8주차(3/3)

역전파 1

파이썬으로 배우는 기계학습

한동대학교
김영섭 교수

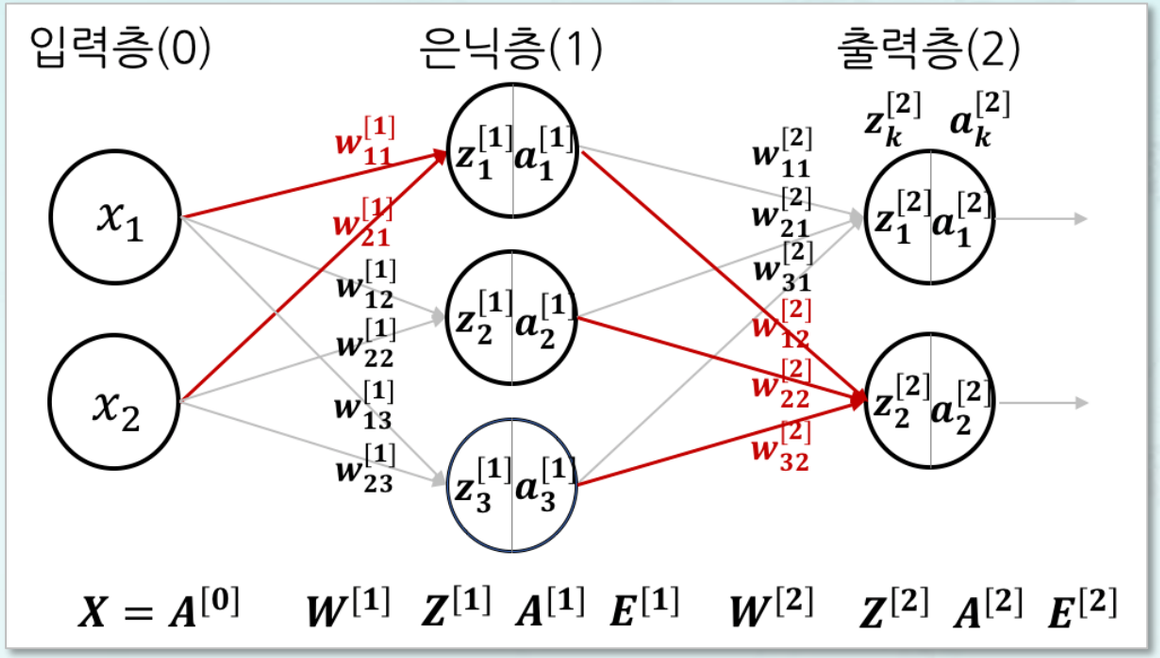
역전파 1

- 학습 목표
  - 역전파 개념의 이해 한다
  - 다층 신경망에서 은닉층 오차를 계산하는 방법을 배운다.
  - 은닉층 오차 계산을 단순화하고 일반화 하는 방법을 배운다.

- 학습 내용
  - 출력층의 오차를 역전파하기
  - 역전파로 은닉층의 오차 계산하기
  - 은닉층 오차 계산의 단순화하고 일반화하기

역전파 1: 신호 처리(복습)

- 다층 구조 인공신경망



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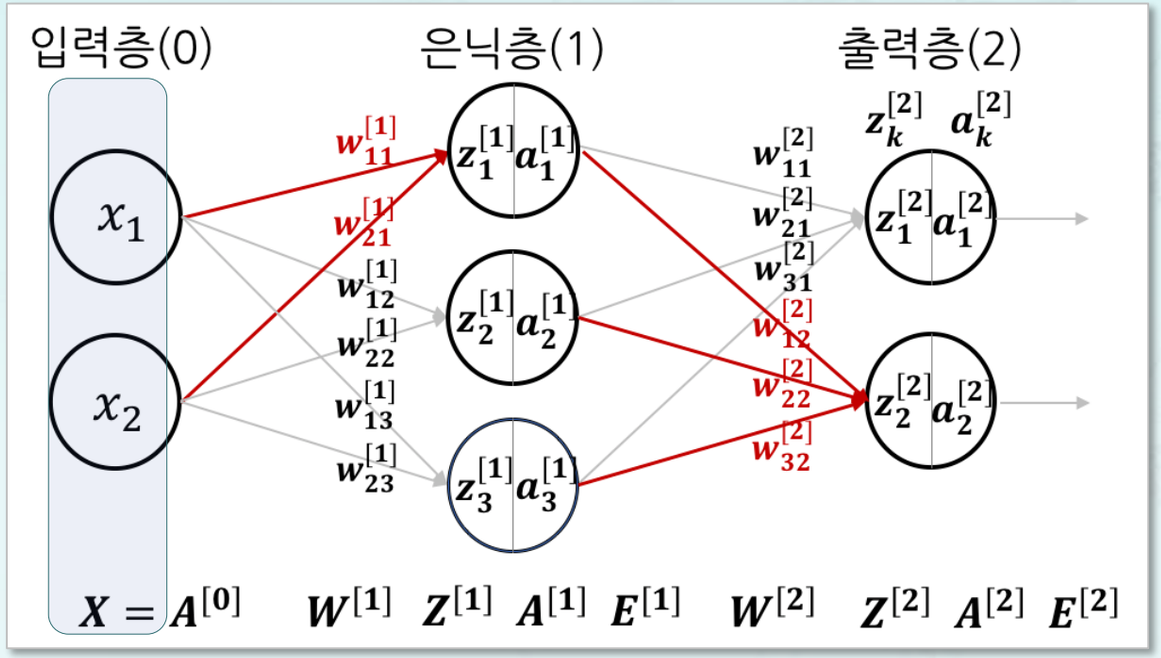

- 신호처리
  - **X** : 입력

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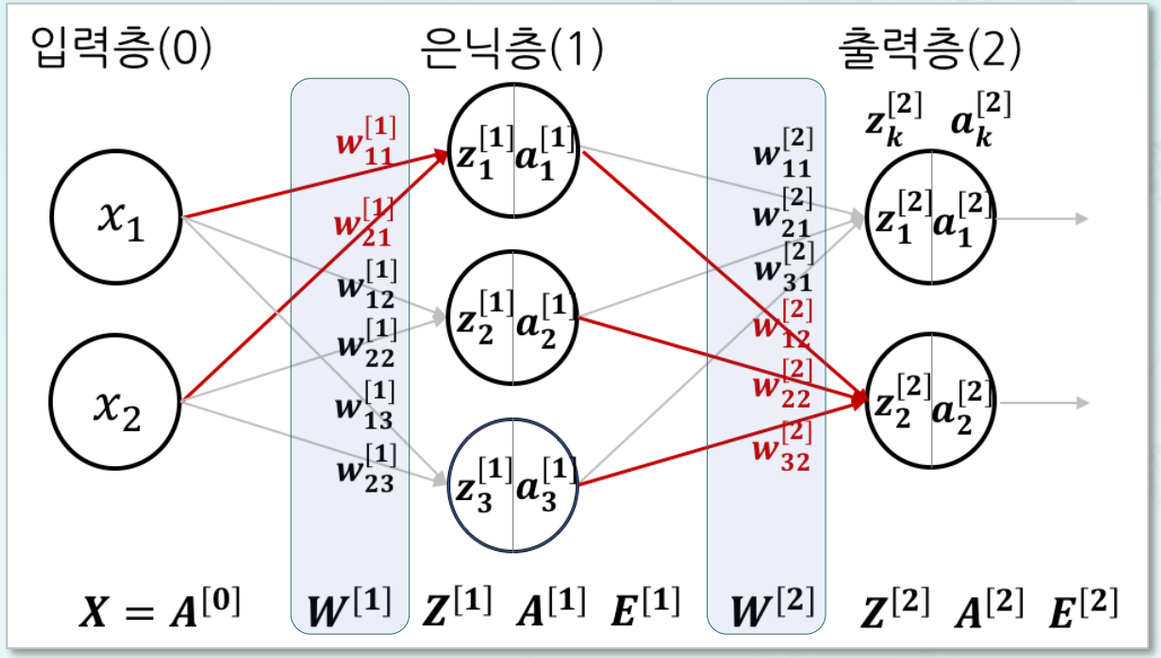

