

Network of Loyalty Programs: A Sequential Formation

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Abstract

Travel rewards are an important component of credit card rewards, and major credit card issuers in the U.S. operate loyalty programs, in which customers may transfer points to the loyalty programs of partner airlines and hotel chains. A credit card issuer's portfolio of transfer partners involves strategic partnerships with other firms and is a significant source of product differentiation. In a network environment, this paper studies how credit card issuers choose their portfolios of airline transfer partners. In particular, the focus is on how the complementarity of resources possessed by an airline and the network of firms affect whether to add, delete, or maintain a transfer partnership with the airline. I model the choice of portfolios as a sequential network formation process, where transfer partnerships between credit card issuers and airlines are determined via a sequence of bilateral meetings. Using a network formation model is necessary because whether or not to form a transfer partnership with an airline may depend on the current state of the credit card issuers's portfolio of transfer partners, the portfolios of other credit card issuers, and partnerships between hotel chains and airlines. This paper conducts a revealed-preference analysis using observations from 2014 to 2018. Estimation results indicate that a credit card issuer is more likely to form a transfer partnership with an airline that better complements its other airline partners. Moreover, the network of firms has a significant effect on partner choice.

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1 Introduction

In 2015, 55 percent of U.S. consumers chose rewards as the most attractive credit card feature, and this rate rose steadily to 79 percent in 2018¹ (Total Systems Services, 2016-2018)². Sharply rising expenditures by credit card issuers confirm the importance of customer rewards. In the fourth quarter of 2016, J.P. Morgan Chase (JPMC) spent up to \$300 mil. on customer rewards³ for a new credit card product. The expenditures of American Express Company (AMEX) on customer rewards increased from \$6.8 bil. in 2016 to \$7.6 bil in 2017, and then to \$9.7 bil. in 2018. For 2017 and 2018, the expenditures accounted for a third of AMEX's total expenses. At the end of 2018, AMEX's liability for unclaimed customer rewards was \$8.4 bil. (American Express Company, 2016-2018).

Major credit card issuers in the U.S. operate loyalty programs to reward customers. Customers may earn points in these programs by signing up for credit card products or making purchases using them. An example is the Membership Rewards program of AMEX, and the firm states

“...through our Membership Rewards program we have partnered with businesses in many industries, including the airline industry, to offer benefits to Card Member participants.”

American Express Company (2014-2018).

Through its partners, a credit card issuer can offer a variety of points redemption options beyond cash, such as gift cards, merchandises, and travel rewards, for its loyalty program.

The partners possessed by a credit card issuer is a significant source of product differentiation; in particular, the differentiation emanates primarily from partners in the travel industry. It is because all points redemption options, except for travel rewards, are similar to redeeming for cash. First, there is hardly any difference in the assortment of non-travel rewards offered by the loyalty programs. Moreover, the redemption rates of loyalty points for non-travel rewards are completely determined by cents-per point rates for the face value of the rewards. Because the cents-per-point rates of each loyalty program are tightly linked to its redemption rate for cash, redeeming for non-travel rewards are essentially redeeming for cash, possibly with a small discount. That is, for non-travel rewards, the loyalty programs are differentiated only by the redemption rates set by the credit card issuers.

In contrast, a redemption for travel rewards is typically done by transferring points to a partner's loyalty program, and the redemption value of a credit card issuer's loyalty program depends on how the customer perceives the portfolio of transfer partners possessed by the credit card issuer. That is, the conversion ratios of points to the loyalty programs of partners, the redemption options offered by the loyalty programs of partners, and the consumer's preferences for the partner firms affect the attractiveness of travel rewards offered by a credit card issuer. The portfolio of travel partners differentiates one loyalty program to another, appealing to heterogeneous preferences of consumers for the partner firms and their loyalty programs. The fact that more credit card issuers

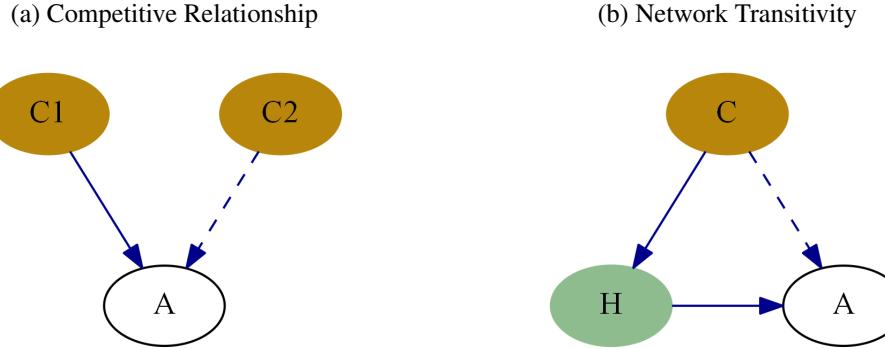
¹In 2018, interest rate was ranked second at 67 percent, and card brand was ranked third at 55 percent.

²Total Systems Services (NASDAQ: TSS) is a U.S.-based payment processing provider with more than \$4 billion revenue in 2018.

³Jennifer Surane and Hugh Son. (2016). “Dimon Says New Sapphire Card Cuts Profit by Up to \$300 Million in Quarter.” Bloomberg. December 6. <https://www.bloomberg.com/news/articles/2016-12-06/dimon-says-new-card-cuts-profit-by-up-to-300-million-in-quarter>.

are launching loyalty programs with points transferable to partners in the travel industry confirms the significance of travel rewards and that the portfolio of travel partners is a significant source of product differentiation.

Figure 1: Network Effects



This figure illustrates the network effects. Sub-figure (a) illustrates competitive relationship. C1 and C2 are two credit card issuers, and A is an airline. A is a transfer partner of C1, and a positive competitive relationship would suggest C2 is more likely to form a transfer partnership with A than otherwise. Sub-figure (b) illustrates network transitivity. C, H, and A are a credit card issuer, a hotel chain, and an airline, respectively. H is a transfer partner of C, and A is a transfer partner of H; a positive network transitivity would suggest C is more likely to form a transfer partnership with A than otherwise.

This paper studies how a credit card issuer chooses its portfolio of transfer partners in the travel industry, in particular, the airline partners. The focus is on how resource complementarity and the network of firms affect partner choice. For a credit card issuer, the resources possessed by an airline is the redemption options for flights, via the airline's loyalty program. Each airline's loyalty program offers different accessibility and points redemption rates for various geographic zones, which account for the differences in flight routes and cost advantages possessed by the airlines. The resource complementarity of an airline is how forming a transfer partnership with the airline adds to redemption options offered by the credit card issuer's loyalty program. Thus resource complementarity not only depends on the resources possessed by the airline but also the set of other airline partners possessed by the credit card issuer. Moreover, this paper studies two network effects: *Competitive Relationship* and *Network Transitivity*. I define Competitive Relationship as whether or not a credit card issuer is more likely to form a transfer partnership with an airline that is a partner of another credit card issuer. I define Network Transitivity as whether or not a credit card issuer is more likely to form a transfer partnership with an airline that is a transfer partner of a partner hotel chain. Figure 1 graphically illustrates these network effects.

The choice of a transfer partner not only depends on the resources possessed by the potential partner but also on how the resources complement the resources possessed by other airline partners. Moreover, the choice of partners by competing credit card issuers and the partnership structure within the travel industry may affect the choice, and vice versa. In other words, the behavior of firms affects the environment, and the environment also affects the behavior of firms. Zeggelink (1996) describes this phenomenon as "The transformation from micro to macro, therefore, is not a simple sum of individual actions." Such externality makes it difficult to use typical

regression models to describe partner choices, because the choices are not independent of each other. For friendship networks, researchers often utilize sequential network formation processes to accommodate such externality, where agents are treated as nodes, and the opportunity to modify a linkage between two agents occurs along some sequence. Linkages are sequentially determined according to the optimizing behavior of agents based on the current state of the network, and the state of the network is updated reflecting their actions. Research utilizing such sequential network formation process includes Christakis et al. (2010) and Snijders et al. (2010), to name a few. This paper's model follows a similar process but accounts for asymmetry in the roles and objectives of firms.

This paper uses a directed network to describe transfer partnerships among loyalty programs. The loyalty programs of credit card issuers, hotel chains, and airlines constitute the node set. A link from a node to another indicates that loyalty points in the source node can be transferred to the target node, and the associated link weight indicates the transfer ratio. In other words, a link indicates a transfer partnership, and the link weight indicates the transfer ratio associated with the transfer partnership. In order to facilitate the transfer of points, the source node purchases points from the target node. Thus, unlike a friendship network, nodes play asymmetric roles as buyers and sellers. Moreover, nodes belonging to different industry sectors - credit card issuers, hotel chains, and airlines - may have different incentives in forming transfer partnerships. I accommodate such features by splitting the network into subnetworks, and linkages are determined by interactions among nodes within a subnetwork, while taking account of the states of other subnetworks. In each subnetwork, I assign distinct roles to nodes either as choosers or bidders. Bidders are potential transfer partners of choosers, and links are always directed from the choosers to the bidders. Thus, subnetworks are uni-directional bipartite. Moreover, I use a sequential authority mechanism to describe the interactions between choosers and bidders. First, bidders submit take-it or leave-it bids to choosers based on pairwise characteristics of bidders and choosers. Second, choosers either accept or reject bids, one at a time. A bid represents how favorable the terms of contract offered by the bidder is to the chooser. I endow all nodes with full information on the entire network but assume that they are myopic.

After receiving bids, choosers sequentially meet bidders to add, remove, or maintain linkages, and the outcomes of the meetings determine the network formation process for a given period. A period denotes the time frame for the transition of the network from an initial state to an ending state, and the the network formation process describes such transition. In each period, (chooser, bidder) pairs meet according to an unknown sequence of bilateral meetings, and in each meeting, the chooser is given the opportunity to modify the linkage to the bidder. The model accommodates transitions of the network over multiple periods by specifying a different sequence of meetings for each period, possibly with a different node set. The sequence of meetings specifies the ordering of meetings (chooser, bidder) pairs across all subnetworks, and I assume that each (chooser, bidder) pair meets exactly once.

The sequence of meetings is a crucial part of the network formation process because the current state of the network at a meeting depends on the outcomes of past meetings. In each meeting, the chooser makes the choice to add, remove, or maintain linkage to the bidder such that the resulting portfolio of partners optimizes the chooser's objective function under the current state of the network. The specification of the chooser's objective function may be different across subnetworks, as choosers belonging to different industry sectors may have different incentives in forming linkages. Note that I model interactions between credit card issuers and airlines and between hotel chains

and airlines, while taking linkages from credit card issuers to hotel chains and linkages among airlines as exogenous. Although the focus of this paper is on how credit card issuers choose their airline partners (linkages from credit card issuers to airlines), modeling the choice of hotel chains (linkages from hotel chains to airlines) is necessary because their choices affect the current state of the network at a meeting between a credit card issuer and an airline.

The primary estimands are the parameters of the chooser's objective function, with credit card issuers as the choosers. I also estimate the parameters of the hotel chain's objective function, but it only briefly discussed. The estimation procedure involves two steps. The first step constructs the bids of airlines to both credit card issuers and to hotel chains. Because bids are observed only for linked pairs (i.e., pairs with transfer partnership), I fitted a hedonic regression model to predict the bids for non-linked pairs. The second step utilizes the method of Christakis et al. (2010) to estimate the parameters of the objective functions, using the predicted bids from the first step. The method involves Markov Chain Monte Carlo iterations and a convergence criterion suggested by Gelman and Rubin (1992).

This paper conducts a revealed-preference analysis using observations on 3 credit card issuers, 7 hotel chains, and 43 airlines from 2014 to 2018. The estimation result indicates the following. Other things equal,

1. A credit card issuer is more likely to form a transfer partnership with an airline that the complements the redemption options of its loyalty program.
2. A credit card issuer is more likely to form a transfer partnership with an airline that is a transfer partner of another credit card issuer. That is, there is statistical evidence for positive Competitive Relationship.
3. A credit card issuer is more likely to form a transfer partnership with an airline that is a transfer partner of its hotel partner. That is, there is statistical evidence for positive Network Transitivity.

In particular, result 1 suggests that a credit card issuer tends to pursue resource complementarity when choosing its airline partners, rather than their individual resources.

The rest of the paper is organized as the following. Section 2 discusses related research. Section 3 explains incentives in partnership formation. Section 4 describes transfer partnerships as a network of loyalty programs. Section 5 presents the model, and section 6 describes the estimation method. Section 7 briefly discusses the data and presents the estimation result. 8 concludes.

2 Related Research

Researchers of strategic business management have studied incentives in partnership formation between firms. Several recent developments include the following. Gulati (1995) analyzes how the network of firms affects the formation of alliances using a panel data of partnerships between firms . Chung et al. (2000) studies how complementarity, status similarity, and the network of firms affect alliance formation among investment banks in the U.S. Rothaermel and Boeker (2008) studies the roles of complementarity and status similarity in the formation of alliances, using a network of pharmaceutical firms. Lin et al. (2009) studies the motives in alliance formation based on how complementarity, status, and the network of firms affect the performance of firms.

Researchers typically model strategic network formation as a collection of pairwise interactions, where the formation of linkage between a pair of agents depends on their characteristics and possibly the characteristics of other agents. Researchers often regard the observed state of the network as an equilibrium outcome, using the pairwise stability of Jackson and Wolinsky (1996) as the equilibrium condition. The pairwise stability extends Nash equilibrium to accommodate pairwise interactions, such that for each pair, neither one of the agents can be made better off by modifying the linkage between the pair. Research that employs the pairwise stability views strategic network formation as a simultaneous-move game.

The simultaneity and possible existence of multiple equilibria complicate estimation. Tamer (2003) explains that naively using regression models may result in biased estimation in the presence of multiple equilibria. Sheng (2018) provides a discussion on multiple equilibria in networks and the problem in parameter identification for network formation models. de Paula et al. (2015) and Sheng (2018) each presents a partial identification method for network formation models based on subnetworks. Jia (2008) utilizes the lattice theory of Tarski (1955) and Topkis (1955) to restrict the set of Nash equilibria in a simultaneous-move game, where Walmart and K-Mart strategically choose their store locations. Nishida (2015) extends the method of Jia (2008) to study strategic location choices of the convenience-store industry in Japan. Miyauchi (2016) extends the method of Jia (2008) and presents an identification method for analyzing the formation of pairwise stable networks. Lee and Fong (2013) develops a two-stage model of strategic network formation, where the first stage determines potential partners and the second stage models linkage formations as bilateral Nash bargaining games.

Christakis et al. (2010) and Snijders et al. (2010) each presents a model of sequential network formation, in which the network transitions from a state to the next state via some unknown sequence of meetings between myopic agents. By construction, their specifications do not allow the existence of multiple equilibria as how the network transitions depends on the ordering of events laid out by the sequence of meetings. However, the model parameters cannot be feasibly estimated because the sequence of meetings is unknown. They employ Bayesian methods to sample the sequence of meetings and estimate the model parameters using Markov Chain Monte Carlo. This paper is built upon their framework but with focus on strategic formation of portfolios of partners, with asymmetric roles of agents in the network.

3 Incentives in Partnership Formation

Major credit card issuers in the U.S. operate loyalty programs to reward customers. For example, American Express Company (AMEX), Citibank (CITI), and J.P. Morgan Chase (JPMC), operate Membership Rewards, ThankYou Rewards, and Ultimate Rewards, respectively. These programs are associated with their flagship credit card products, and customers may earn points by signing up for or making purchases using the products.

The loyalty program of a credit card issuer is an important marketing tool. Wirtz et al. (2007) finds that the attractiveness of a credit card issuer's loyalty program has a positive effect on the consumer's share of wallet⁴, above and beyond the consumer's psychological attachment to the firm. Sharply rising expenditures by credit card issuers confirm the importance of loyalty programs. In

⁴The share of wallet for a credit card issuer is the share of purchases made using the firm's credit card products.

the fourth quarter of 2016, JPMC spent up to \$300 mil. on the Ultimate Rewards program to reward the customers of a new credit card product launched in August 2016⁵. The expenditures of AMEX on the Membership Rewards program increased from \$6.8 bil. in 2016 to \$7.6 bil in 2017, and then to \$9.7 bil. in 2018. For 2017 and 2018, the expenditures accounted for a third of AMEX's total expenses. At the end of 2018, AMEX's liability for unclaimed points in the Membership Rewards program was \$8.4 bil. (American Express Company, 2014-2018).

Credit card issuers possess partners from a diverse pool of industries. Via the partners, they can offer a variety of redemption options for loyalty points beyond cash, such as gift cards, merchandises, and travel rewards. Credit card issuers possess different sets of partners, and who they are partnered with is a significant source of product differentiation. In particular, the differentiation emanates primarily from partners in the travel industry because all redemption options for loyalty points, except for travel rewards, are similar to redeeming for cash. First, there is hardly any difference in the assortment of non-travel rewards offered by the loyalty programs of credit card issuers. Moreover, the redemption rates for non-travel rewards are completely determined by cents-per point rates. Because the cents-per-point rates of each loyalty program are tightly linked to its redemption rate for cash, redeeming for non-travel rewards is essentially the same redeeming for cash, possibly with a small discount. For example, customers of AMEX, CITI, and JPMC may use loyalty points to redeem for merchandises sold at Amazon.com⁶, with redemption rates of 0.7, 0.8, and 0.8 cents per point, respectively⁷. That is, for non-travel rewards, the loyalty programs are differentiated only by the redemption rates set by the credit card issuers.

In contrast, a redemption for travel rewards is typically done by transferring points to a partner's loyalty program⁸, and the redemption value of loyalty points depends on how the customer perceives the portfolio of transfer partners possessed by the credit card issuer. That is, the conversion ratios of points to the loyalty programs of partners, the redemption options offered by the loyalty programs of partners, and the consumer's preferences for the partner firms affect the attractiveness of travel rewards offered by a credit card issuer. The portfolio of partners in the travel industry differentiates one loyalty program to another, appealing to heterogeneous preferences of consumers for the partner firms and their loyalty programs. Figure 8 shows an example of a portfolio of partners possessed by a credit card issuer and how it is used in marketing. The fact that more credit card issuers are launching loyalty programs with points transferable to partners in the travel industry confirms the significance of travel rewards and that the portfolio of transfer partners is a significant source of product differentiation. In particular, Barclays US⁹ and Capital One¹⁰ launched such loyalty programs in 2018, each with a different portfolio of airline partners.

⁵Jennifer Surane and Hugh Son. (2016). "Dimon Says New Sapphire Card Cuts Profit by Up to \$300 Million in Quarter." *Bloomberg*. December 6. <https://www.bloomberg.com/news/articles/2016-12-06/dimon-says-new-card-cuts-profit-by-up-to-300-million-in-quarter>.

⁶One may also view this as redeeming for gift cards at Amazon.com.

⁷For AMEX, CITI, and JPMC, the redemption rates for cash are 0.6, 0.5, and 1 cent per point, respectively

⁸The credit card issuer purchases points from the travel firm to facilitate the transfer of points. In 2014-2015, American Airlines earned 1.3-1.6 cents per point sold to non-airline partners (American Airlines, 2014-2015).

⁹Barclays US. (2018). "Barclays Launches Premier Global Travel Card that Rewards Cardmember Loyalty: Barclays Arrival Premier World Elite Mastercard." April 4. <https://cards.barclaycardus.com/banking/about-us/newsroom/barclays-launches-arrival-premier-world-elite-mastercard/>.

¹⁰Brian Kelly. (2018). "Capital One Venture Card Adds Airline Transfer Partners, Offers Limited Time 75,000-Mile Bonus." *Forbes*. November 13. <https://www.forbes.com/sites/the��sguy/2018/11/13/capital-one-venture-card-lto/#70e949b1383d>.

The partnership between a credit card issuer and a travel firm (i.e., hotel chain or airline) is a form of *brand alliance*. This term was coined by Rao and Ruekert (1994) to describe the joint-branding of firms, where the integration of products and services may also occur. A canonical example of brand alliance is issuing co-branded credit cards. For example, AMEX issues credit card products under the brand name of Delta Air Lines (DL). Joint with DL, AMEX can offer complementary services to customers of DL, including airport lounge access, free checked baggage, and the ability to earn points in DL's loyalty program. On the other hand, DL gains from discounts on payment processing fees and by selling its loyalty points to AMEX¹¹. Such co-brand relationship is mutually beneficial for both firms. AMEX gains by appealing to customers of DL, expanding its customer base. In fact, AMEX reports its co-branded credit card portfolio with DL accounted for 8 percent of its total billed business¹² and 21 percent of its total outstanding card-member loans in 2018 (American Express Company, 2014-2018). DL also gained by \$3.4 bil. in 2018, and it expects the annual benefit to grow to \$7 bil. by 2023¹³. A co-brand partnership usually lasts for ten or more years and is typically exclusive to the credit card issuer¹⁴.

A transfer partnership is similar to a co-brand partnership but with weaker integration, shorter duration, and without exclusivity. The portfolio of transfer partners is updated in as little as three months, and no apparent exclusivity is observed, as credit card issuers often share common transfer partners. Integration occurs through the loyalty programs, as customers of the credit card issuer gain access to the redemption options offered by the partner's loyalty program by transferring points. A transfer partnership is a brand alliance between loyalty programs. The credit card issuer gains by enhancing the attractiveness of its loyalty program, especially for frequent customers of the partner firm. On the other hand, the partner firm not only increases its revenue by selling points to the credit card issuer but also gains from better brand awareness, especially by customers of the credit card issuer.

