

- The condition for ***the epoch of recombination*** can be written as

$$\left(\frac{kT}{I_{\rm H}}\right)^{1.17} e^{I_{\rm H}/kT} = 9.8 \times 10^{15}$$

The solution to this equation is $kT \approx 0.024 I_{\rm H} \approx 0.33\,{\rm eV}$. This corresponds to the following temperature, redshift and mean baryon density:

the epoch of recombination: $T \approx 3800\,{\rm K},\ z \approx 1400,\ \overline{n}_{\rm bary} \approx 690\,{\rm cm}^{-3}$

- ***Dark Ages***
  - The recombination of hydrogen brings in ***the "Dark Ages", which is the period between recombination at z ~ 1400 and the formation of the first stars at z ~ 30***.
  - The Cosmic Background Radiation will have a temperature ranging ***from T ~ 3800 K at z ~ 1400 to T ~ 80 K at z ~ 30***.
  - At the beginning of the Dark Ages, the temperature is about that of an M star. *By the end of the Dark Ages, the photons of the Cosmic Background Radiation would be too low in energy to photo-ionize hydrogen atoms.*
  - ***The reionization of intergalactic hydrogen must be accomplished by UV photons that comes from massive stars and from active galactic nuclei (AGN).***

Epoch of Reionization

- ***The Epoch of Reionization***
  - The reionization is a patchy process. The individual H II regions around early (first) stars and those around AGNs gradually merge to form a single expanse of ionized gas.
  - Thus, ***we cannot speak of an instant of reionization, but rather an epoch of reionization as the patches of ionized gas take over more and more of the universe***.

- We have some observational evidence on the ionization state of the baryonic gas as a function of time during the epoch of reionization, as seen in the quasar spectra at z > 6.

Fan et al. (2006) found that Lyα optical depth as a function of redshift, from a sample of 19 quasars.

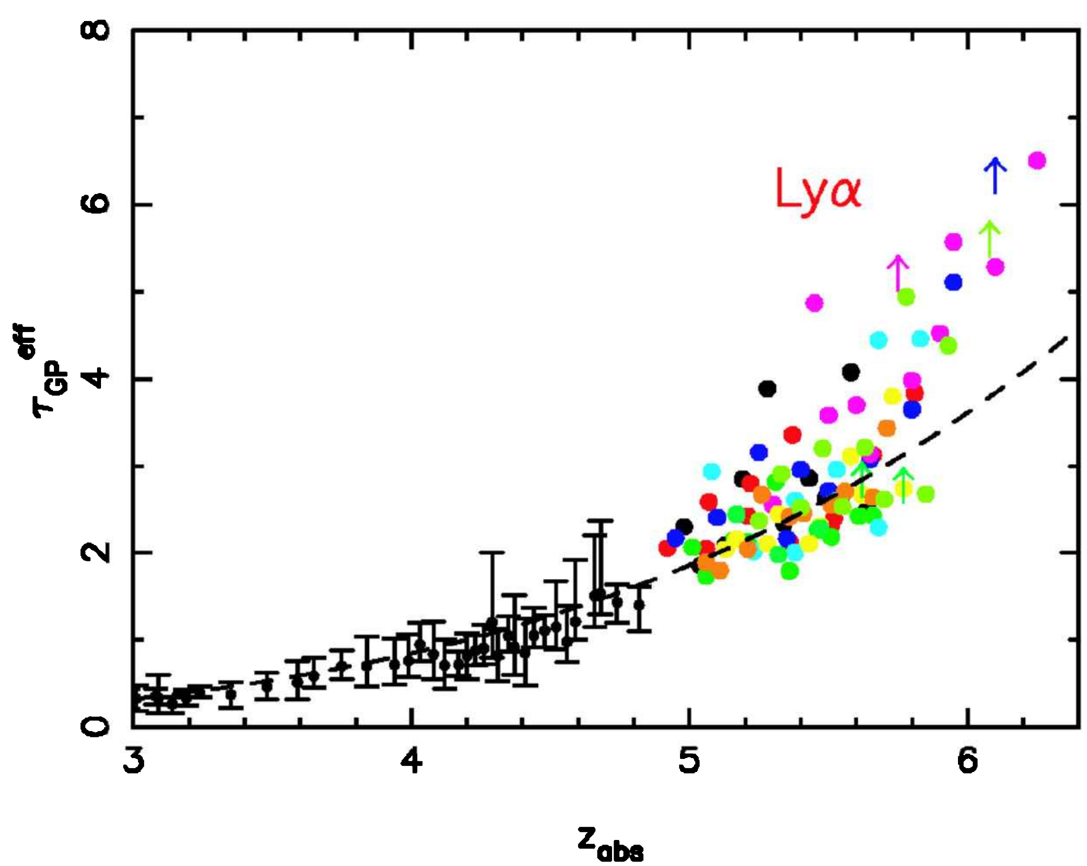
$$\tau_\nu \approx 2.6 \left(\frac{1+z}{6.5}\right)^{4.3} \quad \text{for } z < 5.5$$

We have previously obtained the following equation:

$$\tau_\nu^{\rm GP} \approx 25{,}600\, f_{\rm n}(z)\, (1+z)^{3/2}$$

Combining these two equations, we find ***the neutral fraction is small for z < 5.5***.

$$f_{\rm n}(z) \approx 6.2 \times 10^{-6} \left(\frac{1+z}{6.5}\right)^{2.8}$$



Lyα optical depth as a function of redshift.
[Fig 5, Fan et al. 2006, AJ, 132, 117]
[See also Fan et al. 2006, AR&AA]

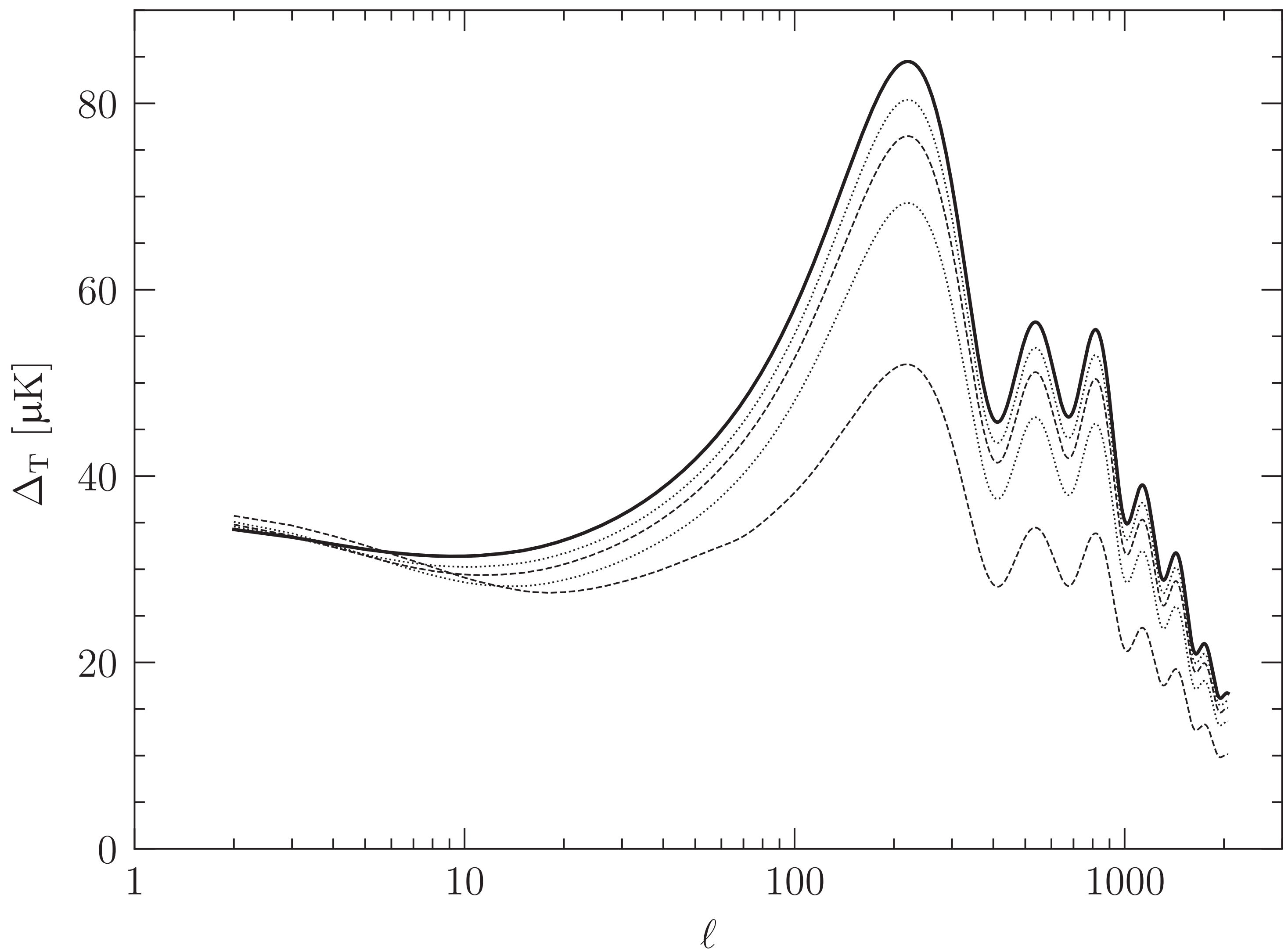
- Limitation of using Gunn-Peterson effect for determining the neutral fraction:
  - *Quasars take us back only to $z \sim 7$.* Thus, the Gunn-Peterson effect can tell us solely about the very late stages of reionization.
  - The optical depth higher than $\tau \sim 5$ (at $z \approx 7$) corresponding to a transmitted fraction $e^{-\tau} \sim 0.007$, which is hard to distinguish from zero in a noisy spectrum.
  - To observe the earlier process of reionization, when the neutral fraction was still close to unity, we have to take a different approach, i.e., the CMB.

- ***The Epoch of Reionization, as probed by the CMB***.
  - The ***Cosmic Microwave Background*** contains information about the epoch of reionization.
  - The reionized gas of the IGM at low redshift provides free electrons between us and the CMB.
    - These free electrons scatter the photons of the CMB via ***Thomson scattering*** with cross-section:

$$\sigma_e = 6.652 \times 10^{-25}\,\mathrm{cm}^2$$

    - *If the optical depth from Thomson scattering were $\tau_e \gg 1$, then the temperature fluctuations of the CMB would be thoroughly smeared out.*

- ***The actual CMB spectrum shows only a modest suppression of the power spectrum of temperature fluctuations on small angular scales*** due to scattering from free electrons in the reionized IGM. Therefore, we expect that the free electrons in the reionized gas provide $\tau_e \ll 1$. ***The Planck results give*** $\tau_e = 0.066 \pm 0.016$.



Effect of reionization on the CMB power spectrum.
Solid line : CMB spectrum without Thomson scattering by reionized gas. The other lines represent the CMB spectrum with different Thomson scattering optical depths $\tau_e = 0.05, 0.1, 0.2, 0.5$, in order of decreasing height of main spectral peak.
[Fig 9.5 Ryden; calculated using CAMB]

- ***Thomson scattering***
  - The optical depth for Thomson scattering can be written as

$$\boxed{\tau_e = c\int_{t_*}^{t_0} n_e(t)\sigma_e dt = c\sigma_e \int_{a(t_*)}^{1} n_e(a)\frac{da}{aH} = c\sigma_e \int_0^{z_*} n_e(z)\frac{dz}{H(z)(1+z)}}$$

Here, t_* ***= time at which the reionization was completed***.