- 신호처리
  - **X** : 입력
  - **W** : 가중치

역전파 1: 신호 처리(복습)

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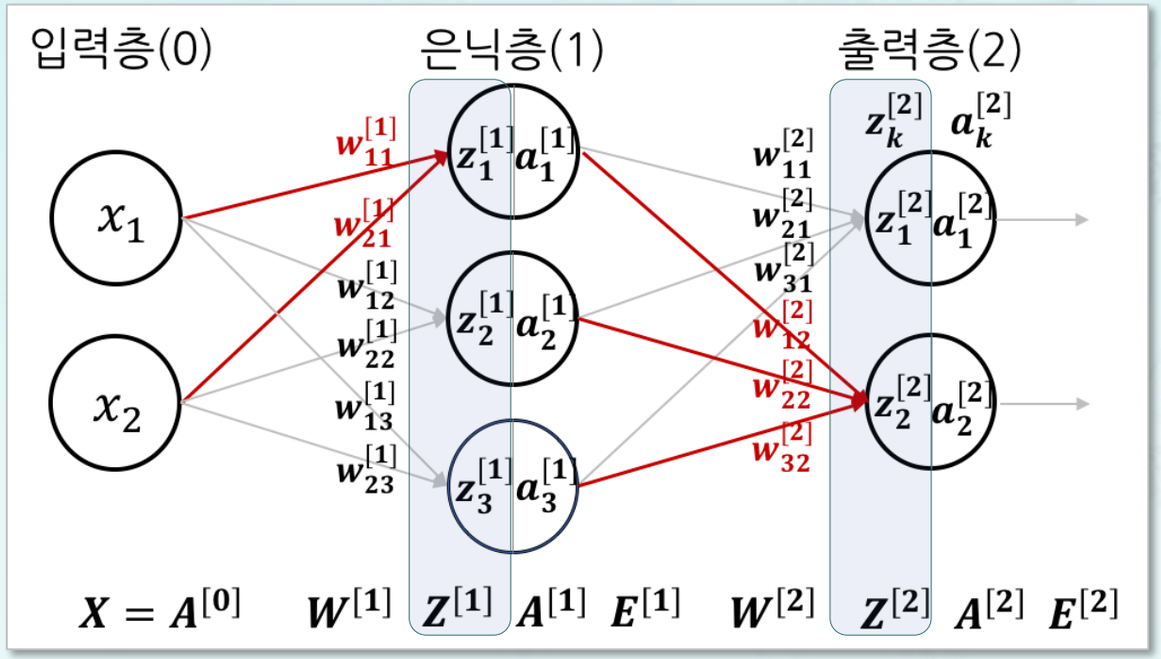

- 신호처리
  - **X** : 입력
  - **W** : 가중치
  - **Z** : 순입력

$$Z^{[l]} = W^{[l]T} \ A^{[l-1]}$$

$$Z^{[2]} = W^{[2]T} \ A^{[1]}$$

역전파 1: 신호 처리(복습)

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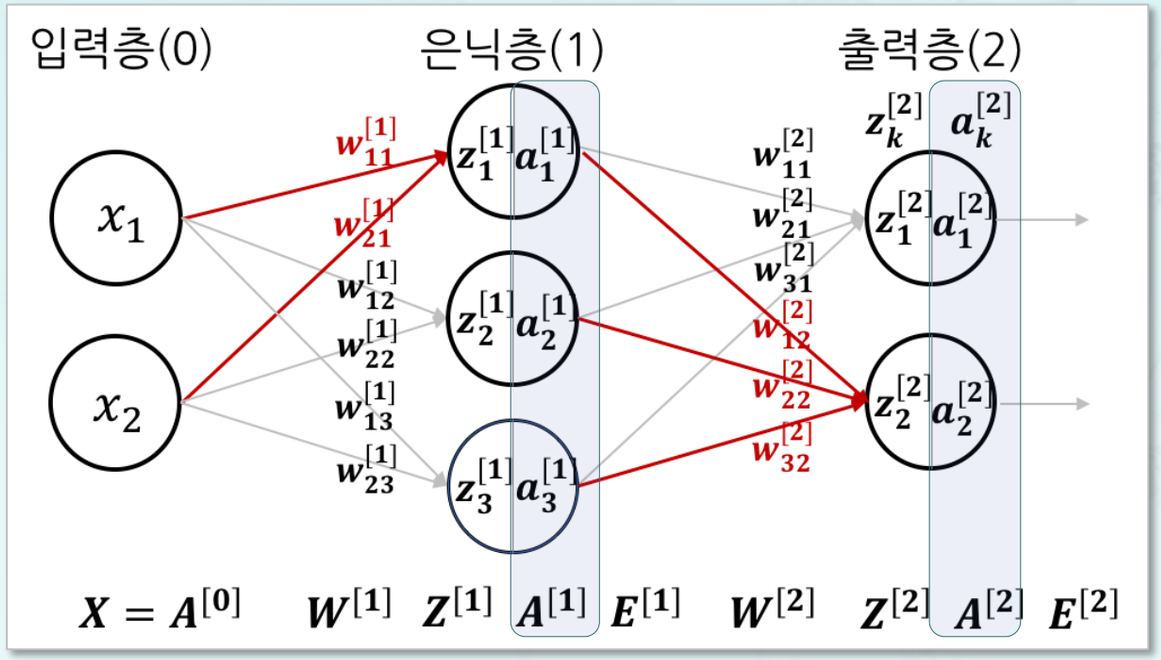

- 신호처리
  - **X** : 입력
  - **W** : 가중치
  - **Z** : 순입력
  - **A** : 출력

$$A^{[l]} = g(Z^{[l]})$$

$$A^{[2]} = sigmoid(Z^{[2]})$$

역전파 1: 신호 처리(복습)

