3 Drift

전기장내에서의 전류

전기장 $\mathcal{E}$에서 전류반송자가 받는 힘 : $F= \pm q\mathcal{E}$
자유공간의 일정한 전기장에 놓인 전자는 일정한 가속도로 운동 $F = m_e a$
+는 양전하, -는 음전하

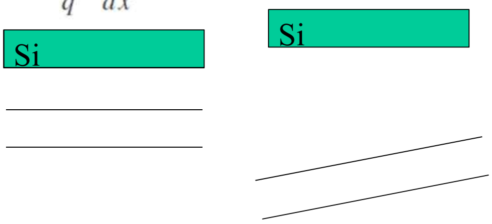
결정내에서는
불완전한 주기성에 의한 전자의 충돌,
결정 원자의 열진동 및 도핑원자에 의해
이동하는 전자끼리의 충돌
→이로 인하여 일정한 평균 속도를 가짐
이를 표동drift 라 하고→ 표동 전류

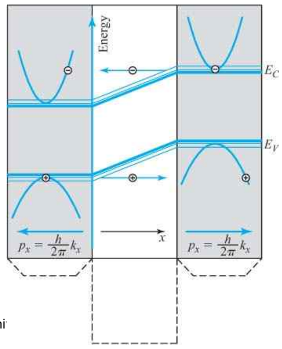
3.1 ENERGY BANDS WITH APPLIED ELECTRIC FIELD

- E_C와 E_V 의 에너지 차이
- 위치에너지와 전위 : E_{pot}=-qφ
- 반도체 양단에 전위(전압) –1V를 가하면 전자의 위치에너지차이는 1 eV 가 됨.

$\mathcal{E}$(전기장)= – dφ/dx

$$E = \frac{1}{q}\frac{dE_{C,V}}{dx}$$





3.1.1 Energy-Band Presentation of Drift Current

$E_{kin} = p^2/2m^*$

- 결정격자 효과를 모두 고려한 것이 반송자의 유효질량 이므로 자유공간처럼 이동한다고 볼 수 있음.
- 산란 센터는
  1. 포논, 도핑원자, 반송자
  2. 결정위치를 중심으로 진동- 주기적 전위에 교란이 발생: 이는 입자처럼 모델링이 되며 → 포논(phonon)이라함.
  3. – 전하를 가진 도핑 원자와의 산란→ 쿨롱 산란
  4. – 반송자끼리의 산란.

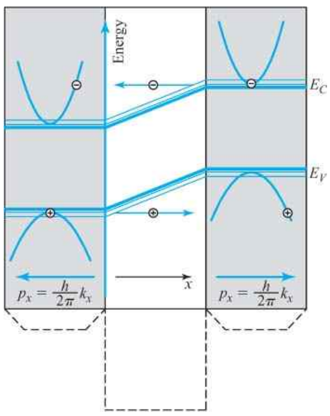


Figure 3.1 The relationship between *E*–*k* and *E*–*x* diagrams with applied electric field.

- 전자가 평균에너지 이상의 운동에너지를 얻게되면 초과하는 에너지를 산란센터에 주게됨.
- 산란후 전자는 전도대의 바닥으로 떨어진다.
- 정공의 경우도 비슷한 과정으로 산란센터에 전달함.
- 전도대가 기울어져 있을 경우 에너지가 낮은 쪽으로 굴러가는 경향- 전자 (구슬) 를 담는 그릇.
- 가전자대는 액체와 공기방울을 담은 그릇으로 설명

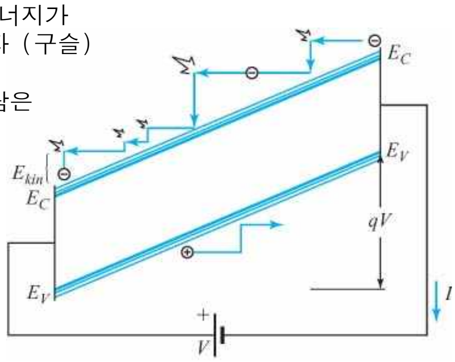


Figure 3.2 Energy-band (E–x) diagram with applied electric field, illustrating the *rolling down* of electrons and *bubbling up* of holes.