The pursuit of resources and reputation possessed by partners can explain how a credit card issuer chooses its transfer partners. This paper defines resources as tangible assets and reputation as intangible assets possessed by a firm. The resource-based view (Penrose, 1959; Barney, 1991) suggests that the incentive in forming a partnership is to gain access to strategic resources possessed by the partner firm. For the loyalty program of a credit card issuer, the resources possessed by an airline is the redemption options for flights, via the airline's loyalty program. Each airline's loyalty program offers different accessibility and different redemption rates of loyalty points to various geographic zones, accounting for differences in flight routes and cost advantages possessed by airlines. Thus, according to the resource-based view, a credit card issuer would favor a transfer partner that enhances the redemption options of its loyalty program. Reputation characterizes the institutional aspects of a firm, such as market share and brand familiarity; it is essentially how consumers perceive a firm. By forming a transfer partnership with a firm, a credit card issuer can appeal to the partner's frequent customers, expanding its customer base. This view is compatible

¹¹Customers of the co-branded credit cards receive points in DL's loyalty program as rewards. AMEX pays DL for the points.

¹²Total billed business is the total amount of card purchases.

¹³Delta Air Lines. (2019). "American Express and Delta Renew Industry-Leading Partnership, Lay Foundation to Continue Innovating Customer Benefits." April 2. <https://news.delta.com/american-express-and-delta-renew-industry-leading-partnership-lay-foundation-continue-innovating>.

¹⁴There are a few exceptions. For example, American Airlines has co-brand partnerships with both Citibank and Barclays US.

with brand spillover effects, as suggested by Simonin and Ruth (1998) and Newmeyer et al. (2013). According to this view, a credit card issuer would favor transfer partners with a stronger market presence.

The pursuit of resources and reputation would suggest that a credit card issuer should choose a large portfolio of transfer partners to enhance the redemption options of its loyalty programs and to broaden its customer base. However, the credit card issuer has other incentives. First, enhancing the redemption options of its loyalty program lowers the revenue of its co-brand partners, as fewer customers sign up for or make purchases using the co-branded credit card products. For major U.S. airlines, the revenue earned from selling loyalty points accounts for a significant share of their operating revenue¹⁵. Second, the cost of operating a loyalty program is realized when customers redeem points; thus, more valuable redemption options induces more redemption of loyalty points, increasing costs for the credit card issuer. With such incentives, it is unclear what characteristics a credit card issuer seeks in its partners, and an empirical revealed-preference analysis would enhance our understanding of partnership formation between credit card issuers and firms in the travel industry. Moreover, one could utilize such understanding to make predictions on what may happen to the partnerships in the event of an exogenous shock.

4 Network of Loyalty Programs

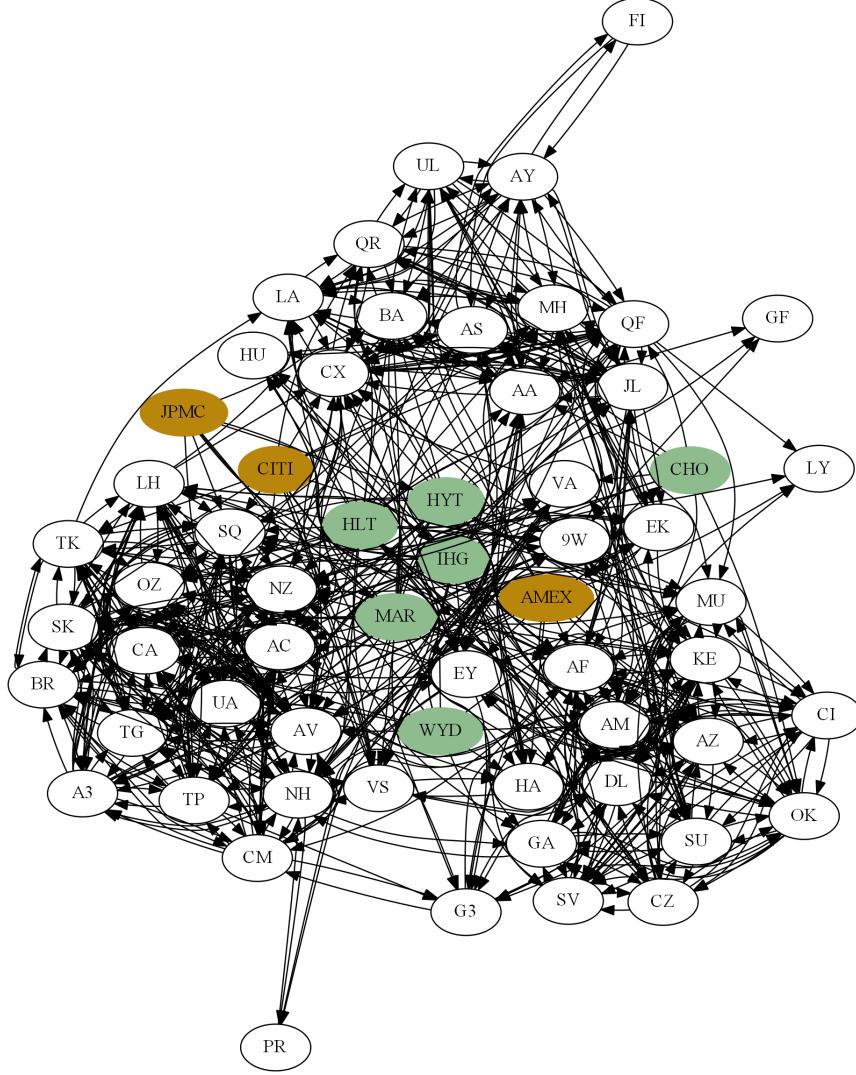
The network of loyalty programs is composed of transfer partnerships among 3 credit card issuers, 7 hotel chains, and 43 airlines. Figure 2 is a snapshot of this network observed in November 2018. A node represents the loyalty program of a firm. In particular, the golden, teal, and white nodes are the loyalty programs of credit card issuers, hotel chains, and airlines, respectively. A directed link indicates a transfer partnership. That is, a directed link from a node to another indicates that loyalty points can be transferred from the source node to the target node. Each link is assigned a weight, which indicates the associated the transfer ratio. A transfer ratio is the number of points in the target node that can be obtained per 1 point in the source node. An exception is the links between airline nodes. A link from an airline node to another indicates that loyalty points in the source node can be used to redeem for flights offered¹⁶ by the target node. The formation of links among airlines is complex, with strategic motives over flight routes. Moreover, a large share of the links are partnerships formed via airline alliances, for which no observed changes are observed since 2014. I take the links among airlines as exogenous, and they are assigned a weight of 1.

In figure 2, nodes are positioned close to each other if they are strongly connected via direct and indirect links. The cluster of airlines on the lower-left is Star Alliance, the cluster on the lower-right is SkyTeam, and the cluster on the top is Oneworld. We can observe that AMEX (American Express Company) is positioned close to SkyTeam, while JPMC (J.P. Morgan Chase Bank) and

¹⁵For example, United Airlines (a co-brand partner of JPMC) earned approximately \$1.2 bil. revenue from selling loyalty points in 2017. It accounted for about 3.2 percent of the airline's total operating revenue (United Airlines, 2014-2018). In the same year, American Airlines (a co-brand partner of CITI) earned \$2.2 bil. revenue from selling loyalty points. It accounted for about 5.2 percent of the airline's total operating revenue (American Airlines, 2014-2018). These are the amounts sold to non-airline third-party partners. The amounts only include recognized revenues, excluding deferred revenues.

¹⁶This is not the same as a flight operated by the partner airline. For example, a codeshare flight is sold by an airline but can be operated by a partner airline. Having a codeshare agreement does not imply being able to use points to redeem for flights offered by the partner airline.

Figure 2: Network Map (Nov. 2018)

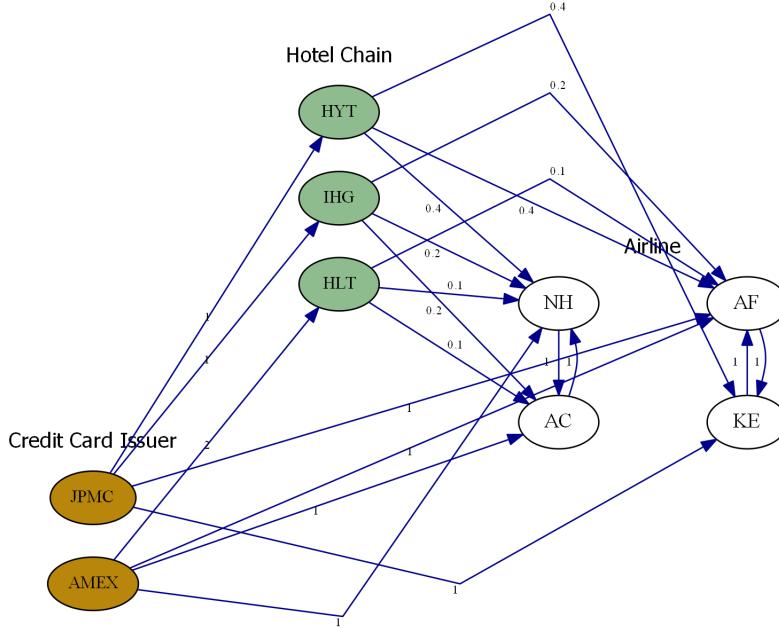


This figure illustrates the network of loyalty programs observed in November 2018. The golden, teal, and white nodes are the loyalty programs of credit card issuers, hotel chains, and airlines, respectively. See table 9 in the appendix for a dictionary of node names. A directed link from a node to another indicates that points may be transferred from the source node to the target node. Nodes are positioned close to each other if they are strongly connected via direct and indirect links. The cluster of airlines on the bottom-left is Star Alliance, the cluster on the bottom-right is SkyTeam, and the cluster on the top is Oneworld. AMEX is positioned close to SkyTeam. On the other hand, JPMC and CITI are positioned close to Star Alliance and Oneworld.

CITI (Citibank) are positioned close to Star Alliance and Oneworld. This paper studies why the linkages between the credit card issuers and the airlines were formed as observed¹⁷. In particular, the goal is to explain how characteristics of firms and network linkages affected the formation of linkage between a credit card issuer and an airline.

Figure 3 visualizes the network of loyalty programs using a subset of the nodes. I remark three critical features. First, except for airlines, there are no links between nodes within the same industry sector¹⁸. This reveals a non-cooperative relationship among credit card issuers and among hotel chains. Second, there are no linked pointing to credit card issuers. Third, there are no links from airlines to hotel chains¹⁹. Such features, along with classification of nodes by industry sector, allow splitting the network into four subnetworks.

Figure 3: Network of Loyalty Programs (Abridged, Nov. 2018)



This figure is an abridged version of figure 2, with only a subset of the nodes. The golden, teal, and white nodes are the loyalty programs of credit card issuers, hotel chains, and airlines, respectively. A directed link from a node to another indicates that points may be transferred from the source node to the target node. Except for the links between airlines, each link weights indicates the transfer ratio per 1 point in the source node. A link between airlines indicate that points in the source node can be used to redeem for flights offered by the target node. All links between airline nodes are assigned a weight of 1.

¹⁷I focus on partnership formation between credit card issuers and airlines, while taking partnerships between credit card issuers and hotel chains, and partnerships between airlines as exogenous. Partnership formation between hotel chains and airlines is modeled but is not a focus. Partnership formation among airlines, such as airline alliances, is beyond the scope of this paper and is taken as exogenous.

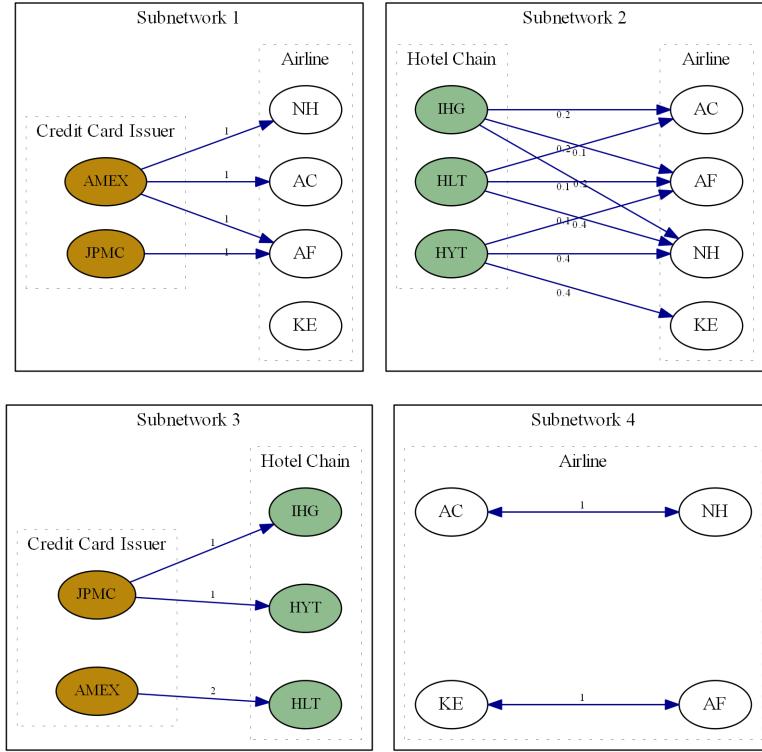
¹⁸A special exception is a merger between hotel chains. For example, the merger of Marriott International and Starwood Hotels and Resorts enabled transfers between their loyalty programs. The loyalty programs were completely integrated shortly after.

¹⁹There are a few exceptions. For example, loyalty points of a few airlines can be transferred to Hilton Hotels & Resorts.

Figure 4 illustrates the splitting. Subnetwork 1 describes linkages from credit card issuers to airlines, and subnetwork 2 describes linkages from hotel chains to airlines. Subnetwork 3 describes linkages from credit card issuers to hotel chains, and subnetwork 4 describes linkages among airlines. In particular, subnetworks 1,2, and 3 describe linkages involving two different industry sectors. These subnetworks are uni-directional bipartite because all links are directed from one industry sector to another, and there are no links within the same industry.

I model only the formation of subnetworks 1 and 2. Although the focus is on the formation of subnetwork 1, modeling subnetwork 2 is necessary because the formation of the subnetworks may depend on each other. Section 5.5 further explains this. As discussed in section 3, transfer partnerships between credit card issuers and hotel chains are determined by co-brand partnerships, and thus the formation of subnetwork 3 is taken as exogenous. Subnetwork 4, which describes partnerships among airlines, is also taken as exogenous, as explained earlier in this section.

Figure 4: Splitting into Subnetworks



The network in figure 3 is split into four subnetworks. Subnetwork 1 (upper left) describes linkages from credit card issuers to airlines. Subnetwork 2 (upper right) describes linkages from hotel chains to airlines. Subnetwork 3 (lower left) describes linkages from credit card issuers to hotel chains. Subnetwork 4 (lower right) describes linkages between airlines.

In the following sections, \mathbf{D} represents the full network, and $\mathbf{D}_1, \mathbf{D}_2, \mathbf{D}_3, \mathbf{D}_4$ represent subnetworks 1,2,3,4, respectively. They are adjacency matrices corresponding to the network and the subnetworks. For example, the $(s, t)^{\text{th}}$ element of \mathbf{D} is equal to 1 if there exists a link from node s to node t and zero if otherwise. The information on link weights (i.e., transfer ratios) is kept separately from the adjacency matrix. In the following, firms, loyalty programs of firms, and nodes are synonymous unless I explicitly distinguish them. For example, I often use “credit card issuer” to denote the “loyalty program of the credit card issuer” or the corresponding node in the network. However, I explicitly distinguish the characteristics of firms (e.g., key performance indicators) and the characteristics of loyalty programs (e.g., redemption rates of loyalty points).

5 The Model

The model assumes that at least two states of the network are observed. The observed states of the network are viewed as snapshots of a discrete-time stochastic process. This specification reflects that contracts (transfer partnerships) between firms are revised at some time interval and the process continues indefinitely. The model describes the transition of the network from an initial state \mathbf{D}^t to the next observed state \mathbf{D}^{t+1} , which I also denote as the ending state.

The sequential network formation model of Christakis et al. (2010) is particularly well-suited to describe the transition of network as a discrete-time process. However, their model assumes that the initial state of the network is the null state (i.e., no links between agents) and can accommodate only one transition. This paper’s model is built upon their work but allows agents to possess links in the initial state and accommodates transitions over multiple periods. From here on, the term *period* t indicates the time frame for the transition from \mathbf{D}^t to \mathbf{D}^{t+1} . The model accommodates transitions of the network over multiple periods by assuming that the model parameters are invariant over time. Section 6.2 provides a more detailed explanation on the specification.

I allow asymmetric roles of agents in the network formation process, by utilizing the unidirectional bipartite structure of the subnetworks 1 and 2. The model simplifies the asymmetric roles of firms as *bidders* and *choosers*, and linkages are determined according to interactions between them via a sequential authority mechanism. First, bidders submit bids to choosers, where a bid represents the terms of contract proposed by the bidder to the chooser. After receiving the bids, choosers choose whether or not to accept the bids. A link from a chooser to a bidder is added or maintained if the bidder’s bid is accepted by the chooser, and an existing link is removed or no linkage is maintained if the bidder’s bid is not accepted by the chooser. In subnetwork 1, airlines are the bidders, and credit card issuers are the choosers. In subnetwork 2, airlines are the bidders and hotel chains are the choosers.

For the rest of the paper, the letters i, j , and k indicate an arbitrary credit card issuer, hotel chain, and airline, respectively. $\mathbb{I}^t, \mathbb{H}^t$, and \mathbb{K}^t respectively denote the set credit card issuers, hotel chains, and airlines for period t . Note that the superscript t indicates that the node sets may vary over the periods. Table 1 provides a reference for notations.

Table 1: Table of Notations

| Notation | Description |
|---------------------------|--|
| Subnetwork 1 | Chooser: credit card issuers, Bidder: airlines |
| Subnetwork 2 | Chooser: hotel chains, Bidder: airlines |
| Subnetwork 3 | Between credit card issuers and hotel chains (exogenous) |
| Subnetwork 4 | Between airlines (exogenous) |
| C_i^t | Key performance indicators (KPIs) of credit card issuer i for period t |
| H_j^t | Key performance indicators of hotel chain j for period t |
| A_k^t | Key performance indicators of airline k for period t |
| \mathbb{K}^t | Set of airlines for period t ; $\mathbb{I}^t, \mathbb{J}^t$ are similar |
| \mathbf{A}^t | Collection of A_k^t for $k \in \mathbb{K}^t$; $\mathbf{C}^t, \mathbf{H}^t$ are similar |
| \mathbf{A}_{-k}^t | \mathbf{A}^t except A_k^t ; $\mathbf{C}_{-i}^t, \mathbf{H}_{-j}^t$ are similar |
| v_i^t | Value of node i 's point for period t ; v_j^t, v_k^t are similar |
| \mathbf{v}^t | Collection of v_i^t, v_j^t, v_k^t for all $i \in \mathbb{I}^t, j \in \mathbb{J}^t, k \in \mathbb{K}^t$ |
| w_{ik}^t | Transfer ratio from i to k for period t ; w_{jk}^t is similar |
| b_{ik}^t | Bid i receives from k for period t ; b_{jk}^t is similar |
| w_i^t | Collection w_{ik}^t for $k \in \mathbb{K}^t$; b_i^t is similar |
| \mathbf{w}^t | Collection of w_i^t for $i \in \mathbb{I}^t$; \mathbf{b}^t is similar |
| ϵ_{ki}^t | Idiosyncratic factor of k specific to i for period t ; $\epsilon_{ik}^t, \epsilon_{jk}^t$ are similar |
| S^t | Sequence of meetings for period t |
| M^t | Length of S^t |
| m | Position of meeting in S^t |
| $(i^{t,m}, k^{t,m})$ | m^{th} meeting in period t if occurs in subnetwork 1 |
| $(j^{t,m}, k^{t,m})$ | m^{th} meeting in period t if occurs in subnetwork 2 |
| $P_i^{t,m}$ | i 's portfolio of partners at beginning of m^{th} meeting in period t ; $P_j^{t,m}$ is similar |
| \mathbf{D}^t | Adjacency matrix of the network at beginning of period t |
| $\mathbf{D}^{t,m}$ | Adjacency matrix of the network at the beginning of m^{th} meeting in period t |
| $\mathbf{D}_{-i}^{t,m}$ | $\mathbf{D}^{t,m}$ excluding node i ; $\mathbf{D}_{-j}^{t,m}$ is similar |
| $\mathbf{D}_1^{t,m}$ | Adjacency matrix of subnetwork 1 at beginning of m^{th} meeting in period t |
| $\mathbf{D}_2^{t,m}$ | Adjacency matrix of subnetwork 2 at beginning of m^{th} meeting in period t |
| \mathbf{D}_3^t | Adjacency matrix of subnetwork 3 for period t (exogenous) |
| \mathbf{D}_4^t | Adjacency matrix of subnetwork 4 for period t (exogenous), also includes information on redemption rates of airline loyalty points |
| $\mathbf{D}_{1,-i}^{t,m}$ | $\mathbf{D}_1^{t,m}$ excluding node i ; $\mathbf{D}_{2,-j}^{t,m}$ is similar |

5.1 Model Outline

This section describes the timeline of the model and the information set available to the nodes (loyalty programs of firms). For each period,

1. The initial state of the network is equal to the state at the end of the previous period. All nodes possess full information over the full network.
2. Nature draws a sequence of meetings. It is unknown to all nodes.
3. All bidders simultaneously submit bids to all choosers. Each chooser observes only the bids submitted to itself.
4. Bilateral meetings between choosers and bidders occur along the sequence of meetings. The sequence of meetings encompasses all (bidder, chooser) pairs for subnetworks 1 and 2. There are three possible outcomes for each meeting. In a meeting, the chooser
 - (a) Adds a new link with the bidder.
 - (b) Removes an exiting link with the bidder.
 - (c) Maintains status quo.
5. The outcome of a meeting is immediately revealed to all nodes. The next meeting occurs.