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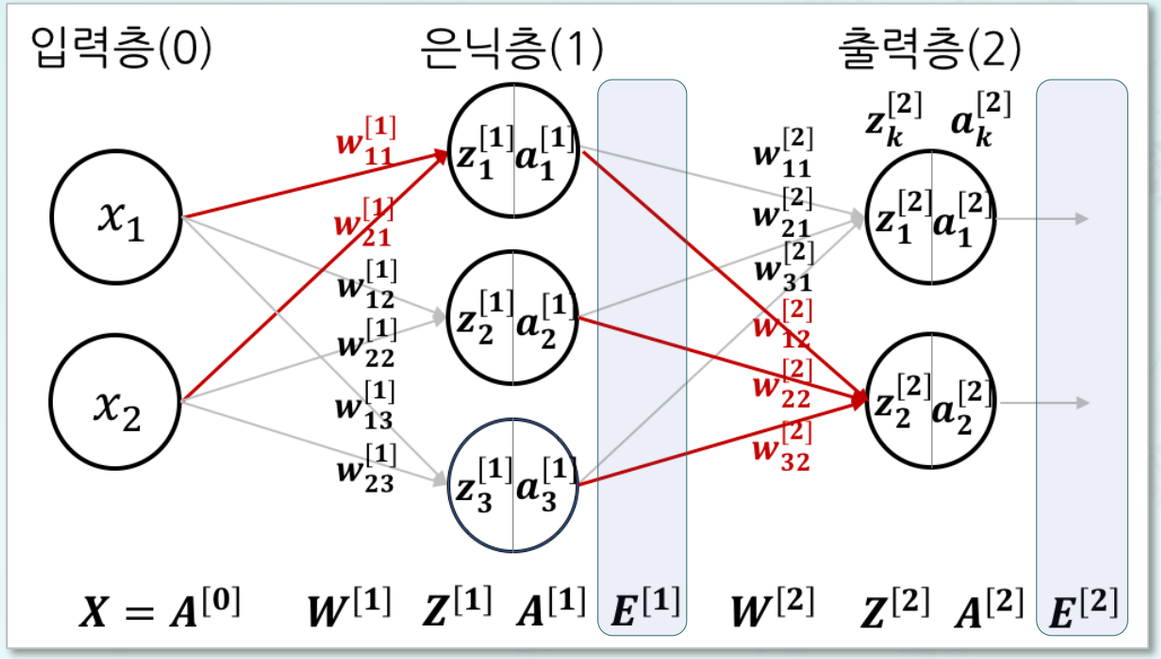
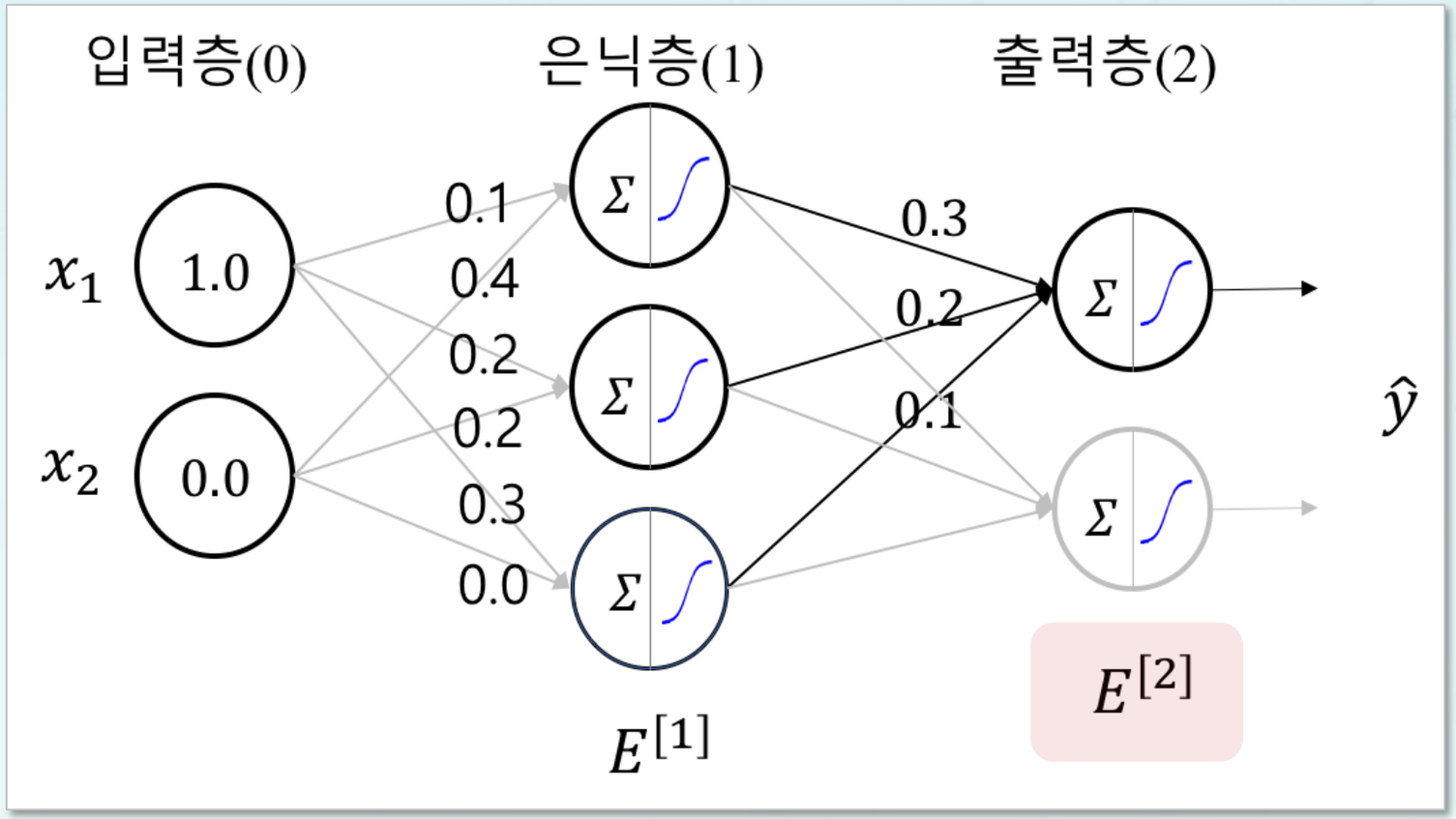


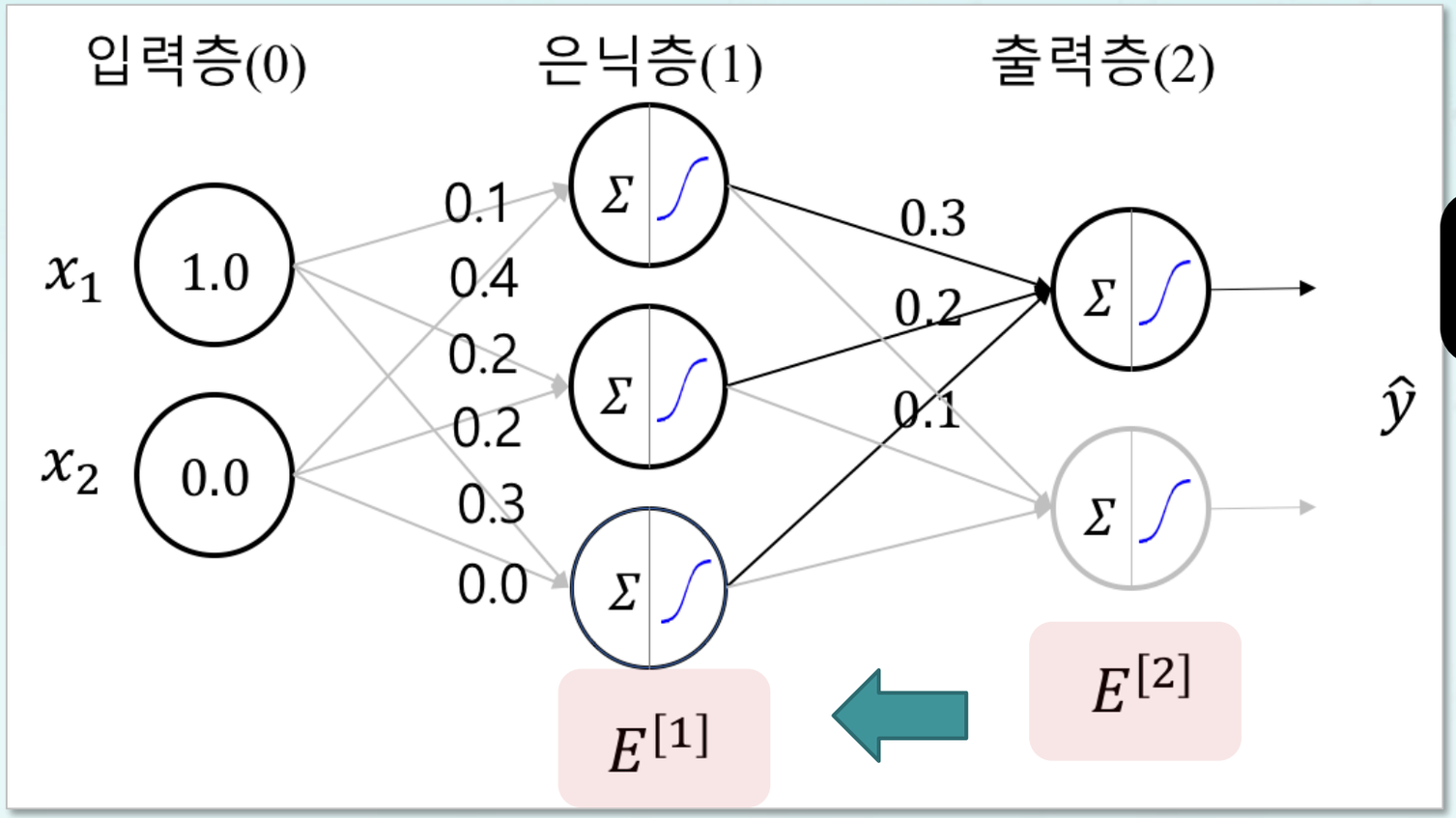
- 신호처리
  - **X** : 입력
  - **W** : 가중치
  - **Z** : 순입력
  - **A** : 출력
  - **E** : 오차

