Rotation-Only Bundle Adjustment

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1542

ROBOTICS

UNIVERSIDAD DE ZARAGOZA

Presentation outline

1. Introduction to Rotation-Only Bundle Adjustment
2. Normalized Epipolar Error (Lee & Civera 2020)
3. Two-view Rotation-Only Optimization (Kneip & Lynen 2013)
4. Rotation-Only Bundle Adjustment
5. Results
6. Limitations and Future work
7. Conclusion

1. Introduction

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(ii) Unlike BA, it is completely independent of the translations and structure.

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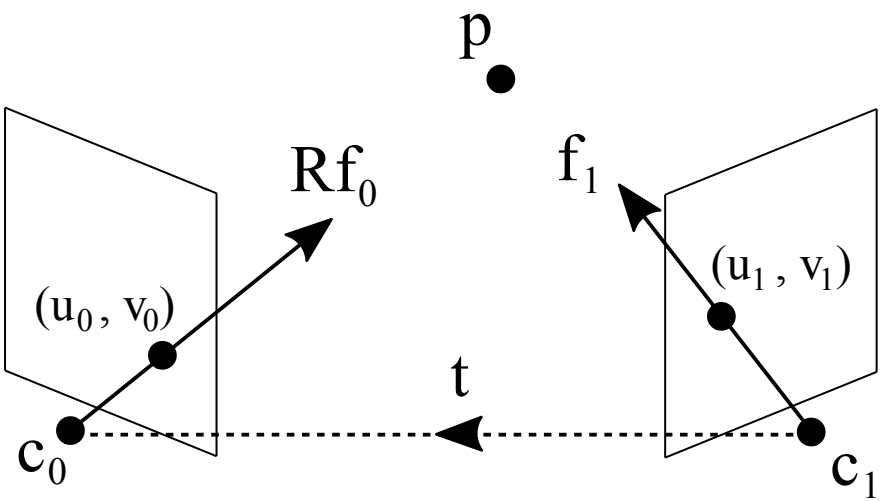

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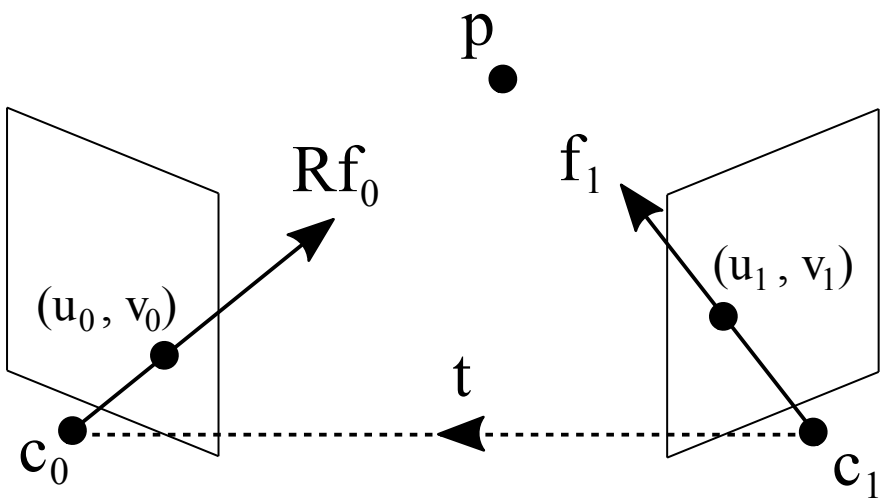

$$f_0 = K_0^{-1} \begin{bmatrix} u_0 \\ v_0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_0/z_0 \\ y_0/z_0 \\ 1 \end{bmatrix},$$ f_1 can be defined analogously.