For the rest of the paper, a credit card issuer (i) serves as the chooser and an airline (k) serves as the bidder. The case with a hotel chain (j) as the chooser and an airline (k) as the bidder is discussed briefly.

5.2 Bidders

To facilitate the transfer of loyalty points, credit card issuers make cash payments to airlines, based on negotiated cents-per-point prices. The bid submitted by airline k to credit card issuer i is a one-dimensional object describing the price and other factors specified in the terms of contract proposed by k . I assume that the transfer ratio from i to k , after accounting for the relative values of loyalty points of i and k , captures the terms of contract. Let b_{ik}^t denote the bid submitted by k to i for period t . It is given by

$$b_{ik}^t = \frac{w_{ik}^t}{v_i^t/v_k^t}, \quad (1)$$

where w_{ik}^t is the transfer ratio from i to k . v_i^t and v_k^t are the values of loyalty points of nodes i and k , respectively. A larger (smaller) w_{ik}^t indicates 1 point in node i can be transferred to obtain a larger (smaller) amount of points in node k . Equation (1) states that a large transfer ratio is observed either because i 's points are more valuable than k 's (large v_i^t/v_k^t) or because i received favorable terms of contract from k (a large b_{ik}^t). This specification is somewhat restrictive because it is possible that a credit card issuer smooths its transfer ratios across its partners. An extreme case would be setting a single transfer ratio for all transfer partners, regardless of the heterogeneity in cost associated with facilitating the transfers of loyalty points.

v_i^t and v_k^t denote the points redemption values of nodes i and k . For the airline k , v_k^t accounts for all redemption options for flights accessible via its loyalty program, including its own flights and the flights of its partners. Thus v_k^t depends on the redemption options of loyalty points offered by k and redemption options granted by its partners. For credit card issuer i , v_i^t only accounts for its own points redemption options, excluding the redemption options accessible via the transfer partners. Appendix C.1 explains the procedure for constructing the node values. Figure 9 and table 10 in the appendix report details on values of nodes for airlines and hotel chains, respectively.

The values serve a tool for comparing loyalty points within the same industry and across different industries, so that the normalized transfer ratios $\frac{w_{ik}^t}{v_i^t/v_k^t}$ are comparable for all (i, k) pairs. For example, the transfer ratio from AMEX to AM is 1.6, while the transfer ratios for most other airlines are 1. The discrepancy exists because the loyalty points of AM are in kilometer units, while the loyalty points of most other airlines are in mile units. These transfer ratios are not comparable unless the difference in units is accounted for.

b_{ik}^t is observed (constructed from observed variables) only when there exists a link from i to k because otherwise w_{ik}^t is not observed. I construct the missing bids using a hedonic regression model

$$b_{ik}^t = \beta_c' C_i^t + \gamma_c' A_k^t + \epsilon_{ki}^t \quad (2)$$

where C_i^t and A_k^t are key performance indicators (KPIs) of the credit card issuer i and the airline k , respectively, in period t (in the empirical analysis, I applied logarithmic transformations to the KPIs except for those in percentage units). The subscript c for coefficients β_c and γ_c indicate that the coefficients are specific to the bids for credit card issuers. Moreover, I assume that the coefficients are invariant over the periods. ϵ_{ki}^t denotes the unobserved idiosyncratic factor of k specific to i . I also impose the following assumption.

Assumption 1. For all t and for all $i \in \mathbb{I}^t, k \in \mathbb{K}^t$

$$\epsilon_{ki}^t | C_i^t, A_k^t \sim i.i.d. \quad (3)$$

C_{-i}^t denotes the collection of KPIs of all credit card issuers in \mathbb{I}^t excluding i , and A_{-k}^t denotes the collection of KPIs of all airlines in \mathbb{K}^t excluding k . This assumption implies that bidders are non-strategic agents and that bids are determined only at pairwise level. The same specification applies with i replaced by j (a hotel chain) and C_i^t replaced with H_j^t (KPIs of j in period t), in which case the coefficients β_h and γ_h carry the subscript h to indicate that the coefficients are specific to the bids for hotel chains.

Table 2: OLS Estimation of Hedonic Regression Model

| Variable | Credit Card Issuer | Hotel Chain |
|----------------------------------|---------------------|---------------------|
| log(Credit Card Purchase) | 0.0504** (0.020) | - |
| Delinquency Rate | -0.0167 (0.106) | - |
| Writeoff Rate | -0.0365 (0.062) | - |
| log(Hotel Revenue) | - (0.005) | 0.0074 |
| log(Number of Hotels) | - (0.004) | 0.0031 |
| log(RevPAR) | - (0.025) | -0.0056 |
| Occupancy Rate | - (0.002) | -0.0005 |
| log(Passenger Revenue) | -0.0136 (0.038) | -0.0206* (0.011) |
| log(RPK) | -0.0069 (0.227) | 0.0854 (0.052) |
| log(ASK) | 0.0496 (0.228) | -0.0447 (0.054) |
| Adj. R-squared | 0.988 | 0.944 |
| No. observations | 79 | 458 |

This table reports OLS estimates of the hedonic regression models for bids. The first column lists the covariates included in the hedonic regression models (see appendix B.3 for definitions). The second column reports parameter estimates associated with the bids for credit card issuers, submitted by airlines. The third column reports parameter estimates associated with the bids for hotel chains, submitted by airlines. Standard errors are reported in parentheses. ** indicates statistical significance at 0.05 level. * indicates statistical significance at 0.1 level.

Table 2 reports the OLS fit of the hedonic regression model (see appendix B.3 for details on the variables; see appendix C.2 for the estimation procedure). The estimation result suggests that a credit card issuer with a larger amount Credit Card Purchase (total amount of credit card purchases processed) tends to receive larger bids from airlines (more favorable to the credit card issuer), other things equal. They also suggest that an airline with a larger Passenger Revenue (revenue from passenger flights) tend to submit smaller bids to hotel chains (less favorable to hotel chains), other things equal. In both estimation of bids (bids for credit card issuers and bids for hotel chains), the reported R^2 is close to 1, indicating that the included KPIs well explain the variations in the bids.

Table 3 reports summary statistics for the observed and the fitted bids. The OLS estimation smooths outliers with extremely small or large bids; however, overall, the distribution of the fitted

bids is similar to the observed bids. The similarity is partially due to the assumption that bidders are non-strategic, and thus whether or not the bidder has links (transfer partnerships) with a certain group of choosers does not affect its bids. I further used the fitted bids and equation (1) to construct the potential transfer ratios for all (chooser, bidder) pairs.

Table 3: Summary Statistics of Bids

| Chooser | N | Mean | SD | 10 th | 25 th | Med | 75 th | 90 th |
|-------------------------------------|-----|-------|-------|------------------|------------------|-------|------------------|------------------|
| Credit Card Issuers | 79 | 0.933 | 0.113 | 0.752 | 0.900 | 0.963 | 1.012 | 1.045 |
| Credit Card Issuers (Fitted) | 381 | 0.915 | 0.055 | 0.844 | 0.881 | 0.917 | 0.958 | 0.990 |
| Hotel Chains | 458 | 0.262 | 0.066 | 0.182 | 0.217 | 0.259 | 0.304 | 0.345 |
| Hotel Chains (Fitted) | 848 | 0.257 | 0.021 | 0.232 | 0.241 | 0.256 | 0.274 | 0.286 |

This table compares fitted bids with observed bids (constructed from observed variables). The OLS estimation smooths outliers with extremely low and extremely large bids. However, the distribution of the fitted bids are overall comparable with the observed bids for both credit card issuers and hotel chains.

5.3 Choosers

After receiving bids, the chooser strategically chooses a portfolio of transfer partners such that the portfolio maximizes its objective function. The objective function of credit card issuer i has the form

$$U_c(P_i^t; b_i^t, w_i^t, \mathbf{A}^t, \mathbf{D}_{1,-i}^t, \mathbf{D}_2^t, \mathbf{D}_3^t, \mathbf{D}_4^t, \epsilon_i^t, \theta_c) = g_c(P_i^t, b_i^t, w_i^t, \mathbf{A}^t, \mathbf{D}_{1,-i}^t, \mathbf{D}_2^t, \mathbf{D}_3^t, \mathbf{D}_4^t, \theta_c) + \sum_{k \in P_i^t} \epsilon_{ik}^t \quad (4)$$

$P_i^t \subseteq \mathbb{K}^t$ denotes the current portfolio of airline partners possessed by i , and $b_i^t = \{b_{ik}\}_{k \in \mathbb{K}^t}$ is the collection of bids i receives in period t . $\{w_i^t\}_{k \in \mathbb{K}^t}$ is the collection of potential transfer ratios for i , and \mathbf{A}^t is the collection of A_k^t for all $k \in \mathbb{K}^t$. $\mathbf{D}_{1,-i}^t$ is the adjacency matrix indicating the current state of the subnetwork 1, excluding the part associated with i . $\mathbf{D}_2^t, \mathbf{D}_3^t$, and \mathbf{D}_4^t are adjacency matrices indicating current states of subnetworks 2,3, and 4, respectively. These four matrices contain the information of \mathbf{D}_{-i}^t , which is the adjacency matrix for the network, excluding the part associated with i . $\epsilon_i^t = \{\epsilon_{ik}^t\}_{k \in \mathbb{K}^t}$ is the collection of unobserved idiosyncratic factors of i specific to k . The vector of model parameters θ_c is specific to credit card issuers and is invariant across periods. A restriction is that ϵ_{ik}^t enters the objective function additively. I also assume the following.

Assumption 2. For all t and for all $i \in \mathbb{I}^t, k \in \mathbb{K}^t$

$$\epsilon_{ik}^t | b_i^t, w_i^t, \mathbf{A}^t, \mathbf{D}^t \sim i.i.d. \quad (5)$$

Equation (5) states that the idiosyncratic factors are independent and identically distributed, conditional on the bids received by the credit card issuer, its potential transfer ratios, the KPIs of airlines, and the current state of the network. For the empirical analysis, I endowed the idiosyncratic factors with the following distribution:

$$\epsilon_{ik}^t | b_i^t, w_i^t, \mathbf{A}^t, \mathbf{D}^t \stackrel{\text{i.i.d.}}{\sim} \text{Logistic}(0, 1). \quad (6)$$

$g_c(\cdot)$ has the following linear form (replacing the notations $\mathbf{D}_{1,-i}^t, \mathbf{D}_2^t, \mathbf{D}_3^t, \mathbf{D}_4^t$ with \mathbf{D}_{-i}^t).

$$\begin{aligned} g_c(P_i^t, b_i^t, w_i^t, \mathbf{A}^t, \mathbf{D}_{-i}^t, \theta_c) = & \theta_{0c} \\ & + \theta_{1c} |P_i^t| + \theta'_{2c} \text{Performance}(P_i^t, b_i^t, \mathbf{A}^t) + \theta_{3c} \text{GeoHub}(P_i^t) \\ & + \theta_{4c} \text{Mileage}(P_i^t, w_i^t, \mathbf{D}_4^t) + \theta_{5c} \text{Routes}(P_i^t, \mathbf{D}_4^t) \\ & + \theta_{6c} \text{Competitor}(P_i^t, \mathbf{D}_{1,-i}^t) + \theta_{7c} \text{Transitivity}(P_i^t, \mathbf{D}_2^t, \mathbf{D}_3^t), \end{aligned} \quad (7)$$

where $\theta_c = (\theta_{0c}, \theta'_{1c}, \theta_{2c}, \theta_{3c}, \theta_{4c}, \theta_{5c}, \theta_{6c}, \theta_{7c})'$. θ_{0c} is the constant term.

The first three components serve as controls. $|P_i^t|$ is the number of airline transfer partners possessed by i , which controls for the cost and benefit associated with the size of the portfolio. *Performance* is a measure of market performance of the airlines in the portfolio. It is equal to

$$\sum_{k \in P_i^t} \frac{b_{ik}^t}{\sum_{k \in P_i^t} b_{ik}^t} A_k^t, \quad (8)$$

which is a weighted average of the KPIs of the airlines in the portfolio, with more importance given to airlines that submitted larger bids. This component serves as a proxy for the overall institutional characteristics of the partner airlines. *GeoHub* is the number of distinct airline hub locations, which is a measure of geographic diverseness possessed by the portfolio (see table 9 for details). The underlying view is that credit card issuers target U.S. consumers with diverse geographic affinity and seek to enhance the attractiveness of their loyalty programs by partnering with airlines with strong presence in various geographic regions.

The fourth component is a measure of redemption options of loyalty points for flights granted by the portfolio (see appendix C.3 for details). Other things equal, a lower value of *Mileage* indicates that the redemption options granted by the portfolio of transfer partners is more cost-effective, in terms of i 's loyalty points, in redeeming for flights to various geographic regions. In other words, a lower value of *Mileage* indicates that the resources of airline partners included in the portfolio well complement each other and that the portfolio of transfer partners grants better redemption options for i 's loyalty program. The fifth component, *Routes*, is the total number reward flight routes granted by the portfolio, which is a measure of diverseness in redemption for reward flights granted by the portfolio of airline partners. *Routes* counts all routes of reward flights, with multiplicity for the same flight route granted by different airline partners. Note that for brevity of notation, I specify that \mathbf{D}_4^t also contains information on the redemption options of airline loyalty points for period t .

The last two components capture the effect of network linkages, which are transfer partnerships of other firms, in partner choice. *Competitor* is the number of i 's airline partners that are also

partners of other credit card issuers. Specifically, it is equal to

$$\left| \left\{ k \in P_i^t : [\mathbf{D}_{1,-i}^t]_{ik} = 1, \forall i \neq k \right\} \right|, \quad (9)$$

where $[\mathbf{D}]_{ik}$ denotes the $(i, k)^{\text{th}}$ element of the matrix \mathbf{D} . *Transitivity* is the number of i 's airline partners that are also partners of i 's hotel partners, which is equal to

$$\left| \left\{ k \in P_i^t : [\mathbf{D}_2^t]_{jk} = 1, \forall j \text{ s.t. } [\mathbf{D}_3^t]_{ij} = 1 \right\} \right|. \quad (10)$$

With hotel chain j as the chooser, the objective function

$$\begin{aligned} U_h(P_j^t, b_j^t, w_j^t, \mathbf{A}^t, \mathbf{D}_1^t, \mathbf{D}_{2,-j}^t, \mathbf{D}_3^t, \mathbf{D}_4^t, \epsilon_j^t, \theta_h) &= g_h(P_j^t, b_j^t, w_j^t, \mathbf{A}^t, \mathbf{D}_1^t, \mathbf{D}_{2,-j}^t, \mathbf{D}_3^t, \mathbf{D}_4^t, \theta_h) \\ &\quad + \sum_{k \in P_j^t} \epsilon_{jk}^t \end{aligned} \quad (11)$$

is specified similarly with a different vector of model parameters θ_h . The subscript h indicates it is associated with hotel chains.

As the portfolio is updated sequentially (explained in the next subsection), by adding, removing, or maintaining links with airlines, the components of θ capture the marginal contributions of variables to the objective function.

5.4 Sequence of Meetings

In each period, bilateral meetings between bidders and choosers occur according to a sequence of meetings. In each meeting, the chooser, after observing all of its bids, decides to accept or reject the bidder's bid. Similar to Christakis et al. (2010), I specify that all (chooser, bidder) pairs meet exactly once in each period. Moreover, I specify that the sequence of meetings is unknown to all nodes in the network. Note that the node set and the sequence of meetings may be different over the periods.

Let

$$S_1^t = \left[(i^{t,1}, k_1^{t,1}), (i^{t,2}, k_1^{t,2}), \dots, (i^{t,M_1^t}, k_1^{t,M_1^t}) \right] \quad (12)$$

$$S_2^t = \left[(j^{t,1}, k_2^{t,1}), (j^{t,2}, k_2^{t,2}), \dots, (j^{t,M_2^t}, k_2^{t,M_2^t}) \right]. \quad (13)$$

S_1^t and S_2^t denote the sequence of meetings between nodes within subnetwork 1 and within subnetwork 2, respectively, in period t . Each element of S_1^t and S_2^t is a unique (chooser, bidder) pair, and the superscripts t, m indicate the m^{th} meeting in period t . M_1^t and M_2^t denote the lengths of the two sequences. S^t denotes the complete sequence of meetings in period t , and it is a mixture of S_1^t and S_2^t with length equal to $M^t = M_1^t + M_2^t$. Thus the m^{th} meeting in S^t is be either $(i^{t,m}, k^{t,m})$ or $(j^{t,m}, k^{t,m})$. Because each pair meets exactly once in a period, a (chooser,bidder) pair can appear only once in S^t .

Without loss of generality, let $(i^{t,m}, k^{t,m})$ be the m^{th} meeting in period t (i.e., the m^{th} meeting is between a credit card issuer and an airline). I assume that the chooser is myopic so that it

maximizes the objective function given the current state of the network, without forming expectations for future states of the network. Formally, let $P_{i^{t,m}}^{t,m}$ and $P_{i^{t,m}}^{t,m+1}$ denote the portfolio of airline partners possessed by $i^{t,m}$ at the beginning and immediately after the m^{th} meeting, respectively. $\mathbf{D}_{-i^{t,m}}^{t,m}$ denotes the state of network, excluding the part associated with $i^{t,m}$, at the beginning of the m^{th} meeting. Note that $P_{i^{t,m}}^{t,m}$ is determined by the outcomes of previous meetings involving $i^{t,m}$, and $\mathbf{D}_{-i^{t,m}}^{t,m}$ is determined by the outcomes of previous meetings not involving $i^{t,m}$. The outcome of the m^{th} meeting satisfies

$$U_c(P_{i^{t,m}}^{t,m+1}; b_{i^{t,m}}^t, w_{i^{t,m}}^t, \mathbf{A}^t, \mathbf{D}_{-i^{t,m}}^{t,m}, \epsilon_{i^{t,m}}^t, \theta_c) \geq U_c(P_{i^{t,m}}^{t,m} \cup \{k^{t,m}\}; b_{i^{t,m}}^t, w_{i^{t,m}}^t, \mathbf{A}^t, \mathbf{D}_{-i^{t,m}}^{t,m}, \epsilon_{i^{t,m}}^t, \theta_c) \quad (14)$$

$$U_c(P_{i^{t,m}}^{t,m+1}; b_{i^{t,m}}^t, w_{i^{t,m}}^t, \mathbf{A}^t, \mathbf{D}_{-i^{t,m}}^{t,m}, \epsilon_{i^{t,m}}^t, \theta_c) \geq U_c(P_{i^{t,m}}^{t,m} \setminus \{k^{t,m}\}; b_{i^{t,m}}^t, w_{i^{t,m}}^t, \mathbf{A}^t, \mathbf{D}_{-i^{t,m}}^{t,m}, \epsilon_{i^{t,m}}^t, \theta_c). \quad (15)$$

Equations (14) and (15) respectively state that $i^{t,m}$ cannot be made better off by adding $k^{t,m}$ to its portfolio (if $k^{t,m}$ is not in the portfolio) or removing $k^{t,m}$ from its portfolio (if $k^{t,m}$ is in the portfolio) immediately after the m^{th} meeting, other things equal. In other words, $i^{t,m}$ cannot be made better off by altering its choice in the m^{th} meeting under the current state of the network. This specification reflects the view that the network evolves constantly, and one can only observe snapshots of the network. There are infinitely many meetings laid out in the future as the number of periods grows, and the chooser cannot predict future states of the network. Instead, the chooser chooses what is optimal under the current state of the network.

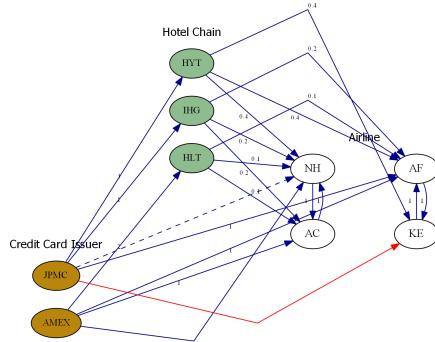
5.5 Dependence on Sequence of Meetings

For each period, the sequence of meetings lays out the ordering of events that constitute the transition of the network. Thus, for a given (chooser, bidder) pair, the current state of the network may be different under a different sequence of meetings. A problem arises because we do not observe the sequence of meetings.

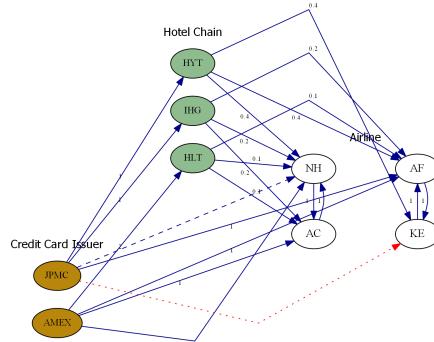
Figure 5 visualizes such differences in the current state of the network. KE was removed from JPMC's portfolio of transfer partners in 2018; however, we do not know if the meetings between JPMC and other airlines occurred before or after the removal of KE. We also do not know if the meetings between other credit card issuers and KE occurred before or after the removal.