역전파 1: 은닉층의 오차 계산



역전파 1: 은닉층의 오차 계산

- 가정: $y_1 = 1, \quad \hat{y}_1 = 0.58$
- 오차: $e_1 = y_1 - \hat{y}_1 = 0.42$



$$e_1 = y_1 - \hat{y}_1 = 1.0 - 0.58$$

역전파 1: 은닉층의 오차 계산

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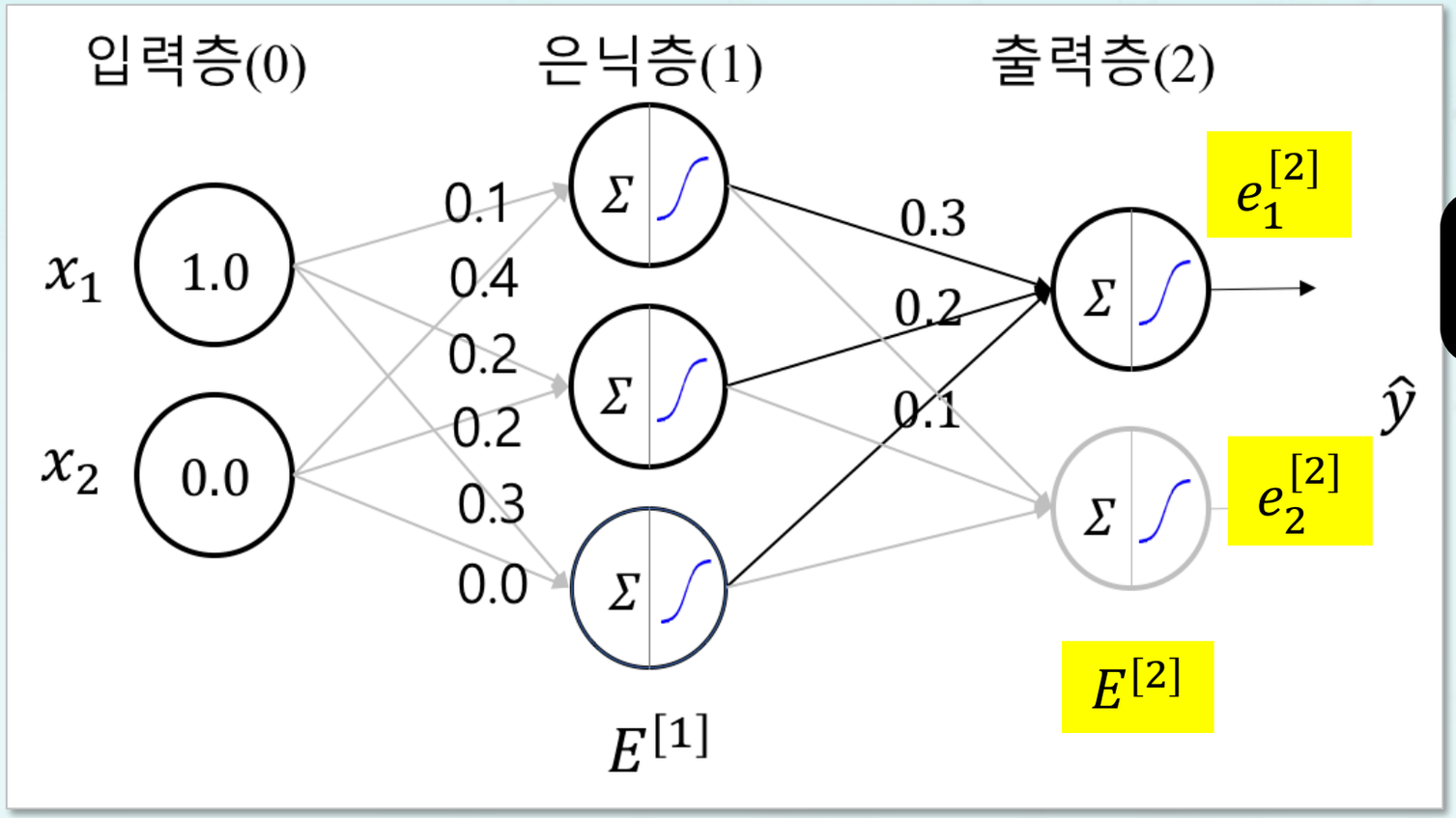

$$e_1 = y_1 - \hat{y}_1 = 1.0 - 0.58$$

$$E^{[2]} = \begin{pmatrix} e_1^{[2]} \\ e_2^{[2]} \end{pmatrix} = \begin{pmatrix} 0.42 \\ \text{미정} \end{pmatrix}$$

역전파 1: 은닉층의 오차 계산

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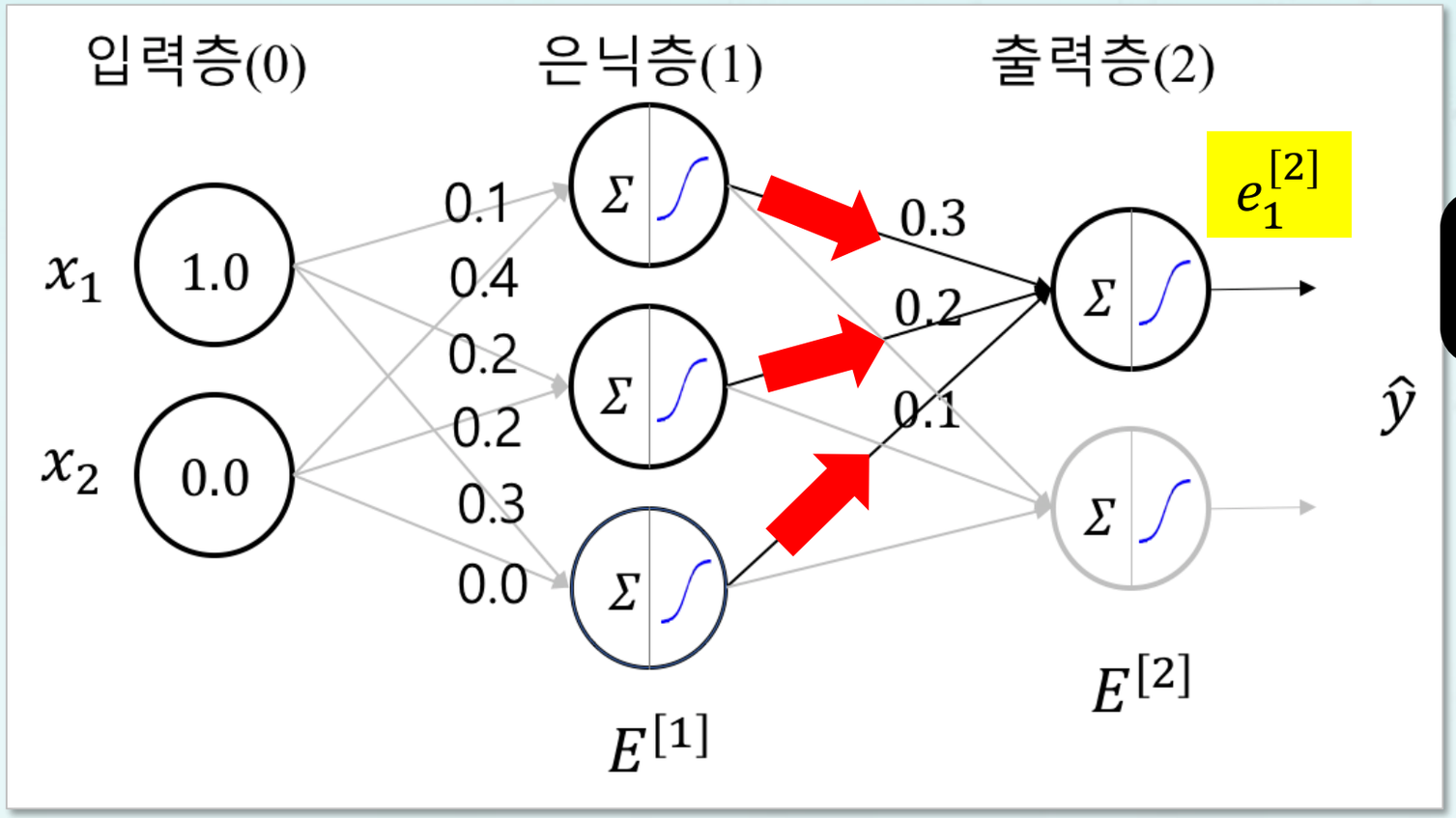

