Stochastic Programming Formulation

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1 Sets

$\mathcal{P}$: product IDs, $\{0, \ldots, 499\}$

$\mathcal{S}$: scenario IDs, $\{0, \ldots, ns - 1\}$

$\mathcal{G}$: set of product ID sets, $\mathcal{G} := \{\mathcal{G}_j\}_{j=0}^{197}$

$\mathcal{G}_j$: product IDs belonging to the substitutability group j, where $j \in \{0, \ldots, 197\}$

$\mathcal{P}_c$: product IDs that capacity limits are defined

2 Parameters

d_i : demand of product $i \in \mathcal{P}$

$\tilde{d}_{i,s}$: demand realization of product $i \in \mathcal{P}$ in scenario $s \in \mathcal{S}$, that is $\tilde{d}_{i,s} := d_i \nu_{i,s}$ where $\nu_{i,s}$ is the sample from the burr12 distribution associated with the product $i \in \mathcal{P}$

ns : the number of scenarios, $ns = |\mathcal{S}|$

a_i : COGS of product $i \in \mathcal{P}$

b_i : selling price of product $i \in \mathcal{P}$, which corresponds to margin+COGS

μ : input parameter, the ratio of aggregated surplus quantities over total (estimated) demand, ranged from 0.1 to 0.5

c_i : capacity limit of product $i \in \mathcal{P}_c$

3 Variables

x_i : surplus of product $i \in \mathcal{P}$, nonnegative continuous

$y_{i,s}$: the amount of demand that cannot be met by the supply of product $i \in \mathcal{P}$ in scenario $s \in \mathcal{S}$, nonnegative continuous

4 Objective Function

$$\max_{x,y} \sum_{i \in \mathcal{P}} \frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} b_i(\tilde{d}_{i,s} - y_{i,s}) - \sum_{i \in \mathcal{P}} a_i(x_i + d_i) \tag{1}$$

The objective function represents the profit margin. The first term of the profit margin is the expected revenue. It is calculated by selling price b_i multiplied by the amount that is sold, which corresponds to $\tilde{d}_{i,s} - y_{i,s}$ for each scenario. The second term represents the overall cost to produce the products. This profit margin is to be maximized.

5 Constraints

Capacity limit constraint

$$x_i \leq d_i c_i, \qquad \forall i \in \mathcal{P}_c \tag{2}$$

Total surplus limit constraint

$$\sum_{i \in \mathcal{P}} x_i \leq \mu \sum_{i \in \mathcal{P}} d_i \tag{3}$$

Substitutability constraint

$$\sum_{i \in \mathcal{G}_j} x_i + d_i \geq \sum_{i \in \mathcal{G}_j} \tilde{d}_{i,s} - y_{i,s} \qquad \forall s \in \mathcal{S}, \forall \mathcal{G}_j \in \mathcal{G} \tag{4}$$