- For a universe made of pure hydrogen, which undergoes complete reionization at the time t_*, the number density of free electrons can be expressed in terms of the Baryon density:

$$n_e = n_{\rm H} = \overline{n}_{\rm bary} = \overline{n}_{\rm bary,0}(1+z)^3$$

- Then after the complete reionization $(t > t_*)$, the optical depth for Thomson scattering is:

$$\tau_e = c\sigma_e\overline{n}_{\rm bary,0}\int_0^{z_e}\frac{(1+z)^2dz}{H(z)} \approx 4.97\times10^{-21}\,{\rm s}^{-1}\int_0^{z_*}\frac{(1+z)^2dz}{H(z)}$$

$$H(z) = H_0\left[\Omega_{\rm m,0}(1+z)^3 + \Omega_{\Lambda,0}\right]^{1/2} \longrightarrow \tau_e = \frac{2}{3}\frac{c\sigma_e\overline{n}_{\rm bary,0}}{H_0\Omega_{\rm m,0}}\left\{\left[\Omega_{\rm m,0}(1+z_*)^3+\Omega_{\Lambda,0}\right]^{1/2} - \left[\Omega_{\rm m,0}+\Omega_{\Lambda,0}\right]^{1/2}\right\}$$

$$\approx \frac{2}{3}\frac{c\sigma_e\overline{n}_{\rm bary,0}}{H_0\Omega_{\rm m,0}}\left\{\left[\Omega_{\rm m,0}(1+z_*)^3+\Omega_{\Lambda,0}\right]^{1/2} - 1\right\}$$

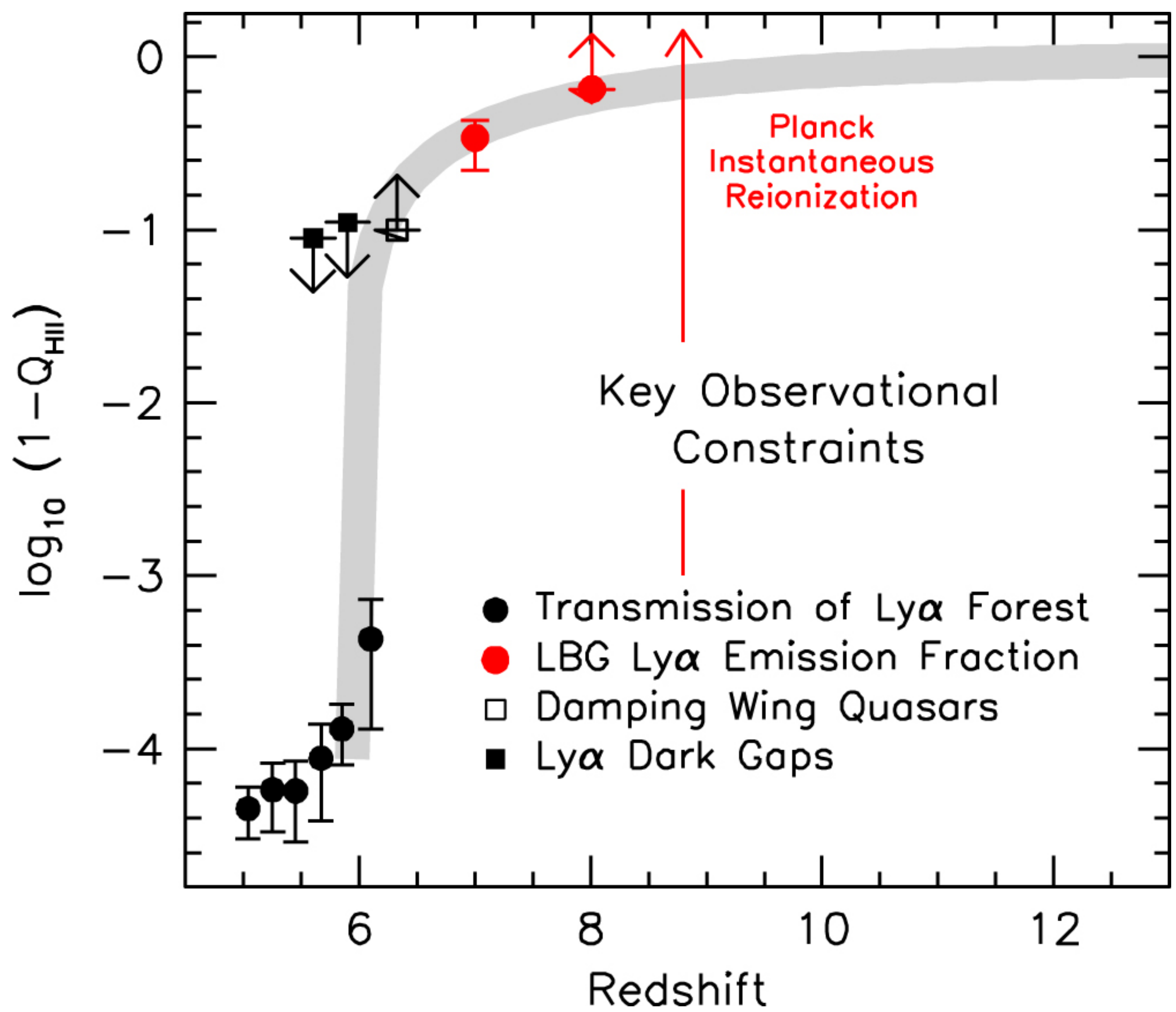
$$\tau_e \approx 0.00486\left\{\left[0.31(1+z_*)^3+0.69\right]^{1/2}-1\right\}$$

- From the observed value of the optical depth, this gives a redshift of reionization:

$$\tau_e \approx 0.066 \quad \rightarrow \quad z_* = 7.8$$

Note that we assumed the pure-hydrogen universe and an instantaneous reionization, in deriving this result.

- In the figure, the filling factor of ionized hydrogen is denoted by Q_{HII}.
- The latest results (Planck Collaboration et al., 2015) places **reionization at z ~ 8.8**, assuming a model in which the universe is instantly reionized.
  - Studies of the cosmic microwave background (CMB) tell us of the column density of ionized material in front of the last scattering surface.
  - Thomson scattering of CMB photons upon free electrons causes the signal to become partially linearly polarized, allowing us to calculate a Thomson optical depth which in turn can be used to estimate when reionization took place.
  - The red arrow shows the instantaneous reionization redshift from Planck Collaboration et al. (2015).
- The gray shaded region schematically follows the evolution in the filling factor.



Summary of constraints on the redshift at which reionization took place. (Bouwens et al. 2015).

The points include Gunn-Peterson and Lyα dark gaps from Fan et al. (2006) and McGreer et al. (2015), quasar damping wings from Schroeder et al. (2013), and Lyα galaxies from Schenker et al. (2014).

Reionization Sources

- We need to find objects that were produce large quantities of ionizing photons with $h\nu > 13.5\,\mathrm{eV}$ at a redshift $z \sim 10$.
  - Suppose that a single source of ionizing photons (either an AGN or a star-forming galaxy) is turned on. Then, a Strömgren sphere will form, with a radius:

$$R(t) = R_s \left(1 - e^{-t/t_{\rm rec}}\right)^{1/3}$$

    - The ionization (and recombination) time $t_{\rm rec}$ is given by

$$\boxed{t_{\rm rec} \equiv \frac{1}{\alpha_{\rm B,H} n_{\rm H}} = \frac{1}{\alpha_{\rm B,H} \overline{n}_{\rm bary,0}} (1+z)^{-3} \approx 0.5\,\mathrm{Gyr} \left(\frac{1+z}{11}\right)^{-3}}$$

see Lecture 07

$\alpha_{\rm B,H} \approx 2.59 \times 10^{-13}\,[\mathrm{cm^3\,s^{-1}}]$

    - The Strömgren radius for the AGN or a star-forming galaxy is

$$R_s \equiv \left(\frac{3Q_0}{4\pi\alpha_{\rm B,H} n_{\rm H}^2}\right)^{1/3} \approx 0.7\,\mathrm{Mpc} \left(\frac{Q_0}{10^{54}\,\mathrm{s^{-1}}}\right)^{1/3} \left(\frac{1+z}{11}\right)^{-2}$$

  - In order for the Strömgren spheres to overlap and fill the universe with ionized hydrogen, ***the number density of photo-ionizing sources*** must be

$$\frac{4\pi R_s^3}{3} n_{\rm source} > 1 \longrightarrow n_{\rm source} > \frac{3}{4\pi R_s^3} \approx 0.6\,\mathrm{Mpc^{-3}} \left(\frac{Q_0}{10^{54}\,\mathrm{s^{-1}}}\right)^{-1} \left(\frac{1+z}{11}\right)^{6}$$

- The above equation gives **the required number density** in physical units; *the number density in comoving length units*, normalized to the present, is

$$n_{\mathrm{source}}\,(1+z)^{-3} = 5\times10^{-4}\,\mathrm{Mpc}^{-3}\left(\frac{Q_0}{10^{54}\,\mathrm{s}^{-1}}\right)^{-1}\left(\frac{1+z}{11}\right)^{3}$$

- Thus, **the rate per comoving volume at which ionizing photons are produced** must be greater than the following critical value:

$$\mathcal{F}_{\mathrm{crit}} \equiv Q_0 n_{\mathrm{source}}\,(1+z)^{-3} = 5\times10^{50}\,\mathrm{Mpc}^{-3}\,\mathrm{s}^{-1}\left(\frac{1+z}{11}\right)^{3}$$

  This is the equivalent of **a dozen O3 main sequence stars per cubic Mpc**, which is not a lot.

- Now examine the OB stars as a candidate for the cosmic reionization. We should notice that the lifetime of an O3 star is only ~ 1 Myr, which is much shorter than the ionization (and recombination) time $t_{\mathrm{rec}} \sim 500\,\mathrm{Myr}\left[(1+z)/11\right]^{-3}$. Thus, we need ***continuous star-formation***.

  - Using the starburst99 code (Leitherer et al. 1999; https://www.stsci.edu/science/starburst99/docs/default.htm), we obtain ***the production rate of ionizing photons, per a unit star-formation rate, for the continuous star-formation***:

$$Q_* = 10^{53.148}\,[\mathrm{s}^{-1}]\,\frac{\mathrm{SFR}}{1\,M_\odot\,\mathrm{yr}^{-1}}$$

adopting the Initial Mass Function of Kroupa (2001), for continuous star-formation

IMF of Kroupa (2001)

$\xi(m)\Delta m = m^{-\alpha}\Delta m$

$\alpha = 0.3$ for $m < 0.08M_\odot$

$\alpha = 1.3$ for $0.08M_\odot < m < 0.5M_\odot$

$\alpha = 2.3$ for $m > 0.5M_\odot$

  - Only a fraction f_{esc} of the photoionizing photons will escape the galaxy and enter the IGM.

- ***Comoving star-formation rate, required to keep the universe ionized*** is then:

$$Q_* f_{\rm esc} \geq \mathcal{F}_{\rm crit} \quad \longrightarrow \quad \boxed{\mathrm{SFR} \gtrsim 0.004\, M_\odot\, \mathrm{yr}^{-1}\, \mathrm{Mpc}^{-3} \left(\frac{1+z}{11}\right)^3 f_{\rm esc}^{-1}}$$

Here, $f_{\rm esc}$ is the escape fraction of ionizing photons (escape out of the host galaxy and enter the IGM).