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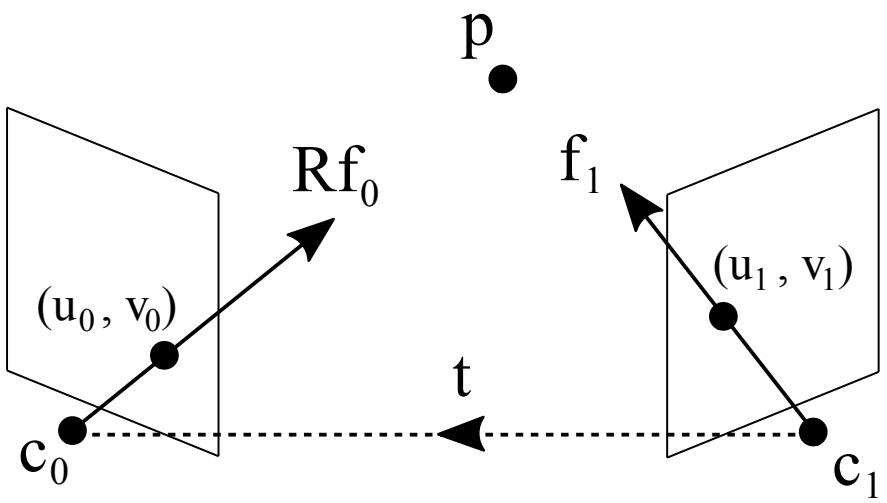

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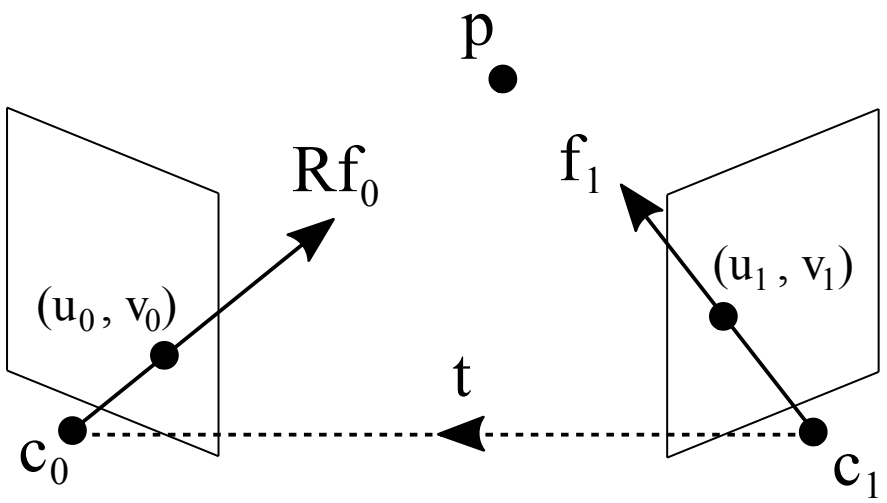

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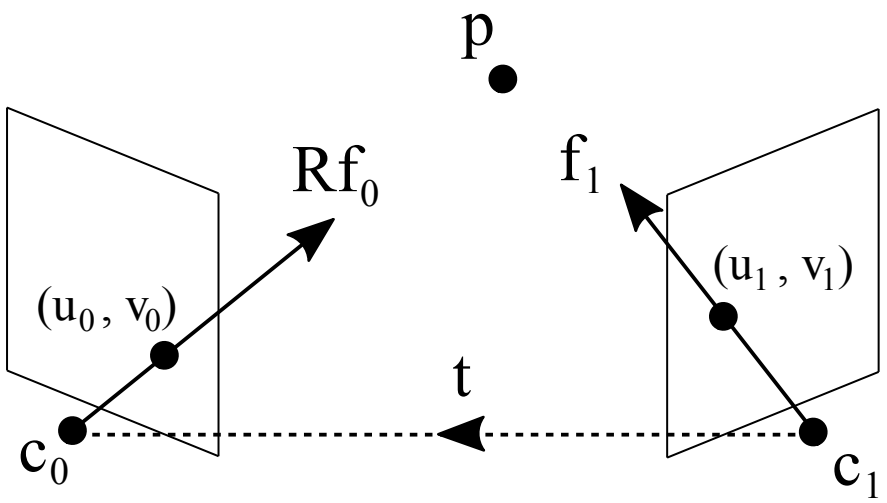