3.1.2 Resistance and Power Dissipation Due to Carrier Scattering

- 반송자의 산란-반송자의 흐름에 저항으로 작용
- 산란에 의해 열에너지로 전환 → 결정의 온도를 증가(결정원자의 진동을 증가)

EXAMPLE 3.1 Resistance and Power Dissipation

The voltage across an N-type semiconductor slab, conducting 10 mA of current, is equal to 1 V. How much energy is delivered to the crystal by every electron that passes through the slab? Relate this energy to the dissipated power.

SOLUTION

$$P = qVN/t = \underbrace{\frac{qN}{t}}_{I} V = IV$$

where $I = qN/t$ is the electric current flowing through the track. The value of the dissipated power is $P = IV = 10$ mW.

3.2 OHM'S LAW, SHEET RESISTANCE, AND CONDUCTIVITY

- If an electric-potential difference ($V = \phi_1 - \phi_0$) is established
- 전류가 흐르게 된다.
- (1) 전위차는 얼마?
- (2) 반도체 저항은 얼마인가?

$$I = \frac{V}{R}$$

3.2.1 Designing Integrated-Circuit Resistors

$$R = \rho \frac{L}{x_j W}$$

rectangular prism with dimensions L, W, and x_j.

$$\sigma = \frac{1}{\rho}$$

R_s : sheet resistance 면저항 $\Omega/\square$,

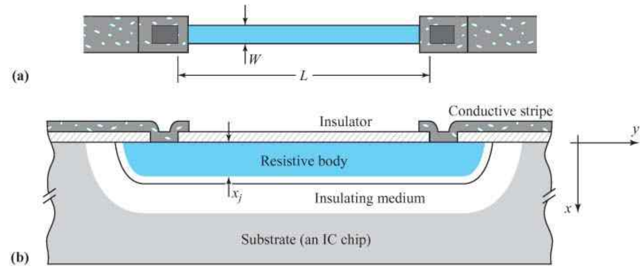
$$R_S = \frac{1}{\sigma x_j} = \frac{\rho}{x_j}$$

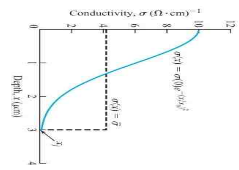
$$R = R_S \frac{L}{W}$$

Figure 3.3 Integrated-circuit resistor: (a) top view and (b) cross section.

$$\overline{\sigma} \approx \frac{1}{x_j}\int_0^{\infty} \sigma(x)\,dx$$

$$R_S = 1/(\overline{\sigma} x_j).$$





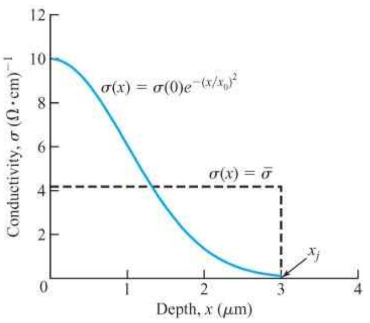


Figure 3.4 A typical variation of conductivity from the surface into the bulk of an integrated circuit resistor (solid line) and the unifor conductivity approximation (dashed line); in the example, $\sigma(0) = 10\ (\Omega \cdot \mathrm{cm})^{-1}$, and $x_0 = \sqrt{2}\ \mu$m.

Metal	ρ at 27°C ($\mu\Omega \cdot$ cm)	$\alpha = \frac{\Delta\rho/\rho}{\Delta T}$ (°C^{-1})
Aluminum	2.82	0.0039
Copper	1.7	0.0039
Gold	2.44	0.0045
Iron	9.7	0.0050
Lead	22	0.0039
Molybdenum	5.2	0.0040
Nichrome (Ni–Cr)	150	0.0004
Nickel	6.9	0.0038
Platinum	11	0.00392
Silver	1.59	0.0038
Tungsten	5.6	0.0045

Table 3.1 Resistivities and Temperature Coefficients for Selected Metals

EXAMPLE 3.2 Resistance, Sheet Resistance, and Resistivity

The thickness of a copper layer, deposited to create the interconnecting tracks in an IC, is 100 nm. The copper resistivity is given in Table 3.1.