The escape fraction is a critical parameter to identify the main source of the cosmic reionization. ***However, the escape fraction of ionizing photon is poorly known at all redshifts.***

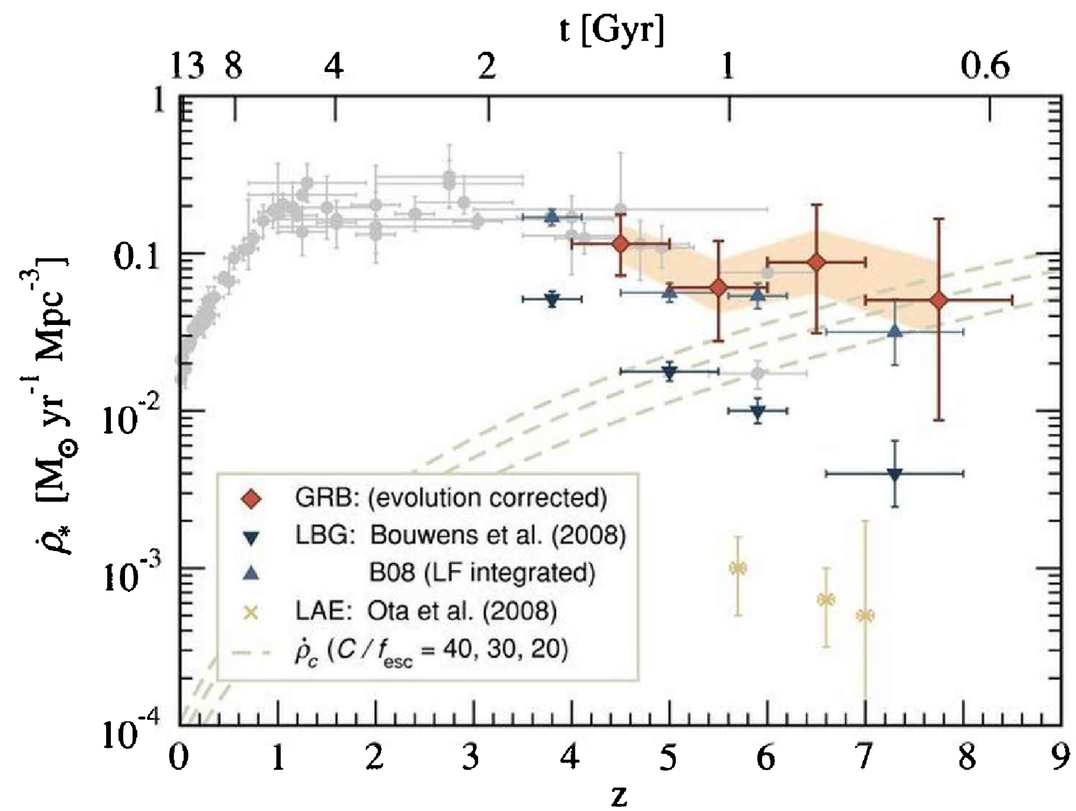
- **Observation of the cosmic star-formation rate**:

Galaxies

- The cosmic star-formation rate has a fairly broad peak in the redshift range $1 < z < 4$.
- The comoving SF rate at $z \sim 10$ was lower than that at $z \sim 2$, but how much lower is not clear.
- The earliest generation of stars (Pop III stars) would definitely play a significant role in ionizing the universe. But, they are still poorly understood.
- ***Galaxies are likely to be the dominant source of the cosmic reionization.***

AGNs

- The observed comoving density of bright quasars has a narrower peak in the redshift range $2 < z < 3$.
- It is known that ***the number of AGNs wasn't enough to reionize the IGM at z ~ 10***.
- However, the faintest AGNs may have been numerous enough to contribute significantly to the reionization. (This scenario is highly unlikely. But, it cannot be completely ruled out.)



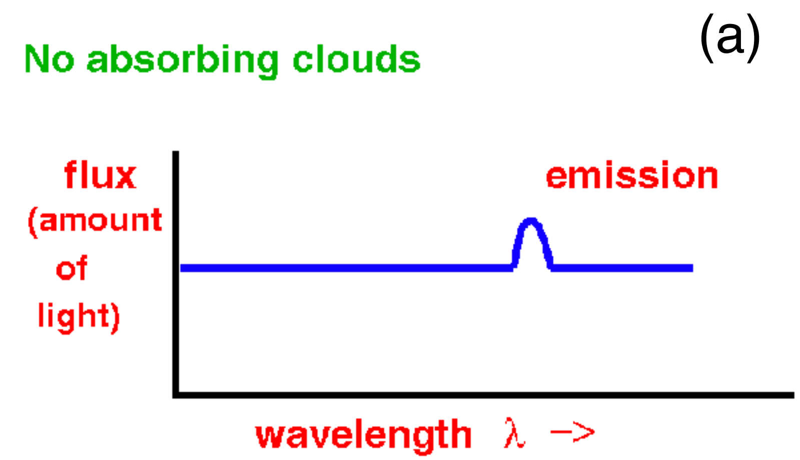
The cosmic star-formation rate using different traces (GRB = Gamma-Ray Bursts; LBG = Luminous Blue Galaxies; LAE = Lyman-Alpha Emitters) [Kistler et al. 2009; Fig 9.6 Ryden]

Lyα Forest

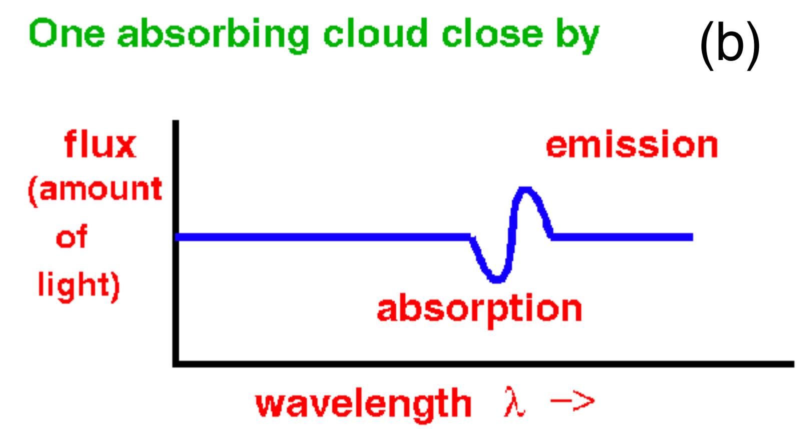
- History
  - A uniformly dark Gunn-Peterson trough is only seen at redshifts $z > 6$.
  - However, at lower redshifts, there exists a “Lyman alpha forest” of absorption lines.
  - The Lyα forest was first discovered by Roger Lynds in 1971.
    - Lynds found many absorption lines in the spectrum of 4C 05.34 (with $z = 2.877$, the largest redshift then known for any quasar), most of which were at wavelengths shorter than the Lyα emission line of the quasar.
    - Lynds concluded that most of the absorption lines that he saw were Lyα lines from hydrogen along the line of sight to the quasar; the other absorption lines were from relatively common heavier elements (such as O, C, N, and Si) at the same redshifts as the absorbing hydrogen.
    - As similar distributions of short-wavelength absorption lines began to be seen in the spectra of additional quasars, astronomers began using the metaphor of a **Lyman alpha “forrest” of absorption lines.**

Lyα Forest

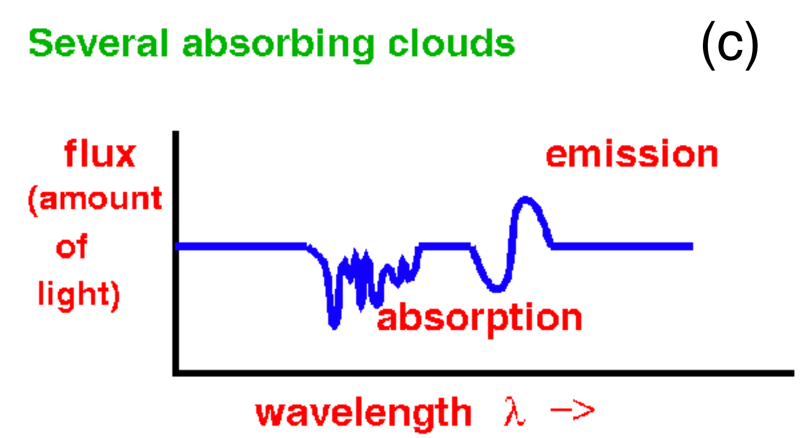
- Figure (a) shows a cartoon of how a quasar spectrum might look like if there were no intervening neutral hydrogen between the quasar and us.
  - The quasar continuum is relatively flat. Broad emission features are produced by the quasar itself (near the black hole and its accretion disk).



- In some cases, gas near the quasar central engine also produces "intrinsic" absorption lines, most notably Lyα, and relatively high ionization metal transitions such as C IV, N V, and O VI.



- However, the vast majority of absorption lines in a typical quasar spectrum are "intervening", produced by gas unrelated to the quasar that is located along the line of sight between the quasar and the Earth.



- Its wavelength is stretched by the expansion of the Universe from what it was initially at the quasar, and, if it had continued to travel to us, it would have been stretched some more from the 1216Å wavelength it had at the absorber.

- The cartoon below shows a quasar with it Lyα emission line redshifted from the UV into the red, and the Lyα absorption lines from four intervening clouds appearing as orange, yellow and green-blue.
- Each structure will produce an absorption line in the quasar spectrum at a wavelength of $\lambda_{\rm obs} = \lambda_{\rm rest}(1 + z_{\rm gas})$, where $z_{\rm gas}$ is the redshift of the absorbing gas and $\lambda_{\rm rest} = 1216\mathring{A}$ is the rest wavelength of the Lyα transition. Since $z_{\rm gas} < z_{\rm quasar}$, the redshift of the quasar, these Lyα absorption lines form a "forest" at wavelengths blueward of the Lyα emission of the quasar.
- The region redward of the Lyα emission will be populated only by absorption through other chemical transitions with longer $\lambda_{Ly\alpha}$.

Definition of redshift:

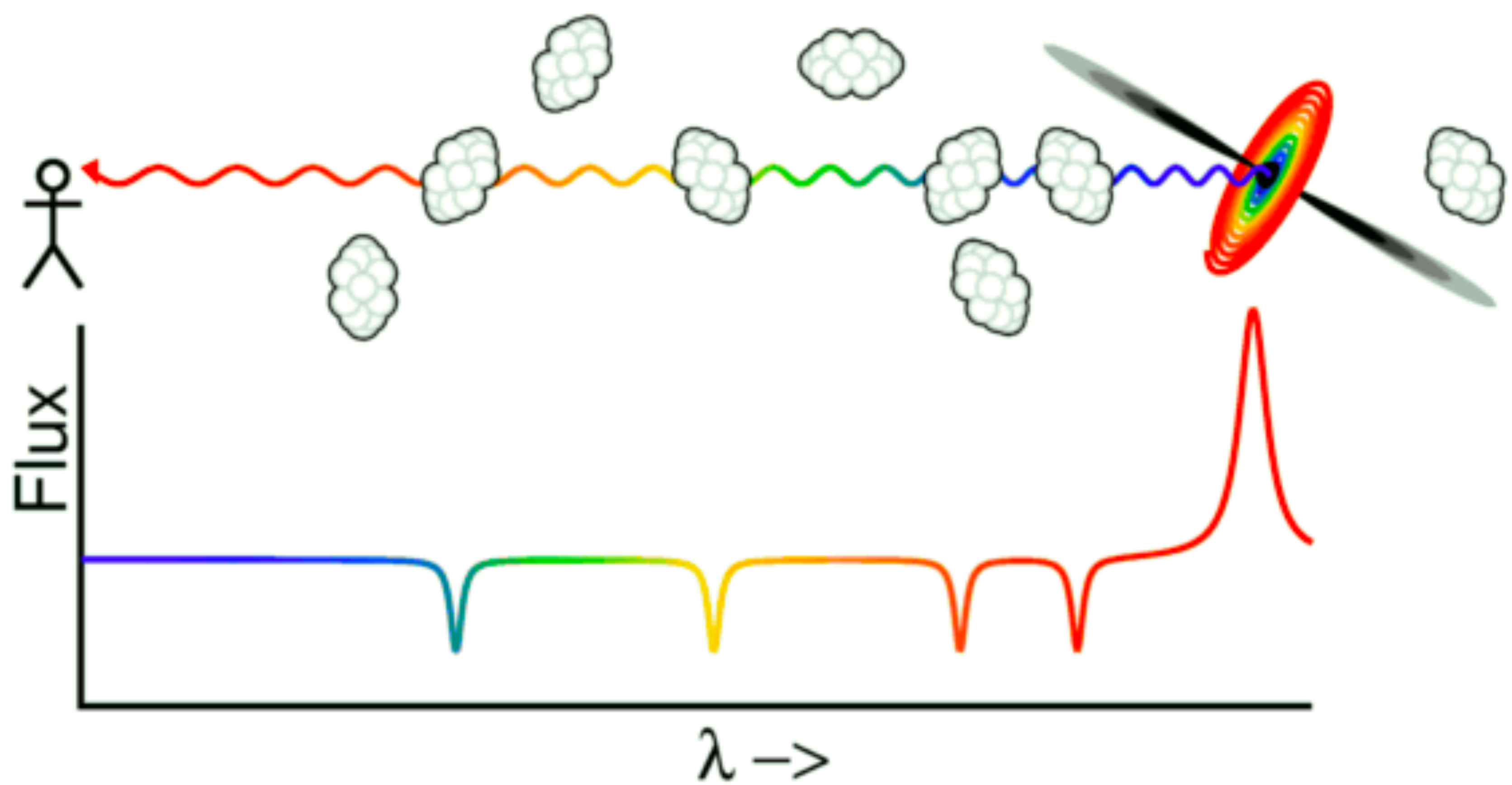
$$z \equiv \frac{\lambda_{\rm obs} - \lambda_{\rm emit}}{\lambda_{\rm emit}}$$