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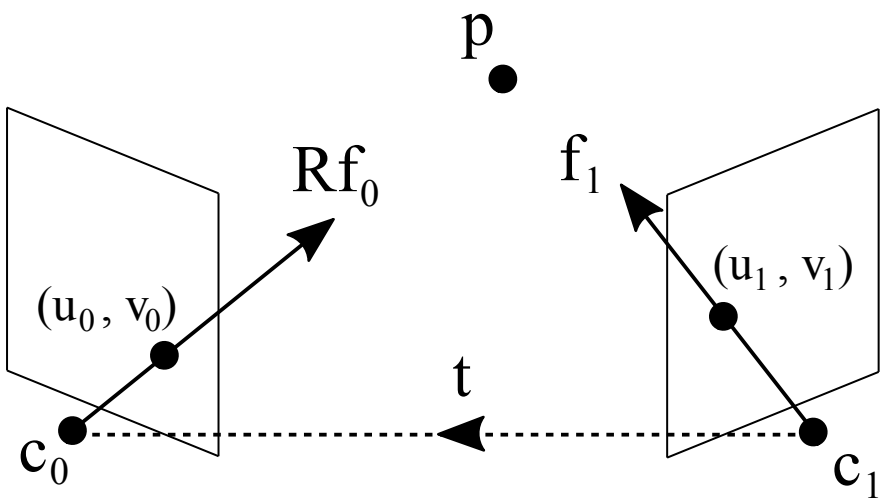

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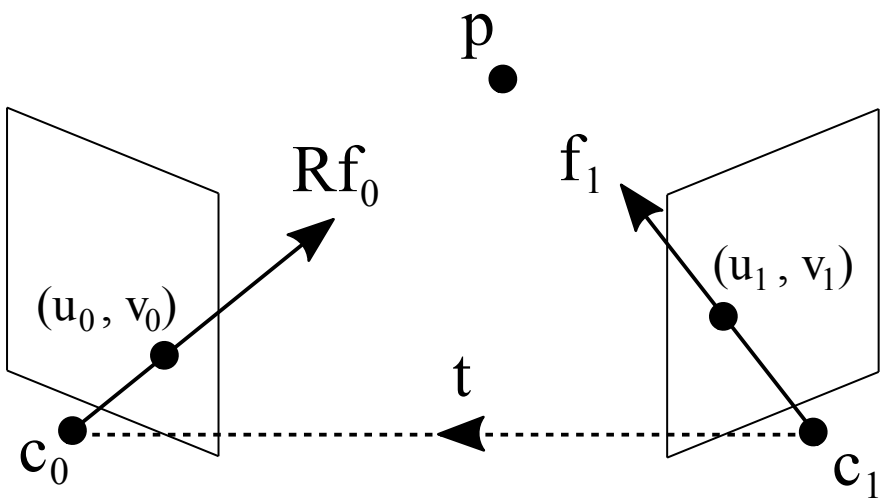

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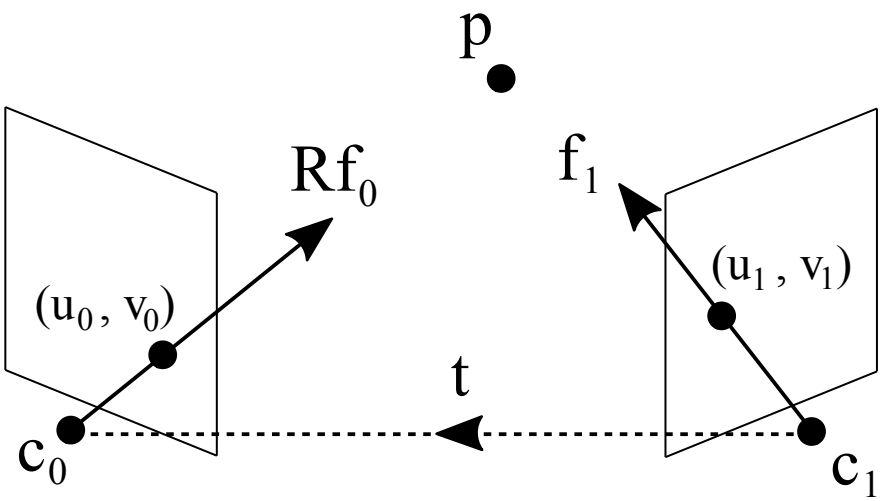