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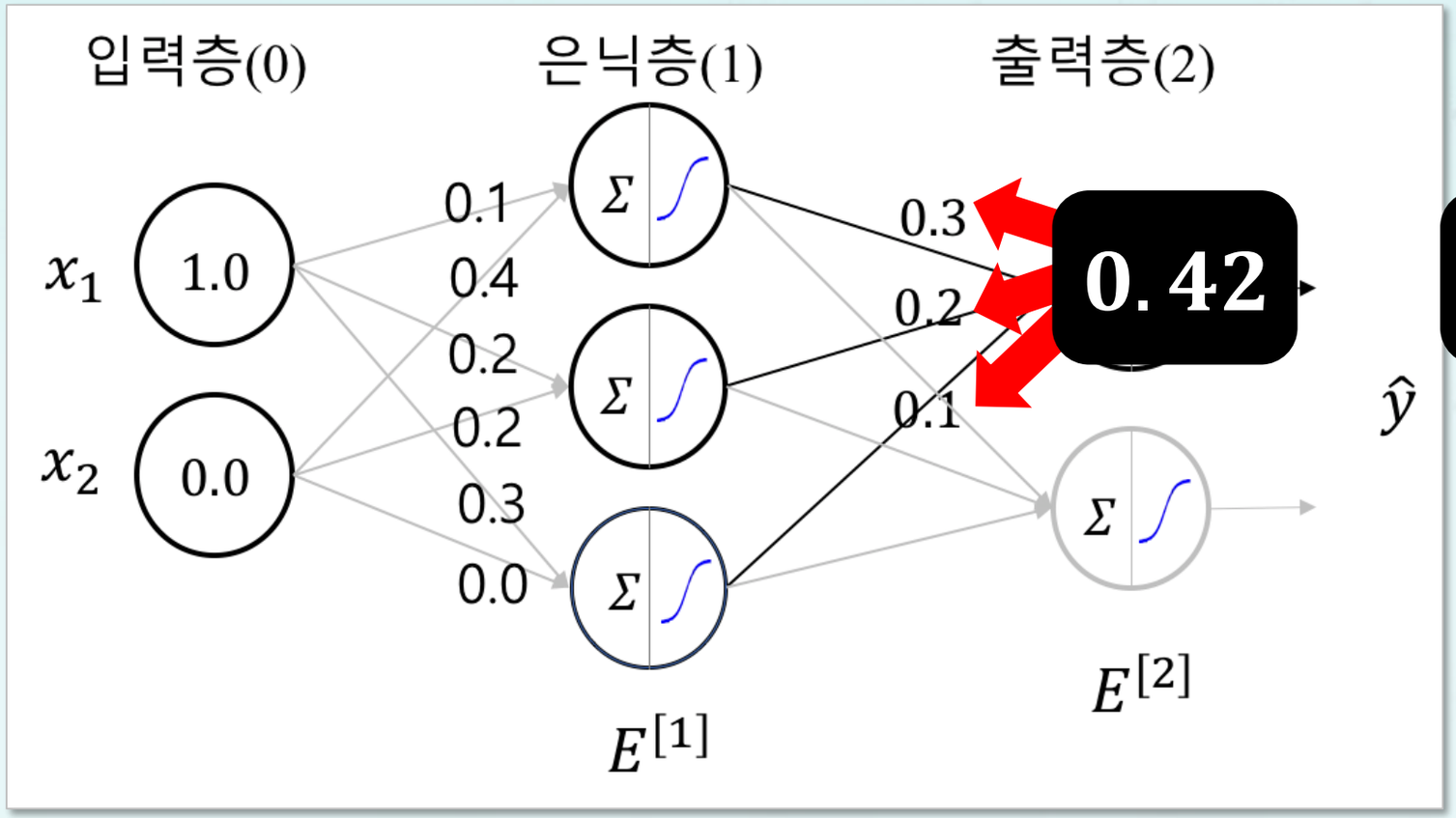

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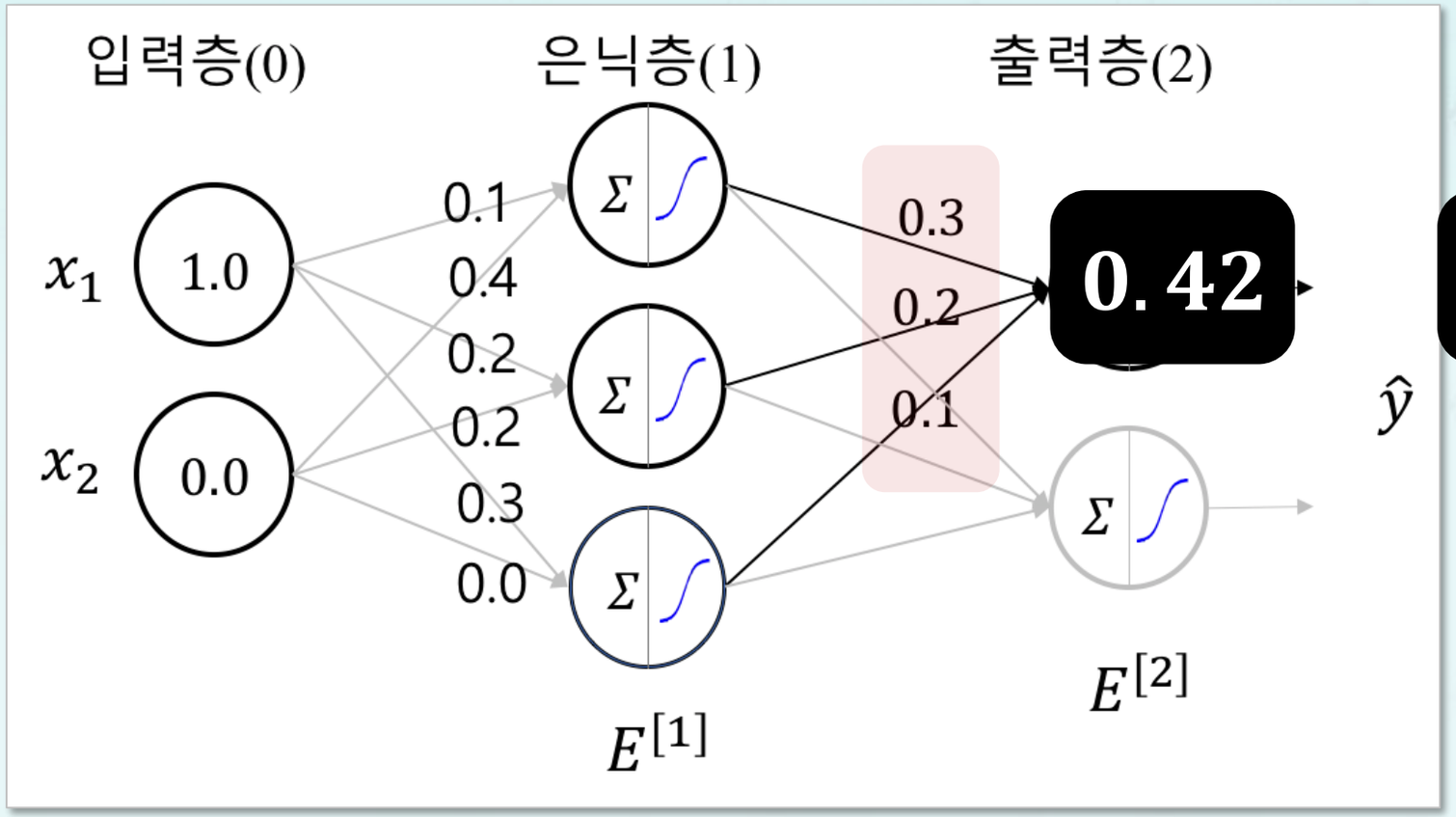

