Closed-Form Optimal Two-View Triangulation Based on Angular Errors

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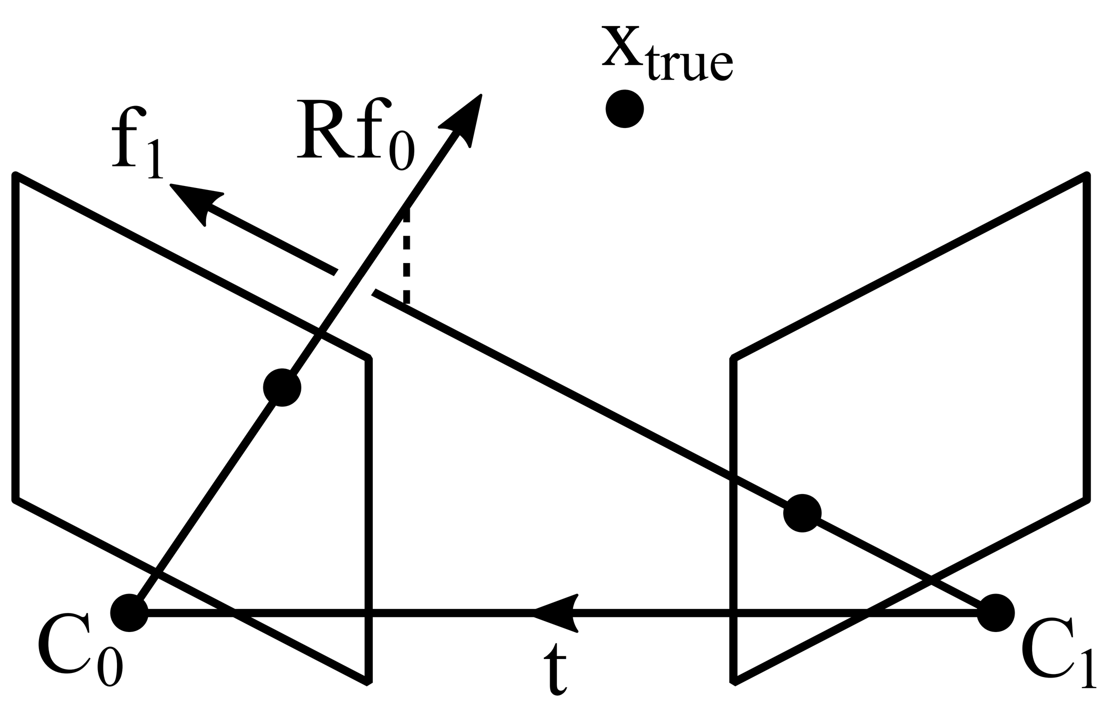
1. Two-View Triangulation

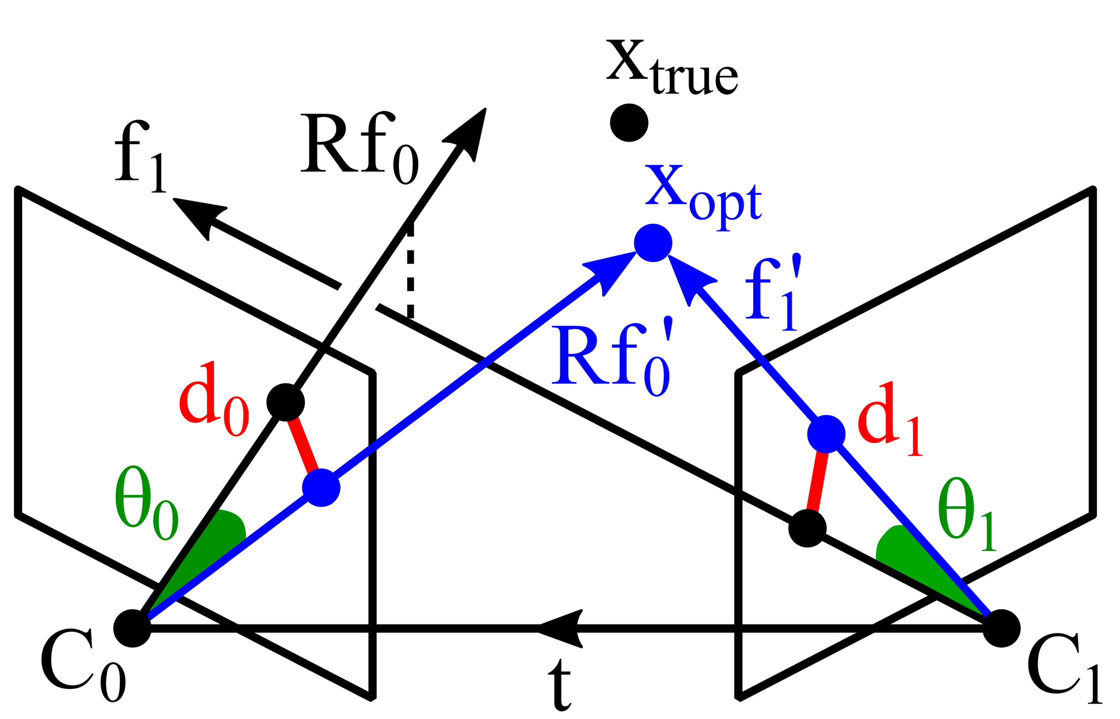
Locating the 3D point given its projections in two views of known calibration and pose.