$$\lambda_{\rm obs} = \lambda_{\rm emit}(1 + z)$$

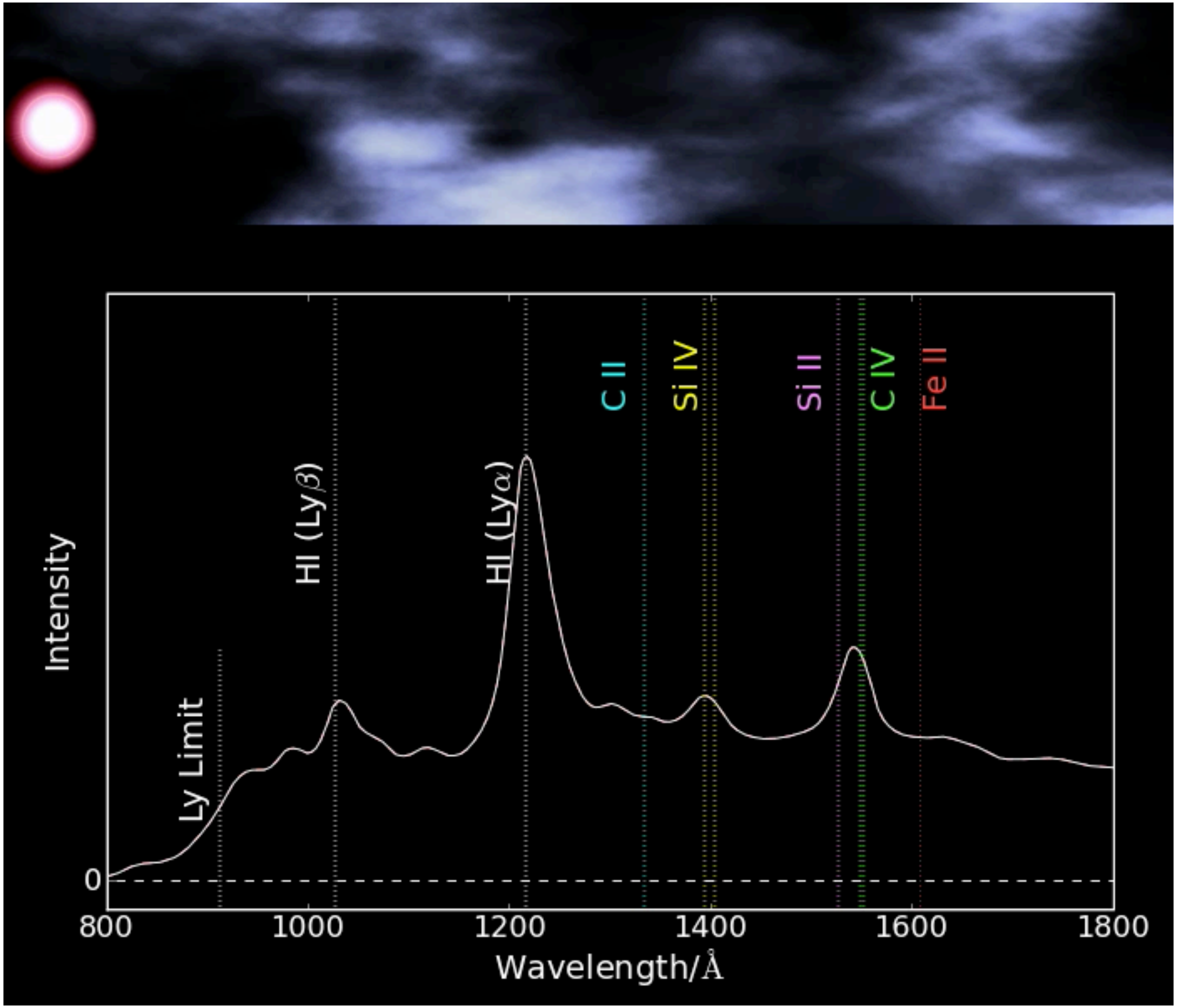
$$\lambda^{\rm gas}_{\rm obs} = \lambda_{\rm rest}(1 + z_{\rm gas})$$

$$\lambda^{\rm quasar}_{\rm obs} = \lambda_{\rm rest}(1 + z_{\rm quasar})$$

$$\therefore \lambda^{\rm gas}_{\rm obs} < \lambda^{\rm quasar}_{\rm obs}$$



A very nice visualization that shows how different systems absorb Lyman-alpha, made by Andrew Pontzen.
To see this movie, please download from http://www.cosmocrunch.co.uk/media/dla_credited.mov

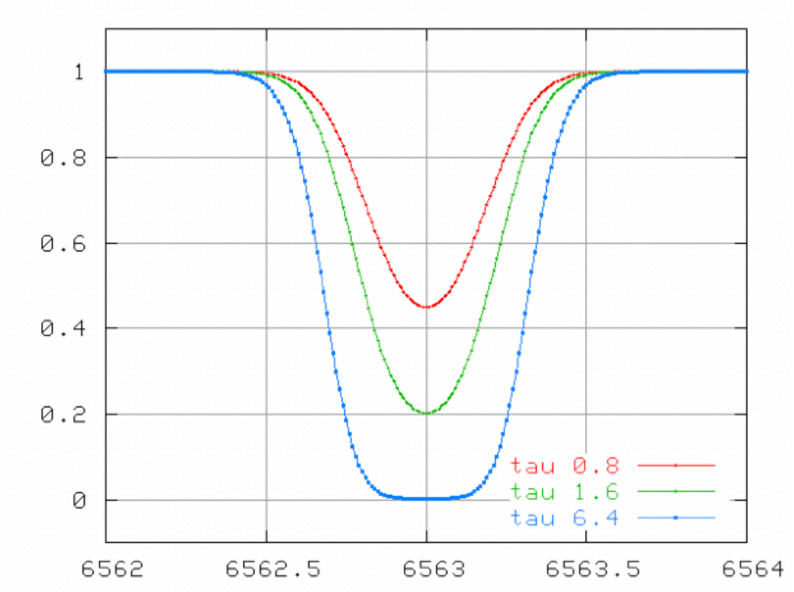


Lyα Absorption System

- A structure along the line of sight to the quasar can be described by its neutral Hydrogen column density $N(\mathrm{H\,I})$, the product of the density of the material and the path length along the line of sight through the gas.
- Classification

$$N(\mathrm{H\,I}) = n_{\mathrm{H}} L$$

$N\,(\mathrm{H\ I}) < 10^{12}\ \mathrm{cm}^{-2}$	Currently not observable
$10^{12} < N\,(\mathrm{H\ I}) < 10^{17}\ \mathrm{cm}^{-2}$	Lyα forest
$10^{17} < N\,(\mathrm{H\ I}) < 2\times10^{20}\ \mathrm{cm}^{-2}$	Lyman limit systems
$2\times10^{20} < N\,(\mathrm{H\ I})$	Dampled Lyα systems



As N (H I) increases, the absorption line depth and width increase.

- A typical temperature of the diffuse IGM is $T \sim 10^5\,\mathrm{K}$ (corresponding to a thermal broadening $b \sim 40\,\mathrm{km\,s^{-1}}$ in Lyα line). The optical depth at line center is then

$$\boxed{\tau_0 \approx 1.9 \left(\frac{b}{40\,\mathrm{km\,s^{-1}}}\right)^{-1} \left(\frac{N_{\mathrm{HI}}}{10^{14}\,\mathrm{cm^{-2}}}\right)}$$

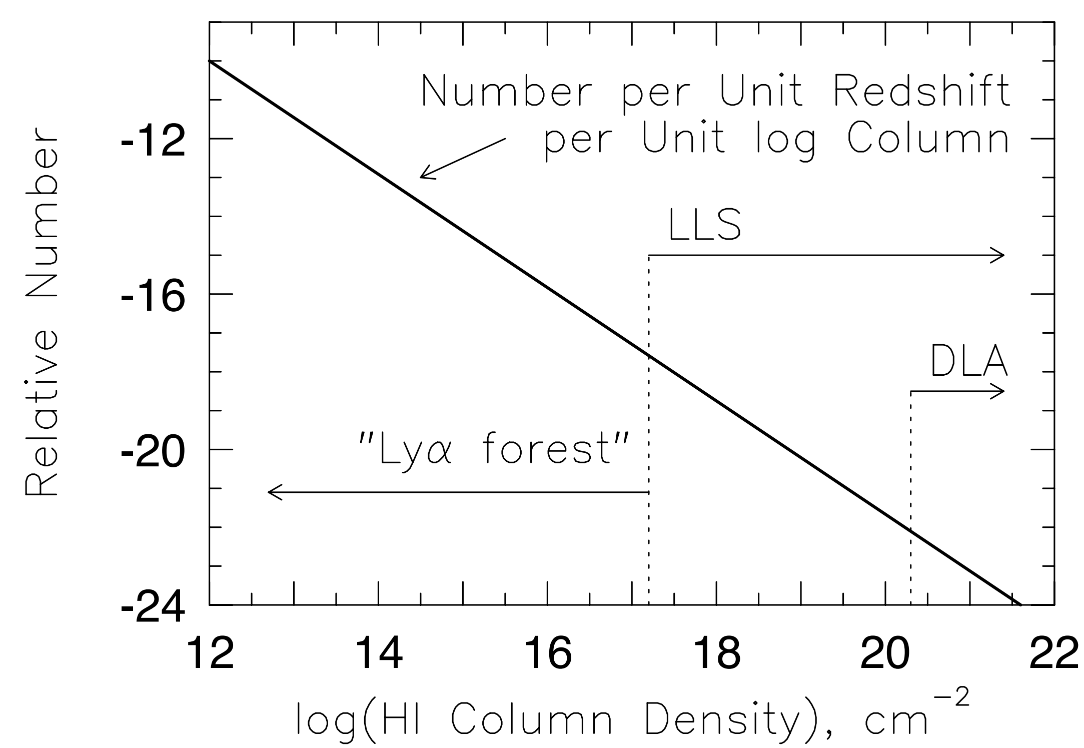
- The name ***"Lyman limit system"*** is given because at these column densities, ***clouds become optically thick to photons with λ < 912Å***, at the Lyman limit. As a consequence, Lyman limit systems are self-shielded from outside ionizing photons.
- ***Damped Lyman alpha systems*** (DLAs) have column densities of neutral hydrogen comparable to a large galaxy like our own.

What are the Lyα absorption systems?