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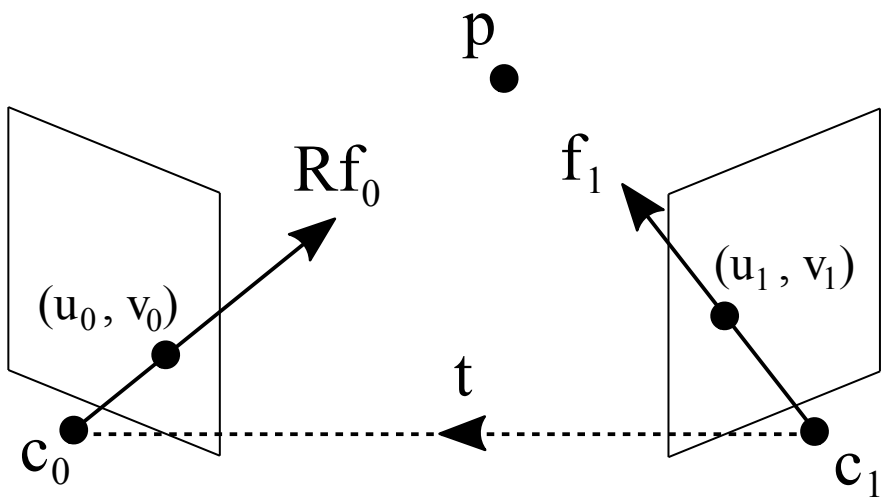

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$$e = \left| \mathbf{f}_1^\top \mathbf{E} \mathbf{f}_0 \right|$$

Purely algebraic quantity.

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It has geometric interpretations.

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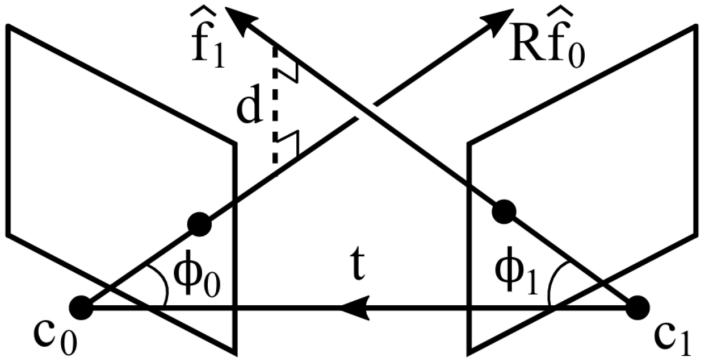

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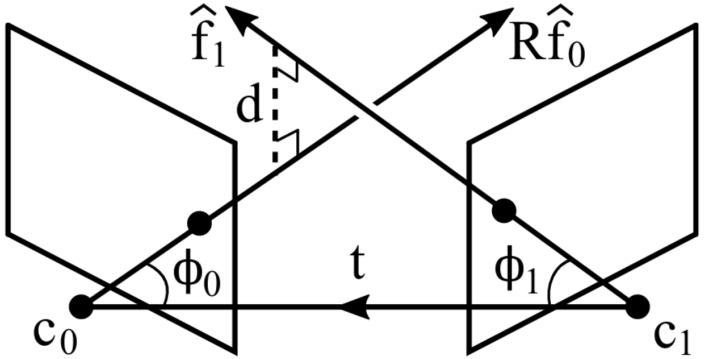

$$\widehat{e} = \frac{\sin(\beta)}{\|\mathbf{t}\|} d.$$

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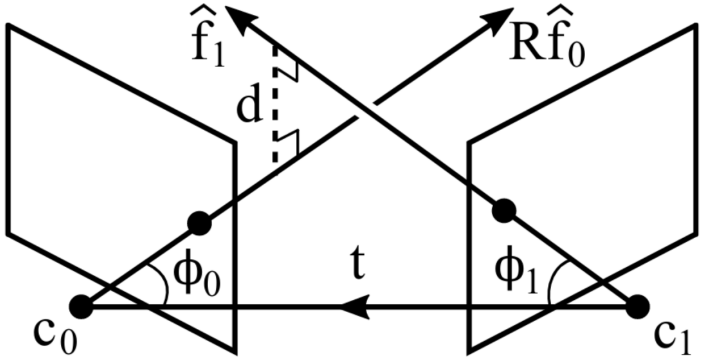

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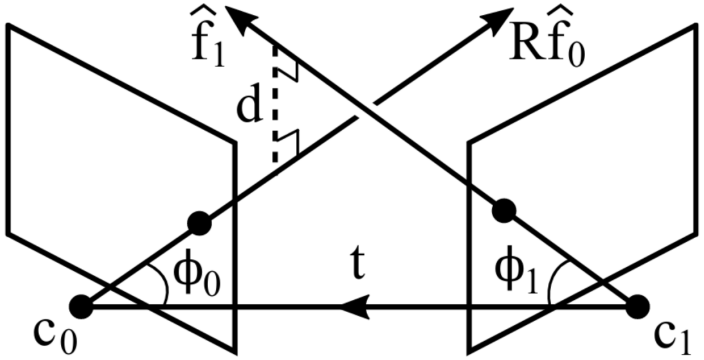