2. Optimal Method

Correct the rays (f_0 and f_1) to make them intersect with a minimal image/angular reprojection cost, e.g.,

- $\mathbf{L_1}$ **norm**: $d_0 + d_1$ [1] or $\theta_0 + \theta_1$ [ours]
- $\mathbf{L_2}$ **norm**: $d_0{}^2 + d_1{}^2$ [1,4] or $\sin^2(\theta_0) + \sin^2(\theta_1)$ [3, ours]
- $\mathbf{L_\infty}$ **norm**: $\max(d_0,\ d_1)$ [2] or $\max(\theta_0,\ \theta_1)$ [ours]



3. L_1 angle minimization

If $\|R\widehat{f_0} \times t\| \leq \|\widehat{f_1} \times t\|$, then

$$Rf_0' = Rf_0 - (Rf_0 \cdot \widehat{n_1})\,\widehat{n_1} \quad \text{with} \quad n_1 = f_1 \times t$$

$$f_1' = f_1$$

Else

$$Rf_0' = Rf_0,$$

$$f_1' = f_1 - (f_1 \cdot \widehat{n_0})\,\widehat{n_0} \quad \text{with} \quad n_0 = Rf_0 \times t$$

4. L_2 sine of angle minimization

$$Rf_0' = Rf_0 - (Rf_0 \cdot \hat{n})\,\hat{n}$$

$$f_1' = f_1 - (f_1 \cdot \hat{n})\,\hat{n}$$

where $\hat{n}$ is the 2nd column of matrix V from

$$USV^T = SVD\left([R\widehat{f_0} \quad \widehat{f_1}]^T\,(I - \hat{t}\,\hat{t}^T)\right)$$

5. L_∞ angle minimization

$$Rf_0' = Rf_0 - (Rf_0 \cdot \hat{n})\,\hat{n}$$

$$f_1' = f_1 - (f_1 \cdot \hat{n})\,\hat{n}$$

where

$$n = \begin{cases} n_a & \text{if } \|n_a\| \geq \|n_b\| \\ n_b & \text{otherwise} \end{cases}$$