Figure 5: Dependence on Sequence of Meetings

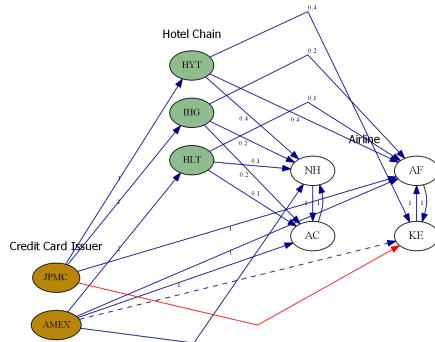
(a) (JPMC, NH) occurs before (JPMC, KE)



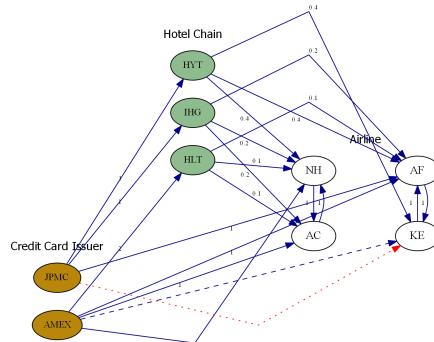
(b) (JPMC, NH) occurs after (JPMC, KE)



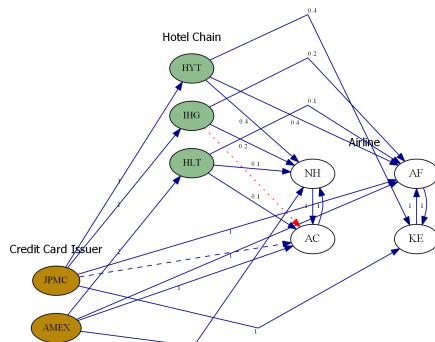
(c) (AMEX, KE) occurs before (JPMC, KE)



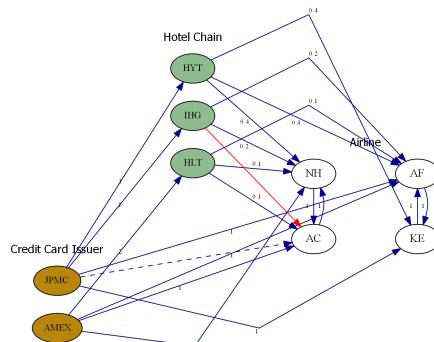
(d) (AMEX, KE) occurs after (JPMC, KE)



(e) (JPMC, AC) occurs before (IHG, AC)



(f) (JPMC, AC) occurs after (IHG, AC)



Sub-figures (a) and (b) illustrate how the order of a credit card issuer's own meetings may affect the current state of the network it faces. In (a), JPMC meets NH before the link from JPMC to KE was removed. In (b), JPMC meets NH after the link from JPMC to KE was removed. (c) and (d) illustrate how the order of another credit card issuer's meetings may affect the current state of the network. In (c), AMEX meets KE before the link from JPMC to KE was removed. In (d), AMEX meets KE after the link from JPMC to KE was removed. (e) and (f) illustrate how the order of a hotel chain's meetings may affect the current state of the network. In (e), JPMC meets AC before the link from IHG to AC was added. In (f), JPMC meets AC after the link from IHG to AC was added.

Sub-figures (a) and (b) illustrate the current state of the network at the beginning of the meeting between JPMC and an airline, occurring before and after the removal, respectively. In sub-figure (a), JPMC has a link to KE when it meets NH, and in sub-figure (b), JPMC does not have a link to KE when it meets NH. For these cases, the current state of the network at the meeting between JPMC and NH vary depending on the ordering of meetings associated with JPMC.

Sub-figures (c) and (d) illustrate the current state of the network at the beginning of the meeting between AMEX and KE, occurring before and after the removal, respectively. In sub-figure (c), JPMC has a link to KE when AMEX meets KE, and in sub-figure (d), JPMC does not have a link to KE when AMEX meets KE. For these cases, the current state of the network at the meeting between AMEX and KE vary depending on the ordering of meetings associated with another credit card issuer, JPMC.

IHG has been a hotel transfer partner of JPMC at least from 2014 to 2018, and IHG added AC to its portfolio of airline transfer partners in 2016. Here, we do not if the meeting between JPMC and AC occurred before or after the addition. Sub-figures (e) and (f) illustrate the current state of the network at the beginning of the meeting between JPMC and AC, occurring before and after the addition, respectively. In sub-figure (e), IHG does not have a link to AC when JPMC meets AC, and in sub-figure (f), IHG has a link to AC when JPMC meets AC. For these cases, the current state of the network at the meeting between JPMC and AC vary depending on the ordering of meetings associated with a partner hotel chain of JPMC.

5.6 Meeting Outcomes

Because each (chooser, bidder) pair meets exactly once in each period, the linkage between a pair is completely determined by the outcome of the meeting between them. Conversely, the initial state and the ending state of the network completely determines the meeting outcomes. Recall that \mathbf{D}^t and \mathbf{D}^{t+1} denote the initial and the ending states of the network for period t . There are four possible cases:

1. $[\mathbf{D}^t]_{i^{t,m} k^{t,m}} = 0$ and $[\mathbf{D}^{t+1}]_{i^{t,m} k^{t,m}} = 0$
 - $i^{t,m}$ does not add $k^{t,m}$ its portfolio at the end of m^{th} meeting (no changes).
 - $i^{t,m}$ is better of with $P_{i^{t,m}}^{t,m}$ than $P_{i^{t,m}}^{t,m} \cup \{k^{t,m}\}$.
2. $[\mathbf{D}^t]_{i^{t,m} k^{t,m}} = 0$ and $[\mathbf{D}^{t+1}]_{i^{t,m} k^{t,m}} = 1$
 - $i^{t,m}$ adds $k^{t,m}$ to its portfolio at the end of m^{th} meeting.
 - $i^{t,m}$ is better of with $P_{i^{t,m}}^{t,m} \cup \{k^{t,m}\}$ than $P_{i^{t,m}}^{t,m}$.
3. $[\mathbf{D}^t]_{i^{t,m} k^{t,m}} = 1$ and $[\mathbf{D}^{t+1}]_{i^{t,m} k^{t,m}} = 0$
 - $i^{t,m}$ removes $k^{t,m}$ from its portfolio at the end of m^{th} meeting.
 - $i^{t,m}$ is better of with $P_{i^{t,m}}^{t,m} \setminus \{k^{t,m}\}$ than $P_{i^{t,m}}^{t,m}$.
4. $[\mathbf{D}^t]_{i^{t,m} k^{t,m}} = 1$ and $[\mathbf{D}^{t+1}]_{i^{t,m} k^{t,m}} = 1$
 - $i^{t,m}$ does not remove $k^{t,m}$ from its portfolio at the end of m^{th} meeting (no changes).
 - $i^{t,m}$ is better of with $P_{i^{t,m}}^{t,m}$ than $P_{i^{t,m}}^{t,m} \setminus \{k^{t,m}\}$.

6 Estimation Method

Given the fitted bids and potential transfer ratios constructed in section 5.2 (they are treated as observed variables), the model specification yields a likelihood function describing the transition of the network, conditional on the sequence of meetings. Following the estimation method suggested by Christakis et al. (2010), I use two iterations of the Metropolis-Hastings algorithm (Metropolis et al., 1953), one for θ and the other for the sequence of meetings. There is a feedback loop between the iterations, such that θ is updated given a sequence of meetings, and the sequence of meetings is updated given a value of θ . $\theta = (\theta'_c, \theta'_h)'$ denotes the full vector of model parameters including the parameters of the credit card issuer's and the hotel chain's objective function.

6.1 Conditional Likelihood Function

Given the sequence of meetings, the conditional probability of the transition from \mathbf{D}^t to \mathbf{D}^{t+1} is constructed by assigning probabilities to the 4 cases in section 5.6. Consider $(i^{t,m}, k^{t,m})$ as the m^{th} meeting in S^t . Define

$$\begin{aligned}\Delta U_{c+}^{t,m} &\equiv U_c \left(P_{i^{t,m}}^{t,m} \cup \{k^{t,m}\}; b_{i^{t,m}}^t, w_{i^{t,m}}^t, \mathbf{A}^t, \mathbf{D}_{-i^{t,m}}^{t,m}, \epsilon_{i^{t,m}}^t, \theta_c \right) \\ &\quad - U_c \left(P_{i^{t,m}}^{t,m}; b_{i^{t,m}}^t, w_{i^{t,m}}^t, \mathbf{A}^t, \mathbf{D}_{-i^{t,m}}^{t,m}, \epsilon_{i^{t,m}}^t, \theta_c \right)\end{aligned}\quad (16)$$

$$\begin{aligned}\Delta U_{c-}^{t,m} &\equiv U_c \left(P_{i^{t,m}}^{t,m} \setminus \{k^{t,m}\}; b_{i^{t,m}}^t, w_{i^{t,m}}^t, \mathbf{A}^t, \mathbf{D}_{-i^{t,m}}^{t,m}, \epsilon_{i^{t,m}}^t, \theta_c \right) \\ &\quad - U_c \left(P_{i^{t,m}}^{t,m}; b_{i^{t,m}}^t, w_{i^{t,m}}^t, \mathbf{A}^t, \mathbf{D}_{-i^{t,m}}^{t,m}, \epsilon_{i^{t,m}}^t, \theta_c \right)\end{aligned}\quad (17)$$

and similarly

$$\begin{aligned}\Delta g_{c+}^{t,m} &\equiv g_c \left(P_{i^{t,m}}^{t,m} \cup \{k^{t,m}\}; b_{i^{t,m}}^t, w_{i^{t,m}}^t, \mathbf{A}^t, \mathbf{D}_{-i^{t,m}}^{t,m}, \theta_c \right) \\ &\quad - g_c \left(P_{i^{t,m}}^{t,m}; b_{i^{t,m}}^t, w_{i^{t,m}}^t, \mathbf{A}^t, \mathbf{D}_{-i^{t,m}}^{t,m}, \theta_c \right)\end{aligned}\quad (18)$$

$$\begin{aligned}\Delta g_{c-}^{t,m} &\equiv g_c \left(P_{i^{t,m}}^{t,m} \setminus \{k^{t,m}\}; b_{i^{t,m}}^t, w_{i^{t,m}}^t, \mathbf{A}^t, \mathbf{D}_{-i^{t,m}}^{t,m}, \theta_c \right) \\ &\quad - g_c \left(P_{i^{t,m}}^{t,m}; b_{i^{t,m}}^t, w_{i^{t,m}}^t, \mathbf{A}^t, \mathbf{D}_{-i^{t,m}}^{t,m}, \theta_c \right).\end{aligned}\quad (19)$$

Equations (16) and (17) correspond to changes to the objective function of $i^{t,m}$ induced by adding $k^{t,m}$ to and removing $k^{t,m}$ from its portfolio, respectively, during the m^{th} meeting. Applying equation (4) yields

$$\Delta U_{c+}^{t,m} = \Delta g_{c+}^{t,m} + \epsilon_{i^{t,m} k^{t,m}}^t \quad (20)$$

$$\Delta U_{c-}^{t,m} = \Delta g_{c-}^{t,m} - \epsilon_{i^{t,m} k^{t,m}}^t. \quad (21)$$

Thus the probability assigned to the 4 cases in section 5.6 are, respectively

$$\mathbb{P}(\Delta U_{c+}^{t,m} \leq 0) = \mathbb{P}(\epsilon_{i^{t,m}k^{t,m}}^t \leq -\Delta g_{c+}^{t,m}) \quad (22)$$

$$\mathbb{P}(\Delta U_{c+}^{t,m} > 0) = \mathbb{P}(\epsilon_{i^{t,m}k^{t,m}}^t > -\Delta g_{c+}^{t,m}) \quad (23)$$

$$\mathbb{P}(\Delta U_{c-}^{t,m} \leq 0) = \mathbb{P}(\epsilon_{i^{t,m}k^{t,m}}^t \geq \Delta g_{c-}^{t,m}) \quad (24)$$

$$\mathbb{P}(\Delta U_{c-}^{t,m} > 0) = \mathbb{P}(\epsilon_{i^{t,m}k^{t,m}}^t < \Delta g_{c-}^{t,m}) \quad (25)$$

The same result holds for hotel chains as choosers, with ΔU_c and Δg_c replaced by ΔU_h and Δg_h , respectively.

Let $S^{t,m}$ denote the m^{th} meeting in S^t . Define $\theta = (\theta'_c, \theta'_h)'$. Under the assumption given by equation (5), the probability of the transition from D^t to D^{t+1} conditional on S^t (the conditional likelihood of θ) is given by the following.

$$\begin{aligned} \mathbb{P}(D^{t+1}|D^t, b^t, A^t, w^t, S^t; \theta) = \\ \prod_{m=1}^{M^{t+1}} \mathbb{1}\{S^{t,m} = (i^{t,m}, k^{t,m})\} \mathbb{P}(\epsilon_{i^{t,m}k^{t,m}}^t \leq -\Delta g_{c+}^{t,m})^{\mathbb{1}\{[D^{t,m}]_{i^{t,m}k^{t,m}}=0, [D^{t+1}]_{i^{t,m}k^{t,m}}=0\}} \\ \times \mathbb{1}\{S^{t,m} = (j^{t,m}, k^{t,m})\} \mathbb{P}(\epsilon_{j^{t,m}k^{t,m}}^t \leq -\Delta g_{h+}^{t,m})^{\mathbb{1}\{[D^{t,m}]_{j^{t,m}k^{t,m}}=0, [D^{t+1}]_{j^{t,m}k^{t,m}}=0\}} \\ \times \mathbb{1}\{S^{t,m} = (i^{t,m}, k^{t,m})\} \mathbb{P}(\epsilon_{i^{t,m}k^{t,m}}^t > -\Delta g_{c+}^{t,m})^{\mathbb{1}\{[D^{t,m}]_{i^{t,m}k^{t,m}}=0, [D^{t+1}]_{i^{t,m}k^{t,m}}=1\}} \\ \times \mathbb{1}\{S^{t,m} = (j^{t,m}, k^{t,m})\} \mathbb{P}(\epsilon_{j^{t,m}k^{t,m}}^t > -\Delta g_{h+}^{t,m})^{\mathbb{1}\{[D^{t,m}]_{j^{t,m}k^{t,m}}=0, [D^{t+1}]_{j^{t,m}k^{t,m}}=1\}} \\ \times \mathbb{1}\{S^{t,m} = (i^{t,m}, k^{t,m})\} \mathbb{P}(\epsilon_{i^{t,m}k^{t,m}}^t \geq \Delta g_{c-}^{t,m})^{\mathbb{1}\{[D^{t,m}]_{i^{t,m}k^{t,m}}=1, [D^{t+1}]_{i^{t,m}k^{t,m}}=0\}} \\ \times \mathbb{1}\{S^{t,m} = (j^{t,m}, k^{t,m})\} \mathbb{P}(\epsilon_{j^{t,m}k^{t,m}}^t \geq \Delta g_{h-}^{t,m})^{\mathbb{1}\{[D^{t,m}]_{j^{t,m}k^{t,m}}=1, [D^{t+1}]_{j^{t,m}k^{t,m}}=0\}} \\ \times \mathbb{1}\{S^{t,m} = (i^{t,m}, k^{t,m})\} \mathbb{P}(\epsilon_{i^{t,m}k^{t,m}}^t < \Delta g_{c-}^{t,m})^{\mathbb{1}\{[D^{t,m}]_{i^{t,m}k^{t,m}}=1, [D^{t+1}]_{i^{t,m}k^{t,m}}=1\}} \\ \times \mathbb{1}\{S^{t,m} = (j^{t,m}, k^{t,m})\} \mathbb{P}(\epsilon_{j^{t,m}k^{t,m}}^t < \Delta g_{h-}^{t,m})^{\mathbb{1}\{[D^{t,m}]_{j^{t,m}k^{t,m}}=1, [D^{t+1}]_{j^{t,m}k^{t,m}}=1\}} \end{aligned} \quad (26)$$

where $b^t = \{b_i^t, b_j^t\}_{i \in \mathbb{I}^t, j \in \mathbb{J}^t}$ and $w^t = \{w_i^t, w_j^t\}_{i \in \mathbb{I}^t, j \in \mathbb{J}^t}$ denote the collection of bids and potential transfer ratios, respectively, encompassing both credit card issuers and hotel chains.

6.2 Two-Period Conditional Likelihood

This subsection extends the previous subsection to construct a two-period conditional likelihood of θ . It is not difficult to further extend it to accommodate transitions of the network over than two periods. I impose the following assumptions.

Assumption 3. For all t ,

$$\begin{aligned} & \mathbb{P}(\mathbf{D}^{t+2} | \mathbf{D}^t, \mathbf{D}^{t+1}, \mathbf{b}^t, \mathbf{b}^{t+1}, \mathbf{w}^t, \mathbf{w}^{t+1}, \mathbf{A}^t, \mathbf{A}^{t+1}, S^t, S^{t+1}; \theta) \\ &= \mathbb{P}(\mathbf{D}^{t+2} | \mathbf{D}^{t+1}, \mathbf{b}^{t+1}, \mathbf{w}^{t+1}, \mathbf{A}^{t+1}, S^{t+1}; \theta) \end{aligned} \quad (27)$$

$$\mathbb{P}(\mathbf{D}^t; \theta) = \mathbb{P}(\mathbf{D}^t) \quad (28)$$

$$\mathbb{P}(\mathbf{C}^t, \mathbf{A}^t, \mathbf{v}^t; \theta) = \mathbb{P}(\mathbf{C}^t, \mathbf{A}^t, \mathbf{v}^t), \quad (29)$$

where $\mathbf{C}^t = \{C_i^t\}_{i \in \mathbb{I}^t}$ is the collection of KPIs of all credit card issuers, and $\mathbf{v}^t = \{v_i^t, v_j^t, v_k^t\}_{i \in \mathbb{I}^t, j \in \mathbb{J}^t, k \in \mathbb{K}^t}$ is the collection of all values of nodes (redemption values of loyalty points).

Equation (27) states that conditional on the state of the network and other observed variables in the previous period, earlier history does not help to explain the transition of the network during the current period. Equation (28) states that unconditional on the previous state of the network and other observed variables, θ cannot explain the probability of observing the current state of the network. Equation (29) states that corporate management and performance does not depend on θ . In particular, it specifies that KPIs of firms and values of nodes are independent of how credit card issuers choose airline transfer partners. Note that this does not state that KPIs of firms or values of nodes are independent of transfer partnerships. I also assume that the sequence of meetings S^t is a random variable independent of θ .

For brevity of notation, let $\mathbf{X}^t = (\mathbf{b}^t, \mathbf{w}^t, \mathbf{A}^t)$, which is the collection of all observed variables except the states of the network \mathbf{D}^t . Conditional on S^{t_0} and S^{t_0+1} , the transition from \mathbf{D}^{t_0} to \mathbf{D}^{t_0+1} and then from \mathbf{D}^{t_0+1} to \mathbf{D}^{t_0+2} has probability

$$\begin{aligned} & \mathbb{P}(\mathbf{D}^{t_0+1}, \mathbf{D}^{t_0+2} | \mathbf{D}^{t_0}, \mathbf{X}^{t_0}, \mathbf{X}^{t_0+1}, S^{t_0}, S^{t_0+1}; \theta) = \\ & \quad \mathbb{P}(\mathbf{D}^{t_0+1} | \mathbf{D}^{t_0}, \mathbf{X}^{t_0}, S^{t_0}; \theta) \mathbb{P}(\mathbf{D}^{t_0+2} | \mathbf{D}^{t_0+1}, \mathbf{X}^{t_0+1}, S^{t_0+1}; \theta). \end{aligned} \quad (30)$$

The posterior of θ is equal to

$$\begin{aligned} & \mathbb{P}(\theta | \mathbf{D}^{t_0}, \mathbf{D}^{t_0+1}, \mathbf{D}^{t_0+2}, \mathbf{X}^{t_0}, \mathbf{X}^{t_0+1}, S^{t_0}, S^{t_0+1}) = \\ & \quad \frac{\mathbb{P}(\mathbf{D}^{t_0+1} | \mathbf{D}^{t_0}, \mathbf{X}^{t_0}, S^{t_0}; \theta) \mathbb{P}(\mathbf{D}^{t_0+2} | \mathbf{D}^{t_0+1}, \mathbf{X}^{t_0+1}, S^{t_0+1}; \theta) \pi_\theta(\theta)}{\mathbb{P}(\mathbf{D}^{t_0+1}, \mathbf{D}^{t_0+2} | \mathbf{D}^{t_0}, \mathbf{X}^{t_0}, \mathbf{X}^{t_0+1}, S^{t_0}, S^{t_0+1})}, \end{aligned} \quad (31)$$

where $\pi_\theta(\cdot)$ is the prior of θ .