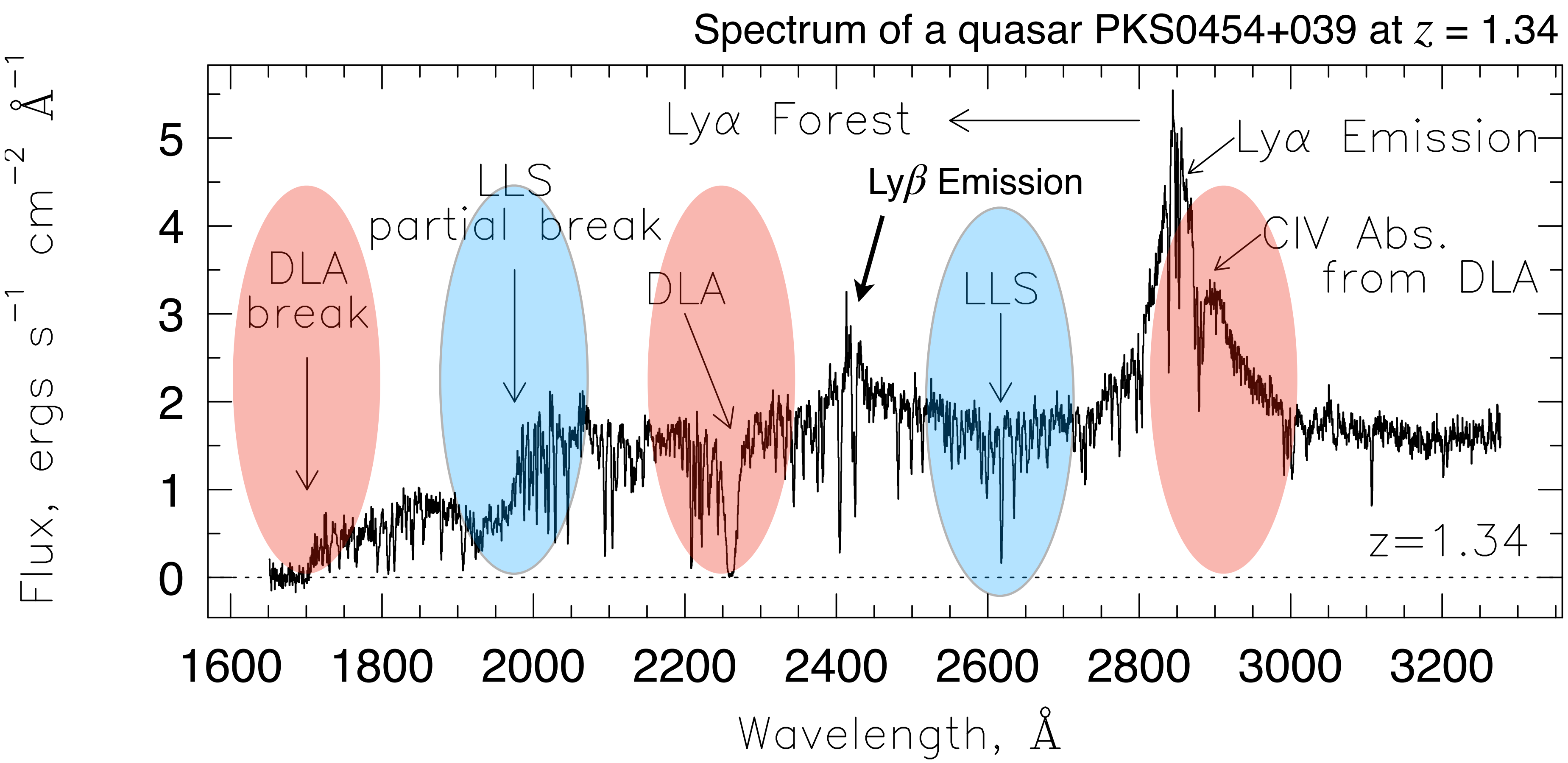
- Metallicity
  - The metallicity of DLAs is typically in the rage $Z \approx 0.01 - 0.3 Z_\odot$.
  - The Lyman alpha forests has a lower metallicity of $Z \approx 0.001 - 0.01 Z_\odot$.
- What are they?
  - ***DLAs can be thought of as gravitationally bound (proto)galaxies***, containing gas (and associated dark matter), but which haven't yet been effective at converting gas into stars.
  - However, the lower column density absorption lines in the Lyman alpha forest, which are vastly more numerous than the DLAs, cannot be associated with individual gravitationally bound gas clouds.
    - ***Densities in the Lyman alpha forests are simply not dense enough to represent gravitationally collapsed, virialized systems*** with a high neutral fraction of hydrogen.
    - Instead, the absorption lines of the Lyman alpha forests are likely produced from highly ionized regions of gas that are broadened primarily by the Hubble flow.
  - The Lyman alpha forest shouldn't be thought as resulting from discrete clouds along the line of sight to a quasar.
    - Instead, ***Lyman alpha forests are more likely to be caused by a smoothly fluctuating density field*** along the line of sight.

- The Lyman alpha absorption systems are generally associated with galaxies, but not always.
  - For instance, 3C 273 lies behind the Virgo cluster of galaxies, and has a couple of absorbers in the cluster's redshift range, but they cannot be clearly identified in position and redshift with specific galaxies in the Virgo cluster.
  - At low redshift, many of the galaxies that are responsible for the DLA absorbers can be directly identified.
    - These galaxies are a heterogeneous population. They are not just the most luminous galaxies, but include dwarf and low surface brightness galaxies.
    - There are even cases where no galaxy has been identified to sensitivity limits.

- ***Column density distribution***
  - There are many more weak lines than strong lines.
  - The column density distribution roughly follows a power-law.



Typical spectrum of a quasar



$$\lambda_{\rm rest}^{\rm Ly\alpha} = 1216\text{Å}$$

$$\lambda_{\rm rest}^{\rm Ly\beta} = 1026\text{Å}$$

$$\lambda_{\rm rest}^{\rm Lyman\ break} = 912\text{Å}$$

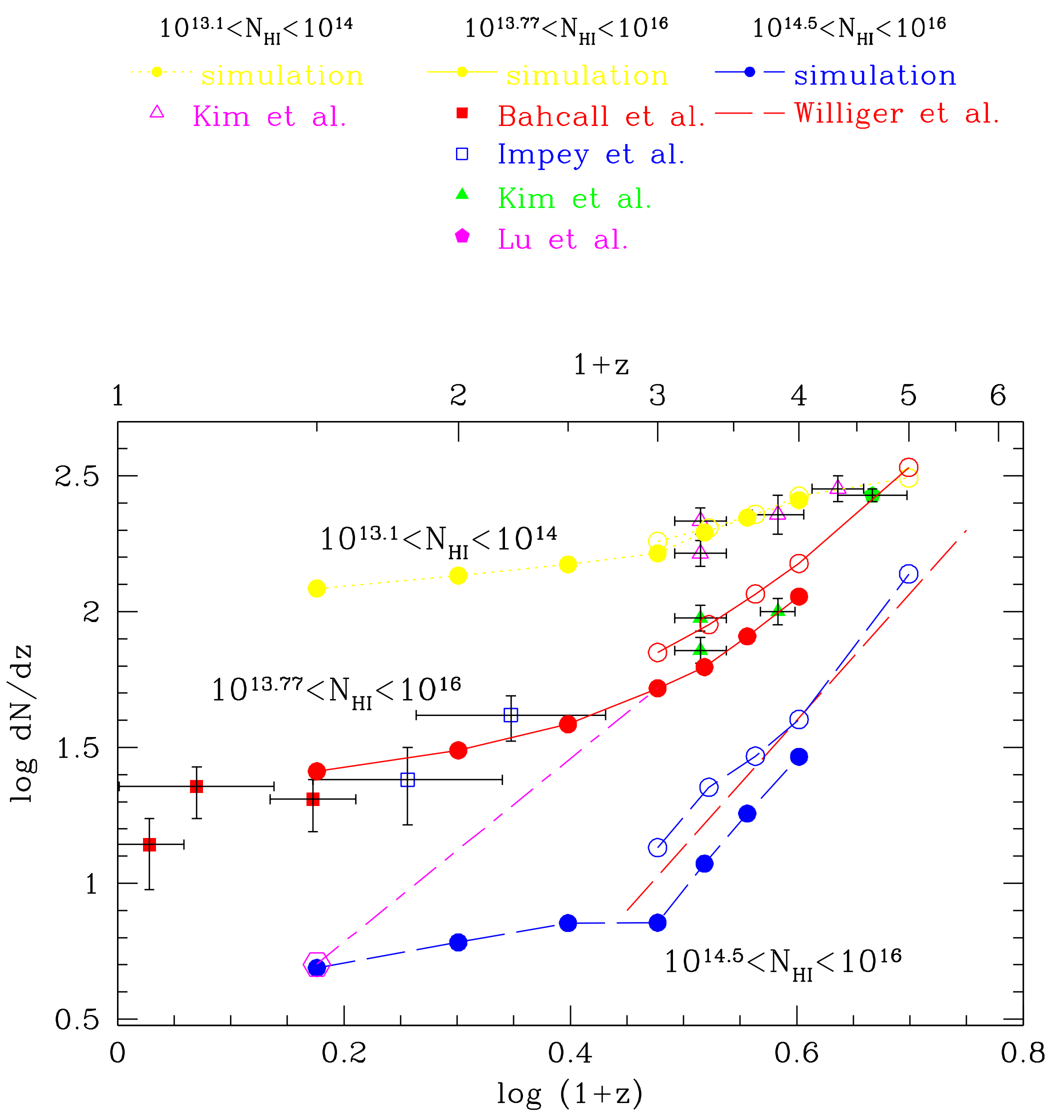
- Typical spectrum of a quasar, showing the quasar continuum and emission lines, and the absorption lines produced by galaxies and IGM that lie between the quasar and the observer.
  - The Lyα forest, absorption produced by various intergalactic clouds, is apparent at wavelengths blueward of the Lyα emission line.
  - The two strongest absorbers, due to galaxies, are a damped Lyα absorber at z = 0.86 ($(1 + 0.86) \times 1216 = 2262\text{Å}$) and a Lyman limit system at z = 1.15 ($(1 + 1.15) \times 1216 = 2614\text{Å}$).
  - The damped Lyα absorber produces a Lyman limit break at ~ 1700Å ($(1 + 0.86) \times 912 = 1696\text{Å}$).
  - The Lyman limit system: a partial Lyman limit break at ~ 1960Å ($(1 + 1.15) \times 912 = 1961\text{Å}$) since the neutral Hydrogen column density is not large enough for it to absorb all ionizing photons.

Evolution of Lyα Absorption Systems

- The Lyα absorption component evolves strongly with cosmic time.
  - We see dramatically ***more absorbers toward higher redshifts***.
  - However, they have not completely disappeared at low redshifts. When the launch of HST provided the first capability of measuring Lyα at low redshifts to the required accuracy, it was found that a few of these absorbers remain in the local Universe.

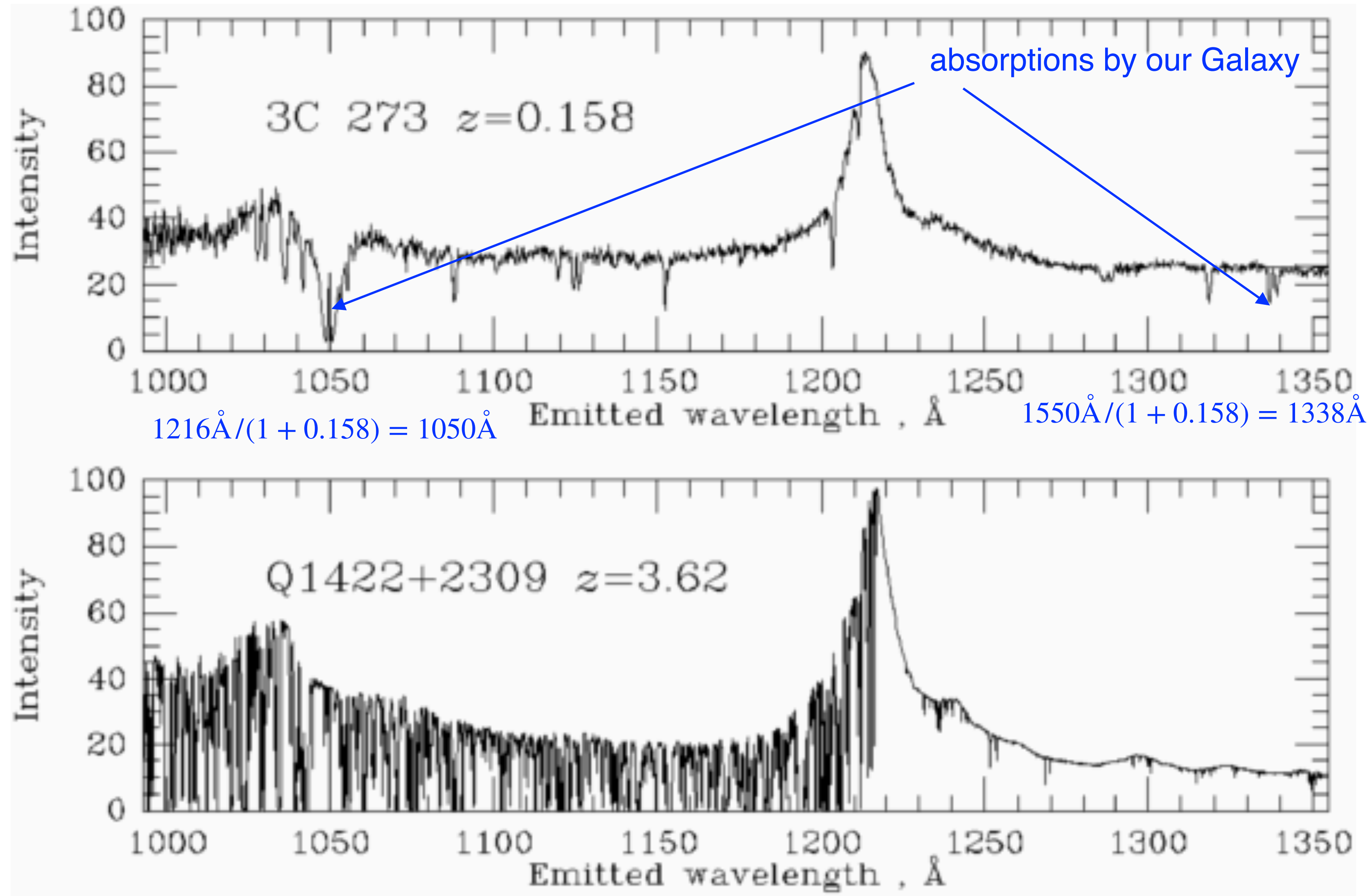
- *The evolution of the Lyα forest may be intimately connected with the history of galaxy formation.*