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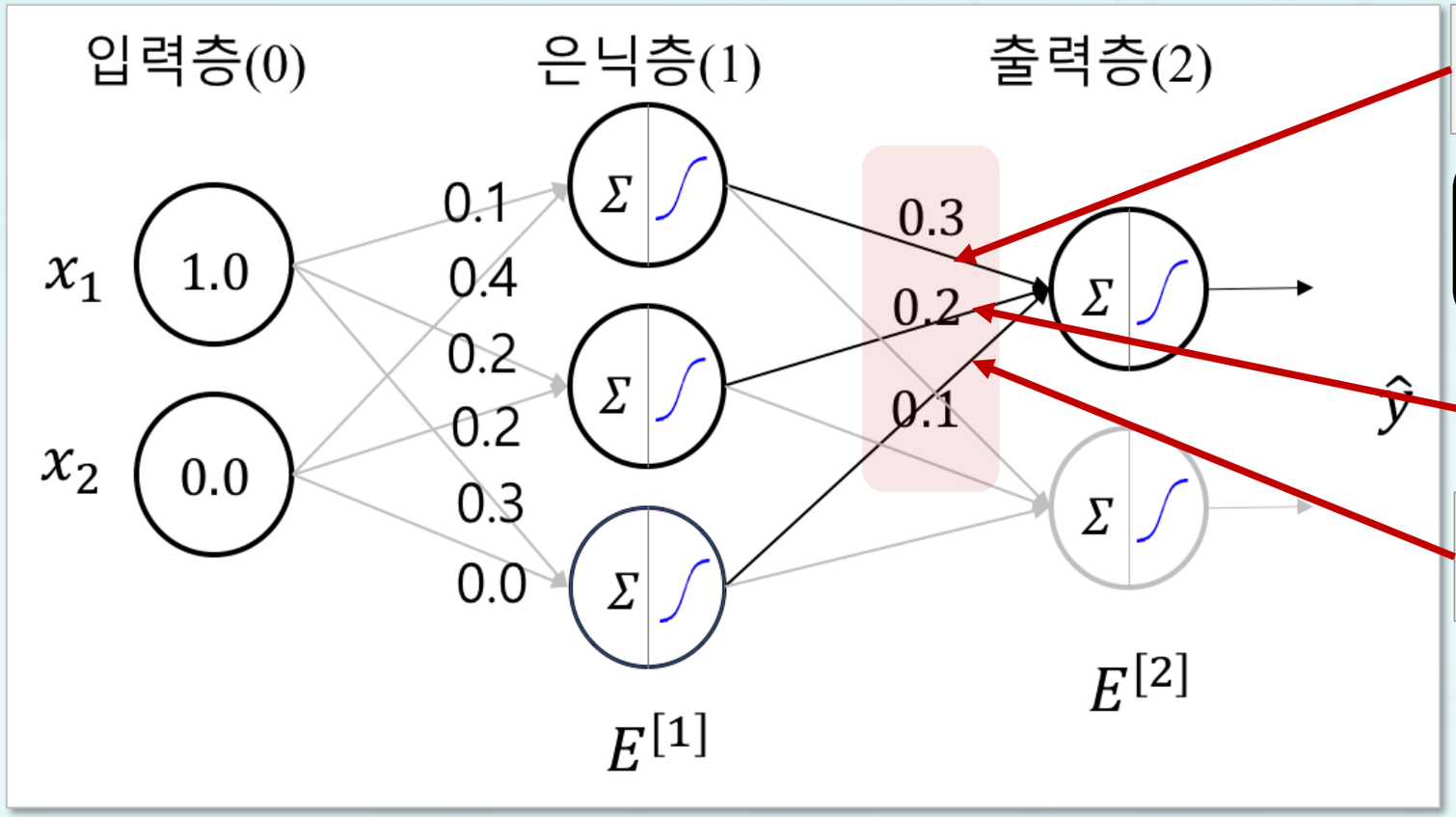


$$0.42 \times \frac{\mathbf{0.3}}{0.3 + 0.2 + 0.1} = 0.21$$

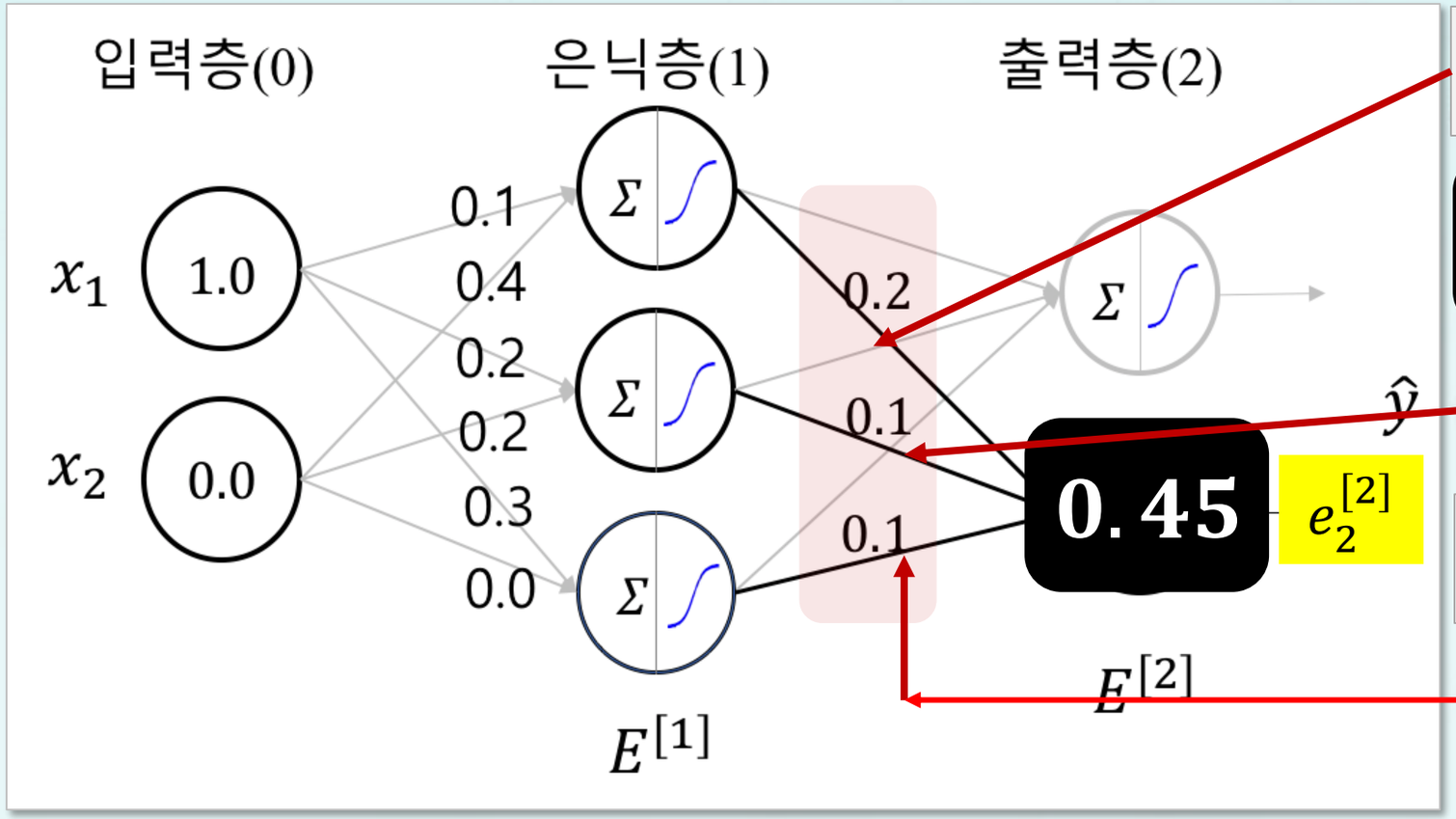
$$\boldsymbol{e_1 = y_1 - \hat{y}_1 = 1.0 - 0.58}$$