6.3 Markov Chain Monte Carlo

Similar to Christakis et al. (2010), I endow S^t with the discrete uniform distribution over its support \mathbb{S}^t . The support is all arrangements of the sequence of (chooser, bidder) pairs in period t . In other words, I assume no prior knowledge on the sequence of meetings, and each sequence of meetings is equally likely. Moreover, S^t is independent of $S^{t'}$ for all $t \neq t'$ and of all other variables. The chained two-period sequence (S^t, S^{t+1}) also has the discrete uniform distribution over the support $\mathbb{S}^t \times \mathbb{S}^{t+1}$. For the two-period setting, the unconditional likelihood of θ (integrated over all sequences of meetings) is equal to

$$\begin{aligned} \mathbb{P}(\mathbf{D}^{t_0+1}, \mathbf{D}^{t_0+2} | \mathbf{D}^{t_0}, \mathbf{X}^{t_0}, \mathbf{X}^{t_0+1}; \theta) = \\ \frac{1}{|\mathbb{S}^{t_0}| \times |\mathbb{S}^{t_0+1}|} \sum_{S^{t_0} \in \mathbb{S}^{t_0}} \sum_{S^{t_0+1} \in \mathbb{S}^{t_0+1}} \mathbb{P}(\mathbf{D}^{t_0+1} | \mathbf{D}^{t_0}, \mathbf{X}^{t_0}, S^{t_0}; \theta) \\ \times \mathbb{P}(\mathbf{D}^{t_0+2} | \mathbf{D}^{t_0+1}, \mathbf{X}^{t_0+1}, S^{t_0+1}; \theta). \quad (32) \end{aligned}$$

Computing the maximum likelihood estimator using equation (32) is infeasible because $|\mathbb{S}^{t_0}|$ and $|\mathbb{S}^{t_0+1}|$ are large. The network of loyalty programs is relatively small with 3 credit card issuers, 7 hotel chains, and 43 airline. Nevertheless, the network contains 430 (chooser, bidder) pairs, and the chained two-period sequence of meetings has a support with cardinality $(430!)^2$.

The estimation of θ utilizes two iterations of the Metropolis-Hastings algorithm: one for sampling θ given a draw of (S^{t_0}, S^{t_0+1}) , the other for sampling (S^{t_0}, S^{t_0+1}) given a draw of θ . With randomly drawn initial values $\theta^{(0)}$ and $S^{(0)}$, the $(l+1)^{\text{th}}$ iteration $\theta^{(l+1)}$ is updated given $(S^{t_0}, S^{t_0+1})^{(l)}$, and $(S^{t_0}, S^{t_0+1})^{(l+1)}$ is updated given $\theta^{(l+1)}$, for $l = 0, 1, 2, \dots$. The goal is to estimate the posterior distribution of θ , unconditional on (S^{t_0}, S^{t_0+1}) .

6.3.1 Iterations of $\theta^{(l)}$

Given $\theta^{(l)}$ and $(S^{t_0}, S^{t_0+1})^{(l)}$, let $\tilde{\theta}$ be the candidate for $\theta^{(l+1)}$ drawn from the proposal distribution

$$q_\theta(\tilde{\theta} | \theta^{(l)}, \mathbf{D}^{t_0}, \mathbf{D}^{t_0+1}, \mathbf{D}^{t_0+2}, \mathbf{X}^{t_0}, \mathbf{X}^{t_0+1}, (S^{t_0}, S^{t_0+1})^{(l)}). \quad (33)$$

For brevity of notation, define $S_2 \equiv (S^{t_0}, S^{t_0+1})$ and suppress the observed variables $\mathbf{D}^{t_0}, \mathbf{D}^{t_0+1}, \mathbf{D}^{t_0+2}, \mathbf{X}^{t_0}, \mathbf{X}^{t_0+1}$. Thus, $q_\theta(\tilde{\theta} | \theta^{(l)}, S_2^{(l)})$ is equivalent to equation (33), and similarly, $\mathbb{P}(\theta | S_2)$ is equivalent to equation (31). The Metropolis-Hastings ratio for updating $\theta^{(l+1)}$ is given by

$$r_\theta(\tilde{\theta}, \theta^{(l)} | S_2^{(l)}) = \min \left\{ 1, \frac{\mathbb{P}(\tilde{\theta} | S_2^{(l)}) q_\theta(\theta^{(l)} | \tilde{\theta}, S_2^{(l)})}{\mathbb{P}(\theta^{(l)} | S_2^{(l)}) q_\theta(\tilde{\theta} | \theta^{(l)}, S_2^{(l)})} \right\}.$$

I endow the proposal distribution for the l^{th} iteration with the Gaussian form

$$q_\theta(\cdot | \theta^{(l)}, S_2^{(l)}) \sim \mathcal{N}(\theta^{(l)}, \Sigma), \quad (34)$$

where Σ is a covariance matrix fitted to attain a target jump rate throughout the iterations. For the empirical analysis, I aimed for a target jump rate of 0.3. This specification implies

$$q_\theta(\tilde{\theta} | \theta^{(l)}, S_2^{(l)}) = q_\theta(\theta^{(l)} | \tilde{\theta}, S_2^{(l)}). \quad (35)$$

Thus, the Metropolis-Hastings ratio simplifies to

$$r_\theta(\tilde{\theta}, \theta^{(l)} | S_2^{(l)}) = \min \left\{ 1, \frac{\mathbb{P}(\tilde{\theta} | S_2^{(l)})}{\mathbb{P}(\theta^{(l)} | S_2^{(l)})} \right\}, \quad (36)$$

and equation (31) yields the computable form

$$\frac{\mathbb{P}(\tilde{\theta}|S_2^{(l)})}{\mathbb{P}(\theta^{(l)}|S_2^{(l)})} = \frac{\mathbb{P}(\mathbf{D}^{t_0+1}|\mathbf{D}^{t_0}, \mathbf{X}^{t_0}, S^{t_0}; \tilde{\theta}) \mathbb{P}(\mathbf{D}^{t_0+2}|\mathbf{D}^{t_0+1}, \mathbf{X}^{t_0+1}, S^{t_0+1}; \tilde{\theta}) \pi_{\theta}(\tilde{\theta})}{\mathbb{P}(\mathbf{D}^{t_0+1}|\mathbf{D}^{t_0}, \mathbf{X}^{t_0}, S^{t_0}; \theta^{(l)}) \mathbb{P}(\mathbf{D}^{t_0+2}|\mathbf{D}^{t_0+1}, \mathbf{X}^{t_0+1}, S^{t_0+1}; \theta^{(l)}) \pi_{\theta}(\theta^{(l)})}. \quad (37)$$

We have

$$\theta^{(l+1)} = \begin{cases} \tilde{\theta} & \text{with probability } r_{\theta}(\tilde{\theta}, \theta^{(l)}|S_2^{(l)}) \\ \theta^{(l)} & \text{with probability } 1 - r_{\theta}(\tilde{\theta}, \theta^{(l)}|S_2^{(l)}) \end{cases} \quad (38)$$

6.3.2 Iterations of $S_2^{(l)}$

Let $\pi_2(\cdot)$ denote the prior of S_2 . Equation (27) and the specification of S^t yield

$$\begin{aligned} & \mathbb{P}(S_2|\mathbf{D}^{t_0}, \mathbf{D}^{t_0+1}, \mathbf{D}^{t_0+2}, \mathbf{X}^{t_0}, \mathbf{X}^{t_0+1}; \theta) \\ &= \frac{\mathbb{P}(\mathbf{D}^{t_0+1}|\mathbf{D}^{t_0}, \mathbf{X}^{t_0}, S^{t_0}; \theta) \mathbb{P}(\mathbf{D}^{t_0+2}|\mathbf{D}^{t_0+1}, \mathbf{X}^{t_0+1}, S^{t_0+1}; \theta) \pi_2(S_2)}{\mathbb{P}(\mathbf{D}^{t_0+1}, \mathbf{D}^{t_0+2}|\mathbf{D}^{t_0}, \mathbf{X}^{t_0}, \mathbf{X}^{t_0+1}; \theta)}. \end{aligned} \quad (39)$$

Given $\theta^{(l+1)}$ and $S_2^{(l)}$, let \tilde{S}_2 be the candidate for $S_2^{(l+1)}$ drawn from the proposal distribution

$$q_2(S_2|S_2^{(l)}, \mathbf{D}^{t_0}, \mathbf{D}^{t_0+1}, \mathbf{D}^{t_0+2}, \mathbf{X}^{t_0}, \mathbf{X}^{t_0+1}; \theta^{(l+1)}). \quad (40)$$

Again, I suppress the notations for observed variables. Thus, $\mathbb{P}(S_2|\theta)$ is equivalent to equation (39), and $q_2(S_2|S_2^{(l)}, \theta^{(l+1)})$ is equivalent to equation (40). The Metropolis-Hastings ratio for updating $S_2^{(l+1)}$ is given by

$$r_2(\tilde{S}_2, S_2^{(l)}|\theta^{(l+1)}) = \min \left\{ 1, \frac{\mathbb{P}(\tilde{S}_2|\theta^{(l+1)}) q_2(S_2^{(l)}|\tilde{S}_2, \theta^{(l+1)})}{\mathbb{P}(S_2^{(l)}|\theta^{(l+1)}) q_2(\tilde{S}_2|S_2^{(l)}, \theta^{(l+1)})} \right\}. \quad (41)$$

The candidate draw \tilde{S}_2 is obtained by randomly permuting a fraction from each of $(S^{t_0}, S^{t_0+1})^{(l)}$. For the empirical analysis, I chose the fraction to attain a target jump rate of 0.3. The specification of \tilde{S}_2 yields $q_2(\tilde{S}_2|S_2^{(l)}, \theta^{(l+1)}) = q_2(S_2^{(l)}|\tilde{S}_2, \theta^{(l+1)})$. Moreover because S_2 is endowed with the discrete uniform distribution, the values of $\pi_2(\cdot)$ are equal over its support. Then the Metropolis-Hastings ratio simplifies to

$$r_2(\tilde{S}_2, S_2^{(l)}|\theta^{(l+1)}) = \min \left\{ 1, \frac{\mathbb{P}(\mathbf{D}^{t_0+1}, \mathbf{D}^{t_0+2}|\mathbf{D}^{t_0}, \mathbf{X}^{t_0}, \mathbf{X}^{t_0+1}, \tilde{S}_2; \theta^{(l+1)})}{\mathbb{P}(\mathbf{D}^{t_0+1}, \mathbf{D}^{t_0+2}|\mathbf{D}^{t_0}, \mathbf{X}^{t_0}, \mathbf{X}^{t_0+1}, S_2^{(l)}; \theta^{(l+1)})} \right\}, \quad (42)$$

, and equation (30) yields a computable form. We have

$$S_2^{(l+1)} = \begin{cases} \tilde{S}_2 & \text{with probability } r_2(\tilde{S}_2, S_2^{(l)}|\theta^{(l+1)}) \\ S_2^{(l)} & \text{with probability } 1 - r_2(\tilde{S}_2, S_2^{(l)}|\theta^{(l+1)}) \end{cases} \quad (43)$$

7 Empirical Analysis: Network of Loyalty Programs

7.1 Setup and Preliminary Data Analysis

The network of loyalty programs is composed of 3 credit card issuers, 7 hotel chains, and 43 airlines, and an additional 10 airlines without firm-level data. The data contain annual observations on transfer partnerships among the firms (snapshots of the network) from the fourth quarter of 2014 to the fourth quarter of 2018 and quarterly observations on their firm-level characteristics, from the first quarter of 2014 to the fourth quarter of 2018. Firm-level characteristics include key performance indicators (KPIs) of firm, characteristics of their loyalty programs, and geographic information for airlines. Table 9 provides the list of included firms. Appendix B provides details on the data collection procedure and definitions of the variables. Among the collected KPIs, the empirical analysis utilizes three KPIs of credit card issuers, four KPIs of hotel chains, and three KPIs of airlines.

In addition to the fitted bids and potential transfer ratios obtained in section 5.2, the empirical analysis on network formation utilizes observations over two periods, where each period contains 8 calendar quarters²⁰. Period 1 denotes the eight quarters from the first quarter of 2015 to the fourth quarter of 2016, and Period 2 denotes the eight quarters from the first quarter of 2017 to the fourth quarter of 2018. The initial and ending states in period 1 are the states of the network observed in the fourth quarter of 2014 and in the fourth quarter of 2016, respectively. The initial and ending states in period 2 are the states of the network observed in the fourth quarter of 2016 and in the fourth quarter of 2018, respectively. For each period, the initial state of the network is equal to the ending state in the previous period. The origin state of the network denotes the initial state in the earliest period (period 1), which is the observation in the fourth quarter of 2014. For all key performance indicators (KPIs) of firms and characteristics of loyalty programs, I averaged the relevant quarterly observations to construct period-level observations. For completeness, this subsection also presents KPIs of firms and characteristics for period 0, which denotes the four quarters of 2014.

Figure 10 in the appendix reports three KPIs of the credit card issuers and their Number of Transfer Partners (size of portfolios of transfer partners), which includes both hotel chain and airline partners. AMEX consistently possessed a larger portfolio of transfer partners than the other two firms although it did not possess the largest Credit Card Purchases (total amount of credit card purchases processed). On the other hand, the customers of AMEX possessed overall better credit quality, indicated by lower Delinquency Rate (fraction of loans past due for 30 or more days) and Writeoff Rate (fraction of loans written off).

Figure 11 in the appendix reports three KPIs of the hotel chains and their Number of Transfer Partners (size of portfolios of transfer partners), which only includes airline partners. MAR consistently possessed the Hotel Revenue (market size of lodging services) and the largest portfolio of transfer partners. On the other hand, HYT had the largest RevPar, indicating that, on average, consumers were willing to pay more for rooms in its hotel properties. Hotel chains generally possessed more transfer partners than credit card issuers, with more than 35 transfer partners for MAR.

Table 11 in the appendix reports summary statistics of KPIs of the airlines. On average, Passenger Revenue (market size of passenger transportation services) fell in Period 1 and then increased

²⁰I chose to use 8 quarters after scrutinizing the history of changes in transfer partnerships of credit card issuers. The history of changes is given in table 5 in the appendix

in Period 2. On average, RPK (total flight distance of sold seats) and ASK (total flight distance of all seats) increased steadily over the periods. RPK is a measure of quantity demanded for flight services, and ASK is a measure of supply or capacity of flight services. For all periods, the distribution of each KPI exhibits a long right tail, indicating that there were a small number of airlines with large market size. The large standard deviation, as well as the $[P_{25}, P_{75}]$ and $[P_{10}, P_{90}]$ percentile ranges, indicate that there was substantial variability in the market size of airlines.

I note that all variables in equation (7) (in section 5.3), except for the constant term (corresponds to θ_{0c}), are endogenous in the network formation process. They are endogenous in the sense that the variables depend on the chooser's and other firms' portfolios of transfer partners, which are endogenous.

7.2 Discussion of Results

Table 4 reports the estimation result for the parameters of the credit card issuer's objective function. Appendix C provides a detailed description of the estimation procedure. Table 12 in the appendix reports the estimation result for the parameters of the hotel chain's objective function; I do not discuss this result because it is not the focus of this paper.

Table 4: Estimation Results (Credit Card Issuer)

| Estimand | Variable | Mean | Median | $[P_{2.5}, P_{97.5}]$ |
|----------------|--|-----------|---------|-----------------------|
| θ_{0c} | Constant term | 0.3141 | 0.0887 | [-0.3203,1.1028] |
| θ_{1c} | Size of Portfolio | -0.5381 | -0.4253 | [-1.8700,0.7010] |
| θ_{21c} | Passenger Revenue (incl. in <i>Performance</i>) | -0.2233 | -0.3045 | [-0.6589,0.3340] |
| θ_{22c} | Revenue Passenger Km (incl. in <i>Performance</i>) | -0.0834 | 0.0025 | [-0.6042,0.2937] |
| θ_{23c} | Available Seat Km (incl. in <i>Performance</i>) | 0.2304 | 0.1218 | [-0.3017,0.9099] |
| θ_{3c} | <i>GeoHub</i> | -0.1431 | -0.1294 | [-0.5185,0.3309] |
| θ_{4c} | <i>Mileage</i> | -0.9683** | -0.9792 | [-1.3119,-0.6260] |
| θ_{5c} | <i>Routes</i> | -0.1999** | -0.2097 | [-0.3370,-0.6021] |
| θ_{6c} | <i>Competitor</i> | 0.5100* | 0.5605 | [-0.0392,1.1371] |
| θ_{7c} | <i>Transitivity</i> | 0.7848** | 0.7398 | [0.1476,1.4663] |

This table reports the result of MCMC iterations. The first column ("Estimand") lists the components of θ_c , the parameter of the credit card issuer's objective function. The second column ("Variable") lists the covariates included in the objective function (see section 5.3 for definitions). The third column ("Mean") reports the mean of the MCMC iterations. The fourth column ("Median") reports the median of the iterations. The last column (" $[P_{2.5}, P_{97.5}]$ ") reports the 2.5 - 97.5 percentile range of the iterations. ** indicates that the 2.5 - 97.5 percentile range does not contain zero. * indicates that the 5 - 95 percentile range does not contain zero. The mean, median, and percentile range were computed after removing the first half of the iterations (after the "burn-in" process).

θ_{0c} corresponds to the constant term, and $\theta_{1c}, \theta_{21c}, \theta_{22c}, \theta_{23c}$, and θ_{3c} correspond to the control

variables. The variables of interest are *Mileage*, *Routes*, *Competitor*, and *Transitivity*. I make inferences based on a credit card issuer's attitude towards adding an airline transfer partner. Consider a meeting between credit card issuer i and airline k_1 , that is not already in i 's portfolio. Now, hold everything fixed and consider a meeting between i and a different airline k_2 , that is also not in i 's portfolio. Assume that k_1 and k_2 are equal in every aspect except for exactly one of the following:

1. Adding k_1 improves redemption options of loyalty points for flights (smaller *Mileage*) offered by i , adding k_2 does not.
2. k_1 has more diverse routes for reward flights (larger *Routes*) than k_2 (I emphasize that adding either airline grants the same redemption options for flights offered by i).
3. k_1 is a partner of another credit card issuer, k_2 is not.
4. k_1 is a partner of i 's partner hotel chain, k_2 is not.

The odds ratio of i adding k_1 relative to adding k_2 is used for interpretation. Similar to equation (18), define

$$\Delta g_{c+}(k; \theta_c) = g_c(P_i \cup \{k\}; b_i, w_i, \mathbf{A}, \mathbf{D}_{-i}, \theta_c) - g_c(P_i; b_i, w_i, \mathbf{A}, \mathbf{D}_{-i}, \theta_c) \quad (44)$$

The probability ratio of adding k_1 relative to adding k_2 is given by

$$\frac{1 + \exp(-\Delta g_{c+}(k_2; \theta_c))}{1 + \exp(-\Delta g_{c+}(k_1; \theta_c))} \quad (45)$$

A probability ratio larger than 1 implies that i is more likely to add k_1 to its portfolio than k_2 . Corresponding to the four cases above,

1. If $\theta_{4c} < 0$, then the probability ratio is larger than 1.
2. If $\theta_{5c} > 0$, then the probability ratio is larger than 1.
3. If $\theta_{6c} > 0$, then the probability ratio is larger than 1.
4. If $\theta_{7c} > 0$, then the probability ratio is larger than 1.

The estimation results indicate the following. Other things equal,

1. There is statistical evidence that a credit card issuer is more likely to add an airline transfer partner that improves the redemption options for flights offered by i .
2. There is statistical evidence that a credit card issuer is less likely to add an airline transfer partner with more diverse routes for reward flights.
3. There is statistical evidence that a credit card issuer is more likely to add an airline transfer partner that is a transfer partner of another credit card issuer.
4. There is statistical evidence that a credit card issuer is more likely to add an airline transfer partner that is a transfer partner of its hotel partner.

Recall that *Routes* counts all routes of reward flights, with multiplicity for the same flight route granted by different airline partners. Thus, results 1 and 2 suggest that a credit card issuer does not necessarily favor an airline partner with more diverse routes for reward flights but instead an airline partner that better complements the redemption options of its other airline partners. In other words, a credit card issuer tends to pursue resource complementarity when choosing its airline partners, rather than their individual resources. While this result does not imply that a credit card issuer chooses the most cost-effective (in terms of redemption rates of loyalty points) portfolio of transfer partners, holding other things constant, it does provide evidence that how it chooses the airline partners aligns with the welfare of their customers. Result 3 suggests that a credit card issuer tends to pursue a transfer partnership with an airline that is a transfer partner of another credit card issuer. This result is not surprising because the credit card issuers already possess different sets of transfer partners via exclusive co-brand partnerships. Moreover, imitating the competition is a commonly observed business strategy. Lieberman and Asaba (2006) provides a survey of academic research on such behavior of firms. Finally, result 4 suggests that a credit card issuer tends to pursue a transfer partnership with an airline that is a transfer partner of its hotel partner. Although I cannot provide details without speculating, the result suggests that there exists certain benefits in sharing a common partner with a partner.

8 Concluding Remarks

A policy discussion by the Federal Reserve Bank of Boston points out that credit card rewards transfer wealth from the poor to the rich, as credit card spending and rewards are positively associated with income (Schuh et al., 2010). Economic theory explains how consumers, with varying degrees of credit card usage, share the cost of issuing credit card rewards. Yet, we understand little about the mechanics of credit card rewards. Credit card rewards are complicated, with stakeholders from a diverse pool of industries, and policy design for credit card rewards must be found on a profound understanding of the relationship among the stakeholders.

This paper took the first step by studying how a credit card issuer chooses its airline partners to provide travel rewards to customers. The study revealed the significance of resource complementarity in partner choice. The study also revealed that the network of firms has a significant effect on partner choice. However, the study lacks insights into why the network effect exists. Moreover, the simple model of bid determination by airlines left opportunities for improvement. Future research should remedy these weaknesses and more thoroughly examine the dynamics of partnerships in the network of loyalty programs.