- This dramatic ***evolution in the number of forest clouds is mostly due to the expansion of the Universe, with a modest contribution from the cosmic structure growth.***

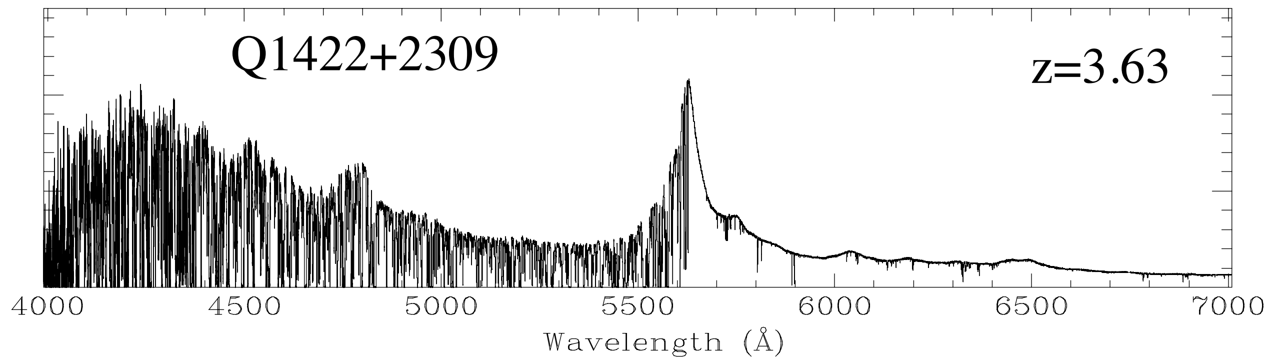


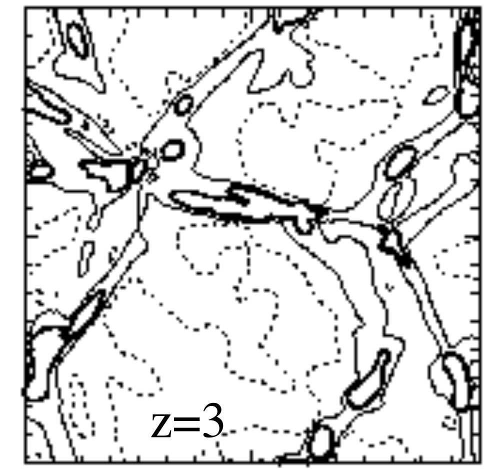
Evolution of the number of lines within a given range of column density obtained from numerical simulations and observations (Efstathiou et al.)

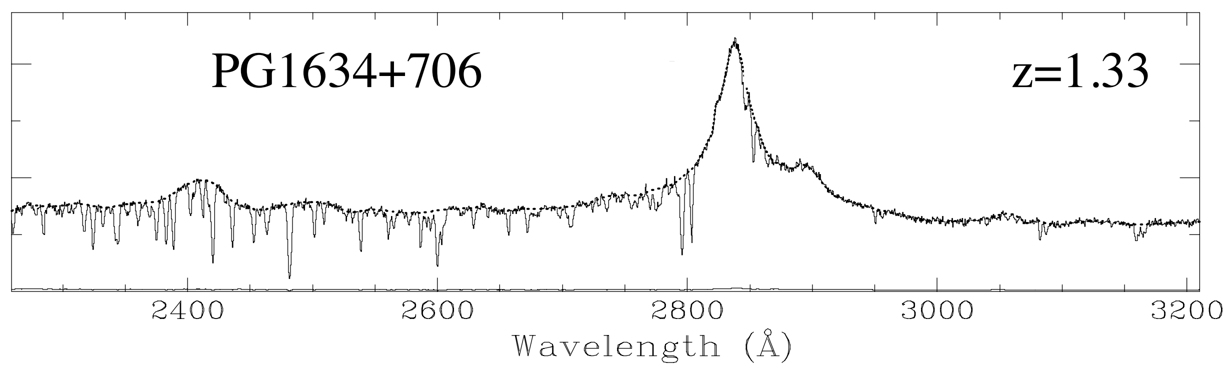
- This figure compares two quasars at very different redshifts, 3C 273 at $z = 0.158$ and 1422+2309 at $z = 3.62$.
- The spectra were shifted to a common scale in emitted wavelength.

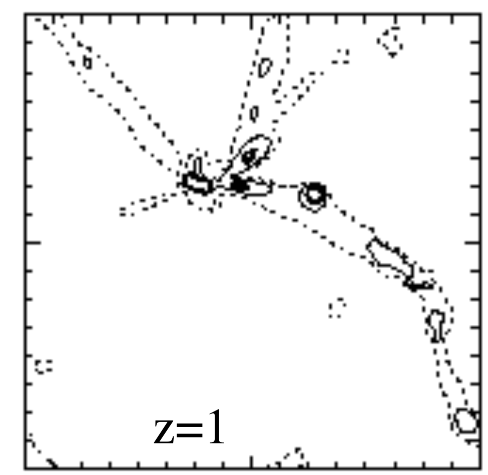


- At low redshift, 3C 273 shows only a handful Lyα absorbers, including the strong and broad absorption from its light intercepting the disk of a foreground spiral galaxy (ours). Our galaxy also produces absorption in the C IV lines around 1550 Å, which appear at 1337 Å in the quasar's emitted frame.
- Hundreds of lines can be identified in the spectrum of 1422+2309, with the densest concentration near the quasar redshift. The strong and broad emission peak is Lyα, which is almost chopped in half by the onset of the Lyα forest in the high-redshift quasar.
- ***This is a very general feature showing how the density of Lyα absorbers decreases with cosmic time (lower z).***









- Illustration of structure evolution of intergalactic gas from high to low redshift.
  - Higher redshift quasars show a much thicker forest of Lyα lines.
- The right-hand panels show slices through N-body/hydrodynamic simulation results at two epochs z = 3 and z = 1.
  - Three contour levels are shown : $10^{11}\ \mathrm{cm}^{-2}$ (dotted lines), $10^{12}\ \mathrm{cm}^{-2}$ (solid lines) and $10^{13}\ \mathrm{cm}^{-2}$ (thick solid lines).
  - Evolution proceeds so that the voids become more empty and even lower column density material is found in filamentary structures at low redshifts.

Correlation between Density and Temperature

- **Density versus Temperature** in the intergalactic gas
  - In simulations of the evolution of intergalactic gas, it is found that there is **a tight *correlation between density and temperature* at $T < 10^5$ K**.
    - The origin of this correlation lies in the *balance between heating and cooling (adiabatic cooling* due to the expansion of the universe)
    - ***Heating***: With only hydrogen present, heating is done by the electrons ejected during the photoionization of hydrogen. The volumetric heating rate is:

$$\mathcal{G}_{\rm pi} = n_{\rm H^0}\zeta_{\rm pi}\langle E\rangle = n_e n_p \alpha_{\rm A,H}\langle E\rangle$$

$n_e = n_p \approx n_{\rm H}$ in a highly ionized hydrogen gas

$$\boxed{\mathcal{G}_{\rm pi} \approx n_{\rm H}^2\alpha_{\rm A,H}\langle E\rangle}$$

$\langle E\rangle$ = the average kinetic energy of an ejected electron. We need to use the Case A recombination rate coefficient in a highly ionized hydrogen gas, responsible to the Lyman alpha forests.

    - ***Cooling***: The regions that give rise to low column density Lyα lines will cool mainly through adiabatic cooling as the universe expands. During adiabatic expansion, the thermal energy density has the dependence $\mathcal{E} \propto V^{-\gamma}$ (V = volume of a gas element). The volumetric cooling rate is then:

$$\mathcal{L}_{\rm adi} = -\frac{d\mathcal{E}}{dt} = \gamma\frac{\mathcal{E}}{V}\frac{dV}{dt} = \left(\gamma\frac{\mathcal{E}}{V}\right)3\frac{V}{a}\frac{da}{dt}$$

$$V \propto a(t)^3$$

$$H(t) \equiv \frac{\dot{a}}{a}$$

$$\boxed{\mathcal{L}_{\rm adi} = 3\gamma\mathcal{E}(t)H(t)}$$

- Energy Balance:

$$\alpha_{\mathrm{A,H}} \propto T^{-0.72} \quad \text{near } T \sim 10^4\,\mathrm{K}$$

$$n_{\mathrm{H}}^2 \alpha_{\mathrm{A,H}} \langle E \rangle = 3\gamma \left(\frac{3}{2} n_{\mathrm{H}} k T\right) H(t) \longrightarrow n_{\mathrm{H}} = \frac{9}{2} \frac{\gamma k T}{\alpha_{\mathrm{A,H}} \langle E \rangle} H(t) \rightarrow n_{\mathrm{H}} \propto \frac{T}{T^{-0.72}}$$

We then obtain

$$n_{\mathrm{H}} \propto T^{1.72} \quad \rightarrow \quad T \propto n_{\mathrm{H}}^{0.58}$$

$$T = T_0 \left(\frac{n_{\mathrm{H}}}{\overline{n}_{\mathrm{bary}}}\right)^{0.58}$$

Here, T_0 is the temperature when $n_{\mathrm{H}} = \overline{n}_{\mathrm{bary}}$

- **Optical Depth versus Density** in the intergalactic gas
  - The balance equation between the photoionization and radiative recombination is given by

$$\zeta_{\mathrm{pi}} n_{\mathrm{H}^0} = n_e n_p \alpha_{\mathrm{A,H}}$$
$$\zeta_{\mathrm{pi}} (1-x) n_{\mathrm{H}} = x^2 n_{\mathrm{H}}^2 \alpha_{\mathrm{A,H}}$$

$\longleftarrow$ fractional ionization $x \equiv n_e / n_{\mathrm{H}}, \; n_e = n_p$

$$1 - x = x^2 \frac{n_{\mathrm{H}} \alpha_{\mathrm{A,H}}}{\zeta_{\mathrm{pi}}}$$

- After the epoch of reionization $z \sim 8$, the neutral fraction $f_{\mathrm{n}} = 1 - x$ will be much smaller than one (i.e., $x \approx 1$). In this limit, the solution for the neutral fraction is

$$f_{\mathrm{n}} \approx \frac{n_{\mathrm{H}} \alpha_{\mathrm{A,H}}(T)}{\zeta_{\mathrm{pi}}}$$

- Then, the number density of neutral hydrogen atoms is

$$n_{\mathrm{H}^0} = f_{\mathrm{n}} n_{\mathrm{H}} \approx \frac{n_{\mathrm{H}}^2 \alpha_{\mathrm{A,H}}(T)}{\zeta_{\mathrm{pi}}}$$

Using the Case A recombination rate coefficient, $\alpha_{\mathrm{A,H}} \approx 4.2 \times 10^{-13}\,\mathrm{cm}^3\,\mathrm{s}^{-1} \left(\frac{T}{10^4\,\mathrm{K}}\right)^{-0.72}$, we find that

$$n_{\mathrm{H}^0} \propto n_{\mathrm{H}}^2 T^{-0.72} \zeta_{\mathrm{pi}}^{-1}$$

- The optical depth for Lyα absorption at a given redshift is proportional to the number density of neutral hydrogen.

$$\tau \propto n_{\mathrm{H}^0} \propto n_{\mathrm{H}}^2 T^{-0.72} \zeta_{\mathrm{pi}}^{-1} \propto n_{\mathrm{H}}^{1.6} \zeta_{\mathrm{pi}}^{-1} \longleftarrow$$

using the density-temperature correlation $T \propto n_{\mathrm{H}}^{0.58}$

- Properly normalizing, we obtain the relation between the density and optical depth:

$$\tau = \overline{\tau} \left(\frac{n}{\overline{n}_{\mathrm{bary}}} \right)^{1.6} \longleftarrow n \propto n_{\mathrm{H}}$$

- The constant $\bar{\tau}$ depends on the assumed cosmology as well as on the amount of ionizing radiation present.
- The above equation is referred to as the ***fluctuating Gunn-Peterson approximation***.