$$0.42 \times \frac{\mathbf{0.2}}{0.3 + 0.2 + 0.1} = 0.14$$

$$0.42 \times \frac{\mathbf{0.1}}{0.3 + 0.2 + 0.1} = 0.07$$

역전파 1: 은닉층의 오차 계산

- 가정: $y_2 = 1, \quad \hat{y}_2 = 0.55$
- 오차: $e_2 = y_2 - \hat{y}_2 = 0.45$



$$0.45 \times \frac{\mathbf{0.2}}{0.2 + 0.1 + 0.1} = 0.23$$

$$\boldsymbol{e_2 = y_2 - \hat{y}_2 = 1.0 - 0.55}$$

$$0.45 \times \frac{\mathbf{0.1}}{0.2 + 0.1 + 0.1} = 0.11$$

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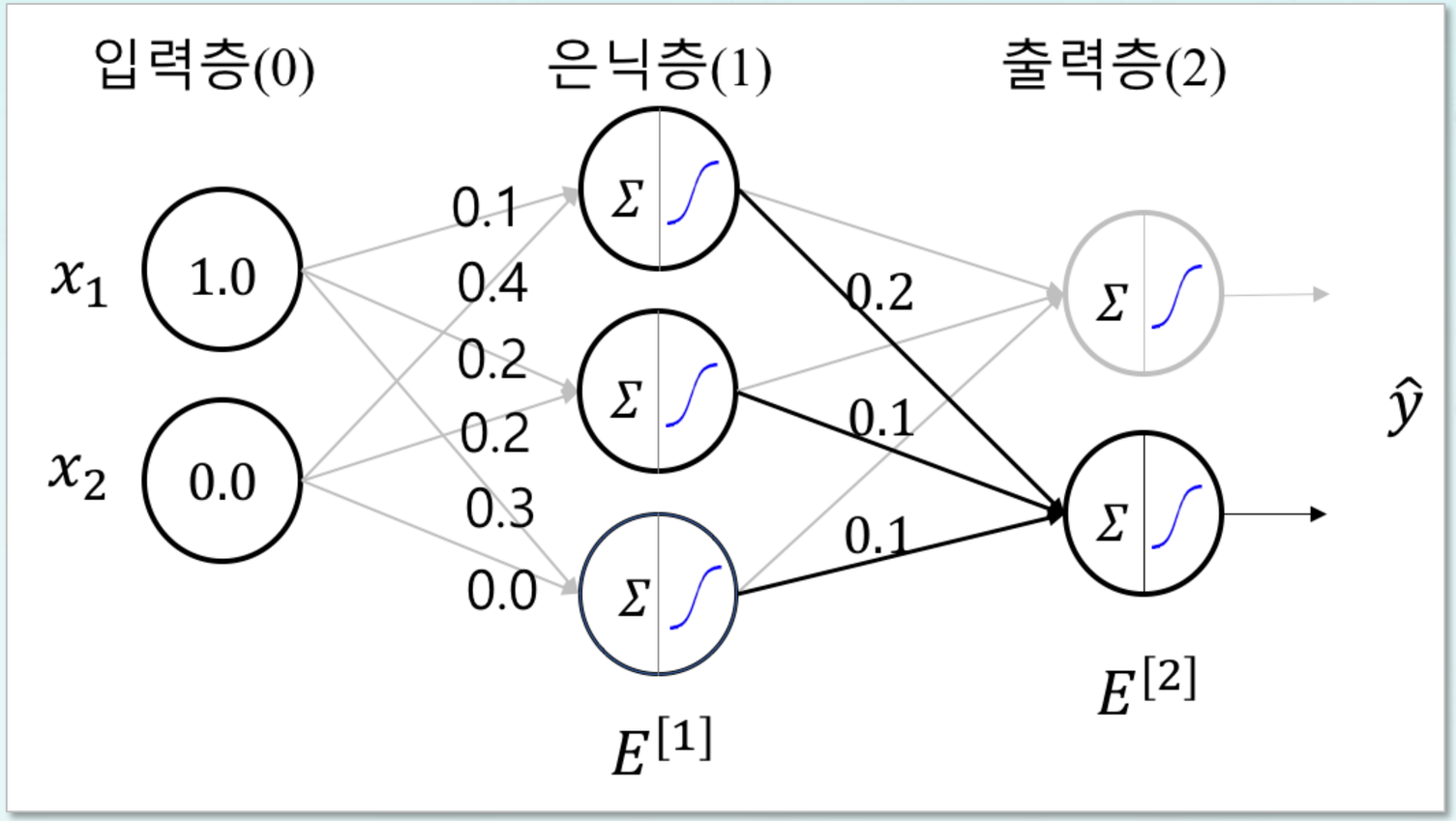
역전파 1: 은닉층의 오차 계산



$$e_1^{[2]} = y_1 - \hat{y}_1$$
$$= 1 - 0.58 = 0.42$$

$$e_2^{[2]} = y_2 - \hat{y}_2$$
$$= 1 - 0.55 = 0.45$$

$$E^{[2]} = \begin{pmatrix} e_1^{[2]} \\ e_2^{[2]} \end{pmatrix} = \begin{pmatrix} 0.42 \\ 0.45 \end{pmatrix}$$

역전파 1: 은닉층의 오차 계산

$$E^{[1]} = \begin{pmatrix} e_1^{[1]} \\ e_2^{[1]} \\ e_3^{[1]} \end{pmatrix} = \begin{pmatrix} e_1^{[2]} \cdot \frac{w_{11}}{\sum_{i=1}^{3} w_{i1}} + e_2^{[2]} \cdot \frac{w_{12}}{\sum_{i=1}^{3} w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{21}}{\sum_{i=1}^{3} w_{i1}} + e_2^{[2]} \cdot \frac{w_{22}}{\sum_{i=1}^{3} w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{31}}{\sum_{i=1}^{3} w_{i1}} + e_2^{[2]} \cdot \frac{w_{32}}{\sum_{i=1}^{3} w_{i2}} \end{pmatrix}$$

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0.42

0.45

$E^{[1]}$ ⟸ $E^{[2]}$

역전파 1: 은닉층의 오차 계산

$$E^{[1]} = \begin{pmatrix} e_1^{[1]} \\ e_2^{[1]} \\ e_3^{[1]} \end{pmatrix} = \begin{pmatrix} e_1^{[2]} \cdot \frac{w_{11}}{\sum_{i=1}^{3} w_{i1}} + e_2^{[2]} \cdot \frac{w_{12}}{\sum_{i=1}^{3} w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{21}}{\sum_{i=1}^{3} w_{i1}} + e_2^{[2]} \cdot \frac{w_{22}}{\sum_{i=1}^{3} w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{31}}{\sum_{i=1}^{3} w_{i1}} + e_2^{[2]} \cdot \frac{w_{32}}{\sum_{i=1}^{3} w_{i2}} \end{pmatrix}$$