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Appendix

A Additional Figures and Tables

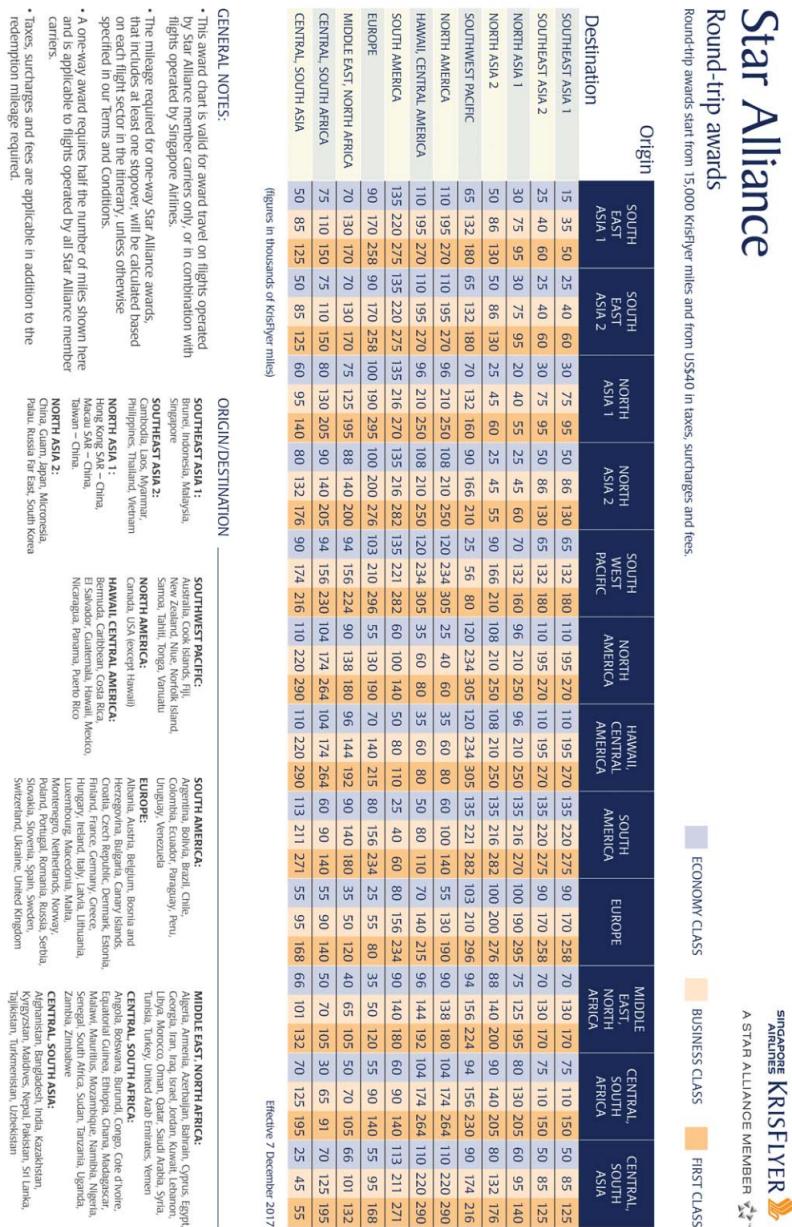
Figure 6: Example of Distance-Based Redemption Chart

The chart displays the points required for redemption based on total trip distance (miles). The columns represent different award classes and sectors. The rows show the distance range and the corresponding points required for each class.

| Total trip distance (mile) | | Until November 19, 2018 Economy class | From November 20, 2018 Economy class | Until November 19, 2018 Premium economy class | From November 20, 2018 Premium economy class | Until November 19, 2018 Business class *1 | From November 20, 2018 Business class *1 | Until November 19, 2018 First class *2 | From November 20, 2018 First class *2 |
|----------------------------|----------|---------------------------------------|--------------------------------------|---|--|---|--|--|---------------------------------------|
| 1 | - 1,000 | 15,000 | 12,000 | 25,000 | 17,000 | 32,000 | 24,000 | 55,000 | 36,000 |
| 1,001 | - 2,000 | 20,000 | 15,000 | 27,500 | 21,000 | 35,000 | 30,000 | 60,000 | 45,000 |
| 2,001 | - 4,000 | 21,000 | 23,000 | 31,500 | 30,000 | 42,000 | 42,000 | 65,000 | 65,000 |
| 4,001 | - 6,000 | 37,000 | 37,000 | 48,500 | 46,000 | 60,000 | 60,000 | 90,000 | 90,000 |
| 6,001 | - 8,000 | 39,000 | 45,000 | 51,000 | 59,000 | 63,000 | 80,000 | 100,000 | 120,000 |
| 8,001 | - 10,000 | 40,000 | 47,000 | 52,500 | 62,000 | 65,000 | 85,000 | 105,000 | 135,000 |
| 10,001 | - 12,000 | 50,000 | 50,000 | 65,000 | 70,000 | 80,000 | 100,000 | 115,000 | 145,000 |
| 12,001 | - 14,000 | 55,000 | 55,000 | 70,000 | 77,000 | 85,000 | 110,000 | 135,000 | 165,000 |
| 14,001 | - 20,000 | 60,000 | 70,000 | 80,000 | 94,000 | 100,000 | 130,000 | 155,000 | 190,000 |
| 20,001 | - 25,000 | 85,000 | 90,000 | 105,000 | 112,000 | 125,000 | 145,000 | 200,000 | 220,000 |
| 25,001 | - 29,000 | 110,000 | 110,000 | 135,000 | 135,000 | 160,000 | 160,000 | 250,000 | 250,000 |
| 29,001 | - 34,000 | 130,000 | 130,000 | 160,000 | 160,000 | 190,000 | 190,000 | 290,000 | 290,000 |
| 34,001 | - 50,000 | 150,000 | 150,000 | 180,000 | 180,000 | 210,000 | 210,000 | 330,000 | 330,000 |

This is a points (mileage) redemption chart of Japan Airlines. It is an example of a distance-based redemption chart. (Japan Airlines. https://www.ar.jal.co.jp/ar/en/jalmile/use/partner_air/p_jmb/jmb_mile_ar.html. Accessed December 15, 2018.)

Figure 7: Example of Zone-Based Redemption Chart



This is a points (mileage) redemption chart of Singapore Airlines for flights accessible via Star Alliance. It is an example of a zone-based points redemption chart. Zone definitions are clearly stated below the chart. (Singapore Airlines. https://www.singaporeair.com/saar5/pdf/ppsclub_krisflyer/charts/StarAlliance_RoundTrip.pdf. Accessed December 16, 2018.)

Figure 8: Example of Airline Transfer Partners

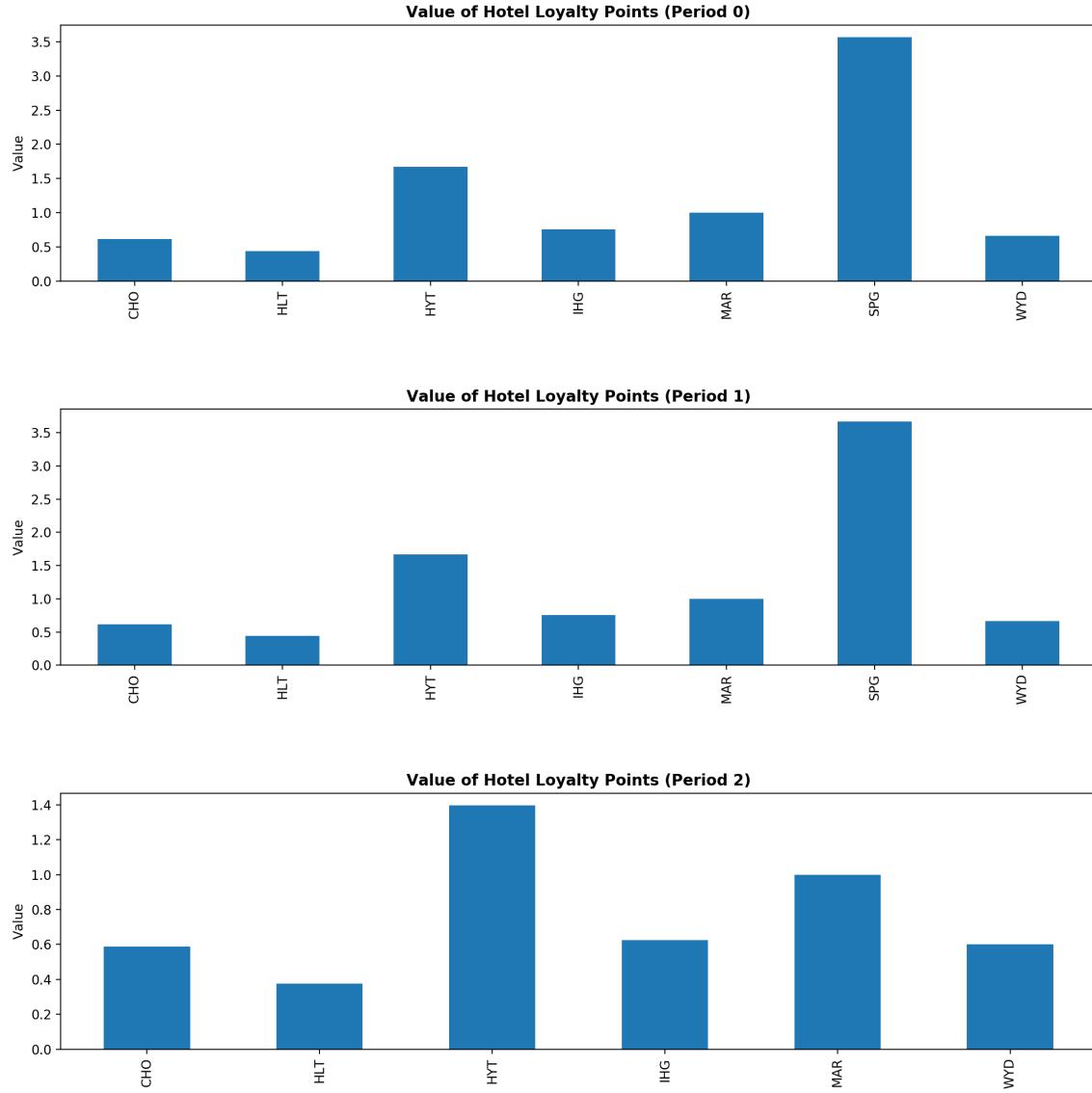
Fly farther with Membership Rewards.

With the most airline points transfer partners of any major U.S. credit card loyalty program, Membership Rewards can take you where you want to go. Learn about our partners and where they fly.



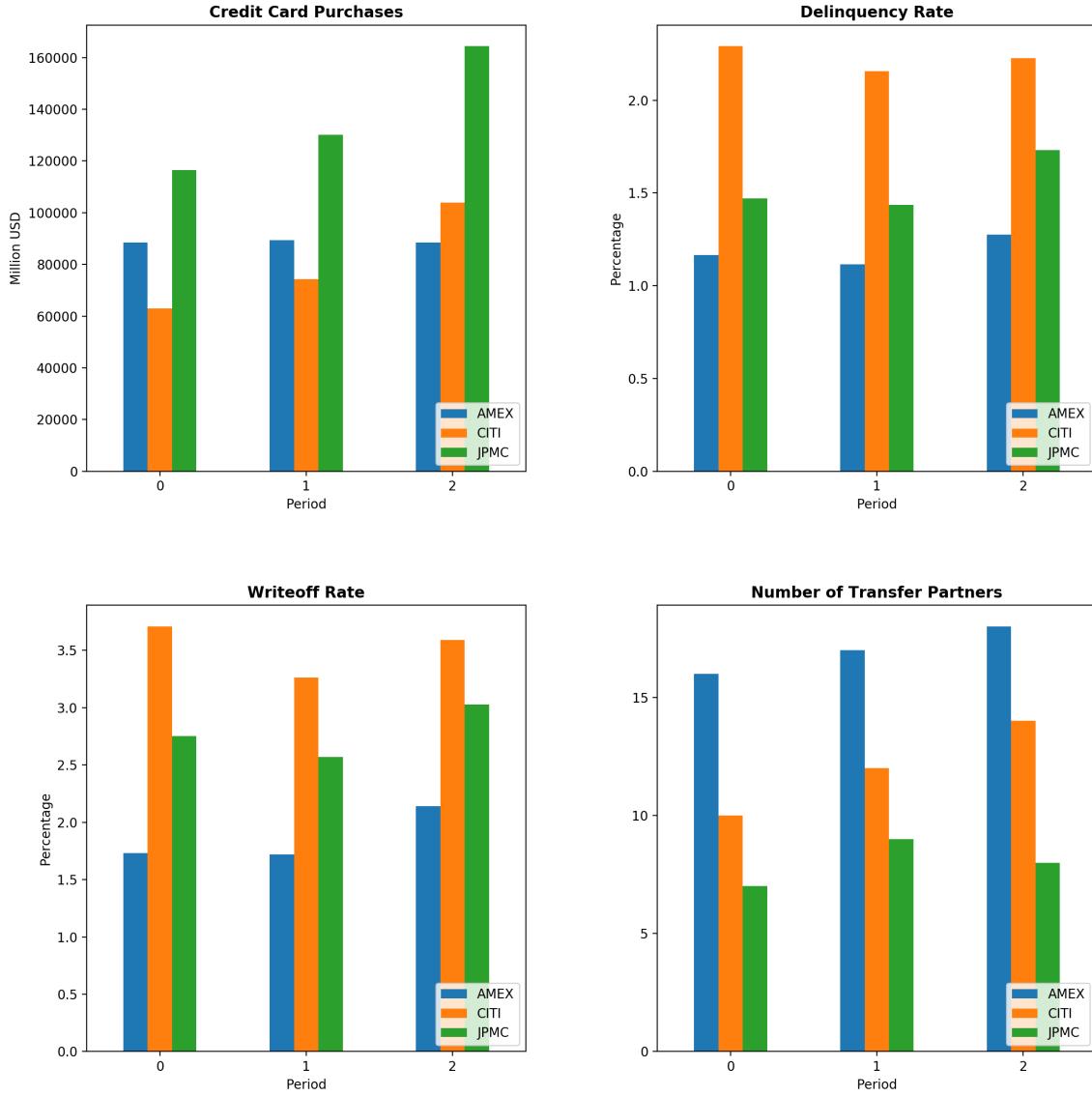
Membership rewards is the loyalty program of American Express Company. The portfolio of transfer partners is actively used as a marketing tool. (American Express Company. <https://www.americanexpress.com/us/rewards/membership-rewards/redeem/airline-partners/partner-airlines.html?linknav=us-loy-mr-mrair-home-airlines>. Accessed September 9, 2019.)

Figure 9: Value of Hotel Loyalty Points



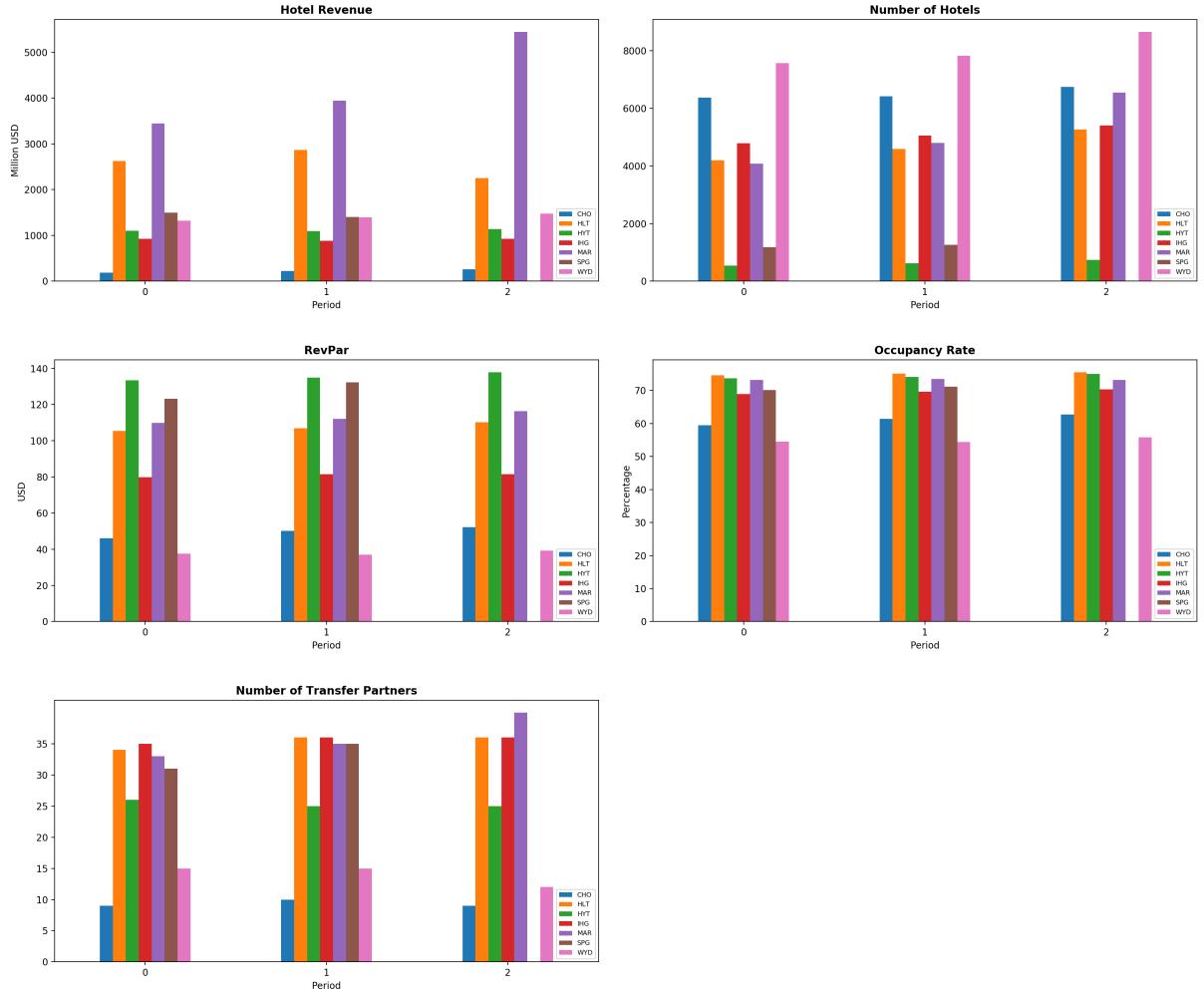
This figure reports summary statistics of the values of hotel chain loyalty points. Periods 0, 1, 2 are the 4 quarters of 2014, the 8 quarters of 2015-2016, and the 8 quarters of 2017-2018, respectively. All measures are averages over the quarters, for each period. All values were normalized using the value of MAR's (Marriott Hotels and Resorts) loyalty points. A value larger (smaller) than 1 indicates that, as an average over 7 categories of hotel properties, it costs more (less) points to redeem for a standard room than MAR. Note that SPG disappears in period 2 because it was acquired by MAR, and their loyalty programs completely integrated.

Figure 10: KPIs for Credit Card Issuers (Quarterly Average)



This figure reports summary statistics of key performance indicators (KPIs) for credit card issuers. Periods 0, 1, and 2 are the 4 quarters of 2014, the 8 quarters of 2015-2016, and the 8 quarters of 2017-2018, respectively. All measures are averages over the quarters, for each period. Credit Card Purchase is the total amount of purchases made using the firm's credit card products. Delinquency Rate is the percentage of outstanding loans that are past due for at least 30 days. Writeoff Rate is the share of net-writeoff in outstanding loans. Only the measures for U.S. consumers were included. Number of Transfer Partners is the number loyalty programs of hotel chains and airlines to which points in the credit card issuer's loyalty program can be transferred to.

Figure 11: KPIs for Hotel Chains (Quarterly Average)



This figure reports summary statistics of key performance indicators (KPIs) for hotel chains. Periods 0, 1, 2 are the 4 quarters of 2014, the 8 quarters of 2015-2016, and the 8 quarters of 2017-2018, respectively. All measures are averages over the quarters, for each period. Hotel Revenue is the total operating revenue of the hotel chain, including revenue from rooms and franchise fees. Number of Hotels is the number of worldwide hotel properties owned and leased by the hotel chain. RevPar (revenue per available room) is room revenue (from both owned and leased properties) divided by the number of available rooms. It is a measure of average consumer willingness to pay for lodging services offered by the hotel chain. Occupancy Rate is the number rooms sold divided by the number of available rooms. It is a measure of demand for the hotel chain's rooms relative to capacity. Number of Transfer Partners is the number loyalty programs of airlines to which points in the hotel chain's loyalty program can be transferred to.