(a) What is the sheet resistance of the copper layer at 27°C (room temperature)?

(a) The sheet resistance incorporates the resistivity ρ and the layer thickness x_j [Eq. (3.6)]:

$$R_S = \rho / x_j = 0.17\ \Omega/\square$$

(b) The minimum width of the interconnecting tracks is set at 0.5 μm. What is the maximum resistance per unit length?

(b) According to Eq. (3.7), the resistance is $R = R_S L/W$, which means that the resistance per unit length is

$$\frac{R}{L} = \frac{R_S}{W} = 3.4 \times 10^5\ \Omega/\text{m} = 0.34\ \Omega/\mu\text{m}$$

Metal	ρ at 27°C ($\mu\Omega \cdot$ cm)	$\alpha = \frac{\Delta\rho/\rho}{\Delta T}$ (°C^{-1})
Aluminum	2.82	0.0039
Copper	1.7	0.0039
Gold	2.44	0.0045
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Lead	22	0.0039
Molybdenum	5.2	0.0040
Nichrome (Ni–Cr)	150	0.0004
Nickel	6.9	0.0038
Platinum	11	0.00392
Silver	1.59	0.0038
Tungsten	5.6	0.0045

(c) The length of a minimum-width track, connecting two components, is 300 μm. What is the resistance of this track?

(c) The total resistance is

$$R = R_S \frac{L}{W} = 102\ \Omega$$

(d) What is the resistance of the track at 75°C?

(d) $R_1 = R_0 [1 + \alpha(T_1 - T_0)] = 121\ \Omega$

Metal	ρ at 27°C ($\mu\Omega \cdot$ cm)	$\alpha = \frac{\Delta\rho/\rho}{\Delta T}$ (°C^{-1})
Aluminum	2.82	0.0039
Copper	1.7	0.0039
Gold	2.44	0.0045
Iron	9.7	0.0050
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Molybdenum	5.2	0.0040
Nichrome (Ni–Cr)	150	0.0004
Nickel	6.9	0.0038
Platinum	11	0.00392
Silver	1.59	0.0038
Tungsten	5.6	0.0045

3.2.2 Differential Form of Ohm's Law

$$j_{dr} = \frac{I}{A}$$

$$E = \frac{V}{L}$$

$$j_{dr} = \sigma E$$

$$E = -d\varphi/dy$$

$$E = \text{const},\ d\varphi/dy = -(\varphi_1 - \varphi_0)/L = -V/L,$$

$$j_{dr} = -\sigma \frac{d\varphi}{dy}, \qquad j = -\sigma \left(\frac{\partial}{\partial x} x_u + \frac{\partial}{\partial y} y_u + \frac{\partial}{\partial z} z_u \right) \varphi = -\sigma \nabla \varphi$$

EXAMPLE 3.5 Current Density Versus Terminal Current

Calculate the maximum current density and the terminal current for the resistor designed in Example 3.4b if a voltage of 5 V is applied to the resistor terminals. Neglect the corner effects.

$$E = V/L = 5/(125 \times 10^{-6}) = 40{,}000 \text{ V/m} = 40 \text{ V/mm.}$$

$$\sigma_{max} = \sigma(0) = 10 \ (\Omega \cdot \text{cm})^{-1}$$

$$j_{dr-max} = \sigma_{max} E = 4 \times 10^{7} \text{ A/m}^2.$$

3.2.3 Conductivity Ingredients

(1) concentration of the carriers available to contribute toward the electrical current mobility of these carriers

→*the conductivity is proportional to the carrier concentration and the carrier mobility.*

$$\sigma = qn\mu_n + qp\mu_p$$

EXAMPLE 3.6 Conductivity and Carrier Concentration

P-type silicon has a resistivity of $0.5\,\Omega \cdot \text{cm}$. Find the following, assuming that $\mu_n = 1450\ \text{cm}^2/\text{V}\cdot\text{s}$ and $\mu_p = 500\ \text{cm}^2/\text{V}\cdot\text{s}$:

(a) the hole and electron concentrations
(b) the maximum change in resistivity caused by a flash of light, if the light creates 2×10^{16} additional electron–hole pairs/cm^3

SOLUTION

(a) In a P-type semiconductor the conductivity due to electrons can be neglected because $p \gg n$. Therefore, $\sigma \approx q\mu_p N_A$. The resistivity is

$$\rho \approx \frac{1}{q\mu_p N_A}$$

The concentration of acceptor ions, and therefore holes, is then $N_A \approx p = 1/(q\mu_p\rho) = 2.5 \times 10^{16}\ \text{cm}^{-3}$. The concentration of electrons is found as

$$n = \frac{n_i^2}{p} = 4.2 \times 10^3\ \text{cm}^{-3}$$

(b) The flash of light produces excess electrons and holes, reducing therefore the resistivity. When the light is removed, the excess electrons and holes will gradually recombine with each other, increasing the resistivity to its original value—that is, the equilibrium value. To find the maximum change in the resistivity, we need to determine the resistivity of the specimen when the light is on. In that case the concentration of holes is $p = 2.5 \times 10^{16} + 2 \times 10^{16} = 4.5 \times 10^{16}\ \text{cm}^{-3}$. The concentration of electrons is $n = 2 \times 10^{16}\ \text{cm}^{-3}$, as generated by the light, and in this specific case it cannot be neglected when compared to the concentration of holes. The conductivity is calculated as

$$\sigma = q\mu_p p + q\mu_n n = 8.24\ (\Omega \cdot \text{cm})^{-1}$$

The corresponding resistivity is $\rho = 1/\sigma = 0.12\ \Omega \cdot \text{cm}$. The maximum difference is, therefore, $\Delta\rho_{max} = 0.5 - 0.12 = 0.38\ \Omega \cdot \text{cm}$.

3.3 CARRIER MOBILITY

3.3.1 Thermal and Drift Velocities

$$E_{kin} = \begin{cases} m^*|\vec{v}|^2/2 = |\vec{p}|^2/2m^* & \text{general case} \\ m^* v_x^2/2 = p_x^2/2m^* & \text{one-dimensional case} \end{cases}$$

$$E_{kin} = \frac{m^* v_{th}^2}{2} = \begin{cases} \frac{3}{2}kT & \text{three-dimensional case} \\ \frac{1}{2}kT & \text{one-dimensional case} \end{cases}$$

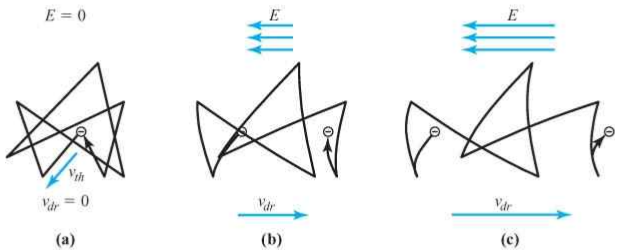


Figure 3.5 The concept of drift velocity (a) No electric field is applied. (b) A small electric field is applied. (c) A larger electric field is applied.

The concept of
drift velocity as the flow of a carrier ➔ *electric current*

단위부피당 전자의 개수: v_{dr} t A
전자의 농도 : n
시간 t 동안 전자수 :nv_{dr} t A
단위시간당 전류는 전하량 x 전자수 : $-qnv_{dr}$ A
표동 속도의 포화

$$I = -qnv_{dr}A$$

$$j_{dr} = \begin{cases} -qnv_{dr} & \text{electrons} \\ qpv_{dr} & \text{holes} \end{cases}$$

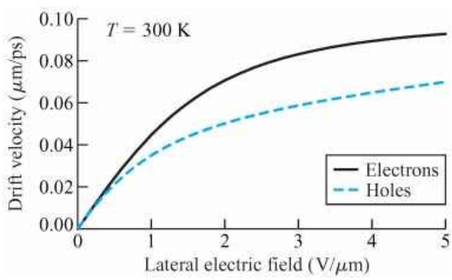


Figure 3.6 Drift velocity versus electric field in Si.