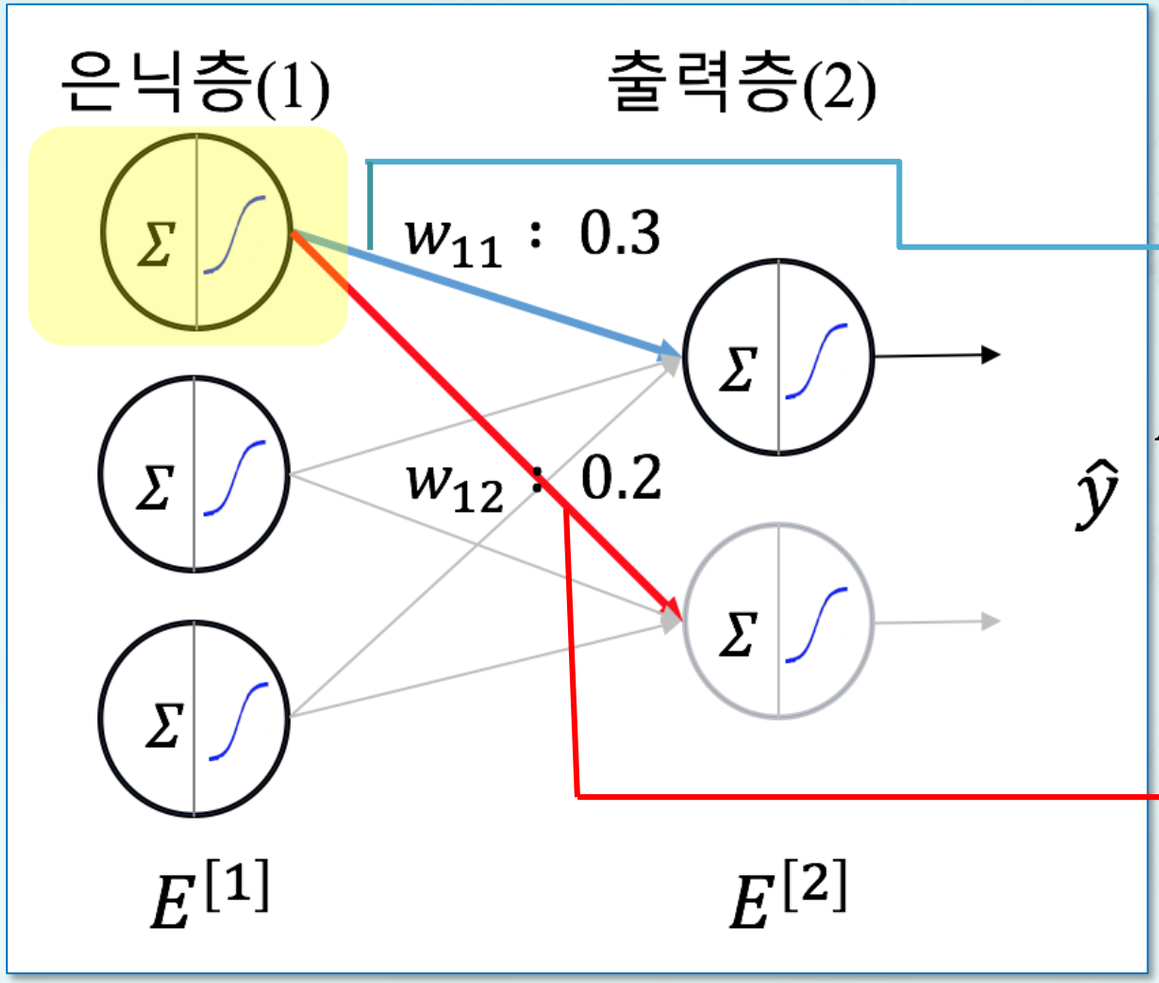
0.42 0.45

$E^{[1]}$ ⇐

$$E^{[2]} = \begin{pmatrix} e_1^{[2]} \\ e_2^{[2]} \end{pmatrix} = \begin{pmatrix} 0.42 \\ 0.45 \end{pmatrix}$$

역전파 1: 은닉층의 오차 계산

- 은닉층의 오차 계산



$$E^{[1]} = \begin{pmatrix} e_1^{[2]} \cdot \frac{w_{11}}{\sum_{i=1}^{3} w_{i1}} + e_2^{[2]} \cdot \frac{w_{12}}{\sum_{i=1}^{3} w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{21}}{\sum_{i=1}^{3} w_{i1}} + e_2^{[2]} \cdot \frac{w_{22}}{\sum_{i=1}^{3} w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{31}}{\sum_{i=1}^{3} w_{i1}} + e_2^{[2]} \cdot \frac{w_{32}}{\sum_{i=1}^{3} w_{i2}} \end{pmatrix}$$

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$$E^{[1]} = \begin{pmatrix} e_1^{[1]} \\ e_2^{[1]} \\ e_3^{[1]} \end{pmatrix} = \begin{pmatrix} e_1^{[2]} \cdot \frac{w_{11}}{\sum_{i=1}^{3} w_{i1}} + e_2^{[2]} \cdot \frac{w_{12}}{\sum_{i=1}^{3} w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{21}}{\sum_{i=1}^{3} w_{i1}} + e_2^{[2]} \cdot \frac{w_{22}}{\sum_{i=1}^{3} w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{31}}{\sum_{i=1}^{3} w_{i1}} + e_2^{[2]} \cdot \frac{w_{32}}{\sum_{i=1}^{3} w_{i2}} \end{pmatrix}$$

$$= \begin{pmatrix} 0.42 \cdot \frac{0.3}{0.3 + 0.2 + 0.1} + 0.45 \cdot \frac{0.2}{0.2 + 0.1 + 0.1} \\ 0.42 \cdot \frac{0.3}{0.3 + 0.2 + 0.1} + 0.45 \cdot \frac{0.2}{0.2 + 0.1 + 0.1} \\ 0.42 \cdot \frac{0.3}{0.3 + 0.2 + 0.1} + 0.45 \cdot \frac{0.2}{0.2 + 0.1 + 0.1} \end{pmatrix}$$

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$$= \begin{pmatrix} 0.42 \cdot \frac{0.3}{0.3+0.2+0.1} + 0.45 \cdot \frac{0.2}{0.2+0.1+0.1} \\ 0.42 \cdot \frac{0.2}{0.3+0.2+0.1} + 0.45 \cdot \frac{0.1}{0.2+0.1+0.1} \\ 0.42 \cdot \frac{0.1}{0.3+0.2+0.1} + 0.45 \cdot \frac{0.1}{0.2+0.1+0.1} \end{pmatrix} = \begin{pmatrix} 0.44 \\ 0.25 \\ 0.18 \end{pmatrix}$$

역전파 1: 오차 계산의 단순화

- 오차 계산의 단순화와 일반화
  - 단순화

$$E^{[1]} = \begin{pmatrix} e_1^{[1]} \\ e_2^{[1]} \\ e_3^{[1]} \end{pmatrix} = \begin{pmatrix} e_1^{[2]} \cdot \frac{w_{11}}{\sum_{i=1}^{3} w_{i1}} + e_2^{[2]} \cdot \frac{w_{12}}{\sum_{i=1}^{3} w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{21}}{\sum_{i=1}^{3} w_{i1}} + e_2^{[2]} \cdot \frac{w_{22}}{\sum_{i=1}^{3} w_{i2}} \\ e_1^{[2]} \cdot \frac{w_{31}}{\sum_{i=1}^{3} w_{i1}} + e_2^{[2]} \cdot \frac{w_{32}}{\sum_{i=1}^{3} w_{i2}} \end{pmatrix}$$

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역전파 1: 오차 계산의 단순화

- 오차 계산의 단순화와 일반화
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$$\boldsymbol{E^{[1]} = W^{[2]T} \cdot E^{[2]}}$$

역전파 1: 오차 계산의 일반화

$$E^{[1]} = \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{pmatrix} \cdot \begin{pmatrix} e_1^{[2]} \\ e_2^{[2]} \end{pmatrix}$$

$$E^{[l]} = \begin{pmatrix} w_{11} & \cdots & w_{1m} \\ \vdots & \ddots & \vdots \\ w_{n1} & \cdots & w_{nm} \end{pmatrix}^T \cdot \begin{pmatrix} e_1^{[l+1]} \\ \vdots \\ e_n^{[l+1]} \end{pmatrix}$$

$$\boldsymbol{E^{[1]}} = W^{[2]\boldsymbol{T}} \cdot \boldsymbol{E^{[2]}}$$

$$E^{[l]} = W^{[l+1]T} \cdot E^{[l+1]}$$

역전파 1

- 학습 정리
  - 출력층으로부터 은닉층의 오차를 계산하기
  - 복잡한 은닉층 오차 계산을 일반화 하기

- 역전파 **2**

8주차(3/3)

역전파 1

파이썬으로 배우는 기계학습

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