Table 5: History of Credit Card Issuers' Transfer Partners

| Time | AMEX | JPMC | CITI |
|---------|--|------------------------------------|--|
| 2013 Q2 | | Adds VS (Jun.) | |
| 2013 Q3 | | | |
| 2013 Q4 | Adds EK (Oct.) | | |
| 2014 Q1 | | | |
| 2014 Q2 | | Adds SQ (May) | Adds CX (Jul.), BR (Jul.), EY (Jul.), GA (Jul.), QR (Jul.), SQ (Jul.), TG (Jul.), MH (Aug.), AF (Aug.) |
| 2014 Q3 | | | |
| 2014 Q4 | | Loses KE (Nov.) | |
| 2015 Q1 | Loses F9 (Jan.) | Adds KE (Jan.) | Adds VS (Jan.), QF (Feb.) |
| 2015 Q2 | | | |
| 2015 Q3 | | | |
| 2015 Q4 | Updates BA ²¹ (Oct.) | | |
| 2016 Q1 | | | |
| 2016 Q2 | Adds EY (Apr.) | Adds AF (May.) | |
| 2016 Q3 | Adds MAR (Sep.); Loses SPG (Sep.) ²² | | |
| 2016 Q4 | | | Adds B6 (Oct.) |
| 2017 Q1 | | | Removes VX ²³ (Jan.) |
| 2017 Q2 | Updates BA ²⁴ (Jul.) | | Adds 9W (Apr.) |
| 2017 Q3 | | | Adds TK (Aug.) |
| 2017 Q4 | Removes VX (Nov.) | Adds EI (Nov.), IB (Nov.) | Adds AV (Nov.); Loses HLT (Dec.) |
| 2018 Q1 | Updates HLT ²⁵ (Jan.) | | |
| 2018 Q2 | | | |
| 2018 Q3 | Adds EI (Aug.) | Loses KE (Aug.); Adds B6 (Aug.) | Updates B6 ²⁶ (Sep.) |
| 2018 Q4 | Adds AV (Nov.) | | |
| 2019 Q1 | Adds QF (May) | | |
| 2019 Q2 | | | |
| 2019 Q3 | | Adds EK (Aug.) | Loses GA (Aug.) |

This table reports the history of changes to transfer partnerships possessed by AMEX, JPMC and CITI, from the second quarter of 2013 to the third quarter of 2019.

Table 6: Preliminary List of Firms

| Industry | Firm |
|-----------------------------|--|
| Credit Card Issuer | American Express Company (AMEX), J.P. Morgan Chase (JPMC), Citibank (CITI) |
| Hotel Chain | Best Western, Choice Hotels, Radisson Hotels, Hilton Hotels and Resorts, Hyatt Hotels, International Hotels Group, Marriott Hotels and Resorts, Starwood Hotels and Resorts, Wyndham Hotels and Resorts |
| Airlines (IATA Code) | 9W, A3, AA, AB, AC, AF, AM, AS, AV, AY, AZ, BA, BR, CA, CI, CM, CX, CZ, DL, EK, EY, FI, G3, GA, GF, HA, HU, JL, KE, LA, LH, LY, MH, MU, NH, NZ, OK, OZ, PR, QF, QR, SA, SK, SQ, SU, SV, TG, TK, TP, UA, UL, VA, VS |

This table reports the preliminary list of firms. Some firms were excluded after defining the market. Small regional airlines and airlines that use loyalty programs of other airlines were excluded. The first column ("Industry") lists the three industry types, and the second column ("Firm") reports the firms in each industry type.

²¹The transfer ratio was updated from 1 to 0.8.

²²MAR acquired SPG. 3:1 transfer between them. SPG was removed from the network of loyalty programs and treated as a part of MAR.

²³AS acquired VX in December 2016.

²⁴The transfer ratio was updated from 0.8 to 1.

²⁵The transfer ratio was updated from 1.5 to 2.

²⁶The transfer ratio was updated from 0.8 to 1.

Table 7: List of Zones and Destinations

| Zone | Departure 1 | Departure 2 | Destination |
|---|-------------|-------------|-------------|
| U.S. (including Alaska) and Canada | JFK | ORD | LAX |
| Hawaii | LAX | JFK | HNL |
| Mexico | LAX | JFK | MEX |
| Caribbean | LAX | JFK | SJU |
| Central America | LAX | JFK | SJO |
| Northern South America | LAX | JFK | BOG |
| Southern South America | LAX | JFK | GRU |
| Western Europe | LAX | JFK | CDG |
| Eastern Europe | LAX | JFK | OTP |
| Middle East | LAX | JFK | DMM |
| North Africa | LAX | JFK | CAI |
| Central Africa | LAX | JFK | NBO |
| South Africa | LAX | JFK | JNB |
| Central Asia (Indian subcontinent) | LAX | JFK | DEL |
| Southeast Asia | LAX | JFK | SIN |
| East Asia | LAX | JFK | ICN |
| Oceania | LAX | JFK | SYD |

The first column ("Zone") reports the 17 geographic zones that were used to compute the number of points required to redeem a reward flight for zone-based redemption charts. The next three columns report the departure and destination airports that were used to compute the number of points required to redeem a reward flight for distance-based charts. The second ("Departure 1") and third ("Departure 2") columns report the departure airports for each zone. For each zone, two departure airports were used to account for the difference in distance to the destination depending on the departure location within the U.S. The fourth column ("Destination") reports the destination airport for each zone. All numbers were computed using U.S. as the departure location.

Table 8: List of Geographic Zones

| Airline Hub Zones | | Flight Redemption Zones | |
|-------------------|--|-------------------------|--|
| Zone | Description | Zone | Description |
| 1 | U.S. (including Alaska and Hawaii) and Canada | 1 | U.S. (including Alaska) and Canada |
| 2 | Mexico, Caribbean, Central America, and Northern South America (e.g. Colombia) | 2 | Hawaii |
| 3 | Southern South America (e.g. Brazil, Chile) | 3 | Mexico, Caribbean, Central America, and Northern South America |
| 4 | North and West Europe (e.g. Finland, Germany, Italy) | 4 | Southern South America |
| 5 | East Europe (e.g. Greece, Russia) | 5 | North and West Europe |
| 6 | Middle East and North Africa (e.g. Egypt, Israel, Saudi Arabia, and Turkey) | 6 | East Europe |
| 7 | Central and South Africa (e.g. Ethiopia, South African Republic) | 7 | Middle East and North Africa |
| 8 | Central Asia (e.g. India, Sri Lanka) | 8 | Central and South Africa |
| 9 | Southeast Asia (e.g. Philippines, Singapore, Thailand) | 9 | Central Asia |
| 10 | East Asia (East of China) | 10 | Southeast Asia |
| 11 | Oceania (e.g. Australia and New Zealand) | 11 | East Asia |
| | | 12 | Oceania |

The "Airline Hub Zones" column reports the 11 airline hub zones used to assign geographic hub locations to airlines. The "Flight Redemption Zones" column reports the 12 geographic zones used to assign required number of points for flights. The required number of points were collected using the 17 zones in table 7 and then aggregated to the 12 zones.

Table 9: List of Firm Names and Abbreviations

| ID | Name | Hub | ID | Name | Hub |
|------|-------------------------------|-------|----|------------------------|-----|
| AMEX | American Express Company | - | FI | Icelandair | 4 |
| CITI | Citibank | - | GA | Garuda Indonesia | 9 |
| JPMC | J.P. Morgan Chase Bank | - | GF | Gulf Air | 6 |
| CHO | Choice Hotels International | - | HA | Hawaiian Airlines | 1 |
| HLT | Hilton Hotels and Resorts | - | HU | Hainan Airlines | 10 |
| HYT | Hyatt Hotels | - | JL | Japan Airlines | 10 |
| IHG | Intercontinental Hotels Group | - | KE | Korean Air Lines | 10 |
| MAR | Marriott Hotels and Resorts | - | LA | LATAM Airlines | 3 |
| SPG | Starwood Hotels and Resorts | - | LH | Lufthansa | 4,5 |
| WYD | Wyndham Hotels and Resorts | - | LY | EI Al Israel Airlines | 6 |
| 9W | Jet Airways | 8 | MH | Malaysia Airlines | 9 |
| A3 | Aegean Airlines | 5 | MU | China Eastern Airlines | 10 |
| AA | American Airlines | 1 | NH | All Nippon Airways | 10 |
| AB | Air Berlin | 4 | NZ | Air New Zealand | 11 |
| AC | Air Canada | 1 | OK | Czech Airlines | 5 |
| AF | Air France/KLM | 4,5,7 | OZ | Asiana Airlines | 10 |
| AM | Aeromexico | 2 | PR | Philippine Airlines | 9 |
| AS | Alaska Airlines | 1 | QF | Qantas Airways | 11 |
| AV | Avianca | 2 | QR | Qatar Airways | 6 |
| AY | Finnair | 4 | SA | South African Airways | 7 |
| AZ | Alitalia | 4 | SK | Scandinavian Airlines | 4 |
| BA | British Airways | 4 | SQ | Singapore Airlines | 9 |
| BR | EVA Air | 10 | SU | Aeroflot | 5 |
| CA | Air China | 10 | SV | Saudia | 6 |
| CI | China Airlines | 10 | TG | Thai Airways | 9 |
| CM | Copa Airlines | 2 | TK | Turkish Airlines | 6 |
| CX | Cathay Pacific Airways | 10 | TP | TAP Air Portugal | 4 |
| CZ | China Southern Airlines | 10 | UA | United Airlines | 1 |
| DL | Delta Air Lines | 1 | UL | SriLankan Airlines | 8 |
| EK | Emirates | 6 | VA | Virgin Australia | 11 |
| EY | Etihad Airways | 6,5 | VS | Virgin Atlantic | 4 |

This table provides a dictionary for abbreviations of firm names. For airlines, their hub zones (see table 8) are also reported.

Table 10: Summary Statistics of Airline Point Values

| Period | N | Mean | SD | 10th | 25th | Med. | 75th | 90th |
|---------------|----------|-------------|-----------|------------------------|------------------------|-------------|------------------------|------------------------|
| 0 | 43 | 1.830 | 6.248 | 0.615 | 0.764 | 0.935 | 1.016 | 1.078 |
| 1 | 43 | 1.828 | 6.248 | 0.612 | 0.773 | 0.917 | 1.016 | 1.069 |
| 2 | 41 | 1.873 | 6.399 | 0.590 | 0.770 | 0.935 | 1.024 | 1.072 |

This table reports summary statistics of the values of airline loyalty points. Periods 0, 1, 2 are the 4 quarters of 2014, the 8 quarters of 2015-2016, and the 8 quarters of 2017-2018, respectively. All measures are averages over the quarters, for each period. All values were normalized using the value of United Airline's loyalty points. A value larger (smaller) than 1 indicates that, as an average over the 12 geographic zones (see "Flight Redemption Zones" in table 8), it costs more (less) points to redeem for flights than United Airlines.

Table 11: Summary Statistics of KPIs for Airlines (Quarterly Average)

| Period | N | Variable | Mean | SD | 10th | 25th | Med. | 75th | 90th |
|---------------|----------|---------------------------------|-------------|-----------|------------------------|------------------------|-------------|------------------------|------------------------|
| 0 | 43 | Passenger Revenue (Million USD) | 2,390 | 2,562 | 515 | 731 | 1,189 | 2,800 | 7,182 |
| | | Revenue Passenger Km (Million) | 23,447 | 22,680 | 6,194 | 7,931 | 14,299 | 27,887 | 56,824 |
| | | Available Seat Km (Million) | 29,206 | 27,286 | 7,838 | 10,200 | 20,742 | 34,642 | 71,323 |
| | 43 | Passenger Revenue (Million USD) | 2,173 | 2,329 | 499 | 667 | 1,078 | 2,593 | 5,623 |
| | | Revenue Passenger Km (Million) | 25,614 | 23,933 | 6,133 | 9,250 | 15,414 | 30,750 | 63,342 |
| | | Available Seat Km (Million) | 31,764 | 28,989 | 8,073 | 11,635 | 21,065 | 38,733 | 74,668 |
| | 41 | Passenger Revenue (Million USD) | 2,562 | 2,682 | 600 | 765 | 1,250 | 2,959 | 6,882 |
| | | Revenue Passenger Km (Million) | 29,747 | 26,276 | 8,176 | 9,707 | 19,750 | 35,765 | 69,758 |
| | | Available Seat Km (Million) | 36,342 | 31,715 | 9,914 | 12,168 | 24,759 | 44,388 | 84,039 |

This table reports summary statistics of key performance indicators (KPIs) for airlines. The first column ("Period") lists the periods. Periods 0, 1, and 2 are the 4 quarters of 2014, the 8 quarters of 2015-2016, and the 8 quarters of 2017-2018, respectively. The second column ("N") reports the number of airlines. Other columns report summary statistics using averages over the quarters, for each period. Passenger Revenue is airline's revenue from scheduled and chartered flights. Revenue passenger Km (RPK) is total flight distance (in kilometers) of paying passengers; it is a measure of demand for an airline's flight services. Available Seat Km (ASK) is total flight distance (in kilometers) of available passenger seats; it is a measure of supply or capacity of an airline's flight services.

Table 12: Estimation Results (Hotel Chains)

| Estimand | Variable | Mean | Median | $[P_{2.5}, P_{97.5}]$ |
|----------------|--|-----------|---------|-----------------------|
| θ_{0h} | Constant term | 0.5152** | 0.5341 | [0.0663,0.8100] |
| θ_{11h} | Size of Portfolio | -1.6273** | -1.7723 | [-2.1376,-0.5865] |
| θ_{12h} | Passenger Revenue (incl. in <i>Performance</i>) | 0.8543** | 0.8323 | [0.5138,1.1762] |
| θ_{13h} | Revenue Passenger Km (incl. in <i>Performance</i>) | -0.2898* | -0.2957 | [-0.5436,0.0214] |
| θ_{2h} | Available Seat Km (incl. in <i>Performance</i>) | 0.0472 | 0.0243 | [-0.1943,0.3308] |
| θ_{3h} | <i>GeoHub</i> | -1.0087** | -1.0148 | [-1.2977,-0.6702] |
| θ_{4h} | <i>Mileage</i> | -0.0386** | -0.0374 | [-0.0817,-0.0018] |
| θ_{5h} | <i>Routes</i> | 0.0638* | 0.0668 | [-0.0070,0.1164] |
| θ_{6h} | <i>Competitor</i> | 2.1022** | 2.1444 | [1.5285,2.5603] |
| θ_{7h} | <i>Transitivity</i> | 1.3304** | 1.3328 | [0.9639,1.7671] |

This table reports the result of MCMC iterations. The first column ("Estimand") lists the components of θ_h , the parameter of the hotel chain's objective function. The second column ("Variable") lists the covariates included in the objective function (see section 5.3 for definitions). The third column ("Mean") reports the mean of the MCMC iterations. The fourth column ("Median") reports the median of the iterations. The last column (" $[P_{2.5}, P_{97.5}]$ ") reports the 2.5 - 97.5 percentile range of the iterations. ** indicates that the 2.5 - 97.5 percentile range does not contain zero. * indicates that the 5 - 95 percentile range does not contain zero. The mean, median, and percentile range were computed after removing the first half of the iterations (after the "burn-in" process).

B Data Collection Procedure

B.1 Initial Screening of Firms

The data collection started in January 2017 by recording the transfer partners of four credit card issuers: American Express Company (AMEX), J.P. Morgan Chase (JPMC), Citibank (CITI), and Diner's Club. Initially, all of their transfer partners and the transfer partners of these partners were included in the study. Except for a few²⁷, all transfer partners were hotel chains or airlines.

I have defined the market scope as credit card issuers that serve U.S. consumers and hotel chains and airlines with significant market presence. Here is the list of firms that were excluded because they did not satisfy the criteria. Diner's Club International was removed because it did not offer credit card products to U.S. consumers. AccorHotels was excluded because it had small presence in the U.S., with about 40 hotel properties. La Quinta Inns and Suites²⁸ was excluded because it had small presence in the worldwide market. Low-cost and ultra low-cost airline carriers such as JetBlue Airways (B6), Southwest Airlines (WN), Virgin America (VX), Frontier Airlines (F9),

²⁷Plenti is an example. Users of Plenti received points by making purchases at selected retailers. It is was subsidiary of AMEX and dissolved in 2018.

²⁸La Quinta Inns and Suites was acquired by Wyndham Hotels and Resorts in May 2018.

and Spirit Airlines (NK) were excluded. Small U.S.-domestic regional airlines, such as Penair (7H), and Great Lakes Airlines (ZK), and small foreign regional airlines such as Flybe (BE), were excluded. Airlines with small market presence, such as Air Mauritius (MK), Interjet (4O), Air Balic (BT), Oman Air (WY) were also excluded.

Airlines with no observed transfer partnerships with any of the credit card issuers or the hotel chains were removed. They were Aerolineas Argentinas (AR), Air India (AI), Azul Brazilian Airlines (AD), Ethiopian Airlines (ET), EgyptAir (MS), Middle East Airlines (ME), Royal Air Maroc (AT), Royal Jordanian Airlines (RJ), S7 Airlines (S7), Ukraine Airlines (PS), Vietnam Airlines (VN), Westjet (WS), and Xiamen Airlines (MF). In addition, Fiji Airways was excluded because enrolling in its loyalty program requires an annual fee. All subsidiary brands of airlines were removed.

Some airlines were removed because they used loyalty programs of other airlines. Air Europa (UX), Kenya Airways (KQ), and TAROM (RO) were removed because they used the loyalty program of Air France²⁹ (AF). LOT Polish Airlines (LO), Austrian Airlines (OS), Brussels Airlines (SN), Eurowings (EW), and Swiss International Air Lines (LX) were removed because they used the loyalty program of Lufthansa (LH). Iberia (IB) and Aer Lingus (EI) were removed because they used the same loyalty points (Avios) as British Airways³⁰. Air Serbia (JU) was removed because it used the loyalty program of Etihad Airways (EY). Note that points in the associated loyalty programs could be used to redeem for flight services offered by these airlines. For example, points in the loyalty program of Air France could be used to redeem for flights offered by Kenya airways. To account for such integration, the geographical hub regions of these airlines were added to the parent airline of the associated loyalty program. Thus, AF, LH, and EY were assigned more than one hub region. Table 6 in the appendix reports the firms remaining after the initial screening.

B.2 Data Source and Availability

I collected two set of data from 2014 to 2018. The first data set contains annual observations on transfer partnerships among firms. The second data set contains quarterly observations on firm-level characteristics. Some firms were further excluded from the study because data were not available.

The first data set contains snapshots of transfer partnerships among the firms for November 2014, 2015, 2016, 2017, and 2018. Each snapshot corresponds to the network of loyalty programs observed in the fourth quarter of the year. For November 2017 and 2018, I obtained the relevant information directly from the official websites of firms. I also collected a snapshot for February 2017. I constructed the snapshot of transfer partnerships for November 2016 by updating the snapshot for February 2017 after scrutinizing official announcements of firms. I constructed the snapshots for November 2014 and 2015 similarly, by updating the snapshot for November 2016. The snapshots also include transfer ratios of points between loyalty programs.

The second data set contains quarterly observations on firm-level characteristics, using the U.S. standard calendar quarters. The first quarter (Q1) is from January 1 to March 31, the second quarter (Q2) is from April 1 to June 30, the third quarter (Q3) is from July 1 to September 30, and the fourth quarter (Q4) is from October 1 to December 31. Some documents reported data

²⁹ Air France (AF) and KLM (KL) are treated as a single entity.

³⁰ Aer Lingus, British Airways, and Iberia belong to the same parent firm, International Airlines Group.

under different formats; I converted them to this quarterly format. For example, monthly data were aggregated to construct quarterly data, and data reported under a different calendar quarters were corrected so that all data entries follow the same format.

For publicly-listed firms in the U.S., I collected firm-level data from their quarterly and annual SEC filings (U.S. Securities and Exchange Commission forms 10Q and 10K). The publicly-listed firms include all three credit card issuers³¹, five hotel chains³², and five airlines³³. For other firms, firm-level data were collected from their quarterly, bi-annual, or annual financial reports, investor presentations, and reports on operating statistics. Geographic locations of airline hubs were also collected from the official websites of airlines.

For hotel chains and airlines, information on the redemption rates of loyalty points were collected from their official websites and images of the official redemption charts found using search engines. For those without a redemption chart, I constructed comparable redemption rates by trying out a number of bookings using loyalty points on their official websites. The bookings were not tried out for the quarters before Q1 of 2017, which are before I started the data collection. Instead, the redemption rates for 2017 were used because there were no observed changes to the redemption rates for those firms. For hotel chains, I recorded the redemption rates for standard rooms. For airlines, I recorded the redemption rates for economy-class seats. Note that for credit card issuers, I did not collect the redemption rates of loyalty points. It was because the portfolio of transfer partners possessed by the credit card issuer determine the value of its loyalty points, as the redemption rates for non-travel rewards are generally inferior to travel rewards.

I was unable to obtain firm-level data for certain firms. For hotel chains, the data for Best Western were not available because it is a private firm. Radisson Hotels publishes quarterly financial reports; however, they do not include data on the firm's operations in the U.S. These two were excluded from the study. For airlines, I was unable to obtain firm-level data for Alitalia (AZ), Gulf Air (GF), Hainan Airlines (HU), Malaysia Airlines (MH), Czech Airlines (OK), Philippine Airlines (PR), Qatar Airlines (QR), Saudia (SV), and TAP Portugal (TP). While data are available for El Al Israel (LY), it was also excluded because its loyalty program had limited redemption options. Note that I still included transfer partnerships associated with these airlines in the first data set.

The data contain annual observations on transfer partnerships among 3 credit card issuers, 7 hotel chains, and 43 airlines and quarterly observations on their firm-level characteristics, from 2014 to 2018. Table 13 lists these firms, and table 9 in the appendix provides full names of firms and geographic hub zones for airlines. The network map in figure 2 illustrates transfer partnerships among these firms and the 10 airlines (without firm-level data) in November 2018. Note that a few firms disappear in the later part of the data due to mergers and financial hardships. For example, SPG, AB, and SA are not present in the network map of 2018.

³¹They are AMEX, JPMC, and CITI.

³²They are Hilton Hotels and Resorts, Hyatt Hotels, Marriott Hotels and Resorts, Starwood Hotels and Resorts, and Wyndham Hotels and Resorts.