EXAMPLE 3.8 Drift Velocity

A uniformly doped semiconductor resistor (doping level $N_D = 10^{16}$ cm^{-3} and cross-sectional area $A = 20\ \mu$m^2) conducts 2 mA of current. The resistor is connected by copper wires with cross section of 0.1 mm^2. Determine and compare the drift velocities of the electrons in the semiconductor and copper regions. The concentration of free electrons in copper is $n_{Cu} = 8.1 \times 10^{22}$ cm^{-3}.

SOLUTION

The relationship between the drift velocity and the current density is given by Eq. (3.21). Therefore, the absolute value of the drift velocity is

$$v_{dr} = \frac{I}{Aqn}$$

where $n = N_D$ and $n = n_{Cu}$ in the semiconductor and the copper wires, respectively.

The drift velocities of electrons in the semiconductor and the copper wires are 62.5 km/s and 1.54 μm/s, respectively. There are so many more electrons in the copper wires that they can move $6.25 \times 10^4/1.54 \times 10^{-6} = 4 \times 10^{10}$ times slower and still supply the necessary current to the resistor.

3.3.2 Mobility Definition

$$j_{dr} = \begin{cases} q\mu_n n E & \text{electrons} \\ q\mu_p p E & \text{holes} \end{cases}$$

$$v_{dr} = \begin{cases} -\mu_n E & \text{electrons} \\ \mu_p E & \text{holes} \end{cases}$$

The **carrier mobility** is the proportionality coefficient in **the dependence of drift velocity on the applied field**.

The unit for mobility is given by **the velocity unit over the electric field unit,** (m/s)/(V/m) = m$_2$/V · s.

3.3.3 Scattering Time and Scattering Cross Section

$$F = -qE$$

$$F = m^* dv/dt.$$

$$v_{dr}/\tau_{sc}$$

$$-qE = m^* v_{dr}/\tau_{sc}$$

$$v_{dr} = -\frac{q\tau_{sc}}{m^*}E$$

$$v_{dr} = -\mu_n E$$

$$\mu_n = \frac{q\tau_{sc}}{m^*}$$

$$\tau_{sc} = \frac{1}{v_{th}\sigma_{sc}N_{sc}}$$

limited mobility
phonon-Coulomb scattering

The reciprocal value of τ_{sc} is the probability that a carrier will be scattered per unit time.

EXAMPLE 3.9 Scattering Cross Section of Phonons

(a) If the phonon-limited scattering time in silicon is $\tau_{sc-ph} = 0.2$ ps, determine the scattering cross section of phonons at 300 K. The effective mass of electrons in silicon is $m^* = 0.26m_0$.

(b) Assuming that the scattering cross section of phonons is proportional to the temperature, determine the temperature dependence of the phonon-limited mobility.

SOLUTION

(a) From Eq. (3.26),

$$\sigma_{sc-ph} = \frac{1}{\tau_{sc-ph}v_{th}N_{sc-phonons}}$$

where N_{sc-ph} is equal to the concentration of silicon atoms (5×10^{22} cm^{-3} according to Example 1.3), and v_{th} can be obtained from the energy-balance equation $m^* v_{th}^2/2 = 3kT/2$:

$$v_{th} = \sqrt{3kT/m^*} = 2.29 \times 10^5 \text{ m/s}$$

With this value for v_{th} and $\tau_{sc-ph} = 0.2$ ps, we obtain $\sigma_{sc-ph} = 4.36 \times 10^{-22}$ m^2 = 4.36×10^{-18} cm^2.