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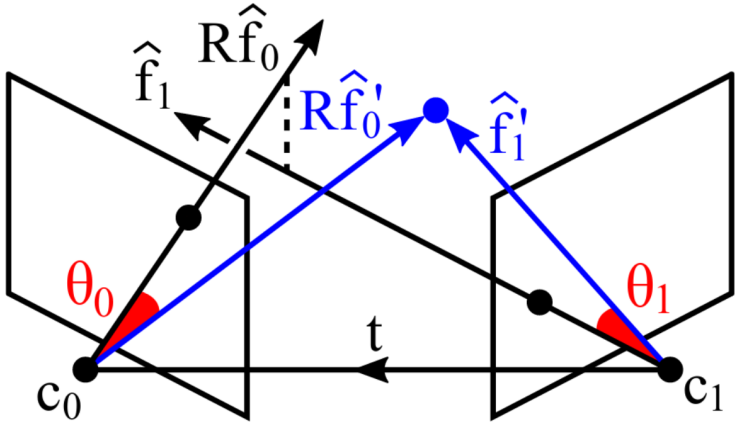

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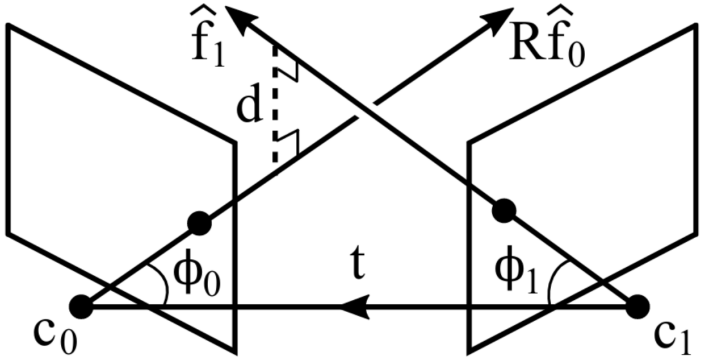

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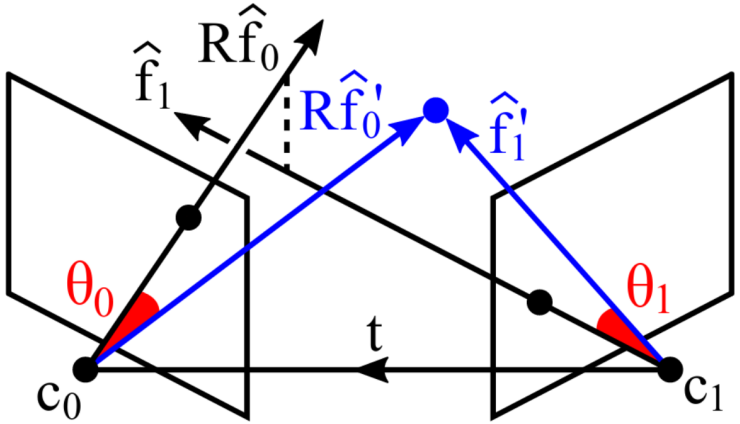

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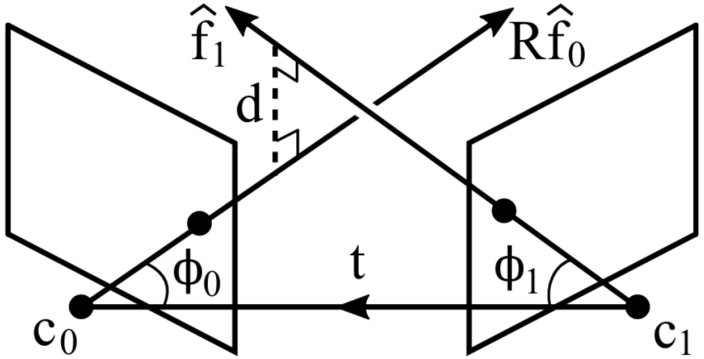