with

$$n_a = \left(R\widehat{f_0} + \widehat{f_1}\right) \times t$$

$$n_b = \left(R\widehat{f_0} - \widehat{f_1}\right) \times t$$

[1] R. Hartley and P. Sturm. *Triangulation*. CVIU. 1997

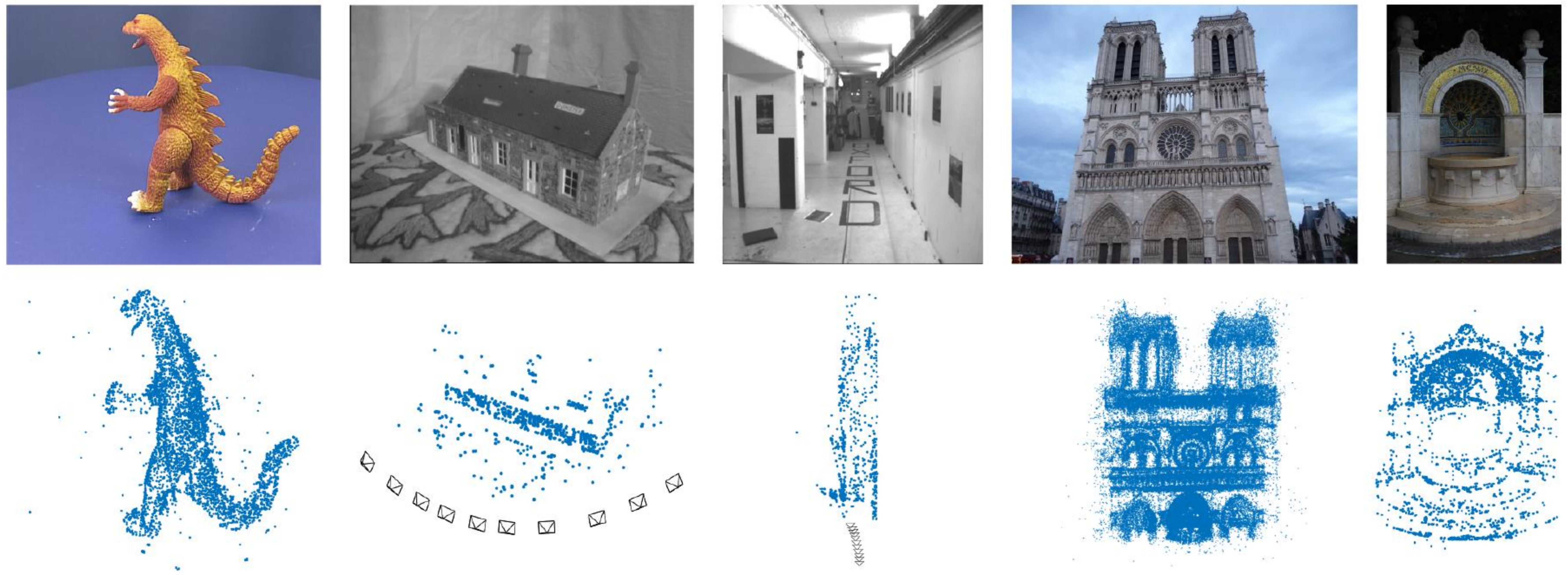
[2] D. Nister. *Automatic Dense Reconstruction from Uncalibrated Video Sequences*. PhD thesis. 2001

[3] J. Oliensis. *Exact Two-Image Structure from Motion*. TPAMI. 2002

[4] P. Lindstrom. *Triangulation made easy*. CVPR. 2010

[5] S. Lee and J. Civera. *Triangulation: Why Optimize?*. BMVC. 2019

6. Qualitative results of the proposed L_1 method (median)



7. Percentage of the total experiments (>5,5M) for which each method yields the lowest error in given criterion

Error Criterion	Midpoint [1]	L_1 img [1]	L_2 img [1]	L_2 img 5 it. [4]	L_∞ img [2]	L_1 ang	L_2 ang	L_∞ ang
$\theta_0 + \theta_1$	-	-	-	-	-	**100 %**	-	-
$\theta_0^2 + \theta_1^2$	-	-	7e-5 %	5e-5 %	-	-	**99.9999 %**	-
$\sin^2(\theta_0) + \sin^2(\theta_1)$	-	-	-	-	-	-	**100 %**	-
$\max(\theta_0, \theta_1)$	-	-	-	-	-	-	-	**100 %**
$d_0 + d_1$	-	**70.84%**	0.002%	0.002%	-	29.16 %	-	-
$d_0^2 + d_1^2$	-	-	23.14 %	**76.86 %**	-	-	-	-
$\max(d_0, d_1)$	-	-	-	-	**100 %**	-	-	-

8. Triangulation speed

	Midpoint [1]	L_1 img [1]	L_2 img [1]	L_∞ img [2]	L_2 img 2 it. [4]	L_2 img 5 it. [4]	L_1 ang	L_2 ang	L_∞ ang
Points/sec	42 M	65 K	92 K	270 K	1.4 M	520 K	29 M	670 K	14 M
Relative Speed	1.0	0.0016	0.0022	0.0064	0.033	0.013	0.71	0.016	0.33

9. Conclusions

- In this work, we derived the exact L_1, L_2 and L_∞ optimal solutions to two-view triangulation based on angular reprojection errors.
- Our methods are extremely simple and fast, and they guarantee global optimality under respective cost functions.

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