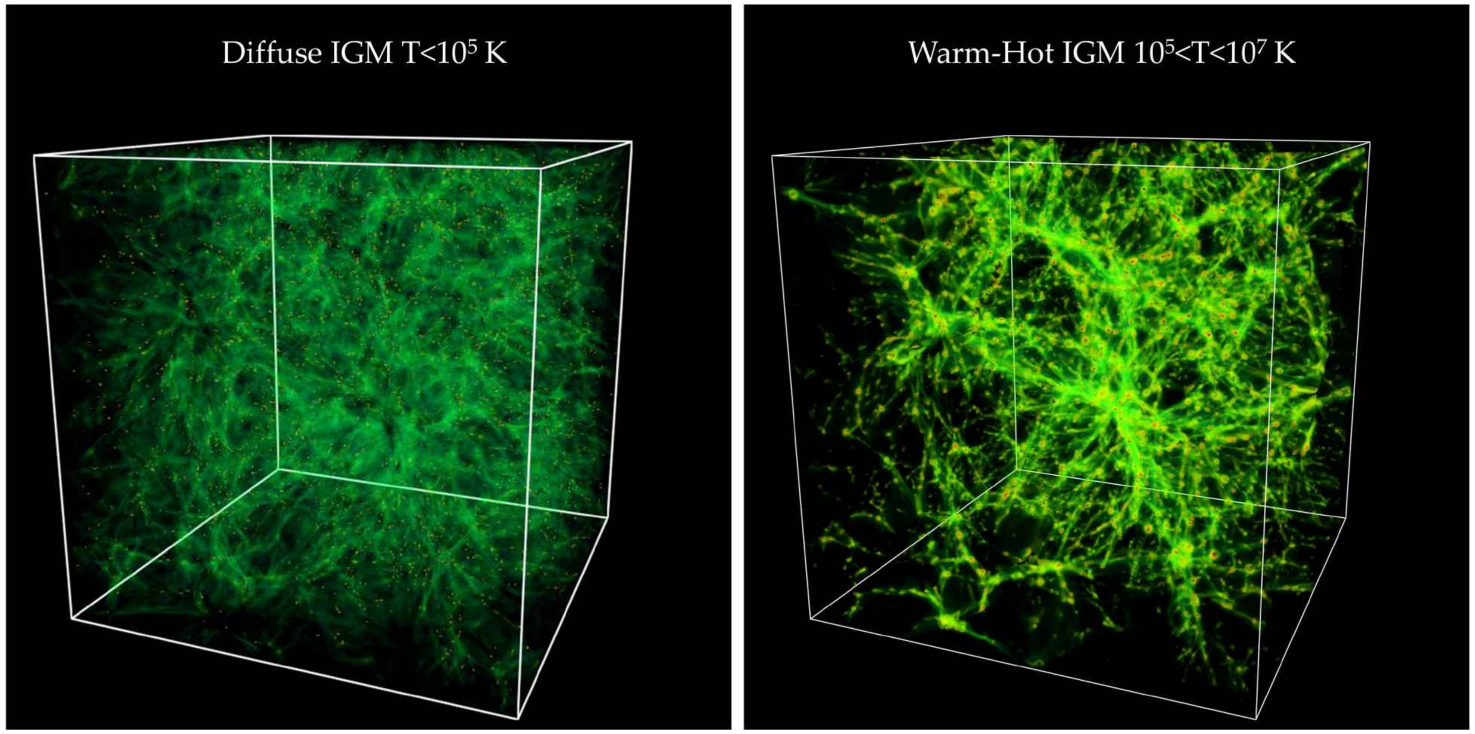
Warm-Hot Intergalactic Medium

- The Warm-Hot Intergalactic Medium (WHIM)
  - The WHIM is at temperature $10^5\,\mathrm{K} < T < 10^7\,\mathrm{K}$,

    and has a density in the range $5 \times 10^{-7}\,\mathrm{cm}^{-3} < n < 5 \times 10^{-5}\,\mathrm{cm}^{-3}$ $\quad (n \approx (2 - 200) \times \overline{n}_{\mathrm{bary},0})$
  - These low densities and relatively high temperatures account for the difficulty of observing the WHIM.
- **Missing baryon problem**
  - The baryonic density has been fairly well known from Big Bang Nucleosynthesis and from early observations of the CMB by the COBE satellite.
  - However, the density in easily detected baryons — stars, interstellar gas, and X-ray emitting gas in clusters — was only $\sim 0.1\bar{n}_{\mathrm{bary},0}$.
  - It is believed that **the unobserved baryons are in a low-density gas spread through intergalactic space**.

Simulations of the WHIM

- Much of what we know about the WHIM comes from numerical cosmological simulations that include gas dynamics.
  - Davé et al. (2001, ApJ, 552, 473) have performed, for the first time, simulations of the intergalactic medium, and found that baryons in the universe reside in four broad phases, defined by their over density $\delta \equiv n/\overline{n} - 1$ and temperature T.
    - Diffuse IGM: $\delta < 1000,\ T < 10^5\,\mathrm{K}$. Photoionized intergalactic gas that gives rise to Lyman alpha absorption.
    - Condensed: $\delta > 1000,\ T < 10^5\,\mathrm{K}$. Stars and cool galactic gas.
    - Hot intracluster medium: $T > 10^7\,\mathrm{K}$. Gas in galaxy clusters and large groups.
    - Warm-Hot: $10^5 < T < 10^7\,\mathrm{K}$. The "warm-hot intergalactic medium."

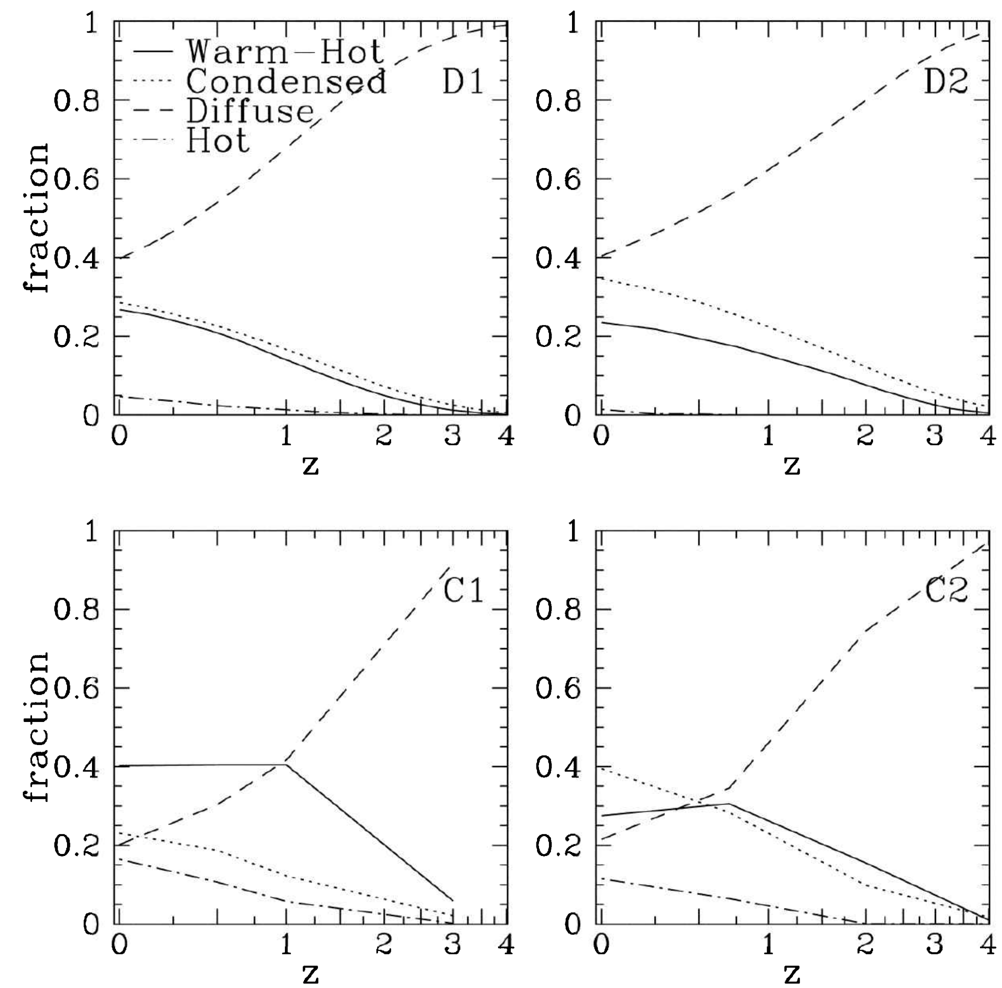


(Left) Distribution of diffuse intergalactic gas at z = 0. (Right) Distribution of warm-hot intergalactic gas at z = 0.

green : $n = 10\,\overline{n}_{\mathrm{bary},0}$
red: $n = 10^4\,\overline{n}_{\mathrm{bary},0}$

[Renyue Cen; Fig 10.1, Ryden]

- Summary:
  - At $z = 4$, shortly after reionization is complete, nearly all the baryonic gas was in the form of photo-ionized gas with $T < 10^5\,\mathrm{K}$.
  - As structure went nonlinear and collapsed, more and more of the baryonic gas became shock-heated to temperatures $10^5\,\mathrm{K} < T < 10^7\,\mathrm{K}$ (WHIM).
  - The WHIM grew steadily with time until it composed 30-40% of the baryonic matter today.
  - “Condensed” gas represents galaxies containing stars, interstellar gas, and circumgalactic gas.
  - “Hot” gas is the intracluster gas at $T > 10^7\,\mathrm{K}$.
- Difference between the DIM & WHIM
  - The diffuse intergalactic medium (DIM) is smoothly distributed.
  - The WHIM is found primarily in long filaments. As it flows along filaments to the clusters, the WHIM is shocked and heated to higher temperatures than the photo-ionized DIM.



Evolution of the fraction of baryonic matter in each of four components.
Four simulation results are shown, with different gas physics and different spatial resolutions.

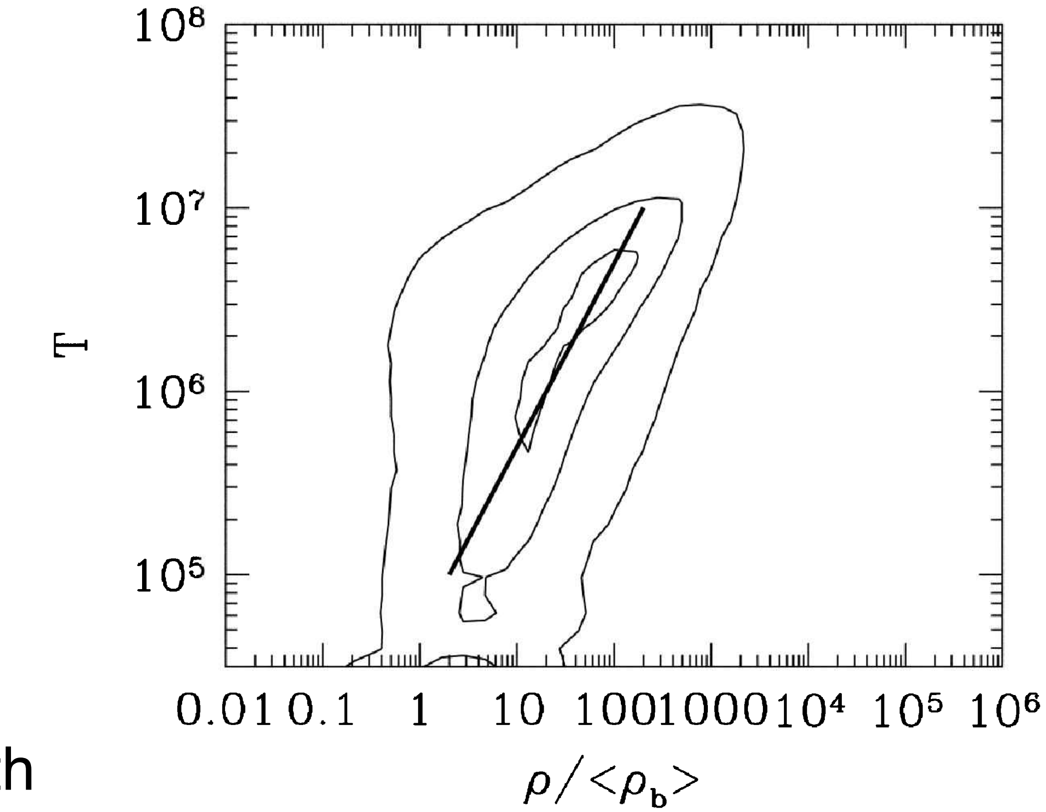
[Fig 10.2, Ryden; Dave et al. 2001]

- Density-Temperature of the WHIM
  - The density is positively correlated with temperature (so there is no pressure equilibrium).

$$\frac{n}{20\,\overline{n}_{\mathrm{bary},0}} = \frac{T}{10^6\,\mathrm{K}}$$

$$20\,\overline{n}_{\mathrm{bary},0} \approx 5\times10^{-6}\,\mathrm{cm}^{-3}$$

- Metallicity
  - The metallicity of intergalactic gas reaches $Z \sim 0.3Z_{\odot}$ primarily in dense virialized clusters, contaminated with gas ejected by supernovae.
  - Along the WHIM filaments, a metallicity $Z \sim 10^{-3}Z_{\odot}$ is more typical.
  - At the low metallicity of the WHIM, bremsstrahlung dominates the cooling down to a temperature as low as $T \sim 10^6\,\mathrm{K}$.
  - The WHIM has a temperature that is typical of the hot interstellar medium of our Galaxy. However, the WHIM has densities that are smaller by 3 orders of magnitude than the HIM ($n_{\mathrm{HIM}} = 4\times10^{-3}\,\mathrm{cm}^{-3}$).