³³They are AA, AS, DL, HA, and UA.

Table 13: List of Firms

| Industry | Firm |
|-----------------------------|--|
| Credit Card Issuer | American Express Company (AMEX), J.P. Morgan Chase (JPMC), Citibank (CITI) |
| Hotel Chain | Choice Hotels (CHO), Hilton Hotels and Resorts (HLT), Hyatt Hotels (HYT), International Hotels Group (IHG), Marriott Hotels and Resorts (MAR), Starwood Hotels and Resorts (SPG), Wyndham Hotels and Resorts (WYD) |
| Airlines (IATA Code) | 9W, A3, AA, AB, AC, AF, AM, AS, AV, AY, BA, BR, CA, CI, CM, CX, CZ, DL, EK, EY, FI, G3, GA, HA, JL, KE, LA, LH, MU, NH, NZ, OZ, QF, SA, SK, SQ, SU, TG, TK, UA, UL, VA, VS |

This table reports the list of firms included in the data. Some firms were excluded after defining the market. Small regional airlines and airlines that use loyalty programs of other airlines were excluded. All private firms were excluded. Certain firms were excluded because data were not available. The first column ("Industry") lists the three industry types, and the second column ("Firm") reports the firms in each industry type.

B.3 Recording Data

This section provides details on how the data entries were recorded. I begin by describing how the snapshots of transfer partnerships were recorded. Each snapshot was recorded using an adjacency matrix with real entries. A transfer partnership indicates that loyalty points can be transferred from the source firm's loyalty program to the target firm's loyalty program. Moreover, the associated transfer ratio indicates the number of points in the target loyalty program that can be obtained per 1 points in the source loyalty program. For a pair of firms (order of the pair matters), the corresponding entry in the adjacency matrix is equal to the transfer ratio if there is a transfer partnership from the source to the target and zero if otherwise. For example, 1 point of AMEX (source) could be transferred to obtain 1.5 points of HLT (target), and I recorded this 1.5, from AMEX to HLT. A single exception is partnerships between airlines. A partnership between two airlines does not explicitly allow the transfer of loyalty points; instead, it allows customers to use points in the source firm's loyalty programs to redeem for flight services offered by the target firm. For partnerships between airlines, I used entered 1 if such redemption of points was possible and zero if otherwise. Note that such relationship is different from a codeshare agreement between airlines. A codeshare agreement allows an airline to market seats on a flight (at least partially) operated by the codeshare partner. Such flight is offered by the airline, not by its codeshare partner.

Next, I describe how I recorded the redemption rates of airline loyalty points, which are often called "miles". Recording the redemption rates was straightforward for airlines that provide zone-based redemption charts, similar to figure 7 in the appendix. I began with 17 geographic zones (see table 7 in the appendix) to accommodate various zone definitions of airlines. Using continental U.S.³⁴, as the departure location, I recorded the required number of points to redeem for a economy-class flight during off-peak seasons to each of the 17 zones. For flights within the continental U.S., I recorded the required number of points for flights between the west coast and the east coast. For each zone, the redemption rates of airline loyalty points is equal to the amount of loyalty points required for a round-trip divided by 2. For each airline, I marked non-accessible zones

³⁴Here, continental U.S. also includes Alaska and Canada. I used this term for brevity of expression.

with zero; a zone is non-accessible if loyalty points of the airline cannot be redeemed for a flight from the continental U.S. to the zone. Due to differences in flight routes and cost over the flight routes, airlines possess different accessibility and redemption rates for the zones. Afterwards, I consolidated the 17 zones into 12 zones (see “Flight Redemption Zones” in table 8 in the appendix) by combining nearby zones. I recorded the average of the redemption rates over the consolidated zones; any zones marked with zero was excluded when computing the average.

For airlines that provide distance-based charts (for example, see figure 6 in the appendix), I determined accessibility and flight distances to the 17 zones by assessing each airline’s own flight routes and flight routes accessible via its partners, where the partnership information is given by the snapshots of the network. Whenever available, I used flight distance calculators on the official websites of airlines in order to accurately assess flight distances to the zones. For airlines that did not provide such tools, I used the flight distance calculator of a partner airline. I find it reasonable because all airlines that did not provide such tools belonged to an airline alliance, and alliance members typically share flight routes with other members. In order to account for the flight distance within the continental U.S., I selected two U.S. international airports - LAX and JFK - as departure locations and selected an international airport as destination (see table 7 in the appendix) for each of the 17 zones except for the continental U.S.. For flights within the continental U.S., I selected JFK and ORD as the departure locations and LAX and the destination. For each zone, I obtained two required number of points, one for each departure location, and then recorded the average as the redemption rate. For all airlines, I marked non-accessible zones with zero; any zones marked with zero was excluded when computing the average.

I used a similar procedure for collecting the redemption rates of loyalty points for hotel chains. Except for CHO, hotel chains provided redemption charts that specify the required number of points to redeem for a standard room (for 1 night) in various tiers of hotel properties. Each hotel chain assigns tiers to its properties based on their prestige, and the number of tiers varied from 1³⁵ to 11. For each hotel chain and for each tier, I selected up to 10 hotel properties and assessed their room rates in dollar amounts. Then I reclassified all hotel tiers into 7 categories based on the room rates so that the categories are comparable across different hotel chains. Finally, I recorded the redemption rates³⁶ of loyalty points for each of the 7 categories for each hotel chain. For some hotel chains, I consolidated multiple tiers into a single category by averaging the required number of points over relevant properties. For some, I expanded a tier into multiple categories. For example, WYD assigned a single tier to all of its hotel properties until Q2 of 2015. Thus, the required number of points was the same for all hotel properties of WYD until Q2 of 2015, and I recorded the same redemption rates for the 7 categories for WYD. For CHO, I tried out bookings for a variety of hotel properties to assess room rates and points redemption rates. Then I assigned categories to its hotel properties so that the room rates for those properties are similar to properties of other hotel chains in the same category. For each category, I recorded the redemption rate for CHO by averaging the required numbers of points over its properties in the category. Note that I assumed the redemption rates for CHO before Q1 of 2017 (before the data collection started) to be the same as 2017, because I could not find an evidence that any changes were made to the required number of points.

³⁵From May 2015 to Apr 2019, one could use 15,000 points to redeem for a standard room in any WYD properties. They had a redemption chart with multiple categories before May 2015.

³⁶For HLT, I recorded the redemption rates for off-peak seasons. Other hotel chains did not distinguish off-peak season until September 2019.

Next, I describe how I collected key performance indicators (KPIs) of firms. Note that I converted all monetary units to million U.S. dollars using the spot exchange rates on the last day of the corresponding quarter. I converted all distance units to million kilometers.

I collected four KPIs for credit card issuers: Credit Card Purchase, Outstanding Loans, Delinquency Rate, and Writeoff Rate. Note that for each firm, I recorded only the data for its credit card divisions that serve U.S. consumers and small businesses to be consistent with the market scope, which limits credit card issuers to those that serve U.S. consumers. For AMEX, I recorded the data for its U.S. Consumer Services (USCS) division³⁷. For CITI, I recorded the data for its credit card division of North America GCB (Global Consumer Banking). For JMPC, I recorded the data for its credit card division excluding commercial (corporate) cards³⁸. Credit Card Purchase is the total amount of purchases made using the firm's credit card products, which is a measure of demand for the firm's credit card products. For AMEX, I aggregated the amounts for credit card products and charge card products; to my knowledge, the other credit card issuers did not offer charge card products to U.S. consumers. Outstanding Loans is the total amount of loans outstanding at the end of the quarter, excluding the loans held for sale. AMEX reported Outstanding Loans separately for its credit card business (reported as "Loans") and charge card business (reported as "Accounts Receivable"); I recorded the aggregated amount. Delinquency Rate is the percentage of Outstanding Loans that were past due for 30 or more days, and Writeoff Rate is the percentage of net write-off³⁹ in Outstanding Loans, in percentage terms. They measure the quality of credit possessed by the credit card issuer's customers. For AMEX, Delinquency Rate and Writeoff Rate were averaged over its credit card business and charge card business, weighted by the business's share of Outstanding Loans.

I collected five KPIs for hotel chains: Revenue, Operating Income, Number of Hotels, RevPar (Revenue per Available Room), and Occupancy Rate. Revenue is equal to the total operating revenue of the hotel chain, including revenue from both owned and leased properties. Operating Income is equal to EBIT (earnings before interest and taxes), which is Revenue subtracted by operating expenses, including depreciation and amortization. Note that I recorded Revenue and Operating Income as reported at the end of the financial period; that is, any amendments reported in later periods were ignored. I recorded all other variables as reported in the hotel chain's financial reports. Number of Hotels is the total number of hotel properties marketed under the hotel chain's brands, including properties owned and leased by the hotel chain. RevPar is computed by dividing the room revenue of each hotel property by its number of rooms and then aggregating over the properties owned and leased by the hotel chain. It is a measure of the consumer's willingness to pay for lodging services offered by the hotel chain. Occupancy rate is computed by dividing the number of sold room by the number of rooms for each hotel property and then aggregating over the properties owned and leased by the hotel chain. It is the percentage of the hotel chain's capacity that were actually consumed. The Only bi-annual observations were available for IHG, and I divided them in half to construct quarterly observations.

Following the guideline of Belobaba and Swelbar (2019), I collected five KPIs for airlines: Revenue, Operating Income, Passenger Revenue, Revenue Passenger Kilometers (RPK), and Available Seat Kilometers (ASK). Revenue is equal to the total operating revenue of the firm, includ-

³⁷From Q2 2018, it was reorganized as the U.S. category of Global Consumer Services Group (GCSG).

³⁸From 2015, commercial card business was reported separately under Corporate and Investment Bank segment.

³⁹Net write-off is debt written-off less of recovery.

ing revenue from chartered flights, cargo, and other transportation-related operations. Operating Income is equal to EBIT (earnings before interest and taxes), which is Revenue subtracted by operating expenses, including depreciation and amortization. I excluded all exceptional items, such as expenses for acquiring aircraft and mergers, from Operating Income. Note that I recorded Revenue and Operating Income as reported at the end of the financial period; that is, any amendments reported in later periods were ignored. Passenger Revenue is the airline's revenue from scheduled passenger flights and chartered flights, which is a measure of the airline's overall size in passenger transportation services. It is slightly different from the guideline of Belobaba and Swelbar (2019), as their definition of passenger revenue excludes revenue from chartered flights. I chose to include both because a number of airlines did not report the revenues separately. RPK is the total flight distance of sold seats, which is equal to the sum of flight distances of all paid passengers. It is a measure of quantity demanded for flight services offered by the airline. ASK is the total flight distance of all seats, and it is a measure of the supply or capacity of flight services produced by the airline. Note that some other KPIs, as discussed in Belobaba and Swelbar (2019), can be constructed using these three. For example, PRASK (passenger revenue per available seat kilometer) can be constructed by dividing Passenger Revenue by ASK. PRASK is a measure of the consumer's willingness to pay for the airline's flight services. For some airlines, I aggregated monthly measures to construct quarterly RPK and ASK. A few airlines only reported bi-annual or annual data; I split them equally to construct quarterly observations.

C Estimation Procedure

The estimation procedure uses transitions of the network over two periods. A period denotes the time frame for the transition of the network from an initial state to an ending state. Period 1 denotes the eight quarters from the first quarter of 2015 to the fourth quarter of 2016, and Period 2 denotes the eight quarters from the first quarter of 2017 to the fourth quarter of 2018. The initial and ending states in period 1 are the states of the network observed in the fourth quarter of 2014 and in the fourth quarter of 2016, respectively. The initial and ending states in period 2 are the states of the network observed in the fourth quarter of 2016 and in the fourth quarter of 2018, respectively. For each period, the initial state of the network is equal to the ending state in the previous period. The origin state of the network denotes the initial state in the earliest period (period 1), which is the observation in the fourth quarter of 2014. Period 0 denotes the four quarters of 2014. For all key performance indicators (KPIs) of firms and characteristics of loyalty programs, I averaged the relevant quarterly observations to construct period-level observations.

C.1 Constructing Values of Nodes

The value of a node (loyalty program of a firm) is measure of the redemption value of its loyalty points. In addition, for each industry sector (airlines and hotel chains), I normalized the values of nodes relative to a pivot node so that the value of a pivot node is equal to 1. The pivot node among the airlines is UA, and the pivot node among the hotel chains is MAR. A value larger (smaller) than 1 indicates that, on average, the loyalty points of the node is more (less) valuable than the pivot node.

For airlines, I constructed the values of nodes using redemption rates of loyalty points for the 12 geographic zones (see table 8) departing from the United States. The precise procedure was

1. For each of the 12 zones, divide the redemption rates of all airlines by the redemption rate of UA.
2. For each airline, average over the 12 zones.
3. Take the inverse.

In step 1, some airlines did not offer redemption for flights to certain zones (marked with zero). For the airlines, those zones were not used when computing the averages in step 2. The variable *Routes* (see section 5.3) accounts for the number of missing zones. Note that UA offered redemption for flights to all 12 zones.

The values of nodes for hotel chains were constructed similarly. The difference is that I used redemption rates for 7 hotel categories, classified by prestige of the hotel property, instead of the geographic zones. For each of the 7 categories, I divided the redemption rates of all hotel chains by the redemption rate of MAR. I then averaged over the 7 categories and then inverted the numbers.

For both airlines and hotel chains, I first computed values of nodes using quarterly observations. They were then averaged over the appropriate quarters to construct period-level observations.

For the credit card issuers, I could not find a reasonable method to derive the values of nodes from their own redemption options. Instead, I constructed the values of nodes by comparing the transfer ratios of loyalty points to the same airlines and hotel chains. In other words, I constructed the values of node using the relative transfer ratios to common transfer partners. For each of JPMC and CITI, I identified common transfer partners and then divided the transfer ratios by the matching transfer ratios of AMEX. I then averaged and inverted them to construct the values of nodes. Thus, a value larger (smaller) than 1 implies that, as an average over common transfer partners, the loyalty points of the credit card issuer is more (less) valuable than AMEX.

C.2 Constructing Bids

I first constructed the observed bids from transfer ratios and values of nodes using equation (1). Then I matched the observed bids with period-level KPIs. Note that I applied logarithmic transformations for all KPIs except for those in percentage terms. Then separately for (credit card issuer, airline) and (hotel chain, airline) as (chooser, bidder) pairs, I estimated the parameters of equation (2) using ordinary least squares (OLS). Note that I pooled the observations for all three periods (periods 0, 1, and 2) to compute the OLS estimates. I then used the parameter estimates and equation (2) to predict bids for all (chooser, bidder) pairs. I also constructed potential transfer ratios for all (chooser,bidder) pairs using equation (1). I used the predicted bids and potential transfer ratios for the Markov Chain Monte Carlo estimation.

C.3 Constructing *Mileage*

Each airline possessed different accessibility and redemption rates for flights to the 12 geographic zones. Accessibility is whether or not points in the airline's loyalty program could be used to redeem for flights to the geographic zones, and redemption rates are equal to the required number

of points to redeem for the flights. *Mileage* is a measure of redemption options, which accounts for both accessibility and redemption rates, granted by the credit card issuer's portfolio of airline partners. A smaller (larger) value of *Mileage* indicates better (worse) redemption options. I employed the following procedure to construct *Mileage*. Given a credit card issuer's portfolio of airline partners,

1. For each airline, multiply the its redemption rates to the 12 geographic zones by the potential transfer ratio from the credit card issuer to the airline. The result gives the required number of points to the geographic zones, denominated in the credit card issuer's loyalty points.
2. Using the result of step 1, for each geographic zone, select three smallest required number of points then compute the average. Any missing redemption options are not counted towards computing the average. For example, if a portfolio of airline partners grants only two redemption options for a zone, compute the average of the two required number of points.
3. Average the result of step 2 over the geographic zones.
4. Count the number of missing redemption options in step 2. Subtract 36 (3 times the number of geographic zones) by the count. Then divide by 36. Take the inverse.
5. Multiply the result of step 3 by the result of step 4.

Note that I calculated *Mileage* whenever the portfolio of airlines was updated in the Markov Chain Monte Carlo estimation.

C.4 Markov Chain Monte Carlo

The Markov Chain Monte Carlo (MCMC) estimation utilizes observations on the state of the network and firm-level characteristics from periods 1 and 2. Note that I applied logarithmic transformations for all components of *Performance* in equation (7). I specified the prior distribution of θ as $\mathcal{N}(0, 100 \times \mathbf{I}_{20})$, where \mathbf{I}_{20} denotes the 20×20 identity matrix. I executed the MCMC estimation in two stages. In the initial stage, I determined the covariance of the proposal distribution for θ (henceforth *covariance*) and fraction of the sequence of meeting to be permuted (henceforth *fraction*) and . In the main stage, I used the covariance and the fraction to sample the sequence of meetings and to compute iterations of θ . I utilized the multi-sequence method of (Gelman and Rubin, 1992) in both stages. In particular, I sampled 10 starting values of θ and separately iterated 10 MCMC sequences in each stage. In the first stage, I computed 1,000 MCMC iterations of θ (after the burn-in process) for each MCMC sequence, and in the second stage, I computed the iterations until the convergence criterion of (Gelman and Rubin, 1992) was satisfied. The following describes the procedure I employed for the MCMC estimation.

Initial Stage:

1. Set the proposal distribution for θ as $\mathcal{N}(\mathbf{0}_{20}, 0.0005 \times \mathbf{I}_{20})$, where $\mathbf{0}_{20}$ denotes the 20×1 vector of zeros. Set the fraction to 0.2. I determined the fraction and the covariance by trial and error to obtain a target jump rate of 0.3, for each of $\theta^{(l)}$ and $S_2^{(l)}$ in the following steps.

2. Sample 10 starting values of $\theta^{(0)}$ from $\mathcal{N}(\mathbf{0}_{20}, 0.05 \times \mathbf{I}_{20})$, which was obtained by multiplying the covariance of the proposal distribution by 100. Generate 10 starting sequences of meetings $S_2^{(0)}$ by permuting a sequence of all (credit card issuer, airline) and (hotel chain, airline) pairs.
3. Compute 2,000 iterations of $\theta^{(l)}$ and $S_2^{(l)}$, for each of the 10 MCMC sequences. Each MCMC sequence corresponds to one of the 10 starting values $(\theta^{(0)}, S_2^{(0)})$.
4. Calibrate the covariance and the fraction using the later 1,000 iterations.
 - (a) Compute the covariance matrix of $\theta^{(l)}$ for each MCMC sequence. Set the covariance of the proposal distribution for θ to $0.25\hat{\Sigma}$, where $\hat{\Sigma}$ denotes the element-wise average of the covariance matrices over the 10 MCMC sequences. I chose the tuning parameter 0.25 by trial and error to obtain a jump rate of 0.3 for $\theta^{(l)}$ in the later 1,000 iterations.
 - (b) Set the fraction to 0.3. I chose it by trial and error to obtain a target jump rate of 0.3 for $S_2^{(l)}$ in the later 1,000 iterations.

Main Stage:

1. Let $\hat{\theta}_0$ be the average of $\theta^{(l)}$ for the later 1,000 iterations from the initial stage (total $10 \times 1,000$ iterations). Let $\hat{\Sigma}$ denote the covariance matrix from step 4.(a) above. Define

$$\hat{p}(\theta) = (2\pi)^{-\frac{\dim(\theta)}{2}} \det(\hat{\Sigma})^{-\frac{1}{2}} \exp\left(-\frac{1}{2} (\theta - \hat{\theta}_0)' \hat{\Sigma}^{-1} (\theta - \hat{\theta}_0)\right) \quad (46)$$

2. Sample 1,000 draws of θ from \hat{p} . Divide each draw by a draw from \mathcal{X}_4^2 and then multiply by 4. They are overdispersed draws from an estimate of the posterior. The underlying density for the overdispersed draws is proportional to

$$p^*(\theta) = \det(\hat{\Sigma})^{-\frac{1}{2}} \left(4 + (\theta - \hat{\theta}_0)' \hat{\Sigma}^{-1} (\theta - \hat{\theta}_0)\right)^{-\frac{\dim(\theta)+4}{2}} \quad (47)$$

3. Let θ^* denote an overdispersed draw from step 2. Compute the importance ratios $\hat{p}(\theta^*) / p^*(\theta^*)$ for each of the 1,000 overdispersed draws. Using the importance ratios, select 10 starting values $\theta^{(0)}$ by sampling from the overdispersed draws without replacement. Generate 10 starting sequence of meetings $S_2^{(0)}$ by permuting a sequence of all (credit card issuer, airline) and (hotel chain, airline) pairs.
4. With the 10 starting values $(\theta^{(0)}, S_2^{(0)})$, separately compute iterations $(\theta^{(l)}, S_2^{(l)})$ for the 10 MCMC sequences using the covariance and the fraction obtained in step 4 of the initial stage. Continue until the criterion suggested by Gelman and Rubin (1992) is satisfied. That is, using the later half of the iterations,
 - (a) Compute the mean of $\theta^{(l)}$ for each of the 10 MCMC sequences. For each component of θ , compute the variance of the means. *Between-Var* denotes the variance of the means.

- (b) For each MCMC sequence, compute the variance of $\theta^{(l)}$ (for each components of θ).
Compute the mean of the variances. *Within-Var* denotes the mean of the variances.
- (c) Continue MCMC iterations until

$$\max \left(\frac{\text{Between-Var}}{\text{Within-Var}} \right) < 0.1. \quad (48)$$

Between-Var/Within-Var is an element-wise division, and the maximum is over the components of θ .