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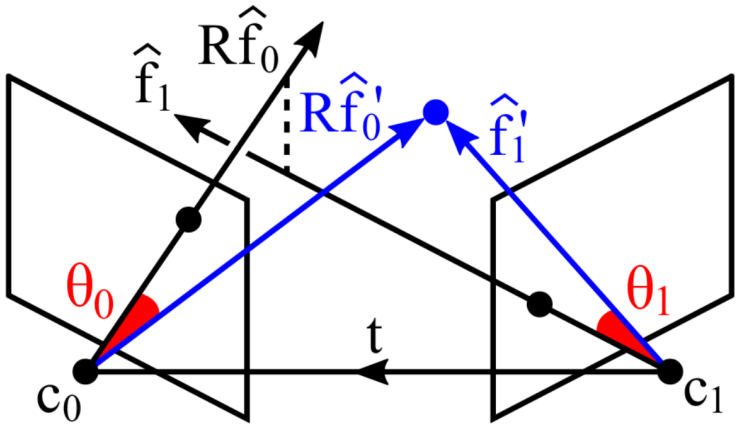

$\theta^*_{L1} = \min(\theta_0 + \theta_1)$ that make the rays intersect.

$$\hat{e} = \sin\left(\max(\phi_0, \phi_1)\right) \sin(\theta^*_{L1}).$$

where $\phi_0 := \angle(\mathbf{R}\hat{\mathbf{f}}_0, \hat{\mathbf{t}}) \in [0, \pi/2],$

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For the derivations and more interpretations, check our arXiv preprint "Geometric Interpretations of the Normalized Epipolar Error" (2020).

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3. Two-View Rotation-Only Optimization (Kneip & Lynen 2013)

- Goal: Find the rotation and translation between camera j and k that minimize the sum of squares of the normalized epipolar errors.

→ $$\underset{\mathbf{R}_{jk},\hat{\mathbf{t}}_{jk}}{\operatorname{argmin}} \sum_{i=1}^{m} (\hat{e}_i)^2_{(j,k)} = \sum_{i=1}^{m} \left| \hat{\mathbf{t}}_{jk} \cdot \left((\hat{\mathbf{f}}_i)_j \times \mathbf{R}_{jk} (\hat{\mathbf{f}}_i)_k \right) \right|^2$$

$$= \hat{\mathbf{t}}_{jk}^{\top} \left[\sum_{i=1}^{m} \left((\hat{\mathbf{f}}_i)_j \times \mathbf{R}_{jk} (\hat{\mathbf{f}}_i)_k \right) \left((\hat{\mathbf{f}}_i)_j \times \mathbf{R}_{jk} (\hat{\mathbf{f}}_i)_k \right)^{\top} \right] \hat{\mathbf{t}}_{jk}$$

$$= \hat{\mathbf{t}}_{jk}^{\top} \mathbf{M}_{jk} \hat{\mathbf{t}}_{jk}$$ (where $\mathbf{M}_{jk}$ is a positive semidefinite matrix that depends only on $\mathbf{R}_{jk}$)

$$\underset{\mathbf{R}_{jk},\widehat{\mathbf{t}}_{jk}}{\operatorname{argmin}} \sum_{i=1}^{m} (\hat{e}_i)^2_{(j,k)} = \underset{\mathbf{R}_{jk},\widehat{\mathbf{t}}_{jk}}{\operatorname{argmin}} \widehat{\mathbf{t}}_{jk}^{\top} \mathbf{M}_{jk} \widehat{\mathbf{t}}_{jk} \ (\mathbf{M}_{jk} \geq 0 \text{ depends only on } \mathbf{R}_{jk})$$

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This transforms a relative pose problem into a rotation-only problem!

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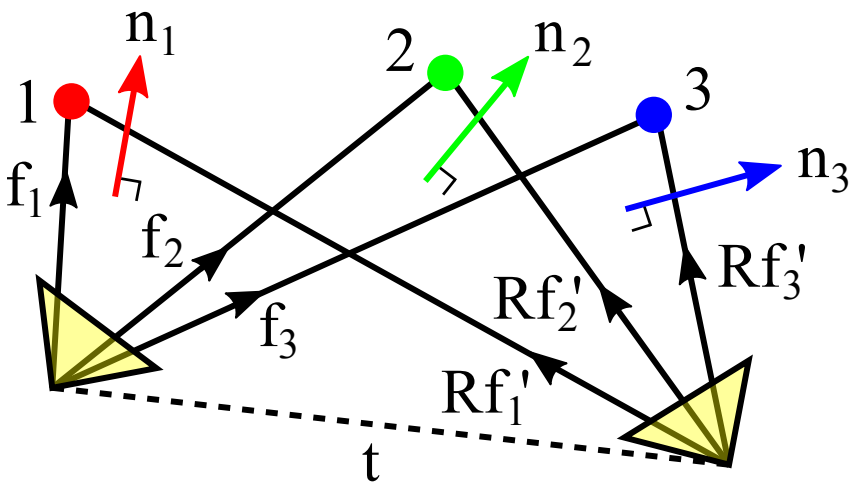