The distribution of WHIM in the density-temperature plane at $z = 0$.

The contours contain 90%, 50%, and 10% of the baryons, from the outermost contour inward.

[Fig 10.3, Ryden; Dave et al. 2001]

Observations

- **Bremsstrahlung**
  - Observing bremsstrahlung emission from the WHIM is “challenging” (impossible).

cooling rate for ICM: $\mathcal{L}_{\rm ff} \approx 1.77 \times 10^{-29}\,{\rm erg\,cm^{-3}\,s^{-1}} \left(\frac{kT}{10\,{\rm keV}}\right)^{1/2} \left(\frac{n_{\rm H}}{10^{-3}\,{\rm cm^{-2}}}\right)^{2}$

cooling rate for WHIM: $\mathcal{L}_{\rm ff} \approx 5.4 \times 10^{-35}\,{\rm erg\,cm^{-3}\,s^{-1}} \left(\frac{kT}{0.1\,{\rm keV}}\right)^{1/2} \left(\frac{n_{\rm H}}{5 \times 10^{-6}\,{\rm cm^{-2}}}\right)^{2}$

  - Note that, in our Galaxy, seeing bremsstrahlung emission from hot bubbles other than the Local Bubble is impossible.

- **Lines from highly ionized heavy elements**.
  - Consider oxygen, for instance, the most abundance element heavier than helium.
    - $T < 3 \times 10^5\,{\rm K}$: O V and O VI become important.
    - $3 \times 10^5\,{\rm K} < T < 2 \times 10^6\,{\rm K}$: The dominant ionization state of oxygen is helium-like O VII.
    - $T > 2 \times 10^6\,{\rm K}$: O VIII and fully ionized O IX become important.

- **O VI emission line**:
  - The WHIM at $T \sim 3 \times 10^6\,\mathrm{K}$ has a low hydrogen number density:

$$n_\mathrm{H} \sim 6\,\overline{n}_{\mathrm{bary},0} \sim 1.5 \times 10^{-6}\,\mathrm{cm}^{-3} \quad \text{at} \quad T \sim 3 \times 10^6\,\mathrm{K}$$

(from the density-temperature relation)

$$\frac{n}{20\,\overline{n}_{\mathrm{bary},0}} = \frac{T}{10^6\,\mathrm{K}}$$

  - For the metallicity $z \sim 0.01\,Z_\odot$, the abundance ratio is $n_\mathrm{O}/n_\mathrm{H} \sim 5 \times 10^{-6}$. This leads to a number density of oxygen:

$$n_\mathrm{O} \sim 8 \times 10^{-12}\,\mathrm{cm}^{-3}$$

  - Even at its maximum relative abundance, at $T \sim 3 \times 10^6\,\mathrm{K}$, O VI accounts for only 25% of all the oxygen:

$$n_\mathrm{OVI} \sim 2 \times 10^{-12}\,\mathrm{cm}^{-3}$$

  - Since the WHIM is concentrated along filaments of the cosmic web, a line of sight passing through a single filament, whose thickness is ~ 1 Mpc, will contribute a column density:

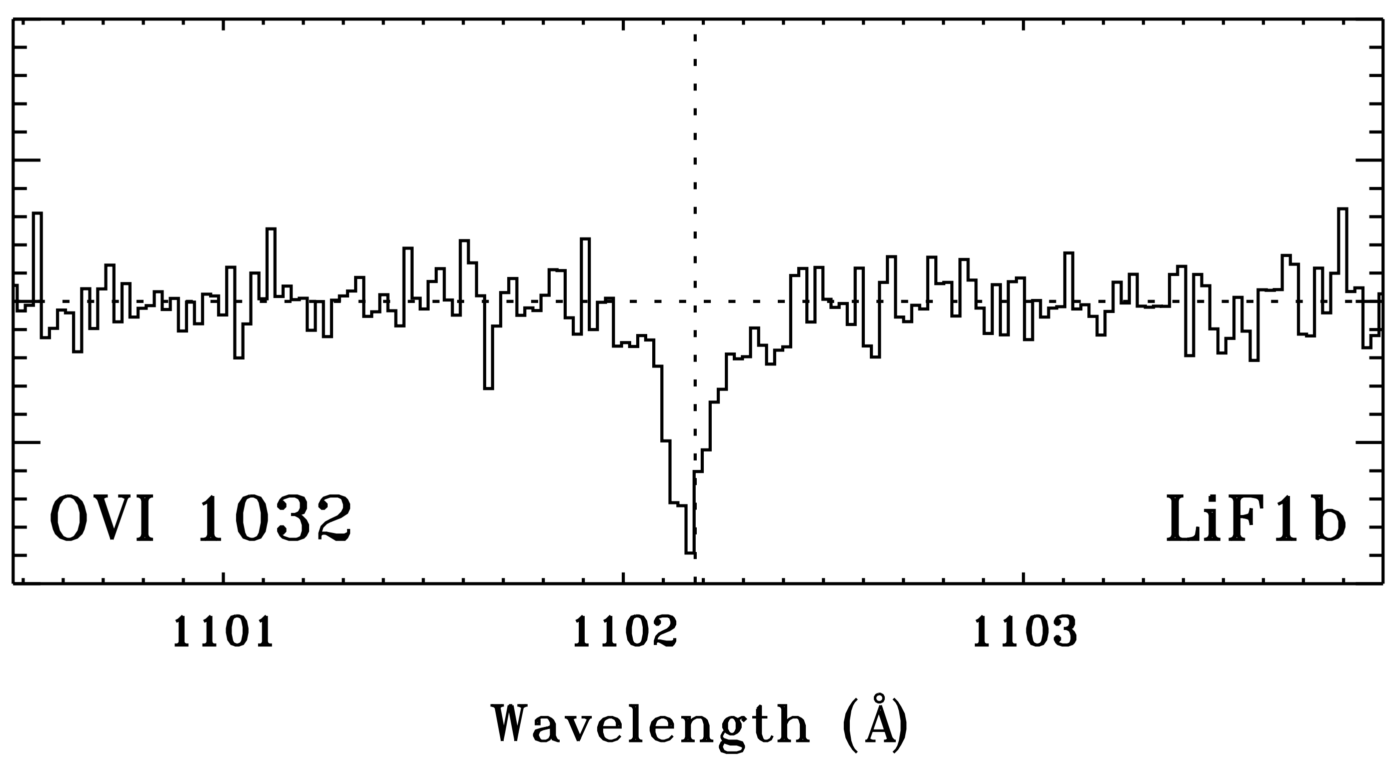
$$N_\mathrm{OVI} \sim n_\mathrm{OVI}\ell \sim 5 \times 10^{12}\,\mathrm{cm}^{-2}\left(\frac{\ell}{1\,\mathrm{Mpc}}\right)$$

  - Measuring a emission line from column density $N_\mathrm{OVI} < 10^{13}\,\mathrm{cm}^{-2}$ is difficult.

- **O VI absorption line**:
  - Danforth & Shull (2008), in the study of UV absorption lines (HST and FUSE) toward bright AGNs, found O VI absorption systems with column densities:

$$N_{\mathrm{OVI}} \sim 8 \times 10^{12} - 5 \times 10^{14}\,\mathrm{cm}^{-2}$$

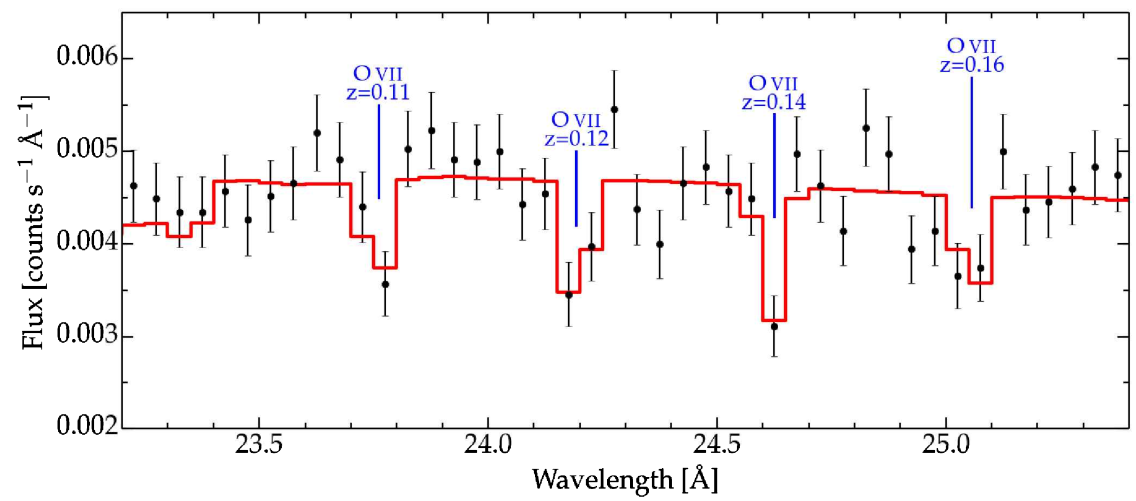
  - The strongest absorption systems, with $N_{\mathrm{OVI}} > 10^{14}\,\mathrm{cm}^{-2}$ had an average Doppler broadening parameter $b \sim 40\,\mathrm{km\,s}^{-1}$, corresponding to $T \sim 10^6\,\mathrm{K}$.
  - They concluded that the WHIM in the temperature range of $10^5\,\mathrm{K} < T < 10^6\,\mathrm{K}$, where O VI absorption is strongest, provides ~ 10% of the baryonic material in the universe.
  - This still leaves a large amount of “missing” baryons in the IGM.



An example spectrum of O VI absorption line in the z = 0.06808 absorber toward PG 0953+414.

Fig 1, Danforth & Shull (2008, ApJ, 679, 194)

- The ***hotter WHIM*** in the range $10^6\,\mathrm{K} < T < 10^7\,\mathrm{K}$ is more difficult.
  - The ion O VII has an X-ray line at $\lambda = 21.6\text{Å}$ (0.57 keV). However, this is not a strong line, and has been found only at a relatively low significance level along a few lines of sight.
  - The right figure shows a simulation of intervening O VII absorption lines at four redshifts along the line of sight to an X-ray bright AGN for a 700 ksec observation (~ 8.1 days) with the Chandra transmission grating instrument.
  - Observations have been very challenging and the results are controversial.



Simulated 700 ksec Chandra LETG/HRC spectrum of B2 1721+34 showing predicted WHIM O VII Kα absorption at redshifts of 0.11, 0.12, 0.14, and 0.16.

[Fig 10.5, Ryden]

- ***The "missing" baryons aren't missing: they just need high-throughput X-ray spectrographs*** with high energy resolution to get their message across to us.