(b) The dependence of phonon-limited mobility on temperature is due to σ_{sc-ph} and v_{th}:

$$\mu_{ph} = \frac{q\tau_{sc-ph}}{m^*} = \frac{q}{m^* v_{th}\sigma_{sc-ph}N_{sc-ph}} \propto \frac{1}{\sigma_{sc-ph}v_{th}}$$

$$\mu_{ph} \propto \frac{1}{T\sqrt{3kT/m^*}} \propto \frac{1}{T^{3/2}}$$

$$\mu_{ph} = A_p T^{-3/2}$$

EXAMPLE 3.10 Cross Section of Coulomb Scattering Centers

The scattering cross section of a donor ion can be related to the spherical region where the thermal energy of a carrier is smaller than the energy associated with the Coulomb attraction.

(a) Estimate the cross section of Coulomb scattering centers in silicon. The dielectric constant of silicon is $\varepsilon_s/\varepsilon_0 = 11.8$.

(b) Determine the temperature dependence of the Coulomb-limited mobility.

SOLUTION

(a) The energy of Coulomb attraction/repulsion is $q^2/(4\pi\varepsilon_s r)$, where r is the distance between the center of the ion and the carrier, whereas the kinetic energy of the carrier is $3kT/2$. Therefore, the radius of the spherical region can be found from the following condition:

$$\frac{q^2}{4\pi\varepsilon_s r} = \frac{3}{2}kT$$

$$r = \frac{q^2}{6\pi\varepsilon_s kT}$$

The cross section of the spherical region (the scattering cross section) is then $\sigma_{sc-C} = \pi r^2 = 3.1 \times 10^{-17}\ \text{m}^{-2} = 3.1 \times 10^{-13}\ \text{cm}^{-2}$.

(b) The temperature-dependent factors are σ_{sc-C} and v_{th}, where $\sigma_{sc-C} = \pi r^2 \propto 1/T^2$:

$$\tau_{sc-C} \propto \frac{1}{\sigma_{sc-C} v_{th}} \propto \frac{1}{(1/T^2)\sqrt{T}} = T^{3/2}$$

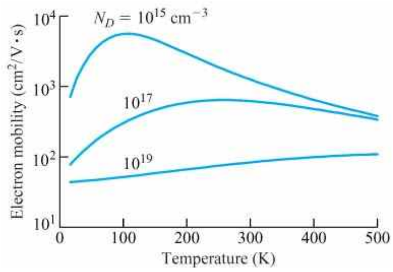
$$\mu_C = A_c T^{3/2}$$

3.3.4 Mathieson's Rule

$$\frac{1}{\tau_{sc}} = \frac{1}{\tau_{sc-ph}} + \frac{1}{\tau_{sc-C}}$$

$$1/\mu = 1/\mu_{ph} + 1/\mu_C$$

$$\frac{1}{\mu} = \sum_{i=1}^{N} \frac{1}{\mu_i} \qquad \text{Mathieson's rule.}$$



– 도핑농도가 10^{15}, 10^{17}, 10^{19} cm^{-3} 온도가 500 K

- 포논과 쿨롱 산란에 대한 온도의 영향이 서로 반대
- 도핑 농도가 낮아지면
  - 저온에서는 약해진 쿨롱 산란으로 이동도가 증가
  - 고온에서는 강해진 포논 산란 으로 이동도가 감소

Figure 3.7 Temperature dependence of mobility for three different doping levels in Si.

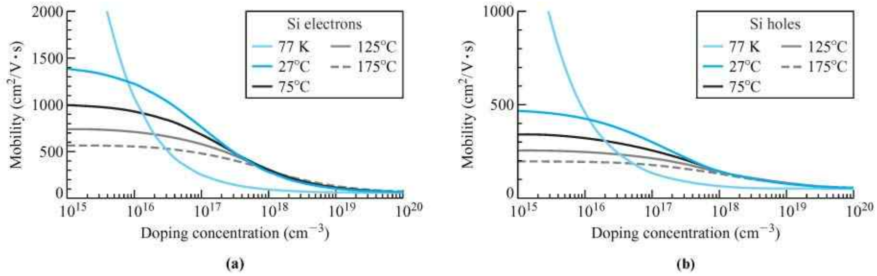


Figure 3.8 (a) Low-field electron and (b) low-field hole mobilities in Si.