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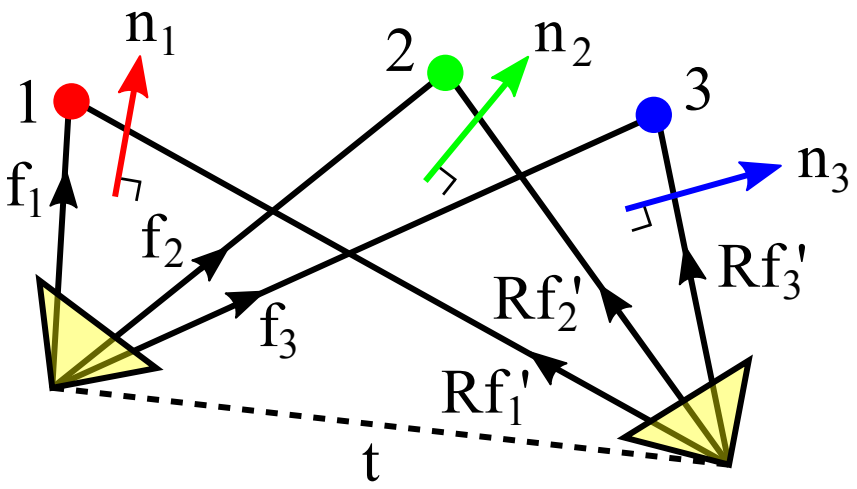

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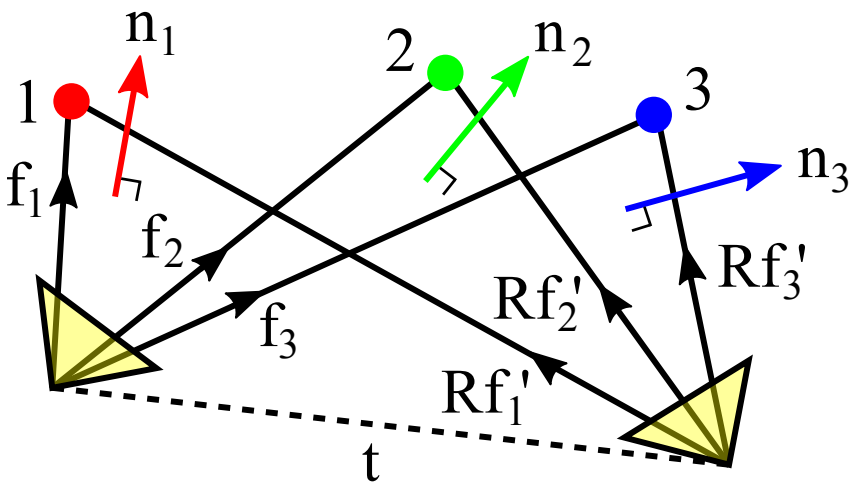

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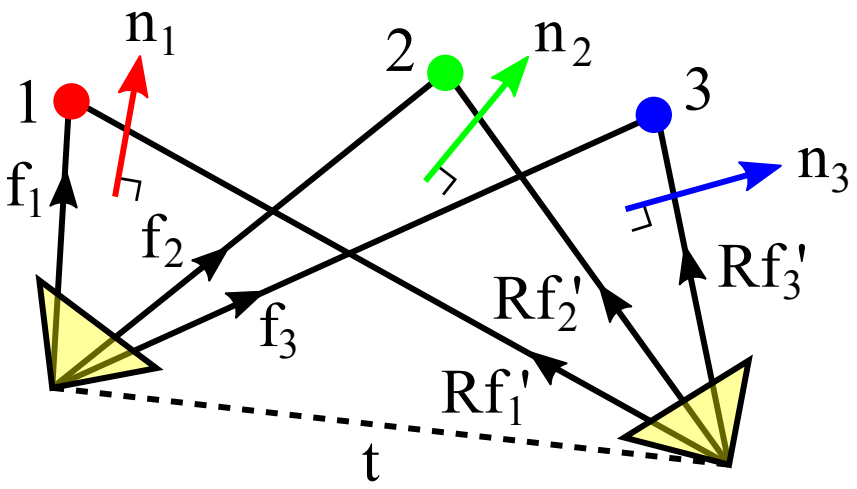

