Stochastic Programming Formulation

Seonho Park

1 Sets

$\mathcal{P}$: product IDs, $\{0, \ldots, 499\}$

$\mathcal{S}$: scenario IDs, $\{0, \ldots, ns-1\}$

$\mathcal{G}$: set of product ID sets, $\mathcal{G} := \{\mathcal{G}_j\}_{j=0}^{197}$

$\mathcal{G}_j$: product IDs belonging to the substitutability group j, where $j \in \{0, \ldots, 197\}$

$\mathcal{P}_c$: product IDs that capacity limits are defined

2 Parameters

d_i : demand of product $i \in \mathcal{P}$

$\tilde{d}_{i,s}$: demand realization of product $i \in \mathcal{P}$ in scenario $s \in \mathcal{S}$, that is $\tilde{d}_{i,s} := d_i \nu_{i,s}$ where $\nu_{i,s}$ is the sample from the burr12 distribution associated with the product $i \in \mathcal{P}$

ns : the number of scenarios, $ns = |\mathcal{S}|$

a_i : COGS of product $i \in \mathcal{P}$

b_i : selling price of product $i \in \mathcal{P}$, which corresponds to margin+COGS

μ : input parameter, the ratio of aggregated surplus quantities over total (estimated) demand, ranged from 0.1 to 0.5

c_i : capacity limit of product $i \in \mathcal{P}_c$

3 Variables

x_i : surplus of product $i \in \mathcal{P}$, nonnegative continuous

$y_{i,s}$: the amount of demand that cannot be met by the supply of product $i \in \mathcal{P}$ in scenario $s \in \mathcal{S}$, nonnegative continuous

4 Objective Function

$$\max_{x,y} \sum_{i \in \mathcal{P}} \frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} b_i(\tilde{d}_{i,s} - y_{i,s}) - \sum_{i \in \mathcal{P}} a_i(x_i + d_i) \tag{1}$$

The objective function represents the profit margin. The first term of the profit margin is the expected revenue. It is calculated by selling price b_i multiplied by the amount that is sold, which corresponds to $\tilde{d}_{i,s} - y_{i,s}$ for each scenario. We assume that even if one product is substituted with another product, the selling price of this product is still the same as before the substitution. The second term represents the overall cost to produce the products. This profit margin is to be maximized.

5 Constraints

Capacity limit constraint

$$x_i \leq d_i c_i, \qquad \forall i \in \mathcal{P}_c \tag{2}$$

Total surplus limit constraint

$$\sum_{i \in \mathcal{P}} x_i \leq \mu \sum_{i \in \mathcal{P}} d_i \tag{3}$$

Substitutability constraint

$$\sum_{i \in \mathcal{G}_j} (x_i + d_i) \geq \sum_{i \in \mathcal{G}_j} (\tilde{d}_{i,s} - y_{i,s}) \qquad \forall s \in \mathcal{S}, \forall \mathcal{G}_j \in \mathcal{G} \tag{4}$$