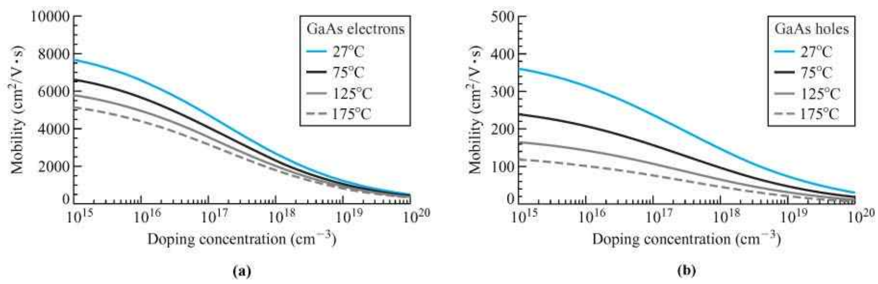


Figure 3.9 (a) Low-field electron and (b) low-field hole mobilities in GaAs.

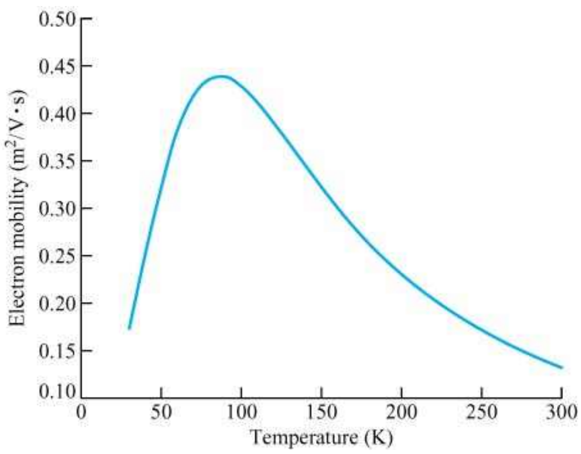


Figure 3.10 The MATLAB plot for Example 3.11a.

*3.3.5 Hall Effect

$$\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}$$

$$qE_x = qv_yB_z \sin\underbrace{[\angle(v_y, B_z)]}_{-90°} \Rightarrow -E_x = v_yB_z$$

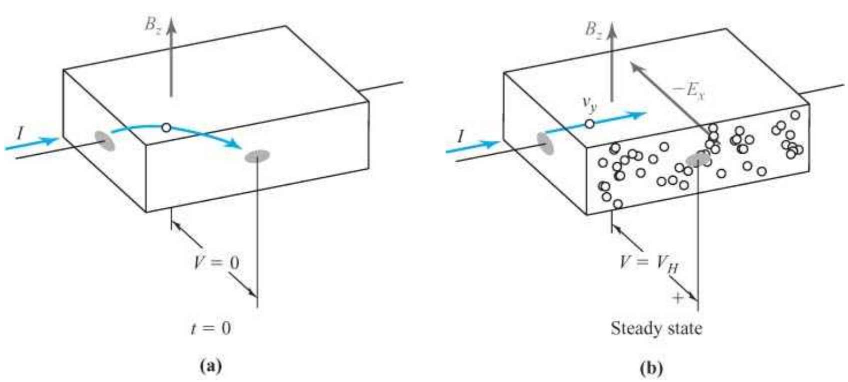


Figure 3.12 Illustration of the Hall effect. (a) The force due to the magnetic field B_z deviates the hole trajectory. (b) Accumulated ho create Hall field $E_H = -E_x$ that counteracts the force from the magnetic field B_z.

$$j_y = qpv_{dr}$$

$$E_H = \frac{j_y}{qp} B_z$$

$$V_H = \frac{1}{qp} \frac{I}{t_s} B_z$$

$$V_H = R_H \frac{I B_z}{t_s}$$

$$R_H = \frac{1}{qp}$$ Hall coefficient $$R_H = \begin{cases} r/qp & \text{for holes} \\ -r/qn & \text{for electrons} \end{cases}$$

$$R_H = \frac{\mu_p}{\sigma_p}$$

the electrons: $(-q)(-v_y)B_z = qv_y B_z$.

$$\sigma_p = q\mu_p p.$$