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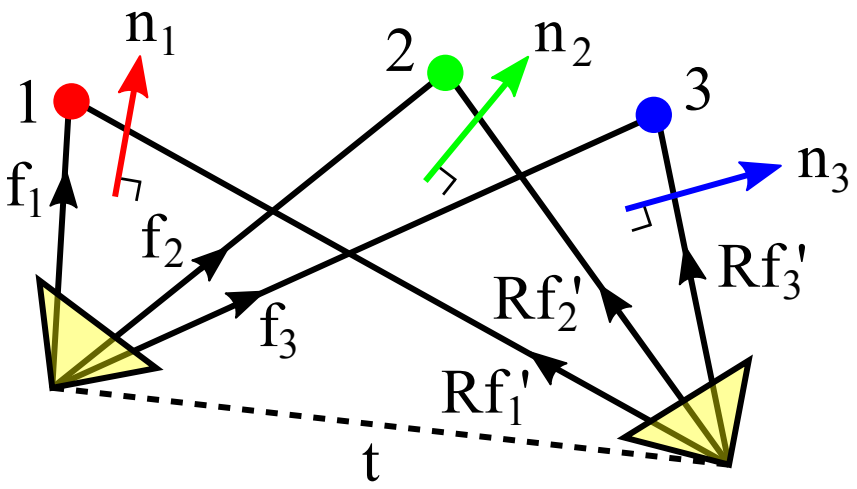

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$$M = \sum_i n_i n_i^T = \left(\hat{f}_i \times R\hat{f}_i'\right)\left(\hat{f}_i \times R\hat{f}_i'\right)^T$$

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1. $\underset{\mathbf{R}_1,\cdots,\mathbf{R}_n}{\operatorname{argmin}} \sum_{(j,k)\in\mathcal{E}} \sqrt{\lambda_{\mathbf{M}}(\mathbf{R}_{jk})}$ where $\mathcal{E}$ is the set of all camera pairs observing more than 10 points in common.
2. Solve it using the Adam optimizer (Kingma & Ba, 2015).

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- Comparing the two approaches:

Rotation Averaging	vs	Rotation Averaging + Rotation-Only Bundle Adjustment
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For rotation averaging, we used the state-of-the-art method by Chatterjee & Govindu, 2018.

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Baseline	2.31	**1.26**
More points	1.78	**1.05**
Fewer views	1.08	**0.33**
More views	4.35	**3.73**
Closer points	5.74	**3.99**
Farther points	0.69	**0.36**
Less noise	2.30	**1.17**
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- It is robust to challenging configurations such as planar scenes and pure rotations.

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- The rotation errors (in deg) are shown in black, and the computation times (in sec) are given in blue.

	RA	RA + ROBA
ALM	4.08 (16)	**2.35** (267)
ELS	2.10 (1)	**1.06** (48)
GDM	6.05 (3)	**2.43** (87)
MDR	6.20 (2)	**4.42** (61)
MND	1.46 (4)	**0.82** (142)
NTD	2.08 (14)	**1.27** (291)
NYC	2.87 (1)	**1.03** (49)
PDP	3.86 (1)	**2.18** (61)
PIC	4.14 (220)	**1.58** (899)
ROF	2.94 (6)	**2.18** (186)
TOL	3.83 (1)	**1.15** (64)
TFG	3.40 (553)	**2.76** (2194)
USQ	5.59 (1)	**3.26** (62)
VNC	6.12 (15)	**4.96** (289)
YKM	3.67 (1)	**1.66** (74)

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- We have not evaluated the resultant impact on the full bundle adjustment.

- We have not thoroughly explored different optimization methods.

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- We formulated the optimization problem by extending the two-view rotation-only method of Kneip & Lynen (2013) to multiple views.

- We solved the problem using the Adam optimizer, and showed that it consistently improves the accuracy when used after rotation averaging.